1 Linear

The effective gravitational potential in Cubic Galileon theory at linear regime reads as (Eq. 29 of 1306.3219)

$$G_{\rm eff} = G\left(1 - \frac{2}{3}\frac{c_3\dot{\varphi}^2}{M_{\rm Pl}\mathcal{M}^3\beta_2}\right) \tag{1}$$

where $M_{\rm Pl}$ is the Planck mass, φ is the Galileon scalar field and $\mathcal{M}^3 \equiv M_{\rm Pl}H_0^2$, β_1 and β_2 are,

$$\begin{aligned}
\beta_1 &= \frac{1}{6c_3}\left[-c_2 - \frac{4c_3}{\mathcal{M}^3}(\ddot{\varphi} + 2H\dot{\varphi}) + 2\frac{\kappa c_3^2}{\mathcal{M}^6}\dot{\varphi}^4\right], \\
\beta_2 &= 2\frac{\mathcal{M}^3 M_{\rm Pl}}{\dot{\varphi}^2}\beta_1. \\
\kappa &= \frac{1}{M_{\rm Pl}^2} = 8\pi G
\end{aligned}$$

In our discussion we set $c_2 = -1$ (see discussion at page 5 of 1709.09135). We also use the tracker solution,

$$\xi \equiv \frac{\dot{\phi}H}{M_{\rm Pl}H_0^2} \tag{2}$$

where ξ is a constant. Following the discussion which leads to E. 18 in 1709.09135 we can deduce that ξ is a constant and can be obtained given c_3,

$$\xi = -\frac{1}{6c_3} \tag{3}$$

As a result we can write,

$$\dot{\varphi} = \frac{\xi M_{\rm Pl}H_0^2}{H} \tag{4}$$

$$\ddot{\varphi} = -\frac{\xi M_{\rm Pl}H_0^2\dot{H}}{H^2} \tag{5}$$

Note that $H = \frac{da}{dt} = \mathcal{H}/a$, where $\mathcal{H}$ is the conformal Hubble factor. Following the discussion presented in 1308.3699 for the background tracker solution we can derive the Hubble expansion rate as a function of a (Eq. 12 of 1308.3699)

$$\mathcal{H}^2 = \frac{\mathcal{H}_0^2}{2}\left[\left(\Omega_{m0}a^{-1} + \Omega_{r0}a^{-2}\right) + a^2\sqrt{\left(\Omega_{m0}a^{-3} + \Omega_{r0}a^{-4}\right)^2 + 4\left(1 - \Omega_{m0} - \Omega_{r0}\right)}\right]. \tag{6}$$

Where $H_0^2 = \frac{8\pi G}{3}$ in MG-evolution unit.Computing $\mathcal{H}'$ results in,

$$\begin{aligned}
\mathcal{H}' = &-\frac{\mathcal{H}_0^2(a\Omega_m + 2\Omega_r)}{4a^2} - \frac{\mathcal{H}_0^2(a\Omega_m + \Omega_r)(3a\Omega_m + 4\Omega_r)}{4a^6\sqrt{4(1-\Omega_m-\Omega_r) + \frac{(a\Omega_m+\Omega_r)^2}{a^8}}} \\
&+ \frac{\mathcal{H}_0^2 a^2}{2}\sqrt{4(1-\Omega_m-\Omega_r) + \frac{(a\Omega_m+\Omega_r)^2}{a^8}}
\end{aligned} \tag{7}$$

We also have,

$$\dot{H} = \frac{\mathcal{H}' - \mathcal{H}^2}{a^2} \tag{8}$$

With all the previous expressions we can obtain the linear $\Delta G/G$ for the Cubic Galileon model.

2 Screening

Following the discussion which leads to Eq. 21 of 1306.3219 we can write,

$$\frac{\Delta G}{G}|_{\rm tot} = \frac{\Delta G}{G}|_{\rm linear} \times \frac{\Delta G}{G}|_{\rm Vainshtein} \tag{9}$$

where $\frac{\Delta G}{G}|_{\rm linear}$ is obtained following the discussion in the previous section and $\frac{\Delta G}{G}|_{\rm Vainshtein}$ can be written in Fourier space as 2003.05927,

$$\frac{\Delta G}{G}|_{\rm Vainshtein} = \frac{\sqrt{1+\epsilon}-1}{\epsilon} \tag{10}$$

where,

$$\epsilon \equiv (\frac{r_V}{r})^3 \rightarrow (\frac{k_*}{k})^3 \tag{11}$$

where r_V the Vainshtein radius and k_* is the corresponding wavenumber in Fourier space. As a result we have two free parameters, namely k_* in the screening and c_3 in the linear modification.