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$a, b, c, d, e$   
E-mail: [..](#), [..](#)

**Abstract.**

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## 1 Introduction

## 2 Modification of Gravity: nDGP

### 2.1 Background

Here we take the same background as  $\Lambda$ CDM or equivalently we consider an artificial dark energy fluid which cancels out the effect of modified gravity in the background level and as a result we see  $\Lambda$ CDM background. This is motivated since we can precisely compare the effect of modified gravity model with standard theory at the perturbation level.

### 2.2 Perturbations

We use the Newtonian flag of Gevolution and modify the Poisson equation as following

$$\nabla\Psi_N = 4\pi G a^2 \left(1 + \frac{\Delta G_{\text{eff}}}{G}\right) \delta\rho \quad (2.1)$$

For the normal branch of DGP (<https://arxiv.org/pdf/1602.02154.pdf>)

$$\frac{\Delta G_{\text{eff}}}{G} = \frac{1}{3\beta(a)} \quad (2.2)$$

where,

$$\beta(a) = 1 + \frac{4}{3} H r_c \left(1 + \frac{A}{2H^2}\right) \quad (2.3)$$

$A = \ddot{a}/a = \dot{H} + H^2$  and  $\Omega_{rc} = \frac{1}{4H_0^2 r_c^2}$ , changing time to conformal time as following,

$$H = \frac{\mathcal{H}}{a}, \quad \dot{H} = \frac{-\mathcal{H}^2 + \mathcal{H}'}{a^2} \quad (2.4)$$

so,

$$\frac{A}{2H^2} = \frac{\dot{H} + H^2}{2H^2} = \frac{\mathcal{H}'}{2\mathcal{H}^2} \quad (2.5)$$

Substituting the values we get

$$\beta(a) = 1 + \frac{2\mathcal{H}(a)}{3\mathcal{H}_0 a \sqrt{\Omega_{rc}}} \left(1 + \frac{\mathcal{H}'}{2\mathcal{H}^2}\right) \quad (2.6)$$

Or probably better to write in terms of  $r_c$ ,

$$\beta(a) = 1 + \frac{4}{3a} \frac{\mathcal{H}}{\mathcal{H}_0} \mathcal{H}_0 r_c \left( 1 + \frac{\mathcal{H}'}{2\mathcal{H}^2} \right) \quad (2.7)$$

Here like what is done in the reference in table 7 we assume different strength of modification  $H_0 r_c$  and read  $\Omega_{rc}$  from that. The GR case is recovered when  $r_c \rightarrow \infty$ , we also take  $\mathcal{H}_0 r_c = 0.5$ ,  $\mathcal{H}_0 r_c = 1.2$  and  $\mathcal{H}_0 r_c = 5.6$ . Specifically we can write,

$$\mathcal{H}_0 r_c = 0.5 \longrightarrow \Omega_{rc} = 1.0000 \quad (2.8)$$

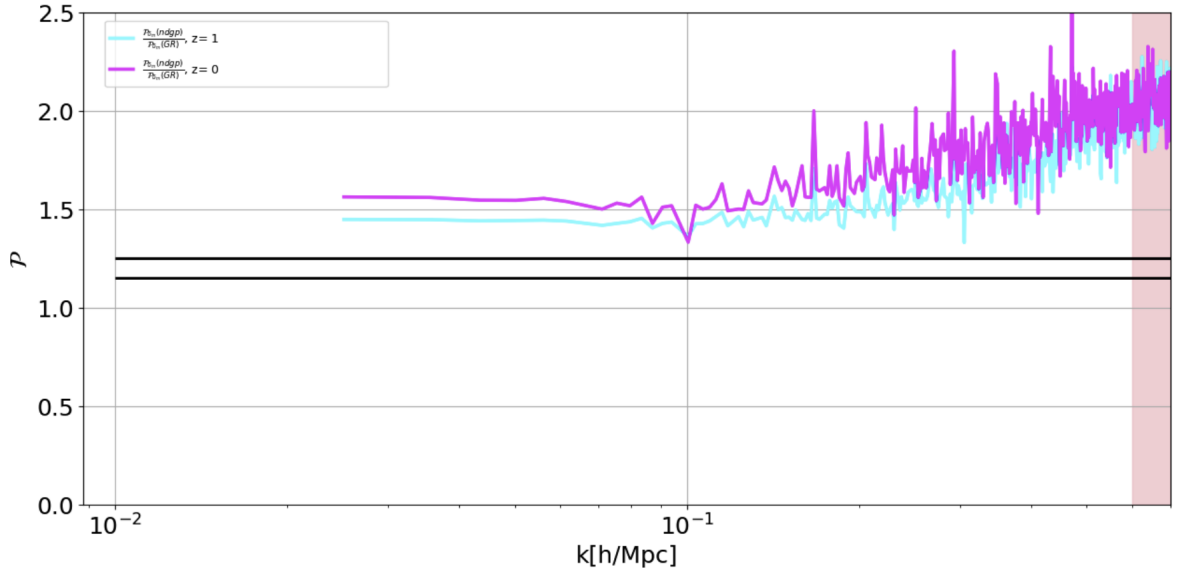
$$\mathcal{H}_0 r_c = 1.2 \longrightarrow \Omega_{rc} = 0.1736 \quad (2.9)$$

$$\mathcal{H}_0 r_c = 0.5 \longrightarrow \Omega_{rc} = 0.0079 \quad (2.10)$$

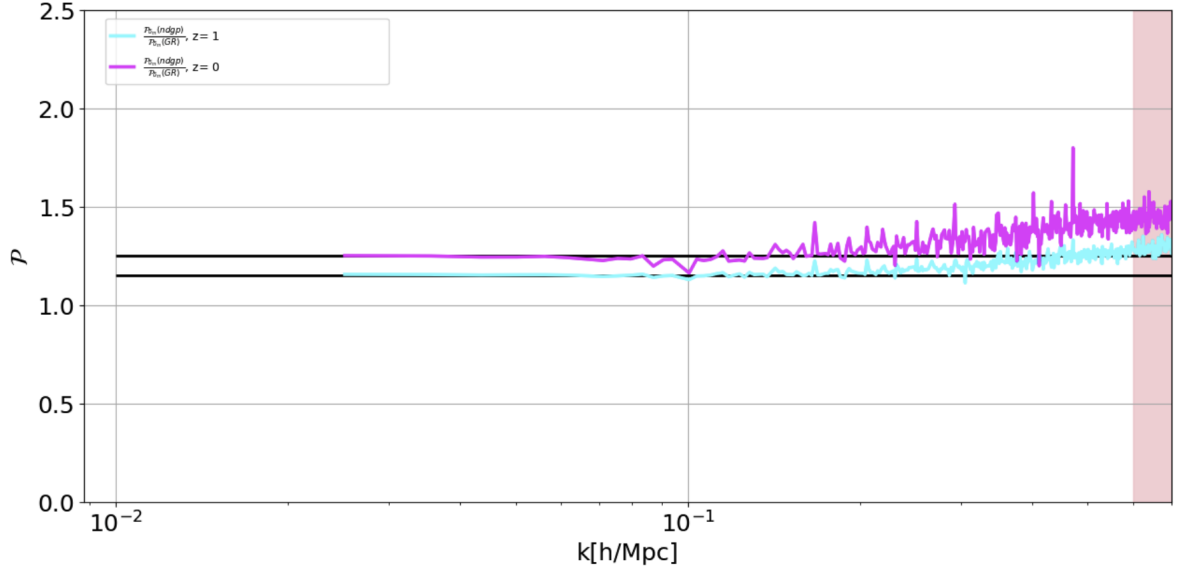
So in the Gevolution we need to compute  $\mathcal{H}'$  and modify the Poisson equation. It is important to fix the cosmological parameters and also  $\sigma_8$  is different for each model! First we thought it means that we need to obtain  $A_s$  corresponds to the relevant  $\sigma_8$  for each set of parameters, so we used BBKS transfer function and integration over that to obtain relevant  $A_s$ . The  $C++$  code is available in github. The relevant  $A_s$  reads as following,

```
# A_s values for LCDM model in Table.7 of arxiv:1602.02154
#sigma(8) = 0.800 (LCDM) ----> A_s = 2.20630e-9 BBKS transfer function integration
#sigma(8) = 0.896 (H0rc=0.5) ----> A_s = 2.76930e-9 BBKS transfer function integration
#sigma(8) = 0.849 (H0rc=1.2) ----> A_s = 2.48530e-9 BBKS transfer function integration
#sigma(8) = 0.812 (H0rc=5.6) ----> A_s = 2.26990e-9 BBKS transfer function integration
```

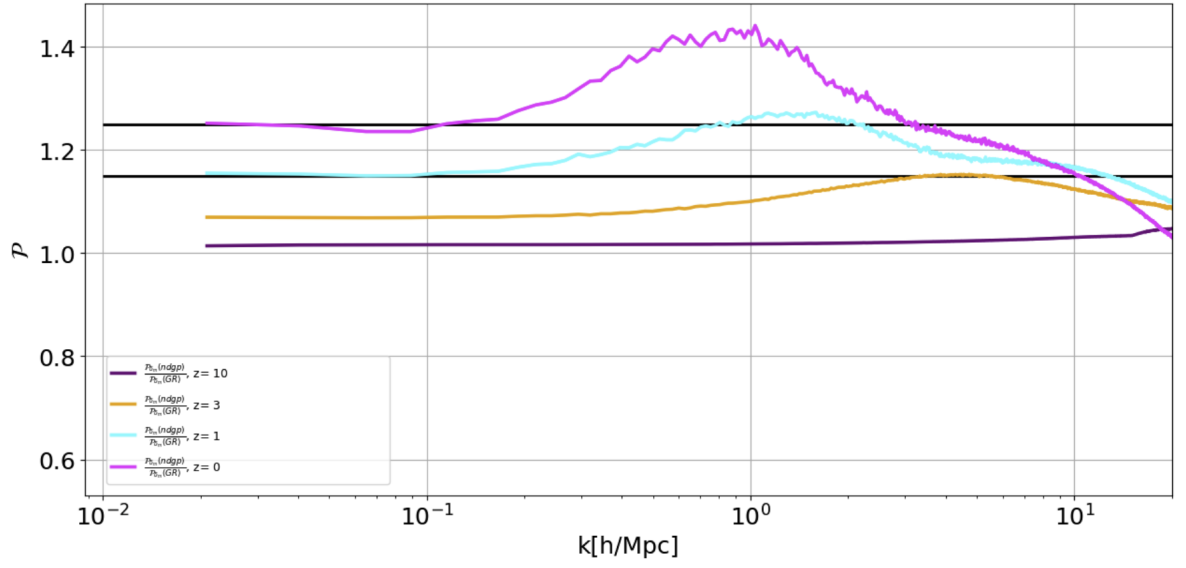
But what we get did not match to the paper results as following,



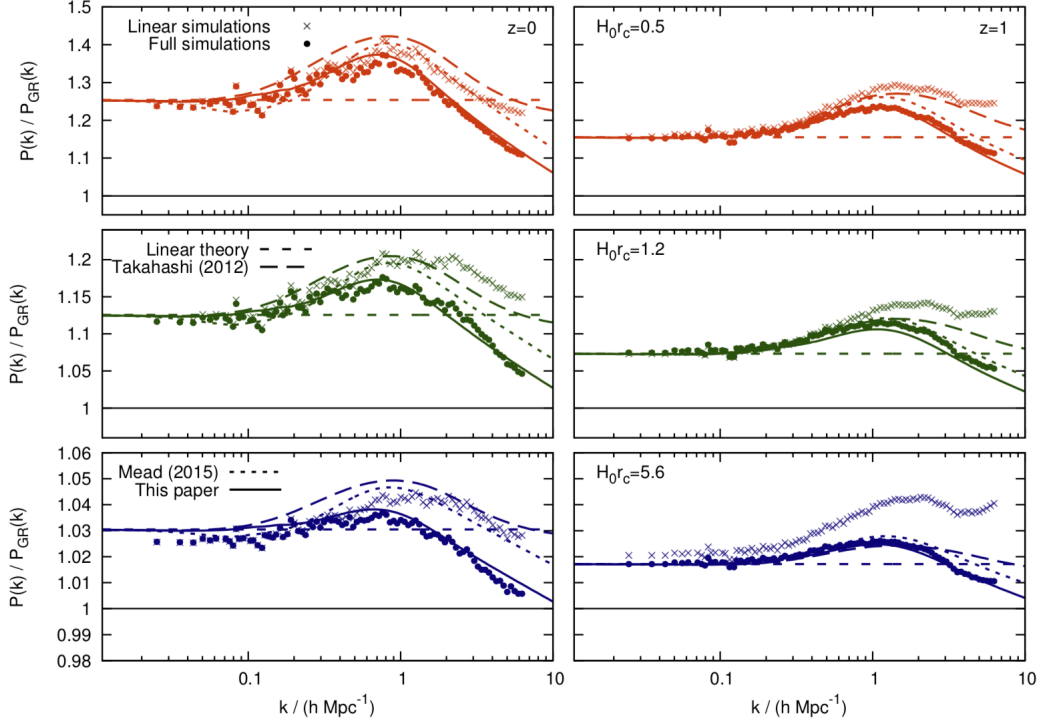
So we found out the meaning of different  $\sigma_8$  comes from just because of different Poisson equation, or modification because of nDGP, which shows itself in the power spectrum and at the end  $\sigma_8$  or  $A_s$ , so now we run for the same cosmological parameters and look at the results,



And because of mode coupling at non-linear regime, we get different grows of perturbations at different scales which is clear in the high-resolution run,



Compared to the paper result seems correct.



### 3 Screening mechanism

Now we are going to add screening to the previous results. We follow the same procedure as <https://arxiv.org/abs/0911.5178> and <https://arxiv.org/pdf/1703.00879.pdf>. Basically because of gravity modification due to scalar field we can write,

$$\Psi = \Psi_N + \varphi/2 \quad (3.1)$$

where,

$$\nabla^2 \Psi_N = 4\pi G a^2 \delta\rho \quad (3.2)$$

$$\nabla^2 \varphi = 8\pi a^2 \Delta G \left( \frac{R}{R_*} \right) \delta\rho \quad (3.3)$$

The modified Poisson equation is written,

$$\nabla^2 \Psi = 4\pi G a^2 \left( 1 + \frac{\Delta G}{G} \right) \delta\rho \quad (3.4)$$

For the nDGP model  $\Delta G$  reads,

$$\frac{\Delta G}{G} = \frac{2}{3\beta(a)} \frac{\sqrt{1+x^{-3}} - 1}{x^{-3}} \quad (3.5)$$

$R_*$  the Vainstein radius is,

$$R_* = \left( \frac{16G\delta M r_c^2}{9\beta^2} \right) \quad (3.6)$$

Moreover,

$$\delta M = 4\pi\delta\rho R^3/3 \quad (3.7)$$

and  $x = \frac{R}{R_*}$ , putting everything together gives,

$$\epsilon = x^{-3} = \left(\frac{R_*}{R}\right)^3 = \frac{8\mathcal{H}_0^2 r_c^2}{9\beta^2} \Omega_m(a) \delta = \frac{8\mathcal{H}_0^2 r_c^2}{9\beta^2} \Omega_{m,0} a^{-3} \delta \quad (3.8)$$

$$\frac{\Delta G}{G} = \frac{2}{3\beta} \frac{\sqrt{1+\epsilon} - 1}{\epsilon} \quad (3.9)$$

$$\nabla^2 \Psi = 4\pi G a^2 \left(1 + \frac{\Delta G}{G}\right) \delta \rho \quad (3.10)$$

In Gevolution we just need to define  $\epsilon$  to implement it into the modified Poisson equation, and  $\beta$  is defined as before,

$$\beta(a) = 1 + \frac{4}{3a} \frac{\mathcal{H}}{\mathcal{H}_0} \mathcal{H}_0 r_c \left(1 + \frac{\mathcal{H}'}{2\mathcal{H}^2}\right) \quad (3.11)$$

Note that the  $\beta$  in the equation. 10 of <https://arxiv.org/pdf/0911.5178.pdf> and equation. 48 of <https://arxiv.org/pdf/1602.02154.pdf> are the same.

### 3.1 Gevolution

The tricky part to implement in Gevolution is how we read  $\delta_m$  in Gevolution.

First note that in Gevolution  $T_0^0(gev) = -a^3 T_0^0 = -a^3(\bar{\rho} + \delta\rho)$  and by definition  $M_{pl}^2 = 1/8\pi G$ . Moreover we have the following identities,

$$\Omega_m = \frac{\rho_m(a)}{\rho_{crit}(a)} = \frac{a^2 \bar{\rho}_m(a)}{3M_{pl}^2 \mathcal{H}^2} \quad (3.12)$$

$$\rho_{crit} = \frac{3M_{pl}^2 \mathcal{H}^2}{a^2} \quad (3.13)$$

$$\rho_{crit}^0 = \frac{3M_{pl}^2 \mathcal{H}_0^2}{a_0^2} = 1 \quad (3.14)$$

$\rho_{crit}^0 = 1$  in Gevolution, i.e.  $\mathcal{H}_0 = H_0 = \frac{8\pi G}{3}$ .

On the other hand,  $\bar{T}_0^0(Gev) = a^3 \bar{\rho}_m$ , so we have,

$$a^3 \bar{\rho}_m = a^3 \frac{\bar{\rho}_m a^{-3}}{\rho_{crit}^0} \rho_{crit}^0 = \Omega_m^0 \quad (3.15)$$

At the end we can write,

$$T_0^0(gev) = \Omega_m^0 (1 + \delta_m) = a^3 \bar{\rho}_m (1 + \delta_m) \quad (3.16)$$

### 3.2 Relativistic

So because of the notation in Gevolution,  $T_0^0$  is the perturbation part plus background so,

$$\delta_m(gev) = \frac{T_0^0(gev) - \bar{T}_0^0(gev)}{\bar{T}_0^0(gev)} = \frac{\text{source}(x) - a^3 \bar{\rho}_m}{a^3 \bar{\rho}_m} = \frac{\text{source}(x) - \Omega_m^0}{\Omega_m^0} \quad (3.17)$$

So  $\delta\rho_m$  is exactly  $T_0^0(gev)$  in Gevolution while it is rescaled by  $a^3$ . To compute  $\epsilon$  in the code we need to use  $\delta_m = \frac{T_0^0(gev) - \bar{T}_0^0(gev)}{\Omega_m^0}$ . So basically having  $\delta_m$  we can compute  $\epsilon$  easily.  $\epsilon$  in Gevolution reads,

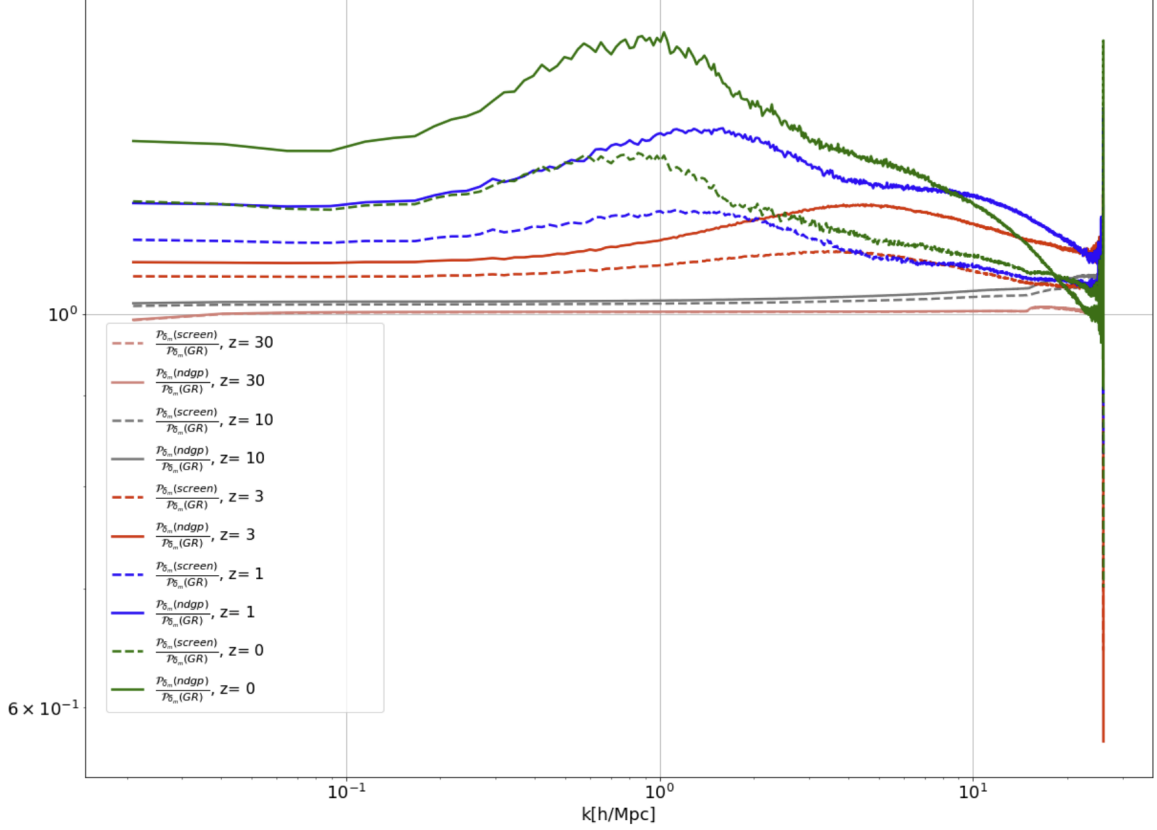
$$\epsilon = \frac{8\mathcal{H}_0^2 r_c^2}{9\beta^2} \Omega_{m,0} a^{-3} \delta_m = \frac{8\mathcal{H}_0^2 r_c^2}{9\beta^2} a^{-3} (T_0^0(gev) - \bar{T}_0^0(gev)) \quad (3.18)$$

### 3.3 Newtonian

In the Newtonian Gevolution we have "source(x)" which is actually  $\bar{\rho}_m \delta_m = \Omega_m^0 \delta_m = \delta \rho_m$ , so  $\epsilon$  reads

$$\epsilon = \frac{8\mathcal{H}_0^2 r_c^2}{9\beta^2} \Omega_{m,0} a^{-3} \delta_m = \frac{8\mathcal{H}_0^2 r_c^2}{9\beta^2} \times \frac{\text{source}(x) - \Omega_m^0}{a^3} \quad (3.19)$$

The result is as following, in which we see an offset in the low wavenumbers,



### 3.4 Analysis

To understand what is happening, for a point on the lattice with some  $\delta$  we study the screening and compare with the code result. The mathematica notebook to do the complete analysis is written. It is not clear from mathematica what really is supposed to happen,, so we follow the below checks,

#### 3.4.1 R

## 4 Conclusions

## Acknowledgements

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## References

- [1] A. Hojjati, L. Pogosian and G. B. Zhao, JCAP **1108** (2011) 005 doi:10.1088/1475-7516/2011/08/005 [arXiv:1106.4543 [astro-ph.CO]].