Cosmology Calculator

In[34]:=

```
Clear["Global`*"];
pdConv[f_]:=TraditionalForm[f/.Derivative[inds__][g_][vars__]:→Apply[Defer[D[g[vars],##]]&
```

$$z \equiv 1/a - 1$$

In[36]:=

```
(*scale factor in terms of the Redshift *)
scalfactor[z_]:= 1/(1+z);
redshift[a_]:= 1/a -1;
```

In[38]:=

```
(*Function to compute conformal time for a given redshift*)
tau[zs_,H0_,Ωm_,Ωd_,w_,Ωr_]:=NIntegrate[H0^-1/Sqrt[Ωd (1+z)^(3(1+w)) +Ωm (1+z)^3+Ωr (1+z)^4],{z,zs,∞}]
```

In[39]:=

```
(*Function to compute Physical time for a given redshift = a'(tau)/a*)
time[zs_,H0_,Ωm_,Ωd_,w_,Ωr_]:=NIntegrate[H0^-1/((1+z) Sqrt[Ωd (1+z)^(3(1+w)) +Ωm (1+z)^3+Ωr (1+z)^4]),{z,zs,∞
```

Friedmann equation: (FROM https://en.wikipedia.org/wiki/Hubble%27s_law)

If the universe is both matter-dominated and dark energy-dominated, then the above equation for the Hubble parameter will also be a function of the equation of state of dark energy. So now:

$$\rho = \rho_m(a) + \rho_{de}(a),$$

where ρ_{de} is the mass density of the dark energy. By definition, an equation of state in cosmology is $P = w\rho c^2$, and if this is substituted into the fluid equation, which describes how the mass density of the universe evolves with time, then

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{P}{c^2}\right) = 0;$$

$$\frac{d\rho}{\rho} = -3\frac{da}{a}(1+w).$$

If *w* is constant, then

$$\ln\rho = -3(1+w)\ln a;$$

implying:

$$\rho = a^{-3(1+w)}.$$

Therefore, for dark energy with a constant equation of state *w*, $\rho_{de}(a) = \rho_{de0}a^{-3(1+w)}$. If this is substituted into the Friedman equation in a similar way as before, but this time set $k = 0$, whic assumes a spatially flat universe, then (see shape of the universe)

In[40]:=

```
(*Physical Hubble Function at a given redshift = ȧ/a *)
PhysicalHubble[z_,H0_,Ωm_,Ωd_,w_,Ωr_]:=H0 Sqrt[Ωd (1+z)^(3(1+w)) +Ωm (1+z)^3+Ωr (1+z)^4]
```

In[41]:=

```
(*Conformal Hubble Function at a given redshift *)
ConformalHubble[z_,H0_,Ωm_,Ωd_,w_,Ωr_]:=H0 Sqrt[Ωd (1+z)^(1+3 w) +Ωm (1+z)+Ωr (1+z)^2]
```

In[49]:=

```
(*Physical Hubble Function at a given redshift = ȧ/a *)
ConformalHubblePrime[z_,H0_,Ωm_,Ωd_,w_,Ωr_]:=D[ConformalHubble[z,H0,Ωm,Ωd,w,Ωr],z]*(-(1+z)
```

In[50]:=

```
(*(*Physical Hubble Function at a given redshift = ȧ/a *)
ConformalHubblePrimePrime[z_,H0_,Ωm_,Ωd_,w_,Ωr_]:=D[ConformalHubblePrime[z,H0,Ωm,Ωd,w,Ωr]
```

The equation to find a''(\tau) which must be consistent with what we find previously. We don't need to do it, instead we can just

To find Ψ and Ψ' we define $N = \ln a$ as a new time coordinate, where we have,

$$\mathcal{H}^2 = H_0^2 \left[\Omega_m e^{-N} + \Omega_r e^{-2N} + \Omega_{kess} e^{-(1+3w)N} \right] \tag{1.6}$$

$$\partial_\tau^2 \phi + 3\frac{\dot{a}}{a}\partial_\tau \phi - \left(8\pi G a^2 \bar{p} + 2K\right)\phi = 4\pi G a^2 \delta p \ .$$

Here we neglect pressure from radiation, meaning that we don' t go deep to the radiation domination and we start our equations from z = 1000 for example

In[51]:=

```
(*Here we neglect pressure from radiation, meaning that we don't go deep to the radiation domin
dNlnH[N_,H0_,Ωm_,Ωd_,w_,Ωr_]:= D[ Log[ConformalHubble[1/E^N-1,H0,Ωm,Ωd,w,Ωr]],N] (*N=ln a*)
Psiequation[ψ_,N_,H0_,Ωm_,Ωd_,w_,Ωr_]:= ψ''[N] +(3+ dNlnH[N,H0,Ωm,Ωd,w,Ωr])ψ'[N]+(2-———
                                                                                  2 Ωm
(*Psiequation[ψ_,N_,H0_,Ωm_,Ωd_,w_,Ωr_]:= ψ''[N] +(3+ dNlnH[N,H0,Ωm,Ωd,w,Ωr])ψ'[N]+(2-———
                                                                                   2 Ωm
(*Eq =ψ''[lna] +(3+dlnHdlna)ψ'[lna]+(2-3Ωm/(2Ωm + 2(1-Ωm)Exp[-3w lna])+dlnHdlna)ψ[lna]== 0*)
```

```
(*s = NDSolve[{((a'[tau] -a[tau]H0√(Ωm a[tau]^-1+(1-Ωm)a[tau]^-(1+3 w)) == 0)/.Ωm→ 3/10/.w→-0.9/
```

RESULTS

The formula for \HH(z)

In[53]:=

```
ConformalHubble[z,H0,Ωm,Ωd,w,Ωr]//pdConv
```

Out[53]//TraditionalForm=

$$\mathrm{H0}\sqrt{\Omega\mathrm{d}\,(z+1)^{3w+1}+\Omega\mathrm{m}\,(z+1)+\Omega\mathrm{r}\,(z+1)^2}$$

The formula for d \HH/dtau (z)

In[55]:=

```
ConformalHubblePrime[z,H0,Ωm,Ωd,w,Ωr]//pdConv
```

Out[55]//TraditionalForm=

$$-\frac{1}{2}\,\mathrm{H0}^2\,(z+1)\left((3\,w+1)\,\Omega\mathrm{d}\,(z+1)^{3\,w}+\Omega\mathrm{m}+2\,\Omega\mathrm{r}\,(z+1)\right)$$

```
(*(* Cosmology setting *)
ΩΛ=0.687;
H0=0.0691023;
Ωm=0.31;
Ωr=9 10^-5;
w=-1;*)
```