1 Non-linear contrubution

Here we write the full equations and try to solve them numerically in Gevolution, and if we got something intreesting or strange we need to solve them in mathematica to see if we did not have make a mistake !
To solve in mathematica we must solve with Non -linear solve command and to exactly compare with Gevolution we can get some symmetric situations. The discussion with Julian is as following,

> I just have a problem with the last part of the note where the time
> solution of discrete Poisson equation is written
> It looked strange to me that for any modes the time dependence of the
> growing mode decreases!

No, epsilon can have either sign. It vanishes for K=Laplace which is the
continuum limit. For K>Laplace epsilon is negative, while it is positive
for K<Laplace

> So I solved the Poisson equation myself and took
> the real part, the solution in mathematica suggest different relation
> than is written in the note. At the end depending on the K and Delta we
> get different behaviour for growing mode. Did I make a mistake somewhere?

Please simplify your expressions and you'll see that you got exactly
what I wrote.

>> P_{\phi'^cH}(k) ~ P_{\phi}(k) * 6.675 * (k/k_Ny)^4
>>
>> where k_Ny is the Nyqvist wavenumber in your simulation
>
> Can you please give me more information about where this formula come from?

The first (and least trivial) step is to compute the explicit
expressions for K and Laplace in discrete Fourier space, given the CIC
and NGP weight functions and discrete derivative operators used in the
code. This allows you to compute the growth rate of each mode individually.

The next step is to do a Taylor series expansion for small k/k_Ny, for
which the first term gives the (k/k_Ny)^4 factor - this factor is easy
to understand because in the continuum limit K~Laplace, and therefore

K/Laplace = 1 + c(n) (k/k_Ny)^2 + ...

is inevitable. c(n) here is a function of the direction of the k-vector,
and due to the symmetries of the grid it can only be a monopole plus an
l=4 spherical harmonic. Explicit computation gives

c(n) = -(4/5) pi^5/2 [Y_0^0(n) - (1/3) Y_4^0(n) - sqrt(5/126) Y_4^4(n) -
sqrt(5/126) Y_4^-4(n)]

So this already gives you the modified growth as a function of n and
k/k_Ny for k/k_Ny << 1. The power spectrum estimator is constructed by
integrating over the directions. This gives the numerical constant in my
final formula, which I quoted as 6.675 but which more precisely is
12 pi^4 / 175.

>> Could you maybe quickly plot this curve to see where this effect becomes
>> relevant?
>
> The result for different redshifts is attached, I assumed the K_Ny=\pi*
> N_grid/Boxsize .

This is excellent. It shows that your resolution was good enough that
this effect only becomes important right at the Nyqvist wavenumber, and
the phi' you get from gevolution should be quite safe from
discretization errors (at first order). Had you chosen Ngrid=1024 or
even 512, the smallest scales would have been quite contaminated, as the
curves would be shifted upwards by a factor of 16 or 256, respectively.

Let me know if you have further questions.

Maybe we can discuss this in the skype meeting. I think one testbed could be to consider plane symmetric configurations first. For these it turns out that K=Laplacian (as an identity) for any k/k_Ny - in other words, my function c(n) vanishes along the principal axes. These modes will therefore have the correct linear growth despite the discretization.

Its a very good idea. Also it looks straightforward to me. I'll try it.

Best,
Farbod

On 26 Apr 2018, at 22:28, Julian Adamek <julian.adamek@qmul.ac.uk> wrote:

Dear Farbod,

Exactly. But unfortunately for my local tests at most I can run with N_grid=256, which the interesting scales are contaminated by the error!

Maybe we can discuss this in the skype meeting. I think one testbed could be to consider plane symmetric configurations first. For these it turns out that K=Laplacian (as an identity) for any k/k_Ny - in other words, my function c(n) vanishes along the principal axes. These modes will therefore have the correct linear growth despite the discretization.

Another advantage would be that you can solve the evolution in Mathematica and compare to your implementation in gevolution.

Best wishes,
Julian

2 Sensitivity to initial conditions

What would be error if we set the initial condition at z=100 to zero?

3 The effect of non linearities on gravitational potential at some different redshifts Ψ and matter power spectrum

4 Trace the average of the perturbation to be consistent

5 Vector elliptic and vector parabolic consistency check