Assignment 2

CSE 330 Numerical Methods

Problem 1: Let $f(x) = \cos x$ on $[-1, 1]$. Interpolate at nodes $\{-\frac{\pi}{4},\ 0,\ \frac{\pi}{4}\}$, and let $p_2(x)$ be the quadratic interpolant.

(a) Show that the interpolation error can be written as

$$f(x) - p_2(x) = \frac{f^{(3)}(\xi)}{3!}\, w(x), \qquad w(x) = \left(x + \tfrac{\pi}{4}\right) x \left(x - \tfrac{\pi}{4}\right),$$

for some $\xi \in [-\frac{\pi}{4}, \frac{\pi}{4}]$.

(b) Find a numerical upper bound for

$$\max_{x \in [-1,1]} |f(x) - p_2(x)|.$$

Problem 2: Consider the Runge function $f(x) = \dfrac{1}{1 + 25x^2}$.

(a) Explain the Runge phenomenon and how Chebyshev nodes help mitigate it.

(b) Compute the degree-3 Chebyshev nodes for the interval $[-2, 4]$, then construct the interpolating polynomial $p_3(x)$ for f in Lagrange form using those nodes.

(c) Let the nodes from part (b) be x_0, x_1, x_2, x_3. Write the Lagrange *basis* polynomial $\ell_1(x)$.

Problem 3: Let $f(x) = e^x \sin x$ and $f'(x) = e^x(\sin x + \cos x)$. Estimate $f'(1)$ with step sizes $h \in \{0.1,\ 0.05,\ 0.025\}$ using the forward, backward, and central difference methods.

(a) For each h, compute the three f-difference estimates of $f'(1)$.

(b) Compute the relative error for each estimate,

$$\text{RelErr} = \frac{|\text{estimate} - f'(1)|}{|f'(1)|},$$

and state which method is most accurate based on your results.