

10.07.2025

(10)

Types of Relation

1. Reflexive Relation

aRa for all $a \in N$

সবগুলির
pair রয়েছে 201

$$A = \{1, 2, 3\}$$

$$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 3)\}$$

2. Irreflexive Relations:

$(a, a) \notin R$

সবগুলির
কোন pair নেই

/ কোন বিপৰীত pair
রয়েছে না

মাত্র not irreflexive

③ Symmetric Relation

$(a, b), (b, a)$

$aRb \quad bRa$ for every element

Ex.

$$A = \{1, 2, 3\}$$

$$R = \{(1, 2), (2, 1), (3, 3)\}$$

Antisymmetric

$aRb, bRa \quad \& \boxed{a=b}$

4.

$(2, 2) \quad (1, 1)$

$$A = \{1, 2, 3, 4\}$$

$$R = \{(1,1), (1,2), (2,3), (1,3), (4,4)\} \rightarrow (\text{Antisymmetric})$$

5. Asymmetric Relation :

$$(a,b) \notin (b,a)$$

$$A = \{1, 2, 3, 4\}$$

$$\times R_1 = \{(1,1), (1,2), (2,3), (1,3), (4,4)\}$$

$$\times R_2 = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)\}$$

$$\checkmark R_3 = \{(1,3), (2,1)\}$$

$$\checkmark R_4 = \emptyset : [\text{empty set, Asymmetric}]$$

$$\times R_5 = A \times A$$

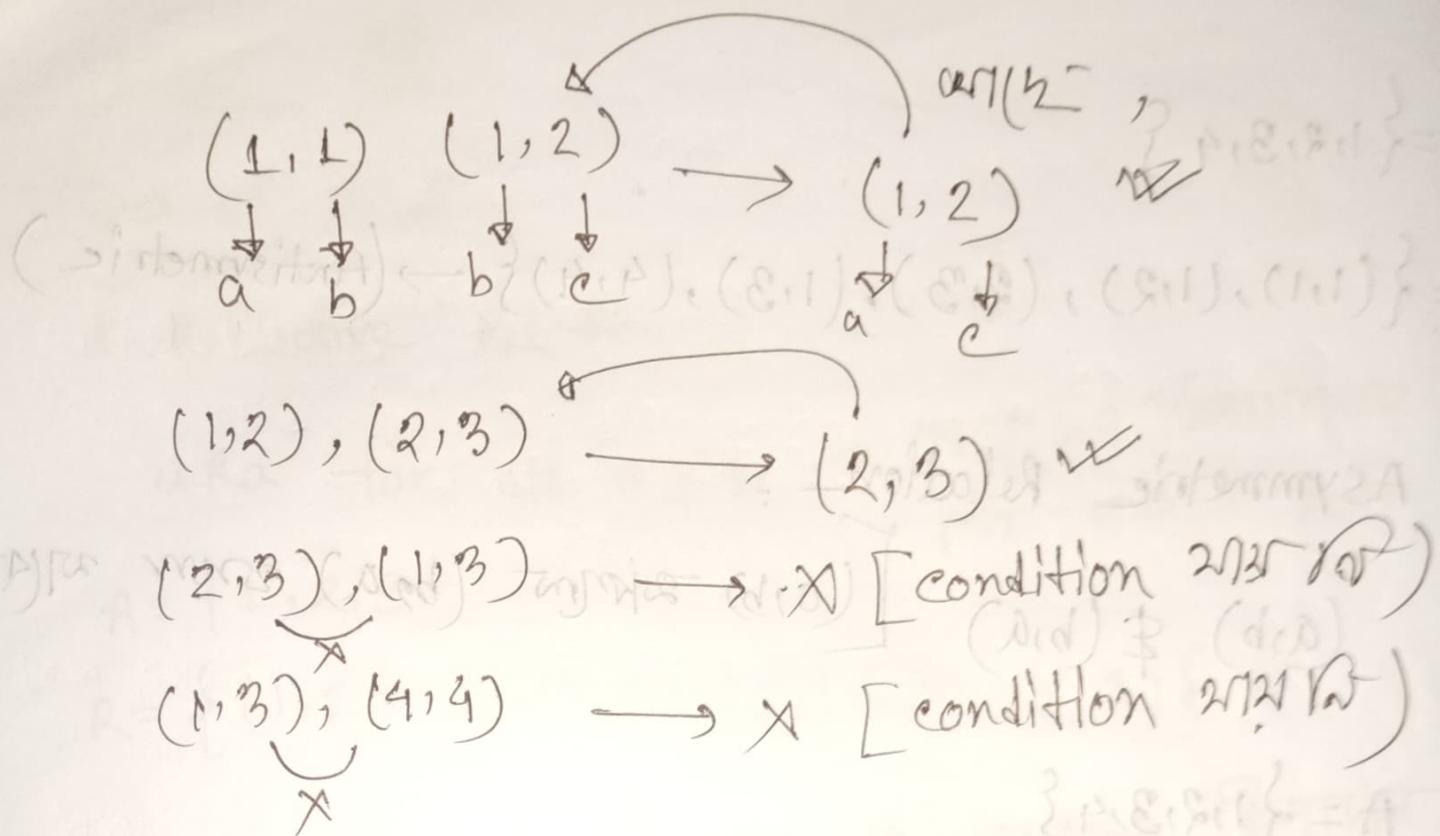
~~(2,1) (1,3) (2,3) X~~
Not transitive

6. Transitive Relations :

$(a,b) \& (b,c)$ then there must be $\boxed{(a,c)}$

$$A = \{1, 2, 3, 4\}$$

$$R = \{(1,1), (1,2), (2,3), (1,3), (4,4)\}$$



so, this is a transitive relation.

—o—

$\{(P, P), (S, S), (C, C), (H, H), (L, L), (M, M)\}$

$\{(1, 1), (2, 2)\} = \{1\}$

$(3, 3)$ is not from result set $\{(1, 1), (2, 2)\}$

$$A = \{1, 2, 3, 4\} = \{S_1 P_1\} \cup \{S_1 P_2\} \cup \{P_1 P_2\}$$

$$R_1 = \{(1,1), (2,2), (2,3), (3,2), (4,2), (4,4)\}$$

~~not reflexive because no pair of the form (a,a)~~

~~(S_1 S_1), (S_1 S_1) & (S_1 S_1), (P_1 P_1)~~

1. Relation R_1 is not reflexive because it does not

contains all order pairs of the form (a,a)

- for every element $a \in A$ i.e., R_1 has only

~~(1,1) (2,2) & (4,4) not (3,3)~~

2. Relation R_1 is not irreflexive because ~~(3,3) (1,1)~~

~~(2,2) and (4,4) is either in R_1 . O = P + R (Q)~~

3. Relation R_1 is not symmetric because for $(4,2)$

there is not $(2,4)$.

~~O = P + R | (P, R) \subseteq Q~~

4. Relation R_1 is not antisymmetric because

- for $(2,3)$ is here and $(3,2)$ is also here
but $2 \neq 3$.

5. Relation R_1 is ~~Not~~ transitive because there

~~(2,2), (2,3) has (2,3) (3,2), (2,3)~~

~~(2,3), (3,2) has (2,2) → (3,3)~~

(4,4), (4,2) has (4,2) $\{1,2,3,4\} = A$

6. Relation R_1 is not an asymmetric relation because of (1,1), (2,2) & (2,3), (3,2) and (4,4).

* problem solving to determine whether relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where, $(x,y) \in R$ if and only if

if a) $x+y=0$. b) $x-y$ is a rational number.

c) $xy=0$

a) $R = \{(x,y) | x+y=0\}$

1. Reflexive: $\forall a \in R (a,a) \in R$

Not reflexive. Only true for $(0,0)$.

2. Symmetric

$$\forall a \forall b \in R ((a,b) \in R \rightarrow (b,a) \in R)$$

if $a+b=0$ then $b+a=0$ i.e. $a=b$ right

Therefore, R is symmetric.

3. Antisymmetric

$$\forall a \forall b \in R ((a,b) \in R \wedge (b,a) \in R) \rightarrow (a=b)$$

So, Not antisymmetric

$$1+(-1)=0 \text{ and } -1+1=0 \text{ but } -1 \neq 1$$

4. Transitive, also if $(a-d)+(d-b) \in R \rightarrow (a,c) \in R$

$$\forall a \forall b \forall c ((a,b) \in R \wedge (b,c) \in R) \rightarrow (a,c) \in R$$

Therefore, not transitive

$$1+(-1)=0 \text{ and } (-1)+1=0 \\ \text{but } 1+1 \neq 0$$

⑤ R = { $(x,y) | x-y$ is a rational no.}

① Reflexive,

$a-a=0$ is a rational number.

2. Symmetric: if $a-b$ is a rational number

then $b-a = -(a-b)$ is also a rational number.

3. Antisymmetric: Not antisymmetric

$3-2$ and $2-3$ are both rational numbers
but $3 \neq 2$

4. Transitive:

if $a-b$ is a rational number and $b-c$ is also a rational number, then $a-c$ is also a rational number.

$a-b = (a-c) + (c-b)$

$(a-c) + (c-b)$ is also a rational number.

$a-c$ is also a rational number.

Ex: $\{x | (kx)\} = A$

wherever ①

more examples ② $\{x | (kx)\} = A$

Closure properties

17.07.2025

① Reflexive - $\rightarrow \text{M}(N)$

$$R \cup \Delta_A$$

$$\checkmark \quad \Delta_A = \{(a, a) : a \in A\}$$

~~$R \cup R^{-1}$~~

$$\{(1,1), (2,2), (3,3)\}$$

② Symmetric - $\rightarrow \text{M}(N)$

$$R \cup R^{-1}$$

Symmetrie (A) :

$$R \cup R^{-1}$$

$$= \{(1,1), (1,3), (3,1), (2,4), (4,2), (3,1), (1,3), (4,3), (3,4)\}$$

reflexive :

$$(R) = R \cup \Delta_A$$

$$= R \cup \{(1,1), (2,2), (3,3)\}$$

$$= \{(1,1), (1,3), (2,4), (3,1), (3,3), (4,3)\} \cup \{(1,1), (2,2), (3,3), (4,4)\}$$

$$= \{(1,1), (1,3), (2,2), (2,4), (3,1), (3,3), (4,3) \cup (4,4)\}$$

transitive - $\rightarrow \text{M}(N)$

$$\left. \begin{array}{l} R \circ R = R^1 \\ R^1 \circ R = R^2 \\ R^2 \circ R = R^3 \end{array} \right\}$$

$$R^1 \circ R = R^2$$

$$R^2 \circ R = R^3 \quad \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\} = R$$

$$\{1, 2, 3, 4\} = A$$

transitive (R) = $R \cup R^2 \cup R^3 \cup \dots \cup R^4$

$$(1,2), (2,3), (3,1) \Rightarrow R = \{1,2,3\}$$

$$R = \{(1,2), (2,3), (3,1)\}$$

$$R^2 = R \circ R = \{(1,3), (2,3), (3,3)\}$$

$$R^3 = R \circ R^2 = \{(1,3), (2,3), (3,3)\}$$

$$R \cup R^2 \cup R^3 = \{(1,2), (2,3), (3,1), (1,3), (2,3), (3,3)\}$$

$$\{(1,2), (1,3), (2,3), (3,1)\}$$

$$(1,2), (2,3), (3,1)$$

Equivalence Relation

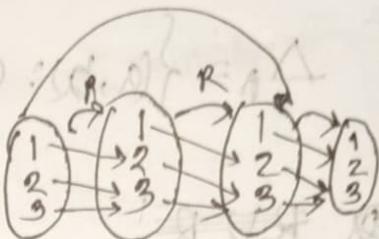
1. aRa (Reflexivity)

2. aRb then bRa

3. If aRb, bRc , then must aRc (Transitivity)

$$A = \{1, 2, 3\}$$

$$R = \{(1,1), (1,2), (2,1), (2,2), (3,3)\}$$



$$(1,3), (2,3), (3,3)$$

$$\emptyset (1,3) (2,3), (3,3)$$

(A) Sufficient

$$-\circ (1,1), (2,2), (3,3), (1,2), (2,1) \}$$

$$\{(1,1), (2,2), (3,3)\}$$

$$R = \{aRa\}$$

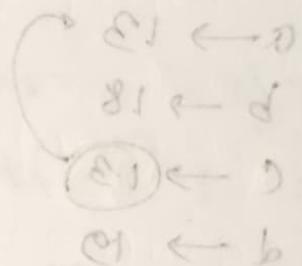
$$aR = R \circ R$$

$$aR = R \circ a$$

Relation R is reflexive, symmetric and transitive
That's why Relation R is equivalence.

iff reflexive iff no reasoning needs of C^o
Between $\exists i$ to $\exists j$ if b_i and b_j are abuttable

: M02 C^o



not abut otherwise there exist no

(abutment → other) C^o

$$d \geq d \wedge d \geq d - \{d, d - \{d\}\} = (d, d) \Delta$$

$$I + (\bar{e}, f) \Delta = (\bar{e}, \bar{f}) \Delta$$

$$I + [I + (\bar{e}, f) \Delta] =$$

$$I + [I + 0] =$$

$$S =$$

- ~~without loss of generality we can assume~~
- To each person on the earth assign the number which corresponds to his age
 - To each country in the world assign the latitude and longitude of its country

a) Soln:

$$\begin{aligned} a &\rightarrow 13 \\ b &\rightarrow 18 \\ c &\rightarrow 13 \\ d &\rightarrow 19 \end{aligned}$$

$$\begin{aligned} a &\rightarrow 13 \\ b &\rightarrow 13 \end{aligned}$$

so, it's not onto function.

b) (onto function)

$$Q(a, b) = \begin{cases} 0 & \text{if } a < b \\ Q(a-b, b) + 1 & \text{if } b \leq a \end{cases}$$

$$\begin{aligned} Q(12, 5) &= Q(7, 5) + 1 \\ &= [Q(2, 5) + 1] + 1 \\ &= [0+1] + 1 \\ &= 2 \end{aligned}$$

Aeurnum:

$A(m, n)$

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a) if $m=0$, then $A(m, n) = \underline{n+1}$

b) if $m \neq 0$ but $n=0$, then $A(m, n) = A(m-1, 1)$

c) if $m \neq 0$ and $n \neq 0$, then $A(m, n) = A(m-1, A(m, n-1))$

$A(1, 3)$

$$A(1, 3) = A(0, A(1, 2))$$

$$= \cancel{A(3)} \Rightarrow A(1, 2) = A(0, A(1, 1))$$

$$A(1, 1) = A(0, \underline{A(1, 0)}) \rightarrow A(0, 2)$$

$$A(1, 0) = A(0, 1)$$

$$= 1 + 1 = 2$$

$$A(1, 1) = A(0, 2) = 2 + 1 = 3$$

$$A(1, 2) = A(0, 3) = 3 + 1 = 4$$

$$A(1, 3) = A(0, 4) = 4 + 1 = 5$$

∴ $\boxed{A(1, 3) = 5}$
(Ans)

T	T	T
T	T	T
T	T	T
T	T	T
T	T	T

PROPOSITION

(C-4)

single proposition: $I + m = (m \cdot m)A$ result: $0 = m$ 4i (a)

$(1, 1-m)A = (x \cdot m)A$ result: $0 = m$ 4d of m 7i (b)

$((1-m, m)A, 1-m)A = (m \cdot m)A$ result: $0 = m$ 4c from 4b (c)

compound proposition: many proposition are composite, that's composed of subpropositions and various connecting devices subsequently. Such composite called proposition called $\perp A$

Basic logical operation:

conjunction ($p \wedge q$) \rightarrow AND

$\begin{matrix} p \\ q \end{matrix}$

$(1, 0)A \leftarrow (0, 1)A \quad (0, 0)A = (0, 1)A$

$p = \text{Roses are red}$

$q = \text{violet are blue}$

$p \wedge q = \text{Roses are red and violet are blue.} \perp A$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

$p = I + \bar{s} = (\bar{s}, 0)A = (\bar{s}, 1)A$

$\bar{s} = I + p = (p, 0)A = (s, 1)A$

$\boxed{\bar{s} = (s, 1)A} \therefore$

(Ans)

disjunction: (OR)

$P \vee Q$

Paris is in France

$\frac{08}{\sqrt{v}}$	$\frac{2+2=4}{q}$
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PV@

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

Negation : $\neg p$

$9 < \sqrt{9}$

(90) : molten 21b

$P \vee Q$

P	Q	$>(P \wedge Q)$				
T	T	T	F	F	T	
T	F	F	T	T	<u>F</u>	error at 21 21309
F	T	T	F	F	T	9
F	F	T	F	F	T	\vee
		①	③	②	$P \vee Q$	

If All output true \rightarrow tautology.

If All u false \rightarrow contradiction.

$P \vee Q$

tautology

P	$\geq P$	$P \vee Q$	$P \wedge Q$	\rightarrow contradiction
T	F	T	(P-F) F	(P < Q)
F	T	T	F	

$P \vee \geq(P \wedge Q)$

P	Q	$P \wedge Q$	$P \vee \geq(P \wedge Q)$	$P \vee \geq(P \wedge Q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	T	F	T	T

$$\neg(p \wedge q) = \neg p \vee \neg q$$

$$p \leftrightarrow q$$

p	$\neg q$	$(p \wedge q)$	$\neg(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F	F	F
F	T	T	F	T
T	F	F	T	T
F	F	T	T	T

$$p \vee q \Leftrightarrow p \leftarrow q$$

1. If is not the case that Roses are red and violets are blue

$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F
F	T	T
T	F	T
F	F	T

2. Roses are not red or violets are not blue

#

$p \rightarrow q = \text{if } p \text{ then } q$ [p implies q]

$p \leftrightarrow q$ (if and only if) ~~p~~ ~~q~~

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

if p true & q false
the $p \rightarrow q$ False

$$\boxed{P \leftrightarrow Q}$$

$$P \wedge Q \Leftrightarrow (P \wedge Q)$$

$P \wedge Q$	P	Q	$P \rightarrow Q$	T
F	T	F	\rightarrow	F
F	F	T	\rightarrow	F
T	T	T	\rightarrow	T
T	F	F	\rightarrow	T

$(P \wedge Q) \vee (P \wedge Q)$	P	Q	P	Q
F	T	F	T	F
F	F	T	F	T
T	T	T	T	T
T	F	F	F	F

$$\boxed{P \rightarrow Q \equiv P \wedge Q}$$

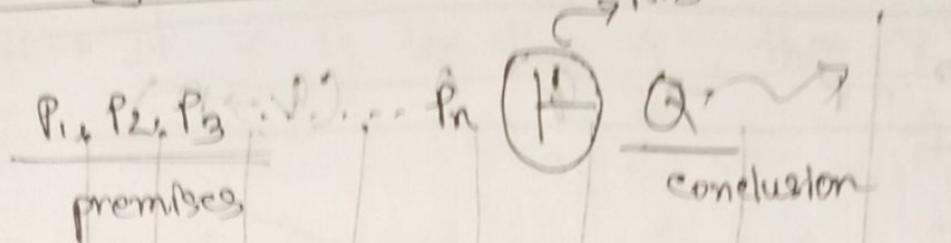
P	Q	$>P$	$>P \wedge Q$
T	F	F	T
F	T	T	F
T	T	T	T
F	F	T	F

[P wiedergibt Q] P wiedergibt Q für $= P \leftrightarrow Q$

$P \leftrightarrow Q (P \text{ wiedergibt } Q)$ $\rightarrow P \leftrightarrow Q$

$P \leftrightarrow Q$	P	Q
F	T	T
F	F	T
T	T	F
T	F	F

$P \leftrightarrow Q$ ist wahr für \leftarrow (7)

II Argument :

$$\frac{(P_1 \wedge P_2 \wedge P_3 \wedge \dots \wedge P_n) \rightarrow Q}{\text{tautology}}$$

$$\frac{P \rightarrow Q \quad Q \rightarrow R}{P \rightarrow R}$$

① ② ③

$$\frac{P \rightarrow Q, Q \rightarrow R}{(P \rightarrow Q) \wedge (Q \rightarrow R)} \rightarrow (P \rightarrow R)$$

P	Q	R	$(P \rightarrow Q)$	$(Q \rightarrow R)$	$(P \rightarrow Q) \wedge (Q \rightarrow R)$	$(P \rightarrow Q) \wedge (Q \rightarrow R) \rightarrow (P \rightarrow R)$			
T	F	T	T	T	T	T			
T	F	F	F	F	F	F			
F	T	T	T	T	T	T			
F	F	F	T	T	T	T			

1. prove $\neg q \rightarrow p$ $\neg q \rightarrow r$

$\neg q \rightarrow p$

$\neg q \rightarrow r$

$\frac{D \leftarrow q}{q \rightarrow p}$
 $\frac{D \leftarrow q}{q \rightarrow r}$
 $\frac{q \rightarrow p \quad q \rightarrow r}{\neg q \rightarrow p \wedge \neg q \rightarrow r}$

$\neg q \leftarrow p$
 $\neg q \leftarrow r$
 $\neg q \leftarrow p \wedge \neg q \leftarrow r$

R	Q	P	P → Q	(P → Q) ∧ (Q → P)
T	T	T	T	T
T	F	F	F	F
T	F	F	F	F
F	T	T	T	T
F	F	F	F	F
F	F	T	T	T

(P → Q) ∧ (Q → P)	(P → P)	H.W.	Q	P	Q
T	T		T	T	T
T	F		F	T	F
F	T		T	F	T
F	F		F	F	F

S1: If a man is unhappy Bachelor, he is unhappy

P

S2: If a man is unhappy, he dies young.

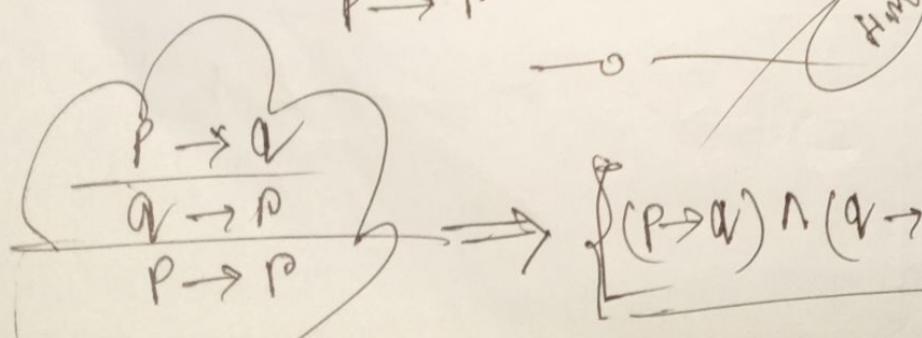
Q

P

S3: Bachelor dies young.

P → P

H.W.



$$\{(P \rightarrow Q) \wedge (Q \rightarrow P)\} \rightarrow P \rightarrow P$$

~~Now~~
S1 → If \exists is less than 4, then \exists is not a prime number.

S2 → \exists is not less than 4 $\neg P \rightarrow Q \rightarrow$

S3: \exists is a prime number $\neg Q \rightarrow \neg P \vee Q$

P	Q	$\neg P$	$P \rightarrow Q$	$(P \rightarrow Q) \wedge (\neg P)$	$\neg Q$	$(P \rightarrow Q) \wedge (\neg Q)$
T	T	F	T	F	F	T
T	F	F	F	F	T	F
F	T	T	T	T	F	F
F	F	T	F	F	F	F

P	Q	$\neg P$	$\neg Q$	$(P \rightarrow \neg Q)$	$(P \rightarrow \neg Q) \wedge (\neg P)$	$(P \rightarrow \neg Q) \wedge (\neg Q)$
T	T	F	F	T	F	T
T	F	F	T	T	T	F
F	T	T	F	F	F	F
F	F	T	T	T	F	F

\rightarrow propositional function:

$P(x) \rightarrow$ true/false for all $x \in A$

$$\neg P \models \{x : P(x)\}$$

$\forall x \models P(x) \rightarrow$ Universal quantifier

\rightarrow true for all element

$$(\forall x \in N)(x+4 > 3) \rightarrow \text{True}$$

$$n=1, 2, 3, \dots$$

$$(\forall x \in N)(x+4 < 7) \rightarrow \text{False}$$

Existential quantifier:

$\exists x P(x) \rightarrow$ true for any element/value at least one.

$$(\exists n \in N)(x+4 < 7) \rightarrow \text{True}$$

$$n=1, 2 \rightarrow \text{True}$$

$$n=3, 4 \rightarrow \text{false}$$

$$(\exists n \in N)(x+4 < 7) \rightarrow \text{false}$$

Negation of those :-

$$> (\forall x \in M) (\underline{x \text{ is male}}) \\ p(x)$$

All math majors are male

It is not the case []

There exists at least one math major who is female

$$(\exists x \in M) (\underline{x \text{ is not male}}) \\ p(x)$$

$$> [\forall x \in A) p(x)] = [\exists x \in A) \neg p(x)]$$

For all positive integers n, we have $n+2 > 8$

$$> [(\forall n \in N) p(n)]$$

$$(\exists n \in N) \neg p(n)$$

There ^{exist} ~~at least~~ one ~~at least~~ one n ,

There exists a positive integer n . Such that

$$n+2 > 8$$

$$\rightarrow [(\exists x \in A) P(x)] = (\forall x \in A) P(x)$$

to rephrase
(there is x) ($M \ni x$)

$$P(x,y) \quad x+y=10$$

$$\forall x \exists y, P(x,y)$$

For All x there exist y , such that $x+y=10$

$$B = \{1, 2, \dots, 9\}$$

this statement true.

$$\exists y \forall x [P(x,y) \wedge \exists_{x,y}] = [\text{true} (\exists x, y)]$$

there exist ay such that for all x , we have

$$x+y=10$$

false

$$\rightarrow [\forall x \exists y, P(x,y)]$$

$$\equiv \exists x, \forall y \rightarrow P(x,y)$$

Note, it is better writing a false statement

Ques. $U = \{1, 2, 3\}$

2019/07/11

a) $\exists x \forall R, x \in R \wedge x < R+1$

for $x=1$,
 $T \left\{ \begin{array}{l} 1 < 1+1 \\ 1 < 1+2 \\ 1 < 1+3 \end{array} \right.$

$$(\checkmark) \wedge (\checkmark) \wedge (\checkmark)$$

So condition is true.

CNF (Conjunctive Normal Form)

b) $\forall x \exists y \boxed{x^y + y^x < 12}$

$$T \left\{ \begin{array}{l} 1^1 + 1^1 < 12 \\ 1^2 + 2^1 < 12 \\ 1^3 + 3^1 < 12 \end{array} \right.$$

$$(q \wedge p \wedge s) \vee (p \wedge q)$$

$$(\checkmark) \wedge (\checkmark) \wedge (\checkmark)$$

$$[(q \wedge p \wedge s) \vee p] \wedge [(q \wedge p \wedge s) \vee q] =$$

So condition true.

$$[(q \vee p) \wedge (p \vee p)] \wedge [(q \vee p) \wedge (p \wedge q) \wedge (q \wedge p)] =$$

c)

$\forall x \forall y \boxed{x^y + y^x < 12}$

false

—o—

NORMAL FORMS

$$(\neg \neg) \wedge (\neg \neg) \wedge (\neg \neg)$$

↓

Bracketed form (1) among others

CNF (conjunctive normal form)

$$(P \wedge Q) \vee (\neg P \wedge Q \wedge R)$$

$$\neg (\neg \neg) \wedge (\neg \neg) \wedge (\neg \neg)$$

$$= [P \vee (\neg P \wedge Q \wedge R)] \wedge [Q \vee (\neg P \wedge Q \wedge R)]$$

$$= [\underbrace{(P \vee \neg P)}_{\text{TRUE}} \wedge (\neg P \wedge Q) \wedge (\neg P \wedge R)] \wedge [\underbrace{(Q \vee \neg P)}_{\text{TRUE}} \wedge (\neg Q \vee Q) \wedge (\neg Q \vee R)]$$

$$= (P \vee Q) \wedge (P \vee R) \wedge (Q \vee \neg P) \wedge Q \wedge (Q \vee R)$$

$$(A \vee B)^c = A^c \wedge B^c$$

$$\neg(P \vee Q) = \neg P \wedge \neg Q$$

(DNF) \rightarrow Disjunctive Normal form [Normal form]

$$(\quad) \vee (\quad) \vee (\quad)$$

Ex- $(\neg p \rightarrow p) \wedge (p \leftrightarrow q)$ DNF? $\left| \begin{array}{l} p \rightarrow q = (p \rightarrow q) \wedge (q \rightarrow p) \\ = (\neg p \vee q) \wedge (\neg q \vee p) \end{array} \right.$

$$\frac{!C}{!(a \rightarrow b)} = q^{\neg p} = (q \wedge r)q$$

$$\frac{!C}{!(a \rightarrow b)} = \frac{!C}{(q \rightarrow p)} = (q \wedge r)q$$

∴ $(\neg p \rightarrow p) \wedge (p \leftrightarrow q) = (\neg p \vee q) \wedge (\neg q \vee p)$

$$\frac{!C}{!(a \rightarrow b)} = \frac{!C}{(q \rightarrow p)} = (q \wedge r)q$$

$$\frac{!C}{!(a \rightarrow b)} = \frac{!C}{(q \rightarrow p)} = \frac{!C}{(q \wedge r)q} = (q \wedge r)q$$

permutation & combination

permutation :-

a, b, c, d

abcd, acbd, adeb

abe, acb, aed

ab, bc

a...

$$P(n, r) = {}^n P_r = \frac{n!}{(n-r)!}$$

$$P(6, 3) = \frac{6!}{(6-3)!} = 120$$

Repetition - arrangement,

RADAR

1 2 1 2

R R A A D

2 1 2 1

R R A A D

$$P(n, r) = \frac{5!}{2! 2!} = \frac{5 \times 4 \times 3 \times 2!}{2 \times 1 \times 2!} = 30$$

(Ans)

combination

a, b, c, d
abe
acd
bcd
abd

combination

$$3! = 6$$

$$3! = 6$$

$$6$$

$$6$$

$$n_{C_p} = \frac{n!}{r!(n-r)!}$$

Suppose repetition are not permitted.

a) How many three digit numbers can be formed from the six digits 2, 3, 9, 5, 6, 7

$$\boxed{6} \quad \boxed{5} \quad \boxed{4} =$$

Options

if, 0, 3, 9, 5, 6, 7

$$\boxed{5} \quad \boxed{5} \quad \boxed{4}$$

[9 → 99]

!01

0 not allowed at first

b) How many of three are less than 400?

$$\boxed{2}$$

$$\boxed{5}$$

$$\boxed{4} =$$

[2 ← 2222]

!8

greater than 400

$$\boxed{4}$$

$$\boxed{5} \quad \boxed{4} =$$

!5 !4

Q) How many of three are even = ?

5 4 2 first three are even

b, c, d, e

odd

5 4 2 =

$\frac{2}{2} \times 10$

360

$\frac{2}{2}$

630

$\frac{2}{2}$

630

$\frac{2}{2}$

630

Word Arrange :

MISSISSI PPI [arrange]

$\frac{10!}{(9-10)!9!} \times 10$

$$= \frac{11!}{4! 9! 2!} \text{ (Arranging for 2nd category)} \\ = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3}{9! 9! 2!} \Rightarrow \frac{11 \times 10 \times 9 \times 7 \times 6 \times 5 \times 4 \times 3}{9! 9! 2!} \text{ Brown Galt} \\ = \boxed{34650}$$

$\Rightarrow \frac{11 \times 10 \times 9 \times 7 \times 6 \times 5 \times 4 \times 3}{9! 9! 2!} \text{ Fizz buzz ext most}$

If P is assume single,

$$\frac{10!}{4! 4!} \quad [PP \rightarrow P]$$

$\boxed{P} \quad \boxed{P} \quad \boxed{P}$

Fizz buzz ext most

4 S must be not be separated

$$\frac{8!}{4! 2!} \quad [SSSS \rightarrow S]$$

$\Rightarrow \boxed{P} \quad \boxed{P} \quad \boxed{S}$

$\Rightarrow \boxed{P} \quad \boxed{P} \quad \boxed{A}$ (IP next category)

A bag contains six white marbles and five red marbles. Find the number of ways four marbles can be drawn from the bag if a) they can be any color,

b) two must be white and two red

c) they must all be of the same color.

a)

$${}^{11}C_4 = \frac{11!}{4!(11-4)!} \rightarrow \text{combination}$$

$$= \frac{11 \cdot 10 \cdot 9 \cdot 8}{4 \cdot 3 \cdot 2 \cdot 1}$$

$$= 330 \text{ ways}$$

b) $\left(\frac{6}{2}\right)\left(\frac{5}{2}\right) = \frac{6 \cdot 5}{1 \cdot 2} \cdot \frac{5 \cdot 4}{1 \cdot 2} = 150 \text{ ways}$

c) $\left(\frac{6}{4}\right) + \left(\frac{5}{4}\right) = 15 + 5 = 20 \text{ ways}$

Q → out of 12 employee a group of four trainers
 is to be sent for "Software testing QA"
 training of one month

a) How many ways can four employee be selected?

$$\rightarrow {}^{12}C_4 = \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}$$

b) What if there are two employee refuse to go together for training?

⊗ ⊙

i) if two are refuse, then

$${}^{10}C_4$$

ii) if A refuse B go

$${}^{10}C_3$$

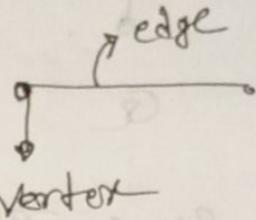
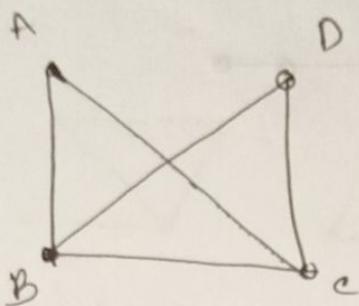
$$= \left(\frac{10}{2}\right) \left(\frac{10}{2}\right)$$

iii) ${}^{10}C_3$ if B refuse A go

So, total way,

$${}^{10}C_4 + {}^{10}C_3 + {}^{10}C_3$$

Graph

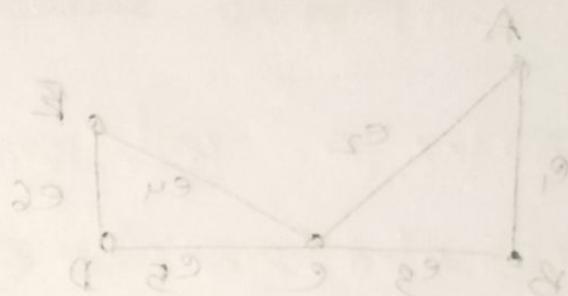
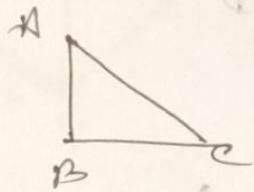
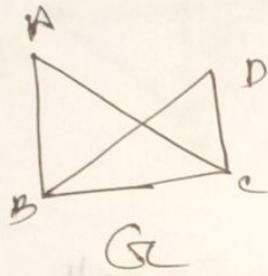


Degree of A = 2

Degree of B = 3

Finite graph:

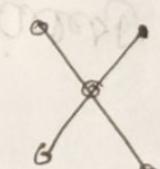
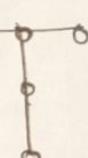
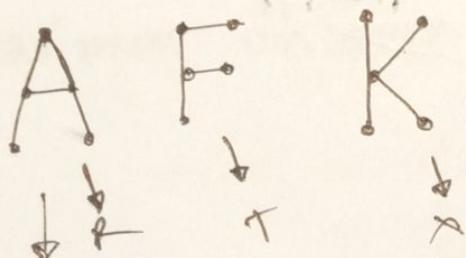
Subgraph:



$G \subset A$

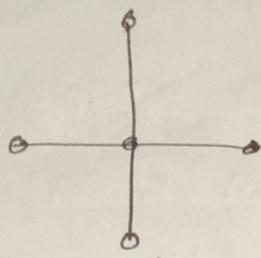
$H \subset A$

Isomorphic Graph:

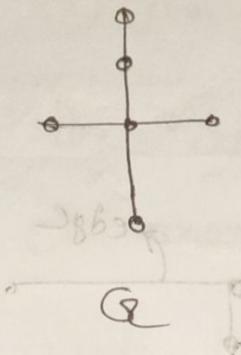


vertex & edge same

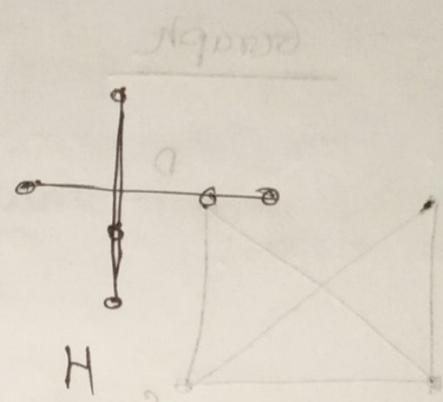
degree ~~match~~ \neq & $(1-1)$ correspond



Base graph



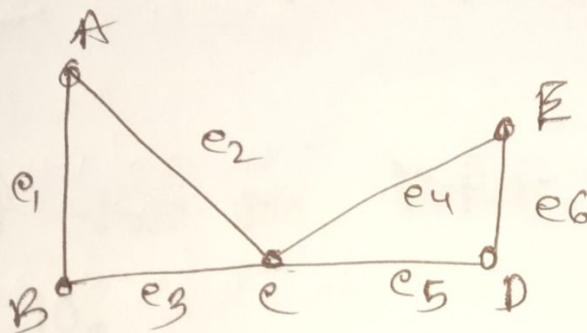
Graph G



Graph H

Base graph Some ~~not~~ homomorphic Graph

Paths & connectivity



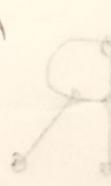
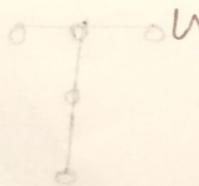
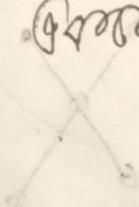
$A \rightarrow D$

$(A, e_1, B, e_3, C, e_5, D)$

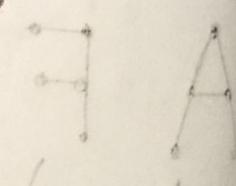
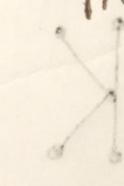
vertex Conn. traverse w/ simple path

edge

graph

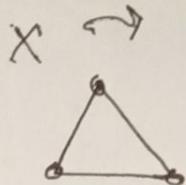
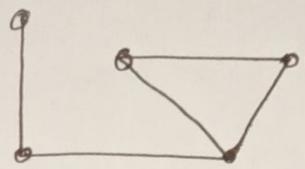


trail.

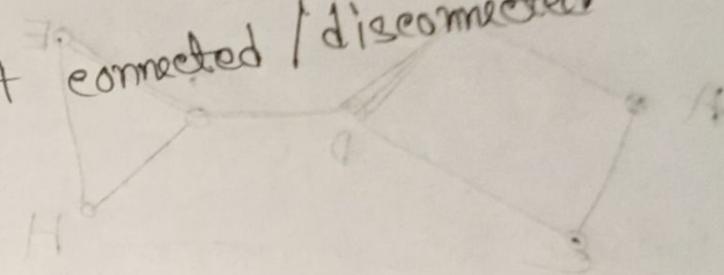


more info & notes

connectivity



\rightarrow Not connected / disconnected

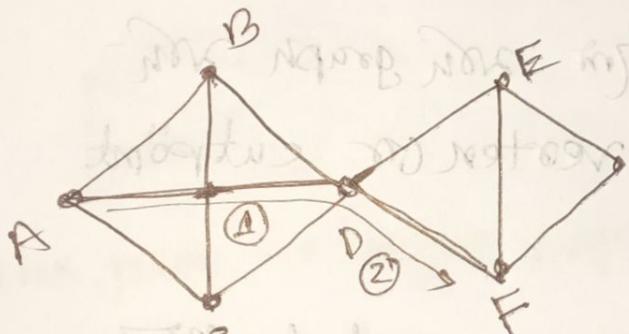


Distance and diameter: Diameter :

$$[P = (H, A) \text{ b}]$$

\rightarrow min vertex (or max vertex) \rightarrow 273220

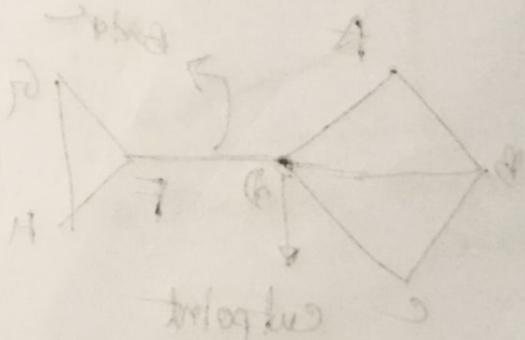
Shortest path called distance

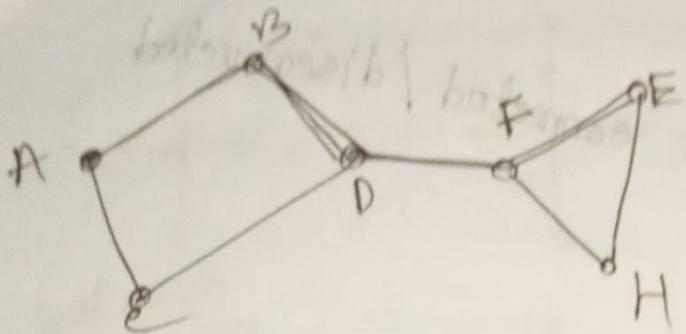


$$d(A, F) = 2$$

$$d(A, H) = 3$$

[Maximum distance = Diameter]



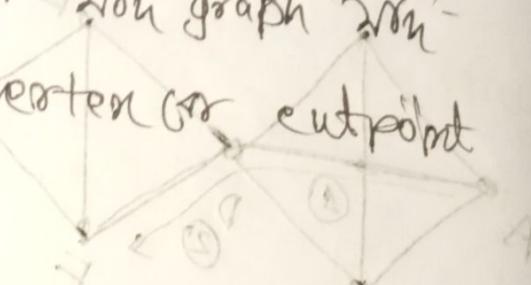


$$[d(A, H) = 4]$$

so diameter ≈ 4

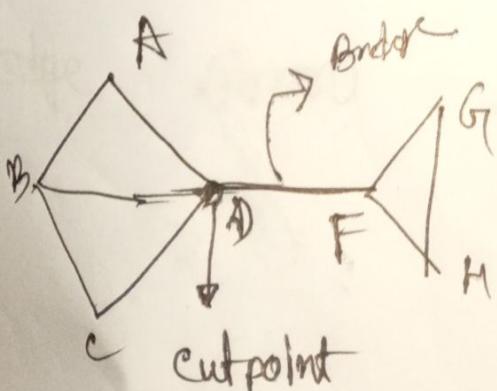
Bridge & Cutpoint

Bridge vertex (or cutpoint) from graph
 If disconnected by 1 vertex or cutpoint

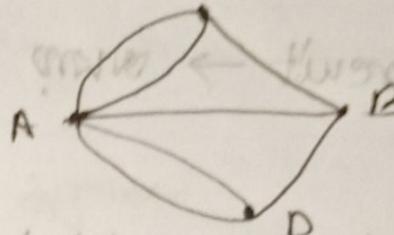
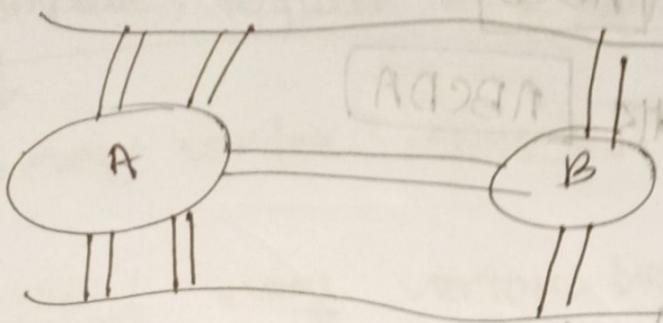


if edge removed from Disconnected graph

exist (Bridge) \rightarrow Bridge \rightarrow $S = (H, D)$ \rightarrow $S = (F, A)$

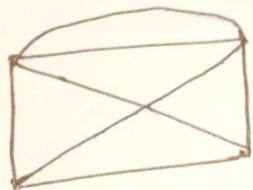


External = external minimum

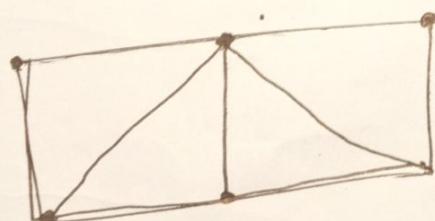


A finite connected graph is Eulerian if and only if each vertex ^{has} even degree.

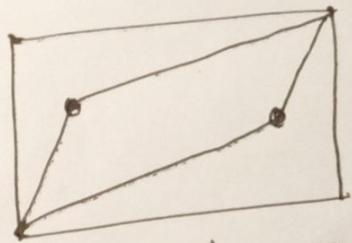
Traversable = odd two degree



- Eulerian graph \rightarrow every edge \in 奇数 traverse - 例 1
- Hamiltonian \rightarrow every vertex \in 奇数 u v



a) Hamilton & non-Eulerian.



b) Eulerian and non-Hamiltonian

Hamilton: if $n \geq 3$ and $n \leq \text{degree}(v)$ for every vertex.

path \rightarrow traverse \rightarrow ABCD

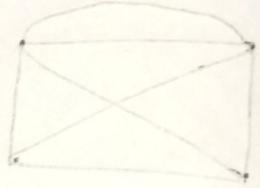
circuit \rightarrow closed loop traversal

Labeled and weighted graph :-

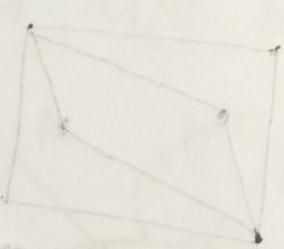
All edges have the same weight 2. Group between stuff
every edge has weight 1. Non adjacent edges out have weight 3.

edges out have weight 3.

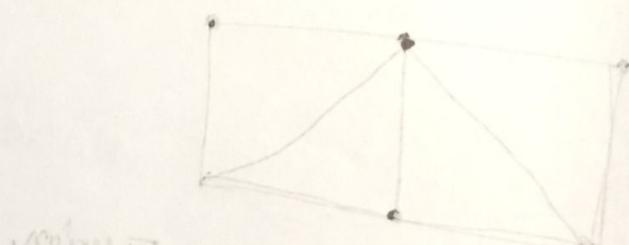
Other definitions



1) first traverse all edges \rightarrow non adjacent edges out have weight 3.



non adjacent edges out have weight 3.



non adjacent edges out have weight 3.

Prove that

$B^k = n$ bro $\in \mathbb{N}$ for $k \geq 1$: not limited

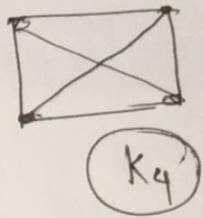
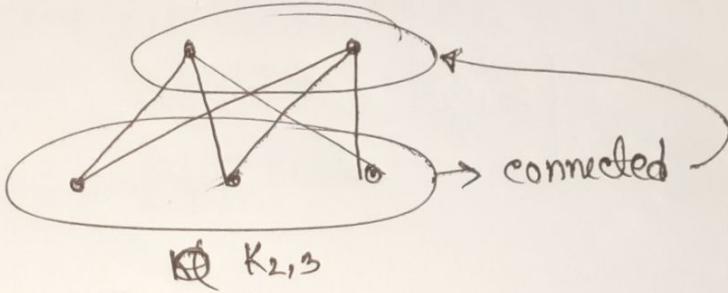
Complete, Regular & Bipartite Graphs

+

① every vertex connected in G

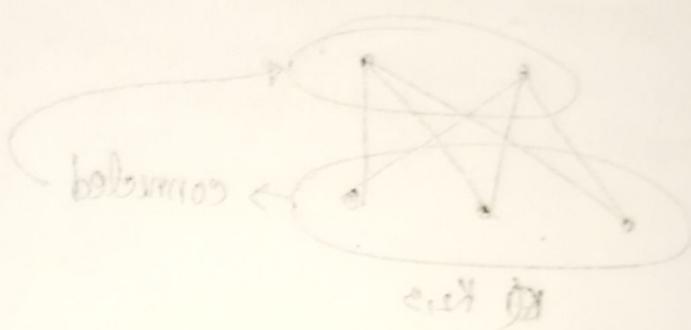
Regular: every vertex has same degree

bipartite:





pt

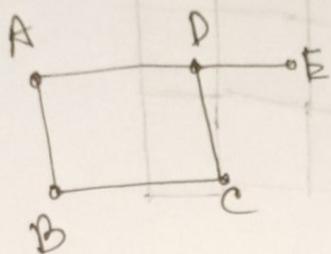


est B



Graphs representation

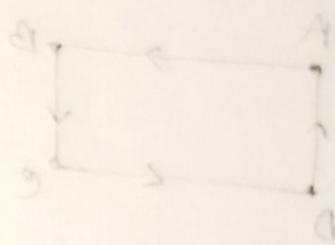
11.09.2025



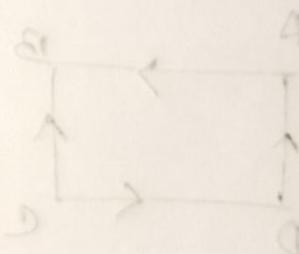
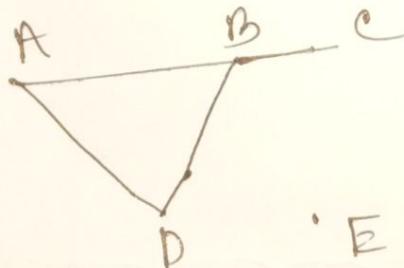
$adj_{ij} = \begin{cases} 1 & \text{if } v_i \text{ vertex adjacent to } v_j \\ 0 & \text{otherwise.} \end{cases}$

	A	B	C	D	E
A	0	1	0	1	0
B	1	0	1	0	0
C	0	1	0	1	0
D	1	0	1	0	1
E	0	0	0	1	0

Matrix of connections between vertices



link list representation



please see

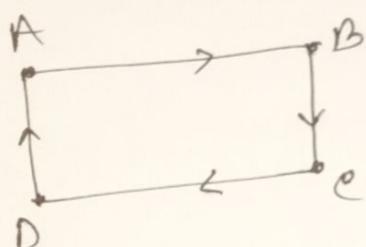
vertex	adj. list
A	B, D
B	A, C, D
C	B
D	A, E
E	O

CSOS-00-11

START

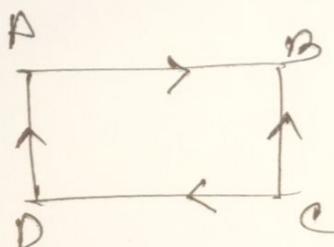
Vertex	1	2	3	4	5	6	7	8
Next v	1	2	3	4	5	6	7	8
PTR								

Strongly connected graph

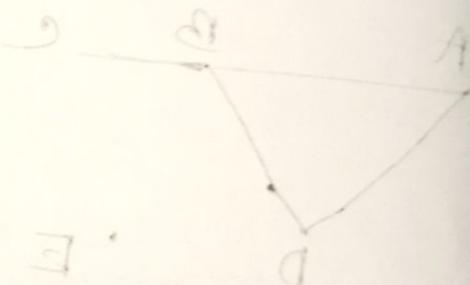


1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
1	0	0	0	1	0	0	0	0
0	1	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0

unilaterally



~~weak~~ weakly



fall - 6/60

Q, S
Q, S @ A

S
S, A
O

notes

A

B

C

D