PHY-111: Physics

Chapter- 2 (Magnetism)

by

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Magnetism

Magnetostatics

- ***Magnetostatics*** is the branch of electromagnetics dealing with the effects of electric charges in steady motion (i.e, steady current or DC).
- The fundamental law of *magnetostatics* is ***Ampere's law of force***.
- *Ampere's law of force* is analogous to *Coulomb's law* in electrostatics.

Two Fundamental Postulates

To study magnetostatics *in free space*, we need only consider the magnetic flux density vector, ***B***. The two fundamental postulates of magnetostatics that specify the divergence and the curl of ***B*** *in free space* are

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

μ_0 is the permeability of free space:

$$\mu_0 = 4\pi \times 10^{-7} \quad (H/m)$$

B is magnetic flux density.

J is current density function in space.

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What is magnetic force with example?

This force causes the magnets to attract or repel one another. Examples of magnetic force is **a compass, a motor, the magnets that hold stuff on the refrigerator, train tracks, and new roller coasters**. All moving charges give rise to a magnetic field and the charges that move through its regions, experience a force.

Torque in magnetism

A magnetic field exerts a torque which tries to align the normal vector of a loop of current with the magnetic field. The size of the torque on a loop of current is torque = (# turns) * (current) * (loop area) * (mag field) * sin(theta) where theta is the angle between the magnetic field and the loop's normal vector.

Ampere's Law

Ampere's circuital law states: The line integral of the magnetic field, over a closed path, or loop, equals μ_0 times the total current enclosed by that closed loop. We express this law through the mathematical expression:

$$\oint \overrightarrow{B} \cdot \overrightarrow{dl} = \mu_0 \, I$$

where ,I is the net current enclosed by the loop 'l';

μ_o = permeability of free space = $4\pi\times10^{-15}$N/A^2

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Hall effect:

If a conductor is placed in a magnetic field such the direction of flow of current is at right angle to the magnetic field, a voltage is developed across the conductor in the direction perpendicular to both current and field.
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The phenomenon by which voltage developed is called Hall effect and voltage itself is Hall voltage.

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Faraday's law of electromagnetic induction:

First law: Whenever the magnetic flux associated with any closed circuit changes, an induced current flows through the circuit which lasts only so long as the change in the magnetic flux produces change lasts. An increase in the magnetic flux produces inverse current, while a decrease of such flux produces a direct current.

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2nd law: The magnetude of the induced emf Produced in a coil is directly Proportional to rate of change of the magnetic flux through the coil.

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

where Φ_B is the magnetic flux through the circuit and the negative sign gives the direction of induced emf.

Negative sign shows that emf induced always oppose the change in flux.

Magnetism

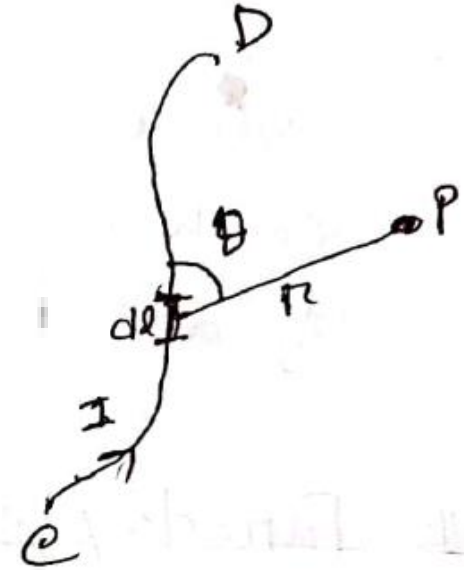
What is Biot Savart's Law?

- The **Biot Savart Law** is an equation describing the magnetic field generated by a constant electric current.
- It relates the magnetic field to the magnitude, direction, length, and proximity of the electric current. Biot–Savart law is consistent with both Ampere's circuital law and Gauss's theorem.
- The Biot Savart law is fundamental to magnetostatics, playing a role similar to that of Coulomb's law in electrostatics.

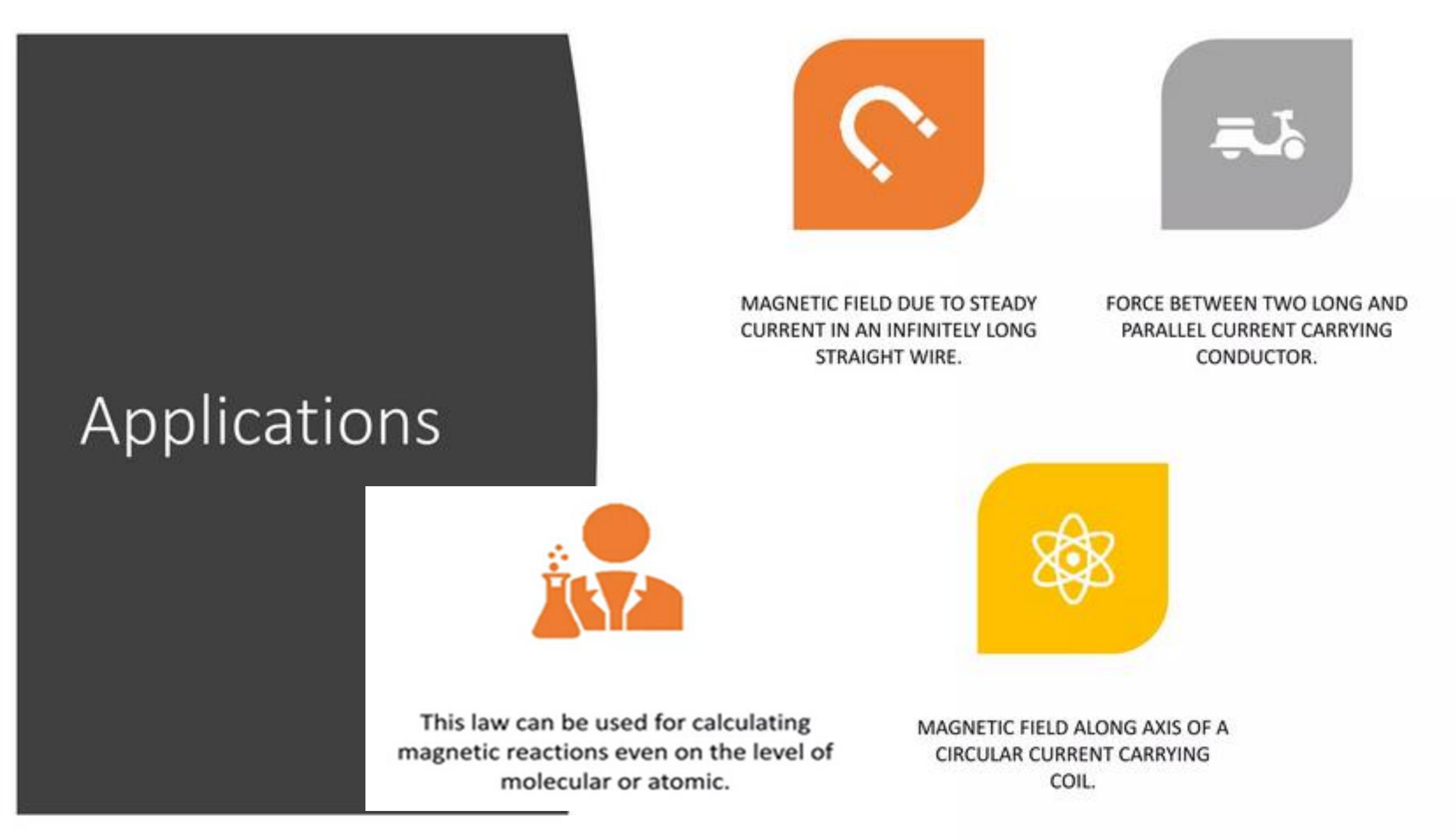
Biot-Savart law:

When current passes through a small length of a conductor then ~~any point~~ magnetic field at any point at the magnetic field produced around it is proportional to its small length, current ~~fa~~ flowing through it, sine angle between the conduction length and the connection line of the point and inversely proportion to the square of the distance between the point and the conductor.

$$\therefore dB \propto \frac{I \cdot dl \cdot \sin\theta}{r^2}$$



Magnetism


Applications
MAGNETIC FIELD DUE TO STEADY CURRENT IN AN INFINITELY LONG STRAIGHT WIRE.
FORCE BETWEEN TWO LONG AND PARALLEL CURRENT CARRYING CONDUCTOR.
This law can be used for calculating magnetic reactions even on the level of molecular or atomic.
MAGNETIC FIELD ALONG AXIS OF A CIRCULAR CURRENT CARRYING COIL.

Magnetism

* **Lenz's law:** An induced electromotive force (emf) always rise to a current whose magnetic field opposes the original change in magnetic flux.

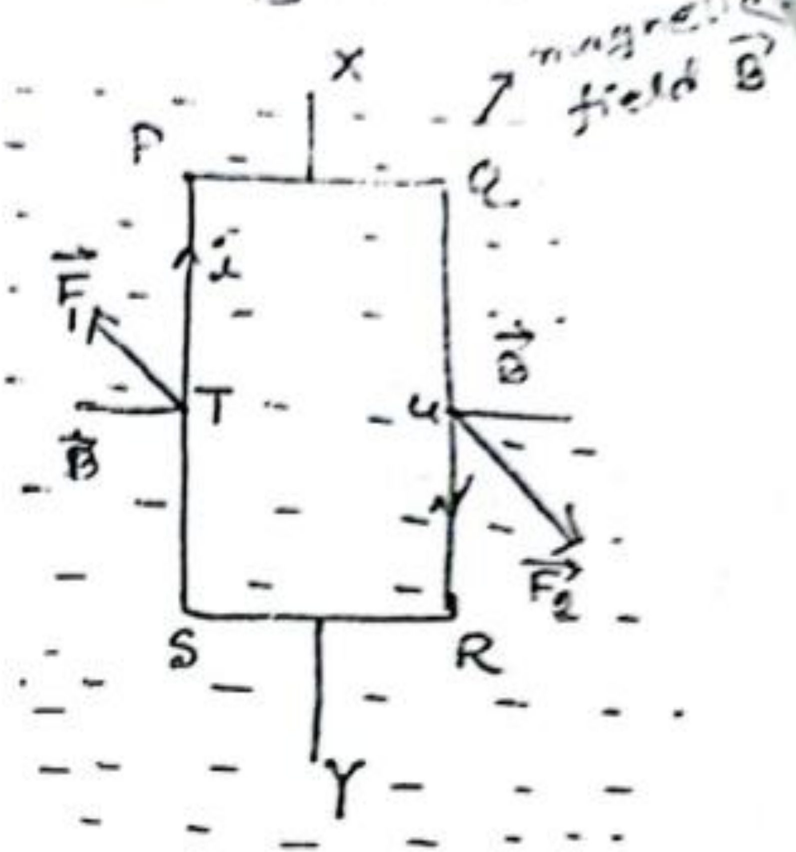
Lenz's law is shown with the minus sign in Faraday's law of induction, $\varepsilon = -N\frac{d\phi}{dt}$.

which indicates that the induced emf (ε) and the change in flux ($d\phi_B$) have opposite sign.

Magnetism

Torque on a current carrying loop:

Consider. PQRS is a small reactangular circuit which is suspended in a uniform field $\vec{B}$ as shown in figure. The direction of $\vec{B}$ being parallel to the plane. The length of PQ and RS is K and they are parallel to $\vec{B}$. Also the length of PS and QR is l and they are perpendicular to $\vec{B}$.



Magnetism

The magnitude of force on the side.

$PQ = I\,\vec{k} \times \vec{B} = 0$ [Because the angle betⁿ $\vec{k}$ and $\vec{B}$ is zero]

Force on the side $RS = I\,\vec{k} \times B = 0$ [" "]

Magnetism

Let the force on side SP is $\vec{F_1}$

$$\therefore \vec{F_1} = I\vec{l}\times\vec{B} = IlB\sin 90^\circ = IlB \text{ — (1)}$$

Again Let the force on side QR is $\vec{F_2}$

$$\therefore \vec{F_2} = I\vec{l}\times\vec{B} = IlB\sin 90^\circ = IlB \rightarrow (2)$$

From the above discussion we can see that the magnitude of $\vec{F_1}$ & $\vec{F_2}$ are equal but in opposite direction.

This force produce a torque. The torque try to circulate the circuit. So the torque is

$$\tau = IlB\times K$$

$$= IlBK$$

$$= IAB \rightarrow (3) \quad [\because l.K = A = \text{area of the circuit}]$$

Now, we draw an axis XY at the middle point of the circuit which is parallel to SP and QR. The direction of τ is along the XY direction.

vector from the equation (3) is

$$\vec{\tau} = \pm \vec{A} \times \vec{B} \longrightarrow (4)$$

The direction of area vectors $\vec{A}$ is perpendicularly downward to the page surface. If the coil has N turns, then the torque is given by,

$$\vec{\tau} = NI\, \vec{A} \times \vec{B} \longrightarrow (5)$$

Let, $NI\vec{A} = \vec{m} \longrightarrow (6)$

where m is the magnetic moment of the circuit, whose direction is along $\vec{A}$

combining eqn (5) and (6), we have

$$\vec{\tau} = \vec{m} \times \vec{B}$$

This coil have magnetic dipole.

Magnetism

Magnetic induction:

The process by which a substance, such as Iron, Steel becomes magnetized by a magnetic field i.e. the process that makes a substance magnetic. The induced magnetism is produced by the force of the field radiating from the poles of a magnet.

$$\vec{F_B} = q\vec{v} \times \vec{B}$$

where, F_B is the magnetic force, q is the moving charge, $\vec{v}$ is the velocity of charge q and $\vec{B}$ is the magnetic field induction.

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Self induction:

The process of generation of an electromotive force (emf) in a circuit by changing the current in that circuit.

Mutual Induction:

The production of an electromotive force (e.mf) in a circuit resulting from a change of current in a neighbouring circuit.

Magnetism

Classification of magnetic materials:

There are three types of magnetic materials. They are-

1. Paramagnetic material.
2. Diamagnetic material.
3. Ferromagnetic material.

Magnetism

ii) Paramagnetic material :

The paramagnetic material are weakly attracked by the magnets when placed in a magnetic field. The paramagnetic matarial feel an attractive force towards the strongest part of the field when they are placed in a non-uniform field.

Example :- Al, Co, $CuSO_4$.

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Diamagnetic material:

The diamagnetic materials are not attracted by magnets, They move from stronger to weaker parts of the magnetic field and characterised by negative susceptibility.

Example:- Cu, Au, Hg, S.

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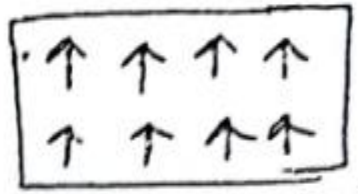
B/. Ferromagnetic material :



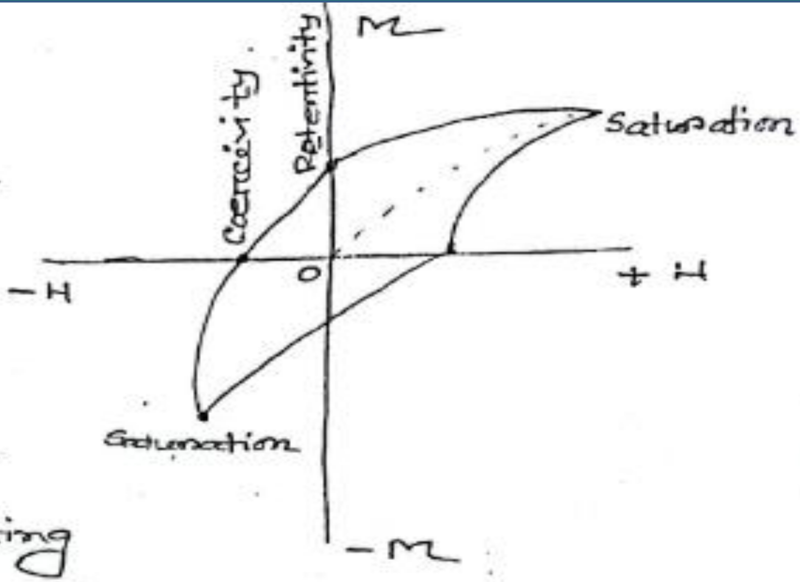
Ferromagnetic

The Ferromagnetic materials have very large permeability. They have the maximum degree of magnetization. These are the magnetic materials which are found to be attracted by magnets or magnetic fields. In this case the magnetic moments are equal and in the same direction.

Example :- Fe, Ni, Co.

Hysteresis :

When a ferromagnetic material is magnitized in one direction, It will not ~~realease~~ relax back to zero magnetization, when the imposed magnetizing field is removed. It must be driven back to zero by a field in the opposite direction. If an alternating magnetic field is applied to the material, its magnetization will trace out a loop called hysteresis loop. The lack of retrace ability of the magnetization curve is the property called hysteresis. The energy which is lost by this process is called hysteresis energy loss.



Hysteresis curve:

A hysteresis curve is a plot, graph or some kind of pictorial representation of the relationship betn the magnetic field and magnetization of the material.

Remanence:/ Retentivity:

The amount of magnetization remain after removing all the applied magnetic field is known as remanence.

Magnetism

Coercivity:

The amount of reverse field required to produce zero magnetization after removing the applied magnetic field. Soft magnetic material has small coercivity and hard magnetic material has large coercivity.

Maxwell's Equations:

1. $\nabla . E = \dfrac{\rho}{\epsilon_0}$ This equation is the same as Gauss's law. It states that the total electric flux through an area is equal to the sum of the charges in that region.

2. $\nabla . B = 0$ This equation states that the total magnetic flux through an area is 0. Hence if we assume that magnetic field is constituted by magnetic charges, in any area, the charges would all cancel each other. Hence, magnetic monopoles do not exist. You can't have only N or only S pole in any area.

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3. $\nabla \mathrm{x} E = -\frac{\partial B}{\partial t}$ This equation states that the change in the magnetic flux through an area is equal to the electric potential developed across it. Furthermore, the electric field is induced in such a way, that it opposes the direction in which magnetic field is increased. Thus, changing magnetic field gives you an induced electric field.

4. $\nabla \mathrm{x} B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$ This equation states that the total magnetic field developed is equal to the current in that region and the rate of increase in the electric field. Thus, changing an electric field gives you a magnetic field.

Here, μ_0 is the absolute permeability of free space, ϵ_0 is the absolute permittivity of free space.

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Maxwell's Equations

$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$	Gauss's Law: The electric field's mapping is equal to the charge density divided by the permittivity of free space. The relationship between electric field and electric charge
$\nabla \cdot B = 0$	Gauss's Law for Magnetism: The net magnetic flux out of any closed surface is zero. There is no such thing as a magnetic monopole
$\nabla \times E = -\frac{\partial B}{\partial t}$	We can make an electric field by changing a magnetic field
$\nabla \times B = u_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$	We can make a magnetic field with a changing electric field or with a current

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Pointing vector

Pointing vector, a quantity describing the magnitude and direction of the flow of energy in electromagnetic waves. It is named after English physicist John Henry Pointing, who introduced it in 1884. The Pointing vector S is defined as to be equal to the cross product $(1/\mu)E \times B$, where μ is the permeability of the medium through which the radiation passes (see magnetic permeability), E is the electric field, and B is the magnetic field. Applying the definition of cross product (see vector) and the knowledge that the electric and magnetic fields are perpendicular to each other gives the magnitude S of the Pointing vector as $(1/\mu)EB$, where E and B are, respectively, the magnitudes of the vectors E and B. The direction of the vector product S is perpendicular to the plane determined by the vectors E and B. For a traveling electromagnetic wave, the Pointing vector points in the direction of the propagation of the wave. The Pointing vector describes a power per unit area, and therefore its units are watts per square meter.

Thanks

For

Your Kind Attention

Magnetism

Magnetism

Maxwell's Equations:

- The equations describing the relations between changing electric and magnetic fields are known as Maxwell's equations.
- Maxwell's equation are extensions of the known work of Gauss, Faraday and Ampere. There are two forms of each Maxwell equation namely Integral form and Differential form(point form).
- Maxwell's equation in the Integral form governs the interdependence of certain field and source quantities(charge and current) associated with regions in space, surfaces and volumes.

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- The Differential form of Maxwell's equations relate characteristics of the field vectors at a given point to one another and to the source densities at that point.
- The Maxwell's equations provides the mathematical background for the study of electromagnetic waves, transmission lines and antenna.

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Maxwell's Equations: Differential and Integral Forms

Name of Law	Differential Form	Integral Form
Gauss's Law	$\nabla \cdot \mathbf{D} = \rho_V$	$\oint_S \mathbf{D} \cdot d\mathbf{s} = \int_v \rho_V \, dv = Q$
Faraday's Law	$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \dfrac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$
Gauss's Law of Magnetics	$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$
Ampere's Law	$\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$	$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{s}$

This version of the integral forms is the most useful for implementation of numerical methods

12.1

Potential difference bet^{n} spherical

$$V = \int dv = \int_a^b E.dr$$

$$= \int_a^b \frac{q}{2\pi \in_0 .rl} dr$$

$$= \frac{q}{2\pi \in_0 l} \int_a^b \frac{dr}{r}$$

$$= \frac{q}{2\pi \in_0 .l} [\ln r]_a^b$$

$$= \frac{q}{2\pi \in_0 .l} \ln \frac{b}{a}$$

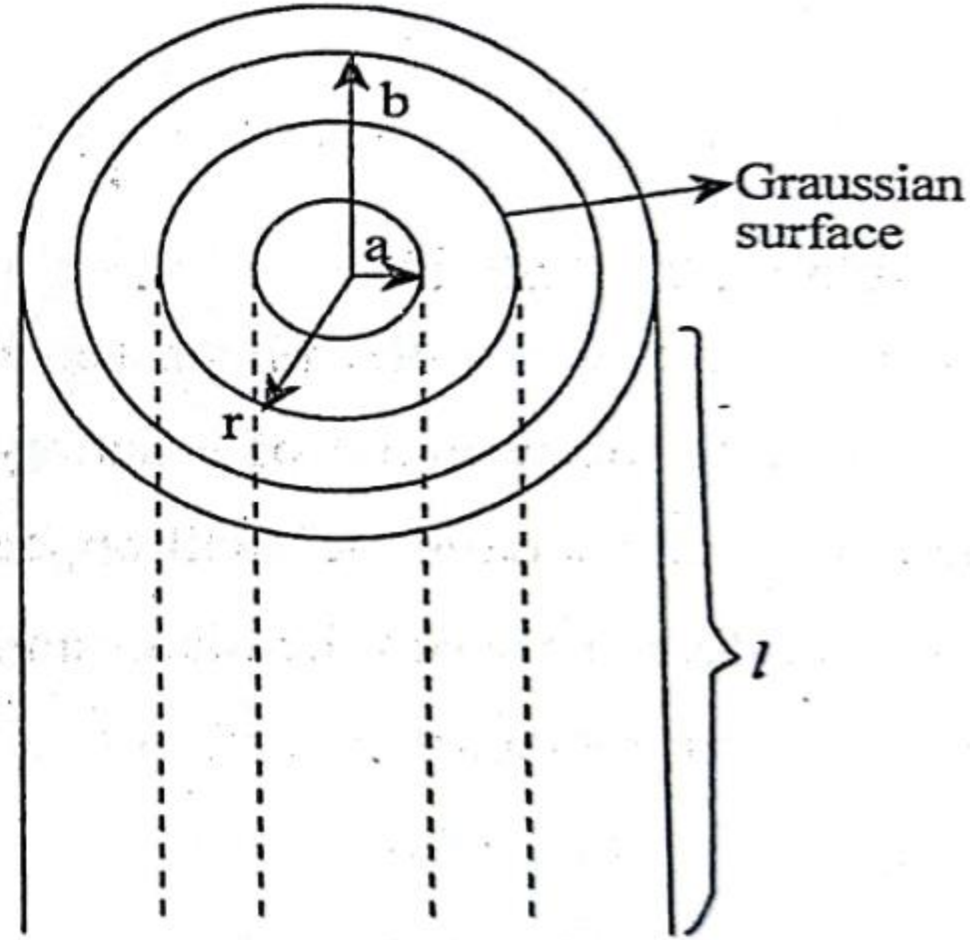

Fig:20

Now, capacitance $C = \frac{q}{v}$ $\Rightarrow C = \frac{q}{\frac{q}{2\pi \in_0 .l} In(b/a)}$ $\therefore C = 2\pi \in_0 . \frac{l}{\ln\left(\frac{b}{a}\right)}$

Q: Energy storage in an electric field of a charge conductor.

Let, C = capacitance of the Capacitor

q = the charge of the Capacitor

V= the raise of potential

Let the work done for charging, the conductor is U.

Therefore, the energy stored in the capacitor is U

Now if we give a small amount of charge dQ, so the energy increased by dU. Then we can write

$$dU = vdQ$$

$$\Rightarrow dU = \frac{Q}{c} dQ \text{..............................} (i)$$

Now when Q = 0 then U=0 and after charging when Q = Q then U = U

Now from-...... (i)

$$\int_0^u dU = \int_0^Q \frac{Q}{c} dQ$$

$$\Rightarrow [U]_0^u = \frac{1}{c}\left[\frac{Q^2}{2}\right]_0^Q \quad \Rightarrow U = \frac{1}{2C} \cdot Q^2 = \frac{1}{2} CV^2 \quad \therefore U = \frac{1}{2} CV^2$$

Charging of the capacitor:

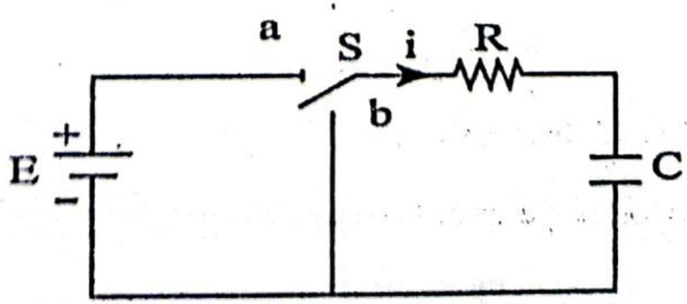

Fig- 31

Let us, consider a circuit contains a resistance 'R' a capacitor 'C' and an ideal battery of e.m.f E. All are connected in series as shown in fig.

When switch 'S' pressed to 'a' charge stored in the capacitor begin to increase, and let, at any time constant t, q is the of charge stored in the capacitor and i is the current in the circuit. We get potential difference across the capacitor, $V = \frac{q}{c}$, current in the circuit $i = \frac{dq}{dt}$.

Electricity

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Applying kirchoff's loop rule to the circuit in clock wise direction, we get.

$$E = iR + \frac{q}{c}$$

$$\Rightarrow E = \frac{dq}{dt}.R + \frac{q}{c}$$

$$\Rightarrow \frac{dq}{dt} = \frac{E}{R} - \frac{q}{Rc} \Rightarrow \frac{dq}{dt} = \frac{EC-q}{Rc}$$

$$\Rightarrow \frac{-dq}{Ec-q} = \frac{-dt}{Rc}$$

$$\Rightarrow \ln(Ec-q) = -\frac{t}{Rc} + \ln A$$

[Integrating both side]

$$\Rightarrow \ln(Ec-q) - \ln A = \frac{-t}{Rc}$$

(A is integrating constant)

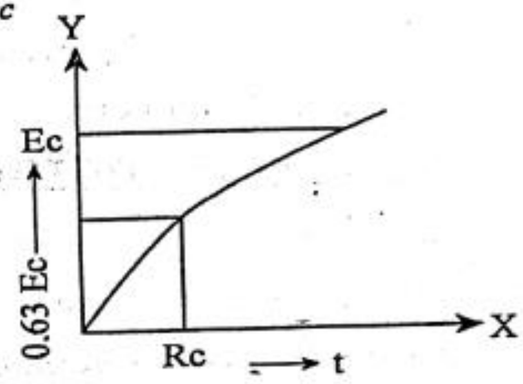

Fig- 32

$$\Rightarrow \ln\frac{Ec-q}{A} = \frac{-t}{Rc}$$

$$\Rightarrow \frac{Ec-q}{A} = e^{-t/Rc} \text{.................(1)}$$

When t =o, q = 0 then A = EC.

Now from (1) $\frac{Ec-q}{Ec} = e^{-t/Rc}$

$$\Rightarrow 1 - \frac{q}{Ec} = e^{-t/Rc}$$

$$\Rightarrow \frac{q}{Ec} = 1 - e^{-t/Rc}$$

$$\Rightarrow q = Ec\left(1 - e^{-t/Rc}\right)$$

This is the equ^n for charging a capacitor. The fig-1 shows that charg grows exponentially with time.

Now current in the circuit,

$$i = \frac{dq}{dt}$$

$$\Rightarrow i = \frac{d}{dt}\left\{Ec\left(1 - e^{-t/Rc}\right)\right\}$$

$$\Rightarrow i = Ec.\frac{e^{-t/Rc}}{Rc}$$

$$\Rightarrow i = \frac{E}{R}.e^{-t/Rc}$$

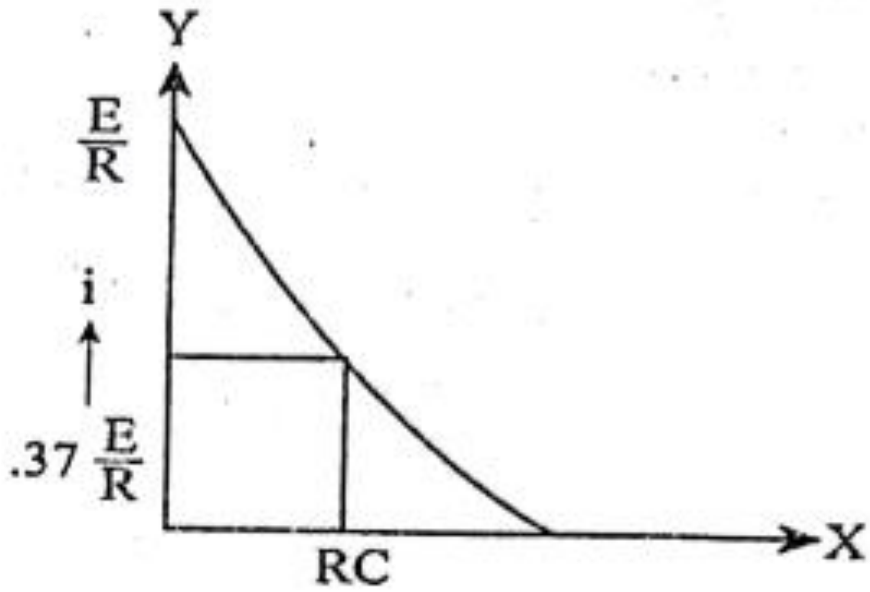

Fig-33

This is the equ^n for current in the R-C circuit. The current decreases exponentially with time as shown in graph.

*41 ~~Show that, capacitance of~~ for two capacitors of ~~are~~ equal capacitance, prove that the equivalent capacitance of the capacitors when in parallel is four times greater than when in series connections.

JCT

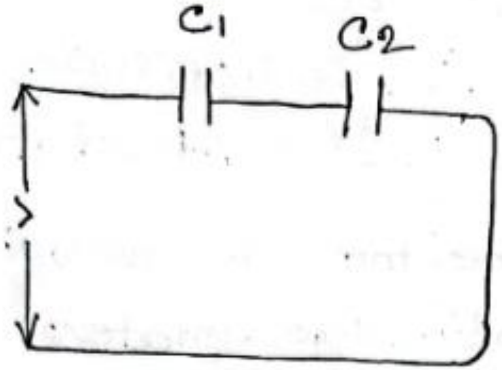

Fig 1. Capacitors in series

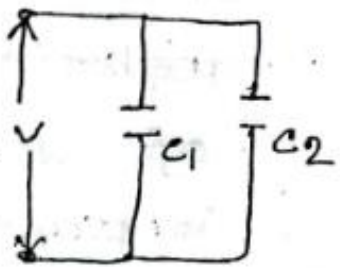

Fig 2. capacitor in parallel

We consider two capacitor with equal capacitance c_1 and c_2 connected in series (in fig: 1) and paralled connections in Fig 2.

we can write the equivalent capacitance for series combination is

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} \longleftarrow (i)$$

when two capacitance is equal. i.e. $C_1 = C_2$ then we can write from eq^n (1)

$$\frac{1}{C_3} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow \frac{1}{C_3} = \frac{1+1}{C_1}$$

$$\Rightarrow C_1 = 2C_3 \longleftarrow (ii)$$

Again we can write the equivalent capacitance for parallel combination:

$$C_p = C_1 + C_2 \quad \text{---} \text{(iii)}$$

When two capacitance is equal. i.e. $C_1 = C_2$ then can write eqn (iii)

$$C_p = C_1 + C_1 = 2C_1$$

$$\therefore C_p = 2C_1$$

$$= 2 \times 2\,C_S \quad [\text{From eq}^n\ 2)$$

$$= 4\ C_S.$$

So, we can write capacitance of two capacitors are equal to the equivalent capacitance of the capacitors when in parallel is four times greater than when in series connection.

A negative point charge of 10^{-6} coul is situated in air at the origin of a rectangular coordinate system. A second negative point charge of 10^{-4} coul is situated on the positive x-axis at a distance of 50 cm from the origin. What is the force on the second charge?

Soln. : from columb force are have,

$$\vec{F} = \frac{1}{4\pi 6o}\frac{z_1 q_2}{r^2}\hat{\gamma}$$

$$= \frac{9\times10^9\times(-10^{-6})\times(-10^{-4})}{(0.5)^2}$$

$$= 3.6\,\hat{\imath}N$$

$$\frac{1}{4\pi\varepsilon o} = 9\times10^9 NM^2 coul$$

$$z_1 = -10^{-6}C$$

$$z_2 = -10^{-4}Col$$

$$\pi = 50cm = 0.5m$$

This there is a force of 3.6 Newton in the positive x-direction on the second charge.

Calculate the force of repulsion between two protons in a nucleus of iron, assuming a separation of $4\times10^{-15}m$

Soln. From columb law we have,

$$F = \frac{1}{4\pi\varepsilon o}\frac{q_1 q_2}{r^2}$$

$$= 9\times10^9\times\frac{(1.6\times10^{-19})^2}{(4\times10^{15})^2}$$

$$= 14\text{ N}$$

$$q_1 = q_2 = q = 1.6\times10^{-19}C$$

$$r = 4\times10^{-15}\text{m}$$

Electricity

Prob: What is the magnitude of the electric field strength $\vec{E}$ such that an electron, placed in the field, would experience an electrical force equal to its weight?

Soln. : We have,

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{mg}{e}$$

$$= \frac{9.1\times10^{-31}\times9.8}{1.6\times10^{-19}}$$

$$= 5.6 \times 10^{-11} N/C$$

$m = 9.1 \times 10^{-31} kg$

$g = 9.8 \text{ ms}^{-2}$

$e = 1.6 \times 10^{-19} \text{c}$

Thank you for your kind attention

Q: Dipole in an electric field or Derive expression for torque and potential energy when an electric dipole is placed in a uniform external electric

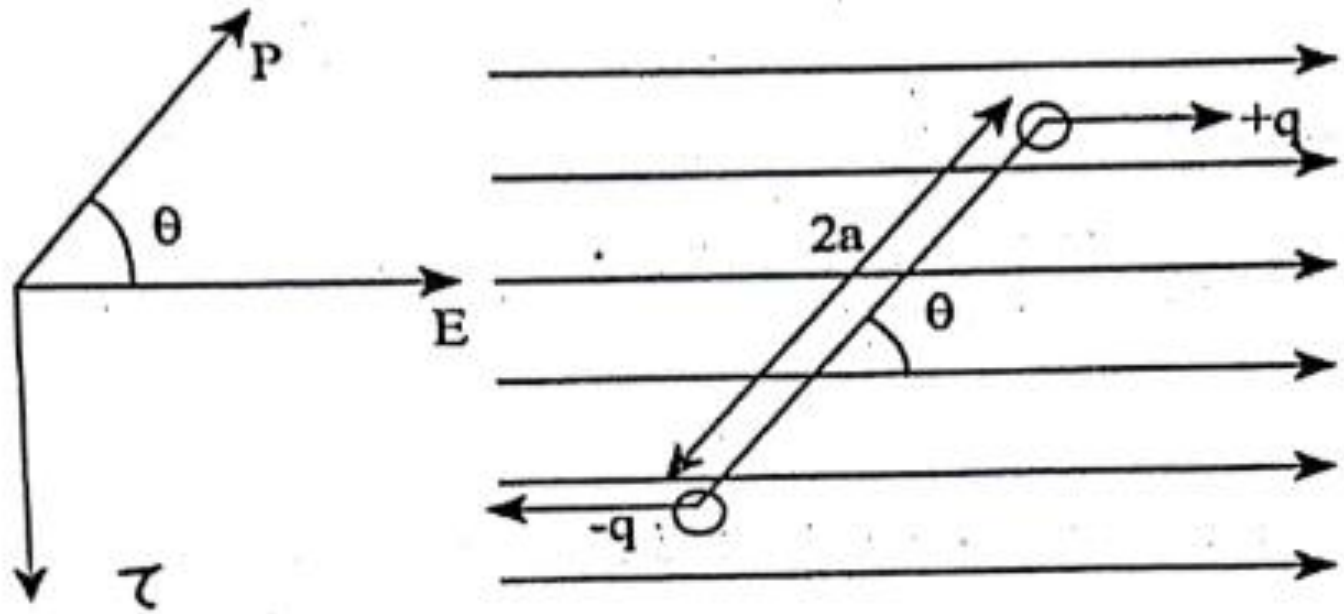

Fig- 5

Electricity

Figure shows an electric dipole formed by placing two charges +q and –q a fixed distance 2a apart. The arrangement is placed in a uniform external electric field E, its dipole moment P making an angle θ with this field. The two equal and opposite forces F and – F making a couple and the moment of the couple or torque is given by-

$$\tau = \text{Force} \times \text{perpendicular distance}$$
$$= F \times 2a \sin\theta$$
$$= qE \times 2a \sin\theta$$
$$= 2qa \times E\sin\theta$$
$$= P\,E\sin\theta$$

Work must be done by an external agent to charge the orientation of an electric dipole. This work is stored as potential energy in the system. We choose 90° as the initial orientation.

$$U = W = \int dw$$

Electricity

$$= \int_{90}^{\theta} \tau . d\theta = \int PE \sin\theta . d\theta$$

$$= PE\left[-\cos\theta\right]_{90}^{\theta}$$

$$= PE(-Cos\theta - Cos90)$$

$$= -PE\, Cos\theta.$$

In vector form $\vec{U} = -\vec{P}.\vec{E}$