Comilla University
Department of Statistics
Course Teacher: Md Safayet Hossain

Chapter :Measures of Central Tendency

Measure of Central Tendency:

When we have a set of quantitative data, it is observed that most of the values of the data set cluster around some central value. This tendency of a set of quantitative data is called Central tendency.

Why it is named so??

Measure of central tendency indicates the location or the general position of the data on the X-axis therefore it is also known as a measure of location or position. It is a summary measure that attempts to describe a whole set of data with a single value that represents the middle/centre of its distribution.

Most Commonly used Measures of Central tendency

1. Mean
2. Median
3. Mode

Types of Mean

1. Arithmetic Mean
2. Geometric Mean
3. Harmonic Mean

Arithmetic Mean or Simply Mean

Definition: Arithmetic mean is the total or sum of the values of a set of observations by the total number of observations.

Ungroup data

Calculate the arithmetic mean for the following the marks obtained by 9 students are given below:

x_i
45
32
37
46
39
36
41
48
36
$\sum_{i=1}^{n} x_i = 360$

$$\frac{\sum_{i=1}^{n} x_i}{n} = \frac{360}{9} = 40$$

Group data

Discrete group data

Calculate the arithmetic mean for the following data given below:

Using formula of direct method of arithmetic mean for grouped data:

$$\frac{\sum_{i=1}^{n} f_i x_i}{n}, \text{ where } \sum_{i=1}^{n} f_i = n$$

Weight (grams)	Frequency
74.5	09
94.5	10
114.5	17
134.5	10
154.5	05
174.5	04
194.5	05

Solution:

(x_i)	Frequency (f_i)	$f_i x_i$
74.5	09	9×74.5=670.5
94.5	10	945.0
114.5	17	1946.5
134.5	10	1345.0
154.5	05	772.5
174.5	04	698.0
194.5	05	972.5
	$\sum_{i=1}^{n} f_i = 60$	$\sum_{i=1}^{n} f_i x_i = 7350.0$

$$\frac{\sum_{i=1}^{n} f_i x_i}{n} = \frac{7350}{60} = 122.5$$

Continuous group data

Short-cut method

Using formula of short cut method of arithmetic mean for grouped data

Weight (grams)	Midpoints (x_i)	Frequency (f_i)	d_i $A = 114.5$, h=20	$f_i\ u_i$
65----84	$(65+84)/2 = 74.5$	09	-2	-18
85----104	94.5	10	-1	-10
105----124	114.5	17	0	0
125----144	134.5	10	1	10
145----164	154.5	05	2	10
165----184	174.5	04	3	12
185----204	194.5	05	4	20
		$\sum_{i=1}^{n} f_i = 60$		$\sum_{i=1}^{n} f_i d_i$ =24

$$\bar{x} = A + \frac{\sum_{i=1}^{n} f_i d_i}{n} \times c \quad \text{, where,} \quad d_i = \frac{x_i - A}{c}$$

$$\bar{x} = 114.5 + \frac{24}{60} \times 20$$

$$\bar{x} = 114.5 + 08$$
$$= 122.5$$

Properties of Arithmetic Mean: The following are the properties of arithmetic mean:

- The mean of a constant is that constant.

  Proof: By definition of arithmetic mean: $\bar{x} = \frac{\sum xi}{n}$

- The sum of deviations from mean is equal to zero. i.e. $\sum (xi - \bar{x}) = 0$

- The sum of squared deviations from the mean is smaller than the sum of squared deviations from any arbitrary value or provisional mean. i.e. $\sum (xi - \bar{x})^2 < \sum (xi - A)^2$

- **Geometric Mean:** "The nth root of the product of "n" positive values is called geometric mean"

$$Geometric\ Mean = \sqrt[n]{product\ of\ "n"\ positive\ values}$$

The following are the formulae of geometric mean:

Ungrouped data	Grouped data
$G = Antilog\left(\frac{\sum logx}{n}\right)$	$G = Antilog\left(\frac{\sum f\ logx}{n}\right)$;Here $n = \sum f$

- **Numerical example of geometric Mean for both grouped and ungrouped data:**
- **Calculate the geometric mean for the following the marks obtained by 9 students are given below:**

x_i	45	32	37	46	39	36	41	48	36

- Using formula of geometric mean for ungrouped data:

$n = 9$

x_i	$\log x_i$
45	$\log 45 = 1.65321$
32	1.50515
37	1.56820
46	1.66276
39	1.59106
36	1.55630
41	1.61278
48	1.62124
36	1.55630
	$\sum_{i=1}^{n} \log x_i = 14.38700$

$$G.M = anti-\log\left(\frac{\sum_{i=1}^{n} \log x_i}{n}\right)$$

$$G.M = anti-\log\left(\frac{14.38700}{9}\right)$$

$$G.M = anti-\log(1.59856)$$

$G.M = 39.68$ (Answer).

- Given the following frequency distribution of weights of 60 apples, calculate the geometric mean for grouped data.

Weights (grams)	65--84	85--104	105--124	125--144	145--164	165--184	185--204
Frequency	09	10	17	10	05	04	05

$$G.M = anti-\log\left(\frac{\sum_{i=1}^{n} f_i \log x_i}{\sum_{i=1}^{n} f_i}\right)$$

Weight (grams)	Midpoints (x_i)	Frequency (f_i)	$\log x_i$	$f_i \log x_i$
65----84	$(65+84)/2 = 74.5$	09	1.8722	16.8498
85----104		10	1.9754	19.7540
105----124	94.5	17	2.0589	35.0013
125----144	114.5	10	2.1287	21.2870
145----164	134.5	05	2.1889	10.9445
165----184	154.5	04	2.2418	8.9672
185----204	174.5	05	2.2889	11.4445
	194.5			
		$\sum_{i=1}^{n} f_i = 60$		$\sum_{i=1}^{n} f_i \log x_i =$ 124.2483

$$G.M = anti-\log\left(\frac{\sum_{i=1}^{n} f_i \log x}{\sum_{i=1}^{n} f_i}\right)$$

$$= anti-\log\left(\frac{124.2483}{60}\right)$$

$$= anti-\log(2.0708)$$

$G.M = 117.7$ grams (Answer).

- **Harmonic Mean:** "*The reciprocal of the Arithmetic mean of the reciprocal of the values is called Harmonic mean*"

$$Harmonic\ Mean = reciprocal\ of\left(\frac{Sum\ of\ reciprocal\ of\ the\ values}{The\ number\ of\ values}\right)$$

The following are formulae of harmonic mean:

Ungrouped data	Grouped data
$H = \frac{n}{\sum\left(\frac{1}{x}\right)}$	$H = \frac{n}{\sum\left(\frac{f}{x}\right)}$; Here $n = \sum f$

- **Numerical example of harmonic Mean for both grouped and ungrouped data:**

- **Calculate the harmonic mean for the following the marks obtained by 9 students are given below:**

x_i	45	32	37	46	39	36	41	48	36

✧ Using formula of harmonic mean for ungrouped data:

$n = 9$

x_i	$1/x_i$
45	0.02222
32	0.03125
37	0.02702
46	0.02173
39	0.02564
36	0.02777
41	0.02439
48	0.02083
36	0.02777
	$\sum_{i=1}^{n} \frac{1}{x_i}$ =0.22862

$$H.M = \frac{n}{\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)}$$

$$H.M = \frac{9}{0.22862}$$

$H.M = 39.36663$ (Answer).

➢ Given the following frequency distribution of weights of 60 apples, calculate the harmonic mean for grouped data.

Weights (grams)	65--84	85--104	105--124	125--144	145--164	165--184	185--204
Frequency	09	10	17	10	05	04	05

$$H.M = \frac{\sum_{i=1}^{n} f_i}{\sum_{i=1}^{n}\left(\frac{f_i}{x_i}\right)}$$

Weight (grams)	Midpoints (x_i)	Frequency (f_i)	f_i/x_i
65----84	$(65+84)/2 = 74.5$	09	0.12081
85----104	94.5	10	0.10582
105----124	114.5	17	0.14847
125----144	134.5	10	0.07435
145----164	154.5	05	0.03236
165----184	174.5	04	0.02292
185----204	194.5	05	0.02571
		$\sum_{i=1}^{n} f_i = 60$	$\sum_{i=1}^{n} \frac{f_i}{x_i}$ =0.53044

$$H.M = \frac{\sum_{i=1}^{n} f_i}{\sum_{i=1}^{n}\left(\frac{f_i}{x_i}\right)} = \frac{60}{0.53044} = 113.11 \text{ grams}$$ (Answer).

❖ **Median:** *"when the observation are arranged in ascending or descending order, then a value, that divides a distribution into equal parts, is called median"*

Median in case of Ungrouped Data	
In this case we first arrange the observations in **increasing** or **decreasing** order then we use the following formulae for Median:	
If "n" is odd	$Median = size\ of \left(\frac{n+1}{2}\right) th\ observation$
If "n" is even	$Median = \frac{size\ of \left\{\left(\frac{n}{2}\right)th + \left(\frac{n}{2}+1\right)th\right\}observation}{2}$

➢ **Numerical example of median for both grouped and ungrouped data:**

If "n" is odd	$Median = size\ of \left(\frac{n+1}{2}\right) th\ observation$

➢ **Calculate the median for the following the marks obtained by 9 students are given below:**

x_i	45	32	37	46	39	36	41	48	36

Arrange the data in ascending order

32, 36, 36, 37, 39, 41, 45, 46, 48. $n = 9$ "n" is odd

$$Median = Size\ of\ \left(\frac{n+1}{2}\right)^{th} observation$$

$$Median = Size\ of\ \left(\frac{9+1}{2}\right)^{th} observation$$

$$Median = Size\ of\ 5^{th} observation$$

$Median = 39$ (Answer).

If "n" is even	$Median = \frac{size\ of \left\{\left(\frac{n}{2}\right)th + \left(\frac{n}{2}+1\right)th\right\}observation}{2}$

- **Calculate the median for the following the marks obtained by 10 students are given below:**

x_i	45	32	37	46	39	36	41	48	36	50

Arrange the data in ascending order

32, 36, 36, 37, 39, 41, 45, 46, 48, 50. $n = 10$ "n" is even

$$Median = \frac{Size\ of\ \left\{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}\right\}\ observation}{2}$$

$$Median = \frac{Size\ of\ \left\{\left(\frac{10}{2}\right)^{th} + \left(\frac{10}{2}+1\right)^{th}\right\}\ observation}{2}$$

$$Median = \frac{Size\ of\ \left\{5^{th} + 6^{th}\right\}\ observation}{2}$$

$$Median = \frac{39+41}{2} = 40$$ (Answer).

- The number of values above the median balances (equals) the number of values below the median i.e. 50% of the data falls above and below the median.

Median in case of continuous Grouped Data

In continuous grouped data, when we are finding median, we first construct the class boundaries if the classes are discontinuous. Then we find cumulative frequencies and then we use the following two steps:

- First we determine the median class using n/2.
- When the median class is determined, then the following formula is used to find the value of median. i.e.

$$Median = l + \frac{h}{f}\left(\frac{n}{2} - C\right); \quad \text{Here } n = \sum f$$

Where l = lower class boundary of the median class
h = width of the median class
f = frequency of the median class
C = cumulative frequency of the class preceding the median class.

- Numerical example: Find the median, for the distribution of examination marks given below:

Marks	30--39	40--49	50--59	60--69	70--79	80--89	90--99
Number of students	08	87	190	304	211	85	20

$$Median = l + \frac{h}{f}\left(\frac{n}{2} - C\right) \quad \text{here} \quad n = \sum_{i=1}^{n} f_i$$

Class boundaries	Midpoints (x_i)	Frequency (f_i)	Cumulative frequency $(c.f)$
29.5---39.5	34.5	8	8
39.5---49.5	44.5	87	95
49.5---59.5	54.5	190	285
59.5---69.5	64.5	304	589
69.5---79.5	74.5	211	800
79.5---89.5	84.5	85	885
89.5---99.5	94.5	20	905
		$\sum_{i=1}^{n} f_i = 905$	

$\frac{n}{2} = \frac{905}{2} = 452.5^{th}$ student which corresponds to marks in the class 59.5---69.5

Therefore

$$Median = l + \frac{h}{f}\left(\frac{n}{2} - C\right) = 59.5 + \frac{10}{304}\left(\frac{905}{2} - 285\right) = 59.5 + \frac{10}{304}(452.5 - 285)$$

$$Median = 59.5 + \frac{1675}{304}, \quad Median = 59.5 + 5.5, \quad Median = 65 \text{ marks} \qquad \text{(Answer).}$$

- Calculate Mode for ungrouped data

$$x_i : 2, 3, 8, 4, 6, 3, 2, 5, 3.$$

Mode = 3 (Answer).

- Calculate Mode in discrete grouped data

No. of assistants	f_i
0	3
1	4
2	6
3	7
4	10
5	6
6	5
7	5
8	3
9	1
	$\sum_{i=1}^{n} f_i = 50$

Mode = 4

- Mode in case of Continuous grouped data:

Class boundaries	Midpoints (x_i)	Frequency (f_i)	Cumulative frequency $(c.f)$
29.5---39.5	34.5	8	8
39.5---49.5	44.5	87	95
49.5---59.5	54.5	190	285
59.5---69.5	64.5	304	589
69.5---79.5	74.5	211	800
79.5---89.5	84.5	85	885
89.5---99.5	94.5	20	905
		$\sum_{i=1}^{n} f_i = 905$	

$$Mode = l + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h \quad = 59.5 + \frac{304 - 190}{(304 - 190) + (304 - 211)} \times 10$$

$$Mode = 59.5 + \frac{114}{114 + 93} \times 10 \quad \Rightarrow Mode = 59.5 + 5.05072$$