PHY-1111: Physics

Chapter- 1 (Mechanics)

by

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Mechanics

Fundamentals of Physics: The building blocks of our universe

Physics, at its core, is the study of matter, energy, motion, and force. It seeks to understand the fundamental laws and principles that govern the behavior of the universe.

Key concepts and principles:

Classical Mechanics:

Newton's Laws of Motion: These laws describe the relationship between force and motion, forming the foundation of classical mechanics.

Conservation Laws: These principles, such as the conservation of energy and momentum, state that certain quantities remain constant in an isolated system.

Gravity: The force of attraction between any two objects with mass.

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Electromagnetism:

Electric charge and fields: The study of electric charges, their interactions, and the electric fields they create.

Magnetism: The study of magnetic fields and their interactions with electric currents and magnetic materials.

Electromagnetic waves: These waves, such as light and radio waves, consist of oscillating electric and magnetic fields.

Thermodynamics:

Heat and Temperature: The study of heat, work, and energy transfer.

Laws of Thermodynamics: These laws describe the behavior of heat and energy.

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Quantum Mechanics:

Atomic and subatomic phenomena: This branch of physics deals with the behavior of matter and energy at the atomic and subatomic levels.

Wave-particle duality: The concept that particles can exhibit both wave-like and particle-like properties.

Relativity:

Special relativity: This theory describes how space and time are relative to the observer's frame of reference.

General relativity: This theory explains gravity as a curvature of space time (a mathematical model that describes how space and time are changed by the presence of matter and energy) caused by mass and energy.

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Why study fundamentals of physics?

Understanding the world around us: Physics helps us understand the fundamental principles that govern the natural world.

Technological advancements: Many technological advancements, from smartphones to spacecraft, are based on principles of physics.

Problem-solving skills: Studying physics develops critical thinking and problem-solving skills that are valuable in many fields.

Intellectual curiosity: Physics explores the fundamental questions about the universe and our place in it.

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Mechanics:

Mechanics is the branch of physics that deals with the study of motion and the forces that cause it. It's a fundamental area of physics that explores how objects move, interact, and change over time.

Key areas of mechanics:

Kinematics: This branch describes the motion of objects without considering the forces that cause the motion. It deals with concepts like position, velocity, acceleration, and time.

Dynamics: This branch investigates the relationship between the motion of objects and the forces acting upon them. It's governed by Newton's laws of motion.

Statics: This branch focuses on objects at rest or in a state of constant velocity (equilibrium). It analyzes the forces acting on these objects to ensure they remain stationary or maintain a uniform motion.

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Why is mechanics important?

Foundation of physics: Mechanics provides the foundation for understanding many other areas of physics, such as thermodynamics, electromagnetism, and even relativity.

Engineering applications: It's crucial for engineering disciplines like mechanical, civil, and aerospace engineering, where understanding the motion and forces acting on structures and machines is essential for design and analysis.

Everyday life: Mechanics helps us understand everyday phenomena, from the motion of a car to the flight of a bird.

Motion in One Dimension

Motion in one dimension/Kinematics:

Motion in one dimension, also known as linear motion, describes the movement of an object along a straight line. This type of motion is the simplest to analyze because it only involves one spatial coordinate.

- Consider motion in one dimension
- Along a straight line
- Motion represents a continual change in an object's position.

Types of Motion

- Translational

An example is a car traveling on a highway.

- Rotational

An example is the Earth's spin on its axis.

- Vibrational

An example is the back-and-forth movement of a pendulum.

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Key factors:

Position: The location of an object along the line of motion. It's often represented by a coordinate (e.g., x).

Displacement: The change in position of an object. It's a vector quantity, meaning it has both magnitude (size) and direction.

Distance: The total length traveled by an object, regardless of direction. It's a scalar quantity, meaning it only has magnitude.

Speed: The rate at which an object covers distance. It's a scalar quantity.

Velocity: The rate of change of displacement. It's a vector quantity, having both magnitude and direction.

Acceleration: The rate of change of velocity. It's also a vector quantity.

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Equations of Motion:

For constant acceleration, the following equations can be used to describe one-dimensional motion:

$$v = u + at$$

$$s = ut + 1/2at^2$$

$$v^2 = u^2 + 2as$$

where:

v = final velocity
u = initial velocity
a = acceleration
t = time
s = displacement

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Applications:

Understanding one-dimensional motion is crucial in various fields, including:

Physics: Analyzing the motion of objects under the influence of forces.

Engineering: Designing and analyzing mechanical systems.

Astronomy: Studying the motion of celestial bodies.

By understanding the concepts of one-dimensional motion, you can put a strong foundation for understanding more complex types of motion, such as two-dimensional and three-dimensional motion.

Distance vs. Displacement – An Example

Assume a player moves from one end of the court to the other and back.

Distance is twice the length of the court

- Distance is always positive

Displacement is zero

- $\Delta x = x_f - x_i = 0$ since $x_f = x_i$

What is two-dimensional motion?

Two-dimensional (2D) motion means motion that takes place in two different directions (or coordinates) at the same time. Two-dimensional motion, as the name suggests, describes the movement of an object in a plane. Unlike one-dimensional motion, which is confined to a straight line, two-dimensional motion allows for movement in two perpendicular directions.

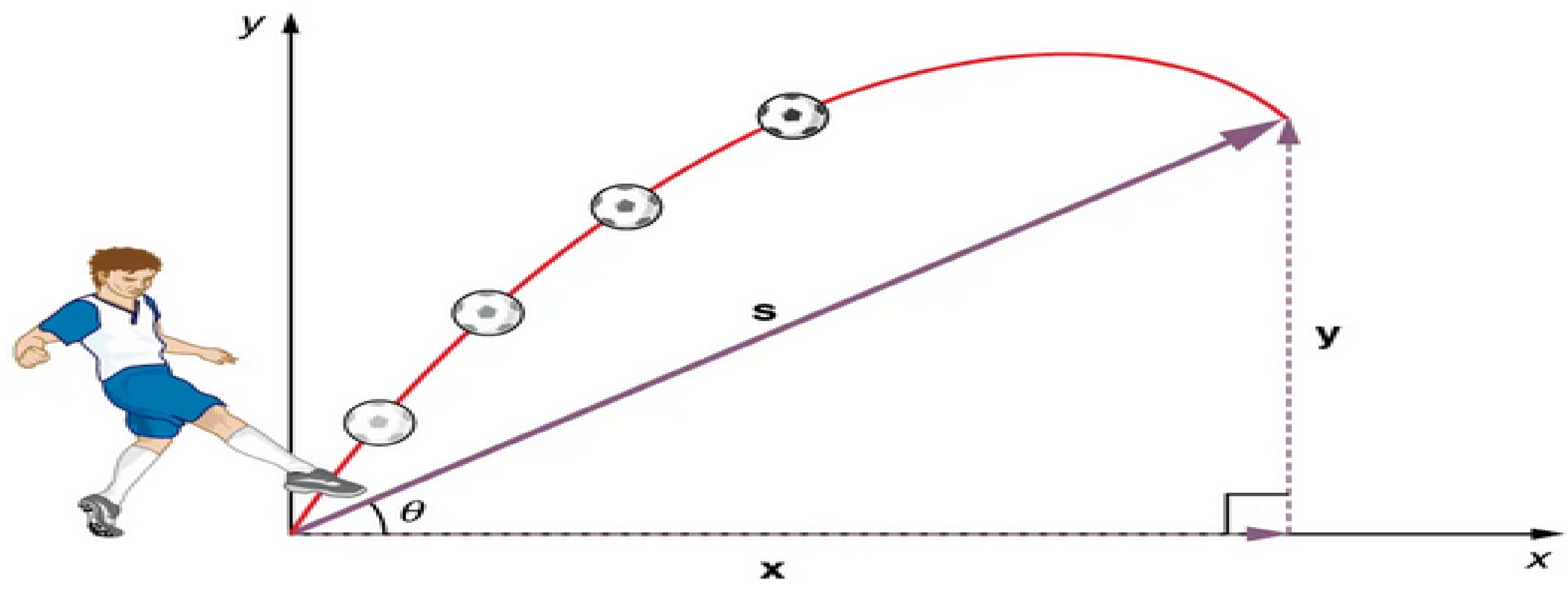

Figure The total displacement **s** of a football ball at a point along its path. The vector **s** has components **x** and **y** along the horizontal and vertical axes. Its magnitude is s, and it makes an angle θ with the horizontal.

What are the components of two-dimensional motion?

Two dimensional motion can be described using the two separate components. The two separate motions are in horizontal and vertical directions respectively. Projectile motion is two-dimensional because it has a horizontal component and a vertical component.

What is the formula for two-dimensional motion?

Few Examples of Two – Dimensional Projectiles

Quantity	**Value**
Time of maximum height	$t_m = v_O \sin\theta_O / g$
Time of flight	$2t_m = 2(v_O \sin\theta_O/g)$
Maximum height of projectile	$h_m = (v_O \sin\theta_O)^2/2g$
Horizontal range of projectile	$R = v_O^2 \sin 2\theta_O/g$

Equation of Motions in Two Dimensions

The three equations of motion in two dimensions x and y are given as:

$$v_x = u_{x0} + a_x t$$

$$\Delta x = u_{x0} t + \frac{1}{2} a_x t^2$$

$$v_x^2 - u_{x0}^2 = 2 a_x \Delta x$$

Similarly, the equations can be written in y directions.

$$v_y = u_{y0} + a_y t$$

$$\Delta y = u_{y0} t + \frac{1}{2} a_y t^2$$

$$v_y^2 - u_{y0}^2 = 2 a_y \Delta y$$

Solved Examples for Two-Dimensional Motion

Example 1: A particle is moving with an initial velocity (2i + 4j) $\frac{m}{s}$ and has constant acceleration of (10i + 2j) $\frac{m}{s^2}$. Calculate the final velocity and displacement after 6 seconds.

Solution: The above problem can be solved by driving velocity and acceleration into one dimension. Thus, along the x-axis, we have initial velocity as 2 $\frac{m}{s}$, acceleration as $10\frac{m}{s^2}$. Using the first equation of motion, we have v=u+at. Substituting the values above in this, we get

$$v = 2 + 10 \times 6$$

$$v = 62ms^{-1}$$

Now, using the second equation of motion, we have $s = ut + \frac{1}{2}at^2$. Substituting the above values we get,

$$s = 2 \times 6 + \frac{1}{2}(10) \times 6^2$$

$$s = 12 + 180$$

$$s = 192 \text{ m}$$

Now, we will calculate the same along the y-direction.
We get the value of velocity as

$$v = 4 + 2 \times 6$$
$$v = 16\ \mathrm{ms}^{-1}$$

Similarly, the displacement will be

$$s = 4 \times 6 + \frac{1}{2} \times 2 \times 6^2$$
$$s = 24 + 36$$
$$s = 60\ \mathrm{m}$$

Thus, final velocity $= v = v_x\hat{i} + v_y\hat{j} = 62\hat{i} + 16\hat{j}$

or

$$v = \sqrt{62^2 + 16^2}$$
$$= v = 64ms^{-1}$$

The final displacement is

$$s = s_x\hat{i} + s_y\hat{j}$$
$$s = 192\hat{i} + 60\hat{j}$$

or

$$s = \sqrt{192^2 + 60^2}$$
$$s = 201.15\ \mathrm{m}$$

Hence, the final velocity and displacement after 6 seconds are $62\hat{i} + 16\hat{j}$ and $201.15\ \mathrm{m}$.

Example 2: An object is moving in a plane with a velocity $v = 4t\hat{i} + 6t^3\hat{j}$. Find the acceleration of the object between time intervals (0,2).

Solution: Acceleration is denoted as $a = \frac{v}{t}$

At 0 time interval, velocity will be zero, whereas at t=2 seconds, velocity will be

$$v = 4 \times 2\hat{i} + 6 \times 2^3\hat{j}$$

$$v = 8\hat{i} + 48\hat{j}$$

Therefore, acceleration will be

$$\Rightarrow \frac{8\hat{i} + 48\hat{j}}{2}$$

$$\Rightarrow 4\hat{i} + 24\hat{j}$$

Hence, the acceleration of the object between time intervals (0,2) is $4\hat{i} + 24\hat{j}$.

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Three-Dimensional Motion: A Journey in Space

Three-dimensional motion describes the movement of an object in three-dimensional space. This is the most general type of motion we encounter in the real world, as it accounts for movement in all directions.

Common Examples:

The flight of a bird: A bird can move forward, sideways, and up or down.

The motion of a planet: Planets orbit the Sun in elliptical paths, moving in three dimensions.

The trajectory of a rocket: Rockets can move in any direction in space, subject to gravitational forces.

Applications:

Physics: Understanding the motion of celestial bodies, fluid dynamics, and other complex phenomena.

Engineering: Designing aircraft, spacecraft, and other vehicles that move in three dimensions.

Computer graphics: Creating realistic simulations of objects moving in virtual environments.

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❖ **Difference between one, two and three dimensional motion:**

One dimensional motion is motion along a straight line. The line used for this motion is often the familiar x-axis, or x number line. The object may move forward or backward along this line: Forward is usually considered positive movement, and this movement is usually considered to be to the right. Remember that the study of one-dimensional motion is the study of movement in one direction, like a car moving from point “A” to point “B.”

Whereas,

In two-dimensional motion, the path the object follows lies in a plane. Two-dimensional motion is the study of movement in two directions, including the study of motion along a curved path, such as projectile and circular motion. Examples are projectile motion where the path is a parabola, or planetary motion where it is an ellipse.

Whereas,

Three-dimensional motion would be a case where the path is more complex and is not confined to a single plane.

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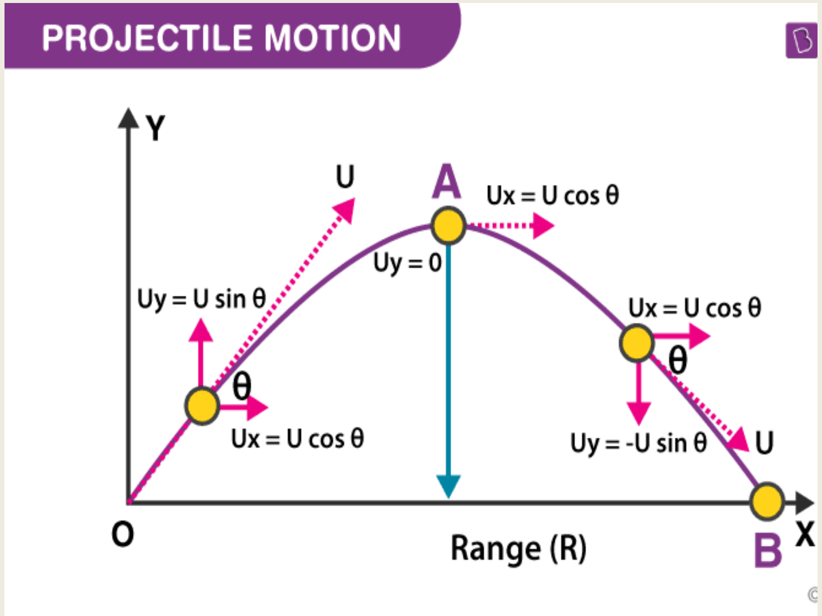
Projectile motion:

Projectile motion is the motion of an object thrown or projected into the air, subject to only the acceleration of gravity. The object is called a projectile, and its path is called its trajectory.

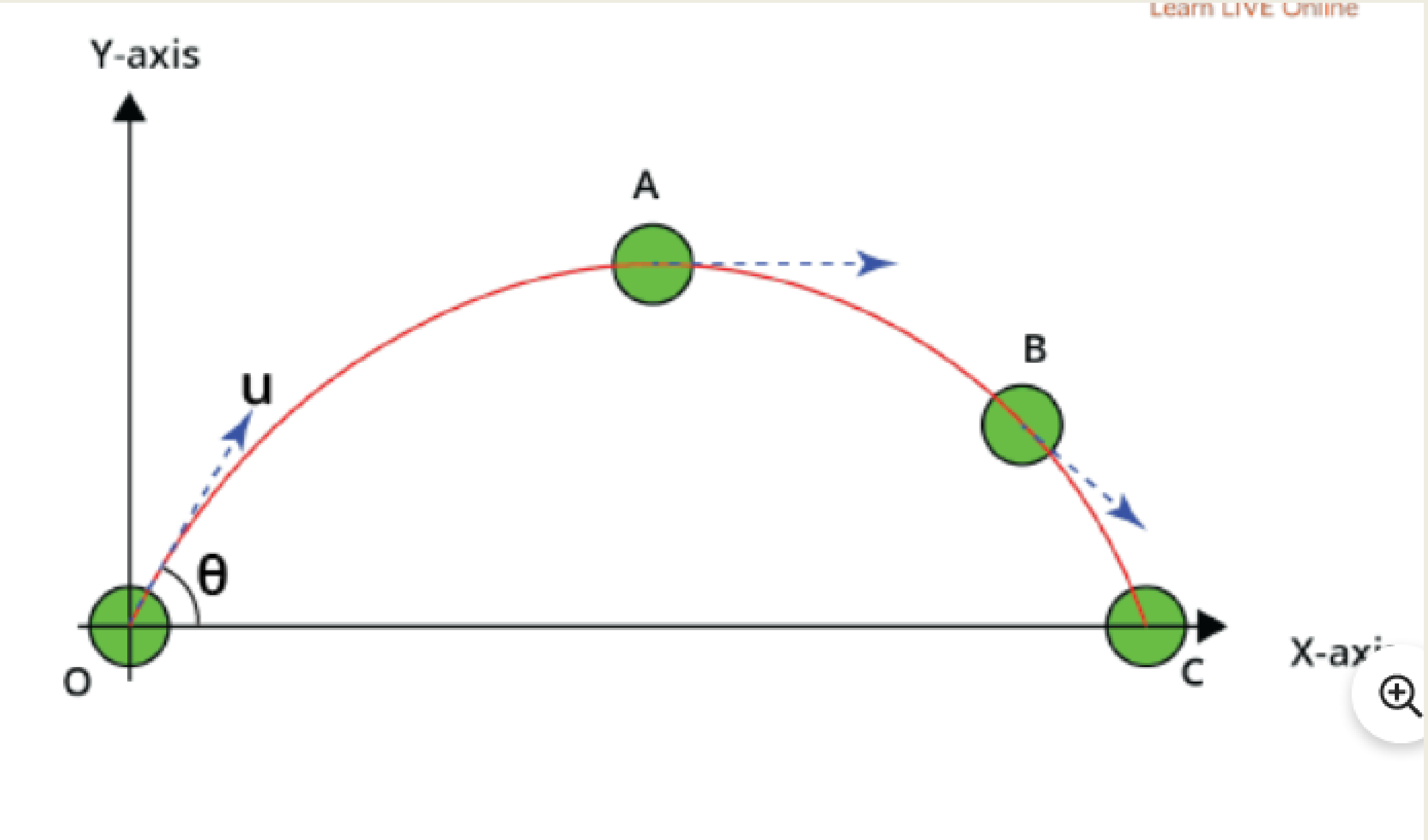


What are the 3 main concepts of projectile motion?

The key components that we need to remember in order to solve projectile motion problems are: Initial launch angle (θ), Initial velocity (u), and Time of flight (T).

Projectile Motion

The motion of an object in a curved path with constant acceleration is known as projectile motion. The motion in this case is along the x-axis and the y-axis.



Projectile Motion

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The acceleration component along the x-axis is 0, whereas in the vertical direction it is -g. However, the component of velocity along the x-axis is $u\cos(\theta)$ and $u\sin(\theta)$, respectively.

The equation of a projectile or the trajectory formula as it is known is given as

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Here, x and y are the horizontal and vertical components, respectively, and u is the initial velocity and g is the acceleration due to gravity. There are other important equations related to the projectile motion which include the time of flight (T), maximum height (H), and the horizontal range of the projectile (R). These equations are

$$T = \frac{2u\sin\theta}{g}$$

$$H = \frac{u^2\sin^2\theta}{2g}$$

$$R = \frac{u^2\sin 2\theta}{g}$$

Here, u is the initial velocity, g is the acceleration due to gravity, and θ at which the projectile was thrown with respect to the horizontal.

These are the equations of motion of the projectile.

What Is Frictional Force?

> Frictional force is the force generated by two surfaces that contact and slide against each other.

A few factors affecting the frictional force:

- These forces are mainly affected by the surface texture and the amount of force impelling them together.
- The angle and position of the object affect the amount of frictional force.
- If an object is placed flat against an object, then the frictional force will be equal to the object's weight.
- If an object is pushed against the surface, then the frictional force will be increased and becomes more than the weight of the object.

Dry Friction

Dry friction describes the reaction between two solid bodies in contact when they are in motion (kinetic friction) and when they are not (static friction). Both static and kinetic friction is proportional to the normal force exerted between the solid bodies. The interaction of different substances is modelled with different coefficients of friction. By this, we mean that certain substances have a higher resistance to movement than others for the same normal force between them. Each of these values are experimentally determined.

Fluid Friction

Fluid Friction is the force that obstructs the flow of fluid. It is a situation where the fluid provides resistance between the two surfaces. If both surfaces offer high resistance, then it is known as high viscous

Examples of Fluid Friction

1. To avoid creaking sounds from doors, we lubricate the door hinges, which leads to the smooth functioning of door hinges.

Work:

Work is the energy transferred to or from an object via the application of force along a displacement. In its simplest form, for a constant force aligned with the direction of motion, the work equals the product of the force strength and the distance traveled. A force is said to do positive work if when applied it has a component in the direction of the displacement of the point of application. A force does negative work if it has a component opposite to the direction of the displacement at the point of application of the force.

For example, when a ball is held above the ground and then dropped, the work done by the gravitational force on the ball as it falls is positive, and is equal to the weight of the ball (a force) multiplied by the distance to the ground (a displacement). If the ball is thrown upwards, the work done by the gravitational force is negative, and is equal to the weight multiplied by the displacement in the upwards direction.

$$W = Fs$$

W = work

F = force

s = Displacement

Energy:

In physics, energy is the quantitative property that is transferred to a body or to a physical system, recognizable in the performance of work and in the form of heat and light. Energy is a conserved quantity—the law of conservation of energy states that energy can be converted in form, but not created or destroyed.

Momentum:

Momentum is the product of the mass and velocity of an object. It is a vector quantity, possessing a magnitude and a direction. If m is an object's mass and v is its velocity, then the object's momentum p is: mv

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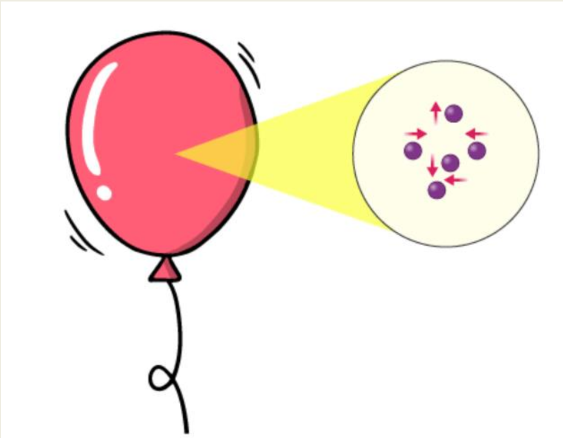
What Is Conservation of Momentum?

Conservation of momentum is a major law of physics which states that the momentum of a system is constant if no external forces are acting on the system. It is embodied in Newton's First Law or The Law of Inertia.

Example of Conservation of Momentum

Consider this example of a balloon, the particles of gas move rapidly colliding with each other and the walls of the balloon, even though the particles themselves move faster and slower when they lose or gain momentum when they collide, the total momentum of the system remains the same.

Hence, the balloon doesn't change in size, if we add external energy by heating it, the balloon should expand because it increases the velocity of the particles and this increases their momentum, in turn, increasing the force exerted by them on the walls of the balloon.



Application of Law of Conservation of Momentum

Having said so the energy of a system is always conserved, one of the best applications of the law of conservation of momentum would be in space travel, there is no medium in space to exert a force on, then how do rockets travel?

Well, they eject matter at a very high speed, so in an isolated system, the momentum should remain constant therefore, the rocket will move in the opposite direction with the same momentum as that of the exhaust.

Work-energy theorem:

The work-energy theorem explains the idea that the net work - the total work done by all the forces combined - done on an object is equal to the change in the kinetic energy of the object. After the net force is removed (no more work is being done) the object's total energy is altered as a result of the work that was done.

According to the theorem,

$$W_{net} = \Delta K = K_f - K_i$$

- W is the total work done
- ΔK is the change in kinetic energy
- K_f is the final kinetic energy
- K_i is the initial kinetic energy

CONCEPT OF MOMENT OF INERTIA

Moment of inertia of a body about an axis is a measure of the difficulty in starting, stopping or changing rotation of the body about that axis.

- It is denoted by I
- The greater the difficulty in starting or stopping , the greater is the moment of inertia of the body about that axis and vice-versa.
- A body rotates under the action of a net external torque.

The Greater the moment of inertia of a body about an axis of rotation, the greater is the torque required to rotate or stop or change rotation of the body that axis and vice-versa.

MOMENT OF INERTIA OF A RIGID BODY

Consider a rigid body rotating about the axis yy^{-1} with an angular speed ω as shown in figure 1 below.

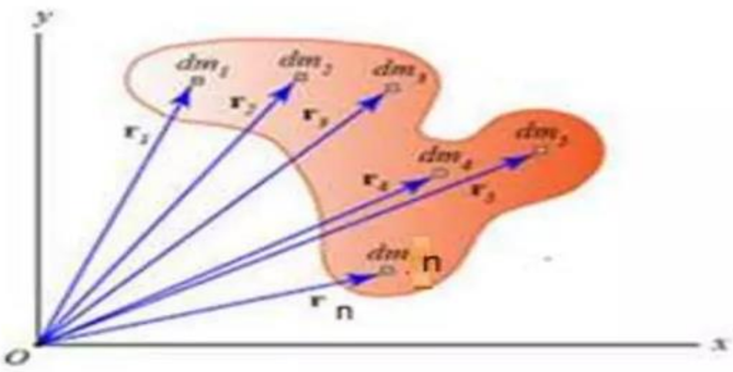


Figure. 1

Suppose the body is made up of a large number (n) of small particles of masses m_1, m_2, m_3 ....m_n situated at perpendicular distances r_1, r_2, r_3 ....r_n respectively from the axis of rotation yy'.

As the body rotates, each particle of the body follows a circular path around the axis.

Although each particle of the body has the same angular speed ω, the linear velocity (v) of each particle depends upon particles distance from the axis of rotation.

Thus particle of mass m_1 follows a circular path of radius r_1. The linear velocity of this particle is v_1

$$V_1 = \omega r_1$$

Rotational kinetic energy of the particles of mass m_1

$$R.K.E = \frac{1}{2} m_1 V_1^2$$

$$R.K.E = \frac{1}{2} m_1 \omega^2 r_1^2$$

$$\therefore R.K.E = \frac{1}{2} m_1 r_1^2 \omega^2$$

The total kinetic energy K_r of the rotating body is the sum of the kinetic energies of all the particles of which the body is composed.

$$K_r = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \frac{1}{2} m_3 r_3^2 \omega^2 - - - -$$

$$K_r = \frac{1}{2} \omega^2 \left[\sum_{i=1}^{i=n} m_i r_i^2 \right]$$

$m_i = mass\ of\ i^{th}\ particle$ and $r_i = its\ perpendicular\ distance\ from\ axis\ of\ ro$

$$\sum_{i=1}^{i=n} m_i r_i^2 = moment\ of\ inertia\ I$$

Hence the total K.E of rotating body

$$K_r = \frac{1}{2} I \omega^2$$

Moment of inertia of a rigid body about a given axis of rotation

Is the sum of the products of the masses of its particles and the squares of their respective perpendicular distance from the axis of rotation.

$$I = \sum_{i=1}^{i=n} m_i r_i^2$$

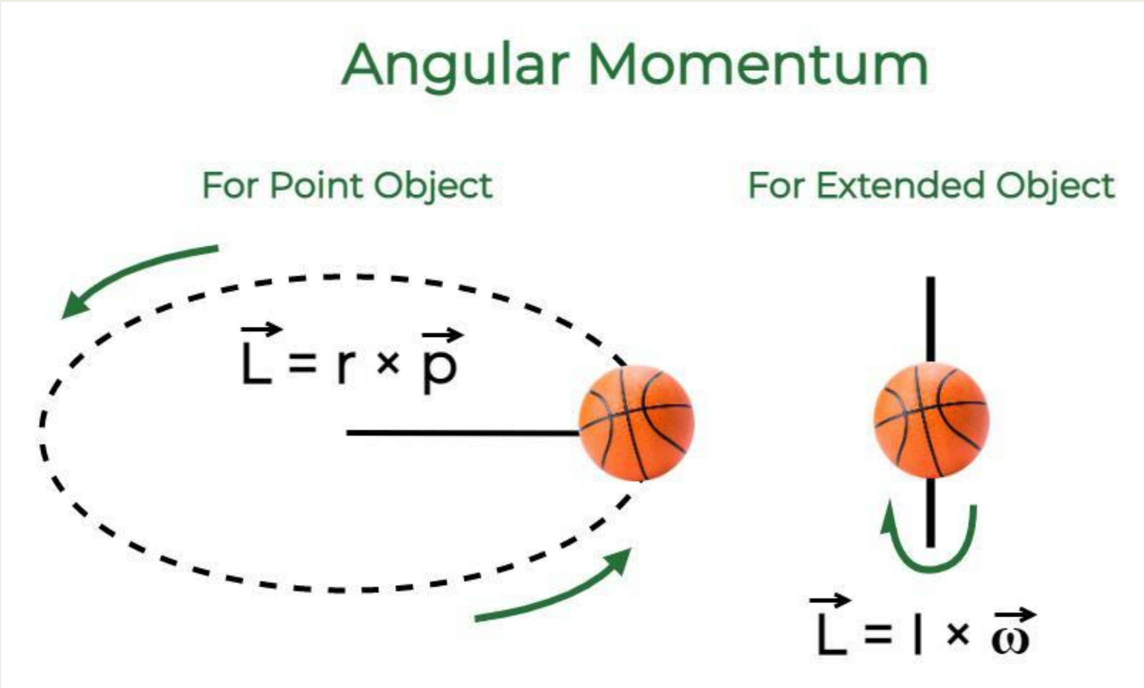
The moment of inertia of a body about an axis of rotation is directly proportional to the total mass of the body.

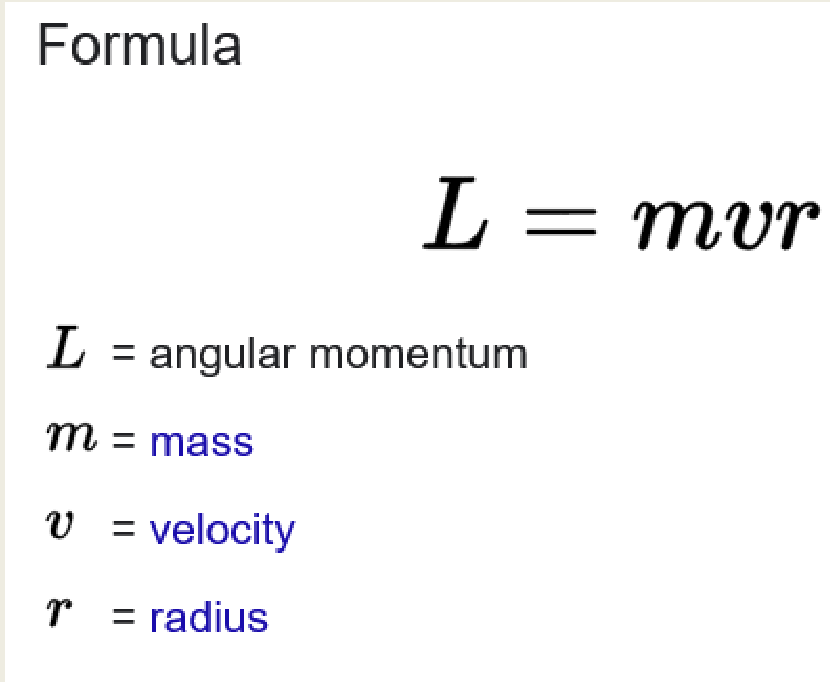
The more massive the body, the more difficult will be to start its rotational motion or stop it from rotating.

For a given mass, the moment of inertia of a body depends upon the distribution of the mass from the axis of rotation. The larger the distance of the mass from the axis rotation the larger will be its moment of inertia. The moment of inertia plays the same role in rotational motion as mass plays in translational motion.

What do you mean by angular momentum?

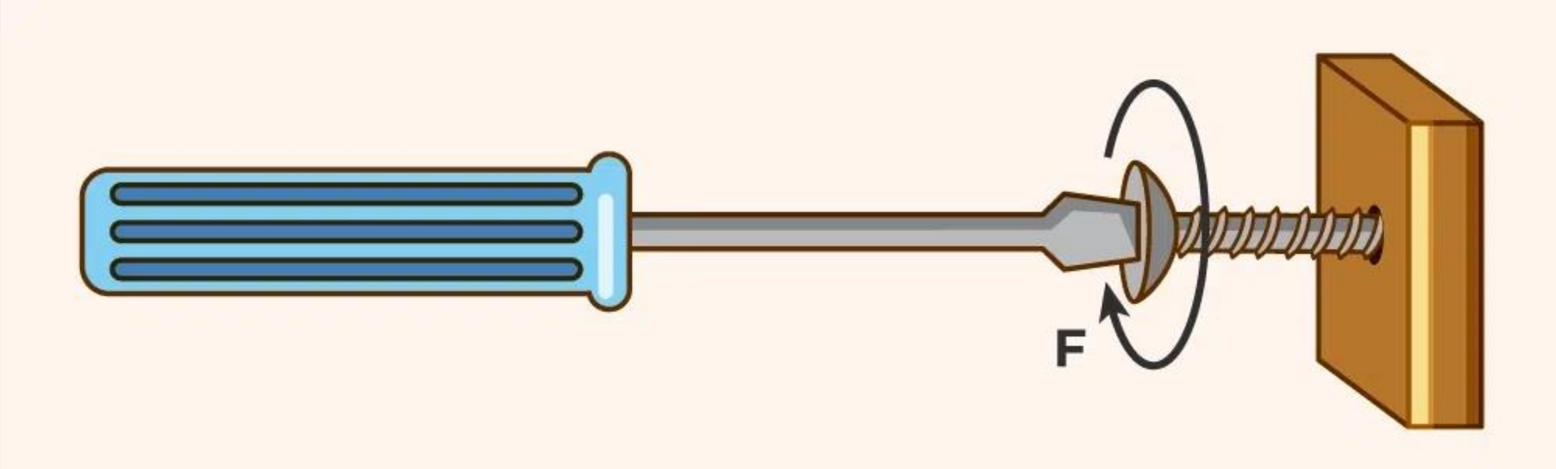
Angular momentum is defined as: The property of any rotating object given by moment of inertia times angular velocity. It is the property of a rotating body given by the product of the moment of inertia and the angular velocity of the rotating object.





Torque:

The force that can cause an object to rotate along an axis is measured as torque. In linear kinematics, force is what drives an object’s acceleration. Similar to this, an angular acceleration is brought on by torque. As a result, torque can be thought of as the rotational counterpart to force. The axis of rotation is a straight line about which an item rotates. Torque in physics is only a force’s propensity to turn or twist.



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Basis of differentiation	Torque	Angular Momentum
Definition	Torque is the force that causes rotational motion	Angular Momentum is the product of a rotating object's moment of inertia and angular velocity
Mathematical Formula	Torque = rF sinθ Here, r is the distance between the object's centre of mass and the point of rotation. F is the force and θ is the angle at which the force is subjected on the object.	Angular Momentum = Iω Here, I is the object's moment of Inertia and w is the angular velocity at which the object is rotating.
Examples	1. When you open a cap of the bottle, you are actually applying a torque good enough to rotate it. 2. When you apply force to open a door, it moves in a circular motion at its hinge. In fact it is the torque acting upon it that causes it to open,	1. When a skater spins her body on the ice turf, it is the angular momentum that prevents her from falling down. 2. The angular momentum prevents the merry-go-round from losing its balance. The same can be said about a giant-wheel as well.

Relation between torque and angular momentum:

The momentum that a rotating object has because of the distance between the object and the perpendicular drawn from the center of rotation is called angular momentum. It is due to the angular momentum that one can able to paddle a bicycle without getting imbalanced. Mathematically, it is defined as the cross product between the linear momentum of an object and the distance between the object's center of mass and center of rotation. This distance is equal to the radius of the circular motion of the object. Therefore, we can calculate the angular momentum of a rotating object from the below formula:

$$\vec{L} = \vec{r} \times \vec{p}$$

Here L is used to denote angular momentum, and r is the radius, i.e., distance from the axis of rotation to the object's center of mass.

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Torque is the force that accelerates the object to follow the rotational motion along its axis of rotation. Torque acts perpendicularly on the distance between the rotational axis. According to the definition of torque, it is the rate at which an object's angular momentum changes. It is equal to the cross product between the distance from the rotational axis and the linear force acting on the object. To find the torque of a rotating object, we can utilize the below equation,

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = rF \sin\theta \hat{n}$$

If we differentiate the angular momentum equation with respect to time then we get

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d}{dt}(\vec{p})$$

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Now, $\frac{d\vec{p}}{dt}$ is the force acting on the object.

As a result, we can write this equation as

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = \vec{\tau}$$

This is the equation where we can say that the rate at which the angular momentum of an object changes can be expressed by torque. From this, we have:

$$\vec{\tau} = \frac{d\vec{L}}{dt} = rF \sin\theta\hat{n}$$

When the angular momentum remains constant, torque is 0. It means that,

$$\vec{L} = \vec{r} \times \vec{p} = \text{constant}$$

Therefore,

$$\vec{\tau} = \frac{d\vec{L}}{dt} = 0$$

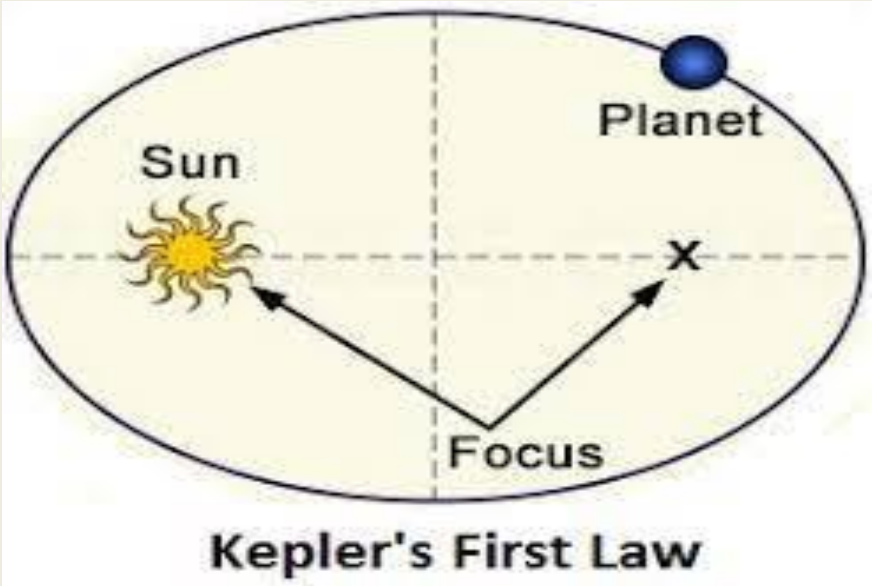
We can use this equation to find the torque and momentum of a rotating body. Last equation can be used to find the relation between torque and angular momentum.

LAWS OF PLANETARY MOTION

Kepler First law – The Law of Orbits

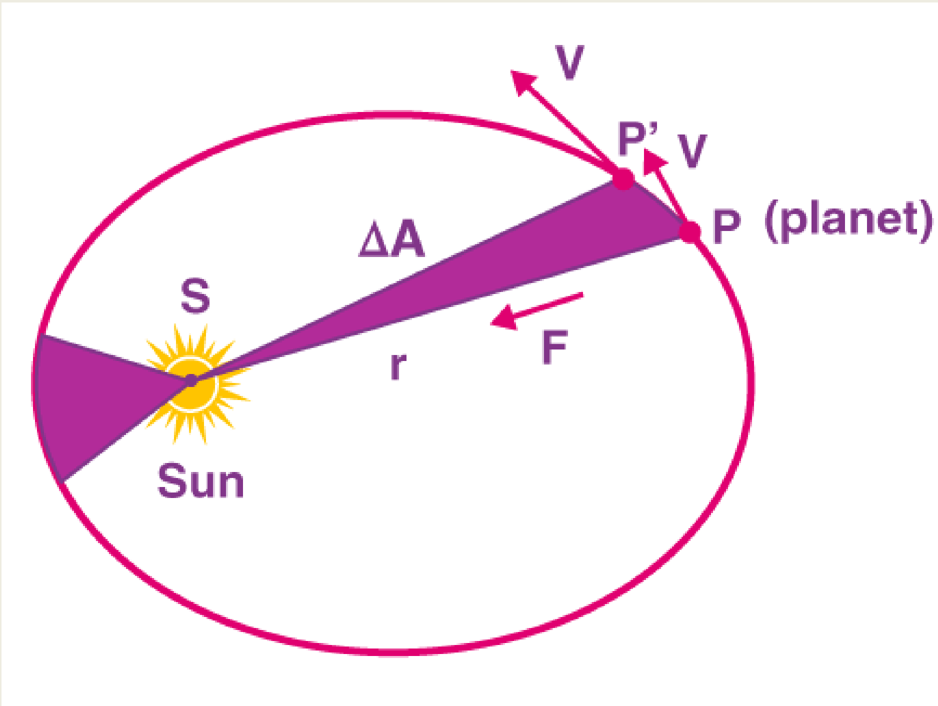
- ❖ According to Kepler's first law, all the planets revolve around the sun in elliptical orbits having the sun at one of the focus". The point at which the planet is close to the sun is known as perihelion, and the point at which the planet is farther from the sun is known as aphelion.

- ➢ It is the characteristic of an ellipse that the sum of the distances of any planet from two focus is constant. The elliptical orbit of a planet is responsible for the occurrence of seasons.



Kepler's Second Law: The Law of Equal Areas

- Kepler's second law states, " The radius vector drawn from the sun to the planet sweeps out equal areas in equal intervals of time".



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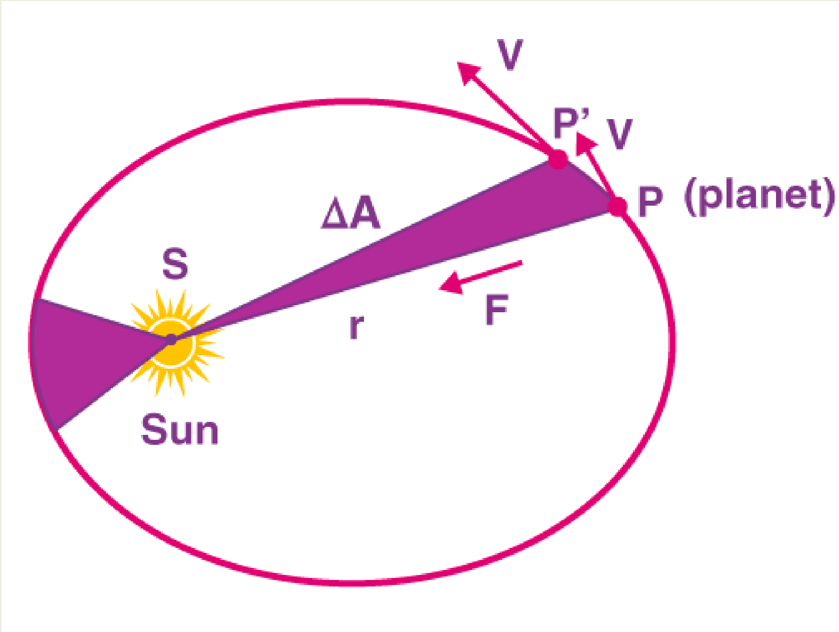
- As the orbit is not circular, the planet's kinetic energy is not constant in its path. It has more kinetic energy near the perihelion, and less kinetic energy near the aphelion implies more speed at the perihelion and less speed (v_{min}) at the aphelion. So, areas swept out by a line from the Sun to planate for both aphelion and perihelion, were equal over the same amount of time.



Kepler’s Third Law: The Law of Periods

According to Kepler’s law of periods, “The square of the time period of revolution of a planet around the sun in an elliptical orbit is directly proportional to the cube of its semi-major axis”.

Explanation: If the mean distance between the sun and the planet "P" is "R" and the period is "T", then according to this law,

$$T^2 \propto R^3$$

If the mean distance between the sun and different planets are R_1, R_2, R_3 and the periods are $T_1, T_2, T_3 \ldots$

then, $\dfrac{T_1^2}{R_1^3} = \dfrac{T_2^2}{R_2^3} = \dfrac{T_3^2}{R_3^3} = \text{constant}.$

Thank you
for your kind attention