**READING LIST:**

## UNIT 1

## NUMERICAL ANALYSIS

**Numerical Analysis** is the study of algorithms for the problems of continuous mathematics (as distinguished from discrete mathematics).

It is concerned with the mathematical derivation, description and analysis of methods of obtaining numerical solution of mathematical problems. It is an area of mathematics and computer science that creates, analyzes and implements algorithms for obtaining numerical solutions to problems involving continuous variables.

**Numerical Method**

Numerical method is a set of rules for solving a problem or problems of a particular type, involving only the operations of arithmetic.

## COMPUTER ARITHMETIC

The decimal number system has the base 10. The decimal integer number 4987 actually means $(4987)_{10} = 4 \times 10^3 + 9 \times 10^2 + 8 \times 10^1 + 7 \times 10^0$ ---------- (1)

which represents a polynomial in the base 10. Similarly, a fractional decimal number 0.6251 means $(0.6251)_{10} = 6 \times 10^{-1} + 2 \times 10^{-2} + 5 \times 10^{-3} + 1 \times 10^{-4}$ -------- (2)

which is a polynomial in $10^{-1}$.

Combining (1) and (2), we may write the number 4987.6251 in decimal system as:

$$4987.6251 = 4 \times 10^3 + 9 \times 10^2 + 8 \times 10^1 + 7 \times 10^0 + 6 \times 10^{-1} + 2 \times 10^{-2} + 5 \times 10^{-3} + 1 \times 10^{-4} \quad (3)$$

**Binary Number System**

Binary number system has base 2 with digits 0 and 1 called bits.

**Example 1:** Find the decimal number corresponding to the binary number $(111.011)_2$

$$111.011_2 = 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}$$

$$= 7.375_{10}$$

**Example 2**: Convert $58_{10}$

| | |
|---|---|
| 2 | 58 |
| 2 | 29 r 0 |
| 2 | 14 r 1 |
| 2 | 7 r 0 |
| 2 | 3 r 1 |
| 2 | 1 r 1 |
| 2 | 0 r 1 |

$58_{10} = 111010_2$

**Example 3:** Convert $0.859375_{10}$ to the corresponding binary fraction.

| | |
|---|---|
| 0 | 0.859375 |
| | x 2 |
| 1 | 0.718750 |
| | x 2 |
| 1 | 0.437500 |
| | x 2 |
| 0 | 0.875000 |
| | x 2 |
| 1 | 0.750000 |
| | x 2 |
| 1 | 0.500000 |
| | x 2 |
| 1 | 0000000 |

The required binary fraction becomes; $0.859375_{10} = 0.110111_2$

**Example 4:** Convert $0.7_{10}$ to the corresponding binary fraction:

$$
\begin{array}{rl}
 & 0.7 \\
 & \underline{\times\ 2} \\
1 & 0.4 \\
 & \underline{\times\ 2} \\
0 & 0.8 \\
 & \underline{\times\ 2} \\
1 & 0.6 \\
 & \underline{\times\ 2} \\
1 & 0.2 \\
 & \underline{\times\ 2} \\
0 & 0.4 \\
 & \underline{\times\ 2} \\
0 & 0.8 \\
 & \underline{\times\ 2} \\
1 & 0.6 \\
 & \underline{\times\ 2} \\
1 & 0.2 \\
 & \underline{\times\ 2} \\
0 & 0.4
\end{array}
$$

Thus we obtain $(0.7)_{10} = (.101100110...)_2$ which is a never ending sequence. If only 7 bits are retained in the binary fraction then the corresponding decimal number becomes

$$0.1011001_2 = 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4} + 0 \times 2^{-5} + 0 \times 2^{-6} + 1 \times 2^{-7}$$

$$= 0.6953125$$

which is not exactly the same as the given number.

The difference

$$0.7 - 0.6953125 = 0.0046875$$

is the round-off error.

**Octal System**

The Octal system has base 8 and uses the digits 0, 1, 2, 3, 4, 5, 6, 7. A binary number can be converted to an Octal number by grouping the bits in groups of three to the right and left of the binary point by adding sufficient zeros to complete the groups and replacing each group of three bits by its Octal equivalent.

Example: Convert the binary number 1101001.1110011 to the octal system. We have

001| 101 | 001 . 111 | 001 | 100

1 5 1 . 7 1 4

$1101001.1110011 = 151.714_8$

**Hexadecimal System**

The hexadecimal system has base 16 and the digits 0 to 9 and A, B, C, D, E, F to represent 10, 11, 12, 13, 14, 15 respectively. To convert a binary number to a hexadecimal number, we form groups of four of the binary bits and replace it by the corresponding digit in the hexadecimal system.

Example: Convert the binary number 1101001.1110011 to the hexadecimal system.

| 0110 | 1001 | 1110 | 0110 |
|---|---|---|---|
| 6 | 9 | E | 6 |

$1101001.1110011 = 69E6_{16}$

The **purposes of numerical analysis** are centered on finding **approximate solutions** to mathematical problems that **cannot be solved analytically** or are too **complex for exact computation**. It provides systematic methods for obtaining accurate and efficient results using computers.

## 1. To Obtain Approximate Solutions

Many mathematical problems—such as nonlinear equations, differential equations, or integrals—do **not have exact analytical solutions**.
Numerical analysis provides **approximation methods** to solve them efficiently.

**Example:**
Finding roots of $e^x = x^2$ or evaluating $\int_0^1 e^{-x^2} dx$ (which has no closed-form solution).

## 2. To Handle Real-World Problems

Most real-world engineering, physical, and scientific problems are too complicated for exact methods.
Numerical analysis allows us to **model and solve** such problems.

**Example:**

- Weather prediction
- Structural analysis
- Heat transfer and fluid flow
- Signal and image processing

## 3. To Minimize and Control Errors

Every numerical method introduces some **errors** (round-off, truncation, etc.).
Numerical analysis helps in **estimating, minimizing, and controlling these errors**, ensuring reliable results.

## 4. To Improve Computational Efficiency

It provides **algorithms** that produce accurate results **with minimal computation time and memory** — important for large-scale simulations and real-time systems.

## 5. To Solve Problems Using Computers

Numerical methods convert mathematical models into **computational algorithms** that can be implemented on digital computers for automatic calculation.

## 6. To Analyze Stability and Convergence

Numerical analysis studies whether a computational method will **converge** to the correct result and how **stable** it is under small changes in input or intermediate results.

## 7. To Support Decision-Making in Engineering and Science

By simulating models and predicting outcomes, numerical analysis supports **design optimization, control, and decision-making** in various fields.

## In essence:

**Numerical analysis bridges the gap between theoretical mathematics and practical computation**, allowing us to **solve complex problems approximately, efficiently, and reliably**.