$A = \{a, b, c, d\}$

$a \in A$

$b \in A$

$e \notin A$

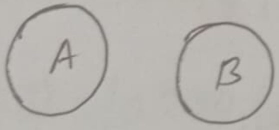
$A = \{a, b\}$

$B = \{1, 2, 3\}$

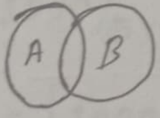
$A \times B = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$

Infinite set $= Q = \{1, 2, 3, \dots\}$

* $n(A \cup B) = n(A) + n(B)$



* $n(A \cup B) = n(A) + n(B) - n(A \cap B)$



* $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C)$
$- n(A \cap C) + n(A \cap B \cap C)$

* $n(F) = 65$ $\quad n(F \cap G \cap R) = 8$

$n(G) = 45$

$n(R) = 42$ $\quad n(F \cup G \cup R) = ?$

$n(F \cap G) = 20$

$n(F \cap R) = 25$

$n(G \cap R) = 15$

⊛ power set

elements of power set = 2^n

⊛ $A = \{1, 2, 3, 4, 5\}$ $\quad U = \{1, \dots, 9\}$

$B = \{4, 5, 6, 7\}$

$C = \{5, 6, 7, 8, 9\}$

$D = \{1, 3, 5, 7, 9\}$

$E = \{2, 4, 6, 8\}$

$F = \{1, 5, 9\}$

1) $A \cap B = \{4, 5\}$

2) $D \cap E = \{\}$ অথবা ϕ

3) $A \setminus B = \{1, 2, 3\}$

4) $D \setminus E = \{1, 3, 5, 7, 9\}$

5) $E \oplus F = (E \setminus F) \cup (F \setminus E)$

$= \{1, 2, 4, 5, 6, 8, 9\}$

6) $(A \setminus E)^c = U - \{1, 3, 5\}$

$= \{2, 4, 6, 7, 8, 9\}$

⊛

Among 1 to 1000,

i) How many of them are not divinible by 3 nor by 5 nor by 7?

ii) How many are not divinible by 5 or 7 but divinible by 3?

i) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(A \cap C) - n(B \cap C)$
$+ n(A \cap B \cap C)$

$n(A) = |1000/3| = 333$

$n(B) = |1000/5| = 200$

$n(C) = |1000/7| = 142$

$n(A \cap B) = |1000/(5\times 3)| = 66$

$n(A \cap C) = |1000/(3\times 3)| = 47$

$n(B \cap C) = |1000/(5\times 7)| = 28$

$n(A\cap B \cap C) = |1000/(5\times 3\times 7)| = 9$

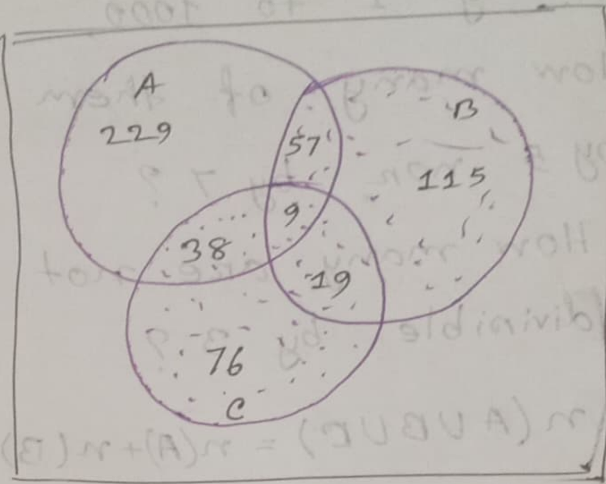
n(A∪B∪C) দ্বারা বুঝায় 3, 5, 7 তিনটির যেকোনো একটি দ্বারা ভাগ যাবেই.

n(A∩B∩C) পুরো n(A∪B∪C) এর ভেতর

$n(\overline{A\cup B\cup C}) = \cancel{833+}\ U - n(A\cup B\cup C)$
$= 1000 - 543$
$= 457$

~~এটা দ্বারা বুঝায়~~

ii)



Among 1 to 300

i) How many of them are not divinible by 3, nor 5, nor 7 ?

ii) How many are not divinible by 5 or 7 but divinible by 3 ?

iii) divinible by 5 and 3 but not divinible by 7

iv) " " 7 or 3 but " " 5

Ans:

i) $n(A) = \lfloor 300/3 \rfloor = 100$

$n(B) = \lfloor 300/5 \rfloor = 60$

$n(C) = \lfloor 300/7 \rfloor = 47$

$n(A \cap B) = \lfloor 300/3\times5 \rfloor = 20$

$n(B \cap C) = \lfloor 300/5\times7 \rfloor = 8$

$n(A \cap C) = \lfloor 300/3\times7 \rfloor = 14$

$n(A \cap B \cap C) = \lfloor 300/3\times5\times7 \rfloor$

$= 2$

$n(A \cup B \cup C) = 162$

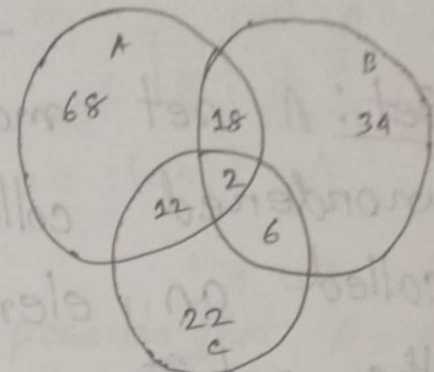
$n(\overline{A \cup B \cup C}) = 300 - 182$

$= 138$

ii) 68

iii) 18

iv) 34



— o —

Types of Relation

i) Reflexive Relation

aRa for all $a \in N$

$A = \{1, 2, 3\}$

$R = \{(1,2), (1,1), (2,2), (3,3), (2,3)\}$

ii) Irreflexive Relation

$(a,a) \notin R$

iii) Symemetric Relation

mine

Set

Set : A set may be viewed as an unordered collection of distinct objects called as elements or members of the set.

$p \in A$ means, p is the element of A,
$p \notin A$ means, p is not the element of A.

N = the set of positive integer: 1,2,3,...
Z = the set of integers. ...., -2,-1,0,1,2,...
Q = the set of rational numbers
R = the set of real number
C = the set of complex number

Universal set : In any application of the theory of sets, the members of all set under investigation usually belongs to some fixed large set called the universal set.

empty set : The set with no elements is called the empty set.

Subset: If every element in a set A is also an element of a set B, then A is called a subset of B.

If $A \subseteq B$ then it is still possible that $A = B$

If $A \subseteq B$ But $A \neq B$ then A is proper subset of B. $A \subset B$.

~~Symmetr~~ Symmetric difference

The ~~s~~ symmetric difference of a sets A and B, denoted by $A \oplus B$,

$$A \oplus B = (A \cup B) \setminus (A \cap B)$$

$$A \oplus B = (A \setminus B) \cup (B \setminus A)$$

⊕ In a survey of 60 people, it was found that

25 read Newsweek week magazine

26 read Time

26 read Fortune

9 read both Newsweek and Fortune

11 read both ~~Time~~ Newsweek and Time

8 read

$R \cup \boxed{\Delta_A}$

$\Delta_A = \{(a,a) : a \in A\}$

$R \cup R^{-1}$

$A = \{1, 2, 3, 4\}$

$R = \{(1,1), (1,3), (2,4), (3,1), (3,3), (4,3)\}$

reflexive $(R) = R \cup \Delta_A$

$= R \cup \{(1,1), (2,2), (3,3), (4,4)\}$

$= \{(1,1), (1,3), (2,2), (2,4), (3,1), (3,3), (4,3), (4,4)\}$

symetric $(R) = R \cup R^{-1}$

$= \{(1,1), (1,3), (3,1), (2,4), (4,2), (3,3), (4,3), (3,4)\}$

$R \circ R = R^2$

$R^2 R = R^3$

transitive $(R) = R \cup R^2 \cup R^3 \cup \ldots \cup R^n$

$A = \{1, 2, 3\}$

$R = \{(1,2), (2,3), (3,3)\}$

transitive $(R) = R \cup R^2 \cup R^3$

$= \{(1,2), (1,3), (2,3), (3,3)\}$

$A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d\}$; $C = \{x, y, z\}$

$R = \{(1,a), (2,d), (3,a), (3,b), (3,d)\}$

$S = \{(b,x), (b,z), (c,y), (d,z)\}$

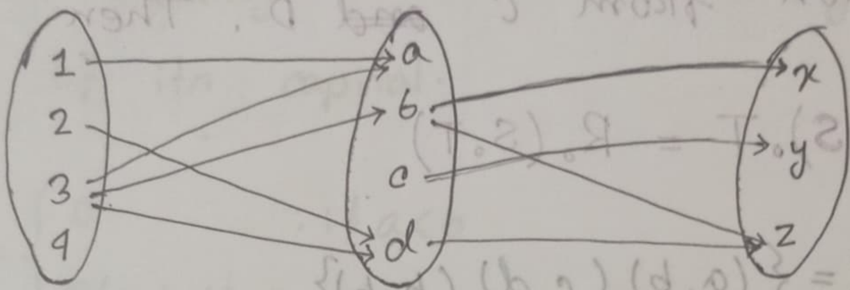


Fig - 1

consider the arrow diagrams of R and S as fig-1. Observe that there is an arrow from 2 to d which is followed by an arrow from d to z. We can view these two arrow as a "Path" which "connects" the element $2 \in A$ to the element $z \in C$. Thus,

$$2(R \circ S)z \quad \text{since } 2Rd \text{ and } dSz$$

Similarly there is a path from 3 to x and a path from 3 to x and a path from 3 to z. Hence

$$3(R \circ S)x \quad \text{and} \quad 3(R \circ S)z$$

No other element of A is connected to an element of C. Accordingly,

$$R \circ S = \{(2,z), (3,x), (3,z)\}$$

Theorem: Let A, B, C and D be sets. Suppose R is a relation from A to B, S is relation from B to C and T is relation from C ~~and~~ D. Then

$$(R \circ S) \circ T = R \circ (S \circ T)$$

Let, $R = \{(a,b),(c,d),(b,b)\}$

$S = \{(d,b),(b,e),(c,a),(a,c)\}$

i) $R \circ S = \{(a,e),(c,b),(b,e)\}$

$S \circ R = \{(d,b),(c,b),(a,d)\}$

note that, $R \circ S \neq S \circ R$

iii) $(S \circ R) \circ R = \{(d,b),(c,d)\}$

iv) $R \circ (S \circ R) = \{(c,b)\}$

Chapter-3

a) To each person on the earth assign the number which corresponds to his age.

b) To each country in the world assign the latitude logitude of its capital.

$$Q(a,b) = \begin{cases} 0 & ; \text{if } a<b \\ Q(a-b,b)+1 & \text{if } b \leq a \end{cases}$$

$$Q(12,5) = Q(7,5)+1$$
$$= [Q(2,5)+1]+1$$
$$= [0+1]+1$$
$$= 2$$

A(m, n)

a) if $m=0$ then $A(m,n) = n+1$

b) if $m \neq 0$ ~~th~~ but $n=0$ then $A(m,n) = A(m-1, 1)$

c) if $m \neq 0$ and $n \neq 0$ then $A(m,n) = A(m-1, A(m, n-1))$

$$A(1,3) = A(0, A(1,2))$$

$$A(1,2) = A(0, A(1,1))$$
$$A(1,1) = A(0, A(1,0))$$
$$A(1,0) = A(0,1)$$
$$A(0,1) = 1+1 = 2$$

$\therefore A(1,0) = A(0,1) = 2$

$\therefore A(1,1) = A(0,2) = 2+1 = 3$

$\therefore A(1,2) = A(0,3) = 3+1 = 4$

$\therefore A(1,3) = A(0,4) = 4+1 = 5$

<u>chapter-4</u>

p	q	$\neg q$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

↓

p	q	$\neg$	$(p$	$\wedge$	$\neg$	$q)$
T	T	T	T	F	F	T
T	F	F	T	T	T	F
F	T	T	F	F	F	T
F	F	T	F	F	T	F

~~$p \vee (p \wedge \neg q)$~~

⊛ $p \vee \neg(p \wedge q)$, ~~Fatu~~ Tautologien or contradictionn

· Logical equivalence

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

1) It in not the cane that ronen are red and violet are blue.

$\neg$ (p $\wedge$ q

2) Ronen are not red or Violet are not blue.

$\neg p$ $\vee$ $\neg q$

if p then q : $p \rightarrow q$ (p implies q)

p if and only if q : $p \leftrightarrow q$ (p biconditional q)

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(1st operand T and 2nd operand F হলে F)

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

✱ $p \rightarrow q \equiv \neg p \vee q$

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

logically equivalent

$P_1, P_2, P_3, \ldots, P_n \vdash Q$

$(P_1 \wedge P_2 \wedge P_3 \wedge \ldots \ldots P_n) \rightarrow Q$

(6) Tautology হলে valid

$p \rightarrow q,\ q \rightarrow r \vdash p \rightarrow r$ (Law of syllogism)

$$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$p \rightarrow r$	$(p\rightarrow q) \wedge (q \rightarrow r) \rightarrow (p\rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	F	F	T	T

S_1: If 7 is less than 4 then 7 is not a prime number

S_2: 7 is not less than 4

S_3: 7 is a prime number

Ans:

p = 7 is less than 4

q = 7 is a prime number

$p \to q, \neg p \vdash \neg q$

$[(p \to q) \wedge (\neg q)] \to \neg q$

p	q	$\neg q$	$(p \to q)$	$(p \to q) \wedge (\neg q)$	$(p \to q) \wedge (\neg q) \to \neg q$
T	T	F	T	F	T
T	F	T	F	F	F
F	T	F	T	F	T
F	F	T	T	T	T

Not tautology no, invalid.

$(\forall_n \in N)(x+4>3) \rightarrow$ True

সকল Natural number এর জন্য $(x+4 \geq 3)$ সত্য,

$(\forall_n \in N)(x+4<3) \rightarrow$ False

সকল Natural number এর জন্য $(x+4<3)$ ~~টি~~ সত্য নয়,

$(\exists_n \in N)(n+4>7) \rightarrow$ True

অন্তত 1, 2 (২-টা value) এর জন্য exist করে.

$(\exists_n \in N)(n+2<0) \rightarrow$ False

কোনো value ই ~~টি~~ নেই $(n+2<0)$ True হওয়ার জন্য

$(\forall_x \in M)(x \text{ is male})$

All math majors are male

$\neg[(\forall_x \in M)(x \text{ is male})] = (\exists_x \in M)(x \text{ is not male})$

It is not case of All math majors are male.

There exist at least one math major who is female not male.

For all positive integer n, we have $\underbrace{n+2>8}_{P(x)}$

$$\neg[(\forall_x \in N)\, P(x)]$$
$$= (\exists_n \in N)\, \neg P(x)$$

There exist positive integer n. such that $n+2 \ngtr 8$

$$\neg[(\forall_x \in A)\ P(x)] = (\exists_x \in A)\ \neg P(x).$$

$$\neg[(\exists_x \in A)\ P(x)] = (\forall_x \in A)\ \neg P(x)$$

$P(x,y) = x+y = 10$

$\forall_x \exists_y , P(x,y)$

→ For all x there exist a y, such that $x+y=10$

$B = \{1, 2, \dots, 9\}$; $x+y=10$

$\forall_x \exists_y , P(x,y)$ true

$\exists_y , \forall_x \; P(x,y)$

⇒ There exist a y, such that for all x, we have $x+y=10$

False

$\neg[\forall_x \exists_y , P(x,y)]$

$= \exists_x \forall_y , \neg P(x,y)$

$U = \{1, 2, 3\}$

a) $\exists_x \forall_y , x^2 < y+1$

There exist atleast a x, such that for all y,

$$T = \begin{cases} 1 \longrightarrow 1^2 < 1+1 \\ \quad\;\; 1^2 < 2+1 \\ \quad\;\; 1^2 < 3+1 \end{cases}$$

$x = 1$

So, True.

⊛ $\forall_x \exists_y \; x^2 + y^2 < 12$ $\qquad U = \{1, 2; 3)$

Over all true

⊛ $\forall_x , \forall_y \; ; \; x^2 + y^2 < 12$ $\qquad U = \{1, 2, 3\}$

False

⊛ $(p \wedge q) \vee (\neg p \wedge q \wedge r)$

$= [p \vee (\neg p \wedge q \wedge r)] \wedge [q \vee (\neg p \wedge q \wedge r)]$

$= [(p \vee \neg p) \wedge (p \vee q) \wedge (p \vee r)] \wedge [(q \vee \neg p) \wedge (q \vee q) \wedge (q \vee r)]$

$= (p \vee q) \wedge (p \vee r) \wedge (q \vee \neg p) \wedge q \wedge (q \vee r)$

* $(\neg p \rightarrow r) \wedge (p \leftrightarrow q)$ dnf ?

p	q	r	$(\neg p \rightarrow r) \wedge (p \leftrightarrow q)$							
T	T	T	F	T	T	T	T	T	T	T
T	T	F	F	T	F	F	F	T	T	T
T	F	T	F	T	T	T	T	T	F	F
T	F	F	F	T	F	F	F	T	F	F
F	T	T	T	F	T	T	F	F	F	T
F	T	F	T	F	T	F	F	F	F	T
F	F	T	T	F	T	T	T	F	T	F
F	F	F	T	F	T	F	T	F	T	F

~~Cap~~ Chapter-1

Set : A set is a well defined collection of distinct object, considered as a single entity.

Complement : Complement of a set A, denoted by A^c, is the set of elements which belong to U ~~and~~ but which do not belong to A.

$$A^c = U - A = U \setminus A$$

Symmetric Difference : The symmetric difference of set A and B, denoted by $A \oplus B$, consists of those elements which belong to A or B but not to both.

$$A \oplus B = (A \cup B) \setminus (A \cap B)$$

$$A \oplus B = (A \setminus B) \cup (B \setminus A)$$

Idempotent laws : $A \cap A = A$

Associative laws : $(A \cap B) \cap C = A \cap (B \cap C)$

$$(A \cup B) \cup C = A \cup (B \cup C)$$

Commutative laws : $A \cap B = B \cap A$

$$A \cup B = B \cup A$$

Distributive laws : $A \cap (B \cap C) = (A \cap B) \cup (A \cap C)$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Identity laws : $A \cap U = A$

$A \cap \emptyset = \emptyset$

$A \cup \emptyset = A$

$A \cup U = U$

Involution laws : $(A^c)^c = A$

Complement laws : $A \cap A^c = \emptyset$

$\emptyset^c = U$

$A \cup A^c = U$

$U^c = \emptyset$

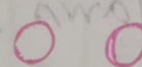
De Morgan's laws : $(A \cap B)^c = A^c \cup B^c$

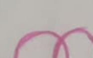
$(A \cup B)^c = A^c \cap B^c$

$A = \{a, b\}$, $B = \{a, c, d\}$

$A \times B = \{a, b\} \times \{a, c, d\}$

$= \{(a,a), (a,c), (a,d), (b,a), (b,c), (b,d)\}$

$n(A \cup B) = n(A) + n(B)$ 

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ 

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

~~Find the number of~~

⊛ Consider the following data for 120 mathematics students at a college concerning the languages French, German and Russian:

45 study German
65 study French
42 study Russian
20 study French and German
25 study French and Russian
15 study German and Russian
8 study all three language

Ans:

Here,

$n(F) = 65,\ n(G) = 45,\ n(R) = 42$

$n(F \cap G) = 20,\ n(F \cap R) = 25$

$n(G \cap R) = 15,\ n(F \cap G \cap R) = 8$

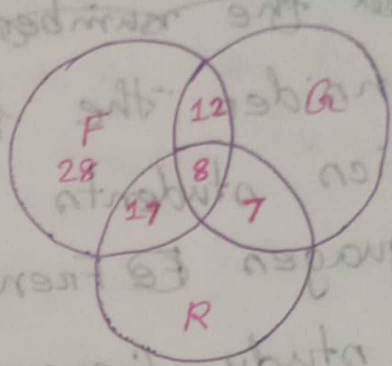
We know,

$$n(F \cup G \cup R) = n(F) + n(G) + n(R) - n(F \cap G) - n(F \cap R) - n(G \cap R) + n(F \cap G \cap R)$$

$$= 65 + 45 + 42 - 20 - 25 - 15 + 8$$

$$= 100$$

8 study all three language



$20 - 8 = 12$ study French and German not Russian

$25 - 8 = 17$ study French and Russian not German

$15 - 8 = 7$ study Russian and German not French.

$65 - 12 - 8 - 17 = 28$ study only French

$45 - 12 - 8 - 7 = 18$ study only German

$42 - 17 - 8 - 7 = 10$ study only Russian

$120 - 100 - 20$ do not study.

Prove: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Ans:

$$n(A \cup B) = n(A \setminus B) + n(B \setminus A) + n(A \cap B)$$

$$= n(A \setminus B) + n(B \setminus A) + n(A \cap B) - n(A \cap B) + n(A \cap B)$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Chapter - Two

$R = \{(a, 3a) \mid a \in I\}$ I = positive integer

$S = \{(a, a+1) \mid a \in I\}$

$R = \{(1,3), (2,6), (3,9), (4,12), (5,15), \dots\}$

$S = \{(1,2), (2,3), (3,4), (4,5), (5,6), (6,7), \dots\}$

a)

$R \cdot S = \{(1,4), (2,7), \dots\}$

$\therefore R \cdot S = \{(a, 3a+1) \mid a \in I\}$

b)

$R \cdot R = \{(1,9), (2,18), \dots\}$

$= \{(a, 9a) \mid a \in I\}$

$18 \times 3 = 54$

c)

$3a-3$

$R \cdot R \cdot R = \{(1,27), (2,54), \dots\}$

$= \{(a, 27a) \mid a \in I\}$

d)

$R \cdot S \cdot R = \{(1,12), (2,21), \dots\}$

$= \{(9a+3) \mid a \in I\}$

Reflexive Relations: A relation R on a set A is reflexive if aRa for every $a \in A$, this is if $(a,a) \in R$ for every $a \in R$.