Sources of Error in Numerical Analysis

In numerical computation, various types of errors can arise from different stages of problem formulation and implementation. The major sources are as follows:

1. Modeling Error

Definition:
Occurs when the **mathematical model** used to represent a real-world system is **inaccurate, oversimplified, or inappropriate**.

Cause:

- Neglecting important physical effects (e.g., friction, air resistance).
- Assuming linear relationships where the system is nonlinear.
- Simplifying boundary or initial conditions.

Example:
Approximating the motion of a projectile **without considering air resistance** introduces a modeling error.

2. Measurement Error

Definition:
Arises from **inaccurate or imprecise measurement of input data or parameters** used in computations.

Cause:

- Limitations of measuring instruments.
- Human observation errors.
- Environmental variations (temperature, pressure, etc.).

Example:
If a temperature sensor reads 99.5°C instead of 100°C, this deviation introduces measurement error.

3. Implementation Error

Definition:
Occurs due to **incorrect algorithm design, programming mistakes, or computational limitations** during implementation.

Cause:

- Wrong algorithm or numerical method selection.

- Programming or coding mistakes.
- Finite precision and rounding errors in digital computation.

Example:
Using an unstable numerical method to solve differential equations, or coding errors in implementing formulas.

4. Simulation Error

Definition:
This is the **error that accumulates during the execution of a numerical model or simulation**. It arises due to **approximations, iterative processes, or propagation of previous errors**.

Cause:

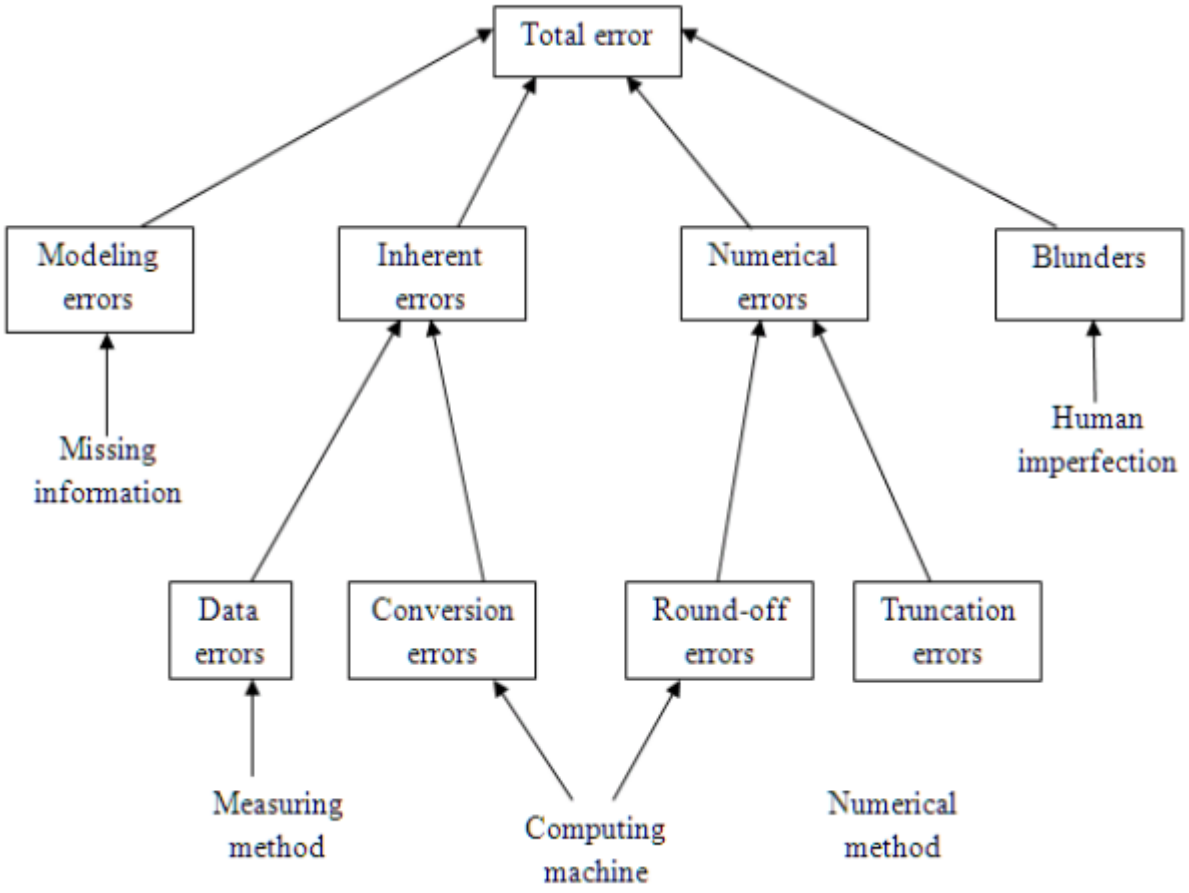
- Repeated use of approximate results.
- Finite time steps or grid sizes in simulation.
- Accumulation of rounding and truncation errors.

Example:
In a long-term weather simulation, small rounding errors can accumulate and significantly affect the final results.

Summary Table

Source of Error	Description	Example
Modeling Error	Using an inappropriate or oversimplified model	Ignoring air resistance in projectile motion
Measurement Error	Inaccurate input data due to poor measurement	Temperature measured as 99.5°C instead of 100°C
Implementation Error	Faulty algorithm or coding mistakes	Wrong formula or unstable method
Simulation Error	Error accumulated during repeated computations	Accumulated rounding/truncation in simulation

Taxonomy of errors:-



N.B: Conversion errors is also called representation errors

Precision and Accuracy in Numerical Analysis

In **numerical analysis** (and measurement systems), the terms **accuracy** and **precision** describe the **quality of computed or measured results**, but they do **not mean the same thing**.

They help assess how close a result is to the true value (**accuracy**) and how consistent repeated results are (**precision**).

1. Accuracy

Definition:
Accuracy refers to **how close a computed or measured value is to the true (exact) value**.

Accuracy=Closeness to the true or accepted value

Interpretation:
A result is said to be **accurate** if it is **close to the true value**, even if the values are not consistent with each other.

Example:
If the true value of gravity is 9.81 m/s^2, and measurements are 9.80,9.82,9.79
then these are **accurate** because they are close to the true value.

2. Precision

Definition:
Precision refers to **how close repeated measured or computed values are to each other**, regardless of how close they are to the true value.

Precision=Closeness among repeated results

Interpretation:
A result is **precise** if repeated measurements give **similar values**, even if those values are far from the true one.

Example:
If repeated measurements of gravity are 9.65,9.66,9.64,
they are **precise** (consistent) but **not accurate** (far from 9.81).