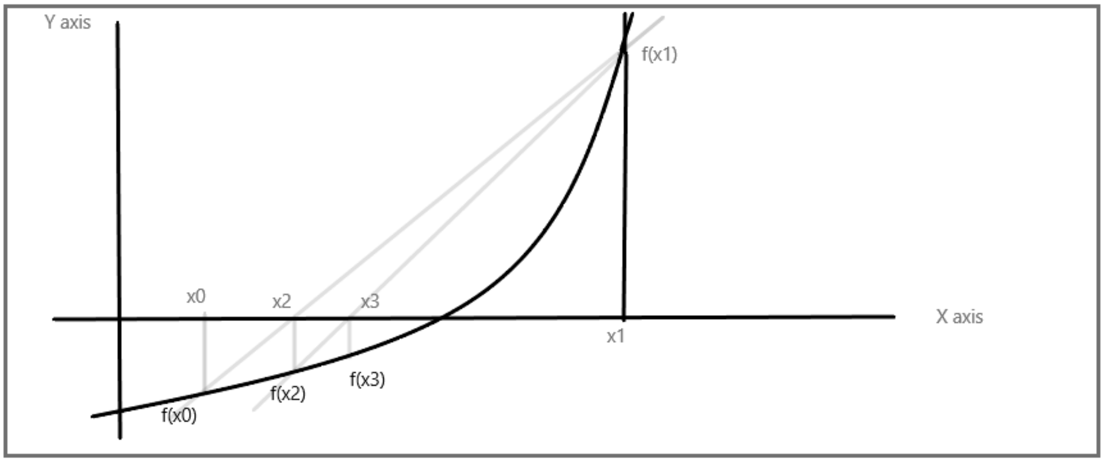
Secant Method of Numerical analysis

Secant method is a recursive method for finding the root of a polynomial by successive approximation.

- In this method, the neighborhood roots are approximated by secant line (A line that intersects the curve at two distinct points) to the function **f(x)**.
- It's similar to the **Regular-falsi** method, but here we don't need to check **$f(x_1)f(x_2) < 0$** again and again after every approximation.
- We do not need to differentiate the given function **f(x)**, as we do in **Newton-Raphson** method.



Secant line

A secant line is a straight line that intersects a curve at two distinct points. In the context of calculus, the secant line can be used to approximate the slope of the curve between those two points. The slope of the secant line is calculated as the change in the y-values divided by the change in the x-values between the two points.

For a function f(x), the secant line through the points (x_0, f(x_0)) and (x_1, f(x_1)) has the equation:

slope of secant line: $\frac{f(x_0)-f(x_1)}{x_0-x_1}$

Formula for Secant Method

Now, we'll derive the formula for secant method. The equation of Secant line passing through two points is :

$$Y-Y_1=m(X-X_1)$$

Here, m=slope

So, apply for **(x_1, f(x_1))** and **(x_0, f(x_0))**

slope of secant line, $m= \frac{f(x_0)-f(x_1)}{x_0-x_1}$

Y - $f(x_1) = [f(x_0)-f(x_1)/(x_0-x_1)]\ (x-x_1)$ **Equation (1)**

As we're finding root of function f(x) so, Y= f(x) =0 in Equation (1) and the point where the secant line cut the x-axis is,

$x= x_1- [(x_0 - x_1)/ (f(x_0) - f(x_1)]f(x_1)$.

We use the above result for successive approximation for the root of function f(x).

Let's say the first approximation is **$x=x_2$:**

$x_2= x_1 - [(x_0 - x_1)/ (f(x_0)-f(x_1))]f(x_1)$

Similarly, the second approximation would be **$x =x_3$**:

$x_3= x_2 - [(x_1-x_2)/ (f(x_1)-f(x_2))]f(x_2)$

And so on, **till k_{th} iteration,**

$$x_{k+1} = x_k - [\frac{(x_{k-1}-x_k)}{(f(x_{k-1})-f(x_k))}]f(x_k)$$

Example: A real root of the equation $f(x) = x^3 - 5x + 1 = 0$ lies in the interval (0, 1). Perform four iterations of the secant method.

Solution: To perform four iterations, we need an initial guess x_0. Let's use x_0= 0.5.

Iteration 1: $x_1 = x_0 - f(x_0) * (x_0 - x_{prev}) / (f(x_0) - f(x_{prev}))$

x_1 = 0.5 - ((0.5^3 - 5 * 0.5 + 1) * (0.5 - 0)) / ((0.5^3 - 5 * 0.5 + 1) - f(0))

x_1= 0.5 - (-0.625 * 0.5) / (-0.625)

x_1 = 0.5 + 0.5

x_1 = 1.0

Iteration 2: (Using x_1 = 1.0 and x_0 = 0.5)

$x_2 = x_1 - f(x_1) * (x_1 - x_0) / (f(x_1) - f(x_0))$

x_2 = 1.0 - (-3 * (1.0 - 0.5)) / (-3 - (-0.625))

x_2 = 1.0 + (3 * 0.5) / -2.375

x_2 = 1.0 - 0.63157

x_2 = 0.368422

Iteration 3: (Using x_2 = 0.368422 and x_1 = 1.0)

$x_3 = x_2 - f(x_2) * (x_2 - x_1) / (f(x_2) - f(x_1))$

x_3 = 0.368422 - (-0.184097) * (0.368422 - 1.0) / (-0.184097 - (-3))

x_3 = 0.368422 + (0.184097 * 0.631578) / 2.815903

x_3 = 0.368422 + 0.041155

x_3 = 0.409577

Iteration 4: (Using x_3 = 0.409577 and x_2 = 0.368422)

$x_4 = x_3 - f(x_3) * (x_3 - x_2) / (f(x_3) - f(x_2))$

x_4 = 0.409577 - (-0.070279) * (0.409577 - 0.368422) / (-0.070279 - (-0.184097))

x_4 = 0.409577 + (0.070279 * 0.041155) / 0.113818

x_4 = 0.409577 + 0.025535

x_4 = 0.435112

So, after four iterations of the secant method, the approximate root is x_4 = 0.435112.