PHY-111: Physics

Chapter- 1 (Electricity)

by

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Electricity

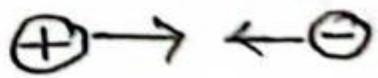
Electric charge:

Electric charge can be defined as a fundamental property of subatomic particles that gives rise to the phenomenon of experiencing force in the presence of electric and magnetic fields. These fields exert influence on charged particles, resulting in observable effects.

Types of electric charg:

Electric charge comes in two main types: positive and negative charges. Positive charges are associated with protons, which are subatomic particles residing in the nucleus of an atom. One the otherhand negative charges are linked to electrons, which orbit to atomic nucleus.

Like charges repel eachother



opposite charges attract eachother

Electricity

Electric current:

Electric current refers to the flow of electricity in an electronic circuit, and to the amount of electricity flowing through a circuit; i.e, An electric current is the physical phenomenon of the displacement of flow of an electric charge, usually of electrons, by means of a conductive material.

Electrostatic:

Electrostatic is the study of the properties and behaviors of electric charges that are stationary or moving slowly. It is a branch of physics that deals with the forces that electric charges exert on each other, and how they attract or repel each other.

Charge density:

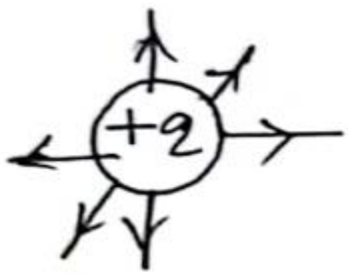
Charge density is the amount of electric charge per unit of length, surface area, or volume. It is a measure of how much electric charge is accumulated in a given space. Charge density can be positive or negative and varies with position.

Charge density can be calculated by dividing the total electric charge (Q) by the volume (V) or area (A) of the material.

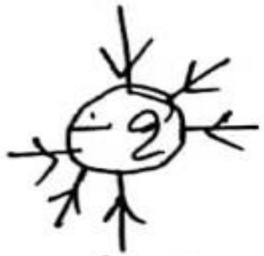
Mathematically, charge density (ρ) is expressed as $\rho = \frac{Q}{V}$ for volume charge density and $\rho = Q/A$ for surface charge density.

Electric field:

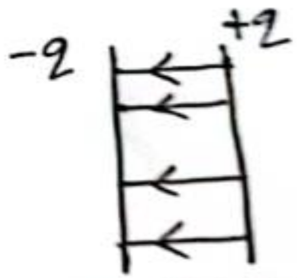
An electric field is the physical field that surrounds electrically charged particles. An electric property associated with each point in space when charge is present in any form. The magnitude and direction of the electric field are expressed by the value of E, called electric field strength or electric field intensity or simply the electric field.



E-field around a positive charge



E-field around a negative charge



E-field inside a parallel plate capacitor.

Electric field, $\vec{E} = \frac{\vec{F}}{q}$

Electric field can never be negative.

Electricity

Electric field : The space surrounding a charged body or a system of charges over which an another charge experiences a force of attraction or repulsion is known as the electric field.

Electric field : When an electric charge is placed at some point in space, this establishes every where a state of electric stress, which is called an electric field. The space where charge, influence can be felt, is called site or electric field.

Electric field strength:

Electric field strength is a quantitative expression of the intensity of an electric field at a particular location. A field strength of 1 V/m represents a potential difference of 1V between points separated by 1 meter.

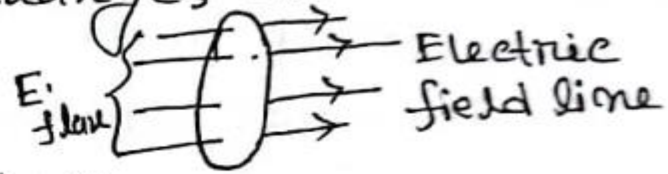
Electric field intensity : The force experienced by a unit positive charge placed at the point at which the field is to be determined.

Mathematical expresion : $\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

Electricity

Electric flux:

Electric flux, property of an electric field that may be thought of as the number of electric lines of force (or electric field lines) that intersect a given area. Electric ~~lines~~ field lines are considered to originate on positive electric charges and to terminate on negative charges.



Is flux always positive?

When the field vectors are going the same direction as the vectors normal to the surface, the flux is positive. When the field vectors are going the opposite direction as the vectors normal to the surface, the flux is negative.

Negative flux

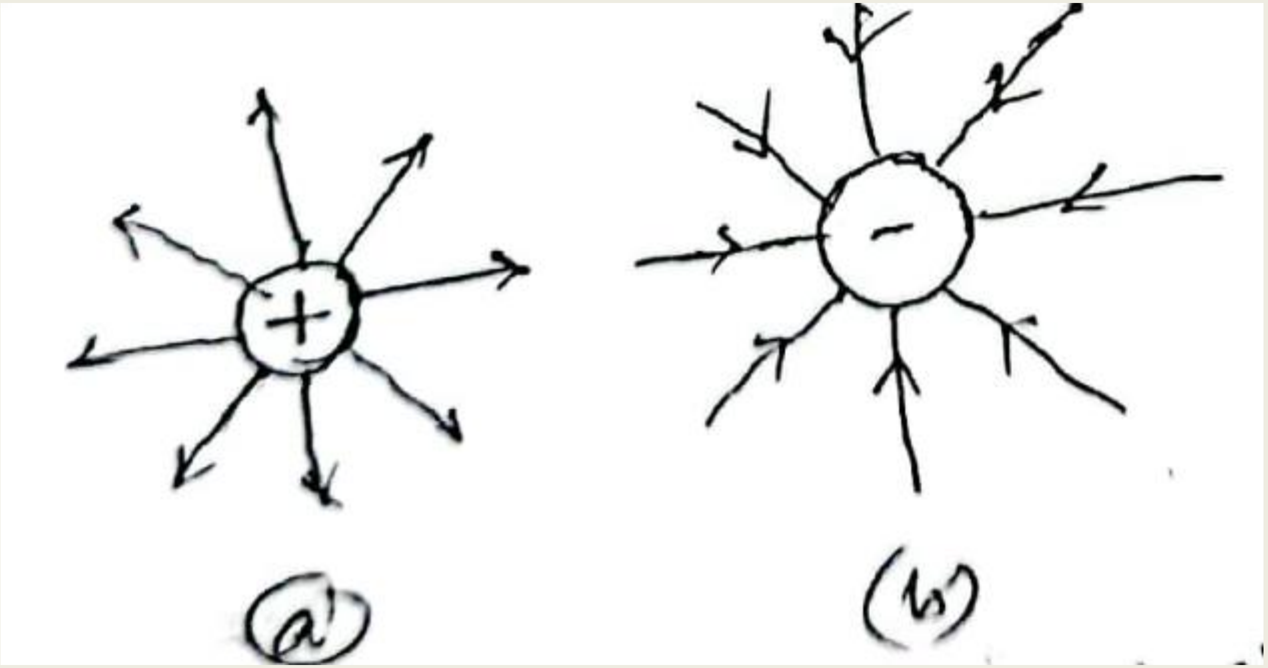
positive field flux.

Electricity

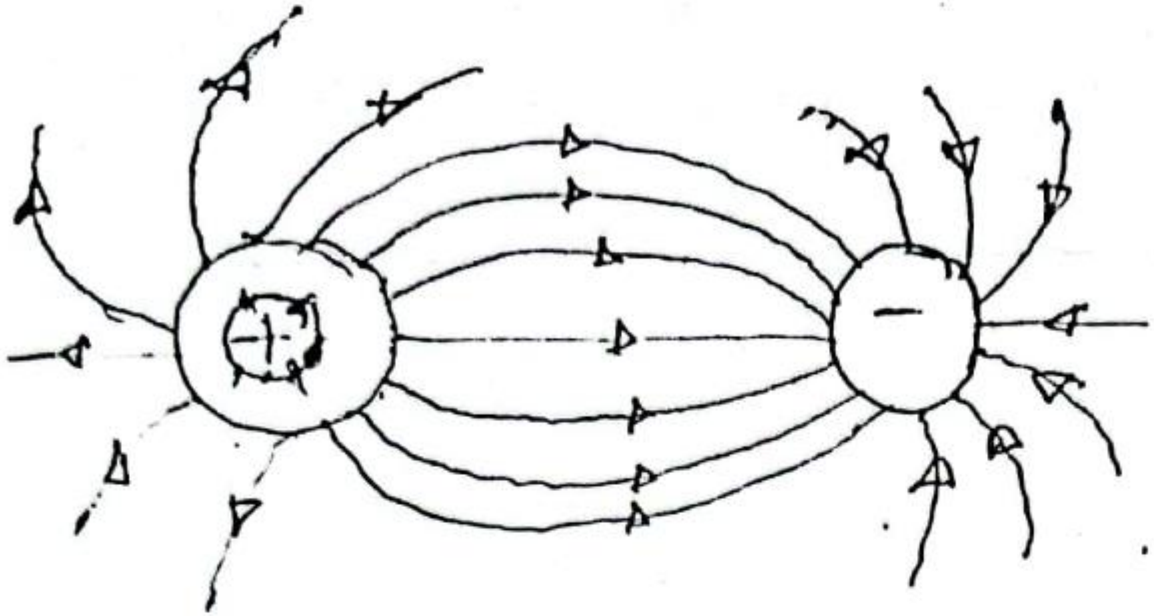
 # What do you understand by electric lines of force ? Write down the properties of electric lines of force. Explain why two lines of force cannot cut each other.

Electric lines of force : If we placed unit, free positive charge in any point of an electric field, then it would follow some imaginary directional line of moving path, which is known as electric lines of force.

For positive charge the direction of electrical lines of force is outward and for negative charge the direction of electric lines of force is inward.

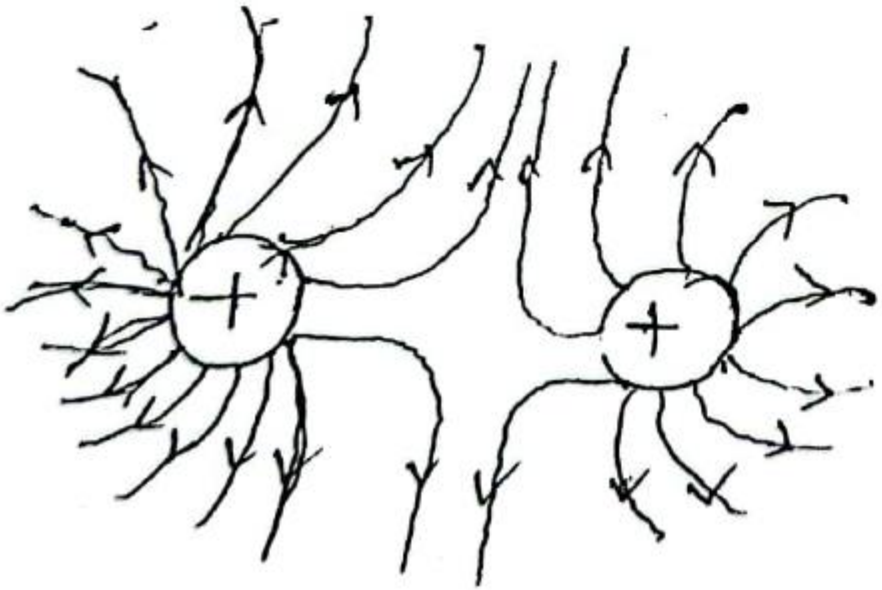


For two equal and oposite charges:



For two equal and oposite charge the electric lines of force Produced from Positive charge and ~~term~~ some of the lines of force terminated to the negative charge.

For two same charges:



For two same types of charges the lines of force bending in such a way in betn two charges that ~~here there are~~ no lines of force exist here. The lines of forces can not cut each other and they imposed repu-sive force to each other.

Electricity

Properties of electric lines of force :

(i) Electrical lines of force s straight line or open bending line.

(ii) Electrical lines of force is produced from positive charge and terminated to negative charge.

(iii) Lines of force cannot cut each other.

(iv) Electrical lines of force repulse each other in side ward direction.

The relationship between the lines of force and electric field.

1. The tangent to a line of force at any point gives the direction of $\vec{E}$ at that point.

2. The number of lines per unit cross-section all area is Proportional to the magnetude of $\vec{E}$ where the lines are close together $\vec{E}$ in large and where they are for apart $\vec{E}$ is small.

Electric Potential : The electric potential at a point in electric field is defined as the work done to bring a unit positive charge from infinity to that point. It is denoted by V.

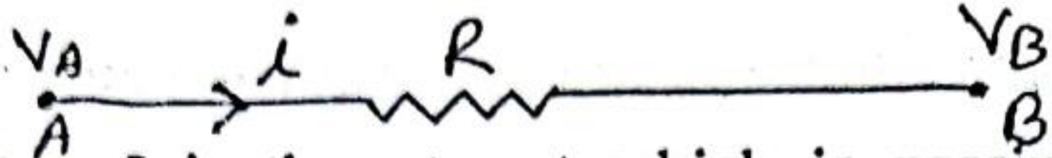
Equipotential Surface : If the potential is same at all points on the surface of a conductor the surface is called the equipotential surface.

Electricity

Ohm's Law:

At constant temperature, the current passing through a conductor is proportional to the potential difference between the two ends of the conductor.

Explanation:



Let, AB is conductor. I is the current which is passing through the conductor. V_A and V_B is the potential between the two ends of the conductor.

Now, according to law,

$$(V_A - V_B) \propto I$$

$$\Rightarrow V \propto I \quad [\because V = V_A - V_B]$$

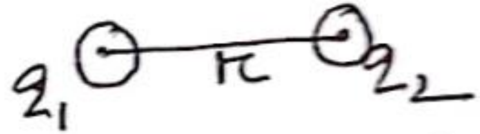
$$\Rightarrow V = RI$$

Where, R is proportional constant. It is called resistance of the conductor

Coulomb's law:

The force of attraction or repulsion between two charged bodies is directly proportional to the product of their charges and inversely proportional to the square of the distance between them. It acts along the line joining the two charges considered ~~to~~ be point charge

$$F \propto \frac{q_1 q_2}{r^2}$$



Faraday's law of induction:

→ **First Law:** → An e.m.f is induced in the loop only when the number of magnetic field lines that passes through the loop changes.

→ **Second Law:** The e.m.f is induced in the loop is equal to the negative of the rate at number of magnetic flux passes through the loop changing with time.

If the e.m.f is E & ϕ_B is the magnetic flux according to Faraday's second law of induction we can write,

$$E = -\frac{d\phi_B}{dt}$$

−ve sign indicates the direction of the e.m.f.

⇒ **Note:** Gate being openned the potential difference betn two electron is known as $\boxed{e.m.f}$.

Gauss law:

Gauss's law states that the net electric flux through any closed surface is equal to $1/\varepsilon_0$ times the net electric charge enclosed within that closed surface. The closed surface is also reffered to as Gaussian surface. If ~~the~~ surface does not enclosed the charge, the flux of $\vec{E}$, i.e., $\oint \vec{E}\cdot\vec{ds}$ equal to zero.

Thus, $\oint \vec{E}\cdot\vec{ds} = q/\varepsilon_0$, q inside the surface.

or, $\varepsilon_0 \oint \vec{E}\cdot\vec{ds} = q$

When q outside the surface,

$$\oint \vec{E}\cdot\vec{ds} = 0$$

Prob: A negative point charge of 10^{-6} coulomb is situated in air at the origin of a rectangular coordinate system. A second negative point charge of 10^{-4}C is situated at a distance of 50cm from the origin. What is the force on the second charge?

Soln: We know,

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

$$= \frac{9\times10^9 \times (-10^{-6})(-10^{-4})}{(0.5)^2}$$

$$= 3.6\ N$$

Here,

$q_1 = -10^{-6}C$

$q_2 = -10^{-4}C$

$r = 50\ cm$

$= 0.5m$

$1/4\pi\epsilon_0 = 9\times10^9\ Nm^{-2}Col^{-2}$

Prob: Calculate the force of repulsion between two protons in a nucleus of iron, assuming a separation of 4×10^{-15} m.

Soln: We know,

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

$$= 9\times10^9 \times \frac{(1.6\times10^{-19})^2}{(4\times10^{-15})^2}$$

$$= 14\ N$$

Here,

$q_1 = q_2 = 1.6\times10^{-19}$ C

$r = 4\times10^{-15}$ m

$F = ?$

Electricity

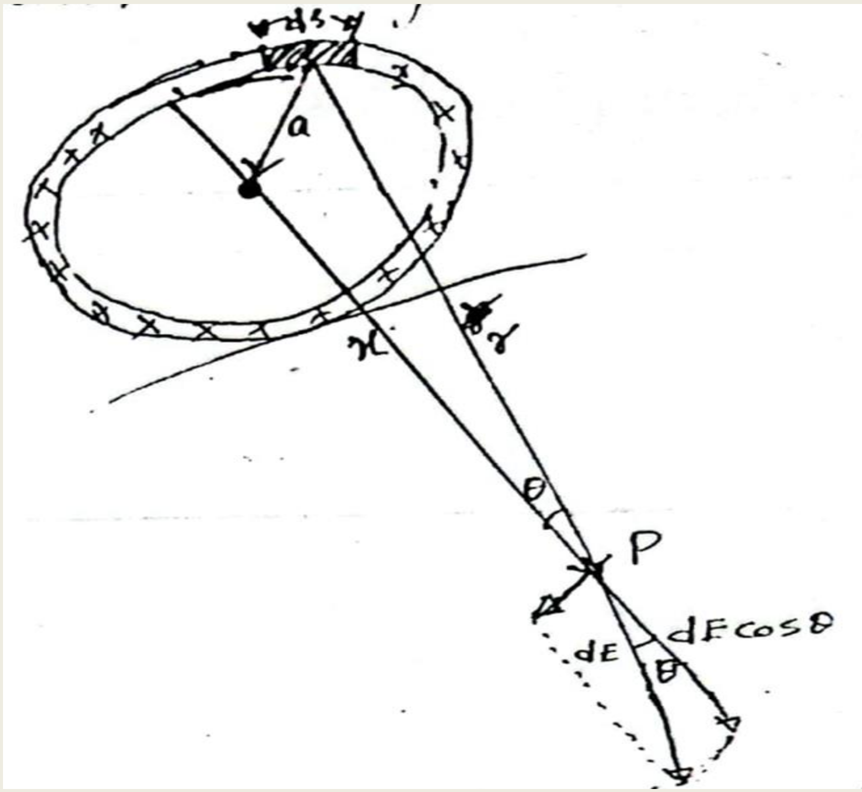
Show that at great enough distance a charged ring behaves like a point charge.

Consider a differential element of the ring of length ds, located at the top of the ring as shown in figure. It contains an element of charge given by

$$dq = q\frac{ds}{2\pi a} \text{———} (1)$$

Where $2\pi a$ is the circumference of the ring. This element sets up a differential electric field dE at point P.

The resultant field $\vec{E}$ at P is found by integrating the effects of all the elements that make up the ring.



$2\pi a = q$

$\Rightarrow 1 = \frac{q}{2\pi a}$

$\Rightarrow ds = q\frac{ds}{2\pi a}$

$\therefore dq = q\frac{ds}{2\pi a}$

Electricity

$$\vec{E} = \int \overrightarrow{dE}$$

become a scalar integral

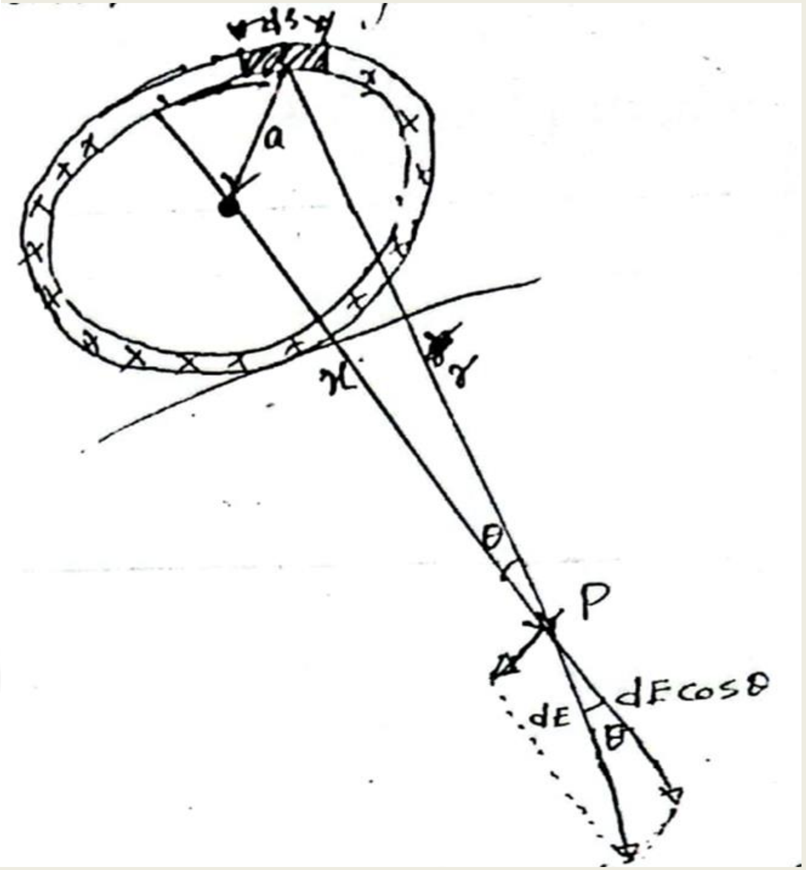
$$E = \int dE\cos\theta \text{ —— } (2)$$

The quantity dE follows

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

$$\Rightarrow dE = \frac{1}{4\pi\varepsilon_0}\left(\frac{q\,ds}{2\pi a}\right)\frac{1}{a^2+x^2} \quad [\because r^2 = a^2 + x^2]$$

From fig. we have $\quad \cos\theta = \frac{x}{r} = \frac{x}{\sqrt{a^2+x^2}}$



Electricity

$\therefore$ from ern (2) we have

$$E = \int dE\cos\theta = \int \frac{1}{4\pi\varepsilon_0}\frac{q\,ds}{(2\pi a)(a^2+x^2)}\cdot\frac{x}{\sqrt{a^2+x^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{qx}{(2\pi a)(a^2+x^2)^{\frac{3}{2}}}\cdot 2\pi a$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{qx}{(a^2+x^2)^{\frac{3}{2}}}$$

For x>> a we can neglect a in the denominator of this equation.

$$E \cong \frac{1}{4\pi\varepsilon_0}\frac{qx}{x^3}$$

$$E \cong \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$$

This an expected result because at great enough distances a charge of a ring behaves like a point charge q.

KIRCHOFFS LAWS:

(i) **First law (Junction rule):** ~~In~~ an electric circuit the algebraic sum of the currents meeting at a junction is zero.

In the fig O is junction. i_2, i_3, i_5, i_8 are currents flowing towards the junction and i_1, i_4, i_6, i_7 are

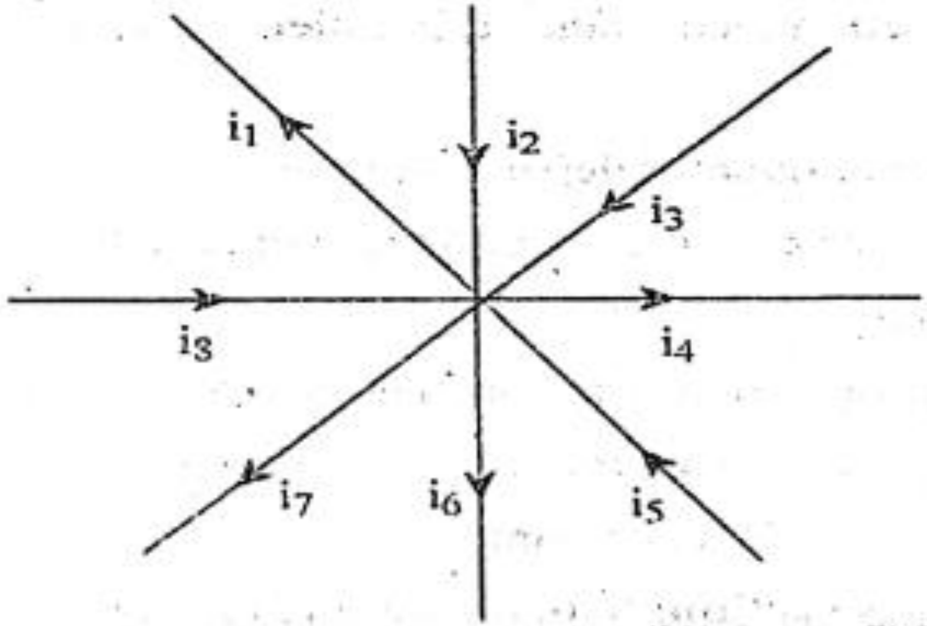


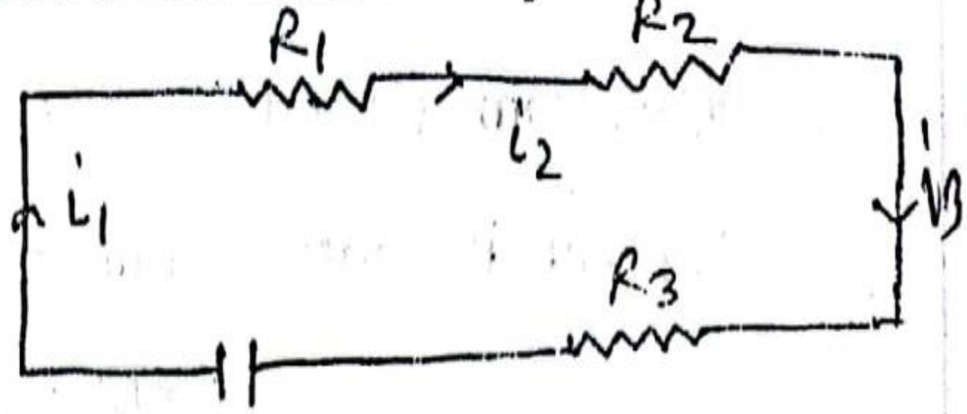
Fig- 28

currents flowing away from currents from the junction. Now according to the law,

$$+i_2 + i_3 + i_5 + i_8 - i_1 - i_4 - i_6 - i_7 = 0$$

i.e, $\Sigma i = 0$.

(ii) Second low (loop rule): For a closed circuit, the algebraic sum of the products of the currents and resistance of each part of the circuit is equal to the total e.m.f of the circuit.

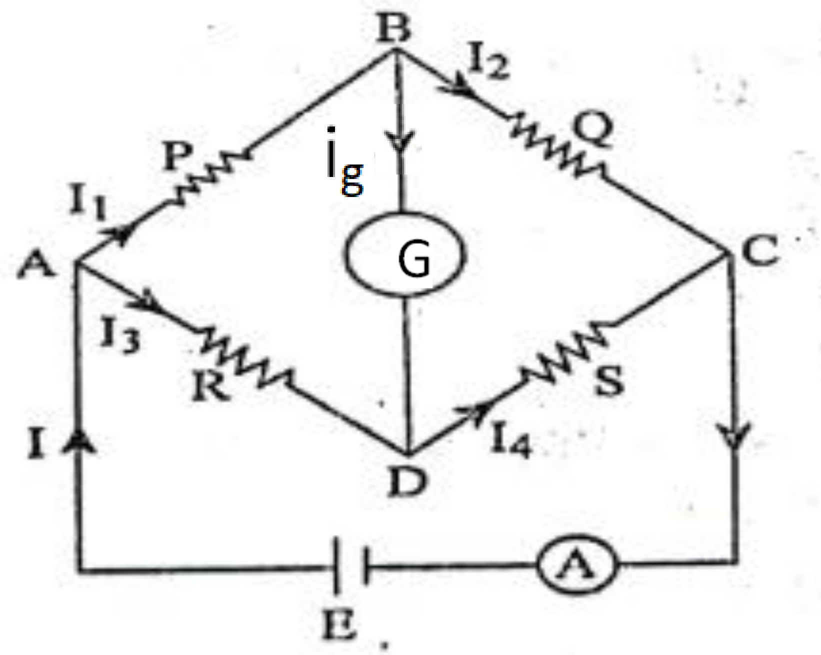


let us, Consider a closed circuit consists of battery E, resistances R_1, R_2, R_3 are connected to wires. So according to loop rule-

$$i_1R_1 + i_2R_2 + i_3R_3 = E$$

$$\therefore E = \Sigma iR.$$

Whetstone's bridge: The whetstone's bridge consists of four resistances P,Q,R,S, arranged as in the figure. A and C point are connected through a battery and B and D point are connected through a galvanometer, of resistance G.



Let I is the current in the battery circuit and $I_1, I_2, I_3, I_4,$ and i_g are the currents in P, Q, R, S and G respectively. Applying kirchoffs first law

at B- $I_1 - I_2 - Ig = 0$

at D- $I_3 + Ig - I_4 = 0$

Electricity

[For a balanced bridge there is no current through the galvanometer.

$$i,e, \quad i_g = 0]$$

Now, from (i) and (ii) $i_1 = i_2$ and $i_3 = i_4$.

Applying kirchoffs second low at ABDA circuit

$$i_1 P + i_g G - i_3 R = 0$$

$$\Rightarrow i_1 P = i_3 R [\because ig = 0]$$

$$\Rightarrow \frac{i_1}{i_3} = \frac{R}{P} \quad \ldots \quad \ldots \quad (iii)$$

At BCDB circuit,

$$i_2 Q - i_4 S - i_g G = 0$$

$$\Rightarrow i_2 Q = i_4 S \; [\because i_g = 0]$$

$$\Rightarrow \frac{i_2}{i_4} = \frac{S}{Q}$$

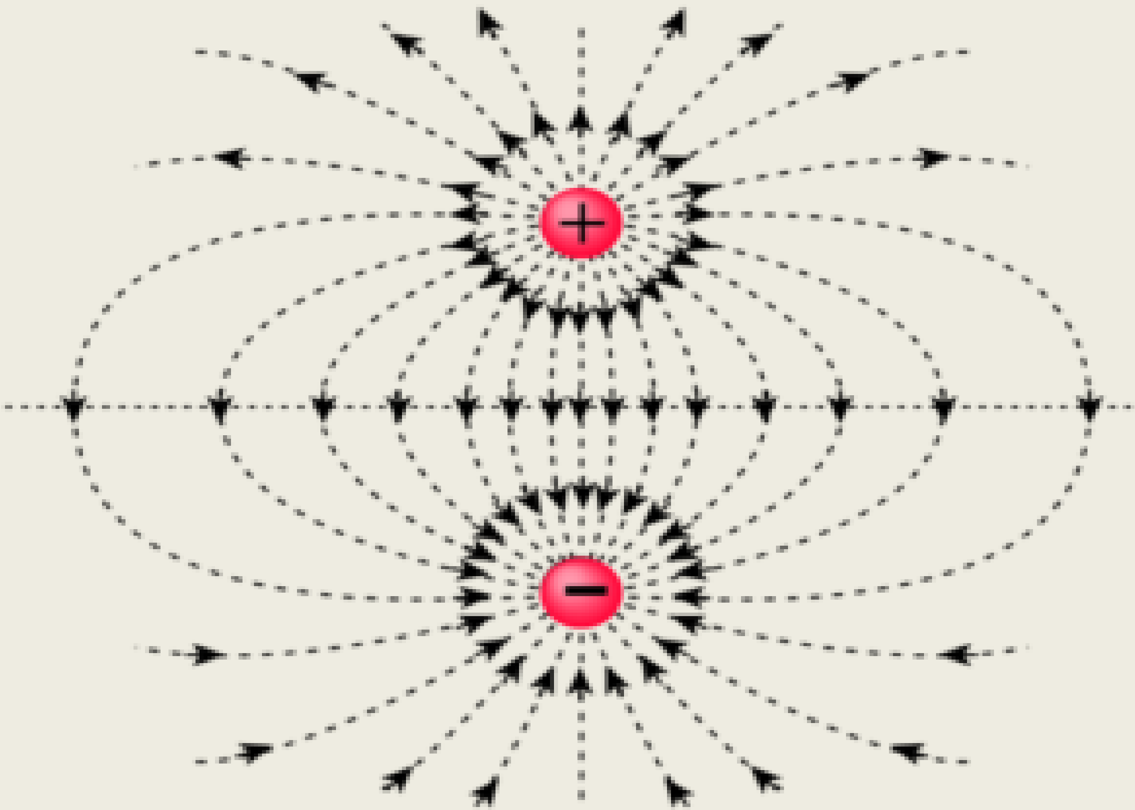
$$\Rightarrow \frac{i_1}{i_3} = \frac{S}{Q} [\because i_1 = i_2 \; and \; i_3 = i_4]$$

from (iv) and (iv) $\frac{R}{P} = \frac{S}{Q}$

$$\Rightarrow \frac{P}{Q} = \frac{R}{S}$$

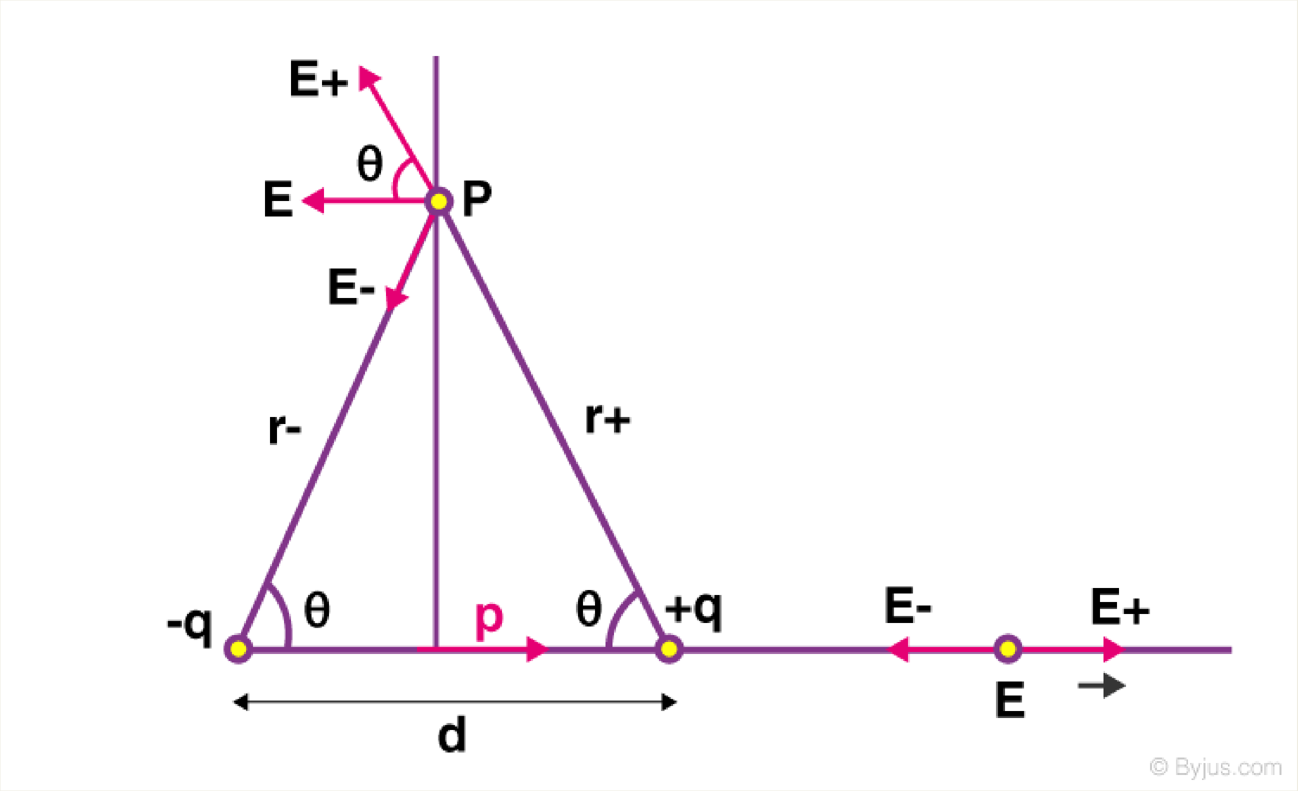
Electric Dipole:

An electric dipole is defined as a couple of opposite charges “q” and “–q” separated by a distance “d”. By default, the direction of electric dipoles in space is always from negative charge “-q” to positive charge “q”. The midpoint “q” and “–q” is called the center of the dipole. It is a useful concept in dielectrics and other applications in solid and liquid materials. These applications involve the energy of a dipole and the electric field of a dipole.



Electricity

Q: Calculate the field E due to a dipole at a point P at a distance r along the perpendicular bisector of the line joining the charges.



Consider an electric dipole with charges +q and -q separated by a distance d. We shall for the sake of simplicity only calculate the fields along symmetry axes, i.e. a point **P** along the perpendicular bisector of the dipole at a distance r from the mid-point of the dipole and a point **Q** along the axis of the dipole at a distance r from the mid-point of the dipole.

Electricity

Along the Perpendicular Bisector (Point P)

The electric fields due to the positive and negative charges (Coulomb's law):

$$E_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+^2}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{(\sqrt{r^2+(\frac{d}{2})^2})^2}$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2+(\frac{d}{2})^2}\right)$$

Similarly,

$$E_- = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_-^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+(\frac{d}{2})^2}$$

The vertical components of the electric field cancel out as P is equidistant from both charges.

$$\implies E = E_+ \cos\theta + E_- \cos\theta$$

$$\implies E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+(\frac{d}{2})^2}\cos\theta + \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+(\frac{d}{2})^2}\cos\theta$$

$$\implies E = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2+(\frac{d}{2})^2}\cos\theta$$

Now,

$$\cos\theta = \frac{\frac{d}{2}}{r_+} = \frac{\frac{d}{2}}{r_-} = \frac{\frac{d}{2}}{\sqrt{r^2 + (\frac{d}{2})^2}}$$

Substituting this value we get,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2 + (\frac{d}{2})^2}\frac{\frac{d}{2}}{\sqrt{r^2 + (\frac{d}{2})^2}} = \frac{1}{4\pi\varepsilon_0}\,\frac{qd}{(r^2 + (\frac{d}{2})^2)^{\frac{3}{2}}}$$

Dipole moment

$$p = q \times d$$

When r >> d, we can neglect the d/2 term. Thus, we have,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}^{\frac{3}{2}}$$
$$\implies E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

The dipole moment direction is defined as pointing towards the positive charge. Thus, the direction of the electric field is opposite to the dipole moment:

$$\overrightarrow{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\overrightarrow{p}}{r^3}$$

Along Axis of Dipole (Point Q)

The electric fields due to the positive and negative charges are:

$$E_+ = \frac{1}{4\pi\epsilon_0}\frac{q}{r_+^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-\frac{d}{2})^2} E_- = \frac{1}{4\pi\epsilon_0}\frac{q}{r_-^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{r+\frac{d}{2}^2}$$

Since the electric fields are along the same line but in opposing directions,

$$E = E_+ \; - \; E_-$$
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-\frac{d}{2})^2} \; - \; \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+\frac{d}{2})^2}$$
$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-\frac{d}{2})^2} \; - \; \frac{1}{(r+\frac{d}{2})^2}\right]$$
$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+\frac{d}{2})^2-(r-\frac{d}{2})^2}{(r^2-(\frac{d}{2})^2)^2}\right]$$
$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{4r\frac{d}{2}}{(r^2-(\frac{d}{2})^2)^2}\right]$$
$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{2rqd}{(r^2-(\frac{d}{2})^2)^2}\right]$$
$$E = \frac{1}{4\pi\varepsilon_0}\;\left[\frac{2rp}{(r^2-(\frac{d}{2})^2)^2}\right]$$

Factoring r^4 from denominator:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^4}\left[\frac{2pr}{(1-(\frac{d}{2r})^2)^2}\right]$$

Now if r >> d, we can neglect the $(d/2r)^2$ term becomes very much smaller than 1. Thus, we can neglect this term. The equation becomes:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{1}{r^4} \left[\frac{2pr}{1^2} \right]$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{2p}{r^3}$$

Since in this case the electric field is along the dipole moment, $E_+ > E_-$,

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{2\vec{p}}{r^3}}$$

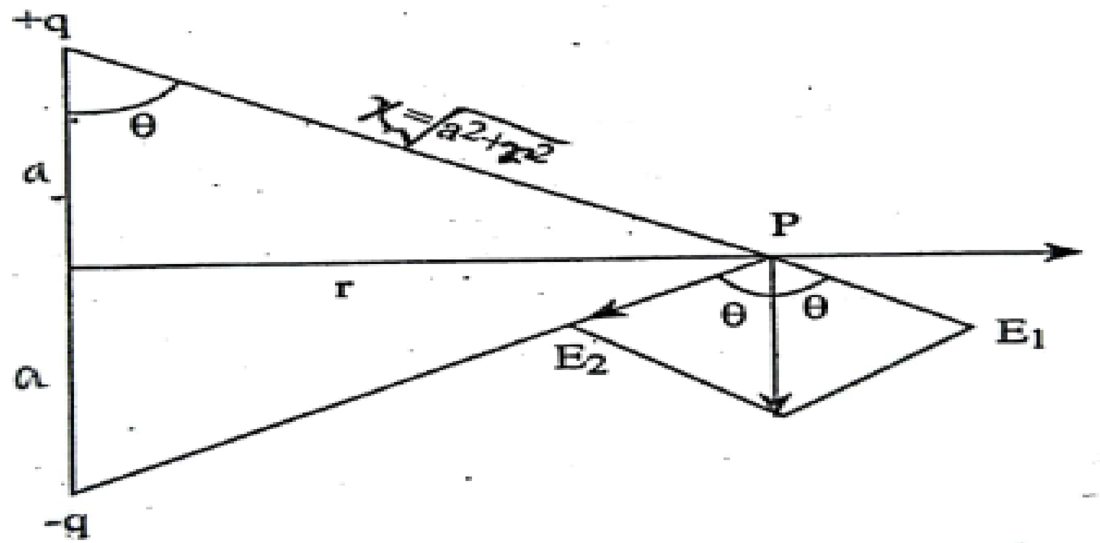
Notice that in both cases the electric field tapers quickly as the inverse of the cube of the distance. Compared to a point charge which only decreases as the inverse of the square of the distance, the dipoles field decreases much faster because it contains both a positive and negative charge. If they were brought to the same point their electric fields would cancel out completely but since they have a small distance separating them, they have a feeble electric field.

Q: Calculate the field E due to a dipole at a point P at a distance r along the perpendicular bisector of the line joining the charges.

Soln: Let us consider a point P, at a distance r along the perpendicular bisector of the line joining the charges. According to figure Electric field

Electric field $\vec{E} = \vec{E}_1 + \vec{E}_2$

Again, $\vec{E}_1 = \vec{E}_2 = \frac{1}{4\pi \epsilon_0} \cdot \frac{q}{x^2} = \frac{1}{4\pi \epsilon_0} \cdot \frac{q}{a^2 + r^2}$



The vector sum of $\vec{E}_1$ and $\vec{E}_2$ points, vertically downwards and has the magnitude. $\therefore\ \vec{E} = 2\vec{E}_1\ Cos\theta = 2.\frac{1}{4\pi\in_0}.\frac{q}{a^2+r^2}.\cos\theta$

$$= 2.\frac{1}{4\pi\in_0}.\frac{q}{a^2+r^2}\times\frac{a}{\sqrt{a^2+r^2}}$$

$$= \frac{1}{4\pi\in_0}.\frac{2aq}{(a^2+r^2)^{3/2}}$$

$$= \frac{1}{4\pi\in_0}.\frac{2aq}{r^3}\ [\text{if } r\rangle\rangle\rangle a]$$

$$= \frac{1}{4\pi\in_0}.\frac{p}{r^3}$$

Where P = 2aq is called the electric dipole moment.

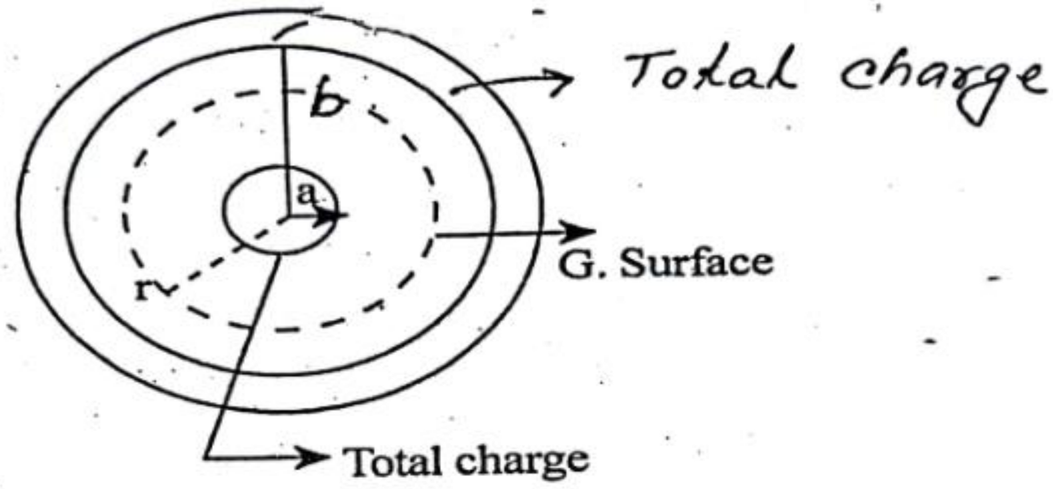
<u>Specific inductive capacity (S.I.C)/Dielectric constant:</u>

The ratio between the capacitance of a capacitor when the space between the plates is filled with a dielectric and the capacitance when the space bet^n the plates is filled with air is called S.I.C and dielectric constant.

Mathematically, $K = \frac{c}{c_0}$

Spherical capacitor capacitance:

Let us consider a central cross-section of a capacitance that consists of two concentric spherical shells of radius 'a' and 'b'. As a Gaussian surface; we draw a sphere of radius r concentric with two shells.



According to Gauss's law

$$\in_0 \oint \vec{E}.\vec{ds} = q$$

$$\Rightarrow \in_0 \oint Eds \cos o^0 \doteq q$$

$$\Rightarrow \in_0 .E\oint ds = q \Rightarrow \in_0 .E(4\pi r^2) = q$$

$$\Rightarrow E. = \frac{q}{4\pi \in_0 .r^2} \left[\because \oint ds = 4\pi r^2\right]$$

Now potential difference bet^n two spherical,

$$V = \int dv = \int_a^b E.dr.$$

$$= \int_a^b \frac{q}{4\pi \epsilon_0 . r^2} dr$$

$$= \frac{q}{4\pi \epsilon_0} \left[-\frac{1}{r} \right]_a^b \quad = -\frac{q}{4\pi \epsilon_0} \left[+\frac{1}{b} - \frac{1}{a} \right]$$

$$= -\frac{q}{4\pi \epsilon_0} \left(\frac{a-b}{ab} \right) = \frac{q}{4\pi \epsilon_0} \left(\frac{b-a}{ab} \right)$$

Again we know,

$$C = \frac{q}{v} = \frac{q}{\frac{q}{4\pi \epsilon_0} \left(\frac{b-a}{ab} \right)}$$

$$\Rightarrow C = 4\pi \epsilon_0 \left(\frac{ab}{b-a} \right) = 4\pi \epsilon_0 \left(\frac{a}{1 - \frac{a}{b}} \right)$$

when b=α and a=R, then $\quad \therefore C = 4\pi \epsilon_0 .R$

Cylinder capacitor:

Let us, consider a cross sectional cylinder capacitor of length l formed by two coaxial cylinders of radius a and b. As a Gaussian surface we choose a cylinder of radius r and length l closed by end cap. According to Gauss's law

$$\epsilon_0 \oint \vec{E}.\vec{ds} = q \Rightarrow \epsilon_0 . \oint E.ds \cos 0^0 = q \quad \Rightarrow \epsilon_0 .E \oint ds = q$$

$$\Rightarrow \epsilon_0 .E.(2\pi rl) = q \quad \Rightarrow E = \frac{q}{2\pi \epsilon_0 .rl}$$

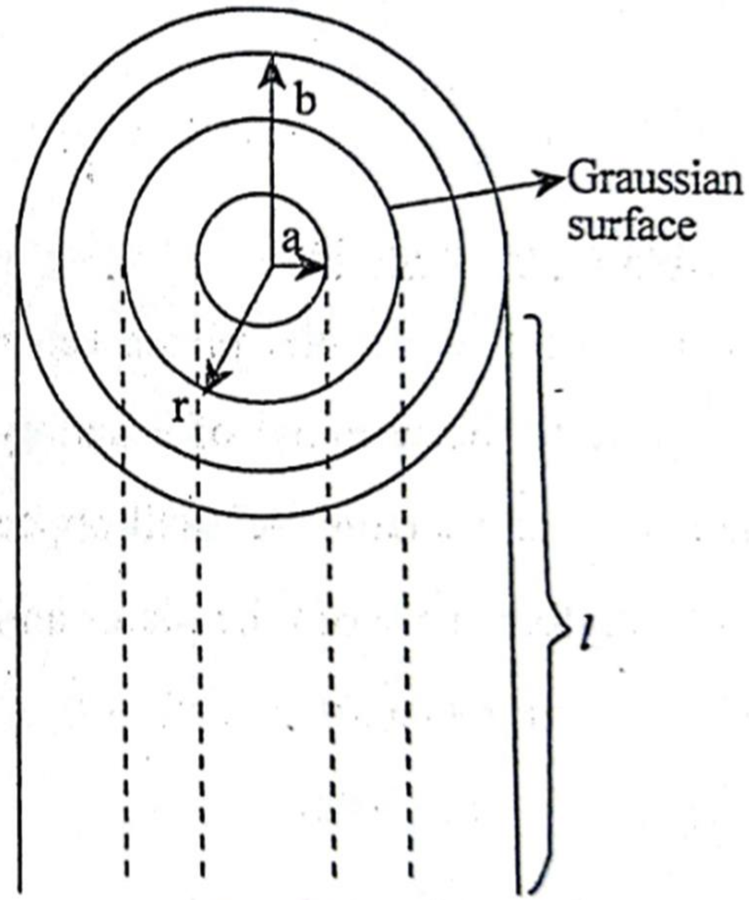


Fig:20

Potential difference bet^n spherical

$$V = \int dv = \int_a^b E.dr$$

$$= \int_a^b \frac{q}{2\pi \epsilon_0 .rl} dr$$

$$= \frac{q}{2\pi \epsilon_0 l} \int_a^b \frac{dr}{r}$$

$$= \frac{q}{2\pi \epsilon_0 .l} [\ln r]_a^b$$

$$= \frac{q}{2\pi \epsilon_0 .l} \ln \frac{b}{a}$$

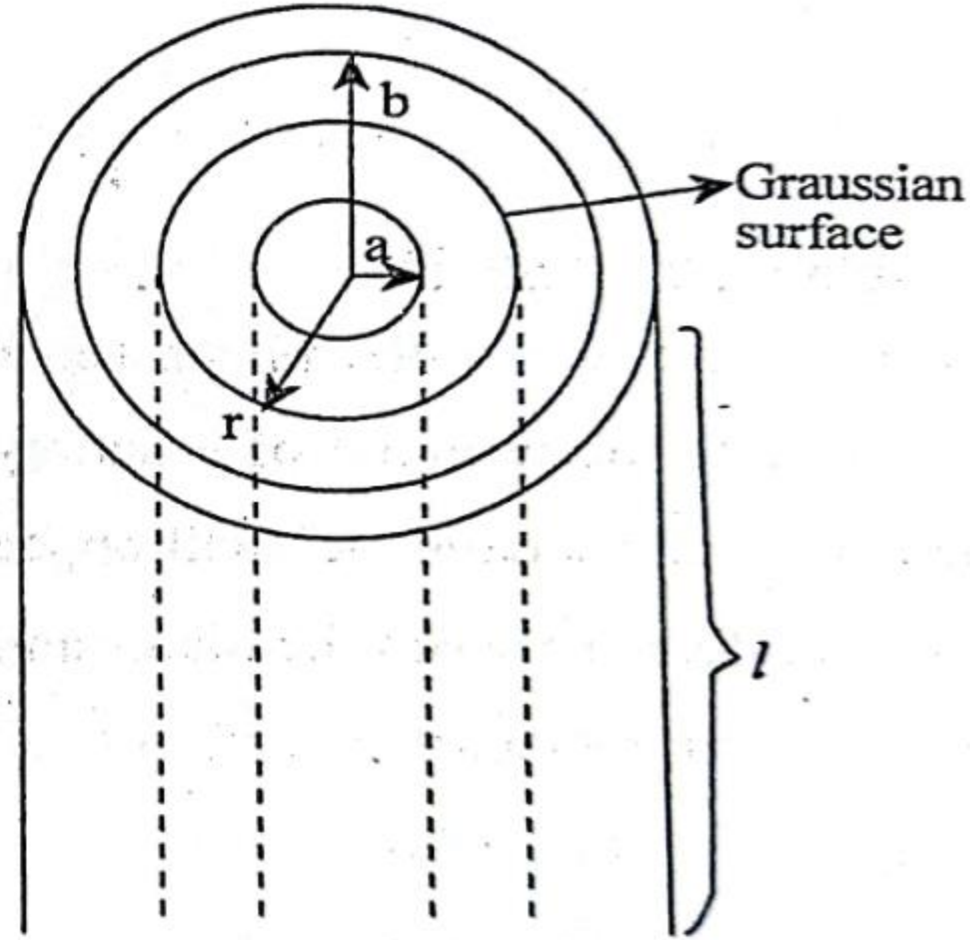


Fig:20

Now, capacitance $C = \frac{q}{v}$ $\Rightarrow C = \frac{q}{\frac{q}{2\pi \epsilon_0 .l} \ln \frac{b}{a}}$ $\therefore C = 2\pi \epsilon_0 . \frac{l}{\ln\left(\frac{b}{a}\right)}$

Q: Energy storage in an electric field of a charge conductor.

Let, C = capacitance of the Capacitor

q = the charge of the Capacitor

V= the raise of potential

Let the work done for charging, the conductor is U.

Therefore, the energy stored in the capacitor is U

Now if we give a small amount of charge dQ, so the energy increased by dU. Then we can write

$$dU = vdQ$$

$$\Rightarrow dU = \frac{Q}{c} dQ \text{..........................} (i)$$

$C = \frac{Q}{V}$

$\Rightarrow V = \frac{Q}{C}$

Now when Q = 0 then U=0 and after charging when Q = Q then U = U

Now from-...... (i)

$$\int_0^U dU = \int_0^Q \frac{Q}{c} dQ$$

$$\Rightarrow [U]_0^u = \frac{1}{c}\left[\frac{Q^2}{2}\right]_0^Q \qquad \Rightarrow U = \frac{1}{2C} \cdot Q^2 = \frac{1}{2} CV^2 \qquad \therefore U = \frac{1}{2} CV^2$$

Charging of the capacitor:

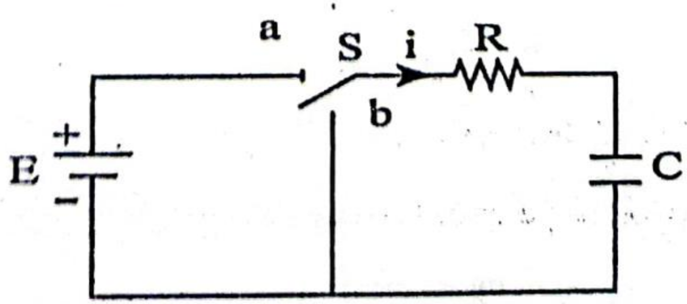
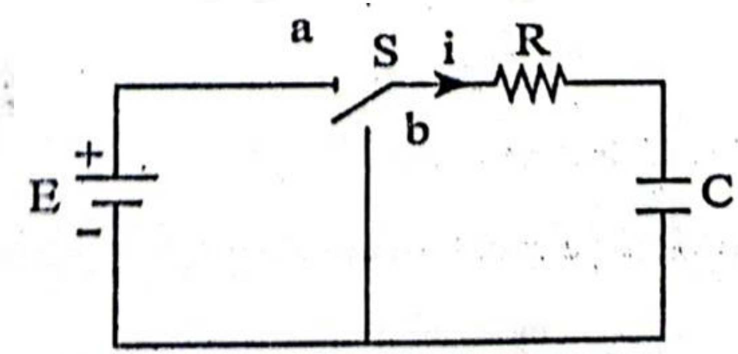


Fig- 31

Let us, consider a circuit contains a resistance 'R' a capacitor 'C' and an ideal battery of e.m.f E. All are connected in series as shown in fig.

When switch 'S' pressed to 'a' charge stored in the capacitor begin to increase, and let, at any time constant t, q is the of charge stored in the capacitor and i is the current in the circuit. We get potential difference across the capacitor, $V = \frac{q}{c}$, current in the circuit $i = \frac{dq}{dt}$.

Applying kirchoff's loop rule to the circuit in clock wise direction, we get.



$$E = iR + \frac{q}{c}$$

$$\Rightarrow E = \frac{dq}{dt}.R + \frac{q}{c}$$

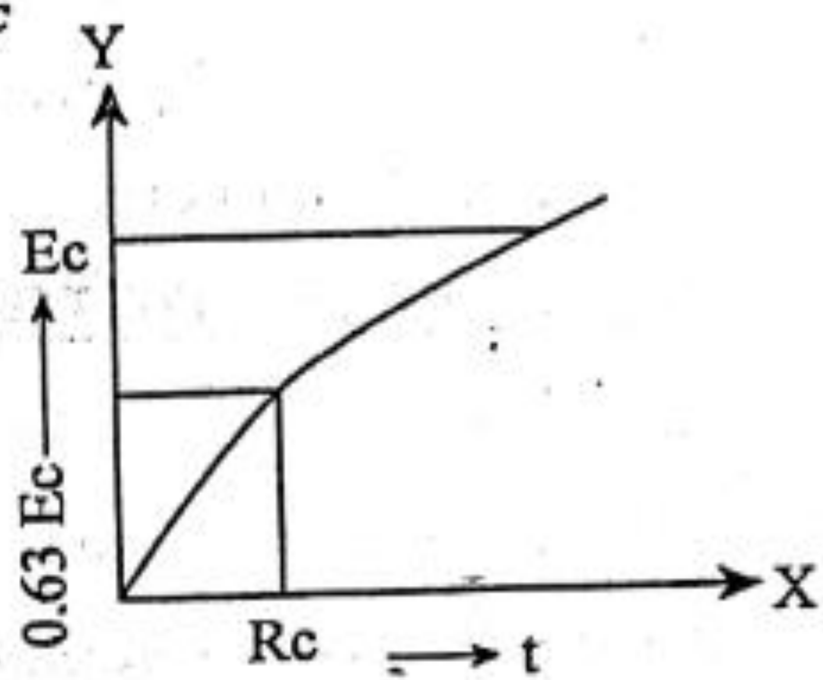
$$\Rightarrow \frac{dq}{dt} = \frac{E}{R} - \frac{q}{Rc} \Rightarrow \frac{dq}{dt} = \frac{EC - q}{Rc}$$

$$\Rightarrow \frac{-dq}{Ec - q} = \frac{-dt}{Rc}$$

$$\Rightarrow \ln(Ec - q) = -\frac{t}{Rc} + \ln A$$

[Integrating both side]

$$\Rightarrow \ln(Ec - q) - \ln A = \frac{-t}{Rc}$$



(A is integrating constant)

$$\Rightarrow \ln \frac{Ec - q}{A} = \frac{-t}{Rc}$$

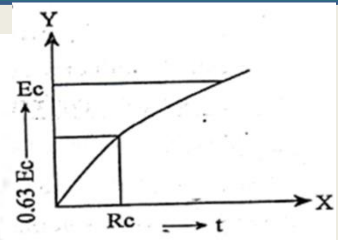
$$\Rightarrow \frac{Ec - q}{A} = e^{-t/Rc} \quad \text{.................}(1)$$

When t =o, q = 0 then A = EC.

Now from (1) $$\frac{Ec - q}{Ec} = e^{-t/Rc}$$

$$\Rightarrow 1 - \frac{q}{Ec} = e^{-t/Rc}$$

$$\Rightarrow \frac{q}{Ec} = 1 - e^{-t/Rc}$$



$$\Rightarrow q = Ec\left(1 - e^{-t/Rc}\right)$$

This is the equ^n for charging a capacitor. The fig-1 shows that charg grows exponentially with time.

Now current in the circuit,

$$i = \frac{dq}{dt}$$

$$\Rightarrow i = \frac{d}{dt}\left\{Ec\left(1 - e^{-t/Rc}\right)\right\}$$

$$\Rightarrow i = Ec.\frac{e^{-t/Rc}}{Rc}$$

$$\Rightarrow i = \frac{E}{R}.e^{-t/Rc}.$$

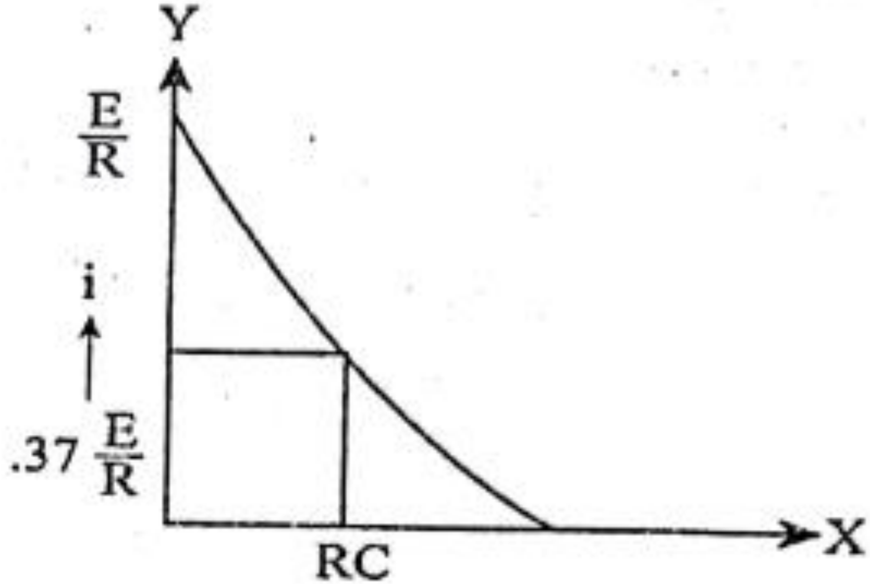


Fig-33

This is the equ^n for current in the R-C circuit. The current decreases exponentially with time as shown in graph.

Electricity

*H ~~Show that, capacitance of~~ for two capacitors ~~are~~ of equal capacitance, prove that the equivalent capacitance of the capacitors when in parallel is four times greater than when in series connections.

Jct

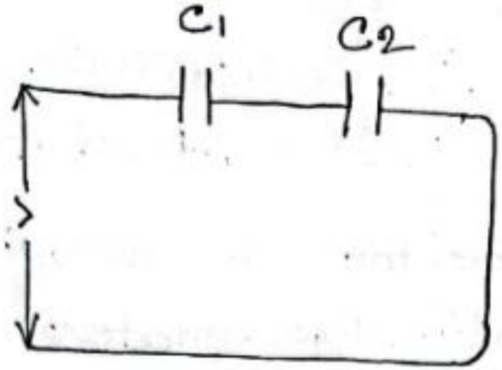


Fig1. Capacitors in series

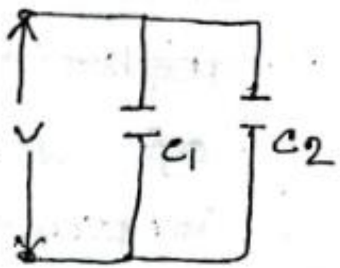


Fig2 capacitor in parallel

We consider two capacitor with equal capacitance c_1 and c_2 connected in Series (in fig:1) and paralled connections in Fig 2.

Electricity

We can write the equivalent capacitance for series combination is

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} \longleftarrow (1)$$

When two capacitance is equal, i.e. $C_1 = C_2$ then we can write from eqn (1)

$$\frac{1}{C_3} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow \frac{1}{C_3} = \frac{1+1}{C_1}$$

$$\Rightarrow C_1 = 2C_3 \longleftarrow (ii)$$

Again we can write the equivalent capacitance for parallel combination

$$C_p = C_1 + C_2 \quad \text{—— (iii)}$$

When two capacitance is equal. i.e. $C_1 = C_2$ then can write eqn (iii)

$$C_p = C_1 + C_1 = 2C_1$$

$$\therefore C_p = 2C_1$$

$$= 2 \times 2C_s \quad [\text{From eq}^n \text{ (2)})$$

$$= 4\ C_s.$$

So, we can write capacitance of two capacitors are equal to the equivalent capacitance of the capacitors when in parallel is four times greater than when in series connection.

A negative point charge of 10^{-6} coul is situated in air at the origin of a rectangular coordinate system. A second negative point charge of 10^{-4} coul is situated on the positive x-axis at a distance of 50 cm from the origin. What is the force on the second charge?

Soln. : from columb force are have,

$$\vec{F} = \frac{1}{4\pi 6o}\frac{z_1 q_2}{r^2}\hat{\gamma}$$

$$= \frac{9\times10^9\times(-10^{-6})\times(-10^{-4})}{(0.5)^2}$$

$$= 3.6\,\hat{\imath}N$$

$$\frac{1}{4\pi\varepsilon o} = 9\times10^9 NM^2 coul$$

$$z_1 = -10^{-6}C$$

$$z_2 = -10^{-4}Col$$

$$\pi = 50cm = 0.5m$$

This there is a force of 3.6 Newton in the positive x-direction on the second charge.

Calculate the force of repulsion between two protons in a nucleus of iron, assuming a separation of $4\times10^{-15}m$

Soln. From columb law we have,

$$F = \frac{1}{4\pi\varepsilon o}\frac{q_1 q_2}{r^2}$$

$$= 9\times10^9\times\frac{(1.6\times10^{-19})^2}{(4\times10^{15})^2}$$

$$= 14\text{ N}$$

$$q_1 = q_2 = q = 1.6\times10^{-19}C$$

$$\text{r} = 4\times10^{-15}\text{m}$$

Electricity

Prob: What is the magnitude of the electric field strength $\vec{E}$ such that an electron, placed in the field, would experience an electrical force equal to its weight?

Soln. : We have,

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{mg}{e}$$

$$= \frac{9.1\times10^{-31}\times9.8}{1.6\times10^{-19}}$$

$$= 5.6\times10^{-11} N/C$$

$m = 9.1\times10^{-31} kg$

$g = 9.8\ ms^{-2}$

$e = 1.6\times10^{-19} c$

Thank you
for your kind attention

Electricity

Electricity

Q: Dipole in an electric field or Derive expression for torque and potential energy when an electric dipole is placed in a uniform external electric

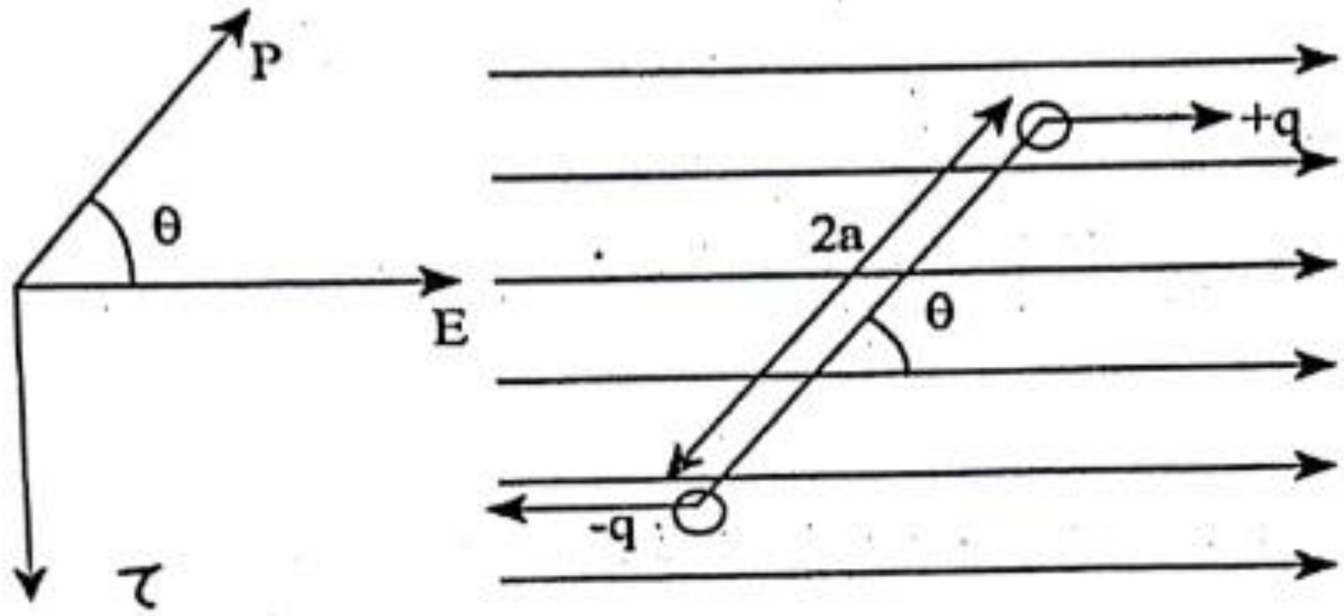


Fig- 5

Electricity

Figure shows an electric dipole formed by placing two charges +q and –q a fixed distance 2a apart. The arrangement is placed in a uniform external electric field E, its dipole moment P making an angle θ with this field. The two equal and opposite forces F and – F making a couple and the moment of the couple or torque is given by-

$$\tau = \text{Force} \times \text{perpendicular distance}$$
$$= F \times 2a \sin\theta$$
$$= qE \times 2a \sin\theta$$
$$= 2qa \times E\sin\theta$$
$$= P\,E\sin\theta$$

Work must be done by an external agent to charge the orientation of an electric dipole. This work is stored as potential energy in the system. We choose 90° as the initial orientation.

$$U = W = \int dw$$

Electricity

$$= \int_{90}^{\theta} \tau . d\theta = \int PE \sin\theta . d\theta$$

$$= PE[-\cos\theta]_{90}^{\theta}$$

$$= PE(-Cos\theta - Cos90)$$

$$= -PE\ Cos\theta.$$

In vector form $\vec{U} = -\vec{P}.\vec{E}$