PHY-1111: Physics

Chapter-2 (Vibrations and Waves)

by

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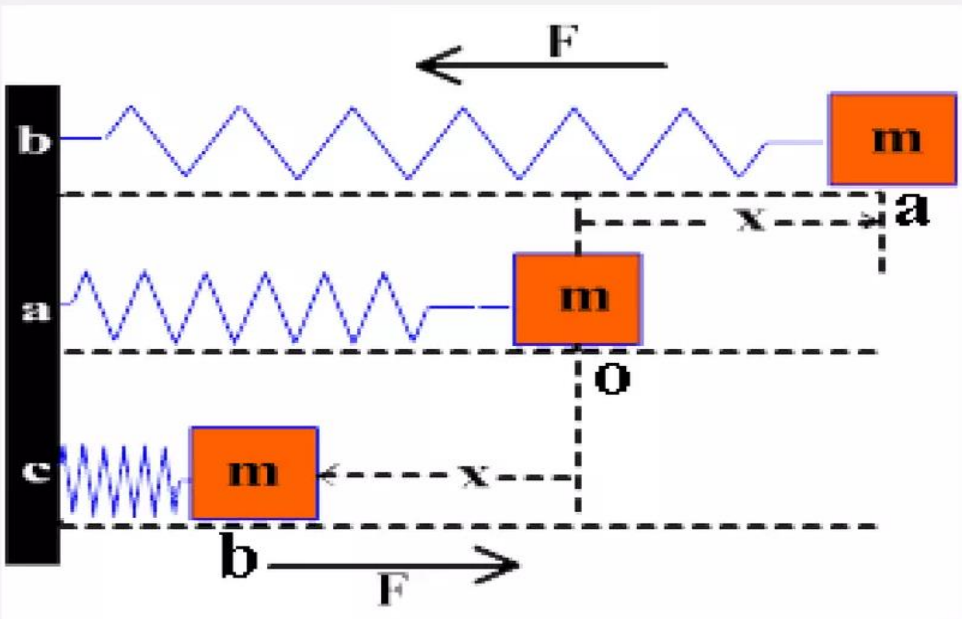
Simple Harmonic Motion (SHM):

Definition:

- Such a motion in which acceleration is directly proportional to the displacement and is directed towards the mean position is called simple harmonic motion(SHM).



Condition FOR SHM:

- The system should haves restoring force.
- The system should have inertia.
- The system should be frictionless.

Hooke's Law

- force that is applied to spring is **directly proportional** to the displacement.
- If the spring is un stretched, there is no net force on the mass or the system is in equilibrium.
- if the mass is displaced from equilibrium, the spring will exert a restoring force, which is a force that tends to restore it to the equilibrium position.

$$F \propto x$$

where,
F → Elastic force
k → Spring constant
x → Displacement

$$\boxed{F = kx}$$

An imaginary circular motion gives a mathematical insight into SHM. Its angular velocity is ω.

The connection between SHM and circular motion'

The time period of the motion, $$T = \frac{2\pi}{\omega}.$$

The frequency of the motion, $$f = \frac{1}{T} = \frac{\omega}{2\pi}.$$

Displacement of the SHM, $$s = A\cos(\omega t).$$

1.3. Differential Equation of SHM

For a particle vibrating simple harmonically, the general equation of displacement is,

$$y = a \sin(\omega t + \alpha) \quad \ldots (1)$$

Here y is displacement and a is the amplitude and α is epoch of the vibrating particle.

Differentiating equation (1) with respect to time

$$\frac{dy}{dt} = a\,\omega \cos(\omega t + \alpha) \quad \ldots (2)$$

Here dy/dt represents the velocity of the vibrating particle.

Differentiating equation (2) with respect to time

$$\frac{d^2 y}{dt^2} = -a\,\omega^2 \sin(\omega t + \alpha)$$

But $\quad a \sin(\omega t + \alpha) = y$

$$\therefore \quad \frac{d^2 y}{dt^2} = -\omega^2 y$$

or

$$\frac{d^2 y}{dt^2} + \omega^2 y = 0 \quad \ldots (3)$$

Here $d^2 y/dt^2$ represents the acceleration of the particle. Equation (3) represents the differential equation of simple harmonic motion.

It also shows that in any phenomenon where an equation similar to equation (3) is obtained, the body executes simple harmonic motion. The general solution of equation (3) is

$$y = a \sin(\omega t + \alpha).$$

Also the time period of a vibrating particle can be calculated from equation (3).

Numerically $$\omega = \sqrt{\frac{d^2y/dt^2}{y}}$$

or $$\omega = \sqrt{\frac{\text{Acceleration}}{\text{Displacement}}}$$

or $$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

1.5. Average Kinetic Energy of a Vibrating Particle

The displacement of a vibrating particle is given by

$$y = a \sin(\omega t + \alpha)$$

$$v = \frac{dy}{dt} = a\omega \cos(\omega t + \alpha).$$

If m is the mass of the vibrating particle, the kinetic energy at any instant

$$= \frac{1}{2} m v^2 = \frac{1}{2} m \, . \, a^2 \omega^2 \cos^2(\omega t + \alpha) \, .$$

The average kinetic energy of the particle in one complete vibration

$$= \frac{1}{T} \int_0^T \frac{1}{2} m a^2 \omega^2 \cos^2(\omega t + \alpha) \, dt$$

$$= \frac{1}{T} \cdot \frac{m a^2 \omega^2}{4} \int_0^T 2 \cos^2(\omega t + \alpha) \, dt$$

$$= \frac{m a^2 \omega^2}{4T} \int_0^T [1 + \cos 2(\omega t + \alpha)] \, dt$$

$$= \frac{m a^2 \omega^2}{4T} \left[\int_0^T dt + \int_0^T \cos 2(\omega t + \alpha) \, dt \right]$$

But $$\int_0^T \cos 2(\omega t + \alpha)\, dt = 0$$

$$\therefore \quad \text{Average K.E.} = \frac{ma^2\omega^2}{4T} \cdot T + 0$$

$$= \frac{ma^2\omega^2}{4} = \frac{ma^2(4\pi^2 n^2)}{4}$$

$$= \pi^2 m a^2 n^2$$

where m is the mass of the vibrating particle, a is the amplitude of vibration and n is the frequency of vibration. Also, the average kinetic energy of a vibrating particle is directly proportional to the square of the amplitude.

Total Energy of a Vibrating Particle

$$y = a \sin(\omega t + \alpha)$$

$$\sin(\omega t + \alpha) = \frac{y}{a}$$

$$\cos(\omega t + \alpha) = \sqrt{1 - \frac{y^2}{a^2}} = \sqrt{\frac{a^2 - y^2}{a^2}}$$

$$= \frac{\sqrt{a^2 - y^2}}{a}$$

Velocity $\quad v = a\,\omega \cos \omega t = \dfrac{a\omega\sqrt{a^2 - y^2}}{a}$

$$= \omega\sqrt{(a^2 - y^2)}$$

$\therefore$ The kinetic energy of the particle at the instant the displacement is y,

$$= \frac{1}{2} m v^2$$

$$= \frac{1}{2} m \,.\, \omega^2 (a^2 - y^2)$$

Potential energy of the vibrating particle is the amount of work done in overcoming the force through a distance y.

Acceleration $\quad = -\omega^2 y$

Force $\quad = -m\omega^2 y$

(The –ve sign shows that the direction of the acceleration and force are opposite to the direction of motion of the virbrating particle.)

$$\therefore \qquad \text{P.E.} = \int_0^y m \cdot \omega^2 y \cdot dy$$

$$= m\omega^2 \cdot \frac{y^2}{2} = \frac{1}{2} m\omega^2 y^2 .$$

Total energy of the particle at the instant the displacement is y

$$= \text{K.E} + \text{P.E.}$$

$$= \frac{1}{2} m\omega^2 (a^2 - y^2) + \frac{1}{2} m\omega^2 y^2$$

$$= \frac{1}{2} m\omega^2 \cdot a^2$$

$$= \frac{1}{2} m (2\pi n)^2 a^2$$

$$= \mathbf{2\pi^2 m a^2 n^2.}$$

As the average kinetic energy of the vibrating particle $= \pi^2 ma^2n^2$, the average potential energy $= \pi^2 ma^2n^2$. The total energy at any instant is a constant.

Other symbols, relationships

Displacement, $s = A\sin(\omega t) = A\sin(2\pi f t)$

At t =0, the object passes through its equilibrium position.

$$v = \frac{d}{dt}[A\sin(\omega t)] = \omega A\cos(\omega t)$$

$$a = \frac{d}{dt}[-\omega A\cos(\omega t)] = -\omega^2 A\sin(\omega t)$$

Phase relationship, ϕ $\quad s = A\sin(\omega t + \phi)$

Example 1.1. *For a particle vibrating simple harmonically, the displacement is 12* cm *at the instant the velocity is 5* cm/s *and the displacement is 5* cm *at the instant the velocity is 12* cm/s. *Calculate (i) amplitude, (ii) frequency and (iii) time period.*

The velocity of a particle executing SHM,

$$v = \frac{dy}{dt} = \omega\sqrt{a^2 - y^2}$$

In the first case,

$$v_1 = \omega\sqrt{a^2 - y_1{}^2}$$

Here $v_1 = 5$ cm/s, $y_1 = 12$ cm.

$$5 = \omega\sqrt{a^2 - 144} \qquad \ldots(1)$$

In the second case

$$v_2 = \omega\sqrt{a^2 - y_2^2}$$

Here $v_2 = 12$ cm/s, $y_2 = 5$ cm

$$12 = \omega\sqrt{a^2 - 25} \qquad \ldots(2)$$

Dividing (2) by (1) and squaring

$$\frac{144}{25} = \frac{a^2 - 25}{a^2 - 144}$$

$$a = 13 \text{ cm}$$

The amplitude is **13 cm.**

Substituting the value of $a = 13$ cm in equation (1)

$$5 = \omega \sqrt{(13)^2 - 144}$$

or $$\omega = 1 \text{ radian/s}$$

The frequency $n = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}$ **hertz**

Time period $T = \dfrac{1}{n} = 2\pi$ **seconds** .

3.1. Free Vibrations

When the bob of a simple pendulum (in vacuum) is displaced from its mean position and left, it executes simple harmonic motion. The time period of oscillation depends only on the length of the pendulum and the acceleration due to gravity at the place. The pendulum will continue to oscillate with the same time period and amplitude for any length of time. In such cases there is no loss of energy by friction or otherwise. In all similar cases, the vibrations will be undamped free vibrations. The amplitude of swing remains constant.

3.2. Undamped Vibrations

For a simple harmonically vibrating particle, the kinetic energy for displacement y, is given by

$$\frac{1}{2} m \left(\frac{dy}{dt}\right)^2$$

At the same instant, the potential energy of the particle is $\frac{1}{2} Ky^2$ where K is the restoring force per unit displacement.

The total energy at any instant,

$$= \frac{1}{2} m \left(\frac{dy}{dt}\right)^2 + \frac{1}{2} Ky^2$$

For an undamped harmonic oscillator, this total energy remains constant.

$$\therefore \quad = \frac{1}{2} m \left(\frac{dy}{dt}\right)^2 + \frac{1}{2} Ky^2 = \text{constant} \quad \ldots (1)$$

Differentiating equation (1) with respect to time,

$$m\frac{d^2y}{dt^2}+Ky=0 \qquad \ldots (2)$$

$$\frac{d^2y}{dt^2}+\left(\frac{K}{m}\right)y=0 \qquad \ldots (3)$$

Equation (3) is similar to the equation

$$\frac{d^2y}{dt^2}+\omega^2 y=0 \qquad \ldots (4)$$

Here $\omega^2=\left(\frac{K}{m}\right)$

The solution for equation (4) is

$$y=a\sin(\omega t-\alpha)$$

$$\therefore \qquad y=a\sin\left[\sqrt{\frac{K}{m}}\,t-\alpha\right]$$

The frequency of oscillation is

$$n=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

Thus, in the case of undamped free vibrations, the differential equation is

$$\frac{d^2y}{dt^2}+\left(\frac{K}{m}\right)y=0 \qquad \ldots (5)$$

This is only an ideal case. In the first chapter, for the motion of a pendulum, loaded spring, LC circuit etc., it has been assumed that the vibrations are free and undamped.

3.3. Damped Vibrations

In actual practice, when the pendulum vibrates in air medium, there are frictional forces and consequently energy is dissipated in each vibration. The amplitude of swing decreases continuously with time and finally the oscillations die out. Such vibrations are called **free damped** vibrations. The dissipated energy appears as heat either within the system itself or in the surrounding medium.The dissipative force due to friction etc. (resistance in *LCR* circuit) is proportional to the velocity of the particle at that instant. Let $\mu \frac{dy}{dt}$ be the dissipative force due to friction etc.

This term is to be introduced in equation (2).

Therefore, the differential equation in the case of Free-damped vibrations is,

$$m \frac{d^2 y}{dt^2} + Ky + \mu \frac{dy}{dt} = 0 \qquad \ldots (6)$$

or
$$\frac{d^2y}{dt^2} + \left(\frac{\mu}{m}\right)\frac{dy}{dt} + \left(\frac{K}{m}\right)y = 0 \qquad \ldots (7)$$

This equation is similar to a general differential equation,

$$\frac{d^2y}{d^2} + 2b\frac{dy}{dt} + k^2 y = 0 \qquad \ldots (8)$$

The solution of this equation is

$$y = ae^{-bt} \sin (\omega t - \alpha) \qquad \ldots (9)$$

The general solution of equation (7) is also given by

$$y = A\ e^{(-b\ +\sqrt{b^2 + k^2})\ t} + B\ e^{(-b - \sqrt{a^2 - k^2})\ t}$$

Here
$$b = \frac{\mu}{2m} \text{ and } k^2 = \frac{K}{m}$$

and
$$\omega = \sqrt{k^2 - b^2}$$

or
$$\omega = \sqrt{\frac{K}{m} - \frac{\mu^2}{4m^2}}$$

or
$$n = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{k^2 - b^2}$$

3.5. Forced Vibrations

The time period of a body executing simple harmonic motion depends on the dimensions of the body and its elastic properties. The vibrations of such a body die out with time due to dissipation of energy. If some external periodic force is constantly applied on the body, it continues to oscillate under the influence of such external forces. Such vibrations of the body are called *forced vibrations*.

Initially, the amplitude of the swing increases, then decreases with time, becomes minimum and again increases. This will be repeated if the external periodic force is constantly applied on the system. In such cases the body will finally be forced to vibrate with the same frequency as that of the applied force. The frequency of the forced vibration is different from the natural frequency of vibration of the body. The amplitude of the forced vibration of the body depends on the difference between the natural frequency and the frequency of

the applied force. The amplitude will be large if difference in frequencies is small and *vice versa*.

Vibrations and Waves

4.1. Wave Motion

Wave motion is a form of disturbance which travels through the medium due to the repeated periodic motion of the particles of the medium about their mean positions, the disturbance being handed over from one particle to the next. When a stone is dropped into a pond containing water, waves are produced at the point where the stone strikes the water in the pond. The waves travel outward, the particles of water vibrate only up and down about their mean positions. Water particles do not travel along with the wave. Similarly when a tuning fork is set into vibration, it produces waves in air. The wave travels from one particle to the next but the particles of air vibrate about their mean positions.

It is essential to understand the concept of wave motion in the study of various branches in Physics. Wave motion, in general, refers to the transfer of energy from one point to another point of the medium. Transference of various forms of energy like sound, heat, light, X-rays, γ-rays, radio-waves etc. takes place in the form of wave motion. For the transference of energy through a medium, the medium must possess the properties of *elasticity, inertia and negligible frictional resistance.*

4.3. Characteristics of Wave Motion

1. Wave motion is a disturbance produced in the medium by the repeated periodic motion of the particles of the medium.

2. Only the wave travels forward whereas the particles of the medium vibrate about their mean positions.

3. There is a regular phase change between the various particles of the medium. The particle ahead starts vibrating a little later than a particle just preceding it.

4. The velocity of the wave is different from the velocity with which the particles of the medium are vibrating about their mean positions. The wave travels with a uniform velocity whereas the velocity of the particles is different at different positions. It is maximum at the mean position and zero at the extreme position of the particles.

There are two types of wave motions :

(*i*) Transverse and (*ii*) Longitudinal.

Sound waves are longitudinal waves and light waves are transverse waves.

4.4. Transverse Wave Motion

In this type of wave motion, the particles of the medium vibrate at right angles to the direction of propagation of the wave.

To understand the propagation of transverse waves in a medium consider nine particles of the medium and the circle of reference (Fig. 4.1). The particles are vibrating about their mean positions up and down and the wave is travelling from left to right. The disturbance takes $T/8$ seconds to travel from one particle to the next.

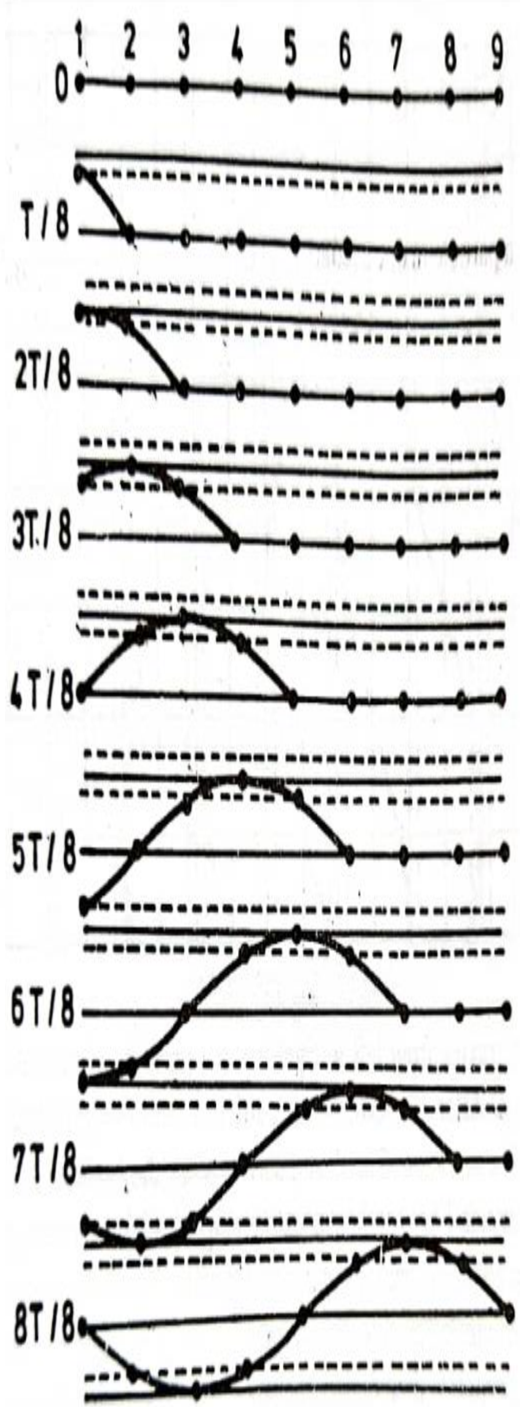
(1) At $t = 0$, all the particles are at their mean position.

(2) After $T/8$ seconds, particle 1 travels a certain distance upward and the disturbance reaches particle 2.

(3) After $2T/8$ seconds, particle 1 has reached its extreme position and the disturbance has reached particle 3.

(4) After $3T/8$ seconds, particle 1 has completed 3/8 of its vibration and the disturbance has reached particle 4. The positions of particles 2 and 3 are also shown in Fig. 4.1.

(5) In this way after $T/2$ seconds, particle 1 has come back to its mean position and the particles 2, 3 and 4 are at the positions shown in the diagram.



The disturbance has reached particle 5.

In this way the process continues and the positions of the particles after *5T/8, 6T/8, 7T/8* and *T* seconds are shown in the diagram.

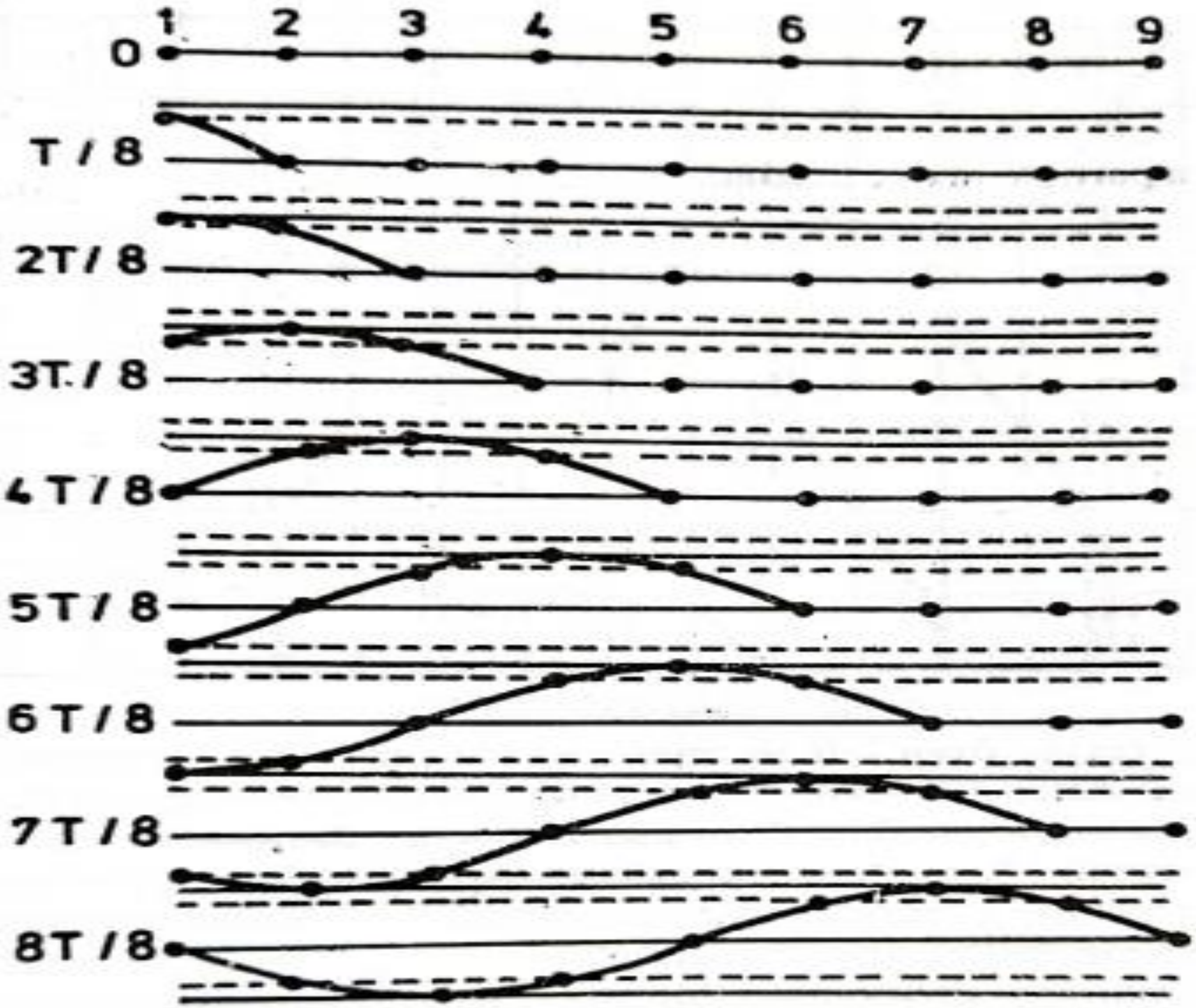


Fig. 4.1.

Vibrations and Waves

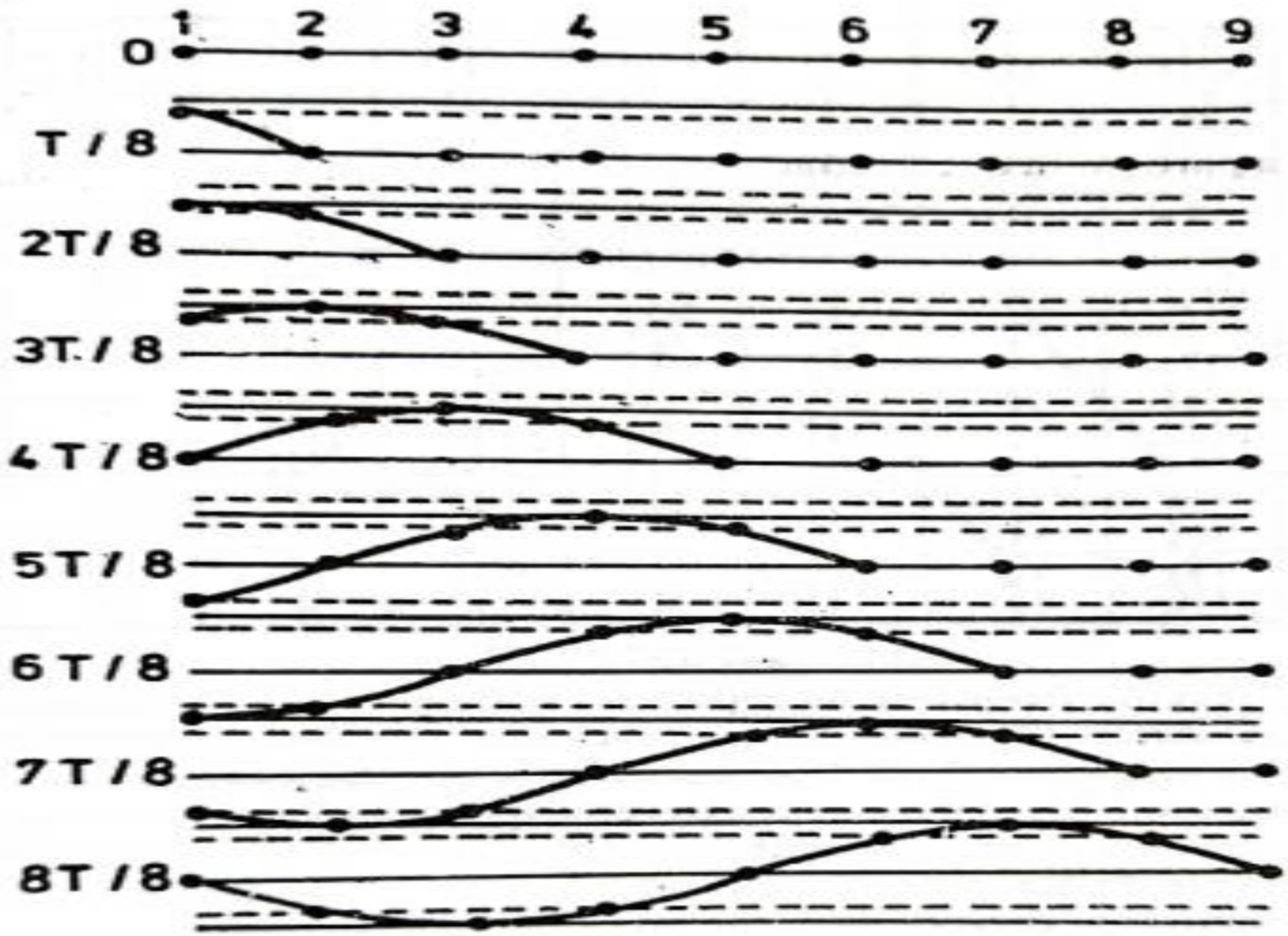


Fig. 4.1.

After T seconds, the particles 1, 5 and 9 are at their mean positions. The wave has reached particle 9. Particles 1 and 9 are in the same phase. The wave has travelled a distance between particles 1 and 9 in the time in which the particle 1 has completed one vibration.

The top point on the wave at the maximum distance from the mean position is called *crest*, while the point at the maximum distance below the mean position is called trough. Thus in a transverse wave, crests and troughs are alternately formed. The contour of the displaced particles of the medium represents the wave. In the case of transverse (or longitudinal) progressive waves, this contour continuously changes position in space and the wave seems to advance in the direction of propagation.

4.5. Longitudinal Wave Motion

In this type of wave motion, particles of the medium vibrate along the direction of propagation of the wave.

Consider nine particles of the medium and the circle of reference (Fig. 4.2).

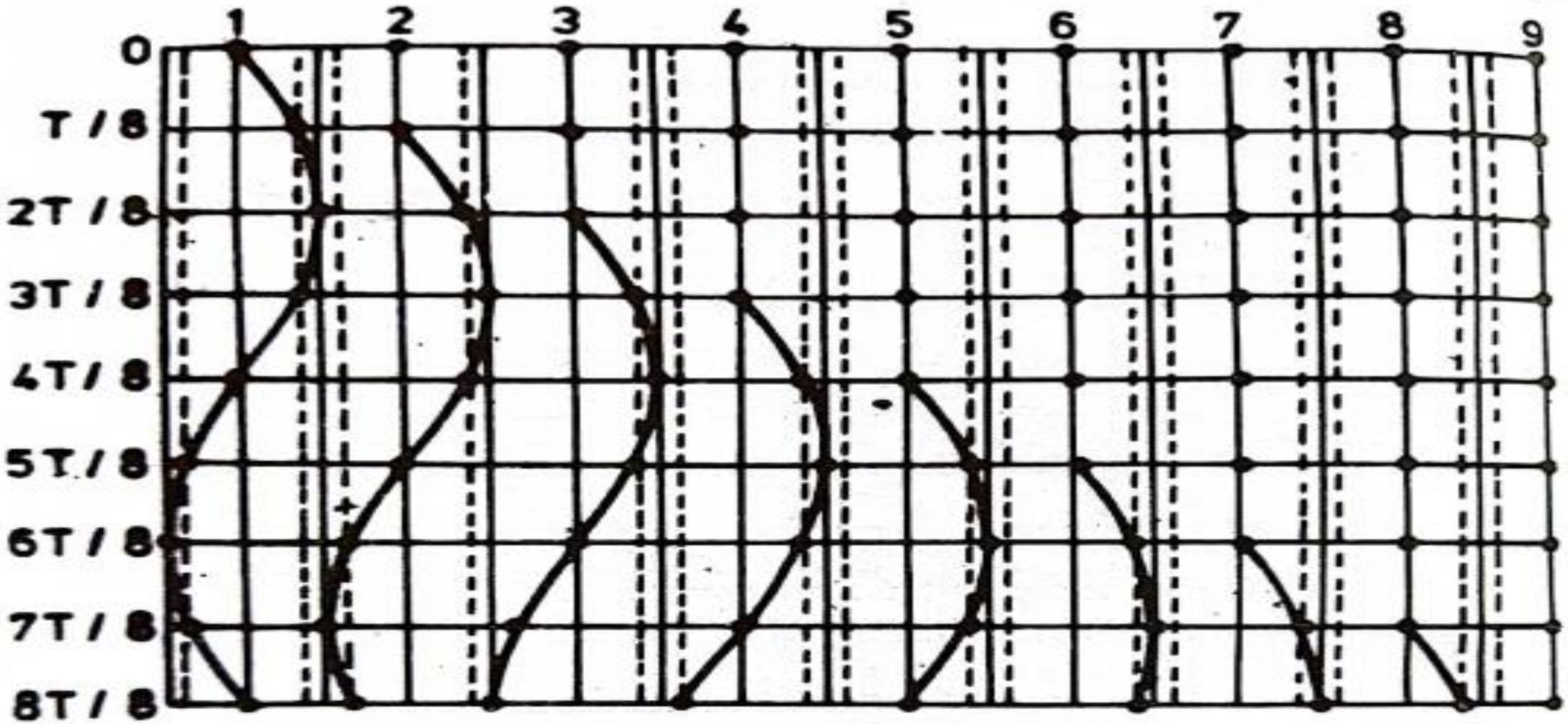
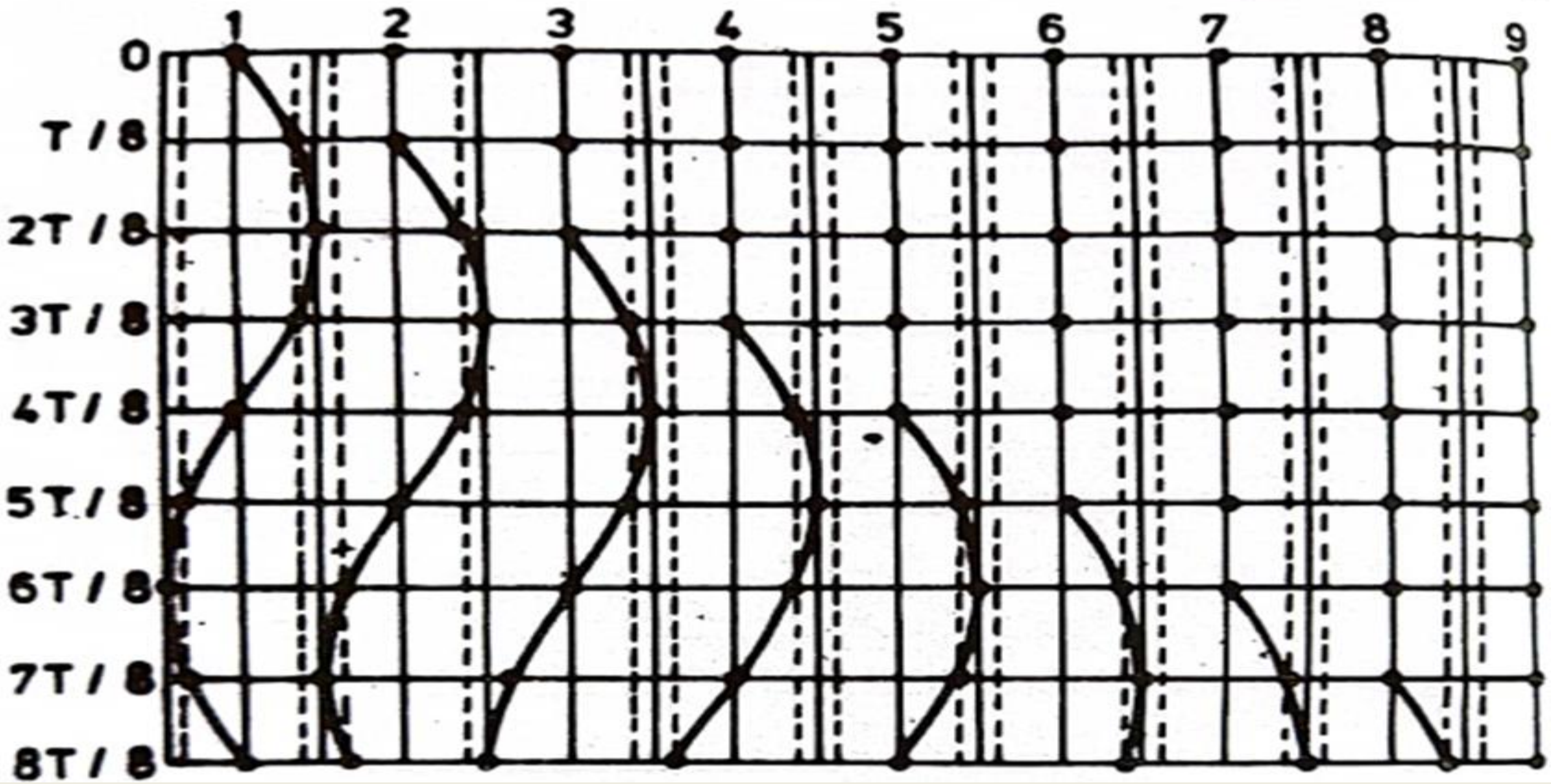


Fig. 4.2.

The wave travels from left to right and the particles vibrate about their mean positions. After *T*/8 seconds, the particle 1 goes to the right and completes 1/8 of its vibration. The disturbance reaches the particle 2. After *T*/4 seconds the particle 1 has reached its extreme right position and completes 1/4 of its vibration and the particle 2 completes 1/8 of its vibration. The disturbance reaches the particle 3. The process continues.

Vibrations and Waves



After one complete time period, the positions of the various particles is as shown in the diagram. The wave has reached particle 9. Here 1 and 9 are again in the same phase. Here particles 1, 5 and 9 are at their mean positions. The particles 1 and 3 are close to the particle 2. This is the position of condensation. Similarly particles 9 and 8 are close to the particle 7. This is also the position of condensation or compression. On the other hand, particles 4 and 6 are far away from the particle 5. This is the position of rarefaction. Hence in a longitudinal wave motion, condensations and rarefactions are alternately formed.

4.6. Definitions

Wavelength. It is the distance travelled by the wave in the time in which the particle of the medium completes one vibration. It is also defined as the distance between two nearest particles in the same phase.

The distance AB (Fig. 4.3) is equal to the wavelength λ.

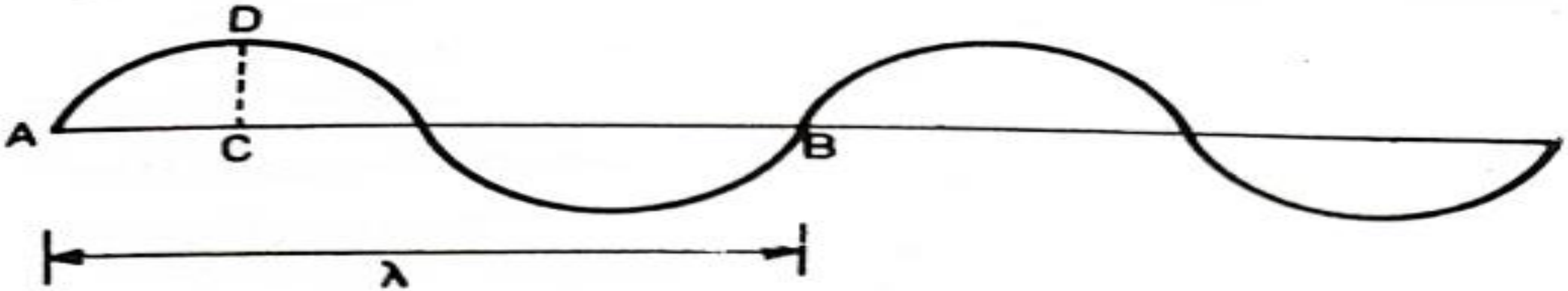


Fig. 4.3.

Frequency. It is the number of vibrations made by a particle in one second.

Amplitude. It is the maximum displacement of the particle from its mean position of rest. In the diagram CD is the amplitude.

Time period. It is the time taken by a particle to complete one vibration.

Suppose frequency $= n$

Time taken to complete n vibrations $= 1$ second.

Time taken to complete 1 vibration $= \frac{1}{n}$ second.

From the definition of time period, time taken to complete one vibration is the time period (T)

$$\therefore \quad T = \frac{1}{n} \quad \text{or} \quad nT = 1$$

$$\therefore \quad \text{Frequency} \times \text{Time period} = 1$$

Vibrations and Waves

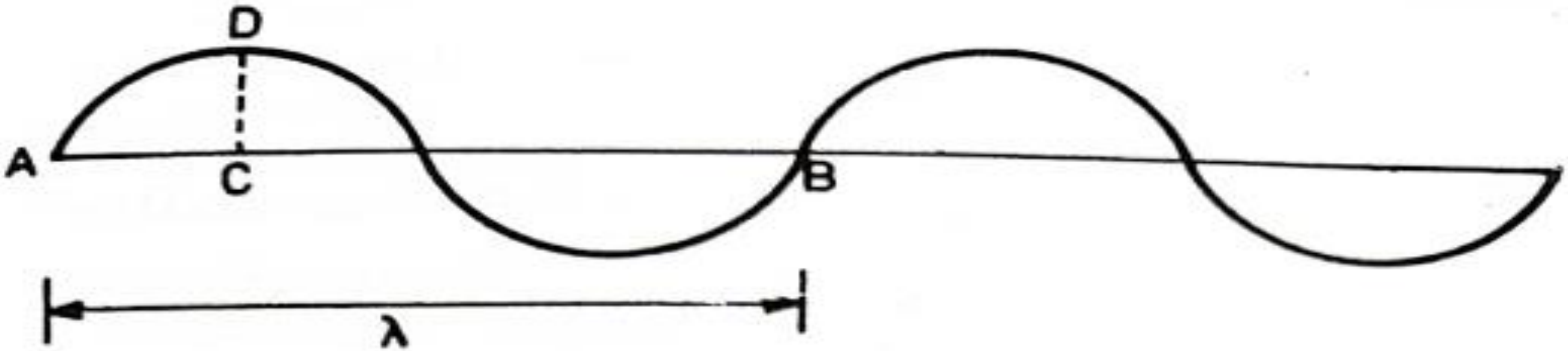


Fig. 4.3.

Vibration. It is the to and fro motion of a particle from one extreme position to the other and back again. It is also equal to the motion of a particle from the mean position to one extreme position, then to the other extreme position and finally back to the mean position.

Phase. It is defined as the ratio of the displacement of the vibrating particle at any instant to the amplitude of the vibrating particle or it is defined as the fraction of the time interval that has elapsed since the particle crossed the mean position of rest in the positive direction or it is also equal to the angle swept by the radius vector since the vibrating particle last crossed its mean position of rest.

4.7. Relation between Frequency and Wavelength

Velocity of the wave is the distance travelled by the wave in one second.

Velocity = Distance

4.7. Relation between Frequency and Wavelength

Velocity of the wave is the distance travelled by the wave in one second.

$$\text{Velocity} = \frac{\text{Distance}}{\text{Time}}$$

Wavelength (λ) is the distance travelled by the wave in one time period *(T)*.

$$\text{Velocity} = \frac{\text{Wavelength}}{\text{Time period}} = \frac{\lambda}{T}$$

But, frequency × time period = 1

$$n \times T = 1$$

$$T = \frac{1}{n}$$

$$v = \frac{\lambda}{T} = \frac{\lambda}{\frac{1}{n}}$$

$$v = n\lambda$$

Example 4.1. *If the frequency of a tuning fork is 400 and the velocity of sound in air is 320 metres/s, find how far sound travels while the fork completes 30 vibrations.*

Here, $n = 400$, $v = 320$ metres/second, $\lambda = ?$

$$v = n\lambda$$

or

$$\lambda = \frac{v}{n} = \frac{320}{400} = \mathbf{0 \cdot 8\ metre}$$

∴ Distance travelled by the wave when the fork completes 1 vibration

$= 0{\cdot}8$ metre

Distance travelled by the wave when the fork completes 30 vibrations

$= 0{\cdot}8 \times 30 =$ **24 metres.**

4.8. Properties of Longitudinal Progressive Waves

1. The particles of the medium vibrate simple harmonically along the direction of propagation of the wave.

2. All the particles have the same amplitude, frequency and time period.

3. There is a gradual phase difference between the successive particles.

4. All the particles vibrating in phase will be at a distance equal to $n\lambda$. Here n =1, 2, 3 etc. It means the minimum distance between two particles vibrating in phase is equal to the wavelength.

5. The velocity of the particle is maximum at their mean position and it is zero at their extreme positions.

6. When the particle moves in the same direction as the propagation of the wave, it is in a region of compression.

7. When the particle moves in a direction opposite to the direction of propagation of the wave, it is in a region of rarefaction.

8. When the particle is at the mean position, it is a region of maximum compression or rarefaction.

9. When the particle is at the extreme position, the medium around the particles has its normal density, with compression on one side and rarefaction on the other.

10. Due to the repeated periodic motion of the particles, compressions and rarefactions are produced continuously. These compressions and rarefactions travel forward along the wave and transfer energy in the direction of propagation of the wave.

4.12. Differential Equation of Wave Motion

The general equation of a simple harmonic wave is,

$$y = a \sin \frac{2\pi}{\lambda} (vt - x) \qquad \ldots (1)$$

Differentiating equation (1) with respect to time,

$$\frac{dy}{dt} = \frac{2\pi a v}{\lambda} \cos \frac{2\pi}{\lambda} (vt - x) \qquad \ldots (2)$$

Differentiating equation (2) with respect to time,

$$\frac{d^2y}{dt^2} = -\frac{4\pi^2 a v^2}{\lambda^2} \sin \frac{2\pi}{\lambda} (vt - x) \qquad \ldots (3)$$

To find the value of compression, differentiate equation (1) with respect to x,

$$\frac{dy}{dx} = -\frac{2\pi a}{\lambda} \cos \frac{2\pi}{\lambda} (vt - x) \qquad \ldots (4)$$

To find the rate of change of compression with respect to distance, differentiate equation (4) with respect to x,

$$\frac{d^2y}{dx^2} = -\frac{4\pi^2 a}{\lambda^2} \sin \frac{2\pi}{\lambda} (vt - x) \qquad \ldots (5)$$

From equations (2) and (4)

$$\frac{dy}{dt} = -v \frac{dy}{dx} \quad \ldots (6)$$

From equations (3) and (5)

$$\frac{d^2 y}{dt^2} = v^2 \frac{d^2 y}{dx^2} \quad \ldots (7)$$

Equation (7) represents the differential equation of wave motion.

The general differential equation of wave motion can be written as

$$\frac{d^2 y}{dt^2} = K \frac{d^2 y}{dx^2} \quad \ldots (8)$$

Here $$K = v^2$$

or $$v = \sqrt{K}$$

Thus, knowing the value of K, the value of the wave velocity can be calculated.

Vibrations and Waves

Particle velocity: Particle velocity is the velocity with which the individual particles of a medium move when traversed by a wave.

How do you find a particles velocity?
The instantaneous velocity v(t) of a particle is the derivative of the position with respect to time. That is, v(t)=dxdt. This derivative is often written as ˙x(t), or simply as ˙x.

wave velocity: distance traversed by a periodic, or cyclic, motion per unit time (in any direction). Wave velocity in common usage refers to speed, although, properly, velocity implies both speed and direction.

Difference between wave velocity and particle velocity:
When the tiny particle is said to travel in a medium, then the velocity associated with the particle is known as the particle velocity. On the other hand, the waves are associated with the oscillatory motion through the medium; therefore, the velocity associated with such waves in space is called wave velocity.

What is difference between wave velocity and particle velocity?

- The velocity with which the wave travels in space is called the wave velocity. It is defined as v=frequency∗wavelength.

- Particle velocity is the velocity with which the particles are vibrating to transfer the energy in form of a wave.

- The wave velocity remains constant (provided the density of medium and frequency of source is constant), whereas particle velocity depends on the time (for a particular particle) or depends on where the particle is (for a particular time).

4.13. Particle Velocity and Wave Velocity

The equation for a simple harmonic wave is given by

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) \qquad \ldots (1)$$

Here v is the velocity of the wave and y is the displacement of the particle. The velocity of the particle $U = dy/dt$.

$\therefore$ Differentiating equation (1) with respect to time t,

$$U = \frac{dy}{dt} = \frac{2\pi a v}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x) \qquad \ldots (2)$$

The maximum value of the particle velocity is

$$U_{max} = \frac{2\pi a v}{\lambda} \qquad \ldots (3)$$

$\therefore$ [Maximum Particle Velocity] $= \frac{2\pi a}{\lambda}$ [Wave Velocity]

To find the particle acceleration, differentiate equation (2) with respect to time

$$f = \frac{d^2 y}{dt^2} = -\frac{4\pi^2 a v^2}{\lambda^2} \sin \frac{2\pi}{\lambda}(vt - x) \qquad \ldots (4)$$

$$f = -\frac{4\pi^2 v^2}{\lambda^2}\left[a \sin \frac{2\pi}{\lambda}(vt - x)\right]$$

$$\therefore \qquad f = -\left(\frac{4\pi^2 v^2}{\lambda^2}\right) y$$

The acceleration is maximum when $y = a$

$$\therefore \qquad f_{max} = -\left(\frac{4\pi^2 v^2}{\lambda^2}\right)a \qquad \ldots (5)$$

The negative sign shows that the acceleration of the particle is directed towards its mean position.

Differentiating equation (1) with respect to x.

$$\frac{dy}{dx} = -\frac{2\pi a}{\lambda}\cos\frac{2\pi}{\lambda}(vt - x) \qquad \ldots (6)$$

dy/dx represents the slope of the displacement curve.

From equations (2) and (6)

$$U = \frac{dy}{dt} = -v\left(\frac{dy}{dx}\right) \qquad \ldots (7)$$

$$\therefore \left[\begin{array}{c}\text{Particle velocity}\\ \text{at any instant}\end{array}\right] = \left[\text{Wave velocity}\right]\left[\begin{array}{c}\text{Slope of the displacement}\\ \text{curve at that instant}\end{array}\right]$$

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Thank you
for your kind attention