

Discrete

Date 28/08/2025

Graph

Degree 2

Isometric

Degree 4

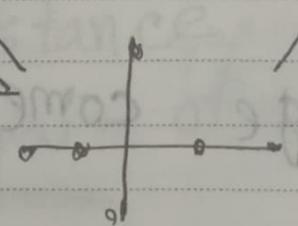
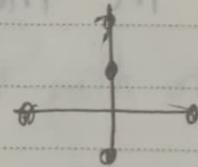
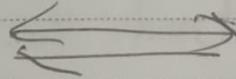
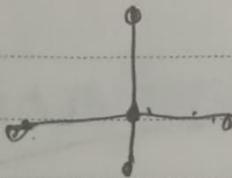
Isoomorphic graph:

→ Vertices same

→ Edges same

→ Degree n

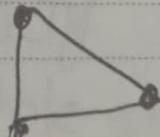
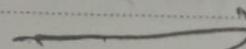
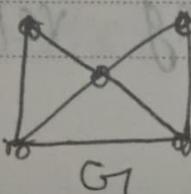
Homomorphic graph



* originate
by deriving
the edges by
more vertex
from a base
graph

Subgraph

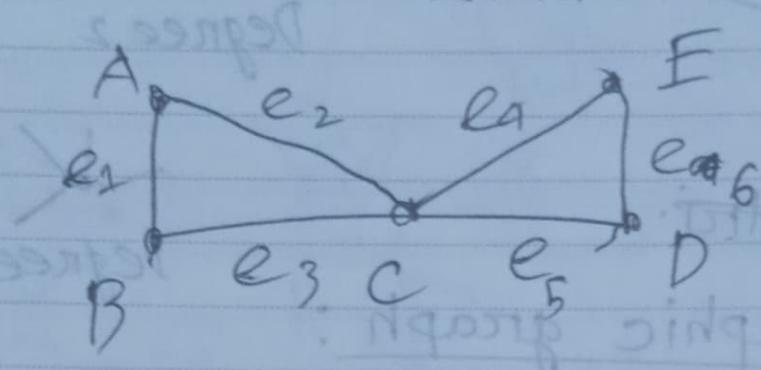
SDG CELL



subgraph of G_7

Path

sequence of vertex, edge from between 2 vertex



A, e₂, C, e₅, D

or
A, e₂, e₃, e₄, E, e₆

simple path

If one vertex comes one time only in the path

Trail path

If one edge comes once in the path.

cycle:

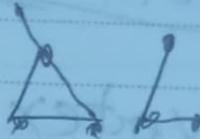
can loop back to same vertex starting vertex.

Connectivity

Can

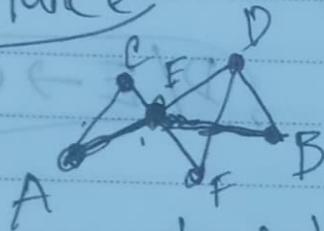


✓



✗

Distance (shortest path)



shortest path = Distance = 2

(AE, EB)

Diameter (max shortest path of any pair)

max Distance possible from
any 2 pair of the graph.

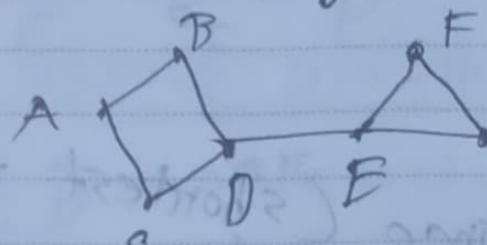
Distance → Between 2 vertex

Diameter → Property of the
whole graph.

~~# Clique~~

cutpoint (A vertex)

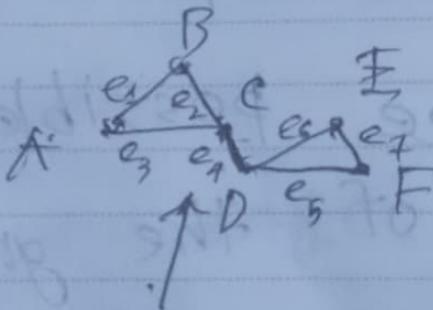
A vertex whose presence affect connectivity.



D/E → Cut point

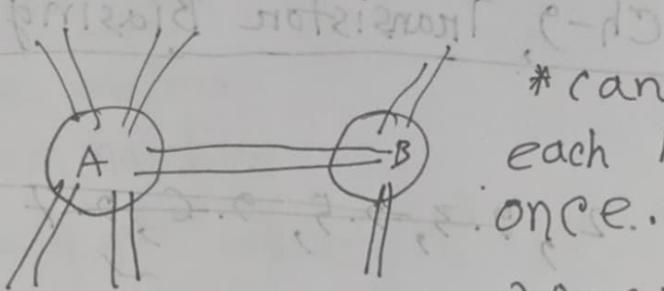
Grease : (An edge)

cutpoint equivalent of for edge

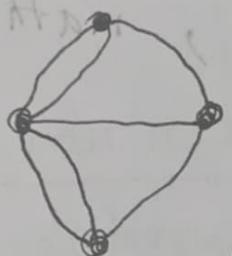


eq grease

04/09/25
Discrete



* can you traverse each bridge ~~at least~~ only once..?
→ Ans: No



* only possible if:

→ All vertex need to have even degree.

or,

→ only 2 odd degree vertex

Eulerian graph:

→ A graph where it is possible to traverse all edges ^{but} only once and reaching starting point.

Hamilton graph:

→ same as Eulerian graph but traverse vertex.

→ only if:
if $n \geq 3$ and $n \leq \text{degree}(\text{vertex})$ for each vertex

circuit

→ Can come back to same node after traversing

Path :

→ Cannot come back to same node but fulfill Eulerian/Hamilton condition.

weighted graph

→ has cost for each vertex

complete graph

→ All node connected with all node

regular graph

→ When all vertex has same degree = k then it called k -regular graph.
denoted by $(k\text{-regular})$

* All complete graph = regular graph
,, regular ,, != complete ,,

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Discrete

Tree

All Tree ~~are~~ ~~is~~ Graph

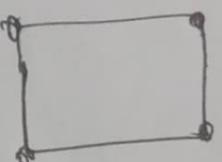
Not all Graph is Tree

if Graph \rightarrow Has cycle

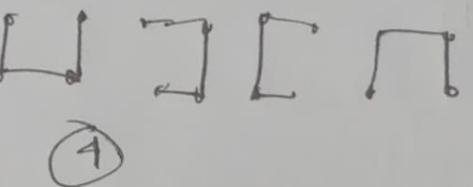
Tree \rightarrow No cycle

\rightarrow ~~No cycle~~ Break any cycle

The number of Tree possible by changing removing edges of graph is called spanning tree



spanning
trees



Spanning tree algorithms (Minimum Spanning tree)

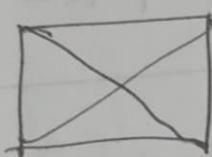
\rightarrow Prim's algorithm \rightarrow Find minimum spanning tree from weighted graph,
~~check~~ check each removable edge.

\rightarrow Kruskal's \rightarrow Same as prim but from reverse, returns only the nonremovable edges.

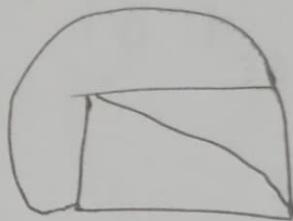
removable edge \rightarrow does not disconnect graph

Planer graph:

→ Can be drawn/redraw without crossing edge.



Planer representation



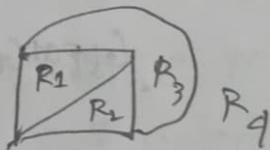
*map:

* A planer ^{graph} representation is called map

*region:

in map

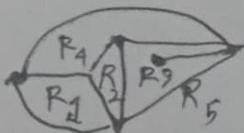
Bounded cycles are called region



* Degree of ~~map~~ region:

→ number of edges that form the boundary of a region

→ number of edge traversal needed to finish a cycle in a bounded region



$$\deg(R_1) = 3$$

$$R_2 = 3$$

$$R_3 = 5$$

$$R_4 = 4$$

$$R_5 = 3$$

* Number of region in a map/planar graph,

Euler's Theorem,

$$V - E + R = 2$$

V → vertex

E → Edge

R → Region

but, condition,

possible

* Condition of planar graph

If, $P \geq 3$

If, $q \geq 3P - 6$

Graph is planar

$P \rightarrow$ vertex
 $q \rightarrow$ edge

Not 100% accurate,
only for simple cases.



$$P = 6 \geq 3$$

$$q = 12 \geq 3P - 6$$

so planar representation

of hexagon is not possible

no two regions share a common boundary

$$P = 6$$

$$q = 12$$

$$6 = 6$$

$$12 = 12$$

$$6 = 6$$

* condition of non-planar graph

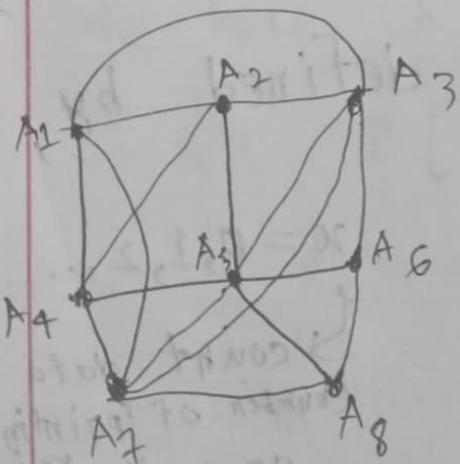
→ If and only if the graph contains
subgraph homomorphic to $K_{3,3}$ and K_5

Graph Coloring

* Adjacent vertex in graph cannot have same color.

* Algorithm
→ Well's Panel's algorithm

↳ Determine minimum number of colors needed to do graph coloring.



Step - 1:
* sort vertex in descending order of degree

$\rightarrow A_5, A_3, A_7, A_1, A_2, A_4, A_6, A_8$

Step - 2:
start coloring for start and also give same color to non connected graph.

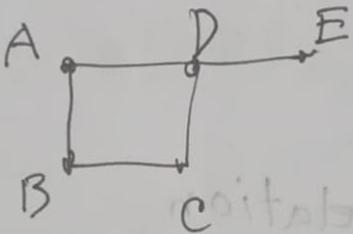
$A_5 \quad A_3 \quad A_7 \quad A_1 \quad A_2 \quad A_4 \quad A_6 \quad A_8$
 $c_1 \quad c_2 \quad c_3 \quad c_1 \quad c_2 \quad c_3 \quad c_2 \quad c_1$

43 colors

22/09/2025

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Graph representation in Data Structure



	A	B	C	D	E
A	0	1	1	0	0
B	1	0	1	0	0
C	0	1	0	1	0
D	1	0	1	0	1
E	0	0	0	1	0

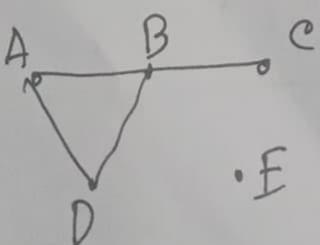
$$a_{ij} = \begin{cases} 1 & \text{if } v_i \text{ is adjacent to } v_j \\ 0 & \text{otherwise} \end{cases}$$

$v_i \rightarrow$ number of edge
 $v_x \rightarrow$ ~~the~~ x^{th} vertex

(1)

* Array representation

② Linked List representation



vertex	adjacent list
A	B, D
B	A, C, D
C	B
D	A, B
E	∅