Mechanics

❖ **What is meant by Fundamentals of Physics?**

The research carried out in the Fundamental Physics theme is focused on discovering the nature of space and time and the proprieties of matter in the universe at the deepest level.

❖ **What is the difference between Physics and Fundamentals of Physics?**

According to the scientist descriptions, "Physics" is suitable for engineers and science majors and "Fundamentals of Physics" is for engineers. So, the last one is aimed at a more applied approach to the material. Hence, which one is "better" depends entirely on what you wish to get out of it.

Mechanics

Mechanics:

Branch of physical sciences concerned with the state of rest or motion of bodies subjected to forces.

Mechanics

- **What are the basic concepts of mechanics?**

Mechanics may be divided into three branches: statics, which deals with forces acting on and in a body at rest; kinematics, which describes the possible motions of a body or system of bodies; and kinetics, which attempts to explain or predict the motion that will occur in a given situation.

Mechanics

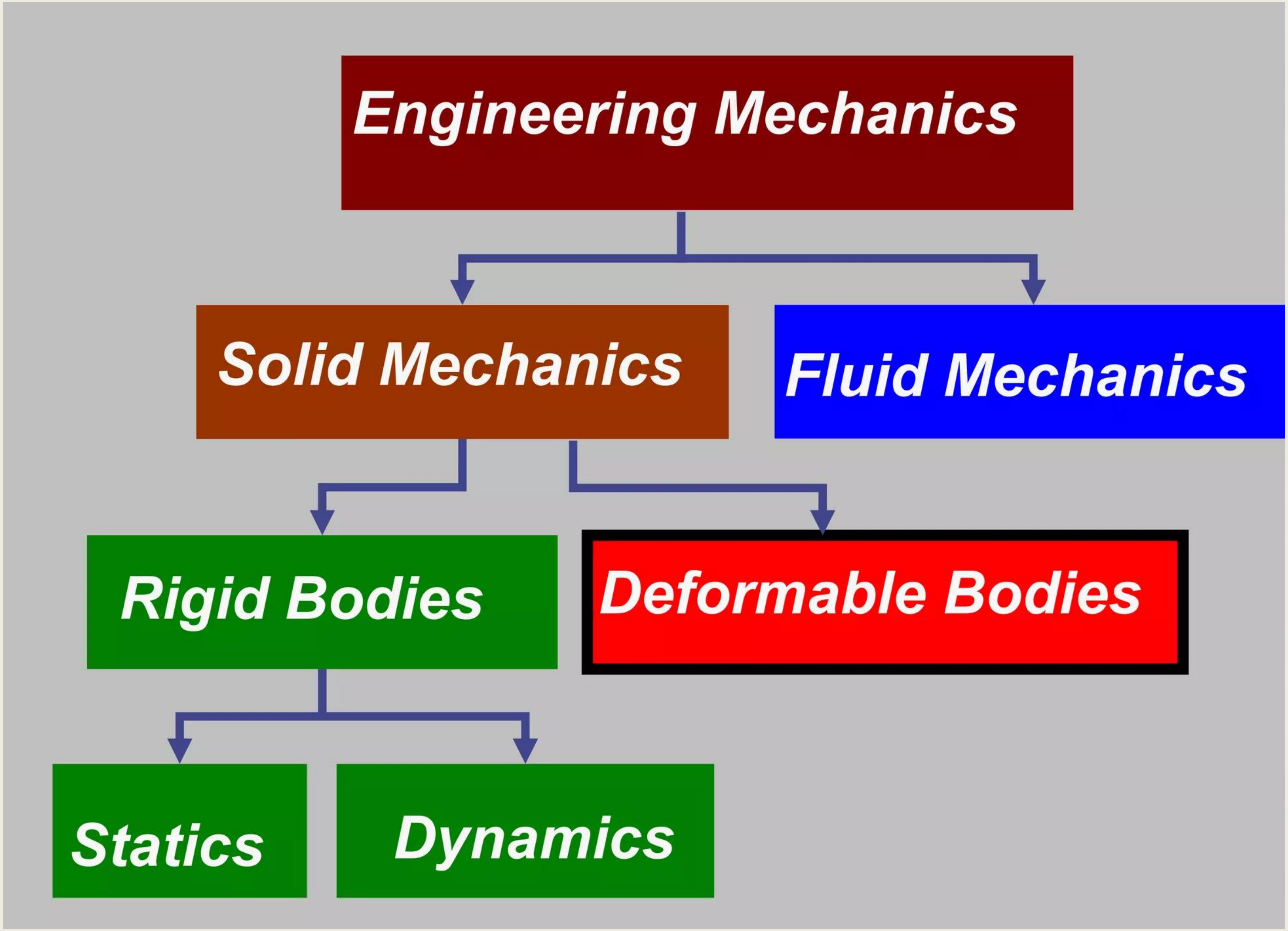
Objectives

1. To provide an introduction to the basic quantities and idealizations of mechanics.
2. To give a statement of Newton's laws of Motion and Gravitation.
3. To review the principles for applying the SI system of units.
4. To examine the standard procedures for performing numerical calculations.
5. To present a general guide for problem solving.

Following are the seven fundamental quantities: Length (meter), Mass (kilogram), Time (second), Electric current (ampere), Thermodynamic temperature (kelvin), Amount of substance (mole), Luminous intensity (candela).

Idealization in Mechanics: The mathematical description of a real engineering problem can become. simplify the application of the theory.

Mechanics



Engineering Mechanics
Solid Mechanics
Fluid Mechanics
Rigid Bodies
Deformable Bodies
Statics
Dynamics

Motion in One Dimension

Kinematics:

Describes motion while ignoring the external agents that might have caused or modified the motion.

- ➢ Consider motion in one dimension
- ➢ Along a straight line
- ➢ Motion represents a continual change in an object's position.

Types of Motion

- ❖ Translational

An example is a car traveling on a highway.

- ❖ Rotational

An example is the Earth's spin on its axis.

- ❖ Vibrational

An example is the back-and-forth movement of a pendulum.

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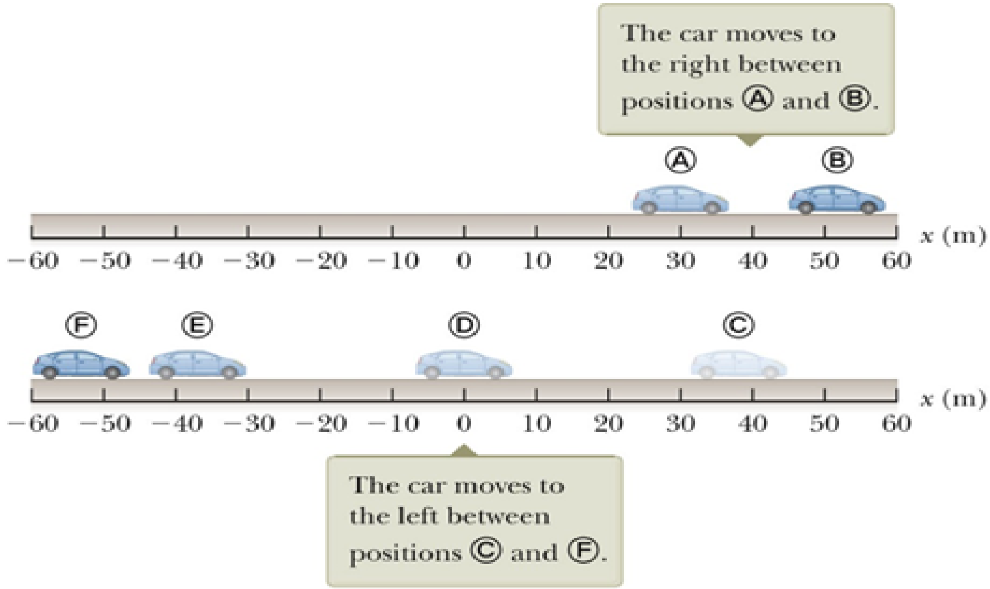
Position

The object's position is its location with respect to a chosen reference point.

- Consider the point to be the origin of a coordinate system.

Only interested in the car's translational motion, so model as a particle



Displacement:

Displacement is defined as the change in position during some time interval.

Represented as Δx

$$\Delta x \equiv x_f - x_i$$

SI units are meters (m)

- Δx can be positive or negative
- Different than distance
- Distance is the length of a path followed by a particle.

Distance vs. Displacement – An Example

Assume a player moves from one end of the court to the other and back.

Distance is twice the length of the court

- Distance is always positive

Displacement is zero

- $\Delta x = x_f - x_i = 0$ since $x_f = x_i$

Mechanics

Kinematic Equations:

- The kinematic equations can be used with any particle under uniform acceleration.
- The kinematic equations may be used to solve any problem involving one-dimensional motion with a constant acceleration.
- You may need to use two of the equations to solve one problem.
- Many times there is more than one way to solve a problem.

Kinematic Equations, 1

For constant a_x,

$$v_{xf} = v_{xi} + a_x t$$

Can determine an object's velocity at any time t when we know its initial velocity and its acceleration

Kinematic Equations, 2

For constant acceleration,

$$v_{x,avg} = \frac{v_{xi} + v_{xf}}{2}$$

The average velocity can be expressed as the arithmetic mean of the initial and final velocities.

- This applies only in situations where the acceleration is constant.

Kinematic Equations, 3

For constant acceleration,

$$x_f = x_i + v_{x,avg}\, t = x_i + \frac{1}{2}\left(v_{xi} + v_{fx}\right) t$$

This gives you the position of the particle in terms of time and velocities.

Doesn't give you the acceleration

Kinematic Equations, 4

For constant acceleration,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

Gives final position in terms of velocity and acceleration

Doesn't tell you about final velocity

Kinematic Equations, 5

For constant *a*,

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Gives final velocity in terms of acceleration and displacement

Does not give any information about the time

When a = 0

When the acceleration is zero,

- $v_{xf} = v_{xi} = v_x$
- $x_f = x_i + v_x t$

The constant acceleration model reduces to the constant velocity model.

Freely Falling Objects

- Freely falling object is any object moving freely under the influence of gravity alone.
- It does not depend upon the initial motion of the object.
- Dropped – released from rest
- Thrown downward
- Thrown upward

Mechanics

What is two-dimensional motion?

Two-dimensional (2D) motion means motion that takes place in two different directions (or coordinates) at the same time. The simplest motion would be an object moving linearly in one dimension. An example of linear movement would be a car moving along a straight road or a ball thrown straight up from the ground.

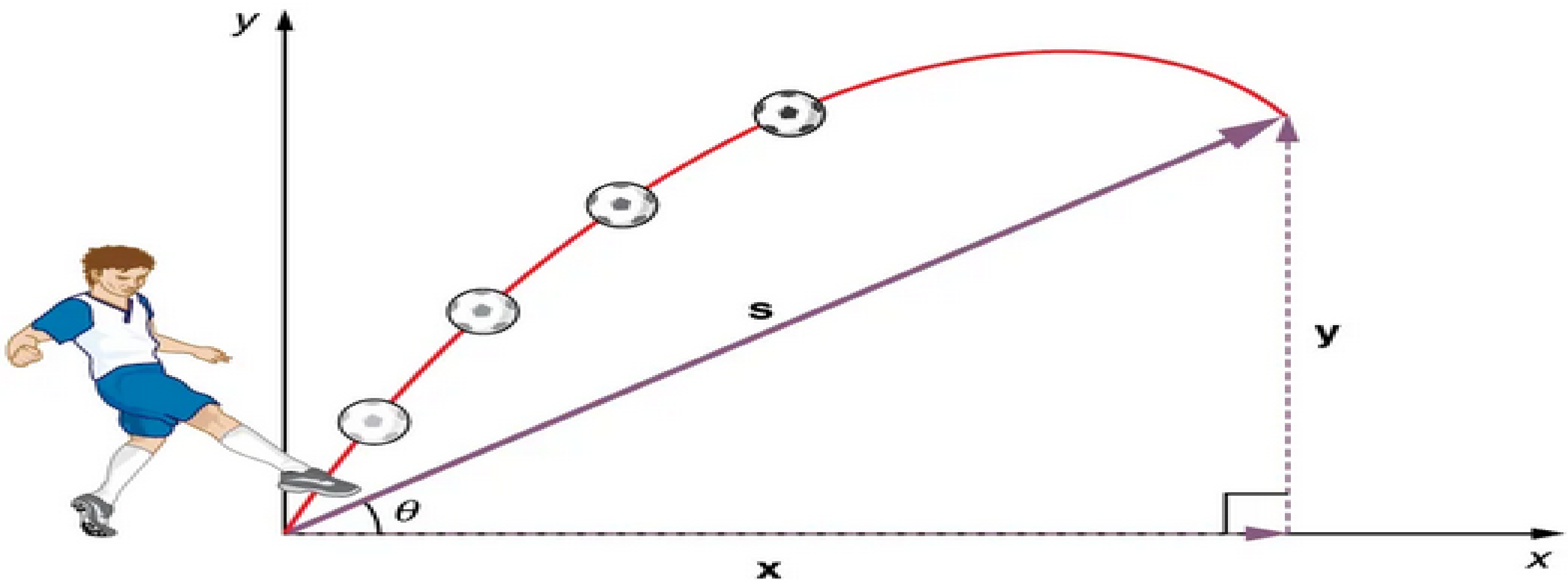


Figure 3.34 The total displacement **s** of a soccer ball at a point along its path. The vector **s** has components **x** and **y** along the horizontal and vertical axes. Its magnitude is s, and it makes an angle θ with the horizontal.

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What are the components of two-dimensional motion?

Two dimensional motion can be described using the two separate components. The two separate motions are in horizontal and vertical directions respectively. Projectile motion is two-dimensional because it has a horizontal component and a vertical component.

What is the formula for two-dimensional motion?

Few Examples of Two – Dimensional Projectiles

Quantity	Value
Time of maximum height	$t_m = v_o \sin\theta_o / g$
Time of flight	$2t_m = 2(v_o \sin\theta_o/g)$
Maximum height of projectile	$h_m = (v_o \sin\theta_o)^2/2g$
Horizontal range of projectile	$R = v_o^2 \sin 2\theta_o/g$

Equation of Motions in Two Dimensions

The three equations of motion in two dimensions x and y are given as:

$$v_x = u_{x0} + a_x t$$

$$\Delta x = u_{x0} t + \frac{1}{2} a_x t^2$$

$$v_x^2 - u_{x0}^2 = 2 a_x \Delta x$$

Similarly, the equations can be written in y directions.

$$v_y = u_{y0} + a_y t$$

$$\Delta y = u_{y0} t + \frac{1}{2} a_y t^2$$

$$v_y^2 - u_{y0}^2 = 2 a_y \Delta y$$

Solved Examples for Two-Dimensional Motion

Example 1: A particle is moving with an initial velocity (2i + 4j) $\frac{m}{s}$ and has constant acceleration of (10i + 2j) $\frac{m}{s^2}$. Calculate the final velocity and displacement after 6 seconds.

Solution: The above problem can be solved by dividing velocity and acceleration into one dimension. Thus, along the x-axis, we have initial velocity as 2 $\frac{m}{s}$, acceleration as $10\frac{m}{s^2}$. Using the first equation of motion, we have v=u+at. Substituting the values above in this, we get

$$v = 2 + 10 \times 6$$
$$v = 62ms^{-1}$$

Now, using the second equation of motion, we have $s = ut + \frac{1}{2}at^2$. Substituting the above values we get,

$$s = 2 \times 6 + \frac{1}{2}(10) \times 6^2$$
$$s = 12 + 180$$
$$s = 192 \text{ m}$$

Now, we will calculate the same along the y-direction.
We get the value of velocity as

$$v = 4 + 2 \times 6$$

$$v = 16\ \mathrm{ms}^{-1}$$

Similarly, the displacement will be

$$s = 4 \times 6 + \frac{1}{2} \times 2 \times 6^2$$

$$s = 24 + 36$$

$$s = 60\ \mathrm{m}$$

Thus, final velocity = $v = v_x\hat{i} + v_y\hat{j} = 62\hat{i} + 16\hat{j}$

or

$$v = \sqrt{62^2 + 16^2}$$

$$= v = 64ms^{-1}$$

The final displacement is

$$s = s_x\hat{i} + s_y\hat{j}$$

$$s = 192\hat{i} + 60\hat{j}$$

or

$$s = \sqrt{192^2 + 60^2}$$

$$s = 201.15\ \mathrm{m}$$

Hence, the final velocity and displacement after 6 seconds are $62\hat{i} + 16\hat{j}$ and $201.15\ \mathrm{m}$.

Example 2: An object is moving in a plane with a velocity $v = 4t\hat{i} + 6t^3\hat{j}$. Find the acceleration of the object between time intervals (0,2).

Solution: Acceleration is denoted as $a = \frac{v}{t}$

At 0 time interval, velocity will be zero, whereas at t=2 seconds, velocity will be

$$v = 4 \times 2\hat{i} + 6 \times 2^3\hat{j}$$

$$v = 8\hat{i} + 48\hat{j}$$

Therefore, acceleration will be

$$\Rightarrow \frac{8\hat{i} + 48\hat{j}}{2}$$

$$\Rightarrow 4\hat{i} + 24\hat{j}$$

Hence, the acceleration of the object between time intervals (0,2) is $4\hat{i} + 24\hat{j}$.

❖ **Difference between one, two and three dimensional motion:**

One dimensional motion is motion along a straight line. The line used for this motion is often the familiar x-axis, or x number line. The object may move forward or backward along this line: Forward is usually considered positive movement, and this movement is usually considered to be to the right. Remember that the study of one-dimensional motion is the study of movement in one direction, like a car moving from point “A” to point “B.”

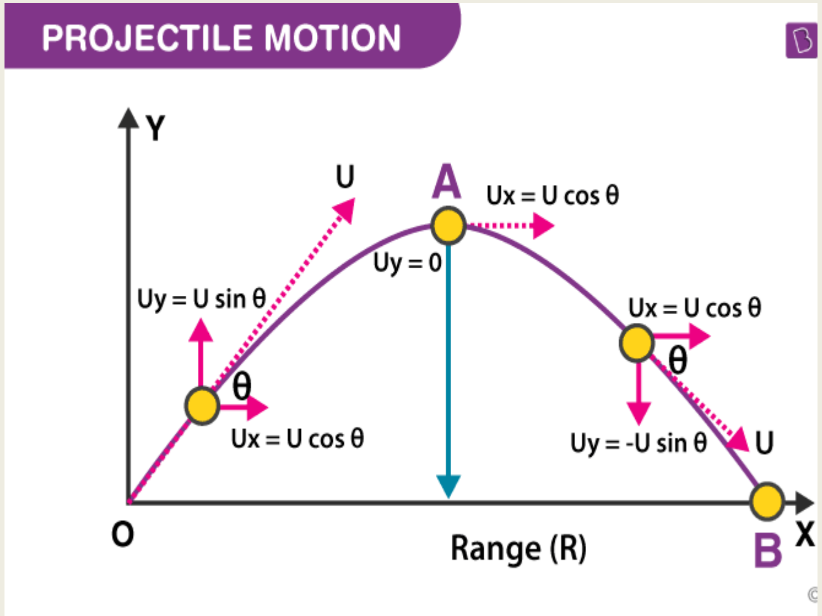
Whereas,

In two-dimensional motion, the path the object follows lies in a plane. Two-dimensional motion is the study of movement in two directions, including the study of motion along a curved path, such as projectile and circular motion. Examples are projectile motion where the path is a parabola, or planetary motion where it is an ellipse.

Whereas,
Three-dimensional motion would be a case where the path is more complex and is not confined to a single plane.

Projectile motion:

Projectile motion is the motion of an object thrown or projected into the air, subject to only the acceleration of gravity. The object is called a projectile, and its path is called its trajectory.

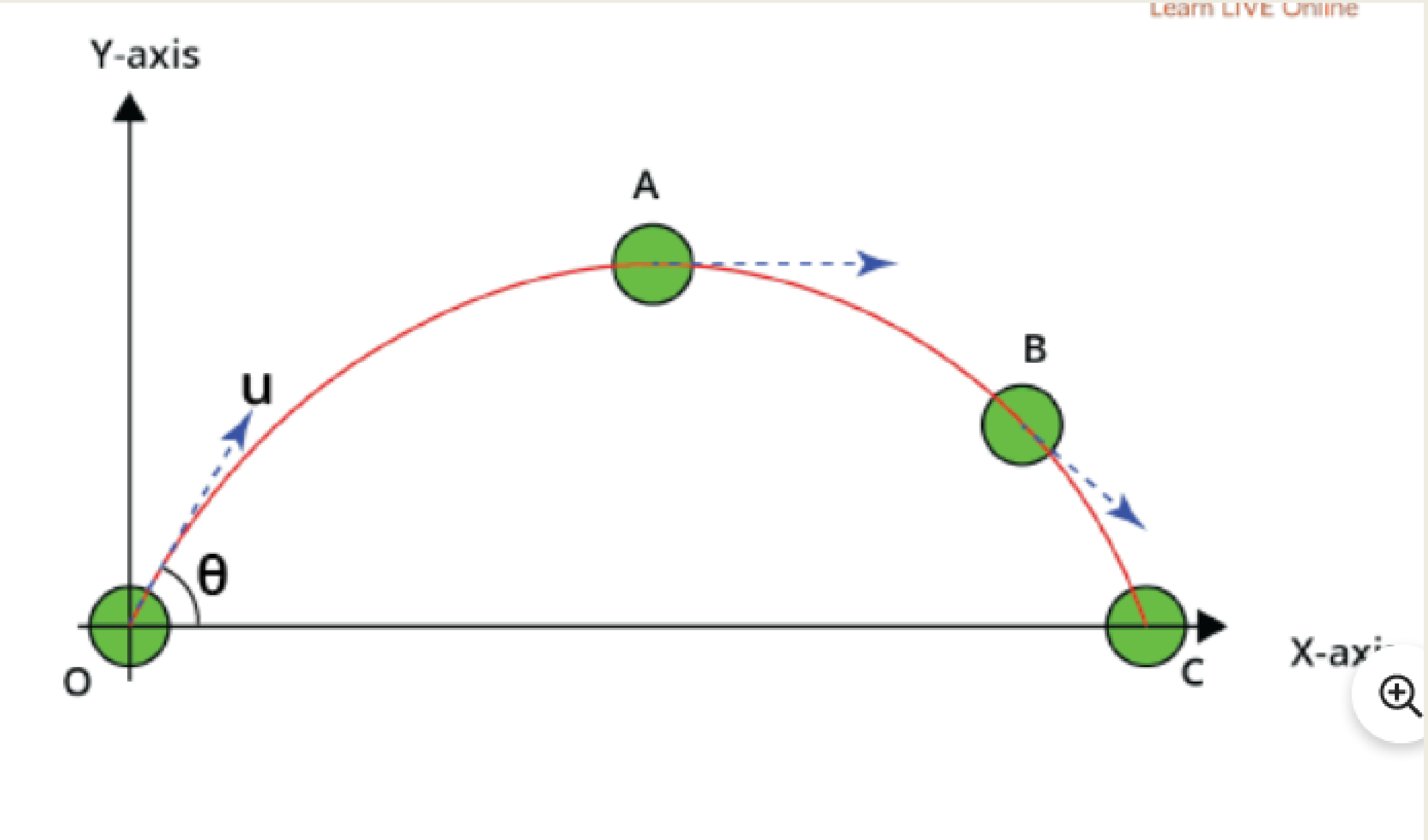


What are the 3 main concepts of projectile motion?

The key components that we need to remember in order to solve projectile motion problems are: Initial launch angle (θ), Initial velocity (u), and Time of flight (T).

Projectile Motion

The motion of an object in a curved path with constant acceleration is known as projectile motion. The motion in this case is along the x-axis and the y-axis.



Projectile Motion

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The acceleration component along the x-axis is 0, whereas in the vertical direction it is -g. However, the component of velocity along the x-axis is $u\cos(\theta)$ and $u\sin(\theta)$, respectively.

The equation of a projectile or the trajectory formula as it is known is given as

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Here, x and y are the horizontal and vertical components, respectively, and u is the initial velocity and g is the acceleration due to gravity. There are other important equations related to the projectile motion which include the time of flight (T), maximum height (H), and the horizontal range of the projectile (R). These equations are

$$T = \frac{2u\sin\theta}{g}$$

$$H = \frac{u^2\sin^2\theta}{2g}$$

$$R = \frac{u^2\sin 2\theta}{g}$$

Here, u is the initial velocity, g is the acceleration due to gravity, and θ at which the projectile was thrown with respect to the horizontal.

These are the equations of motion of the projectile.

What Is Frictional Force?

> Frictional force is the force generated by two surfaces that contact and slide against each other.

A few factors affecting the frictional force:

- These forces are mainly affected by the surface texture and the amount of force impelling them together.
- The angle and position of the object affect the amount of frictional force.
- If an object is placed flat against an object, then the frictional force will be equal to the object's weight.
- If an object is pushed against the surface, then the frictional force will be increased and becomes more than the weight of the object.

Dry Friction

Dry friction describes the reaction between two solid bodies in contact when they are in motion (kinetic friction) and when they are not (static friction). Both static and kinetic friction is proportional to the normal force exerted between the solid bodies. The interaction of different substances is modelled with different coefficients of friction. By this, we mean that certain substances have a higher resistance to movement than others for the same normal force between them. Each of these values are experimentally determined.

Fluid Friction

Fluid Friction is the force that obstructs the flow of fluid. It is a situation where the fluid provides resistance between the two surfaces. If both surfaces offer high resistance, then it is known as high viscous

Examples of Fluid Friction

1. To avoid creaking sounds from doors, we lubricate the door hinges, which leads to the smooth functioning of door hinges.

Work:

Work is the energy transferred to or from an object via the application of force along a displacement. In its simplest form, for a constant force aligned with the direction of motion, the work equals the product of the force strength and the distance traveled. A force is said to do positive work if when applied it has a component in the direction of the displacement of the point of application. A force does negative work if it has a component opposite to the direction of the displacement at the point of application of the force.

For example, when a ball is held above the ground and then dropped, the work done by the gravitational force on the ball as it falls is positive, and is equal to the weight of the ball (a force) multiplied by the distance to the ground (a displacement). If the ball is thrown upwards, the work done by the gravitational force is negative, and is equal to the weight multiplied by the displacement in the upwards direction.

$$W = Fs$$

W = work

F = force

s = Displacement

Energy:

In physics, energy is the quantitative property that is transferred to a body or to a physical system, recognizable in the performance of work and in the form of heat and light. Energy is a conserved quantity—the law of conservation of energy states that energy can be converted in form, but not created or destroyed.

Momentum:

Momentum is the product of the mass and velocity of an object. It is a vector quantity, possessing a magnitude and a direction. If m is an object's mass and v is its velocity, then the object's momentum p is: mv

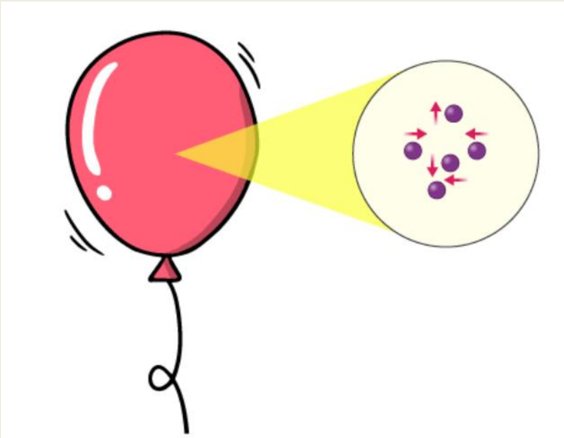
What Is Conservation of Momentum?

Conservation of momentum is a major law of physics which states that the momentum of a system is constant if no external forces are acting on the system. It is embodied in Newton's First Law or The Law of Inertia.

Example of Conservation of Momentum

Consider this example of a balloon, the particles of gas move rapidly colliding with each other and the walls of the balloon, even though the particles themselves move faster and slower when they lose or gain momentum when they collide, the total momentum of the system remains the same.

Hence, the balloon doesn't change in size, if we add external energy by heating it, the balloon should expand because it increases the velocity of the particles and this increases their momentum, in turn, increasing the force exerted by them on the walls of the balloon.



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Application of Law of Conservation of Momentum

Having said so the energy of a system is always conserved, one of the best applications of the law of conservation of momentum would be in space travel, there is no medium in space to exert a force on, then how do rockets travel?

Well, they eject matter at a very high speed, so in an isolated system, the momentum should remain constant therefore, the rocket will move in the opposite direction with the same momentum as that of the exhaust.

Work-energy theorem:

The work-energy theorem explains the idea that the net work - the total work done by all the forces combined - done on an object is equal to the change in the kinetic energy of the object. After the net force is removed (no more work is being done) the object's total energy is altered as a result of the work that was done.

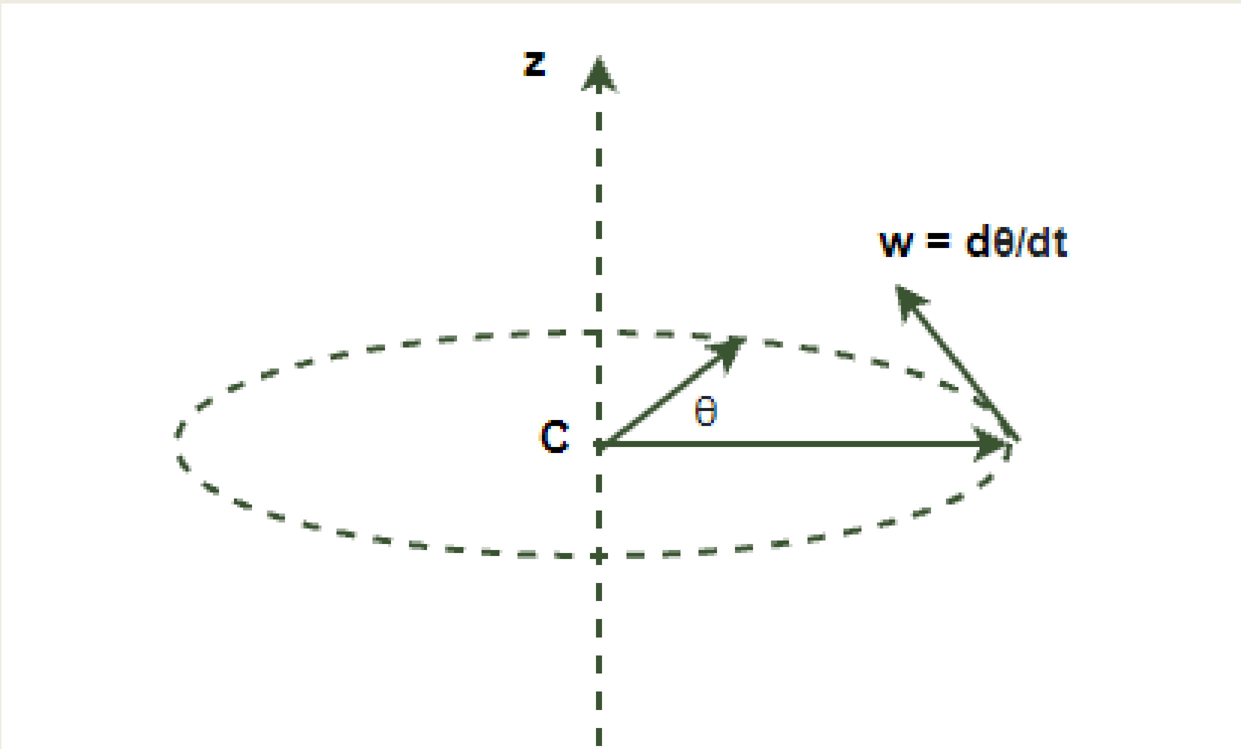
According to the theorem,

$$W_{net} = \Delta K = K_f - K_i$$

- W is the total work done
- ΔK is the change in kinetic energy
- K_f is the final kinetic energy
- K_i is the initial kinetic energy

Rotation of a rigid body:

Rigid body rotation is a motion that occurs when a solid body moves in a circular path around something. The rotational motion can be broken down into two types of rotation – Rotation about a fixed axis and rotation about a fixed point.



Linear inertia

Is the tendency of a body to resist change in its linear velocity.

In other words; objects do not change their state of linear motion unless acted upon by some not external force.

Rotational inertia

Is the tendency of a body to resist change in its angular velocity.

- In other words, objects do not change their rotational motion unless acted upon by some not external torque.
- It is also called moment of inertia.

CONCEPT OF MOMENT OF INERTIA

Moment of inertia of a body about an axis is a measure of the difficulty in starting, stopping or changing rotation of the body about that axis.

- It is denoted by I
- The greater the difficulty in starting or stopping , the greater is the moment of inertia of the body about that axis and vice-versa.
- A body rotates under the action of a net external torque.

The Greater the moment of inertia of a body about an axis of rotation, the greater is the torque required to rotate or stop or change rotation of the body that axis and vice-versa.

MOMENT OF INERTIA OF A RIGID BODY

Consider a rigid body rotating about the axis yy^{-1} with an angular speed ω as shown in figure 1 below.

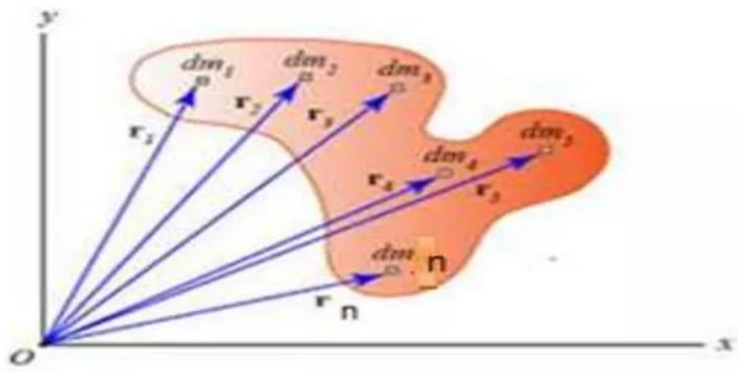


Figure. 1

Suppose the body is made up of a large number (n) of small particles of masses m_1, m_2, m_3 ....m_n situated at perpendicular distances r_1, r_2, r_3 ....r_n respectively from the axis of rotation yy'.

As the body rotates, each particle of the body follows a circular path around the axis.

Although each particle of the body has the same angular speed ω, the linear velocity (v) of each particle depends upon particles distance from the axis of rotation.

Thus particle of mass m_1 follows a circular path of radius r_1. The linear velocity of this particle is v_1

$$V_1 = \omega r_1$$

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Rotational kinetic energy of the particles of mass m_1

$$R.K.E = \frac{1}{2}m_1V_1^2$$

$$R.K.E = \frac{1}{2}m_1\omega^2r_1^2$$

$$\therefore R.K.E = \frac{1}{2}m_1r_1^2\omega^2$$

The total kinetic energy K_r of the rotating body is the sum of the kinetic energies of all the particles of which the body is composed.

$$K_r = \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \frac{1}{2}m_3r_3^2\omega^2 - - - -$$

$$K_r = \frac{1}{2}\omega^2\left[\sum_{i=1}^{i=n} m_ir_i^2\right]$$

$m_i = mass\ of\ i^{th}\ particle$ and $r_i = its\ perpendicular\ distance\ from\ axis\ of\ ro$

$$\sum_{i=1}^{i=n} m_ir_i^2 = moment\ of\ inertia\ I$$

Hence the total K.E of rotating body

$$K_r = \frac{1}{2}I\omega^2$$

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Moment of inertia of a rigid body about a given axis of rotation

Is the sum of the products of the masses of its particles and the squares of their respective perpendicular distance from the axis of rotation.

$$I = \sum_{i=1}^{i=n} m_i r_i^2$$

The moment of inertia of a body about an axis of rotation is directly proportional to the total mass of the body.

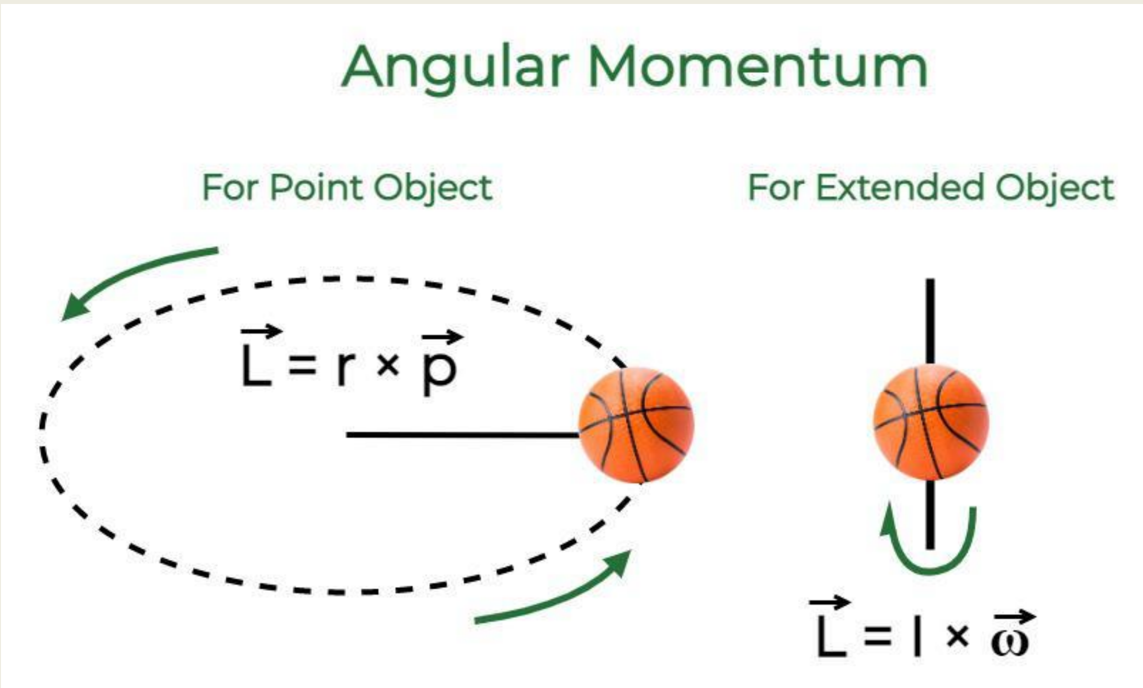
The more massive the body, the more difficult will be to start its rotational motion or stop it from rotating.

For a given mass, the moment of inertia of a body depends upon the distribution of the mass from the axis of rotation. The larger the distance of the mass from the axis rotation the larger will be its moment of inertia. The moment of inertia plays the same role in rotational motion as mass plays in translational motion.

Mechanics

What do you mean by angular momentum?

Angular momentum is defined as: The property of any rotating object given by moment of inertia times angular velocity. It is the property of a rotating body given by the product of the moment of inertia and the angular velocity of the rotating object.



Formula

$$L = mvr$$

L = angular momentum

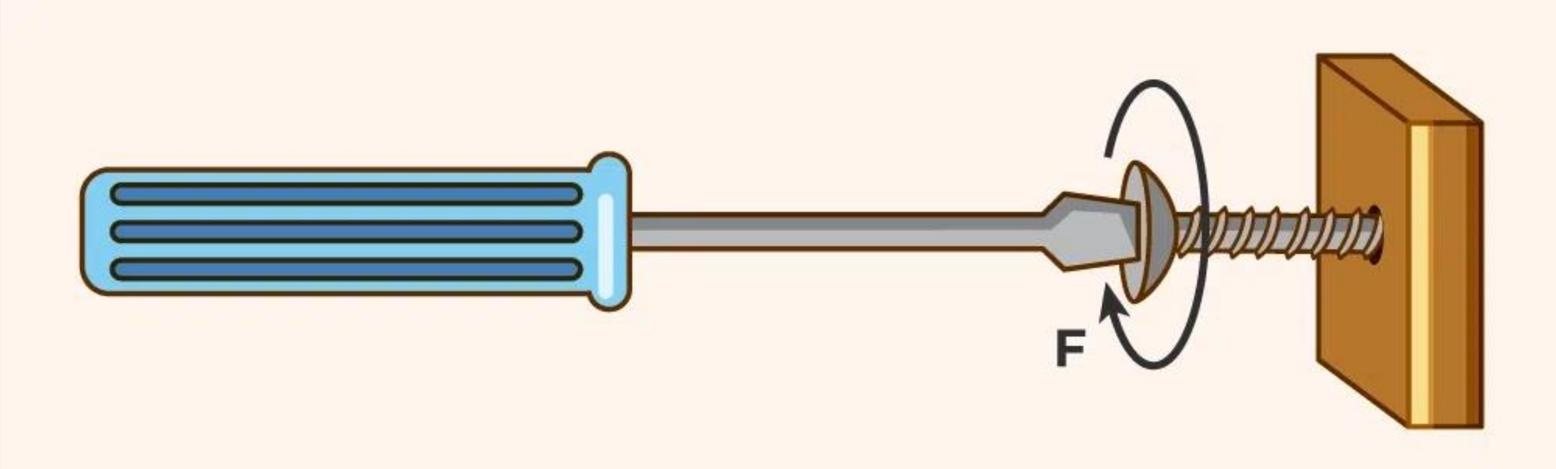
m = mass

v = velocity

r = radius

Torque:

The force that can cause an object to rotate along an axis is measured as torque. In linear kinematics, force is what drives an object's acceleration. Similar to this, an angular acceleration is brought on by torque. As a result, torque can be thought of as the rotational counterpart to force. The axis of rotation is a straight line about which an item rotates. Torque in physics is only a force's propensity to turn or twist.



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Basis of differentiation	Torque	Angular Momentum
Definition	Torque is the force that causes rotational motion	Angular Momentum is the product of a rotating object's moment of inertia and angular velocity
Mathematical Formula	Torque = rF sinθ Here, r is the distance between the object's centre of mass and the point of rotation. F is the force and θ is the angle at which the force is subjected on the object.	Angular Momentum = Iω Here, I is the object's moment of Inertia and w is the angular velocity at which the object is rotating.
Examples	1. When you open a cap of the bottle, you are actually applying a torque good enough to rotate it. 2. When you apply force to open a door, it moves in a circular motion at its hinge. In fact it is the torque acting upon it that causes it to open,	1. When a skater spins her body on the ice turf, it is the angular momentum that prevents her from falling down. 2. The angular momentum prevents the merry-go-round from losing its balance. The same can be said about a giant-wheel as well.

Relation between torque and angular momentum:

The momentum that a rotating object has because of the distance between the object and the perpendicular drawn from the center of rotation is called angular momentum. It is due to the angular momentum that one can able to paddle a bicycle without getting imbalanced. Mathematically, it is defined as the cross product between the linear momentum of an object and the distance between the object's center of mass and center of rotation. This distance is equal to the radius of the circular motion of the object. Therefore, we can calculate the angular momentum of a rotating object from the below formula:

$$\vec{L} = \vec{r} \times \vec{p}$$

Here L is used to denote angular momentum, and r is the radius, i.e., distance from the axis of rotation to the object's center of mass.

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Torque is the force that accelerates the object to follow the rotational motion along its axis of rotation. Torque acts perpendicularly on the distance between the rotational axis. According to the definition of torque, it is the rate at which an object's angular momentum changes. It is equal to the cross product between the distance from the rotational axis and the linear force acting on the object. To find the torque of a rotating object, we can utilize the below equation,

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = rF \sin \theta \hat{n}$$

If we differentiate the angular momentum equation with respect to time then we get

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d}{dt}(\vec{p})$$

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Now, $\frac{d\vec{p}}{dt}$ is the force acting on the object.

As a result, we can write this equation as

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = \vec{\tau}$$

This is the equation where we can say that the rate at which the angular momentum of an object changes can be expressed by torque. From this, we have:

$$\vec{\tau} = \frac{d\vec{L}}{dt} = rF \sin\theta \hat{n}$$

When the angular momentum remains constant, torque is 0. It means that,

$$\vec{L} = \vec{r} \times \vec{p} = \text{constant}$$

Therefore,

$$\vec{\tau} = \frac{d\vec{L}}{dt} = 0$$

We can use this equation to find the torque and momentum of a rotating body. Last equation can be used to find the relation between torque and angular momentum.

What is meant by center of mass?

The center of mass is a position defined relative to an object or system of objects. It is the average position of all the parts of the system, weighted according to their masses. For simple rigid objects with uniform density, the center of mass is located at the centroid. For example, the center of mass of a uniform disc shape would be at its center. Sometimes the center of mass doesn't fall anywhere on the object. The center of mass of a ring for example is located at its center, where there isn't any material.

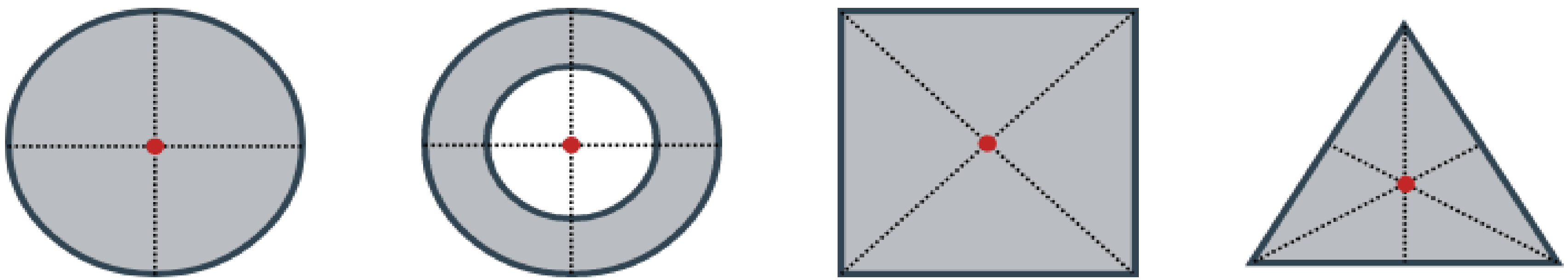
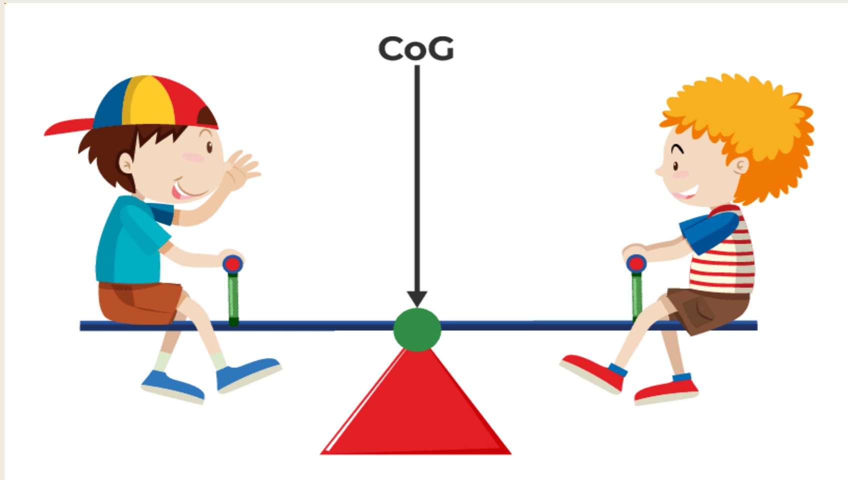


Figure 1: Center of mass for some simple geometric shapes (red dots).

Center of gravity:

Centre of gravity is an imaginary point in a body of matter where, the total weight of the body may be thought to be concentrated. The concept is sometimes useful in designing static structures (e.g., buildings and bridges) or in predicting the behaviour of a moving body when it is acted on by gravity.

In a uniform gravitational field the center of gravity is identical to the center of mass. The two do not always coincide, however. For example, the Moon's center of mass is very close to its geometric center (it is not exact because the Moon is not a perfect uniform sphere), but its center of gravity is slightly displaced toward Earth because of the stronger gravitational force on the Moon's near side.

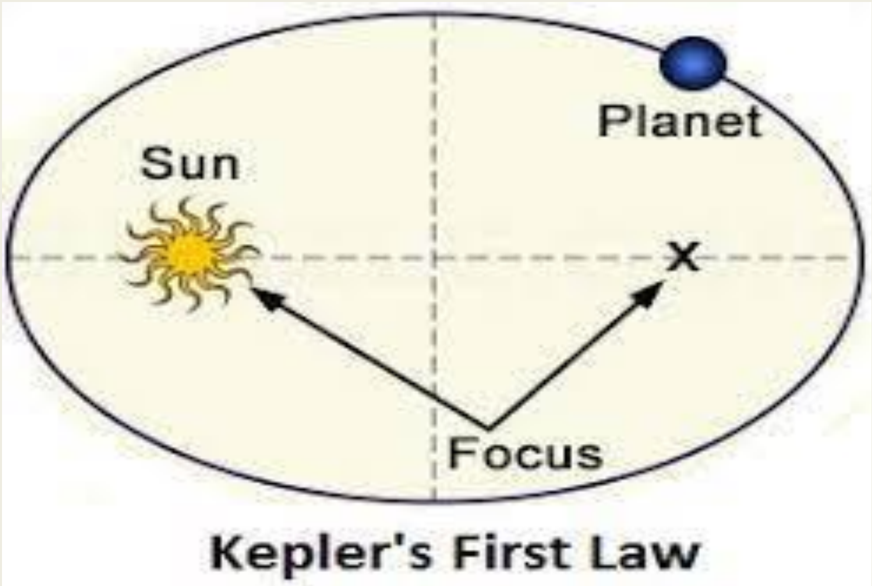


LAWS OF PLANETARY MOTION

Kepler First law – The Law of Orbits

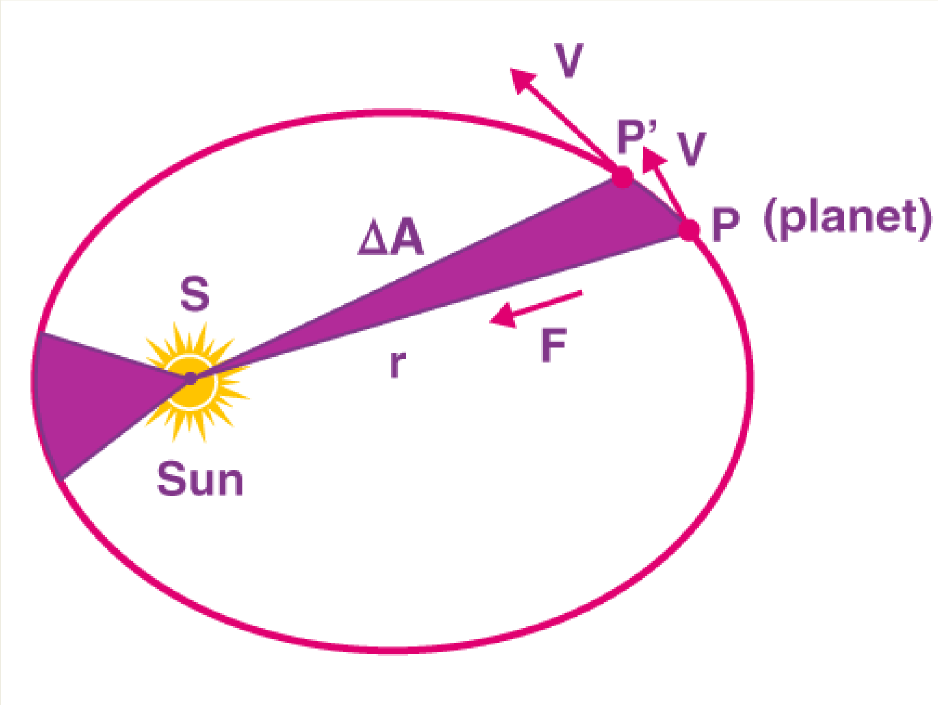
❖ According to Kepler's first law, all the planets revolve around the sun in elliptical orbits having the sun at one of the focus". The point at which the planet is close to the sun is known as perihelion, and the point at which the planet is farther from the sun is known as aphelion.

➢ It is the characteristic of an ellipse that the sum of the distances of any planet from two focus is constant. The elliptical orbit of a planet is responsible for the occurrence of seasons.



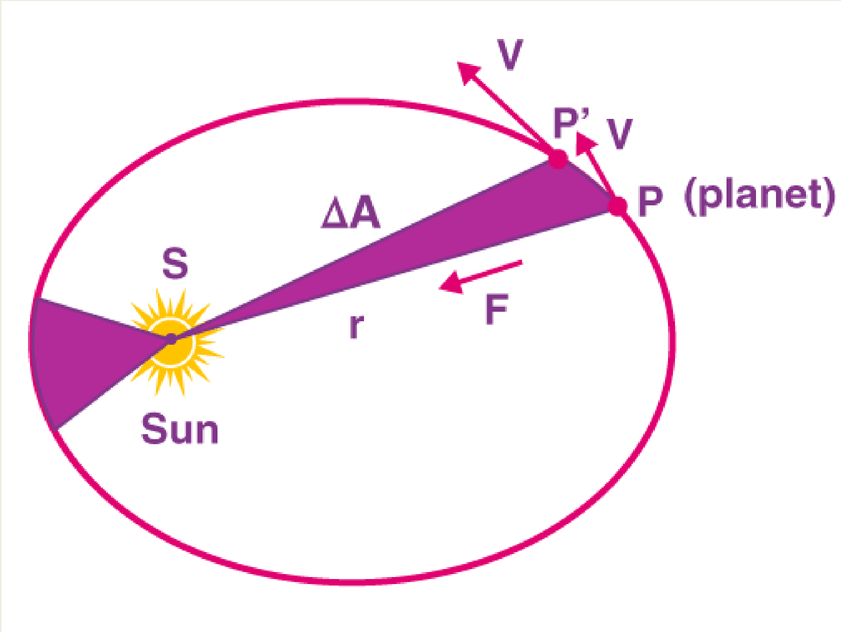
Kepler's Second Law: The Law of Equal Areas

- ❖ Kepler's second law states, " The radius vector drawn from the sun to the planet sweeps out equal areas in equal intervals of time".



Mechanics

- As the orbit is not circular, the planet's kinetic energy is not constant in its path. It has more kinetic energy near the perihelion, and less kinetic energy near the aphelion implies more speed at the perihelion and less speed (v_{min}) at the aphelion. So, areas swept out by a line from the Sun to planate for both aphelion and perihelion, were equal over the same amount of time.



Kepler's Third Law: The Law of Periods

According to Kepler's law of periods, "The square of the time period of revolution of a planet around the sun in an elliptical orbit is directly proportional to the cube of its semi-major axis".

Explanation: If the mean distance between the sun and the planet ~~R~~ "P" is "R" and the period is "T", then according to this law,

$$T^2 \propto R^3$$

If the mean distance between the sun and different planets are R_1, R_2, R_3 and the periods are $T_1, T_2, T_3 \ldots$

then, $$\frac{T_1^2}{R_1^3} = \frac{T_2^2}{R_2^3} = \frac{T_3^2}{R_3^3} = \text{constant}.$$

Thank you
for your kind attention