

Discrete

Date 28 08 2025

Graph

Degree 2

Isometric

Isomorphic graph:

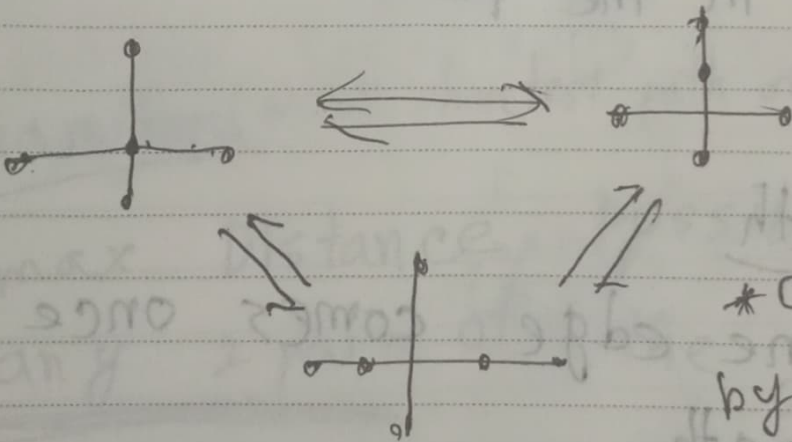
Degree 4

→ Vertex same

→ Edge "

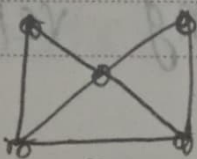
→ Degree "

Homomorphic graph

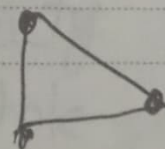
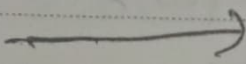


* Originate
by deviding
the edges by
one more vertex
from a base
graph

subgraph



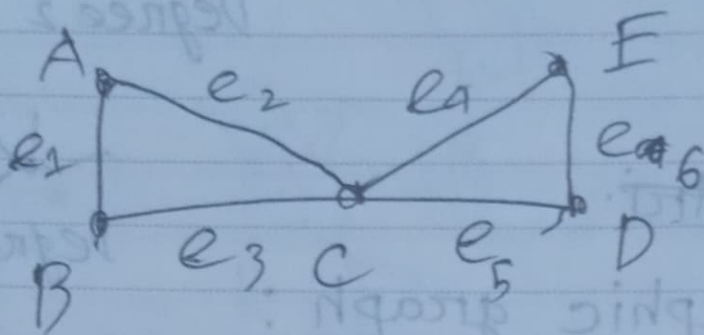
G_1



subgraph of G_1

Path

sequence of vertex, edge ~~from~~ between 2 vertex



A, e_2, C, e_5, D or

$A, e_2, C, e_4, E, e_6, D$

simple path

If one vertex comes one time only in the path

Trail path

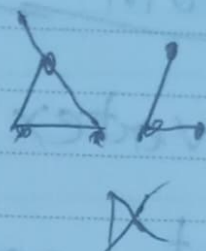
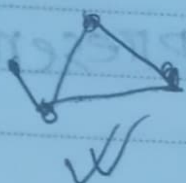
If one edge comes once in the path.

Cycle

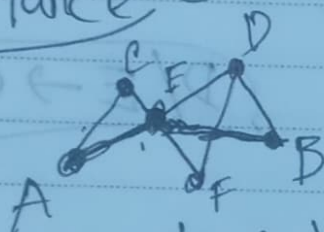
can loop backs to same vertex starting vertex.

Connectivity

Can



Distance (shortest path)



shortest path = Distance = 2

(AE, EB)

Diameter (max shortest path of any pair)

max Distance possible from
any 2 pair of the graph.

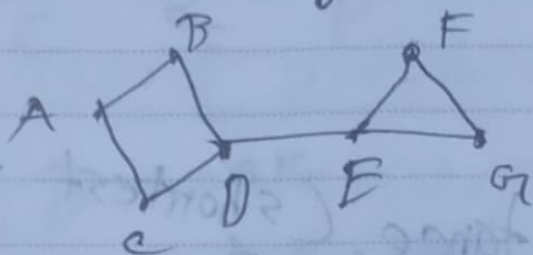
Distance \rightarrow Between 2 vertex

Diameter \rightarrow Property of the
whole graph.

~~# Grease~~

Cutpoint (A vertex)

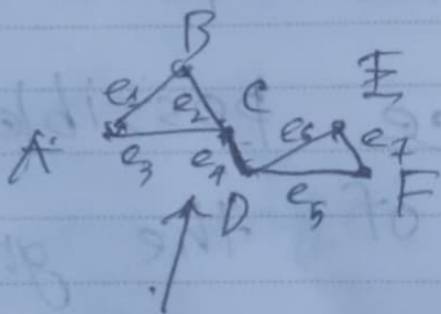
A vertex whose presence affect connectivity.



{D/E} → Cut point

Grease: (An edge)

cutpoint equivalent of for edge

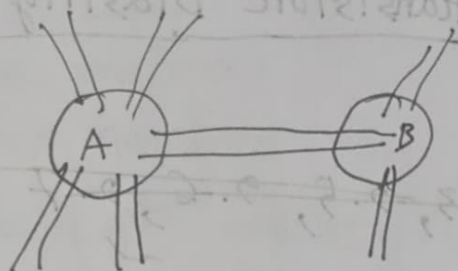


e4 grease

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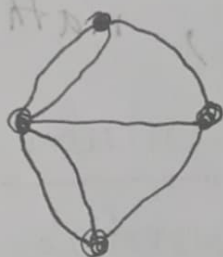
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* can you traverse each bridge ~~at least~~ only once.?

→ Ans: NO



* only possible if:

→ All vertex need to have even degree.

or,

→ only 2 odd degree vertex

Eulerian graph:

→ A graph where ^{it is possible to} ~~only one~~ traversing all ~~vertex~~ ^{edge} ^{but} only once each and ~~reaching~~ starting point.

Hamilton graph:

→ Same as Eulerian graph but ~~traverse~~ vertex.

→ only if:
if $n \geq 3$ and $n \leq \text{degree}(\text{vertex})$ for each vertex

Circuit

→ Can come back to same node after traversing

Path :

→ Cannot come back to same node but fulfill Eulerian/Hamilton condition.

weighted graph

→ has cost for each vertex

complete graph

→ All node connected with all node

Regular graph

→ when all vertex has same degree = k
the it called k -regu graph.
denoted by $(k\text{-regular})$

* All complete graph \equiv regular graph
,, regular ,, \equiv complete ,,

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Tree

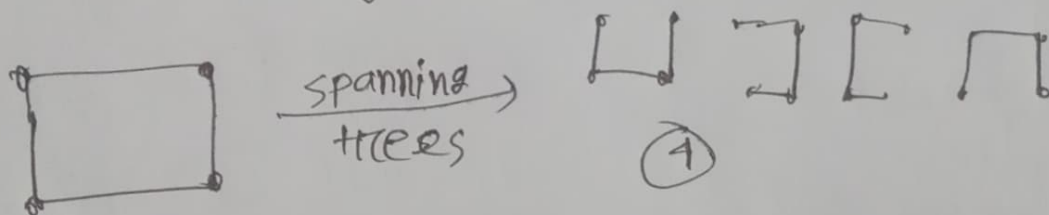
All Tree are Graph
Not all Graph are Tree

ei Graph \rightarrow Has cycle

Tree \rightarrow No cycle

\rightarrow No cycle Break any cycle

The number of Tree possible by
changi ~~removing~~ ^{removing} edges of graph is
called spanning tree



Spanning tree algorithms (Minimum Spanning tree)

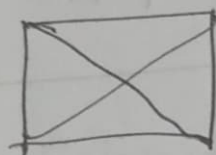
\rightarrow Prim's algorithm \rightarrow Find minimum spanning tree from weighted graph, ~~check~~ check each removable edge.

\rightarrow Kruskal's \rightarrow Same as prim but from reverse, returns only the nonremovable edges.

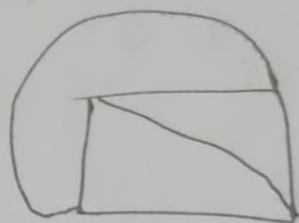
removable edge \rightarrow does not disconnect graph

Planer graph:

→ Can be redrawn/redrawn without crossing edge.



Planer
representation



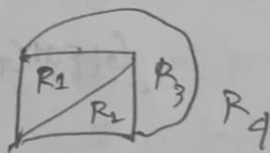
* map:

* A planer ^{Graph} representation is called map

* region:

in map

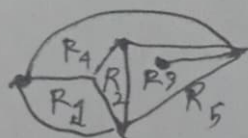
∴ Bounded cycles are called region



* Degree of map region:

→ ~~number of edges that form the boundary~~
of a region

→ number of edge traversal needed to finish a cycle in a bounded region



deg($R_1 = 3$

$R_2 = 3$

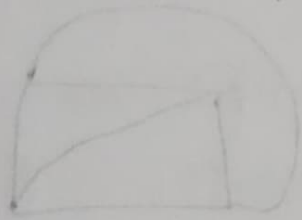
$R_3 = 5$

$R_4 = 4$

$R_5 = 3$

* Number of region in a map/Planer graph,

Euler's Theorem,



$$V - E + R = 2$$

$V \rightarrow$ vertex

$E \rightarrow$ Edge

$R \rightarrow$ Region

but, condition,

possible

* Condition of planer graph

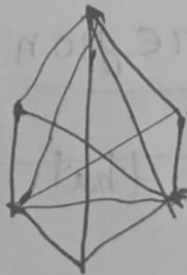
if, $P \geq 3$

if, $E \geq 3P - 6$

Graph is planer

Not 100% accurate, only for simple cases.

$P \rightarrow$ vertex
 $E \rightarrow$ edge



$$P = 6 \geq 3$$

$$E = 12 \geq 3P - 6$$

So planer representation possible

* condition of non-planar graph

→ If and only if the graph contains subgraph homomorphic to $K_{3,3}$ and K_5

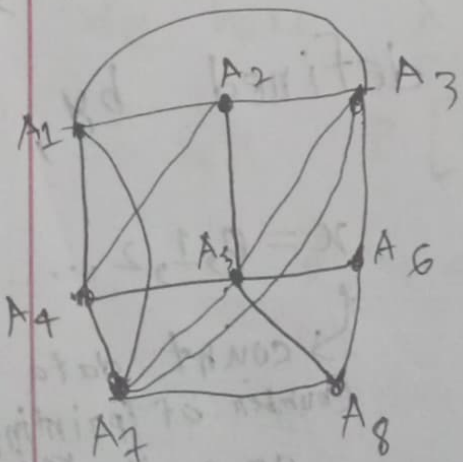
Graph Coloring

* Adjacent vertex in graph cannot have same color.

* Algorithm

→ Well's Panel's algorithm

↳ Determine minimum number of ~~color~~ colors needed to do graph coloring.



Step-1:

* sort vertex in descending order of degree

→ $A_5, A_3, A_7, A_1, A_2, A_4, A_6, A_8$

Step-2:

start coloring for start and also give same color to non connected graph.

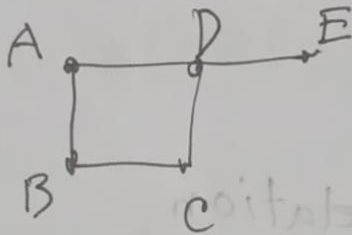
$A_5 \quad A_3 \quad A_7 \quad A_1 \quad A_2 \quad A_4 \quad A_6 \quad A_8$
 $c_1 \quad c_2 \quad c_3 \quad c_1 \quad c_3 \quad c_2 \quad c_3 \quad c_2$

3 colors

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Graph representation in Data Structure



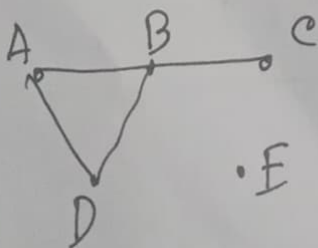
$$a_{ij} = \begin{cases} 1 & \text{if } v_i \text{ is adjacent to } v_j \\ 0 & \text{otherwise} \end{cases}$$

	A	B	C	D	E
A	0	1	1	0	0
B	1	0	1	0	0
C	0	1	0	1	0
D	1	0	1	0	1
E	0	0	0	1	0

$v_i \rightarrow$ number of edge
 $v_x \rightarrow x^{\text{th}}$ vertex

①
 * Array representation

② * Linked List representation



vertex	adjacent list
A	B, D
B	A, C, D
C	B
D	A, B
E	\emptyset