A relation R on a net A in called to be reflexive if $\forall a \in A, (a,a) \in R$

Ex: $A = \{a, b, c\}$

$R = \emptyset$ Not Reflexive

$R = A \times A$ Reflexive

$R = \{(a, a), (b, b), (c, c)\}$ Reflexive

$R = \{(a, b), (b, c), (a, a)\}$ Not Reflexive

$R = \{(a, b), (a, c), (a, a), (b, c), (b, b), (c, c), (b, a)\}$ Reflexive

Irreflexive Relation:

A relation R on a net A in called to be reflexive if $\forall a \in A, (a,a) \notin R$.

Ex: $A = \{a, b, c\}$

$R = \emptyset$ Irreflexive

$R = A \times A$ Not Irreflexive

$R = \{(a, a), (b, b), (c, c)\}$ Not Irreflexive

$R = \{(a, b), (b, c), (a, a)\}$ Not Irreflexive

$R = \{(a, b), (a, c), (a, a), (b, c), (b, b), (c, c), (b, a)\}$ Not Irreflexive

$R = \{(a, c), (b, a), (b, c)\}$ Irreflexive

Symmetric Relations

A Relation R on a set A is called to be symmetric if $\forall a, b \in A$, $(a,b) \in R$, then $(b,a) \in R$

* $A = \{a, b, c\}$ ~~Symmetric~~

$R = \{(a,b),(b,a)\}$ Symmetric

$R = \{(b,c),(c,b),(b,b),(c,c)\}$ Symmetric

$R = \{(a,a),(b,b),(c,c)\}$ Symmetric

$R = \phi$ Symmetric

$R = A \times A$ Symmetric

$R = \{(a,b),(b,c),(a,c)\}$ Not symmetric

$R = \{(a,b),(b,a),(a,c)\}$ Not symmetric

Antisymmetric Relations

A relation R on a set A is called to antisymmetric if $\forall a, b \in A$, $(a,b) \in R$, $(b,a) \in R$, then $a = b$

* $A = \{a, b, c\}$

$R = \{(a,b),(b,a),(a,c)\}$ Not Antisymmetric

$R = \{(a,b),(a,a),(b,b)\}$ Antisymmetric

$R = \{(a,a),(b,b),(c,a)\}$ Antisymmetric

$R = \{(a,b),(b,a),(b,c),(c,c)\}$ Not Antisymmetric

$R = \phi$ Antisymmetric

$R = A \times A$ Not Antisymmetric

$R = \{(a,b),(b,c),(a,c),(c,a),(a,a),(c,c)\}$ Not Antisymmetric

Closure Properties:

Reflexive closure of $R = R \cup \Delta_A$

where $\Delta_A = \{(a,a) : a \in A\}$

⊛ $A = \{1, 2, 3, 4\}$

$R = \{(1,1), (1,3), (2,4), (3,1), (3,3), (4,3)\}$

reflexive $(R) = R \cup \Delta_A$

$= \{(1,1), (1,3), (2,4), (3,1), (3,3), (4,3)\} \cup \{(1,1), (2,2), (3,3), (4,4)\}$

$= \{(1,1), (2,2), (3,3), (4,4), (1,3), (2,4), (3,1), (4,3)\}$

symmetric closure of $R = R \cup R^{-1}$

⊛ $A = \{1, 2, 3\}$

$R = \{(1,1), (1,3), (1,2), (2,2)\}$

$R^{-1} = \{(1,1), (3,1), (2,1), (2,2)\}$

symmetric $(R) = R \cup R^{-1}$

$= \{(1,1), (1,3), (1,2), (1,2), (2,2)\} \cup \{(1,1), (3,1), (2,1$ $(2,2)\}$

$= \{(1,1), (1,3), (1,2), (1,2), (3,1), (2,1)\}$

Trannitive clonuren of $R = R \cup R^2 \cup \dots \cup R^n$

Here $n =$ number of element.

$R^n = R^{n-1} \cdot R$

* $A = \{1, 2, 3\}$

$R = \{(1,2), (2,3), (3,3)\}$

Here, ~~are~~ number of the element $n = 3$

$R^2 = R \circ R = \{(1,3), (2,3), (3,3)\}$

$R^3 = R^2 \cdot R = \{(1,3), (2,3), (3,3)\}$

$\therefore$ Trannitive $(R) = R \cup R^2 \cup R^3$

$= \{(1,2), (2,3), (3,3)\} \cup \{(1,3), (2,3), (3,3)\} \cup \{(1,3), (2,3), (3,3)\}$

$= \{(1,2), (2,3), (3,3), (1,3)\}$

Eg) Equivalence Relation

A relation R on S in an equivalence relation if R in Reflexive, symmetric, and trannitive.

⊛ $A = \{1, 2, 3\}$

$R = \{(1,1), (2,2), (3,3)\}$ R✓ S✓ T✓ → Equivalence

$R = \{(1,1), (2,2), (3,3), (2,1), (1,2)\}$ R✓ S✓ T✓ → Equivalence

$R = \{(1,1), (2,2), (3,3), (3,2), (1,3)\}$ R✓ S× T✓ → NoT Equivalence

$R = \emptyset$ R× S✓ T✓ → Equivalence

chapter-3

function: A function is a relation between sets that associates to every element of a first set exactly one element of the second set.

$f: X \to Y$ and $g: Y \to Z$

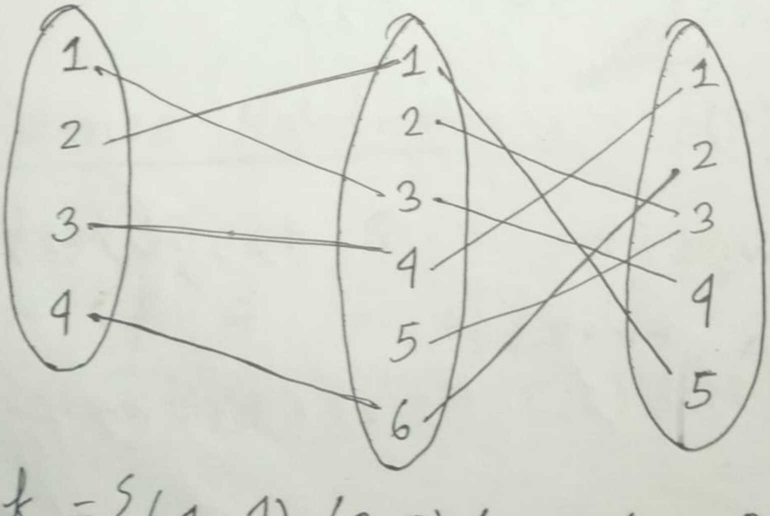
$$\begin{tikzcd} X \arrow[r, "f"] \arrow[rr, bend right] & Y \arrow[r, "g"] & Z \end{tikzcd}$$

$h(x) = g(f(x))$

✱ Composition of functions on a finite set If $f = \{(1,3),(2,1),(3,4),(4,6)\}$ and $g = \{(1,5),(2,3),(3,4),(4,1),(5,3),(6,2)\}$ $g \cdot f = ?$

Ans:

$g \cdot f = g(f(x))$



$g \cdot f = \{(1,4),(2,5),(3,1),(4,2)\}$

✱ if $g(x) = 1-x$, $h(x) = x/(x-1)$, then $g(h(x))/h(g(x))$ is :

$$g(x) = 1 - x$$

$$h(x) = \frac{x}{x-1}$$

$$g(h(x)) = 1 - \frac{x}{x-1} = \frac{x-1-x}{x-1} = \frac{-1}{x-1}$$

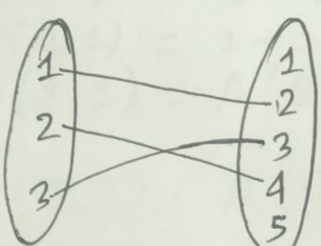
$$h(g(x)) = \frac{1-x}{1-x-1} = \frac{x-1}{x}$$

$$\therefore g(h(x))/h(g(x)) = \frac{\frac{-1}{x-1}}{\frac{x-1}{x}} = \frac{-1}{x-1} \times \frac{x}{x-1}$$

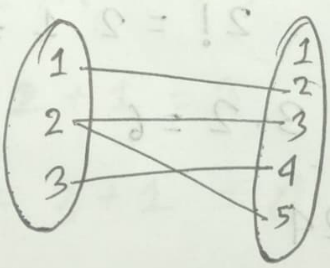
$$= \frac{-x}{(x-1)^2}$$

One-one function :

A function $f : A \to B$ is called one-to-one if different elements in the domain A have distinct images in B.



one-one



Not one-one

Onto function:

A function f from set A to set B is onto if each element of B is mapped to atleast one element of A.

Factorial Function

a) If $n = 0$ then $n! = 1$

b) If $n > 0$ then $n! = n(n-1)!$

Calculate: $4!$

Ans:

1) $4! = 4 \cdot 3!$
2) $3! = 3 \cdot 2!$
3) $2! = 2 \cdot 1!$
4) $1! = 1 \cdot 0!$
5) $0! = 1$
6) $1! = 1 \cdot 1 = 1$
7) $2! = 2 \cdot 1 = 2$
8) $3! = 3 \cdot 2 = 6$
9) $4! = 4 \cdot 6 = 24$

Febonacci sequence :

a) If $n=0$ or $n=1$ then $F_n = n$

b) If $n>1$ or $F_n = F_{n-2} + F_{n-1}$

Ackermann Function

a) If $m=0$ then $A(m,n) = n+1$

b) If $m \neq 0$ but $n=0$ then $A(m,n) = A(m-1, 1)$

c) If $m \neq 0$ and $n \neq 0$ then $A(m,n) = A(m-1, A(m, n-1))$

$A(1,3)$

$$A(1,3) = A(1-1, A(1, 3-1))$$
$$= A(0, A(1,2))$$

$$A(1,2) = A(1-1, A(1, 2-1))$$
$$= A(0, A(1,1))$$

$$A(1,1) = A(1-1, A(1, 1-1))$$
$$= A(0, A(1,0))$$

$$A(1,0) = \cancel{0+1 = 1}\ A(1-1, 1) = A(0,1)$$

$$A(0,1) = 1+1 = 2$$

$$\therefore A(1,1) = A(0,2) = 2+1 = 3$$

$$\therefore A(1,2) = A(0,3) = 3+1 = 4$$

$$\therefore A(1,3) = A(0,4) = 4+1 = 5$$

Chapter - 4

Conjunction, $p \wedge q$ (AND)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction $p \vee q$ (OR)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation (NOT)

p	$\neg p$
T	F
F	T

Conditional and Biconditional Statement

$p \rightarrow q$ means "p implies q" / "p only if q"

$p \leftrightarrow q$ means "p if and only if q"

$$p \rightarrow q \equiv \cancel{\neg(p \vee q)}\ \neg p \vee q$$

$$p \leftrightarrow q \equiv pq + \bar{p}\bar{q} = (p \wedge q) \vee (\neg p \wedge \neg q)$$

Arguments

The arguments $P_1, P_2, \ldots, P_n \vdash Q$ is valid if and only if the proposition $(P_1 \wedge P_2 \wedge P_3 \wedge \ldots \wedge P_n) \rightarrow Q$ is Tautology.

* Test the validity of each argument:

If it rains, Erik will be sick

It did not rain

Erik was not sick

Ans:

Here,

p = it rain

q = Erik will be sick

$\neg p$ = it did not rain

$\neg q$ = Erick was not sick

according to context,

$$p \rightarrow q, \neg p \vdash \neg q$$

so, $(p \to q) \wedge \neg p \to \neg q$

test:

p	q	(p	→	q)	∧	¬p	→	¬q
T	T	T	T	T	F	F	T	F
T	F	T	F	F	F	F	T	T
F	T	F	T	T	T	T	F	F
F	F	F	T	F	T	T	T	T

The ~~ten~~ argument is not valid.
The proposition is not a tautology.

Universal Quantifier

Let $p(x)$ be a propositional function defined on a set A.

$\forall x\, P(x)$ means For every x in A, $P(x)$ is a true statement.
simply, "For all x, $P(x)$"

- $(\forall n \in N)(n+4 > 3)$ is true
  $\{n : n+4 > 3\} = \{1, 2, 3, \ldots\}$
- $(\forall n \in N)(n+2 > 8)$ is False
  $\{n : n+2 > 8\} = \{7, 8, 9, \ldots\}$

Exintential Quantifier

Let $p(x)$ be a proponitional function defined on a net A, connider the exprennion.

$\exists x ; P(x)$ or $(\exists x \in A)\, P(x)$ There exintn atleant an x in A such that $p(x)$ in a ture ntatement.

- The proponition $(\exists n \in N)(n+4 < 7)$ in true

$$\{n : n+4<7\} = \{1, 2\} \neq \emptyset$$

- The proponition $(\exists n \in N)(n+6<4)$ in Falne

$$\{n : n+6<4\} = \emptyset$$

Negation of Quantified Statement

- $\neg(\forall x \in A)\, P(x) \equiv (\exists x \in A) \neg P(x)$
- $\neg(\exists x \in A)\, P(x) \equiv (\forall x \in A) \neg P(x)$

Negating:

$\forall x\, \exists y : P(x,y)$ in $\exists x\, \forall y : \neg P(x,y)$

- $\forall x\, \exists y\, (x * y = 3)$

$= \neg \forall x\, \exists y (x * y = 3)$

$= \exists x\, \forall y\, \neg(x * y = 3)$

- $\exists x\, \forall a\, \exists y\, (F(x,y) \wedge A(y,a))$

$\Rightarrow \neg \exists x\, \forall a\, \exists y\, (F(x,y) \wedge A(y,a))$

$\Rightarrow \forall x\, \exists a\, \forall y\, \neg(F(x,y) \wedge A(y,a))$

$= \forall x \, \exists a \, \forall y \, (\neg F(x,y) \wedge \neg A(y,a))$

OFF topic

p	q	r	$(\neg p$	$\longleftrightarrow$	$r)$	$\wedge$	$(p$	$\longleftrightarrow$	$q)$
T	T	T	F	T	T	T	T	T	T
T	T	F	F	T	F	T	T	T	T
T	F	T	F	T	T	F	T	F	F
T	F	F	F	T	F	F	T	F	F
F	T	T	T	T	T	F	F	F	T
F	T	F	T	F	F	F	F	F	T
F	F	T	T	T	T	T	F	T	F
F	F	F	T	F	F	F	F	T	F

$cnF = p \wedge q \wedge r$

✻ Suppose repetition are not permitted.

a) How many three digit numbers can be formed from the six digits 2, 3, 9, 5, 6, 7

b) How many of them are less 400?

c) How many of them are even?

Ans:

a)

$$\underset{5}{\boxed{6}} \quad \underset{5}{\boxed{5}} \quad \underset{4}{\boxed{4}}$$

b)

$$\underset{4}{\boxed{2}} \quad \underset{5}{\boxed{5}} \quad \underset{4}{\boxed{4}}$$

c)

$$\underset{5}{\boxed{5}} \quad \underset{4}{\boxed{4}} \quad \underset{4}{\boxed{2}}$$

✻ MISSISSIPPI

$$\frac{11!}{4!\,4!\,2!} = \frac{\cancel{1}\cdot\cancel{2}\cdot\cancel{3}\cdot\cancel{4}\cdot 5\cdot \overset{3}{\cancel{6}}\cdot 7\cdot \overset{2}{\cancel{8}}\cdot 9\cdot \overset{5}{\cancel{10}}\cdot 11}{\cancel{1}\cdot\cancel{2}\cdot\cancel{3}\cdot\cancel{4}\times 1\cdot\cancel{2}\cdot\cancel{3}\cdot\cancel{4}\times 1\cdot\cancel{2}}$$

$$= \frac{5\cdot 7\cdot 9\cdot 11}{1}$$

$$= 34650$$

⊛ A bag contains 6 white marbles and five red marbles. Find the number of ways four marbles can be drawn from the bag if a) they can be any color

b) two must be white and two red.

c) they must all be of the same color.

<u>Ans:</u>

a)

$$\binom{11}{4} = \frac{11!}{4!\,7!} = \frac{11\cdot 10\cdot \cancel{9}^{3}\cdot \cancel{8}^{4}\cdot 7!}{1\cdot \cancel{2}\cdot \cancel{3}\cdot \cancel{4}\cdot 7!} = 330$$

b)

$$\binom{6}{2} \times \binom{5}{2}$$

c)

$$\binom{6}{4} + \binom{5}{4}$$

⊛ Out of 12 employees, a group of four trainees to be sent for 'Software testing and QA' training of one month.

a) How many ways can the four employee be selected?

b) What if there are two employees who refuse to go together for training?

<u>Ans:</u>

a) $^{12}C_4 = \dfrac{12!}{4! \times 8!}$

b)

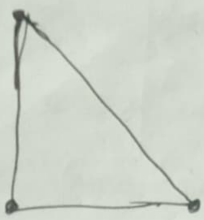
Both A and B do not go = ${}^{10}C_4$

A select, B refunen = ${}^{10}C_3$

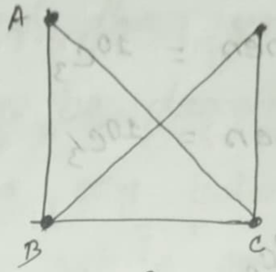
B select, A refunen = ${}^{10}C_3$

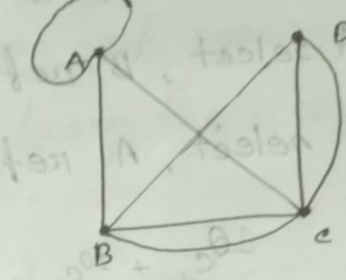
$${}^{10}C_4 + {}^{10}C_3 + {}^{10}C_3$$

Graph



H

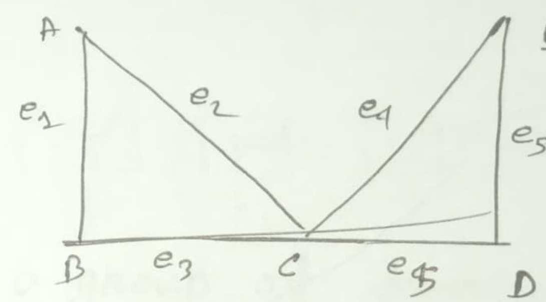




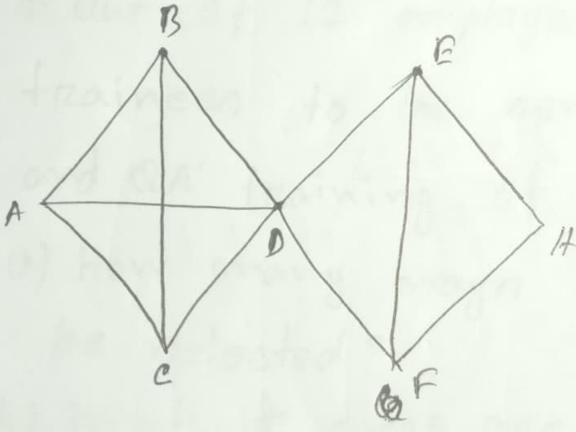
G

H, G এর Subgraph

$A \rightarrow D$



A, e_1, B, e_3, C, e_5, D



$d(A,F) = 2$

$d(A,H) = 3$

A, H = 3

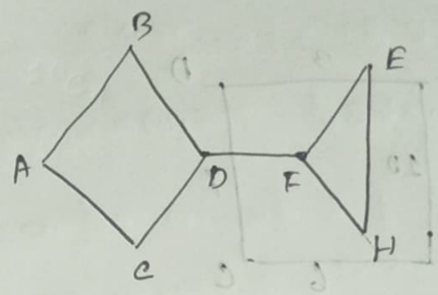
A, E = 2

A, F = 2

C, H = 3

B, H = 3

$d(A, H) =$



যেটি vertex বাদ দিলে disconnect হয় তা cutpoint.

যেটি edge বাদ দিলে disconnect হয়, তা bridge.

a)

$f \cdot g = f(g(x)) = y$

$y = f(2, 1, 3) = \{(1, 3), (2, 2), (3, 1)\}$

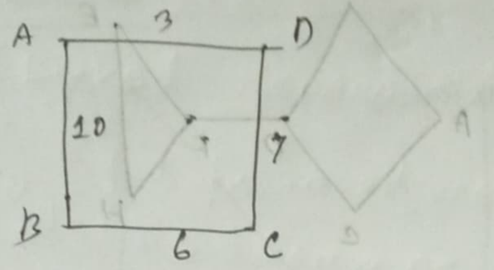
$g \cdot f = g(f(x))$

$f(x) = X = 1,\ y = 2,\ X = 2,\ y = 3,\ X = 3,\ y = 1$

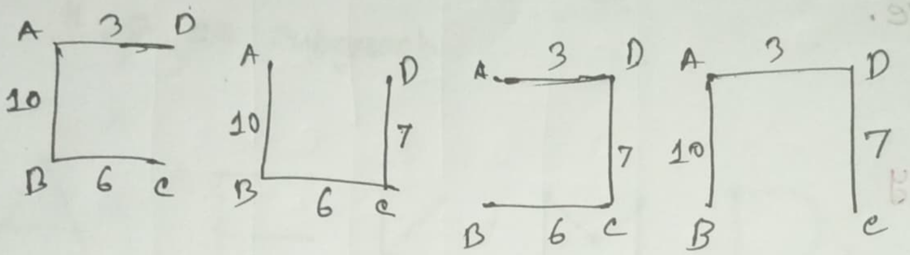
$Y = g(y) = \{(1, 1), (2, 3), (3, 2)\}$

$f \cdot g \cdot h = f(g(h(x)))$

$h(x) = X = 1\ \ y =$



Spanning tree



minimum spaning tree : ~~19~~ $3+7+6=16$

~~prim~~ prim's Algorithm :

9 8 7 6 5 4 4 4 3 3 2 1

× × × × × ✓ × ✓ ✓ ✓ ✓ ✓

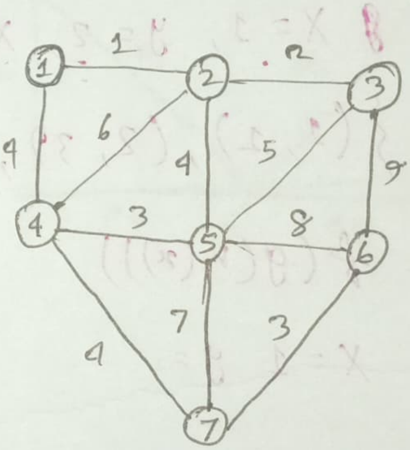
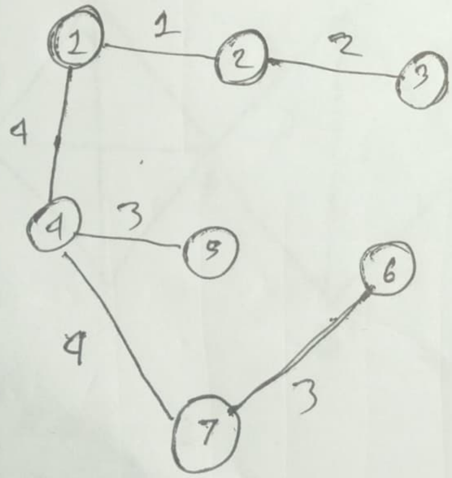


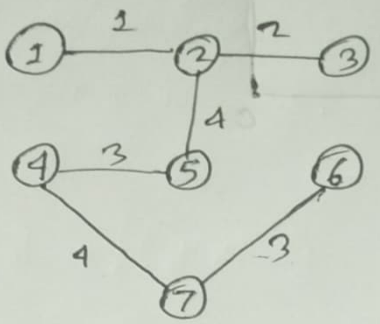
Fig-1



Kruskal Algorithm : une Fig - 1

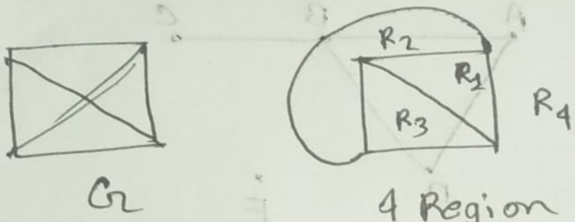
1 . 2 3 3 4 4 4 5 6 7 8 9

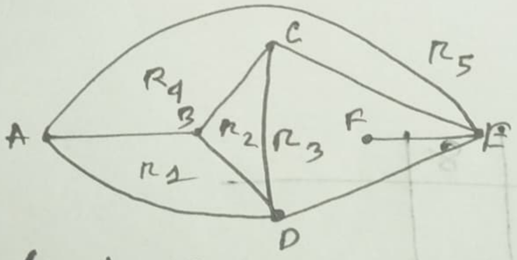
✓ ✓ ✓ ✓ ✓ × ✓ × × × × ×



Planar graph

map





$deg(r_1) = 3$

$deg(r_2) = 3$

$deg(r_3) = 5$

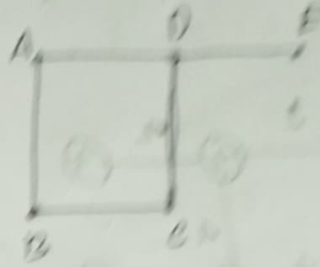
$deg(r_4) = 4$

$deg(r_5) = 3$

Euler Formula

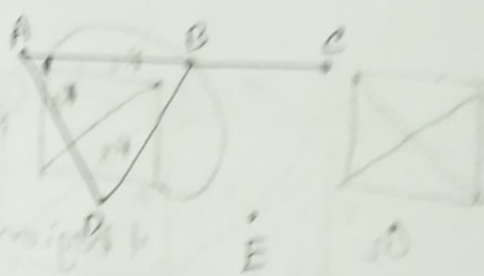
Linked Representation of a graph G

$a_{ij} = \begin{cases} 1 & \text{if } v_i \text{ is adjacent to } v_j \\ 0 & \text{, otherwise} \end{cases}$



	A	B	C	D	E
A	0	1	0	1	0
B	1	0	1	0	0
C	0	1	0	1	0
D	1	0	1	0	1
E	0	0	0	1	0

Vertex	adj list
A	B, D
B	A, C, D
C	B
D	A, B
E	φ



	1	2	3	4	5	6	7	8
vertex	A	B	C	D	E	F		
Next-V								
PTR								