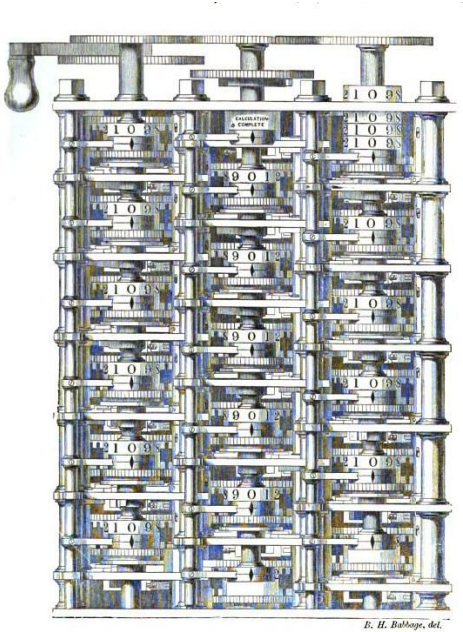

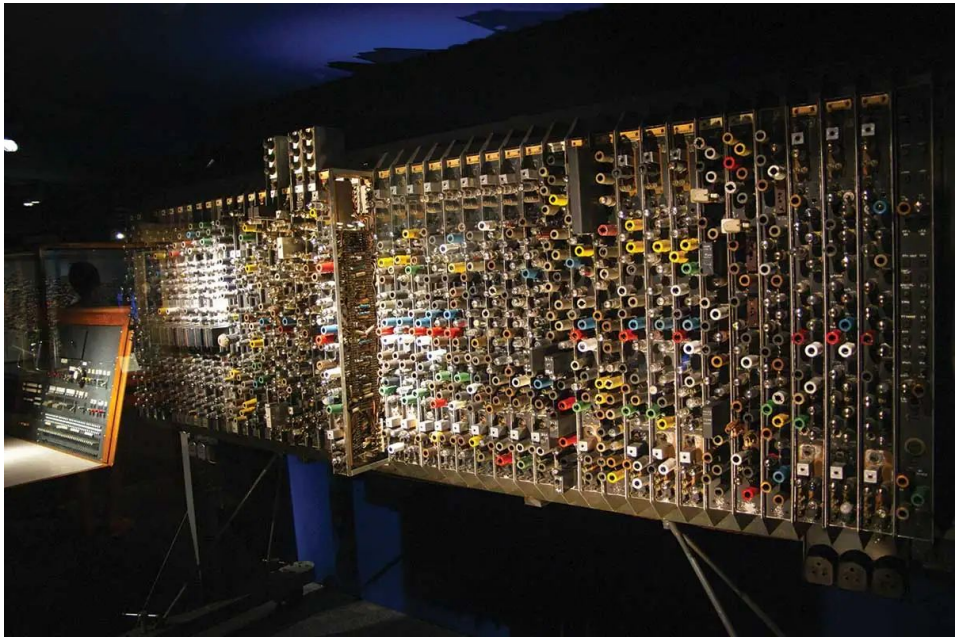

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A Brief History of Programming Languages

of the last 100 years

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0. Before the beginning

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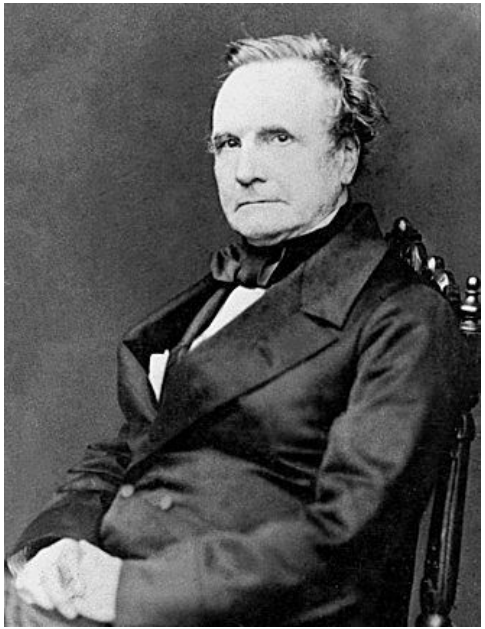
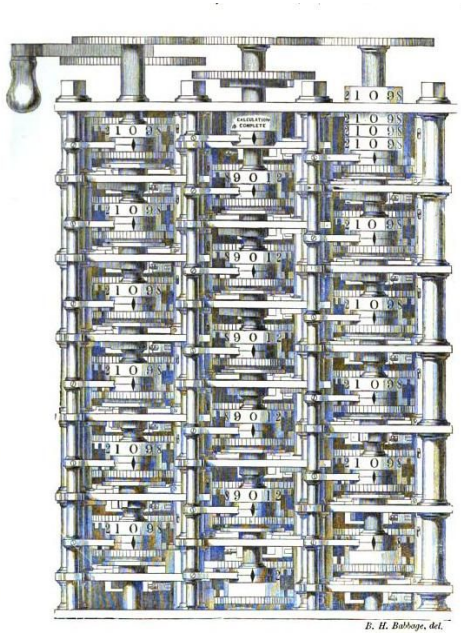
Charles Babbage and his analytical engine





- First mechanical computers
- Costed £17,000, was terminated before completion after 10 years.
- British Treasury lost confidence.
- Goal was to compute polynomial functions.

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Ada Lovelace and her tabulation method

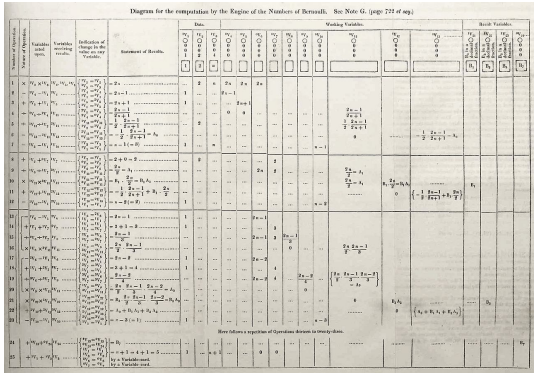

Diagram for the computation by the Engine of the Numbers of Bernoulli. See Note G. (page 722 et seq.)

- Recognized the potential of Babbage's analytical engine, and its far reaching consequences.
- Developed one of the first algorithms for the analytical engine to compute the Bernoulli number sequence.

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Ada Lovelace: deep learning and generated music

[The Analytical Engine] might act upon other things besides number, were objects found whose mutual fundamental relations could be expressed by those of the abstract science of operations, and which should be also susceptible of adaptations to the action of the operating notation and mechanism of the engine…

Supposing, for instance, that the fundamental relations of pitched sounds in the science of harmony and of musical composition were susceptible of such expression and adaptations, the engine might compose elaborate and scientific pieces of music of any degree of complexity or extent.

– Countess of Lovelace Augusta Ada King

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1. Discovery of Computation

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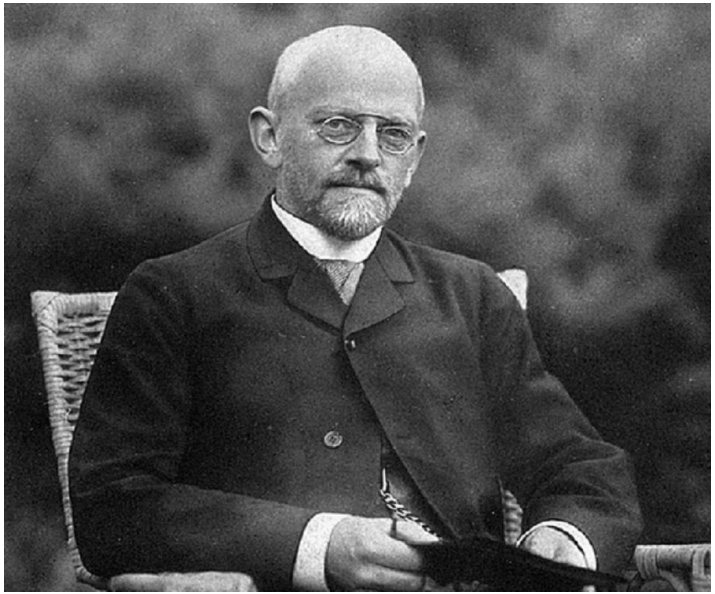
1900: David Hilbert



He is recognized as one of the most influential and universal mathematicians of the 19th and early 20th centuries.

Wikipedia

In 1900, Hilbert formulated a collection of mathematical problems for the community to work on. Collectively, they are known as the Hilbert's problems.

Hilbert's 10th Problem

- Diophantine equation is an equation whose left-hand side is a polynomial with only integer coefficients and finitely many unknowns to the integer power. The right-hand side is zero.
- For example: $x^2 + 78\ xy - y^6z = 0$
- Hilbert asks if there is a procedure to decide if there are integer solutions of an arbitrarily given diophantine equation.
- It's worth noting that if we relax the solution space to real numbers, then it can be decided. See SymPy for Python.

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Hilbert's Entscheidungsproblem in 1928

Definition:

Given first order logic and a set of consistent axioms, is it possible to always decide if a sentence is true or false.

Before the 10th problem was better understood, Hilbert posed a far more grand problem known as the Entscheidungsproblem in 1928.

It questions whether solutions to mathematical problems can be procedurally derived (always) if the mathematical system is properly well-defined.

1931: Kurt Gödel

Considered along with Aristotle and Gottlob Frege to be one of the most significant logicians in history, Gödel had an immense effect upon scientific and philosophical thinking in the 20th century.

Wikipedia

Kurt Gödel, in 1931, three years after Hilbert's formulation of the Entscheidungsproblem came up with the negative result.

It is possible to assert a set of consistent axioms, and pose a question in first-order logic that cannot be decided (to be true or false). Thus the logical sentence is unsolvable.

Such axioms are called incomplete.

Gödel's Number

- Gödel not only proved a negative result to the Entscheidungsproblem, he did so by construction.
- It's been well-known that integer arithmetics is axiomatizable in first-order logic.
- Gödel formulated a logical sentence that describes a specific property for a special number known as the Godel's number.
- He showed that such number cannot be derived by logic deduction, and yet it exists.

Thus ***Integer Arithmetic*** is an incomplete logical system. See discussion.

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Revisiting the diophantine equation, Hilbert's 10th Problem

Diophantine Equation:

Polynomials with integer coefficients and integer roots

Integer Arithmetic is incomplete:

- Many problems involving integer arithmetics can be solved easily.
- But **some** cannot be solved within mathematical logic.

Surprise:

The general diophantine equation problem is undecidable.

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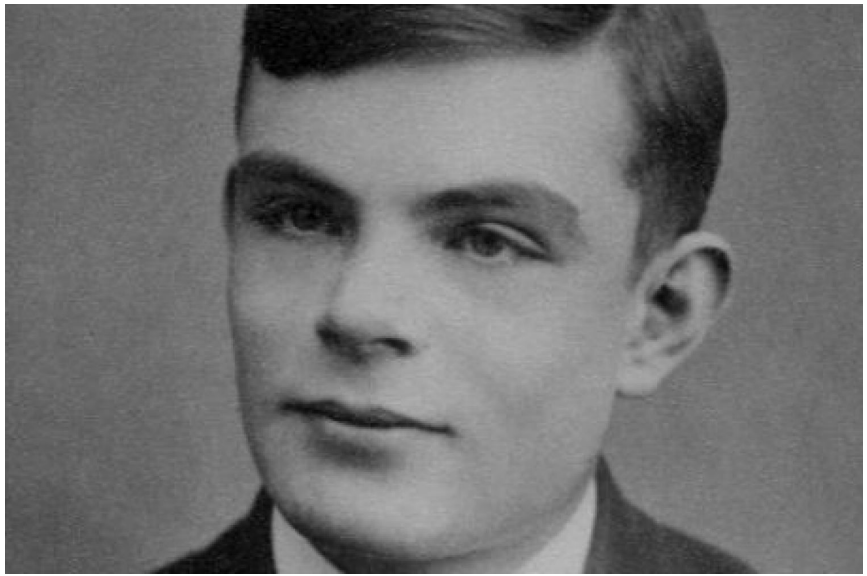
1936: Alan Turing, *Boy Genius*



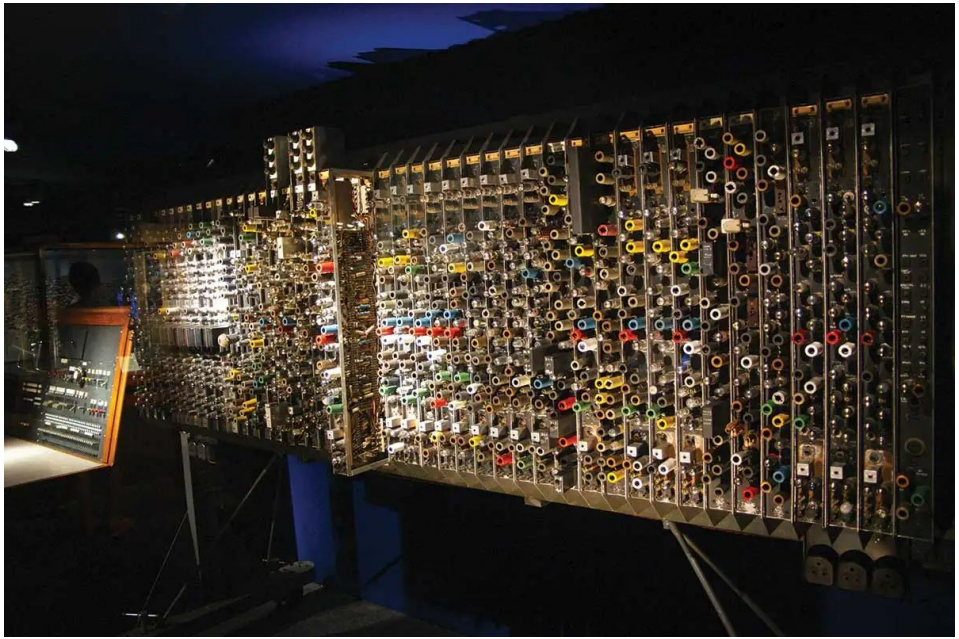
He is widely considered to be the father of theoretical computer science and artificial intelligence

Wikipedia

1936: Alan Turing (at 24) formalized formal reasoning as *computation*, which is further concretely defined as a mechanical device, known today as the *Turing Machine* (TM).

The Turing Machine

With TM, Turing was able to reproduce the incompleteness result of Godel. Turing's proof of incompleteness is much closer to today's concept of **programming**.



1936: Alonzo Church

He is best known for the ***lambda calculus***, the Church–Turing thesis, proving the unsolvability of the ***Entscheidungsproblem***. Wikipedia

Alonzo Church in 1936 (the same year as TM) proposed a string rewrite system for formally reasoning about mathematical functions, their definitions and actions. The rewrite system is known las Lambda Calculus (LC).

By the way, Church was Turing's PhD advisor at Princeton University before WWII.

Lambda Calculus

- It was soon discovered that LC and TM are equivalent.
- Together, Church and Turing seem to agree that TM and LC model the most powerful computation possible. So, TM and LC are the universal upper bound on the ability of computation. This bound is called Turing Completeness.
- The belief that we cannot go beyond Turing Completeness is called Church-Turing Thesis.