

$$T(n) = 2T(n/2) + n^4$$

$$a = 2, b = 2, f(n) = n^4$$

$$n^{\log_b a} = n^{\log_2 2} = n$$

$$n^4 = \Omega(n^{\log_2 2 + 3})$$

$$af(n/b) \leq cf(n) \text{ for some constant } c < 1$$

case 3 of the master theorem applies

$$T(n) = \Theta(n^4)$$

$$T(n) = T(7n/10) + n$$

$$a = 1, b = 10/7, f(n) = n$$

$$n^{\log_b a} = n^{\log_{10/7} 1} = n^0 = 1$$

$$n = \Omega(n^{\log_{10/7} 1 + 1})$$

$$af(n/b) \leq cf(n) \text{ for some constant } c < 1$$

case 3 of the master theorem applies

$$T(n) = \Theta(n)$$

$$T(n) = 16T(n/4) + n^2$$

$$a = 16, b = 4, f(n) = n^2$$

$$n^{\log_b a} = n^{\log_4 16} = n^2$$

$$n^2 = \Theta(n^{\log_4 16})$$

case 2 of the master theorem applies,

$$T(n) = \Theta(n^2 \lg n)$$

$$T(n) = 2T(n/4) + \sqrt{n}$$

$$a = 2, b = 4, f(n) = \sqrt{n},$$

$$n^{\log_b a} = n^{\log_4 2} = \sqrt{n}.$$

$$\sqrt{n} = \Theta(n^{\log_4 2})$$

case 2 of the master theorem applies

$$T(n) = \Theta(\sqrt{n} \lg n)$$