Arithmetics

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Adding Church Numbers

The **Add** expression is a LC expression with two parameters:

Add = \m.\n. _____

We will using **Succ** expression to build up the **Add** expression.

Observe:

```
m + n = n + 1 + 1 + 1 + ... + 1
      = succ( ... succ(succ(succ(n)))))   m-times
      = ((m Succ) n)
```

Add	= \m.\n. (m **Succ** n)
	= \m.\n. (m (\n.\f.\x. f (n f x)) n)

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Multiplication

We can continue with the same type of iterative definition to build up **Mult** from **Add**.

Mult = \m.\n. _____

m * n = (0 + n + n + n + n ... + n)

We need an auxiliary function:

h(x) = x + n = (n **Succ** x)

As an LC expression,

h = \x. (n **Succ** x)

m * n = (0 + n + n + n + n ... + n)
= h(...h(h(h(0)))) *m-times*
= m h 0

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Multiplication

We can continue with the same type of iterative definition to build up **Mult** from **Add**.

Mult = \m.\n. _____

m * n = (0 + n + n + n + n ... + n)
= h(...h(h(h(0)))) *m-times*
= m h 0

where h = \x. (n **Succ** x)

Mult = \m.\n. (m (\x. n **Succ** x) 0)

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Power

We can further the exercise to define exponentiation: m^n

How do you define an iterative form of mn in terms of multiplication?

Power = \m.\n. _____

Power(m,n)

= 1 * n * n * n ...

= ...

Complete the rest as a challenge.