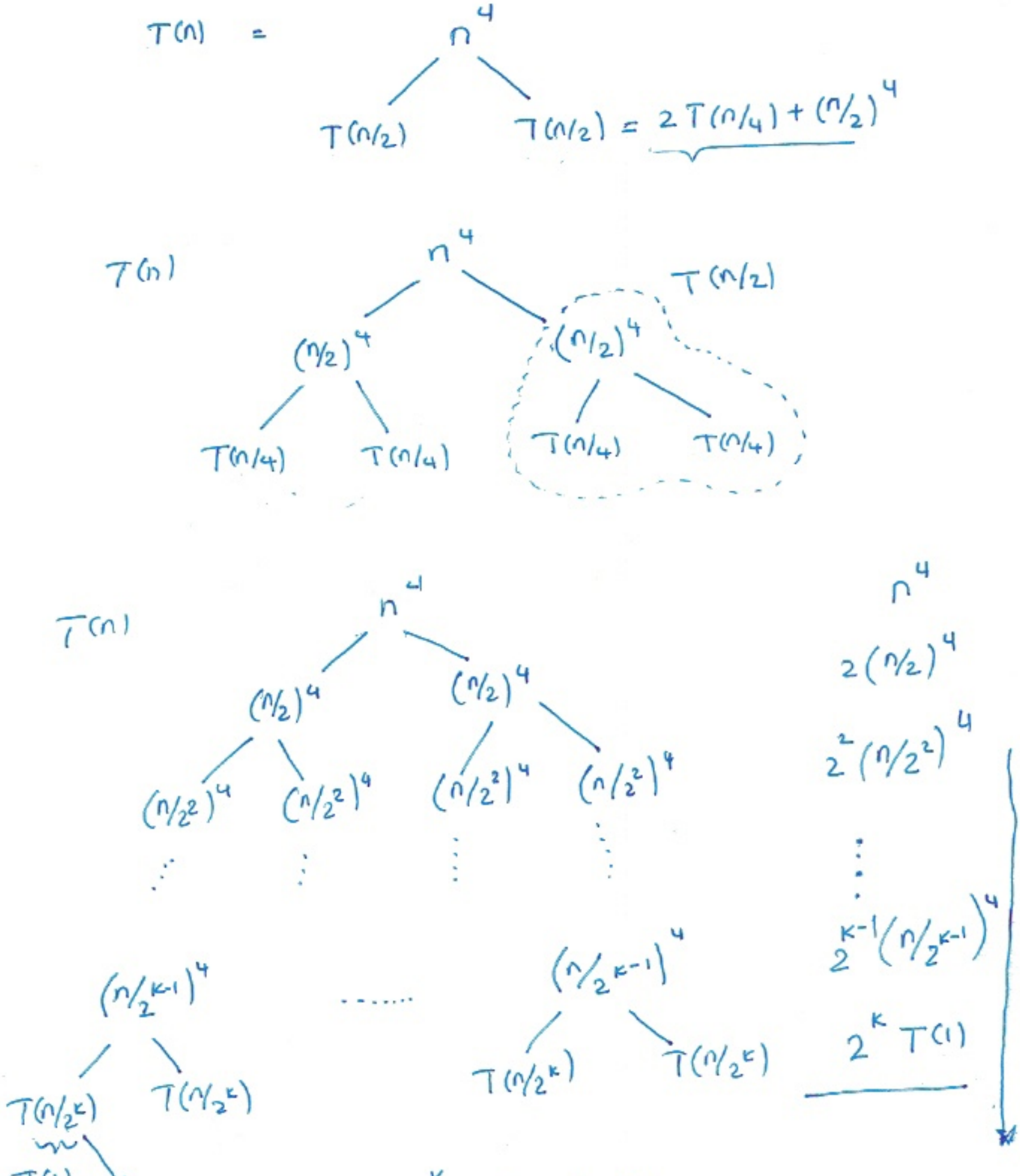
$T(n) = 2T(n/2) + n^4$

$T(1) = 1$

Recurrence Eq.

Goal: Find $T(n)$ in terms of n

Recursion Tree:



$n/2^k = 1 \Rightarrow n = 2^k \Rightarrow \boxed{k = \log_2 n}$

$$T(n) = n^4 + 2^1\left(\frac{n}{2^1}\right)^4 + 2^2\left(\frac{n}{2^2}\right)^4 + \cdots + 2^{k-1}\left(\frac{n}{2^{k-1}}\right)^4 + 2^k\,T(1)$$

$$(k = \log_2^n)$$

$$= n^4\left(1 + \frac{1}{2^3} + \frac{1}{2^6} + \cdots + \left(\frac{1}{2^{k-1}}\right)^3\right) + 2^k$$

$$= n^4\left(\frac{1-\left(\frac{1}{2^3}\right)^k}{1-\left(\frac{1}{2}\right)^3}\right) + 2^k$$

$$= n^4\left(\frac{1-\left(\frac{1}{2^3}\right)^{\log_2^n}}{1-\left(\frac{1}{2}\right)^3}\right) + 2^{\log_2^n}$$

$$\boxed{a^{\log_b^n} = n^{\log_b^a}}$$

$$= n^4\,\frac{\left(1-\left(\frac{1}{n^3}\right)^{\overbrace{\log_2^2}^{1}}\right)}{1-\left(\frac{1}{2}\right)^3} + 2^{\overbrace{\log_2^n}^{1}}$$

$$= n^4\,\frac{(1-n^{-3})}{1-\left(\frac{1}{2}\right)^3} + n^1 \qquad d = 4$$

$$\in \Theta(n^4)$$