

Towards Subtraction

Pairs

Pairs of Numbers

How do we encode data structures using LC expression?

Let's look at pairs: (a, b).

$\text{Pair}(a, b) = \lambda z. z a b$

Think of z as a selector function that can be used to pick a or b.

Let $p = \lambda z. z a b$.

Then observe:

- $p \text{ True} = a$
- $p \text{ False} = b$

Iterating over pairs of $(n-1, n)$

- Start with **Pair(0, 0)**.
- We want to have a generator of the following:
 $(0, 0) \rightarrow (0, 1) \rightarrow (1, 2) \rightarrow (2, 3) \rightarrow \dots \rightarrow (n-1, n)$

```
h = \p.  
  IfZero p[1] Then  
    Pair(0, 1)  
  Else  
    Pair(Succ(p[0]), Succ(p[1]))
```

```
h = \p. IfZero (p False) (\z. z 0 1) (\z. z (Succ p True) (Succ p False))
```

Extracting predecessor from $(n-1, n)$

So, we can compute $(n-1, n)$ with the help of h :

$$\lambda n. n \mathbf{h} (\lambda z. z \ 0 \ 0)$$

Now, we can extract the first component which is $(n-1)$:

$$\mathbf{Pred} = \lambda n. (n \mathbf{h} (\lambda z. z \ 0 \ 0) \mathbf{True})$$