Digital pendulum - Control Experiments

33-935/936-1V60

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Control in a MATLAB® Environment

Digital Pendulum

33-935/936-1V60 (For 33-005 Matlab 6 version)

Control Experiments

Feedback

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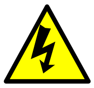

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1 Introduction

One of the simplest problems in robotics is that of controlling the position of a single link using a steering force applied at the end. Pole-balancing systems are impressive demonstration models of missile stabilisation problems. The crane used at shipping ports is an another example of non-linear electromechanical systems having a complex dynamic behaviour and creating challenging control problems. Mathematically both are just a pendulum in a stable or unstable position.

The **pendulum-cart set-up** consists of a pole mounted on a cart in such a way that the pole can swing free only in vertical plane. The cart is driven by a DC motor. To swing and to balance the pole the cart is pushed back and forth on a rail of limited length.

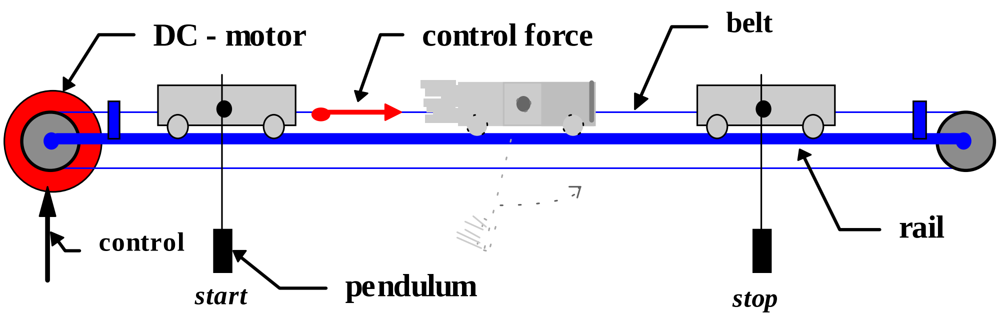


Figure 1: The pendulum / cart arrangement

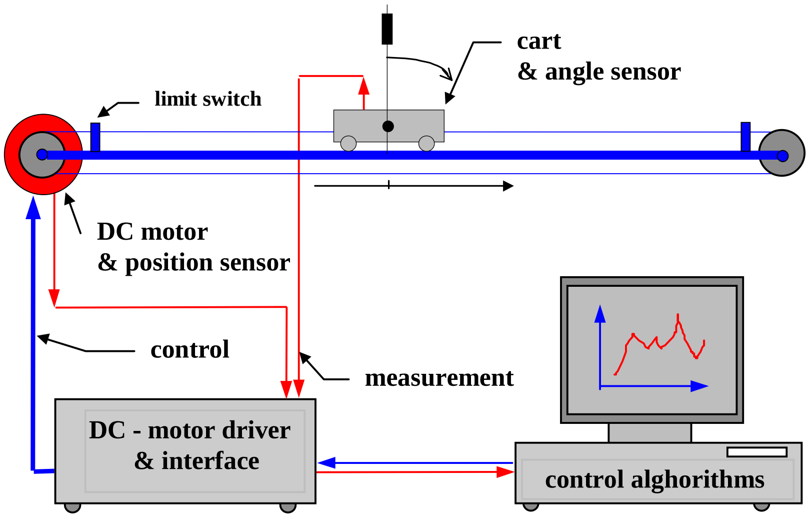


Figure 2: Pendulum Control System

The vertical stationary positions of the pendulum (upright and down) are equilibrium positions when no force is being applied. In the upright position a small deviation from it results in an unstable motion.

Generally the pendulum control problem is to bring the pole to one of the equilibrium positions and preferably to do so as fast as possible, with few oscillations, and without letting the angle and velocity become too large. After the desired position is reached, we would like to keep the system in this state despite random perturbations.

Manual control of the cart-pole system is possible only for simple tasks e.g. for moving the cart from one place on the rail to another. For more complicated tasks (such as stabilising the pole in an upright position) a feedback control system must be implemented as shown in Figure 2.

The purpose of the inverted pendulum control algorithm is to apply a sequence of forces of constrained magnitude to the cart, such that the pole starts to swing with an increasing amplitude without the cart overriding the ends of the rail.

Firstly the pole is swung up into the vicinity of its upright position and then, once this has been accomplished, the controller maintains the pole vertically and at the same time brings the cart back to the centre of the rail. Therefore two independent control algorithms are implemented for this purpose:

- a swinging algorithm, and
- a stabilising algorithm.

Only one of two control algorithms is active in each control zone. These zones are shown in Figure 3.

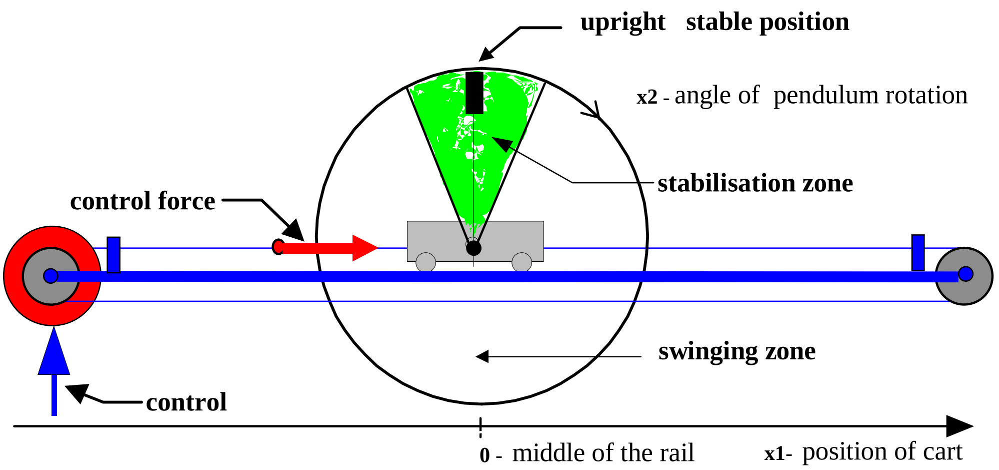


Figure 3: Activity zones of two control algorithms

The swinging control algorithm is a heuristic one, based on energy rules. The algorithm steers the pole up thus increasing its total energy. There is a trade-off between two tasks: to *swing* the pendulum to the upright position and to *centre* the cart on the rail.

Due to the presence of disturbances and parameter uncertainties, a robust behaviour is more important than the optimal character of the control strategy. The switching moments are calculated according to a simple rule. The characteristic feature of control is its “bang-bang” character. Swinging up the pole may result in over-reaching the upper unstable equilibrium point.

To achieve a “soft” landing in the vicinity of the upright position (the stabilisation zone in Figure 3), a routine called the soft landing arbiter checks whether the kinetic energy of the pole, minus the energy loss due to friction, is sufficient to raise the centre of gravity of the pole to its upright position. If the condition is satisfied then the control is set to zero and the "bang-bang" character of the control is finished. After the pole has entered the stabilisation zone, the system can be treated as linear and the control is switched to the stabilising algorithm.

Due to the limited length of the rail a routine called “length control” is introduced, to reinforce the centring of the cart and prevent over-running the ends of the rail. The rule is very simple. When the position given by the parameter "length" is reached, then the maximal force is applied to the cart steering it back away from this position.

2 The Feedback Digital Pendulum System

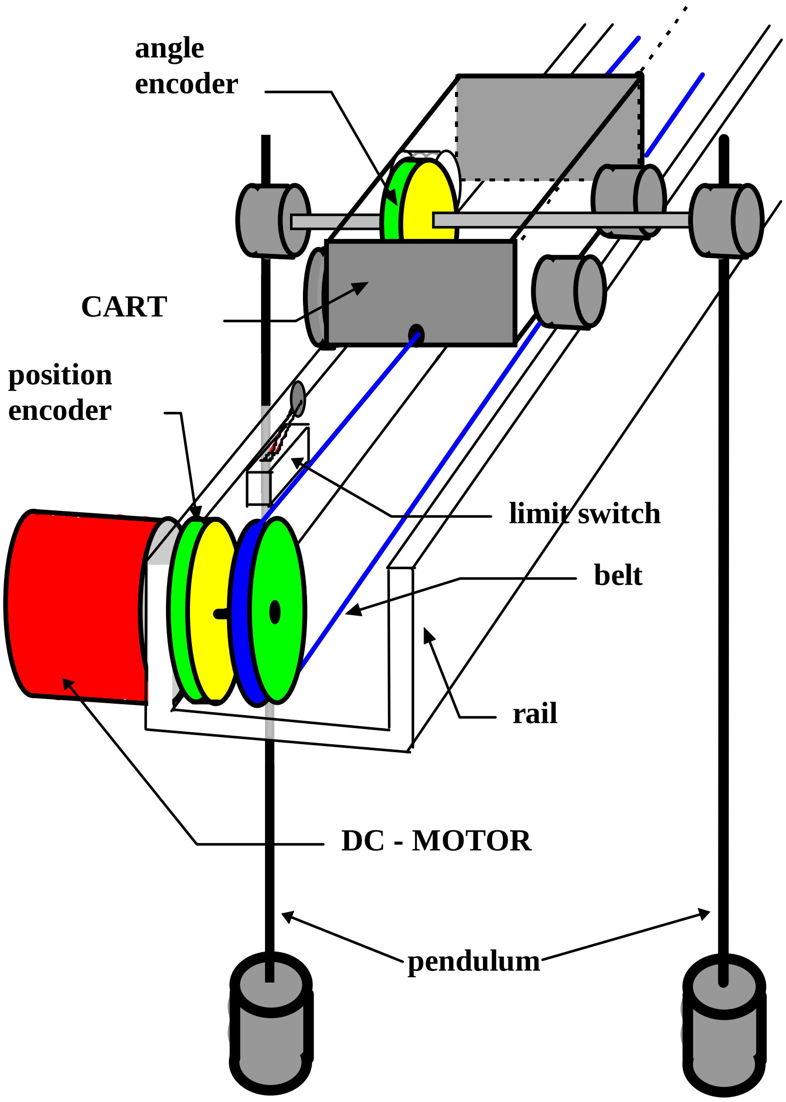


Figure 4: Mechanical part and sensors of pendulum-cart set-up

The pendulum-cart set-up uses incremental encoders. Figure 5 illustrates the principle of determining the direction of rotation for an incremental optical encoder.

The light beams emitted by two light sources (A and B) go through two rings of slits on the disc. The slits have a phase difference, so that the electric output of the receivers (A and B) are rectangular waves with a phase difference. The sign of the phase difference allows the direction of rotation to be determined.

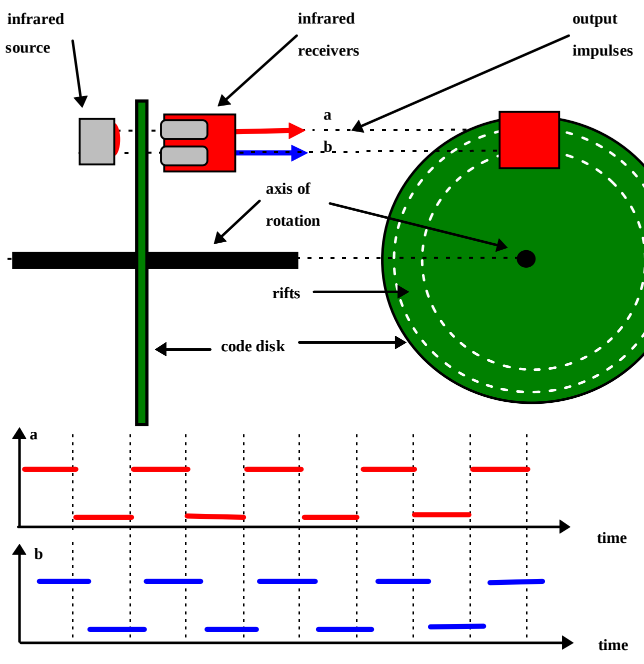


Figure 5: Position Sensor

The following point should be noted when using an incremental encoder. After each experiment or power switching the cart should be moved to the centre of the rail before starting the next experiment, to allow the zero value of the position to be set.

The control signal flows from the computer through the D/A converter of the data acquisition board. The D/A output is wired to the power amplifier input which drives the DC motor. The power amplifier and encoder interface are located in the *Digital Pendulum Controller* box. This box is equipped with two switches: the main power switch and the switch for cutting off the DC motor power. At the rail ends there are two limit-switches which cut off the DC motor power when the cart overruns the limit points (Figure 6).

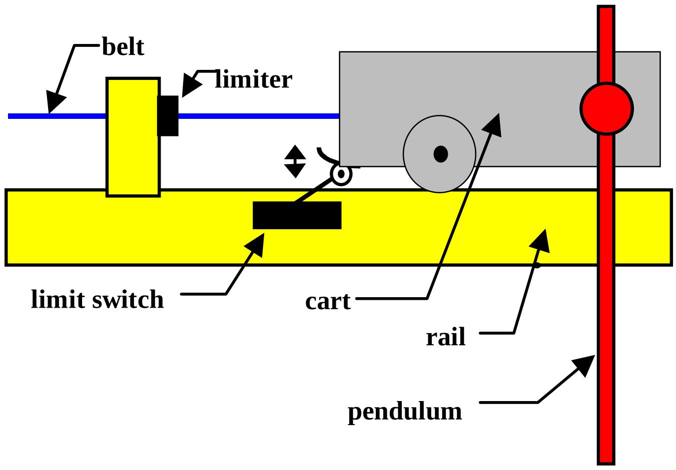


Figure 6: Sensors of the limit points

Notes

3 Model and Parameters

3.1 Introduction

The parameters of the pendulum model can be divided into three groups:

Constant (not to be identified)

Well identified

Poorly identified.

For example the pendulum and the cart are produced with production tolerances and so the masses are assumed to be identical.

A different case is with the moment of inertia of the rotating pendulum. It is slightly different in different plants. The reason is simple: the pendulum is assembled by the user in the laboratory and so small differences in the dimensions of the assembled set may occur. Unfortunately, in a real system there is usually a group of poorly identified parameters, characterised by random behaviour. In the pendulum-cart system, parameters related to the cart friction belong to this group.

Chapter 3.2 describes the basic physics of the pendulum cart, and shows how the derived parameters are calculated.

The parameters listed in Chapter 3.33 are a combination of measured values and derived values obtained by identification experiments. For example the various friction parameters which could be used in a Simulink model must be identified as they will vary fro one plant to another.

.

System Dynamics

The pendulum-cart system can illustrate several complex and nontrivial problems of control theory. The following characterisation is suggested for this system:

four state variables and one control variable

the dynamics are described by one ordinary differential equation (ode) of the fourth order or by four ordinary differential equations of the first order.

the system is non-linear

the control is bounded

the length of the rail is limited and the cart position is bounded

Control problems

crane	steering from point to point with minimum oscillation of the pendulum
pendulum	swinging up to the vicinity of the upright position and stabilisation in the upright position

Quality criteria

minimisation of the Integral Absolute Error (IAE)

minimum cost (energy)

time-optimal

3.2 The Physical Model

Figure 3-1 shows a schematic of the pendulum cart system, and Figure 3-2 shows the forces acting on the cart.

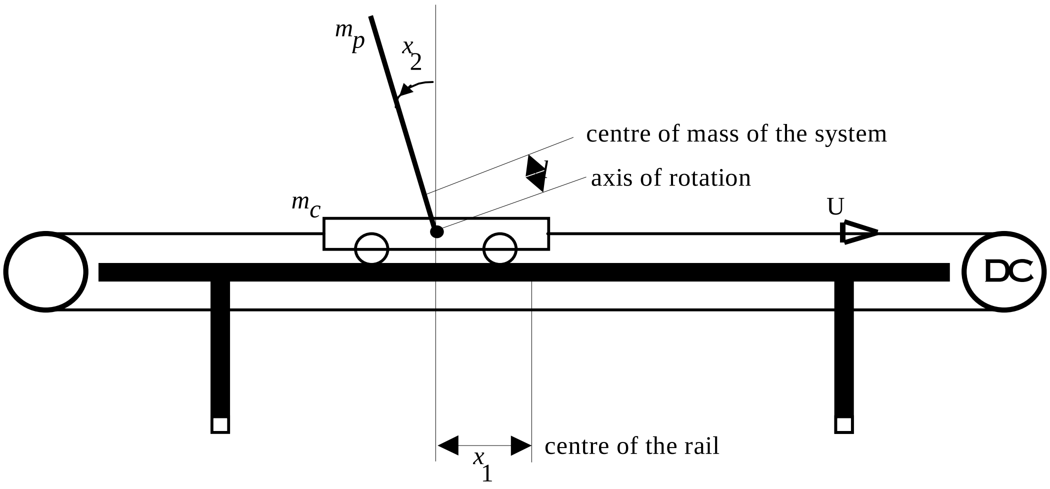


Figure 3-1: Laboratory model of Pendulum-Cart System

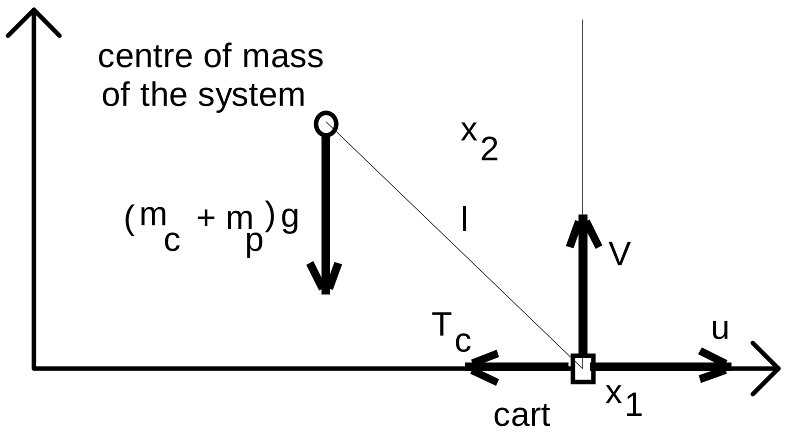


Figure 3-2: Showing the forces acting on the cart

The pendulum rotates in a vertical plane around an axis located on a cart. The cart can move along a horizontal rail whose long axis lies in the plane of rotation. A control force U, parallel to the rail, is applied to the cart. The mass of the cart is denoted by m_c and the mass of the pendulum, by m_p. l is the distance from the axis of rotation to the centre of mass of the pendulum-cart system. J is the moment of inertia of the pendulum-cart system with respect to the centre of mass.

The state of the system is the vector $x = \mathrm{col}(x_1, x_2, x_3, x_4)$ where:

x_1 is the cart position (distance from the centre of the rail)

x_2 is the angle between the upward vertical and the ray pointing at the centre of mass, measured counter-clockwise from the cart ($x_2 = 0$ for the upright position of the pendulum)

x_3 is the cart velocity

x_4 is the pendulum angular velocity

T_c denotes the friction in the motion of the cart
D_p is the moment of friction in the angular motion of the pendulum, proportional to the angular velocity: $D_p = f_p x_4$

The force of reaction of the rail V acts vertically on the cart. As the horizontal co-ordinate of the centre of mass is equal to $x_1 - l\sin x_2$ and the vertical to $l\cos x_2$, the equations of motion are as follows:

$$(m_c + m_p)(x_1 - l\sin x_2)'' = F - T_c,$$

$$(m_c + m_p)(l\cos x_2)'' = V - (m_c + m_p)g,$$

$$J x_2'' = (F - T_c)\, l\cos x_2 + Vl\sin x_2 - D_p.$$

(.)" denotes the second derivative with respect to time t and (.)' denotes the first derivative with respect to time t. The first two equations describe the translation of the centre of mass, while the third describes the rotation of the whole system around the centre of mass. After the elimination of V and simple calculations we obtain the state equations (for $t \geq 0$)

$$x_1' = x_3,$$

$$x_3' = \frac{a(F - T_c - \mu x_4^2 \sin x_2) + l \cos x_2 (\mu g \sin x_2 - f_p x_4)}{J + \mu l \sin^2 x_2}$$

$$x_2' = x_4,$$

$$x_4' = \frac{l \cos x_2 (F - T_c - \mu x_4^2 \sin x_2) + \mu g \sin x_2 - f_p x_4}{J + \mu l \sin^2 x_2} \tag{3.1}$$

where

$$a = l^2 + \frac{J}{m_c + m_p} , \quad \mu = (m_c + m_p)l. \tag{3.2}$$

The admissible controls are bounded

$$| F(t) | \le M. \tag{3.3}$$

Cart friction

The cart friction T_c in the model is a non linear function of the cart velocity x_3, as shown in Figure 3-3.

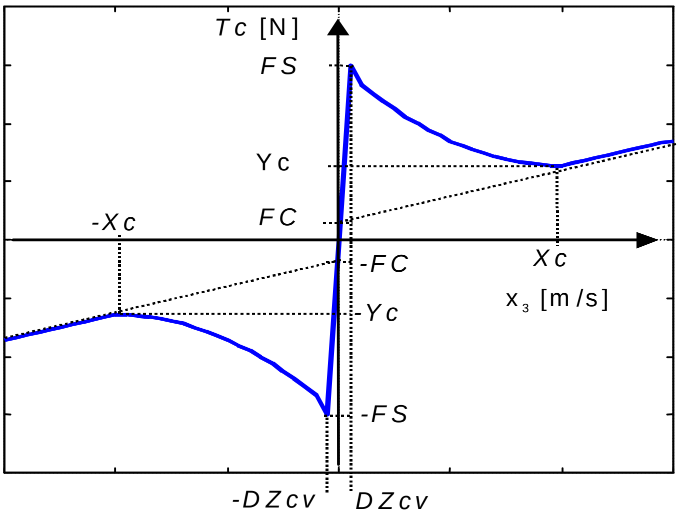


Figure 3-3: Cart friction T_c vs. cart velocity x_3 (for a large range of x_3)

The following notation is used in Figure 3-3.

FS	-	static friction,
FC	-	dynamic or Coulomb friction ,
Xc	-	cart velocity - beginning of the linear

		dependence zone,
Yc	-	friction value for *Xc* point,
DZ_{CV}		dead zone of cart velocity.

Consider Figure 3-3. The diagram represents the experimental relationship between cart friction and cart velocity.

Static friction exists in the dead zone of the cart, and this requires a force to be applied on order to start movement ($0< x_3 <DZ_{CV}$). Having overcome this initial friction, however, the static friction reduces, with an approximately quadratic relation to velocity, but the dynamic friction increases almost linearly with velocity.

The static friction eventually reaches a constant value of zero (at $x_3 = x_C$), at which point the total friction becomes equal to the dynamic or Coulomb friction.

Since friction always acts in a direction opposite to the direction of motion, the function is mirrored for negative values of x_3 with the friction force changed in sign.

In order to obtain some analytical results for the total friction force acting on the cart, we divide the friction function into 5 zones.

$x_3 < -x_C$	Zone 1
$-x_C < x_3 < -DV_{CV}$	Zone 2
$-DV_{CV} < x_3 < DV_{CV}$	Zone 3
$DV_{CV} < x_3 < x_C$	Zone 4
$X_3 > x_C$	Zone 5

We shall only consider cases in zones 3, 4, and 5 since zones 1 and 2 are mirror images of zones 3 and 4.

Method of deriving the equations for the linear and quadratic parts of the friction function

The relationship for the linear and quadratic parts of T_C, the friction function, are derived as follows:

Linear

Two points are needed for a solution:

The standard equation for a line passing through the points (x1, y1) and (x2, y2) is

$$y-y1 = \frac{(y2-y1)}{(x2-x1)}.(x-x1) \qquad 3.4$$

Quadratic

Either three points are needed for a solution, ***or*** two points ***and*** a known gradient at a point. Use the following assumptions:

The equation is of the form $y= \mathbf{a}x^2 + \mathbf{b}x + \mathbf{c}$. i)

The curve passes through (x_1 , y_1) and (x_2, y_2) ii)

The gradient at (x_2, y_2) is G. iii)

From i) $y' = 2ax + b$

From iii) $2ax_2+b=G$, and therefore $b=G-2ax_2$ iv)

Therefore from i) $y=ax2 + (G-2ax_2)x + c$ v)

From ii) and iv) $y1=ax_1^2 +(G-2ax_2)x_1 + c$ and vi)

and $y2=ax_2^2 +(G-2ax_2)x_2+ c$ vii)

Subtracting vi) from vii) gives

$$y_2-y_1 = a(x_2^2 - x_1^2) + (G-2ax_2)(x_2-x_1) \qquad viii)$$

Hence **a** can be found from viii), **b** from iv) and **c** from vi)

From Figure 3-3 replace x_1,y_1 by 0, f_S. replace x_2,y_2 by x_C, y_C and note that the gradient at this point is zero (g=0 at $x=x_C$). also at x=0 the friction is the static friction only (f_S) as the speed (x_3) is zero, - hence $c=f_S$., and from iv) $b=-2ax_C$

From viii) $a= \dfrac{(y_2-y_1)}{(x_1-x_2)^2}$ which is equivalent to $\dfrac{(y_C-Fs)}{(x_C)^2}$

Zone 3

The Friction linearly increases in the range $DV_{CV} < x_3 < DV_{CV}$

From Figure 3-3 the value of the Friction at $x_3 = \pm DV_{CV}$ is $\pm F_S$

Using the linear method and equation 3.4, and noting that this linear part passes through the origin, we derive the relationship

$$T_C = \frac{F_S.X_3}{DZ_{CV}}$$

Zone 4

The static friction has a quadratic relation with velocity, decreasing as the velocity increases and eventually reaching zero, and the dynamic friction has a quadratic relation with velocity. The total friction, which is the sum of the static and dynamic elements, therefore has a quadratic relation with velocity.

We note that the static friction reaches a constant value of zero at $x_3=x_C$. This implies that its gradient at this point is zero. Using the quadratic method described above, with G=0, gives the following relation for T_C

$$T_C = \frac{(F_S - Y_C)}{X_C2} \cdot (X_3)^2 \; -2\,\frac{(F_S - Y_C)}{X_C} \cdot X_3 + F_S$$

Zone 5

The static friction has reached zero and the dynamic or Coulomb friction increases linearly with velocity.

Using the linear method and equation 2.4, and noting that this linear part passes through the point (xc, yc) we derive the relationship

$$T_C = \frac{(Y_C - F_C) . x_3 - F_C}{X_C}$$

Moment of Inertia

The figure depicted in Figure 3-4 is used to illustrate calculations of the Moment of Inertia of the rotating pendulum

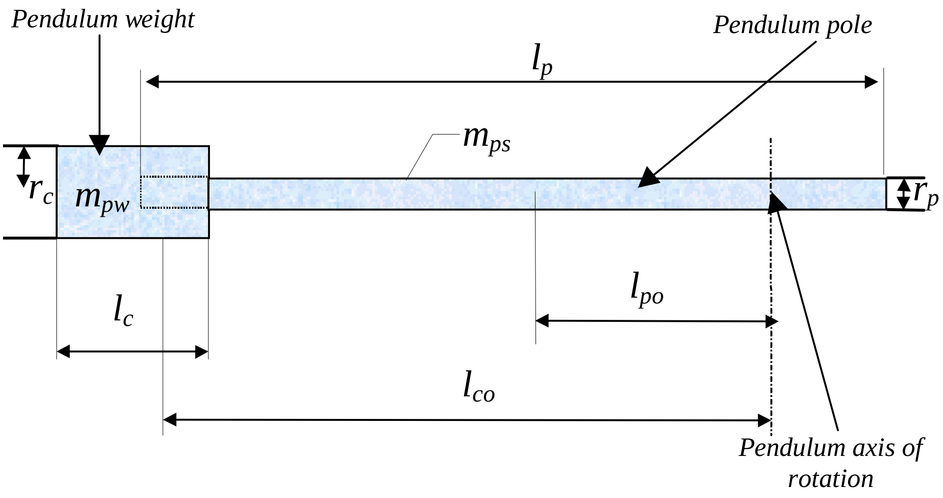


Figure 3-4: View of the pendulum

The following notation is used in Figure 3-4.

m_{pw} - mass of the load pendulum weight[kg]

m_{ps} - mass of the pendulum pole [kg]

l_p - length of the pole [m]

l_{po} - distance between centre of the pole mass and the pendulum rotation axis [m]

l_c - length of load [m]

l_{co} - distance between centre of the load mass and the pendulum rotation axis [m]

r_p	-	radius of the pole [m]
r_c	-	radius of the load [m]

The moment of inertia of the pendulum related to the pendulum rotation axis is given by

$$Jp = \left[\frac{1}{12}m_{pw}l_c^2 + \frac{1}{4}m_{pw}r_c^2 + m_{pw}l_{co}^2\right] + \left[\frac{1}{12}m_{ps}l_p^2 + \frac{1}{4}m_{ps}r_p^2 + m_{ps}l_{po}^2\right]x \qquad (3.4)$$

We shall make the following assumptions in order to simplify the model.

The pendulum is a uniform **thin** cylinder of length L metres and mass m_p

The pivot position is at the mass centre of the cart and at one end of the pole

The cart behaves as a point mass M_c, at the pivot point

Consider the pendulum cart system and consider the moment of inertia about the mass centre of the system

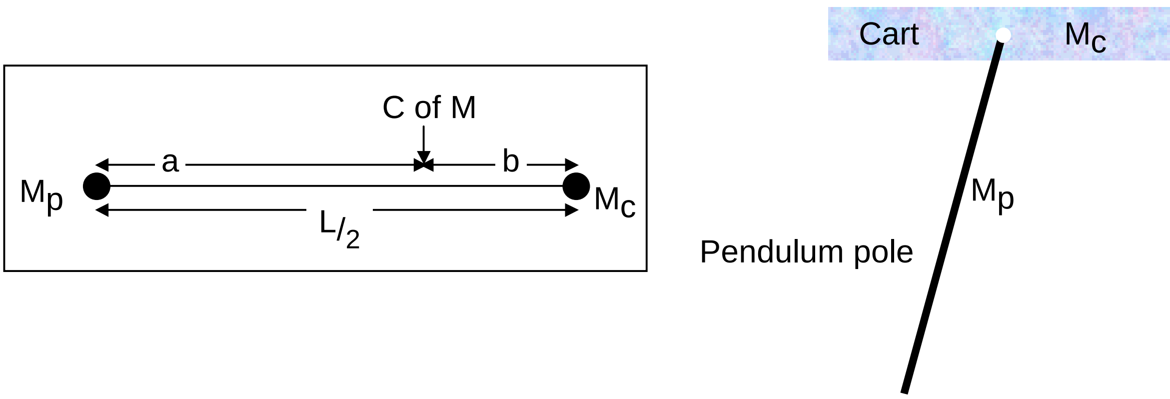


Figure 3-5

Now it is a well known principle that if J is the Moment of Inertia about a first axis through the mass centre and J_p is the Moment of Inertia about a second parallel axis separated by d from the first then

$$J_p = J + Md^2 \qquad 3.5)$$

Referring to Figure 3-5, the centre of mass (C of M)of the system is such that

$M_p a = M_c b$, and $a+b = L/2$... (3.6)

Hence the total Moment of Inertia about the mass centre (from (3.5)) is given by

$$J = M_p a^2 + M_p b^2 + \frac{M_p}{12} L^2$$

and $a = \dfrac{LM_c}{2(M_p + M_c)}$ and $b = \dfrac{LM_p}{2(M_p + M_c)}$ from (3.6) (3.8)

From (3.7) and (3.8) the Moment of Inertia is

$$J = \frac{m_p(4m_c + m_p)L^2}{12(m_c + m_p)} \qquad (3.9)$$

J can be expressed in terms of J_p , since there is no rotation of the cart about its own mass centre

$$J = Jp - l^2(m_c + m_p) \qquad (3.10)$$

The distance from the centre of mass to the axis of rotation equals

$$l = \frac{0.5 m_p L}{m_c + m_p} \qquad \text{(from 3.8)} \qquad (3.11)$$

3.3 The system model in SIMULINK

A simulation model of the dynamics of the pendulum-cart system (*model.m in Simulink)* is shown in Figure 3-6, Figure 3-7 and Figure 3-8.

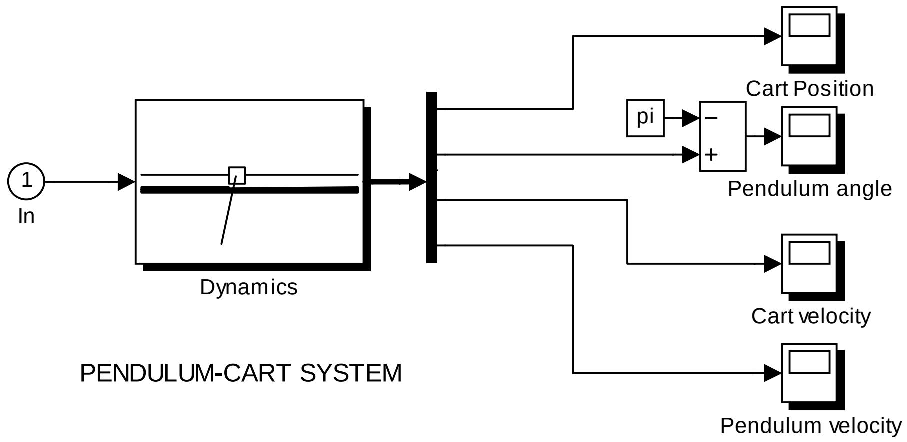


Figure 3-6: Pendulum-cart model *model.m*

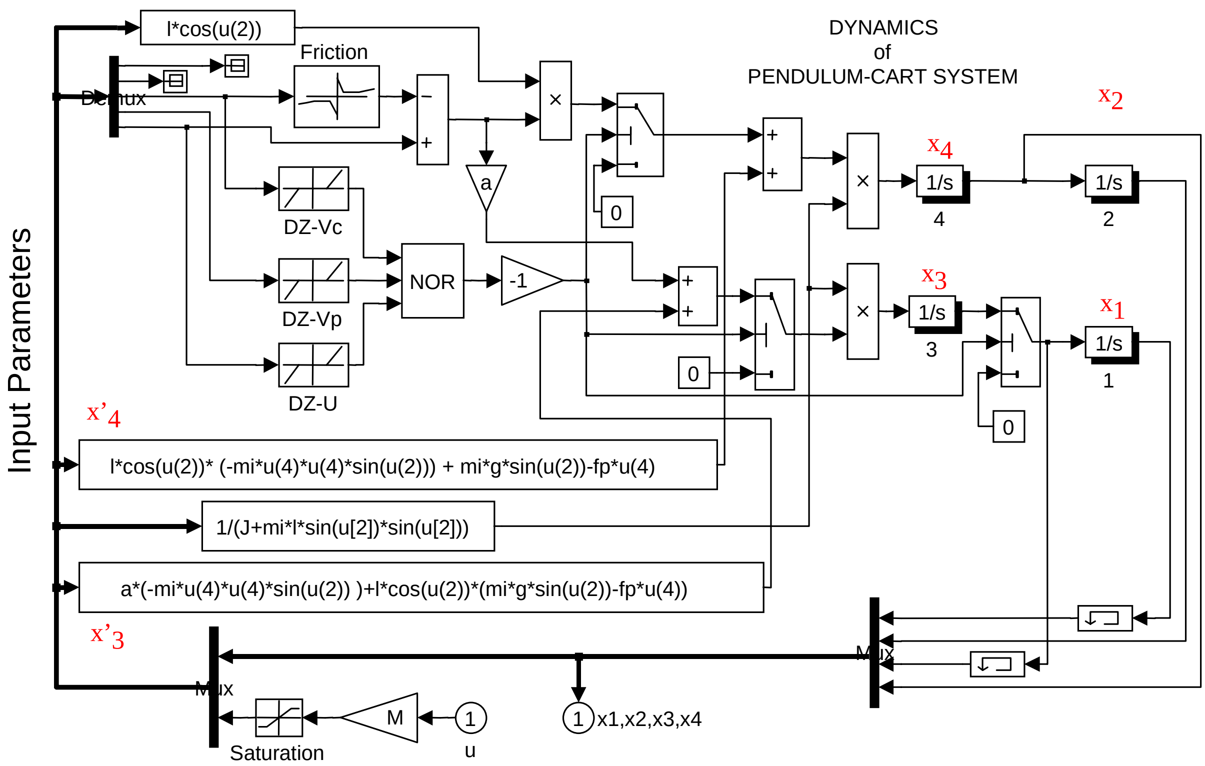


Figure 3-7: Dynamics block

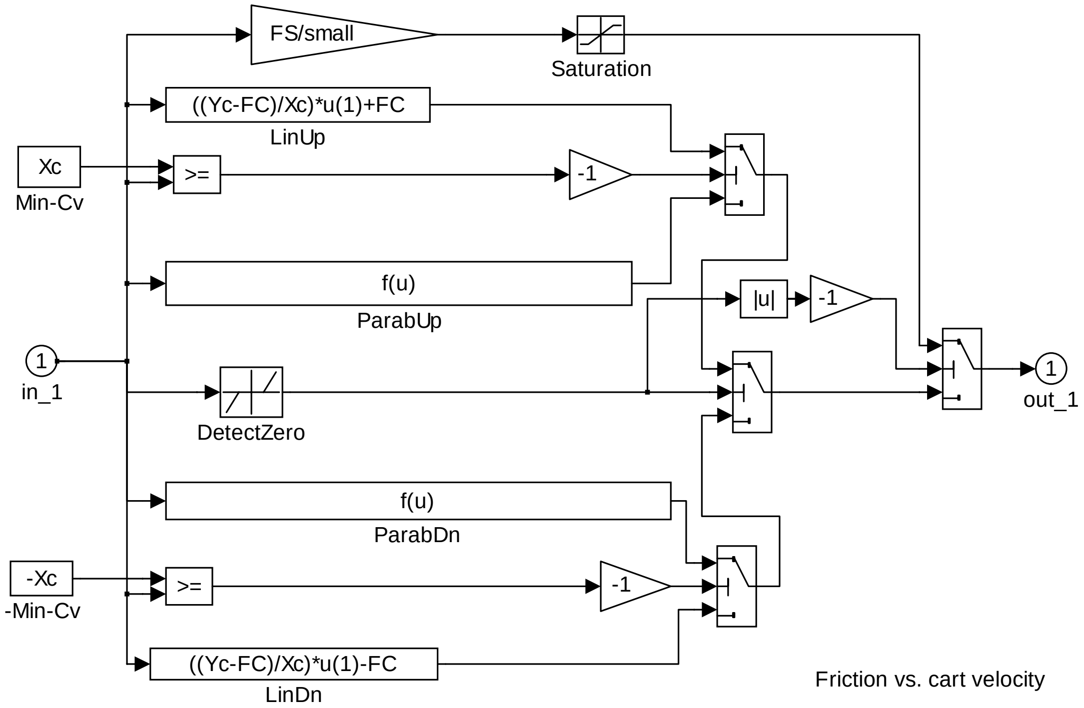


Figure 3-8: Friction block

The cart-pendulum dynamics (equations from (3.1) to (3.5)) are modelled in the masked block *Dynamics.* The unmasked block *Dynamics* is given in Figure 3-7. It contains several specialised blocks from the SIMULINK library such as *Dead zone* and *Saturation*, and the *Friction* block dedicated to friction representation.

Friction is the most important block for the identification procedure. The interior of this block is shown in Figure 3-8. Omitting or simplifying the *Friction* block results in poor compatibility between the real and simulation models responses. Two memory blocks are introduced to avoid the generation of algebraic loops. There are three *Dead zone* blocks to make the model sensitive to small values of velocities and control.

The default simulation parameters are given in Figure 3-9 and Figure 3-10.

Notice, that the fifth order Runge Kutta integration method with the constant step 0.01 [s] is used.

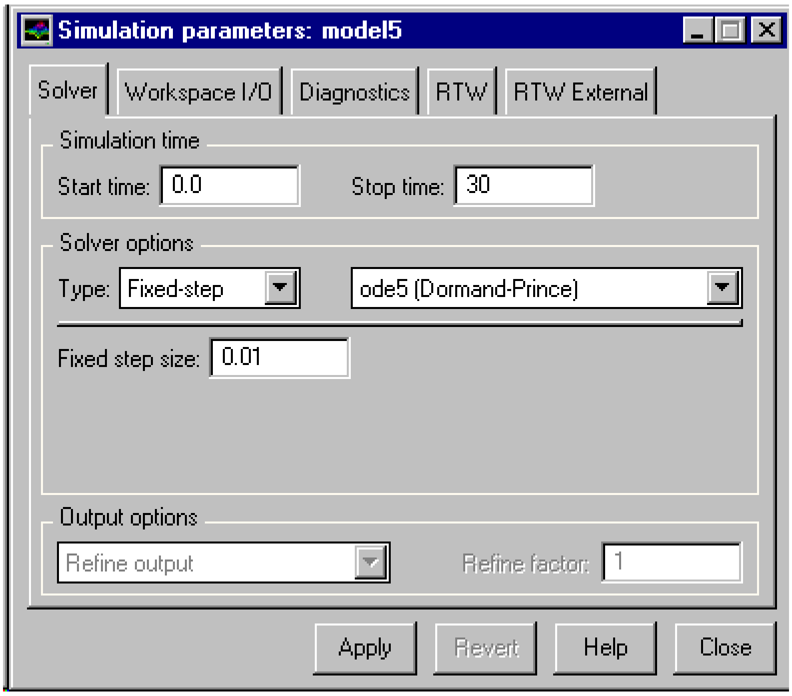


Figure 3-9: Parameters of simulation - Solver

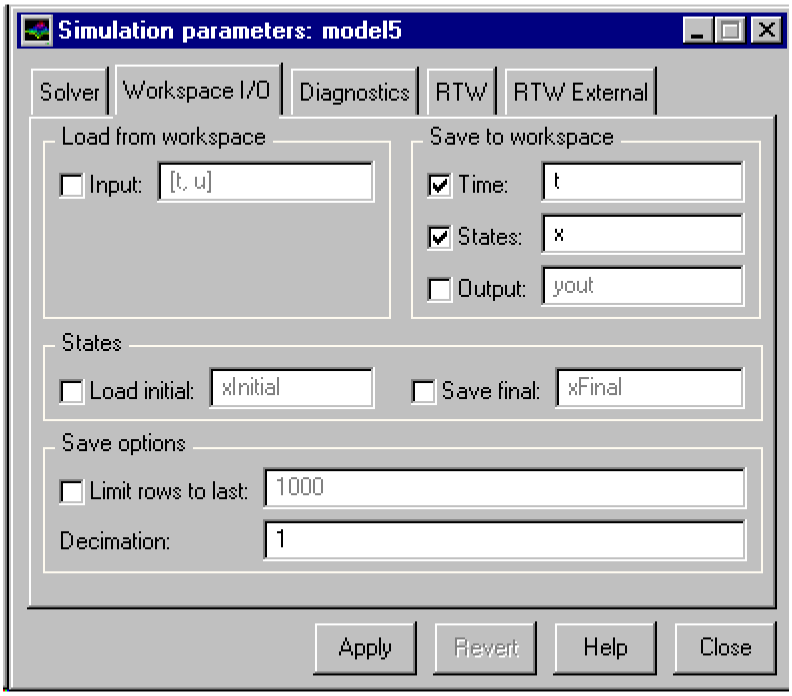


Figure 3-10: Parameters of simulation - Workspace I/O

Running the Simulink Model

The Simulink model is located in the same folder as the actual Simulink control programs described later in this document. This will usually be in the folder

`matlabroot\feedback\pendulum`

(unless you have placed your Simulink files in another folder)

The difference is that the model does not access the hardware and therefore the mode of running is different

To run the Simulink model either

Type in `model` into the MATLAB command window

or Open the model from the MATLAB file menu

or Find the model with Windows explorer and double click its icon

Having opened the Simulink model, start its execution by clicking `Start` in the `Simulink` menu in the title bar of the window containing the graphical model.

Note

Do not use the External mode for running Simulation models. External mode is intended to run models which actually control hardware.

3.4 Physical Parameters

PARAMETER DESCRIPTION	SYMBOL	VALUE
Cart mass [kg] :	mc	1.12
Cart mass without positioners [kg] :	mc_	1.045
Pendulum weight mass [kg] :	mpw	0.095
Pendulum stick mass [kg] :	mps	0.025
Rail length [m] :	Rl	1
Pendulum stick length [m] :	lp	0.402
Distance lpo [m] :	lpo	0.146
Length of load[m] :	lc	0.041
Distance lco [m] :	lco	0.347
Radius of the load [m] :	rc	0.02
Radius of the stick [m] :	rp	0.006
Theoretic pend. moment of inertia [kg*m2]:	Jpt	0.0123321
Static friction [N] :	FS	2.28133
Coulombic friction [N] :	FC	2.53165
Maximum control [N] :	M	17.463
Minimum control [N] :	DZu	1.37918
Minimum cart velocity [m/s] :	DZcv	-0.00793711
Minimum pend. velocity [rad/s] :	DZpv	3.37476
Pendulum friction [kg*m2/s] :	fp	0.000107443
Pendulum period [s] :	T	1.16
Pendulum moment of inertia [s] :	Jp	0.0139231
Moment of inertia [kg*m2] :	J	0.0135735
Distance: axis of rotation-mass centre [m] :	l	0.0167903

Close

Figure 3-11: System parameters as measured or derived

Summary of all parameters with symbols used in the System Dynamics chapter

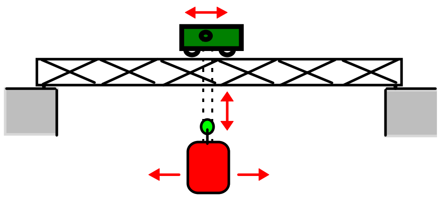
Name	Description	Unit
m_c	cart mass	[kg]
m_{c-}	cart mass without positioners for pendulum	[kg]
m_{ps}	pole mass	[kg]
m_{pw}	load mass	[kg]
R_l	rail length	[m]
l_p	length of pole	[m]
l_{po}	distance between centre of pole mass and rotation axis	[m]
l_c	length of load	[m]
l_{co}	distance between centre of load mass and rotation axis	[m]
r_c	radius of the load	[m]
r_p	radius of the stick	[m]
J_{pt}	Theoretic pendulum moment of inertia	[kg·m^2]
FS	cart static friction	[N]
FC	cart dynamic friction coefficient	[kg/s]
M	maximal force	[N]
DZu	minimal force to move a cart	[N]
DZcv	dead zone of cart velocity	[m/s]
DZpv	dead zone of pendulum velocity	[rad/s]
f_p	pendulum friction constant	[kg·m^2/s]
T	pendulum period	[s]
J_p	moment of inertia related to the axis of rotation	[kg·m^2]
J	moment of inertia related to the mass centre	[kg·m^2]
l	distance between axis of pendulum rotation and the centre of pendulum-cart mass	[m]

Table 1 Names of all parameters

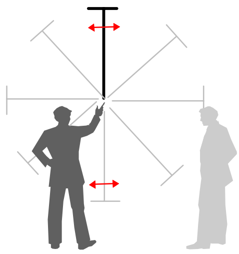
Notes

4 Control Algorithms

Three control problems are presented:

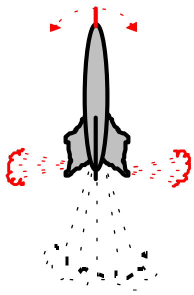


crane control (**C**)



swing-up control (**SU**)

and



self erecting control (**SE**)

Figure 12

A human or a computer may operate as a controller. A human operator may be fairly successful in the C problem, but completely fails in the other two problems. The SU and SE problems requires a fast operating controller and only a computer can manage such fast and complex tasks.

Notes

5 Safety Instructions

Read these instructions carefully

Ensure you are acquainted with the safety Instructions in the Preface to this Manual

Do not enter the workspace of the Equipment whilst it is in operation.

In the event of an emergency. the control effort should immediately be discontinued by hitting the Red Stop button on the Pendulum Control Unit.

All users of this equipment should be familiar with and trained in good Laboratory Practice where electrical machinery is used.

When running any control experiment ensure that the Real Time control program is running before pressing the Start button on the equipment control unit

Notes

6 Experiment 1 – Double PID Crane Control

Theory and Model Description

The crane problem is illustrated in Figure 1 where the pendulum is hanging in the down (equilibrium) position from the cart. Swing is induced in the pendulum as the cart is moved back and forth by the DC motor. Generally, the goal of the crane control problem is to move the cart between positions; preferably as fast as possible, with few oscillations, and without letting the angle and velocity of the pendulum swing become too large or the duration of the swing too prolonged. This problem is named for the similarity to the problem of controlling overhead cranes used at shipping ports and construction sites to move cargo and supplies.

The PID control rule is very common in control systems. It is the basic tool for solving most process control problems. The PID controllers are usually standard building blocks for industrial automation. The most basic PID controller has the form

$$u(t)=K_p e(t)+K_i\int_0^t e(\tau)d\tau + K_d\frac{d}{dt}(e(t))$$

where $u(t)$ is the control output

and the error, $e(t)$, is defined as

> $e(t)$ = desired value – measured value of quantity being controlled.

The control gains

> K_p, K_d, and K_i

determine the weight of the contribution of the error, the integral of the error, and the derivative of the error to the control output and will dictate the response of the closed-loop system to the initial conditions and inputs.

There are a number of tuning methods for PID controllers. Some of them are based on transient response experiments e.g., the Ziegler-Nichols step response tuning rule, other are based on relay feedback when the parameters of a PID controller are determined from features of the limit cycle of the closed-loop system, some others are based on frequency analysis. In the crane system under consideration, the candidate quantities for PID control are the cart position and pendulum angle.

We shall consider the pendulum-cart control system containing two PID controllers. The first operates based on the angle of the pendulum and the second operates based on the position of the cart. The outputs of the PID controllers are added to produce the final control value for the D/A converter, and as a result the output motor torque.

A schematic of the proposed double PID control system for the pendulum-cart system is shown in Figure 13, constructed as a Simulink block diagram. It is used to control the pendulum system as described in the following text.

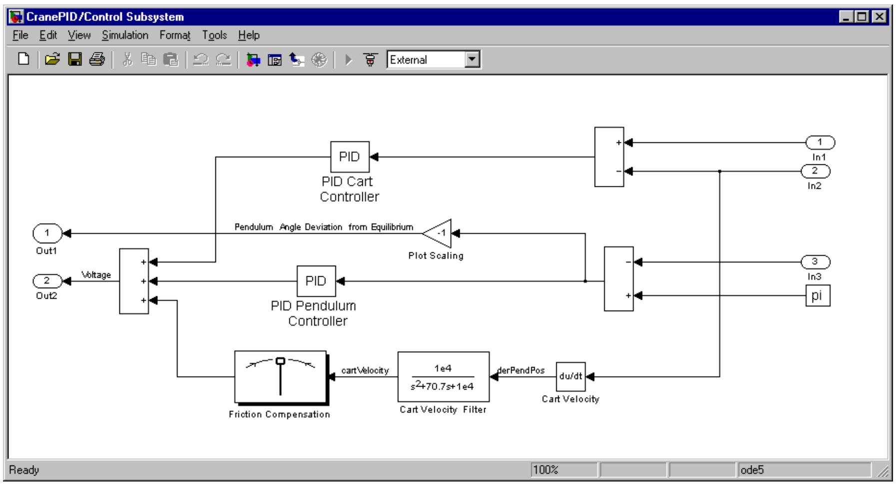


Figure 13: Block Diagram of Double Crane PID controller

Loading the RTWT Block Diagram

To load and execute the RTWT block diagram for the crane control, follow the procedure outlined below:

Start MATLAB

Type the following commands into the MATLAB command window

```
CranePID
```

You can also open it from the MATLAB File menu, or double click its icon before starting MATLAB.

The RTWT block diagram for the Crane PID, appears and is shown in Figure 14.

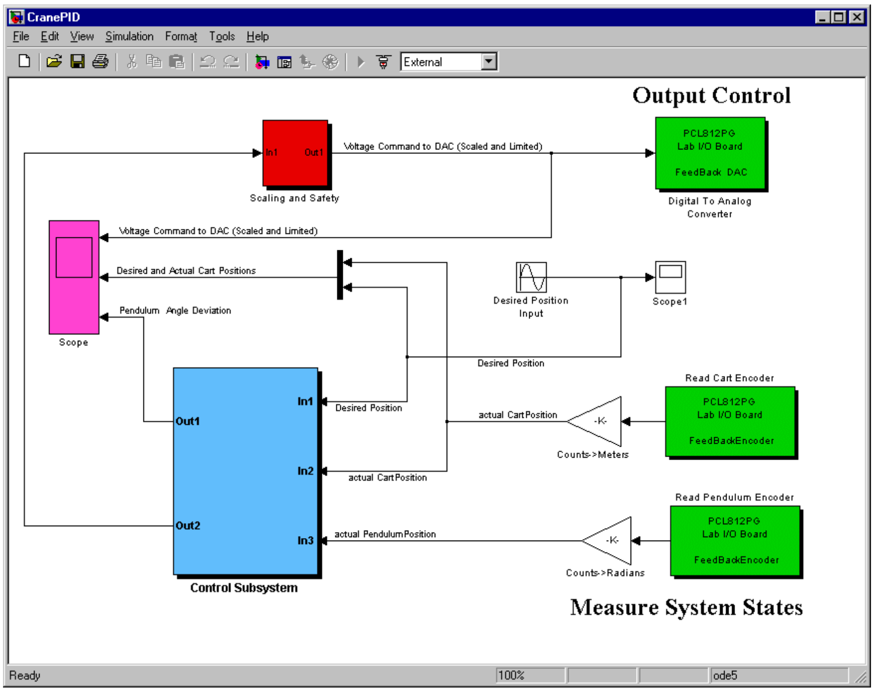


Figure 14: RTWT Block Diagram for Crane Control Experiment

Simulink Block Diagram - Description of Block Functions

In Figure 14, the green blocks denote input/output operations or hardware and the red block denotes the safety subsystem. The user has full access to the control algorithm and safety settings. The control gains should be changed only after their affect on the stability of closed-loop system is understood. The user must read the RTWT User’s Manual in great detail before modifying the hardware and safety blocks.

Input Output Setup

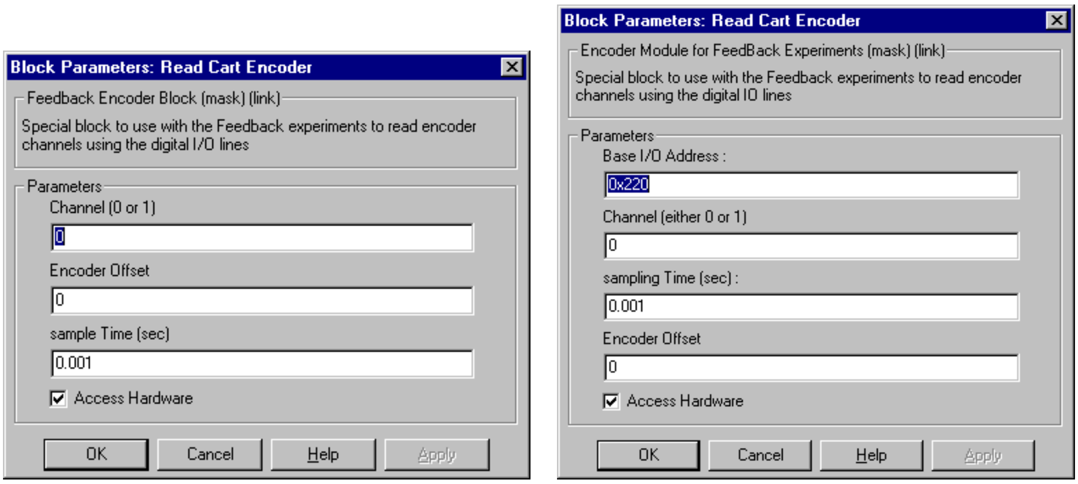
In Figure 14, the green blocks, labeled “Read Cart Encoder” and “Read Pendulum Encoder”, represent the input of the cart and pendulum positions as encoder counts from two incremental encoders.

Note

Prior to the start of a control experiment the system must be at the reference position. This may entail the user depressing the "STOP" button and manually moving the cart to the center of travel and manually stabilizing the pendulum in the down position.

These cart and pendulum positions will be referred to as 0 meter and π radian, respectively. Motion of the cart to the right from the reference position will be considered motion in the positive direction and clockwise rotation of the pendulum will be considered positive.

The input blocks and the attached scaling blocks must be configured so that the startup initializes these positions and the feedback has the correct direction sense.



PCI ISA

Figure 15: Cart encoder input configuration dialogue box

Double click on the “Read Cart Encoder” to open the dialog box shown in
PCI ISA
Figure 15.
There are the following edit windows for entering

Base I/O Address (ISA only)

Channel

Sampling Time

Encoder Offset

You will notice that there are preset values already entered into the edit windows. The Base I/O address is set by means of switches when the Advantech PCL812 board is inserted into the PC. The appropriate value should be entered into the edit window in Hex, and the model saved. It should not be necessary to then change this value unless another I/O interface board is used.

Note

The Read Encoder windows for the PCI versions of the Simulink models are a little different from the ISA version, since the Advantech PCI1711 PCI I/O card has its base address automatically set by Windows, and therefore it is therefore not necessary for the user to set it.

In the following experiments the following values are set

Base I/O Address is set to 220 Hex or 544 Decimal

Encoder Channel is set to 0

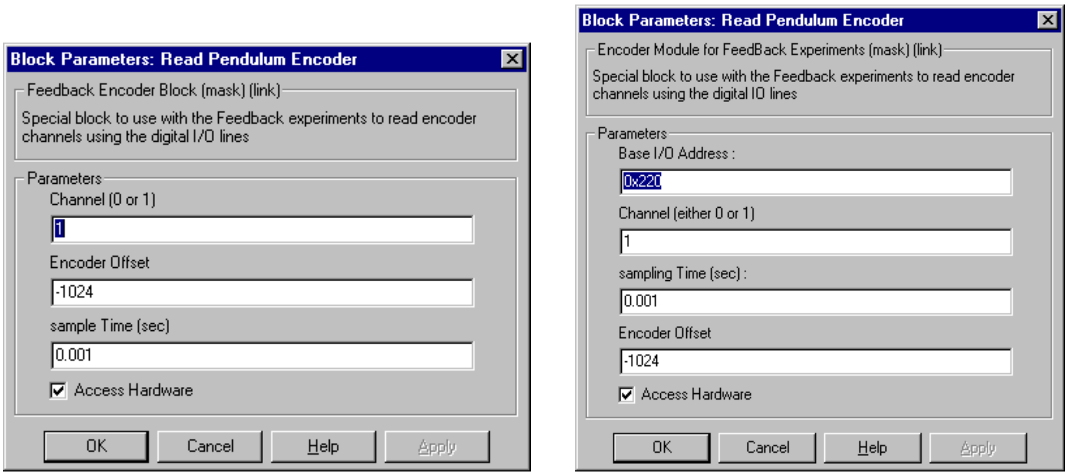
Sampling Time is set to 0.001

Encoder Offset is set to 0 to initialize the cart encoder input to zero encoder counts at the start of the simulation, thus naming the start position zero.

Note that the **Access Hardware** is enabled.

The scaling block converts from counts of the incremental encoder turning with the motor, to units of meters for the movement of the cart along the rails.

Read Pendulum Encoder Block



PCI ISA

Figure 16: Pendulum encoder input configuration dialogue box

Double click on the “Read Pendulum Encoder” to open the dialog box shown in Figure 16.

In the following experiments the following values are set

Base I/O Address is set to 220 Hex or 544 Decimal

encoder **Channel** is set to 1

Sampling Time is set to 0.001

Encoder Offset is set to -1024. This sets the pendulum reference position to π according to the following calculation.

$$\text{Reference Position} = -1024 \text{ count} \times \frac{1 \text{ pendulum revolution}}{2048 \text{ count}} \times \frac{-2\pi \text{ radian}}{1 \text{ pendulum revolution}}$$

$$= \pi \text{ radian}$$

where the –2π term is contributed by the scaling block. The sign on the scaling blocks create the proper direction sense as described above.

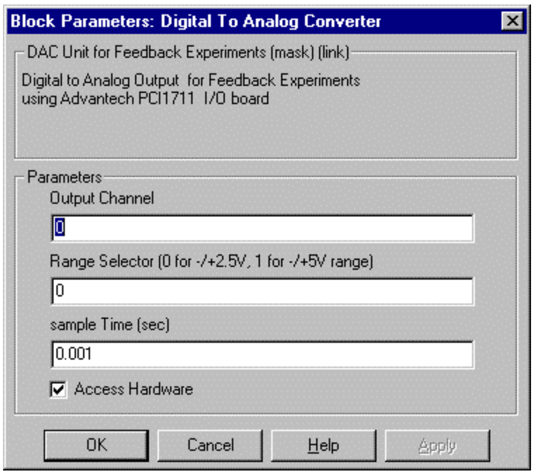
Note that the **Access Hardware** is enabled.

Digital to Analogue Converter Block

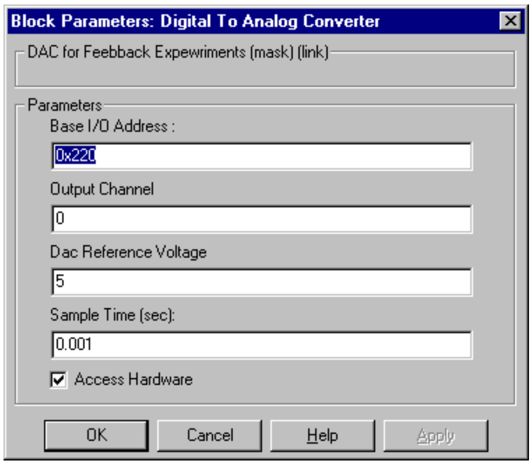
The output command must match the capabilities of the hardware.

The Advantech board is capable of outputting a 0-5V signal. This signal is shifted in the amplifier to create a +/-2.5V capability required to command the drive motor in both directions. The shifting is transparent to the user at this level of use, so it is only necessary to ensure that the output command is in the range +/-2.5V.

The saturation block found in “Scaling and Safety” ensures that this constraint is met. Scaling is set to 1 so that a positive input creates motor torque that acts to move the cart in the positive direction.



PCI



ISA

Figure 17: Digital to Analogue converter configuration dialogue box

Double click on the “Digital to Analogue Converter Block” to open the dialog box shown in Figure 17.

Base I/O Address is set to 220 Hex or 544 Decimal

Output Channel is set to 0 (i.e. the first D/A channel on the Advantech Board)

Dac Reference Voltage is set to 5

Sample Time is set to 0.001

Control Subsystem Block

The Control Subsystem is shown as a blue Block in the lower left section of Figure 14.

Double click on this block to open the PID control subsystem shown in Figure 18.

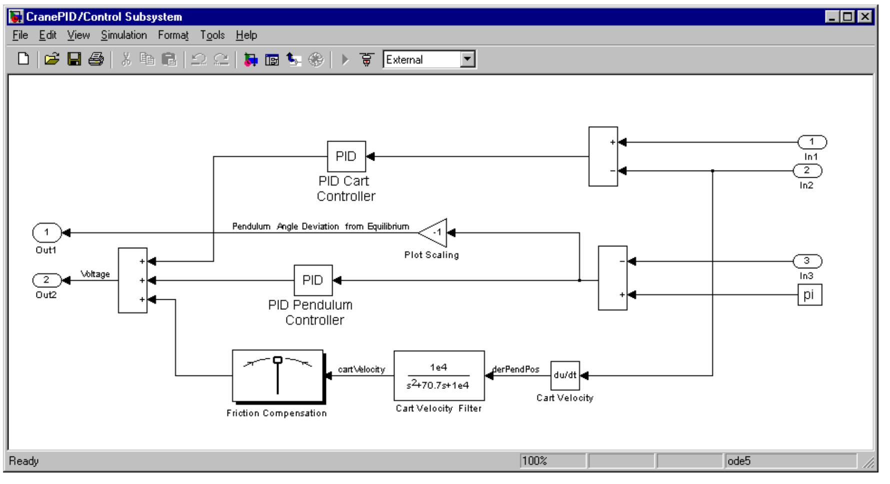


Figure 18: Crane mode PID subsystem

The “PID Cart Controller” and “PID Pendulum Controller” blocks are masked.

To reveal the structure either right click the mouse and select "`Look under mask`" or type (Ctrl+u) as shown in Figure 19 and Figure 20.

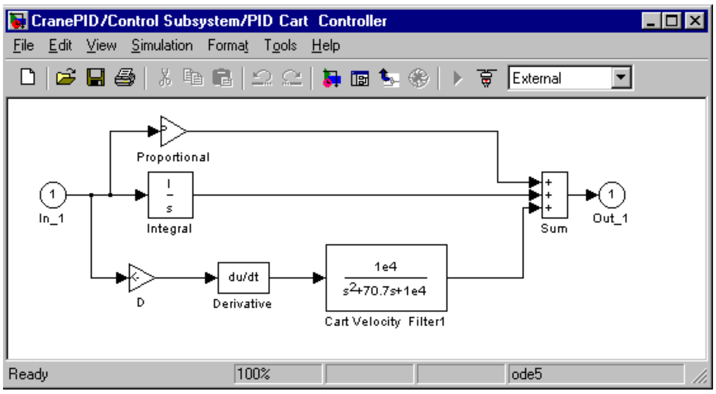


Figure 19: PID Cart Controller

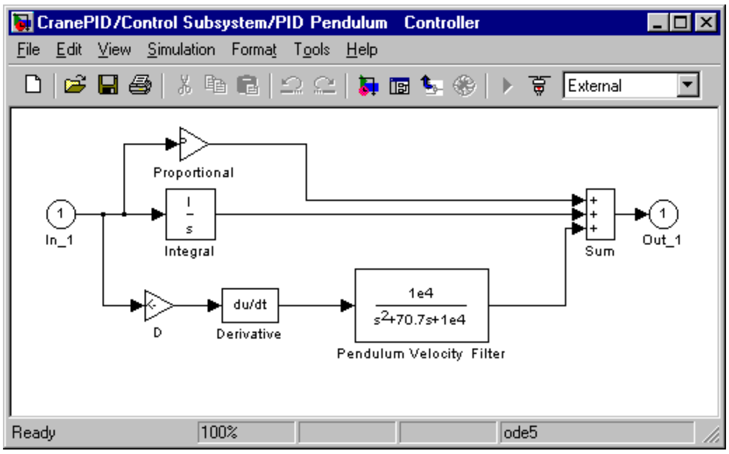


Figure 20: PID Pendulum Controller

Double click on these same blocks to reveal the changeable control gains as shown in Figure 21 and Figure 22.

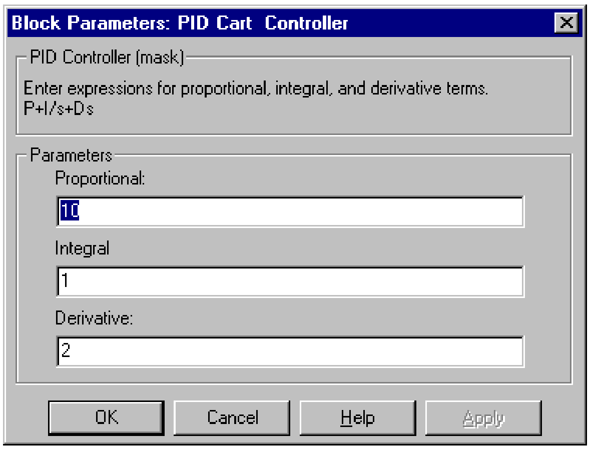


Figure 21: PID Cart Controller Parameters

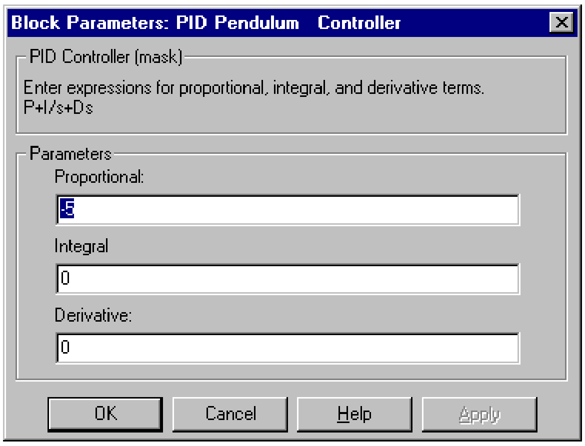


Figure 22: PID Pendulum Controller Parameters

Friction

An addition block "Friction Compensation" is used to compensate for the nonlinear static friction.

The diagram in Figure 23 represents the experimental relationship between cart friction and cart velocity.

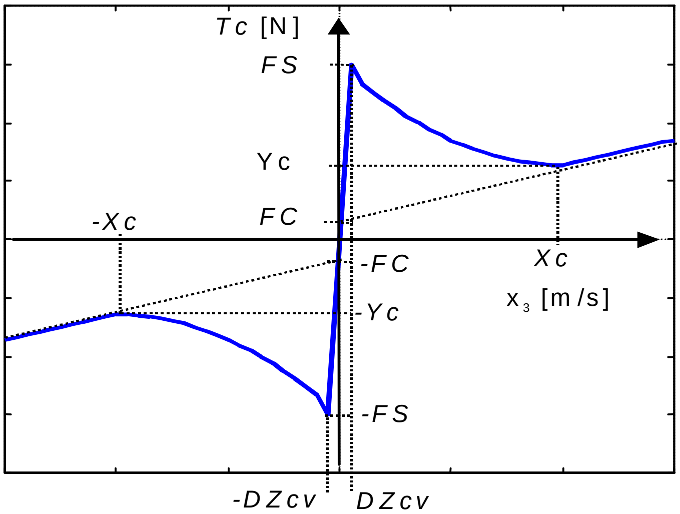


Figure 23: Typical cart friction T_C versus cart velocity x_3

Static friction exists in the dead zone of the cart, and this requires force to be applied on order to start movement (0< cart velocity <DZ cv).

Having overcome this initial friction, however, the static friction reduces, with an approximately quadratic relation to velocity, but the dynamic friction increases almost linearly with velocity.

The static friction eventually reaches a constant value of zero (at cart velocity = x_c), at which point the total friction becomes equal to the dynamic or Coulomb friction.

Since friction always acts in a direction opposite to the direction of motion, the function is mirrored for negative values of cart velocity with the friction force changed in sign.

The compensation used here represents a simplified approach in which the cart velocity is used to indicate the direction of a constant force offset.

The output of this function shown in Figure 25 produces a force in the direction of the sign of the velocity. It is clear that this can only partially compensate for the actual friction shown in Figure 23.

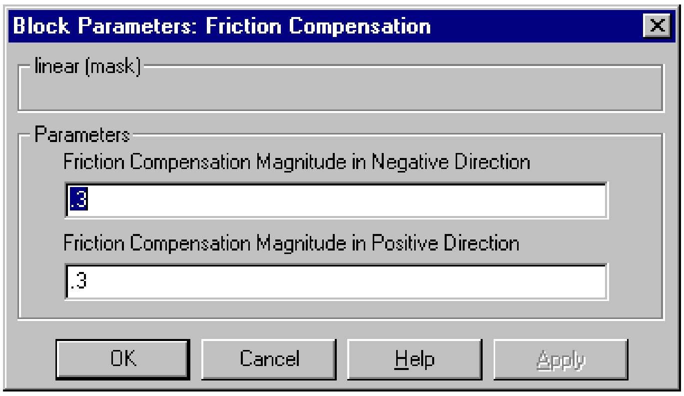


Figure 30: Friction Block parameters

The magnitude of the constant offset was obtained though applying the friction compensation as the only output (achieved by setting all PID gains for the cart and pendulum to zero) then varying the magnitude through trial and error until it feels that the cart glides freely. The user may adjust this magnitude slightly in order to better match the friction in a given system (Figure 30)

The user should continue through this manual and run the defined experiments before attempting to adjust the friction in this manner.

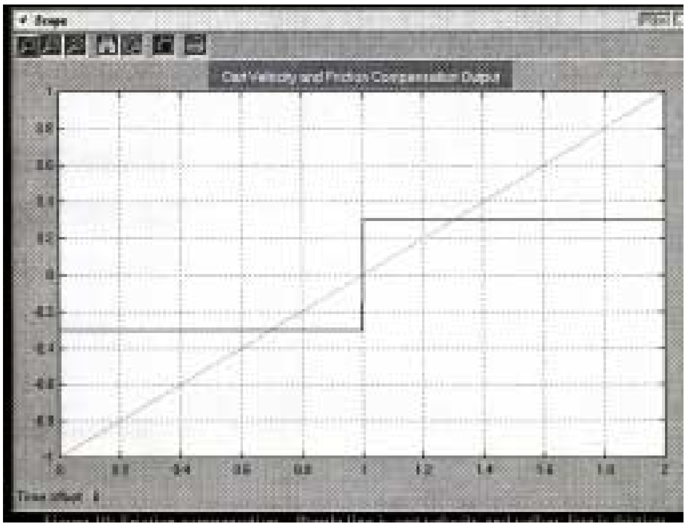


Figure 25: Friction Compensation

Display

The purple “Scope” block in Figure 14 provides the user with a record of the system states. This window can be opened, by double-clicking on the block *before* running the control, in order to see a live display. It can also be opened *after* the control has stopped in order to see a plot of systems states.

The user is referred to Simulink help for the options to scale, zoom, and print using this standard scope block.

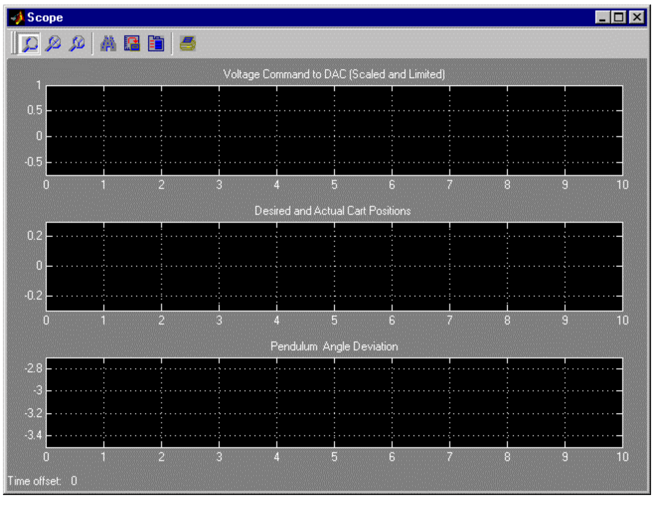


Figure 32: Scope Block

Building the Model

Build Options

Before initiating the subsequent automatic build process (that is using Real Time Workshop to generate, compile and link the actual C code to run the control model), it is necessary to ensure that the build options are set correctly.

To this end, select **Tools** from the Simulink model menu bar and then selecting **Real-Time Workshop** followed by **Options** from the pull-down menu. The **Simulation Parameters** window appears as shown in Figure 33.

From this window, click on the **Real-Time Workshop** tab as shown in Figure 33.

If you have followed the installation sequence described in both this manual and the introductory manual 33-000-1 the Simulation Parameters window should look exactly as in Figure 33.

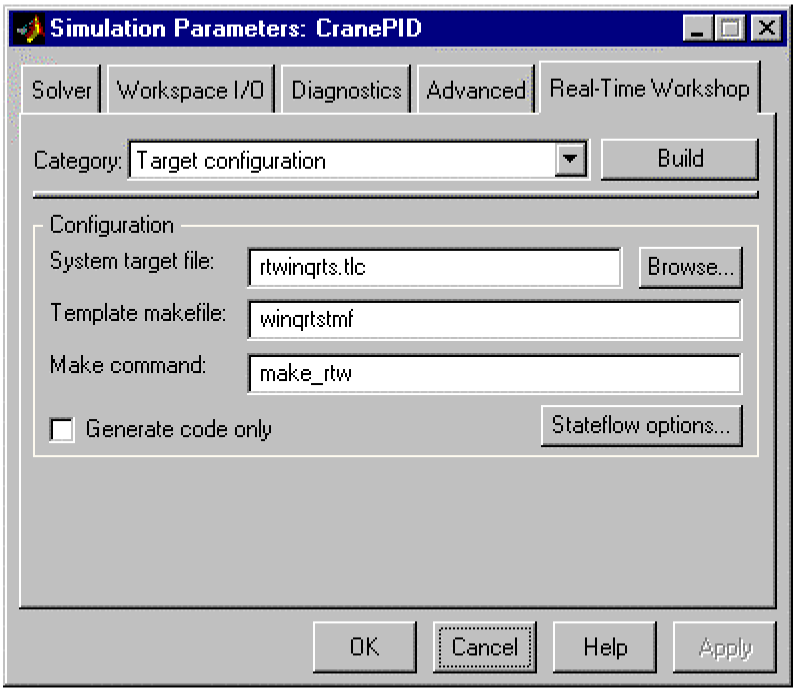


Figure 33: Simulation Parameter Options

Non standard

Installations

If the text in the

`System target file`

`Template makefile`

`Make command`

edit boxes is not the same as shown in Figure 33 then follow the following sequence

Click on the **Browse** button.

From the resulting **System Target File Browser** window, select

```
rtwinqrts.tlc   Real-Time Windows Target with
                QRTS Extensions
```

as shown in Figure 34.

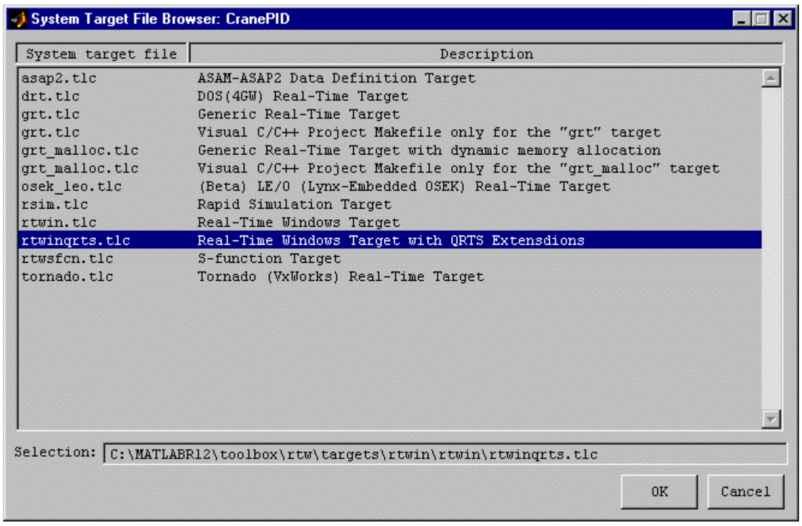


Figure 34: System Target File Browser

When the Simulation Parameters window is correctly set up, click the `Build` tab.

Real Time Workshop will now generate, compile and link the code necessary to run the model.

After selecting **Real-Time Workshop / Build** for *feedbackCraneDPID.mdl*, the following files will be automatically generated and saved in the directory c:\Feedback\Pendulum\

> *feedbackCraneDPID.bat, feedbackCraneDPID.c, feedbackCraneDPID.dt,*
>
> *feedbackCraneDPID.h, feedbackCraneDPID.mk, feedbackCraneDPID.obj,*
>
> *feedbackCraneDPID.prm, feedbackCraneDPID.reg, feedbackCraneDPID.rwd.*

To connect to the target and execute the control program, follow the following sequence

Select `Tools` from the Simulink model menu bar.

Select `External Mode Control Panel` from the pull-down menu. The window shown in Figure 35 appears.

Click on the `Connect` button.

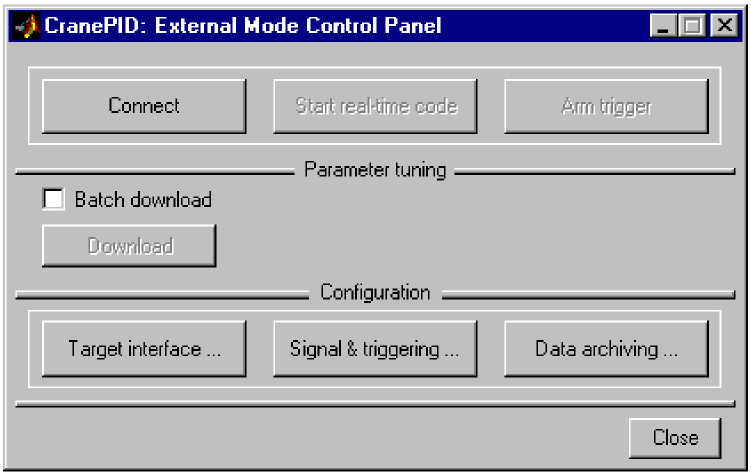


Figure 35: External Mode Control Panel

The `Connect` Button changes to `Disconnect` and the `Start real-time code` button is made available.

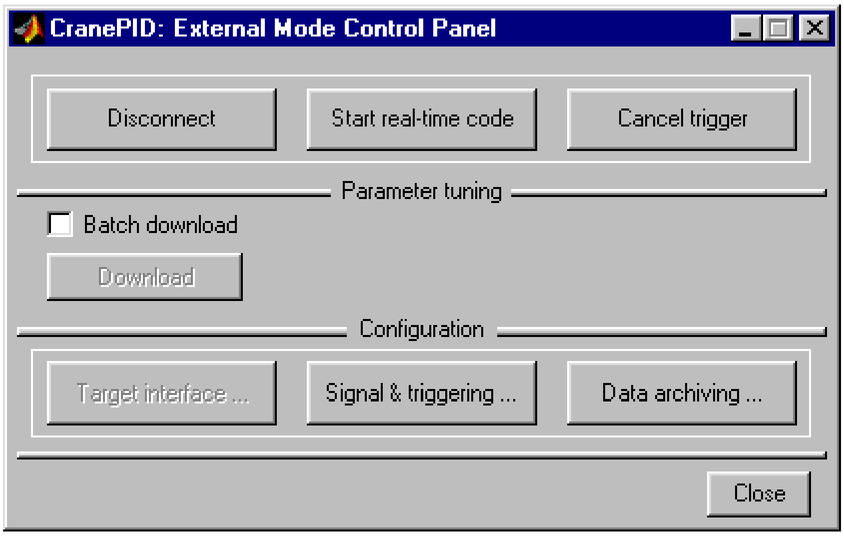


Figure 36: External Mode Control Panel

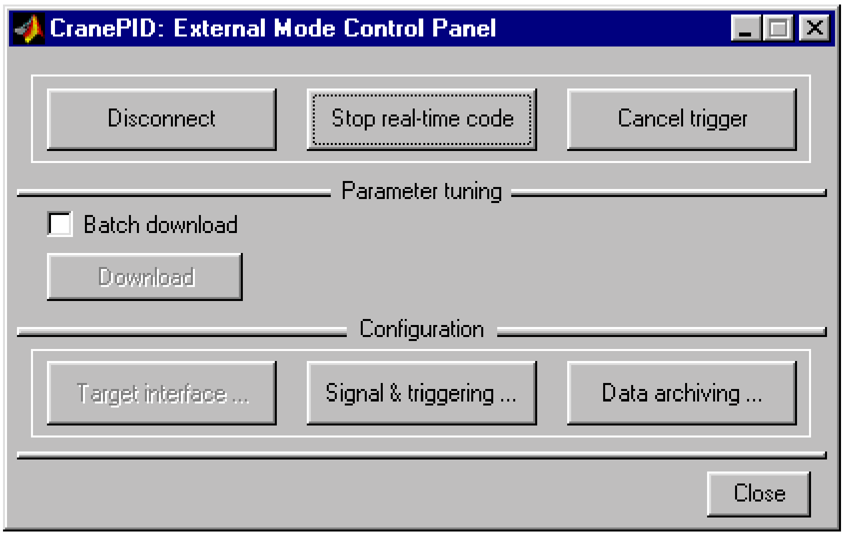


Figure 37: External Mode Control Panel

Note

The reference position is established during `connection` to the real-time target; thus the system must be moved to the reference position *before* the `Connect` button is activated.

If the system is not at the reference position, indicated by a vertical bar label on the cart track, click on the `Disconnect button`, move the cart to the reference position, and click on the `Connect` button.

Control Program Execution

Starting

To **begin** execution of the real time Control Program, click the

`Start real-time code` button

The control software is now active (the Advantech hardware adapter has been started)

Press the Green Start button on the pendulum control unit

Stopping

To **stop** execution of the real-time target click either the

`Stop real-time code` button or the
`Disconnect` button shown in Figure 37.

Press the Red Stop button on the pendulum control unit

In the `External Mode Control Panel`, the check box

`batch download`

under the

`Parameter tuning`

group title, controls the behaviour of the parameter tuning. If the

`batch download`

check box is enabled, then the new parameters will not be downloaded automatically to the real-time target until you click the

`Download`

button .

This feature is useful in a situation where you need to change *multiple* parameters and desire that all of these changes take effect instantaneously. Note, however, that there are some limitations to this capability.

Changing Real Time Parameters

To change parameters follow the following sequence;

Double Click the Simulink Block containing the parameters, make the changes, and then click the `Apply` Button followed by `OK.`

Note

Some of the parameters can be changed during program execution, others dealing with *changes* to the model will necessitate a new system build and new c code generation.

Real Time Workshop will inform you if a new build is necessary.

For example, the P I and D coefficients values can be changed in real time without re-compilation as they are simply constants read by the real time program, whereas a change of the exciting waveform from square to sinusoidal, will require a new-build.

System Operation

Before starting any experiments, perform the following steps:

Make sure that the green *Start* button on the controller box is in its **off** position. (not lit)

Switch-on the *power* switch on the controller box

Bring the cart to the centre of the rail and stop oscillations of the pole.

Start the controlling software as described above (e.g. the Simulink simulation) for an experiment.

Press the green button motor start switch on the controller box

Typical results

Figure 38 shows the graph of the cart velocity and friction compensation output. The line with the step function is the friction compensation added to the control based on the velocity, and the continuous line represents the cart velocity.

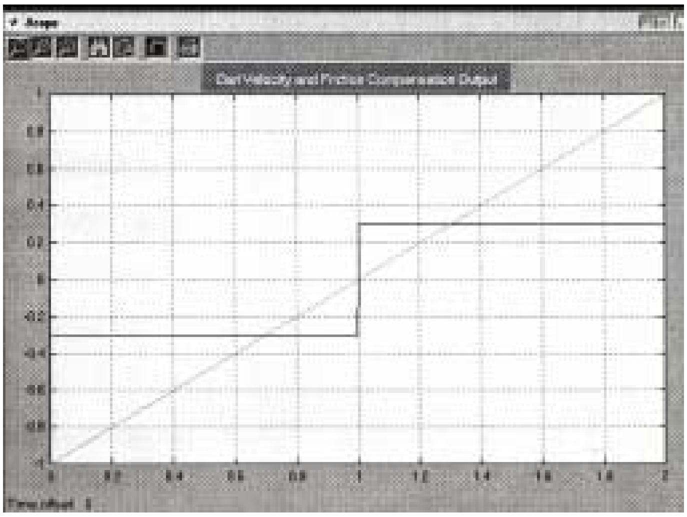


Figure 38: Cart friction versus velocity chart

Note that if the friction compensation parameters are set too high, then the cart velocity will be higher than expected leading to possible oscillations of the cart about the desired position.

Figure 39:

The first trial, plotted in Figure 39, moves the cart between two points and attempts to reduce the swing of the pendulum at these points.

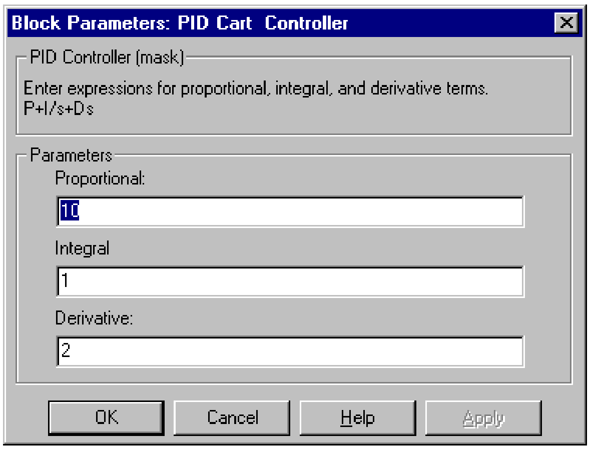


Figure 40: PID Cart Controller Gains

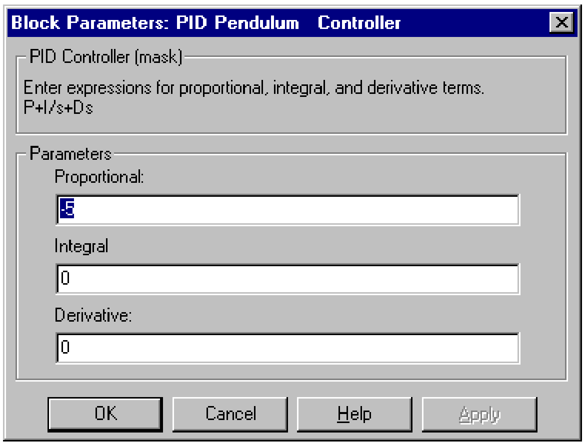


Figure 41: PID Pendulum Controller Gains

The gains are shown in Figure 40 and Figure 41. The effectiveness of the control is highlighted by setting all gains shown in Figure 41 to zero in the PID Pendulum Controller" to yield the results shown in Figure 42.

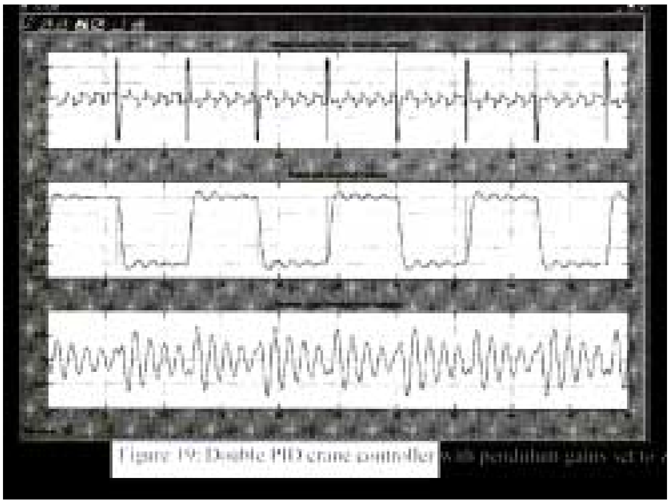


Figure 42

It can be seen that the cart is following the desired trajectory but that the pendulum continuously oscillates about the equilibrium point.

Notes

7 Experiment 2 - Inverted Pendulum - Swing Up and Stabilise

Theory and Model description

This experiment demonstrates the stabilization of the inverted pendulum. The sequence of operations for building the model, starting and stopping the program and changing parameters are the same as those described in Experiment 1.

Loading the RTWT Block Diagram

To load and execute the block diagram for the inverted pendulum control, follow the procedure outlined below:

Start MATLAB

Type the following commands into the MATLAB command window

```
Invertedpd
```

You can also open it from the MATLAB File menu, or double click its icon before starting MATLAB.

The Simulink block diagram for the Inverted Pendulum, appears and is shown in Figure 43.

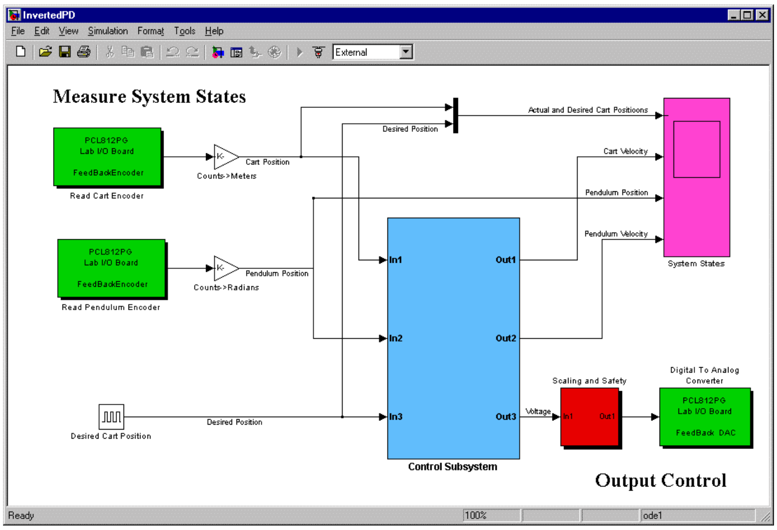


Figure 43: Simulink Block Diagram for Inverted Pendulum

The goal of this controller is to first move the pendulum near enough to a vertical region, see Figure 44, that a PD control can be switched in to stabilize the pendulum in the inverted position.

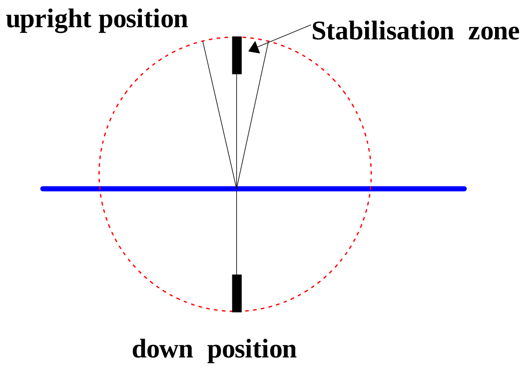


Figure 44: Upper Stabilisation Zone

Double Click on the Blue Control System Block in Figure 43 to reveal the detail shown in Figure 45

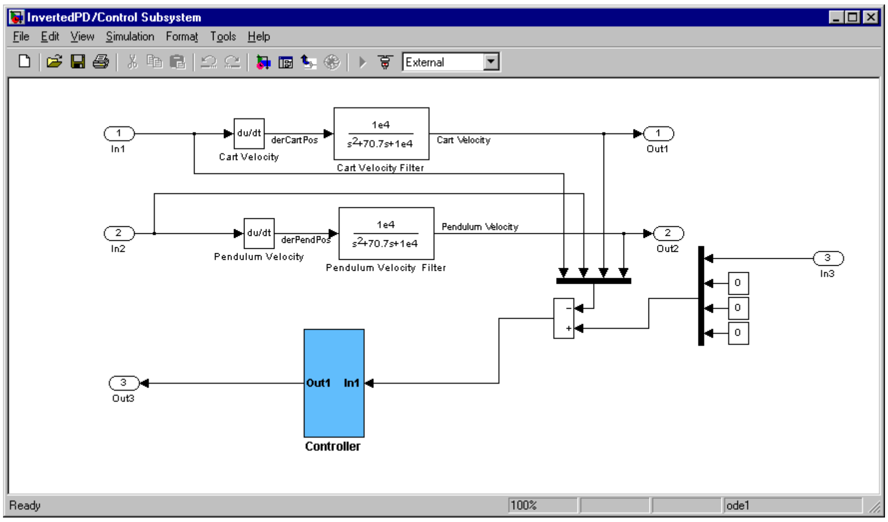


Figure 45

Out 1 and Out 2 are outputs to the Simulink scope for cart and pendulum velocity. Out 3 is the control output to the motor.

Double click on the blue Controller block in Figure 45 to reveal the details of the controllers, shown in Figure 46.

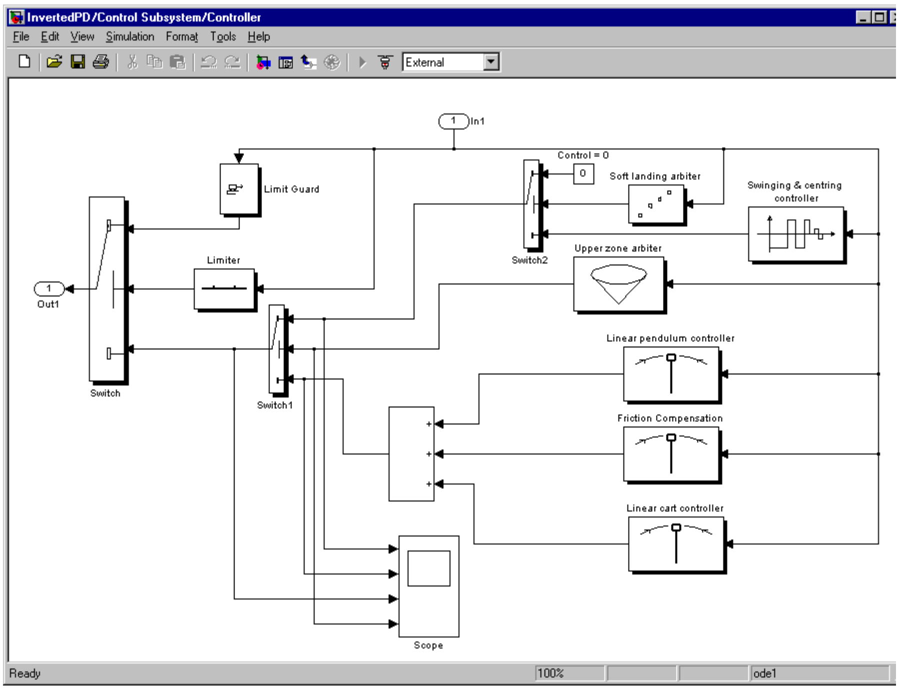


Figure 46: Inverted Pendulum Controllers

The block “Upper Zone Arbiter” determines the point at which the stabilizing control is activated.
The "Soft Landing Arbiter" calculates the kinetic energy KE of the pendulum at the lowest vertical point of travel, and the potential energy PE at the highest vertical point of travel. If KE is greater than PE then the pendulum will rotate *through* the vertical point. The function of this block is to limit the swing so that the kinetic energy of the pendulum when entering the stabilisation zone is almost zero.

The “Limiter” block switches the control to a safe mode that moves the cart in the direction away from a travel limit if the cart position approaches either end of travel.

Between these zones the “Swinging Controller” will move the cart in a manner such that the pendulum rotates near the equilibrium position.

The input, output, and friction compensation procedures are identical to the first experiment. The control gains are shown in Figure 48 to Figure 50.

The experiment is very sensitive to the values in Figure 48; that is, if alpha and beta are too small the cart will run into the overtravel as it attempts to swing up and if they are too large the pendulum will not rotate to within the stabilization zone. The difference between “too large” and “too small” is very little and you may need to make slight adjustments for a given electromechanical system.

The result of this control is shown in Figure 47 where it can be seen from the third plot that the pendulum is rotated until it can be stabilized near the zero position (vertical). A cart disturbance is added after the pendulum has balanced as shown in the first plot of Figure 47.

Note that the input-output setup, implementation, and execution are the same for both Control Experiments 1 and 2; hence, these topics are not discussed again in this section.

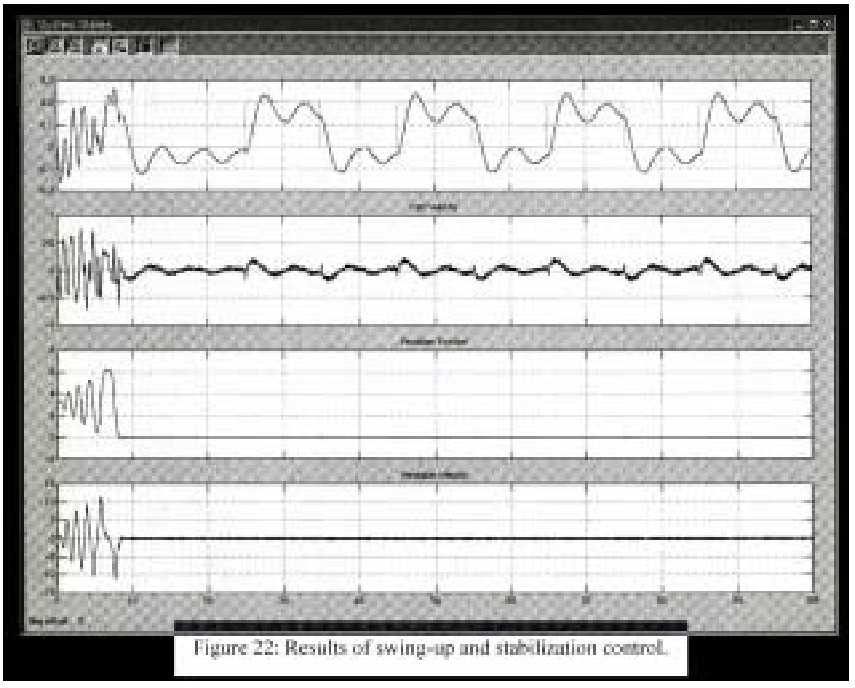


Figure 47: Results of Swing up and stabilisation control

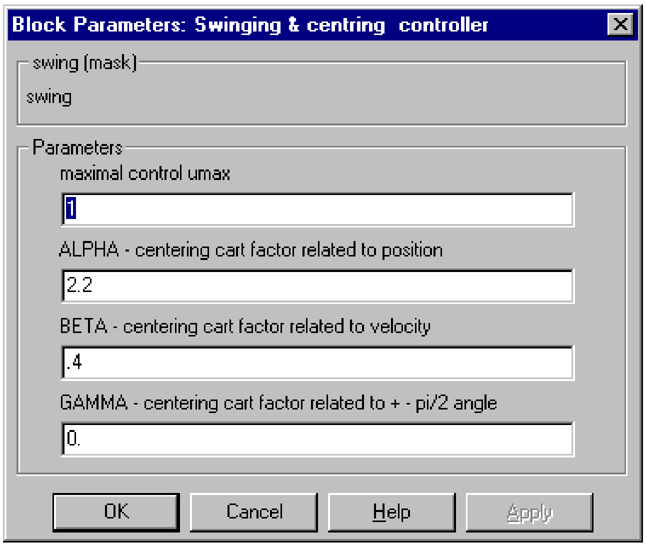


Figure 48: Parameters to control swing behaviour

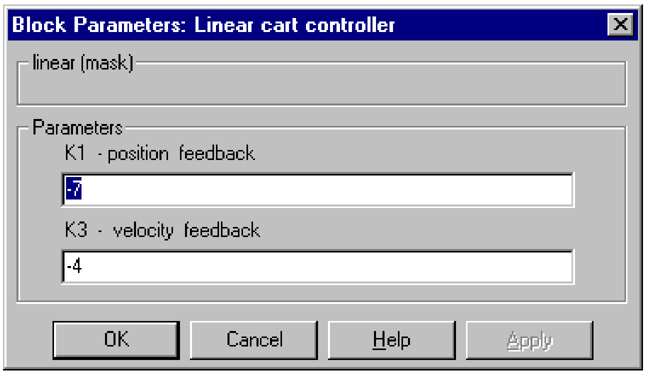


Figure 49: Cart PD gains

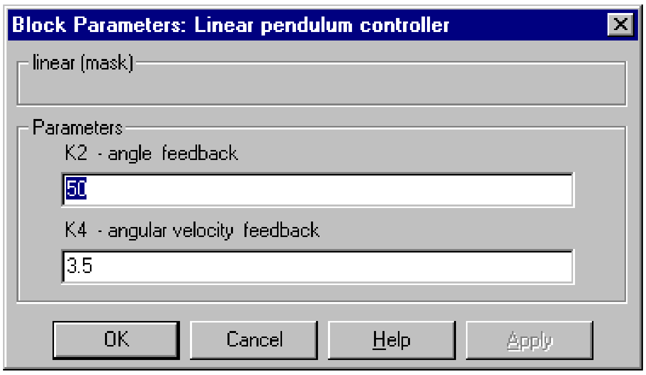


Figure 50

Notes

8 Experiment 3 - UpDownDemo

This experiment combines both the CranePID and the InvertedPd control experiments, by switching between them on a periodic basis.

Start MATLAB

Type the following commands into the MATLAB command window

```
UpDownDemo
```

You can also open it from the MATLAB File menu, or double click its icon before starting MATLAB.

The Simulink block diagram for the UpDownDemo, appears and is shown in Figure 51.

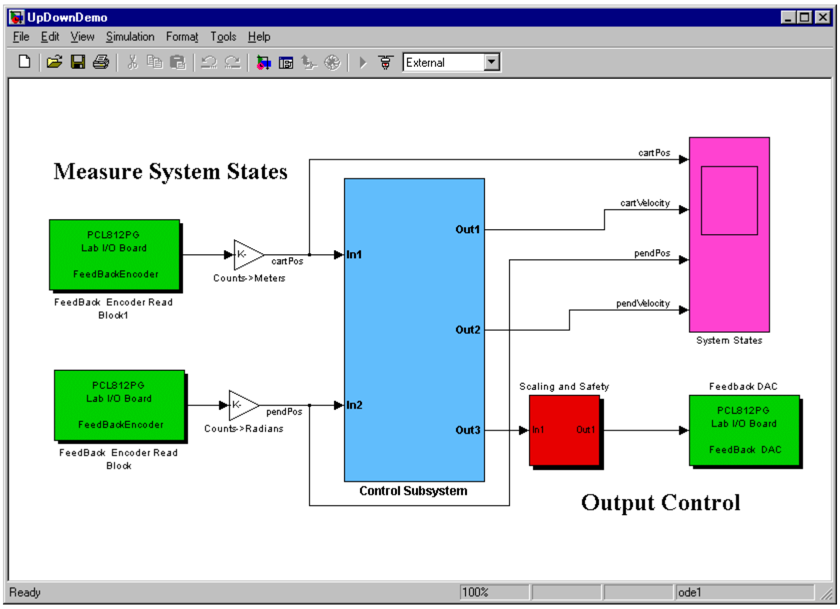


Figure 51:UpDownDemo

Double click the blue Control subsystem block to reveal its structure with the two controllers for each timed interval.

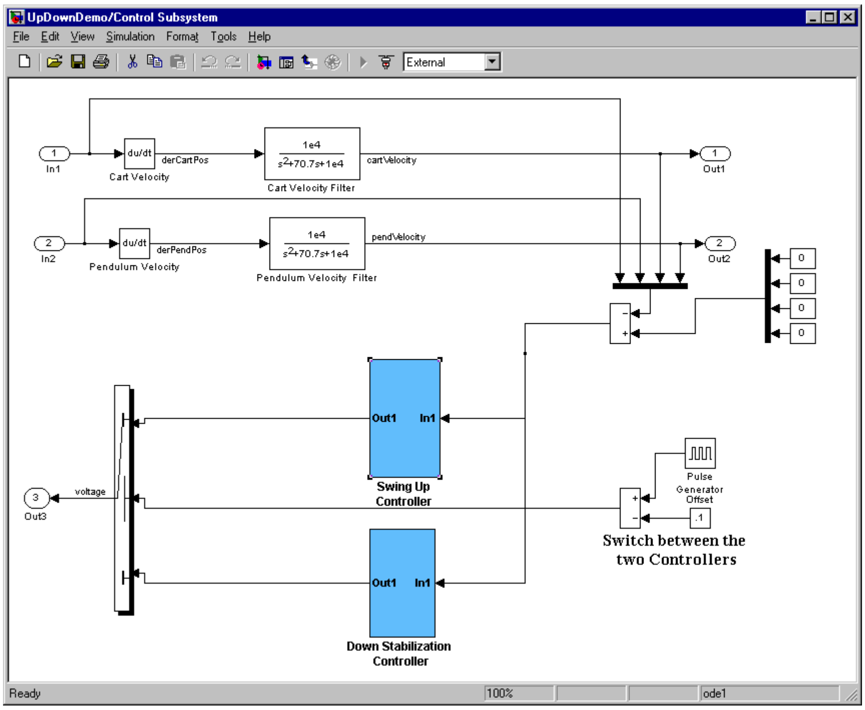


Figure 52: UpDownDemo

9 Customised Control

RTWT combines the powerful functionality of MATLAB, Simulink and Real-Time Workshop and allows users to implement any kind of control algorithm.

If you wish to implement a different kind of controller you may copy one of the supplied block diagrams, and simply replace the existing control blocks with customized control blocks (or subsystems).

However, before you start designing and implementing customized control algorithms, it is strongly recommended that you refer to the RTWT User's Manual and the Simulink User's Guide.

Notes

10 Some Common Problems

CranePID

1 If the Friction Compensation Parameter is set to high you may notice high frequency oscillations of the cart about its desired position. To remove these you should gradually reduce the friction compensation parameter value shown in Figure 30 until the oscillation stops.

Inverted PD

2 If the pendulum never reaches a vertically upright position, the parameter umax, the maximum power applied to the motor, may be too low (Figure 48). Try increasing this in small increments, until the pendulum swings with sufficient speed to reach this position.

3 If you wish to change other parameters (such as alpha or beta), then choose the parameter to vary and make small positive and negative increments to the initial value. Find the optimum value of *this* parameter which gives the "best" quality of control. Then, using this value and the same procedure find the optimum value of another parameter. After changing relevant parameters, you can repeat the sequence starting with the first parameter if necessary.

Notes