Digital Pendulum
Control Experiments

33-936S

Product Use

All users must familiarise themselves with the following information.

This product is marked as CE compliant. This means that it complies with the relevant European Directives for this product. In particular the Directives cover Low Voltage, EMC, Machinery, Pressure and electronic waste disposal.

The equipment, when used in normal or prescribed applications and within the parameters set for its mechanical and electrical performance, should not cause any danger or hazard to health or safety.

If, in specific cases, circumstances exist in which a potential hazard may be brought about by careless or improper use, these will be pointed out and the necessary precautions emphasised.

This equipment is designed for use by students as part of the learning process who must be under the supervision of a suitably qualified and experienced person in a laboratory environment where safety precautions and good engineering practices are applied.

By the nature of its intrinsic teaching functionality, parts are visible and accessible that might normally be covered up or encased in an industrial or domestic product. For this reason students attention should be drawn to the need to operate the equipment only in the manner prescribed in the accompanying documentation and supervisors must make students aware of any particular risk. The equipment should not be operated by any person alone.

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CAUTION -
RISK OF
DANGER

CAUTION -
RISK OF
ELECTRIC SHOCK

CAUTION -
ELECTROSTATIC
SENSITIVE DEVICE

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This equipment has been designed to comply with the essential requirements of the Directive. However, because of the intrinsic teaching function it cannot be electromagnetically shielded to the same extent as equipment designed for industrial or domestic use. For this reason the equipment should only be operated in a teaching laboratory environment where electromagnetic emissions in the immediate area might not be expected to cause adverse effects. In the same way users should be aware that operating the equipment near to an electromagnetic source may cause the experimental results to be outside the range expected.

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If this equipment is disposed of it must be in accordance with the regulations regarding the safe disposal of electronic and electrical items and not placed with ordinary domestic or industrial waste.

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We maintain a policy of continuous product improvement by incorporating the latest developments and components into our equipment, even up to the time of dispatch.

All major changes are incorporated into up-dated editions of our manuals and this manual was believed to be correct at the time of printing. However, some product changes which do not affect the teaching capability of the equipment, may not be included until it is necessary to incorporate other significant changes.

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In order to maintain compliance with the Directives all replacement components must be identical to those originally supplied.

Operating Conditions

WARNING:

This equipment must not be used in conditions of condensing humidity.

This equipment is designed to operate under the following conditions:

Operating Temperature	10°C to 40°C (50°F to 104°F)
Humidity	10% to 90% (non-condensing)

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ACKNOWLEDGEMENTS

Table of Contents

Manual overview

The following manual refers to the Feedback Instruments Digital Pendulum Control application. It serves as a guide for the control tasks and provides useful information about the physical behaviour. In the manual a model is proposed and linearisation is introduced. Identification algorithms are proposed and the models obtained are compared with phenomenological models. Control algorithms are developed and tested on the pendulum models and then implemented in a real time application.

Throughout the manual various exercises are proposed to bring the user closer to the pendulum control problem. Depending on the knowledge level of the user some of the sections and exercises can be skipped. The more advanced users can try to model, identify and control the pendulum on their own from the beginning.

If any of the identification or controller design exercises appear to be too difficult or the results are not satisfactory, you can use the ready-made control applications that are supplied and test them by changing the parameters of the controllers.

The relative difficulty of each exercise is indicated using the following icons:

- easy level,

- medium level,

- expert level.

Introduction

The pendulum workshop can be divided into two separate control problems. First is the crane control problem, in which the goal is to move the cart into a desired position with as little oscillation of the load (pendulum arms) as possible. The other is to stabilize the inverted pendulums in an upright position. The crane control problem is very often encountered in industrial applications where load movement is incorporated. It is especially difficult to realise when cranes are placed on ships and the effect of waves is considered.

The inverted pendulum task can be seen as a self-erecting control problem, which is present in missile launching and control applications. Furthermore the pendulum application involves a swing-up control aspect if initially the pendulum hangs freely in the vertical position.

These two control problems (inverted pendulum and crane control) have one very important difference, which is the stability. The pendulum serving as a crane is stable without a working controller. Due to energy loss through friction and air resistance it will always end up at an equilibrium point.

The inverted pendulum is inherently unstable. Left without a stabilizing controller it will not be able to remain in an upright position when disturbed.

Pendulum set description

The description of the pendulum setup in this section refers mainly to the control problems. For connection, interface and an explanation of how the signals are measured and transferred to the PC, refer to the '*Installation & Commissioning*' manual.

As shown in Figure 1 the pendulum setup consists of a cart moving along the 1 metre length track. The cart has a shaft to which two pendulums are attached and are able to rotate freely. The cart can move back and forth causing the pendulums to swing.
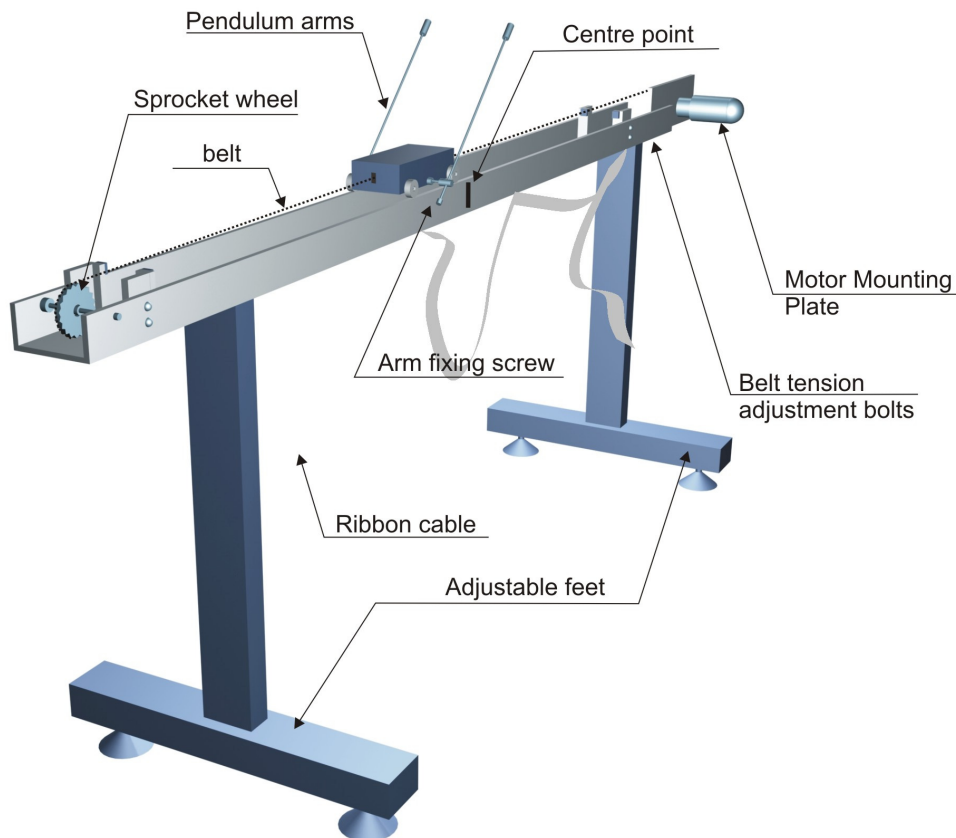
Figure 1 Digital Pendulum mechanical unit

The movement of the cart is caused by pulling the belt in two directions by the DC motor attached at the end of the rail. By applying a voltage to the motor we control the force with which the cart is pulled. The value of the force depends on the value of the control voltage. The voltage is our control signal. The two variables that are read from the pendulum (using optical encoders) are the pendulum position (angle) and

the cart position on the rail. The controller's task will be to change the DC motor voltage depending on these two variables, in such a way that the desired control task is fulfilled (stabilizing in an upright position, swinging or crane control).

Figure 2 presents how the control system is organised.
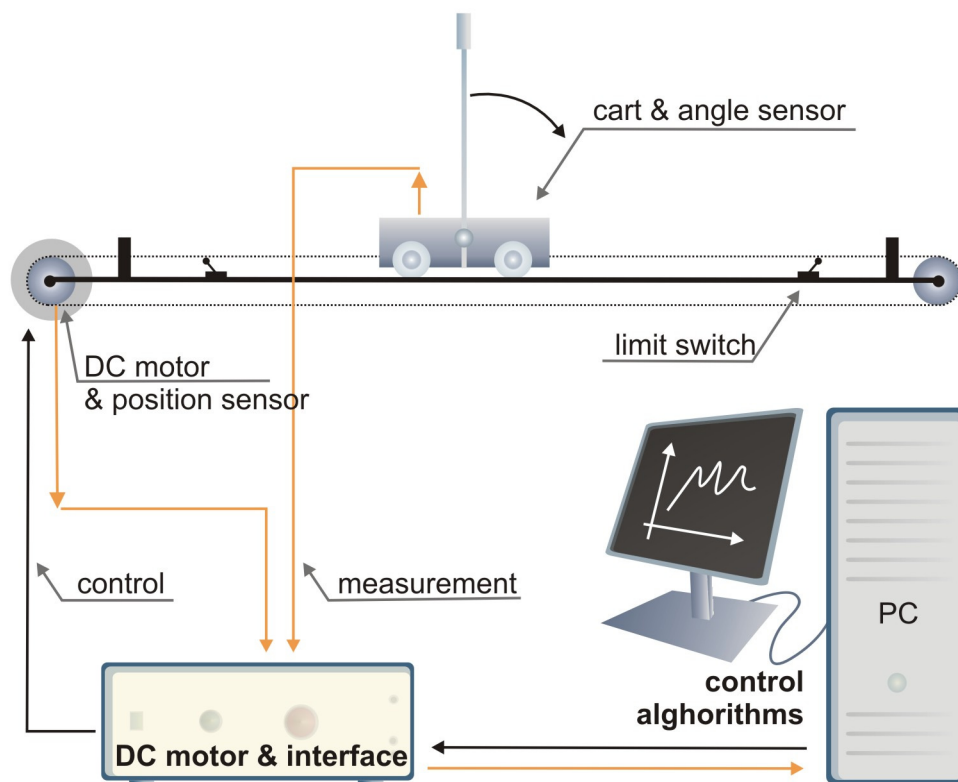
Figure 2 Pendulum control system

In order to design any control algorithms one must understand the physical background behind the process and carry out identification experiments. The next section explains the modelling process of the pendulum.

Pendulum model

Every control project starts with plant modelling, so as much information as possible is given about the process itself. The mechanical model of the pendulum is presented in Figure 3.
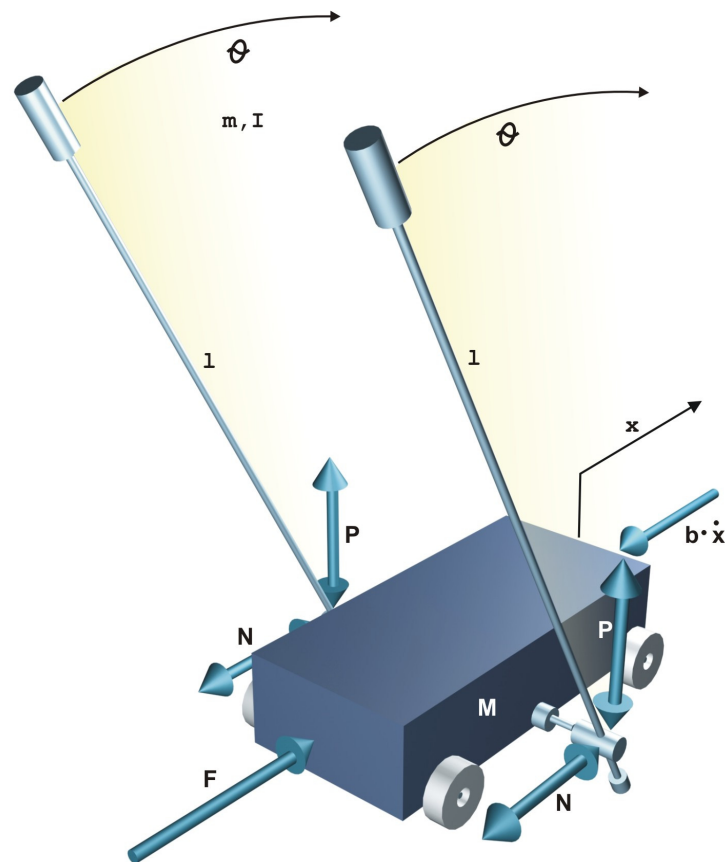
Figure 3 Pendulum phenomenological model

The phenomenological model of the pendulum is nonlinear, meaning that at least one of the states (x and its derivative or θ and its derivative) is an argument of a nonlinear function. For such a model to be presented as a transfer function (a form of linear plant dynamics representation used in control engineering), it has to be linearised.

Equations of motion

Summing the forces acting on the pendulum and cart system and the moments we obtain the following nonlinear equations of motion:

$$(m + M)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \tag{1}$$

$$(I + ml^2)\ddot{\theta} - mgl\sin\theta + ml\ddot{x}\cos\theta + d\dot{\theta} = 0 \tag{2}$$

Very often control algorithms are tested on such nonlinear models. However for the purpose of controller design the models are linearised and presented in the form of transfer functions. Such a linear equivalent of the nonlinear model is valid only for small deviations of the state values from their nominal value. Such a nominal value is often called the equilibrium point. The pendulum has two of these, one is when *θ = 0* (inverted pendulum) and the other when θ = π (hanging freely – crane control).

The inverted pendulum is an unstable system which, in terms of behaviour, means that the plant left without any controller reaches an unwanted, very often destructive state. Thus for such plants it is useful to carry out simulation tests on the models before approaching the real plant.

To complete the model given by motion equations (1) and (2), we must introduce the values of all parameters. The following table gives these values:

Parameter	Value
g- *gravity*	*9.81 m/s²*
l- *pole length*	*0.36 to 0.4 m* *- depending on the configuration*
M- *cart mass*	*2.4 kg*
m- *pole mass*	*0.23 kg*
I- *moment of interia of the pole*	*about 0.099 kg·m²* *- depends on the configuration*
b - *cart friction coefficient*	*0.05 Ns/m*
d - *pendulum damping coefficient*	*although negligible,necessary* *in the model- 0.005 Nms/rad*

Figure 4 Pendulum parameters

Two things have to be kept in mind when designing the controllers. Both the cart position and the control signal are bounded in a real time application. The bound for the control signal is set to [–2.5V .. +2.5V] and the generated force magnitude of

around [–20.0N .. +20.0N]. The cart position is physically bounded by the rail length and is equal to [-0.5m .. +0.5m].

The pendulum is a SIMO plant – single input multiple output (Figure 5). The model described by equations (1) and (2) is still missing the translation between the force *F* and the actual control signal, which is the control voltage *u* that we supply with the PC control card. Assuming that the relation between the control voltage *u* and the generated cart velocity is linear, we might add the velocity vector generated by the motor to the model and ignore the *F* vector, or translate the control voltage *u* to the generated force *F* under the assumption that constant voltage will cause the cart to move with constant velocity:

$$F = k_{Fu} \cdot \frac{du}{dt},$$ (3)

where k_{Fu} is the gain between the *u* voltage derivative and the *F* force. However one must remember that derivative introduction in models especially in Simulink may cause simulation problems.
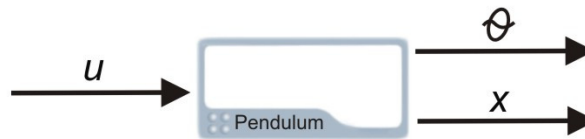
Figure 5 Pendulum model

Exercise 1 – Nonlinear model

Introduction
For the initial exercise the user has been provided with the pendulum model described by equations (1) and (2). The model shown in Figure 5 can be opened in Simulink. The model is called *pendmod_nonlin.mdl* and it can be run by navigating using the Windows *start* menu (Figure 6):

Start menu→All Programs→Feedback Instruments→Feedback 33-936…→Simulink Models

Figure 6 Start menu

Matlab will run and a Simulink menu will be displayed (Figure 7). Double click *Pendulum Simulation Models* then double click the appropriate model in the next selection window.
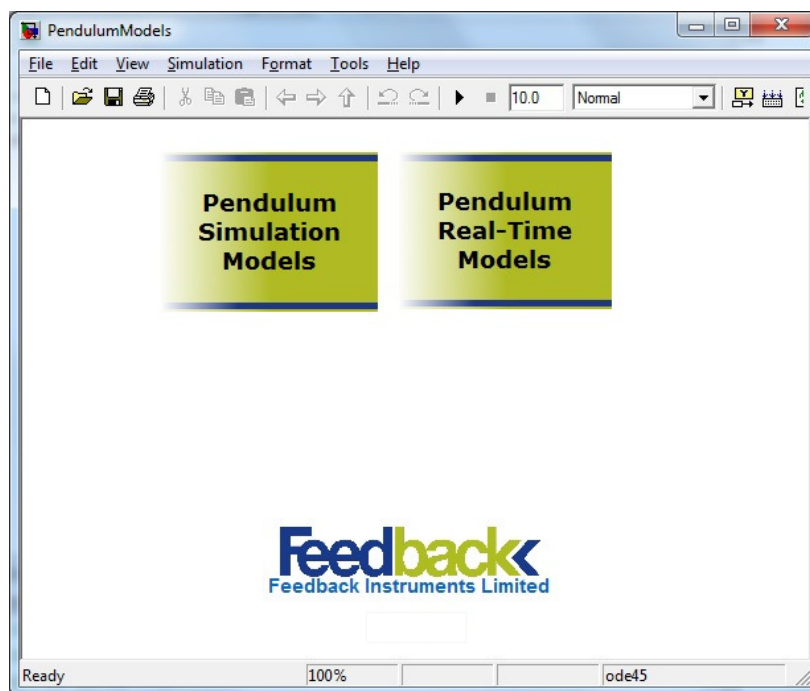
Figure 7: Model category window

Task

To begin with the user is advised to check the responses of the model in the situation where zero *u* voltage is applied (i.e. the cart is not driven). You can change the value of θ_0 (the initial pendulum angle) and see how the pendulum responds. To change the initial pendulum angle double click the 'Pendulum with DC motor' block then double click the 'Pendulum' block to gain access to its parameters. If you wish to explore the pendulum model further, right click on the block and select 'Look under mask'.

Example results and comments

Two values of the angle θ_0 are particularly interesting: $\theta_0 = 0$ and $\theta_0 = \pi$. The first is an equilibrium point for which a small disturbance of an open loop system will cause the pendulum to fall down, swing and finally settle down with $\theta = \pi$. The small disturbance can be introduced by small initial angle selection, for example $\theta_0 = 0.001$. The second $\theta_0 = \pi$ is an equilibrium in which the pendulum will always settle when no *F* force is applied.

The results for initial θ_0 value of $\theta_0 = 0.001$, are presented in Figure 8. Because of the friction forces the pendulum swings until it settles at $\theta = \pi$.
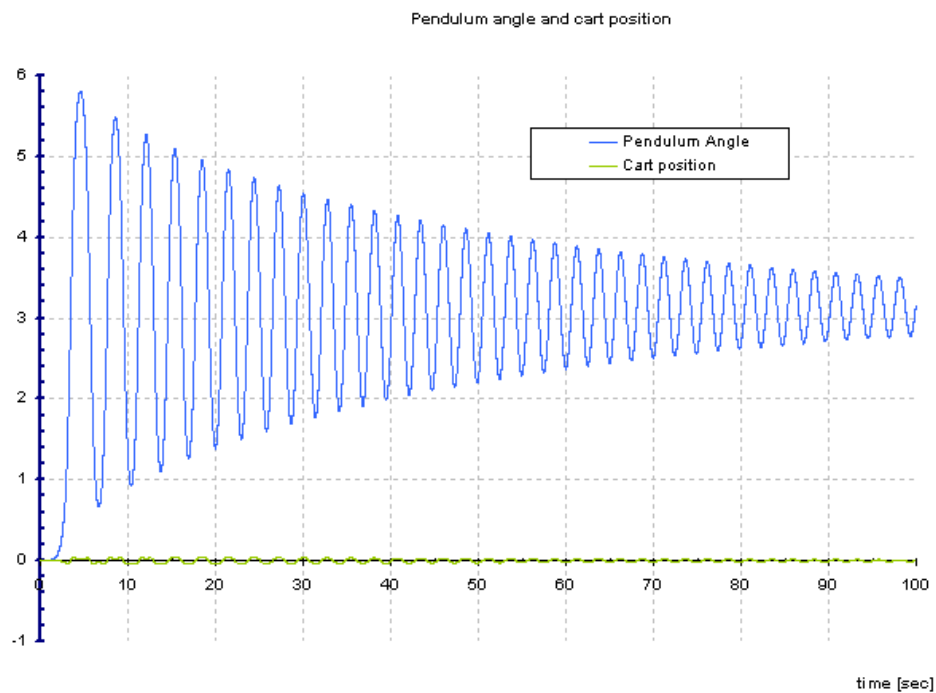
Figure 8 Pendulum model results for $\theta_0 = 0.001$

Open the scope then run the simulation. Click the binocular icon in the scope window if necessary to rescale the plots. Try different values of θ_0. Examine the response of the pendulum and the cart.

Model linearisation

To carry out analysis of the model dynamics for open loop[1] systems using techniques such as Bode plots, poles and zeros maps, Nyquist plots, root locus (for closed loop[2] systems only), the model has to be linearised. Linearisation of a given phenomenological model can be carried out for the pendulum and in equations (1) and (2) we could substitute the nonlinear functions (sine and cosine) with their linear equivalent. Such a linearisation of a working point is done with a Taylor approximation of the nonlinear functions. For small angle deviations around an equilibrium point of $\theta = 0$ (inverted pendulum) we can assume that the following functions can be linearised:

$$\sin\theta \cong \theta, \tag{4}$$

$$\cos\theta \cong 1, \tag{5}$$

$$\dot{\theta}^2 = 0. \tag{6}$$

Thus the motion equations (1) and (2) take the form:

$$(m+M)\ddot{x}+b\dot{x}+ml\ddot{\theta}=F, \tag{7}$$

$$(I+ml^2)\ddot{\theta}-mgl\theta+ml\ddot{x}+d\dot{\theta}=0. \tag{8}$$

One must remember that the equations (7) and (8) will only be valid around the position where $\theta = 0$. For the position where $\theta = \pi$ (crane control) the following substitutions have to be made:

$$\sin\theta \cong -\theta, \tag{9}$$

$$\cos\theta \cong -1, \tag{10}$$

$$\dot{\theta}^2 = 0. \tag{11}$$

Thus the motion equations (1) and (2) take the form:

$$(m+M)\ddot{x}+b\dot{x}-ml\ddot{\theta}=F, \tag{12}$$

$$(I+ml^2)\ddot{\theta}+mgl\theta-ml\ddot{x}+d\dot{\theta}=0. \tag{13}$$

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[1] Open loop system – the plant without a controller
[2] Closed loop system – the plant and controller with a negative feedback loop, see "Control" section for more information.

The linear model of the pendulum, just as the nonlinear model, has one input which is the force *F,* and two outputs which are the pendulum angle *θ* and the cart position *x* (Figure 5)*.* However, in the inverted pendulum task we are mostly interested in the *θ* angle stabilisation thus we may treat the cart position as an uncontrolled output. With one input *F* and one output *θ,* two linear models in the form of transfer functions can be obtained, each for small deviations of the *θ* angle from the two equilibrium points of *θ* = [0, π]. Remember that the translation between the control voltage and the force should be added (equation 3).

Exercise 2 – Linear models

Introduction

Two of the linear models described by the equations (7),(8) and (12),(13) have been created for you called *pendmod_lin_unstable.mdl* (inverted pendulum) and *pendmod_lin_stable.mdl* (crane) respectively. The user is advised to transform (7),(8) and (12),(13) into transfer function form as an exercise.

One must remember that both of these transfer functions resemble the behaviour of the real plant only for small deviations of the *θ* angle. The models *pendmod_lin_stable.mdl* and *pendmod_lin_unstable.mdl* hold the appropriate transfer functions:

$$G(s) = \frac{\theta(s)}{F(s)} = \frac{s}{d_3 \cdot s^3 + d_2 \cdot s^2 + d_1 \cdot s + d_0} \qquad (14)$$

For both of these models the signs of the denominator parameters will differ. The *x* position output is calculated based upon the output of the *G(s)* transfer function, which is *θ.* For the *stable* transfer function (i.e. the representation of the pendulum behaviour in the region where *θ* = *π*), an offset of o_{ffset} = *π* has to be added to the output of the transfer function. That is our initial condition.
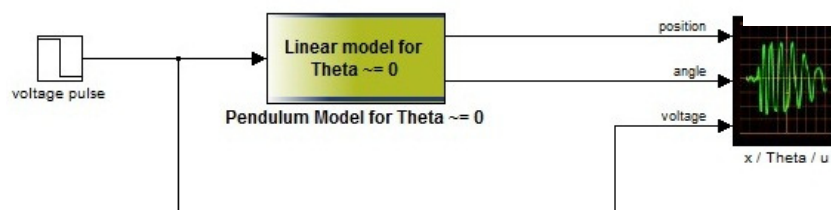

Figure 9 Example of the linear Simulink model

Task

Run the linear (*pendmod_lin_unstable.mdl*) & nonlinear (*pendmod_nonlin.mdl*) pendulum models in an open loop system (no controller) and compare the responses. Also, with the use of Matlab, the Bode diagrams, zeros and poles maps can be drawn to carry out initial dynamic response analysis of the pendulum.

In the transfer function form of the model the initial condition is equal to zero thus in order to see the linear model response we have to stimulate it with a control voltage pulse to see the reaction. Inspection of the *voltage pulse* block in either model will show that the system is given a momentary 0.1V 'kick' at time t=0 so that the response can be observed.

Example results and comments

The dynamic response of the linearised unstable transfer function *pendmod_lin_unstable.mdl* will be different from the nonlinear system *pendmod_nonlin.mdl*. When disturbed by a small voltage impulse *u,* the unstable system will not settle at equilibrium with $\theta = \pi$. The linearised transfer function is only valid for small deflections of θ thus it is unstable and the response grows to infinity without any control action.

The stable (crane) transfer function *pendmod_lin_stable.mdl* is also only valid for small deviations of θ but for an equilibrium of $\theta = \pi$, it will behave similarly to the nonlinear model around the point $\theta = \pi$.

Model identification

The phenomenology analysis delivers a model that we 'think' fits the pendulum the best. However we know that it is just some approximation. We might have made mistakes analysing the phenomenology and, for example, chosen the wrong model structure or incorrect parameter values. To have an adequate model we could tune the phenomenological model. Because of the fact that it is nonlinear, the identification and tuning of that model can be a very difficult task (gradient methods). To simplify the identification, modelling and control, the control algorithms will be designed based upon the dynamics of the linear models. They will be tested however on the nonlinear model and plant. Furthermore the identified models will be discrete as such models are obtained in the course of the Least Mean Square identification methods implemented in Matlab. Any of the continuous models can also be transformed for comparison purposes into the discrete form. The discrete models obtained can be also transformed into the continuous equivalent.

Plant identification theory is very broad and solves numerous problems. In the identification experiments of the linear pendulum models it is convenient however to use the Matlab identification tool.

Before any model identification procedures of the pendulum setup are carried out, we have to describe, through a simple real-time simulation, the character of the dead zone. The dead zone is a nonlinearity, which could influence the model validity. Because of the static friction forces (also called *stiction*), with very small values of control signal the cart will not move. Furthermore the static friction force may not be symmetrical. We can compensate for that simply by adding an additional voltage offset value to the cart control signal. Measurement of the value of the offset is the purpose of Exercise 3.

### Exercise 3 – Static friction compensation

Introduction
Since the control signal is the voltage that we supply to drive the cart motor, the dead zone value will be expressed in volts. As explained previously, it is likely to be different for the two directions of movement. It has to be identified because the tension of the pendulum motor belt can be set up differently by the user and thus cause significantly different static friction forces. The measurement of static friction can be carried out using the real-time model *PendulumFriction.mdl*. In the simulation the control voltage is increased until the cart moves in the positive direction. The cart is stopped. Then the control is reversed and increased until the cart moves in the negative direction. The voltage values for which the cart begins to move are recorded and can be used in the controller to compensate for static friction.

Task
Run the friction identification simulation on the pendulum. Unlike the previous models which were non real-time simulations and did not require access to real hardware, this model runs in real-time and interfaces with the Pendulum Mechanical unit using an I/O card installed in the PC and the Digital Pendulum Controller.

The model is called *PendulumFriction.mdl* and it can be run from the Simulink menu by double clicking the *Pendulum Real-time models* block.

The real-time models provided with the Feedback 33-936 software must be built before they can be executed in real-time. If the model is not already built, select the *Tools* menu then *Real-Time Workshop* and *Build Model* (Figure 10).
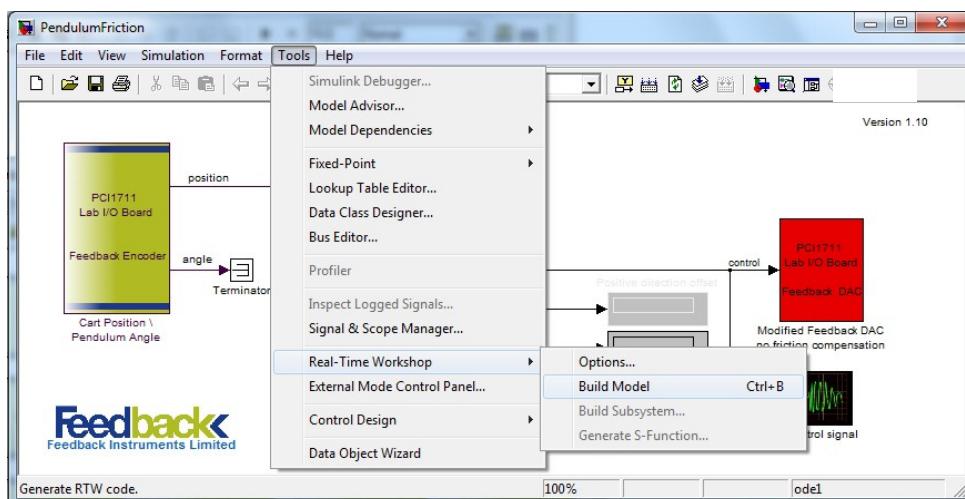
Figure 10 Building a model

Go to the Matlab command window to observe the progress of the build. When the build is complete, check that it was successful (i.e. no error messages were displayed in the command window).

You are now ready to run the model.

Running a real-time model

The following sequence is very important and should always be followed when preparing to execute a real-time model.

1. **Make sure that before you run the application, the Digital Pendulum Controller POWER switch is ON but the green START pushbutton is OFF (the green pushbutton lamp is not illuminated). If necessary, press the red STOP pushbutton.**

2. **Place the cart in the centre of the track (over the marker) and stop the pendulum from swinging.**

3. **Connect the application with the target by pressing the *Connect to Target* button 🔧 or using the *External Mode Control Panel* from the *Tools* menu of the Simulink window. The application will not run yet but**

the PCI-1711 card will be initialised correctly. Confirm that the application is connected by the scrolling activity indicator at the bottom of the Simulink model window. Now press the green START pushbutton on the Digital Pendulum Controller unit.

4. **If the area around the pendulum is clear you can start the application by pressing the *Start Real Time Code* button ▶ or using the *External Mode Control Panel*.**

5. **When the simulation is complete, press the red STOP button on the Digital Pendulum Controller.**

6. **If you wish to stop a simulation part way through, press the red STOP switch on the Digital Pendulum Controller first. Then stop the simulation using the Simulink controls. To restart, follow the steps above again.**

Follow the steps above and run the *PendulumFriction.mdl* model. The signal to the cart motor will be increased until the cart just moves and then its value will be recorded. The sequence is then repeated in the opposite direction.

Example results and comments

The results of the static friction measurements will be presented in two displays. You can use these measured values to correct the friction compensation value in all future simulations. The compensation values are stored in the '*Friction Compensation*' block (Figure 11, Figure 12), which is located within the '*Feedback DAC*' block of each controller model. However if the result of the test is significantly greater than ±0.2V it may appear to be biased by other nonlinearities and should be investigated further.
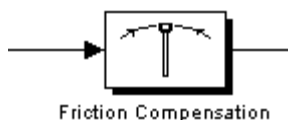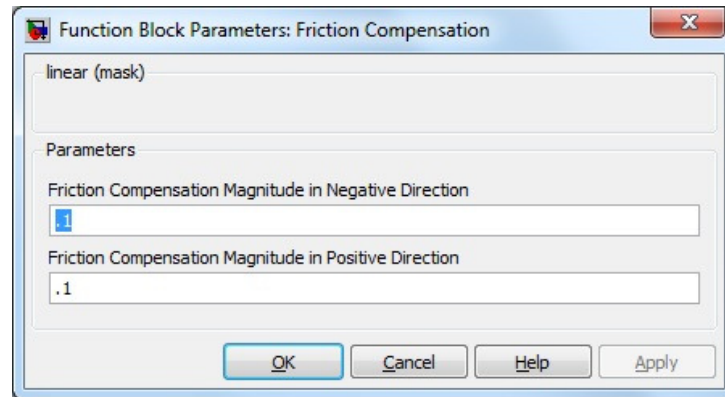

Figure 11 Friction compensation block

Figure 12 Friction compensation block parameters

Dynamic Model

With the static friction identification carried out we can try to identify the first dynamic model of the pendulum. Although the most interesting signal relation is the one between the control signal u and the angle θ we can first try to identify the linear model between the control signal u and the cart position x. That model will be used for first PID controller design and tested in the *Pendulum control* section.

There are a few important things that the control system designer has to keep in mind when carrying out an identification experiment:

- *Stability problem* – if the plant that is identified is unstable, the identification has to be carried out with a working controller which introduces additional problems that will be discussed further on. If the plant is stable and does not have to work with a controller the identification is much simpler.
- *Structure choice* – a very important aspect of the identification. For the linear models it comes down to the choice of the numerator and denominator order of the transfer function. It applies for both the continuous and discrete systems.
  As far as the discrete models are concerned the structures are also divided in terms of the error term description: ARX, ARMAX, OE, BJ[3].
- *Sampling time* – the sampling time choice is important for both identification and control. It cannot be too short nor can it be too long. Too short sampling time might influence the identification quality

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[3] More information about these structures can be obtained as part of a System Identification course.

because of the quantisation effect introduced by the A/D converter. Furthermore the shorter the sampling time the faster the software and hardware has to be and more memory is needed. However short sampling time will allow for elimination of aliasing effects and thus anti aliasing filters[4] may not be needed. Long sampling times will not allow for including all of the dynamics.

- *Excitation signal* – for linear models the excitation choice is simple. Very often designers use white noise, however in industrial applications it is often disallowed. It is attractive because it holds a very broad frequency content thus the whole dynamics of the plant can be identified. If the dynamics are not too complex, several sinusoids with different frequencies can be summed to produce a satisfactory excitation signal.
- *Identification method* – usually two methods are used, the Least Mean Square (LMS) method and the Instrumental Variable method. The LMS method is the most popular and is implemented in Matlab. This method minimizes the error between the model and plant output. The optimal model parameters, for which the square of the error is minimal is the result of the identification.

Cart model identification

The following exercise takes into account the facts mentioned above and provides an identification experiment, which results in a discrete model of the moving cart due to the application of the control voltage. At this point the pendulum is ignored; its movement is treated as a distortion. It would be best to immobilize the pendulum to reduce the disturbance.

Exercise 4 – Cart model identification

Introduction

All of the control real-time simulations are carried out with a sampling time of $Ts = 0.001$[s]. In the identification experiments the sampling time varies. For this identification the Matlab identification interface is used. Here the sampling time is set to $Ts = 0.05$[s].

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[4] These are the basics of a Digital Signal Processing course. For more insight the user is asked to study more on signal processing and digital control.

The identification experiment is carried out with the model *CartIdent.mdl*. The excitation signal is composed of several sinusoids and an optional random signal block, which output can be added to the excitation. You can check the effect of the additional random signal on the identification results. However one must be careful with the choice of random signal parameters as extensive random signals are destructive for the actuator. The experiment lasts 20 seconds and signals are collected in a form of vectors and are available in the Matlab workspace. The signals are cart position, pendulum angle, motor control voltage and a time reference.

Task

Carry out the identification experiment by running *CartIdent.mdl* and collecting the data. When the run is complete, press the red STOP pushbutton on the Digital Pendulum Controller unit.
Reduce the slider gain value if the cart hits the limit switch during the identification experiment or improve stiction compensation.

Analyse the results using the Matlab identification interface (part of the System Identification Toolbox). You will notice that a new workspace variable *simout* has been created which contains the signals listed above.

For simplicity assign the input and output data to the *u* and *y* vectors respectively:

u = simout(:,3);
y = simout(:,1);

For instructions on identification experiments refer to the '*Matlab Guide*' manual.

One of the possible model structure and order choices is the oe241.
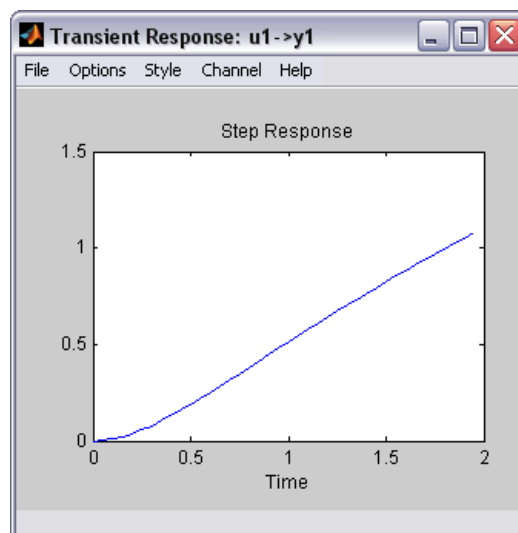
Figure 13 Step response

Example results and comments

The step response of the identified system should be similar to the one presented in Figure 13. If the model is transferred into the Workspace it can be compared against the discrete equivalent of the continuous transfer function. In order to obtain the discrete form use the '*c2d*' command. Make sure you specify the proper sampling time.

Compare the step responses or bode plots of the two systems (using '*step*' and '*bode*' commands). You can also transform the discrete models into continuous equivalent with the means of '*d2c*' command.

The model obtained is used in the first PID control exercise.

Crane identification

A similar identification experiment can be carried out for the Crane mode to identify the transfer function between the control voltage *u* and the pendulum angle *θ*. As presented in the *Model linearisation* section, two linear models can be identified. The first which holds for small deviations of angle *θ* around the *θ=0* point, and the other when *θ=π.* In this section the crane function of the pendulum is considered, thus *θ=π.*

Exercise 5 – Crane linear model identification

Introduction

The crane linear discrete model can be identified in the same way as the cart model (Exercise 4), using the Matlab System Identification Toolbox. The identification experiment is carried out with the model *CraneIdent.mdl*. The excitation signal is composed of several sinusoids. The experiment lasts 100 seconds in total. The excitation signal is applied for the first 10 seconds then the pendulum is allowed to swing freely for 80 seconds to identify the damping effect. Signals are collected in a form of vectors which are available in the Matlab workspace. The signals are cart position, pendulum angle, motor control voltage and a time reference. The sampling time is set at *Ts = 0.05* [s].

Task

Carry out the identification experiment by running *CraneIdent.mdl* and collecting the data. When the run is complete, press the red STOP pushbutton on the Digital Pendulum Controller unit.

For simplicity assign the input and output data to the *u* and *y* vectors respectively:

u = simout(:,3);
y = simout(:,2);

With the use of the Matlab identification interface, identify a discrete model. Import the data and run the estimation as described in *Matlab Guide* manual. Select the proper structure of the model (e.g. OE 2 6{or higher} 1). You can check the quality of the response of the identified model by the step response analysis, transient response, pole and zeros map, frequency response and model residuals.

Example results and comments

The response of the model is compared to the pendulum output in Figure 14. If the model is transferred into the Workspace it can be compared against the discrete equivalent of the continuous transfer function (14). In order to obtain the discrete form use the '*c2d*' command. Make sure you specify the proper sampling time.

The models you obtain may strongly depend on the mounting of the belt and the tightness of all fixings.
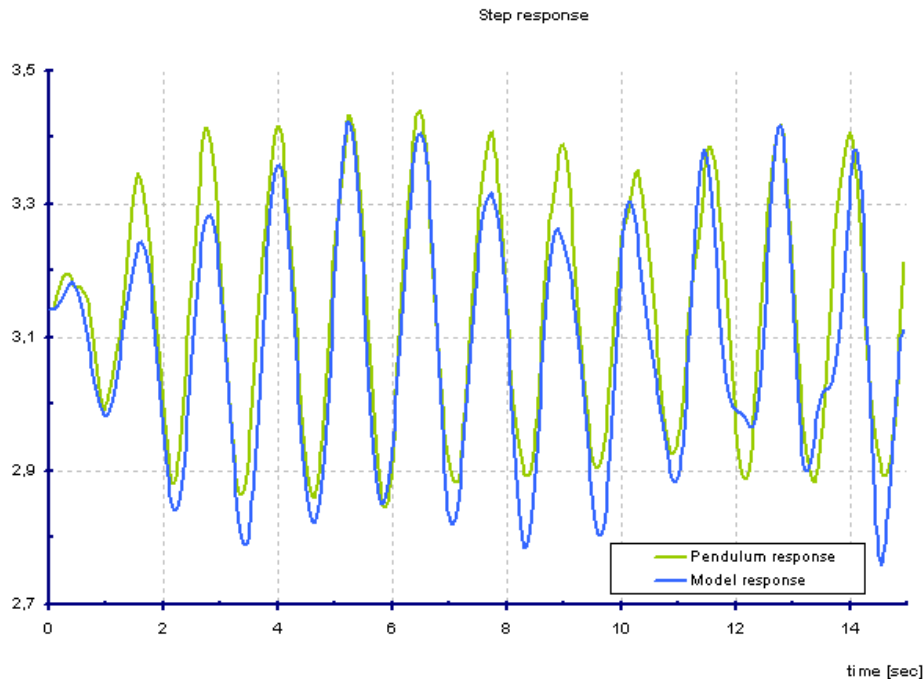
Figure 14 Identified model and pendulum response to excitation

Drag the oe241 model icon to the *To Workspace* box to export it to a workspace variable. Save the session using the *File→ Save session as..* menu.
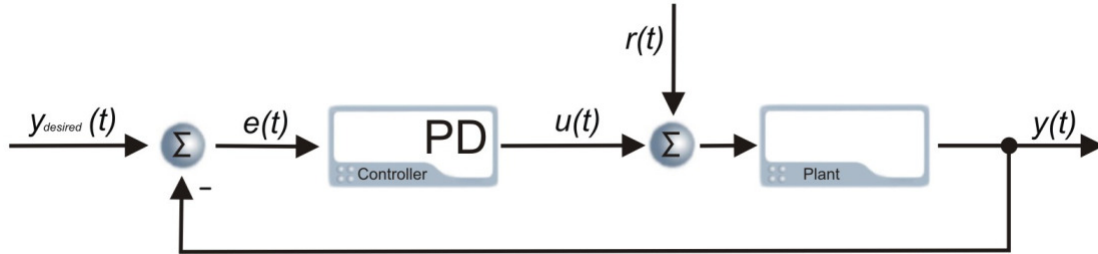
Inverted pendulum identification

Inverted pendulum model identification is a difficult task mainly for one aspect – stability. The inverted pendulum is unstable and has to be identified with a running, stabilising controller[5] i.e. closed loop identification[6]. The controller introduces output noise and control signal correlation, which leads to model corruption. This correlation can be broken by introducing an additional excitation signal *r*, which is added to the control signal u (Figure 15).

If the power of the signal *r* is substantial compared to the noise power *n,* the proper model should be identified.

---

[5] The pendulum control aspect is explained in the '*Pendulum Control*' section.

[6] Closed loop system identification is a broad topic and more advanced users are advised to refer to identification literature to get more insight.

Figure 15 Unstable system identification

Such an approach will only allow for the linear model identification of the transfer function between the voltage control signal u and the pendulum angle θ, for small deviations of the angle around the equilibrium point of $\theta = 0$.

More intelligent identification methods should be applied for complete nonlinear pendulum model identification (gradient methods).

A closed-loop system describing the relation between signal r and y is identified:

$$T(z^{-1}) = \frac{G(z^{-1})}{1 + C(z^{-1}) \cdot G(z^{-1})}, \tag{15}$$

where $T(z^{-1})$ is the complete system discrete transfer function describing the relation between, $C(z^{-1})$ is the discrete controller transfer function and $G(z^{-1})$ is the pendulum model discrete transfer function. The relation (15) can be transformed to yield the pendulum model:

$$G(z^{-1}) = \frac{T(z^{-1})}{1 - C(z^{-1}) \cdot T(z^{-1})} \tag{16}$$

Such approach is presented in Exercise 6 to yield a complete inverted pendulum model.

Exercise 6 – Inverted pendulum linear model identification

Introduction
The linear discrete model of the inverted pendulum can be identified only with a controller. This is a major difference compared to the previous exercises.

The identification experiment is carried out with the model *InvPendIdent.mdl*. The excitation signal is composed of several sinusoids. The experiment lasts 40 seconds in total. Signals are collected in a form of vectors which are available in the Matlab workspace. The signals are pendulum angle, motor control voltage, cart position and a time reference. The sampling time is set at *Ts = 0.001* [s].

It is IMPORTANT that before the identification experiment is started, you place the cart in the ZERO position and settle the pendulum in the bottom vertical position, $\theta = \pi$.

Task

Carry out the identification experiment by running *InvPendIdent.mdl* and collecting the data.

Caution: Keep well clear of the pendulum when running the model

When the run is complete, press the red STOP pushbutton on the Digital Pendulum Controller unit. The pendulum response to the excitation will be as presented in Figure 16.
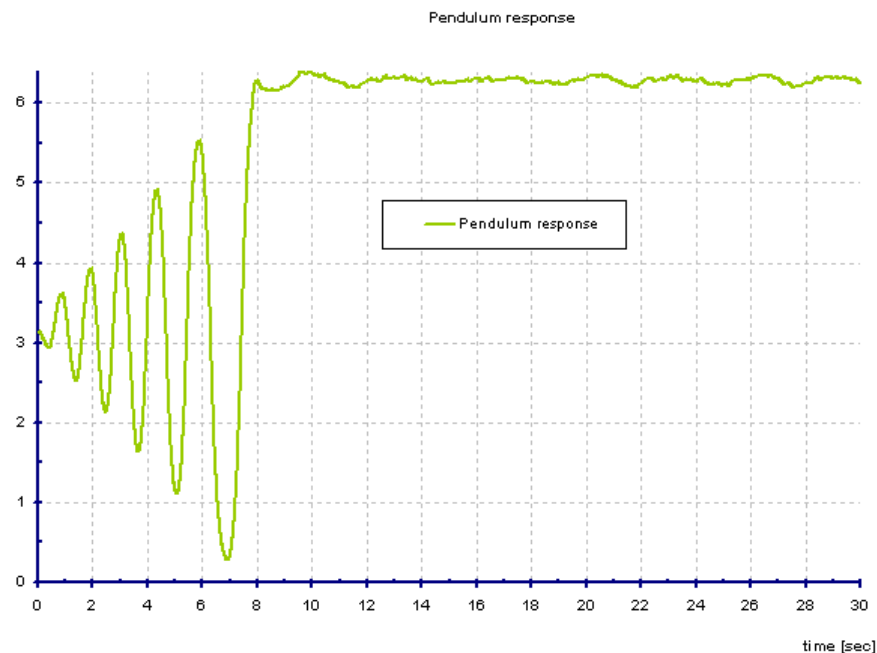
Figure 16 Pendulum response

For simplicity assign the input and output data to the *u* and *y* vectors respectively:

u = simout(:,5);
y = simout(:,1);

For instructions on identification experiments refer to the '*Matlab Guide*' manual.

With the use of the Matlab identification interface, identify a discrete model of the whole system together with controller - $T(z^{-1})$. Specify the proper sampling time which is *Ts=0.001*s for this example. Select the proper structure of the model (e.g. OE 3 6 1 or higher) then estimate.

After identifying the transfer function of the whole system with controller extract the plant model according to equation (16). Use the *Mextract.m* file to perform that transformation. Type '*help Mextract*' to view help on this file. The model is also transformed into a continuous form.

Make sure that for the model identification you use the data from the time when the pendulum is in the upright position. Referring to the example plot (Figure 16), it is safe to use data from around the 9 seconds onwards in this case. Different mechanical units will have a different swing-up time so either plot the position vs. time using:

plot (simout(:,4), simout(:,1)) to check, or use a larger figure (e.g. 15 seconds) to be sure to avoid the swing-up region.

Import the data as usual using a start time of 0. The time range of the imported data can then be selected using *Preprocess ->Select Range…* Enter the start time (say 15) in the *Time span* box then hit *Enter* then select *Insert* and *Close*. The new dataset will appear. Drag the new dataset into the *Working Data* box.

You can check the quality of the response of the identified model by the step response analysis, transient response, pole and zeros map, frequency response and model residuals.

Example results and comments

The response of the identified closed loop system $T(z^{-1})$ can be easily compared with the plant response to express the identification quality (Figure 17). It is impossible to compare the model of the unstable plant itself unless nonparametric methods are incorporated in the identification task.

The model obtained will be used in the '*Pendulum Control*' section for the controller design.
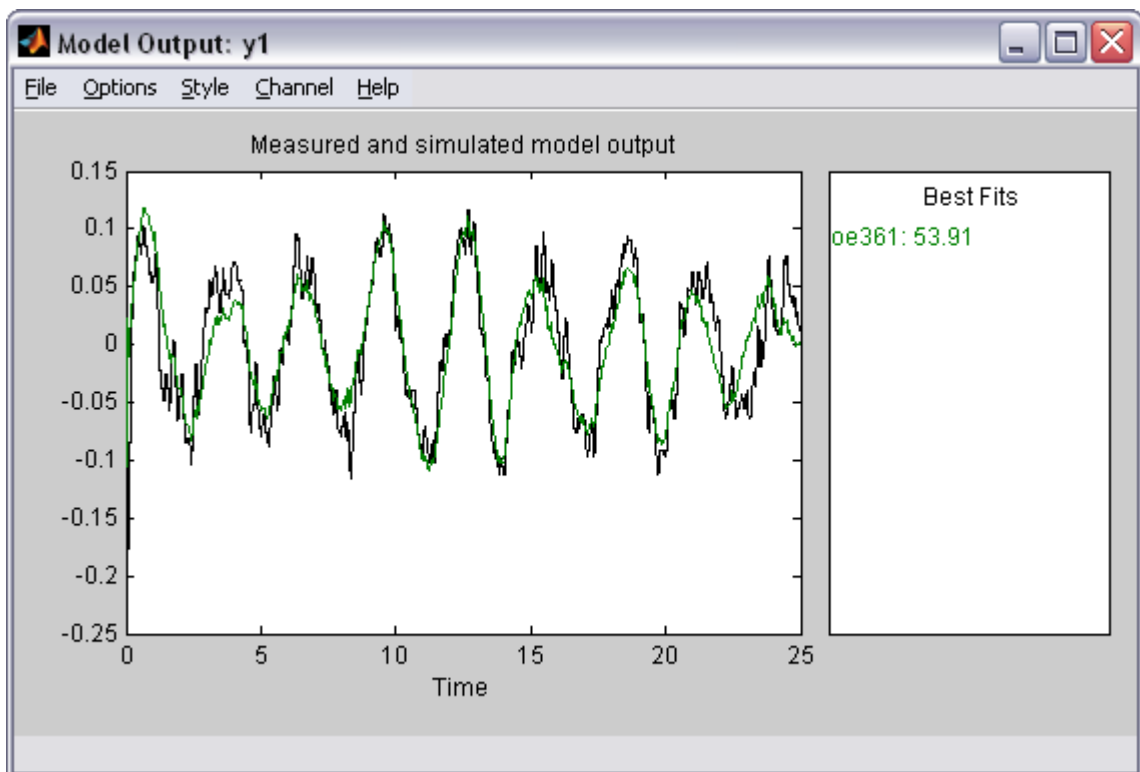

**Figure 17 Inverted pendulum model vs plant response**

Pendulum setup control

The pendulum control aspect covers three control areas:

- crane control,
- inverted pendulum control,
- swing up control.

Each of these control problems has its particular characteristic. As mentioned in the introduction the pendulum is a nonlinear plant, thus control over a wide range of angles by one simple linear controller is impossible. The nonlinear pendulum model has been linearised (*Model linearisation* section) to give two separate models for the upright and hanging pendulum position. Corresponding models have been also obtained in the course of the identification experiments as described in the *Model identification* section. Based on these linear models, simple control algorithms can be designed, one of which is the very widely used PID controller.

The plant itself is called an *open loop system* (Figure 18) that means it has no controller. The pendulum can be treated as a SISO or SIMO system depending on how many states we want to control.
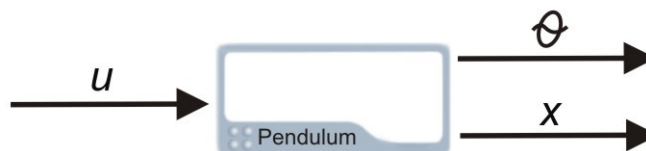
Figure 18 Pendulum

Matlab provides various analysis methods for linear systems as far as dynamics are concerned (root locus, frequency analysis tools – Bode diagrams, Nyquist plots, pole and zero maps etc.). Controllers can be designed using the information which Matlab provides about the dynamics of the system. The following sections explain how the PID controller works and how it can be tuned.

Plant control

There are numerous control algorithms however PID control is the most popular because of its simplicity. A general schematic of a simple closed loop control system is presented in Figure 19.
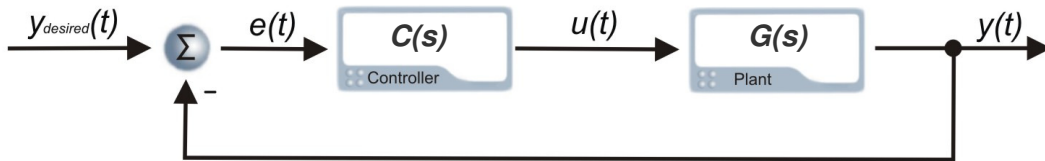

Figure 19 Simple closed-loop control system

Assuming that the plant is represented by its linear model its transfer function can be described as:

$$G(s) = \frac{B(s)}{A(s)} \tag{17}$$

where s is the Laplace operator. The idea of control algorithms is to find such a controller (transfer function, discrete transfer function, any nonlinear), which will fulfil our requirements (certain dynamic response, certain frequency damping, good response to the dynamic changes of the desired value etc.). The controller input is the $e(t)$ error signal. Sometimes disturbance signals are also measured. Depending on the present and past values of the error signal, the controller performs such an action (changes the $u(t)$ control signal) so that the $y(t)$ is as close as possible to the $y_{desired}(t)$ value at all times.

There are many controller design and tuning methods. All of them consider the behaviour of the *closed loop system* (plant with a controller as in Figure 19) and provide controller parameters according to the assumed system characteristics. With the known plant transfer function $G(s)$ it is possible to find satisfactory parameters of the $C(s)$ controller such that the closed loop system will have the desired characteristics described by the transfer function $T_c(s)$:

$$T_c(s) = \frac{C(s) \cdot G(s)}{1 + C(s) \cdot G(s)} \tag{18}$$

PID controller

A PID controller consists of 3 blocks: Proportional, Integral and Derivative. The equation governing the PID controller is as follows:

$$u(t) = P \cdot e(t) + I \cdot \int e(t)dt + D \cdot \frac{de(t)}{dt} \tag{19}$$

$$e(t) = y_{desired}(t) - y(t) \tag{20}$$

By means of the Laplace transform such a structure can be represented as a transfer function:

$$U(s) = (P + \frac{I}{s} + D \cdot s) \cdot E(s) \tag{21}$$

$$C(s) = \frac{U(s)}{E(s)} = (P + \frac{I}{s} + D \cdot s) = \frac{Ds^2 + Ps + I}{s} \tag{22}$$

Each block of the PID controller plays an important role. However for some applications, the integral or derivative part has to be excluded to give satisfactory results. The proportional block is mostly responsible for the speed of the system reaction. However for oscillatory plants it might increase the oscillations if the value of P is set to be too large.

The integral part is very important and assures zero error value in the steady state, which means that the output will be exactly what we want it to be. Nevertheless the integral action of the controller causes the system to respond more slowly to the desired value changes and for systems where fast reaction is very important it has to be omitted. Certain nonlinearities will also cause problems for the integration action.

The derivative part is introduced to make the response faster. However it is very sensitive to noise and may cause the system to react very nervously. Thus very often it is omitted in the controller design. Filtering of the derivative output may reduce the nervous reaction but also slows down the response of the controller and sometimes undermines the benefit of using the derivative part. Very often filtering can help to reduce the high frequency noise without degrading the control system performance in the lower frequency band.

There are several PID tuning techniques. Most often Ziegler-Nichols rules are used or a relay experiment is undertaken. Very often the closed loop system roots are

analysed and set in the desired position by means of correctly chosen *P*, *I* and *D* components. The Matlab Control System Toolbox provides a root locus tool which helps in such design. To gain some experience with root locus controller design and its effects on the system, the following exercise is proposed to design a PID controller for the cart position.

Exercise 7 – PID control of cart model position

Introduction
In order to design a PID controller a model of the plant is needed. For this purpose we can use a discrete model and design a discrete PID controller, or use a continuous model and design a continuous PID controller, however when a controller is implemented in some kind of a control unit it has to be used in a discrete form.

To begin with the linear continuous model of the cart will be used:

$$G(s) = \frac{X(s)}{F(s)} \tag{23}$$

which means that our goal will be to follow a certain path with the cart. The pendulum oscillations at that time are treated as disturbance.

Task
Design a PID controller for the above transfer function. If you want to use the phenomenological continuous linear model refer to the modelling section of this manual. Input the transfer function in Matlab using the '*tf*' command. You may also use the model that you have identified.

The discrete model obtained in *Cart Identification* section can be transformed into a continuous model before the controller design. You will need the following commands for that purpose. Load the OE model obtained in Exercise 4 *Cart Identification* into the workspace then create a discrete transfer function Gd(z):

Gd = filt(oe241.b, oe241.f, 0.05)

Convert from discrete Gd to continuous form Gc:

Gc = d2c(Gd, 'zoh')

To aid design of the controller you can use the root locus tool which is part of the Control System Toolbox. Open it by entering the *rltool* command in the Matlab workspace.

Refer to the '*Matlab Guide'* manual for PID design guide.

The first controller tests have to be done on the models. The file *PID_CartModelN.mdl* has been created for test purposes.

Example results and comments

The root locus of the identified model with the PID control with *P = 27.84, I =50, D = 3.9* is presented in Figure 20.
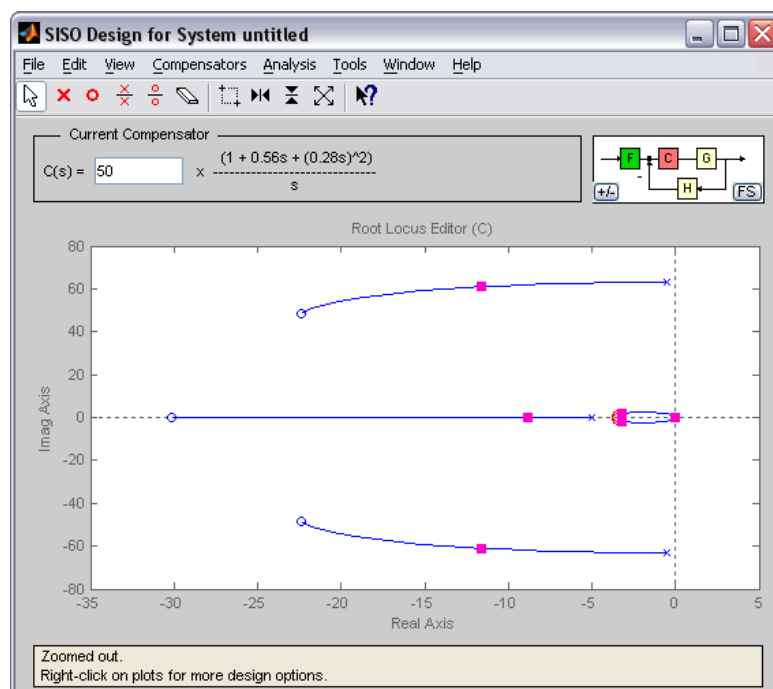
Figure 20 Root locus with PID controller

You can move the poles, zeros and the gain to obtain for example faster step response of the closed loop system. Then you can export the controller into Workspace and test it either using the offline simulation of the nonlinear model of the cart with *PID_CartModelN.mdl,* or on the model that you have identified.

Use a sinusoidal signal for the set value of the cart position $x_{desired}(t)$. Change the frequency and see how the output follows the desired value. Vary the values of the proportional, integral and derivative gains in the PID controller

and see how this influences the tracking of the desired value. You should observe the influence that is described at the beginning of this section: the Integral part assures zero steady state error but slows the system reaction, the Derivative part makes the system faster but causes the controller to behave nervously.

Before you test the designed controller on the Digital Pendulum make sure you that the D component is the filtered derivative of the *x* cart position. Such a controller is prepared in the following exercise.

The controller that has been obtained in the above exercise can be tested on the pendulum setup. The following exercise will guide you through it.

Exercise 8 – Real time PID control of cart position

Introduction

Just as in exercise 7 only the position of the cart will be controlled. This time it will be tested in real time. For this exercise use the *CartControl.mdl* presented in Figure 21.
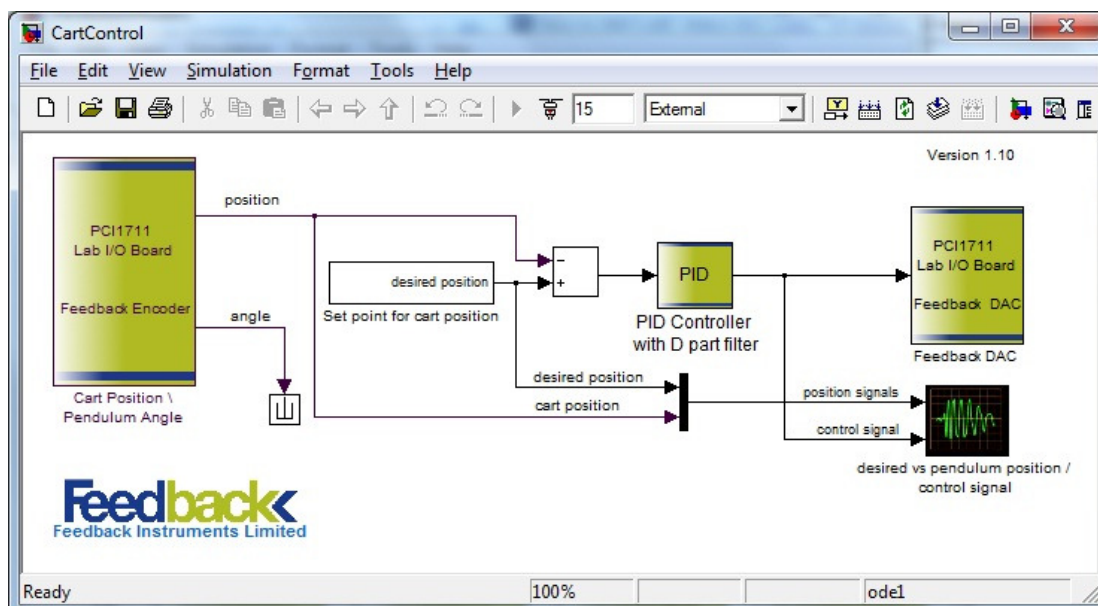

Figure 21 Real-time cart position control

As you open the *CartControl.mdl* you will notice that this simulation is directed to an external module, which is indicated in the upper part of the simulation

window. The PID controller has been already designed, however you can change its parameters according to the results obtained in Exercise 7.

Before you run the real time simulation make sure the pendulum setup is properly connected and that the digital pendulum controller is turned off. Refer to the section *Running a real-time model* for more instructions on real time simulations.

Task

The task is to vary the controller *P*, *I* and *D* values and observe how the signal $x_{desired}(t)$ is tracked by the system. Open the *CartControl.mdl* model. Remember to open the oscilloscope block before running the model. Three waveforms are displayed: the cart control voltage, the desired cart position and the actual cart position.

Open the PID controller block and make any desired changes **before** running the model. Make sure the changes of the *P*, *I* and *D* gains will not have a destructive effect on the pendulum. You should already have gained some experience in Exercise 7 on how the values of these gains can be changed.

Example results and comments

Figure 22 and Figure 23 present real-time simulation results of the cart PID control. Figure 22 uses correctly chosen I and D values. The simulation illustrates that an increase of I value may slow the system down and cause more overshoot. You should also notice that the larger the *D* value the faster the response but the more nervous the control signal *u* becomes. Very energetic control signal changes can often lead to break down of the control unit.
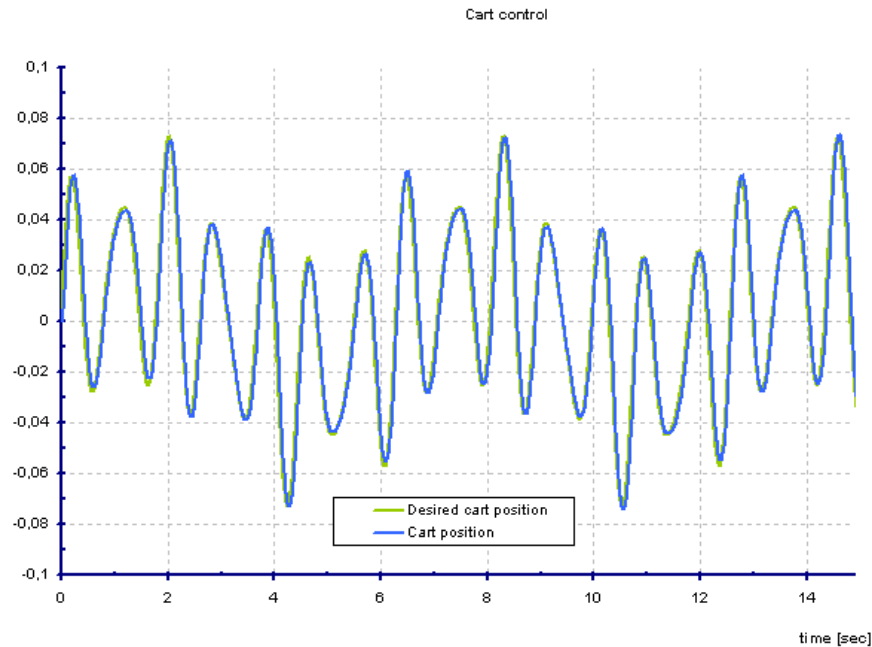
**Figure 22 PID control with P = 27.84, I =50, D = 3.9**

The result presented in Figure 22 proves how efficient controllers can be designed when an identification experiment is incorporated. Figure 23 shows how wrong controller tuning may degrade the system performance.
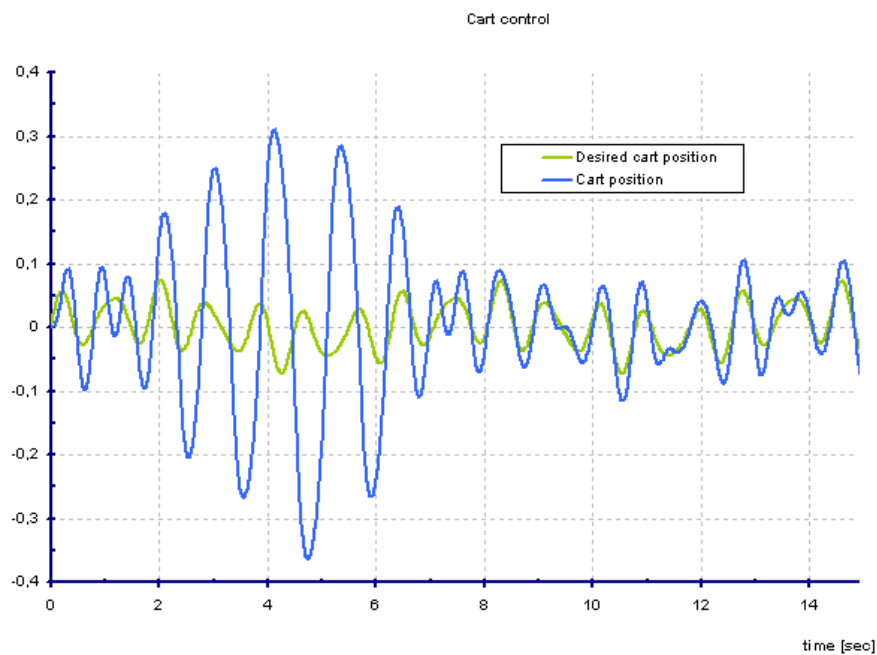
Figure 23 Real-time cart control results, with greater I gain and smaller D gain

Inverted pendulum control

Inverted pendulum control is divided into two control problems. First is the swing up control, which allows the pendulum to reach the upright position ($\theta = 0$) and the second is the pendulum PID control around that equilibrium point (Figure 24).
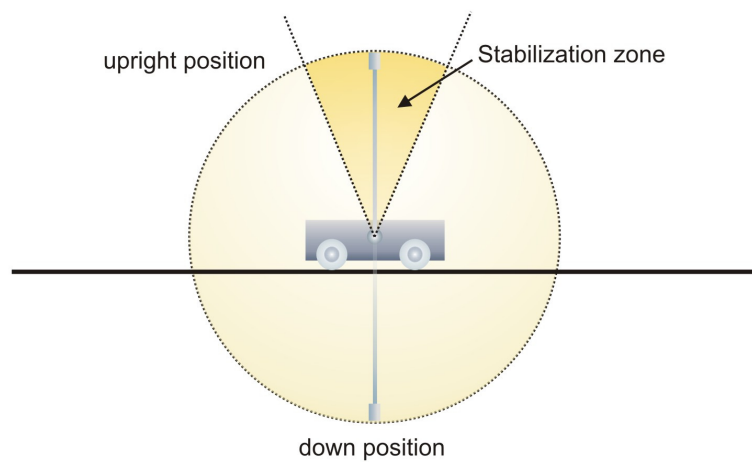
Figure 24 Pendulum stabilisation and swing up zones

Before the inverted pendulum control aspect is discussed the swing-up strategy is considered in the following section.

Control of pendulum swing-up

If the acceleration is unbounded it is possible to bring the pendulum to the upright position in a single swing. However because of safety it is better to swing up the pendulum in a robust way, which will assure that the pendulum will end up in a vertical position where $\theta = 0$. The motor control signal is bounded to ± 2.5 V (the actual range is 0V to +5V) and ± 20.0 N in terms of force so these are the maximal values that can be transferred. The physical bound of the track has also to be considered.

To begin with, the swinging alone is considered. The goal of the control is to bring the pendulum to an upright position with as minimal angular velocity as possible. This will ensure that the stabilizing controller, which takes over from the swing-up controller, will have an easy task. A large velocity value at the point of controller changeover could cause the pendulum to over swing or cause very nervous reactions of the cart or even cause it to hit the end of the track.

There are many algorithms used for swing-up control, however their robustness is in trade-off with their time performance. In the early stages of the controller design it is better to have a robust controller and then optimize it for time duration. One swinging strategy comes from the pendulum kinetic and potential energy analysis and can be summed up in few rules. It is presented graphically in Figure 25.

- If the θ is smaller than π/2 and 3·π/2 the pendulum is in the lower zone => apply a force until the pendulum velocity reaches $\dot{\theta} = 0$. If so, reverse the direction of the force.
- If crossing upper-lower zone border reverse the direction of the applied force.
- If the pendulum velocity reaches zero ($\dot{\theta} = 0$) switch the direction of the force again. If entering the lower zone switch it once more.
- Continue force switching according to the above scheme until the angle of the pendulum is within say *0.2* radians of the vertical position i.e. within the stabilisation zone.
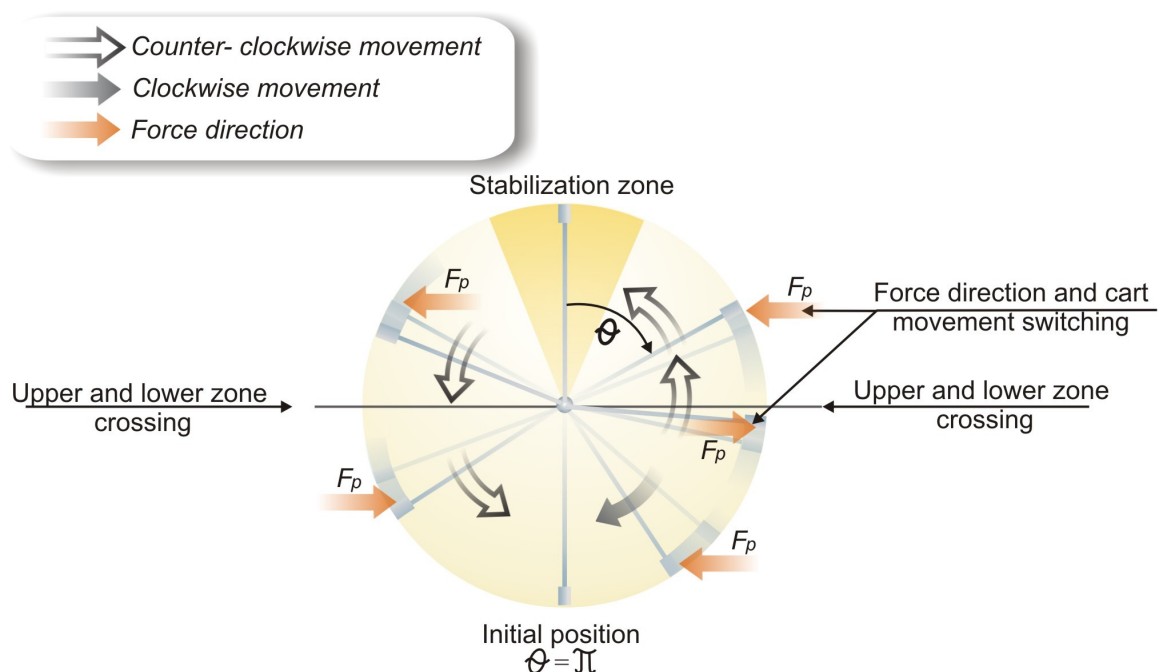

Figure 25 Pendulum swing-up principle

In order to impose forces as shown in Figure 25 the cart has to be moved the opposite way to the force arrow.

Exercise 9 – Model swing up control

Introduction

Before the swing up control is tested in real-time, just as with any algorithm it should be tested on a model first. You are advised to design the swing up controller on your own however the strategy described above has been designed for you in *ModelSwingUp.mdl*. A constant control voltage u_a is applied to the motor to drive it in one direction and then reversed according to the strategy described in the previous section.

Task

Check how the swing-up is influenced when the values of the parameters u_a (control voltage value) and the angle comparison value θ_s (absolute angle switching value – angle compare value) are changed. The u_a increase will cause the cart to swing faster, however this might force the cart to hit the limit switch. The simulation stops when the angle reaches the upper position $\theta=0$ or $\theta=2\cdot\pi$ within $\pm\theta_s$.

Example results and comments

Such a strategy, if programmed correctly, should ensure that the pendulum swings to the vertical position where $\theta=0$. Increasing the u_a parameter will cause the swing up to be shorter, however it may cause the angular velocity to be too large at the moment when the control algorithm switches to stabilisation mode.

Exercise 10 – Real time pendulum swing up control

Introduction

For this exercise you can use the swing up control developed in Exercise 9 and modify it to produce a real-time application. You can also use the real-time model *PendSwingUp.mdl* which has already been designed for you to run the swing up control on the pendulum.

Task

Run the swing up control on the pendulum in real time. Change the values of the parameters u_a (control voltage) and θ_s (absolute switching angle) and observe the behaviour. **Caution: Do not set u_a to greater than 1.**

Example results and comments

You may observe that excessively large or small control voltage values cause the cart to reach the end of the track before swing-up is complete.

Inverted pendulum stabilisation

Many stabilisation control schemes can be applied, however two of the simplest will be studied. You can approach the inverted pendulum stabilisation in two ways. Either design an LQ controller, which will include the multidimensionality of the pendulum (SIMO – single input multiple output) or design two separate PID controllers and sum their outputs to produce the final control signal. The first approach is more difficult as far as control system understanding is concerned however the second approach could lead to an unsatisfactory result because of the possibility of interaction between the two controllers if designed incorrectly. Only the LQ control is explained here which is more appropriate for advanced users.

LQ controller

The LQ controller control signal is determined in the following way:

$$u = -(K_1 \cdot e_1 + K_2 \cdot e_2 + K_3 \cdot e_3 + K_4 \cdot e_4) \tag{22}$$

where

$K_1 \cdot e_1$ – cart position error multiplied by controller gain K_1
$K_2 \cdot e_2$ – cart velocity error multiplied by controller gain K_2
$K_3 \cdot e_3$ – pendulum angle error multiplied by controller gain K_3
$K_4 \cdot e_4$ – pendulum velocity error multiplied by controller gain K_4

The values of the gain vector $K = [K_1, K_2, K_3, K_4]$ can be obtained through the quadratic cost function minimisation

$$J = \frac{1}{2}\int_0^\infty (e^T Q e + u^T R u)dt \tag{23}$$

The errors are defined as:

$$e = [z_{desired} - z_{measured}], \tag{24}$$

where *z* is a state vector $z = [x, \dot{x}, \theta, \dot{\theta}]$.

The solution of such problem, the *K* vector, is expressed by the solution *P* of the Riccati equation:

$$A^T P + PA + Q - PBR^{-1}B^T P = 0, \qquad (25)$$

$$K = R^{-1}B^T P. \qquad (26)$$

assuming that the system dynamics are described by the following state space representation:

$$\dot{z} = Az + Bu, \qquad (27)$$

$$y = z, \qquad (28)$$

and properly chosen weighting matrixes *Q* and *R*.

PD control

PD and LQ control can be combined if two PD controllers are designed (Figure 26). Just as in the LQ controller the control signal will be the weighted sum of the errors of four pendulum states : x - cart position, $\dot{x}$ - cart velocity, θ - pendulum angle, $\dot{\theta}$ - pendulum angular velocity. Later on, integration action can be introduced to see how it influences the inverted pendulum control.

Just as in the first PID controller design (*PID controller* section and Exercise 7) the *P* and *D* gains can be chosen using the root locus technique, Ziegler-Nichols rules or any other technique leading to PD controller design. The model that has to be used for such controller design is the linear pendulum model for the *θ = 0* (pendulum in vertical upward position). The model can come from the nonlinear model linearisation or the identification experiment presented in the early sections of this manual ('*Pendulum model*' section).
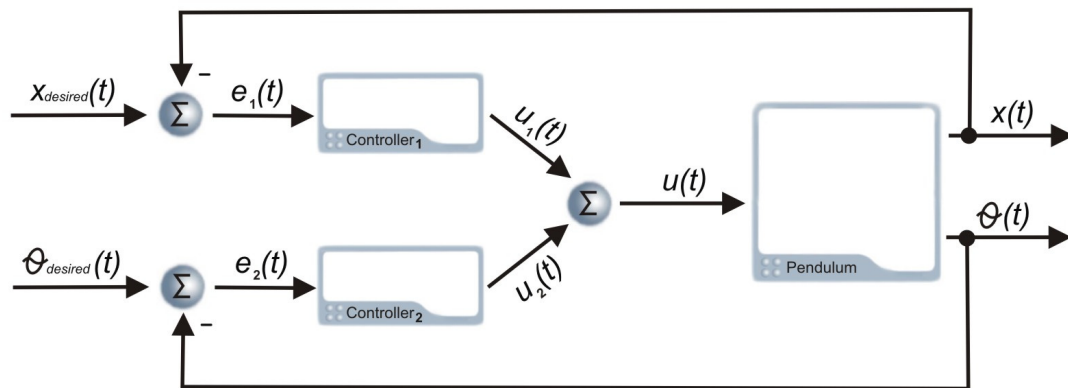
Figure 26 Pendulum control scheme

Exercise 11 – Pendulum stabilisation using the nonlinear model

Introduction

Just as in the Exercise 7 you can test the pendulum control algorithm firstly on the model using *ModelInvertedPD.mdl*. A nonlinear model is used for the pendulum stabilisation.

Task

Change the controller *P*, *D* and *I* gains to see how they influence the control action. Vary the initial value of the θ_0 angle to see how the controllers perform. For the θ_0 initial angle choose different values ranging from *0* to *0.5* radians.

Example results and comments

Depending on the *P, D* and *I* gains you should observe the same reaction as described in the '*PID controller*' section. The initial angle value, if increased, will cause more problems for the controllers due to the nonlinearity.

Exercise 12 – Pendulum real time stabilisation

Introduction

For this exercise you can use the real-time model *PendStabPD.mdl*. This model implements PD stabilisation control but has no swing-up controller. Make sure the control starts with the pendulum hanging freely and not moving otherwise the wrong initial value of the angle will be read by the controller.

Task

Start the simulation and turn the pendulum slowly by hand in the **counter-clockwise direction** (when viewed with the motor to your left) until you reach

the vertical position. The control action will start automatically when you move the pendulum within the border value of $\theta_b=0.2$ radians. This parameter can be varied to see how the control system reacts.

Change the controller *P*, *D* and *I* gains to see how they influence the control action.

You can design you own PID controller using the identified model of the inverted pendulum. The design can be carried out according to the instructions given in the *Matlab Guide* manual in the '*PID controller design*' section.

Example results and comments

Depending on the *P*, *D* and *I* gains you should observe the same reaction as described in the '*PID controller*' section. The θ_b value, if increased, will cause more problems for the controllers due to the nonlinearity.

Crane control

The crane control problem for the pendulum-cart setup also refers to a SIMO plant description. The goal is to travel with the crane from one point to another keeping the pendulum swinging as little as possible (Figure 27).
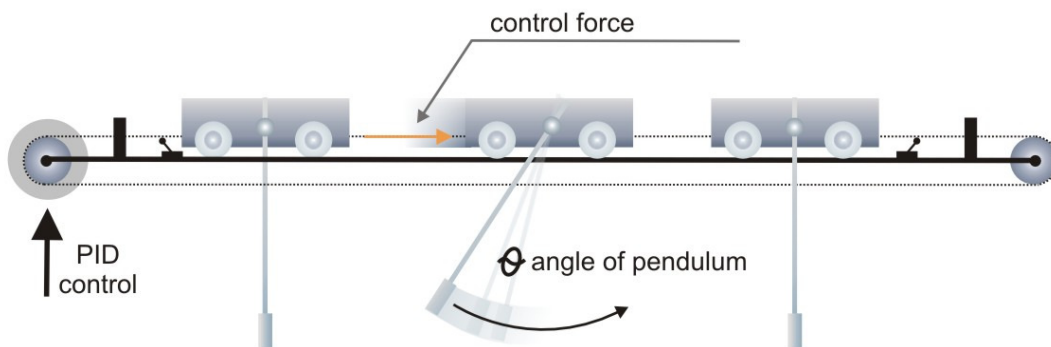
Figure 27 Crane control

The problem can be solved with many control engineering algorithms. Again the simplest method is presented here.

As with the inverted pendulum stabilisation, PD and LQ controllers can be used as they will actually have the same structure. They both use weighted errors of four state variables of the pendulum: x (cart position), $\dot{x}$ (cart velocity), θ (pendulum angle) and $\dot{\theta}$ (pendulum angular velocity). For both of the controllers, the linear model of the pendulum is used which has been obtained either from linearisation of the nonlinear model for $\theta = \pi$ radians or from identification of the crane model.

To simplify the design process the following exercises with Simulink demonstrations have been prepared, however more advanced users should be able to design the crane control system based on the information already given.

Exercise 13 – Crane control

Introduction

For this exercise you can use the *CraneModelStab.mdl* for model simulation and *CraneStab.mdl* for real time crane control. Make sure the control starts with the pendulum hanging freely and not moving.

Task

First run the offline simulation *CraneModelStab.mdl.* Start the simulation and observe the crane following the desired trajectory. Change the desired trajectory to see how the pendulum reacts.

Make sure the trajectory value is inside the range of cart movement.
Change the controller *P*, *D* and *I* gains to see how they influence the control action.

Now run the real-time model *CraneStab.mdl* and experiment with *P*, *D* and *I* gains to see how they influence the real system.

Example results and comments

Depending on the *P*, *D* and *I* gains you should observe the same reaction as described in the '*PID controller*' section.

Combined control techniques

This section combines the control techniques into a complete pendulum control application. The tasks are divided into exercises in order of increasing difficulty.

Swing up and hold

This application combines the swing up control algorithm with the pendulum LQ or PD control in the upright position. You can try to combine two of these control actions. Remember that excessive pendulum velocity when entering the stabilisation zone might cause the pendulum to over swing.

Exercise 14 – Swing up and hold control

Introduction

For this exercise you can use the *SwingHoldModel.mdl* for model simulation and *SwingHoldPendulum.mdl* for real time pendulum control. Make sure the control starts with the pendulum hanging freely and not moving.

Task

Run the offline model *SwingHoldModel.mdl.* Observe the swing-up controller bring the pendulum close to the stabilisation zone where the stabilisation controller takes over.

Change the controller *P*, *D* and *I* gains to see how they influence the stabilisation action.

Change the swing up parameters to see how that influences the whole control task.

Run the real-time model *SwingHoldPendulum.mdl* and observe the above effects on the real system.

Example results and comments

You may have noticed that the swing-up control sometimes lets the cart move out of bounds and it trips the limit switches. Exercise 15 will show how this can be improved. You can also use the improved real-time model *SwingHoldPendulumExtra.mdl* which has additional cart position control and observe how this improves the swing up. The extra position control block is described in Figure 42. Further information on how the additional position control is designed is given in the last section of the manual where the most advanced application is described *UpDownPendulum.mdl.*

Depending on the *P*, *D*, and *I* gains you should observe the same reaction as described in the '*PID controller*' section.

Changing the swing-up parameter may either cause the pendulum never to reach the upper zone or cause the pendulum to have excessive velocity when entering the stabilisation zone and thus cause problems for the stabilizing controller.

Up and down

This application adds fall and crane stabilisation of the pendulum to the previous swing up and hold controller. The pendulum will be swung up and stabilised in the inverted position for some time and then released to be stabilised and returned to the initial point where crane control is applied.

Exercise 15 – Up and down

Introduction

For this exercise you can use the *UpDownPendulum.mdl* for real time inverted pendulum control combined with crane control. Make sure the control starts with the pendulum hanging freely and not moving.

Task

Start the simulation and observe the swing-up controller bring the pendulum close to the stabilisation zone where the stabilisation controller takes over. After a period of time the stabilizing controller will be turned off and the pendulum will fall to be stabilised in the $x = 0$ and $\theta = \pi$ position. The cart is then made to follow a sinusoidal path (crane control). After a further period of time the control scheme is repeated until the time runs out.

You can experiment with the parameters of the control actions used within the model. Refer to the previous exercises for their explanation.

Example results and comments

Figure 28 shows the angle and the control signal of the pendulum during up and down task. Figure 29 shows the cart position.
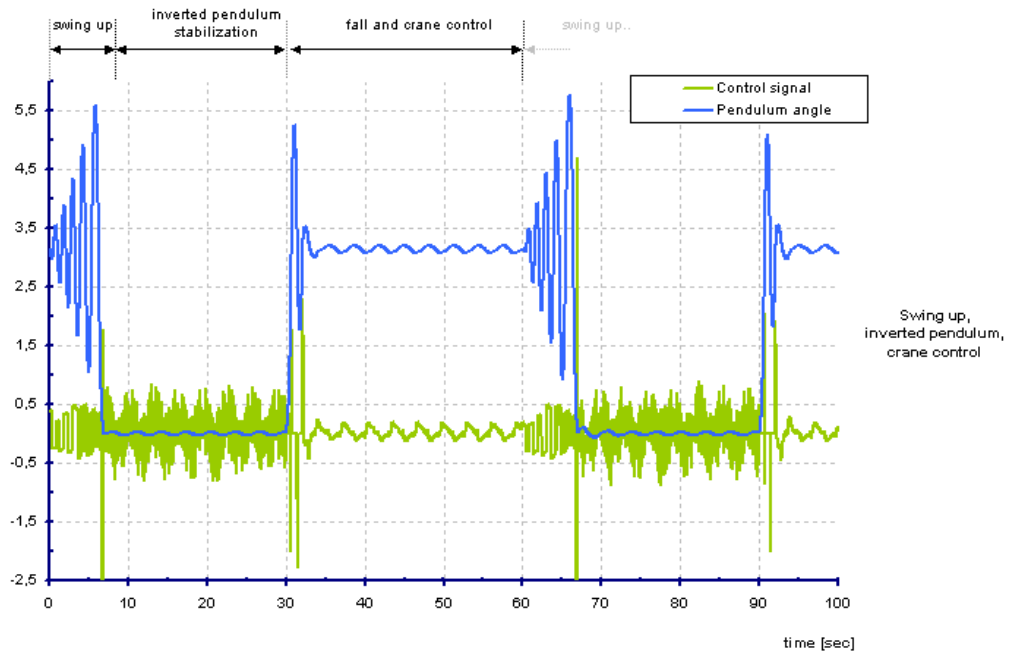
Figure 28 Control signal and pendulum angle
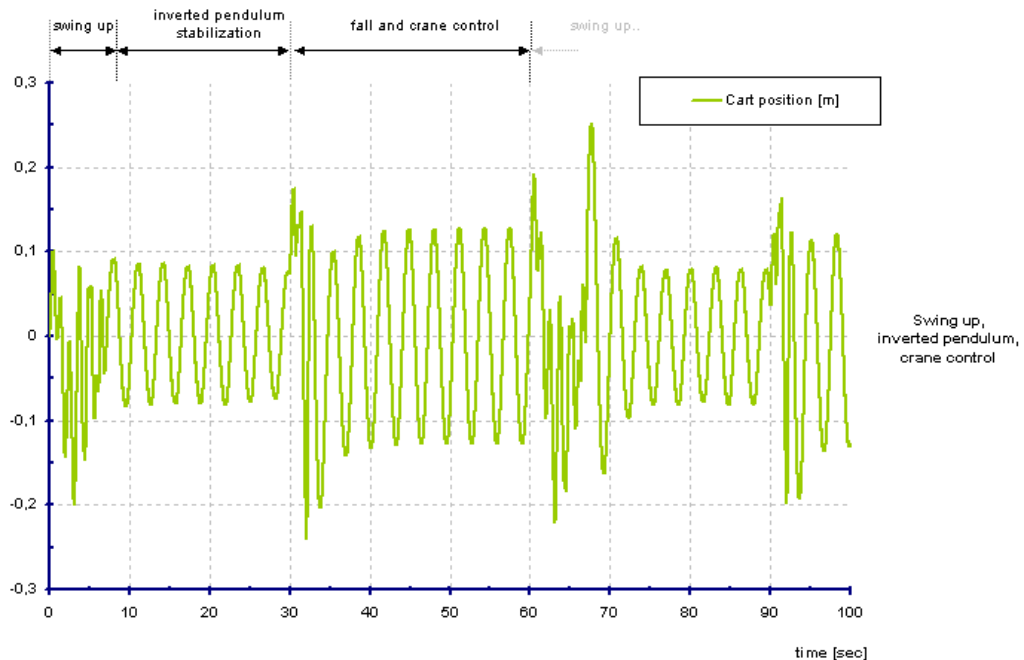
Figure 29 Cart position

The Up and Down demo includes some necessary control algorithm improvements resulting in robustness of the application. The improvements and the way each of the control block works is explained in the following subsection.

Detailed description of the Up/Down model

In case you have not designed all of the applications step by step on your own you might find this section particularly interesting. Here the blocks created in 'Up and Down' demonstration are described. Detailed information on each signal is given.

▪ Two most important blocks that have already been described in the "*Installation & Commissioning*" manual are the '*Feedback encoder for Pendulum*' block and the '*Feedback DAC for Pendulum*' block (Figure 30).

Figure 30 External equipment blocks

The outputs of the encoder block are the cart relative position in metres with respect to the initial starting point. The second is the pendulum relative angle in radians with respect to the initial starting point. The input of the '*Feedback DAC for Pendulum*' block is the control voltage for the pendulum motor. Inside the block, static friction compensation is added together with a safety block restricting the amplitude of the input control voltage to [-2.5 V .. +2.5 V]. The block converts the ±2.5V input signal to 0 to +5V when writing to the DAC as the PCI1711 I/O card has a unipolar (0V→+5V) DAC output.

▪ Another important block is the block presented in Figure 31.

Figure 31 Cart movement safety block

You can look inside this block and view the simple movement bounding function.

▪ The '*Task switching*' block (Figure 32) is used for switching the control actions. A pulse generator is used to switch between the two tasks: the swing up and stabilisation control, and the fall and crane control mode.
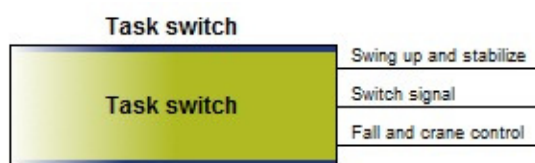

Figure 32 Task switching block

Block outputs: '*swing and stabilize*' when high, enables the swing and stabilize task, '*Pulse*' output for switching control signals, '*fall and crane control*' signal when high, enables the fall and crane control task.

▪ The '*Swing up and stabilisation control*' block (Figure 33) is responsible for swing up and inverted pendulum stabilisation. It is enabled by the 'Task switching' block.
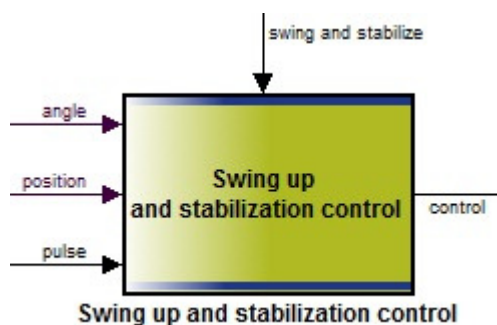
Figure 33 Swing up and stabilisation control block

Block inputs: *angle* of the pendulum, *position* of the cart, *pulse* (a delayed enabling signal for swing up initiation).
Block output: control signal.

Inside the '*Swing up and stabilisation control*' block two main control actions are designed, which are the swing up control

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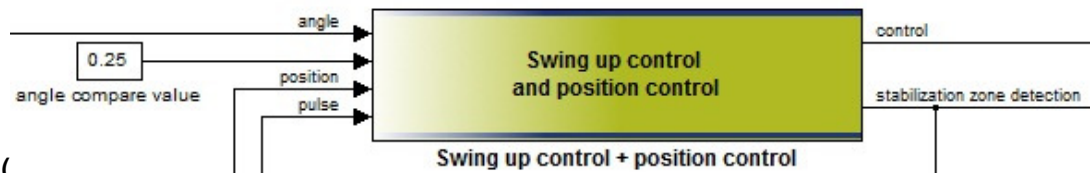
Figure 34) and the inverted pendulum stabilisation (Figure 35).
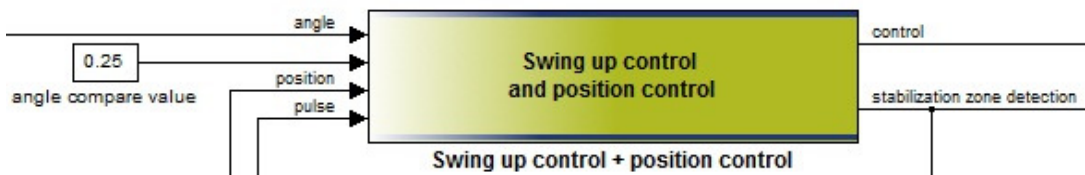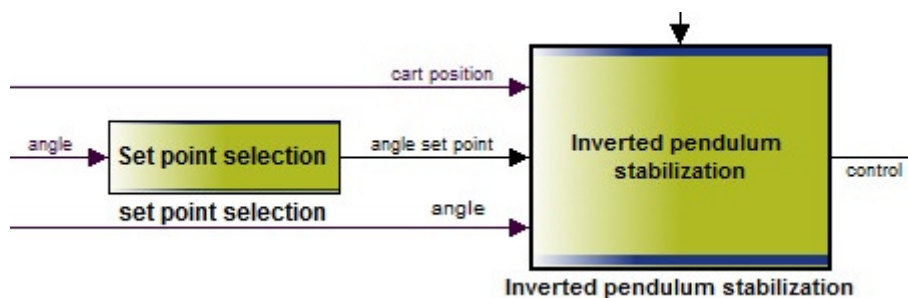

Figure 34 Swing up control

Figure 35 Inverted pendulum control

'*Swing up control*' block

(

- Figure 34).

Block inputs: *angle* of the pendulum, *angle compare value* (explained below), *cart position, pulse (a* delayed enabling signal for swing up initiation). **Block outputs:** *control (*control voltage), *stabilisation zone detection* – goes high when the pendulum enters the stabilisation zone.

Inside this block several control actions have been programmed.

o The system presented in Figure 36 is responsible for the proper control signal application depending on the values of its inputs. It utilizes the swing up scheme described in the '*Swing up control*' section. The block is enabled by the *pulse* signal.

Block inputs: *angular velocity* of the pendulum, *control voltage* – value of applied control voltage, *upper / lower zone detection* – high value signals upper half zone.
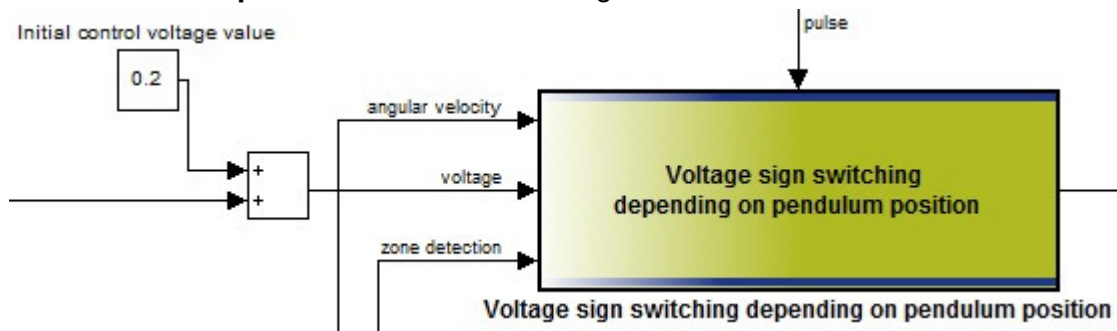**Block outputs:** *control* – control voltage.

Figure 36 Voltage sign switching block for swing up

o Figure 37 presents the angular velocity calculation blocks. It consists of a derivative and low-pass filter block.
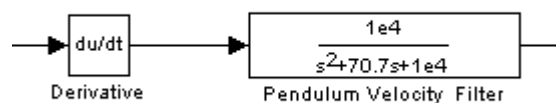
Figure 37 Angular velocity calculation

o The block presented in Figure 38 gives a high output signal when the pendulum enters the upper half zone.
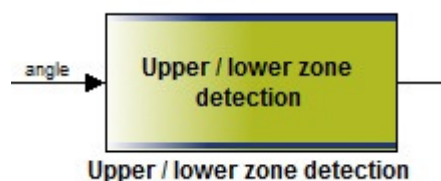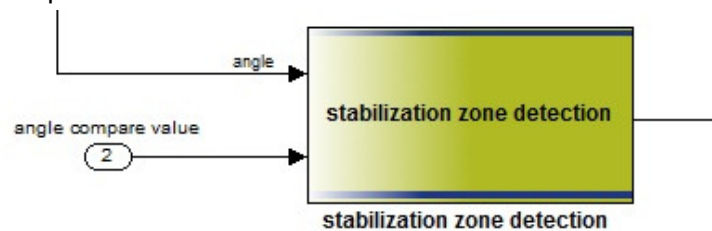

Figure 38 Upper zone detection

o If the pendulum angle is within the *angle compare value* of the vertical position, the block presented in Figure 39 gives a high output signal. This means the pendulum has entered the stabilisation zone.

Figure 39 Stabilisation zone detection system

o First movement of the cart for the purpose of swing up is initiated by the '*First move*' block presented in Figure 40.


Figure 40 First move system

The next group of control subsystems has been created in the '*Swing up control*' block to improve the control performance and robustness.

o The subsystem presented in Figure 41 is responsible for control value calculation in the swing up control.

Block inputs: *angle* – depending on its value the extra control signal value is generated. Its value is calculated by a $\sin(\max|angle\text{-}\pi|)$ function. This assures a soft start and slow entry into the stabilisation zone. You may see the block functioning by observing its signals with Simulink scopes.

**Block outputs:** *extra control* – extra control voltage.
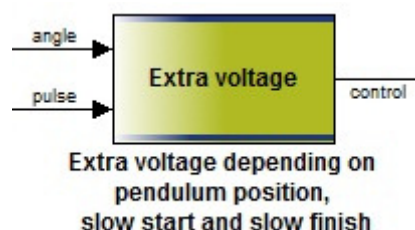
Figure 41 Extra voltage block

o   To ensure that the cart will be kept within the admissible range, a PI controller is added to the swing up controller. The block responsible for the position control action is presented in Figure 42.
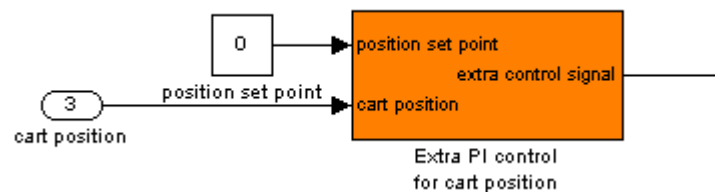
Figure 42 Position PI control block

▪   Inside the '*Inverted pendulum stabilisation*' block (Figure 35) two PID control algorithms are designed: one for the cart position control and the other for pendulum angle control. The desired value for the cart position controller is set to *0.* The set value for the angle is calculated by the '*set point selection*' block presented in Figure 43. The output of this block is either set to *2·π* or *0* depending on the side from which the pendulum has entered the stabilisation zone.
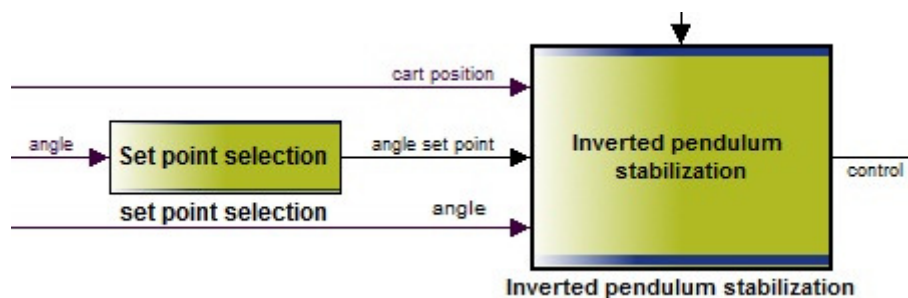
Figure 43 Vertical position knockout block

▪   In the main simulation window a '*knock from upright position*' block is placed (Figure 44).

o   The block is enabled with the '*fall and crane control*' signal. Its inputs are the pendulum '*angle*' and the '*theta set point*'. The set point gives the value around which the inverted pendulum has been stabilised: either *2·π* or *0.* The output is the control signal which is actually just a pulse of 0.5 V. Inside this block, the subsystem presented in Figure 44 gives a control voltage pulse dropping the pendulum in the direction which will ensure that the angle will again be *θ=π* in the lower vertical position. The falling is then initiated.
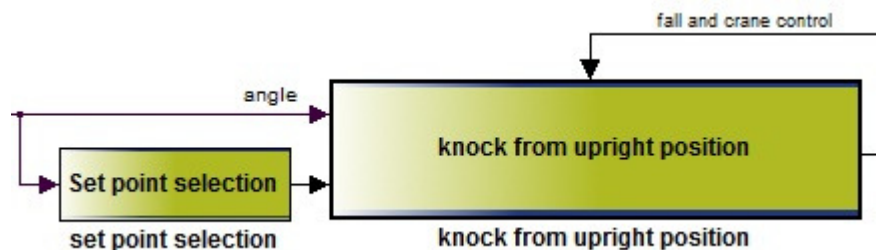
Figure 44 Pendulum fall initiation

- The 'Lower zone stabilisation and crane control' block presented in Figure 45 provides pendulum fall control and crane stabilisation.
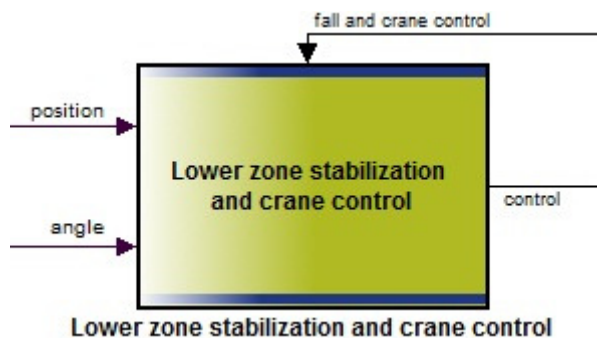

Figure 45 Lower zone and crane control block

Inside the block, two subsystems are designed: 'stabilisation zone detection' (Figure 46) and 'crane control' block (Figure 47).

- The 'stabilisation zone detection' system gives a high output when the angle is within the 'angle compare value' from the θ=π position.
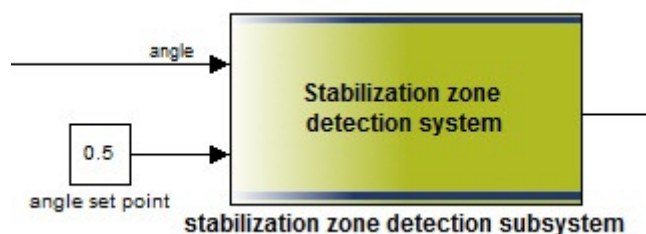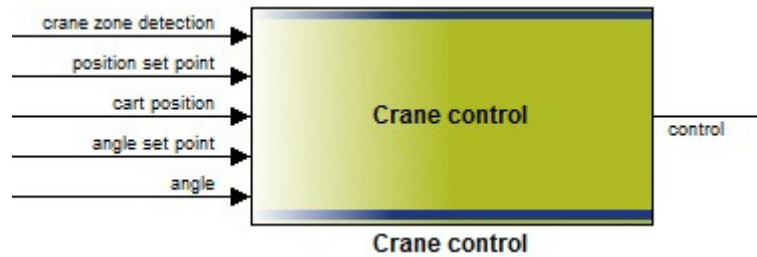

Figure 46 Stabilisation zone detection

o The '*crane control*' block utilizes the crane control algorithm. Two PID controllers are designed: one for pendulum angle and the other for cart position control.

Figure 47 Crane control subsystem