A Basic SISO Example.

Given the uncertain plant, defined in real factored form with parametric uncertainty

$$P(s) = k\frac{s+a}{1+2\zeta\, s/\omega_n + s^2/\omega_n^2},$$
$$k \in [2,5],\quad a \in [1,3],\ \zeta \in [0.1, 0.6],\ \omega_n \in [4,8],$$

close the loop with a feedback compensator $G(s)$, and a prefilter $F(s)$ such that the following specifications are satisfied:

- $M \leq 10\%$
- $t_s \leq 1.5$ sec.
- $S(j\omega) = |1/(1+L(j\omega))| \leq 6$ dB

Plant Definition

In the new object oriented QFT toolbox the plant is defined in an m-file or by the command line as

```
k=qpar('k',2,2,5,8);
a=qpar('a',3,1,3,8);
z=qpar('z',0.6,0.3,0.6,8);
wn=qpar('wn',4,4,8,8);

num = [k*wn*wn k*wn*wn*a];
den = [1 2*z*wn wn*wn];
P = qplant(num,den);
```

Each of the uncertain parameters is defined as a `qpar` object. A qpar object such as k is defined using the syntex

`par = qpar(name,nom,lbnd,ubnd,cases)`.

where `name` is a string; `nom`, `lbnd`, `ubnd` are scalaric numbers describing the nominl value, lower bound, and upper bound, respectively; `cases` is an optional input agrument which specifies the number of uncertain cases, i.e. the number of grid points. For exmaple

```
k = qpar('k',2,2,5,8)
```

```
k = 
  qpar with properties:

           name: 'k'
        nominal: 2
          lower: 2
          upper: 5
          cases: 8
       discrete: []
    description: []
```

Two or more qpar objects can be combined together into a qexpression, which stores the parametric description along with a list of all envolded qpar objects. For exmaple

```
exp = k*wn*wn
```

```
exp = 
  qexpression with properties:

    expression: '(k * wn) * wn'
          pars: [2×1 qpar]
```

The numerator an denomenator are defined using a raw vector of qpar, qexpression, or numeric objects. The first element is the n-order coefficient and so on. The result is a qpoly element with all coefficients and parameters

```
den = [1 2*z*wn wn*wn]
```

```
den = 
qpoly with coefficients

    s2: 1
    s1: '(2 * z) * wn'
    s0: 'wn * wn'
```

Finally, P is an instance of a qplant class, defined by two qpoly elemetns definning the numerator and denomenator.

```
P = qplant(num,den)
```

```
P = 
  qplant with properties:

          num: [1×1 qpoly]
          den: [1×1 qpoly]
        delay: []
         pars: [4×1 qpar]
     unstruct: []
       uncint: []
    templates: []
      nominal: []
         info: 'generated from [num,den] data on: 08-Feb-2019 22:17:44'
```

Note that the preperties 'templates' and 'nominal' are empty. They require computation. The nominal case is computed by the command cnom as follows

```
w_nom = logspace(-2,2,200);
P.cnom(w_nom)
```

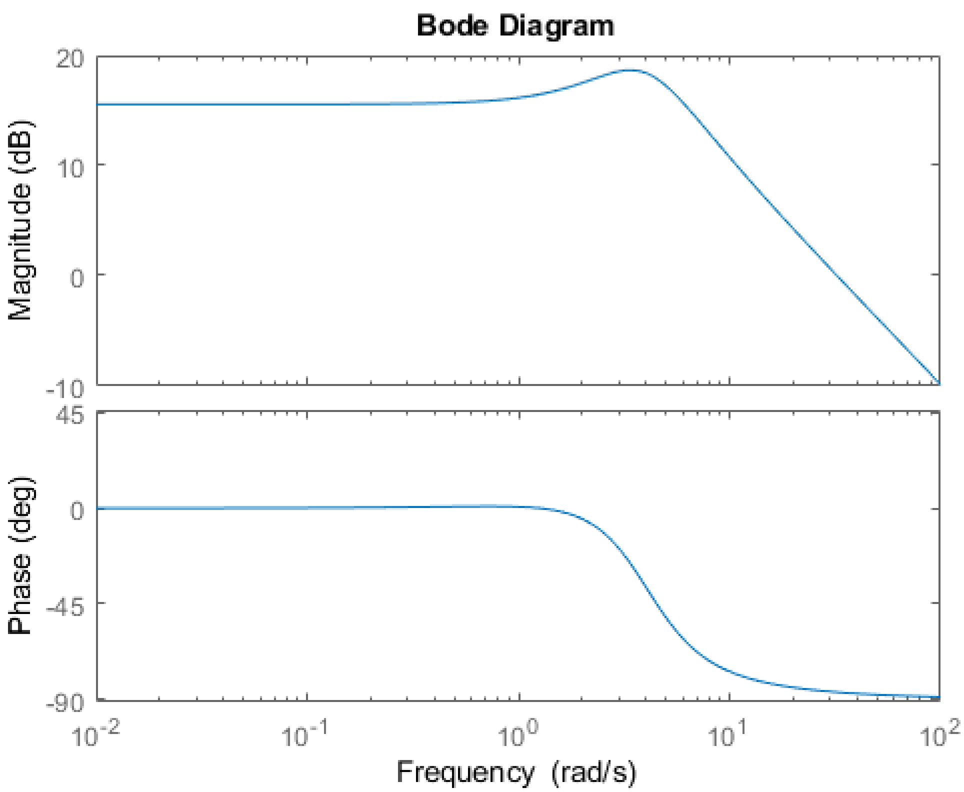
```
ans = 
  qplant with properties:

          num: [1×1 qpoly]
          den: [1×1 qpoly]
        delay: []
```

```
        pars: [4×1 qpar]
    unstruct: []
      uncint: []
   templates: []
     nominal: [1×1 qfr]
        info: 'generated from [num,den] data on: 08-Feb-2019 22:17:44'
```

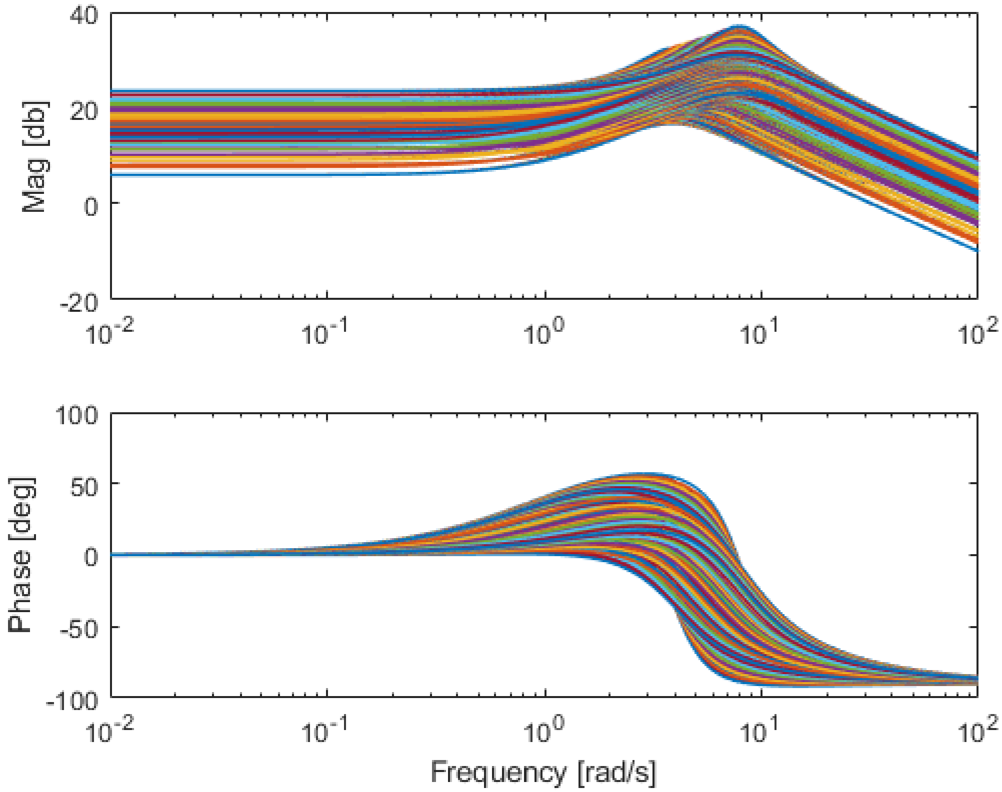
The nominal case that is computed is a `qfr` object. It behaves very simailarly to Matlab's LTI object. For exmaple, a Bode plot is drawn by

```
figure, P.nominal.bode()
```



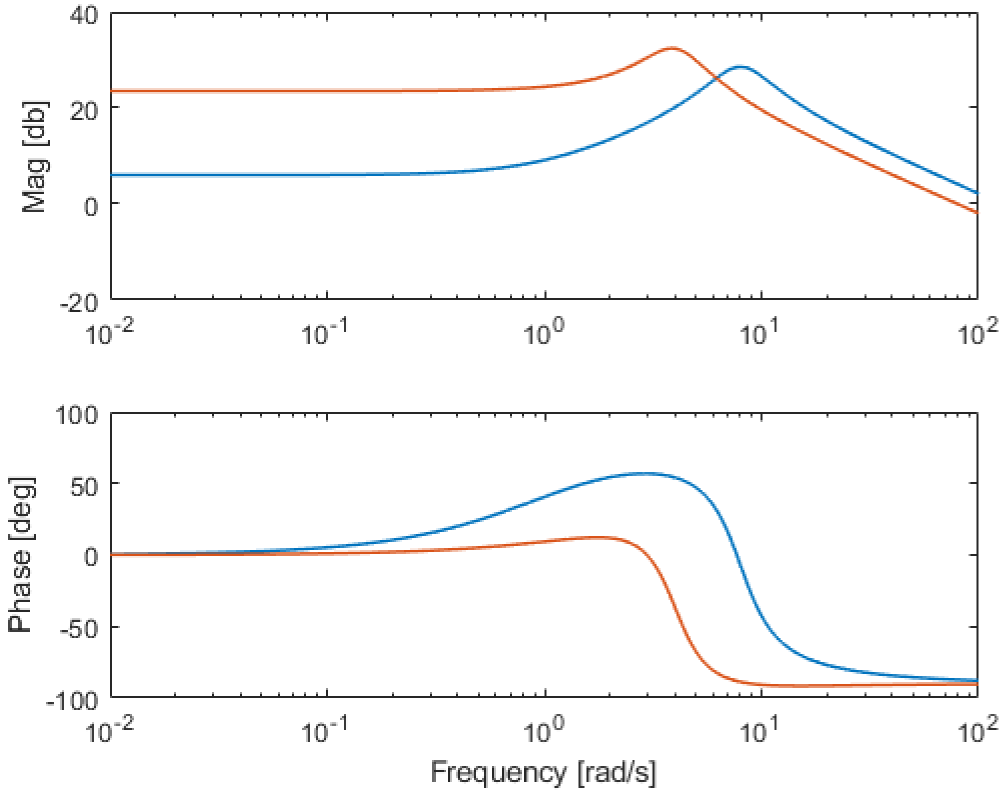
If one wishes to plot other cases (not the noiminal), the commands `bodcases` and `niccases` are useful. For exmaple, to plot the Bode for uniform grid of parameter cases

```
figure, P.bodcases()
```



A specific set of cases is shown by

```
pars = [1 3; 2 5; 8 4; 0.3 0.3];
figure, P.bodcases(pars)
```



Template Computation

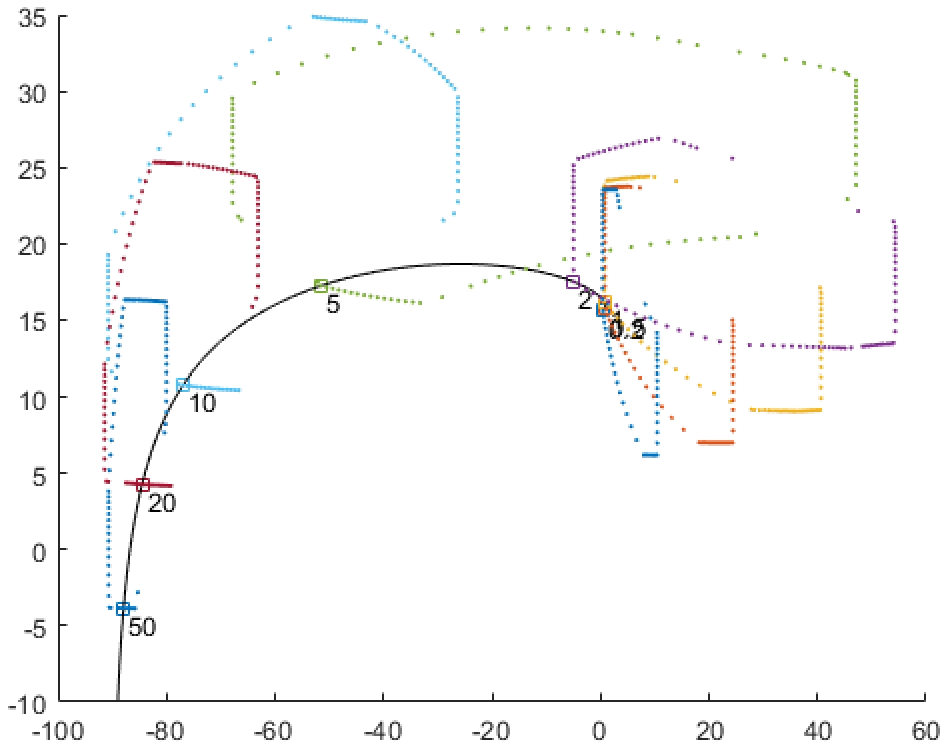
The templates are computed using `ctpl`. To compute using recurcive edge grid ('aedgrid') at selected frequencies and show them in figure,

```
w = [0.2 0.5 1 2 5 10 20 50];
P.ctpl('aedgrid',w)
```

```
Calculating templates by recurcive edge grid
--> for w=0.2 [rad/s]
--> for w=0.5 [rad/s]
--> for w=1 [rad/s]
--> for w=2 [rad/s]
--> for w=5 [rad/s]
--> for w=10 [rad/s]
--> for w=20 [rad/s]
--> for w=50 [rad/s]
ans =
  qplant with properties:

          num: [1×1 qpoly]
          den: [1×1 qpoly]
        delay: []
         pars: [4×1 qpar]
     unstruct: []
       uncint: []
    templates: [8×1 qtpl]
      nominal: [1×1 qfr]
         info: 'generated from [num,den] data on: 08-Feb-2019 22:17:44'
```

```
P.showtpl()
```



Specifications

It is now time for the specifications. The 6dB sensitivity specs. are defined as a `qsps` object

```
spec1 = qspc('odsrs',w,6)
```

```
spec1 =
  qspc with properties:

         name: 'odsrs'
    frequency: [0.2000 0.5000 1 2 5 10 20 50]
        upper: [6 6 6 6 6 6 6 6]
        lower: []
      timespc: []
      timeres: []
```

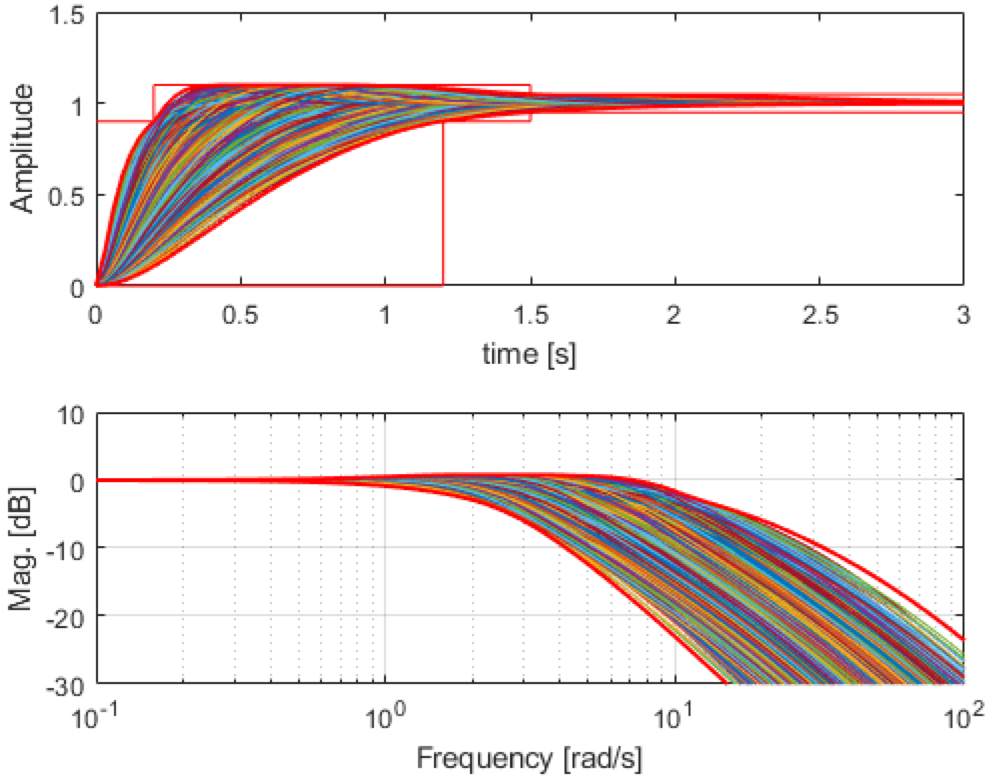
The servo specifications are defined by

```
spec2 = qspc.rsrs([1.2 0.2],10,1.5,[],logspace(-1,2),2.85,3.1)
```

```
Creating array of size 4x6000
Reducing to 4x2916
Reducing to 4x2286
Reducing to 4x1547
Reducing to 4x1216
```

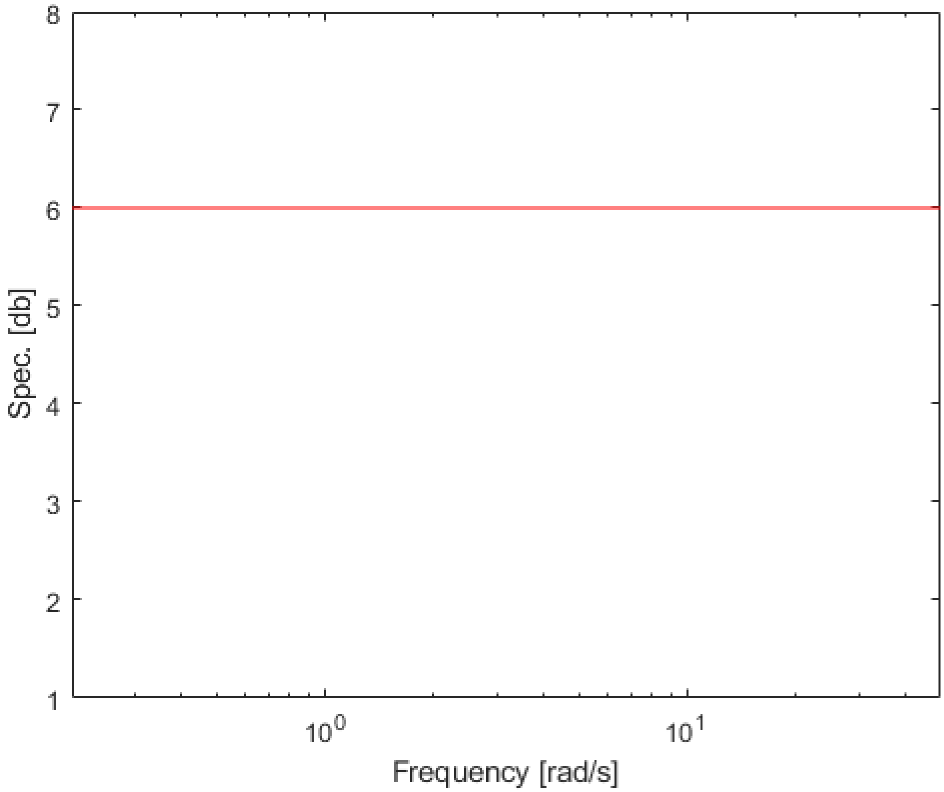
```
Reducing to 4x983
Reducing to 4x974
Reducing to 4x960
Number of good step-responses: 960
spec2 = 
  qspc with properties:

         name: 'rsrs'
    frequency: [50×1 double]
        upper: [50×1 double]
        lower: [50×1 double]
      timespc: [8×3 double]
      timeres: [109×3 double]
```



which does the translation from time-domain to frequency domain. The two computed specs. can be shown using the command `show`. For example

```
spec1.show();
```



Horowitz-Sidi bounds computation

With the templates and frequency domain specifications we can now compute the Horowitz-Sidi (H-S) bounds. First we create a `qdesign` object which will facilitates the bounds creation and loop shaping design. The bounds can than be computed using the command `cbnd(spcname)` with the specification name. The bounds are shown by `showbnd(spcname)`.

For the sensitivity specification:

```
des = qdesign(P,[spec1 spec2])
```

```
You now have a QFT loop desgin object. Compute bounds using CBND
des =
  qdesign with properties:

    tpl: [8×1 qtpl]
    nom: [1×1 qfr]
    spc: [1×2 qspc]
    bnd: []
    col: [8×3 double]
```

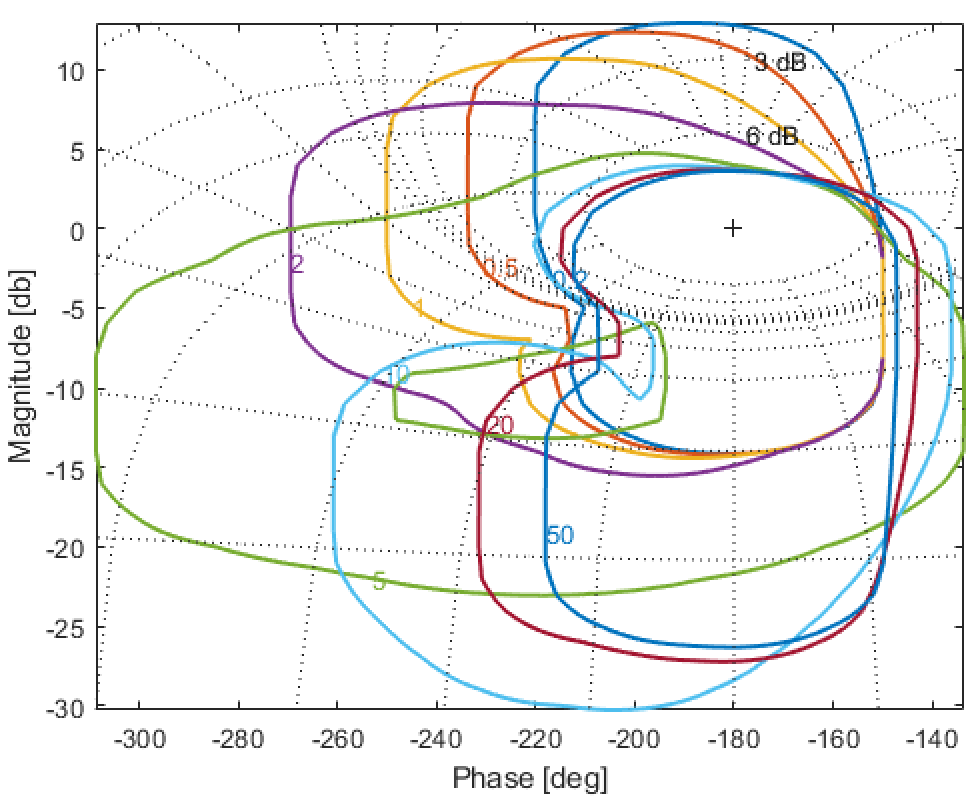
```
des.cbnd('odsrs')
```

```
Calculating bounds for odsrs
--> w(1) = 0.2 [rad/s]
--> w(2) = 0.5 [rad/s]
--> w(3) = 1 [rad/s]
--> w(4) = 2 [rad/s]
```

```
--> w(5) = 5 [rad/s]
--> w(6) = 10 [rad/s]
--> w(7) = 20 [rad/s]
--> w(8) = 50 [rad/s]
ans =
  qdesign with properties:

    tpl: [8×1 qtpl]
    nom: [1×1 qfr]
    spc: [1×2 qspc]
    bnd: [1×1 struct]
    col: [8×3 double]
```

```
des.showbnd('odsrs')
```



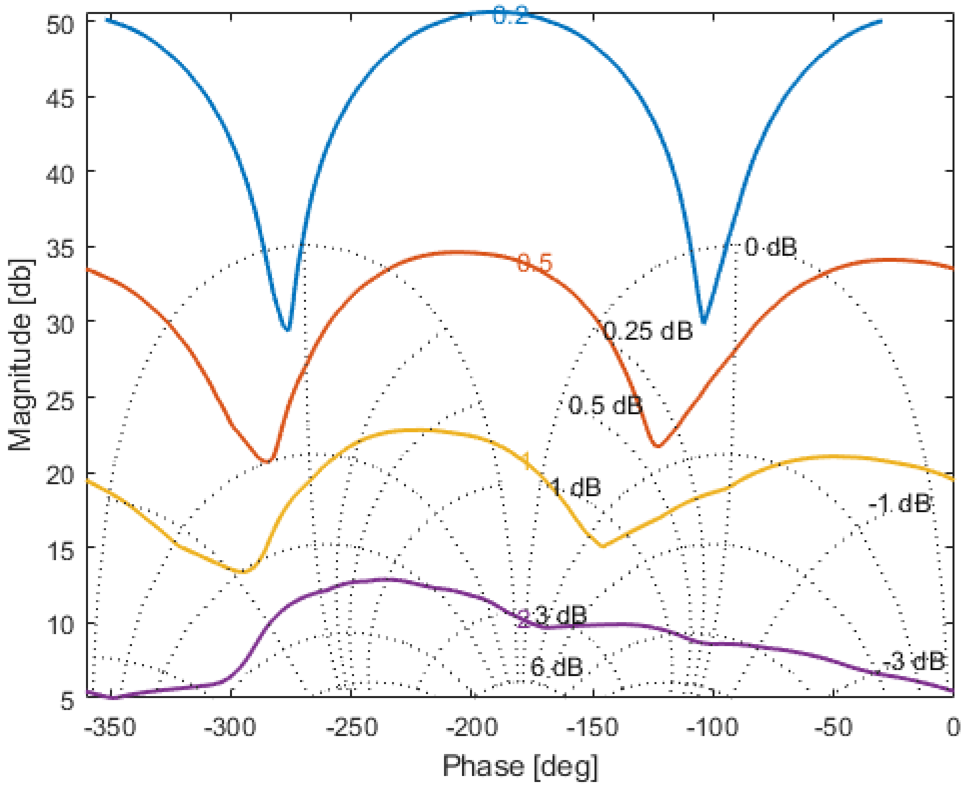
Similarly for the servo specifications:

```
des.cbnd('rsrs')
```

```
Calculating bounds for rsrs
--> w(1) = 0.2 [rad/s]
--> w(2) = 0.5 [rad/s]
--> w(3) = 1 [rad/s]
--> w(4) = 2 [rad/s]
--> w(5) = 5 [rad/s]
--> w(6) = 10 [rad/s]
--> w(7) = 20 [rad/s]
--> w(8) = 50 [rad/s]
ans =
  qdesign with properties:
```

```
tpl: [8×1 qtpl]
nom: [1×1 qfr]
spc: [1×2 qspc]
bnd: [1×2 struct]
col: [8×3 double]
```

```
des.showbnd('rsrs')
```



Loop Shaping

Now we can do loop shaping!

We use `showbnd` to plot the dominant bounds. Note the use of the figure handle `h` to show all bounds on the same chart. The feedback compensator can be defined by e.g. `zpk` from the Control Systems Toolbox.

```
h = des.showbnd('odsrs',[],[5 10 20 50]);
des.showbnd('rsrs',h,[0.2 0.5 1 2]);

% define the compensator G(s):
s = zpk(0,[],1);
set(s,'DisplayFormat','Frequency')
G = 2.5*(1+s/6)*(1+2*0.6*s/4+s^2/16)/s/(1+s)/(1+s/3.2)/(1+s/26)
```

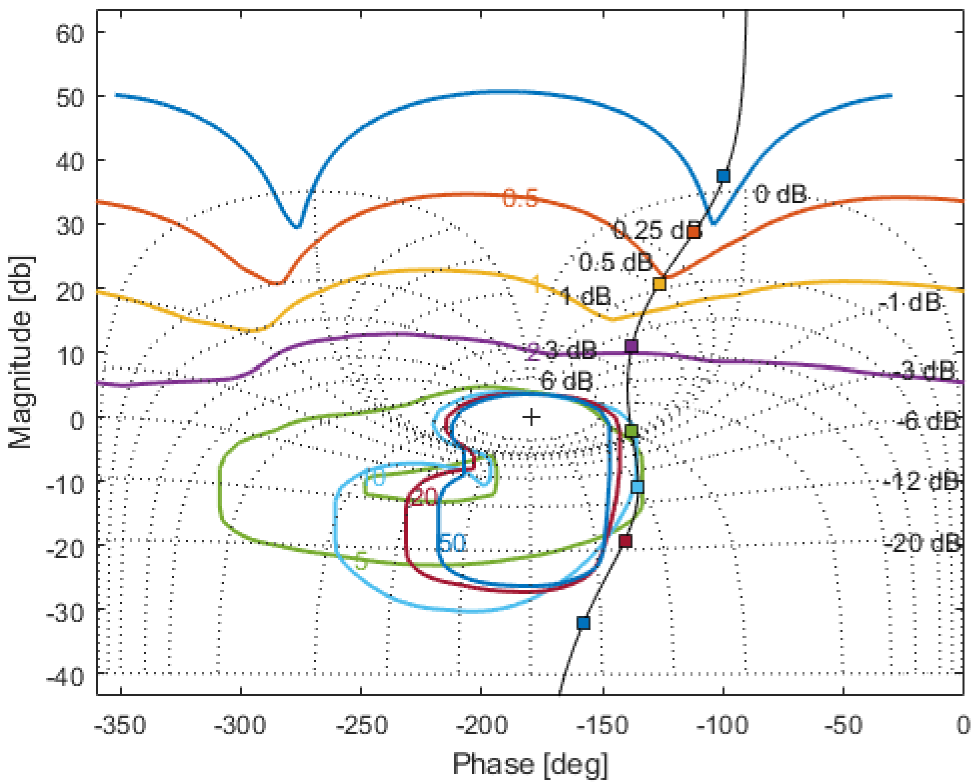
```
G =

  2.5 (1+s/6) (1 + 1.2(s/4) + (s/4)^2)
```

```
    -----------------------------------
         s (1+s) (1+s/3.2) (1+s/26)

Continuous-time zero/pole/gain model.
```

```
des.loopnic(G) % plot G*Pnom on NC
```

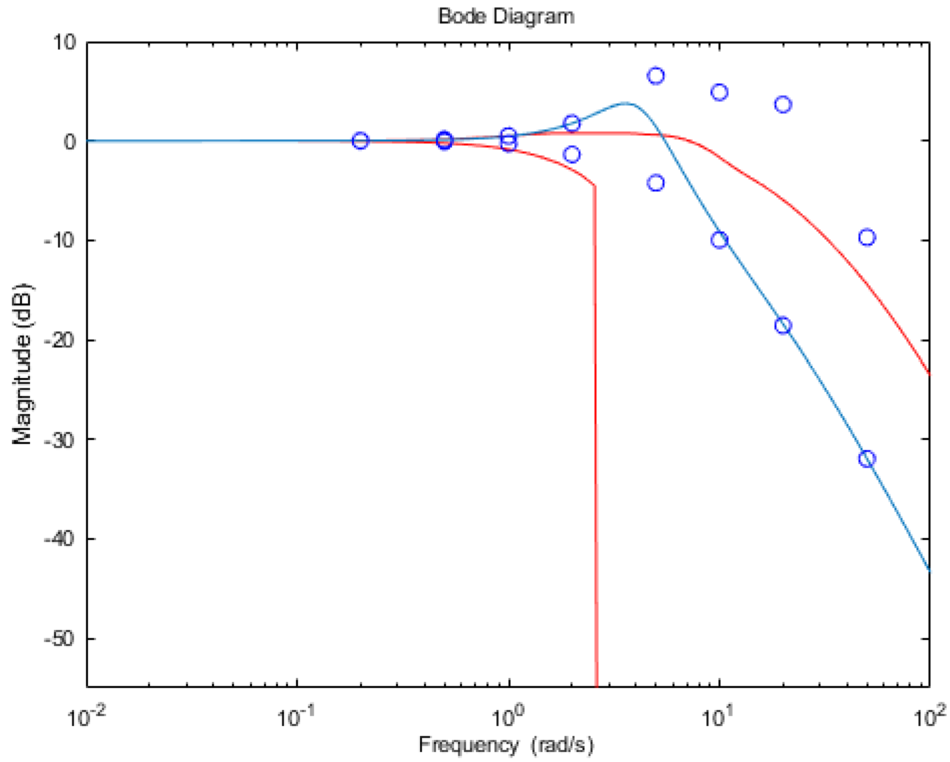


Note that each frequency has a dedicated color in which the bound and nominal point are drawn

Prefilter

The prefilter is now designed. To view the closed loop time response with $F(s) = 1$, the command `clmag(G,1)` is used

```
spec2.show('freq');
des.clmag(G,1)
ylim([-55 10])
```



With a properly designed $F(s)$ we have

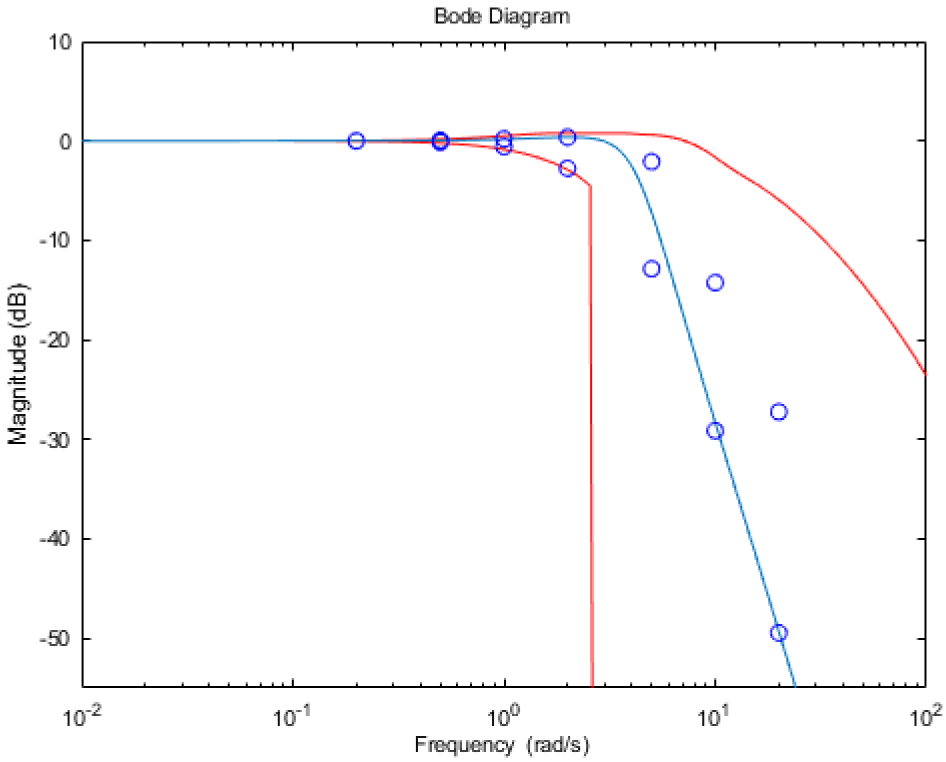
```
F = 1/(1+2*0.83*s/3.4+s^2/3.4^2)
```

```
F =

                 1
  -----------------------------
  (1 + 1.66(s/3.4) + (s/3.4)^2)

Continuous-time zero/pole/gain model.
```

```
spec2.show('freq');
des.clmag(G,F)
ylim([-55 10])
```


Bode Diagram
10
0
-10
-20
-30
-40
-50
Magnitude (dB)
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)