VASICEK MODEL AND BOND YIELD CALIBRATION

Abstract

In the following report we will assume the interest rate follows an Ornstein-Uhlenbeck process, the idea behind it is that contrarily to stock prices the interest rate cannot rise or fall forever, if it did it would hamper the economy, so we model the interest rate as if it were a mean-reverting process, in the long run, it will get back to some mean value which is its long time average .

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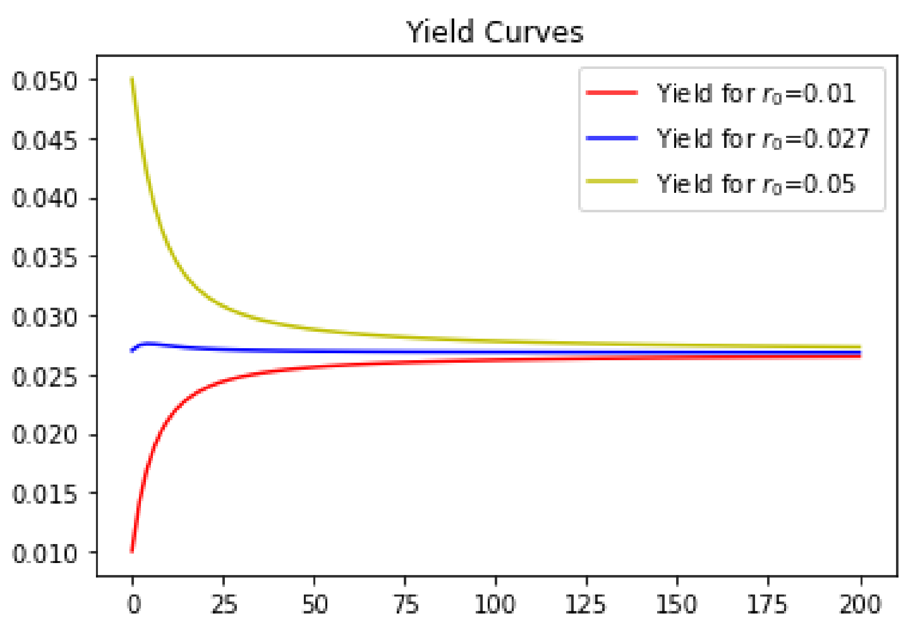
Part 1: Yield curves at t=0

The goal of this section is to show that the future structure produced by the model assumes three different curve shapes depending on the initial value of the interest rate.

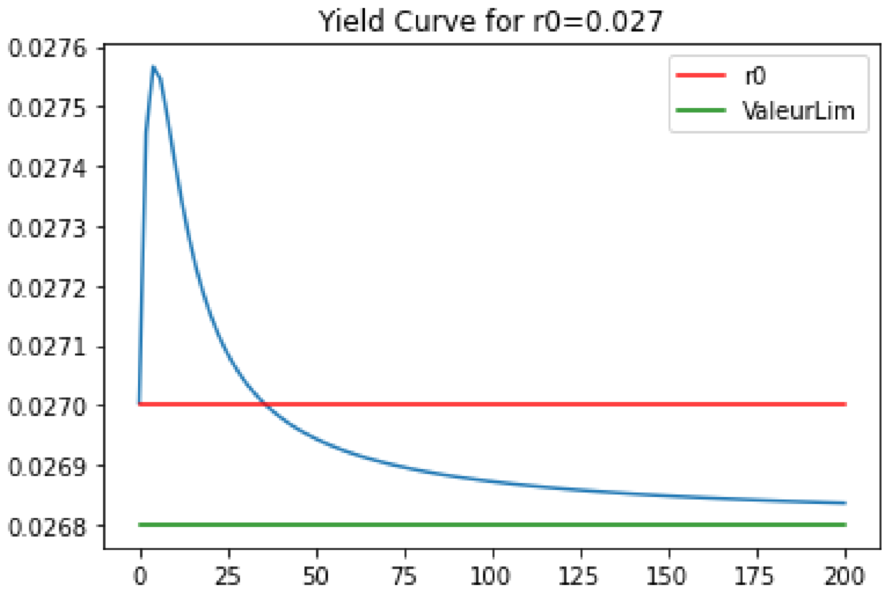
We will use the exact solution of the bond price to find its forward rate Y at t=0.

We will then verify the asymptotic values of the forward rate when the maturity tends to zero and to infinity.



We have chosen the long term mean to be 0.03 (η/γ), hence if the starting interest rate is 0.05 the curve will be inverted because the interest rate will "stabilize" around the long term mean which is 0.03 in the long term. The same reasoning applies if the starting interest rate is 0.01, the forward rate will stabilize around the long-term mean which is η/γ, of course this only applies if the volatility is small enough, for high values of volatility this reasoning break.

Let's take a closer look at the yield curve for 0.027 as a starting interest rate:



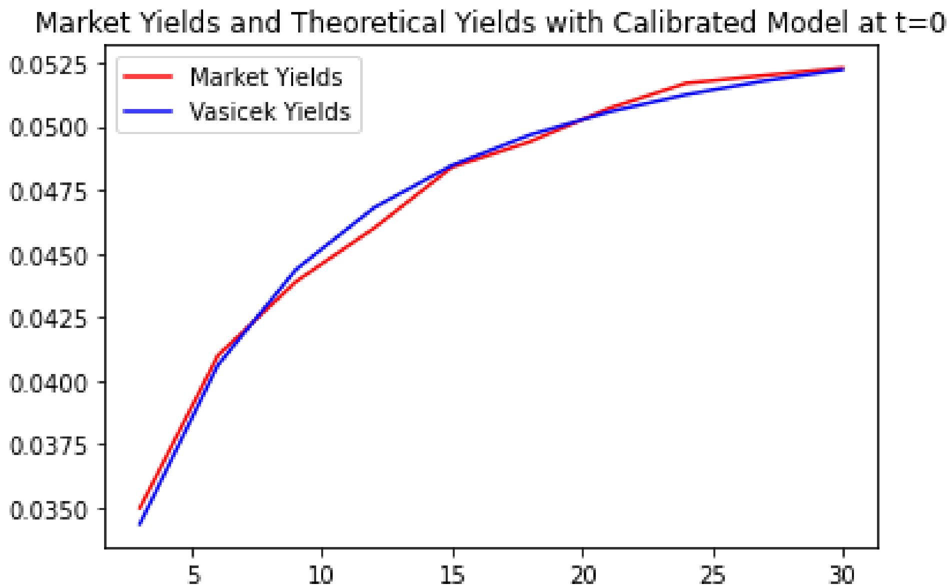
We notice that it's humped at the start, I would say it's due to the fact the long term mean is 0.03 as well as the fact the Brownian "noise" didn't have enough time to affect the interest rate, taking a closer look at the formula giving the Yield when the maturity tends to infinity gives a good idea of how the Brownian's variance affects the interest rate, now since at the start the values of T are small, the variance of the Brownian noise will be small as well and hence it will be offset by the mean-reversion, this is why we have the slight hump.

Part2: Calibration to the market yields

We are looking for three parameters minimizing the distance between the market yields and the theoretical Vasicek yields for a given maturity, we will apply a non-linear least squares algorithm to find those parameters.

We will take 0.1 for each parameter as a starting value, I tried taking 1 but the algorithm didn't converge, in fact it gave me rather strange results with negative volatility values coming into the equation… probably because it overshoots the next step.

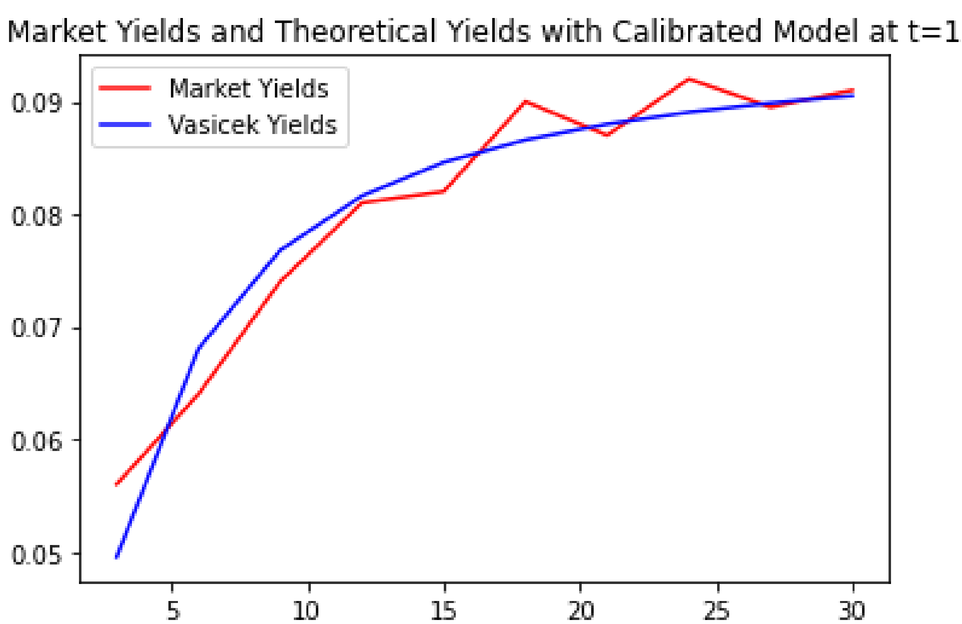
It seems to work fine for smaller initial values of parameters, although I'm guessing it would also work fine if we took a smaller lambda value.



These are the plots for both yields, the values I found were:

[Etha,Sigma,Gamma]=[0.01530756325354006, 0.03743028216, 0.2141660224339749]

We then recalibrate with the new market yields at t=1, which gives us the following plot:

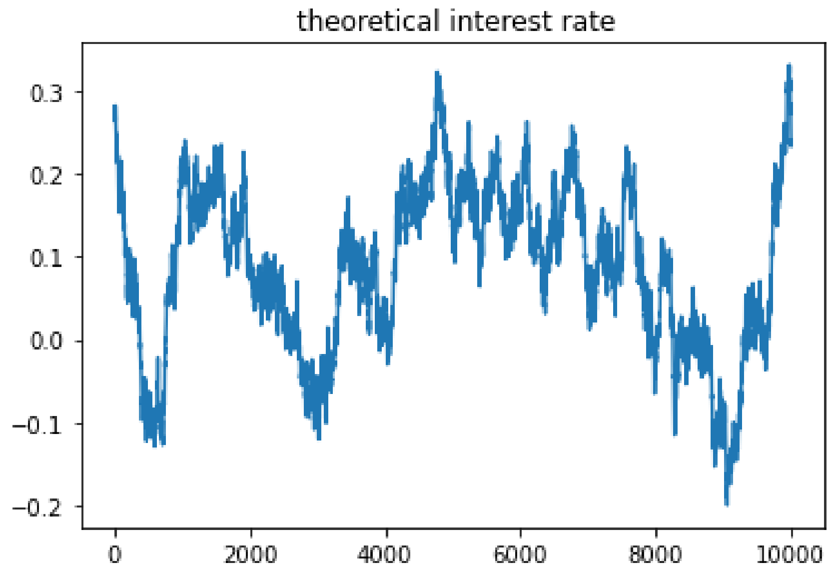


The previous plot looked smoother, I might have made a mistake and I will look into it.

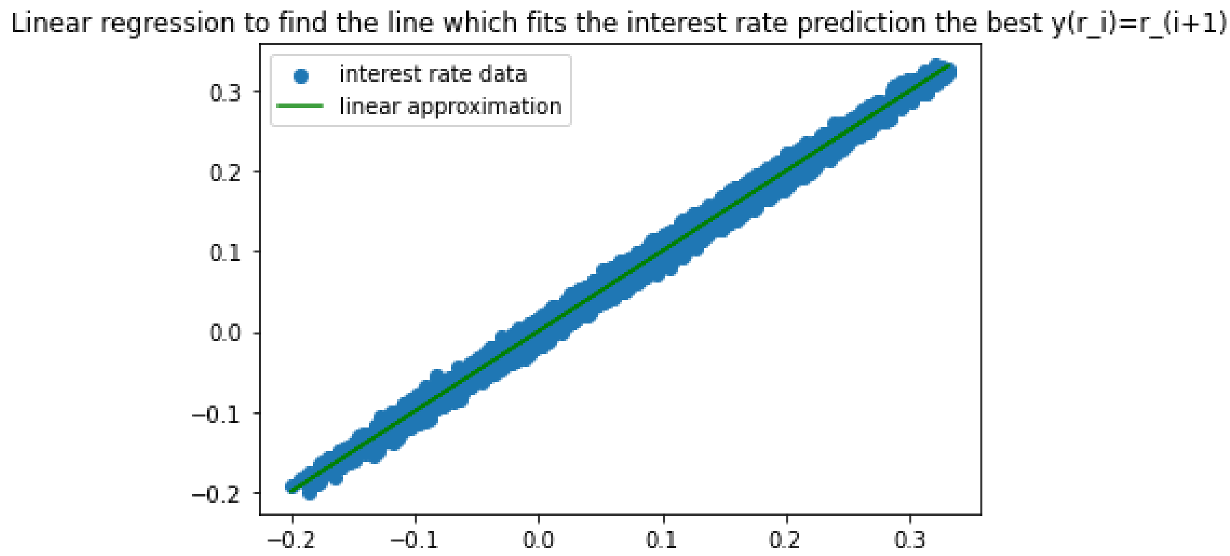
Part 3: Simulation of the market data and linear regression

First, we have to simulate the interest rate using the theoretical formula, this will provide us with the data we need to fit our linear model.

We took N=10000



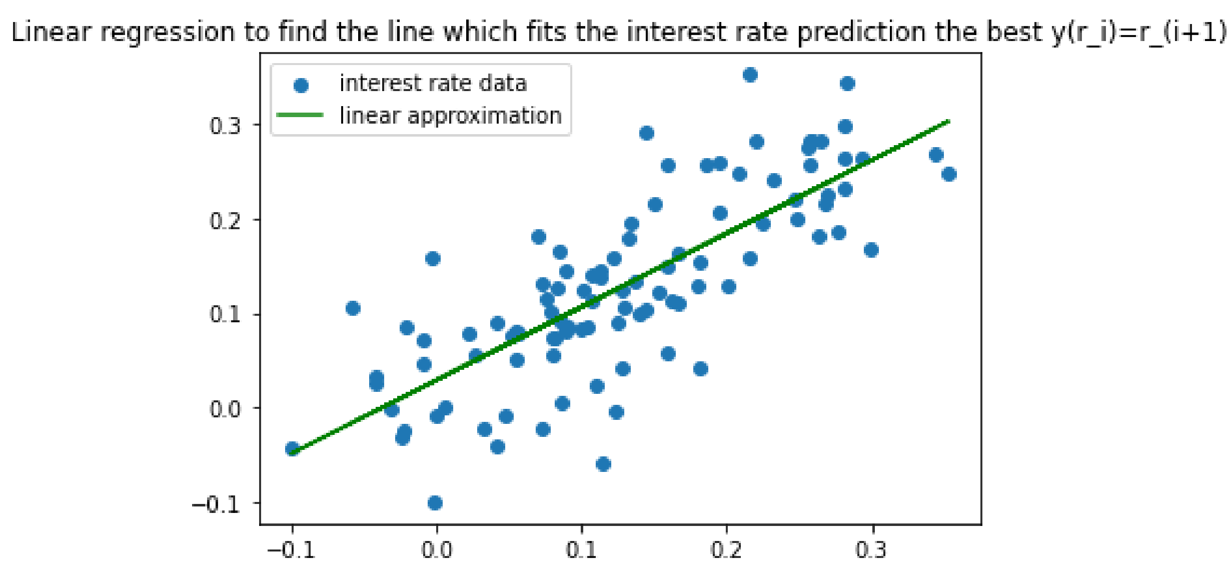
Next we will identify the next interest rate as the image of the previous interest rite by some linear function of unknown parameters, we will use the same algorithm we used before to find those parameter, in fact I don't think it's needed to use non-linear least squares in this case, Gauss-Newton or Gradient descent would work just as fine, but since we already implemented it, might as well use it, it just takes modifying the Jacobian and we are set the go!



It gives us the following linear model, the reason we have so many points is that I took a high number of steps, N=10000.

Our parameters a and b are [0.9974438829353508, 0.00021735923744558457]

Which is almost y=x, it might be because I took a number of steps that is too high, if I take N=100 it gives the following linear model:



With different parameters, in this case our parameters a and b are [0.7774258732942564, 0.028727833512522862].

In this case the variance is equal to 0.0038078008743730836.

We will use this value to compute the theoretical parameters, remember that the parameters we used for the simulation are given by:

[Etha,Gamma,Sigma]=[0.6,4,0.08]

The theoretical Vasicek values in this case are given by:

[Etha,Gamma,Sigma]= [0.6499156008827799, 5.035339585766761, 0.31133891459548513]

Once again, I might have done something wrong, because the values are quite far off, except for Etha.