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# CHPC & NITHECS CODING SUMMER SCHOOL

# Probability & Statistics

## Numerical Data & Regression

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# NUMERICAL DATA & REGRESSION

## EXAMPLE: Arachnophobia

- **In a study on arachnophobia (the fear of spiders), 24 arachnophobes (persons fearing spiders) had to interact with spiders of different sizes.**

- **During each interaction, the level of anxiety of the arachnophobe was measured through galvanic skin response (GSR).**

EXAMPLE adapted from:

Field, A. (2009). *Discovering Statistics using SPSS (and sex and drugs and rock 'n' roll)*, SAGE Publications Ltd, London, UK.

- **Consider the following two variables for $i = 1, 2, \ldots, 24$:**

  $y_i$: **The GSR measurement for the level of anxiety of the $i^{\text{th}}$ arachnophobe.**

  $x_i$: **The size of the spider in centimeters (cm) for the $i^{\text{th}}$ arachnophobe.**

- **Calculate the correlation coefficient between the sizes of the spiders and the GSR measurements.**

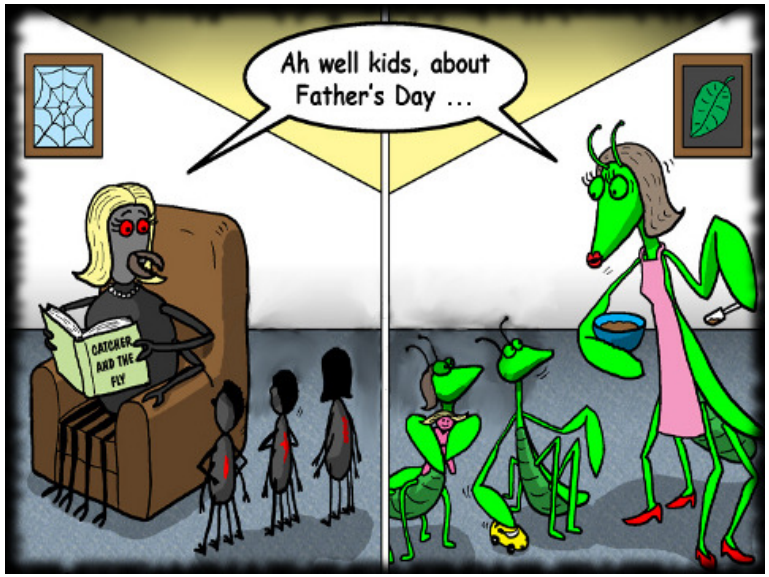
$$r = 0.89$$

- **Characteristics of the correlation coefficient:**
  - $r$ is only a measure of LINEAR dependence.
  - $-1 \leq r \leq 1$
  - $r_{x,y} = r_{y,x}$
  - $r$ is independent of the units and the scale in which $x$ and $y$ are measured.
  - If $x$ and $y$ are independent, then $r = 0$.
  - But if $r = 0$, then $x$ and $y$ are not necessarily independent.
  - $r$ cannot be used to describe cause-and-effect relationships.

- **Correlation analysis vs regression analysis:**

  - **With CORRELATION ANALYSIS the strength of the linear relation between two variables is measured.**

  - **With REGRESSION ANALYSIS the population mean value of the response variable is estimated in terms of known values of the explanatory variable.**



- **Simple linear regression model**
  - **Population regression line:**

$$y_i = \beta_0 + \beta_1 x_i$$

$x$: explanatory or predictor variable

$y$: response variable

$\beta_0$: intercept parameter which gives the mean value of $y$ for $x = 0$

$\beta_1$: slope parameter which gives the change in the mean value of $y$ for a unit increase in the value of $x$

- **Sample regression line:**

$$\hat{y}_i = b_0 + b_1 x_i$$

$\hat{y}$: estimator of the mean value of $y$ for a given value of $x$

$b_0$: point estimate of the intercept parameter $\beta_0$

$b_1$: point estimate of the slope parameter $\beta_1$

- **Fit a linear regression model using the sizes of the spiders to explain the GSR measurements.**

$$\hat{y}_i = 3.5 + 2.8\, x_i$$

- **Interpret the parameter estimates.**

$$b_0 = 3.5$$

**The mean GSR measurement for a spider of 0 cm is 3.5.**

$$b_1 = 2.8$$

**If the size of the spider that the arachnophobe has to interact with is increased by 1 cm, the mean GSR measurement will increase by 2.8.**

## Predicted values and residuals

- Fitted regression line:

$$\hat{y}_i = b_0 + b_1 x_i$$

- Difference between the observed and the predicted values of the response variable:

$$e_i = y_i - \hat{y}_i$$

$y_i$: observed value of the response variable

$\hat{y}_i$: predicted value of the response variable

$e_i$: residual

- **Predict the GSR measurement for Nosnow Cannotski who had to interact with a spider of 13 cm and calculate the corresponding residual.**

$$\hat{y}_5 = 3.5 + 2.8\,x_5 \qquad (\text{note that } x_5 = 13)$$
$$= 3.5 + 2.8 \times 13$$
$$= 39.9$$

$$e_5 = y_5 - \hat{y}_5$$
$$= 35 - 39.9$$
$$= -4.9$$

**Since $e_5 < 0$, the GSR measurement of Nosnow Cannotski is <u>OVERESTIMATED</u> by the fitted regression model.**

- **Least squares regression line**
  - We want the residuals to be as small as possible.
  - Sum of the residuals:

    $\sum e_i = 0$: cannot minimize this sum…
  - Sum of the absolute residuals:

    $\sum |e_i|$: mathematically possible but practically not helpful to minimize this sum…
  - Sum of the squared residuals:

    $\sum e_i^2$: minimizing this sum gives the **LEAST SQUARES REGRESSION LINE**

- **Total variation:** $\sum(y_i - \overline{y})^2$
- **Explained variation:** $\sum(\hat{y}_i - \overline{y})^2$
- **Unexplained variation:** $\sum(y_i - \hat{y}_i)^2 = \sum e_i^2$
- **Percentage of the total variation in the response variable explained by the fitted regression line:**

$$R^2 = \frac{\sum(\hat{y}_i - \overline{y})^2}{\sum(y_i - \overline{y})^2} = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \overline{y})^2}$$

- $0 \leq R^2 \leq 1$

- **Calculate and interpret $R^2$ for the fitted regression line.**

$$R^2 = 0.8$$

**80% of the variation in the GSR measurements is explained by the fitted regression line with the sizes of spiders as explanatory variable.**

# ON YOUR OWN (EXERCISE): South African Animals

- Consider the data from the PnP cards on South African animals.
- Subset the data to study only the mammals (or only the birds).
- Analyse the relation between the weight in kilograms and the size in centimeters.
- Because the relation between these two variables is non-linear, a linear regression model cannot be fitted.
- Apply log transformations to the variables to obtain a linear relation and fit then the linear regression model.