Student ID: ______________________________

February 14, 2018 – 9:00 AM
Duration: 50 minutes

ENEL 471 - Winter 2018
1st Midterm Exam

Notes:

- This exam is closed book and closed notes.
- Non-programmable calculators are allowed.
- The exam duration is 50 minutes.
- The exam is composed of 2 Problems and 3 pages. All the problems are independent.
- Please write your name and ID# in each page

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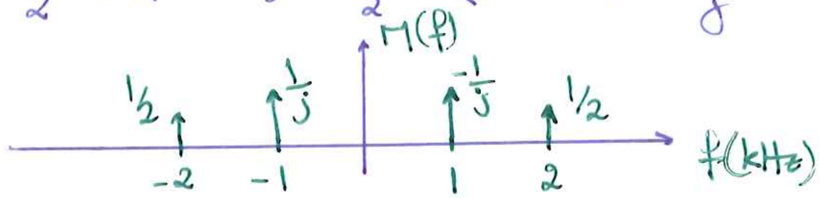
Problem 1 [10 pts]

A lower sideband SSB-SC signal is generated by modulating a 100 kHz cosine carrier by the message signal $m(t) = \cos(4000\pi t) - 2\sin(2000\pi t)$. The amplitude of the carrier is $A_c = 1$.

1. Determine the expression of the frequency domain representation of this lower-sideband SSB-SC signal
2. Sketch the frequency spectrum of this lower-sideband SSB-SC signal. Show all frequencies and amplitudes of interest.
3. Determine the time domain expression for this lower-sideband SSB-SC signal.
4. Propose a demodulator to recuperate the message m(t) from this lower sideband SSB-SC signal. Provide the expression of all the input and output signals and the cutoff frequencies of any filter used.

3pts

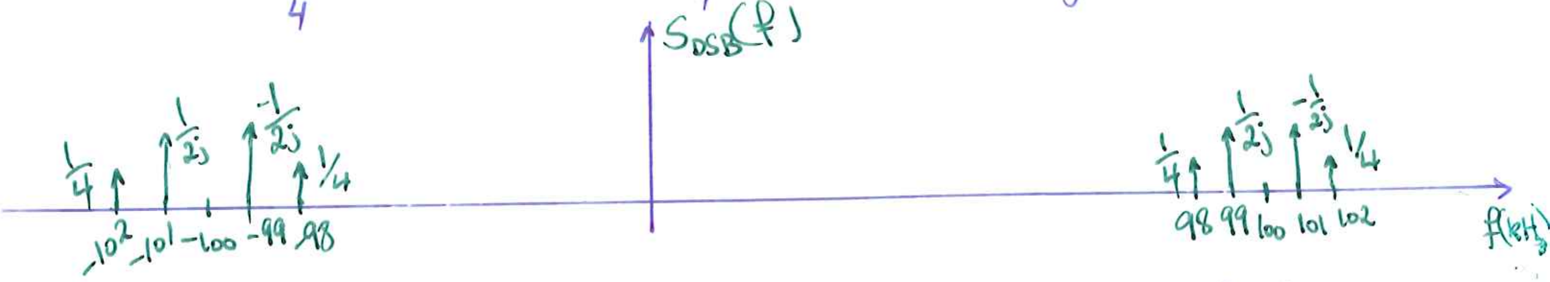
1- $M(f) = \frac{1}{2}\,\delta(f - 2\,\text{kHz}) + \frac{1}{2}\,\delta(f + 2\,\text{kHz}) - \frac{1}{j}\,\delta(f - 1\,\text{kHz}) + \frac{1}{j}\,\delta(f + 1\,\text{kHz})$



the DSB-SC signal is:

$$S_{DSB}(f) = \frac{1}{4}\,\delta(f - 102\,\text{kHz}) + \frac{1}{4}\,\delta(f - 98\,\text{kHz}) - \frac{1}{2j}\,\delta(f - 101\,\text{kHz}) + \frac{1}{2j}\,\delta(f - 99\,\text{kHz})$$

$$+ \frac{1}{4}\,\delta(f + 98\,\text{kHz}) + \frac{1}{4}\,\delta(f + 102\,\text{kHz}) - \frac{1}{2j}\,\delta(f + 99\,\text{kHz}) + \frac{1}{2j}\,\text{[illegible]}$$



the LSB signal is obtained from the DSB signal by removing the upper sideband.

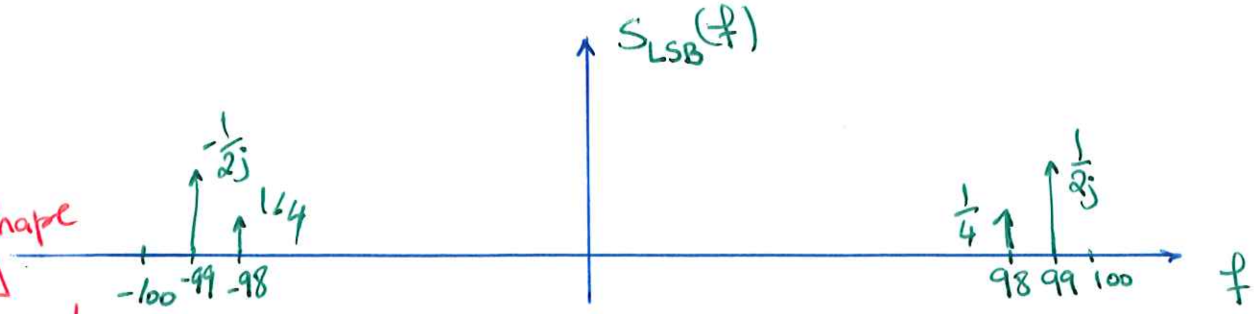
$$\boxed{S_{LSB}(f) = \frac{1}{4}\,\delta(f - 98\,\text{kHz}) + \frac{1}{2j}\,\delta(f - 99\,\text{kHz}) + \frac{1}{4}\,\delta(f + 98\,\text{kHz}) - \frac{1}{2j}\,\delta(f + 98\,\text{kHz})}$$

1pt on using impulses (knowin that the F.T of cos() & sin() are impulses)

1pt on frequencies. (understanding that LSB is obtained by shifting M(f) by ±fc then removing the high freq.)

1pt on amplitudes

2.



3 pts

$\boxed{1\,pt}$ on the shape [impulses]

$\boxed{1\,pt}$ on amplitudes

$\boxed{1\,pt}$ on frequencies

3. 2 pts

$$\boxed{s_{SSB}(t) = \frac{1}{2}\cos(2\pi f_1 t) + \sin(2\pi f_2 t)}$$

with $f_1 = 98$ kHz

$f_2 = 99$ kHz

$\boxed{1\,pt}$: on the inverse Fourier of the cosine

→ 0.5 on cos(.)

→ 0.5 on amplitude

$\boxed{1\,pt}$ on the inverse Fourier of the sine

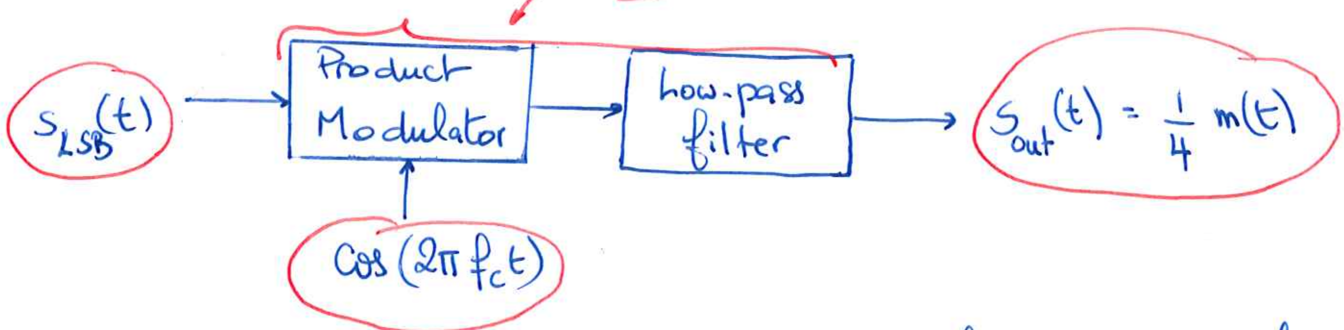
→ 0.5 on sin(.)

→ 0.5 on amplitude

4. 2 pts

A coherent demodulator can be used to demodulate this LSB signal.

or $\boxed{1\,pt}$ on the demodulator [either coherent or block diagram]



the cut-off frequency of the low-pass filter should verify:

$$\boxed{2\,kHz \leq f_{cut\text{-}off} \leq 198\,kHz}$$

$\boxed{1\,pt}$ on the inputs/outputs & cut-off frequency.

→ 0.25 (×3) on each input/output. Please note that they can use a carrier with any amplitude: $A_c' \cos(2\pi f_c t)$. If the output has the format $k.m(t)$ give the 0.25

→ 0.25 on the cutoff freq. Any value in this range give full mark

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Problem 2 [10 pts]
An AM signal has the form:

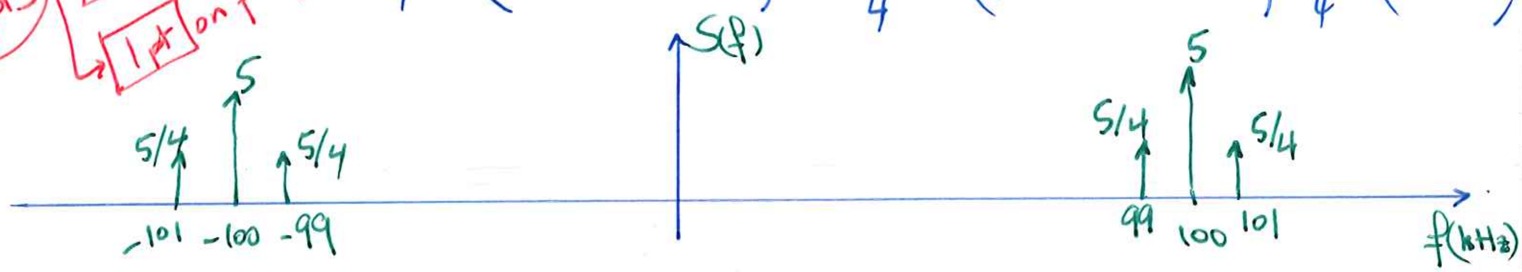
$$s(t) = \left[10 + 5\cos(2000\pi t)\right]\cos(2\pi f_c t)$$

where $f_c = 10^5\ Hz$. For calculating the power, assume a unity resistance (R = 1Ω).

1. Sketch the spectrum of s(t)
2. Determine the modulation index
3. Determine the sidebands' power, the total power, and the ratio of the sidebands power to the total power (the power efficiency of this modulation)
4. This signal is received by an AM receiver using an envelope detector. The average noise power per unit bandwidth measured at the receiver input is 10^{-5} Watt per Hertz. Determine the input and output signal-to-noise ratios (SNR_{in} and SNR_{out}) of the system.
5. By how many decibels is this system inferior to a DSB-SC modulation system?

1- $S(f) = 5\,\delta(f-f_c) + 5\,\delta(f+f_c) + \frac{5}{4}\,\delta\left(f-(f_c+1kHz)\right) + \frac{5}{4}\,\delta\left(f-(f_c-1kHz)\right) + \frac{5}{4}\,\delta\left(f+(f_c+1kHz)\right) + \frac{5}{4}\,\delta\left(f+(f_c-1kHz)\right)$

2pts → 1pt on amplitudes, 1pt on frequencies



2. The modulation index is: $\mu = \max_t |k_a\, m(t)|$

2pts → $\boxed{\mu = 0.5 \text{ or } 50\%}$

↳ give 1pt if they know the formula but they did the wrong calculation / application.

3- the sideband power is: $P_{side\,band} = (5)^2 \times \frac{1}{2} \times \frac{1}{2} = \boxed{\frac{25}{4}}$ or $\boxed{6.25W}$ 0.5 pt

2pts

the total power is: $P_{total} = P_{sideband} + P_{carrier} = \frac{25}{4} + \frac{(10)^2}{2}$

$\boxed{P_{total} = \frac{225}{4} = 56.25W}$ ← 1pt → 0.5 on knowing that, → 0.5 on calculation | give full mark if final result is correct

$$\eta_{AM} = \frac{P_{sideband}}{P_{total}} = \frac{25/4}{225/4} \longrightarrow \boxed{\eta_{AM} = \frac{1}{9} = 0.11} \leftarrow \boxed{0.5}$$

4- * $\boxed{SNR_{in} = \frac{P_{total}}{P_{noise\_in}}}$ and $\boxed{1pt}$

$$P_{noise\_in} = 10^{-5}\ \text{Watt/Hz} \cdot \underbrace{2000\ \text{Hz}}_{\text{bandwidth of the modulated signal}} = 20\text{mW}$$

$$SNR_{in} = \frac{56.25\ \text{W}}{2 \cdot 10^{-2}\ \text{W}} = \boxed{2812.5} \quad \text{or} \quad \boxed{34.5\ \text{dB}}$$

* SNR_{out} =

method 1:

$$\boxed{SNR_{out} = \frac{P_{s\,out}}{P_{n\,out}} = \frac{\cancel{2}\, P_{sideband}}{\cancel{2}\, P_{noise\ in}}} \quad \boxed{1pt}$$

$$SNR_{out} = \frac{6.25}{2 \cdot 10^{-2}} = \boxed{312.5} \text{ or } \boxed{24.94\text{dB}}$$

method 2:

$$FoM = \frac{\mu^2}{2+\mu^2} = \frac{0.25}{2.25} = \frac{1}{9}$$

$$\boxed{SNR_{out} = \frac{SNR_{in}}{9}} = \boxed{312.5} \text{ or } \boxed{24.94\text{dB}}$$

$\boxed{1pt}$

5. this system is inferior to a DSB-SC modulation system by: $\boxed{9.5\text{dB}}$

2pts