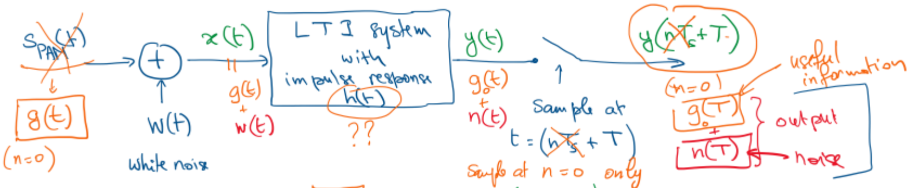
Online Lecture # 11 - Digital Baseband Modulation - Performance of PAM in the Presence of Noise

Wednesday, April 15, 2020

8:52 AM



objective is to choose $h(t)$ so that $y(\cancel{nT_s} + T)$ has the highest ratio of signal to noise : $\boxed{\eta = \frac{|g_o(T)|^2}{E[n^2(t)]}}$ ← should be maximized by the choice of $h(t)$

To do the analysis in time-domain of the effect of noise on PAM demodulation.

we will consider only one pulse $g(t)$

pulse of duration T and amplitude A.

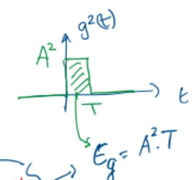
$$g_o(t) = g(t) * h(t) = \int_{-\infty}^{\infty} g(t-\tau) \cdot h(\tau)\, d\tau$$

$$\boxed{n(t) = w(t) * h(t)}$$

$$y(t) = g_o(t) + n(t)$$

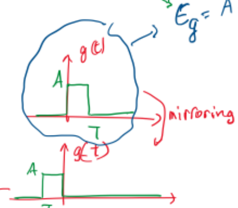
$$y(T) = g_o(T) + n(T)$$

$$y(T) = \underbrace{\int_{-\infty}^{\infty} g(T-\tau)\, h(\tau)\, d\tau}\ + n(T)$$



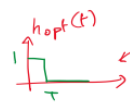
objective: choose $h(t)$ so that to maximize: $\eta = \frac{|g_o(T)|^2}{E[n^2(t)]}$

We can prove mathematically that $|g_o(T)|^2$ is maximized if :

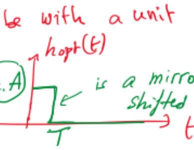


matched filter → $\boxed{h_{opt}(t) = k \cdot g(T-t)}$

constant can be $= \frac{1}{A}$ → $h_{opt}(t)$



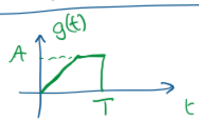
In general, $h_{opt}(t)$ does not need to be with a unit amplitude but has to be constant vs. time



should be constant for all the pulses.

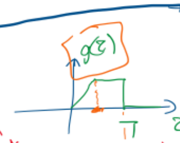
is a mirrored and time shifted by T version of $g(t)$

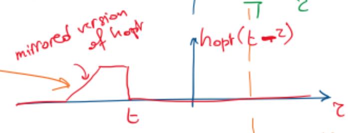
Example of a non-ideal rectangular pulse

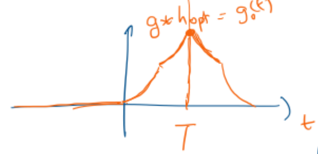


$h_{opt}(t) = k \cdot g(T-t)$

$g_o(t) = ?$







Using a matched filter for an ideal rectangular pulse, we obtain:

$$g_o(t) = g(t) * h_{opt}(t) = \int_{-\infty}^{\infty} g(\tau)\cdot h_{opt}(t-\tau)\, d\tau$$

$$g_o(t) = k\int_{-\infty}^{\infty} g(\tau)\cdot g\big(T-(t-\tau)\big)\, d\tau$$

at $t = T$

$$g_o(T) = k\int_{-\infty}^{\infty} g(\tau)\, g(\cancel{T}-\cancel{T}+\tau)\, d\tau = \boxed{k\underbrace{\int_{-\infty}^{\infty} g^2(\tau)\, d\tau}_{E_g}}$$

$$\eta = \frac{|g_o(T)|^2}{E[n^2(t)]} = \frac{k^2 E_g^2}{E[n^2(t)]}$$

we have: $n(t) = w(t) * h_{opt}(t)$

$$S_n(f) = \underbrace{S_w(f)}_{\frac{N_0}{2}} \cdot |H_{opt}(f)|^2$$

$$E[n^2(t)] = \int_{-\infty}^{\infty} S_n(f)\, df = \frac{N_0}{2}\underbrace{\int_{-\infty}^{\infty}|H_{opt}(f)|^2\, df}_{E_h = k^2 E_g}$$

$$E[n^2(t)] = \frac{N_0}{2}\cdot k^2 E_g$$

$$\eta = \frac{\cancel{k^2} E_g^{\cancel{2}}}{\cancel{k^2}\cancel{E_g}\cdot\frac{N_0}{2}} = \boxed{\frac{2E_g}{N_0}}$$

peak-pulse-to-noise ratio