ENEL 476 – Assignment #2 W2020

Due on Wednesday March 5 at 5 pm in D2L Dropbox

1. The electric field of a uniform plane wave is given by:

$$\mathbf{E}_s(z)=10e^{j0.2z}\,\mathbf{a}_y \text{ V/m.}$$

If the phase velocity of the wave is 1.5×10^8 m/s and the relative permeability of the medium is $\mu_r=2.4$, find the following:

- wavelength (λ)
- frequency (f)
- relative permittivity (ε_r)
- magnetic field ($\mathbf{H}(z,t)$).

2. Dry soil is characterized by $\varepsilon_r=2.5$, $\mu_r=1$ and $\sigma=10^{-4}$ S/m. At each of the following frequencies, determine if soil may be considered a good conductor, then calculate α, β, λ, η and v_p (phase velocity).

- 60 Hz
- 1 MHz
- 1 GHz

3. The electric field of a UPW propagating in a non-magnetic ($\mu_r=1$) medium is given by:

$$\mathbf{E}(x,t)=25\,e^{-30x}\cos(2\pi x10^9 t-40x)\,\mathbf{a}_z \text{ V/m}$$

Find $\mathbf{H}(x,t)$.

4. You are investigating monitoring hydration with microwave sensors placed in contact with the arm. The field in the arm (incident field) is modeled as a uniform plane wave propagating in tissue ($\varepsilon_r=40$, $\mu_r=1$, $\sigma=0.9$ S/m). The frequency of operation is 2.45 GHz. The electric field amplitude in the arm (tissue) and adjacent to the transmitting sensor is 15 V/m. The electric field is oriented in the –x direction and propagates in the z direction.

a) Is the tissue a good conductor?
b) Calculate the attenuation constant, α.
c) Calculate the phase constant, β.
d) Calculate the intrinsic impedance, η.
e) Find an expression for the electric field ($\mathbf{E}(z,t)$).
f) Find an expression for the magnetic field ($\mathbf{H}(z,t)$).
g) Calculate the time-averaged Poynting vector in the tissue ($\mathbf{P}_{av}(z)$). If the wave travels 7 cm through the tissues, by how much is the power density reduced?

1. $\vec{E}_s(z) = 10e^{j0.2z}\,\vec{a}_y$ V/m → assume $\sigma = 0$ S/m (no attenuation term)

$v_p = 1.5\times10^8$ m/s

$\mu_R = 2.4$

↳ λ

↳ f

↳ ϵ_R

↳ $\vec{H}(z,t)$

$$v_p = \omega/\beta$$

$$1.5\times10^8 = \frac{\omega}{0.2}$$

$$\omega = 0.3\times10^8$$

$$= 3\times10^7 \text{ rad/s}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.2}$$

$$\boxed{\lambda = 10\pi \text{ m}}$$

$$f = 0.48\times10^7 \text{ Hz}$$

$$\boxed{f = 4.8\text{ MHz}}$$

$\epsilon_R \rightarrow$ $\beta = \omega\sqrt{\mu_R\mu_0\epsilon_R\epsilon_0}$

$$0.2 = \frac{3\times10^7}{3\times10^8}\sqrt{(2.4)(\epsilon_R)}$$

$$2 = \sqrt{2.4\epsilon_R}$$

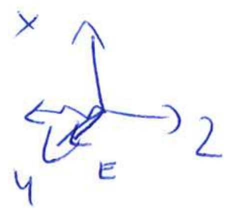
$$4 = 2.4\epsilon_R$$

$$\boxed{\epsilon_R = 1.67}$$

$\vec{H}(z,t) \Rightarrow$ $\eta = \sqrt{\frac{\mu_R\mu_0}{\epsilon_R\epsilon_0}}$

$$= \left(\sqrt{\frac{2.4}{1.67}}\right)(120\pi)$$

$$= 452.2\ \Omega$$



$$\vec{H}(z,t) = \frac{10}{452.2}\cos(3\times10^7 t + 0.2z)\,\vec{a}_x$$

$$= 0.022\cos(3\times10^7 t + 0.2z)\,\vec{a}_x \quad \text{A/m}$$

2. $\epsilon_r = 2.5$

$\mu_r = 1$

$\sigma = 10^{-4}$ S/m

$f_1 = 60$ Hz

$$\frac{\sigma}{\omega\epsilon} = \frac{10^{-4}}{2\pi \times 60 \times 2.5 \times \frac{1}{36\pi} \times 10^{-9}}$$

$$= \frac{3 \times 10^5}{25}$$

$= 1.2 \times 10^4 \Rightarrow$ good conductor

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$= \sqrt{\frac{2\pi \times 60 \times 4\pi \times 10^{-7} \times 10^{-4}}{2}}$$

$$= \sqrt{(\pi^2)(240)(10^{-11})}$$

$= 1.54 \times 10^{-4}$ Np/m

$\beta = 1.54 \times 10^{-4}$ rad/m

$$\lambda = \frac{2\pi}{\beta}$$

$= 40825$ m

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \measuredangle 45^\circ$$

$$|\eta| = \sqrt{\frac{2\pi \times 60 \times 4\pi \times 10^{-7}}{10^{-4}}}$$

$$= \sqrt{\pi^2 \times 480 \times 10^{-3}}$$

$\eta = 2.176 \measuredangle 45^\circ$

$v_p = \omega/\beta \Rightarrow v_p = 2\pi \times 60 / 1.54 \times 10^{-4} \Rightarrow v_p = 2.45 \times 10^6$ m/s

$f_z = 1 \times 10^6$ Hz

$$\frac{\sigma}{\omega \epsilon} = \frac{10^{-4}}{(2\pi \times 10^6)(2.5)\left(\frac{1}{36\pi} \times 10^{-9}\right)}$$

$= 1.8 / 2.5$

$= 0.72 \rightarrow$ lossy material

$$\alpha = 2\pi \times 10^6 \sqrt{\frac{2.5\mu_0\epsilon_0}{2}\left[\sqrt{1+(0.72)^2} - 1\right]}$$

$$= \frac{2\pi \times 10^6}{3 \times 10^8}\sqrt{1.25\,[0.23]}$$

$= 0.01128$

$= 1.128 \times 10^{-2}$ Np/m

& $\beta = 3.5 \times 10^{-2}$ rad/m

$\lambda = 2\pi / \beta$

$= 179.5$ m

$\eta = 2.14.7 \angle 0.31$ rad

$v_p = \omega / \beta$

$= 1.79 \times 10^8$ m/s

$f_3 = 1 \times 10^9 \, Hz$

$$\frac{\sigma}{\omega \epsilon} = \frac{10^{-4}}{(2\pi \times 10^9)(2.5)(\frac{1}{\cancel{36\pi}_{18}} \times 10^{-9})}$$

$$= \frac{(10^{-4})(18)}{2.5}$$

$= 7.2 \times 10^{-4}$ → reasonable insultator
(can approximate with lossless eq'ns with small error)

$\alpha = 0.01192$
$\simeq 0$

$\beta \simeq \omega \sqrt{\mu_0 (2.5) \epsilon_0}$
$= 33.1 \text{ rad/m}$

$\lambda = 2\pi / \beta$

$= 0.19 \, m$

$\eta \simeq \sqrt{\frac{\mu_0}{\epsilon_r \epsilon_0}}$
$= 238.4 \quad \angle 0.0004 \text{ rad}$

$v_p = \omega / \beta$

$= 1.9 \times 10^8 \, m/s$

(5)

3. $\underline{E}(x,t) = 25e^{-30x}\cos(2\pi\times10^9 t - 40x)\vec{a}_z$

$\alpha = 30$ $\quad\quad$ $\omega = 2\pi\times10^9$

$\beta = 40$ $\quad\quad$ $\eta = ? \rightarrow \sigma, \epsilon = ?$

$$\alpha = \omega\sqrt{\frac{\mu_0\epsilon_R\epsilon_0}{2}\left[\underbrace{\sqrt{1+(\sigma/\omega\epsilon_R\epsilon_0)^2}}_{a} - 1\right]}$$

$$30 = \frac{2\pi\times10^9}{3\times10^8\sqrt{2}}\left[\sqrt{b\,(a-1)}\right]$$

(*) this term also contains b but this approach is easier to keep track of variables, etc.

$$\frac{9\sqrt{2}}{2\pi} = \sqrt{b(a-1)} \Rightarrow (2.026)^2 = b(a-1)$$

$$b = \frac{(2.026)^2}{a-1}$$

$$\beta \Rightarrow 40 = \frac{2\pi\times10^9}{3\times10^8\sqrt{2}}\left[\sqrt{b(a+1)}\right]$$

$$(2.7)^2 = b(a+1)$$

$$= (2.026)^2\left(\frac{a+1}{a-1}\right)$$

$$\Rightarrow a = 3.62 \Rightarrow [2.7^2 - 2.026^2]a = (2.026)^2 + (2.7)^2$$

$$b = \frac{(2.026)^2}{2.62}$$

$$= 1.57 \Rightarrow \boxed{\epsilon_R = 1.57}$$

$$\sqrt{1+\left(\frac{\sigma}{\omega\epsilon_R\epsilon_0}\right)^2} = 3.62$$

$$1+\left(\frac{\sigma}{\omega\epsilon}\right)^2 = (3.62)^2$$

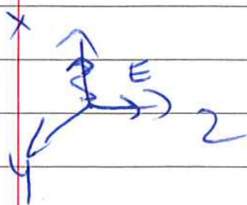
$$\left(\frac{\sigma}{\omega\epsilon_0\epsilon_R}\right)^2 = (3.62)^2 - 1$$

$$\sigma = \omega\epsilon_R\epsilon_0\sqrt{3.62^2-1}$$

$$\boxed{\sigma = 0.35\ \mathrm{S/m}}$$

Now $\eta = 158.96 \angle 0.643$ rad (from full eq'ns)

So, $\vec{H}(x,t) = \frac{-25}{158.96} e^{-30x} \cos(2\pi\times10^{9} t - 40x - 0.643)\,\vec{a}_y$



4. $\epsilon_r = 40$
$\mu_r = 1$
$\sigma = 0.9$ S/m
$f = 2.45\times10^{9}$

a) $\frac{\sigma}{\omega\epsilon} = \frac{0.9}{(2\pi\times2.45\times10^{9})(40)(\frac{1}{36\pi}\times10^{-9})}$ 18

$= \frac{(0.9)(18)}{(2.45)(40)}$

$= 0.165$ $\Rightarrow$ not a good conductor

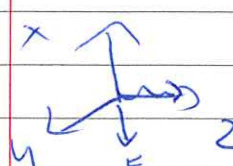
b) $\alpha = 26.71$ Np/m

c) $\beta = 3.258\times10^{2}$ rad/m

d) $\eta = 59.17 \angle 0.082$ rad

e) $\vec{E}(z,t) = -15 e^{-26.71z} \cos(4.9\pi\times10^{9} t - 3.258\times10^{2} z)\,\vec{a}_x$

f) $\vec{H}(z,t) = \frac{-15}{59.17} e^{-26.71z} \cos(4.9\pi\times10^{9} t - 3.258\times10^{2} z - 0.082)\,\vec{a}_y$



g) $\vec{P}_{AV}(z) = \frac{(15)^2}{2(59.17)} e^{-53.42z} \cos(0.082)\,\vec{a}_z$

$e^{-53.42(0.07)} = 0.024 \Rightarrow \vec{P}_{AV}(z=0.07) = 0.024\,\vec{P}_{AV}(z=0)$

$\Rightarrow$ power is reduced by 97.6%