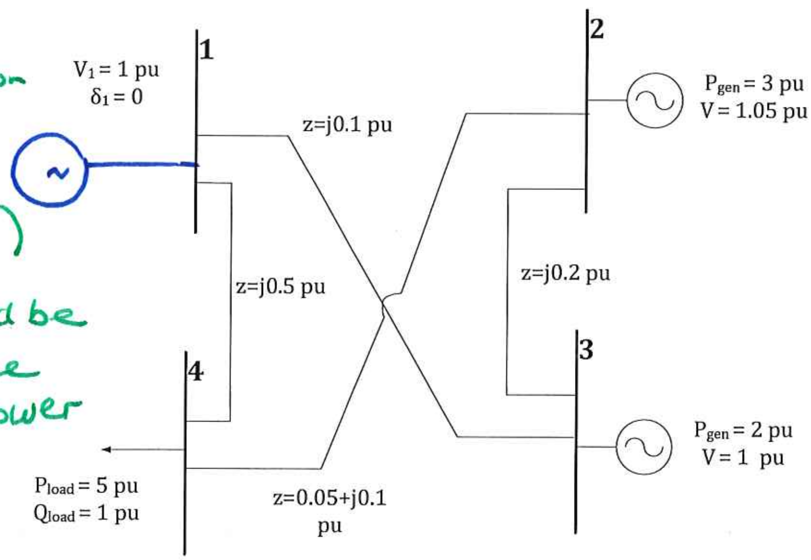
Tutorial 7

Consider the following system. The impedances (branches) between busses could represent lines or transformers in per unit.

this is a better representation for bus 1 →
(i.e. include gen symbol)

slack bus should be able to produce & consume power



- Identify bus types in this system
- Set up the X vector
- Set up Y_{bus}
- Write the equation f_1 (Real power flow equation at bus 1)

Power flow equations with Y_{bus} expressed in rectangular coordinates:

$$f_i = P_{gen,i} - P_{load,i} - \sum_{k=1}^{N} V_i V_k G[i,k]\cos(\delta_i - \delta_k) - \sum_{k=1}^{N} V_i V_k B[i,k]\sin(\delta_i - \delta_k)$$

$$f_{N+i} = Q_{gen,i} - Q_{load,i} - \sum_{k=1}^{N} V_i V_k G[i,k]\sin(\delta_i - \delta_k) + \sum_{k=1}^{N} V_i V_k B[i,k]\cos(\delta_i - \delta_k)$$

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- We organize power flow equations as:

$$F(x) = \begin{bmatrix} f_i \\ \hdashline f_{N+i} \end{bmatrix} \begin{matrix} \}N \\ \}N \end{matrix} = \begin{bmatrix} P_{ean,i} \\ \hdashline Q_{ean,i} \end{bmatrix}$$

$i = 1, \dots, N$

- Bus type:

Bus #	type	unknown
1	Slack	$P_{gen,1}$ & $Q_{gen,1}$
2	Gen/PV	δ_2 & $Q_{gen,2}$
3	Gen/PV	δ_3 & $Q_{gen,3}$
4	Non-gen	δ_4 & V_4

- $P_{load,2} = Q_{load,2} = 0$. Bus 2 would still be a gen bus even if $P_{load,2}$ or $Q_{load,2} \neq 0$. Same applies for bus 3.
- Bus 4 would still be a non-gen (load) bus even if $P_{load,4} = Q_{load,4} = 0$ (as long as there is no gen connected to it).
- The X vector is organized as follows:

$$X = \left[P_{gen,1} \quad \delta_2 \quad \delta_3 \quad \delta_4 \;\vdots\; Q_{gen,1} \quad Q_{gen,2} \quad Q_{gen,3} \quad V_4 \right]^T$$

- Setting up Y_{bus} :

$$Y_{1,3} = \frac{1}{Z_{1,3}} = \frac{1}{j0.1} = -j10$$

$$Y_{1,4} = \frac{1}{Z_{1,4}} = \frac{1}{j0.5} = -j2$$

$$Y_{2,3} = \frac{1}{Z_{2,3}} = \frac{1}{j0.2} = -j5$$

$$Y_{2,4} = \frac{1}{Z_{2,4}} = \frac{1}{0.05 + j0.1} = 4 - j8$$

Σ of all admittances connected to bus 1 $\qquad\qquad -Y_{1,4}$

$$Y_{bus} = \begin{bmatrix} -j12 & 0 & +j10 & +j2 \\ 0 & 4-j13 & j5 & -4+j8 \\ j10 & j5 & -j15 & 0 \\ j2 & -4+j8 & 0 & 4-j10 \end{bmatrix}$$

therefore,

$$G_{bus} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & -4 & 0 & 4 \end{bmatrix} \quad \& \quad B_{bus} = \begin{bmatrix} -12 & 0 & 10 & 2 \\ 0 & -13 & 5 & 8 \\ 10 & 5 & -15 & 0 \\ 2 & 8 & 0 & -10 \end{bmatrix}$$

no j in the matrix!

$Y = G + jB$

- f_1 : real power flow eq at bus 1

$$f_1 = P_{eqn,1} = P_{gen,1} - \cancelto{0}{P_{load,1}} - V_1 V_1 \cancelto{0}{G[1,1]} \cos(\delta_1 - \delta_1)$$

$$- V_1 V_2 \cancelto{0}{G[1,2]} \cos(\delta_1 - \delta_2) - V_1 V_3 \cancelto{0}{G[1,3]} \cos(\delta_1 - \delta_3)$$

$$- V_1 V_4 \cancelto{0}{G[1,4]} \cos(\delta_1 - \delta_4) - V_1 V_1 B[1,1] \cancelto{0}{\sin(\delta_1 - \delta_1)}$$

$$- V_1 V_2 \cancelto{0}{B[1,2]} \sin(\delta_1 - \delta_2) - V_1 V_3 B[1,3] \sin(\delta_1 - \delta_3)$$

$$- V_1 V_4 B[1,4] \sin(\delta_1 - \delta_4)$$

unknowns in red

$$= P_{gen,1} - (1)(1)(10) \sin(0 - \delta_3) - (1) V_4 (2) \sin(0 - \delta_4)$$

$$= P_{gen,1} - 10 \sin(-\delta_3) - 2 V_4 \sin(-\delta_4)$$

To complete the mismatch vector, repeat this for bus 2-4 & also write Q_{eqn} for bus 1 - 4 (f_5 to f_8)

$$F(x) = \begin{bmatrix} P_{gen,1} - 10 \sin(-\delta_3) - 2 V_4 \sin(-\delta_4) \\ \vdots \\ \\ \\ \end{bmatrix}$$

mismatch vector

we can <u>not</u> write this as $A \cdot X = B$ (linear sys of equations).

So, start with an initial guess/estimate for X :

$$X^0 = \left[\begin{array}{cccc|cccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]^T$$

flat start : Set unknown $V = 1$ pu

" $\delta = 0°$

" $P_{gen}, Q_{gen} = 0$ pu

then, - Calculate $F(x^0)$. Likely $\| F(x^0) \| \neq 0$

- improve x^0 by using the update term from NR come up with new vector x^1.
- Repeat until we find optimal solution x^*.