Then,

$$\boxed{C_{an} = \frac{2\pi\varepsilon}{\ln \frac{D}{D_{SC}}}}$$ for symm alignment

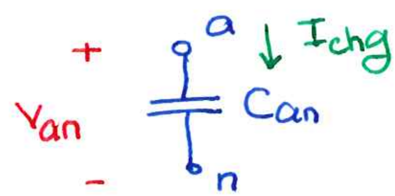
$$\boxed{C_{an} = \frac{2\pi\varepsilon}{\ln \frac{D_{eq}}{D_{SC}}}}$$ for asymm alignment with transposition

where D is the distance between bundle centers

charging current was the last cocenpt in Topic 5, Part 1. We didn't get to cover it in Week 7

- Current supplied to line capacitance is <u>charging current</u>

$$\bar{I}_{chg} = \frac{\bar{V}_{an}}{Z_c} = \frac{\bar{V}_{an}}{\frac{1}{j\omega C_{an}}} = j\omega \cdot C_{an} \cdot \bar{V}_{an}$$



$$\underbrace{Q_{chg}}_{\text{reactive power from } C_{an}} = \frac{V_{an}^2}{X_{cap}} = \frac{V_{an}^2}{\frac{-1}{\omega \cdot C_{an}}} = -\omega \cdot C_{an} \cdot V_{an}^2$$

Transmission line voltage and current equations

Transmission line models used in previous topics looked like: 

$\Omega/m \times l$ — Line length

Z = R+jX = R+jωL

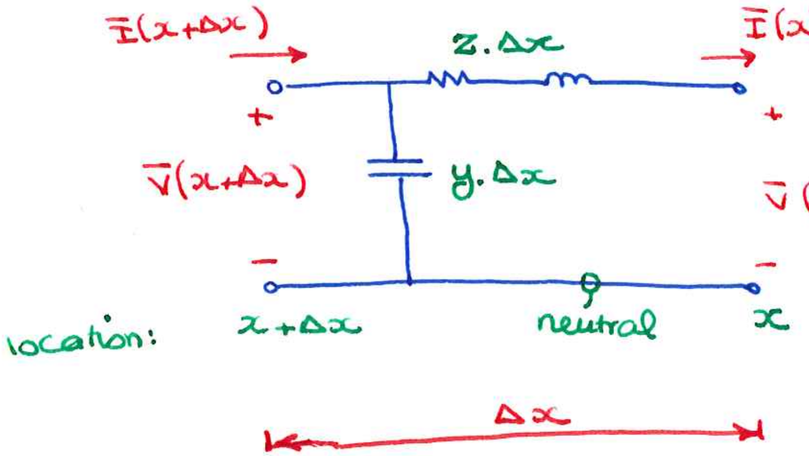
$L\,(H/m) \times l$

We ignored the line capacitance in these models.

In part 1 of topic 5, we came up with equations to calculate the distributed inductance (H/m) and distributed capacitance (F/m)

Objective of part 2: Determine how to model (lump) this all together.

Consider a small portion of the line with length Δx:



lower case

z = R + j ωL where:
R is in Ω/m &
L is in H/m

y = j ωC where
C is in F/m

lower case

From KVL: $\mathbf{V}(x+\Delta x) = \mathbf{V}(x) + z.\Delta x.\mathbf{I}(x)$ Therefore: $\underbrace{\lim_{\Delta x \to 0} \frac{V(x+\Delta x)-V(x)}{\Delta x}}_{\frac{d\bar{V}(x)}{dx}} = z.\mathbf{I}(x)$

$$\frac{d\bar{V}(x)}{dx} = z.\bar{I}(x) \qquad \text{(i)}$$

From KCL: $\mathbf{I}(x+\Delta x) = \mathbf{I}(x) + \overbrace{\underbrace{\mathbf{V}(x+\Delta x)}_{\text{voltage across cap}}\ \underbrace{y.\Delta x}_{\text{admittance of cap}}}^{\bar{I}\text{ in cap}}$ Therefore: $\underbrace{\lim_{\Delta x \to 0} \frac{I(x+\Delta x)-I(x)}{\Delta x}}_{\frac{d\bar{I}(x)}{dx}} = y.\mathbf{V}(x+\Delta x)$

$\Delta x \to 0$

$$\frac{d\bar{I}(x)}{dx} = y.\bar{V}(x) \qquad \text{(ii)}$$

Solve DEQ from combining (i) & (ii) to get: $V(x) = K_1.\cosh(\gamma x) + K_2.\sinh(\gamma x)$ where $\boxed{\gamma = \sqrt{y.z}}$ propagation constant (1/m)

If we know $\bar{V}$ and $\bar{I}$ at one end, e.g. receiving end (x=0): $\mathbf{V}_R = \mathbf{V}(0)$ and $\mathbf{I}_R = \mathbf{I}(0)$, we can find K_1 and K_2 :

$\boxed{\boldsymbol{V}(x) = \boldsymbol{V}_R.\cosh(\gamma x) + \boldsymbol{I}_R Z_c.\sinh(\gamma x)}$ (1) where $\boxed{Z_c = \sqrt{z/y}}$ characteristic impedance (Ω)

Similar derivation for $\mathbf{I}(x)$ gives: $\boxed{\boldsymbol{I}(x) = \boldsymbol{I}_R.\cosh(\gamma x) + \frac{\boldsymbol{V}_R}{Z_c}.\sinh(\gamma x)}$ (2)

Eq (1) and (2) give us voltage and current at any point x along the line (x is the distance from the receiving end)

Note: voltage and current in above expressions are phasors.

• addition to the "Transmission Line Eq" handout:

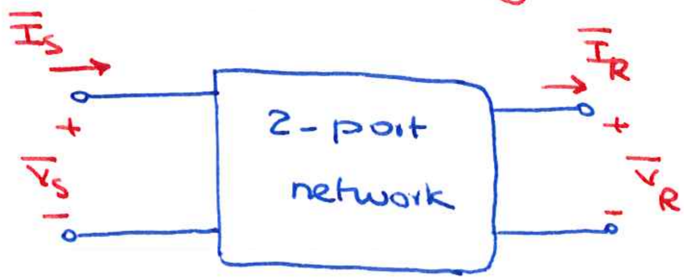
combining (i) & (ii) gives us 2^{nd} order DEQ: $\dfrac{d^2\bar{V}(x)}{dx^2} - z.y.\bar{V}(x) = 0$

Transmission Line Models

. we are often interested in terminal characteristics of the line:

$\bar{V}_R$, $\bar{I}_R$, $\bar{V}_S$, $\bar{I}_S$ ← sending end

• don't care about $\bar{V}(x)$ & $\bar{I}(x)$ for x between two terminals



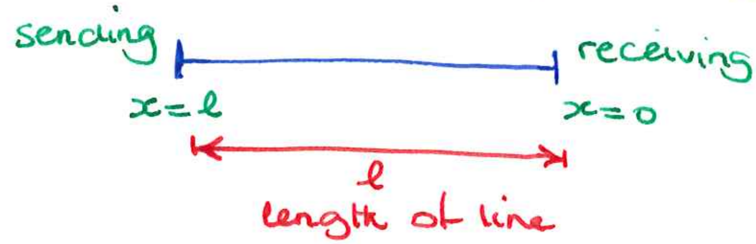
where:

$$\begin{bmatrix} \bar{V}_S \\ \bar{I}_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \bar{V}_R \\ \bar{I}_R \end{bmatrix}$$

$$\Updownarrow$$

$$\begin{cases} \bar{V}_S = A.\bar{V}_R + B.\bar{I}_R \\ \bar{I}_S = C.\bar{V}_R + D.\bar{I}_R \end{cases}$$

need to find ABCD parameters for transmission lines! Let's use $\bar{V}(x)$ and $\bar{I}(x)$ equations



$$\bar{V}_S = \overbrace{\bar{V}(x)}^{eq(1)}\Big|_{x=\ell} = \underbrace{\cosh(\gamma\ell)}_{A}.\bar{V}_R + \underbrace{Z_c.\sinh(\gamma\ell)}_{B}.\bar{I}_R$$

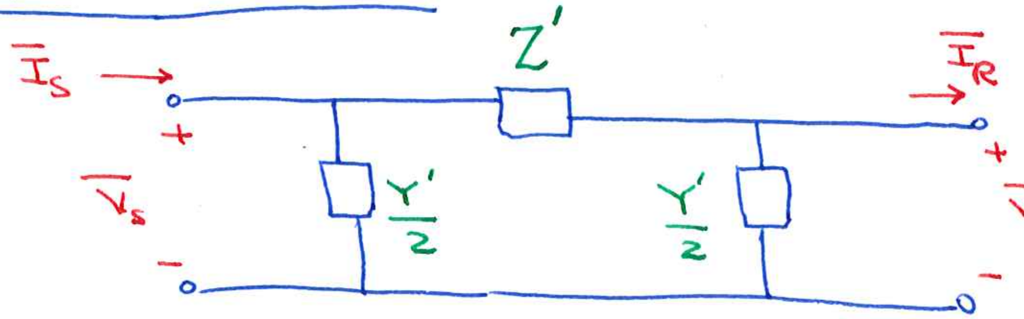
$$\bar{I}_S = \underbrace{\bar{I}(x)}_{eq(2)}\Big|_{x=\ell} = \underbrace{\cosh(\gamma\ell)}_{D}.\bar{I}_R + \underbrace{\frac{1}{Z_c}\sinh(\gamma\ell)}_{C}.\bar{V}_R$$

$$\therefore \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma\ell) & Z_c.\sinh(\gamma\ell) \\ \frac{1}{Z_c}\sinh(\gamma\ell) & \cosh(\gamma\ell) \end{bmatrix}$$

• these are the exact ABCD parameters for two-port network representation of a line.

- For modelling lines in simulations/circuit analysis, need to model the line using circuit elements. Common representation is

Equivalent π Model :



notice upper case Z & Y.

objective : find Z' & Y' such that this model has the same behaviour as the 2-port network?

- skipping the derivation (writing KCL & KVL for eq π model and equating $\overline{V}_s$ & $\overline{I}_s$ equations with 2-port network)

$$\boxed{Z' = Z \cdot \frac{\sinh(\gamma \ell)}{\gamma \ell}}$$

where $Z = z \cdot \ell$

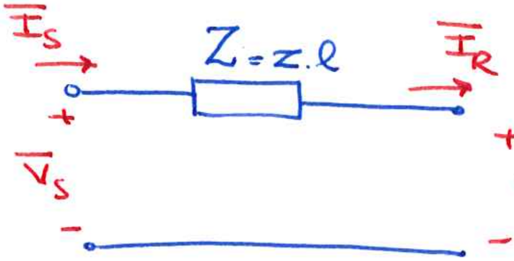
total impedance (Z); distributed impedance (Ω/m) (z)

$$\boxed{\frac{Y'}{2} = \frac{Y}{2} \frac{\tanh\left(\frac{\gamma\ell}{2}\right)}{\frac{\gamma\ell}{2}}}$$

where $Y = y \cdot \ell$

distributed admittance (y)

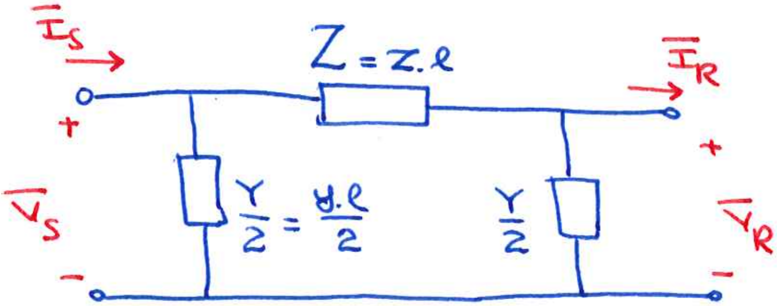
• For <u>short lines</u> ($\ell < 80$ km), use $Z' = Z$ & $\frac{Y'}{2} = 0$



$$\begin{cases} \overline{V}_S = Z.\overline{I}_R + \overline{V}_R & \text{from KVL} \\ \overline{I}_S = \overline{I}_R \end{cases}$$

$$\therefore \begin{bmatrix} \overline{V}_S \\ \overline{I}_S \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}}_{\text{ABCD parameters for short lines}} \begin{bmatrix} \overline{V}_R \\ \overline{I}_R \end{bmatrix}$$

• For <u>medium lines</u> ($80 < \ell < 250$ km) use $Z' = Z$ & $\frac{Y'}{2} = \frac{Y}{2}$

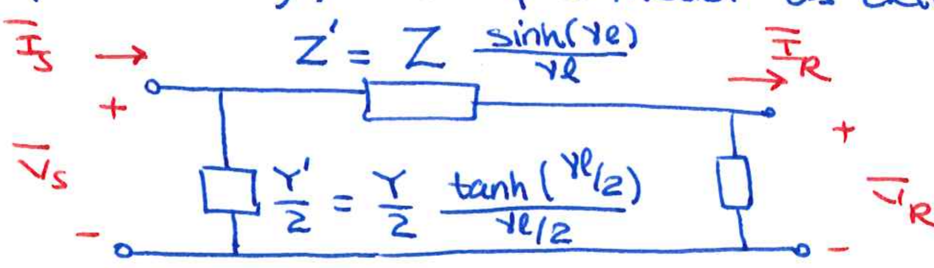


Nominal π Circuit

by using KVL & KCL, we can arrive at ABCD parameters for this model

$$\begin{bmatrix} \overline{V}_S \\ \overline{I}_S \end{bmatrix} = \underbrace{\begin{bmatrix} 1 + \frac{YZ}{2} & Z \\ Y\left(1 + \frac{YZ}{4}\right) & 1 + \frac{YZ}{2} \end{bmatrix}}_{\text{ABCD parameters for medium lines}} \begin{bmatrix} \overline{V}_R \\ \overline{I}_R \end{bmatrix}$$

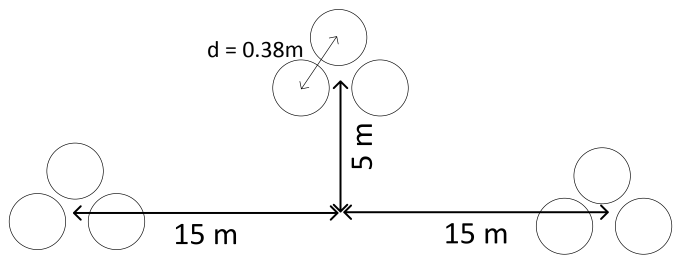
• For <u>long lines</u> ($\ell > 250$ km), use Eq. π model as defined previously.



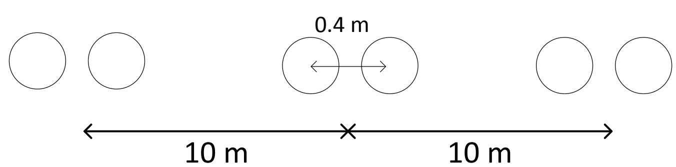
Tutorial 6

Problems 1-2 are from Topic 5 - Part 1 (Transmission line parameters)

Problem 1: Find resistance per phase, inductance per phase and current carrying capacity (ampacity) per phase for 275km long Condor conductor in the following configuration. R is 0.072 Ω/km.



Problem 2: Figure below shows the conductor configuration of a completely transposed, three phase, 345 kV, 60 Hz transmission line with a two conductor bundle of 795 kcmil conductors. Bundle spacing is 0.4 m. Flat horizontal spacing is retained, with 10 m between adjacent bundle centers. Length of line is 200 km. (GMR of a 795 kcmil conductor is 0.0114 m, and radius of conductor is 0.0141m.)



a) Calculate the total capacitance-to-neutral of one phase in F and the admittance-to-neutral in S.

b) If the line voltage is 345 kV, determine the charging current in kA per phase and the total (three phase) reactive power in MVAr supplied by the line capacitance.

The following problems are from Topic 5 - Part 2 (Transmission line models).

Problem 3: 765 kV rated line. V_R = 765 kV (line to line). $\mathbf{S}_R$ = 2000 + j1000 MVA . z = 0.0201 + j0.535 Ω/km, y = j7.75x10^{-6} S/km. Write the expression for V(x).

Problem 4: 500 kV rated line. R = 0.02 Ω/km, x = 0.335 Ω/km, y = j4.807x10^{-6} S/km, length = 300 km. Find Z_c, γl, exact ABCD parameters, equivalent π model.

Tutorial 6

1) From table A.4, $D_S = 0.0368$ ft, ampacity $= 900$ A

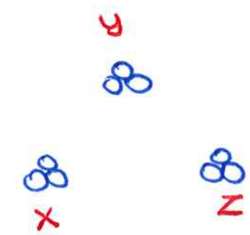
$= 0.0112$ m

R : 3 conductors per phase connected in parallel.

$$\therefore R_{eq} = \frac{1}{\frac{1}{0.072} + \frac{1}{0.072} + \frac{1}{0.072}} = \frac{0.072}{3} = 0.024 \ \Omega/km$$

$$R_{total} = R_{eq} \times 275 \text{ km} = 6.6 \ \Omega$$

L : $L = 2\times10^{-7} \ln \frac{D_{eq}}{D_{SL}}$



where $D_{eq} = \sqrt[3]{\underbrace{D_{XY}}_{\sqrt{15^2+5^2}} \cdot \underbrace{D_{YZ}}_{\sqrt{15^2+5^2}} \cdot \underbrace{D_{XZ}}_{30\,m}}$

$= 19.57$ m

$$D_{SL} = \sqrt[3]{D_S \cdot d^2} = \sqrt[3]{0.0112 \times (0.38)^2} = 0.1174 \text{ m}$$

$$\therefore L = 2\times10^{-7} \ln \frac{19.57}{0.1174} = 1.02\times10^{-6} \text{ H/m}$$

total inductance $L_{total} = L \times 275\,000 = 0.28$ H

total reactance $X_{total} = \omega . L = (2\pi \times \underset{\text{assumed N. American}}{60}) \times L = 106.08 \ \Omega$

(not needed)

ampacity : 3 conductors per bundle $\therefore$ total ampacity per phase

$= 2700$ A

2) $C_{an} = \dfrac{2\pi\varepsilon}{\ln \dfrac{D_{eq}}{D_{sc}}}$ (F/m)

where $D_{eq} = \sqrt[3]{10 \times 10 \times 20}$

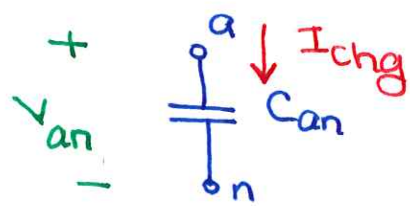
$D_{sc} = \sqrt{r \cdot d} = \sqrt{0.0141 \times 0.4} = 0.075\ \text{m}$

$C_{an_{total}} = C_{an} \times 200\,000 = \dfrac{2\pi \times 8.85 \times 10^{-12}}{\ln \dfrac{12.6}{0.075}} \times 200\,000$

$= 2.17 \times 10^{-6}$ F

$Y_{an_{total}} = j\omega C_{an_{total}} = j\,(2\pi \times 60)(2.17 \times 10^{-6}) = j\,8.19 \times 10^{-4}$ S

part b).



$I_{chg} = \dfrac{V_{an}}{Z_{cap}} = V_{an} \cdot Y_{cap}$

$= \dfrac{345\ \text{kV}}{\sqrt{3}} \cdot (8.19 \times 10^{-4}) = 0.163$ kA

$Q_{chg,3\phi} = 3 \times Q_{chg,1\phi} = 3\,(V_{an}^2 \cdot B_{an_{total}})$

$= 3\left(\dfrac{345\ \text{kV}}{\sqrt{3}}\right)^2 \cdot (8.19 \times 10^{-4}) = 97.5$ MVAr

$Z_c = \dfrac{1}{j\omega C} = \cancel{R}^{\,0} + jX$

$Y_c = j\omega C = \cancel{G}^{\,0} + jB$

$Q = I^2 \cdot X = \dfrac{V^2}{X}$

$= \dfrac{I^2}{B} = V^2 \cdot B$

since $R = G = 0$

$X = \dfrac{1}{B}$

3) $\overline{V}(x) = \overline{V}_R \cdot \cosh(\gamma x) + \overline{I}_R \cdot Z_c \cdot \sinh(\gamma x)$

given

need to calculate

need to calculate based on $\overline{V}_R$ & $\overline{S}_R$

need to calculate

$$\gamma = \sqrt{z \cdot y} = \sqrt{4.15 \times 10^{-6} \angle 177.8^\circ} = 2.036 \times 10^{-3} \angle 88.9^\circ \quad 1/\text{km}$$

$\sqrt{\ }$ of magnitude

divide phase by 2

$$Z_c = \sqrt{\frac{z}{y}} = 262.7 \angle -1.1^\circ \quad \Omega$$

$$V_R = \frac{765 \text{ kV}}{\sqrt{3}} = 441.7 \text{ kV}$$

← $\overline{V}_R, \overline{V}_S, \overline{V}(x)$ are line-to-neutral quantities in line models

$$\overline{V}_R = 441.7 \angle 0^\circ \quad \text{kV}$$

arbitrarily assigned

$$\overline{S}_{R_{1\phi}} = \overline{V}_{R_{l\text{-}n}} \cdot \overline{I}_R^* \quad \therefore \quad \overline{I}_R = \frac{\overline{S}_{R_{1\phi}}^*}{\overline{V}_{R_{l\text{-}n}}^*} = \frac{\frac{1}{3}(2000 + j1000)^*}{441.7 \angle 0^\circ} = 1688 \angle -26.6^\circ$$

Finally, $\overline{V}(x) = 441.7 \angle 0^\circ \cosh\left(2.036 \times 10^{-3} \angle 88.9^\circ \cdot x\right)$

$+ 443.44 \angle -27.7^\circ \sinh\left(2.036 \times 10^{-3} \angle 88.9^\circ \cdot x\right)$

this is the <u>line-to-neutral</u> voltage in <u>kV</u> at any point x measured from receiving end.

4) $Z = R + jX = 0.02 + j\,0.335 = 0.336 \angle 86.6^\circ \quad \Omega/\text{km}$

$y = j\,4.807 \times 10^{-6} \quad S/\text{km} = 4.807 \times 10^{-6} \angle 90^\circ \quad S/\text{km}$

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{0.336 \angle 86.6^\circ}{4.807 \times 10^{-6} \angle 90^\circ}} = 264.4 \angle -1.7^\circ \quad \Omega$$

$$\gamma . \ell = \sqrt{z . y} \, . \, \ell = 0.0113 + j\,0.381$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma\ell) & Z_c . \sinh(\gamma\ell) \\ \frac{1}{Z_c} \sinh(\gamma\ell) & \cosh(\gamma\ell) \end{bmatrix}$$

$A = D = \cosh(\gamma\ell) = \cosh(0.0113 + j\,0.381)$

using $\cosh(\alpha + j\beta) = \cosh(\alpha).\cos(\beta) + j \sinh(\alpha).\sin(\beta)$,

$A = D = \cosh(0.0113).\cos(0.381) + j \sinh(0.0113).\sin(0.381)$

$= 0.9285 + j\,0.00418$

$= 0.9285 \angle 0.26^\circ$

$B = Z_c . \sinh(\gamma\ell) = 264.4 \angle -1.7^\circ \sinh(0.0113 + j0.381)$

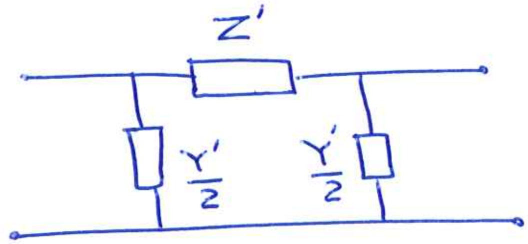
using $\sinh(\alpha + j\beta) = \sinh(\alpha).\cos(\beta) + j \cosh(\alpha).\sin(\beta)$,

$B = 264.4 \angle -1.7^\circ \,(0.3716 \angle 88.39^\circ)$

$= 98.25 \angle 86.69^\circ \quad \Omega$

$C = \frac{1}{Z_c} \sinh(\gamma\ell) = 1.405 \times 10^{-3} \angle 90.09^\circ \quad S$

Eq π model :



$$Z' = Z \cdot \frac{\sinh(\gamma l)}{\gamma l} = 98.26\angle 86.69^\circ$$

$$z.l = (0.336\angle 86.6^\circ) \times 300$$

$$\frac{Y'}{2} = \frac{Y}{2} \cdot \frac{\tanh\left(\frac{\gamma l}{2}\right)}{\frac{\gamma l}{2}} = 6.37\times10^{-7} + j\,7.3\times10^{-4} \text{ S}$$

$$y.l = (j\,4.807\times10^{-6}) \times 300$$