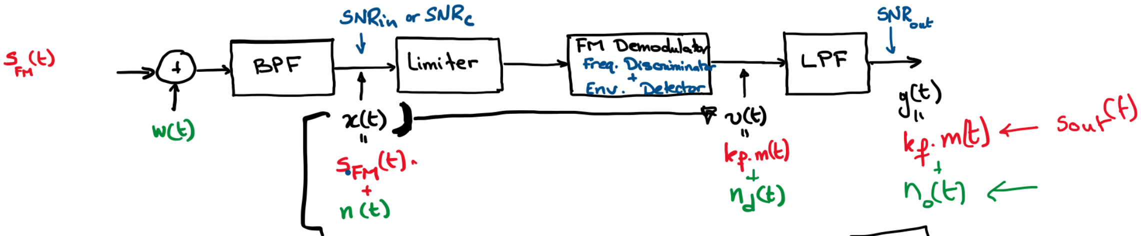
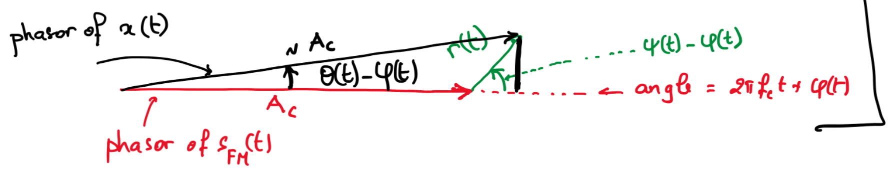
Online Lecture # 06 - FM Demodulation in the Presence of Noise Part II

Wednesday, April 1, 2020
8:42 AM





$$s_{FM}(t) = A_c \cos\Big(2\pi f_c t + \underbrace{2\pi k_f \int_0^t m(\tau)\,d\tau}_{\varphi(t)}\Big).$$

$$n(t) = \underbrace{r(t)}_{\text{amplitude variation}} \cdot \cos\big(2\pi f_c t + \underbrace{\psi(t)}_{\text{phase variation}}\big)$$

$$\boxed{x(t) \simeq A_c \cdot \cos\big(2\pi f_c t + \theta(t)\big)}$$

$$\boxed{\theta(t) = \varphi(t) + \frac{r(t)}{A_c} \cdot \sin\big(\psi(t) - \varphi(t)\big)}$$

$$v(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} = k_f\, m(t) + n_d(t)$$

$$n_d(t) = \frac{1}{2\pi}\frac{d}{dt}\left[\frac{r(t)}{A_c}\sin\big(\psi(t) - \varphi(t)\big)\right]$$

$$n_d(t) = \frac{1}{2\pi A_c}\frac{d}{dt}\Big[r(t)\sin\big(\underbrace{\psi(t) - \varphi(t)}\big)\Big]$$

We can assume that + PSD of $n_d(t)$ is the same as the PSD of: $\dfrac{1}{2\pi A_c} \cdot \dfrac{d}{dt}\Big[\underbrace{r(t)\sin(\psi(t))}_{n_Q(t)}\Big]$

$$S_{n_d}(f) = \frac{1}{(2\pi A_c)^2}\,|j2\pi f|^2 \cdot S_{n_Q}(f)$$

$$S_n(f) = \mathcal{F}(R_n(\tau)) = \mathcal{F}\left(E[n(t)\cdot n(t-\tau)]\right)$$

$$S_{2n}(f) = \mathcal{F}(R_{2n}(\tau)) = \mathcal{F}\left(E[2n(t)\cdot 2n(t-\tau)]\right)$$
$$= \mathcal{F}\left(4\, E[n(t)\cdot n(t-\tau)]\right)$$
$$= 4\cdot \mathcal{F}\left(E(n(t)\cdot n(t-\tau))\right) = 4\, S_n(f)$$

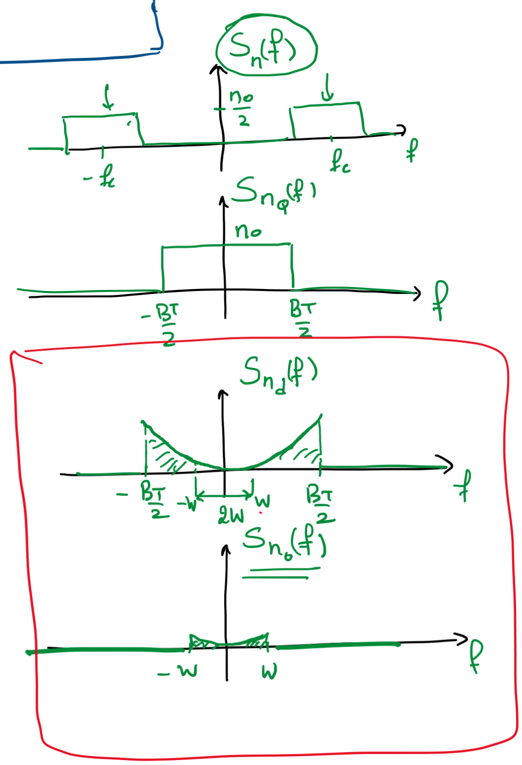
$$n(t) = r(t)\cos(2\pi f_c t + \psi(t))$$
$$= \underbrace{r(t)\cos(\psi)}_{n_I(t)\ \text{in phase component}}\cdot \cos(2\pi f_c t) - \underbrace{r(t)\sin(\psi(t))}_{n_Q(t)\ \text{quadrature component}}\sin(2\pi f_c t)$$

$$\boxed{S_{n_Q}(f) = 2\cdot S_n(f-f_c)}$$

$$S_{n_d}(f) = \frac{1}{(2\pi A_c)^2}(2\pi f)^2 \cdot \underbrace{2\cdot S_n(f-f_c)}_{=\begin{cases} n_0 & \text{for } |f| < \frac{B_T}{2} \\ 0 & \text{elsewhere}\end{cases}}$$

$$S_{n_d}(f) = \begin{cases} \boxed{\dfrac{f^2}{A_c^2}\cdot n_0} & \text{for } |f| < \frac{B_T}{2} \\ 0 & \text{elsewhere} \end{cases}$$

$$S_{n_o}(f) = \begin{cases} \dfrac{f^2}{A_c^2}\cdot n_0 & \text{for } |f| < W \\ 0 & \text{elsewhere} \end{cases}$$



At the output of the FM receiver:

$$P_{S_{out}} = \boxed{k_f^2\, P_m}$$

$$P_{n_o} = \int_{-\infty}^{\infty} S_{n_o}(f)\, df = \int_{-W}^{W} \frac{f^2}{A_c^2}\cdot n_0\, df$$

$$P_{n_o} = \frac{n_0}{A_c^2}\left[\frac{f^3}{3}\right]_{-W}^{W} = \boxed{\frac{n_0}{A_c^2}\,\frac{2}{3}\,W^3}$$

$$SNR_{out} = \frac{P_{S_{out}}}{P_{n_{out}}} = \frac{k_f^2 P_m}{\frac{n_0}{A_c^2}\cdot\frac{2}{3}W^3}$$

$$SNR_{out} = \frac{3}{2}\,\frac{k_f^2 A_c^2 P_m}{n_0 W^3}$$

At the input of the FM receiver:

$$P_{S_{FM}} = \frac{A_c^2}{2}$$

$$P_{n_{in}} = \frac{n_o}{2} \cdot 2W = n_o \cdot W$$

$$SNR_{in} = SNR_c = \frac{P_{S_{FM}}}{P_{n_{in}}} = \boxed{\frac{-{}_c^2}{n_o W}}$$

$$FOM_{FM} = \frac{SNR_{out}}{SNR_{in}} = \frac{3}{\cancel{2}} \frac{k_f^2 \cancel{A_c^2} P_m}{\cancel{n_o} W^3} \cdot \frac{\cancel{2}\cancel{n_o} \cdot W}{\cancel{A_c^2}}$$

$$\boxed{FOM_{FM} = 3 \frac{k_f^2 P_m}{W^2}}$$

Example: $m(t) = A_m \cos(2\pi f_m t)$

$$P_m = \frac{A_m^2}{2} \quad ; W = f_m$$

$$FOM_{FM} = 3 \, k_f^2 \frac{A_m^2}{2} \cdot \frac{1}{f_m^2} = \frac{3}{2} \underbrace{\left(\frac{k_f A_m}{f_m}\right)^2}$$

$$\boxed{FOM_{FM} = \frac{3}{2} \beta^2}$$

Example: if $\beta = 4 \rightarrow FOM_{FM} = \frac{3}{2}(4)^2 = \boxed{24}$.

this wideband FM receiver would have $SNR_{out} = 24 \cdot SNR_{in}$

this FM R_x is 24 better than DSB R_x.