University of Calgary

Schulich School of Engineering

Department of Electrical and Computer Engineering

ENEL 476 – Electromagnetic Waves and Applications

Quiz 1

Winter Session 2016
Thursday February 4, 2016
12:30-1:10 pm

ST 148

Student Name or ID number:

Dr. Fear

Question 1. **(13 marks)**

A magnetic flux density is described by:

$$\vec{B_s}(y) = 5e^{-j5y}\vec{\mathbf{a}}_x \ \mu\text{Wb/m}^2$$

The flux density is located in free space ($\varepsilon=\varepsilon_o$, $\mu=\mu_o$ and $\sigma=0$). Assume a source-free region ($\mathbf{J}=0$, $\rho_v=0$).

a) Find the displacement current density in phasor form ($\mathbf{J}_{ds}(y)$).

$$\vec{J}_{ds}(y) = j\omega\epsilon_0\vec{E}_s(y)$$

(1) $$\nabla\times\vec{H}_s(y) = \vec{J}_{ds}(y)$$

$$\vec{H}_s(y) = \frac{5}{\mu_0}e^{-j5y}\vec{a}_x \ 10^{-6}$$

(1) $$\nabla\times\vec{H}_s = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_{sx} & H_{sy}^{\nearrow 0} & H_{sz}^{\nearrow 0} \end{vmatrix}$$

$$= -\vec{a}_y\left(-\frac{\partial}{\partial z}H_{sx}\right)^{\nearrow 0} + \vec{a}_z\left(-\frac{d}{dy}H_{sx}\right)$$

$$\nabla\times\vec{H}_s = \frac{-d}{dy}\left(\frac{5}{\mu_0}e^{-j5y}\right)\vec{a}_z$$

$$= -\frac{5}{\mu_0}(-j5)e^{-j5y}\vec{a}_z$$

(1) $$= \frac{j25}{\mu_0}e^{-j5y}\vec{a}_z$$

$$\therefore \vec{J}_{ds}(y) = \frac{j25}{\mu_0}e^{-j5y}\vec{a}_z \ \mu A/m^2$$

(1)

b) Find the electric field in phasor form ($\mathbf{E}_s(y)$). Keep your expression in terms of ω, ε and μ.

(1) $$\vec{E}_s(y) = \frac{\vec{J}_{ds}(y)}{j\omega\epsilon_0}$$

(1) units

$$= \frac{j25}{j\mu_0\omega\epsilon_0}e^{-j5y}\vec{a}_z$$

(1) $$\boxed{\vec{E}_s(y) = \frac{25}{\omega\mu_0\epsilon_0}e^{-j5y}\vec{a}_z \ \mu V/m}$$

c) Find the frequency of the fields (ω).

(1) $\nabla \times \vec{E}_s(y) = -j\omega\mu_0 \vec{H}_s(y)$

$$\nabla \times \vec{E}_s = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 0 & 0 & E_{sz} \end{vmatrix}$$

$$= \vec{a}_x\left(\frac{d}{dy}E_{sz}\right) - \vec{a}_y\left(\cancel{\frac{d}{dx}E_{sz}}^{\,0}\right)$$

$$= \frac{d}{dy}\left[\frac{25}{\omega\mu_0\epsilon_0}e^{-j5y}\right]\vec{a}_x$$

(1) $$= \frac{-j125}{\omega\mu_0\epsilon_0}e^{-j5y}\,\vec{a}_x$$

$\therefore$ (1) $$\frac{\cancel{-j}\cancel{125}^{\,25}}{\omega\mu_0\epsilon_0}\cancel{e^{-j5y}} = \cancel{-j}\omega\cancel{\mu_0}\frac{5}{\cancel{\mu_0}}\cancel{e^{-j5y}}$$ (1)

$$\omega^2 = \frac{25}{\mu_0\epsilon_0} \Rightarrow \omega = \frac{5}{\sqrt{\mu_0\epsilon_0}} \Rightarrow \omega = 5\times3\times10^8$$
$$\omega = 1.5\times10^9 \text{ Rad/s}$$

d) Find an expression for the electric field in time-domain form (**E**(y,t)). Substitute in values and simplify.

$$\vec{E}(y,t) = \frac{25}{(1.5\times10^9)(4\pi\times10^{-7})\left(\frac{1}{36\pi}\times10^{-9}\right)}\cos(1.5\times10^9t - 5y)\,\vec{a}_z \ \mu\text{V/m}$$

$$= (25)\left(\frac{2}{3}\right)(9)^{3}\cos(1.5\times10^9t - 5y)\vec{a}_z \times10^7\times10^{-6} \text{ V/m}$$

$$= 1500\cos(1.5\times10^9t - 5y)\vec{a}_z \text{ V/m}$$

$$= 1.5\cos(1.5\times10^9t - 5y)\vec{a}_z \text{ kV/m}$$

($\frac{1}{2}$) ($\frac{1}{2}$) ($\frac{1}{2}$) ($\frac{1}{2}$)

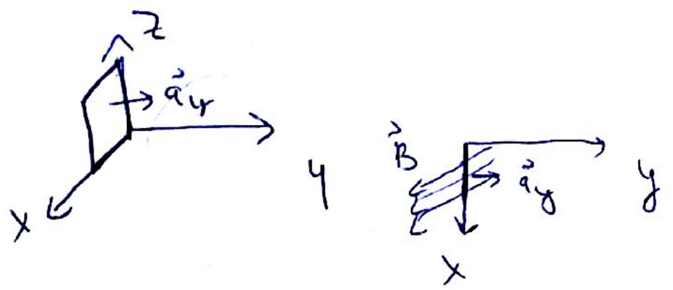
Question 2. **(15 marks)**

A rectangular loop of wire contains a resistor of 20 Ω. The loop is moving in an externally applied magnetic flux density:

$$\vec{B}(t) = (10\vec{a}_x - 5\vec{a}_y)\cos(2\pi x 10^3 t)\ \text{mWb/m}^2$$

The loop has surface normal in the $\mathbf{a}_y$ direction (i.e. **ds** is in $+\mathbf{a}_y$) and is located in the y=0 plane. The loop extends from x=0 to x=2.5 cm and z=0 to z=4 cm.

a) Sketch the loop and flux density (a 2D sketch in the xy plane is suggested).



(1) loop
(1) orientation of flux

b) Find the total flux (Φ) passing through the surface of the loop.

(1) $\phi = \int \vec{B}\cdot d\vec{s}$

$\vec{B}\cdot d\vec{s} = \left[(10\vec{a}_x - 5\vec{a}_y)\cos 2\pi\times10^3 t\right]\cdot dx\,dz\,\vec{a}_y$

$= -5\cos(2\pi\times10^3 t)\,dx\,dz$

(1.) $\phi = \int_0^{0.04}\int_0^{0.025} -5\cos(2\pi\times10^3 t)\,dx\,dz$

$= -5\cos(2\pi\times10^3 t)(0.04)(0.025)\ \times10^{-3}$

less ↑ (1) $= \boxed{-5\cos(2\pi\times10^3 t)\ \mu\text{Wb}}$

c) Find the EMF (V_{emf}).

(1) $V_{emf} = -\frac{d}{dt}\phi$

$= -\frac{d}{dt}(-5\cos(2\pi\times10^3 t)\times10^{-6})$

$V_{emf} = -10\pi\times10^3 \sin(2\pi\times10^3 t)$

4

$= -10\pi \sin(2\pi\times10^3 t)$ mV

(1)

$$V_{emf} = -\frac{d}{dt}\phi$$

$$= -\frac{d}{dt}\left(-5\cos(2\pi\times10^3 t)\right) \times 10^{-6}$$

$$= 5\,\frac{d}{dt}\left(\cos(2\pi\times10^3 t)\right) \times 10^{-6}$$

$$= -(5)(2\pi\times10^3)\sin(2\pi\times10^3 t) \times 10^{-6}$$

$$= -10\pi\sin(2\pi\times10^3 t)\times 10^{-3} \quad \checkmark$$

~~units~~

d) Find the current induced in the loop.

(1) $$I = V/R$$

$$= \frac{-10\pi\sin(2\pi\times10^3 t)\times10^{-3}}{20}$$

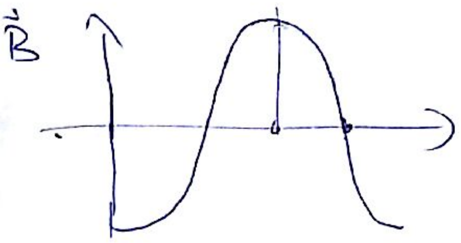
(1) $$\boxed{I = -1.57\sin(2\pi\times10^3 t)\ \text{mA}}$$

e) Sketch 1 period of the flux density in $\vec{a}_y$ and induced current in order to show time variation. Sketch the loop. From t=0.5 ms to t=0.75 ms, show the direction of induced current flow. Discuss how this direction satisfies Lenz's law.

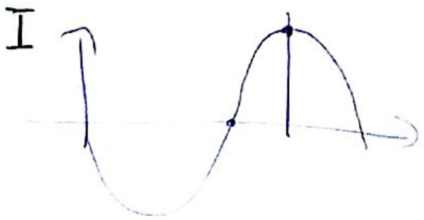
$f = 1\times10^3$

$T = 1\,\text{ms}$

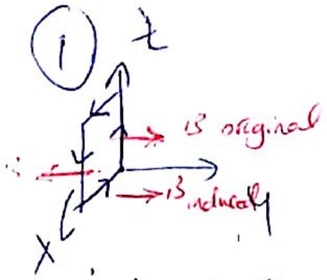
(1)



(1)



(1)



→ induced flux in the $\vec{a}_y$

(1) → $\vec{B}$ in decreases ∴ counters this change

Question 3. **(4 marks)**

a) A material has ε_r=24 and σ=2 Sm. At 2 GHz, the ratio between conduction and displacement current is:

(2)

- 0.75
- 1.25
- 1.5 x 10^{11}
- 6.6 x 10^{-3}
- not possible to calculate

b) Consider dropping a magnet down a copper tube, as in the Lab 1 demo. Which of the following statements are true:

1/2 each

- X There is no current to consider, as the copper tube is not connected to any sources.
- X Because we are using a permanent magnet, there is no time rate of change of magnetic flux.
- ✓ The magnet falls more slowly than it would due to gravity alone.
- ✓ Currents are induced as the location of the magnet changes in the tube.
- ✓ There is an induced magnetic flux related to the induced currents. This induced flux influences the velocity of the magnet.
- X Lenz's law is violated in this experiment, so it is more of a suggestion than a law.
- ✓ In an experiment like this, the magnet typically reaches a constant velocity (given a long enough tube and strong enough magnet).

Name	
Q1	
Q2	
Q3	
Total	