ENEL 487 | Winter 2019 | Quiz 3

Name: ID:

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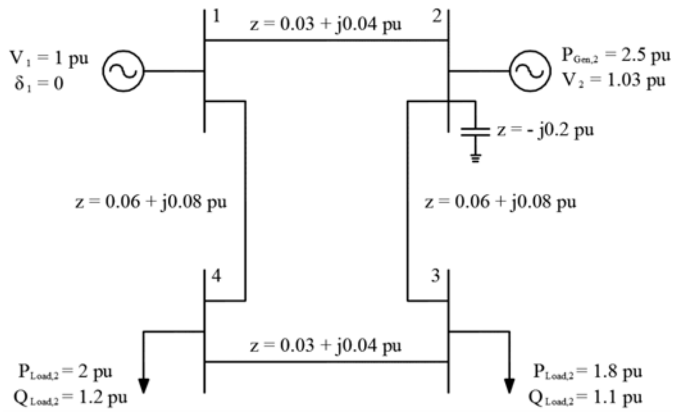
1) Consider the system below:



a. Fill out the following table for this system. The answer(s) should be selected from terms in the brackets. [3 marks]

Bus Number	Bus Type (Load, Gen, Slack)	Known Variable (Pgen, Qgen, V, δ, Pload, Qload)	Unknown Variable (Pgen, Qgen, V, δ, Pload, Qload)
1	Slack	V, δ, Pload, Qload	Pgen, Qgen
2	Gen	Pgen, V, Pload, Qload	Qgen, δ
3	Load	Pgen, Qgen, Pload, Qload	V, δ
4	Load	Pgen, Qgen, Pload, Qload	V, δ

b. Create the X vector for this system (in the form of X = [-----]^T). [4 marks]

$$X = [P_{gen,1} \quad \delta_2 \quad \delta_3 \quad \delta_4 \quad Q_{gen,1} \quad Q_{gen,2} \quad V_3 \quad V_4]^T$$

c. Create the admittance matrix Y_{bus} for system. Also, provide the B_{bus} and G_{bus} matrices. [4 marks]

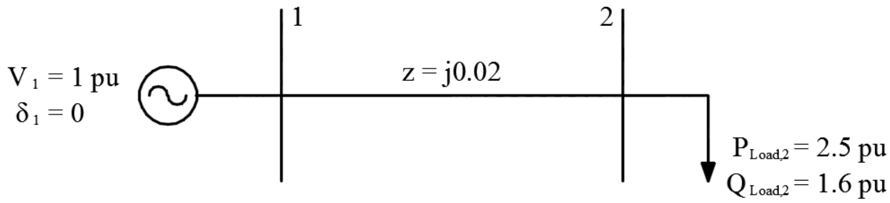
$$Y_{bus} = \begin{bmatrix} 18-j24 & -12+j16 & 0 & -6+j8 \\ -12+j16 & 18-j19 & -6+j8 & 0 \\ 0 & -6+j8 & 18-j24 & -12+j16 \\ -6+j8 & 0 & -12+j16 & 18-j24 \end{bmatrix}$$

$$B_{bus} = \begin{bmatrix} -24 & 16 & 0 & 8 \\ 16 & -19 & 8 & 0 \\ 0 & 8 & -24 & 16 \\ 8 & 0 & 16 & -24 \end{bmatrix} \qquad G_{bus} = \begin{bmatrix} 18 & -12 & 0 & -6 \\ -12 & 18 & -6 & 0 \\ 0 & -6 & 18 & -12 \\ -6 & 0 & -12 & 18 \end{bmatrix}$$

d. If all the branches in the system were <u>lossless</u>, how much <u>real power</u> is injected by the slack bus? [1 mark]

If there are no losses in the branches, $\sum P_{gen} = \sum P_{load} \rightarrow P_{gen,1} = P_{load,3} + P_{load,4} - P_{gen,2} = 2 + 1.8 - 2.5 = 1.3$ pu

2) Consider the two bus system below:



a. Create the Ybus for this system. [1 mark]

$$Y_{bus} = \begin{bmatrix} -j50 & j50 \\ j50 & -j50 \end{bmatrix}$$

b. Create the X vector for this system. [1 mark]

$$X = [P_{gen,1} \quad \delta_2 \quad Q_{gen,1} \quad V_2]^T$$

c. Write the real and reactive power flow equations for bus 2. Plug in all the known values and simplify as much as possible. [3 marks]

$$f_2 = P_{gen,2} - P_{load,2} - \sum_{k=1}^{2} V_2 V_k G[2,k]\cos(\delta_2 - \delta_k) - \sum_{k=1}^{2} V_2 V_k B[2,k]\sin(\delta_2 - \delta_k)$$
$$= 0 - 2.5 - V_2 V_1 G[2,1]\cos(\delta_2 - \delta_1) - V_2 V_2 G[2,2]\cos(\delta_2 - \delta_2) - V_2 V_1 B[2,1]\sin(\delta_2 - \delta_1) - V_2 V_2 B[2,2]\sin(\delta_2 - \delta_2)$$
$$= -2.5 - 50V_2 \sin(\delta_2)$$

$$f_4 = Q_{gen,2} - Q_{load,2} - \sum_{k=1}^{2} V_2 V_k G[2,k]\sin(\delta_2 - \delta_k) + \sum_{k=1}^{2} V_2 V_k B[2,k]\cos(\delta_2 - \delta_k)$$
$$= 0 - 1.6 - V_2 V_1 G[2,1]\sin(\delta_2 - \delta_1) - V_2 V_2 G[2,2]\sin(\delta_2 - \delta_2) + V_2 V_1 B[2,1]\cos(\delta_2 - \delta_1) + V_2 V_2 B[2,2]\cos(\delta_2 - \delta_2)$$
$$= -1.6 + 50V_2 \cos(\delta_2) - 50(V_2)^2$$

d. If the optimal value of $\delta_2 = -0.05211$ radians, what is the optimal voltage at bus 2? [3 marks]

Need to find the value of V_2 that "solves" the power flow equation.
If using f_2, solve for V_2 such that: $-2.5 - 50.V_2.\sin(-0.05211) = 0$ This gives $V_2 = 0.96$ pu
If using f_4, solve for V_2 such that: $-1.6 + 50.V_2.\cos(-0.05211) - 50(V2)^2 = 0$ Solving this quadratic equation gives:
$V_2 = 0.965$ pu or 0.03 pu (Voltage of 0.03 pu, however, is not acceptable as it indicates the load has gone "dark"!)
Using either of the above equations is acceptable for this problem!