6.18 $x^3 + 8x^2 + 2x - 40 = 0$

$x(0) = 1$

$$J(i) = \frac{df}{dx}\bigg|_{x=x(i)} = [3x^2 + 16x + 2]_{x=x(i)}$$

In the general form:

$$x(i+1) = x(i) + \frac{1}{3x(i)^2 + 16x(i) + 2} \times (-x(i)^3 - 8x(i)^2 - 2x(i) + 40)$$

Using $x(0) = 1$, we arrive at

i	0	1	2	3	4
x	1	2.381	1.9675	1.9116	1.9106
ε	1.381	0.1737	0.0284	5.19E-4	

After 4 iterations, $\varepsilon < 0.001$.

$\therefore x = 1.9106$

The textbook uses $\frac{x(i+1)-x(i)}{x(i)} < \varepsilon$ as the stopping criterion. You can also use $x(i+1) - x(i) < \varepsilon$ or $f(x(i+1)) < \varepsilon$ as discussed in class.

6.23

$$x^0 = \begin{bmatrix} 1.0 \\ 0.5 \end{bmatrix} \qquad \text{stopping condition: } \varepsilon = 0.005$$

$$F(x) = \begin{bmatrix} e^{x_1 x_2} - 1.2 \\ \cos(x_1 + x_2) - 0.5 \end{bmatrix} \qquad J(x) = \begin{bmatrix} x_2 e^{x_1 x_2} & x_1 e^{x_1 x_2} \\ -\sin(x_1 + x_2) & -\sin(x_1 + x_2) \end{bmatrix}$$

Find the next value of X using:

- $J(X^{(K)}) \cdot (X^{(K+1)} - X^{(K)}) = -F(X^{(K)})$. Solve the system of linear equations to get $(X^{(K+1)} - X^{(K)})$. Then, using values of $X^{(K)}$, you can calculate $X^{(K+1)}$
- Alternatively, you can find the inverse of the Jacobian, and use: $X^{(K+1)} = X^{(K)} - J(X^{(K)})^{-1} F(X^{(K)})$ to calculate $X^{(K+1)}$ directly.

K	$X^{(k)}$	$F(X^{(k)})$	$J(X^{(k)})$	$X^{(k+1)}$	$\|F(X^{(k)})\|$	Another iteration?
1	$\begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.4487 \\ -0.4293 \end{bmatrix}$	$\begin{bmatrix} 0.8244 & 1.6487 \\ -0.9975 & -0.9975 \end{bmatrix}$	$\begin{bmatrix} 0.6836 \\ 0.386 \end{bmatrix}$	0.4487	ANOTHER ONE
2	$\begin{bmatrix} 0.6836 \\ 0.386 \end{bmatrix}$	$\begin{bmatrix} 0.102 \\ -0.0196 \end{bmatrix}$	$\begin{bmatrix} 0.5026 & 0.8901 \\ -0.877 & -0.877 \end{bmatrix}$	$\begin{bmatrix} 0.8956 \\ 0.1518 \end{bmatrix}$	0.102	ANOTHER ONE
3	$\begin{bmatrix} 0.8956 \\ 0.1518 \end{bmatrix}$	$\begin{bmatrix} -0.0544 \\ -0.0001 \end{bmatrix}$	$\begin{bmatrix} 0.1739 & 1.026 \\ -0.8661 & -0.8661 \end{bmatrix}$	$\begin{bmatrix} 0.8315 \\ 0.2157 \end{bmatrix}$	0.054	ANOTHER ONE
4	$\begin{bmatrix} 0.8315 \\ 0.2157 \end{bmatrix}$	$\begin{bmatrix} -0.0036 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.2580 & 0.9949 \\ -0.866 & -0.866 \end{bmatrix}$	$\begin{bmatrix} 0.8267 \\ 0.2205 \end{bmatrix}$	0.0036	ANOTHER ONE
5	$\begin{bmatrix} 0.8267 \\ 0.2205 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.2646 & 0.992 \\ -0.866 & -0.866 \end{bmatrix}$	$\begin{bmatrix} 0.8267 \\ 0.2206 \end{bmatrix}$	0.000	Nope!

b) Same problem but setting $x^0 = \begin{bmatrix} 1.0 \\ 2.0 \end{bmatrix}$

K	$X^{(k)}$	$F(X^{(k)})$	$J(X^{(k)})$	$X^{(k+1)}$	$\|F(X^{(k)})\|$	Another iteration?
1	$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$	$\begin{bmatrix} 6.1891 \\ -1.49 \end{bmatrix}$	$\begin{bmatrix} 14.7781 & 7.3891 \\ -0.1411 & -0.1411 \end{bmatrix}$	$\begin{bmatrix} 10.7207 \\ -18.2791 \end{bmatrix}$	6.189	ANOTHER ONE
2	$\begin{bmatrix} 10.7207 \\ -18.2791 \end{bmatrix}$	$\begin{bmatrix} -1.2 \\ -0.2087 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0.9566 & 0.9566 \end{bmatrix}$			

Cannot proceed anymore since J(x) is not invertible.

6.28

a) $Y[1,1] = 2 - j4 + 3 - j6 = 5 - j10 = 11.18 \angle -63.43°$
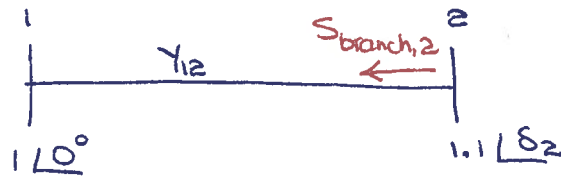
$Y[2,2] = 2 - j4 = 4.47 \angle -63.43°$

$Y[3,3] = 3 - j6 = 6.71 \angle -63.43°$

$Y[2,1] = Y[1,2] = -2 + j4 = 4.47 \angle 116.57°$

$Y[1,3] = Y[3,1] = -3 + j6 = 6.71 \angle 116.57°$

$Y[2,3] = Y[3,2] = 0$

b) at bus 2,

$$P_{gen,2} - P_{load,2} - P_{branch,2} = 0$$

$$1.5 - 0 - P_{branch,2} = 0$$

$$S_{branch,2} = \bar{V}_2 \cdot \bar{I}_{12}^*$$

$$= \bar{V}_2 \cdot \left[(\bar{V}_2 - \bar{V}_1) \cdot Y_{12} \right]^*$$

$$= \bar{V}_2 \cdot Y_{12}^* \cdot (\bar{V}_2 - \bar{V}_1)^*$$

$$= \bar{V}_2 \cdot Y_{12}^* \cdot \bar{V}_2^* - \bar{V}_2 \cdot Y_{12}^* \cdot \bar{V}_1^*$$

$$= (V_2)^2 \cdot Y_{12} \angle \theta_{12} - V_2 \angle \delta_2 \cdot Y_{12} \angle -\theta_{12} \cdot V_1 \angle -\delta_1$$

$$= (V_2)^2 \cdot Y_{12} \angle -\theta_{12} - V_1 \cdot V_2 \cdot Y_{12} \angle \delta_2 - \delta_1 - \theta_{12}$$

$$\therefore P_{branch,2} = (V_2)^2 (Y_{12}) \cdot \cos(-\theta_{12}) - V_1 \cdot V_2 \cdot Y_{12} \cos(\delta_2 - \delta_1 - \theta_{12})$$

This is the Pbranch equation with Y expressed in polar coordinates.
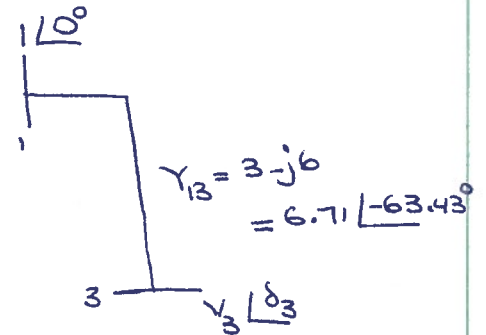
Please note the difference between Y_{12} and $Y[1,2]$

reminder $Y_{12} = 2 - j4 = 4.47 \angle -63.43°$

$\therefore \ 0 = 1.5 - 0 - (1.1)^2 (4.47) \cos(+ 63.43°) + (1)(1.1)(4.47) \cos(\delta_2 + 63.43°)$

$\text{solve for } \delta_2 \quad : \quad \delta_2 = 15.795°$

c) Similar to part b),

$P_{branch, 3} = (V_3)^2 (Y_{13}) \cdot \cos(-\theta_{13})$

$\qquad - V_1 \cdot V_3 \cdot Y_{13} \cdot \cos(\delta_3 - \delta_1 - \theta_{13})$

$Y_{13} = 3 - j6$
$\qquad = 6.71 \ \underline{/-63.43°}$

$\therefore \quad -1.5 - 0 - (V_3)^2 (6.71) \cos(63.43°) + (1) V_3 (6.71) \cos(\delta_3 + 63.43°) = 0$

(i)

$Q_{branch, 3} = (V_3)^2 (Y_{13}) \sin(-\theta_{13}) - V_1 V_3 Y_{13} \sin(\delta_3 - \delta_1 - \theta_{13})$

$\therefore \quad 0.8 - 0 - (V_3)^2 (6.71) \sin(63.43°) + (1) V_3 (6.71) \sin(\delta_3 + 63.43°) = 0$

(ii)

Solve (i) & (ii) to get V_3 & δ_3, painfully!

$V_3 = 0.9723 \ \text{pu}$

$\delta_3 = -15.1°$

d) $P_{branch, 1} = (V_1)^2 (Y_{12}) \cos(-\theta_{12}) - V_1 \cdot V_2 \cdot Y_{12} \cos(\delta_1 - \delta_2 - \theta_{12})$

$\qquad + (V_1)^2 (Y_{13}) \cos(-\theta_{13}) - V_1 \cdot V_3 \cdot Y_{13} \cos(\delta_1 - \delta_3 - \theta_{13})$

plug in all values,

$P_{branch, 1} = 0.39 \ \text{pu}$

$P_{gen, 1} - P_{load, 1} - P_{branch, 1} = 0$

$\therefore \ P_{gen, 1} = 0.39 \ \text{pu}$

e) $P_{loss} = P_{gen} - P_{load}$

$\qquad = P_{gen,1} + P_{gen,2} - P_{load,3}$

$\qquad = 0.39 + 1.5 - 1.5$

$\qquad = 0.39 \ pu$

6.30

$6.25 - j18.695$	$-5.00 + j15.00$	$-1.25 + j3.75$	0	0
$-5.00 + j15.00$	$12.9167 - j38.665$	$-1.6667 + j5.00$	$-1.25 + j3.75$	$-5.00 + j15.00$
$-1.25 + j3.75$	$-1.6667 + j5.00$	$8.7990 - j32.2294$	$-5.8824 + j23.5294$	0
0	$-1.25 + j3.75$	$-5.8824 + j23.5294$	$9.8846 - j36.4037$	$-2.7523 + j9.1743$
0	$-5.00 + j15.00$	0	$-2.7523 + j9.1743$	$7.7523 - j24.1443$

6.31 First, we need to find the per-unit shunt admittance of the added capacitor.

75 Mvar $= 0.75$ p.u.

To find Y, we use the relation

$S = V^2 Y$

$Y = \dfrac{S}{V^2} = \dfrac{0.75}{1^2} = 0.75$ p.u.

Now, we can find Y_{44}

$$Y_{44} = \frac{1}{0.08 + j0.24} + \frac{1}{0.01 + j0.04} + \frac{1}{0.03 + j0.10}$$

$$+ j\frac{0.05 + 0.01 + 0.04}{2} + j0.75$$

$$= 9.8846 - j35.6537$$