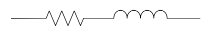
Transmission line voltage and current equations

Transmission line models used in previous topics looked like: 

Z = R+jX = R+jωL

We ignored the line capacitance in these models.

In part 1 of topic 5, we came up with equations to calculate the <u>distributed</u> inductance (H/m) and <u>distributed</u> capacitance (F/m)

Objective of part 2: Determine how to model (lump) this all together.

Consider a small portion of the line with length Δx:

z = R + j ωL where:
R is in Ω/m &
L is in H/m

y = j ωC where
C is in F/m

From KVL: $\mathbf{V}(\mathrm{x}+\Delta\mathrm{x}) = \mathbf{V}(\mathrm{x}) + \mathrm{z}.\Delta\mathrm{x}.\mathbf{I}(\mathrm{x})$ Therefore: $\frac{\mathbf{V}(\mathrm{x}+\Delta\mathrm{x})-\mathbf{V}(\mathrm{x})}{\Delta\mathrm{x}} = \mathrm{z}.\mathbf{I}(\mathrm{x})$

From KVL: $\mathbf{I}(\mathrm{x}+\Delta\mathrm{x}) = \mathbf{I}(\mathrm{x}) + \mathbf{V}(\mathrm{x}+\Delta\mathrm{x})\,\mathrm{y}.\Delta\mathrm{x}$ Therefore: $\frac{\mathbf{I}(\mathrm{x}+\Delta\mathrm{x})-\mathbf{I}(\mathrm{x})}{\Delta\mathrm{x}} = \mathrm{y}.\mathbf{V}(\mathrm{x}+\Delta\mathrm{x})$

Combine (i) and (ii) to get: $\boldsymbol{V}(x) = K_1.\cosh(\gamma x) + K_2.\sinh(\gamma x)$ where $\gamma=\sqrt{\mathrm{y.z}}$ propagation constant (1/m)

If we know V and I at one end, e.g. receiving end (x=0): $\mathbf{V}_R = \mathbf{V}(0)$ and $\mathbf{I}_R = \mathbf{I}(0)$, we can find K_1 and K_2 :

$\boldsymbol{V}(x) = \boldsymbol{V}_R.\cosh(\gamma x) + \boldsymbol{I}_R Z_c.\sinh(\gamma x)$ where $\mathrm{Z_c}=\sqrt{\mathrm{z/y}}$ characteristic impedance (Ω)

Similar derivation for **I**(x) gives: $\boldsymbol{I}(x) = \boldsymbol{I}_R.\cosh(\gamma x) + \frac{\boldsymbol{V}_R}{Z_c}.\sinh(\gamma x)$

Eq (1) and (2) give us voltage and current at any point x along the line (x is the distance from the receiving end)

Note: voltage and current in above expressions are phasors.