ENEL 471 – Winter 2020

Assignment 2 - Solutions

Problem 3.4

Consider the square-law characteristic:

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t) \qquad (1)$$

where a_1 and a_2 are constants. Let

$$v_1(t) = A_c \cos(2\pi f_c t) + m(t) \qquad (2)$$

Therefore substituting Eq. (2) into (1), and expanding terms:

$$v_2(t) = a_1 A_c \left[1 + \frac{2a_2}{A_1} m(t)\right] \cos(2\pi f_c t) \qquad (3)$$

$$+ a_1 m(t) + a_2 m^2(t) + a_2 A_c^2 \cos^2(2\pi f_c t)$$

The first term in Eq. (3) is the desired AM signal with $k_a = 2a_2/a_1$. The remaining three terms are unwanted terms that are removed by filtering.

Let the modulating wave $m(t)$ be limited to the band $-W \le f \le W$, as in Fig. 1(a). Then, from Eq. (3) we find that the amplitude spectrum $|V_2(f)|$ is as shown in Fig. 1(b). It follows therefore that the unwanted terms may be removed from $v_2(t)$ by designing the tuned filter at the modulator output of Fig. P2.2 to have a mid-band frequency fc and bandwidth $2W$, which satisfy the requirement that $f_c > 3W$.

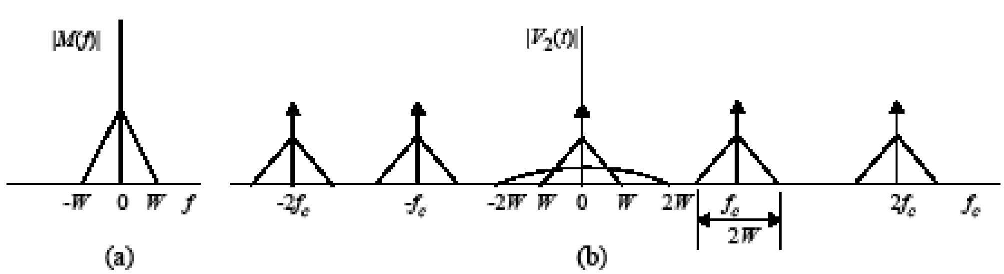


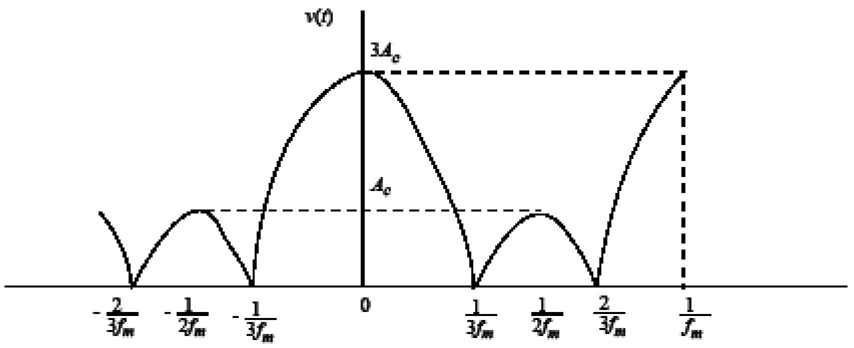
Figure 1

Problem 3.5

(a) The envelope detector output is

$$v(t) = A_c\left|1 + \mu\cos(2\pi f_m t)\right|$$

which is illustrated below for the case when $\mu = 2$.



We see that $v(t)$ is periodic with a period equal to f_m, and an even function of t, and so we may express $v(t)$ in the form:

$$v(t) = a_0 + 2\sum_{n=1}^{\infty} a_n \cos(2n\pi f_m t)$$

where

$$a_0 = 2f_m\int_0^{1/2f_m} v(t)dt$$

$$= 2A_c f_m\int_0^{1/3f_m}[1 + 2\cos(2n\pi f_m t)]dt + 2A_c f_m\int_{1/3f_m}^{1/2f_m}[-1-2\cos(2n\pi f_m t)]dt$$

$$= \frac{A_c}{3} + \frac{4A_c}{\pi}\sin\left(\frac{2\pi}{3}\right) \qquad (1)$$

$$a_n = 2f_m\int_0^{1/2f_m} v(t)\cos(2n\pi f_m t)dt$$

$$= 2A_c f_m \int_0^{1/3f_m} [1 + 2\cos(2\pi f_m t)] \cos(2n\pi f_m t) dt$$

$$+ 2A_c f_m \int_{1/3f_m}^{1/2f_m} [-1-2\cos(2\pi f_m t)] \cos(2n\pi f_m t) dt$$

$$= \frac{A_c}{n\pi}\left[2\sin\left(\frac{2n\pi}{3}\right) - \sin(n\pi)\right] + \frac{A_c}{(n+1)\pi}\left\{2\sin\left[\frac{2\pi}{3}(n+1)\right] - \sin[\pi(n+1)]\right\}$$

$$+ \frac{A_c}{(n-1)\pi}\left\{2\sin\left[\frac{2\pi}{3}(n-1)\right] - \sin[\pi(n-1)]\right\} \qquad (2)$$

For $n = 0$, Eq. (2) reduces to that shown in Eq. (1).

(b) For $n = 1$, Eq. (2) yields

$$a_1 = A_c\left(\frac{\sqrt{3}}{2\pi} + \frac{1}{3}\right)$$

For $n = 2$, it yields

$$a_2 = \frac{A_c\sqrt{3}}{2\pi}$$

Therefore, the ratio of second-harmonic amplitude to fundamental amplitude in $v(t)$ is

$$\frac{a_2}{a_1} = \frac{3\sqrt{3}}{2\pi + 3\sqrt{3}} = 0.452$$

Problem 3.6

Let

$$v_1(t) = A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t)$$

a- The output of the square-law device is given by:

$$
\begin{aligned}
v_2(t) &= a_1 v_1(t) + a_2 v_1^{\,2}(t) \\
&= a_1\left(A_c\left[1+k_a m(t)\right]\cos\left(2\pi f_c t\right)\right) + a_2\left(A_c\left[1+k_a m(t)\right]\cos\left(2\pi f_c t\right)\right)^2 \\
&= a_1 A_c\left[1+k_a m(t)\right]\cos\left(2\pi f_c t\right) + \frac{a_2 A_c^{\,2}}{2}\left[1+2k_a m(t)+k_a^{\,2} m^2(t)\right]\left(1+\cos\left(4\pi f_c t\right)\right) \\
&= \underbrace{\frac{a_2 A_c^{\,2}}{2}\left[1+2k_a m(t)+k_a^{\,2} m^2(t)\right]}_{\text{at 0 Hz}} + \underbrace{a_1 A_c\left[1+k_a m(t)\right]\cos\left(2\pi f_c t\right)}_{\text{at } f_c} \\
&\qquad + \underbrace{\frac{a_2 A_c^{\,2}}{2}\left[1+k_a m(t)\right]^2\cos\left(4\pi f_c t\right)}_{\text{at } 2\,f_c}
\end{aligned}
$$

b- After low-pass filtering the component that will be maintained in the received signal is:

$$
v_o(t) = \frac{a_2 A_c^{\,2}}{2}\left[1+2k_a m(t)+k_a^{\,2} m^2(t)\right] = \frac{a_2 A_c^{\,2}}{2}\left[1+k_a m(t)\right]^2
$$

If the modulation sensitivity is set in order to avoid over-modulation ($\left[1+k_a m(t)\right]\geq 0$ for all t), then the message can be recuperated using a square rooting system followed by an amplitude shifting system.

If the modulation sensitivity is chosen to be low ($\left|k_a m(t)\right| << 1$), then the square term $k_a^{\,2} m^2(t)$ is very low ($k_a^{\,2} m^2(t) << 2k_a m(t)$) and can be neglected. The output is then given by:

$$
v_o(t) \approx \frac{a_2 A_c^{\,2}}{2}\left[1+2k_a m(t)\right]
$$

And the message can be recovered by just an amplitude shifting system.