

①

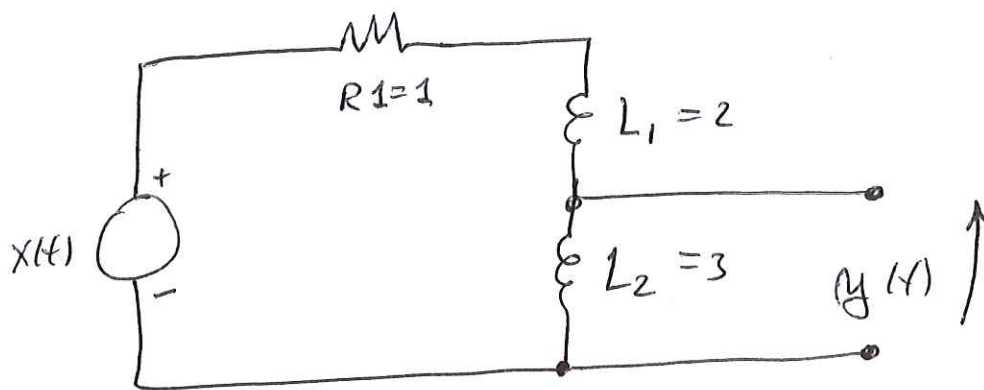
Unit 2 problemsQuiz 2&3 prep

- Transfer functions of
 - electrical
 - translational
 - rotational
 - motors
 - gears
 - mixed systems

Quiz 2

Quiz 3

P1 Find transfer function of electrical network

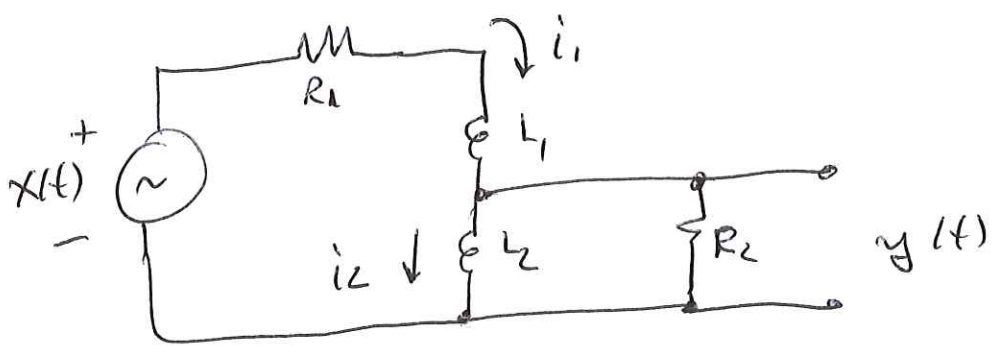


$$y(0^-) = 0$$

$$\dot{y}(0^-) = 0$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{sL_2}{sL_2 + sL_1 + R_1} = \frac{3s}{5s + 1}$$

P2 Find state space system matrix



define two currents i_1 and i_2 as state variables.

$$L_1 \frac{di_1}{dt} + L_2 \frac{di_2}{dt} = x_1 - R_1 i_1$$

$$L_2 \frac{di_2}{dt} = (i_1 - i_2) R_2$$

$$\begin{bmatrix} L_1 & L_2 \\ 0 & L_2 \end{bmatrix} \begin{bmatrix} di_1/dt \\ di_2/dt \end{bmatrix} = \begin{bmatrix} -R_1 & 0 \\ R_2 & -R_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} x$$

take inverse $\Rightarrow \frac{1}{L_1 L_2} \begin{bmatrix} L_2 & -L_2 \\ 0 & L_1 \end{bmatrix}$

(3)

State space transfer

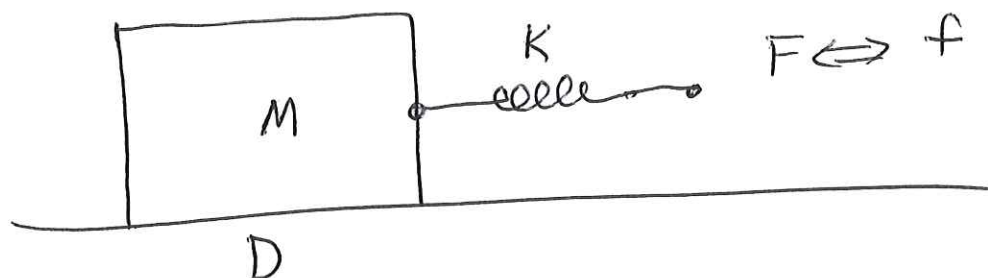
$$\begin{bmatrix} di_1/dt \\ di_2/dt \end{bmatrix} = A \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + Bx$$

$$A = \frac{1}{L_1 L_2} \begin{bmatrix} L_2 & -L_2 \\ 0 & L_1 \end{bmatrix} \begin{bmatrix} -R_1 & 0 \\ R_2 & -R_2 \end{bmatrix}$$

$$B = \frac{1}{L_1 L_2} \begin{bmatrix} L_2 & -L_2 \\ 0 & L_1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

P3 transfer

Solve for the transfer $x(t)$. Assume force of $f(t) = v(t)$.

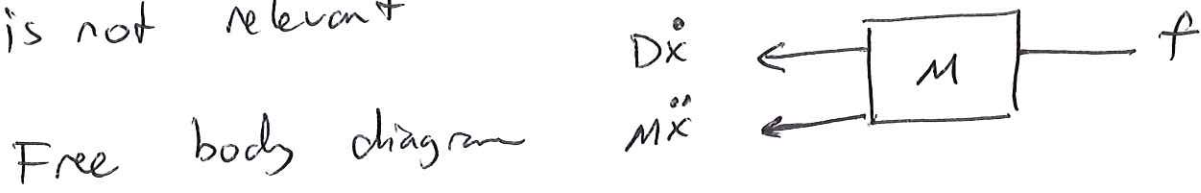


\curvearrowright D is the coefficient of friction such that the resistance force is $f_{\text{friction}} = D \dot{x}$

Note that the spring does not contribute to the problem as the stretch of the spring is:

$$\Delta x = \frac{f}{K}$$

is not relevant



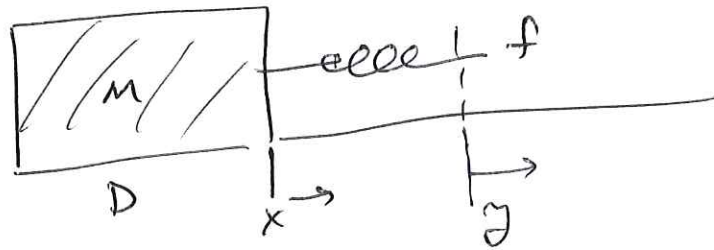
Therefore we have

$$(sD + s^2M) X(s) = F(s)$$

Transfer function $\frac{X(s)}{F(s)} = \frac{1}{s(D + sM)} = \frac{1/M}{s(s + D/M)}$

Pole at $s = -D/M$

Now compute the transfer function between $f(t)$ and the displacement of the tip of the spring (where f is applied)



$$y = x + \frac{f}{K}$$

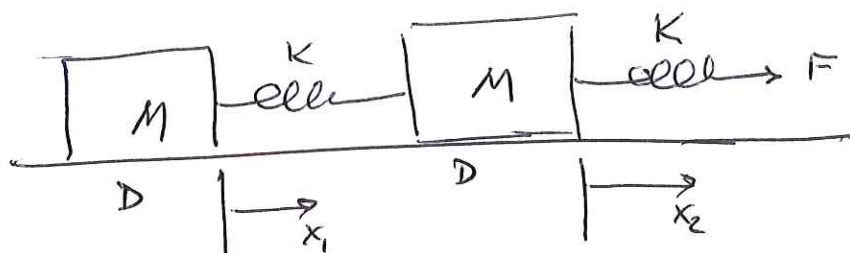
Therefore

$$Y(s) = X(s) + \frac{F(s)}{K}$$

$$= \left(\frac{X(s)}{F(s)} + \frac{1}{K} \right) F(s)$$

$$\frac{Y(s)}{F(s)} = \frac{1/M}{s(s + D/M)} + \frac{1}{K}$$

P4



- ① Find the transfer function of F to x_1
- ② State space formulae.

- Start with body diagrams

- note that the right spring is not necessary to include into calculation.

We have state variables

$$\begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}$$

↑ state variable vector.

From first body diagram

$$D\dot{x}_1 + M\ddot{x}_1 = K(x_2 - x_1)$$

$$Dz_2 + M\dot{z}_2 = Kz_3 - Kz_1$$

$$\textcircled{1} \quad \dot{z}_2 = -\frac{K}{M} z_1 - \frac{D}{M} z_2 - \frac{K}{M} z_3$$

From second body diagram

$$D\dot{x}_2 + M\ddot{x}_2 + K(x_2 - x_1) = F$$

$$Dz_4 + M\dot{z}_4 + Kz_3 - Kz_1 = F$$

$$\textcircled{2} \quad \dot{z}_4 = -\frac{K}{M} z_1 - \frac{K}{M} z_3 - \frac{D}{M} z_4 + \frac{F}{M}$$

we also have

$$\dot{z}_1 = z_2 \quad \textcircled{3}$$

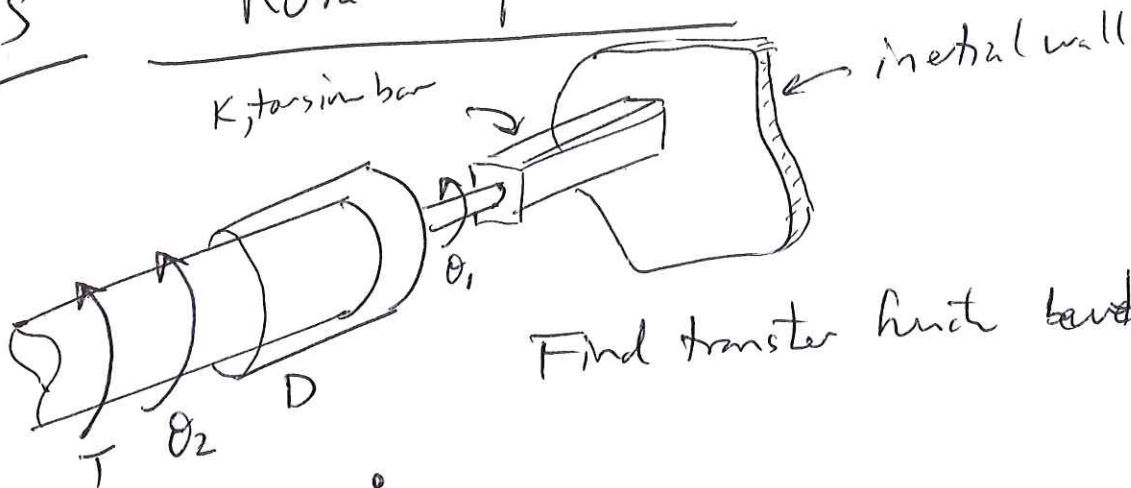
$$\dot{z}_3 = z_4 \quad \textcircled{4}$$

Combining these 4 equations

(8)

$$\frac{d}{dt} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1/m & -D/m & -1/m & 0 \\ 0 & 0 & 0 & 1 \\ -1/m & 0 & -1/m & -D/m \end{bmatrix}}_A \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_B F$$

PS Rotation problem



Find transfer function between T and w_2

$$w_2 = \dot{\theta}_2$$

$$T = (\dot{\theta}_2 - \dot{\theta}_1) D \rightarrow \text{through coupling}$$

$$T = K \theta_1 \rightarrow \text{torque at torsion bar (same as there is no inertia)}$$

$$\dot{T} = K \dot{\theta}_1$$

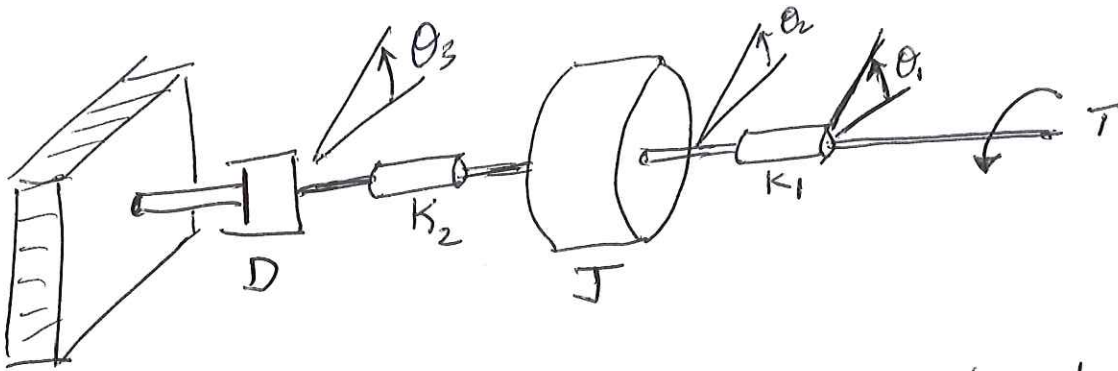
$$T = \dot{\theta}_2 D - \frac{\dot{T}}{K} D$$

$$w_2 = \frac{T}{D} + \frac{\dot{T}}{K}$$

(9)

P6

Determine the system of coupled DEQ's for the following rotational system

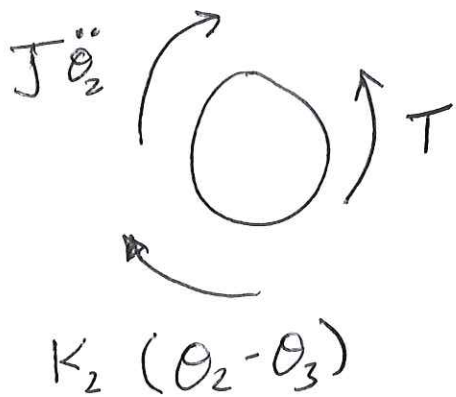


First note that K_1 is not relevant except

that $T = K_1 (\theta_1 - \theta_2)$

K_1 does not change the torque applied to the Flywheel of inertia J .

Diagram for flywheel



$$T = J\ddot{\theta}_2 + K_2(\theta_2 - \theta_3)$$

$$D\dot{\theta}_3 = K_2(\theta_2 - \theta_3)$$

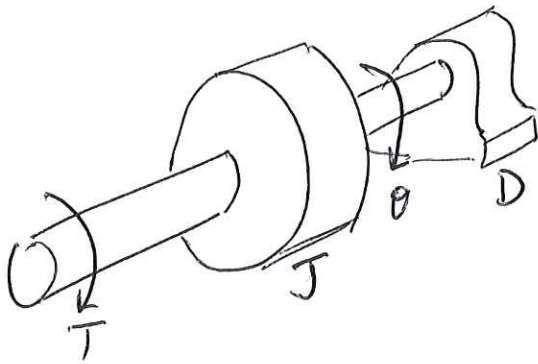
let state variables be $x = \begin{bmatrix} \dot{\theta}_2 \\ \theta_2 \\ \theta_3 \end{bmatrix}$

Then

$$\frac{d}{dt} \begin{bmatrix} \dot{\theta}_2 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{k_2}{J} & \frac{k_2}{J} \\ 1 & 0 & 0 \\ 0 & \frac{k_2}{D} & -\frac{k_2}{D} \end{bmatrix} \begin{bmatrix} \dot{\theta}_2 \\ \theta_2 \\ \theta_3 \end{bmatrix} + \begin{bmatrix} T/J \\ 0 \\ 0 \end{bmatrix}$$

What is the system of equations written in State space format.

P7 Find the response of the system for T to θ



$$T = J\ddot{\theta} + D\dot{\theta}$$

$$\frac{\Theta(s)}{T_L(s)} = \frac{1}{Js^2 + Ds}$$



note that
 $T(s) \leftrightarrow T_L(s)$

(11)

Repeat P7 but with output of w instead of θ

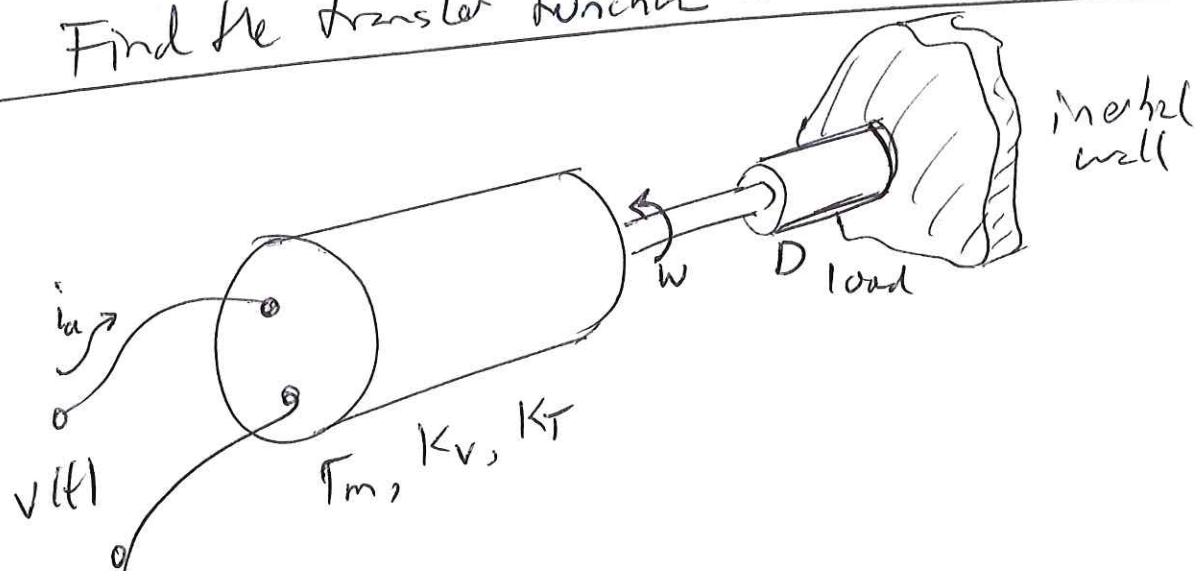
$$w = \dot{\theta} \quad \therefore T = J\dot{w} + Dw$$

$$\frac{N(s)}{T(s)} = \frac{1}{Js + D}$$

Note that $N(s) = s\Theta(s)$

$$\therefore \frac{N(s)}{T(s)} = s \frac{\Theta(s)}{T(s)} = \frac{s}{Js^2 + Ds} = \frac{1}{Js + D}$$

P8) Find the transfer function from $v(t)$ to $w(t)$



i_a - armature current

$$i_a = \frac{v - K_v w}{r_m}$$

$$T = i_a K_T$$

$$T = Dw$$

$$i_a = -\frac{K_v}{r_m} w + \frac{1}{r_m} v$$

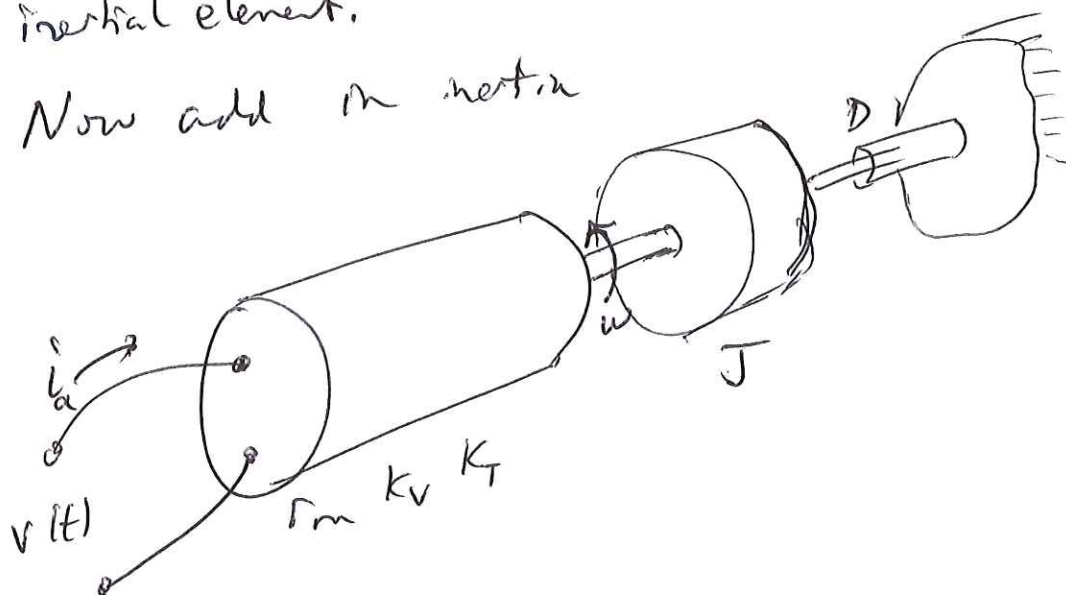
$$T = -\frac{K_v K_T}{r_m} w + \frac{K_T}{r_m} v$$

$$\left(D + \frac{K_v K_T}{r_m}\right) w = \frac{K_T}{r_m} v$$

$$W = \frac{(K_T D / r_m)}{D + \frac{K_V K_T}{r_m}} V$$

Note this result indicates that w is proportional to v .
There is no DEQ involved as there is no energy storage or inertial element.

Now add inertia



$$K_T \left(\frac{V - K_V W}{r_m} \right) = D \dot{w} + \underbrace{J \ddot{w}}_{\text{new}}$$

Transfer function replace $D \rightarrow D + Js$

$$W(s) = \frac{K_T D / r_m}{Js + D + \frac{K_V K_T}{r_m}} V(s)$$

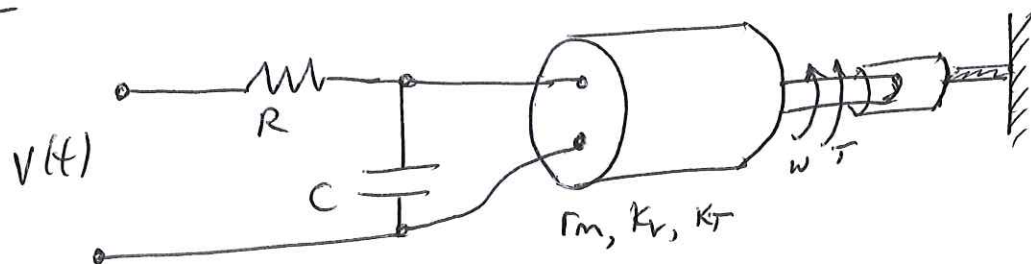
Now we have a low pass response with a pole

$$\text{at } s = -\frac{1}{J} \left(D + \frac{K_V K_T}{r_m} \right)$$

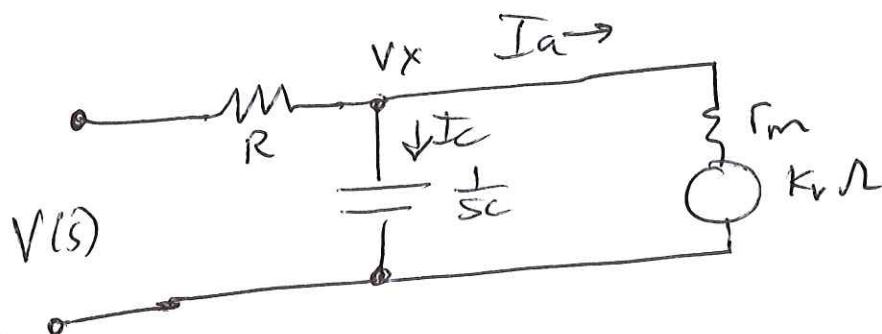
Pg

Find transfer function between $v(t)$ and $w(t)$

(13)



Re draw the diagram and start with a new variable V_x, I_c



$$V_x = V - R(I_a + I_c)$$

$$V_x = V - R I_a - R V_x sC \quad (1)$$

$$V_x = I_a r_m + K_v \Omega \quad (2)$$

$$(2) = (1) \Rightarrow \text{eliminates } V_x$$

$$I_a r_m + K_v \Omega = \frac{V - R I_a}{1 + R s C}$$

$$I_a \left(r_m + \frac{R}{1 + R s C} \right) = -K_v \Omega + \frac{V}{1 + R s C}$$

Torque $T = K_T I_a = D \Omega$

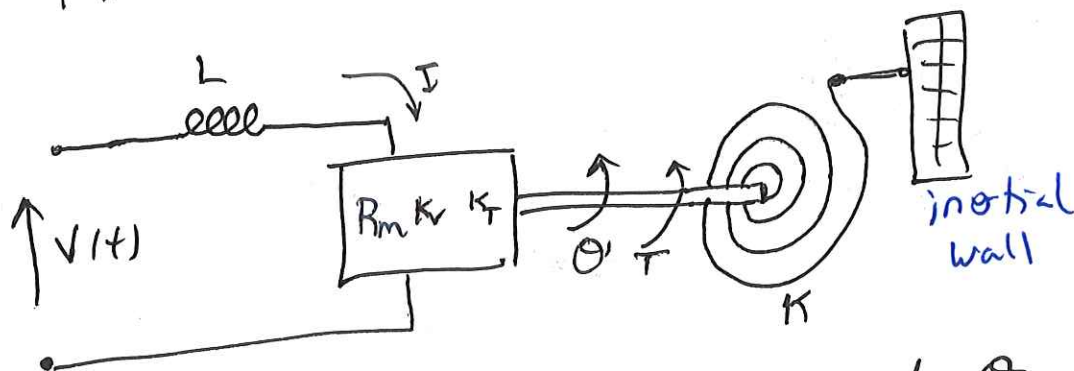
$$\frac{D\Omega}{K_T} \left(\Gamma_m + \frac{R}{1+RCS} \right) = \pm K_V \Omega + \frac{V}{1+RCS}$$

$$\Omega \left(\frac{\Gamma_m D}{K_T} + \frac{D}{K_T} \frac{R}{1+RCS} + K_V \right) = \frac{V}{1+RCS}$$

Which gives the net rate that can be further simplified.

P 10) Find transfer function between $V(t)$ and $\Theta(t)$

(15)



Find the relation between V and Θ

$$V = sLI + R_m I + K_v s \Theta$$

$$T = K_T I = K \Theta \Rightarrow I = \frac{K \Theta}{K_T}$$

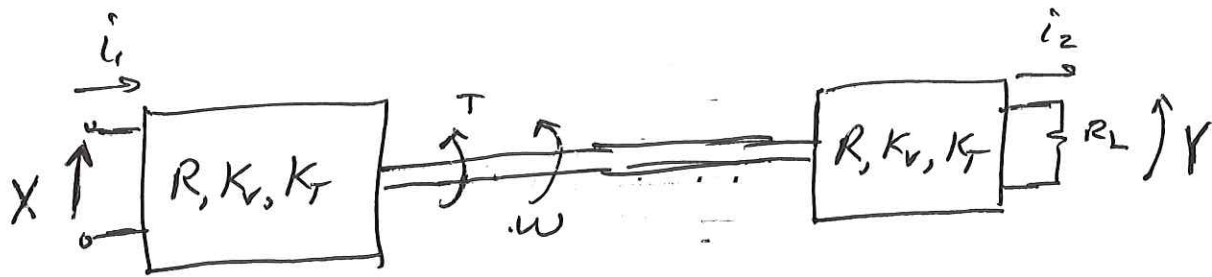
$$V = (sL + R_m) \frac{K \Theta}{K_T} + K_v s \Theta$$

$$V = \left(s \left(\frac{L K}{K_T} + K_v \right) + \frac{R_m K}{K_T} \right) \Theta$$

$$\frac{\Theta(s)}{V(s)} = \frac{1}{\left(\frac{L K}{K_T} + K_v \right) \left(s + \frac{R_m K}{K_T \left(\frac{L K}{K_T} + K_v \right)} \right)}$$

P11)

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Voltage $x(t) \Leftrightarrow X(s)$ applied to motor

find output voltage $y(t) \Leftrightarrow Y(s)$

$i_1 = i_2$ as motor torque and generator torque the same.

$$I_1 K_t = T$$

$$I_1 R + K_v \Omega = X$$

$$I_2 (R_L + R) = \Omega K_v \Rightarrow \Omega = \frac{I_2 (R_L + R)}{K_v}$$

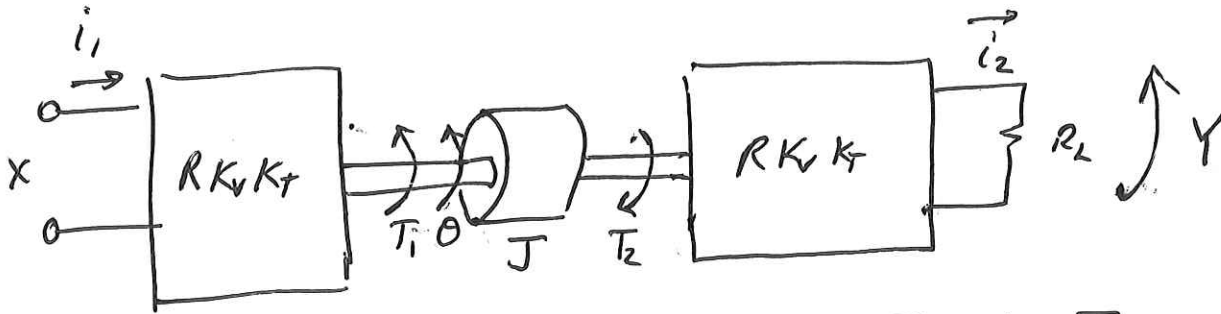
$$I_1 R + \frac{K_v I_1 (R_L + R)}{K_v} = X \quad (\text{use } I_1 = I_2)$$

$$I_1 (2R + R_L) = X$$

$$Y = I_2 R_L = I_1 R_L = \frac{R_L}{2R + R_L} X$$

P12) Now Add a rotational inertial unit.

(17)



$$i_1 \neq i_2 \quad \text{since} \quad T_1 \neq T_2$$

$$I_1 K_T = T_1$$

$$I_1 R + K_v \Omega = X \quad (1)$$

$$I_2 (R_L + R) = \Omega K_v \quad \therefore \Omega = \frac{I_2 (R_L + R)}{K_v} \quad (2)$$

$$T_2 \left(\ddot{\theta} \right) \left(\text{rotational inertia} \right) T_1$$

$$T_1 = T_2 + J \ddot{\theta}$$

$$I_1 K_T = I_2 K_T + J s \Omega$$

$$\therefore I_1 = I_2 + \frac{J}{K_T} s \Omega \quad (3)$$

Put (2) and (3) into (1)

$$\therefore \left(I_2 + \frac{J}{K_T} s \Omega \right) R + K_v \left(\frac{I_2 (R_L + R)}{K_v} \right) = X$$

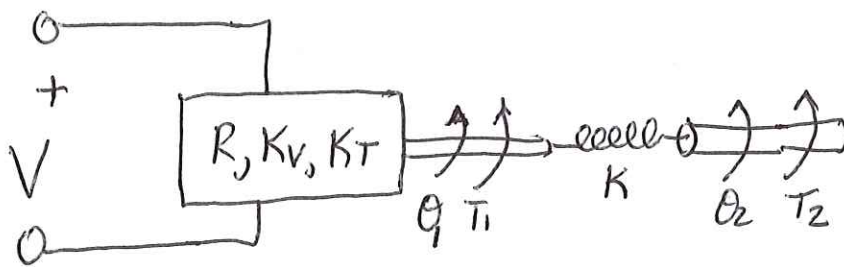
Use (2)

$$I_2 \left(R + \frac{J}{K_T} s \frac{(R_L + R)}{K_v} R \right) + I_2 (R_L + R) = X$$

$$I_2 \left[(2R + R_L) + s \left(\frac{J}{K_T K_V} R (R_L + R) \right) \right] = X$$

$$Y(s) = \frac{R_L}{2R + R_L + s \left(\frac{J}{K_T K_V} R (R_L + R) \right)} X(s)$$

P13)

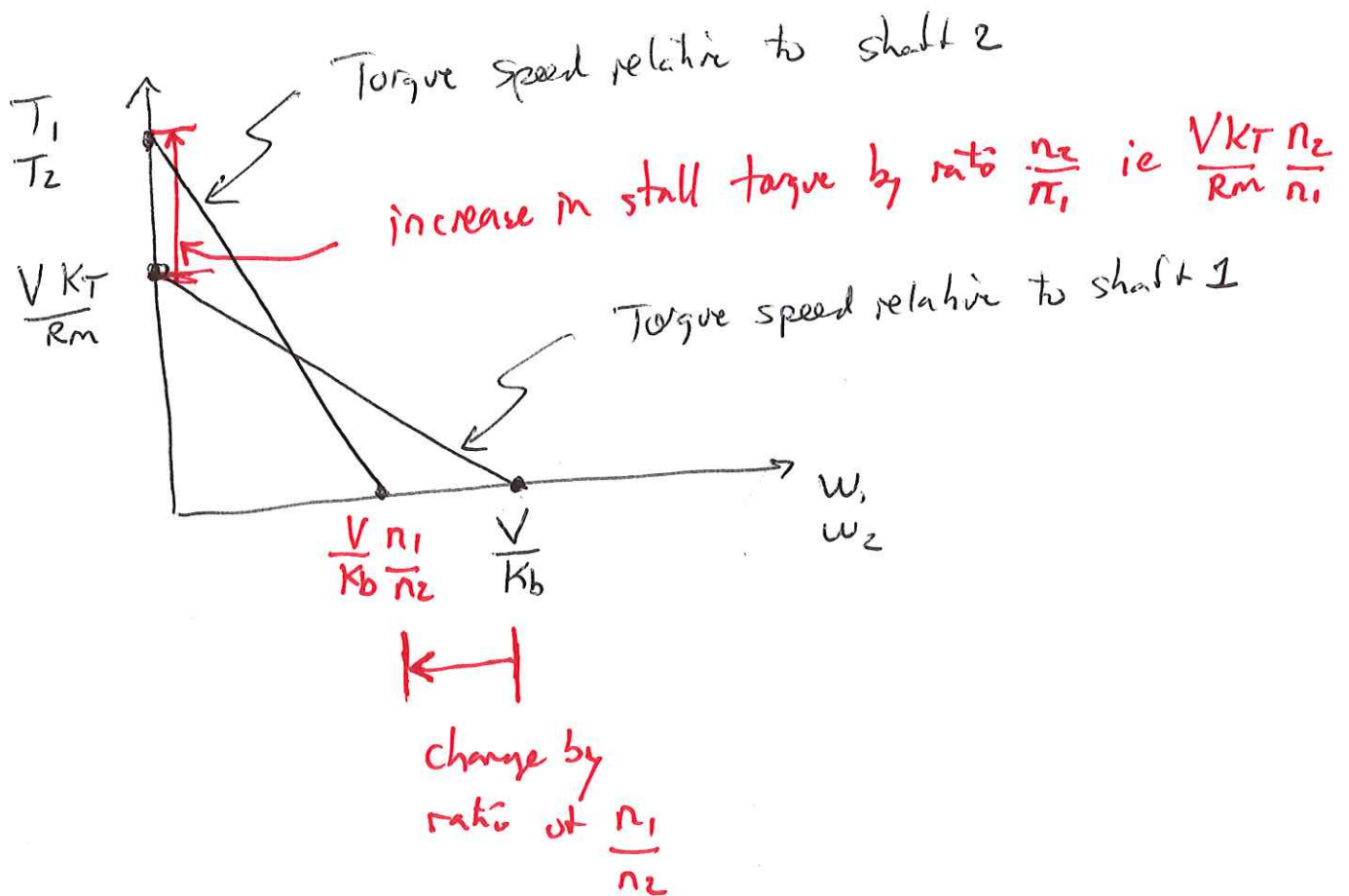
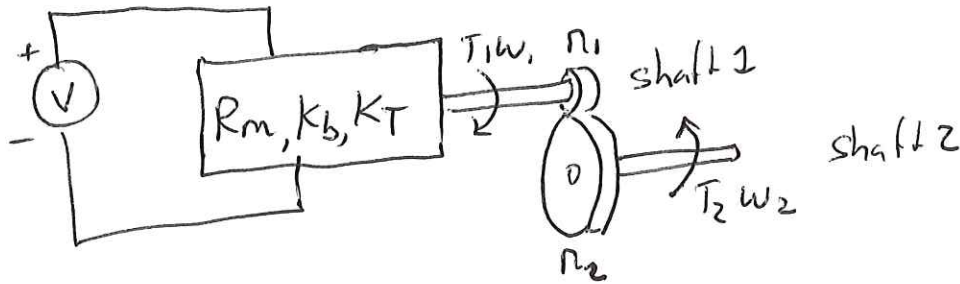


Find transfer from \$V(s)\$ to \$\theta_2(s)\$

There is no load on second shaft so \$T_1 = T_2 = 0\$

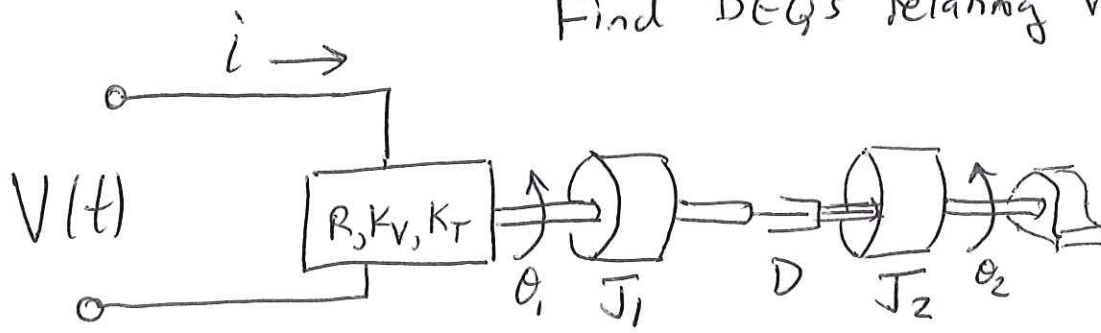
$$\therefore \ddot{\theta}_2 = \frac{V}{K_V}, \quad \theta_1 = \theta_2$$

P14) Show the effect of a gear ratio of $\frac{n_1}{n_2}$ on the motor torque speed curve



(P15) Mixed Problem

Find DEQ's relating $V(t)$, $\dot{\theta}_1$ and $\dot{\theta}_2$



Shaft 1

$$D(\dot{\theta}_1 - \dot{\theta}_2) \uparrow J_1 \ddot{\theta}_1 \uparrow \text{Torque}$$

$$T = J_1 \ddot{\theta}_1 + D(\dot{\theta}_1 - \dot{\theta}_2) \quad (1)$$

Shaft 2

$$J_2 \ddot{\theta}_2 \uparrow D(\dot{\theta}_1 - \dot{\theta}_2)$$

$$\ddot{\theta}_2 = \frac{D}{J_2} \dot{\theta}_1 - \frac{D}{J_2} \dot{\theta}_2 \quad (2)$$

① Get rid of T

$$\dot{i} = \frac{V - K_V \dot{\theta}_1}{R} = \frac{T}{K_T}$$

$$\frac{V K_T}{R} - \frac{K_V K_T}{R} \dot{\theta}_1 = J_1 \ddot{\theta}_1 + D \dot{\theta}_1 - D \dot{\theta}_2$$

(21)

① becomes

$$\ddot{\theta}_1 = \frac{-D}{J_1} \dot{\theta}_1 + \frac{D}{J_1} \dot{\theta}_2 + \frac{k_T}{R} V - \frac{k_V k_T}{R} \dot{\theta}_1$$

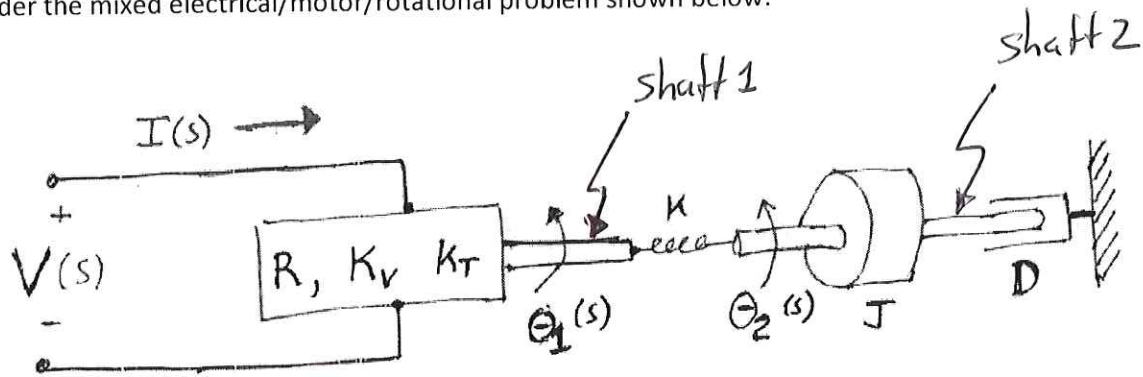
① and ② relate $\dot{\theta}_1$ and $\dot{\theta}_2$ to V .

2015 Quiz 4

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ENEL441 QUIZ 4 Name _____ UCID _____

Consider the mixed electrical/motor/rotational problem shown below:



$\theta_1(t) \Leftrightarrow \Theta_1(s)$ rotation angle of shaft 1

$\theta_2(t) \Leftrightarrow \Theta_2(s)$ rotation angle of shaft 2

1.(5) Draw the two rotational body diagrams for shaft 1 and shaft 2 in terms of the variables and parameters shown in the figure above

$$V = IR + K_v \omega_1$$

$$T = I K_T$$

$$T = \frac{V - K_v \omega_1}{R}$$

Free Body Diagrams for shaft 1 and shaft 2

$$K(\theta_1 - \theta_2) \quad T \quad J\ddot{\theta}_2 + D\dot{\theta}_2$$

2. (5) Based on the two rotational body diagrams derived in part 1 state the two equations relating $\Theta_1(s)$ and $\Theta_2(s)$ to $V(s)$ and the parameters K , J and D

$$K(\theta_1 - \theta_2) = \frac{V - K_v \dot{\theta}_1}{R}$$

$$K(\theta_1 - \theta_2) = J \ddot{\theta}_2 + D \dot{\theta}_2$$

Two Equations

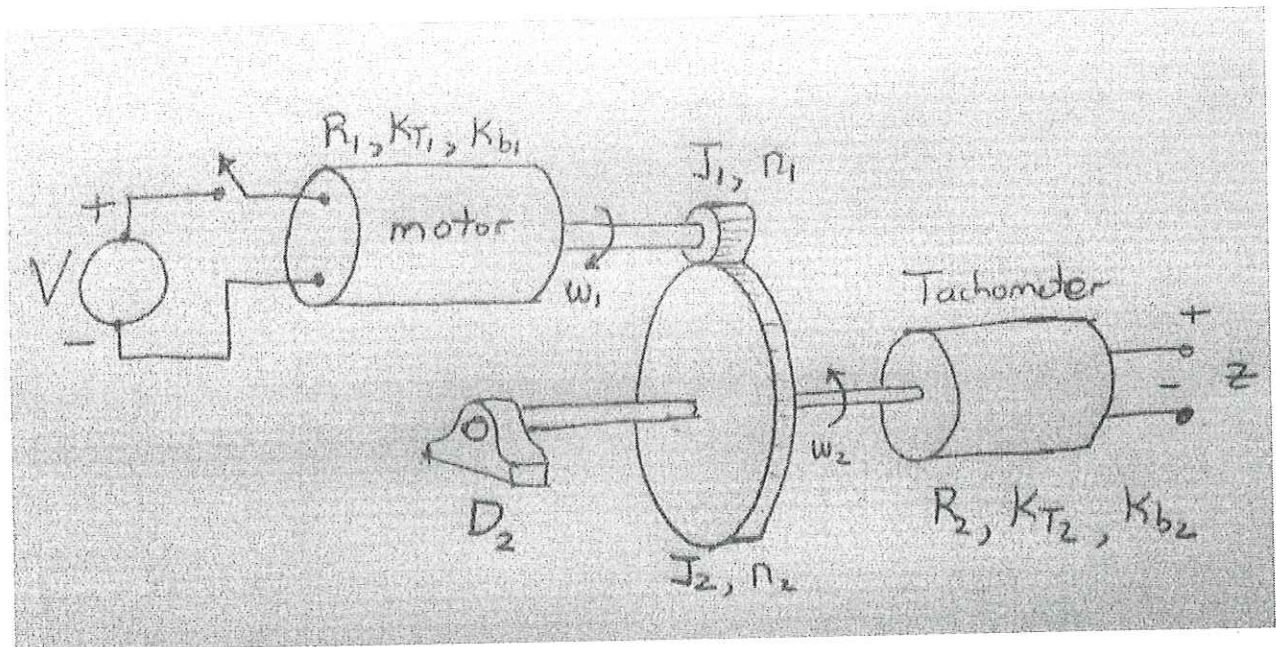
Solution

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35 minutes

Consider the model of an electric motor driving a set of gears and a tachometer. A tachometer is just an electric motor run as a generator with no electrical load as shown in the figure below.



Assume that the voltage V is applied to the motor as represented as

$$v(t) = V_o u(t)$$

The motor is represented with a linear model with coefficients of $\{R_1, K_{T1}, K_{b1}\}$ which drives a primary gear of n_1 teeth and a moment of inertia of J_1 . This small gear drives a bigger gear with n_2 teeth and a moment of inertia of J_2 . The secondary shaft of the big gear is supported by a bearing with frictional loss of D_2 and then drives the tachometer which is assumed to have no friction and no inertia. The output z is the open circuit voltage of the tachometer.

Q1 (15) Determine $z(t)$ as a function of the parameters given.

Q2 (5) Suppose that a load resistor was applied to the tachometer output terminals. Describe in a sentence or two discuss what would change in the output response of $z(t)$ and why.

Solution

Q1 (15) Start with the tachometer which is an open circuit so no current flows and hence it does not consume any torque. Hence we have $z(t) = \omega_2 K_{b2}$. Write this in terms of the primary shaft as

$$z(t) = \frac{\omega_1 K_{b2} n_1}{n_2}$$

Now the inertia of the primary shaft is $J_1 + J_2 \left(\frac{n_1}{n_2} \right)^2$ and the loss is $D_2 \left(\frac{n_1}{n_2} \right)^2$. Since the tach does not consume torque we have

$$\left(J_1 + J_2 \left(\frac{n_1}{n_2} \right)^2 \right) \dot{\omega} + D_2 \left(\frac{n_1}{n_2} \right)^2 \omega = T_1 = K_{T1} \frac{V - \omega K_{b1}}{R_1}$$

which reduces to

$$\left(\left(J_1 + J_2 \left(\frac{n_1}{n_2} \right)^2 \right) s + \left(D_2 \left(\frac{n_1}{n_2} \right)^2 + \frac{K_{b1} K_{T1}}{R_1} \right) \right) \Omega(s) = V(s) \frac{K_{T1}}{R_1}$$

Putting this transfer function into the form of

$$H(s) = G \frac{1}{as + 1}$$

we have a DC gain of

$$G = \frac{\frac{K_{T1}}{R_1}}{D_2 \left(\frac{n_1}{n_2} \right)^2 + \frac{K_{b1} K_{T1}}{R_1}}$$

and a pole of

$$a = - \frac{\left(D_2 \left(\frac{n_1}{n_2} \right)^2 + \frac{K_{b1} K_{T1}}{R_1} \right)}{\left(J_1 + J_2 \left(\frac{n_1}{n_2} \right)^2 \right)}$$

and a step response of

$$z(t) = G \left(1 - e^{-\frac{t}{a}} \right)$$

Q2 (5) Suppose that a load resistor was applied to the tachometer output terminals. Then a current will flow that is proportional to ω_1 . But this will generate a torque that is equivalent to a loss term. Hence it is equivalent to increasing D_2 . This will lower the gain G and will actually make the response a bit faster

Aid Sheet

One sided Laplace Transform $F(s) = \int_0^{\infty} f(t) e^{-st} dt$, $f(t) = \frac{1}{2\pi j} \int F(s) e^{st} ds$

| $f(t)$ | $F(s)$ | $f(t)$ | $F(s)$ |
|---|----------------------|------------------------------------|---|
| 1 | $\delta(s)$ | \dot{f} | $sF(s) - f(0^-)$ |
| $\delta(t)$ | 1 | \ddot{f} | $s^2 F(s) - sf(0^-) - \dot{f}(0^-)$ |
| $u(t)$ | $1/s$ | \ddot{f} | $s^3 F(s) - s^2 f(0^-) - s\dot{f}(0^-) - \ddot{f}(0^-)$ |
| $t^m u(t)$ | $m! / s^{m+1}$ | $\int f(t) dt$ | $F(s)/s$ |
| $e^{-at} u(t)$ | $1/(s+a)$ | $\lim_{t \rightarrow \infty} f(t)$ | $\lim_{s \rightarrow 0} sF(s)$ |
| $\frac{1}{(m-1)!} t^{m-1} e^{-at} u(t)$ | $1/(s+a)^m$ | $\sin(at) u(t)$ | $\frac{a}{s^2 + a^2}$ |
| $f(t-T)$ | $F(s)e^{-sT}$ | $\cos(at) u(t)$ | $\frac{s}{s^2 + a^2}$ |
| $tf(t)$ | $-\frac{d}{ds} F(s)$ | $x(t) * y(t)$ | $X(s)Y(s)$ |

* implies convolution as $x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau) y(t-\tau) d\tau$

Poles of second order system $s^2 + 2D\omega_n s + \omega_n^2 = 0$,

$$s = -\sigma \pm j\omega_d \quad \sigma = D\omega_n \quad \omega_d = \omega_n \sqrt{1-D^2} \quad \text{rise time } \tau_r = \frac{2.2}{\text{pole}}$$

State Space $\dot{x} = Ax + Br \quad y = Cx + Dr \quad H(s) = C(sI - A)^{-1} B + D$

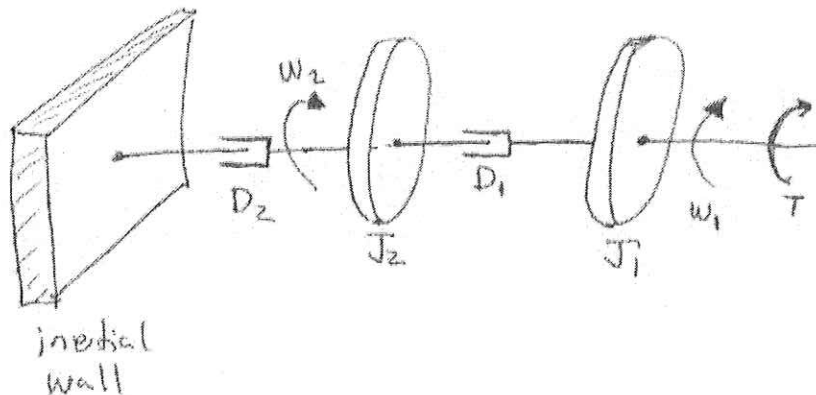
Electric motor with parameters $\{R, K_T, K_b\}$, R internal resistance, current I flowing through motor gives torque $T = K_T I$, ω is rotation rate then induced back EMF voltage is $V_b = K_b \omega$

roots of quadratic $ax^2 + bx + c = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

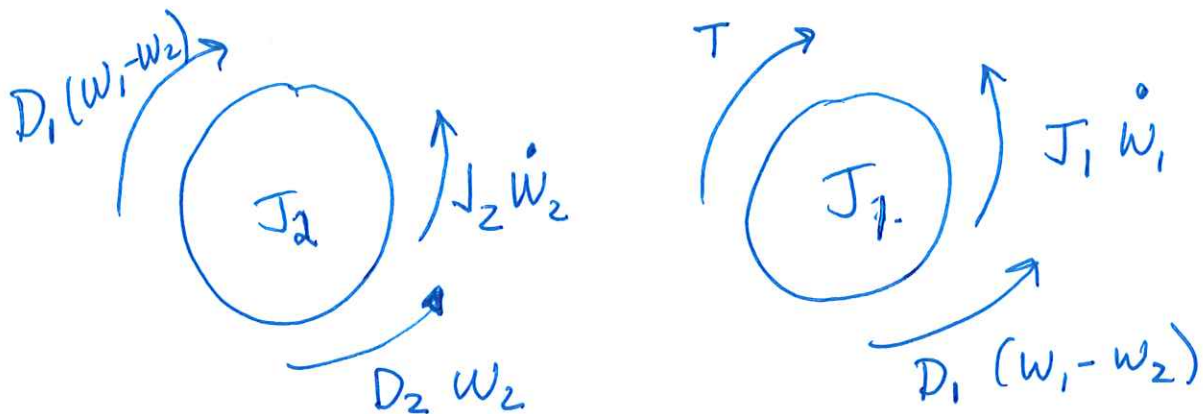
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ENEL441 QUIZ 3 Name SOLUTION UCID _____

Consider the rotational system in the diagram below. Assume $T(t)$ is an applied torque that results in rotations of the two flywheels with angular rotation rates of $\omega_1(t)$ and $\omega_2(t)$ as indicated.



Q1.(4) Determine the two free body diagrams showing the rotational forces for this system.



Q2. (4) Determine the two coupled DEQ's of this rotational system that relates $T(t)$ to $\omega_1(t)$ and $\omega_2(t)$.

$$T(t) = J_1 \dot{\omega}_1 + D_1 (\omega_1 - \omega_2)$$

$$D_1 (\omega_1 - \omega_2) = D_2 \omega_2 + J_2 \dot{\omega}_2$$

Q3. (2) Transform the DEQ's of **Q2** assuming Laplace pairs of

$$T(t) \Leftrightarrow T_s(s) \quad \omega_1(t) \Leftrightarrow \Omega_1(s) \quad \omega_2(t) \Leftrightarrow \Omega_2(s)$$

$$T_s(s) = s J_1 \Omega_1(s) + D_1 (\Omega_1(s) - \Omega_2(s))$$

$$D_1 \Omega_1(s) - D_1 \Omega_2(s) = D_2 \Omega_2(s) + J_2 s \Omega_2(s)$$

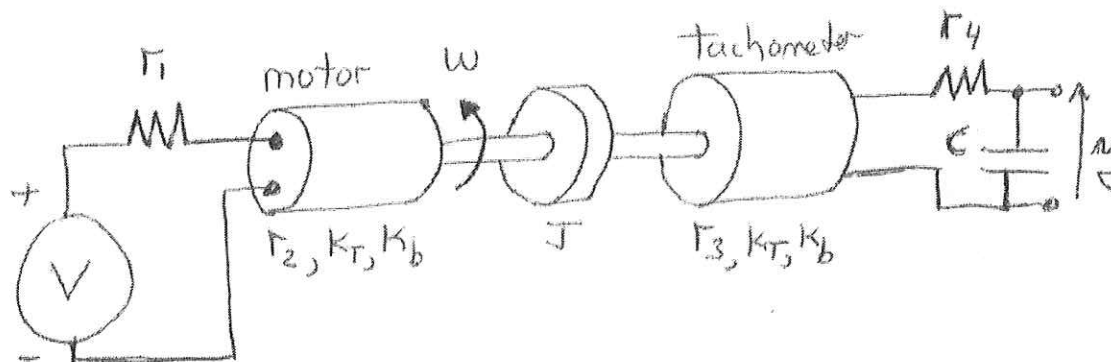
Aid Sheet

One sided Laplace Transform $F(s) = \int_0^{\infty} f(t) e^{-st} dt$, $f(t) = \frac{1}{2\pi j} \oint F(s) e^{st} ds$

| $f(t)$ | $F(s)$ | $f(t)$ | $F(s)$ |
|---|----------------------|------------------------------------|---|
| 1 | $\delta(s)$ | \dot{f} | $sF(s) - f(0^-)$ |
| $\delta(t)$ | 1 | \ddot{f} | $s^2 F(s) - sf(0^-) - \dot{f}(0^-)$ |
| $u(t)$ | $1/s$ | $\ddot{\ddot{f}}$ | $s^3 F(s) - s^2 f(0^-) - s\dot{f}(0^-) - \ddot{f}(0^-)$ |
| $t^m u(t)$ | $m! / s^{m+1}$ | $\int f(t) dt$ | $F(s)/s$ |
| $e^{-at} u(t)$ | $1/(s+a)$ | $\lim_{t \rightarrow \infty} f(t)$ | $\lim_{s \rightarrow 0} sF(s)$ |
| $\frac{1}{(m-1)!} t^{m-1} e^{-at} u(t)$ | $1/(s+a)^m$ | $\sin(at) u(t)$ | $\frac{a}{s^2 + a^2}$ |
| $f(t-T)$ | $F(s)e^{-sT}$ | $\cos(at) u(t)$ | $\frac{s}{s^2 + a^2}$ |
| $tf(t)$ | $-\frac{d}{ds} F(s)$ | $x(t) * y(t)$ | $X(s)Y(s)$ |

* implies convolution as $x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau) y(t-\tau) d\tau$

Consider the model of a motor driving a tachometer as shown. The objective is to determine the filtered tachometer output $y(t)$ as a function of the input voltage $v(t)$. J models the rotational inertia of the motor.



Determine the state space system describing this model assuming the state variables of

$$x(t) = \begin{bmatrix} \omega(t) \\ y(t) \end{bmatrix}$$

That is determine the state space matrices A , B , C and D for this problem. Recall that

$$\frac{dx(t)}{dt} = Ax(t) + Bv(t) \quad \text{and} \quad y(t) = Cx(t) + Dv(t)$$

Solution

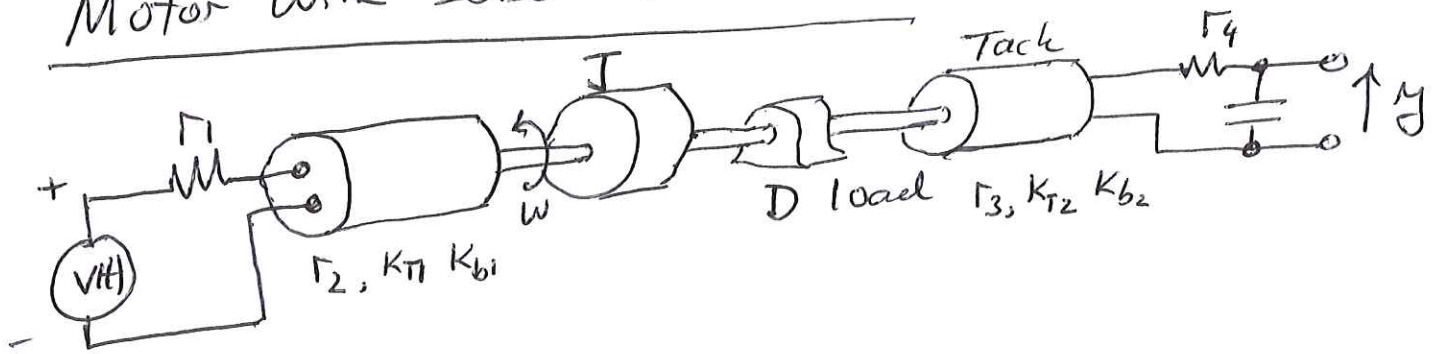
One possibility is to first write the currents of the motor and generator in terms of the state variables and input as

$$i_1 = \frac{v - k_b \omega}{r_1 + r_2}$$

$$i_2 = \frac{k_b \omega - y}{r_3 + r_4}$$

Then write the derivatives of the state variables in terms of the currents as

Motor with sensor Feedback

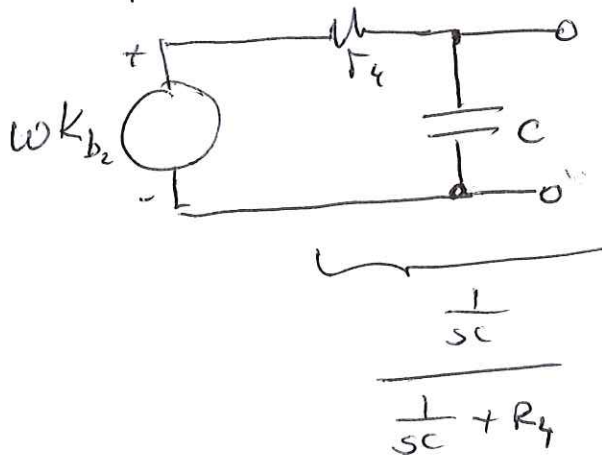


Transfer function from $v(t)$ to $y(t)$

$$\left(\frac{V - K_{b1} \omega}{R_1 + R_2} \right) K_{T1} = J \dot{\omega} + D \omega + T_T$$

Torque T_{ack} ignore

Splits up problem



Split up problem ① Transfer function v to w
② w to y