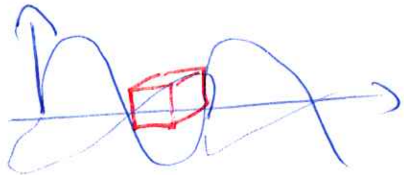
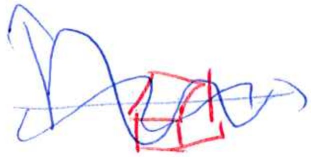
Poynting theorem





$$\boxed{\oint (\vec{E}\times\vec{H})\cdot d\vec{s}}$$

$\Rightarrow$ (A) $\nabla\times\vec{E} = -\mu\dfrac{\partial\vec{H}}{\partial t}$

(B) $\nabla\times\vec{H} = \sigma\vec{E} + \epsilon\dfrac{\partial\vec{E}}{\partial t}$

$\hookrightarrow$ (B) $\rightarrow$ $\vec{E}\cdot$(B) $\Rightarrow$ $\vec{E}\cdot\nabla\times\vec{H} = \sigma\vec{E}^2 + \vec{E}\cdot\epsilon\dfrac{\partial\vec{E}}{\partial t}$

$\Downarrow$

vector identify

$$\vec{H}\cdot(\nabla\times\vec{E}) + \nabla\cdot(\vec{H}\times\vec{E}) = \sigma\vec{E}^2 + \vec{E}\cdot\epsilon\frac{\partial\vec{E}}{\partial t}$$

$$\vec{H}\cdot -\mu\frac{\partial\vec{H}}{\partial t} - \nabla\cdot(\vec{E}\times\vec{H}) = \sigma\vec{E}^2 + \frac{1}{2}\epsilon\frac{\partial\vec{E}^2}{\partial t}$$

$$\frac{1}{2}\mu\frac{\partial\vec{H}^2}{\partial t}$$

$$\Rightarrow \nabla\cdot(\vec{E}\times\vec{H}) = -\left[\sigma\vec{E}^2 + \frac{1}{2}\epsilon\frac{\partial\vec{E}^2}{\partial t} + \frac{1}{2}\mu\frac{\partial\vec{H}^2}{\partial t}\right]$$

$$\oint_s (\vec{E}\times\vec{H})\cdot d\vec{s} = -\frac{\partial}{\partial t}\left[\int_v \underbrace{\left[\frac{1}{2}\epsilon\vec{E}^2 + \frac{1}{2}\mu\vec{H}^2\right]}_{\text{stored energy}} dv\right] - \int\sigma\vec{E}^2 dv$$

total energy leaving surface

$\hookrightarrow$ time derivative $\Rightarrow$ decrease in stored energy

$\hookrightarrow$ dissipated energy

$\vec{E}\times\vec{H} \rightarrow$ Poynting vector

$\rightarrow$ ~~fora~~

For lossy material,

$\vec{P} = \vec{E} \times \vec{H}$

$\uparrow$ Poynting $\rightarrow$ instantaneous

$\vec{E}(z,t) = E^+ e^{-\alpha z} \cos(\omega t - \beta z)\vec{a}_x$

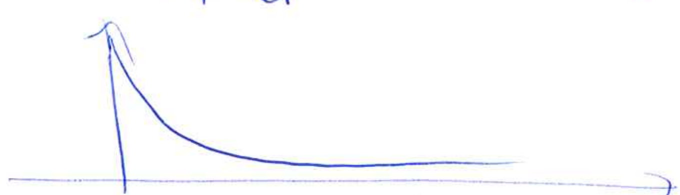
$\vec{H}(z,t) = \frac{E^+}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\vec{a}_y$

$$\vec{P}_{Avg}(z) = \frac{1}{2} Re\{\vec{E}_s \times \vec{H}_s^*\}$$

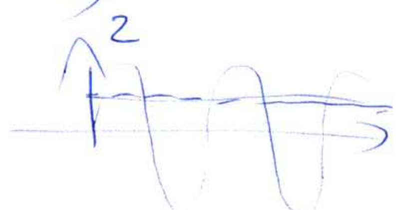
$$= \frac{1}{2} Re\left\{E^+ e^{-\alpha z} e^{-j\beta z} \vec{a}_x \times \frac{E^+}{|\eta|} e^{-\alpha z} e^{j\beta z} e^{j\theta_\eta} \vec{a}_y\right\}$$

$$= \frac{1}{2} Re\left\{\frac{E^{+2}}{|\eta|} e^{-2\alpha z} e^{j\theta_\eta} \vec{a}_z\right\}$$

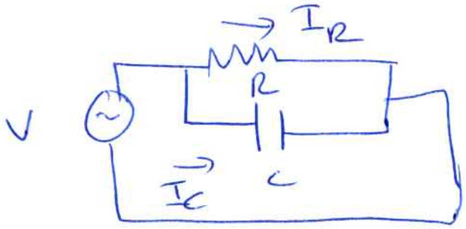
$$= \frac{E^{+2}}{2|\eta|} e^{-2\alpha z} \cos(\theta_\eta)\vec{a}_z$$



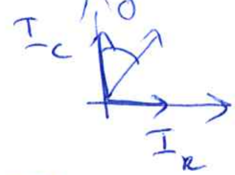
Lossless: $\vec{P}_{Avg}(z) = \frac{E^{+2}}{2\eta}\vec{a}_z$



Microwave heating



$I_C = \omega V C$



$I_R = I \sin\delta \rightarrow I_R = I_C \tan\delta$

$Q = V I_R$

$= V I_C \tan\delta$

$= \omega V^2 C \tan\delta$

Permittivity : ϵ_R $\Rightarrow$ complex permittivity $\epsilon = \epsilon' - j\epsilon''$

$= \epsilon_0(\epsilon_R' - j\,\epsilon_R'')$

$$\epsilon_R'' = \underbrace{\epsilon_{Rd}''}_{\text{dipole rotation}} + \underbrace{\frac{\sigma}{\omega\epsilon_0}}_{\text{current}}$$

$\tan\delta \rightarrow$ loss tangent

$$\tan\delta = \epsilon_R''/\epsilon_R'$$

$$Q = \omega V^2 C \tan\delta \longrightarrow \epsilon_R''/\epsilon_R'$$

$V \rightarrow (Ed)^2$, $C \rightarrow \dfrac{\epsilon_0\epsilon_R' A}{d}$

$$= \omega\epsilon_0\epsilon_R'' E^2 A d$$

$P = \omega\epsilon_0\epsilon_R'' E^2$ $\rightarrow$ power dissipated per unit volume

$\hookrightarrow$ $\vec{E} \times \vec{H} = \dfrac{-\partial}{\partial t}\left(\tfrac{1}{2}\epsilon\vec{E}^2\right) - \sigma\vec{E}^2$

$\hookrightarrow$ related to heating? $\omega\epsilon_0\epsilon_R'' E^2 = \rho C_p \dfrac{\Delta T}{\Delta t}$

ρ : density

C_p : specific heat capacity

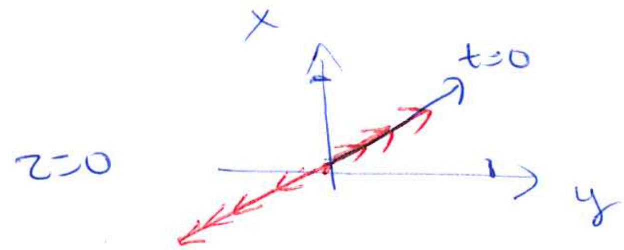
$\dfrac{\Delta T}{\Delta t}$ $\rightarrow$ increase in temp/time

Polarization

→ pattern traced out by $\vec{E}$ with time (considering single point in space)

Case 1: linear polarization

e.g. $\vec{E}(z,t) = E_x \underbrace{\cos(\omega t - \beta z)}_{-1 \text{ to } +1} \vec{a}_x + \underbrace{E_y}_{\text{no phase shift}} \cos(\omega t - \beta z)\vec{a}_y$



Case 2: Elliptical polarization

$\vec{E}(z,t) = E_x \cos(\omega t - \beta z + \phi_x)\vec{a}_x + E_y \cos(\omega t - \beta z + \phi_y)\vec{a}_y$

↳ $\phi_x = \phi_y \Rightarrow$ linear

↳ $|E_x| = |E_y|$ and $|\phi_x - \phi_y| = 90^\circ \Rightarrow$ circular

⇒ all other cases are elliptical

→ right & left handed elliptical & circular polarization

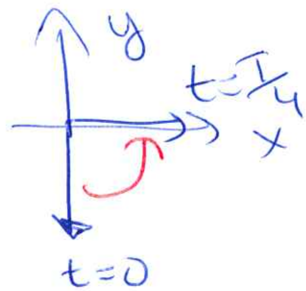
⇒ thumb in direction of propagation & fingers curl in direction of rotation

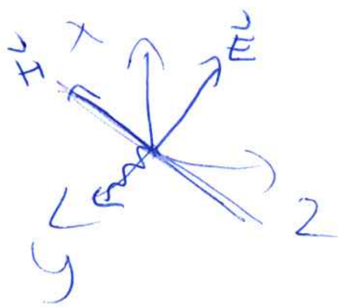
↳ RCP ⇒ y lags x

↳ LCP ⇒ y lead x

e.g. $\vec{E}(z,t) = 10\cos(\omega t - \beta z - \pi/2)\vec{a}_x + 10\cos(\omega t - \beta z - \pi)\vec{a}_y$

$z=0, t=0 \Rightarrow \vec{E} = -10\vec{a}_y$ $\qquad$ $z=0, t=T/4 \Rightarrow \vec{E} = 10\vec{a}_x$


y
t = T/4
x
t = 0


H
x
E
y
z