From Faraday's Law : $V_1 = N_1 \dfrac{d\phi_m}{dt}$, $V_2 = N_2 \dfrac{d\phi_m}{dt}$

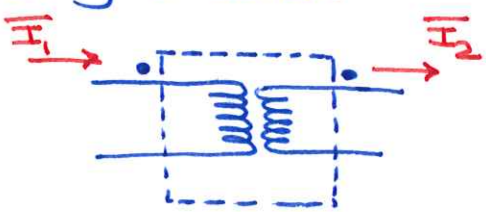
$$\therefore \frac{d\phi_m}{dt} = \frac{V_1}{N_1} = \frac{V_2}{N_2} \quad \therefore \boxed{\frac{V_1}{V_2} = \frac{N_1}{N_2} = a}$$

turns ratio

since $S_1 = V_1 I_1$ & $S_2 = V_2 I_2$ and $S_1 = S_2$

$$\therefore \boxed{\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a}}$$

- Winding direction is not always visible. Solution: dot convention

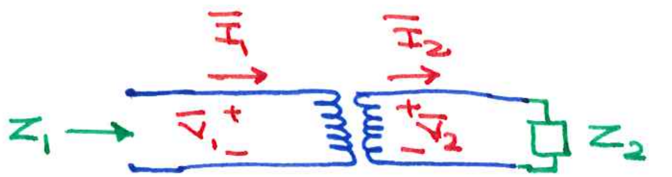


when current enters a winding at the dotted terminal, flux is in the direction of the dot

$\therefore$ when current enters dotted terminal from one side & leaves dotted terminal on other side, those currents are in phase

i.e.. $\overline{I_1}$ & $\overline{I_2}$ are in phase, just scaled up/down by $1/a$

Referring Impedances



if Z_2 is connected to winding 2, Z_1 (impedance seen from winding 1) is :

$$Z_1 = \frac{\overline{V_1}}{\overline{I_1}} = \frac{\frac{N_1}{N_2} \cdot \overline{V_2}}{\frac{N_2}{N_1} \cdot \overline{I_2}} = \left(\frac{N_1}{N_2}\right)^2 \cdot \underbrace{\frac{\overline{V_2}}{\overline{I_2}}}_{Z_2} = \left(\frac{N_1}{N_2}\right)^2 \cdot Z_2$$

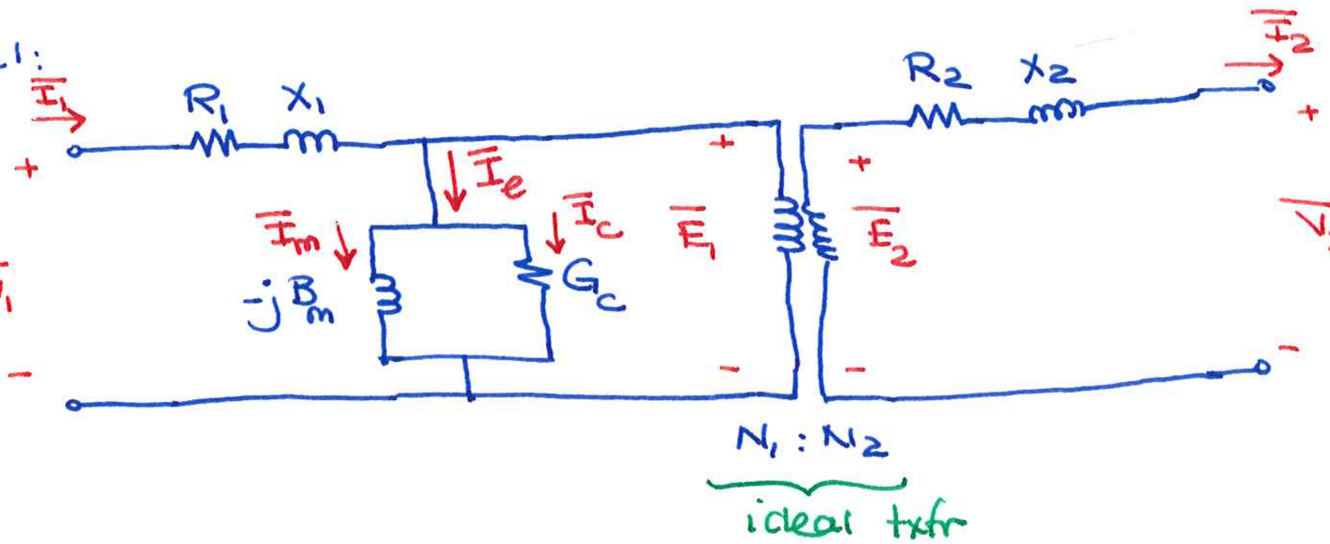
- Z_1 is Z_2 referred to winding 1 :

$Z_1 = \left(\dfrac{N_1}{N_2}\right)^2 \cdot Z_2$

Real (Non-ideal) Transformers

- Have losses (winding resistance & core losses)
- Have leakage flux
- Have finite μ

Model:



- R_1 & R_2 to model winding resistance
- X_1 & X_2 to model leakage flux (ϕ from one winding that doesn't link with the other)
- G_c (and $\bar{I}_c$) to model core losses (hysteresis & eddy current losses)
- B_m (and $\bar{I}_m$) to model magnetizing current
- Excitation current $I_e = I_m + I_c$

Reminder: Impedance, Admittance, etc.

$$\underbrace{Y}_{\text{admittance}} \triangleq \frac{1}{Z} = \frac{1}{R+jX} = \underbrace{G}_{\text{conductance}} + j\underbrace{B}_{\text{susceptance}}$$

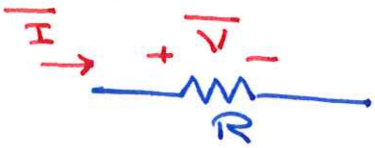
if $R=0$, $Y = \frac{1}{jX} = -j\frac{1}{X} = -jB$

if $X=0$, $Y = \frac{1}{R} = G$

- Admittances in parallel add

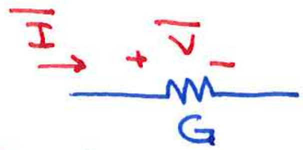
$$\therefore \quad G_c \parallel -jB_m = \underbrace{G_c - jB_m}_{\text{equivalent admittance of exc'n branch}}$$

- To represent a resistor in a circuit, we can use:



$\overline{V} = \overline{I}.R$

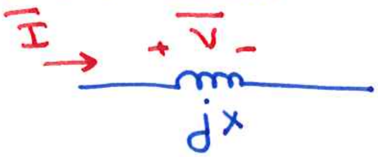
or



$Y = G = \frac{1}{R}$

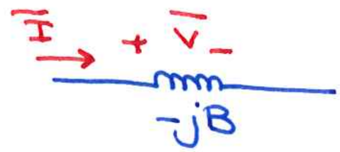
$\overline{I} = \overline{V}.G$

- for an inductor



$\overline{V} = Z.\overline{I} = (jX)\overline{I}$

or



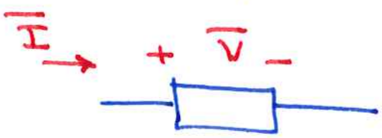
$B = \frac{1}{X}$

$Y = -jB$

$\overline{I} = Y.\overline{V}$

$= (-jB).\overline{V}$

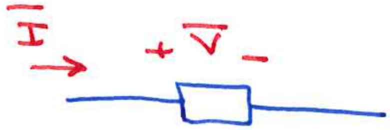
- for a complex impedance



$Z = R + jX$

$\overline{V} = Z.\overline{I}$

or



$Y = \frac{1}{Z} = G + jB$

$\overline{I} = Y.\overline{V}$

- Please note that:

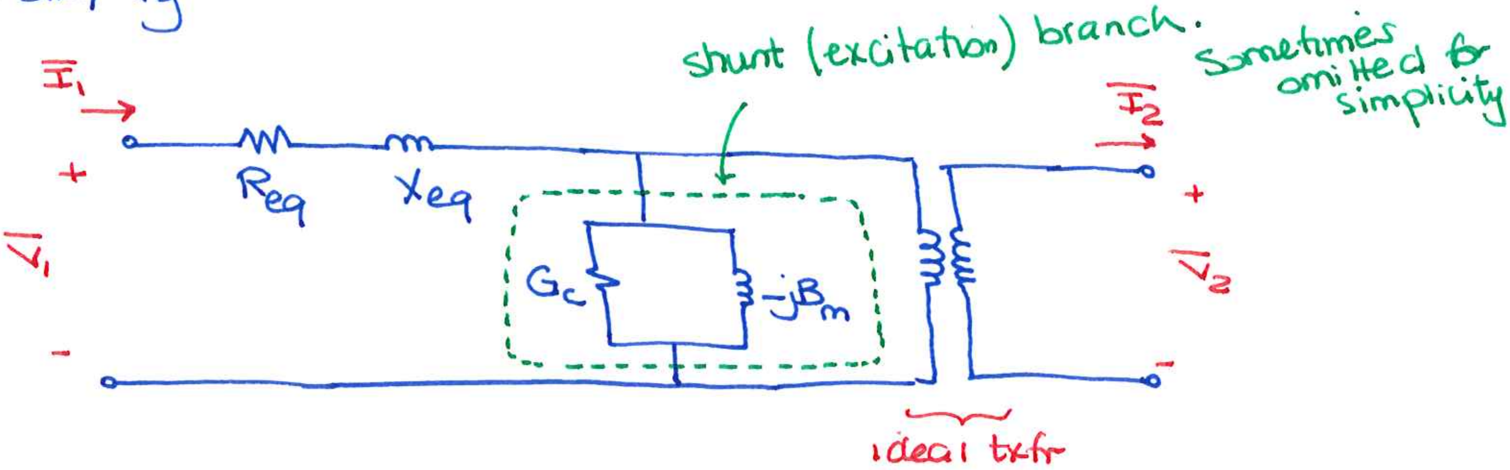
$$Y \triangleq \frac{1}{Z} = \frac{1}{R+jX} \cdot \frac{R-jX}{R-jX} = \frac{R-jX}{R^2 - \underbrace{j^2}_{-1} X^2}$$

$$= \frac{R-jX}{R^2+X^2} = \underbrace{\frac{R}{R^2+X^2}}_{G} + j\underbrace{\frac{-X}{R^2+X^2}}_{B}$$

can't use $G = \frac{1}{R}$ or $B = \frac{-1}{X}$ for complex Z

Back to the non-ideal transformer model:

- Simplify the model by referring all impedances to one side:

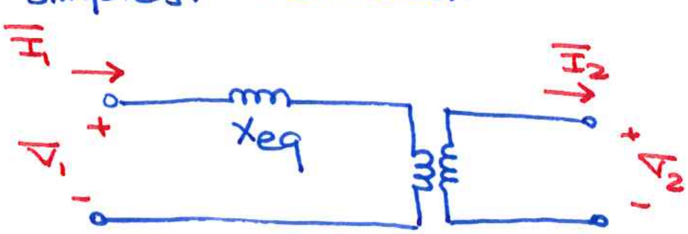


ENGL 487 non-ideal txfr model

$$R_{eq} \simeq R_1 + \left(\frac{N_1}{N_2}\right)^2 \cdot R_2$$

$$X_{eq} \simeq X_1 + \left(\frac{N_1}{N_2}\right)^2 \cdot X_2$$

- In the simplest non-ideal txfr model, R_{eq} is also omitted.



Transformer Rated Values

- Each txfr has:
  - rated voltage for HV & LV side
  - rated current for HV & LV sides
  - rated power

- For 1ϕ txfr: $\quad I_{rated} = \dfrac{S_{rated}}{V_{rated}}$

- For 3ϕ txfr: $\quad I_{rated} = \dfrac{S_{rated}}{\sqrt{3}\, V_{rated}}$

3ϕ power

line current

line-to-line voltage

Ex : Txfr at Enmax Substation #34 (feeds Campus & adj 'hood)

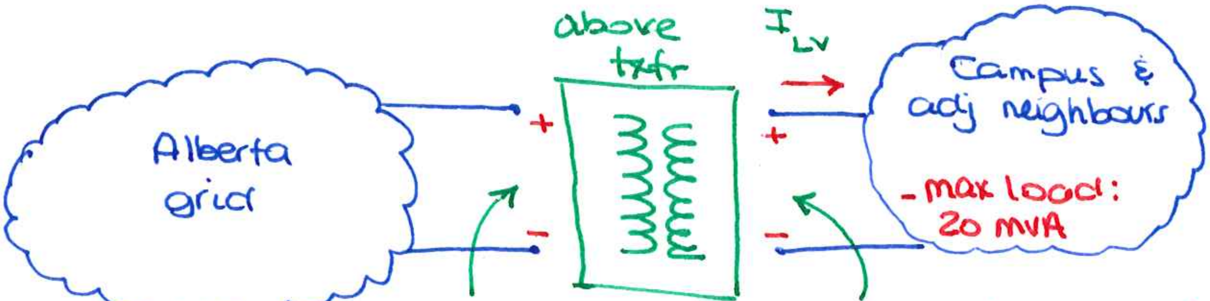
$$3\phi \ , \ \underbrace{69}_{V_{rated_{HV}}} : \underbrace{13.2}_{V_{rated_{LV}}} \text{ kV} \ , \ \underbrace{25 \text{ MVA}}_{S_{rated}}$$

V_{rated} : . Voltage that a winding & its insulation is designed to handle

. Typically, same as system nominal / rated voltage

$$I_{rated_{HV}} = \frac{25 \text{ MVA}}{\sqrt{3} \times 69 \text{ kV}} = 209 \text{ A}$$ ← max line current that HV side can handle

$$I_{rated_{LV}} = \frac{25 \text{ MVA}}{\sqrt{3} \times 13.2 \text{ kV}} = 1093 \text{ A}$$



operating V_{ll} (at or near) 69 kV

expects operating V_{ll} (at or near) 13.2 kV

. For a load of 20 MVA, operating LV voltage = 13.2 kV,

then $$I_{LV} = \frac{20 \text{ MVA}}{\sqrt{3} \times 13.2 \text{ kV}} = 879 \text{ A} < I_{LV_{rated}}$$ ok!

. Could we have used a 3ϕ, $\underbrace{138 : 26.4 \text{ kV}}$, 25 MVA txfr instead?

provides the same voltage transformation

if $V_{HV_{operating}} = 69$ kV

then $$V_{LV_{operating}} = 69 \times \frac{26.4}{138} = 13.2 \text{ kV}$$

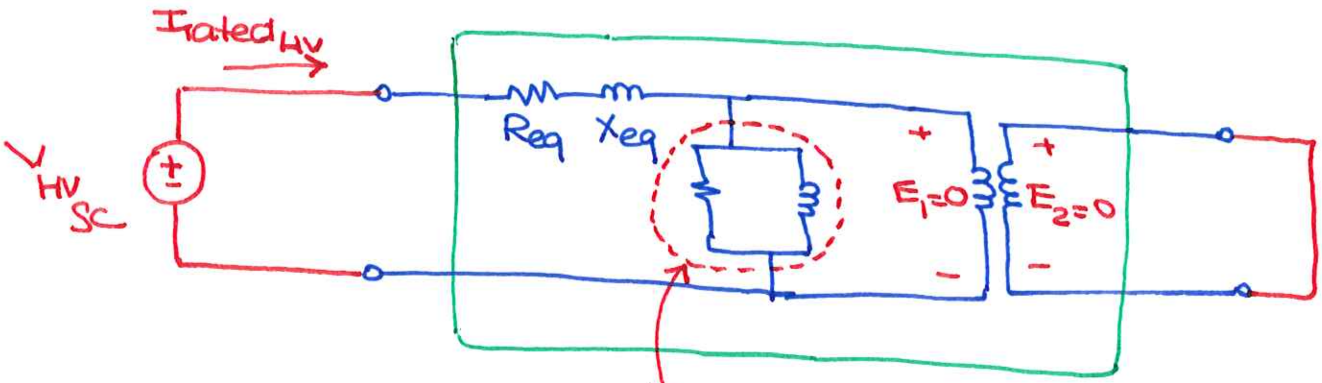
we still get the same voltage.

. while the voltages are ok, we will exceed I_{rated} if load > 12.5 MVA

Measuring Txfr Model Parameters: R_{eq}, X_{eq}, G_c, B_m

1) Short Circuit Test (Load Loss Test)

- Short LV side
- Apply voltage to HV side until $I_{rated_{HV}}$ flows on that side
- Measure applied voltage ($V_{HV_{sc}}$) & real power consumed (P_{sc})



shorted out since $E_1 = 0$

$$P_{sc} = I_{rated_{HV}}^2 \cdot R_{eq} \quad \therefore \quad \boxed{R_{eq} = \frac{P_{sc}}{I_{rated_{HV}}^2}}$$

From KVL & Ohm's Law: $\overline{V}_{HV_{sc}} = \overline{I}_{rated_{HV}} \cdot \underbrace{Z_{eq}}_{R_{eq} + jX_{eq}}$

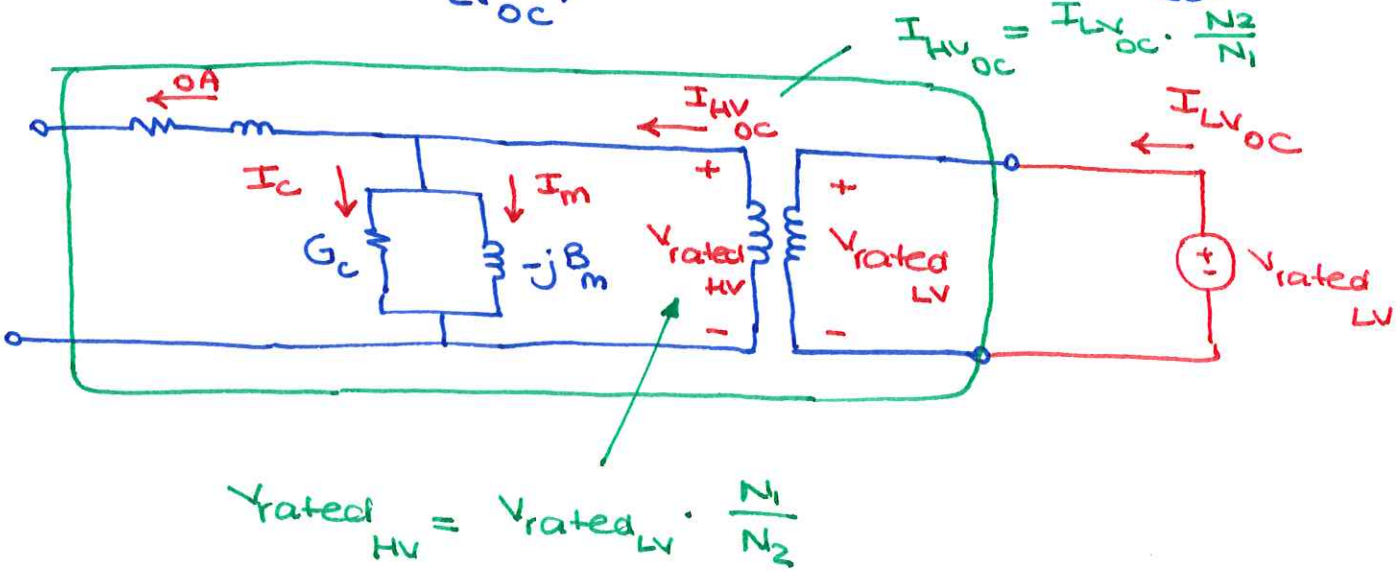
in magnitudes only: $V_{HV_{sc}} = I_{rated_{HV}} \cdot |Z_{eq}|$

we can solve for $|Z_{eq}| = \dfrac{V_{HV_{sc}}}{I_{rated_{HV}}}$

Finally, $\boxed{X_{eq} = \sqrt{|Z_{eq}|^2 - R_{eq}^2}}$

2) Open Circuit Test (Core Loss Test, No Load Loss Test)

- Open HV side
- Apply rated voltage to LV terminals
- Measure current ($I_{LV_{OC}}$) & power consumed (P_{OC})



- we can calculate $V_{rated_{HV}}$ & $I_{HV_{OC}}$ in this test

$$P_{OC} = (V_{rated_{HV}})^2 \cdot G_c \quad \therefore \quad \boxed{G_c = \frac{P_{OC}}{(V_{rated_{HV}})^2}}$$

$$\bar{I}_{HV_{OC}} = \bar{I}_c + \bar{I}_m = (\bar{V}_{rated_{HV}} \cdot G_c) + (\bar{V}_{rated_{HV}} \cdot -jB_m)$$

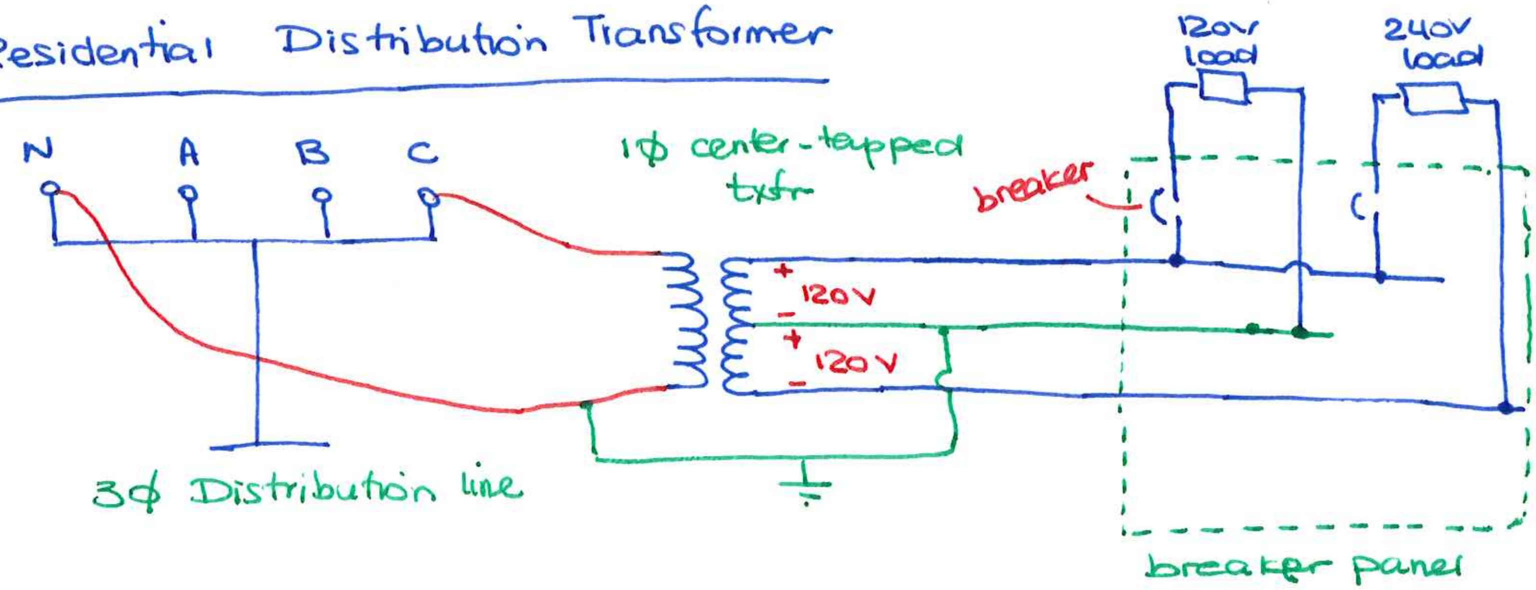
$$= \bar{V}_{rated_{HV}} \underbrace{(G_c - jB_m)}_{\text{total admittance } (Y_{eq}) \text{ of exc'n branch}}$$

in magnitudes only, $I_{HV_{OC}} = V_{rated_{HV}} \cdot |Y_{eq}|$

$$\therefore \quad |Y_{eq}| = \frac{I_{HV_{OC}}}{V_{rated_{HV}}}$$

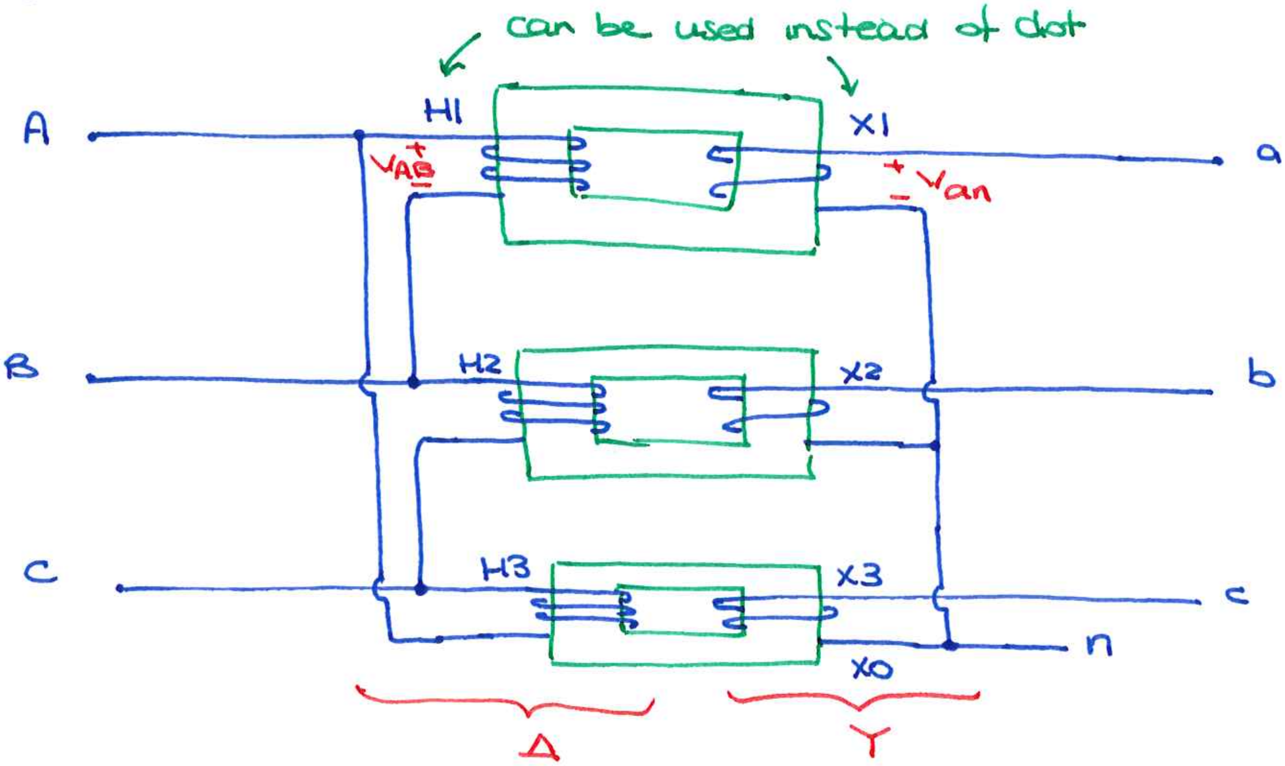
finally, $\boxed{B_m = \sqrt{|Y_{eq}|^2 - G_c^2}}$

Residential Distribution Transformer

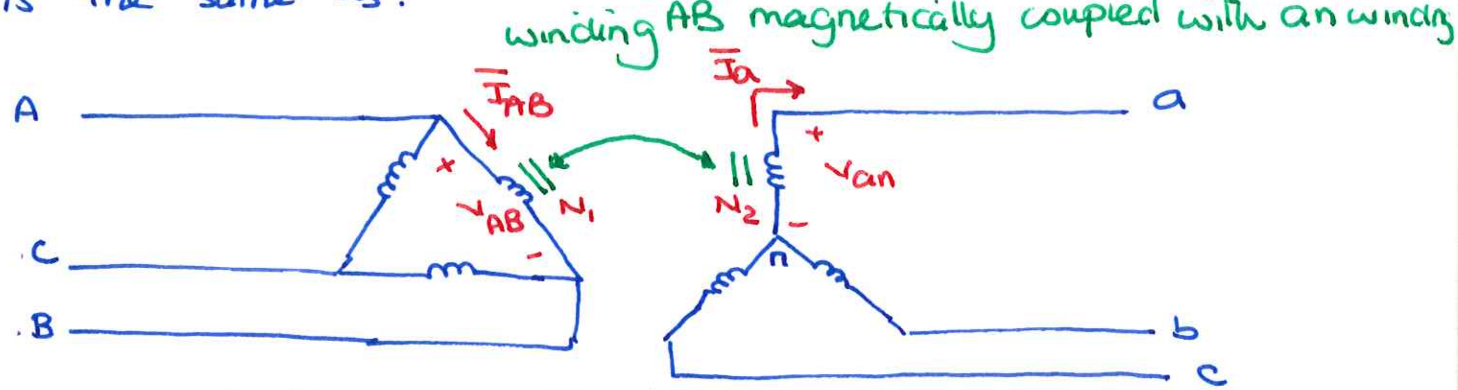


Three Phase Transformers

- 3 windings on each side: 3 on HV side, 3 on LV side.
  Each side can be connected as Y or Δ ∴ 4 possible connection types: YY, ΔΔ, ΔY, YΔ



This is the same as:



because of the magnetic coupling,

$$\overline{V}_{an} = \frac{N_2}{N_1} \cdot \overline{V}_{AB} \quad : \quad \overline{V}_{an} \text{ in phase with } \overline{V}_{AB}\text{, scaled down by } \frac{N_2}{N_1}$$

$$\overline{I}_a = \frac{N_1}{N_2} \cdot \overline{I}_{AB} \quad : \quad \overline{I}_a \text{ in phase with } \overline{I}_{AB}\text{, scaled up by } \frac{N_1}{N_2}$$

- Reminder : we are often interested in line-to-line voltages & line currents on either side of the txfr.

Relationships between $\overline{V}_{\ell\ell}$ & $\overline{I}_\ell$ on either side depends on the connection types.