Assignment 3 Solution

4.10

$$D = 4\,\text{ft}$$

$$r' = e^{\frac{-1}{4}}\left(\frac{.5}{2}\right)\left(\frac{1\,\text{ft}}{12\,\text{in}}\right)$$

$$r' = 1.6225\times10^{-2}\ \text{ft}$$

$$L_1 = 2\times10^{-7}\,Ln\left(\frac{D}{r'}\right)\frac{\text{H}}{\text{m}}$$

$$L_1 = 2\times10^{-7}\,Ln\left(\frac{4}{1.6225\times10^{-2}}\right)$$

$$L_1 = \underline{\underline{1.101\times10^{-6}}}\ \frac{\text{H}}{\text{m}}$$

$$X_1 = \omega L_1 = (2\pi60)(1.101\times10^{-6})(1000) = \underline{\underline{0.4153}}\,\Omega/\text{km}$$

4.18

$$D_{eq} = \sqrt[3]{8\times8\times16} = 10.079\,\text{m}$$

$$\text{From Table A.4, } D_S = (0.0403\,\text{ft})\frac{1\,\text{m}}{3.28\,\text{ft}} = 0.0123\,\text{m}$$

$$L_i = 2\times10^{-7}\ln\left(D_{eq}/D_s\right) = 2\times10^{-7}\ln\left(\frac{10.079}{0.0123}\right) = 1.342\times10^{-6}\ \text{H/m}$$

$$X_1 = 2\pi(60)L_1 = 2\pi(60)1.342\times10^{-6}\,\frac{\Omega}{\text{m}}\times\frac{1000\,\text{m}}{1\,\text{km}} = 0.506\,\Omega/\text{km}$$

4.25

D_{eq}

$$\text{GMD} = \left[(41.76)(80)(41.76)\right]^{1/3},\ \text{using } \sqrt{40^2+12^2} = 41.76.$$
$$= 51.78\,\text{ft}.$$

[Note: from Table A.4, conductor diameter. = 1.196 in.; $r = \frac{1.196}{2}\times\frac{1}{12} = 0.0498\,\text{ft}$.] and conductor GMR = 0.0403 ft.

$$\text{GMR for the bundle}: 1.091\left[(0.0403)(1.667)^3\right]^{1/4} = 0.7171\ \text{ft}.$$

Please notice the units here.
1 mile = 1609.34 m

$$\therefore X = 0.2794 \log\left(\frac{51.87}{0.7171}\right) = 0.5195\,\Omega/\text{mi} \leftarrow$$

Rated current carrying capacity for each conductor in the bundle, as per Table A.4, is 1010 A; since it is a 4-conductor bundle, rated current carrying capacity of the overhead line is

$$1010 \times 4 = 4040\text{A} \leftarrow$$

4.34 $$C_1 = \frac{2\pi\varepsilon_0}{Ln\left(\frac{D}{\text{r}}\right)} = \frac{2\pi\left(8.854\times10^{-12}\right)}{Ln\left(\frac{4}{0.25/12}\right)} = \underline{\underline{1.058\times10^{-11}}}\,\frac{\text{F}}{\text{m}}$$

$$\overline{Y}_1 = j\omega C_1 = j\left(2\pi 60\right)\left(1.058\times10^{-11}\right)\left(1000\right)$$

$$= \underline{\underline{j3.989\times10^{-6}}}\,\frac{\text{S}}{\text{km}}$$

4.39 $$D_{eq} = \sqrt[3]{8\times8\times16} = 10.079\,\text{m}$$

For Table A.4, $r = \frac{1.196}{2}\text{in}\left(\frac{0.0254\,\text{m}}{1\,\text{in}}\right) = 0.01519\,\text{m}$

$$C_1 = \frac{2\pi\varepsilon_0}{\ln\left(\frac{D}{r}\right)} = \frac{2\pi\left(8.854\times10^{-12}\right)}{\ln\left(\frac{10.079}{0.01519}\right)} = 8.565\times10^{-12}\ \text{F/m}$$

$$\overline{Y}_1 = j\omega c_1 = j2\pi\left(60\right)8.565\times10^{-12}\left(1000\right) = j3.229\times10^{-6}\ \text{S/km}$$

For a 100 km line length

$$I_{chg} = Y_1 V_{LN} = \left(3.229\times10^{-6}\times100\right)\left(230/\sqrt{3}\right) = 4.288\times10^{-2}\ \ \text{kA/Phase}$$

5.2 (a) $\bar{A} = \bar{D} = 1 + \frac{\bar{Y}\bar{Z}}{2} = 1 + \frac{1}{2}\left(3.33\times10^{-6}\times200\angle90^\circ\right)\left(0.08 + j0.48\right)\left(200\right)$

$= 1 + \left(0.0324\angle170.5^\circ\right) = 0.968 + j0.00533 = 0.968\angle0.315^\circ \text{pu}$

$\bar{C} = \bar{Y}\left(1 + \frac{\bar{Y}\bar{Z}}{4}\right) = \left(6.66\times10^{-4}\angle90^\circ\right)\left(1 + .0.0162\angle170.5^\circ\right)$

$= 6.553\times10^{-4}\angle90.155^\circ \text{ S}$

$\bar{B} = \bar{Z} = 97.32\angle80.54^\circ\ \Omega$

(b) $\bar{V}_R = \frac{220}{\sqrt{3}}\angle0^\circ = 127\angle0^\circ \text{kV}_{\text{L-N}}$

$\bar{I}_R = \frac{P_R\angle-\cos^{-1}(pf)}{\sqrt{3}V_{R_{L-L}}(pf)} = \frac{250\angle-\cos^{-1}0.99}{\sqrt{3}(220)(0.99)} = 0.6627\angle-8.11^\circ \text{kA}$

Same as:
$|I|=S/(\sqrt{3}.V_{ll})$ & $\theta_I = \cos^{-1}(pf)$

$\bar{V}_S = \bar{A}\bar{V}_R + \bar{B}\bar{I}_R = \left(0.968\angle0.315^\circ\right)\left(127\angle0^\circ\right) + \left(97.32\angle80.54^\circ\right)\left(0.6627\angle-8.11^\circ\right)$

$= 142.4 + j62.16 = 155.4\angle23.58^\circ \text{ kV}_{\text{L-N}}$

$\bar{V}_{S_{L-L}} = 155.4\sqrt{3} = 269.2 \text{ kV}$

$\bar{I}_S = \bar{C}\bar{V}_R + \bar{D}\bar{I}_R = \left(6.553\times10^{-4}\angle90.155^\circ\right)\left(127\right) + \left(0.968\angle-0.315^\circ\right)\left(0.6627\angle-8.11^\circ\right)$

$= 0.6353 - j3.786\times10^{-3} = 0.6353\angle-0.34^\circ \text{kA}$

5.14 (a) $\bar{Z}_C = \sqrt{\frac{\bar{z}}{\bar{y}}} = \sqrt{\frac{0.03 + j0.35}{4.4\times10^{-6}\angle90^\circ}} = 282.6\angle-2.45^\circ\ \Omega$

(b) $\bar{\gamma}l = \sqrt{\bar{z}\,\bar{y}}(l) = \sqrt{\left(0.35128\angle85.101^\circ\right)\left(4.4\times10^{-6}\angle90^\circ\right)}\left(400\right)$

$= 0.4973\angle87.55^\circ = 0.02126 + j0.4968 \text{ pu}$

(c) $\bar{A} = \bar{D} = \cosh\bar{\gamma}l = \cosh\left(0.02126 + j0.4968\right)$

$= \left(\cosh 0.02126\right)\left(\cos 0.4968 \text{ radians}\right) + j\left(\sinh 0.02126\right)\left(\sin 0.4968 \text{ radians}\right)$

$= \left(1.00023\right)\left(0.87911\right) + j\left(0.02126\right)\left(0.47661\right)$

$= 0.87931 + j0.01013 = 0.8794\angle0.66^\circ \text{ pu}$

$\sinh\bar{\gamma}l = \sinh\left(0.02126 + j0.4968\right)$

$= \sinh\left(0.02126\right)\cos\left(0.4968 \text{ radians}\right) + j\left(\cosh 0.02126\right)\left(\sin 0.4968 \text{ radians}\right)$

$= (0.02126)(0.87911) + j(1.00023)(0.47661)$

$= 0.01869 + j0.4767 = 0.4771\angle87.75^\circ$

$\bar{B} = \bar{Z}_C \sinh\left(\bar{\gamma}l\right) = \left(282.6\angle-2.45^\circ\right)\left(0.4771\angle87.75^\circ\right)$

$= 134.8\angle85.3^\circ\Omega$

$\bar{C} = \frac{1}{\bar{Z}_C}\sinh\left(\bar{\gamma}l\right) = \frac{0.4771\angle87.75^\circ}{282.6\angle-2.45^\circ} = 1.688\times10^{-3}\angle90.2^\circ \text{S}$

5.26 (a) $\bar{Z}_C = \sqrt{\dfrac{\bar{z}}{\bar{y}}} = \sqrt{\dfrac{j0.34}{j4.5\times10^{-6}}} = 274.9\angle 0^\circ = 274.9\,\Omega$

(b) $\bar{\gamma} l = \sqrt{\bar{z}\,\bar{y}}\,(l) = \sqrt{(j0.34)(j4.5\times10^{-6})}\,(300) = j0.3711$

5.41 (a) $\bar{Z} = \bar{z}l = (0.088 + j0.465)100 = 8.8 + j46.5\,\Omega$

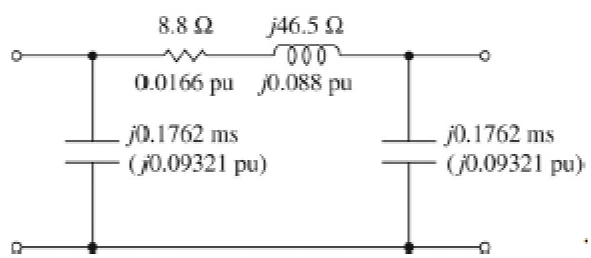
$$\frac{\bar{Y}}{2} = \frac{\bar{y}l}{2} = (j3.524\times10^{-6})100/2 = j0.1762\,\text{mS}$$

$$Z_{base} = V_{L\,base}^2 / S_{3\phi\,base} = \frac{(230)^2}{100} = 529\,\Omega$$

$$\therefore \bar{Z} = (8.8 + j46.5)/529 = 0.0166 + j0.088\,\text{pu}$$

$$\frac{\bar{Y}}{2} = j0.1762/(1/0.529) = j0.09321\,\text{pu}$$

The nominal π circuit for the medium line is shown below:



(b) $S_{3\phi\,rated} = V_{L\,rated}\, I_{L\,rated}\sqrt{3} = 230(0.9)\sqrt{3} = 358.5\,\text{MVA}$

(c) $\bar{A} = \bar{D} = 1 + \dfrac{\bar{Z}\bar{Y}}{2} = 1 + (8.8 + j46.5)(0.1762\times10^{-3}) = 0.9918\angle 0.1^\circ$

$$\bar{B} = \bar{Z} = 8.8 + j46.5 = 47.32\angle 79.3^\circ\,\Omega$$

$$\bar{C} = \bar{Y} + \frac{\bar{Z}\bar{Y}^2}{4} = 0.351 \quad \angle 90.04^\circ\,\text{mS}$$