

**University of Calgary**  
**Schulich School of Engineering**  
**Department of Electrical and Computer Engineering**  
  
**ENEL 476 – Electromagnetic Waves and Applications**

**Final Examination**  
**Winter Session 2017**  
**April 22, 2017 at 8:00 am**  
**Red Gym**  
**Instructor: Elise Fear**

**3 hours**  
**Closed book**

Student name:

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**Instructions**

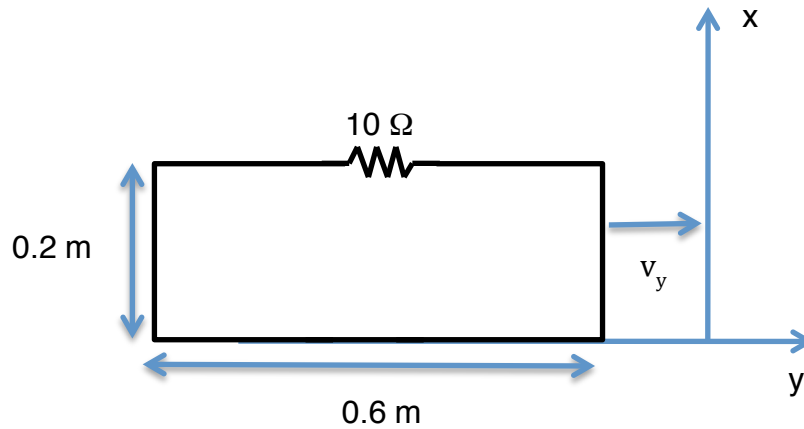
- (1) This is a closed book exam. No texts or notes are allowed.
- (2) Show as much of your reasoning as time permits. Write your answers in the examination booklets.
- (3) Calculators are permitted.
- (4) Formulas are provided at the end of the question pages.
- (5) Hand in all pages. If you detach any pages(s), write your name and UCID number on the detached page(s).
- (6) If you write anything you do not want marked, put a large X through it and write “Rough work” beside it.

**Question 1 (15 marks).**

The loop shown below moves from an area of zero external magnetic flux density ( $y < 0$ ) into an external magnetic flux density ( $y > 0$ ). In the region  $y > 0$ , the flux density is described by:

$$\mathbf{B} = -0.3\mathbf{a}_z \text{ Wb/m}^2$$

The loop moves with velocity of  $2\mathbf{a}_y$  cm/s and the location of the right side of the loop at  $t=0$  is  $y=0$ .

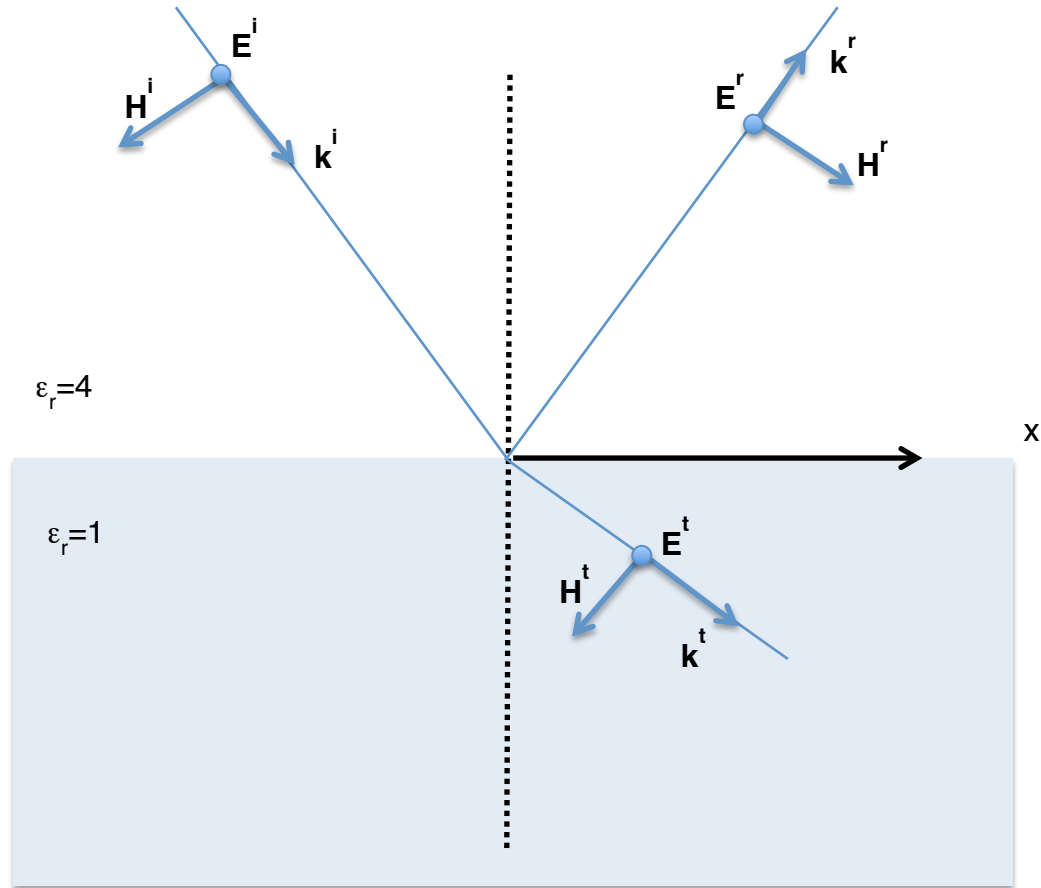


- a) Find the total magnetic flux ( $\phi$ ) passing through the surface of the loop (5 marks).

- b) Find the induced EMF (3 marks).
- c) Find the induced current flowing in the loop (2 marks).
- d) Explain how the induced current satisfies Lenz's law (3 marks).
- e) The external flux density in the region  $y > 0$  changes to:
- $$\mathbf{B} = -0.3 \cos(2\pi \times 10^3 t) \mathbf{a}_z \text{ Wb/m}^2$$
- How does the induced EMF change (2 marks)?

**Question 2 (15 marks).**

A uniform plane wave is obliquely incident on the interface as shown below. Assume that the planar interface is infinite in extent. Assume that  $\mu_r=1$  and  $\sigma=0$  S/m for both materials. The angular frequency is  $\omega=3\times 10^9$  rad/s. The orientation of the electric field is out of the page.



- a) Find the propagation constant ( $\beta$ ) in region 1 ( $\epsilon_r=4$ ) (2 marks).
- b) If the angle of incidence is  $45^\circ$  and the amplitude of the electric field is 10 V/m, find an expression for the incident electric field ( $\mathbf{E}^i$ ) (3 marks).

- c) Find the reflection coefficient ( $\Gamma$ ) (4 marks).
- d) What is the critical angle for this scenario? (2 marks)
- e) Find an expression for the reflected electric field ( $\mathbf{E}^r$ ) (3 marks).
- f) Describe the transmitted electric field ( $\mathbf{E}^t$ ) (1 mark).

**Question 3 (20 marks).**

**A Smith Chart with  $z_L$  and  $z_{in}$  already plotted is provided for this question.**

Show your work on the Smith chart and summarize answers here.

- a) Given that the transmission line has characteristic impedance of  $75\Omega$ , what is the load impedance ( $Z_L$ )? (2 marks)
  
- b) Find the reflection coefficient at the load and the reflection coefficient at  $z_{in}$ . (4 marks)
  
- c) Find the voltage standing wave ratio (s, SWR or VSWR) (2 marks).
  
- d) What is the value of  $Z_{in}$ ? How far from the load is  $Z_{in}$  located? (5 marks)
  
- e) Find the shortest distance from  $z_{in}$  to a voltage minimum. (2 marks)
  
- f) Indicate the location of the open and short on the Smith Chart. (2 marks)
  
- g) If you were to match the load to the line with a quarter-wave transformer, how far from the load would you attach the line and what impedance would you select? (3 marks)



**Question 4 (25 marks).**

You are designing a microwave imaging system to investigate the brain. The incidence field is modeled as a uniform plane wave propagating in free space ( $\epsilon_r=1$ ,  $\mu_r=1$ ,  $\sigma=0$  S/m). The electric field is given by:

$$\mathbf{E}(x,t)=10\cos(3 \times 10^8 t + \beta x) \mathbf{a}_y \text{ V/m}$$

Find:

- a) The phase constant,  $\beta$ . (1 mark)
- b) The wavelength,  $\lambda$ . (1 mark)
- c) The phase velocity,  $v_p$  or  $u$ . (1 mark)
- d) The intrinsic impedance,  $\eta$ . (1 mark)
- e) The polarization. (1 mark)
- f) The magnetic field in phasor form,  $\mathbf{H}_s(x)$ . (2 marks)

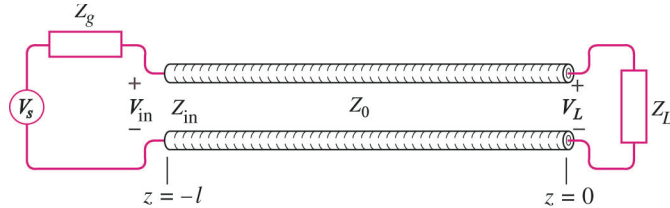
The field is normally incident on an interface located at  $x=0$ . The material in the region  $x<0$  represents skull and has  $\epsilon_r=25$ ,  $\mu_r=1$ ,  $\sigma=0.25$  S/m.

- g) Find the phase constant ( $\beta$ ) in the material (1.5 marks)
  
  
  
  
  
  
  
  
  
  
- h) Find the attenuation constant ( $\alpha$ ) in the material (1.5 marks).
  
  
  
  
  
  
  
  
  
  
- i) Find the intrinsic impedance ( $\eta$ ) of the material (2.5 marks).
  
  
  
  
  
  
  
  
  
  
- j) Find the reflection coefficient ( $\Gamma$ ) (2.5 marks).
  
  
  
  
  
  
  
  
  
  
- k) Find the transmission coefficient ( $T$ ) (2.5 marks).

- l) Find an expression for the reflected electric field ( $\mathbf{E}^r(x,t)$ ). (2 marks)
- m) Find an expression for the transmitted electric field ( $\mathbf{E}^t(x,t)$ ). (2.5 marks)
- n) Calculate the time-average Poynting vector ( $\mathbf{P}_{av}(x)$ ) in the region  $x < 0$ . For microwave imaging, energy must interrogate the object of interest and then propagate to the receivers. Based on your calculations, do you think that this scenario holds promise for microwave imaging of the head? (3 marks)

**Question 5 (20 marks)**

A transmission line with  $Z_0=50\ \Omega$  has length of  $0.3\lambda$ . A load of  $Z_L=20-j20\ \Omega$  is connected at one end, and a generator with  $Z_g=50\ \Omega$  is connected at the other end. The frequency of operation is 1 GHz, and the velocity of propagation on the line is  $0.9\ c$  (where  $c$  is the speed of light in free space). The generator supplies a signal with 10 V peak-to-peak.



- Using the transmission line formula, find the reflection coefficient at the load ( $\Gamma_L$ ). (2 marks)
- Using the transmission line formula, find the input impedance ( $Z_{in}$ ) looking into the  $0.3\lambda$  line terminated with the load. (2 marks)
- Check the value that you calculated for  $Z_{in}$  with the Smith chart (2 marks).

- d) Find the power available to the load ( $P_L$  or  $P_{in}$ ). (2 marks)
- e) Design a series stub tuner to match the load to the line. Specify the location of the stub and length of the open circuited stub required to match the load. Use both wavelengths and physical distance. (6 marks)
- f) Design a shunt stub tuner to match the load to the line. Specify the location of the stub and length of the open-circuited stub required to match the load. Use both wavelengths and physical distance. (6 marks)

**Question 6 (10 marks).**

- a) An electric field is given by:

$$\mathbf{E}(y,t)=40 \cos(2\pi \times 10^6 t - \beta y) \mathbf{a}_x \text{ mV/m}$$

The field is in a dielectric material with  $\epsilon_r=2.4$ . Find the associated displacement current density ( $\mathbf{J}_d$ ). (2 marks)

- b) Consider a time-varying field and a perfect electric conductor. Boundary conditions indicate that (circle the correct statement/s): (1 mark)
- Both inside and outside the PEC, the electric field is purely tangential, while the magnetic field is purely normal.
  - Both inside and outside of the PEC, the magnetic field is tangential to the surface of the PEC, while the electric field is normal.
  - Outside of the PEC, the electric field is purely tangential, while the magnetic field is purely normal.
  - Outside of the PEC, the magnetic field is tangential to the surface of the PEC, while the electric field is normal.
  - The fields are zero everywhere.

- c) Consider the Yagi-Uda antenna from Lab 3. Briefly comment on each of the following statements. (3 marks)

The antenna pattern is omnidirectional, so the antenna orientation does not matter when receiving signals.

Small objects completely block signals from the transmitter.

The dimensions of the antenna (active element, director, reflector and spacing) are important.

- d) A distortionless transmission line has the following parameters:  
 $Z_0=50 \, \Omega$ ,  $\alpha=40 \, \text{mNp/m}$  and  $v_p=2.5 \times 10^8 \, \text{m/s}$ . At 250 MHz, find: (4 marks)

- R
- L
- G
- C

Question	Mark
1	15
2	15
3	20
4	25
5	20
6	10
Total	