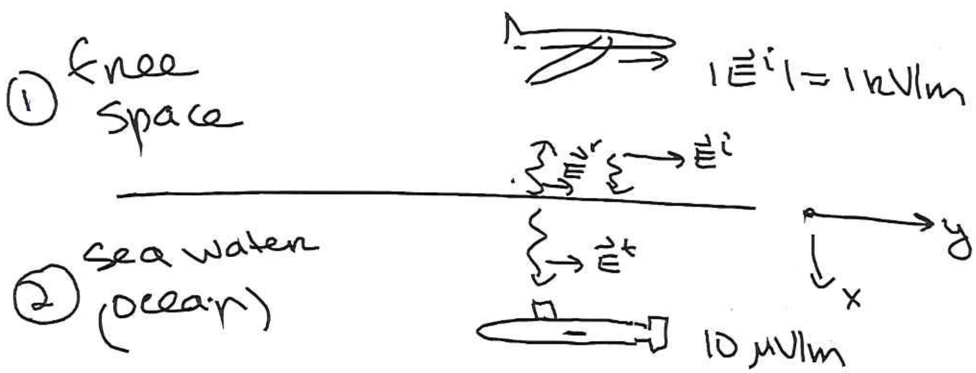
EX// Consider a plane flying over the surface of the ocean + transmitting a 1 MHz signal using a long wire antenna.

a) Assume that the wave transmitted by the antenna is a UPW of $|\vec{E}^i| = 1\,kV/m$.

If submarine's receiver requires minimum signal of $10\,\mu V/m$, determine max. depth for communication.




$\Rightarrow |\vec{E}^t| = 10\,\mu V/m$

$= (T)(1\,kV/m)e^{-\alpha_2 x}$

$\epsilon_R = 81$

$\sigma = 4\,S/m$

$\mu_R = 1$

$$\boxed{\frac{\sigma}{\omega\epsilon} = \frac{4}{(2\pi\times 10^6)(81)(\frac{1}{36\pi}\times 10^{-9})} \approx 900}$$

$T = \dfrac{2\eta_2}{\eta_2 + \eta_1}$ $\eta_1 \rightarrow 120\pi$

$\hookrightarrow \alpha_2 = \beta_2 = 3.97$

$\hookrightarrow \eta_2 = 1.4 \angle 45^\circ$ $+ \eta_1 = 120\pi\ \Omega$

$T = 7.43\times 10^{-3} \angle 44.8^\circ$

$\Gamma = 0.995 \angle 180^\circ$

UDW



$$|\vec{E}^t| = (1000)(7.43\times10^{-3})\,e^{-3.97x}$$

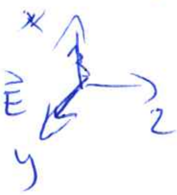
↑ $E^{inc}$

$$= 1\times10^{-5} \Rightarrow 7.43\,e^{-3.97x} = 1\times10^{-5}$$

$$\Rightarrow x = \frac{1}{3.97}\ln(7.43\times10^{5})$$

$$\boxed{x = 3.41\,m}$$

b) $\vec{E}^i(x,t) = 1000\cos(2\pi\times10^6 t - 2.09\times10^{-2}x)\,\vec{a}_y$




$\vec{H}^i(x,t) = \frac{1000}{120\pi}\cos(2\pi\times10^6 t - 2.09\times10^{-2}x)\,\vec{a}_z$

$|T|E^{inc}$ $\quad\beta_2\quad \angle T$

$\vec{E}^t(x,t) = (7.43)\,e^{-3.97x}\cos(2\pi\times10^6 t - 3.99x + \pi/4)\,\vec{a}_y$

$\vec{H}^t(x,t) = \frac{7.43}{1.4}\,e^{-3.97x}\cos(2\pi\times10^6 t - 3.97x + \pi/4 - \pi/4)\,\vec{a}_z$

$\eta_2$ $\quad|\Gamma|E^{inc}$ $\quad$ change direction $\rightarrow \angle T \rightarrow \angle\eta_2$

$\vec{E}^r(x,t) = -995\cos(2\pi\times10^6 t + 2.09\times10^{-2}x)\,\vec{a}_y$

↑ 180° phase shift


$\vec{H}^r(x,t) = \frac{995}{120\pi}\cos(2\pi\times10^6 t + 2.09\times10^{-2}x)\,\vec{a}_z$

c) Power density at surface in water ($x=0$) & power density dissipated in 1 skin depth.

$$\vec{P}_{AV}(x) = \frac{1}{2}\frac{|\vec{E}^t|^2}{|\eta|}\,e^{-2\alpha x}\cos(\theta_\eta)\,\vec{a}_x$$

$P(x,t) = \vec{E}(x,t)\times\vec{H}(x,t)$

⇓

$\vec{P}_{AV}(x) = \frac{1}{2}\,Re\{\vec{E}_s\times\vec{H}_s^*\}$

• $\vec{P}_{AV}(x=0) = \frac{1}{2}\frac{(7.43)^2}{1.4}\cos(45^\circ)\,\vec{a}_x$

$= 13.9\,a_x\ W/m^2$

• skin depth

↳ field is $\frac{1}{e}$ of initial value

B

skin depth $\delta = \frac{1}{\alpha}$

$= \frac{1}{3.97}$

$= 0.252\ m$

Power density dissipated $= \vec{P}_{AV}(x=0) - \vec{P}_{AV}(\delta)$

$= \vec{P}_{AV}(x=0)\left[1 - e^{-2\alpha_2 \delta}\right]$

$= 13.9\left[1 - e^{-2(3.97)(0.252)}\right]$

$= 12\ W/m^2$

Standing wave ratio $\rightarrow$ $S = \frac{1+|\Gamma|}{1-|\Gamma|}$

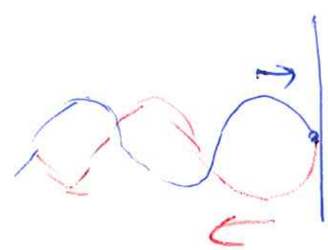
$1 \leq S < \infty$

↳ PEC in region 2

$\rightarrow \eta_2 = 0$

$\rightarrow \Gamma = -1$

$\rightarrow \tau = 0$



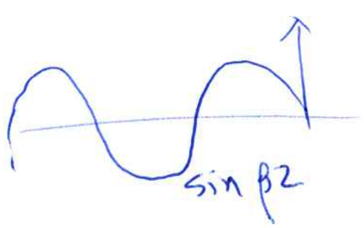
$\rightarrow$ lossless dielectric in region 1

$\vec{E}^{total}(z,t) = \vec{E}^{i}(z,t) + \vec{E}^{r}(z,t)$

$= E_x \cos(\omega t - \beta z)\vec{a}_x - E_x \cos(\omega t + \beta z)\vec{a}_x$

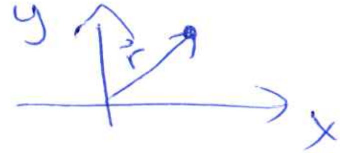
$= E_x\left[\cos(\omega t - \beta z) - \cos(\omega t + \beta z)\right]\vec{a}_x$

$= E_x\left[2 \sin(\omega t) \sin(\beta z)\right]\vec{a}_x$



# Propagation of waves in general directions

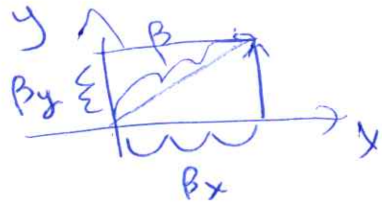
↳ $\vec{r} = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z \Rightarrow$ position vector




↳ $\vec{k} = k_x\vec{a}_x + k_y\vec{a}_y + k_z\vec{a}_z \Rightarrow$ propagation or wave number vector

$= \sqrt{k_x^2 + k_y^2 + k_z^2}$

$= \omega\sqrt{\mu\epsilon}$ (lossless)



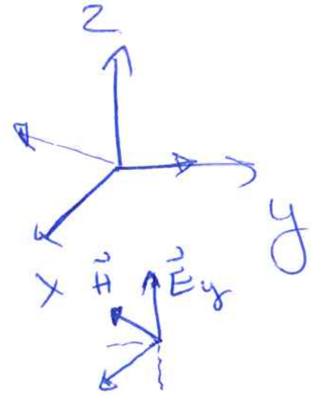

↳ $\vec{E}_s(\vec{r}) = \vec{E}_0 e^{-j(\vec{k}\cdot\vec{r})}$

$\uparrow$ vector

e.g. $\vec{k} = k_x\vec{a}_x + k_z\vec{a}_z$

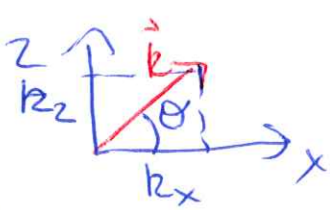
$\vec{r} = x\vec{a}_x + z\vec{a}_z$

$\vec{E}_s(\vec{r}) = E_0 e^{-j(k_x x + k_z z)}\vec{a}_y$




↳ $\theta \rightarrow$ angle of propagation w.r.t. x-axis

$\theta = \tan^{-1}(k_z/k_x)$




↳ $\lambda$ + $v_p$ depend on direction

$\rightarrow$ in direction of $\vec{k}$: $\lambda = \dfrac{2\pi}{|\vec{k}|}$

$\rightarrow$ in x-direction $\lambda_x = \dfrac{2\pi}{k_x}$