Transmission Line Models Tutorial Questions for Lecture 2

1. The sending-end voltage, current, and power factor of a three-phase transmission line spanning from Red Deer to Calgary are found to be 260 kV, 300 A, and 0.9 lagging, respectively. The *ABCD* constants are:

$$A = D = 0.8904\angle 1.34^o$$

$$B = 186.82\angle 79.45^o\ \Omega$$

$$C = 1.131 \times 10^{-3}\angle 90.41^o \mathrm{S}$$

Find:

(i) The corresponding receiving-end phase voltage and current

(ii) Receiving-end power factor.

In-Class Example Lecture 2 Solution

1. **Given:**

$|I_S| = 300$ A; $\cos\theta_S = 0.9$ lag; $ABCD$ constants

Required: $[\,V_R, I_R, \cos\theta_R]$

The sending-end phase voltage, $|V_S| = 260/\sqrt{3} = 150.111$ kV

The sending-end current is:

$I_S = |I_S|\langle -\cos^{-1} 0.9 = 300\langle -25.84^o$ A; We have used negative sign because the power factor is lagging

Recall,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \frac{1}{AD-BC}\begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$$

But $AD - BC = 1$

Therefore, $\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$

This means that:

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} 0.8904\angle 1.34^o & -186.82\angle 79.45^o \\ -1.131 \times 10^{-3}\angle 90.41^o & 0.8904\angle 1.34^o \end{bmatrix} \begin{bmatrix} 150.111 \times 10^3 \\ 300\angle -25.84^o \end{bmatrix}$$

$$V_R = (0.8904\angle 1.34^o)(\,150.111 \times 10^3) + (\,-186.82\angle 79.45^o)(300\angle -25.84^o)$$

$$= (133658.8\angle 1.34^o) + (\,-56046\angle 53.61^o)$$

$$= (133622.3 + j3125.65) + (-33250.9 - j45116.9)$$

$$= (100371.4 - j41991.25)$$

$$V_R = \mathbf{108.8\angle -22.7^o\ kV}$$

$$I_R = (-1.131 \times 10^{-3}\angle 90.41^o)(\,150.111 \times 10^3) + (0.8904\angle 1.34^o)(300\angle -25.84^o)$$

$$= (-169.77\angle 90.41^o) + (\,267.12\angle -24.5^o)$$

$$= (1.215 - j169.76) + (243.07 - j110.77)$$

$$= (244.28 - j280.53)$$

$$I_R = \mathbf{372\angle -48.95^o\ A}$$

(ii) The receiving-end power factor is:

$$\cos\theta_R = \cos(\theta_{VR} - \theta_{IR})$$

$$= \cos(\mathbf{-22.7^o + 48.9^o}) = 0.897\ \textbf{lagging}$$