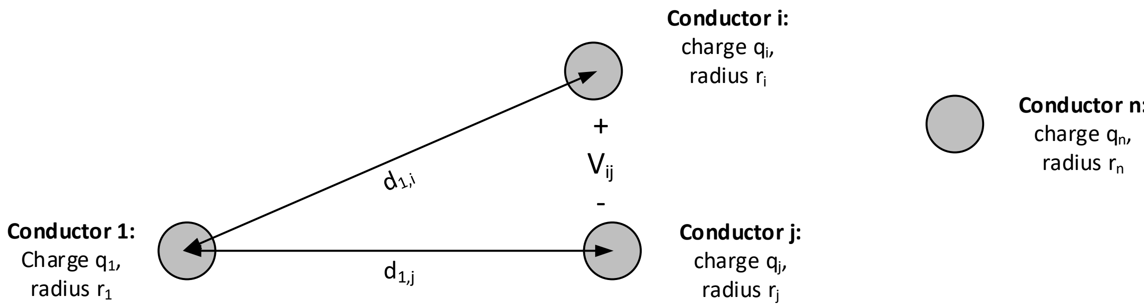
(Distributed) Capacitance of solid and stranded conductors

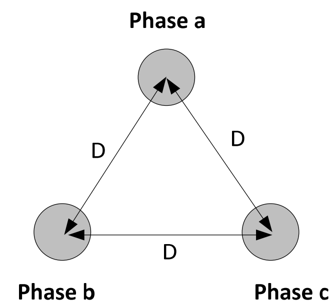
For n conductors with AC charge q_k (C/m) uniformly distributed along the conductor:



Voltage V_{ij} due to the electric fields from all n conductors: $V_{ij} = \frac{1}{2\pi\varepsilon} \sum_{k=1}^{n} q_k . \ln\frac{d_{j,k}}{d_{i,k}}$ (i)

For a conductor in free space, permittivity $\varepsilon = \varepsilon_0 = 8.85 \times 10^{-12}$ F/m

<u>For a balanced 3ϕ line with symmetric alignment</u>:



Even though the equation above was derived for solid conductors, the electric field of a stranded conductor with outside radius r is almost identical to the electric field of a solid conductor with radius r. So, the ensuing equations are valid for solid or stranded conductors.

Re-write eq(i) for a balanced three phase system with symmetric alignment:

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left(q_a.\ln\frac{D}{r} + q_b.\ln\frac{r}{D} + q_c.\ln\frac{D}{D}\right) = \frac{1}{2\pi\varepsilon}\left(q_a.\ln\frac{D}{r} + q_b.\ln\frac{r}{D}\right) \quad \text{(ii)}$$

Similarly, $$V_{ac} = \frac{1}{2\pi\varepsilon}\left(q_a.\ln\frac{D}{r} + q_c.\ln\frac{r}{D}\right) \quad \text{(iii)}$$

Also know that: $V_{an} = \frac{1}{3}(V_{ab} + V_{ac})$

$$\therefore\ V_{an} = \frac{1}{3}\frac{1}{2\pi\varepsilon}\left(2q_a.\ln\frac{D}{r} + (q_b + q_c).\ln\frac{r}{D}\right) = \frac{1}{3}\frac{1}{2\pi\varepsilon}\left(2q_a.\ln\frac{D}{r} - q_a.\ln\frac{r}{D}\right) = \frac{1}{2\pi\varepsilon}\left(q_a.\ln\frac{D}{r}\right)$$

Using $C_{an} = \frac{q_a}{V_{an}}$ and the equation above, we can arrive at an expression for the line-to-neutral capacitance of a solid or stranded conductor:

$$C_{an} = \frac{2\pi\varepsilon}{\ln\frac{D}{r}}$$

$C_{an} = C_{bn} = C_{cn}$ in a balanced system