ENEL 476 - 2017 - Assignment #2

10.18 $\mu = 750\mu_0$

$\epsilon = 5\epsilon_0$

$\sigma = 10^{-6}$ S/m

$f = 10$ MHz

a) $\dfrac{\sigma}{\omega\epsilon} = \dfrac{10^{-6}}{(2\pi\times10^{7})(5)(\frac{1}{36\pi}\times10^{-9})}$

$= (10^{-4})(18/5)$

$\ll 1 \Rightarrow$ consider lossless

b) $\beta = \omega\sqrt{\mu\epsilon}$

$= \dfrac{(2\pi\times10^{7})}{3\times10^{8}}\sqrt{(750)(5)}$

$= 12.8$ rad/m

$\lambda = \dfrac{2\pi}{\beta}$

$= 2\pi/12.8$

$= 0.49$m

c) $\beta z = (12.8 \text{ rad/m})(2\text{m})$

$= 25.6$ rad

d) $\eta = \sqrt{\mu/\epsilon}$

$= 120\pi\sqrt{\dfrac{750}{5}}$

$= 4617\ \Omega$

$\approx 4.62\text{k}\ \Omega$

②

10.32 $\mu = \mu_0$

$\epsilon = 24\epsilon_0$

$\sigma = 4$ S/m

$f = 2$ GHz

$e^{-\alpha z} = 10^{-5}$

$-\alpha z = \ln(10^{-5})$

$z = \dfrac{\ln(10^{-5})}{-\alpha}$

$\alpha \Rightarrow \dfrac{\sigma}{\omega\epsilon} = \dfrac{\cancel{4}}{(2\pi)(2\times10^{9})(\cancel{24})\left(\frac{1}{36\pi}\times10^{-9}\right)}$

$= \dfrac{3}{2}$

$\alpha = (2\pi\times10^{9})\sqrt{\dfrac{24\mu_0\epsilon_0}{2}\left[\sqrt{1+\left(\tfrac{3}{2}\right)^2}-1\right]}$

$= 130$ Np/m

$z = \dfrac{\ln(10^{-5})}{-130}$

$= 0.089$ m

10.34 $\vec{E} = 2\sin(\omega t - \beta x)\,\vec{a}_y - 5\sin(\omega t - \beta x)\,\vec{a}_z$ V/m

in phase $\Rightarrow$ linear

10.60 μ_0

$\epsilon = 4\epsilon_0$

$$\vec{E} = 12\cos(\omega t - 40\pi x)\vec{a}_z \text{ V/m}$$

a) $\omega = ? \Rightarrow \beta = 40\pi$

$$\beta = \omega\sqrt{4\epsilon_0\mu_0}$$

$$\omega = \frac{40\pi}{\sqrt{4\epsilon_0\mu_0}}$$

$$= (20\pi)(3\times10^8)$$

$$= 60\pi\times10^8$$

$$= 6\pi\times10^9 \text{ rad/s}$$

b)

R1	R2
$\mu = \mu_0$	$\mu = \mu_0$
$\epsilon = 4\epsilon_0$	$\epsilon = 3.2\epsilon_0$

$$\eta_1 = \sqrt{\frac{\mu_0}{4\epsilon_0}} = 60\pi$$

$$\eta_2 = \sqrt{\frac{\mu_0}{3.2\epsilon_0}} = 210.7\ \Omega$$

$$\Gamma = \frac{210.7 - 188.5}{(210.7 + 188.5)}$$
$$= 0.056$$

$$T = 1 + \Gamma$$
$$= 1.056$$

$$\vec{E}^r = (12)(0.056)\cos(6\pi \times 10^9 t + 40\pi x)\vec{a}_z$$
$$= 0.67\cos(6\pi \times 10^9 t + 40\pi x)\vec{a}_z$$

$$\vec{E}^t = (12)(1.056)\cos(6\pi \times 10^9 t - \beta_2 x)\vec{a}_z$$

$$\beta_2 = \omega\sqrt{3.2\mu_0\epsilon_0}$$
$$= \frac{\overset{2}{\cancel{6}}\pi \times 10^{\cancel{9}}}{\cancel{3 \times 10^8}}(\sqrt{3.2})$$
$$= 112.4$$

$$\therefore \vec{E}^t = 12.67\cos(6\pi \times 10^9 t - 112.4x)\vec{a}_z$$

Additional question

$\epsilon_r = 45$

$\sigma = 0.9$ S/m

$\mu_r = 1$

$f = 404$ MHz

- $|\vec{E}| = 10$ V/m next to antenna
- propagation $+z$
- $\vec{H}$ in $+x$

a) $\alpha \Rightarrow$ $$\frac{\sigma}{\omega\epsilon} = \frac{0.9}{(2\pi)(404\times10^6)(45)\left(\frac{1}{36\pi}\times10^{-9}\right)}$$

$= 0.89 \Rightarrow$ use full formulas

$\alpha = 23.38$ Np/m

b) $\beta = 61.41$ rad/m

c) $$\eta = \frac{\sqrt{\dfrac{(\mu_0)}{45\epsilon_0}}}{\left[1+(0.89)^2\right]^{1/4}}$$

$= 48.5\ \Omega$

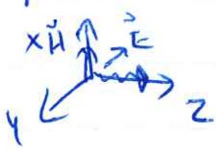
$\tan 2\theta_\eta = 0.89$

$\theta_\eta = 20.83^\circ$

$= 0.36$ rad

d) assuming $z=0$ at antenna,

$$\vec{E}(z,t) = -10e^{-23.38z}\cos(2\pi\times404\times10^6 t - 61.41z)\vec{a}_y$$



e) $\vec{H}(z,t) = \frac{10}{48.5} e^{-23.38z} \cos(2\pi \times 404 \times 10^{6} t - 61.41z - 0.36)\vec{a}_x$

f) $\vec{P}_{AV}(z) = \frac{1}{2}\frac{(10)^2}{48.5}\cos(0.36)\, e^{-46.8z}\,\vec{a}_z$

$= 0.965\, e^{-46.8z}\,\vec{a}_z$

g) skin | air

$45\epsilon_0$ | ϵ_0

μ_0 | μ_0

$\sigma = 0.9$ | $\sigma = 0$

3mm | $\eta = 120\pi$

z=0 | z=3mm

$$T = \frac{2(120\pi)}{(120\pi + 49.5\angle 0.36)}$$

$$= 1.78\angle -0.04$$

h) $\vec{E}^{t} = ?$ At interface, $|\vec{E}^{inc}| = -10e^{-(23.38)(0.003)}$

$= -9.32$

$\vec{E}^{inc} = -9.32\cos(2\pi \times 404 \times 10^{6} t - (61.41)(0.003))\vec{a}_y$

$\vec{E}^{t} = -\vec{a}_y(9.32)(1.78)\left[\cos\left((2\pi)(404\times10^{6})t - 8.46(z-0.003) - 0.188\right)\right]$

$-0.188 \rightarrow (-61.41)(0.003) - 0.04$

$\vec{E}^{t} = -16.6\cos\left[(2\pi \times 404 \times 10^{6})t - 8.46(z-0.003) - 0.188\right]\vec{a}_y$

i) $\vec{P}_{AV}(z) = \frac{1}{2} \frac{(16.6)^2}{120\pi} \vec{a}_z$ in air

$= 0.366 \text{ W/m}^2$

$\vec{P}_{AV}(z=0) = 0.965 \text{ W/m}^2$

$\therefore$ ~38% of power near antenna