ENEL441 QUIZ Aid Sheet

Aid Sheet

One sided Laplace Transform $F(s) = \int_{-\infty}^{\infty} f(t)e^{-st}dt$, $f(t) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} F(s)e^{st}ds$

$$f(t) \qquad F(s) \qquad f(t) \qquad F(s)$$

$$1 \qquad \delta(s) \qquad \dot{f} \qquad sF(s) - f(0^{-})$$

$$\delta(t) \qquad 1 \qquad \ddot{f} \qquad s^{2}F(s) - sf(0^{-}) - \dot{f}(0^{-})$$

$$u(t) \qquad 1/s \qquad \ddot{f} \qquad s^{3}F(s) - s^{2}f(0^{-}) - s\dot{f}(0^{-}) - \ddot{f}(0^{-})$$

$$t^{m}u(t) \qquad m!/s^{m+1} \qquad \int f(t)dt \qquad F(s)/s$$

$$e^{-at}u(t) \qquad 1/(s+a) \qquad \lim_{t \to \infty} f(t) \qquad \lim_{s \to 0} sF(s)$$

$$\frac{1}{(m-1)!}t^{m-1}e^{-at}u(t) \qquad 1/(s+a)^{m} \qquad \sin(at)u(t) \qquad \frac{a}{s^{2}+a^{2}}$$

$$f(t-T) \qquad F(s)e^{-sT} \qquad \cos(at)u(t) \qquad \frac{s}{s^{2}+a^{2}}$$

$$tf(t) \qquad -\frac{d}{ds}F(s) \qquad x(t)*y(t) \qquad X(s)Y(s)$$
* implies convolution as $x(t)*y(t) = \int_{-\infty}^{\infty} x(\tau)y(t-\tau)d\tau$

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$$x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau) y(t-\tau) d\tau$$

Poles of second order system $s^2 + 2D\omega_n s + \omega_n^{\ 2} = 0$,

$$s = -\sigma \pm j\omega_{\scriptscriptstyle d} \quad \sigma = D\omega_{\scriptscriptstyle n} \quad \omega_{\scriptscriptstyle d} = \omega_{\scriptscriptstyle n} \sqrt{1-D^2} \quad \text{ rise time } \tau_{\scriptscriptstyle r} = \frac{2.2}{pole}$$

State Space
$$\dot{x} = Ax + Br$$
 $y = Cx + Dr$ $H(s) = C(sI - A)^{-1}B + D$

Electric motor with parameters $\left\{R,K_{T},K_{b}\right\}$, R internal resistance, current I flowing through motor gives torque $T=K_TI$, $\,\omega\,$ is rotation rate then induced back EMF voltage is $V_{\scriptscriptstyle b}=K_{\scriptscriptstyle b}\omega\,$

roots of quadratic
$$ax^2 + bx + c = 0$$
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$