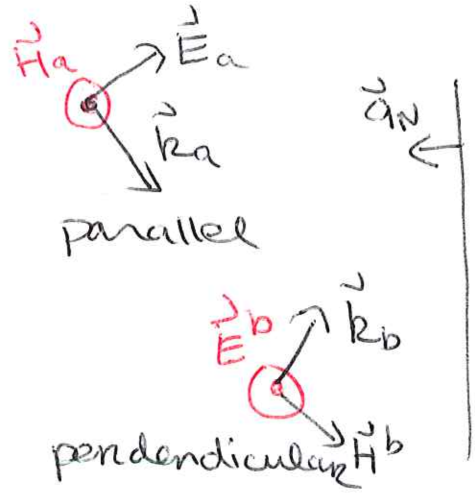
Oblique Incidence tx|Rx

① 2020

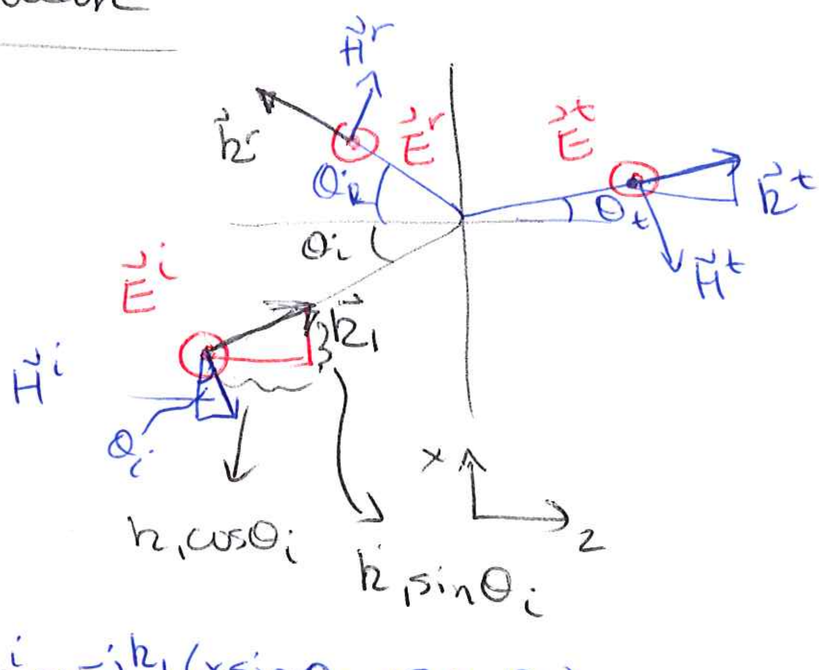
↳ plane of incidence → $\vec{k}_i + \vec{a}_N$

↳ polarization = orientation of $\vec{E}$ w.r.t. POI

paper = POI



Perpendicular



$|\vec{k}_1| = \omega\sqrt{\mu_1\epsilon_1}$

$$\vec{E}^i_s = E^i e^{-jk_1(x\sin\theta_i + z\cos\theta_i)}\,\vec{a}_y$$

$$\vec{H}^i_s = \frac{E^i}{\eta_1} e^{-jk_1(x\sin\theta_i + z\cos\theta_i)}\left[-\cos\theta_i\,\vec{a}_x + \sin\theta_i\,\vec{a}_z\right]$$

$$\vec{E}^r_s = \Gamma E^i e^{-jk_1(x\sin\theta_R - z\cos\theta_R)}\,\vec{a}_y$$

$$\vec{E}^t_s = \tau E^i e^{-jk_2(x\sin\theta_t + z\cos\theta_t)}\,\vec{a}_y$$

$\vec{E}_{1,tan} = \vec{E}_{2,tan}$ & $\vec{H}_{1,tan} = \vec{H}_{2,tan}$

↳ $\theta_i = \theta_r$

↳ $n_1 \sin\theta_i = n_2 \sin\theta_t$ → Snell's Law

↳ $n_1 = c\sqrt{\mu_1 \epsilon_1}$

$= \dfrac{c \rightarrow \text{in vacuum}}{v_p \rightarrow \text{in material}}$

$$\Rightarrow \Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$T_\perp = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

Ex// $\vec{E}^i = 8\cos(\omega t - 4x - 3z)\vec{a}_y$ V/m

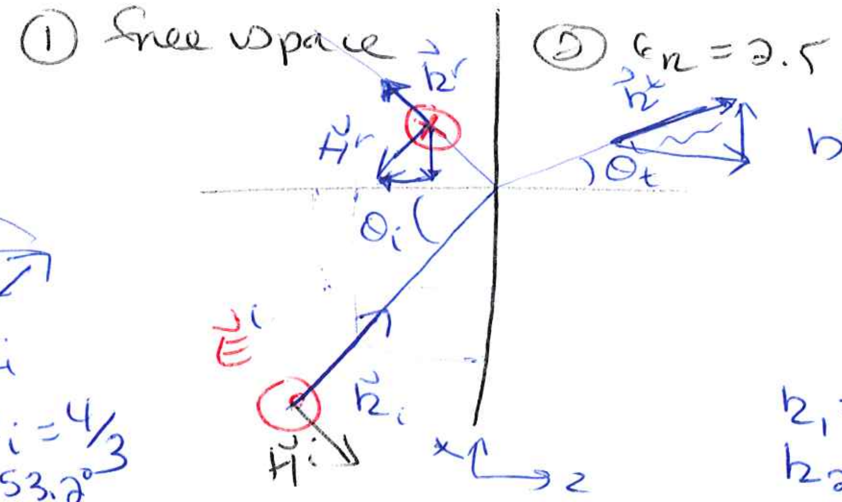
→ incident on interface at $z=0$ with $\mu_r = 1$, $\sigma = 0$, $\epsilon_r = 2.5$ from free space

a) angle of incidence ⇒ $\vec{k}_i$

b) reflected electric field → $\Gamma_\perp$ → θ_t ⇒ Snell's law → $\Gamma_\perp$

c) transmitted electric field → $T_\perp$, $\vec{k}_2$

$$\boxed{\begin{aligned} k &= |\vec{k}| \\ &= \beta \\ &= \omega\sqrt{\mu_0 \epsilon_0 \epsilon_r \mu_r} \end{aligned}}$$



b) $\eta_1 = 120\pi\ \Omega$

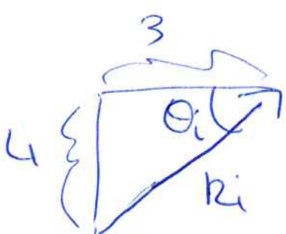
$\eta_2 = \sqrt{\dfrac{\mu_0}{2.5\epsilon_0}} = 238.4\ \Omega$

$\theta_t \Rightarrow$

$k_1 \sin\theta_i = k_2 \sin\theta_t$

$k_1 = \omega\sqrt{\mu_0 \epsilon_0}$

$k_2 = \omega\sqrt{\mu_0 2.5\epsilon_0}$



$\tan\theta_i = 4/3$

$\theta_i = 53.2°$

$$\Gamma_\perp = \frac{(238.4)\cos(53.2^\circ) - (120\pi)\cos(30.4)}{(238.4)\cos(53.2^\circ) + (120\pi)\cos(30.4)}$$

$$= -0.39$$

$$\sin\theta_t = \frac{k_1}{k_2}\sin\theta_i = \frac{\sin\theta_i}{\sqrt{2.5}}$$

$$\theta_t = 30.4^\circ$$

$$\vec{E}^r = (8)(\underbrace{-0.39}_{\Gamma_\perp})\cos(\omega t - 4x + 3z)\vec{a}_y$$

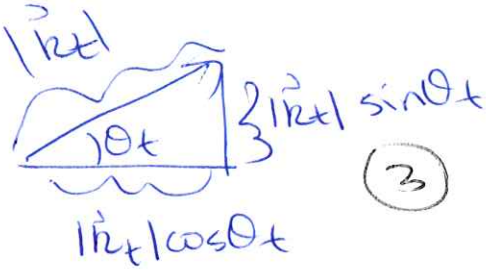
c) $\vec{E}^t \rightarrow T_\perp = 1 + \Gamma_\perp = 0.61$

$$\vec{E}^t = (8)(0.61)\cos(\omega t - \text{—} - \text{—})\vec{a}_y$$

$$|\vec{k}_t| = |\vec{k}_2| = \omega\sqrt{\mu_0 2.5\epsilon_0} \quad (2)$$

$$|\vec{k}_i| = \omega\sqrt{\mu_0\epsilon_0}$$

$$\vec{k}_i = 4\vec{a}_x + 3\vec{a}_z \quad (1)$$



$$|\vec{k}_i| = 5$$

$$\omega = 1.5\times10^9 \text{ rad/s}$$

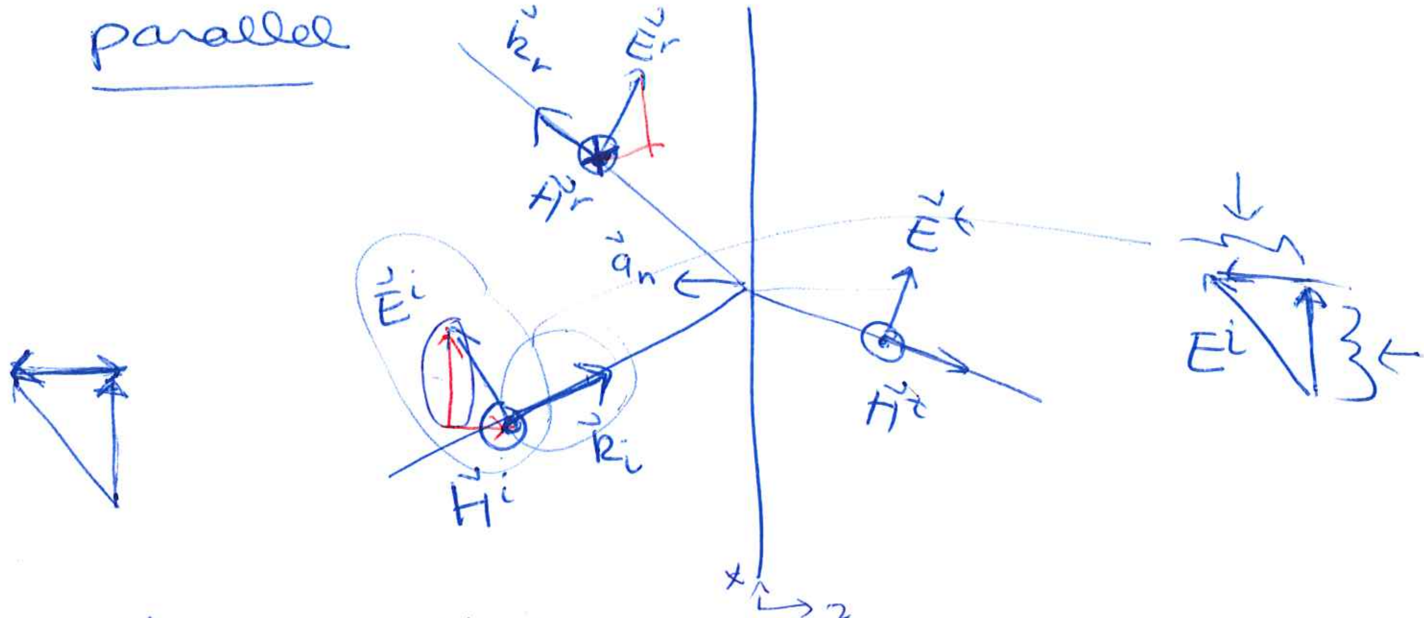
$$|\vec{k}_t| = \frac{1.5\times10^9}{3\times10^8}\sqrt{2.5} = 7.91 \text{ rad/m}$$

$$|\vec{k}_t|\sin\theta_t = 7.91\sin(30.4^\circ) = 4$$

$$|\vec{k}_t|\cos\theta_t = 6.82$$

$$\vec{E}^t = \underbrace{4.9}_{E T_\perp}\cos(1.5\times10^9 t - \underbrace{4x}_{|\vec{k}_t|\sin\theta_t} - (\underbrace{6.82}_{|\vec{k}_t|\cos\theta_t} z))\vec{a}_y$$

parallel



$$\vec{E}^i = E^i e^{-jk_1(x\sin\theta_i + z\cos\theta_i)}\left(\boxed{\cos\theta_i \vec{a}_x} - \sin\theta_i \vec{a}_z\right)$$

$$\vec{E}^r = \Gamma E^i e^{-jk_1(x\sin\theta_r - z\cos\theta_r)}\left(\cos\theta_r \vec{a}_x + \sin\theta_r \vec{a}_z\right)$$

$$\vec{E}^t = T E^i e^{-jk_2(x\sin\theta_t + z\cos\theta_t)}\left(\cos\theta_t \vec{a}_x - \sin\theta_t \vec{a}_z\right)$$

$\Rightarrow$ boundary conditions

$$\Gamma_{\parallel} = \frac{n_2\cos\theta_t - n_1\cos\theta_i}{n_2\cos\theta_t + n_1\cos\theta_i}$$

$$T_{\parallel} = \frac{2 n_2 \cos\theta_i}{n_2\cos\theta_t + n_1\cos\theta_i}$$

Special case #1: Brewster angle

↳ transmission = 1 + reflection = 0

↳ $\Gamma_{\parallel} = 0$ with $\mu_{r_1} = \mu_{r_2} = \mu_0$ + $\sigma = 0$

($\Gamma_{\perp} \to \mu_{r_1} \neq \mu_{r_2}$; $\epsilon_{R1} = \epsilon_{R2}$)

$$\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = 0$$

$$\Rightarrow \eta_2 \cos\theta_t = \eta_1 \cos\theta_i$$

$$\frac{\cos\theta_t}{\sqrt{\epsilon_{R2}}} = \frac{\cos\theta_i}{\sqrt{\epsilon_{R1}}} \quad \theta_i = ?$$

$$\frac{\cos^2\theta_t}{\epsilon_{R2}} = \frac{\cos^2\theta_i}{\epsilon_{R1}}$$

$$\frac{1 - \sin^2\theta_t}{\epsilon_{R2}} = \frac{1 - \sin^2\theta_i}{\epsilon_{R1}}$$

$\Rightarrow$ Snell's law: $k_1 \sin\theta_i = k_2 \sin\theta_t$

$$\Rightarrow \sin^2\theta_t = (\sin^2\theta_i)\left(\epsilon_{R1}/\epsilon_{R2}\right)$$

$$\Rightarrow \tan(\theta_i) = \sqrt{\frac{\epsilon_{R2}}{\epsilon_{R1}}}$$

$$\Rightarrow \theta_B = \tan^{-1}\left(\sqrt{\frac{\epsilon_{R2}}{\epsilon_{R1}}}\right)$$