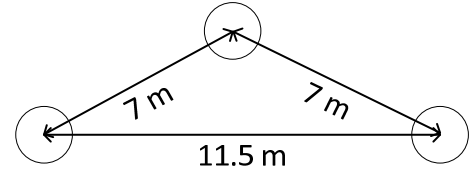Name: ID:

Problem 1

a) The Figure below shows the arrangement of a single-circuit three-phase transmission line being operated at 60 Hz. If the GMR (geometric mean radius) of the conductor used is 11.4 mm, find:
- The inductance per kilometer per phase. [3.5 marks]
- The inductive reactance per kilometer per phase. [1.5 marks]

Geometric mean distance of the conductors (GMD), $D_{eq} = \sqrt[3]{D_{12}.D_{23}.D_{31}}$

$D_{eq} = \sqrt[3]{7*7*11.5} = 8.260 \text{ m}$

$L = 2 \times 10^{-7} \times \ln(\frac{D_{eq}}{D_s})$ [H/m]

$\therefore L = 2 \times 10^{-7} \times \ln\left(\frac{8.260}{0.0114}\right) = 1.317 \times 10^{-6} \text{ H/m}$

$X_L = 2\pi f L$

Thus $X_L = 2\pi * 60 * 1.317 \times 10^{-6} * 1000 = 0.4965 \, \Omega/\text{km}$

b) A three-phase 60-Hz transmission line has its conductors arranged in a triangular formation such that two of the distances between conductors are 7.62 m and the third is 12.80 m. The conductors are ACSR Osprey. If the line is 200 km long, find the total capacitance to neutral in microfarads. The diameter of the Osprey line is 22.33 mm, and the permittivity of the medium is 8.85 × 10^{-12} F/m. [5 marks]

Geometric mean distance of the conductors (GMD), $D_{eq} = \sqrt[3]{D_{12}.D_{23}.D_{31}}$

$D_{eq} = \sqrt[3]{7.62*7.62*12.80} = 9.058 \text{ m}$

$C_n = 2\pi\epsilon / \ln(D_{eq}/r)$ [F/m]

$r = \frac{0.02233}{2} = 0.011165 \text{ m}$

$\therefore C_n = \frac{2\pi \times 8.854 \times 10^{-12}}{\ln\left(\frac{9.058}{0.011165}\right)} = 8.305 \times 10^{-6} \, \mu F/m$

For a 200 km line, the capacitance to neutral is: $C_n = 200 \times 10^3 \times 8.305 \times 10^{-6} = 1.66 \, \mu F$

Problem 2

The sending-end voltage, current, and power factor of a three-phase transmission line are found to be 260 kV, 300A, and 0.9 lagging, respectively. The ABCD parameters are:

A = D = 0.8904∠1.34⁰   B = 186.82∠79.45⁰ Ω   C=1.131×10−3∠90.41⁰ S

a) Calculate the corresponding receiving-end voltage and current [9 marks]

Hint: $\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \frac{1}{AD-BC}\begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$

Hint 2: AD − BC = 1 for this line

b) Calculate receiving-end power factor [1 mark]

The sending-end phase voltage, V_S is: $260/\sqrt{3} = 150.111$ kV

The sending-end current is: $I_S = 300∠\text{-cos}^{-1}(0.9 \text{ lagging}) = 300∠−25.84^{\circ}$ A

$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \frac{1}{AD-BC}\begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$ But $AD − BC = 1$ Therefore, $\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$

This means that:

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1}\begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix}\begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} 0.8904∠1.34^{o} & -186.82∠79.45^{o} \\ -1.131 \times 10^{-3}∠90.41^{o} & 0.8904∠1.34^{o} \end{bmatrix}\begin{bmatrix} 150.111 \times 10^{3} \\ 300∠-25.84^{o} \end{bmatrix}$$

$\bar{V}_R = (0.8904∠1.34^{o})(150.111 \times 10^{3}) + (−186.82∠79.45^{o})(300∠−25.84^{o})$
$\bar{V}_R = \mathbf{108.8∠−22.7^{o}}$ **kV**

$\bar{I}_R = (−1.131 \times 10^{-3}∠90.41^{o})(150.111 \times 10^{3}) + (0.8904∠1.34^{o})(300∠−25.84^{o})$
$\bar{I}_R = \mathbf{372∠−48.9^{o}}$ **A**

b) The receiving-end power factor is:

$$\text{Cos } \theta = \cos(\theta_{VR} − \theta_{IR})$$

$$= \cos(−\mathbf{22.7^{o}} + \mathbf{48.9^{o}}) = 0.897 \text{ lagging}$$