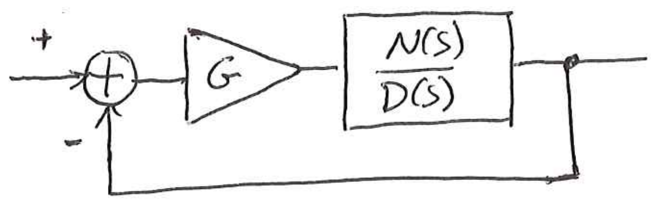
Part B Bode and Nyquist



Root locus determines closed loop poles as loop gain is varied.

need poles / zeros of $H_{OL}(s) = \frac{N(s)}{D(s)}$

what if $H_{OL}(s)$ not in polynomial form?

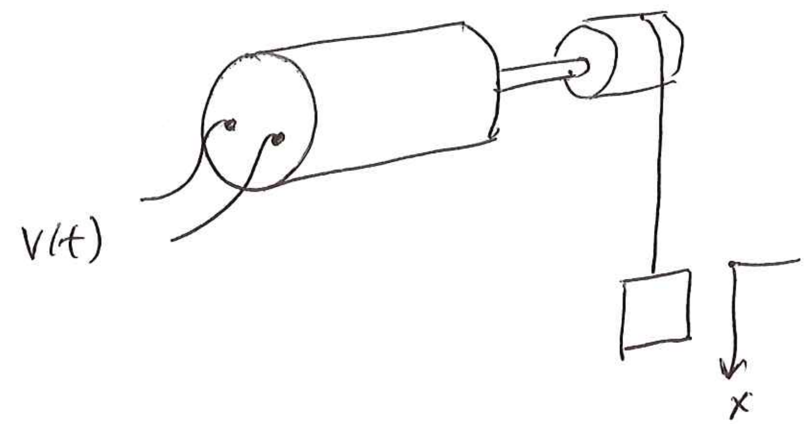
Suppose $H_{OL}(s) = e^{-sT_d} \frac{N(s)}{D(s)}$

or what if $H_{OL}(s)$ only measured frequency response?

Say we only have $H_{OL}(j2\pi f_n)$

$$f_n = f_0 + n\Delta f$$

Typical Application



measure $x(t)$ based on applied $v(t)$

$$v(t) = V_0 \sin(2\pi f_n t)$$

$$x(t) = c_n \sin(2\pi f_n t + \phi_n)$$

Tabulate measured data f_n, $\frac{c_n}{V_0}$, ϕ_n

gives us $H_p(j f_n)$ for $\underbrace{n = 1, 2, \dots N}_{N \text{ measurements}}$

does not give us the general $H_p(s)$.

Can not apply root locus directly.

Possible approach

$$H_p(jf_n) \Rightarrow \left. \frac{\sum_{m=0}^{M} b_m s^m}{\sum_{n=0}^{N} a_n s^n} \right|_{s=j2\pi f_n}$$

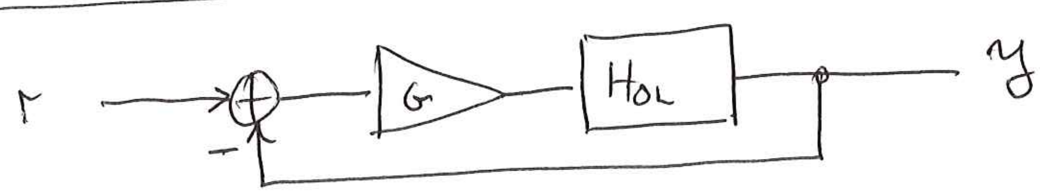
find a_n, b_m assuming $\{a_n, b_n\} \in \mathbb{R}$

What order of polynomial model do we use?

Then use root locus

Another direct approach use Bode and Nyquist plots.

Consider a feedback loop with gain G



If $\angle G H_{OL}(j\omega) = 180^\circ + n\, 360^\circ$ $\quad n = 0, \pm 1, \pm 2, \ldots$

The the phase around loop is $m\, 360^\circ$

$m = 0, \pm 1, \pm 2 \ldots$

If $|G H_{OL}(s)| > 1$ then loop will go unstable. Hence from measured values of

$H_{OL}(j 2\pi f_n) \quad n = 1, 2 \ldots N$

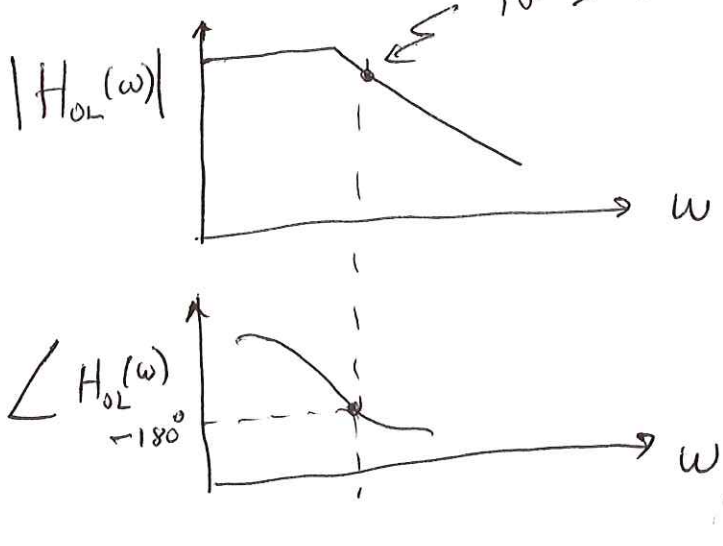
We can interpolate and determine values of G where instability will occur.

Closed loop transfer function $\quad H_{CL}(s) = \dfrac{G H_{OL}(s)}{\underbrace{1 + G H_{OL}(s)}}$

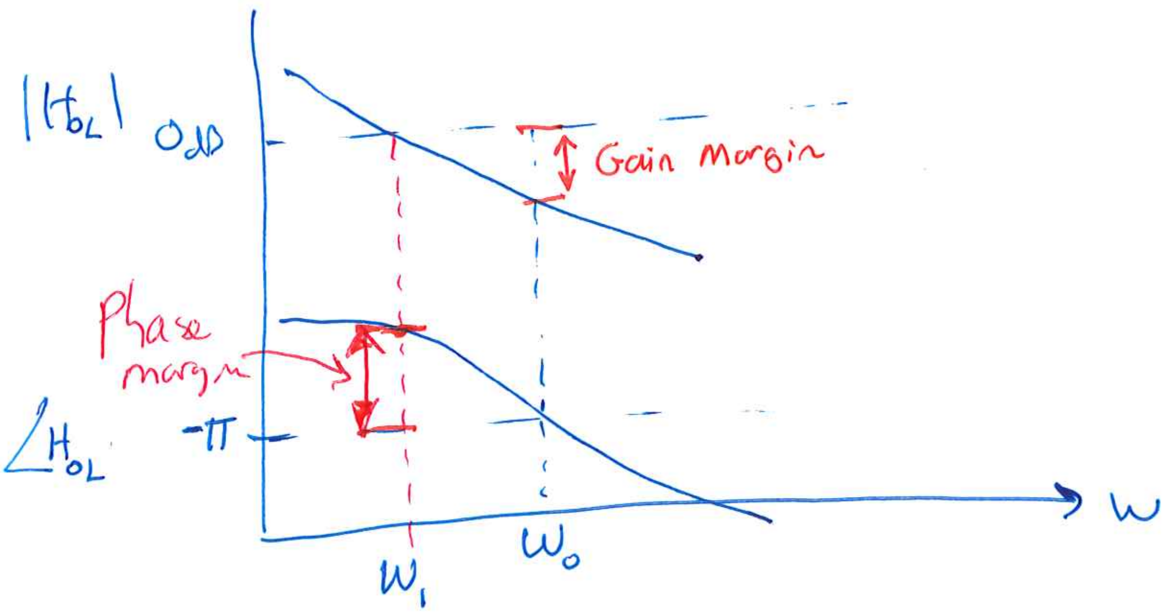
instability when denominator goes to 0

$G H_{OL}(s) = -1$

if > 0 dB then closed loop will be unstable.



Gain Margin & Phase Margin



Gain Margin - how much the loop gain can be increased at the frequency ω_0 where $\angle H_{OL} = -\pi$ before $|H_{OL}(\omega_0)|$ gets to be 0 dB

Phase Margin When $|H_{OL}(\omega_1)| \to 0$ dB

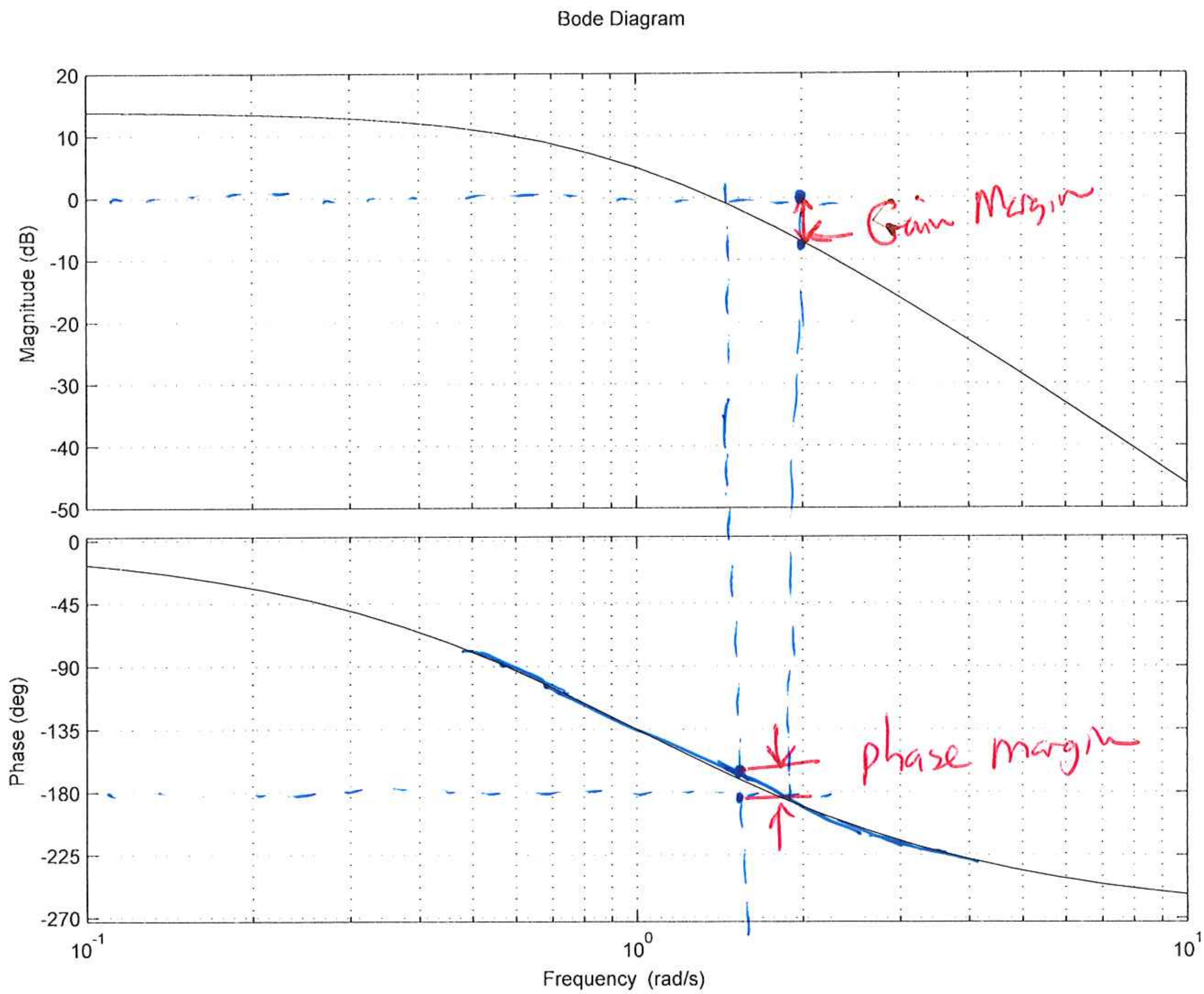
Phase margin is $\angle H_{OL} + \pi$.

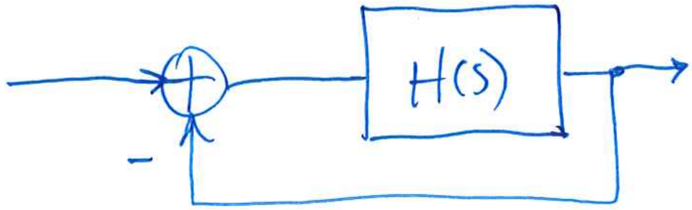
Example $H_{OL}(s) = \dfrac{5}{(s+1)^3}$

Example

$$H(s) = \frac{5}{(s+1)^3}$$

```
H = zpk([], [-1;-1;-1], 5)
bode(H)
```

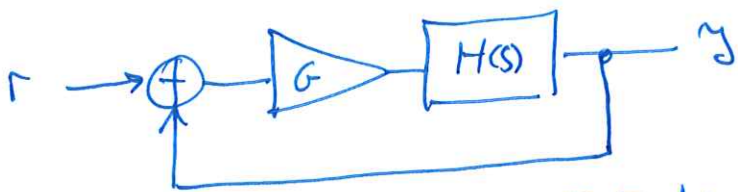




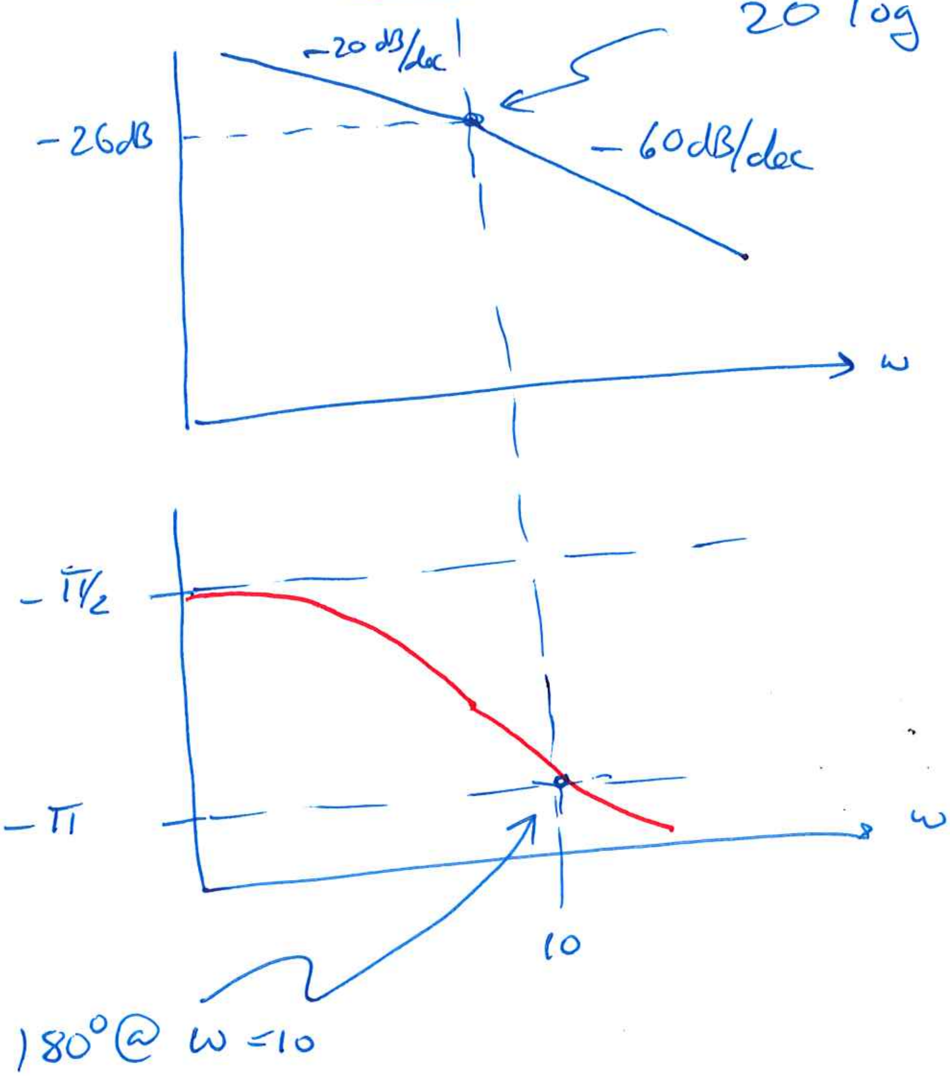
Example

Find the maximum loop gain that results in a stable response

$$H(s) = \frac{100}{(s+10)^2 s}$$



$$20 \log \left(\frac{100}{200 \cdot 10} \right) = -26\,dB$$



G can be up to 26 dB before instability

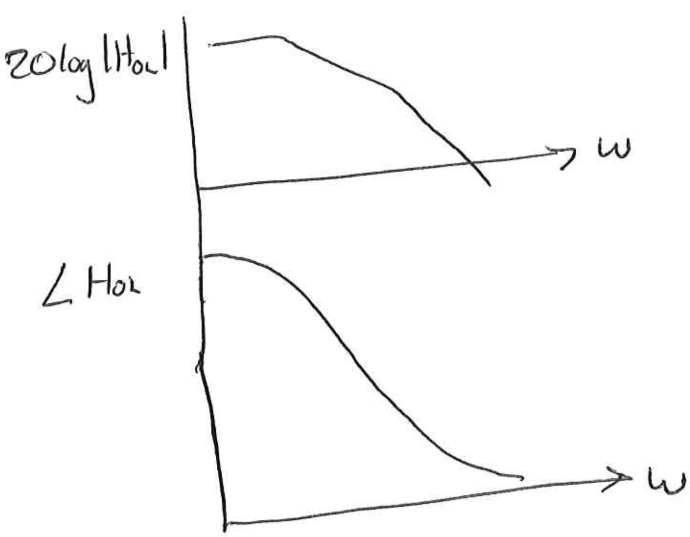
Practical application of Bode Plot where Root locus cannot help.

H_{OL} given as data samples

w	$\lvert H_{OL}(w) \rvert$	$\angle H_{OL}(w)$
0.1	10	-.1
0.5	9	-.5
1	8.5	-.6
1.1	8.4	-.65
1.5	–	–
2	–	.
10	0.1	-3.6

Can be irregulary Sampled.

Interpolate data and draw bode plot



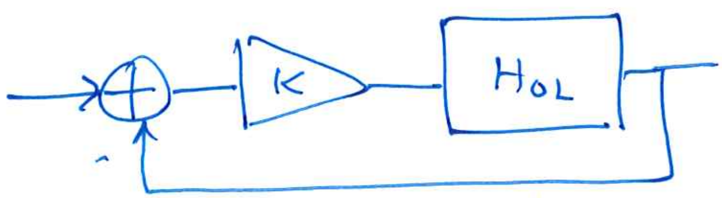
determine the phase and gain margin

In chemical plants, $H_{OL}(w)$ can often be measured in real time and it can be varying.
Need to adjust compensator with time. Bode is suitable as it gives a visualization of what is going on

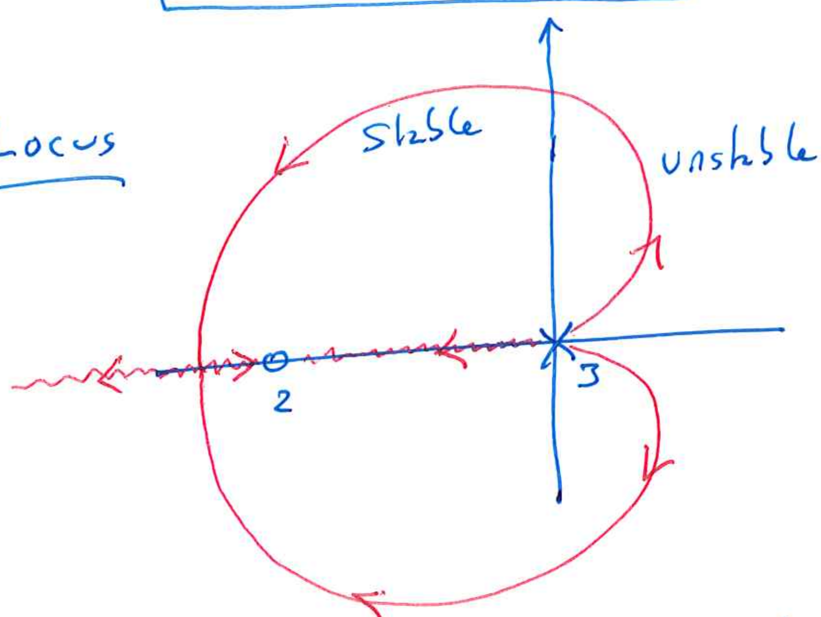
A confusion with Bode Phase and Gain margin

If phase slope is increasing at point where it crosses $-\pi$ This does not indicate an instability.

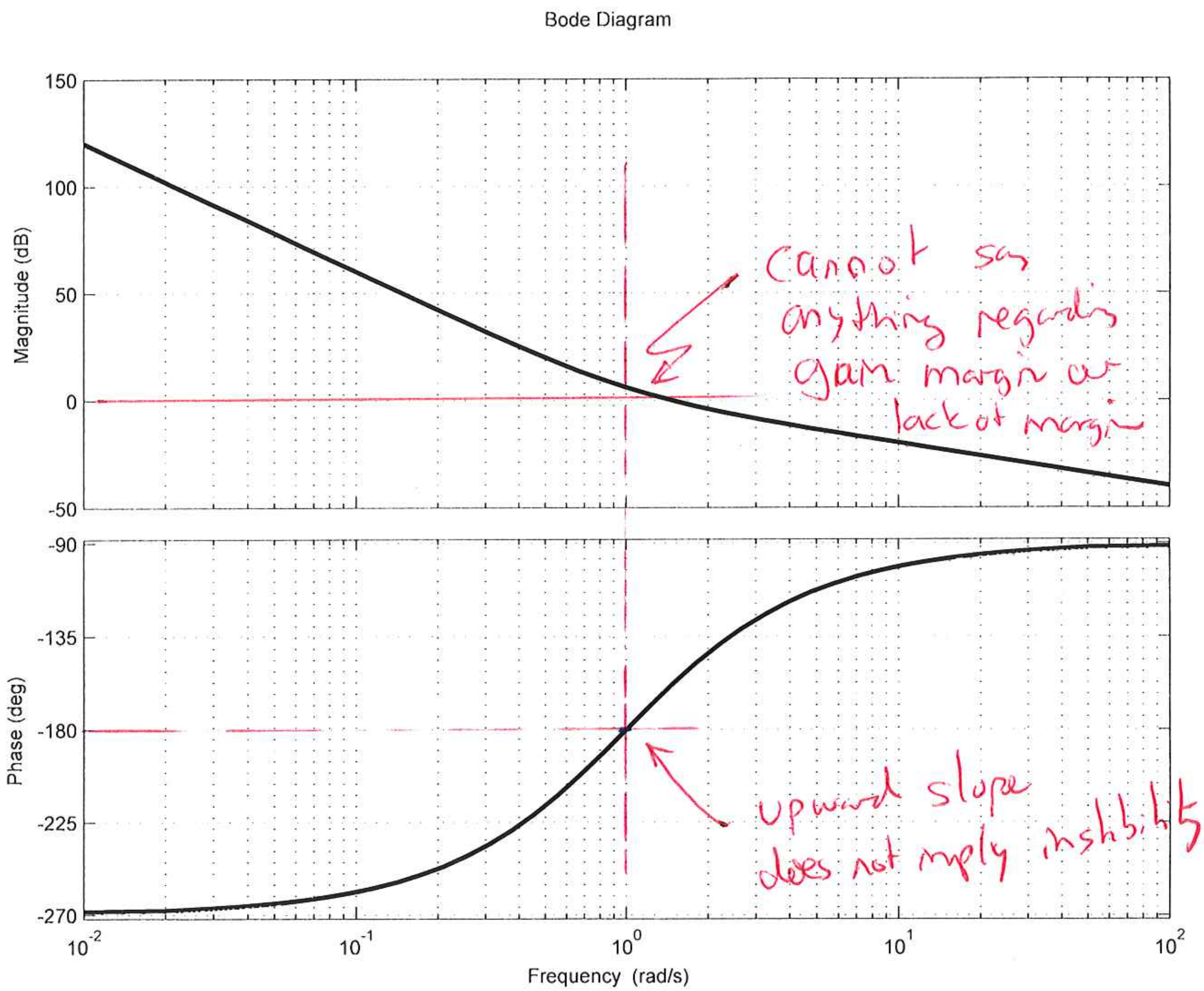
Example $H_{OL}(s) = \frac{(1+s)^2}{s^3}$ as in PLL lab.



Root Locus



unstable for small K, stable for large K

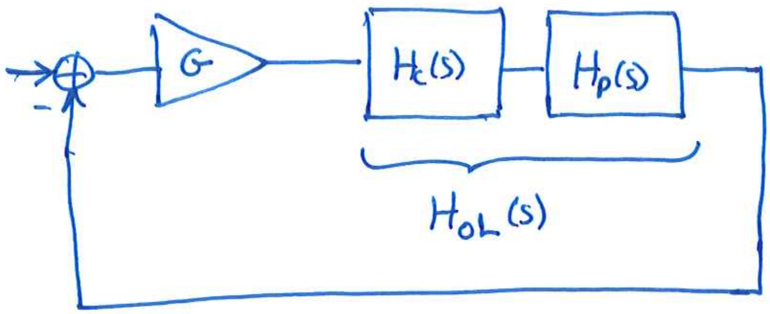


For this type of problem you have to use Root Locus or Nyquist.

Nyquist Fundamentals

Nise 10.3-10.6

Define our feedback loop as:



The closed loop response from any input to any output is:

$$H_{CL} = \frac{\sim}{1 + G\,H_{OL}(s)}$$

just means same function of s in numerator that is not important here.

Marginal stability

$$s = j\omega$$

$$1 + G\,H_{OL}(j\omega) = 0$$

Nyquist Plot

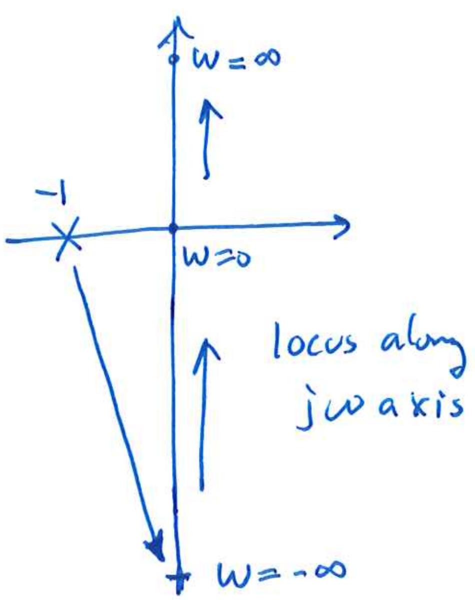
$H_{OL}(j\omega)$ for $-\infty < \omega < \infty$

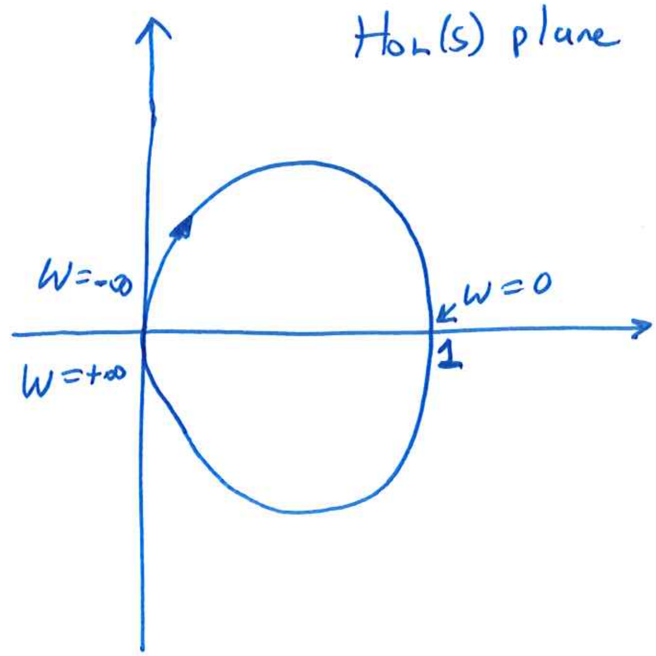
How do we use Nyquist?

find G such that $\underbrace{-\frac{1}{G} = H_{OL}(j\omega)}_{\text{condition for marginal stability.}}$

Example

$$H_{OL}(s) = \frac{1}{s+1}$$





Start at $s = -j\infty$ $\quad |H_{OL}| = 0$

move up $j\omega$ axis note phasor angle changes from $-90°$ towards zero as $\omega \to 0$

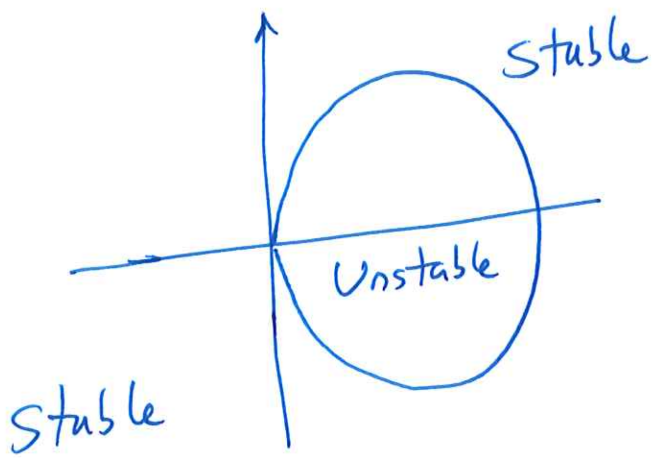
Hence locus starts at $+90°$ and goes towards $0°$

(See example 10.4 pg 555-557 in Nise.)

Plug H_{OL} into Matlab and get a Nyquist Plot.

```
HOL = tf(1, [1, 1])
Nyquist (HOL)
```

Back to Nyquist Plot of $H_{OL}(s) = \frac{1}{s+1}$



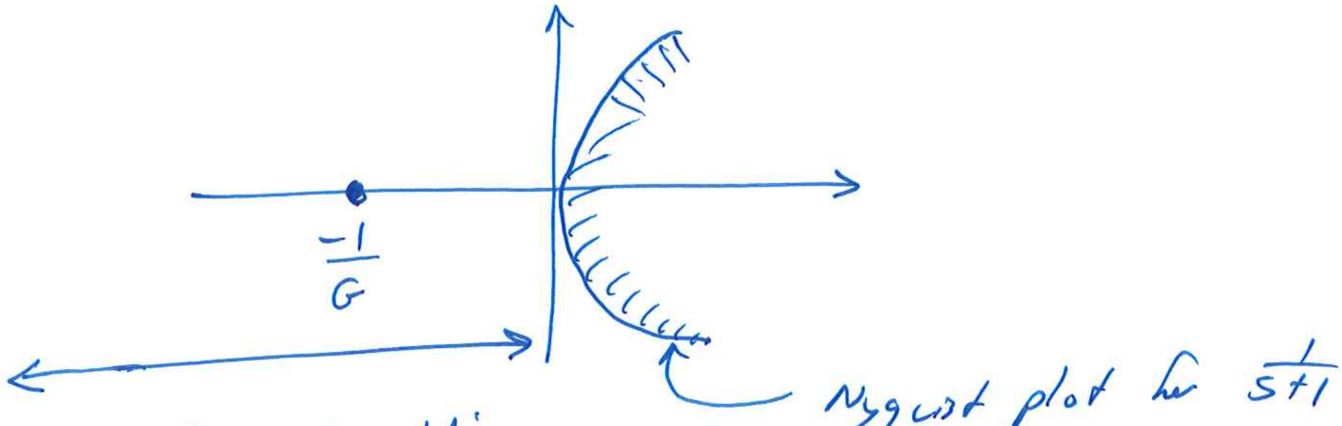
<u>Nyquist stability condition</u>

The negative feedback loop is stable if the point $-1/G$ is outside any encirclement of the Nyquist plot.

Hence the labelling for stable and unstable regions in plot above.

Stability for region of $G \in$ Real

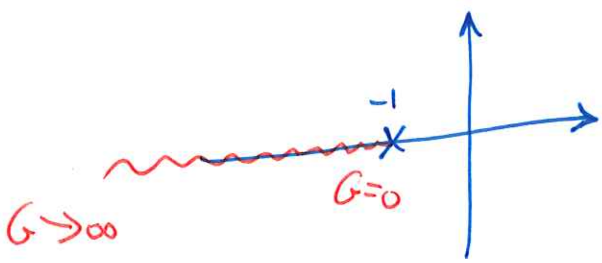
$G > 0$



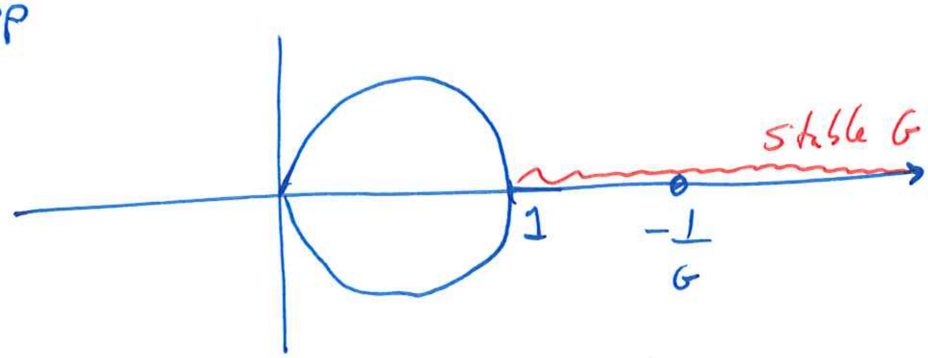
G stable in this region

$0 < G < \infty$

Note this is consistent with Root Locus for $\frac{1}{s+1}$



But there is another region where G results in a stable loop



$-1 < G < 0$

This range of $G < 0$ is not covered by the Root locus we have looked at.

$G > 0$ & negative feedback, 180° root locus
8 - rules
familiar with this

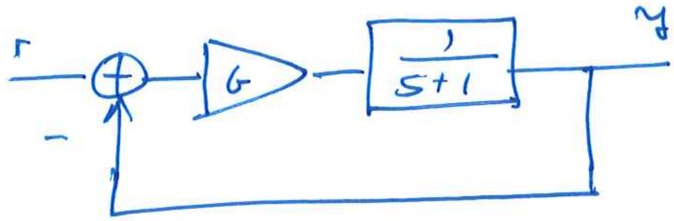
$G < 0$ & negative feedback
$G > 0$ & positive feedback
- 0° root locus
- new rules
- less common
- have not studied in 441.

In matlab
rlocus (H_{OL}) 180° root locus
rlocus ($-H_{OL}$) 0° root locus.

Going back to the Nyquist plot we have the two ranges of G giving stable behavior:

$G > 0$
$-1 < G < 0$
combine these regions
$G > -1$
$G \in$ Real

Let us do a direct calculation of stability region by calculating the closed loop pole directly

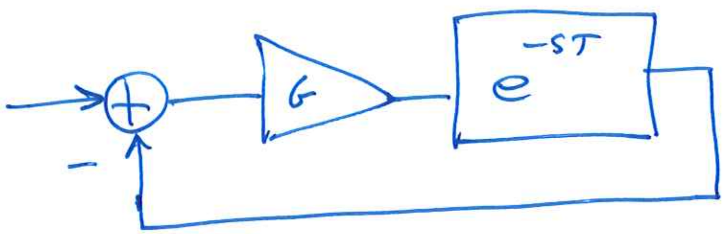


$$H_{CL}(s) = \frac{\frac{G}{s+1}}{1+\frac{G}{s+1}} = \frac{G}{s+(1+G)}$$

$G > -1$ for stability $\Rightarrow$ agrees with Nyquist plot.

Now consider $H_{OL}(s)$ to be a pure delay.

$H_{OL}(s) = e^{-sT} \Longleftrightarrow \delta(t-T)$ (delay of T)

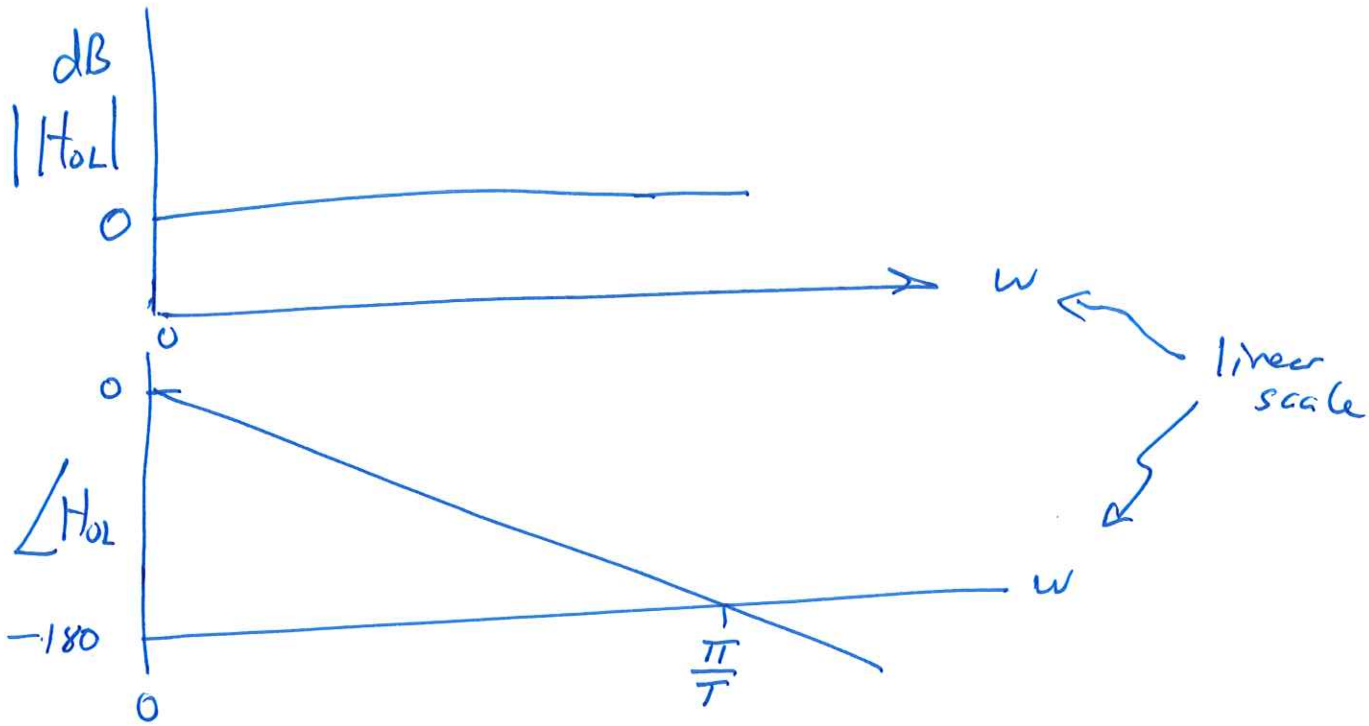


Nyquist plot of $e^{-j\omega T}$

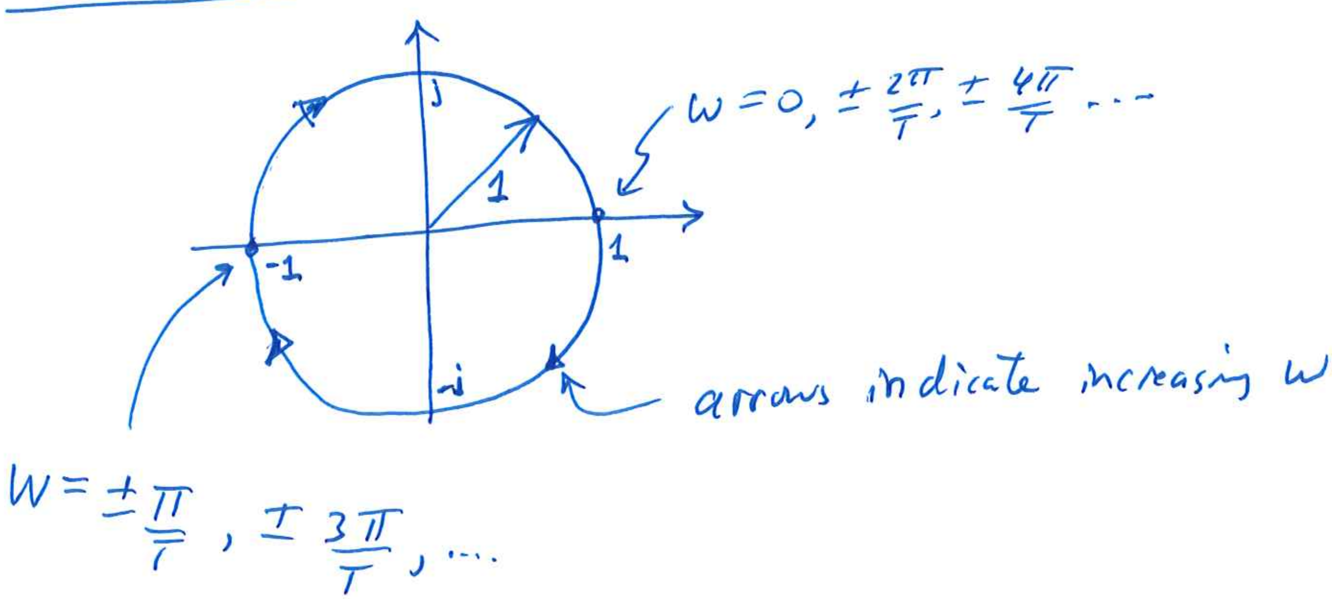
$$\left| e^{-j\omega T} \right| = 1 \quad \text{for all } \omega$$

$$\angle e^{-j\omega T} = -\omega T$$

Bode Plot of $H_{OL}(j\omega) = e^{-j\omega T}$



Nyquist Plot of $H_{OL}(j\omega) = e^{-j\omega T}$



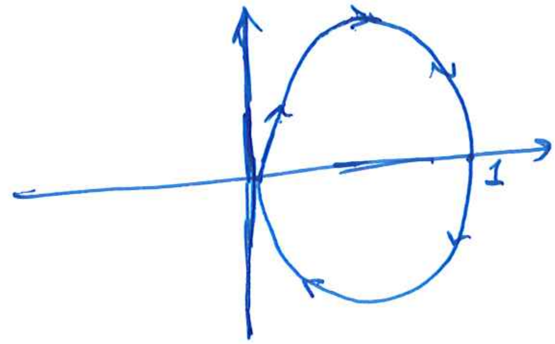
Again outside of region that is encircled is stable. Hence $|G| < 1$ for stability as point $-\frac{1}{G}$ will be outside encirclement of Nyquist plot.

An advantage with Nyquist over root locus is that a pole-zero model of $H_{OL}(s)$ is not required.

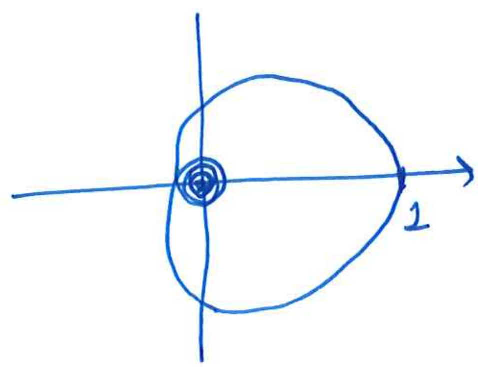
Can accomodate delays directly in Nyquist.

eg. $H_{OL}(s) = \dfrac{e^{-sT}}{s+1}$

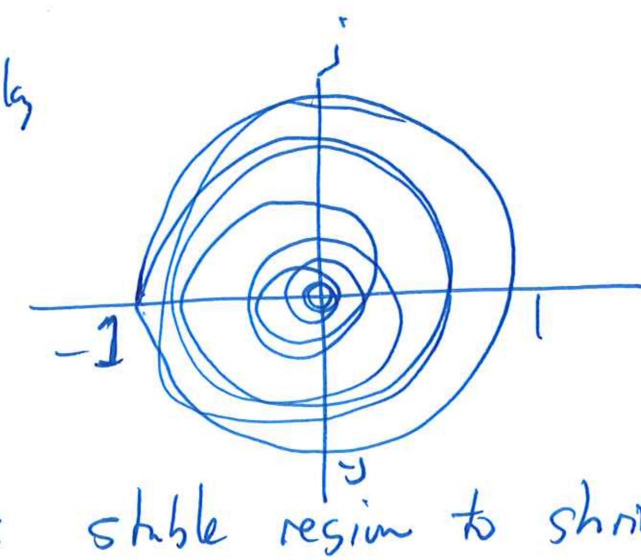
$H_{OL}(jw)$ for no delay



$H_{OL}(jw)$ for small delay $T \ll 1$



$H_{OL}(jw$ for very large delay $T \gg 1$

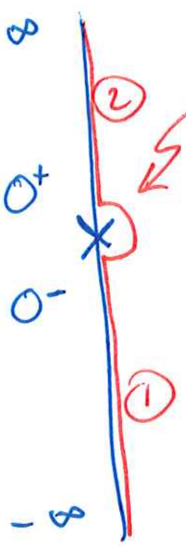


Note how delay causes stable region to shrink and unstable region to grow.

Issue with poles on the $j\omega$ axis

Consider $H_{OL}(s) = \frac{1}{s}$

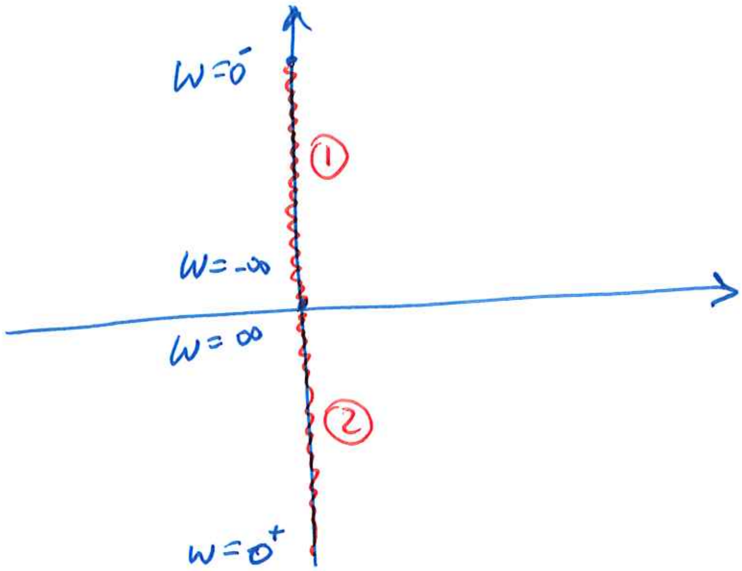
A problem with application of Nyquist is that the contour in ω has to be on the right side of any open loop poles. In this case we have a pole at $s=0$



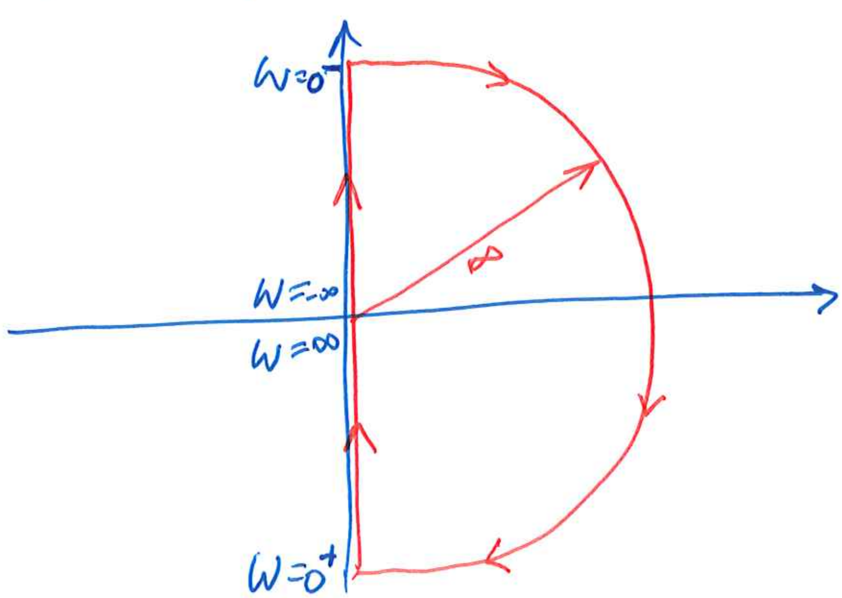
contour in s makes a small detour around pole such that it is to the left of the contour.

Draw Nyquist of segment $\omega = -\infty$ to $\omega = 0^-$ and the segment of $\omega = 0^+$ to $\omega = \infty$.

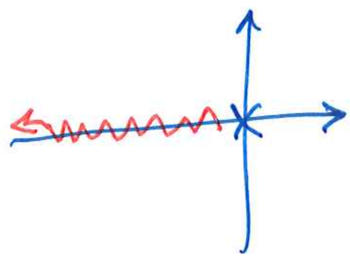
Then consider the portion of the contour from $\omega = 0^-$ to $\omega = 0^+$.



Then consider the contour from 0^- to 0^+ and note the pole is the only contribution to the Nyquist plot. We go in an infinitesimal semi circle. for 180° ccw Therefore the Nyquist plot should do the opposite. Have an infinite radius and 180° cw. Then we can draw the complete locus as below:



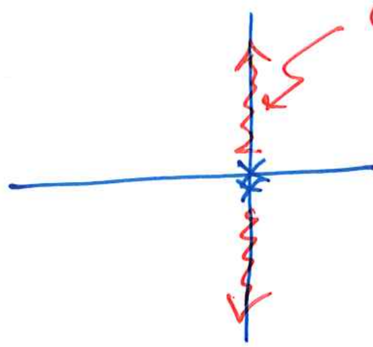
Now consider the stability based on the Nyquist plot. It reveals that $-\infty < -\frac{1}{G} < 0$ or $0 < G < \infty$. Note this agrees with Root locus.



<u>Example</u> $H_{OL}(s) = \frac{1}{s^2}$

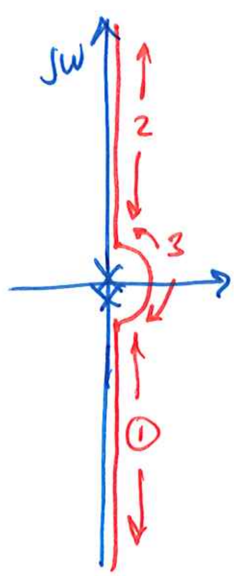
From root locus we have that this is marginally stable closed loop for all positive values of G. What does Nyquist say?

First root locus

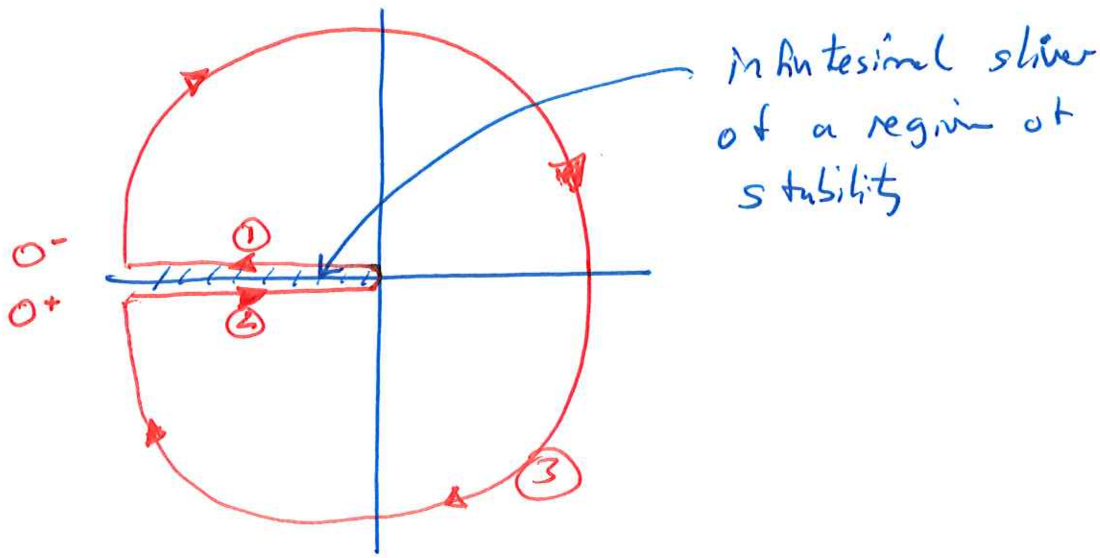


closed loop poles on $j\omega$ axis for all $G>0$ marginally stable.

<u>Nyquist</u>



Contour segments	
①	$-\infty < \omega < 0^-$
②	$0^+ < \omega < \infty$
③	0^- to 0^+



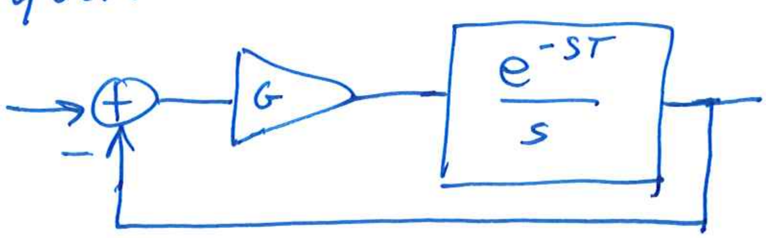
Only a sliver of infintesimal width is not enclosed and can be considered stable.

However as the region is infintesimal it is considered marginally stable.

Note we detour around two poles at $s=0$. Arc in splane is $180°$ but times two poles is $360°$. Hence for the $180°$ ccw contour we do $360°$ cw in Nyquist.

Phase Margin

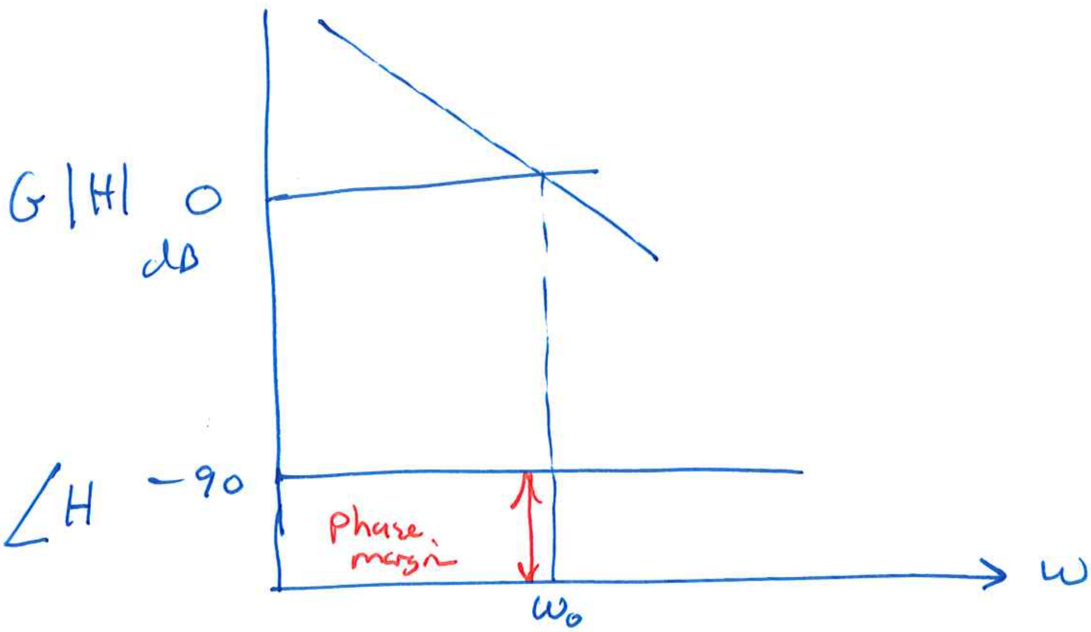
Consider the problem of an integrator with a delay. The question is how much delay can we tolerate?



We group the delay with the gain as

$$\underbrace{G e^{-j\omega T}}_{G_0} \cdot \underbrace{\frac{1}{s}}_{H(s)}$$

Consider Bode plot for $T = 0$

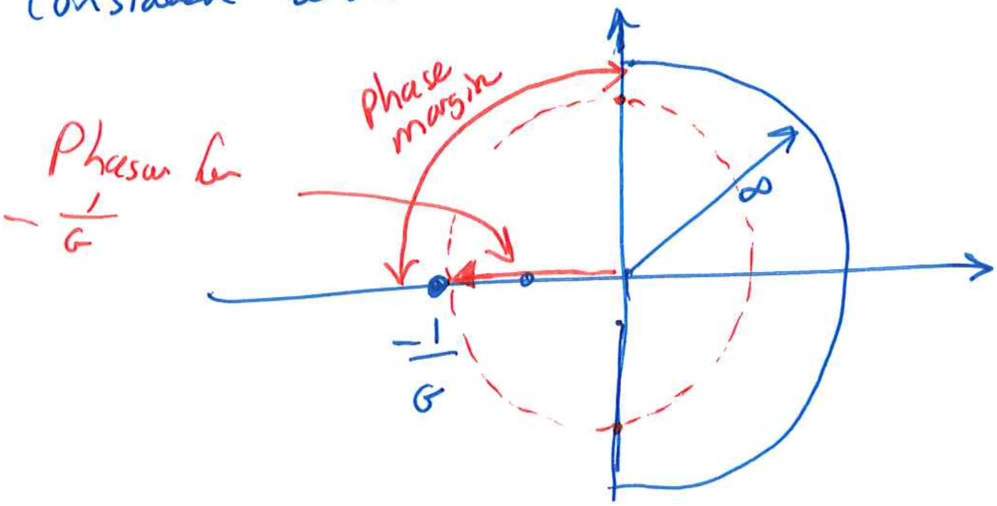


We have a phase margin of 90° as the open loop gain drops to 0 dB at ω_0

Hence we can tolerate a delay of $T\omega_0 = \frac{\pi}{2}$ before instability sets in.

What does Nyquist say?

The Nyquist plot for $H_{OL}(s) = \frac{1}{s}$ was considered before as below:



now we have a complex gain G. The phase that can be applied to G before instability is shown in the Nyquist plot by the rotation of the phasor at $-\frac{1}{G}$.

The phase margin is the angle of rotation that needs to be applied to this phasor such that it intercepts the Nyquist plot encirclement.

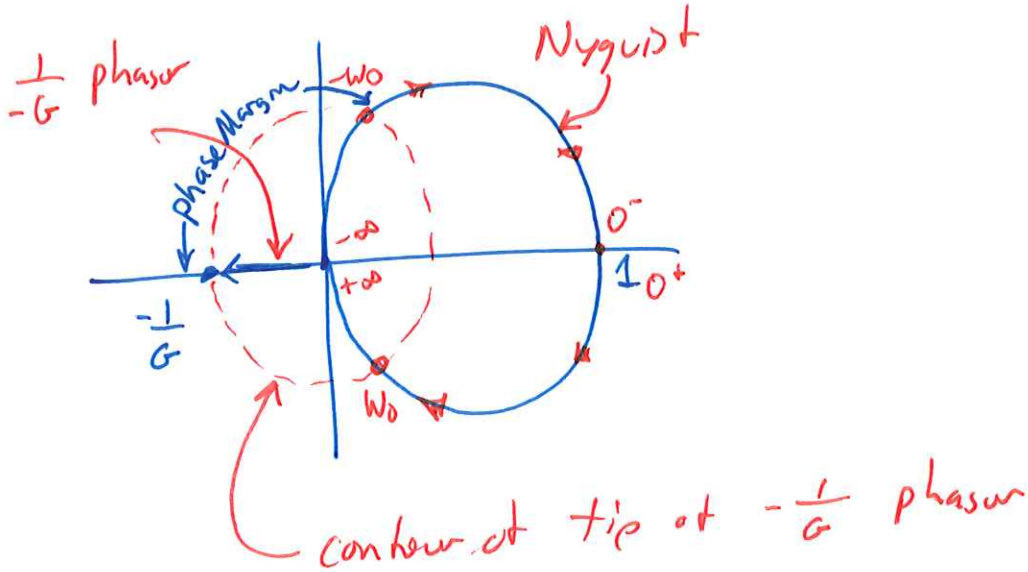
This phase margin is indicated in the diagram above.

Note in this example the phase margin is 90 degrees as for the bode plot.

Another example on phase margin

Suppose we have $H_{OL}(s) = \frac{e^{-sT}}{s+1}$

Sketch the Nyquist and show the phase margin.



① Select G, $G \in$ Real $G > 0$

② phasor of $-\frac{1}{G}$ rotated to form circle.

③ find points of interception of Nyquist plot. This gives w_0 the frequency corresponding to the intercept point.

④ Determine Phase Margin as in diagram

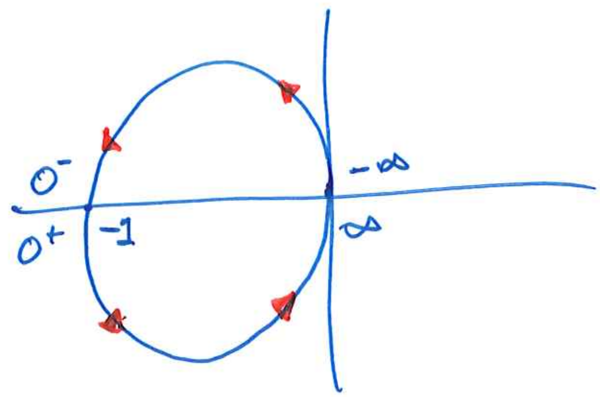
⑤ Determine maximum delay that can be tolerated as

phase margin $= w_0 \cdot$ delay that can be tolerated

Example — Unstable open loop pole.

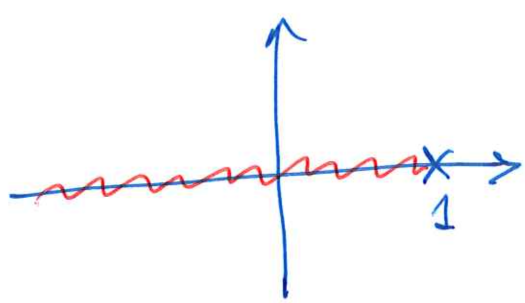
$$H_{OL}(s) = \frac{1}{s-1}$$

Nyquist plot is shown below



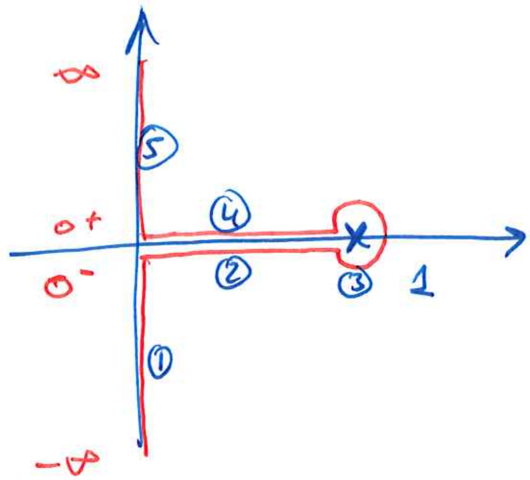
This would indicate that the closed loop is stable for $-\infty < G < 0$ and $0 < G \leq 1$

But this is not valid as per root locus

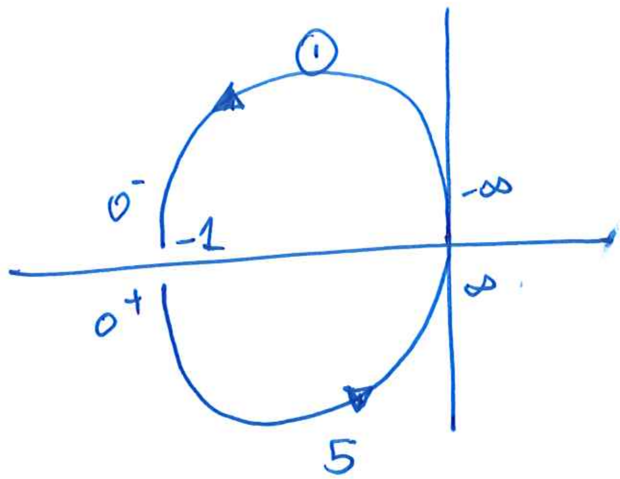


Stable for
$1 < G < \infty$
Just the opposite.

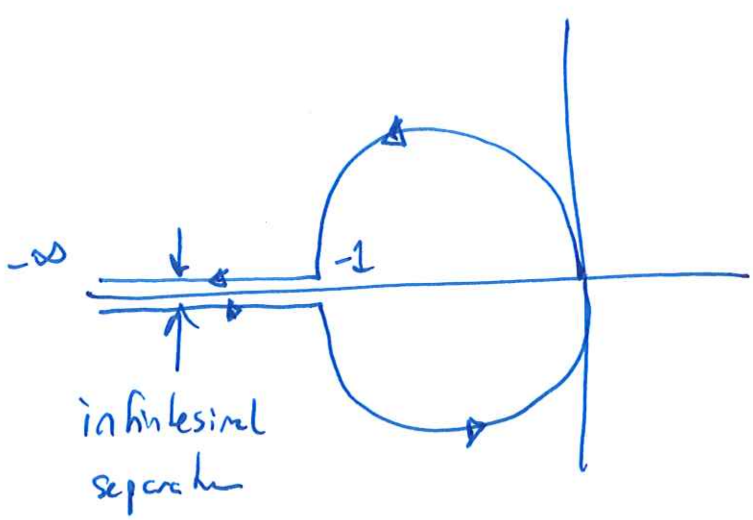
But we have violated the rule that in order to apply Nyquist we must have contour on Right side of all poles.



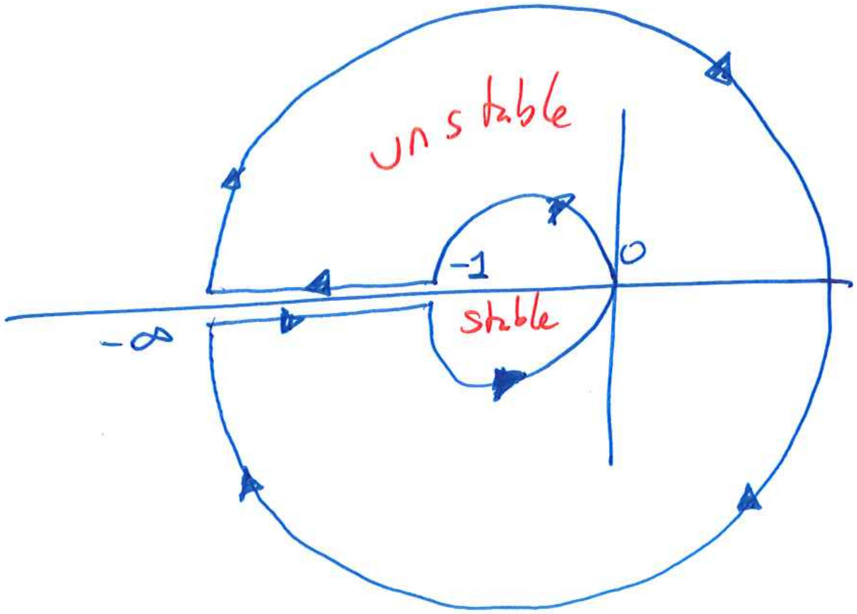
Go back an draw the contours corresponding to 1 and 5



now add in the trajectory segments 2 and 4

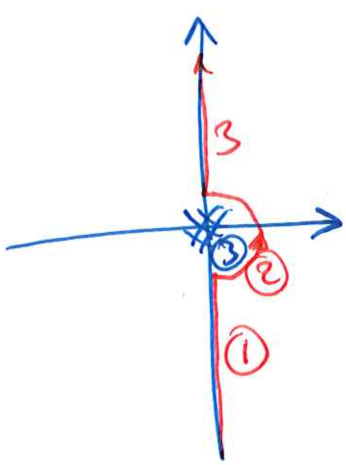


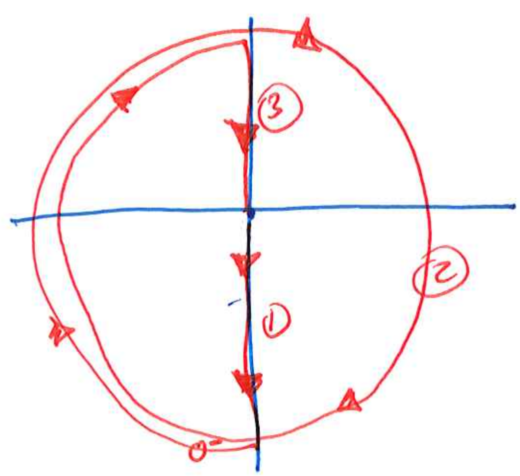
Finally add in the contour for 3 which is counter clockwise so that we need a clockwise rotation in Nyquist at 360°



Now the remarkable observation is that the disk between 0 and −1 is the only region of stability. Every where else it is enclosed by a loop and is unstable.

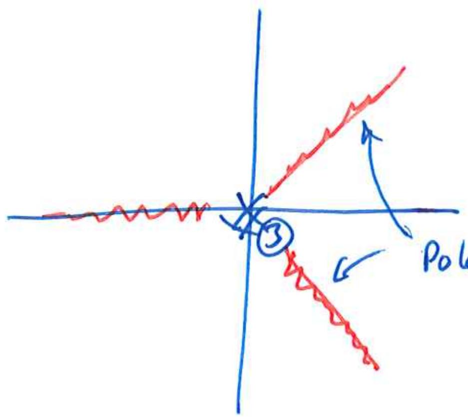
<u>Example</u> $H_{OL}(s) = \dfrac{1}{s^3}$





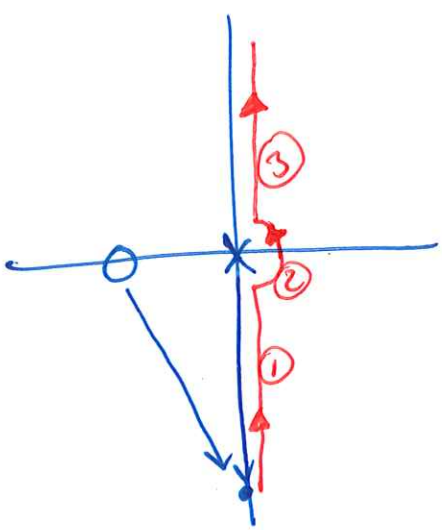
As we see the Nyquist contour encircles the entire complex plane and therefore no value of G is in a stable region.

This is consistent with root locus



<u>Example</u>

$$H_{OL}(s) = \frac{s+1}{s}$$



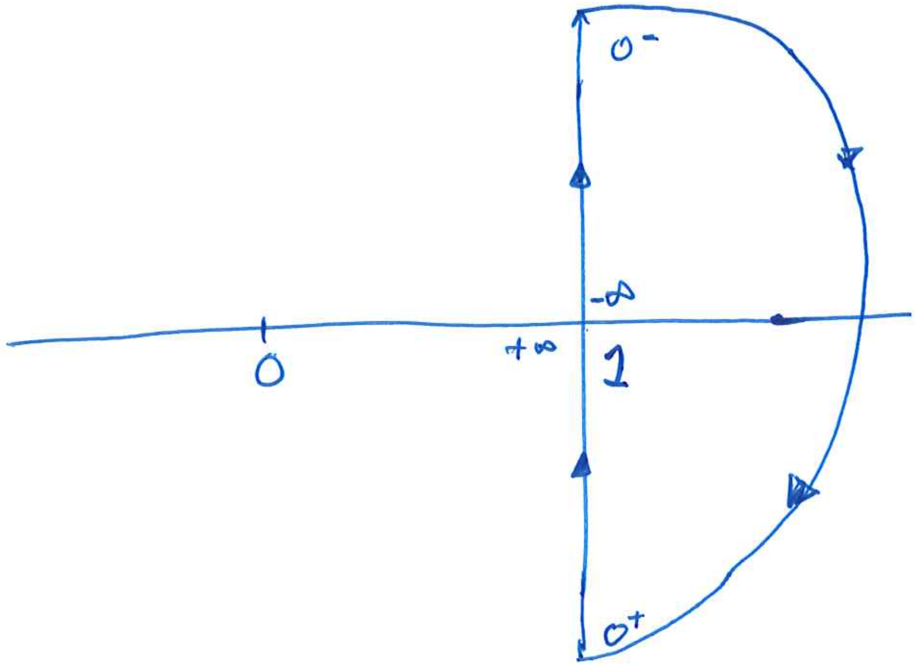
Just like in Bode plots we have to determine the phasors of multiple poles and zeros and determine what they do.

Write as

$$H_{OL}(j\omega) = \frac{j\omega}{j\omega} + \frac{1}{j\omega}$$

$$= 1 + \underbrace{\frac{1}{j\omega}}$$

only variation along imaginary axis at offset of 1.



Nyquist stability

$$\underset{0 < G < \infty}{\longleftrightarrow} \;\Big|\; \underset{-1 < G < 0}{\longleftrightarrow}$$

. stable for $-1 < G < \infty$

<u>Show this directly.</u>

closed loop $\dfrac{G\frac{s+1}{s}}{1+G\frac{s+1}{s}} = \dfrac{G(s+1)}{s(1+G)+1}$

Stable if $\dfrac{1}{1+G} > 0$ $\quad G > -1$

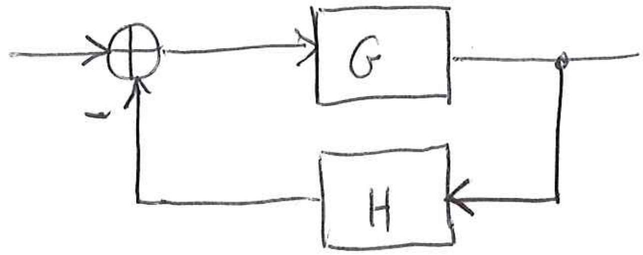
Extra Reading Not covered in Lecture

(Will not be on Quiz or Final Exam)

Nyquist Stability Criterion (How it works)

A stability test for time invariant linear systems can also be derived in the frequency domain. It is known as Nyquist stability criterion. It is based on the complex analysis result known as *Cauchy's principle of argument.* Note that the system transfer function is a complex function. By applying Cauchy's principle of argument to the *open-loop system* transfer function, we will get information about stability of the closed-loop system transfer function and arrive at the Nyquist stability criterion (Nyquist, 1932).

The importance of Nyquist stability lies in the fact that it can also be used to determine the relative degree of system stability by producing the so-called phase and gain stability margins. These stability margins are needed for frequency domain controller design techniques.



We present only the essence of the Nyquist stability criterion and define the phase and gain stability margins. The Nyquist method is used for studying the stability of linear systems with pure time delay.

For a SISO feedback system the closed-loop transfer function is given by

$$M(s) = \frac{G(s)}{1 + H(s)G(s)}$$

where $G(s)$ represents the system and $H(s)$ is the feedback element. Since the system poles are determined as those values at which its transfer function becomes infinity, it follows that the closed-loop system poles are obtained by solving the following equation

$$1 + H(s)G(s) = 0 = \Delta(s)$$

which, in fact, represents the *system characteristic equation.*

In the following we consider the complex function

$$D(s) = 1 + H(s)G(s)$$

whose zeros are the closed-loop poles of the transfer function. In addition, it is easy to see that the poles of $D(s)$ are the zeros of $M(s)$. At the same time the poles of $D(s)$ are the open-loop control system poles since they are contributed by the poles of $H(s)G(s)$, which can be considered as the open-loop control system transfer function—obtained when the feedback loop is open at some point. The Nyquist stability test is obtained by applying the Cauchy principle of argument to the complex function $D(s)$. First, we state Cauchy's principle of argument.

4

Cauchy's Principle of Argument

Let $F(s)$ be an analytic function in a closed region of the complex plane s given in Figure 4.6 except at a finite number of points (namely, the poles of $F(s)$). It is also assumed that $F(s)$ is analytic at every point on the contour. Then, *as s travels around the contour in the s-plane in the clockwise direction, the function* $F(s)$ *encircles the origin in the* $(Re\{F(s)\}, Im\{F(s)\})$*-plane in the same direction N times (see Figure 4.6), with N given by*

$$N = Z - P$$

where Z and P stand for the number of zeros and poles (including their multiplicities) of the function $F(s)$ *inside the contour.*

The above result can be also written as

$$\arg\{F(s)\} = (Z - P)2\pi = 2\pi N$$

which justifies the terminology used, "the principle of argument".

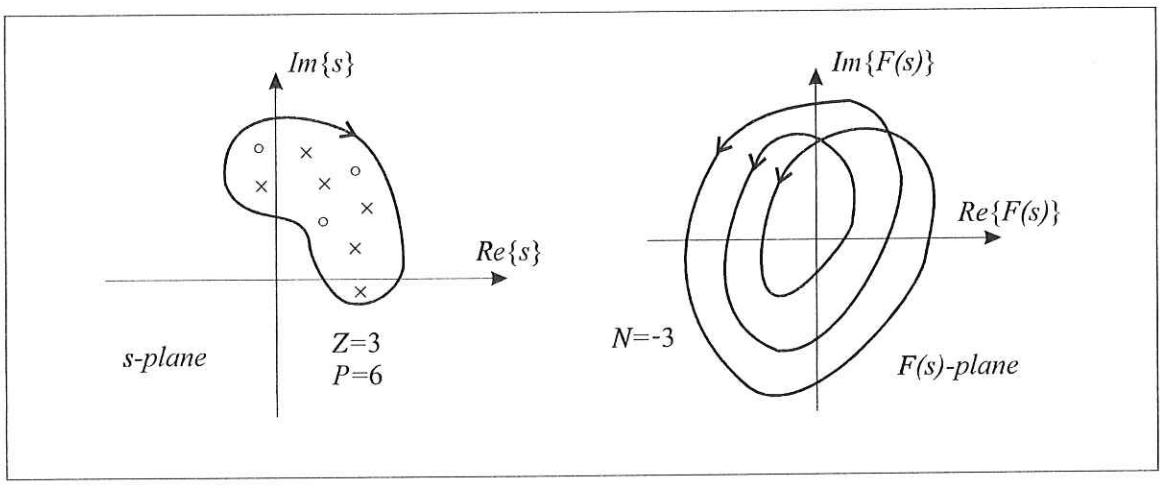


Figure 4.6: Cauchy's principle of argument

Nyquist Plot

The Nyquist plot is a polar plot of the function $D(s) = 1+G(s)H(s)$ when s travels around the contour given in Figure 4.7.

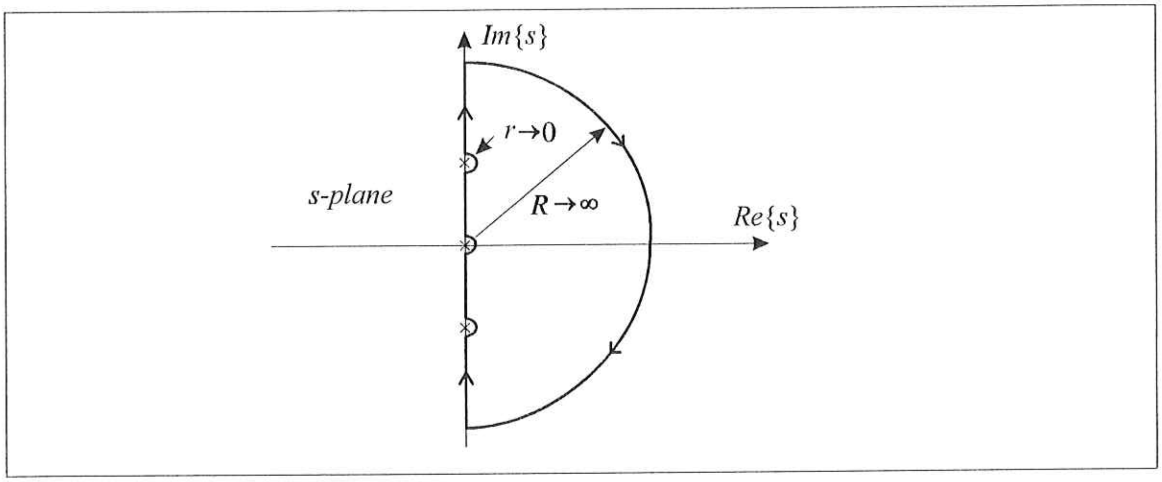


Figure 4.7: Contour in the s-plane

The contour in this figure covers the whole unstable half plane of the complex plane s, $R \to \infty$. Since the function $D(s)$, according to Cauchy's principle of argument, must be analytic at every point on the contour, the poles of $D(s)$ on the imaginary axis must be encircled by infinitesimally small semicircles.

Nyquist Stability Criterion

It states that the number of unstable closed-loop poles is equal to the number of unstable open-loop poles plus the number of encirclements of the origin of the Nyquist plot of the complex function $D(s)$.

This can be easily justified by applying Cauchy's principle of argument to the function $D(s)$ with the s-plane contour given in Figure 4.7. Note that Z and P represent the numbers of zeros and poles, respectively, of $D(s)$ in the unstable part of the complex plane. At the same time, *the zeros of* $D(s)$ *are the closed-loop system poles*, and *the poles of* $D(s)$ *are the open-loop system poles* (closed-loop zeros).

The above criterion can be slightly simplified if instead of plotting the function $D(s) = 1 + G(s)H(s)$, we plot only the function $G(s)H(s)$ and count encirclement of the Nyquist plot of $G(s)H(s)$ around the point $(-1, j0)$, so that the modified Nyquist criterion has the following form.

The number of unstable closed-loop poles (Z) is equal to the number of unstable open-loop poles (P) plus the number of encirclements (N) of the point $(-1, j0)$ *of the Nyquist plot of* $G(s)H(s)$, *that is*

$$Z = P + N$$

Phase and Gain Stability Margins

Two important notions can be derived from the Nyquist diagram: *phase and gain stability margins.* The phase and gain stability margins are presented in Figure 4.8.

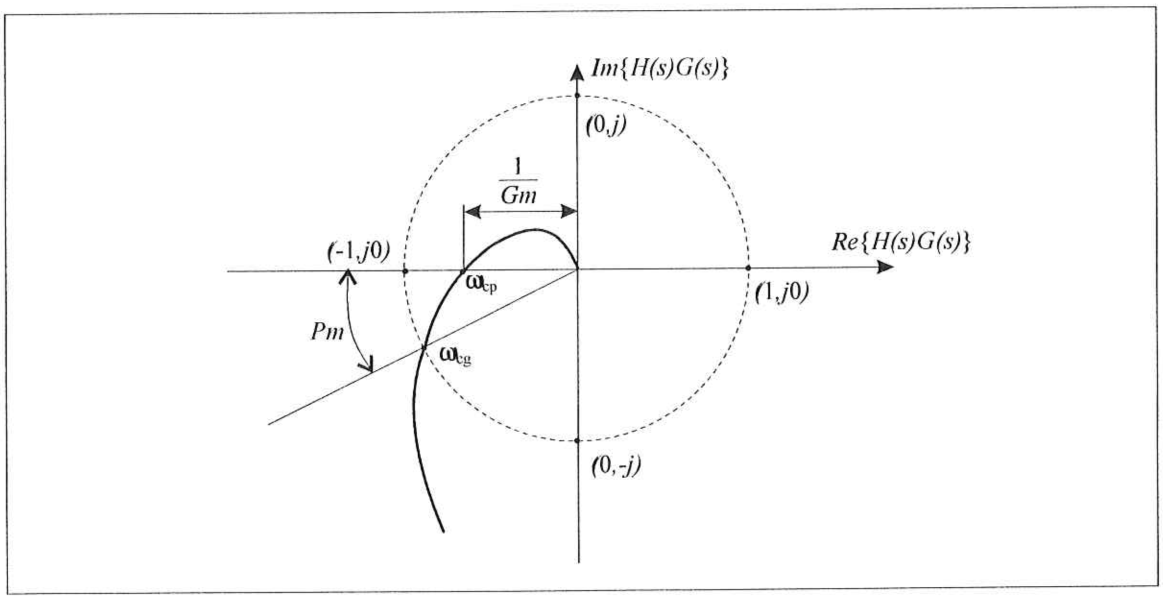


Figure 4.8: Phase and gain stability margins

They give the degree of relative stability; in other words, they tell how far the given system is from the instability region. Their formal definitions are given by

$$Pm = 180^\circ + \arg\{G(j\omega_{cg})H(j\omega_{cg})\}$$

$$Gm\,[dB] = 20\log\frac{1}{|G(j\omega_{cp})H(j\omega_{cp})|}\,[dB]$$

where ω_{cg} and ω_{cp} stand for, respectively, the *gain and phase crossover frequencies*, which from Figure 4.8 are obtained as

$$|G(j\omega_{cg})H(j\omega_{cg})| = 1 \;\Rightarrow\; \omega_{cg}$$

and

$$\arg\{G(j\omega_{cp})H(j\omega_{cp})\} = 180^\circ \;\Rightarrow\; \omega_{cp}$$

Example 4.23: Consider a control system represented by

$$G(s)H(s) = \frac{1}{s(s+1)}$$

Since this system has a pole at the origin, the contour in the s-plane should encircle it with a semicircle of an infinitesimally small radius. This contour has three parts (a), (b), and (c). Mappings for each of them are considered below.

(a) On this semicircle the complex variable s is represented in the polar form by $s = Re^{j\Psi}$ with $R \to \infty$, $-\frac{\pi}{2} \leq \Psi \leq \frac{\pi}{2}$. Substituting $s = Re^{j\Psi}$ into $G(s)H(s)$, we easily see that $G(s)H(s) \to 0$. Thus, the huge semicircle from the s-plane maps into the origin in the $G(s)H(s)$-plane (see Figure 4.9).

12

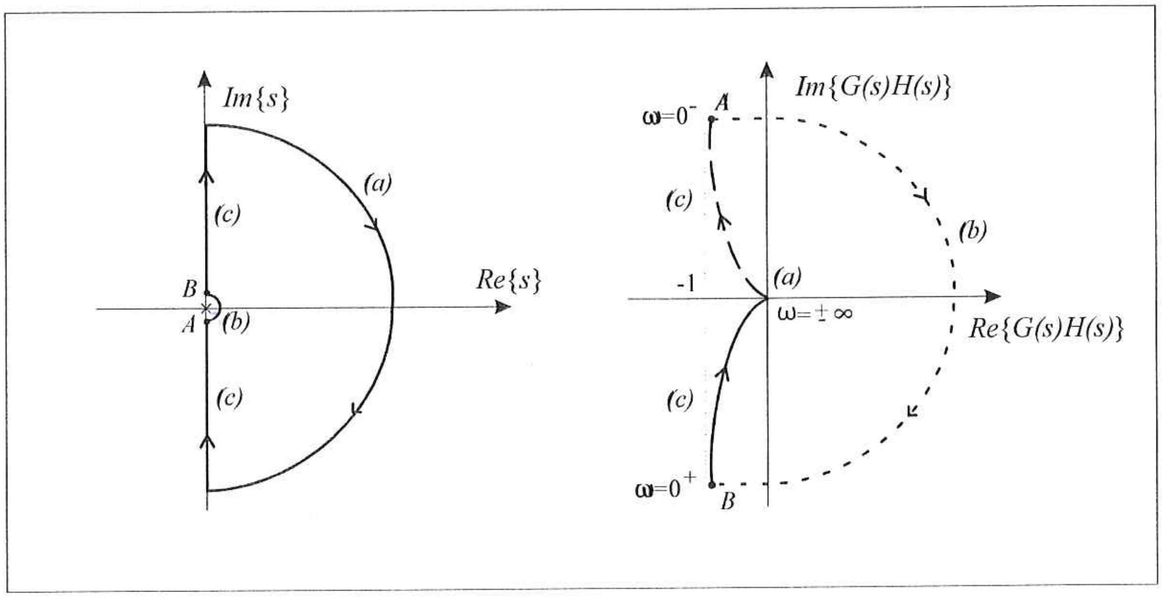


Figure 4.9: Nyquist plot for Example 4.23

(b) On this semicircle the complex variable s is represented in the polar form by $s = re^{j\Phi}$ with $r \to 0$, $-\frac{\pi}{2} \le \Phi \le \frac{\pi}{2}$, so that we have

$$G(s)H(s) \to \frac{1}{re^{j\Phi}} \to \infty \times \arg(-\Phi)$$

Since Φ changes from $-\frac{\pi}{2}$ at point A to $\frac{\pi}{2}$ at point B, $\arg\{G(s)H(s)\}$

will change from $\frac{\pi}{2}$ to $-\frac{\pi}{2}$. We conclude that the infinitesimally small semicircle at the origin in the s-plane is mapped into a semicircle of infinite radius in the $G(s)H(s)$-plane.

(c) On this part of the contour s takes pure imaginary values, i.e. $s = j\omega$ with ω changing from $-\infty$ to $+\infty$. Due to symmetry, it is sufficient to study only mapping along $0^+ \leq \omega \leq +\infty$. We can find the real and imaginary parts of the function $G(j\omega)H(j\omega)$, which are given by

$$Re\{G(j\omega)H(jw)\} = \frac{-1}{\omega^2 + 1}$$

$$Im\{G(j\omega)H(jw)\} = \frac{-1}{\omega(\omega^2 + 1)}$$

From these expressions we see that neither the real nor the imaginary parts can be made zero, and hence the Nyquist plot has no points of intersection with the coordinate axis. For $\omega = 0^+$ we are at point B and since the plot at $\omega = +\infty$ will end up at the origin, the Nyquist diagram corresponding to part (c) has the form as shown in Figure 4.9. Note that the vertical asymptote of the Nyquist plot in Figure 4.9 is given by $\boldsymbol{Re}\{G(j0^{\pm})H(j0^{\pm})\} = -1$ since at those points $\boldsymbol{Im}\{G(j0^{\pm})H(j0^{\pm})\} = \mp\infty$.

From the Nyquist diagram we see that $N = 0$ and since there are no open-loop poles in the left half of the complex plane, i.e. $P = 0$, we have $Z = 0$ so that the corresponding closed-loop system has no unstable poles.

15

The Nyquist plot is drawn by using the MATLAB function `nyquist`

```
num=1; den=[1  1  0];
nyquist(num,den);
axis([-1.5  0.5  −10  10]);
axis([-1.2  0.2  1  1]);
```

The MATLAB Nyquist plot is presented in Figure 4.10. It can be seen from Figures 4.8 and 4.9 that $\mathbf{1}/Gm = \mathbf{0}$, which implies that $Gm = \infty$. Also, from the same figures it follows that $\omega_{cp} = \infty$. In order to find the phase margin and the corresponding gain crossover frequency we use the MATLAB function `margin` as follows

```
[Gm,Pm,wcp,wcg]=margin(num,den)
```

producing, respectively, gain margin, phase margin, phase crossover frequency, and gain crossover frequency. The required phase margin and gain crossover frequency are obtained as $\boldsymbol{Pm = 53.4108^\circ}$, $\omega_{cg} =$ $\mathbf{0.7862\,rad/s}$.

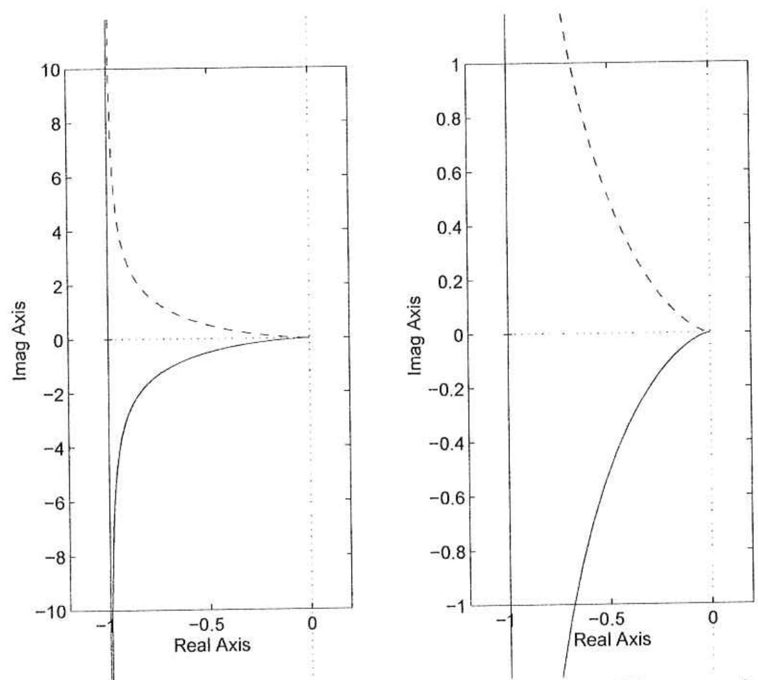


Figure 4.10: MATLAB Nyquist plot for Example 4.23

Example 4.24: Consider now the following system, obtained from the one in the previous example by adding a pole, that is

$$G(s)H(s) = \frac{1}{s(s+1)(s+2)}$$

The contour in the s-plane is the same as in the previous example. For cases (a) and (b) we have the same analyses and conclusions. It remains to examine case (c). If we find the real and imaginary parts of $G(j\omega)H(j\omega)$, we get

$$Re\{G(j\omega)H(j\omega)\} = \frac{-3}{9\omega^2 + (2-\omega^2)^2}$$

$$Im\{G(j\omega)H(j\omega)\} = \frac{-(2-\omega^2)}{\omega\left[9\omega^2 + (2-\omega^2)^2\right]}$$

(18)

It can be seen that an intersection with the real axis happens at $\omega = \sqrt{2}$ at the point $R_e\left\{G\left(j\sqrt{2}\right)H\left(j\sqrt{2}\right)\right\} = -1/6$. The Nyquist plot is given in Figure 4.11. The corresponding Nyquist plot obtained by using MATLAB is given in Figure 4.12.

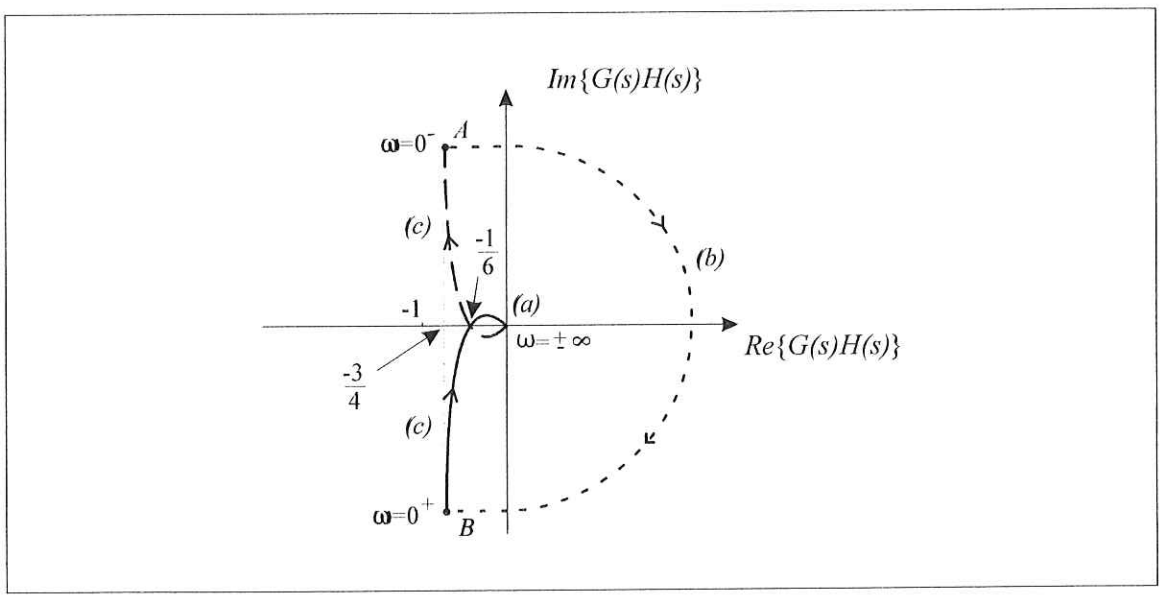


Figure 4.11: Nyquist plot for Example 4.24

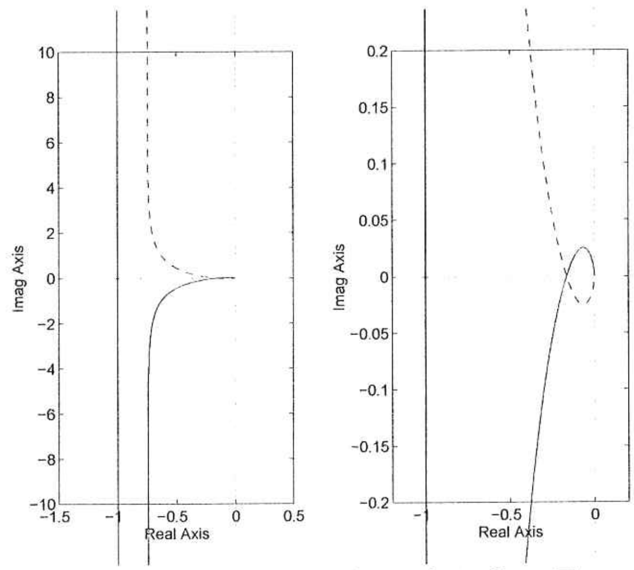


Figure 4.12: MATLAB Nyquist plot for Example 4.24

Note that the vertical asymptote is given by $\boldsymbol{Re\{G(j0)H(j0)\}} = \boldsymbol{-3/4}$. Thus, we have $\boldsymbol{N = 0, P = 0}$, and $\boldsymbol{Z = 0}$ so that the closed-loop system is stable. The MATLAB function `margin` produces

$$\boldsymbol{Gm = 6\,\mathrm{dB}, \quad Pm = 53.4108^{\circ}}$$

$$\boldsymbol{\omega_{cg} = 0.4457\,\mathrm{rad/s}, \quad \omega_{cp} = 1.4142\,\mathrm{rad/s}}$$