University of Calgary

Schulich School of Engineering

Department of Electrical and Computer Engineering

ENEL 476 – Electromagnetic Waves and Applications

Final Examination

Winter Session 2011

April 19, 2011

3 hours (8 am – 11 am)

Student name: ______________________________

Instructions

(1) This is a closed book exam. No texts or notes are allowed.
(2) Show as much of your reasoning as time permits, except for question 1. For question 1, fill in your answers on the bubble sheet. For questions 2-6, write your answers in the examination booklets.
(3) The sanctioned Schulich School of Engineering calculator is permitted.
(4) Formulas are provided at the end of the question pages.
(5) Hand in all pages. If you detach any pages(s), write your name and UCID number on the detached page(s).
(6) If you write anything you do not want marked, put a large X through it and write "Rough work" beside it.

EXAMINATION RULES AND REGULATIONS

STUDENT IDENTIFICATION

Each candidate must sign the Seating List confirming presence at the examination. All candidates for final examinations are required to place their University of Calgary I.D. cards on their desks for the duration of the examination. (Students writing mid-term tests can also be asked to provide identity proof.) Students without an I.D. card who can produce an acceptable alternative I.D., e.g., one with a printed name and photograph, are allowed to write the examination.

A student without acceptable I.D. will be required to complete an Identification Form. The form indicates that there is no guarantee that the examination paper will be graded if any discrepancies in identification are discovered after verification with the student's file. A Student who refuses to produce identification or who refuses to complete and sign the Identification Form is not permitted to write the examination.

EXAMINATION RULES

(1) Students late in arriving will not normally be admitted after one-half hour of the examination time has passed.

(2) No candidate will be permitted to leave the examination room until one-half hour has elapsed after the opening of the examination, nor during the last 15 minutes of the examination. All candidates remaining during the last 15 minutes of the examination period must remain at their desks until their papers have been collected by an invigilator.

(3) All inquiries and requests must be addressed to supervisors only.

(4) Candidates are strictly cautioned against:

(a) speaking to other candidates or communicating with them under any circumstances whatsoever;

(b) bringing into the examination room any textbook, notebook or memoranda not authorized by the examiner;

(c) making use of calculators and/or portable computing machines not authorized by the instructor;

(d) leaving answer papers exposed to view;

(e) attempting to read other student's examination papers.

The penalty for violation of these rules is suspension or expulsion or such other penalty as may be determined.

(5) Candidates are requested to write on both sides of the page, unless the examiner has asked that the left hand page be reserved for rough drafts or calculations.

(6) Discarded matter is to be struck out and not removed by mutilation of the examination answer book.

(7) Candidates are cautioned against writing in their answer book any matter extraneous to the actual answering of the question set.

(8) The candidate is to write his/her name on each answer book as directed and is to number each book.

(9) A candidate must report to a supervisor before leaving the examination room.

(10) Answer books must be handed to the supervisor-in-charge promptly when the signal is given. Failure to comply with this regulation will be cause for rejection of an answer paper.

(11) If during the course of an examination a student becomes ill or receives word of a domestic affliction, the student should report at once to the supervisor, hand in the unfinished paper and request that it be cancelled. If physical and/or emotional ill health is the cause, the student must report at once to a physician/counsellor so that subsequent application for a deferred examination is supported by a completed Physician/Counsellor Statement form. Students can consult professionals at University Health Services or University Counselling Services during normal working hours or consult their physician/counsellor in the community.

Should a student write an examination, hand in the paper for marking, and later report extenuating circumstances to support a request for cancellation of the paper and for another examination, such a request will be denied.

(12) Smoking during examinations is strictly prohibited.

Question 1 (20 marks): Indicate the response that best answers the question on the bubble sheet.

Part 1: The magnetic field of an electromagnetic wave traveling through air can be expressed in phasor form as:

$$\mathbf{H} = 0.1e^{j10y}\,\mathbf{a}_x \text{ A/m}$$

The complex electric field vector of the wave is given by:

(A) $\mathbf{E} = 265\,e^{-j10y}\,\mathbf{a}_z\ \mu\text{V/m}$
(B) $\mathbf{E} = 37.7\,e^{-j10y}\,\mathbf{a}_x$ V/m
(C) $\mathbf{E} = 37.7\,e^{j10y}\,\mathbf{a}_z$ V/m
(D) $\mathbf{E} = -37.7\,e^{j10y}\,\mathbf{a}_z$ V/m
(E) $\mathbf{E} = -265\,e^{j10y}\,\mathbf{a}_y\ \mu\text{V/m}$

Part 2: A lossy medium has parameters ε, μ_0, and σ. The conduction current density at a point is given by $J(t) = J_0 \sin \omega t$. The displacement current density at that point is of the following form (J_0, ω, and J_{d0} are positive constants):

(A) $J_d(t) = J_{d0} \sin \omega t$
(B) $J_d(t) = -J_{d0} \sin \omega t$
(C) $J_d(t) = J_{d0} \cos \omega t$
(D) $J_d(t) = J_{d0}$
(E) None of the above.

Part 3: An electromagnetic wave has an instantaneous electric field intensity vector given by:

$$\mathbf{E}(x, t) = [2 \cos(\omega t + \beta x)\,\mathbf{a}_y - \sin(\omega t + \beta x)\,\mathbf{a}_z] \text{ V/m}$$

where ω and β are the angular frequency and phase coefficient of the wave. The polarization of the wave is:

(A) linear
(B) circular
(C) elliptical
(D) parallel
(E) perpendicular

Part 4: A time-harmonic wave travels with attenuation and phase coefficients α and β, respectively, along a transmission line with small losses. The RMS voltage of the wave at the load terminals is V^i. As an approximation, the characteristic impedance of the line can be taken to be purely real and equal to Z_0. The line is terminated in a matched load, so that there is no reflected wave on the line. In a cross section of the line that is at a distance l from the load terminals, the time-average power carried by the wave equals:

(A) $P = \dfrac{(V^i)^2}{Z_o}$

(B) $P = \dfrac{(V^i)^2}{Z_o} e^{2\alpha l}$

(C) $P = \dfrac{(V^i)^2}{Z_o} e^{-2\alpha l}$

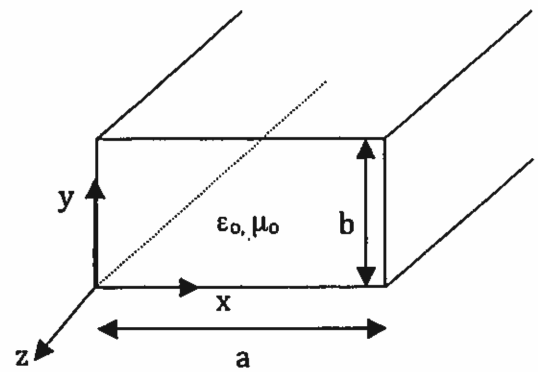
(D) $P = \dfrac{(V^i)^2}{Z_o} e^{-4\alpha l}$

(E) $P = \dfrac{(V^i)^2}{Z_o} e^{-2\alpha l} \cos(\beta l)$

Part 5: An antenna with a purely real input impedance equal to $R_A = 400\ \Omega$ needs to be impedance-matched to a transmission line having a characteristic impedance of $Z_{01} = 100\ \Omega$ at a frequency of f = 300 MHz. This can be done using an air-filled lossless transmission-line section:

(A) of length l = 75 cm and characteristic impedance $Z_{02} = 100\ \Omega$.
(B) of length l = 50 cm and characteristic impedance $Z_{02} = 200\ \Omega$.
(C) of length l = 25 cm and characteristic impedance $Z_{02} = 250\ \Omega$.
(D) of length l = 25 cm and characteristic impedance $Z_{02} = 200\ \Omega$.
(E) More than one of the combinations above.

Part 6: Consider the field configuration of an arbitrary TE_{mn} wave mode propagating along the rectangular metallic waveguide shown in the figure below.



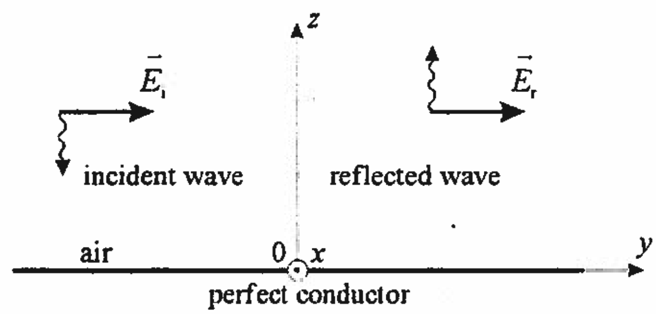
The integers m and n (if nonzero) equal the number of:

(A) wavelengths along the x- and y-axes that fit into a and b, respectively.
(B) half-wavelengths along the x- and y-axes that fit into a and b, respectively.
(C) half-wavelengths along the x-axis for electric and magnetic fields, respectively.
(D) wavelengths along the x-axis for the electric and y-axis for the magnetic field.
(E) None of the above.

Part 7: An air-filled rectangular metallic waveguide has a = 5 cm and b = 2.5 cm, so that the cutoff frequency of the dominant mode is f_c = 3 GHz. The following is a complete list of modes that can propagate along this waveguide at an operating frequency of f = 4.2 GHz:

(A) TE_{10} .
(B) TE_{10} and TM_{10} .
(C) TE_{10}, TE_{01}, and TE_{20}.
(D) TE_{10}, TE_{01} , TM_{10} , and TM_{01} .
(E) TE_{10} , TE_{01} , TE_{20} , TE_{11} , and TM_{11}.

Part 8: A time-harmonic uniform plane wave in air has a wavelength λ and impinges normally on a perfectly conducting plane, as shown in the figure before. We wish to receive information contained in the wave using a short dipole antenna (electric probe). The signal induced in the antenna will be maximal if the dipole is positioned



(A) parallel to the x-axis and at height $z = \lambda/4$.
(B) parallel to the y-axis and at height $z = \lambda/4$.
(C) parallel to the z-axis and at height $z = \lambda/4$.
(D) parallel to the x-axis and at height $z = \lambda/2$.
(E) parallel to the y-axis and at height $z = \lambda/2$.

Part 9: A circular wire loop of radius a = 30 cm in free space carries a time-harmonic current with an amplitude that varies significantly along the loop. Which of the following are possible frequencies of this current?

(A) 0 Hz (dc), 60 Hz, and 100 kHz.
(B) 60 Hz and 100 kHz.
(C) 60 Hz, 100 kHz, and 1 GHz.
(D) 100 kHz and 1 GHz.
(E) 1 GHz.

Part 10: Consider a uniform plane time-harmonic electromagnetic wave of frequency f traveling through an arbitrary lossy medium of parameters ε, μ, and σ. The phase velocity of this wave, v_p, is:

(A) a constant, equal to 3×10^8 m/s.
(B) a constant, different from 3×10^8 m/s.
(C) a function of frequency (f).

Question 2 (20 marks)

A lossless transmission line of Z_o=50 Ω is terminated with an unknown impedance, Z_L. The standing wave ratio of the line is 3. Successive voltage minima are 20 cm apart, and the first minimum is located 5 cm from the load.

Show your work on the Smith chart, labeling steps clearly.

a) What is Z_L?

b) Find the reflection coefficient at the load.

c) Find the location and length of a short-circuited shunt stub required to match the load. If you are not able to solve part a, use Z_L=40+j20 Ω.

Question 3 (20 marks)

Mr. Dr. Fear is designing a fish finder to assist in his ice fishing expeditions. He estimates that the properties of fresh water are ε_r=80 and σ=0.01 S/m at 100 MHz. Fish may be represented as a lossless dielectric with ε_r=50. The antenna used in the fish finder is immersed in water. To estimate performance of the device, uniform plane wave behaviour is assumed to describe the fields.

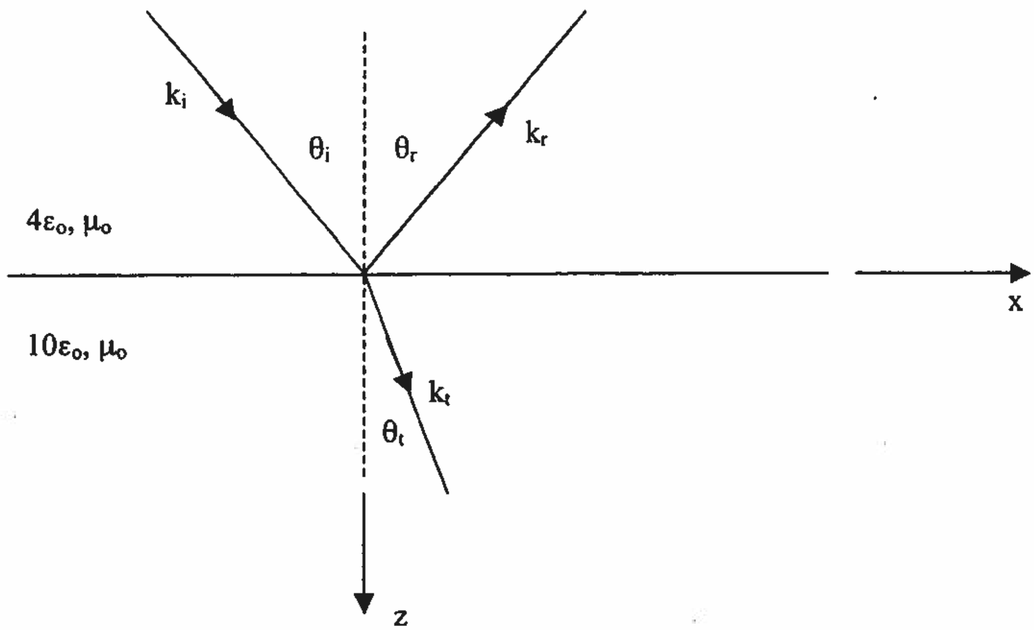
(a) If the incident electric field has an amplitude of 1 kV/m at the surface of and in the water, then find expressions for the incident electric and magnetic fields. Sketch the co-ordinate system that you use.
(b) Find an expression for the reflected electric field. You may model the fish as infinite in extent, assume that the field in incident normally on the fish, and that the fish is located 5 m below the surface of the water.
(c) If the receiver can detect a signal of 0.1 mW/m^2, will the fish finder be successful?

Question 4 (20 marks)

A uniform plane wave has the electric field given by:

$$\mathbf{E}_i=25(0.866\mathbf{a}_x-0.5\mathbf{a}_z)e^{-j(5x+8.66z)}$$

This wave is incident on a dielectric interface between a medium of dielectric constant $\varepsilon_1=4\varepsilon_o$ and another of dielectric constant $\varepsilon_2=10\varepsilon_o$, as shown in the figure below.



Find:

a) The direction of propagation of incident field.

b) The frequency ω

c) The angle of incidence

d) The angle of transmission

e) The reflection and transmission coefficients.

f) An expression for the reflected electric field.

g) An expression for the transmitted electric field.

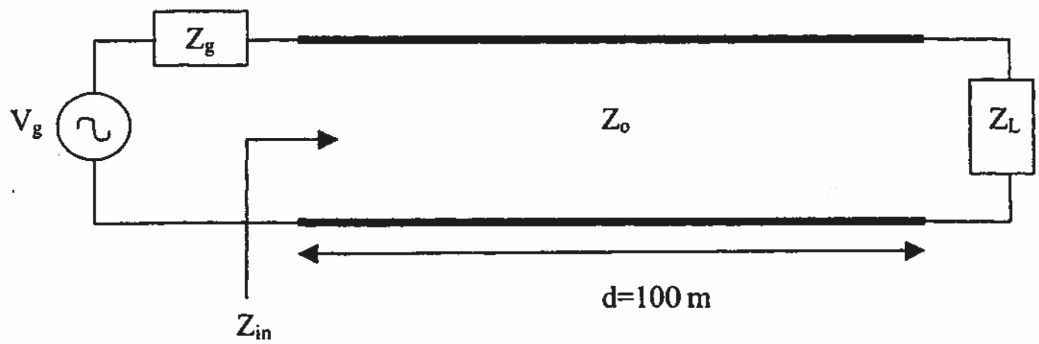
Question 5 (20 marks)

A transmission line operates at 2 MHz and is 100-m long. The transmission line has the following distributed constants: R=153 Ω/km, L=1.4 mH/km, C=88 nF/km and G=0.8 mS/km.

Find:

a) The characteristic impedance of the line (Z_o)
b) The propagation constant ($\alpha + j\beta$) of the transmission line
c) The percentage of power lost along the line (i.e. assume 100% power at the "start" of the line and calculate the fraction remaining at the "end" of the line; assume no reflections along the line)

Now consider the transmission line as lossless and with an impedance of $Zo=125\ \Omega$. The phase constant may be considered to be 14 rad/km. An antenna is connected to the transmission line, and has $Z_L=75+j150$. A transmitter (generator) is connected to the other end of the transmission line. The output voltage of the transmitter is $V_g=100e^{j0}$ and its internal impedance is 75 Ω.



Find:

d) The input impedance of the transmission line terminated by the antenna.
e) The total voltage on the line where the generator connects to the transmitter (V_{in}).

Question	Mark
Q1	
Q2	
Q3	
Q4	
Q5	
Total	

$\mu_o = 4\pi \times 10^{-7}$ H/m
$1/\mu_o = 8 \times 10^5$ m/H
$\eta_o = 120\pi\ \Omega$
$\varepsilon o = 8.85 \times 10^{-12}$ F/m

$\mathbf{a}_r \cdot \mathbf{a}_x = \sin\theta\cos\phi$	$\mathbf{a}_\theta \cdot \mathbf{a}_x = \cos\theta\cos\phi$	$\mathbf{a}_\varphi \cdot \mathbf{a}_x = -\sin\phi$
$\mathbf{a}_r \cdot \mathbf{a}_y = \sin\theta\sin\phi$	$\mathbf{a}_\theta \cdot \mathbf{a}_y = \cos\theta\sin\phi$	$\mathbf{a}_\varphi \cdot \mathbf{a}_y = \cos\phi$
$\mathbf{a}_r \cdot \mathbf{a}_z = \cos\theta$	$\mathbf{a}_\theta \cdot \mathbf{a}_z = -\sin\theta$	$\mathbf{a}_\varphi \cdot \mathbf{a}_z = 0$

$\mathbf{a}_x \cdot \mathbf{a}_\rho = \cos\phi$	$\mathbf{a}_x \cdot \mathbf{a}\phi = -\sin\phi$	$\mathbf{a}_x \cdot \mathbf{a}_z = 0$
$\mathbf{a}_y \cdot \mathbf{a}_\rho = \sin\phi$	$\mathbf{a}_y \cdot \mathbf{a}\phi = \cos\phi$	$\mathbf{a}_y \cdot \mathbf{a}_z = 0$
$\mathbf{a}_z \cdot \mathbf{a}_\rho = 0$	$\mathbf{a}_z \cdot \mathbf{a}\phi = 0$	$\mathbf{a}_z \cdot \mathbf{a}_z = 1$

$x = \rho\cos\phi$

$y = \rho\sin\phi$

Cartesian	Cylindrical	Spherical
$dx\,a_x + dy\,a_y + dz\,a_z$	$d\rho\,a_\rho + \rho\,d\phi\,a_\phi + dz\,a_z$	$dr\,a_r + r\,d\theta\,a_\theta + r\sin\theta\,d\phi\,a_\phi$
$(dy\,dz)a_x$ $(dx\,dz)a_y$ $(dx\,dy)a_z$	$(\rho\,d\phi\,dz)a_\rho$ $(d\rho\,dz)a_\phi$ $(\rho\,d\rho\,d\phi)a_z$	$(r^2\sin\theta\,d\theta\,d\phi)a_r$ $(r\sin\theta\,dr\,d\phi)a_\theta$ $(r\,dr\,d\theta)a_\phi$
$dx\,dy\,dz$	$\rho\,d\rho\,d\phi\,dz$	$r^2\sin\theta\,dr\,d\theta\,d\phi$

$r = \sqrt{x^2 + y^2 + z^2}$ $\qquad x = r\sin\theta\cos\phi$

$\theta = \cos^{-1}\dfrac{z}{\sqrt{x^2+y^2+z^2}}$ $\qquad y = r\sin\theta\sin\phi$

$\phi = \tan^{-1}\dfrac{y}{x}$ $\qquad z = r\cos\theta$

$$\nabla V = \frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z$$

$$\nabla V = \frac{\partial V}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\bar{a}_\phi + \frac{\partial V}{\partial z}\bar{a}_z$$

$$\nabla V = \frac{\partial V}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\bar{a}_\phi$$

$$\nabla\cdot\bar{E} = \left[\frac{\partial(E_x)}{\partial x} + \frac{\partial(E_y)}{\partial y} + \frac{\partial(E_z)}{\partial z}\right]$$

$$\nabla\cdot\bar{E} = \left[\frac{1}{\rho}\frac{\partial(\rho E_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial(E_\phi)}{\partial \phi} + \frac{\partial(E_z)}{\partial z}\right]$$

$$\nabla\cdot\bar{E} = \left[\frac{1}{r^2}\frac{\partial(r^2 E_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta E_\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial(E_\phi)}{\partial \phi}\right]$$

$$\nabla^2 V = \left[\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}\right]$$

$$\nabla^2 V = \left[\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + + \frac{\partial^2 V}{\partial z^2}\right]$$

$$\nabla^2 V = \left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}\right]$$

$$\nabla\times\bar{\mathbf{A}} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \mathbf{A}_x & \mathbf{A}_y & \mathbf{A}_z \end{vmatrix}$$

$$\nabla\times\bar{\mathbf{A}} = \frac{1}{\rho}\begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ \mathbf{A}_\rho & \rho\mathbf{A}_\phi & \mathbf{A}_z \end{vmatrix}$$

$$\nabla\times\bar{\mathbf{A}} = \frac{1}{r^2\sin\theta}\begin{vmatrix} a_r & r a_\theta & r\sin\theta a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ \mathbf{A}_r & r\mathbf{A}_\theta & r\sin\theta\mathbf{A}_\phi \end{vmatrix}$$

ENEL 475/6 Formula Sheet 2 Revised 10/02/09

Magnetostatics	materials
$\vec{H} = \int_L \frac{Id\vec{l} \times \vec{a}_R}{4\pi R^2}$ $W_M = \frac{1}{2}\int \vec{H} \bullet \vec{B} dv$ **T=m x B** $L = \frac{N\Psi}{I}$ $M_{12} = \frac{N_1 \Psi_{12}}{I_2}$ $\vec{F} = \oint I d\vec{l} x \vec{B}$	$\vec{M} = \chi_m \vec{H}$ $\nabla \times \vec{M} = \vec{J}_M$ $\mu_r = 1 + \chi_m$
Lorentz force **F**=q(**E**+**v**x**B**)	Continuity $\nabla \bullet \vec{J} = -\frac{\partial \rho_v}{\partial t}$
Electrostatics	Materials
$\vec{E}(\vec{r}) = \int \frac{\rho_v dv}{4\pi\varepsilon_0\varepsilon_r R^2}\vec{a}_R$ $V(\vec{r}) = \int \frac{\rho_v(\vec{r}')dv'}{4\pi\varepsilon_r\varepsilon_0 \lvert\vec{r}-\vec{r}'\rvert} + C$ $V = -\int \vec{E} \bullet d\vec{l} + C$ $\vec{E} = -\nabla V$ $W_E = \frac{1}{2}\int \vec{E} \bullet \vec{D} dv$ $\nabla^2 V = -\frac{\rho_v}{\varepsilon}$ $\nabla^2 V = 0$ C=Q/V	$R = l/(\sigma S)$ $\vec{P} = \chi_e \varepsilon_o \vec{E}$ $\nabla \bullet \vec{P} = -\rho_{pv}$ $\vec{P} \bullet \vec{a}_n = \rho_{ps}$ $\varepsilon_r = 1 + \chi_e$

Boundary conditions		
$\mathbf{D}_{1n}-\mathbf{D}_{2n}=\rho s$ $\mathbf{E}_{t1}=\mathbf{E}_{t2}$ $\mathbf{B}_{1n}-\mathbf{B}_{2n}=0$ $\mathbf{a}_{21}$ x $(\mathbf{H}_1-\mathbf{H}_2)=\mathbf{J}_s$	$\mathbf{a}_{21} \cdot (\mathbf{D}_1-\mathbf{D}_2)=\rho s$ $\mathbf{a}_{21}$ x $(\mathbf{E}_1-\mathbf{E}_2)=0$ $\mathbf{a}_{21} \cdot (\mathbf{B}_1-\mathbf{B}_2)=0$ $\mathbf{a}_{21}$ x $(\mathbf{H}_1-\mathbf{H}_2)=\mathbf{J}_s$	J_s is surface current (also denoted as **K**)

Maxwell's equations	
$\oint_c \vec{E} \bullet d\vec{l} = -\frac{\partial}{\partial t}\int_s \vec{B} \bullet d\vec{s}$ $\oint_s \vec{B} \bullet d\vec{s} = 0$ $\oint_s \varepsilon_r \varepsilon_0 \vec{E} \cdot d\vec{s} = \int_v \rho_v dv$ $\oint_c \vec{H} \bullet d\vec{l} = \int_s \vec{J} \bullet d\vec{s} + \frac{\partial}{\partial t}\int_s \vec{D} \bullet d\vec{s}$ $\nabla \bullet \vec{B} = 0$ $\nabla \bullet \vec{D} = \rho_v$ $\nabla \times \vec{H} = \vec{J} + \frac{\partial}{\partial t}\vec{D}$ $\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$ $V_{emf} = -\int_s \frac{\partial \vec{B}}{\partial t} \bullet d\vec{s} + \oint_L (\vec{v} x \vec{B}) \bullet d\vec{l}$	Note: for static fields, time derivatives are zero. In phasor form, time derivatives become $j\omega$ terms.

Time-varying fields: UPW	
$\nabla^2 \vec{E}_s - \gamma^2 \vec{E}_s = 0$ $\nabla^2 \vec{H}_s - \gamma^2 \vec{H}_s = 0$ $\gamma = \alpha + j\beta$	Vector wave equations for time-harmonic fields in lossy medium. With lossless medium or free space, α=0.
$\lambda = \frac{2\pi}{\beta}$ $\beta = \omega\sqrt{\mu_r \mu_o \varepsilon_r \varepsilon_o}$ $T = 1/f$ $\lvert E \rvert / \lvert H \rvert = \eta = \sqrt{\frac{\mu_r \mu_o}{\varepsilon_r \varepsilon_o}}$ $v_p = \frac{\omega}{\beta}$	Uniform plane wave in lossless medium. For free space, μr=1 and εr=1.
$\vec{E}(z,t) = E_o e^{-\alpha z}\cos(\omega t - \beta z)\vec{a}_x$ $\vec{H}(z,t) = \frac{E_o}{\lvert\eta\rvert} e^{-\alpha z}\cos(\omega t - \beta z - \theta_\eta)\vec{a}_y$	One example of E and H fields in lossy medium.
$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$ $\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$ $\lvert\eta\rvert = \frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}$ $\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon}$ $\delta = 1/\alpha$ $\lambda = \frac{2\pi}{\beta}$	Parameters describing UPW in lossy medium.
$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$ $\eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^o$	good conductor: $(\sigma/\omega\varepsilon >> 1)$
$\vec{P}_{avg}(z) = \frac{1}{2}\mathrm{Re}(\vec{E}_s(z) \times \vec{H}_s^*(z))$ **P**(z,t)=**E**(z,t) x **H**(z,t)	Poynting vector
$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ $T = \frac{2\eta_2}{\eta_2 + \eta_1}$	Transmission and reflection coefficients: normal incidence
$\Gamma_\parallel = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$ $T_\parallel = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$ $\Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$ $T_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$	Transmission and reflection coefficients: oblique incidence $\theta_i = \theta_r$ $\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}$

ENEL 476 Formula Sheet 3

Revised 01/28/11

Waves and T/R – continued

$$s = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\sin\theta_B = \sqrt{\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}}$$

$$\tan\theta_B = \sqrt{\varepsilon_2/\varepsilon_1}$$

$$\sin\theta_c = \sqrt{\varepsilon_2/\varepsilon_1}$$

$$R_{ac} = L/(\sigma\delta w)$$

Transmission lines:

$$V_s(z) = V^+e^{-\gamma z} + V^-e^{\gamma z}$$

$$I_s(z) = \frac{1}{Z_o}\left[V^+e^{-\gamma z} - V^-e^{\gamma z}\right]$$

$$Z_o = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$\gamma = \sqrt{(G+j\omega C)(R+j\omega L)} = \alpha + j\beta$$

$$P_{ave} = \frac{|V_o^+|^2}{2Z_o}e^{-2\alpha z}\cos\theta$$

Distortionless transmission lines:

R/L=G/C

$$\alpha = \sqrt{RG}$$
$$\beta = \omega\sqrt{LC}$$
$$R_o = \sqrt{\frac{R}{G}}$$

Lossless line:

$$Z_{in} = Z_o\frac{Z_L + jZ_o\tan(\beta l)}{Z_o + jZ_L\tan(\beta l)}$$

$$Z_{in\_max} = Z_o\frac{1+|\Gamma|}{1-|\Gamma|} = Z_o \cdot SWR$$

$$\Gamma_l = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_{in\_min} = \frac{Z_o}{SWR}$$

$$\Gamma(l) = \Gamma(0)e^{-j2\beta l}$$

$$SWR = \left|\frac{V_{max}}{V_{min}}\right| = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Waveguides:

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$v_p = u_p = \frac{u'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$v_g = u_g = u'\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$v_p v_g = u'^2$$

$$\alpha_d = \frac{\sigma\eta'}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\alpha_c|_{TE10} = \frac{2R_s}{b\eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left(0.5 + \frac{b}{a}\left(\frac{f_c}{f}\right)^2\right)$$

$$\alpha_c|_{TE} = \frac{2R_s}{b\eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left(\left(1+\frac{b}{a}\right)\left(\frac{f_c}{f}\right)^2 + \frac{\frac{b}{a}\left(\frac{b}{a}m^2+n^2\right)}{\frac{b^2}{a^2}m^2+n^2}\left(1-\left(\frac{f_c}{f}\right)^2\right)\right)$$

$$\alpha_c|_{TM} = \frac{2R_s}{b\eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left(\frac{\left(\frac{b^3}{a^3}m^2+n^2\right)}{\frac{b^2}{a^2}m^2+n^2}\right)$$

Table 12.1 Important Equations for TM and TE Modes

TM Modes	TE Modes
$E_{xs} = -\frac{j\beta}{h^2}\left(\frac{m\pi}{a}\right) E_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$	$E_{xs} = \frac{j\omega\mu}{h^2}\left(\frac{n\pi}{b}\right) H_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$
$E_{ys} = -\frac{j\beta}{h^2}\left(\frac{n\pi}{b}\right) E_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$	$E_{ys} = -\frac{j\omega\mu}{h^2}\left(\frac{m\pi}{a}\right) H_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$
$E_{zs} = E_o \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$	$E_{zs} = 0$
$H_{xs} = \frac{j\omega\varepsilon}{h^2}\left(\frac{n\pi}{b}\right) E_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$	$H_{xs} = \frac{j\beta}{h^2}\left(\frac{m\pi}{a}\right) H_o \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$
$H_{ys} = -\frac{j\omega\varepsilon}{h^2}\left(\frac{m\pi}{a}\right) E_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$	$H_{ys} = \frac{j\beta}{h^2}\left(\frac{n\pi}{b}\right) H_o \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$
$H_{zs} = 0$	$H_{zs} = H_o \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$
$\eta = \eta'\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$	$\eta = \frac{\eta'}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$\lambda_c = \frac{u'}{f_c}$$

$$\beta = \beta'\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$u_p = \frac{\omega}{\beta} = f\lambda$$

$$\text{where } h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2,\ u' = \frac{1}{\sqrt{\mu\varepsilon}},\ \beta' = \frac{\omega}{u'},\ \eta' = \sqrt{\frac{\mu}{\varepsilon}}$$

Resonators:

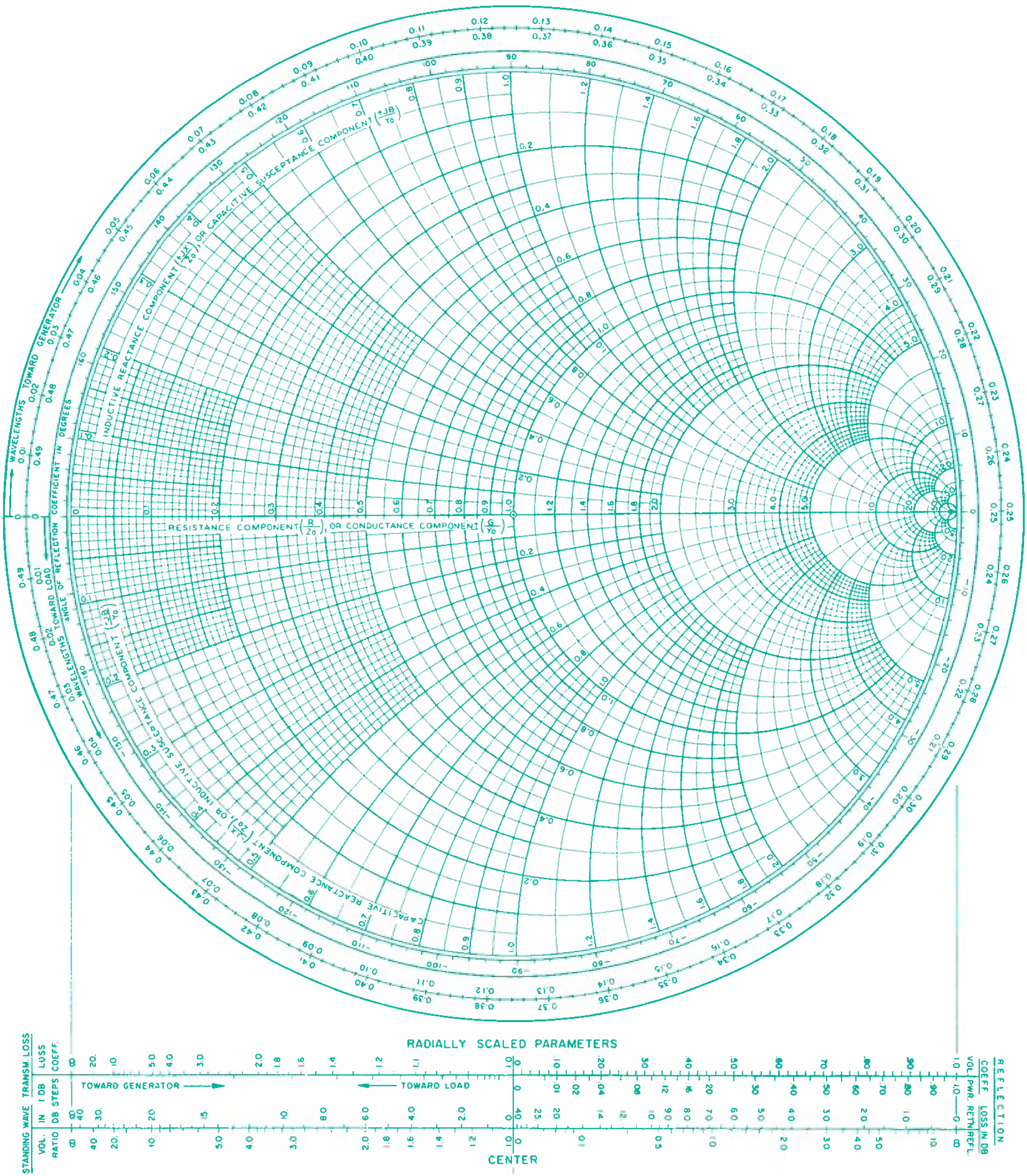
$$f_r = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

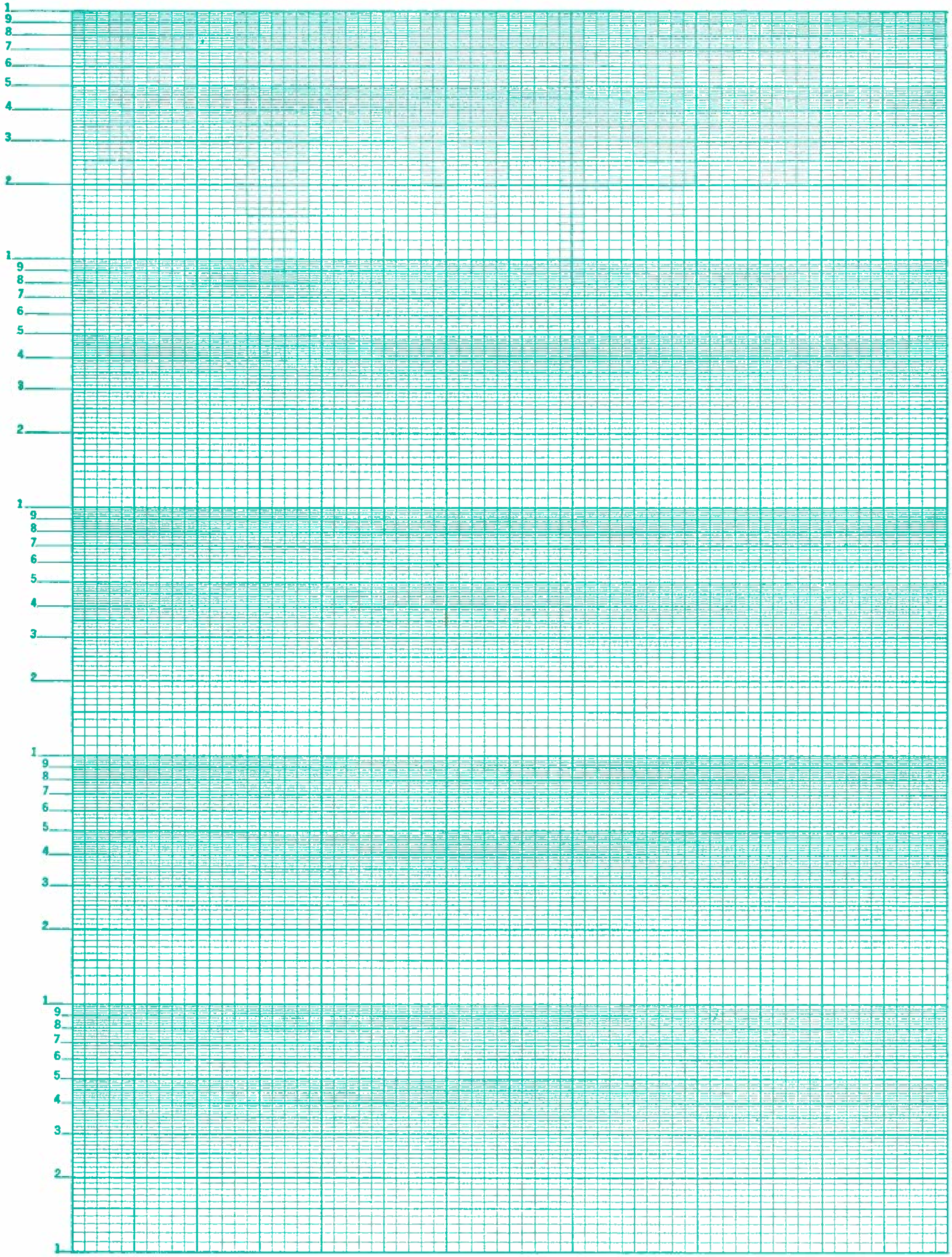
$$Q|_{TE101} = \frac{(a^2+c^2)abc}{\delta[2b(a^3+c^3)+ac(a^2+c^2)]}$$

$$\lambda_r = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}}$$

$$Q = \omega\frac{W}{P_L}$$

IMPEDANCE OR ADMITTANCE COORDINATES

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