3.)

Given:

$$z = 0.524\angle 79.04^\circ\ \Omega/km\ ;\quad y = 3.172\times10^{-6}\angle 90^\circ\ S/km$$

$$P_R = 130\,MW\ ;\quad V_{R(L-L)} = 220\,kV\ ;\quad \cos\theta_R = 1$$

$$l = 350\,km$$

Required: [$V_s$, $I_s$, $P_s$, Voltage Regulation]

$$|V_R| = \frac{|V_{R(L-L)}|}{\sqrt{3}} = \frac{220\,kV}{\sqrt{3}} = \underline{127.02\,kV}$$

$$|I_R| = \frac{P_R}{\sqrt{3}\,|V_{R(L-L)}|\cos\theta_R} = \frac{130\times10^6}{\sqrt{3}\times 220000\times 1} = \underline{341.16\,A}$$

$$I_R = |I_R|\angle\cos^{-1}1 = \underline{341.16\angle 0^\circ\,A}$$

$$\gamma l = \sqrt{zy}\cdot l = \left[\sqrt{0.524\angle 79.04^\circ\times 3.172\times10^{-6}\angle 90^\circ}\right]\times 350$$

$$\gamma l = 1.289\times10^{-3}\angle 84.52^\circ\times 350$$

$$\gamma l = 0.45115\angle 84.52^\circ = 0.0431 + j0.449$$

But $\gamma l = (\alpha + j\beta)l \Rightarrow$ $\left.\begin{aligned}\alpha l &= 0.0431\\ \beta l &= 0.449\end{aligned}\right\}$ in radians

$$Z_c = \sqrt{\frac{zl}{yl}} = \sqrt{\frac{z}{y}} = \sqrt{\frac{0.524\angle 79.04^\circ}{3.172\times10^{-6}\angle 90^\circ}}$$

$$Z_c = \sqrt{\frac{0.524}{3.172\times10^{-6}}\angle 79.04^\circ - 90^\circ}$$

$$Z_c = \sqrt{\frac{0.524}{3.172 \times 10^{-6}}} \angle \frac{79.04^\circ - 90^\circ}{2}$$

$$Z_c = \underline{\underline{406.44 \angle -5.48^\circ\ \Omega}}$$

Now $\cos h(\gamma l) = \cos h(\alpha + j\beta) l$

$$\cancel{= \cos(\alpha l) \cos\beta}$$

$$= \cos h(\alpha l) \cos(\beta l) + j \sin h(\alpha l) \sin(\beta l)$$

$$= \cos h(0.0431) \cos(0.449) + j \sinh(0.0431) \sin(0.449)$$

$$= 0.9017 + j\, 0.0187$$

$$= 0.9018 \angle 0.021 \text{ rad} \quad \text{or} \quad \underline{\underline{0.9018 \angle 1.118^\circ}}$$

$$\sin h(\gamma l) = \sin h(\alpha + j\beta) l$$

$$= \sin h(\alpha l) \cos(\beta l) + j \cos h(\alpha l) \sin(\beta l)$$

$$= \sinh(0.0431) \cos(0.449) + j \cosh(0.0431) \sin(0.449)$$

$$= 0.0388 + j\, 0.43446$$

$$= 0.4361 \angle 1.48 \text{ rad} \quad \text{or} \quad \underline{\underline{0.4361 \angle 84.8^\circ}}$$

$$A = \cos h(\gamma l) = D = \underline{\underline{0.9018 \angle 1.118^\circ}}$$

$$B = Z_c \sin h(\gamma l) = (406.44 \angle -5.48^\circ)(0.4361 \angle 84.8^\circ)$$

$$B = \underline{\underline{177.25 \angle 79.32^\circ\ \Omega}}$$

$$C = \left(\frac{1}{Z_c}\right) \sin h(\gamma l) = \frac{0.4361 \angle 84.8^\circ}{406.44 \angle -5.48^\circ}$$

$$= \underline{\underline{1.073 \times 10^{-3} \angle 90.28^\circ\ S}}$$

$$V_s = V_R \cosh(\gamma l) + I_R Z_c \sinh(\gamma l)$$

$$V_s = A V_R + B I_R$$

$$V_s = (0.9018\angle 1.19^\circ)(127020\angle 0^\circ) + (177.25\angle 79.32^\circ)(341.16\angle 0^\circ)$$

$$= 114546.63\angle 1.19^\circ + 60470.61\angle 79.32^\circ$$

$$= 114521.9 + j2378.9 + 11206.6 + j59423.1$$

$$= 125728.5 + j61802$$

$$V_s = 140097\angle 26.176^\circ$$

$$\underline{V_s = 140.1\angle 26.176^\circ \text{ KV}} \quad \text{where } |V_s| = \underline{140.1 \text{ kV}}$$

$$\theta_{V_s} = \underline{26.176^\circ}$$

$$I_s = \frac{V_R}{Z_c}\sinh(\gamma l) + I_R \cosh(\gamma l)$$

$$I_s = C V_R + D I_R$$

$$I_s = (1.073\times 10^{-3}\angle 90.28^\circ)(127020\angle 0^\circ) + (0.9018\angle 1.118^\circ)(341.16\angle 0^\circ)$$

$$= 136.29\angle 90.28^\circ + 307.66\angle 1.118^\circ$$

$$= -0.666 + j136.288 + 307.60 + j6$$

$$= 306.934 + j142.288$$

$$= \underline{338.31\angle 24.87^\circ \text{A}} \quad \text{where } |I_s| = \underline{338.31 \text{ A}}$$

$$\theta_{I_s} = 24.87^\circ$$

$$P_s = \left[\sqrt{3}\,|V_s|\,|I_s| \cos(\theta_{V_s} - \theta_{I_s})\right]$$

$$P_s = \left[\sqrt{3} \times (140.01 \times 10^3) \times (338.31) \cos(26.176^\circ - 24.87^\circ)\right]$$

$$\underline{P_s = 142.06\ MW}$$

$$\% V.R = \frac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} \times 100$$

But $|V_{R,FL}| = |V_R| = \underline{127020}\ V$

To compute $|V_{R,NL}|$ ; we recall that

$$V_s = A V_R + B I_R$$

At no-load, $I_R = 0$

$$\Rightarrow \quad V_s = A V_{R,NL}$$

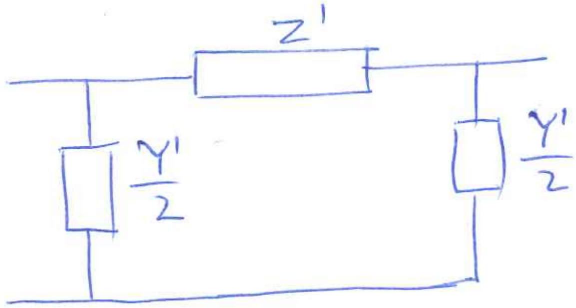
$$\therefore \quad |V_{R,NL}| = \frac{|V_s|}{|A|}$$

$$\Rightarrow \quad \% V.R = \frac{\frac{|V_s|}{|A|} - |V_{R,FL}|}{|V_{R,FL}|} \times 100$$

$$= \frac{\frac{|140010|}{|0.9018|} - |127020|}{|127020|} \times 100$$

$$\underline{\% V.R \cong 22.23\ \%}$$

4.] Equivalent $\pi$ – model




$$Z' = (zl)\,\frac{\sinh(\gamma l)}{\gamma l}$$

$$= \frac{(0.524\angle 79.04^\circ \times 350)\,\sinh(\gamma l)}{0.45115\angle 84.52^\circ}$$

$$= \frac{183.4\angle 79.04^\circ \times 0.4361\angle 84.8^\circ}{0.45115\angle 84.52^\circ}$$

$$= 177.28\angle 79.32^\circ\ \Omega$$ ← compare with 'B' or 'Z' computed using the long-line model

$$\frac{Y'}{2} = \left(\frac{yl}{2}\right)\frac{\tanh(\gamma l/2)}{\gamma l/2} = \left(\frac{y}{\gamma}\right)\tanh(\gamma l/2)$$

$$= \frac{3.172\times 10^{-6}\angle 90^\circ}{1.289\times 10^{-3}\angle 84.52^\circ}\tanh(\gamma l/2)$$

$$Y'/2 = \left[2.46\times 10^{-3}\angle 5.48^\circ\right]\tanh(\gamma l/2)$$

attenuation constant

Phase constant

$$\text{Now}\quad \tanh\left(\gamma l/2\right) = \frac{\sinh(\gamma l/2)}{\cosh(\gamma l/2)} = \frac{\sinh\left(\frac{\alpha l}{2} + j\frac{\beta l}{2}\right)}{\cosh\left(\frac{\alpha l}{2} + j\beta l/2\right)}$$

From computations done in question (3),

$$\frac{\alpha l}{2} = 0.02155 \quad ; \quad \frac{\beta l}{2} = 0.2245$$

$$\Rightarrow \sinh\left(\frac{\alpha l}{2} + j\frac{\beta l}{2}\right) = \sinh\left(\frac{\alpha l}{2}\right)\cos\left(\beta l/2\right) + j\cosh\left(\alpha l/2\right)\sin\left(\beta l/2\right)$$

$$= \sinh(0.02155)\cos(0.2245) + j\cosh(0.02155)\sin(0.2245)$$

$$= 0.0210 + j\,0.22267$$

$$= 0.2236\angle 1.4767\text{ rad or } \underline{\underline{0.2236\angle 84.61^\circ}}$$

$$\cosh\left(\frac{\alpha l}{2} + j\beta l/2\right) = \cosh\left(\frac{\alpha l}{2}\right)\cos\left(\beta l/2\right) + j\sinh\left(\frac{\alpha l}{2}\right)\sin\left(\beta l/2\right)$$

$$= \cosh(0.02155)\cos(0.2245) + j\sinh(0.02155)\sin(0.2245)$$

$$= 0.9751 + j\,4.798\times10^{-3}$$

$$= 0.9751\angle 4.92\times10^{-3}\text{ rad or } \underline{0.9751\angle 0.282^\circ}$$

$$\therefore \frac{Y'}{2} = \left(2.46\times10^{-3}\angle 5.48^\circ\right)\left(\frac{0.2236\angle 84.61^\circ}{0.9751\angle 0.282^\circ}\right)$$

$$\underline{\underline{\frac{Y'}{2} = 5.64\times10^{-4}\angle 89.81^\circ\ S}}$$

Compare with $\left(\frac{yl}{2}\right)$

Alternatively,

$$\frac{Y'}{2} = \frac{Y}{2}\,\frac{\tanh\left(\frac{\gamma l}{2}\right)}{\gamma l/2} = \left[\frac{Y}{2}\right]\frac{\cosh(\gamma l) - 1}{\gamma l/2\ \sinh(\gamma l)}$$

$$\frac{Y'}{2} = \frac{yl}{2\left(\frac{\gamma l}{2}\right)} \left[ \frac{\cos h(\gamma l) - 1}{\sin h(\gamma l)} \right]$$

$$\frac{Y'}{2} = \frac{y}{\gamma} \left[ \frac{(0.9018 \angle 1.118^\circ) - 1}{0.4361 \angle 84.8^\circ} \right]$$

$$= \frac{3.172 \times 10^{-6} \angle 90^\circ}{1.289 \times 10^{-3} \angle 84.52^\circ} \left[ \frac{0.9017 + j0.0187 - 1}{0.0388 + j0.43446} \right]$$

$$= 2.46 \times 10^{-3} \angle 5.48^\circ \left[ \frac{-0.0983 + j0.0187}{0.0388 + j0.43446} \right]$$

$$= 2.46 \times 10^{-3} \angle 5.48^\circ \left[ \frac{0.1 \angle 169.23^\circ}{0.4361 \angle 84.896^\circ} \right]$$

$$= \left(2.46 \times 10^{-3} \angle 5.48^\circ\right)\left(0.229 \angle 84.33^\circ\right)$$

$$\frac{Y'}{2} = 5.63 \times 10^{-4} \angle 89.81^\circ \ \text{S}$$