In-Class Example Lecture 2 Solution

1. **Given:**

$|I_S| = 300$ A; $\cos\theta_S = 0.9$ lag; $ABCD$ constants

Required: $[\, V_R, I_R, \cos\theta_R, \%\,\mathrm{V.R}]$

The sending-end phase voltage, $|V_S| = 260/\sqrt{3} = 150.111$ kV

The sending-end current is:

$I_S = |I_S|\langle -\cos^{-1} 0.9 = 300\langle -25.84^o$ A; We have used negative sign because the power factor is lagging

From the ABCD form of the medium line model, we can write:

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

Recall that the inverse of a 2 by 2 matrix is given by,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \frac{1}{AD-BC}\begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$$

But $AD - BC = 1$

Therefore, $\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$

This means that:

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} 0.8904\angle 1.34^o & -186.82\angle 79.45^o \\ -1.131\times 10^{-3}\angle 90.41^o & 0.8904\angle 1.34^o \end{bmatrix} \begin{bmatrix} 150.111\times 10^3 \\ 300\angle -25.84^o \end{bmatrix}$$

$$V_R = (0.8904\angle 1.34^o)(\,150.111\times 10^3) + (\,-186.82\angle 79.45^o)(300\angle -25.84^o)$$

$$= (133658.8\angle 1.34^o) + (\,-56046\angle 53.61^o)$$

$$= (133622.3 + j3125.65) + (-33250.9 - j45116.9)$$

$= (100371.4 - j41991.25)$

$V_R = \mathbf{108.8\angle - 22.7^o\ kV}$

$|V_R| = 108.8\ \mathbf{kV}\ ;\ \theta_{VR} = \mathbf{-22.7^o}$

$I_R = (-1.131 \times 10^{-3}\angle 90.41^o)(\ 150.111 \times 10^3) + (0.8904\angle 1.34^o)(300\angle - 25.84^o)$

$= (-169.77\angle 90.41^o) + (\ 267.12\angle - 24.5^o)$

$= (1.215 - j169.76) + (243.07 - j110.77)$

$= (244.28 - j280.53)$

$I_R = \mathbf{372\angle - 48.95^o\ A}$

$|I_R| = 372\ \mathbf{A}\ ;\ \theta_{IR} = \mathbf{-48.95^o}$

(ii) The receiving-end power factor is:

$\cos\theta_R = \cos(\theta_{VR} - \theta_{IR})$

$= \cos(\mathbf{-22.7^o + 48.9^o}) = 0.897\ \mathbf{lagging}$

2. % Voltage Regulation is:

$\%\ V.R = (|V_{R(NL)}| - |V_{R(FL)}|/|V_{R(FL)}|) \times 100$

$|V_{R(FL)}| = |V_R| = 108.8\ \mathrm{kV}$

We find $|V_{R(NL)}|$ by setting $I_R = 0$ (i.e. no-load condition) in the ABCD form of equation for the medium line. Thus, from the ABCD form, we know that:

$V_S = \mathrm{A}V_R + BI_R$

With $I_R = 0; V_R = V_{R(NL)}$

Therefore, $V_S = \mathrm{A}V_{R(NL)}$ or $|V_S| = |A|.|V_{R(NL)}|$

$|V_{R(NL)}| = |V_S|/|A| = 150.111/0.8904 = 168.6\ \mathrm{kV}$

Side Note: Can we notice something? The no-load voltage (measured at the receiving end) is greater than the sending-end voltage. Where did the extra voltage come from? Marveled? You do not need to. The extra voltage came from the line capacitance. When the line is energized, the line capacitance draws a leading charging current which introduces a voltage drop across the line inductance which is in-phase with the (or adds to the) sending-end voltage thereby resulting in a no-load voltage (measured at the receiving-end) that is greater than the sending-end voltage. This phenomenon is called **Ferranti Effect**. The effect occurs when the line is not loaded or is lightly loaded. The effect is more pronounced in long lines due to large line capacitance caused by increased line length. The overvoltage introduced by Ferranti effect can destroy transmission equipment. In order to mitigate this effect, utility companies introduce a line reactor (which is essentially an inductor) in series with long lines when they are not loaded or lightly loaded. The function of the inductor (or reactor) is to draw a lagging current which neutralizes the leading current drawn by the line capacitance thereby reducing the in-phase voltage drop across the line inductance caused by the line capacitance.

Now, $\% \, V.R = ((|V_S|/|\mathrm{A}| - |V_{R(FL)}|)/|V_{R(FL)}|) \times 100$

$$\% \, V.R = \left(\frac{\frac{150.111}{0.8904} - 108.8}{108.8}\right) \times 100 = 54.95 \,\%$$