Faraday's Law

$$\text{EMF} = -\frac{d}{dt}\Phi \quad \leftarrow \text{always works}$$

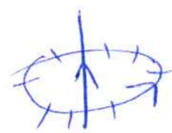
$$= -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$$

$$= \oint(\vec{v}\times\vec{B})\cdot d\vec{l} - \int \frac{\partial}{\partial t}\vec{B}\cdot d\vec{s}$$

motional transformer

Ampere's Law

↳ 475 $\Rightarrow \oint \vec{H}\cdot d\vec{l} = I_{encl}$



$$\nabla\times\vec{H} = \vec{J}$$

$$\nabla\cdot(\nabla\times\vec{H}) = 0$$

$$\nabla\cdot\vec{J} = -\frac{\partial \rho_v}{\partial t} \neq 0 \text{ for time-varying fields}$$

↳ continuity equation

So, $\nabla\times\vec{H} = \vec{J} + \vec{J}_d \rightarrow$ displacement current density

Now ~~that~~ $\nabla\cdot\nabla\times\vec{H} = 0 \Rightarrow \nabla\cdot(\vec{J}+\vec{J}_d) = 0$

or $\nabla\cdot\vec{J} + \nabla\cdot\vec{J}_d = 0$

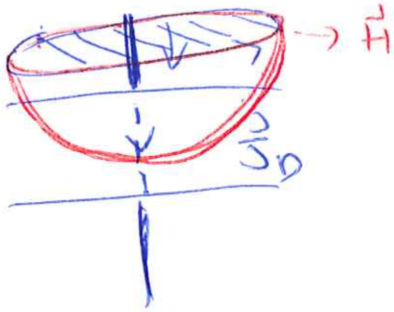
$$\Rightarrow \nabla\cdot\vec{J}_d = -\nabla\cdot\vec{J} = \frac{\partial\rho_v}{\partial t}$$

$(\nabla\cdot\vec{D} = \rho_v)$
Gauss' law

$$\text{So, } \nabla \cdot \vec{J}_d = \frac{\partial}{\partial t}(\nabla \cdot \vec{D})$$
$$= \nabla \cdot \left(\frac{\partial}{\partial t}\vec{D}\right)$$

$$\Rightarrow \vec{J}_d = \frac{\partial}{\partial t}\vec{D}$$
$$= \frac{\partial}{\partial t}\epsilon\vec{E}$$

$$\Rightarrow \nabla \times \vec{H} = \vec{J} + \frac{\partial}{\partial t}\epsilon\vec{E}$$



$$\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{s} + \int \vec{J}_d \cdot d\vec{s}$$

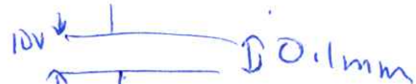
$\vec{J}_D \rightarrow$ displacement current density (A/m²)

$I_D = \int \vec{J}_D \cdot d\vec{s} \rightarrow$ displacement current (A)

Ex:
- Capacitor filled with teflon ($\epsilon_R = 2.4$)
- capacitor connected to generator operating at 10MHz + suppling 10V.
- Capacitor has plate areas of 1.5cm² and separation between plates of 0.1mm.

Find maximum values of $\vec{J}_D$ + I_D.

$|\vec{E}| = V/d$



$V(t) = 10\cos(2\pi\times10^{7}t)$

$E(t) = 1\times10^{5}\cos(2\pi\times10^{7}t)$ V/m

$D(t) = (2.4)(\epsilon_0)(1\times10^{5})\cos(2\pi\times10^{7}t)$

$\epsilon_0 = 8.85\times10^{-12}$

$\vec{J}_d = \frac{\partial}{\partial t}\vec{D} \Rightarrow \vec{J}_d = -(2.4)\epsilon_0(1\times10^{5})(2\pi\times10^{7})\sin(2\pi\times10^{7}t)$

$|\vec{J}_d|_{max} = 133.3$ A/m^2

$I_{d,max} = (133.3)(1.5\,cm^2)$
$= 0.02$ A

Source-free region & time varying fields

↳ $\rho_v = 0$

↳ $\vec{J} = 0$

$\nabla\cdot\vec{D} = \cancel{\rho_v}\; 0$ $\qquad \nabla\times\vec{E} = -\frac{\partial}{\partial t}\vec{B}$

$\nabla\cdot\vec{B} = 0$ $\qquad \nabla\times\vec{H} = \cancel{\vec{J}}\; 0 + \frac{\partial\vec{D}}{\partial t}$

Ex $\vec{E}(x,t) = 20\cos(\omega t - 50x)\vec{a}_y$

(*) free space

(*) source-free region

Find a) $\vec{J}_D(x,t)$

b) $\vec{H}(x,t)$ including ω

a) $\vec{J}_D(x,t) = \frac{\partial}{\partial t}\left[\epsilon_0\, 20\cos(\omega t - 50x)\vec{a}_y\right]$

$= -20\epsilon_0\omega\sin(\omega t - 50x)\vec{a}_y$

b) $\nabla\times\vec{E} = -\frac{\partial}{\partial t}\mu_0\vec{H}$

(4)

$$\nabla \times \vec{E} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} \qquad E_y = 20\cos(\omega t - 50x)$$

$$= \vec{a}_x\left(-\underbrace{\frac{\partial}{\partial z}E_y}_{0}\right) - \vec{a}_y(0) + \vec{a}_z\left(\frac{\partial}{\partial x}E_y\right)$$

$$= 1000\sin(\omega t - 50x)\vec{a}_z$$

$$-\frac{\partial}{\partial t}\mu_0\vec{H} = 1000\sin(\omega t - 50x)\vec{a}_z$$

$$\frac{\partial}{\partial t}\vec{H} = -\frac{1000}{\mu_0}\sin(\omega t - 50x)\vec{a}_z$$

$$\Rightarrow \vec{H} = \frac{1000}{\omega\mu_0}\cos(\omega t - 50x)\vec{a}_z$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \Rightarrow \omega = ?$$

$$\begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_z \end{vmatrix} = -\vec{a}_y\left(\frac{\partial}{\partial x}H_z\right)$$

$$\left.\begin{aligned} \nabla \times \vec{H} &= -\frac{50\,000}{\omega\mu_0}\sin(\omega t - 50x)\vec{a}_y \\ \vec{J}_D &= -20\epsilon_0\omega\sin(\omega t - 50x)\vec{a}_y \end{aligned}\right\} \nabla \times \vec{H} = \vec{J}_D$$

$$\Rightarrow -\frac{50000}{\omega\mu_0} = -20\epsilon_0\omega \qquad c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3\times10^8\ m/s$$

$$\Rightarrow \quad \omega^2 = \frac{2500}{\mu_0 \epsilon_0}$$

$$\omega = \sqrt{\frac{2500}{\mu_0 \epsilon_0}}$$

$$= (50)(3 \times 10^8)$$

$$= 1.5 \times 10^{10} \text{ rad/s}$$

$$\nabla \cdot \vec{D} = 0 \checkmark$$

$$\nabla \cdot \vec{B} = 0 \checkmark$$

valid expression

start with curl of vector that is known

$$① \quad \nabla \times \vec{E} = -\frac{\partial}{\partial t} \mu_0 \vec{H}$$

$$② \quad \nabla \times \vec{H} = \frac{\partial}{\partial t} \epsilon_0 \vec{E} \checkmark$$

$\Downarrow$

unknown parameter ω

$\Rightarrow$ time-harmonic fields

$$\vec{E}(x,t) = 20 \cos(\omega t - 50x)\vec{a}_y$$

$$\Rightarrow \vec{E}_s(x) = 20 e^{-j50x} \vec{a}_y$$