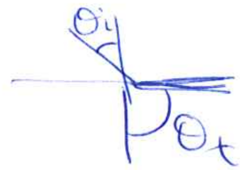
<u>Oblique tx/rx</u> → special case #2
↳ total reflection

$k_1 \sin\theta_i = k_2 \sin\theta_t \qquad (\mu_1, \mu_2 = \mu_0)$

$$\sin\theta_t = \frac{k_1}{k_2}\sin\theta_i \qquad k_1 = \omega\sqrt{\mu_0\epsilon_0\epsilon_{R1}}$$
$$k_2 = \omega\sqrt{\mu_0\epsilon_0\epsilon_{R2}}$$
$$= \frac{\sin\theta_i}{\sqrt{\epsilon_{R2}/\epsilon_{R1}}}$$

↳ $\sin\theta_t = 1 \Rightarrow \theta_t = 90^\circ \Rightarrow$ total reflection



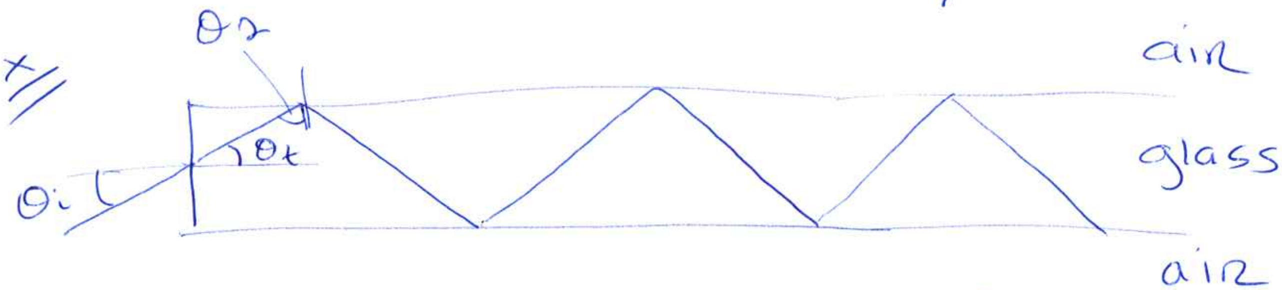
$$\Rightarrow \sin\theta_i = \sqrt{\epsilon_{R2}/\epsilon_{R1}}$$

↳ $\theta_i = \theta_c \Rightarrow$ critical angle

$\Rightarrow$ if $\theta_i \geq \theta_c$, then total reflection

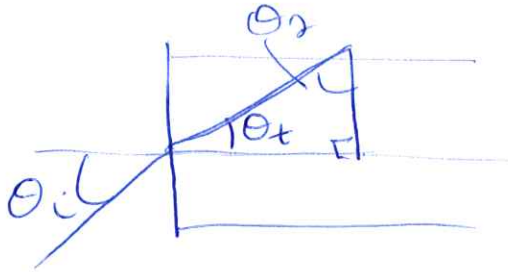
$$\theta_c = \sin^{-1}\left(\sqrt{\epsilon_{R2}/\epsilon_{R1}}\right) \quad (*)\ \epsilon_{R1} > \epsilon_{R2}$$

Ex//



↳ what is θ_i such that $\theta_2 > \theta_c$?

↳ what range of ϵ_{R1} provides $\theta_2 > \theta_c$ for any θ_i?



$$\sin\theta_2 \geq \sin\theta_c$$

$$\theta_2 = \frac{\pi}{2} - \theta_t$$

$$\sin(\pi/2 - \theta_t) \geq \sin\theta_c$$

$$\cos\theta_t \geq \sin\theta_c$$

θ_t vs θ_i ?

$\rightarrow$ Snell's law $\rightarrow$ $\dfrac{\sin\theta_i}{\sin\theta_t} = \sqrt{\epsilon_R}$

$$\sin\theta_t = \frac{\sin\theta_i}{\sqrt{\epsilon_R}}$$

$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t}$$

$$= \sqrt{1 - \frac{\sin^2\theta_i}{\epsilon_R}}$$

$$\Rightarrow \sqrt{1 - \frac{\sin^2\theta_i}{\epsilon_R}} \geq \sin\theta_c \quad \Rightarrow \sin\theta_c = \frac{1}{\sqrt{\epsilon_R}}$$

$$\sqrt{1 - \frac{\sin^2\theta_i}{\epsilon_R}} \geq \frac{1}{\sqrt{\epsilon_R}}$$

$$\epsilon_R \geq 1 + \underbrace{\sin^2\theta_i}_{} \rightarrow \max = 1$$

$$\epsilon_R \geq 2 \Rightarrow \theta_2 \geq \theta_c \text{ for } \theta_i$$

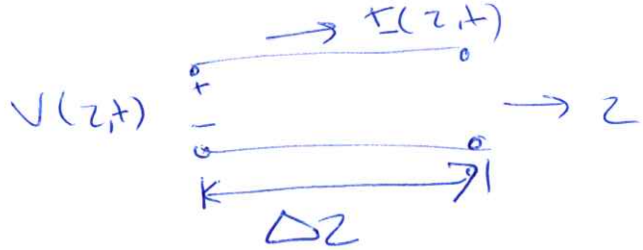
Transmission Lines

↳ 2+ parallel conductors

↳ describe with distributed circuit parameters

→ lumped element ⇒ component with particular R, L, C

→ distributed ⇒ per unit length



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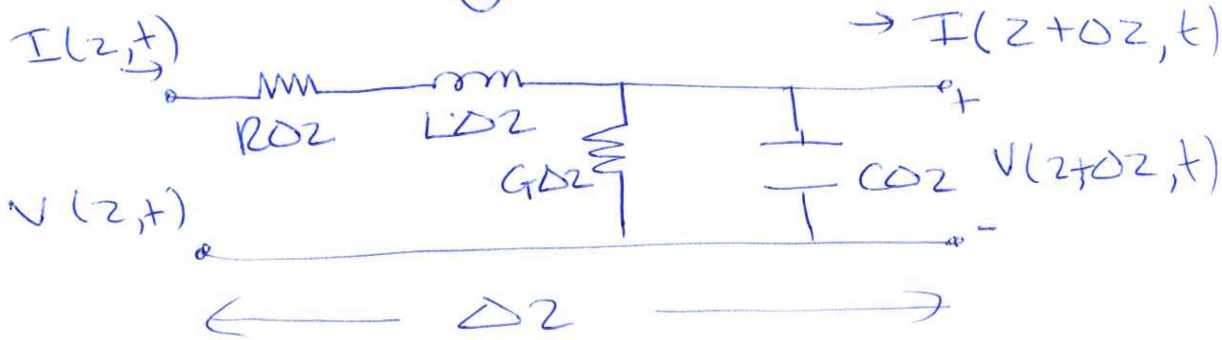


R → series resistance/unit length (Ω/m)

↳ finite conductivity of conductors

L → series inductance/unit length (H/m)

↳ inductance of 2 lines

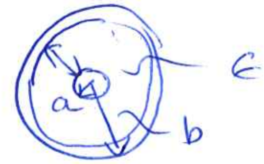
C → shunt capacitance/unit length (F/m)

↳ close proximity of conductors

G → shunt conductance/unit length (S/m)

↳ loss in material in between conductors

R, L, G, C for coax



Recall: $C = Q/V = \dfrac{\epsilon \oint \vec{E} \cdot d\vec{s}}{-\int \vec{E} \cdot d\vec{l}}$

for coax $\quad C = \dfrac{2\pi\epsilon}{\ln(b/a)}$

$$R = \frac{-\int \vec{E} \cdot d\vec{l}}{\sigma \int \vec{E} \cdot d\vec{s}}$$

$$RC = \epsilon/\sigma \Rightarrow \frac{C}{G} = \frac{\epsilon}{\sigma} \Rightarrow G = C\frac{\sigma}{\epsilon}$$

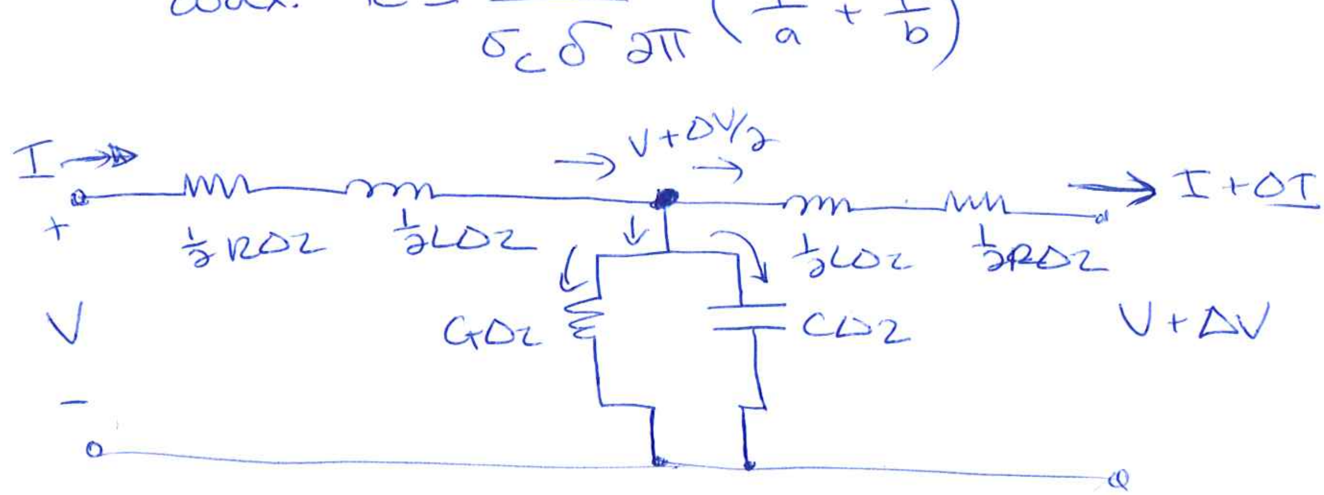
for coax: $\quad G = \dfrac{2\pi\sigma}{\ln(b/a)}$ → material between conductors

↳ $LC = \mu\epsilon \Rightarrow L = \mu\epsilon/C$

for coax: $\quad L = \dfrac{\mu}{2\pi}\ln(b/a)$

↳ $R \rightarrow R_{ac} = \dfrac{l}{\sigma\delta w} \qquad w = 2\pi a$

for coax: $\quad R = \dfrac{1}{\sigma_c \delta 2\pi}\left(\dfrac{1}{a} + \dfrac{1}{b}\right)$



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$\rightarrow$ KVL: $V = \frac{1}{2} RI\Delta z + \frac{1}{2} L\Delta z \frac{\partial I}{\partial t} + \frac{1}{2} L\Delta z \left(\frac{\partial I}{\partial t} + \frac{\partial \Delta I}{\partial t} \right)$

$+ \frac{1}{2} R (I + \Delta I) \Delta z + V + \Delta V$

$$\frac{\Delta V}{\Delta z} = \left(-RI + L\frac{\partial I}{\partial t} + \frac{1}{2} L \frac{\partial \Delta I}{\partial t} + \frac{1}{2} R \Delta I \right)$$

$$\Delta I = \frac{\partial I}{\partial z} \Delta z \quad \& \quad \Delta V = \frac{\partial V}{\partial z} \Delta z$$

$$\Rightarrow \frac{\partial V}{\partial z} = -\left(1 + \underbrace{\frac{\Delta z}{2} \frac{\partial}{\partial z}}_{\Delta z \Rightarrow 0} \right) \left(RI + L\frac{\partial I}{\partial t} \right)$$

As $\Delta z \Rightarrow 0$, $\boxed{\frac{\partial V}{\partial z} = -\left(RI + L\frac{\partial I}{\partial t} \right)}$ (A)

KCL $\rightarrow$ $I = I_g + I_c + I + \Delta I$