Faraday’s law (9.2 and 9.3)

- Relation between fields and sources from ENEL 475?
- Time-varying fields – what is different?
- Faraday’s experiment and Faraday’s law
  - EMF
  - Lenz’s law
- Transformer and motional EMF

Faraday’s law

- Faraday’s experiment
- https://en.wikipedia.org/wiki/Michael_Faraday#/media/File:Induction_experiment.png

Faraday’s experiment demo

- http://www.youtube.com/user/CarletonPhysics
- https://www.youtube.com/watch?v=7MTTyWzX1EY

Time-varying fields

↳ 2 charged particles at rest act on each with a force

$\Rightarrow$ Coulomb's law

$$\vec{F} = Q\vec{E}$$

↳ 2 charged particles with uniform velocities act on each other with magnetic force

$$\Rightarrow \vec{F} = Q\vec{v} \times \vec{B}$$

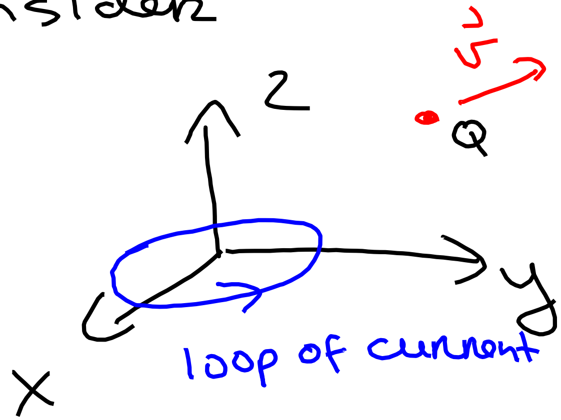
↳ accelerated particle $\Rightarrow$ another force exerted on stationary or moving particle

$\Rightarrow$ small compared to $\vec{F} = Q\vec{E}$ but many charges in conductor $\Rightarrow$ measureable

$\Rightarrow$ same form as $\vec{F} = Q\vec{E}$ but $\vec{E}$ is different

$\Rightarrow \vec{E}_{ind}$ (induced electric field strength)

Consider

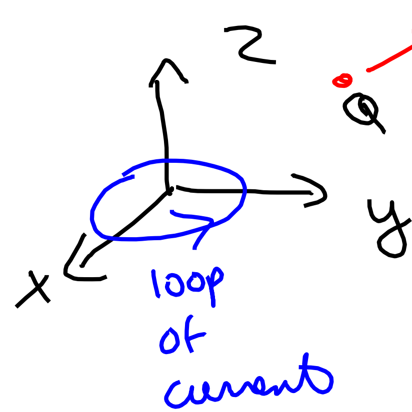


→ record force acting on particle
$\vec{F} = Q\vec{v} \times \vec{B}$

→ if $\vec{v} = 0$, no force

⇒ magnetic field, no electric field

$$\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})$$



(*) frame of reference moves with particle

→ observer moving with particle ⇒ looks stationary

→ particle experiences force ⇒ $\vec{F} = Q\vec{E}$ because particle isn't moving

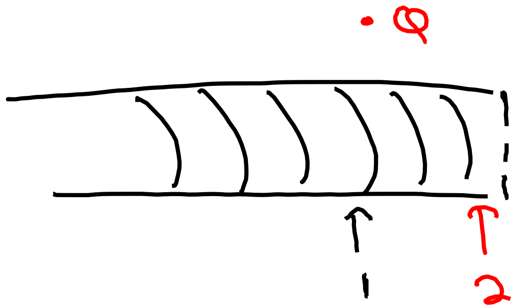
<u>but</u> $\vec{E}$ is time-varying <u>and</u> magnetic field is time varying

⇒ different conclusions from same scenario with different observers

⇒ $\vec{F} = Q\vec{v} \times \vec{B}$ vs. $\vec{F} = Q\vec{E}$ ⇒ but time-vary magnetic field accompanied by time-varying electric field

⇒ $\vec{E}_{ind}$ (time-varying electric field)

Consider solenoid with charged particle



→ move solenoid between pos'ns 1+2

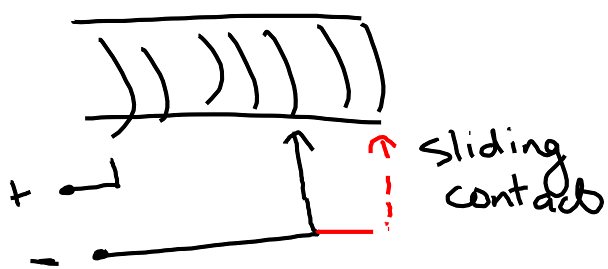
↳ observe from particle

⇒ see changing field ⇒ $\vec{F} \propto Q\tilde{E}$

↳ observe from solenoid

⇒ see moving Q ⇒ $\vec{F} \propto Q\vec{v} \times \tilde{B}$

→ switch solenoid



→ magnetic field changes in same way <u>but</u> cause of change is different

→ see same effect on particle

→ regardless of cause of time-varying magnetic field, get induced electric field

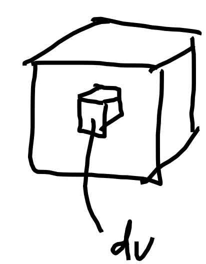
Charge in Static + induced field experiences:

$$\vec{F} = Q(\vec{E}_{Static} + \vec{E}_{ind})$$

? e.g. $\vec{E}_{ind} = \vec{v} \times \vec{B}$
(charged particle moving with $\vec{v}$ in field $\vec{B}$)

<u>but</u> time-vary currents are also "sources" of $\vec{E}_{ind}$

$$\vec{E}_{ind} = -\frac{\partial}{\partial t}\left(\frac{\mu_0}{4\pi}\int_V \frac{\vec{J}\,dv}{r}\right) \quad (V/m)$$



$\vec{J}\,dv$ ↑ volume

- $\vec{J}\,ds$ → surface
- $i\,d\vec{l}$ → line
- $Q\,d\vec{v}$ → charge

Recall:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

$$\vec{E} = -\nabla V$$

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{Q\vec{v}}{R} \Rightarrow$$ magnetic vector potential

↳ due to elementary source of magnetic field

$$\vec{B} = \nabla \times \vec{A}$$

total flux

$$\Phi = \int \vec{B}\cdot d\vec{s}$$

$$= \int (\nabla \times \vec{A})\cdot d\vec{s}$$

$$= \oint_C \vec{A}\cdot d\vec{l}$$

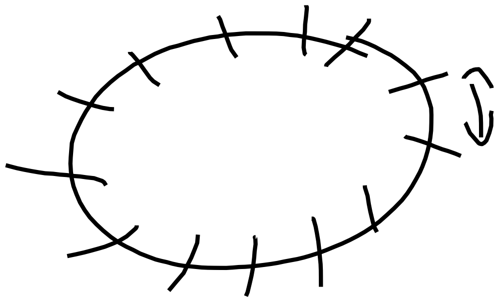
→ In region with free charge carriers (e.g. wire), $\vec{E}_{ind}$ acts on carriers with $\vec{F} = Q\vec{E}_{ind}$

↳ charge of free e^-

→ line integral of $\vec{E}_{ind}$ between 2 points M+N ⇒ EMF (electomotive force) induced in line

$$EMF = \int_M^N \vec{E}_{ind}\cdot d\vec{l}$$

wire in time-varying field



→ each small section ⇒ generator

→ time-varying fields are source of EMF