University of Calgary

Schulich School of Engineering

Department of Electrical and Computer Engineering

ENEL 476 – Electromagnetic Waves and Applications

Quiz #2

Winter Session 2016
Monday March 7, 2016
2:00-2:50 pm

ICT 102

Student Name or ID number:

Dr. Fear
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Question 1. (26 marks)

A uniform plane wave is propagating in the –y direction and in a material with ε_r=10, σ=2 S/m and μ_r=2. The frequency of the wave is 100 MHz. The electric field is oriented in the +x direction and has magnitude of 1 kV/m at y=0.

Find:

a) phase constant (β)

(1) $\frac{\sigma}{\omega\epsilon} = \frac{2}{(2\pi)(10^8)(10)(\frac{1}{36\pi}\times10^{-9})}$

$= 36$

(1) ∴ good conductor

or (1) for σ formula in (c)

(1) $\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$

$= \sqrt{\frac{(2\pi\times10^8)(2)(4\pi\times10^{-7})(2)}{2}}$

$= \sqrt{160\pi^2}$

(1) $\boxed{\beta = 39.7 \text{ rad/m}}$

b) velocity of propagation (v_p or u)

(1) $V_p = \omega/\beta$

$= \frac{2\pi\times10^8}{39.7}$

(1) $\boxed{V_p = 1.58\times10^7 \text{ m/s}}$

c) attenuation constant (α)

(1) $\alpha = \beta$

(1) $\boxed{\alpha = 39.7 \text{ Np/m}}$

Note: full formulas give

$\alpha = 39.2$ Np/m

$\beta = 40.3$ rad/m

d) skin depth (δ)

(1) $\delta = \frac{1}{\alpha}$

(1) $\boxed{\delta = 0.0252 \text{ m}}$

e) wavelength (λ)

(1) $\lambda = \frac{2\pi}{\beta}$

$= \frac{2\pi}{39.7}$

(1) $\boxed{\lambda = 0.1583 \text{ m}}$

f) the electric field in the time domain (**E**(y,t))

$$\vec{E}(y,t) = 1e^{+39.7y}\cos(2\pi\times10^{8}t + 39.7y)\,\vec{a}_x \ \text{kV/m}$$

(1) (1) (1) (1) (1)

kV

g) intrinsic impedance of the medium (η)

$$(1)\ |\eta| = \sqrt{\frac{\omega\mu}{\sigma}}$$

$$= \sqrt{\frac{(2\pi\times10^{8})(4\pi\times10^{-7})(2)}{2}}$$

$$= \sqrt{80\pi^2}$$

(1) (1)

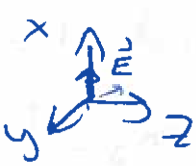
$$= 28.1\,\Omega \quad \angle 45^\circ$$

full formula:

$|\eta| = 28.1$

$\angle\eta = 0.7715$ rads

h) magnetic field in the time domain (**H**(y,t))



$$\vec{H}(y,t) = \frac{1}{28.1}e^{39.7y}\cos(2\pi\times10^{8}t + 39.7y - \tfrac{\pi}{4})\,\vec{a}_z \ \text{kA/m}$$

(1)

$$= 0.036e^{39.7y}\cos(2\pi\times10^{8}t + 39.7y - \tfrac{\pi}{4})\,\vec{a}_z \ \text{kA/m}$$

(1) (1)

i) the time-averaged Poynting vector (**P**$_{av}$(y)).

$$(1)\ \vec{P}_{av}(y) = -\frac{(1000)^2}{2(28.1)}\cos(\tfrac{\pi}{4})\,e^{2(39.7)y}\,\vec{a}_y \ \text{W/m}^2$$

$$= -(1.78\times10^{4})(\cos(\tfrac{\pi}{4}))\,e\ \ \vec{a}_y$$

$$\boxed{\vec{P}_{av}(y) = -126\,e^{79.4y}\,\vec{a}_y \ \text{kW/m}^2}$$

(1) (1) (1)

dirn

Question 2. **(14 marks)**

A uniform plane wave is normally incident on a planar interface located at y=0. The incident field is located in the region y<0 and the electric field of this uniform plane wave is described by:

$$\mathbf{E}^i(y,t)=5\cos(2\pi \times 10^8 t - 4.2y)\,\mathbf{a}_z \quad \text{V/m}$$

The material in the region y<0 has $\varepsilon_r=2$, $\sigma=0$ and $\mu_r=2$. The material in the region y>0 is a dielectric with $\varepsilon_r=25$, $\sigma=0$, $\mu_r=1$.

(1) | (2)

$\epsilon_r = 1$ | $\epsilon_r = 25$

$\mu_r = 2$

a) Calculate the reflection coefficient (Γ). 5

(1) $\eta_1 = \sqrt{\dfrac{\mu_r \mu_0}{\epsilon_0 \epsilon_r}} = \sqrt{\dfrac{2}{2}\dfrac{\mu_0}{\epsilon_0}}$

(1) $= 120\pi\ \Omega$

$\eta_2 = \sqrt{\dfrac{\mu_r \mu_0}{25\epsilon_0}} = 120\pi/5\ \Omega \Rightarrow \eta_2 = 24\pi\ \Omega$ (1)

$\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ (1)

$= \dfrac{24\pi - 120\pi}{24\pi + 120\pi}$

$= \dfrac{-96}{144}$

$\boxed{\Gamma = -0.67}$ (1)

b) Calculate the transmission coefficient (T). 2

(1) $T = 1 + \Gamma$

$= 1 - 0.67$

(1) $= 0.33$

Check: $T = \dfrac{2\eta_2}{\eta_2 + \eta_1} = \dfrac{48}{144}$

$T = 0.33$

c) Find the reflected electric field ($\mathbf{E}^r(y,t)$). 2

$\vec{E}^r(y,t) = (-0.67)(5)\cos(2\pi \times 10^8 t + 4.2y)\,\vec{a}_z$

$\vec{E}^r(y,t) = -3.35\cos(2\pi \times 10^8 t + 4.2y)\,\vec{a}_z$ V/m

(1) (1)

d) Find the transmitted electric field ($\mathbf{E}^t(y,t)$). 3

$$\vec{E}^t(y,t) = (0.33\times5)\cos(2\pi\times10^8 t - \beta_2 y)\vec{a}_z \quad V/m$$

1

$$\beta_2 = \omega\sqrt{\mu_r\mu_0\epsilon_0\epsilon_r} = \omega\sqrt{25\mu_0\epsilon_0} \rightarrow \beta_2 = \frac{2\pi\times10^8}{3\times10^8}(5)$$

e) Find the transmitted magnetic field ($\mathbf{H}^t(y,t)$). 2

$$\beta_2 = \frac{10\pi}{3}$$

$$\boxed{\beta_2 = 10.47} \quad rad/m$$

$$\vec{E}^t(y,t) = 1.65\cos(2\pi\times10^8 t - 10.47y)\vec{a}_z$$

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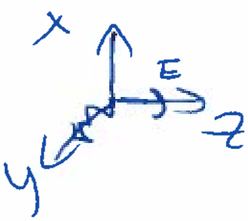
e) $$\vec{H}^t(y,t) = \frac{1.65}{24\pi}\cos(2\pi\times10^8 t - 10.47y)\vec{a}_x$$



$$= 0.022\cos(2\pi\times10^8 t - 10.47y)\vec{a}_x \quad A/m$$

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Name	
Q1	
Q2	
Total	