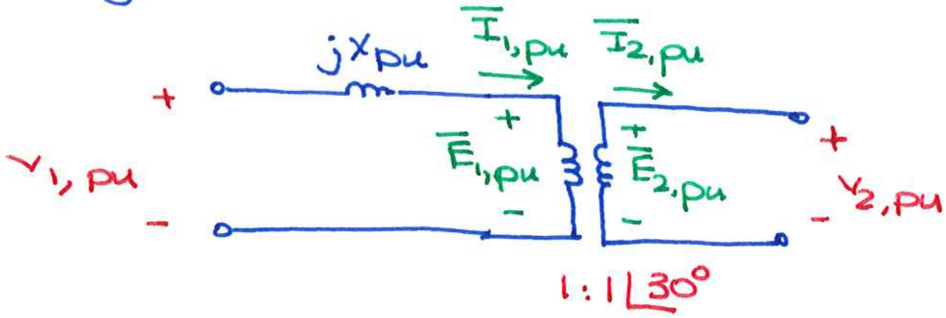
2) Δ-Y : 30° phase shift between 2 sides. P.U. only deals with magnitudes:



$\bar{I}_{2,pu} = \bar{I}_{1,pu} \angle 30^\circ$

$\bar{E}_{2,pu} = \bar{E}_{1,pu} \angle 30^\circ$

3) Y-Δ : identical to Δ-Y but with a $1:1\angle{-30^\circ}$ ideal txfr

- We will ignore the phase shifts for Δ-Y & Y-Δ transformers in this course.

Motors & Generators (machines)

- on the SLD

Gen Ⓝ— Motor Ⓜ—

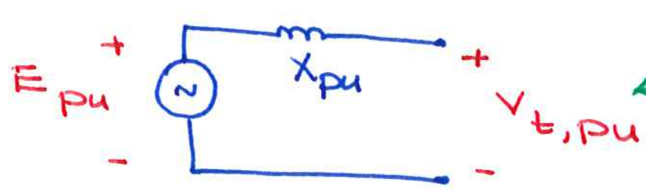
operating voltage & power may be different { 13.8 kV ← rated voltage (L-L)

10 mVA ← rated power (3ϕ)

$X = 10\%$ ← pu reactance

- In PU circuit (impedance diagram), machines are shown as an EMF behind an impedance:

operating internal EMF voltage →



← operating voltage at terminals

value in PU circuit

$$X_{pu} = X_{pu,old} \times \frac{V_{rated}^2}{V_{base}^2} \times \frac{S_{base}}{S_{rated}}$$

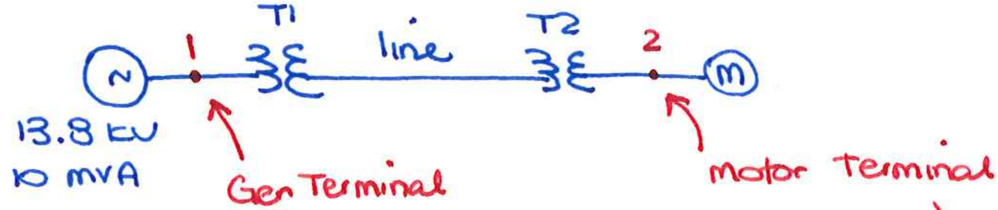
V_{rated}: $V_{base,old}$ (13.8 kV in our example)

$X_{pu,old}$: value based on rated V, S (10% in our example)

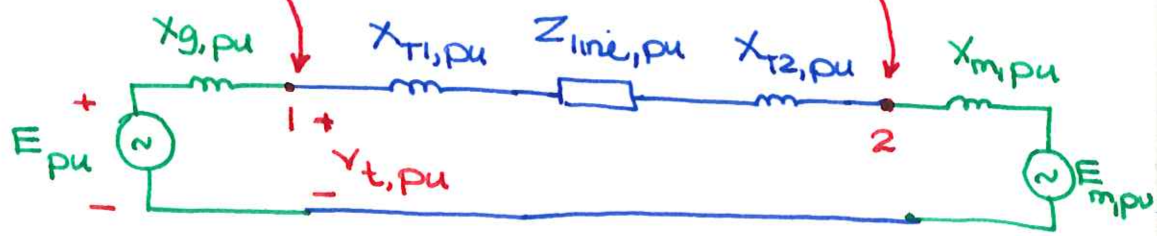
V_{base}: our choice for V_{base}

S_{rated}: $S_{base,old}$ (10 mVA in our ex.)

. SLD



. PU Circuit (Impedance Diagram)



. How to get $V_{t,pu}$ (or E_{pu}) ?

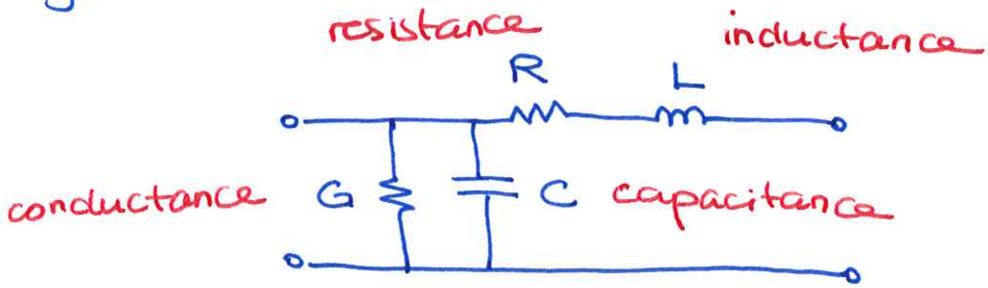
Depends on available information, i.e. system operating conditions

Given info	To get $V_{t,pu}$
. "Machine is operating at rated voltage". $\Rightarrow$	$V_{t,pu} = \dfrac{\text{operating } V}{V_{base}} = \dfrac{V_{rated}}{V_{base}}$ Not necessarily equal to V_{rated}! we choose this value. . in our ex above, $V_{t,pu} = \dfrac{13.8\text{ kV}}{V_{base}}$
. Machine terminal voltage is given. e.g. Gen terminal = 13.7 kV $\Rightarrow$	$V_{t,pu} = \dfrac{\text{operating } V}{V_{base}}$. in our ex above, $V_{t,pu} = \dfrac{13.7\text{ kV}}{V_{base}}$
. Current, voltage, power somewhere else in the circuit $\Rightarrow$	Solve PU circuit to find $V_{t,pu}$

Topic 5: Transmission Lines

Part 1: Line Parameters

objective: Come up with following parameters to model a line:



Note: these are <u>distributed</u> parameters, e.g Ω/km, F/km, etc.

Resistance: R per unit of length. $R = \dfrac{\rho_T}{A}$

where ρ_T: conductor resistivity at temperature T

A: conductor cross section

Conductance: To account for real power losses between conductor and ground due to leakage current, etc

- Usually neglected.

Inductance

- For inductance of a <u>solid</u> conductor, see handout

L for stranded conductors

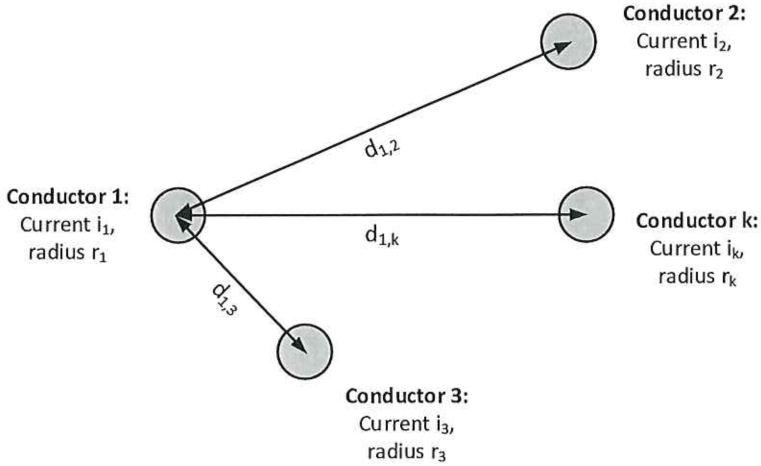
- Let's suppose each stranded conductor is made of 4 solid conductors:

(Distributed) Inductance of solid conductors

Reminder: Transmission (and distribution) lines are not made of a *single* solid conductor. They are made from stranded conductors: a number of solid conductors (strands) wrapped together.

To determine the inductance of stranded conductors, let's start with the inductance of solid conductors. We can then expand this to stranded conductors.

For n solid conductors with current i_k:



If $\sum_{k=1}^{n} i_k = 0$, flux linkage of conductor 1 is: $\lambda_1 = \frac{\mu_0}{2\pi}\left(i_1.\ln\frac{1}{r_1'} + i_2.\ln\frac{1}{d_{1,2}} + \cdots + i_n.\ln\frac{1}{d_{1,n}}\right)$

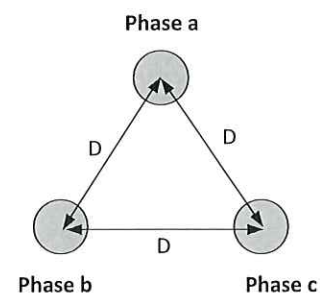
where: μ_0 is permeability of free space = $4\pi \times 10^{-7}$ H/m

$r' = e^{-1/4}.r = 0.7788\ r$

self - inductance

mutual inductance

Since λ = L.i, we can write this as: $\lambda_1 = L_{1,1}.i_1 + L_{1,2}.i_2 + \cdots + L_{1,n}.i_n$



For a balanced 3ϕ line made from solid conductors and with symmetric alignment:

$$\lambda_a = \frac{\mu_0}{2\pi}\left(I_a.\ln\frac{1}{r'} + I_b.\ln\frac{1}{D} + I_c.\ln\frac{1}{D}\right)$$

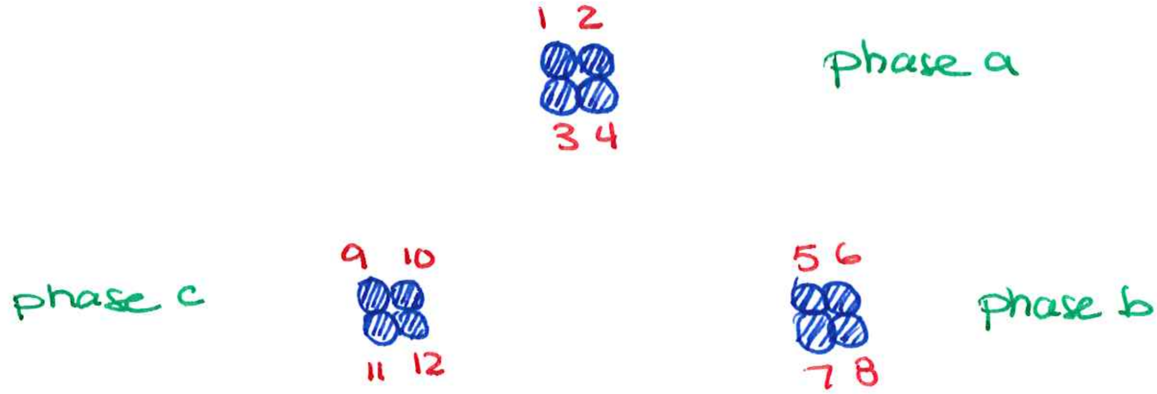
$$= 2\times10^{-7}\left(I_a.\ln\frac{1}{r'} + \underbrace{(I_b + I_c)}_{= -I_a}.\ln\frac{1}{D}\right)$$

in a balanced system, $I_a + I_b + I_c = 0$

$$= 2\times10^{-7}\left(I_a.\ln\frac{1}{r'} - I_a.\ln\frac{1}{D}\right) = 2\times10^{-7}\left(I_a.\ln\frac{D}{r'}\right) \rightarrow L_a = \frac{\lambda_a}{I_a} = \boxed{2\times10^{-7}\ln\frac{D}{r'}}$$

unit: H/m

This is the distributed inductance for a solid conductor in a balanced three phase system with symmetric alignment. Values for L_b and L_c are identical.



- 4 strands (sub-conductors) per phase
- Each strand is a solid conductor

Using L_a expression for solid conductors (from the handout) as a starting point, we can arrive at an expression for stranded conductors. To do that, define:

1) D_s : Geometric Mean Radius (GMR) of phase conductors:

$$D_s \triangleq \left(r' \cdot \underbrace{d_{1,2}}_{\text{distance between strands 1 \& 2}} \cdot d_{1,3} \cdot d_{1,4} \right)^{1/4}$$

for 4 strands in one phase

For conductors with n strands:

$$D_s \triangleq \left(r' \cdot d_{1,2} \cdot \ \cdots \ d_{1,n} \right)^{1/n}$$

r' for one strand

$\boxed{\text{See Table A.4 in the appendix for GMR values}}$

i.e. we don't need to use the equations above!

2) $D_{1,b}$: Geometric mean Distance (GMD) between strand 1 & phase b

$$\triangleq (d_{1,5} \cdot d_{1,6} \cdot d_{1,7} \cdot d_{1,8})^{1/4}$$

we can note that $D_{1,b} \simeq D_{2,b} \simeq D_{3,b} \simeq D_{4,b}$

therefore, let's use $D_{a,b}$ distance between conductor centers

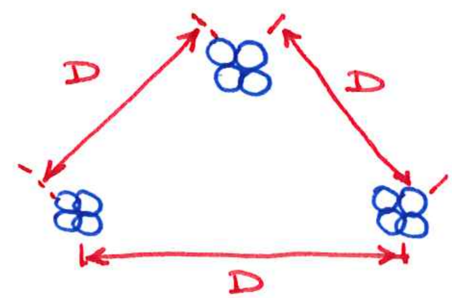
- Minimum distance between phases is governed by local safety codes

- Now, we can write expressions for inductance :

i) For 3ϕ stranded conductors with symmetric alignment :

$$D_{a,b} = D_{b,c} = D_{c,a} = D$$

$$\boxed{L_a = L_b = L_c = 2 \times 10^{-7} \ \ln \frac{D}{D_s}}$$ unit: H/m

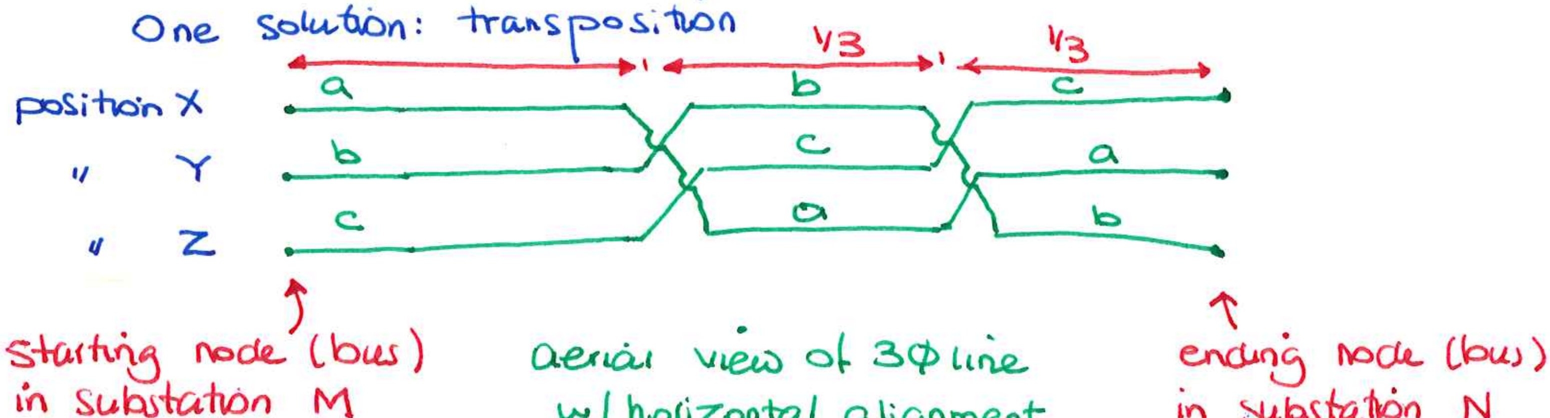


ii) For non-symmetric alignment

e.g. horizontal alignment

This results in unbalance between phases : $L_a \neq L_b \neq L_c$

One solution: transposition



For non-symmetric alignment with transposition:

$$\boxed{L_a = L_b = L_c = 2\times10^{-7} \ln \frac{D_{eq}}{D_s}}$$

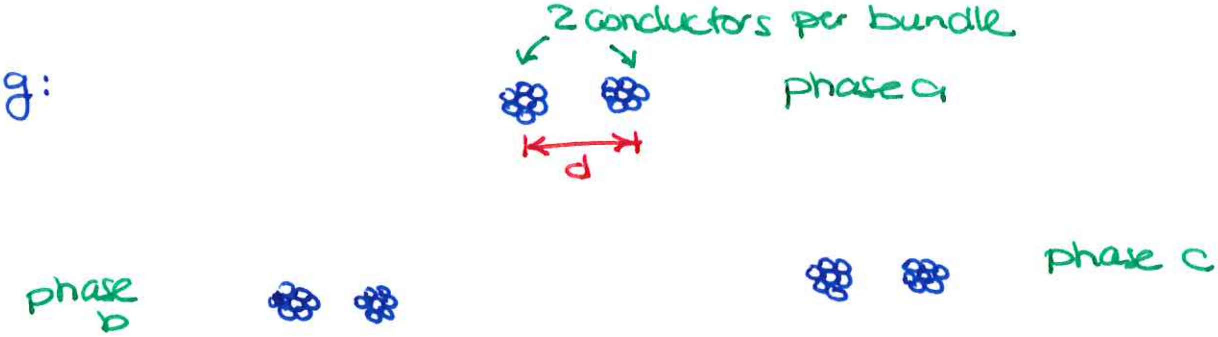
where $\underbrace{D_{eq}}_{}$ $= (D_{XY} \cdot D_{YZ} \cdot D_{XZ})^{1/3}$

Geometric Mean of distance between positions

L for bundled conductors

- To increase capacity of the line & to reduce losses, we can have more than one stranded conductor per phase:

e.g:

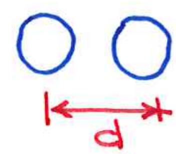


- GMR of the bundle is defined as $(r' \cdot d_{1,2} \cdots d_{1,2n})^{1/2n}$ for a 2 conductor bundle with n strands in each conductor
- if $\underbrace{d \gg \text{strand radius}}_{\text{always in ENEL 487}}$, we use equivalent GMR (D_{SL}):

  - For 2C bundle : $D_{SL} = \sqrt{D_s \cdot d}$

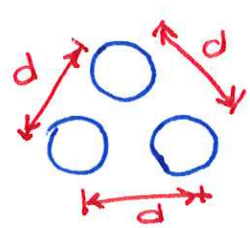
    conductor

    GMR of one conductor

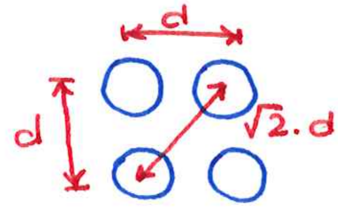


  - For 3C bundle

$$D_{SL} = \sqrt[3]{D_s \cdot d^2}$$



· For 4C bundle



$$D_{SL} = \sqrt[4]{D_s \cdot d^2 \cdot (\sqrt{2}d)}$$
$$= 1.091 \sqrt[4]{D_s \cdot d^3}$$

Then,

i) For symmetric alignment bundled conductor

$$\boxed{L_a = L_b = L_c = 2\times10^{-7} \ln \frac{D}{D_{SL}}}$$

unit: H/m

D: distance between bundle centers

ii) For non-symmetric alignment bundled conductors with transposition, replace D with Deq:

$$D_{eq} = \sqrt[3]{\underbrace{D_{XY}}_{\text{Distance between bundle centers in positions X \& Y.}} \cdot D_{YZ} \cdot D_{XZ}}$$

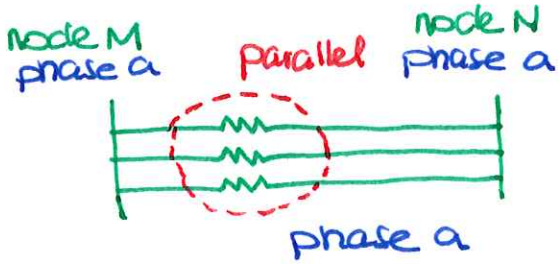
Ex: 795 kcmil Drake conductor, R = 0.1288 Ω/mile

Ampacity (current carrying capacity) = 900 A

what is R & ampacity for 3C bundle?

ampacity = 3x 900 = 2700 A

$$R_{bundled} = \frac{0.1288}{3} = 0.043 \ \Omega/\text{mile}$$



we can calculate $L_{bundled}$ using equations above.

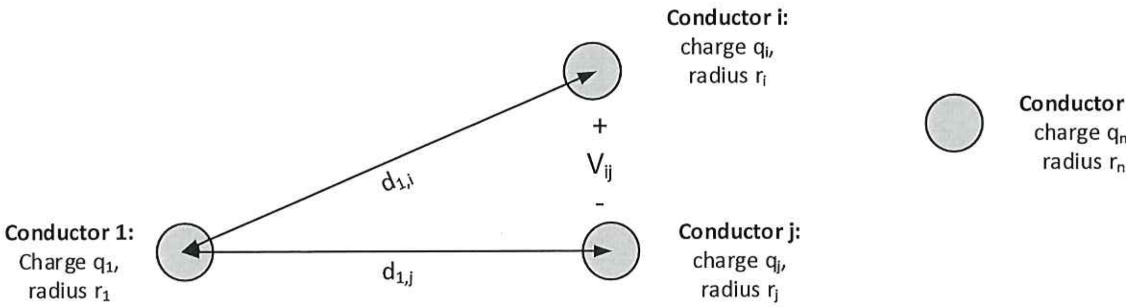
In the SLD for this system:

$$\underbrace{(r_{bundled} + j\omega \cdot l_{bundled})}_{\Omega/\text{mile}} \times \underbrace{\text{length}}_{\text{mile}}$$

· ignored line capacitance in the model here

(Distributed) Capacitance of solid and stranded conductors

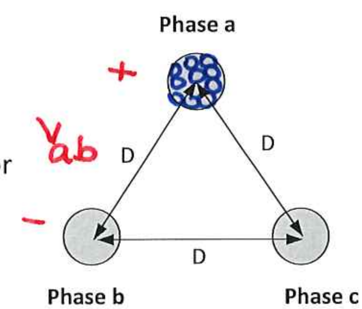
For n conductors with AC charge q_k (C/m) uniformly distributed along the conductor:



Voltage V_{ij} due to the electric fields from all n conductors: $V_{ij} = \frac{1}{2\pi\varepsilon} \sum_{k=1}^{n} q_k . \ln\frac{d_{j,k}}{d_{i,k}}$ (i)

For a conductor in free space, permittivity $\varepsilon = \varepsilon_0 = 8.85 \times 10^{-12}$ F/m

<u>For a balanced 3ϕ line with symmetric alignment</u>:



Even though the equation above was derived for solid conductors, the electric field of a stranded conductor with outside radius r is almost identical to the electric field of a solid conductor with radius r. So, the ensuing equations are valid for solid or stranded conductors.

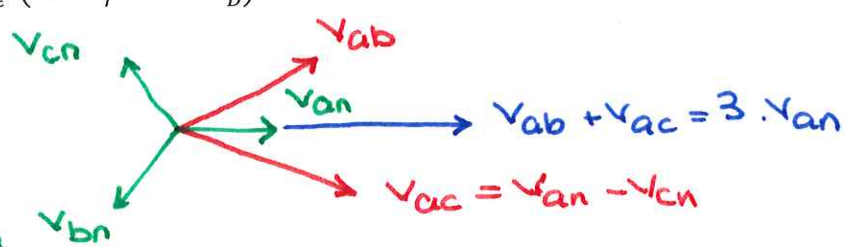
Re-write eq(i) for a balanced three phase system with symmetric alignment:

$d_{a,b}$

$$V_{ab} = \frac{1}{2\pi\varepsilon}\left(q_a . \ln\frac{D}{r} + q_b . \ln\frac{r}{D} + \overbrace{\cancel{q_c . \ln\frac{D}{D}}}^{0}\right) = \frac{1}{2\pi\varepsilon}\left(q_a . \ln\frac{D}{r} + q_b . \ln\frac{r}{D}\right) \quad \text{(ii)}$$

$d_{a,a}$

Similarly, $$V_{ac} = \frac{1}{2\pi\varepsilon}\left(q_a . \ln\frac{D}{r} + q_c . \ln\frac{r}{D}\right) \quad \text{(iii)}$$



Also know that: $V_{an} = \frac{1}{3}(V_{ab} + V_{ac})$

$q_a + q_b + q_c = 0$

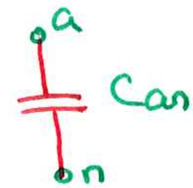
$\therefore$

$$\therefore\ V_{an} = \frac{1}{3}\frac{1}{2\pi\varepsilon}\left(2q_a . \ln\frac{D}{r} + \overbrace{(q_b + q_c)}^{=-q_a} . \ln\frac{r}{D}\right) = \frac{1}{3}\frac{1}{2\pi\varepsilon}\left(2q_a . \ln\frac{D}{r} - q_a . \ln\frac{r}{D}\right) = \frac{1}{2\pi\varepsilon}\left(q_a . \ln\frac{D}{r}\right)$$

Using $C_{an} = \frac{q_a}{V_{an}}$ and the equation above, we can arrive at an expression for the line-to-neutral capacitance of a solid or stranded conductor:

$$\boxed{C_{an} = \frac{2\pi\varepsilon}{\ln\frac{D}{r}}}$$

units: F/m



$C_{an} = C_{bn} = C_{cn}$ in a balanced system

Line capacitance

- See handout for capacitance of solid & stranded conductors.
- The mechanism for calculating C is similar to L. There is one distinction:
  - For L, the internal flux is not zero so we used r' and GMR (r', the equivalent radius of a hollow conductor with zero internal flux)
  - For C, the internal electric field is zero, so we use r.

- For asymm alignment with transposition, replace D with D_{eq}

$$\boxed{C_{an} = \frac{2\pi\varepsilon}{\ln \frac{D_{eq}}{r}}}$$

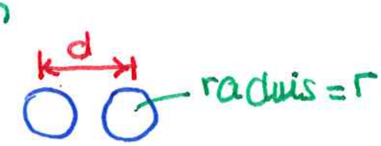
- For bundled conductors:

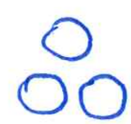
  Define D_{sc} (equivalent GMR of a phase bundle):

$$= \left(r \cdot d_{1,2} \cdots d_{1,n}\right)^{1/n}$$ for n conductor bundle

used conductor GMR for L calculation

$$D_{sc} = \begin{cases} \sqrt{r \cdot d} & \text{for 2C bundle} \\ \sqrt[3]{r \cdot d^2} & \text{for 3C bundle} \\ 1.091\sqrt[4]{r \cdot d^3} & \text{for 4C bundle} \end{cases}$$





Then

$$\boxed{C_{an} = \frac{2\pi\varepsilon}{\ln \frac{D}{D_{sc}}}}$$ for symm alignment

$$\boxed{C_{an} = \frac{2\pi\varepsilon}{\ln \frac{D_{eq}}{D_{sc}}}}$$ for asymm alignment w/ transposition