ENEL 476

Winter 2012

Quiz #1 **Feb 1, 2012** **EDC 179, 10:00-10:50 am**

Name or ID ____________________

Instructions

- Closed book and closed notes.
- Formula sheet is provided.
- Complete the quiz on the quiz paper. If you run out of paper, request additional blank paper from the invigilators.
- Only the solution of the multiple-choice questions will be evaluated. Clearly circle only one choice.
- Programmable calculators may be used.

Question 1. (7 marks)

An electric field in phasor form is given by:

$\vec{E}_s = 50e^{j\pi/4}e^{-j\beta z}\vec{a}_x$

The angular frequency ω=2 x 10^9 rad/s and the field is in free space (ε= ε_o, μ= μ_o, σ=0)

(A) Find an expression for $\vec{H}_s$

1 equation $\nabla \times \vec{E}_s = -j\omega\mu_o\vec{H}_s$

(*) $\nabla \times \vec{E}_s = \frac{\partial}{\partial z}E_x \vec{a}_y$

1 and

$\nabla \times \vec{E}_s = (-j\beta)(50)e^{j\pi/4}e^{-j\beta z}\vec{a}_y$

$\vec{H}_s = \frac{(-j\beta)(50)e^{j\pi/4}e^{-j\beta z}\vec{a}_y}{-j\omega\mu_0}$

$\boxed{\vec{H}_s = 50\left(\frac{\beta}{\omega\mu_0}\right)e^{j\pi/4}e^{-j\beta z}\vec{a}_y}$

1 result

(B) Find β.

(+) $\nabla \times \vec{H}_s = j\omega\varepsilon_0\vec{E}_s$

$\nabla \times \vec{H}_s = -\frac{\partial}{\partial z}H_y\vec{a}_x$

$= (j\beta)(50)\left(\frac{\beta}{\omega\mu_0}\right)e^{j\pi/4}e^{-j\beta z}\vec{a}_x$

1 equating LHS & RHS

$\therefore (\cancel{j}\beta)(5\cancel{0})\left(\frac{\beta}{\omega\mu_0}\right) = \cancel{j}\omega\varepsilon_0(5\cancel{0})$

$\beta^2 = \omega^2\mu_0\varepsilon_0$

$\beta = \omega\sqrt{\mu_0\varepsilon_0}$

$= \frac{2\times10^9}{3\times10^8}$

1 result $\boxed{\beta = 20/3 \text{ Rad/m}}$

(C) Find an expression for $\vec{H}(t)$

$\vec{H}_s = (50)\left(\frac{20}{3}\right)\left(\frac{e^{j\pi/4}e^{-j\beta z}}{(2\times10^9)(4\pi\times10^{-7})}\right)\vec{a}_y$

sub in $= \frac{5}{12\pi}e^{j\pi/4}e^{-j\beta z}\vec{a}_y$

$\boxed{\vec{H}(t) = \frac{5}{12\pi}\cos(\underset{2\times10^9}{\omega} t - \frac{20}{3}z + \frac{\pi}{4})\vec{a}_y}$

1 phasor → time

(*) $\nabla \times \vec{E}_s = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix}$

$= \vec{a}_x(0) - \vec{a}_y\left(-\frac{\partial}{\partial z}E_x\right) + \vec{a}_z\left(-\cancelto{0}{\frac{\partial}{\partial y}E_x}\right)$

(+) $\nabla \times \vec{H}_s = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & H_y & 0 \end{vmatrix}$

$= -\frac{\partial}{\partial z}H_y\vec{a}_x$

Question 2. (2 marks)

A spherical capacitor consists of two concentric spherical shells separated by a space that is filled with a lossless dielectric (σ=0) with relative permittivity ε_r=2. Assume that inner shell has outer radius of a, and the outer shell has inner radius of b. The capacitor is connected to a low frequency voltage given by:

$$v(t)=10\cos(\omega t) \text{ V/m}$$

and the electric field between the plates is:

$$\vec{E}(t) = \frac{ab \quad v(t)}{(b-a)r^2}\vec{a}_r \text{ V/m}$$

The displacement current density between the shells is:

(A) $\vec{J}_d = -10\omega\sin(\omega t)\vec{a}_\phi$

(B) $\vec{J}_d = \frac{-20\omega\varepsilon_o ab\sin(\omega t)}{(b-a)r^2}\vec{a}_r$

(C) $\vec{J}_d = \frac{-20\omega\varepsilon_o ab\sin(\omega t)}{(b-a)r^2}\vec{a}_\phi$

(D) $\vec{J}_d = \frac{-60\omega\varepsilon_o ab\cos(\omega t)}{(b-a)r^3}\vec{a}_r$

$$\vec{J}_d = \frac{\partial}{\partial t}\epsilon_R\epsilon_0\vec{E}(t)$$

$$= \frac{\partial}{\partial t} 2\epsilon_0\left(\frac{ab}{b-a}\right)\frac{(10\cos wt)}{r^2}\vec{a}_r$$

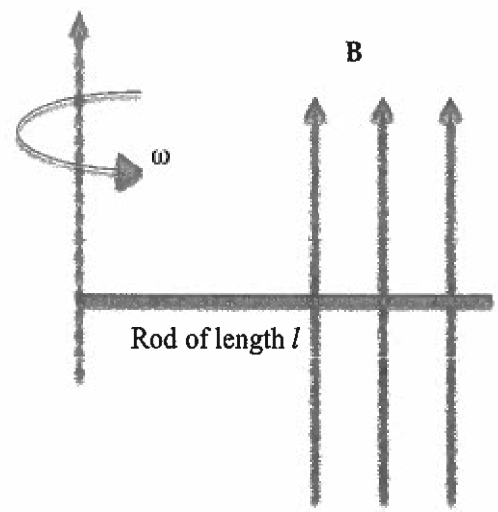
$$= \left[\frac{2\epsilon_0 ab}{(b-a)r^2}\right]\left[-10w\sin wt\right]\vec{a}_r$$

$$= \left(-20w\epsilon_0\left(\frac{ab}{(b-a)r^2}\right)\sin(wt)\right)\vec{a}_r$$

Question 3. (2 marks)

A conducting rod is placed in a magnetic flux density, **B**. The rod rotates with velocity ω about an axis that is located at one end of the rod. The axis of rotation is perpendicular to the length of the rod, and the magnetic flux density is parallel to the axis of rotation. This is illustrated in the figure below. Note that magnetic flux density **B** is uniform throughout space, however only 3 lines are shown for simplicity.

Axis of rotation



The EMF induced in the rod is:

(A) $\frac{-\omega B l^2}{2}$

(B) $\frac{\omega B l^2}{2}$

(C) $\rho\omega B l$

(D) Zero

$$EMF = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

$\vec{v} = ?$ $\Rightarrow$ rod is rotating

$$\rho \frac{d\phi}{dt} = \rho\omega$$

$\vec{v}$ is in $\vec{a}_\phi$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \vec{a}_\rho & \vec{a}_\phi & \vec{a}_z \\ 0 & \rho\omega & 0 \\ 0 & 0 & B \end{vmatrix}$$

$$= \vec{a}_\rho (\rho\omega B)$$

$$EMF = \int_0^l (\vec{v} \times \vec{B}) \cdot d\rho\, \vec{a}_\rho$$

$$= \int_0^l \rho\omega B\, d\rho$$

$$= \frac{\rho^2}{2}\Big|_0^l \omega B$$

$$= \frac{l^2 \omega B}{2}$$

Question 4. (2 marks)

A coaxial line has the following parameters:

R= 6.5 Ω/m

L=3.4 μH/m

G= 8.4 mS/m

C=21.5 pF/m

At a frequency of 2 MHz,

(A) Z_o=893.5+j5067 Ω, α=0.043 Np/m, β=0.36 rad/m

(B) Z_o=55.1+j45.9 Ω, α =0.45 Np/m, β =0.4 rad/m

(C) Z_o=45.9+j55.1 Ω, α =0.45 Np/m, β =0.4 rad/m

(D) Z_o=55.1+j45.9 Ω, α =0.4 Np/m, β =0.45 rad/m

$$Z_o = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$= \sqrt{\frac{(6.5+j(2\pi)(2\times10^6)(3.4\times10^{-6})}{(8.4\times10^{-3})+j(2\pi)(2\times10^6)(21.5\times10^{-12})}}$$

$$= \sqrt{\frac{6.5+j42.73}{8.4\times10^{-3}+j2.7\times10^{-4}}}$$

$$= \sqrt{\frac{43.22\angle 81.35^\circ}{0.0084\angle 1.84^\circ}}$$

$$= \sqrt{5145.24\angle 80^\circ}$$

$$= 71.73\angle 39.76^\circ$$

$$\boxed{Z_o = 55.1+j45.9}$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{(43.22\angle 81.35^\circ)(0.0084\angle}$$

$$= \sqrt{0.363\angle 83.19^\circ}$$

$$= 0.45+j0.4$$

$\therefore \alpha = 0.45$ Np/m

$\beta = 0.4$ rad/m