UPW propagation, transmission & reflection 2020 (1)

Poynting vector: $\vec{P}(z,t) = \vec{E}(z,t) \times \vec{H}(z,t)$

↳ power density associated with wave

$$\Rightarrow \vec{P}_{Avg}(z) = \frac{1}{2} Re\{\vec{E}_s \times \vec{H}_s^*\}$$

$$= \frac{E_{xo}^2}{2|\eta|} e^{-2\alpha z} \cos(\theta_\eta) \vec{a}_z$$

↳ direction of propagation

lossless: $\vec{P}_{Avg}(z) = \frac{E_{xo}^2}{2\eta} \vec{a}_z$

Ex mashed potatos → 27 MHz

↳ $\alpha = 7.15$ Np/m

↳ $\beta = 8.94$ rad/m

↳ $|\eta| = 18.62\ \Omega$

↳ $\theta_\eta = 0.675$ rad

If $|\vec{E}| = 250$ V/m at $x=0$ & wave propagates in $+x$, & $\vec{E}$ is in $\vec{a}_y$, find $\vec{E}$, $\vec{H}$ & $\vec{P}_{Avg}$. Find δ.

$$\vec{E}(x,t) = 250 e^{-7.15x} \cos(2\pi \times 27\times10^6 t - 8.94x)\vec{a}_y \text{ V/m}$$

$$\vec{H}(x,t) = \frac{250}{18.62} e^{-7.15x} \cos(2\pi \times 27\times10^6 t - 8.94x - 0.675)\vec{a}_z \text{ A/m}$$

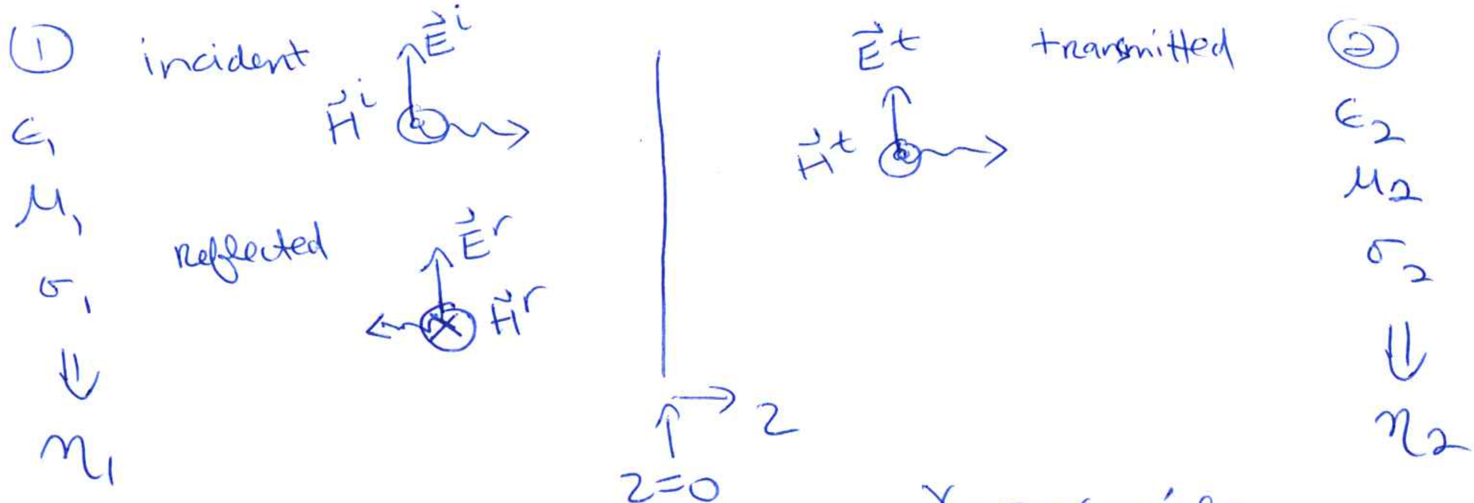
$$\vec{P}_{Avg}(x) = \frac{(250)^2}{2(18.62)} e^{-2(7.15)x} \cos(0.675)\vec{a}_x \text{ W/m}^2$$

$$\delta = \frac{1}{\alpha} = 0.14 \text{ m}$$

Temperature increase: $\omega \epsilon_0 \epsilon_R'' |E|^2_{rms} = \rho C_p \frac{\Delta T}{\Delta t}$

power deposition; $\epsilon = \epsilon_R' - j\epsilon_R''$; density; specific heat capacity

Reflection & transmission at normal incidence



$\gamma_1 = \alpha_1 + j\beta_1$ $\quad\quad$ $\gamma_2 = \alpha_2 + j\beta_2$

incident: $\vec{E}_s^i = E^{inc} e^{-\gamma_1 z} \vec{a}_x \rightarrow e^{-\alpha_1 z} e^{-j\beta_1 z}$

$\vec{H}_s^i = \frac{E^{inc}}{\eta_1} e^{-\gamma_1 z} \vec{a}_y$

reflected: $\vec{E}_s^r = E^{refl} e^{\gamma_1 z} \vec{a}_x$

$\vec{H}_s^r = -\frac{E^{refl}}{\eta_1} e^{\gamma_1 z} \vec{a}_y$

transmitted:

$\vec{E}_s^t = E^{trans} e^{-\gamma_2 z} \vec{a}_x$

$\vec{H}_s^t = \frac{E^{trans}}{\eta_2} e^{-\gamma_2 z} \vec{a}_y$

Apply boundary conditions: $\vec{E}_{1,tan} = \vec{E}_{2,tan}$

$E^{inc} + E^{refl} = E^{trans}$ (1)

at $z=0$

$\tilde{H}_{1,tan} = \tilde{H}_{2,tan}$

$$\frac{E^{inc}}{\eta_1} - \frac{E^{refl}}{\eta_1} = \frac{E^{trans}}{\eta_2} \quad (2)$$

Define $\Gamma = \dfrac{E^{refl}}{E^{inc}}$ (reflection coefficient)

$T = \dfrac{E^{trans}}{E^{inc}}$ (transmission coefficient)

$$(2) \Rightarrow \frac{\eta_2}{\eta_1}\left(E^{inc} - E^{refl}\right) = E^{trans}$$

↳ sub into (1)

$$E^{inc} + E^{refl} = \frac{\eta_2}{\eta_1}\left(E^{inc} - E^{refl}\right)$$

$$E^{inc}\left(\frac{\eta_2 - \eta_1}{\eta_1}\right) = E^{refl}\left(\frac{\eta_1 + \eta_2}{\eta_1}\right)$$

$$\Gamma = \frac{E^{refl}}{E^{inc}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

Similarly, $T = \dfrac{2\eta_2}{\eta_2 + \eta_1}$

Ex: UPW at 3GHz is incident on a planar interface at $z=0$. The region $z<0$ is free space. The region $z>0$ has $\epsilon_r = 4$, $\mu_r = 1$, $\sigma = 0$.

a) Find Γ & T.

$\eta_1 = 120\pi\ \Omega \qquad \eta_2 = \sqrt{\dfrac{\mu_0}{4\epsilon_0}} = 60\pi\ \Omega$

$$\Gamma = \frac{60\pi - 120\pi}{60\pi + 120\pi}$$

$$= \frac{-60}{180}$$

$$\Gamma = -\frac{1}{3} \rightarrow 0 \le |\Gamma| \le 1$$

$$T = \frac{2\eta_2}{\eta_1 + \eta_2}$$

$$= \frac{2(60\pi)}{60\pi + 120\pi}$$

$$T = 2/3 \rightarrow T = 1 + \Gamma$$

b) If $\vec{E}^i(z,t) = 3\cos(6\pi\times10^9 t - \beta_0 z)\,\vec{a}_x$, find $\vec{H}^i(z,t)$.
Find $\vec{E}^r(z,t)$, $\vec{H}^r(z,t)$ and $\vec{E}^t(z,t)$ & $\vec{H}^t(z,t)$

$$\vec{E}^i(z,t) \Rightarrow \beta_0 = \omega\sqrt{\mu_0\epsilon_0}$$

$$= \frac{6\pi\times10^9}{3\times10^8}$$

$$= 20\pi \text{ rad/m}$$

E^{inc}

$$\vec{E}^i(z,t) = 3\cos(6\pi\times10^9 t - 20\pi z)\,\vec{a}_x$$

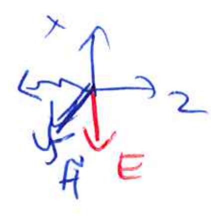
$$\vec{H}^i(z,t) = \frac{3}{120\pi}\cos(6\pi\times10^9 t - 20\pi z)\,\vec{a}_y$$

change direction of propagation

$$\vec{E}^r(z,t) = \underbrace{\left(-\tfrac{1}{3}\right)}_{\Gamma}\underbrace{(3)}_{E^{inc}}\cos(6\pi\times10^9 t + 20\pi z)\,\vec{a}_x$$

$$= -\cos(6\pi\times10^9 t + 20\pi z)\,\vec{a}_x$$

$$\vec{H}^r(z,t) = \frac{1}{120\pi}\cos(6\pi\times10^9 t + 20\pi z)\,\vec{a}_y$$



$$\vec{E}^t(z,t) = \left(\frac{2}{3}\right)(3)\cos(6\pi\times10^9 t - \beta_2 z)\,\vec{a}_x$$

T Einc

Region 2

$$\beta_2 = \omega\sqrt{4\mu_0\epsilon_0}$$

$$= \frac{6\pi\times10^9\,(2)}{3\times10^8}$$

$$= 40\pi \text{ rad/m}$$

$$\vec{E}^t(z,t) = 2\cos(6\pi\times10^9 t - 40\pi z)\,\vec{a}_x$$

$$\vec{H}^t(z,t) = \frac{2}{60\pi}\cos(6\pi\times10^9 t - 40\pi z)\,\vec{a}_y$$

η_2

$$\vec{E}_{1,tan} = \vec{E}_{2,tan} \Rightarrow 3 - 1 = 2 \quad \checkmark$$

$$\vec{H}_{1,tan} = \vec{H}_{2,tan} \Rightarrow \frac{3}{120\pi} + \frac{1}{120\pi} = \frac{2}{60\pi} \quad \checkmark$$