Question 1. (26 marks)

A uniform plane wave is propagating in a material with ε_r=14, σ=0.2 S/m and μ_r=1.6. The frequency of the wave is 100 MHz. The wave is propagating in the +x direction and the magnetic field is oriented in the +y-direction. The amplitude of the magnetic field is 0.1 A/m at x=0.

Calculate the following quantities:

a) attenuation constant (α)
b) phase constant (β)
c) skin depth (δ)
d) velocity of propagation (v_p or u)
e) wavelength (λ)
f) magnetic field in the time domain (**H**(x,t))
g) intrinsic impedance of the medium (η)
h) electric field in the time domain (**E**(x,t))
i) electric field in phasor form (**E**$_s$(x))
j) Poynting vector.

a) $\frac{\sigma}{\omega\epsilon} = \frac{0.2}{(2\pi\times 10^8)(\cancel{14}^{7})(\frac{1}{36\pi}\times 10^{-9})\frac{1}{10}}$

1 $= \frac{18}{7}$ $\therefore$ use full formulas

1 $\alpha = 2\pi\times 10^8 \sqrt{\underbrace{\frac{\overset{\mu_0}{(4\pi\times 10^{-7})}\overset{\mu_R}{(1.6)}\overset{\epsilon_R}{(14)}\overset{\epsilon_0}{(\frac{1}{36\pi}\times 10^{-9})}}{2}}_{\left[\frac{(1.6)(7)}{9}\right]\times 10^{-16}}\left[\sqrt{1+(18/7)^2}-1\right]}$

1 $\boxed{\alpha = 9.3 \text{ Np/m}}$

1 b) $\boxed{\beta = 13.6 \text{ rad/s}}$ 1 $\beta = 2\pi\times 10^8\sqrt{\frac{(1.6)(7)}{9}\times 10^{-16}\left[\sqrt{1+(18/7)^2}+1\right]}$

c) $\delta = \frac{1}{\alpha}$

$\frac{1}{2}$ 1 $\boxed{\delta = 0.11\text{m}}$

d) $v_p = \frac{\omega}{\beta}$ $\frac{1}{2}$

$= \frac{2\pi\times 10^8}{13.6}$

1 $\boxed{v_p = 4.62\times 10^7 \text{ m/s}}$

e) $\lambda = \frac{2\pi}{\beta}$ $\frac{1}{2}$

$= 2\pi/13.6$

$\boxed{\lambda = 0.462\,\cancel{m}}$ 1

9.5

f) $\vec{H}(x,t) = 0.1e^{-9.3x}\cos(2\pi\times10^{8}t - 13.6x)\vec{a}_y$

g) $|\eta| = \dfrac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\sigma/\omega\varepsilon\right)^2\right]^{1/4}}$

$= \dfrac{\sqrt{\dfrac{(4\pi\times10^{-7})(1.6)}{(\frac{1}{36\pi}\times10^{-9})(14)}}}{\left[1+(18/7)^2\right]^{1/4}}$

$= 76.8\ \Omega$

$\measuredangle\eta \Rightarrow \tan 2\theta_\eta = \dfrac{\sigma}{\omega\varepsilon}$

$\theta_\eta = \measuredangle 34.37^\circ$

$= 0.6$ rad

h) $\vec{E}(x,t) = -7.68e^{-9.3x}\cos(2\pi\times10^{8}t - 13.6x + 0.6)\vec{a}_z$

i) $\vec{E}_s(x) = -7.68e^{-9.3x}e^{-j(13.6x - 0.6)}\vec{a}_z$

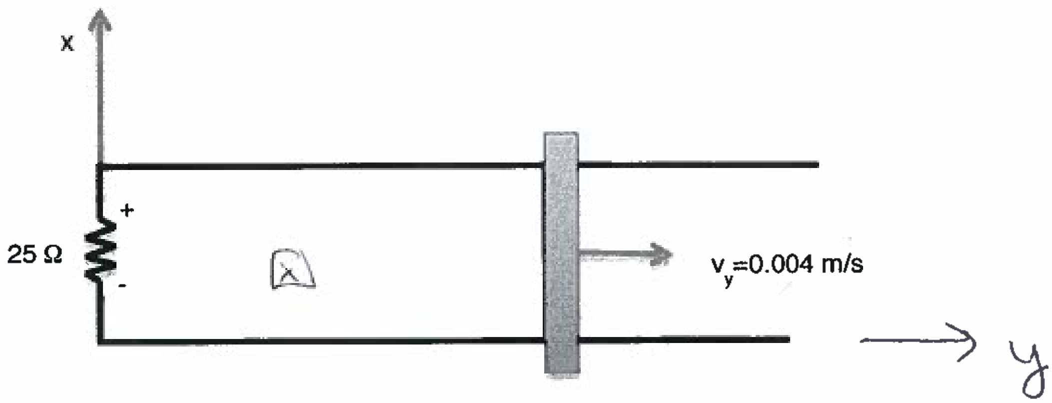
j) $\vec{P}_{AV}(x) = \dfrac{1}{2}\dfrac{|\vec{E}|^2}{|\eta|}\cos(\theta_\eta)\vec{a}_x$

$= \dfrac{1}{2}\left(\dfrac{-7.68e^{-9.3x}}{76.8}\right)^2\cos(0.6)\vec{a}_x$

$= 0.32e^{-18.6x}\vec{a}_x$

Question 2. **(14 marks)**

Consider a bar sliding on a set of parallel rails, as shown in the figure below. The separation between the bars in the x-direction is 20 cm. At time t=0, assume that the bar is at y=0.



The rails and bars are placed in a magnetic field with flux density

$$\mathbf{B}=5\cos(120\pi t)\ \mathbf{a}_z\ \text{mWb/m}^2$$

Find the following quantities:

a) total flux through the loop (ϕ).

b) EMF (V_{emf})

c) induced current. Indicate the initial direction of current flow ~~during the first quarter period~~.

d) Solve the problem by considering the sum of motional and transformer EMF.

a) $\Phi = \int \vec{B}\cdot d\vec{s}$ 1

$= \int_0^{l}\int_0^{0.2} 5\cos(120\pi t)\,dx\,dy$ $\frac{1}{2}$ $\frac{1}{2}$

1 $= 5(l)(0.2)\cos 120\pi t$

$= (1)(l)\cos(120\pi t)$

$= 0.004t\cos(120\pi t)$ mWb

$\boxed{\Phi = 4t\cos(120\pi t)\ \mu\text{Wb}}$

$l = ?$

$\frac{dy}{dt} = 0.004$

$y = 0.004t + c$

$t=0, y=0$

$\therefore y = 0.004t$

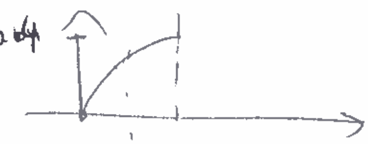
b) $V_{emf} = -\frac{d}{dt}\Phi$ 1

1 $= -4[\cos(120\pi t) - t\sin(120\pi t)(120\pi)]$

$$V_{emf} = -4\cos(120\pi t) + 4t\sin(120\pi t)(120\pi)$$

1 $$= -4\cos(120\pi t) + \overset{1.51}{\underbrace{480\pi t}_{0 \to 0.40}}\,\sin(120\pi t) \times 10^{-6}\ V$$

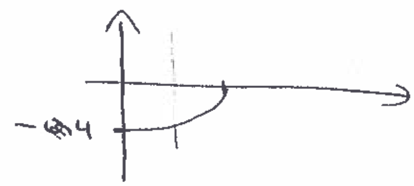
c) $t=0 \to t = T/4$



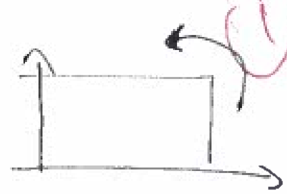
$T = \frac{1}{f}$

$= \frac{1}{60}$

$= 0.017s$



$T/4 = 0.00425\ s$



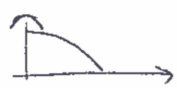
1 initial direction of current flow

$I = \frac{V_{emf}}{R}$ 1

$= \frac{-4\cos(120\pi t)}{25}$

$+ \frac{480\pi t}{25}\sin(120\pi t)$

→ induced flux in $-\vec{a}_z$ direction

→  → cos term decreases

but area of loop increases

⇓

dominant effect

d) $\oint \vec{v}\times\vec{B}\cdot d\vec{l} = V_{motional}$ 1

$-\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s} = V_{transformer}$ 1

$V_{emf} = V_{motional} + V_{transformer}$

$$\vec{v}\times\vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 0 & 4\times10^{-3} & 0 \\ 0 & 0 & B_z \end{vmatrix} \Rightarrow \int_{0.2}^{0} 20\cos(120\pi t)\,dx$$

$= \vec{a}_x(4B_z)$ 1

$= -4\cos(120\pi t)\ \mu V$ 1

$\frac{d\vec{B}}{dt} = 600\pi\sin(120\pi t) \times 10^{-6}$ 1

$-\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$ 1

$= 600\pi\sin(120\pi t)(0.2)l$

$= 120\pi(0.004t)\sin(120\pi t)$

$= 480\pi t\sin(120\pi t)\ \mu V$

Question 3. **(14 marks)**

a) Consider a field normally incident on a planar interface located at y=0. The material in the region y<0 is a dielectric with ε_r=4, σ=0, μ_r=1. The material in the region y>0 has ε_r=6.4, σ=0 and μ_r=2.5. Calculate the reflection coefficient (Γ) and the transmission coefficient (T).

$$\eta_1 = 60\pi$$

$$\eta_2 = \sqrt{\frac{(2.5)\mu_0}{6.4\ \varepsilon_0}} = \frac{.5}{.8}\overset{(15)}{120\pi} = 75\pi$$

$$\Gamma = \frac{75\pi - 60\pi}{75\pi + 60\pi} = \frac{15}{135} = 0.11$$

1 Γ= 0.11 1 T= 1.11

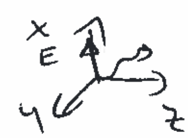
b) For the same scenario as the previous question, the incident electric field is given by:

$$\mathbf{E}^i(y,t) = 10\cos(10^8 t - 0.67y)\ \mathbf{a}_x$$

Find the reflected electric field ($\mathbf{E}^r(y,t)$), reflected magnetic field ($\mathbf{H}^r(y,t)$) and transmitted electric field ($\mathbf{E}^t(y,t)$).

$\mathbf{E}^r(y,t)$= $1.1\cos(10^8 t + 0.67y)\vec{a}_x$

1 1



$\mathbf{H}^r(y,t)$= $\frac{1.1}{60\pi}\cos(10^8 t + 0.67y)\vec{a}_z$

1 1

$$\beta_2 = \omega\sqrt{\mu_{r2}\mu_0\varepsilon_{r2}\varepsilon_0} = \frac{10^8}{3\times10^8}\sqrt{(2.5)(6.4)} = 4/3$$

$\mathbf{E}^t(y,t)$= $11.1\cos(10^8 t - \frac{4}{3}y)\vec{a}_x$

1 1

c) A parallel plate capacitor is filled with a material having $\varepsilon_r=5$, $\mu_r=1$ and $\sigma=0$. The plate area is 5 cm^2 and separation between the plates is 4 mm. If a voltage of 5 cos(100πt) V is applied to the plates, find the amplitude of the displacement current density, $|\mathbf{J}_d|$ and the total displacement current, I_d.

$$|\vec{J}_d| = \omega \varepsilon_R \varepsilon_0 \vec{E} \qquad |\vec{E}| = V/d$$

$$= 5/4\times10^{-3}$$

$$= (100\pi)\left(\frac{5}{4}\right)(10^{3})(5)\left(\frac{1}{36\pi}\times10^{-9}\right)$$

$$= 17.36\times10^{-6}$$

$|\mathbf{J}_d|$ = 17.36 μA/m^2

(1) (½)

I_d = 8.68 $\times 10^{-9}$ A → 8.68 nA

(1) (½)

d) Provide a definition of the polarization of an electromagnetic wave.

→ trace of electric field at a location in space

(1) Wrt time

The electric field associated with a uniform plane wave is described by:

$$\mathbf{E}(z,t) = 20\cos(\omega t-\beta z+\pi/4)\mathbf{a}_x + 60\cos(\omega t-\beta z-\pi/4)\mathbf{a}_y$$

What is the polarization of this wave? Provide a sketch to illustrate.

(1) ↳ elliptical

(1)