Unit 2 Models of mechanical and mechatronic systems

In this unit we will be looking at modelling transfer functions of mechanical and mechatronic systems. We will start with the review of the basis of modelling simple mechanical devices for displacement and rotational motion. We will then consider electric motor drives and gears. From this we can get a basic set of tools to model mechatronic systems. We can then expand into more complex systems that involve a number of components. This is organized systematically with state space analysis.

The outline is as follows:

1. Transfer function of mechanical systems of displacement motion

2. Transfer function of mechanical systems of rotational motion

3. Mapping of rotational properties through gears

4. Linear models of electric motors

5. Using state space to systematically combine several interacting mechanical and electrical components

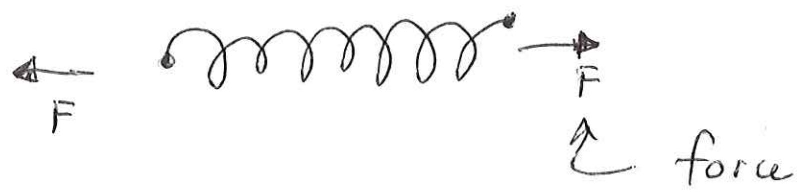
We could look at systems modelling in other fields with the goal of generating a mathematical model that can be used for the basis of a feedback control system design. Other fields could be purely electronic such as a feedback control for a sound system or a power plant. It could be a chemical system where the plant is a contained chemical reaction such as in a refinery. Other area are epidemic control, fiscal management and so forth. Obviously we only have time to consider a small subset of applications.

2.4 electrical
2.5 translation
2.6 rotational
2.7 Gears
2.8 motors

Nise textbook sections.

Displacement Motion

Consider an ideal spring that generates a force depending on how far it is stretched or compressed

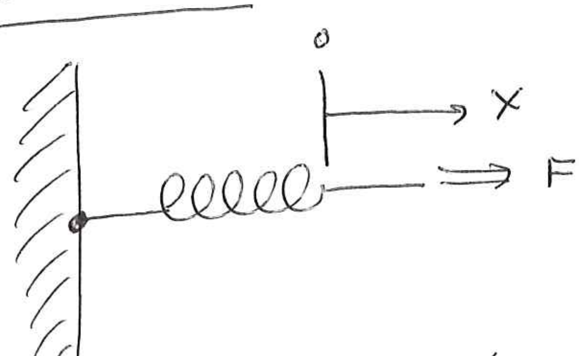


Linearized law $F = KX$

K - Spring constant

X - how far it is stretched.

Inertial wall



Wall fixed to "static" coordinate system

Equation of Motion

$$F = KX$$

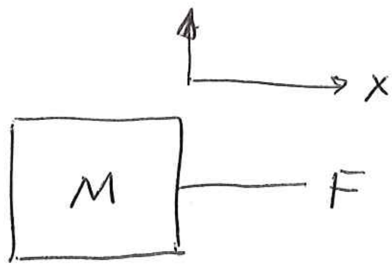
Spring approximated as being massless and linear.

Mass

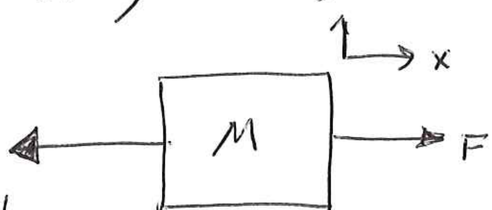
Newtons law

$$F = M \times \text{acceleration}$$

$$F = M \frac{d^2x}{dt^2}$$



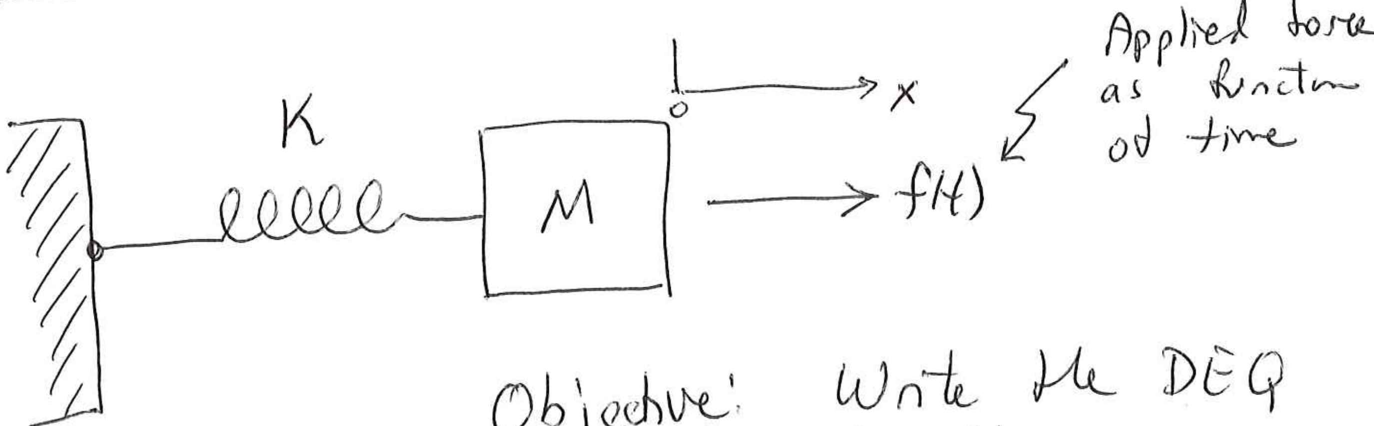
Free body diagram



Inertial force

$$M \frac{d^2x}{dt^2} = M\ddot{x} \qquad \ddot{x} = \frac{d^2x}{dt^2}$$

System of Spring and Mass



inertial wall

Objective: Write the DEQ for this system

Start with free body diagram centered on rigid body.

<u>DEQ</u> Σ forces on left = Σ forces on right

$$K X + M\ddot{X} = f(t)$$

$$M \frac{d^2 x}{dt^2} + K X = f(t)$$

Convert this to Laplace domain to find transfer function.

$$\mathcal{L}\left(\frac{d^2 x}{dt^2}\right) = s^2 X(s) - s X(0) - \dot{x}(0)$$

$$\left(M s^2 + K\right) X(s) = F(s) + s x(0) + \dot{x}(0)$$

Assume that mass is at rest for $t < 0$

Then $x(0) = 0$
$\dot{x}(0) = 0$

$$X(s) = \frac{F(s)}{Ms^2 + K}$$

$$\frac{X(s)}{F(s)} = H(s) = \frac{1}{Ms^2 + K}$$

$$h(t) = \mathcal{L}^{-1}\left(\frac{1/M}{s^2 + K/M}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{\sqrt{K/M}}{s^2 + K/M}\right)\frac{1}{\sqrt{KM}}$$

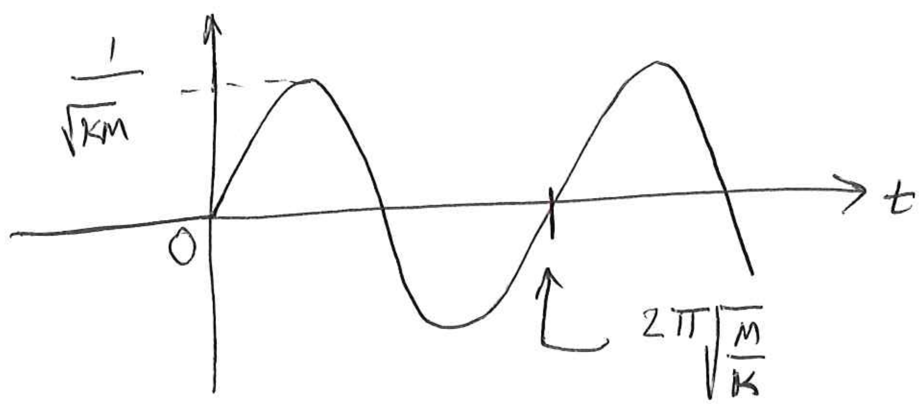
$$= \sin\left(\sqrt{\frac{K}{M}}\,t\right)\frac{1}{\sqrt{KM}}\,u(t)$$

from table

$$\sin(\omega t)u(t) \Leftrightarrow \frac{\omega}{s^2+\omega^2}$$

Suppose $f(t) = \delta(t)$ then

$$x(t) = \frac{1}{\sqrt{KM}}\sin\left(\sqrt{\frac{K}{M}}\,t\right)\,u(t).$$



Does this agree with intuition?

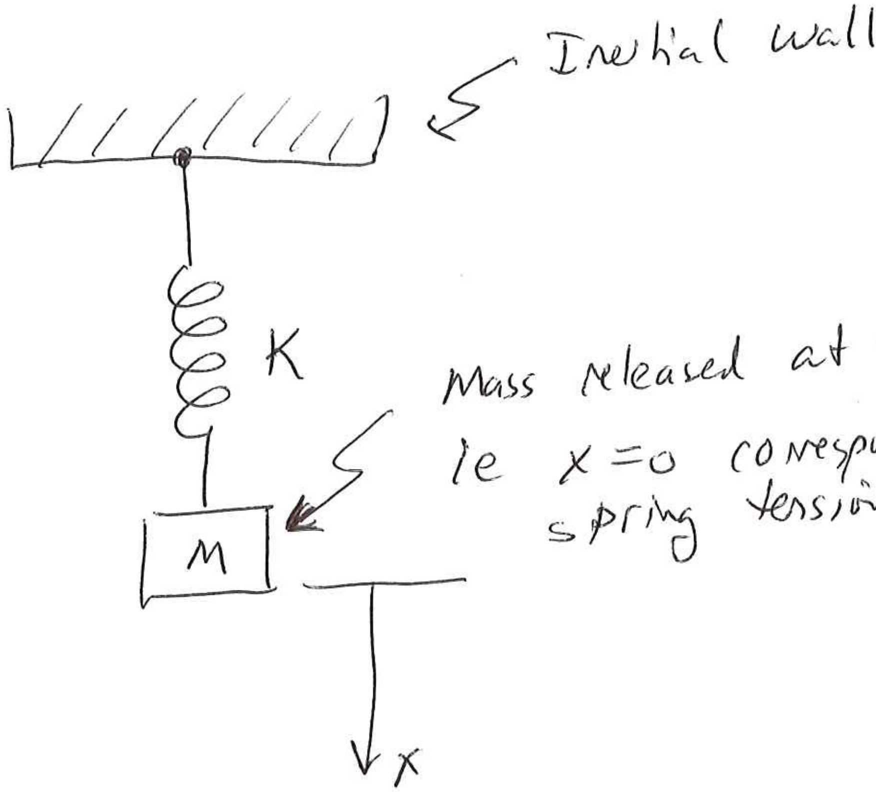
1) Oscillation response starts from $x=0$ at $t=0$
Continues forever without attenuation.
Energy put into system with $f(t)$. No energy dissipated

2) Period of $2\pi\sqrt{\frac{M}{K}}$, the bigger M is the slower the oscillation. Stiff spring (K larger) results in faster oscillation $\Rightarrow$ period smaller.

3) Amplitude of $\frac{1}{\sqrt{KM}}$, Bigger mass then for same energy input movement smaller. Also larger K is the less the excursion.

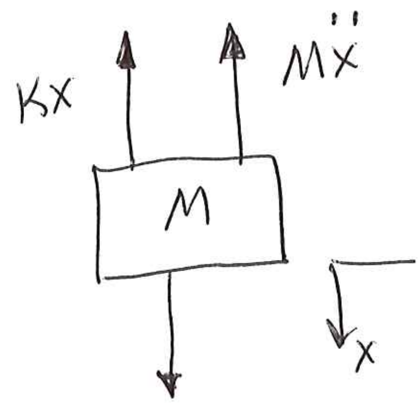
Example

Weight held in a gravitational field and released at $t=0$. Find the response.



Mass released at $t=0$
ie $x=0$ corresponds to no spring tension,

<u>body diagram</u>



$Mg\,u(t) \Longleftarrow$ mass released at $t=0$

$$\underbrace{M\ddot{x}}_{Kg\frac{m}{s^2}} + \underbrace{Kx}_{\frac{N}{m}m} = \underbrace{Mg\,u(t)}_{Kg\frac{m}{s^2}}$$

<u>Units</u> $Kg\frac{m}{s^2}$ $\quad \frac{N}{m}m$ $\quad Kg\frac{m}{s^2}$

N

$$X(s)\left(Ms^2 + K\right) = Mg\,\frac{1}{s}$$

$$X(s) = Mg\,\frac{1}{s\left(Ms^2+K\right)}$$

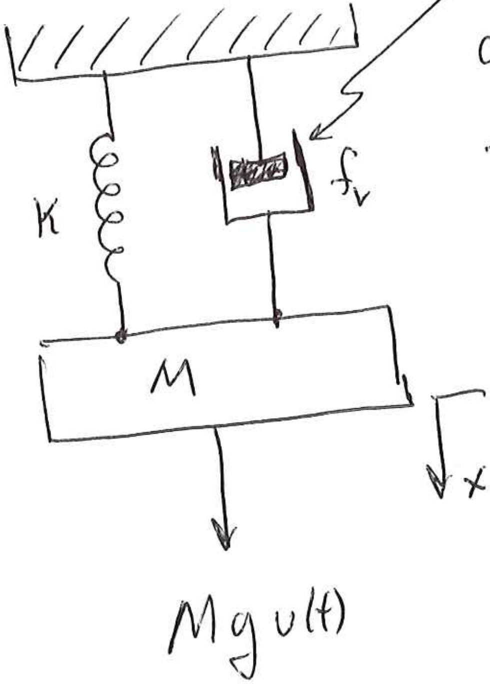
Put this into Matlab get response

```
x(t) = ilaplace( Mg/(s*(K+MS^2)), 's', 't')
```

or use

```
X = tf( Mg, [M, 0 K 0])
impulse (X)
```

Add a Damping Loss to Model

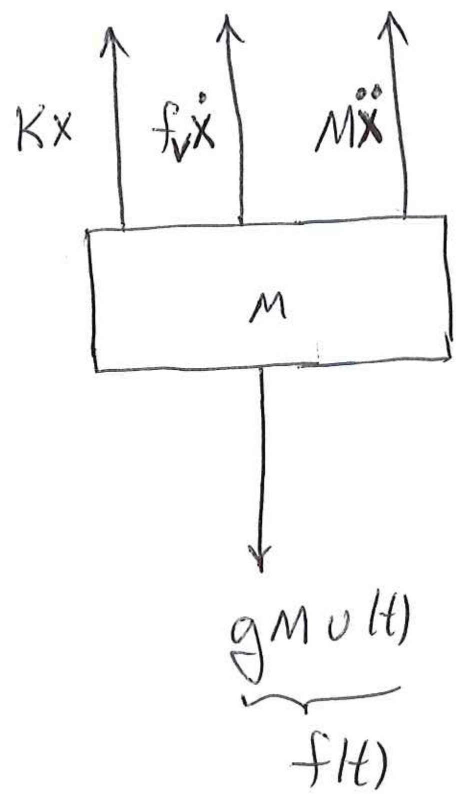


symbol for lossy damping device

force of damper is $f_v \dot{x}$

(linearized model)

Again set up free body diagram



$$Kx + f_v\dot{x} + M\ddot{x} = f$$

$$\left(K + f_v s + Ms^2\right) X(s) = F(s)$$

$$H(s) = \frac{1}{Ms^2 + f_v s + K}$$

if $f(t) = Mg\,u(t) \Rightarrow F(s) = \frac{Mg}{s}$

$$X(s) = H(s)\,F(s)$$

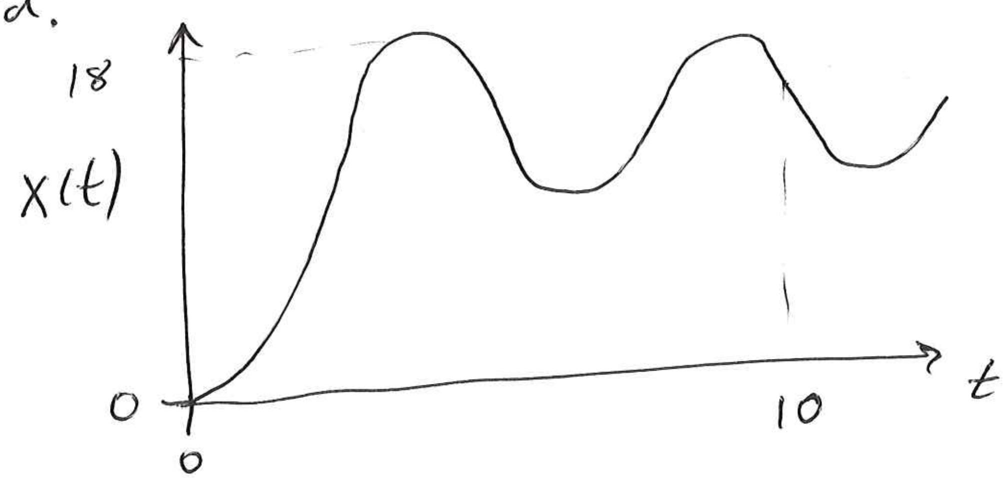
$$= \frac{Mg}{s(Ms^2 + f_v s + K)}$$

Solve with Matlab

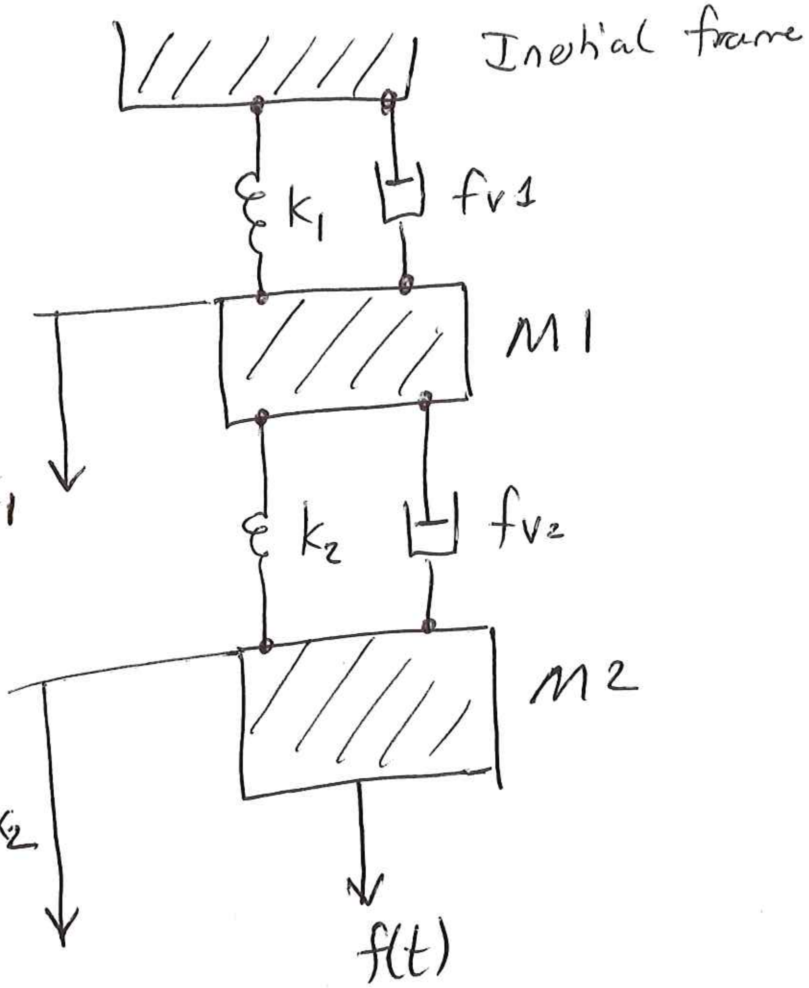
```
X = ilaplace(Mg/(s*(M*s^2 + fv*s + k)))
ezplot(subs(X, {M,k,g,fv}, {1, 1, 10, 1/10}))
```

Variables in subs given as cell array

Suggest also putting a range into ezplot( ) say [0, 10] as X does not include heaviside(t). Hence plot for $t < 0$ is not valid.



Example More Complex Problem

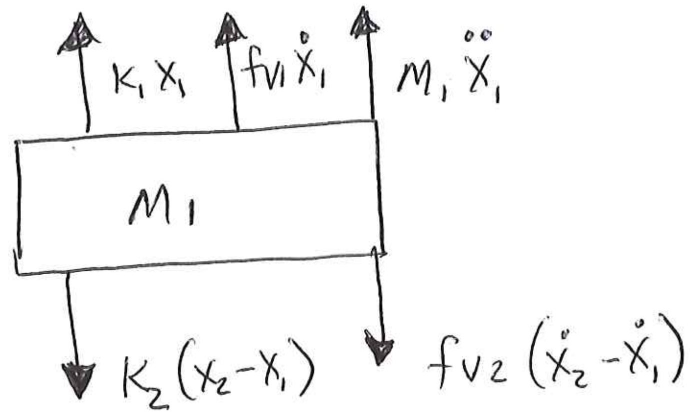


find the responses of $x_1(t)$ and $x_2(t)$ to the applied force $f(t)$.

Start with the FBD (free body diagrams)

And write down the DEQ for it.

first Mass

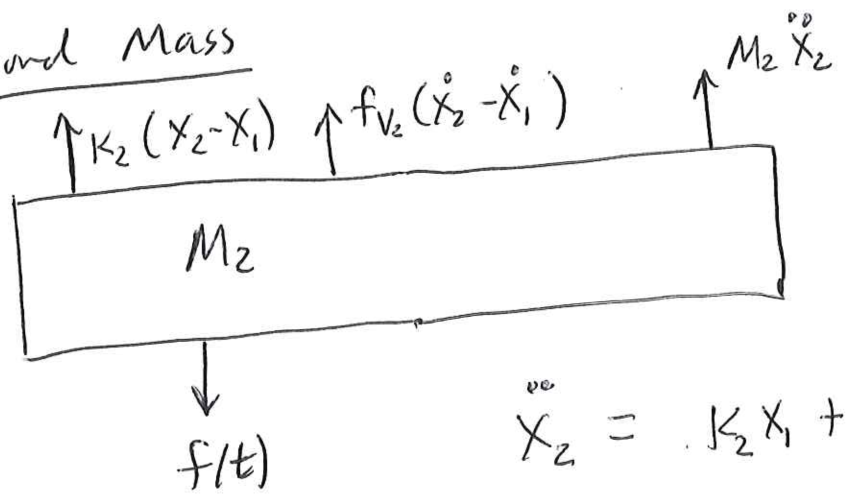


$$(K_1+K_2)X_1 + (f_{v_1}+f_{v_2})\dot{X}_1 + M_1\ddot{X}_1 - K_2X_2 - f_{v_2}\dot{X}_2 = 0$$

Define some state variables $z_1 = X_1$, $z_3 = X_2$
$z_2 = \dot{X}_1$, $x_4 = \dot{X}_2$

$$\dot{z}_2 = -\frac{(f_{v_1}+f_{v_2})}{M_1} z_2 - \left(\frac{K_1+K_2}{M_1}\right) z_1 + \frac{K_2}{M_1} z_3 + \frac{f_{v_2}}{M_1} z_4$$

Second Mass



$$\ddot{X}_2 = K_2X_1 + f_{v_2}\dot{X}_1 - K_2X_2 - f_{v_2}\dot{X}_2 + f$$

$$\dot{z}_4 = \frac{K_2}{M_2} z_1 + \frac{f_{v_2}}{M_2} z_2 - K_2 z_3 - f_{v_2} z_4 + f$$

Now we can put these into a state space form.

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1-k_2}{m_1} & \frac{-f_{v_1}-f_{v_2}}{m_1} & -k_2 & f_{v_2} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{M_2} & \frac{f_{v_2}}{M_2} & -k_2 & -f_{v_2} \end{bmatrix}}_{A} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{B} f$$

Identify the A and B state matrices.

If the output is X_1 then

$C = [1\ 0\ 0\ 0]$ $\quad D = [0]$

Then use ss2tf(A B C D) to get the transfer function of the force $f(t) \Leftrightarrow F(S)$ to $x(t) \Leftrightarrow X_1(S)$.

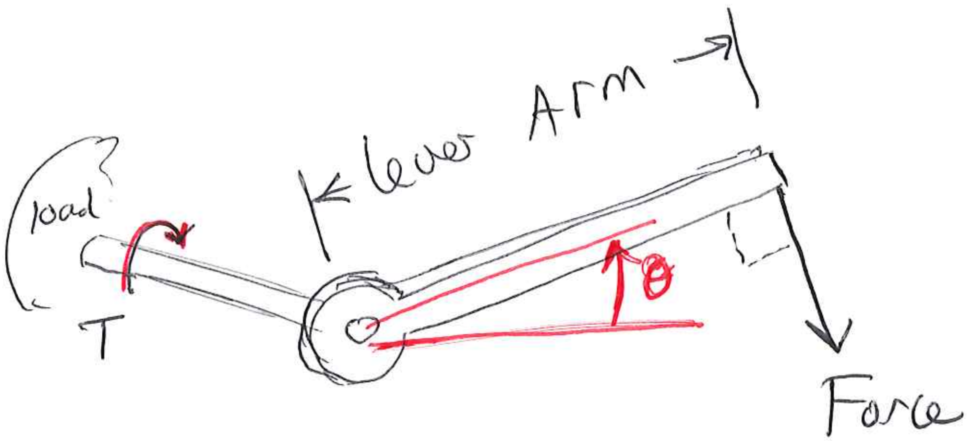
Rotation Systems

Section (2,6)

Two types of mechanical motion

- displacement
- rotation

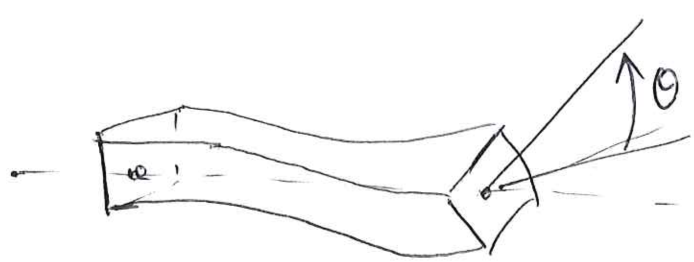
What is Torque?



$$T = (\text{lever arm}) \cdot (\text{Force})$$

(meters) (newtons)

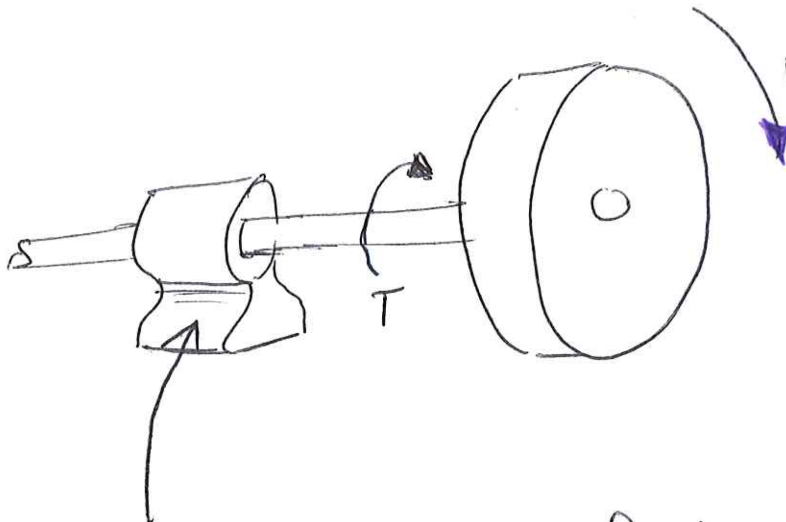
Spring Torsion Bar



$$T = K\theta$$

Torque required to twist torsion bar through an angle θ.

Friction



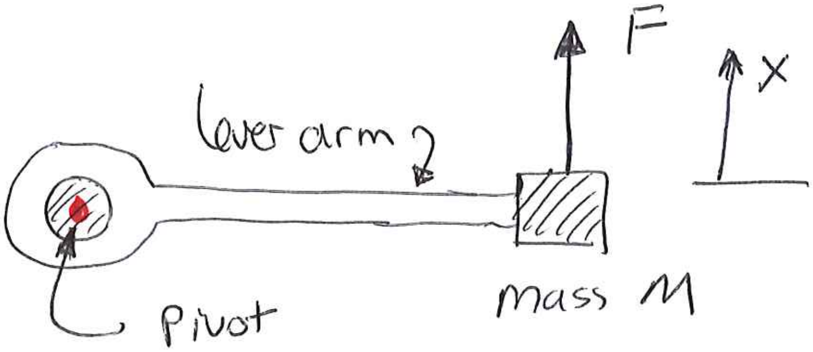
$$\omega = \frac{\partial \theta}{\partial t}$$

rotation rate

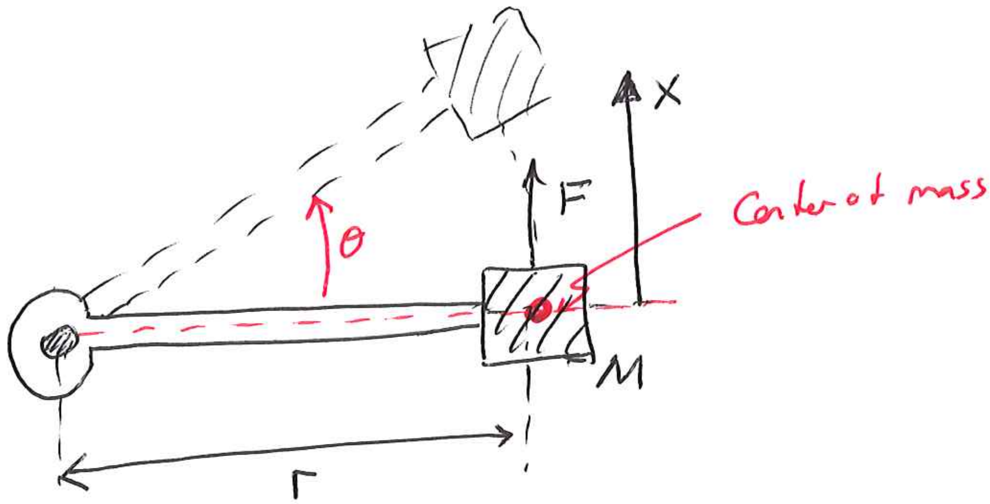
Bearing with friction

$T = \omega D$ $\quad$ D - coefficient of rotational friction.

Inertia



$$F = M \frac{d^2x}{dt^2}$$



Small θ $|\theta| \ll 1$ $r\theta \approx x$

$$r\,d\theta = dx$$

$$r\frac{d\theta}{dt} = r\omega = \frac{dx}{dt}$$

$$r\frac{d^2\theta}{dt^2} = r\dot{\omega} = \frac{dx^2}{dt^2}$$

$$@\ \theta \approx 0 \quad r\ddot{\theta} = \frac{F}{M}$$

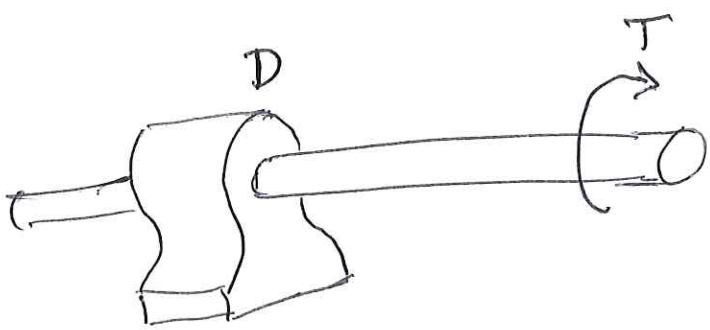
$$\text{But } T = \underbrace{F}_{r\ddot{\theta}M} \cdot r = r^2 M \ddot{\theta}$$

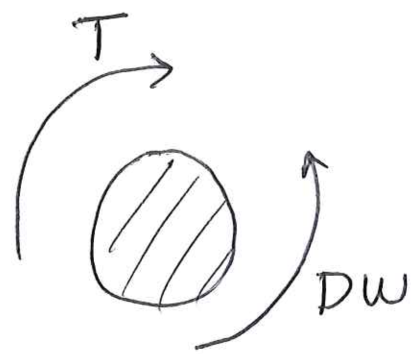
$r^2 M$ - moment of inertia $\equiv J$

$$\boxed{T = J\ddot{\theta}}$$

Problem

How Fast does a shaft rotate
with { coefficient friction of D
Torque applied T



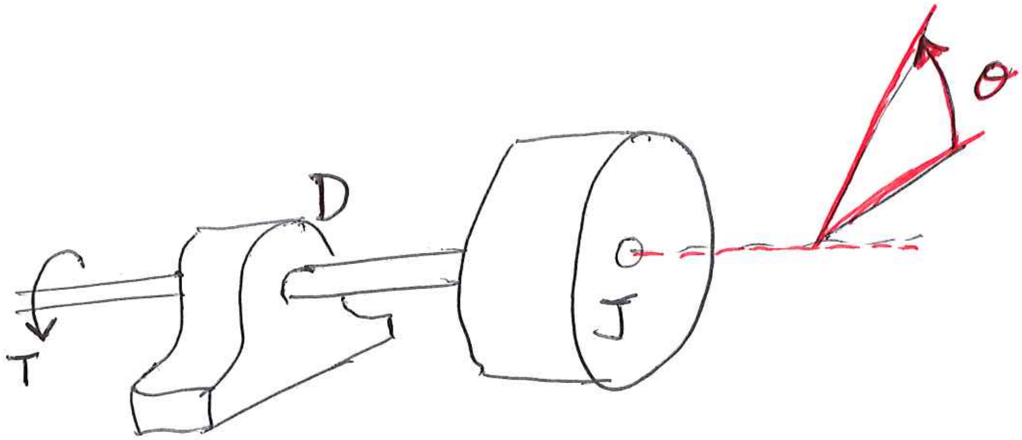


Draw a diagram

$T = DW$

shaft rotations/sec $\Rightarrow \frac{T}{2\pi D}$

Problem with inertial-rotation load.



θ = angle of rotation

$$\frac{d\theta}{dt} = \omega$$

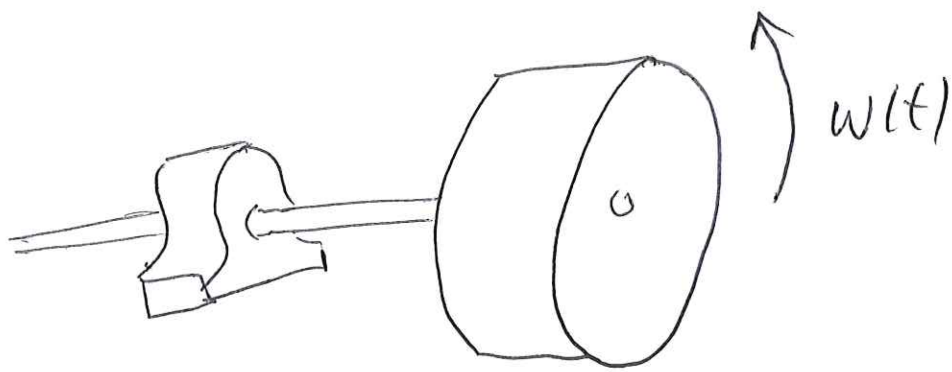
D - coefficient of friction of bearing

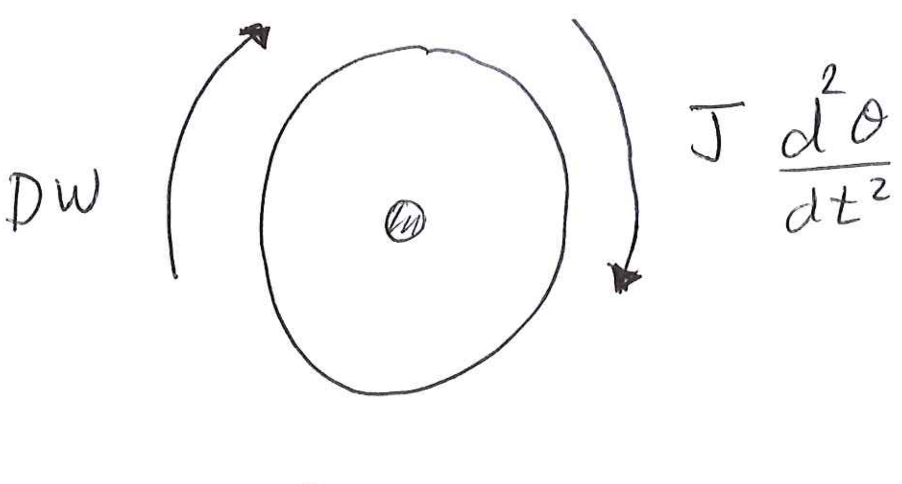
Torque required to overcome friction $= D\omega$
$= D\dot{\theta}$

rotational inertia J

Torque required to overcome inertia $= J\dot{\omega}$
$= J\ddot{\theta}$

Assume a shaft is rotating at rate w_0 at $t=0$. If it has inertia J and friction D, find $w(t)$.





$$J\frac{d^2\theta}{dt^2} + DW = 0$$

$$J\dot{w} = -DW \quad \text{with IC } W(0) = W_0$$

Solution $\quad w(t) = b e^{-at}$

a, b constants.

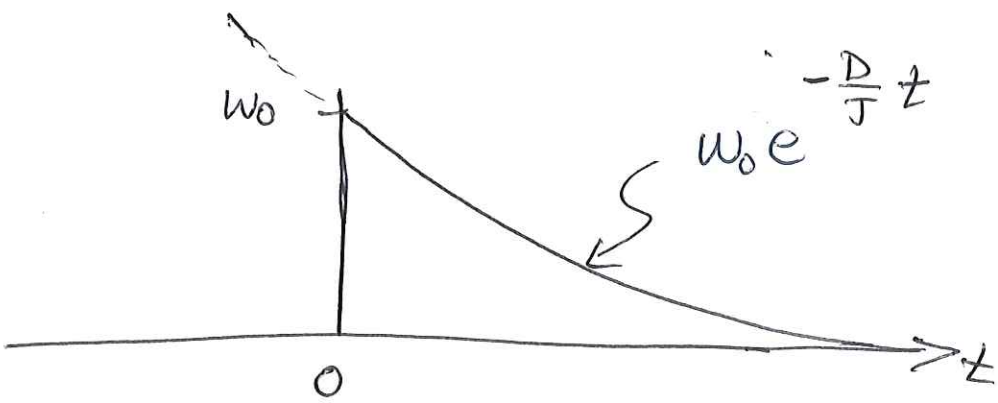
$$J \underbrace{\cancel{b}(-a)\cancel{e^{-at}}}_{\dot{w}} = -D \underbrace{\cancel{b}\cancel{e^{-at}}}_{w}$$

$$\therefore a = \frac{D}{J}$$

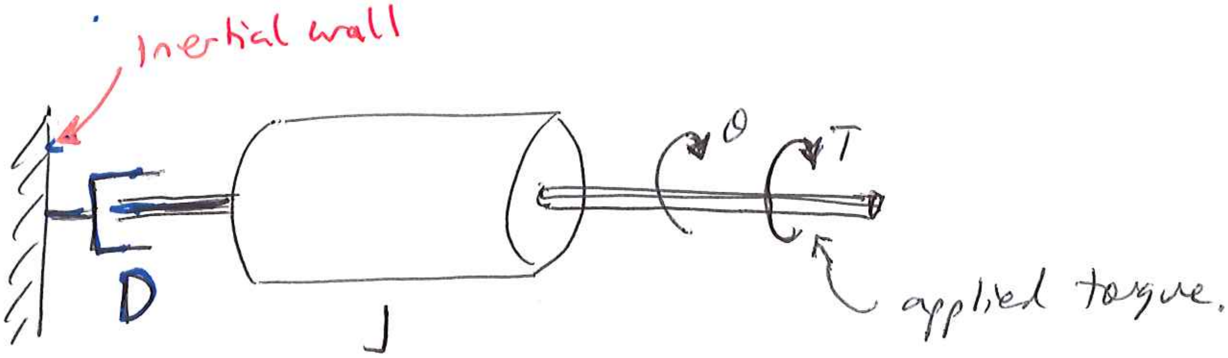
also at $t = 0 \qquad w(0) = w_0$

$$\therefore b = w_0$$

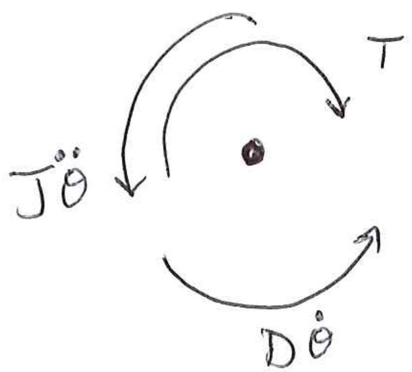
$$w(t) = w_0 e^{-\frac{D}{J}t}$$



Find response to applied torque of System



Body Diagram for torque



$$J\ddot{\theta} + D\dot{\theta} = T$$

Notation

$T(t)$ torque as function of time

$T_L(s)$ Torque in Laplace domain

let $T_L = \frac{1}{s}$ torque applied as step function at t

$$T(t) = u(t)$$

$$J\ddot{\theta} + D\dot{\theta} = u(t)$$

$$J\dot{\omega} + D\omega = u(t)$$

$$J s\Omega(s) + D\Omega(s) = \frac{1}{s}$$

$w(t) \Longleftrightarrow \Omega(s)$

$$\Omega(s) = \frac{1}{s}\,\frac{1}{Js + D}$$

where $\Omega(s) \Longleftrightarrow \omega(t)$

$$\Omega(s) = \frac{A}{s} + \frac{B}{Js + D}$$

$$AJs + AD + sB = 1$$

$$A = \frac{1}{D}$$

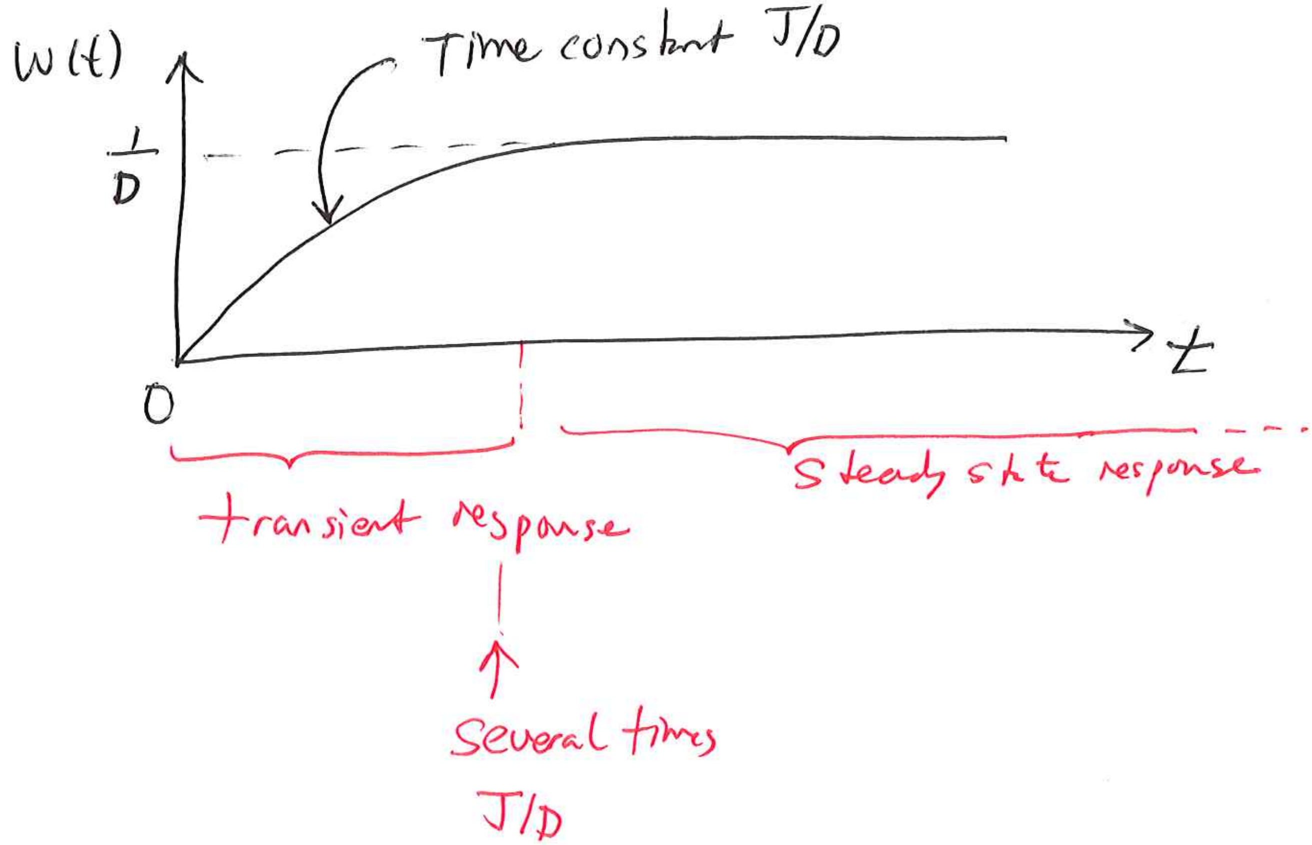
$$\frac{1}{D}J + B = 0 \quad \therefore\ B = -\frac{1}{D}J$$

$$\Omega(s) = \frac{\frac{1}{D}}{s} - \frac{\frac{J}{D}}{Js+D}$$

inverse laplace

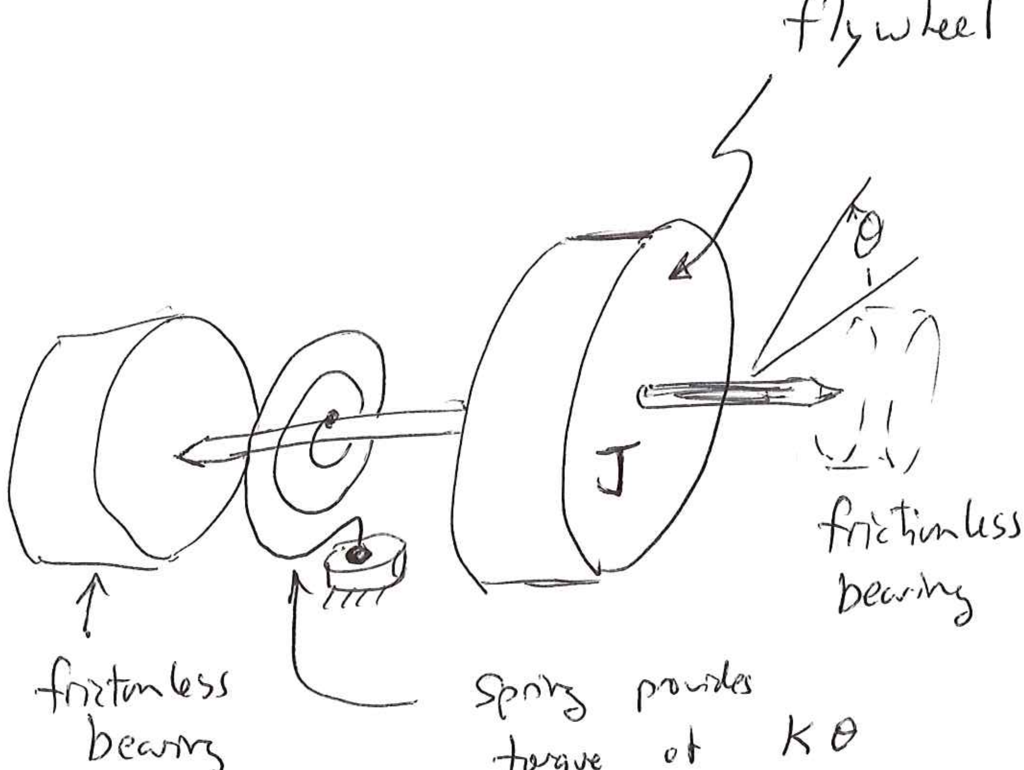
$$w(t) = \frac{1}{D}u(t) - \frac{1}{D}u(t)e^{-\frac{D}{J}t}$$

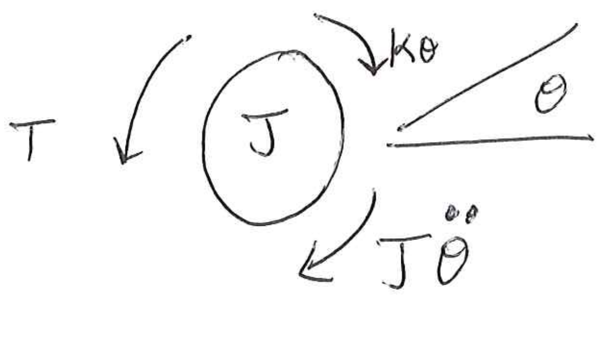
$$w(t) = \frac{u(t)}{D}\left(1 - e^{-\frac{D}{J}t}\right)$$



Another example

Mechanical Watch Flywheel.





From body diagram

$$T = K\theta + J\ddot{\theta}$$

let $T(t) = \delta(t)$ impulse of torque

$$\theta(t) \Leftrightarrow \Theta(s)$$

$$T(t) \Leftrightarrow T_L(s)$$

$$T_L(s) = (K + s^2 J)\,\Theta(s)$$

$$\Downarrow$$

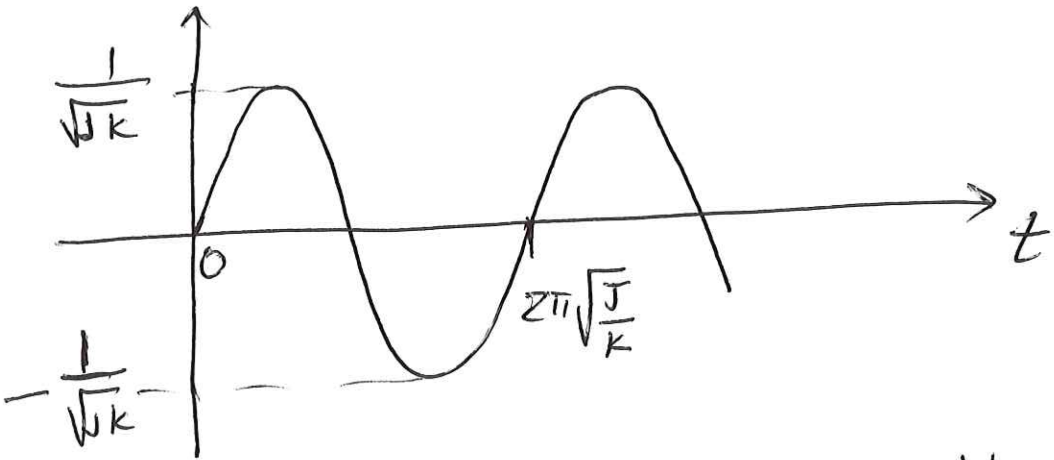
$$1$$

$$\Theta(s) = \frac{1}{K + s^2 J}$$

Table pg 36 $\quad \sin(\omega t)\,u(t) \Leftrightarrow \frac{\omega}{s^2+\omega^2}$

$$\Theta(s) = \frac{\frac{1}{J}}{s^2 + \frac{K}{J}} = \frac{\overbrace{\sqrt{\frac{K}{J}}}^{\omega}\ \frac{1}{\sqrt{JK}}}{s^2 + \underbrace{\frac{K}{J}}_{\substack{\omega \\ \omega^2}}}$$

$$\theta(t) = \frac{1}{\sqrt{JK}} \sin\left(\sqrt{\frac{K}{J}}\,t\right) u(t)$$

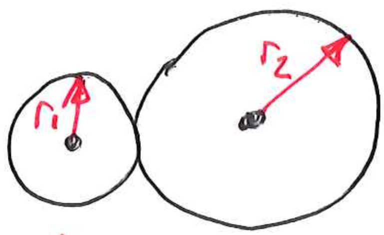


Impulse response of flywheel attached to inertia wall via a torsion rod.

Equivalences between Displacement and Rotational motion

Rotational θ		Displacement or Translational X	
Torsion spring	$T = K\theta$	displacement spring	$F = KX$
Damping	$T = D\frac{d\theta}{dt}$	Damping	$F = f_v\dot{X}$
Rotational Inertia	$T = J\ddot{\theta}$	Translational inertia	$F = M\ddot{X}$

Gears



gear 2
radius r_2
number of teeth N_2

gear 1
radius r_1
number of teeth N_1

$$\frac{r_1}{r_2} = \frac{N_1}{N_2}$$

$$\text{gear ratio} = \frac{N_1}{N_2}$$

Suppose gear 1 rotates by angle θ_1
" 2 " " " θ_2

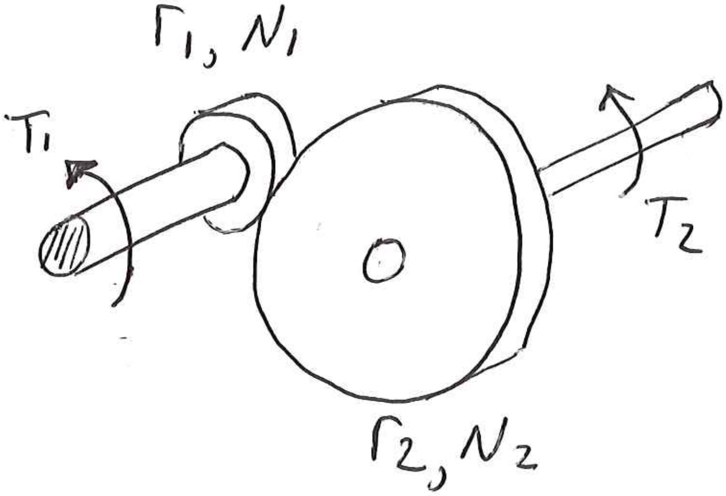
$$\theta_1 = \frac{N_2}{N_1}\theta_2$$

or $\boxed{\theta_1 N_1 = \theta_2 N_2}$

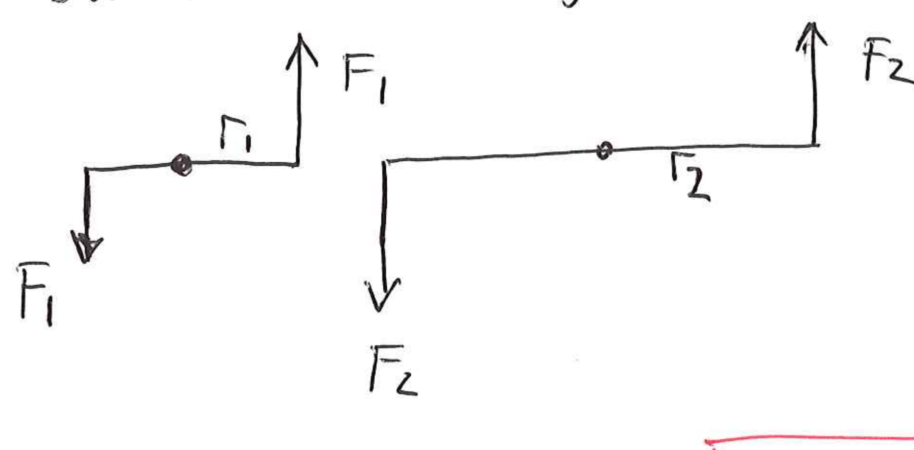
Take derivative

$$\dot{\theta}_1 N_1 = \dot{\theta}_2 N_2 \Rightarrow \boxed{\omega_1 N_1 = \omega_2 N_2}$$

Torque on a gear (Assume gears frictionless)



Set T_2 such that the torque from T_1 is balanced and the gears do not rotate.



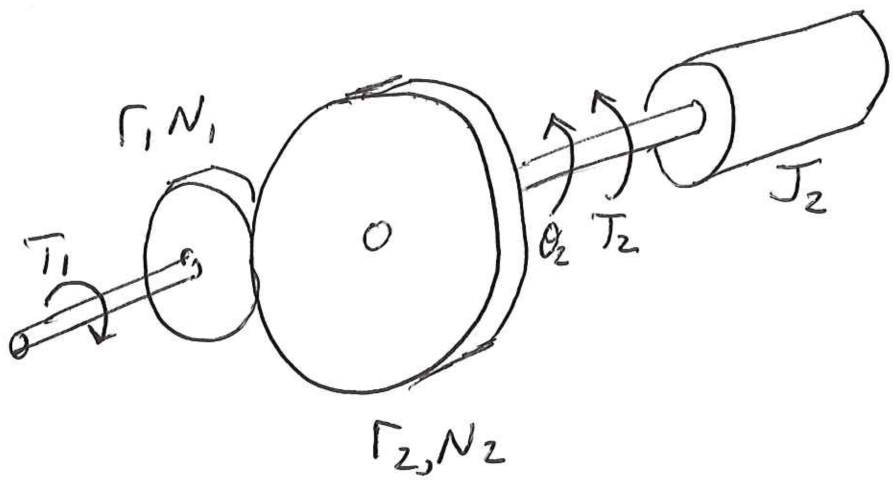
$$F_1 = F_2$$

$$\frac{F_1 r_1}{r_1} = \frac{F_2 r_2}{r_2}$$

$$\frac{T_1}{r_1} = \frac{T_2}{r_2}$$

$$\boxed{\frac{T_1}{T_2} = \frac{r_1}{r_2} = \frac{N_1}{N_2}}$$

Mapping Inertia



$$T_2 = \ddot{\theta}_2 J_2$$

Find T_1 . and equivalent moment of inertia as referenced to shaft 1.

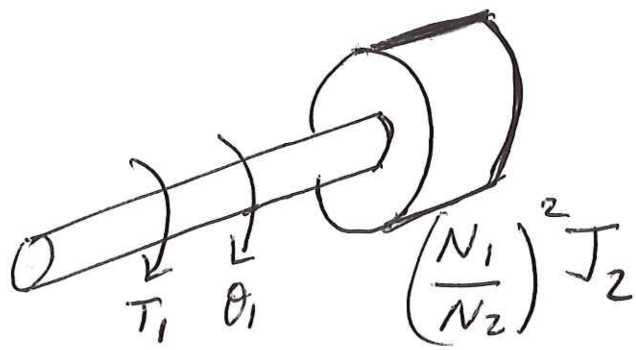
$$T_1 = \frac{N_1}{N_2} \cdot T_2$$

$$\ddot{\theta}_2 = \ddot{\theta}_1 \frac{N_1}{N_2}$$

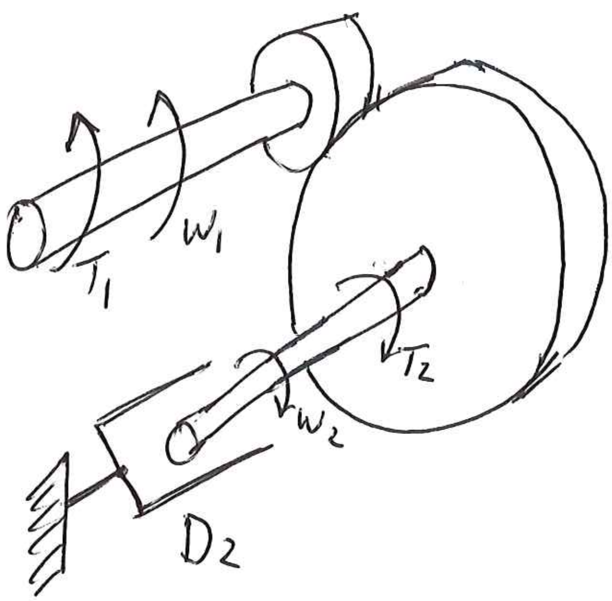
$$\therefore T_1 \frac{N_2}{N_1} = \ddot{\theta}_1 \frac{N_1}{N_2} J_2$$

$$\boxed{T_1 = \ddot{\theta}_1 \left(\frac{N_1}{N_2}\right)^2 J_2}$$

Equivalent Mechanism



Next consider rotational friction



$$T_2 = D_2 w_2$$

$$w_2 = w_1 \frac{n_1}{n_2}$$

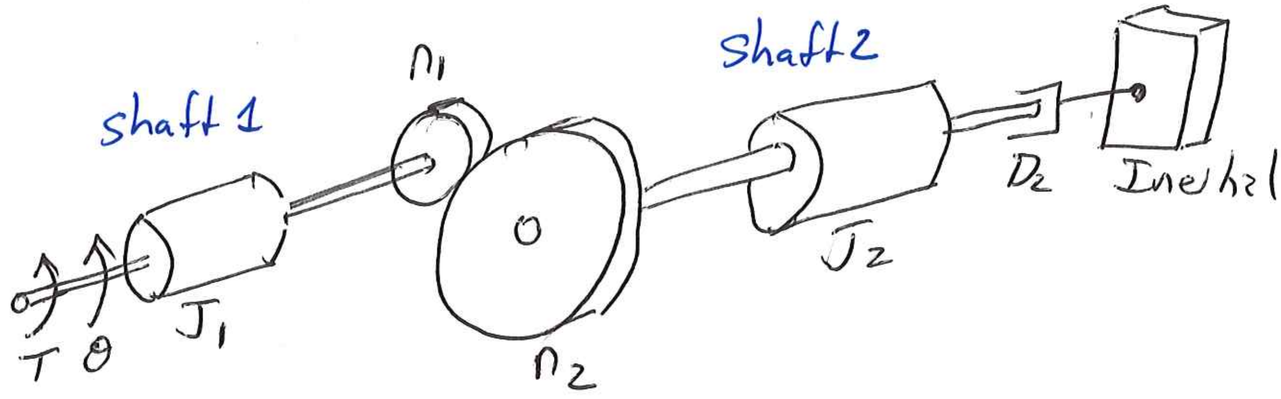
sub into

$$T_1 = T_2 \frac{n_1}{n_2}$$

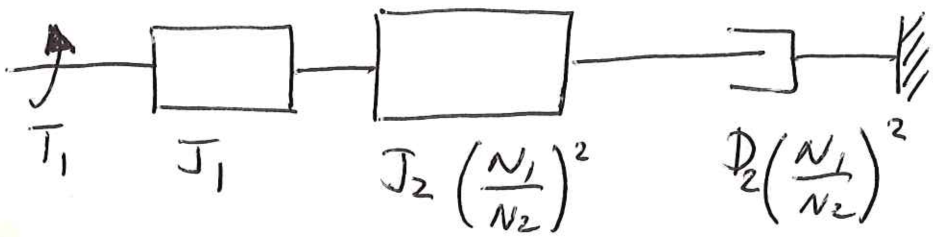
$$\therefore \; T_1 \frac{n_2}{n_1} = D_2 \, w_1 \frac{n_1}{n_2}$$

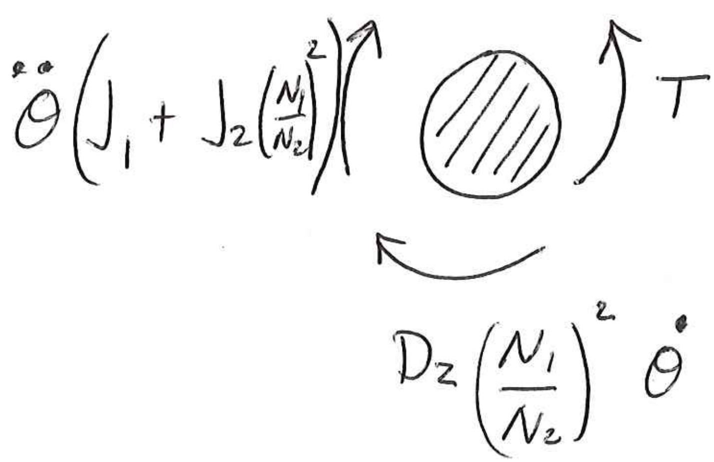
$$\boxed{T_1 = \underbrace{\left(\frac{n_1}{n_2}\right)^2 D_2}_{\text{equivalent friction}} w_1}$$

Solve the DEQ representing this system



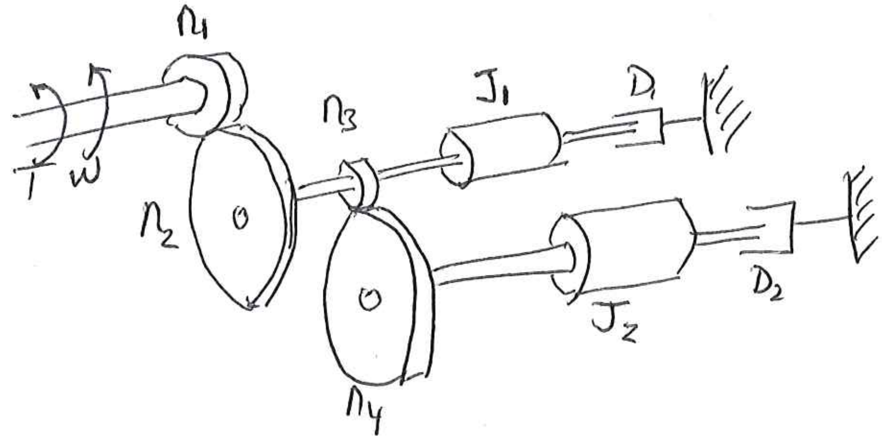
Simply translate the J_2 and D_2 to shaft 1





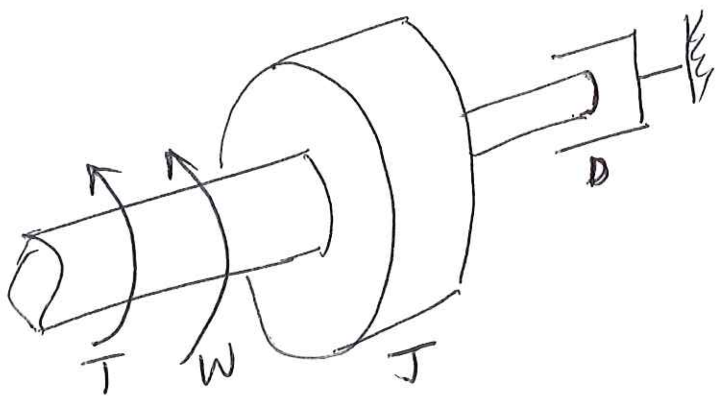
$$T(t) = \left(J_1 + J_2\left(\frac{N_1}{N_2}\right)^2\right)\ddot{\theta} + D_2\left(\frac{N_1}{N_2}\right)^2\dot{\theta}$$

Another Gear Example



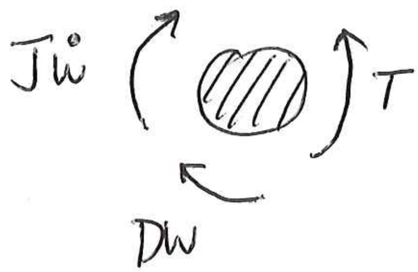
Find the transfer function between $T_s(s)$ and $\Omega(s)$

Transform the inertial and damping loads.



$$J = J_1 \left(\frac{n_1}{n_2}\right)^2 + J_2 \left(\frac{n_1 n_3}{n_2 n_4}\right)^2$$

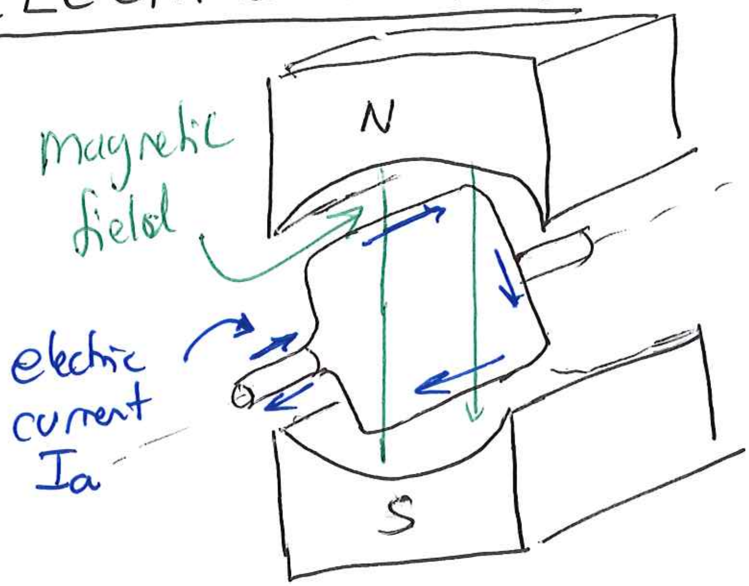
$$D = D_1 \left(\frac{n_1}{n_2}\right)^2 + D_2 \left(\frac{n_1 n_3}{n_2 n_4}\right)^2$$



$$T_L(s) = (D + sJ)\,\Omega(s)$$

$$\frac{\Omega(s)}{T_L(s)} = \frac{1}{D + sJ}$$

ELECTRIC MOTORS



$$\boxed{T = K_t\, I_a}$$

motor torque

torque constant

armature current

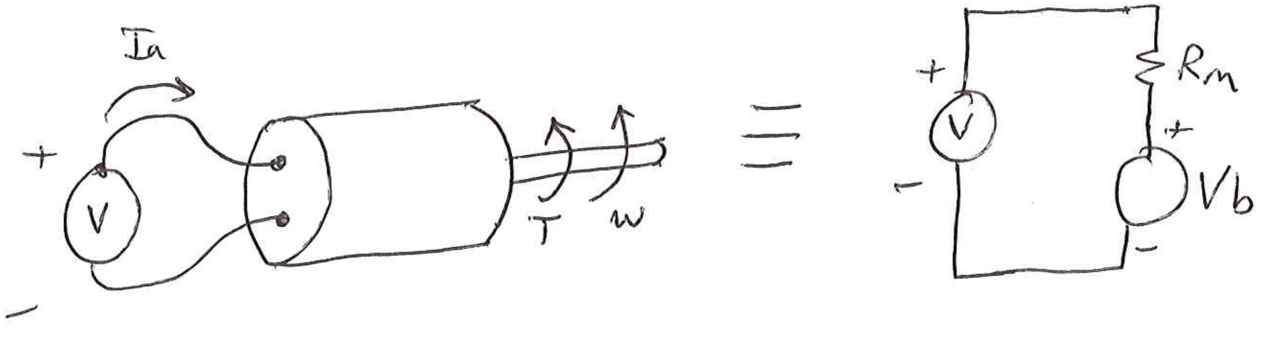
Spin motor at some rate w (rad/sec) and it will generate a voltage of

$$V_b = K_b w$$

↑ back emf (electromotive force) voltage

There is an internal resistance R_m in the windings.

<u>Model of motor</u>



Relation between applied voltage V, back emf voltage V_b, shaft rotation rate w and produced torque T

$$T = K_T I_a \qquad V_b = K_b w$$

$$I_a = \frac{V - V_b}{R_m} \qquad \therefore T = \frac{K_T}{R_m}(V - V_b)$$

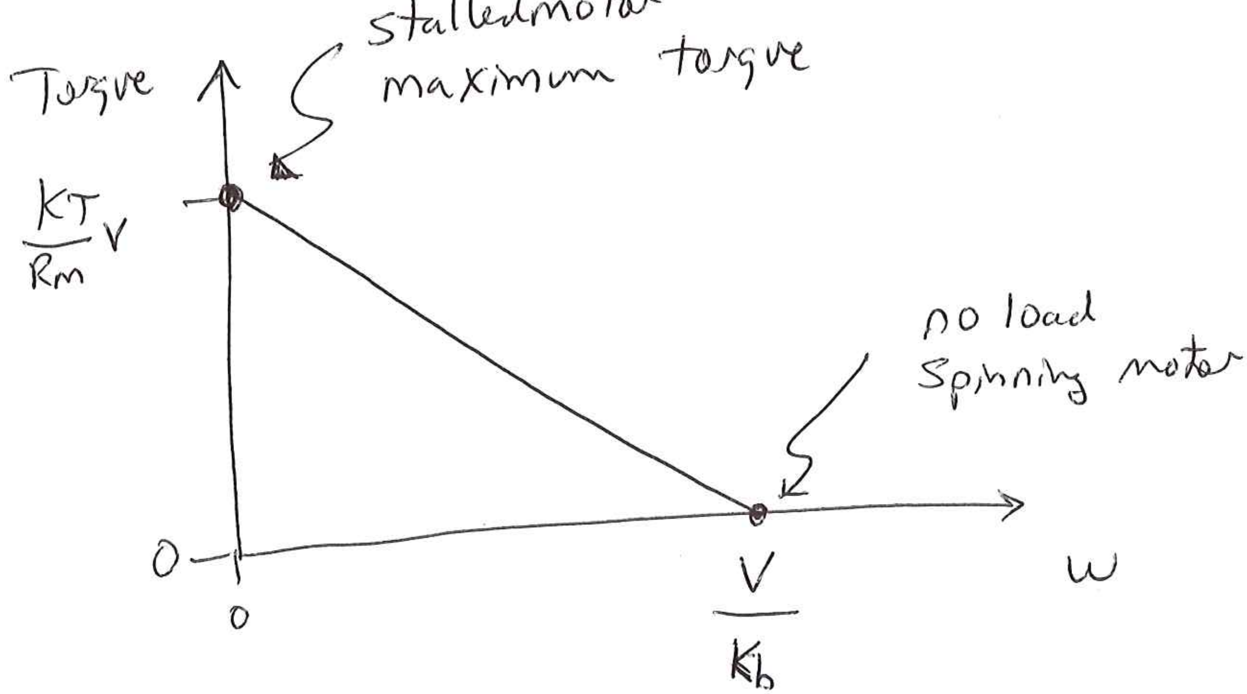
$$\text{or } T = \frac{K_T}{R_m}(V - K_b w)$$

Torque, Rotation Rate Curve

When $\omega = 0$ torque is $T = \frac{K_T}{R_m} V$

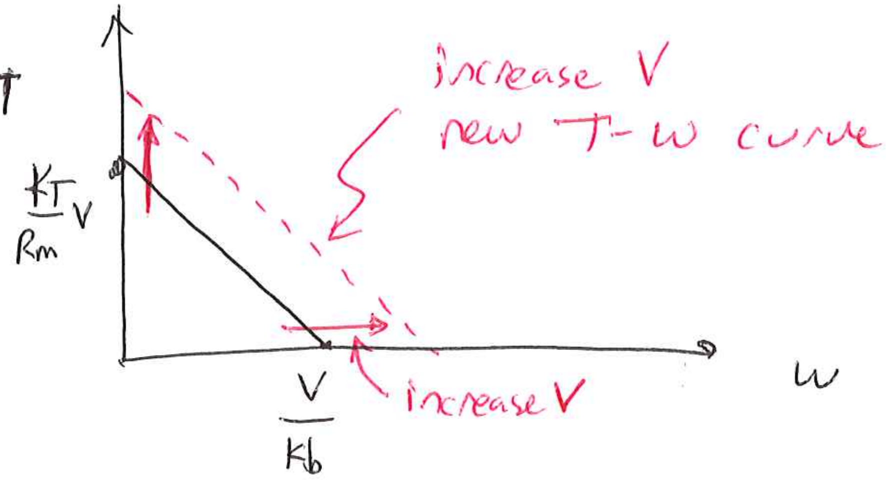
When $T = 0$ ω is $\omega = \frac{V}{K_b}$

$$T = \frac{K_T}{R_m}\left(V - K_b\,\omega\right)$$ a linear function of ω

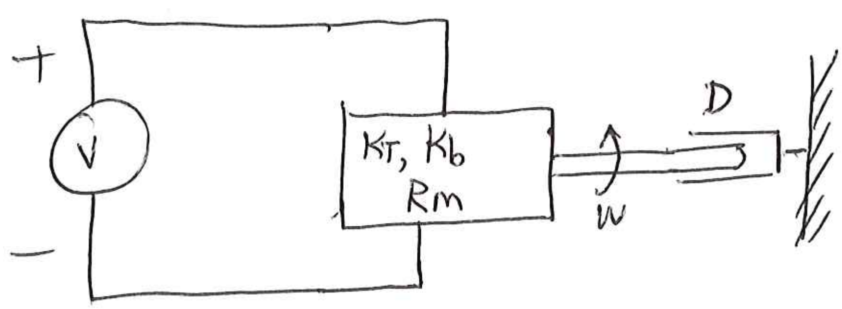


This is ideal linear model of motor. Generally R_m, K_T, K_b not directly available but can be measured. Torque-ω curve experimentally measured and parameters derived from this.

Effect of increasing V



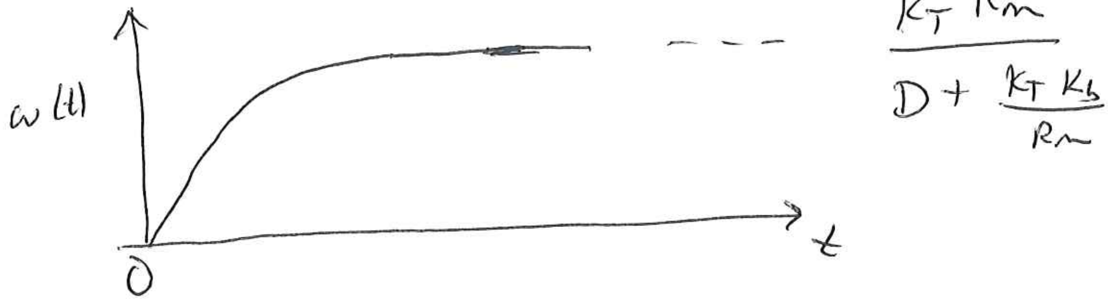
Example Find w when V applied to motor
motor shaft output friction D



$$T = \frac{K_T}{R_m}(V - K_b w) = wD$$

$$\frac{K_T}{R_m}V = w\left(D + \frac{K_T K_b}{R_m}\right)$$

$$w = V \frac{\frac{K_T}{R_m}}{D + \frac{K_T K_b}{R_m}}$$



What is final value?

table of Laplace identities

final value theorem $\quad \omega(\infty) = \lim\limits_{s \to 0} \left(s\,\Omega(s) \right)$

$$\lim_{s \to 0} s\,\Omega(s) = \frac{K_T / R_m}{D + \frac{K_T K_b}{R_m}}$$

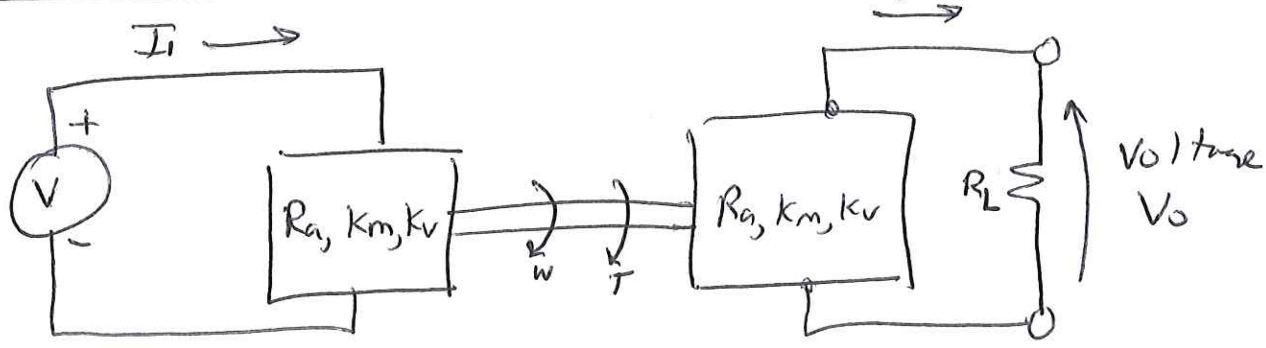
What is another way to determine the $\omega(\infty)$ for $v(t) = u(t)$?

Use DC response of impulse response.

$$\frac{\Omega(s)}{V(s)} = \frac{K_T / R_m}{Js + D + \frac{K_T K_b}{R_m}} = H(s)$$

$$\underbrace{H(0)}_{\text{DC response}} = \frac{K_T / R_m}{D + \frac{K_T K_b}{R_m}} \qquad \text{same as before.}$$

Motor Generator



Simplification, Motor same as generator.

$T = k_m I_1$ generated by motor

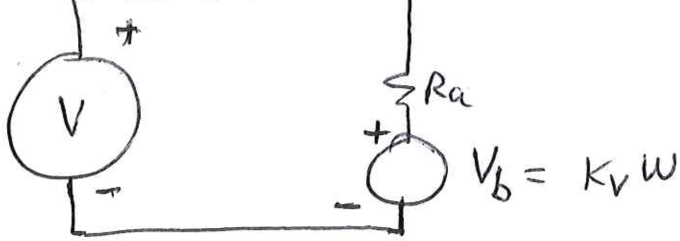
$T = k_m I_2$ resistance torque

$\therefore I_1 = I_2$ regardless of R_L

$R_L \to \infty$ no load

$I_2 = 0$, $I_1 = 0$

$I_1 \rightarrow$



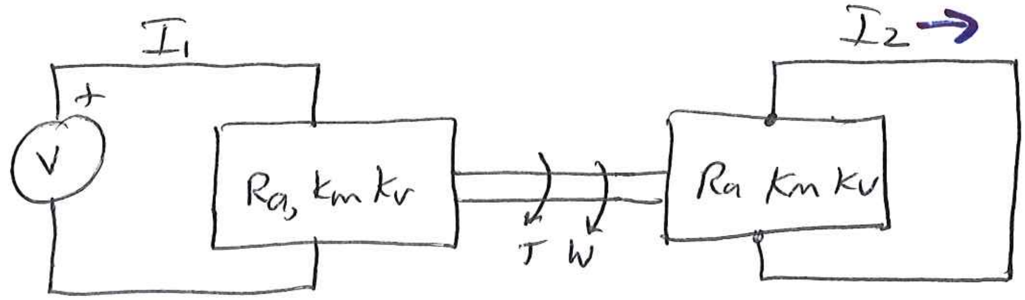
$I_1 = 0$

$\therefore V_b = V$

$\therefore V = k_v w$

Output $V_o = V = k_v w$

$R_L \to 0$ shorted generator



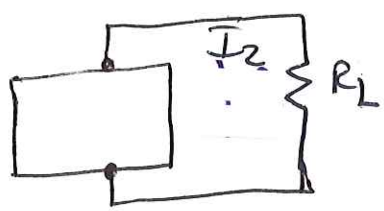
$$I_1 = I_2 = \frac{V - k_v w}{R_a}$$

also for generator $I_2 = \dfrac{k_v w}{R_a}$

$$\therefore \frac{k_v w}{R_a} = \frac{V}{R_a} - \frac{k_v w}{R_a}$$

or $2 k_v w = V$ $\boxed{w = \dfrac{V}{2k_v}}$

Finite R_L



$$I_1 = \frac{V - k_v w}{R_a} \qquad I_2 = \frac{k_v w}{R_a + R_L}$$

As $I_1 = I_2$ we have

$$\frac{V - K_V W}{R_a} = \frac{K_V W}{R_a + R_L}$$

$$V - K_V W = K_V W \frac{R_a}{R_a + R_L}$$

$$V = K_V W \left(1 + \frac{R_a}{R_a + R_L} \right)$$

$$V = K_V W \left(\frac{2R_a + R_L}{R_a + R_L} \right)$$

$$W = V \left(\frac{R_a + R_L}{K_V (2R_a + R_L)} \right)$$

<u>Output current</u>

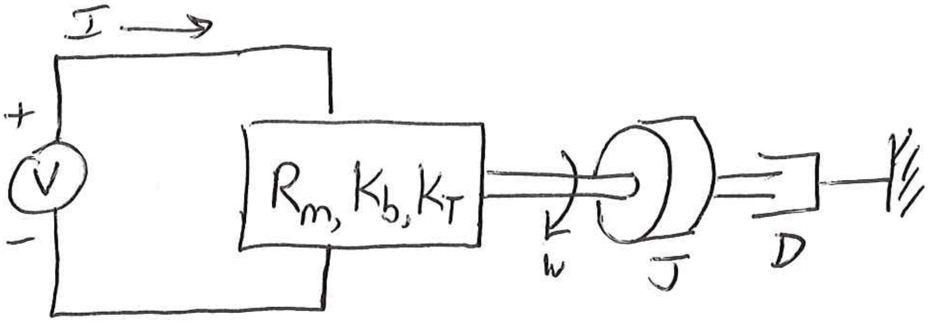
$$I_1 = \frac{V - W K_V}{R_a} = I_2$$

<u>Output voltage</u>

$$V_0 = I_2 R_L$$

Mixed Models

Solve for the transfer function between V and ω



Electrical side step 1

$$T = k_T I = k_T \frac{V - \omega k_b}{R_m} \quad (1)$$

Mechanical side step 2

$$T = J\dot{\omega} + D\omega \quad (2)$$

Combine (1) and (2) step 3

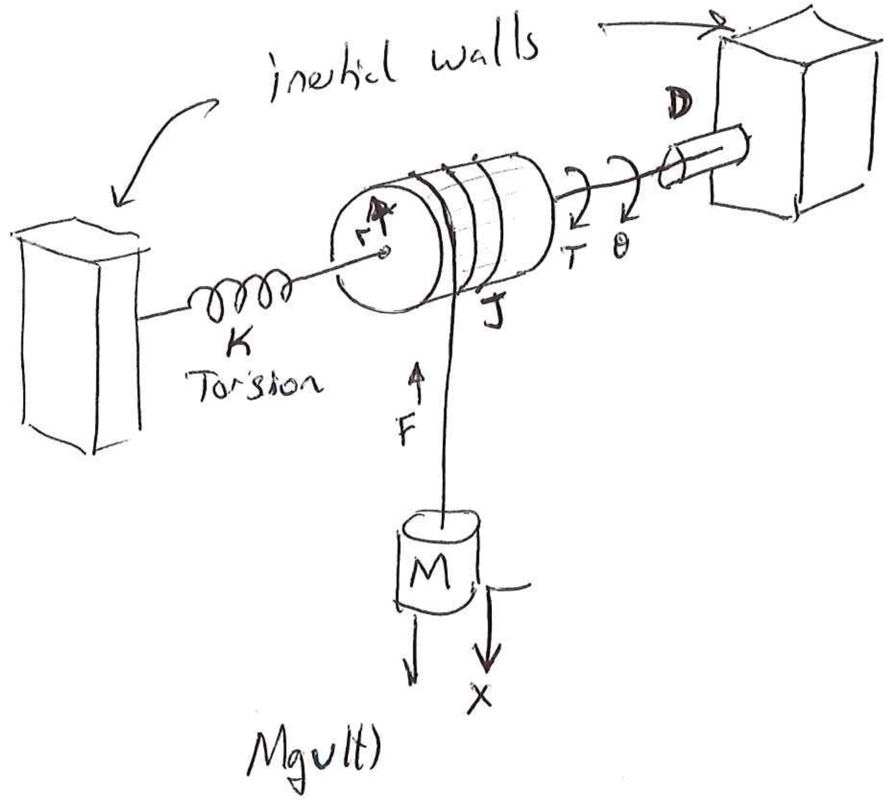
$$(Js + D)\Omega(s) = \frac{K_T}{R_m} V(s) - \frac{k_T k_b}{R_m}\Omega(s)$$

$$\Omega(s) = \frac{\frac{K_T}{R_m} V(s)}{Js + D + \frac{k_T k_b}{R_m}}$$

Transfer function

$$H(s) = \frac{K_T/R_m}{Js + \left(D + \frac{k_T k_b}{R_m}\right)}$$

Mixed Problem of Rotation and Translation

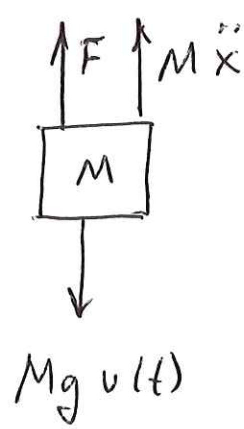


at $t<0$, $x=0$ $\quad \dot{x}=0$ $\quad \theta=0$ $\quad \dot{\theta}=0$

at $t=0$ gravity switched on and weight released.

find $\theta(t)$

Free body diagram of weight

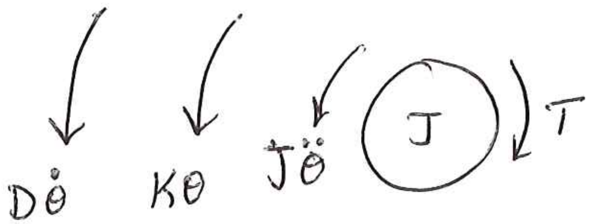


F = tension on cable

$Fr = T$ ← Torque on drum

r ← radius of drum

Free body diagram of Drum and shaft.



Also $x = r\theta$

$$\dot{x} = r\dot{\theta}$$

$$\ddot{x} = r\ddot{\theta}$$

From Mass diagram

$$F + M\ddot{x} = Mg\,u(t)$$

$$\frac{T}{r} + Mr\ddot{\theta} = Mg\,u(t)$$

Wheel equation

$$T = J\ddot{\theta} + K\theta + D\dot{\theta} \qquad (2)$$

Combine (1) and (2)

$$\frac{J}{r}\ddot{\theta} + \frac{K}{r}\theta + \frac{D}{r}\dot{\theta} + Mr\ddot{\theta} = Mg\,u(t)$$

$$\left(\frac{J}{r} + Mr\right)\ddot{\theta} + \frac{D}{r}\dot{\theta} + \frac{K}{r}\theta = Mg\,u(t)$$

Let $\frac{J}{r} + Mr = 10$

$\frac{D}{r} = 0.1$

$\frac{K}{r} = 0.1$

$Mg = 10$

$$10\,\ddot{\theta} + 0.1\,\dot{\theta} + .1\theta = 10\,u(t)$$

$$\Theta(s) = \left(\frac{10}{s}\right) \frac{1}{10s^2 + 0.1s + 0.1}$$

H = tf (10, [10, 0.1, 0.1])

step (H)