Chapter III – Amplitude Modulation

ENEL 471 – Introduction to Communications Systems and Networks

Chapter Objectives

- At the end of this chapter, you will be able to:
  - Define and analyze the time domain and frequency domain representations of amplitude modulated signals
  - Perform and analyze the amplitude demodulation of signals in the absence of noise
  - Perform and analyze the amplitude demodulation of signals in the presence of channel noise

Outline

- Amplitude Modulation (AM)
  - Theory of Amplitude Modulation
  - Amplitude Demodulation
- Double Side-band Supressed Carrier (DSB-SC) Modulation
  - Theory of DSB-SC Modulation
  - DSB-SC Demodulation: Coherent Detection
  - Costas Receiver
- Single Side-band (SSB) and Vestigial Side-band (VSB) Modulation
  - Single Side-band Modulation
  - Vestigial Side-band Modulation
- Noise in AM receivers
  - Noise in DSB-SC Receivers
  - Noise in AM Receivers

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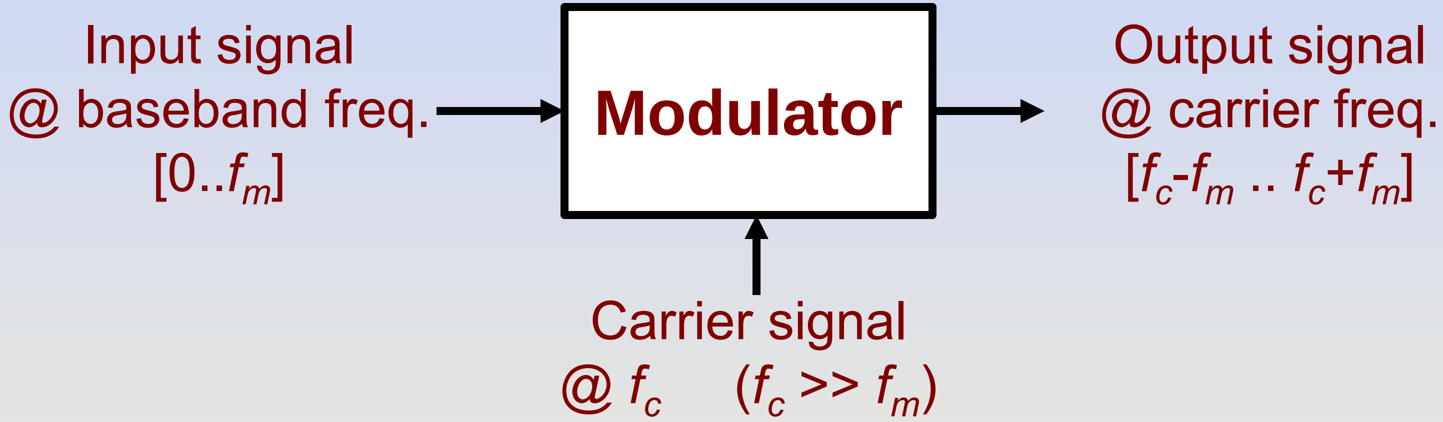
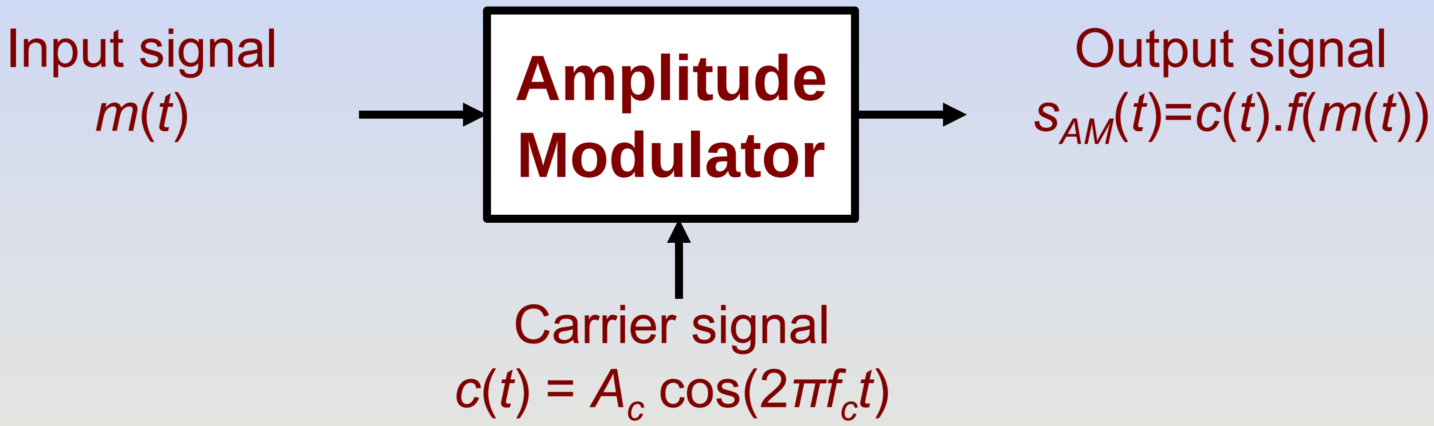
Definition of Modulation



- Modulation can be defined as:
  - A process by which some characteristic of a carrier is varied in accordance with a modulating wave (signal)
  - During this process, the information in the input signal is encoded in a carrier signal
- In wireless communication, the carrier is generally a sinusoidal signal with a frequency (carrier frequency) $f_c >> f_m$, where f_m is the highest frequency component in the input signal.

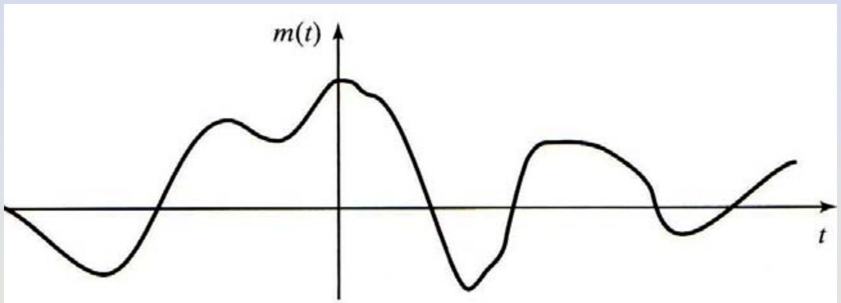
Definition of Amplitude Modulation



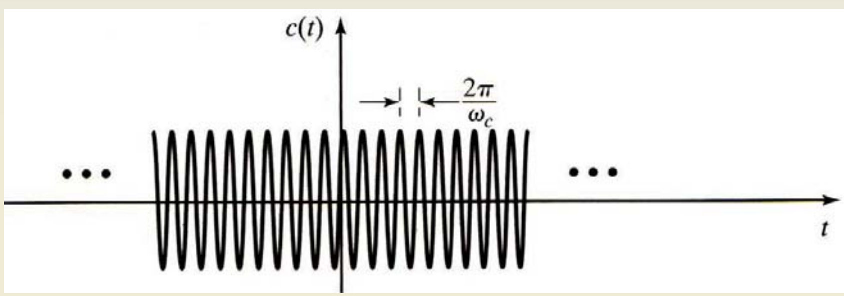
- For amplitude modulation :
  - The amplitude of the carrier is the characteristic of the carrier that is varied in accordance with the modulating wave (signal)
  - → The information in the input signal is encoded in the amplitude of the carrier signal

The Simplest Amplitude Modulation

$$s_{AM}(t) = m(t) \cdot c(t) = A_c \cdot m(t) \cdot \cos(2\pi f_c t)$$



✖ Multiplication of $m(t)$ by $c(t) = cos(2\pi f_c t)$



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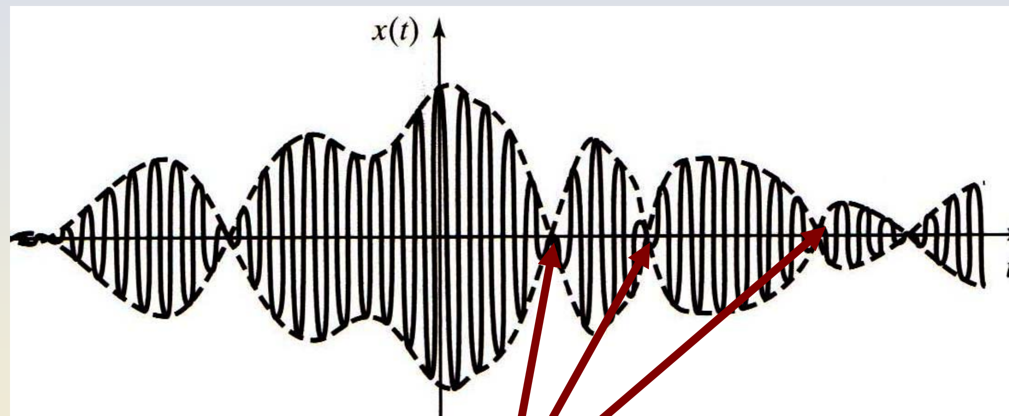


Phase reversal

- The phase reversal leads to envelope distortion

→ A simple envelope detector will not be sufficient to restore the information

Conventional AM : Time Domain Analysis

- In order to simplify the demodulation process (using a simple envelope detector, the phase reversal should be avoided
- The amplitude modulation can be expressed by:

$$s_{AM}(t) = \left(1 + k_a m(t)\right) \cdot c(t) = A_c \cdot \left(1 + k_a m(t)\right) \cdot \cos\left(2\pi f_c t\right)$$

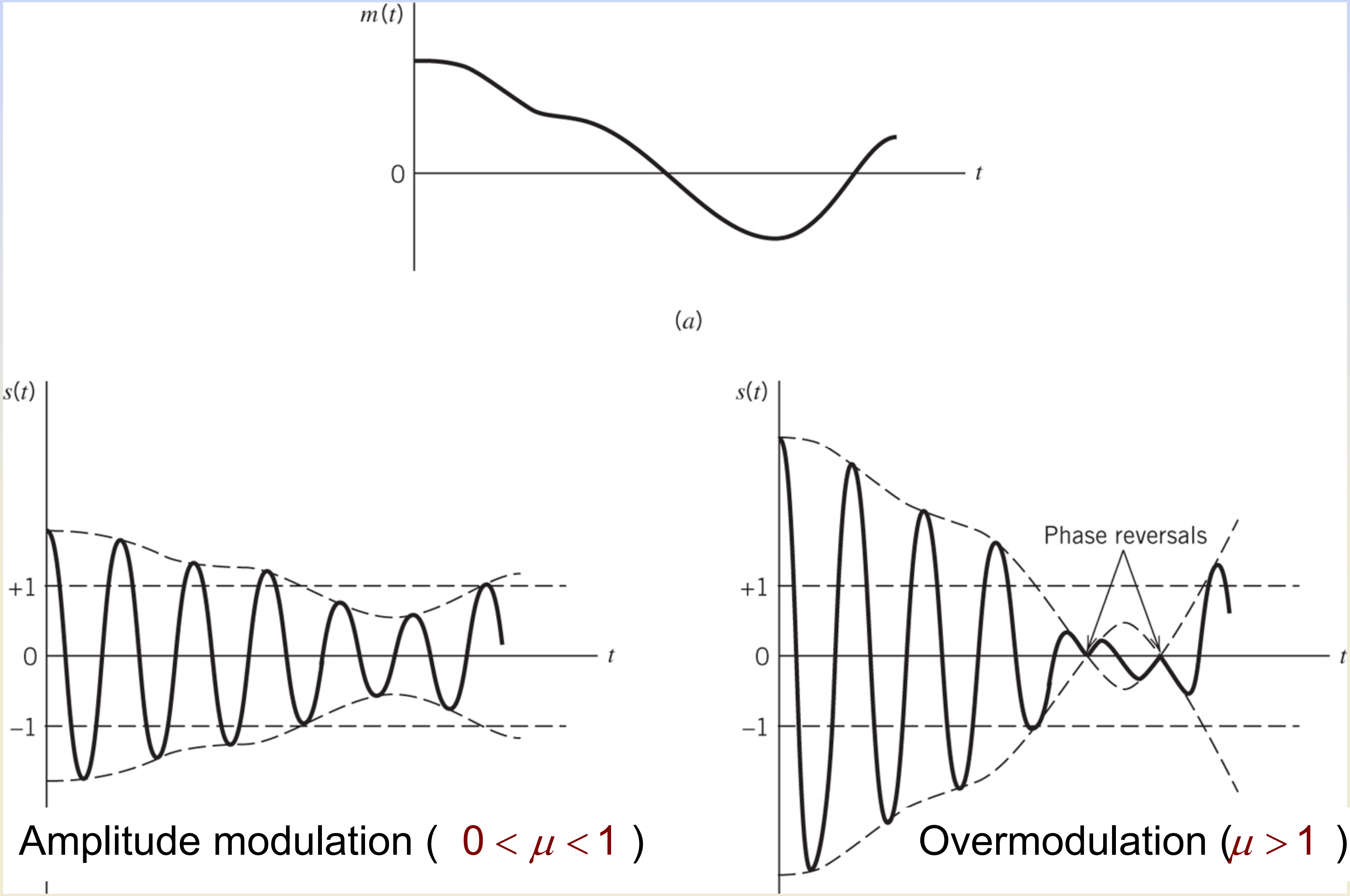
- k_a is a constant called <u>amplitude sensitivity</u>
- To avoid phase reversal, $1 + k_a m(t)$ should be positive or alternatively:

$$\left|k_a m(t)\right| \leq 1, \qquad \text{for all } t$$

- Modulation factor (or percentage modulation):

$$\mu = \max_t \left(\left|k_a m(t)\right|\right)$$

Conventional AM : Time Domain Analysis



Amplitude modulation ($0 < \mu < 1$)

Overmodulation ($\mu > 1$)

Conventional AM : Time Domain Analysis

Modulation factor (μ)	Minimum envelope (A_{min})	Maximum enve.ope (A_{max})	Interpretation
0	A_c	A_c	No modulation
$0 < \mu < 1$	$0<A_{min}<A_c$	$A_c<A_{max}<2A_c$	Amplitude modulation
1	$A_{min}=0$	$A_{max}=2A_c$	100% modulation
$\mu > 1$	$A_{min}<0$	$A_{max}>2A_c$	Overmodulation

Conventional AM : Frequency Domain Analysis

- By applying the Fourier transform to the expression of the amplitude modulation:

$$s_{AM}(t) = \left(1 + k_a m(t)\right) \cdot c(t) = A_c \cdot \left(1 + k_a m(t)\right) \cdot \cos(2\pi f_c t)$$

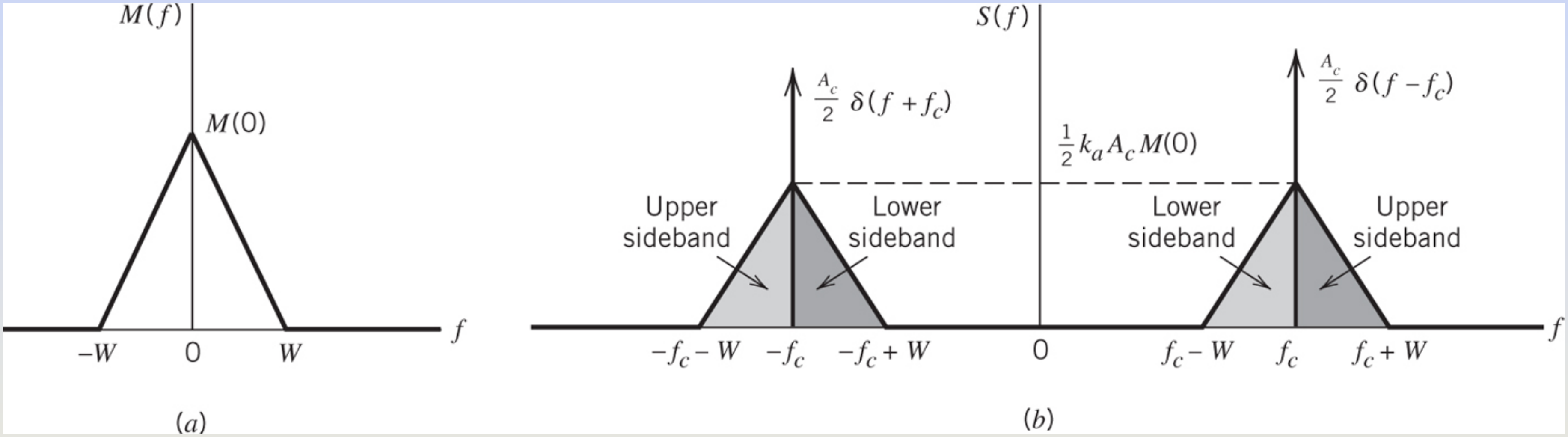
we can write:

$$S_{AM}(f) = \mathscr{F}\left(s_{AM}(t)\right) = \mathscr{F}\left(A_c \cdot \left(1 + k_a m(t)\right) \cdot \cos(2\pi f_c t)\right)$$

or:

$$S_{AM}(f) = \frac{A_c}{2}\left(\delta(f - f_c) + \delta(f + f_c)\right) + k_a \frac{A_c}{2}\left(M(f - f_c) + M(f + f_c)\right)$$

Conventional AM : Frequency Domain Analysis



- The modulation process shifts the message spectrum to high frequencies (around f_c and $-f_c$)
- The amplitude modulated spectrum is composed of 3 parts:
  - Upper sideband (USB): frequencies outside $\pm f_c$
  - Lower sideband (LSB): frequencies inside $\pm f_c$
  - A carrier frequency that does not carry any information
- The amplitude modulated signal requires a transmission bandwidth of 2W

Efficiency the Conventional AM Modulation

- The time domain representation can be rewritten as:

$$s_{AM}(t) = \underbrace{A_c \cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{k_a A_c m(t) \cos(2\pi f_c t)}_{\text{sidebands}}$$

- The power in the carrier is given by: $E\left[A_c^{\,2}\cos^2(2\pi f_c t)\right] = \dfrac{A_c^{\,2}}{2}$
- The power in the sidebands is:

$$E\left[A_c^{\,2}k_a^{\,2}m^2(t)\cos^2(2\pi f_c t)\right] = \frac{A_c^{\,2}k_a^{\,2}}{2}E\left[m^2(t)\right]$$

- Power efficiency of conventional AM:

$$\eta_{AM} = \frac{\text{power in the sidebands}}{\text{total power of the AM signal}} = \frac{\dfrac{A_c^{\,2}k_a^{\,2}}{2}E\left[m^2(t)\right]}{\dfrac{A_c^{\,2}}{2} + \dfrac{A_c^{\,2}k_a^{\,2}}{2}E\left[m^2(t)\right]} = \frac{k_a^{\,2}E\left[m^2(t)\right]}{1 + k_a^{\,2}E\left[m^2(t)\right]}$$

Conventional AM : Advantages and Drawbacks

- Advantages:
  - No distortion to the amplitude of the modulated signal (no phase reversal)
  - Ease of implementation of the receiver (envelope detector is sufficient to detect the original message)

- Drawbacks:
  - Low power efficiency: high total transmit power but only part of it is used for the actual information (sidebands)
  - High transmission bandwidth requirements: the amplitude modulated signal requires a transmission bandwidth of 2W (twice the message bandwidth

Example: Single Tone Modulation

- A transmitter uses conventional amplitude modulation to modulate a message having the form of a single tone:

$$m(t) = A_m \cos(2\pi f_m t)$$

The carrier signal is :

$$c(t) = A_c \cos(2\pi f_c t)$$

a. What is the time-domain representation of the modulated signal
b. Calculate the frequency domain representation of the modulated signal and plot its frequency spectrum
c. Determine the power efficiency of the modulation

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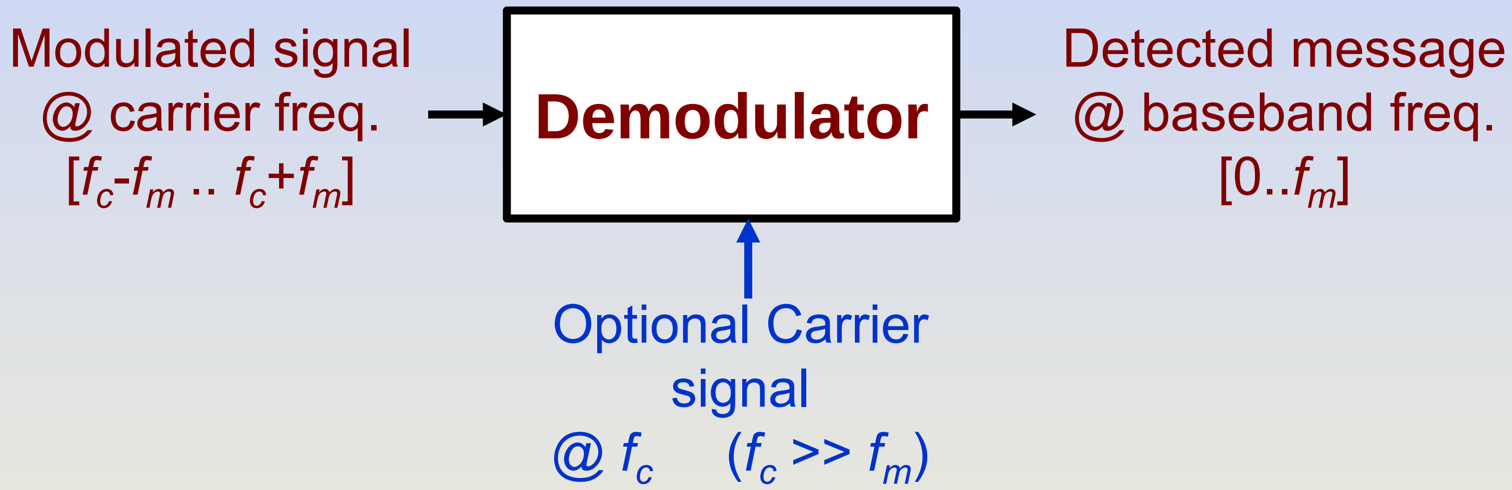
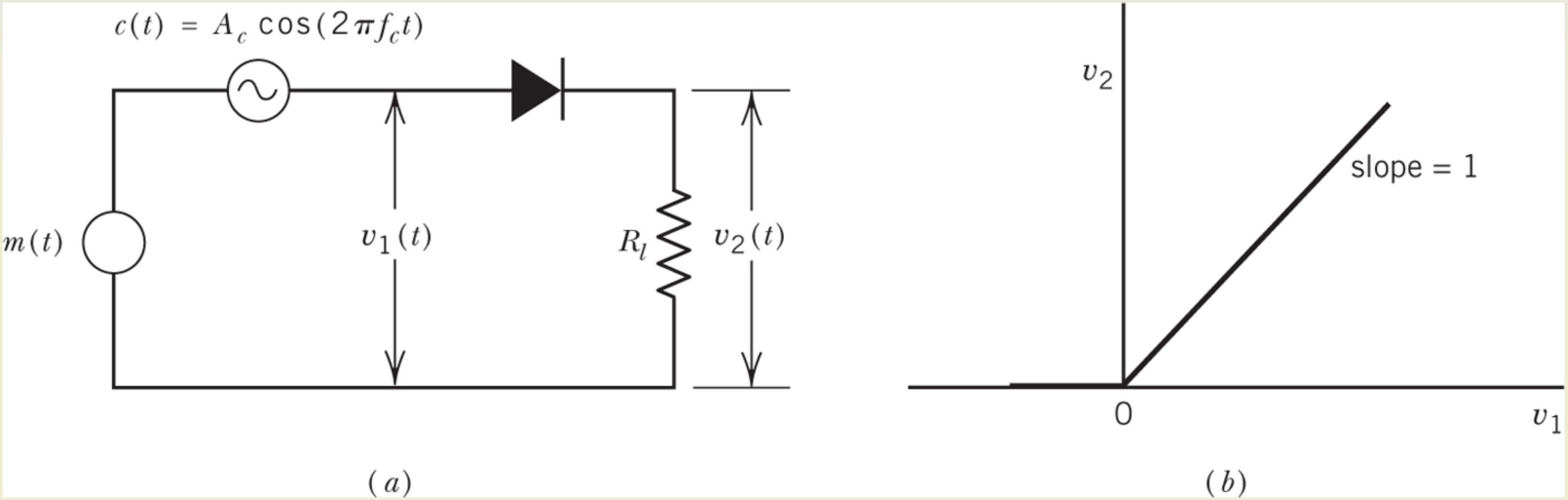
Definition of Demodulation



- Demodulation can be defined as:
  - A process by which the original message is recovered from the modulated signal (inverse of the modulation)
  - During this process, the information in the input signal is decoded/extracted from the carrier signal
- There are generally 2 types of demodulators:
  - Non coherent : doesn’t require a carrier recovery
  - Coherent : requires carrier recovery

Example of Implementation of Conventional AM

- The generation of AM signals can be done using different devices.
- One option is to use a **switching modulator**
- The circuit uses a diode that acts as an ideal switch: presents a zero impedance when forward biased ($c(t) > 0$) and infinite impedance when reverse biased ($c(t) < 0$).
- [It assumes that the amplitude of the carrier A_c is much larger than of $m(t)$]



Switching Modulator: Principle of Operation

- The input voltage is the sum of the carrier and the message signal

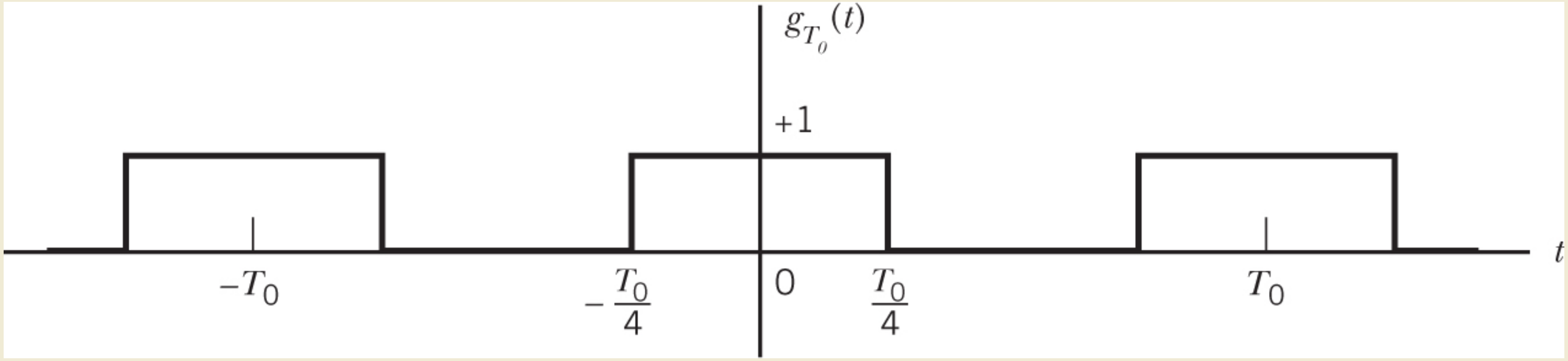
$$v_1(t) = m(t) + c(t) = m(t) + A_c \cos(2\pi f_c t)$$

- If A_c >> $|m(t)|$, the diode switches as a function of the sign of the carrier. This is equivalent to a multiplication with a periodic pulse train $g_{T_o}(t)$

$$v_2(t) \approx \left[m(t) + A_c \cos(2\pi f_c t)\right] \cdot g_{T_o}(t)$$

- Using the Fourier Series, the periodic pulse train can be expressed as:

$$g_{T_o}(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos\left(2\pi f_c t(2n-1)\right)$$



Switching Modulator: Principle of Operation

- The signal at the output of the switching modulator is the sum of 2 components:
  - The desired AM modulated signal, given by:

$$\frac{A_c}{2}\left[1+\frac{4}{\pi A_c}m(t)\right]\cos(2\pi f_c t)$$

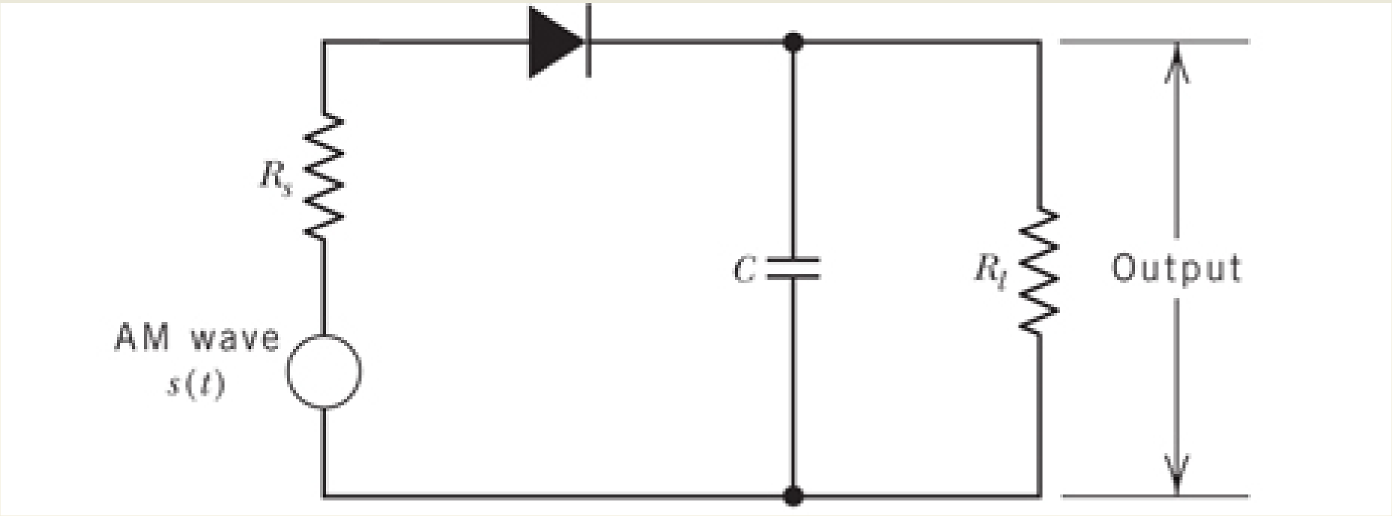
  - Unwanted components at even harmonics (delta functions) and odd harmonics (spectrums of width 2W)
- The unwanted components should be filtered out using a band-pass filter at the output of the switching modulator
- The sensitivity of the AM modulated signal is

$$k_a = \frac{4}{\pi A_c} << 1$$

→ The efficiency of the modulation is very low

Example of Implementation of Conventional Amplitude Demodulator – Envelope Detector

- The demodulation can be achieved using an envelope modulator circuit.
  → Objective, detect the envelope of the modulated signal $A_c \cdot \left(1 + k_a m(t)\right)$
- The circuit uses a diode that acts as an ideal switch: presents a small resistance r_f when forward biased and infinite impedance when reverse biased.
- We assume that the amplitude modulated signal is supplied by a voltage source of internal resistance R_s



Example of Implementation of Conventional Amplitude Demodulator – Envelope Detector

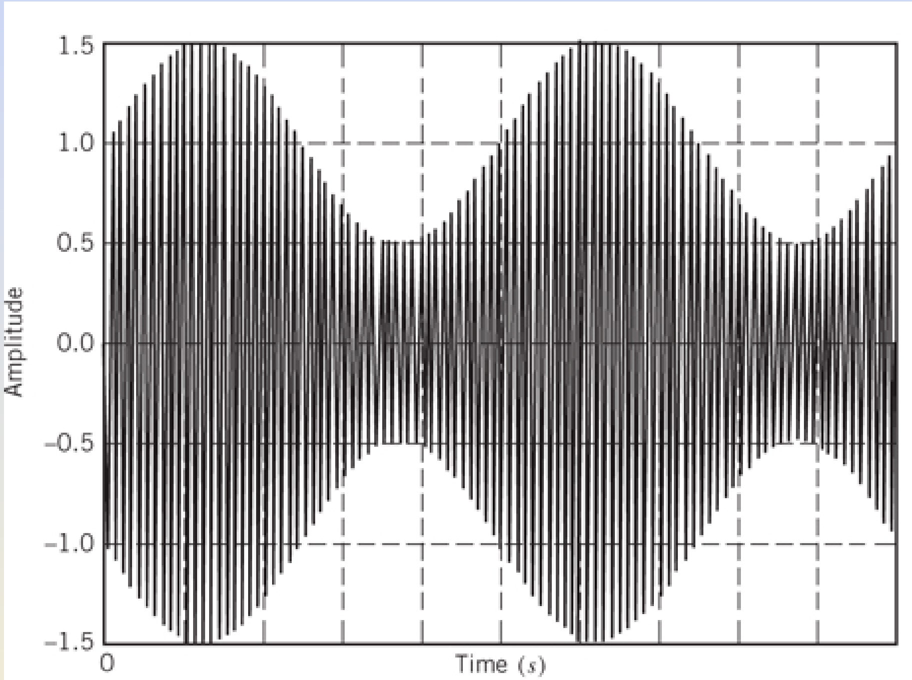
- Principle of operation:
  - On a positive half cycle of the modulated signal, the diode is forward biased. The capacitor *C* is charged
  - On a negative half cycle of the modulated signal, the diode is reverse biased. The capacitor *C* is discharged in the load resistor R_l
- Conditions for proper operation:
  - The charging time constant should be short compared to the carrier period

$$\left(r_f + R_s\right)C << \frac{1}{f_c}$$

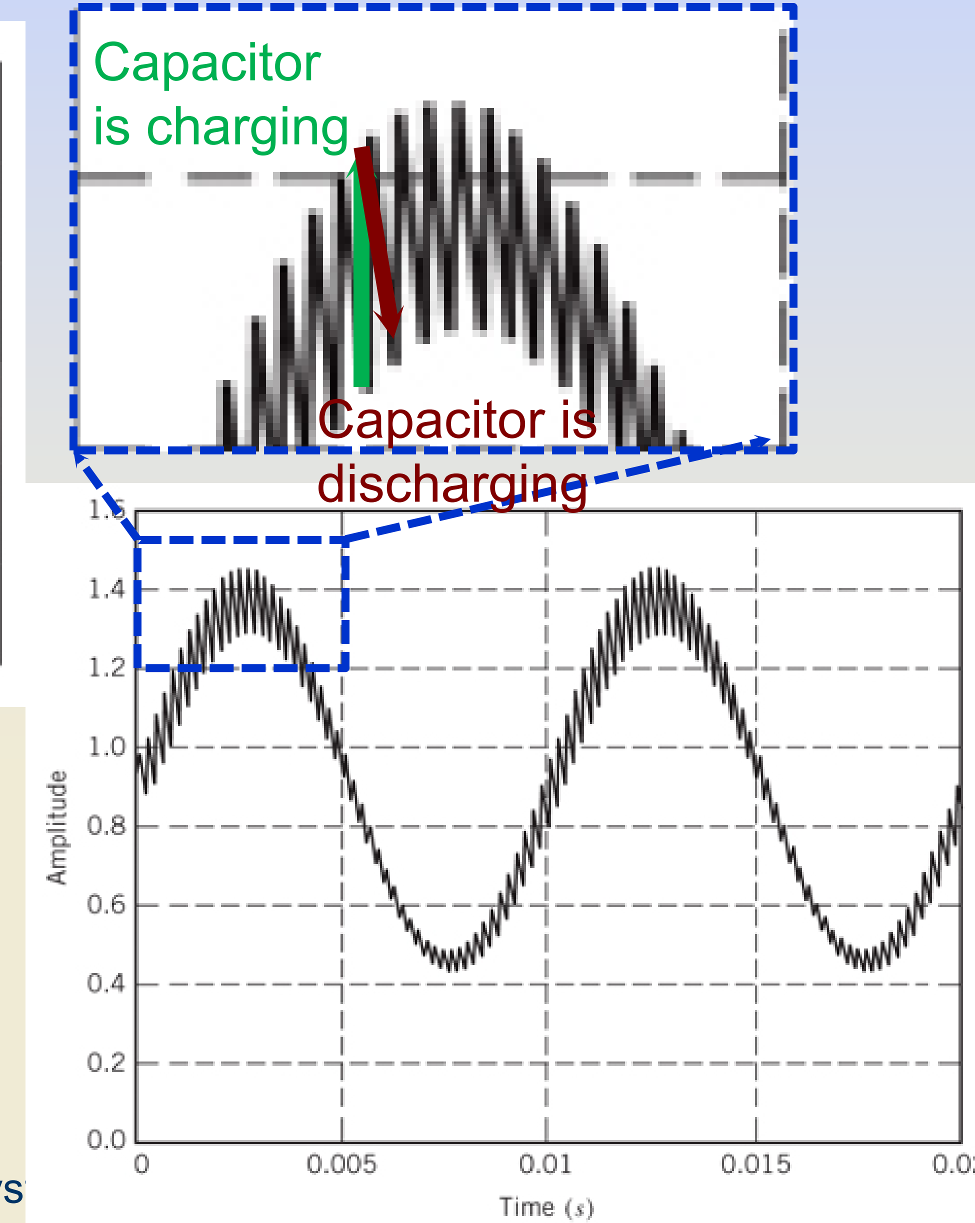
  - The discharging time constant should be long compared to the carrier period and short compared to the envelope variation

$$\frac{1}{f_c} << R_l C << \frac{1}{W}$$

Example of Implementation of Conventional Amplitude Demodulator – Envelope Detector



AM signal



Demodulated signal

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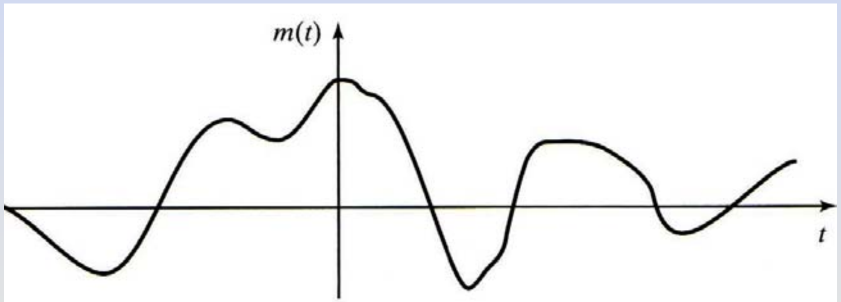
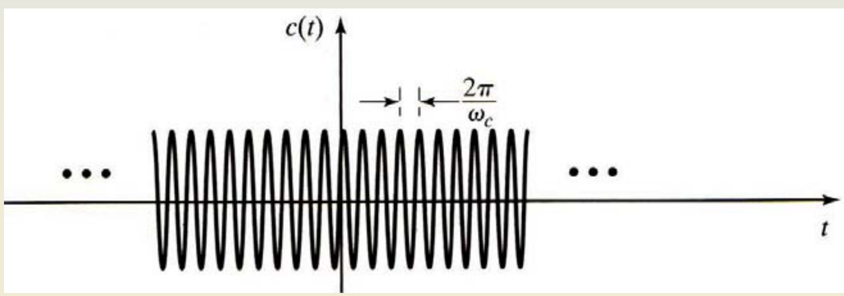
Double Sideband – Supressed Carrier Modulation (DSB-SC): Time Domain Analysis

$$s_{AM}(t) = m(t) \cdot c(t) = A_c \cdot m(t) \cdot \cos(2\pi f_c t)$$



✖ Multiplication of $m(t)$ by $c(t)=cos(2\pi f_c t)$



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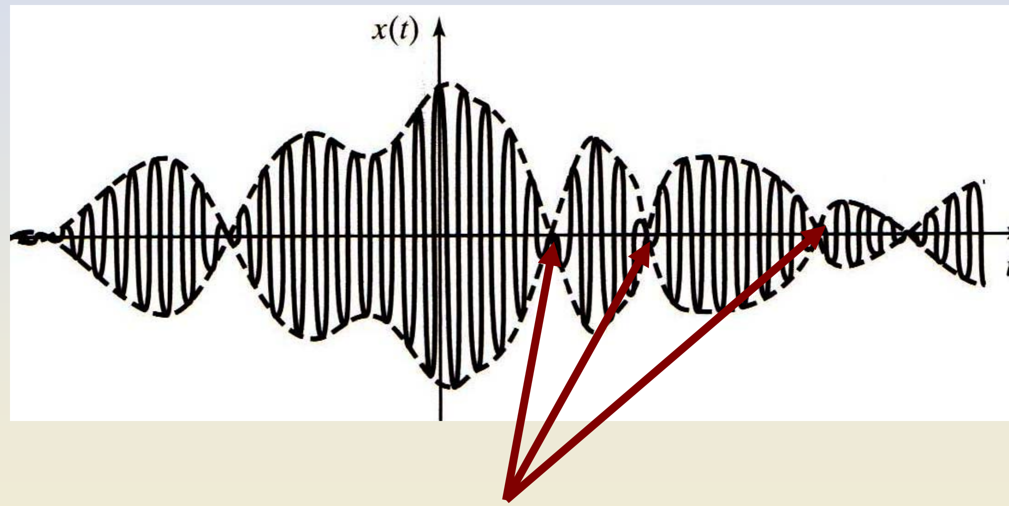


Phase reversal

- By supressing the carrier, there will be no wasted energy on transmitting a power that carrier no information (the carrier)

→ The phase reversal will require more complicated circuits in the transmitter and receiver

DSB–SC : Frequency Domain Analysis

- By applying the Fourier transform to the expression of the amplitude modulation:

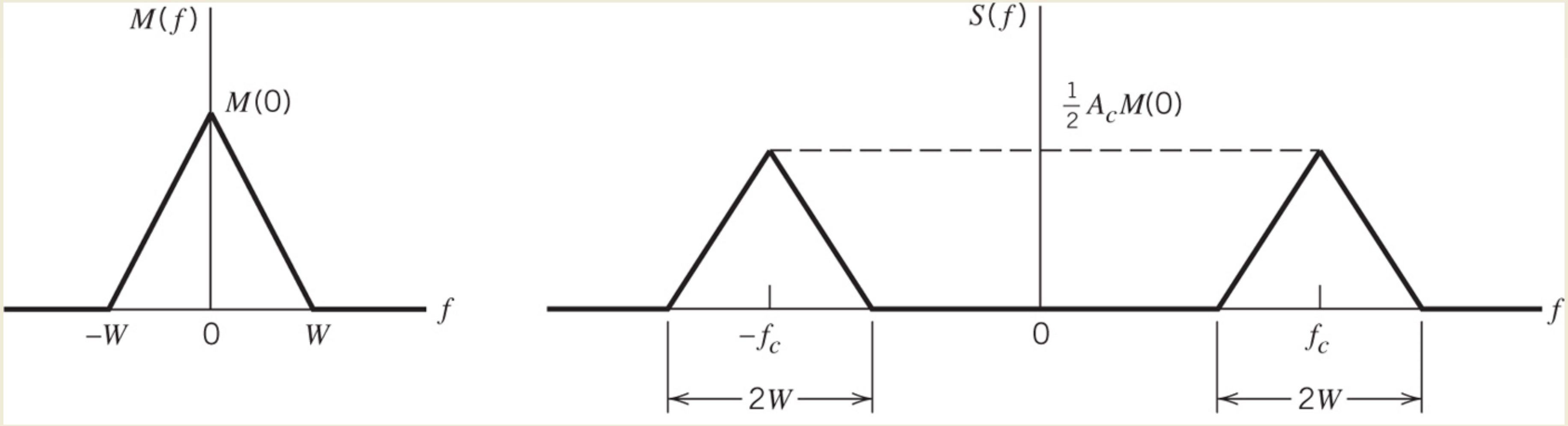
$$s_{AM}(t) = m(t) \cdot c(t) = A_c \cdot m(t) \cdot \cos(2\pi f_c t)$$

we can write:

$$S_{AM}(f) = \mathscr{F}(s_{AM}(t)) = \mathscr{F}(A_c \cdot m(t) \cdot \cos(2\pi f_c t))$$

or:

$$S_{AM}(f) = \frac{A_c}{2}(M(f - f_c) + M(f + f_c))$$



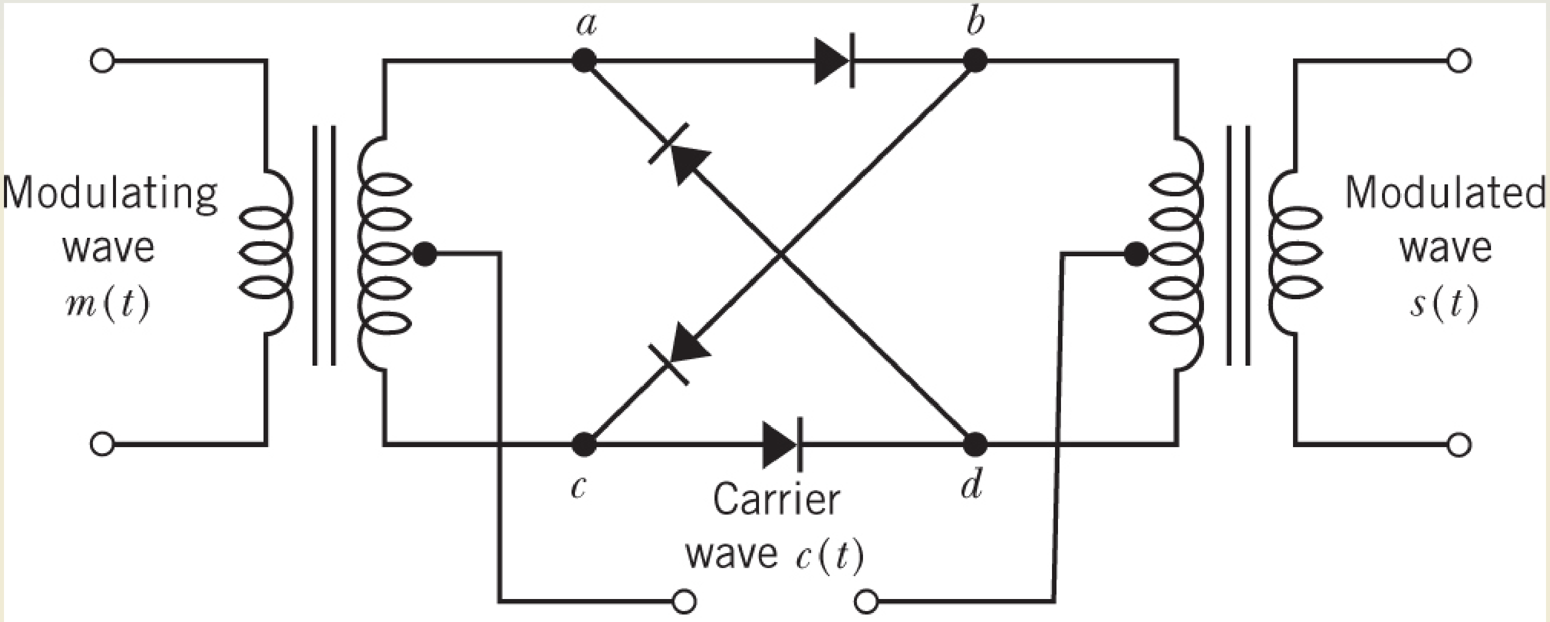
DSB–SC : Advantages and Drawbacks

- Drawbacks:
  - Require more complex implementation than the conventional AM (envelope detector is not sufficient to detect the original message)
  - High transmission bandwidth requirements: the amplitude modulated signal requires a transmission bandwidth of 2W (twice the message bandwidth

- Advantages:
  - Good power efficiency: No wasted power in transmitting the carrier

Example of Implementation of a DSB-SC – Ring Modulator

- The objective is to provide a circuit that will multiply a varying message signal $m(t)$ with a carrier signal $c(t)$
- One of the most useful product modulator or DSB-SC modulators is the ring modulator
- It includes four diodes all of them placed in the same direction to form a ring

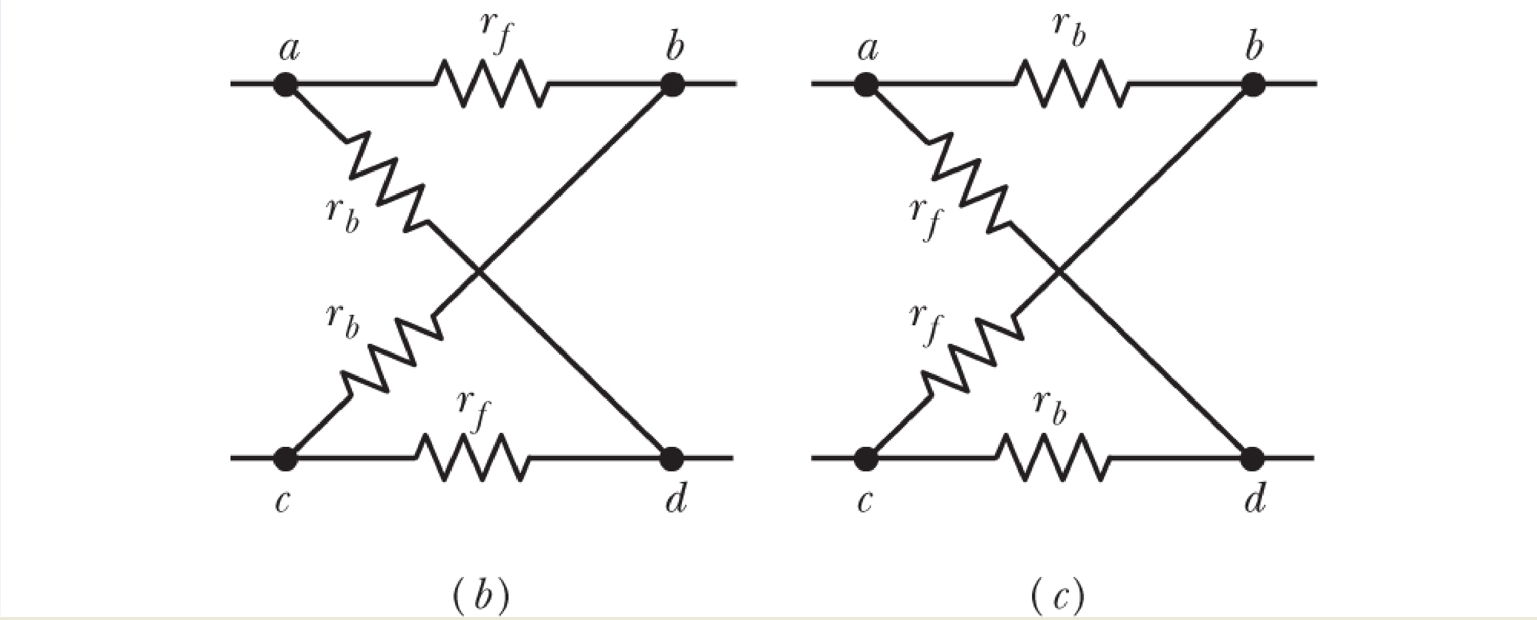


Example of Implementation of a DSB-SC – Ring Modulator

- We assume that the diodes are identical and are ideal:
  - when they are forward biased they have a resistance r_f
  - When they are reverse biased they have a very high resistance r_b
- The diodes are controlled by a square wave carrier signal $c(t)$ of frequency f_c , applied by means of 2 center tapped transformers
- The principle of operation is as follows:
  - If the carrier signal is positive, the four-diode equivalent circuit is given by figure b (see next slide) and the output is then m(t)
  - If the carrier signal is negative, the four-diode equivalent circuit is given by figure c (see next slide) and the output is then inverted, $-m(t)$

→ The circuits acts as a commutator

Example of Implementation of a DSB-SC – Ring Modulator



- The output is then given by:

$$s(t) = \begin{cases} m(t) & \text{if } c(t) > 0 \\ -m(t) & \text{if } c(t) < 0 \end{cases}$$

- Or simply:

$$s(t) = m(t) \cdot c(t)$$

Ring Modulator – Time Domain Analysis

- The modulated signal is given by:

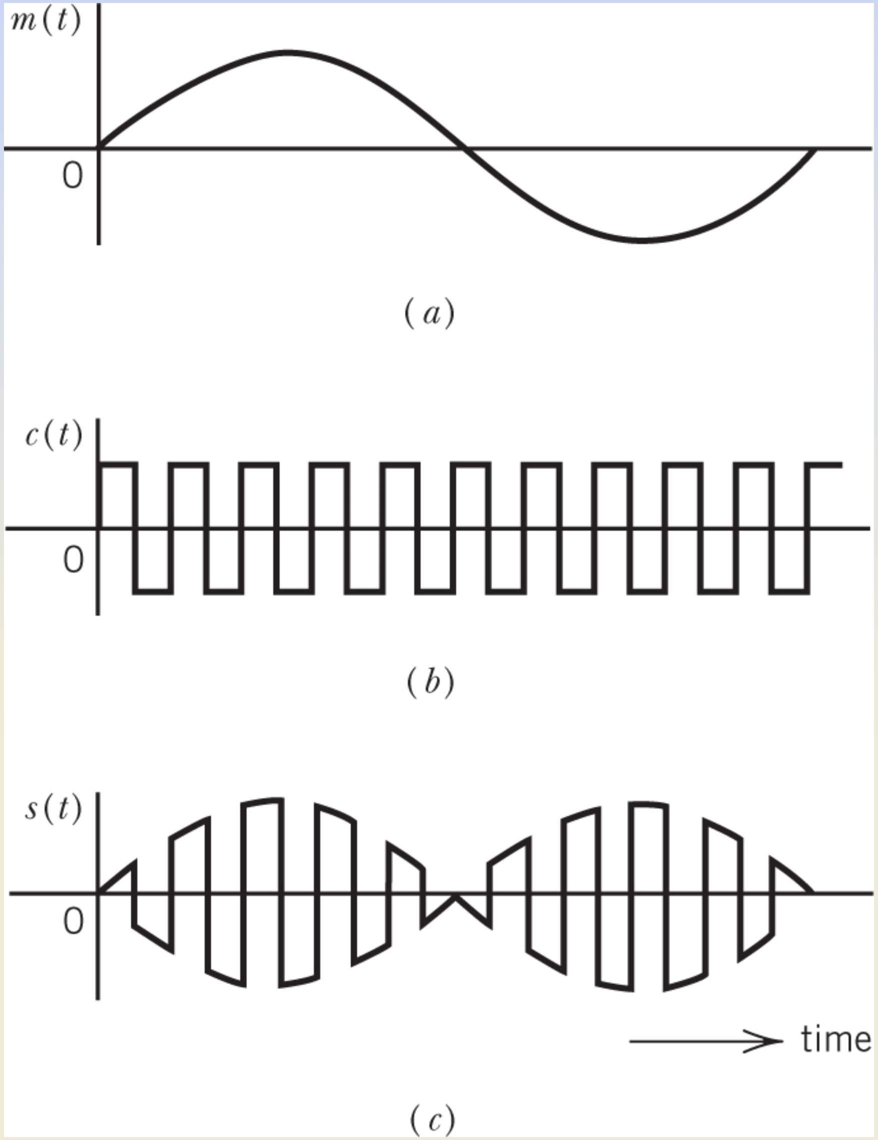
$$s(t) = m(t) \cdot c(t)$$

- Using Fourier series the square wave carrier can be expressed as:

$$c(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left(2\pi f_c t (2n-1)\right)$$

- The modulated signal can then be expressed by:

$$s(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left(2\pi f_c t (2n-1)\right) \cdot m(t)$$

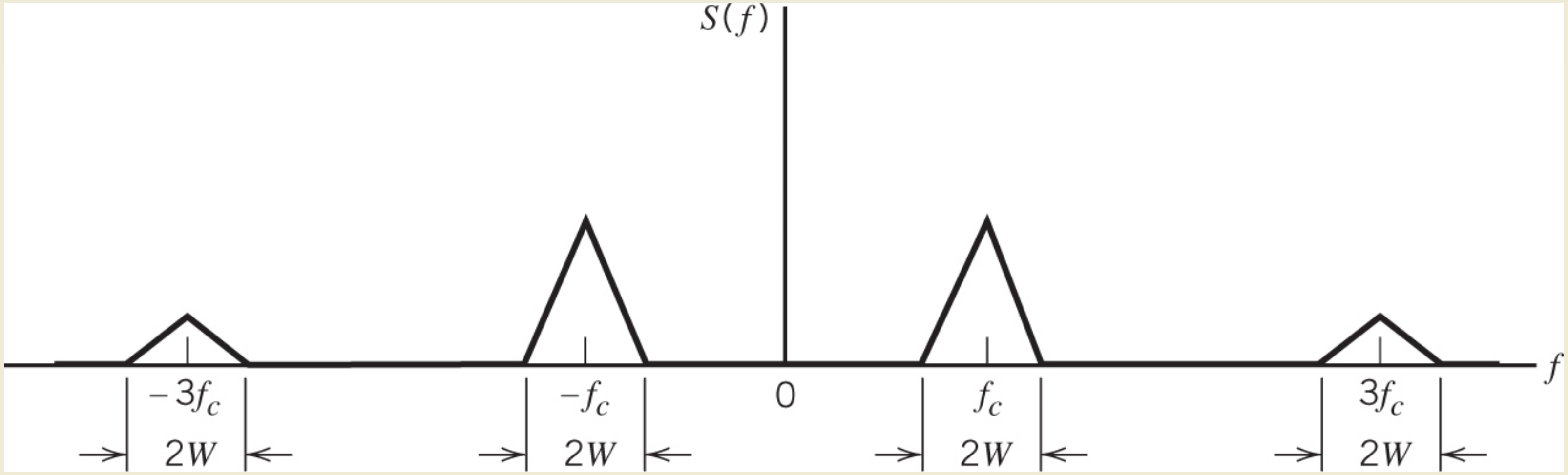


Ring Modulator – Frequency Domain Analysis

- Using the Fourier transform, the spectrum of the modulated signal can then be expressed by:

$$S(f) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\left[M\left(f-(2n-1)f_c\right)+M\left(f+(2n-1)f_c\right)\right]$$

- The modulated signal is composed of:
  - A useful signal around the carrier frequency f_c
  - Non useful harmonics at odd multiples of f_c, they can be removed by a low-pass filtering



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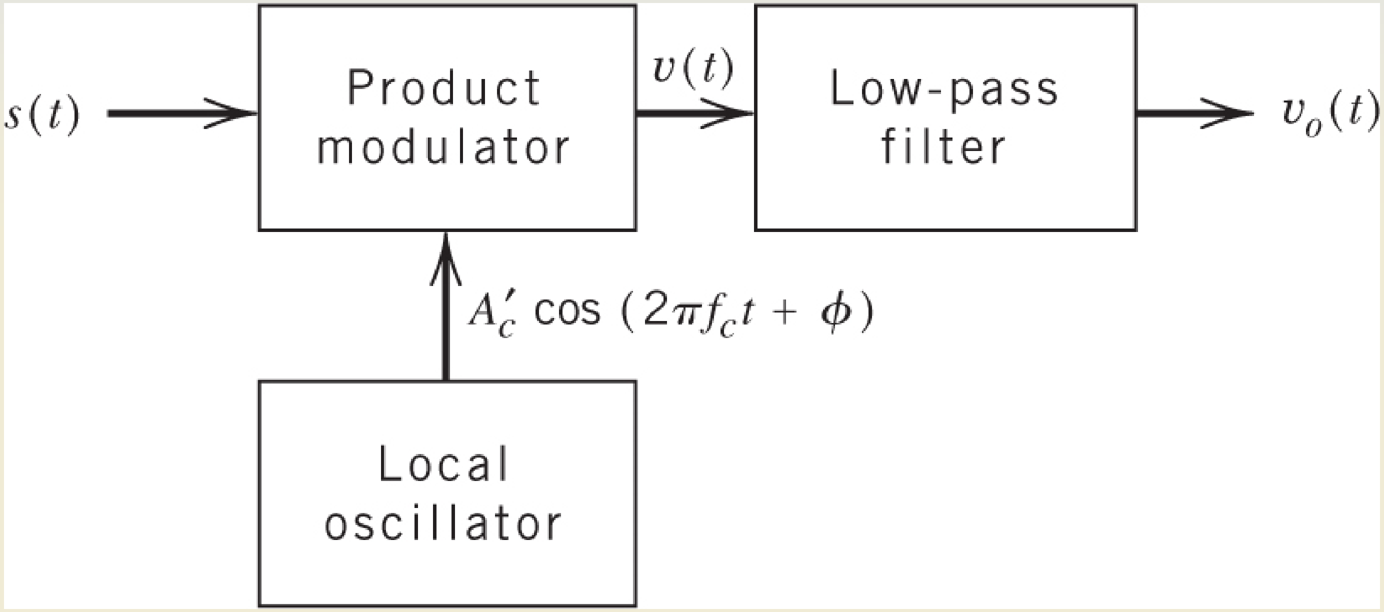
DSB-SC Demodulation – Coherent Detection

- The message signal $m(t)$ can be uniquely recovered from the DSB-SC signal $s(t)$ by:
  - Multiply $s(t)$ with a cosine wave
  - Apply a low-pass filter
- We assume that the cosine wave in the demodulator is perfectly synchronized in phase and frequency with the carrier wave



DSB-SC Demodulation – Coherent Detection

- The signal at the output of the product modulator is given by:

$$v(t) = s(t) \cdot A_c' \cos(2\pi f_c t + \phi)$$

$$v(t) = A_c \cos(2\pi f_c t) \cdot A_c' \cos(2\pi f_c t + \phi) \cdot m(t)$$

$$v(t) = \frac{A_c A_c'}{2} \cos(4\pi f_c t + \phi) \cdot m(t) + \frac{A_c A_c'}{2} \cos(\phi) \cdot m(t)$$

- After low-pass filtering, the signal becomes:

$$v_o(t) = \frac{A_c A_c'}{2} \cos(\phi) \cdot m(t)$$

- If the phase shift, ϕ , is equal to zero, the output is maximum
- If the phase shift, ϕ , is equal to 90^{o}, the output is zero

→ It is important to maintain synchronization in both the phase and amplitude of the receiver carrier compared to the transmitter carrier

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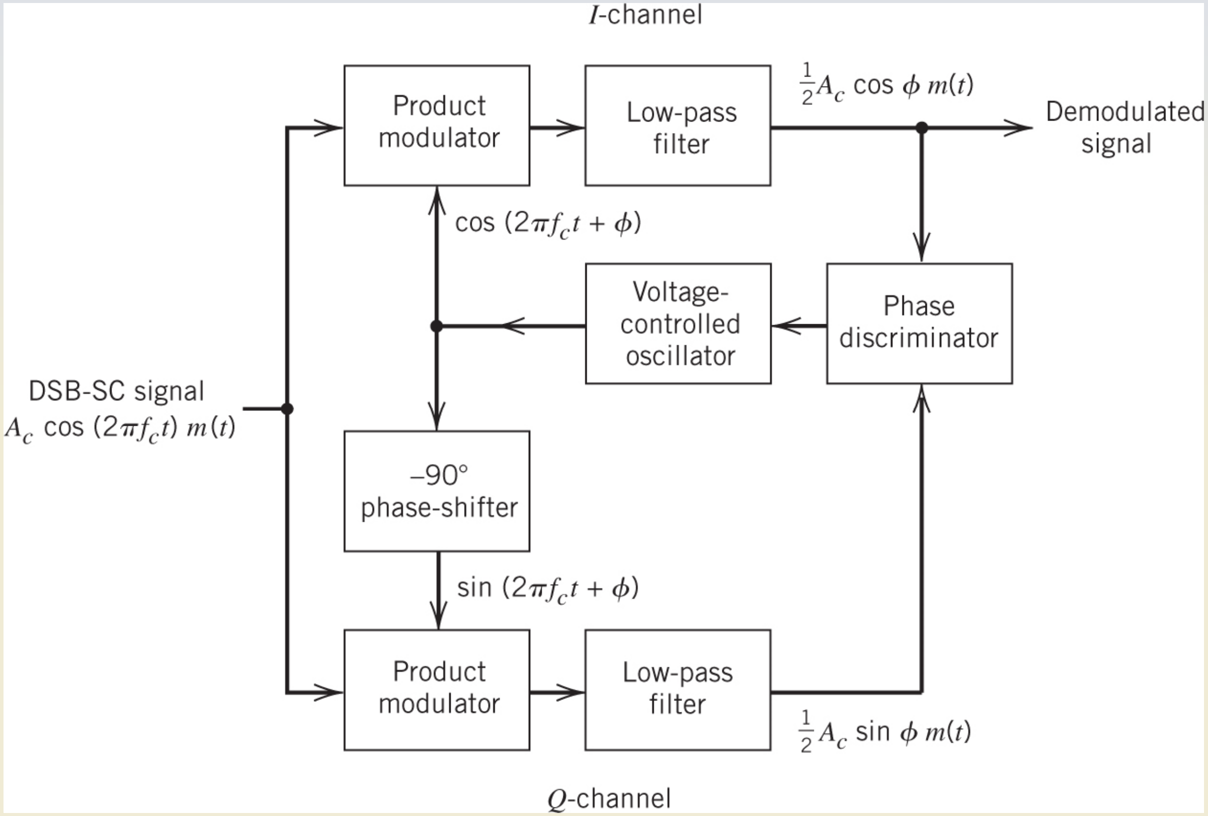
Costas Receiver

- In order to synchronize the phase of the transmitting and receiving carriers, two receiving paths are used
- The carrier is shifted by 90° between the two paths (sine and cosine)
- The two outputs are compared and used for estimating and cancelling the phase offset.



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Singe Sideband – Supressed Carrier Modulation (SSB-SC)

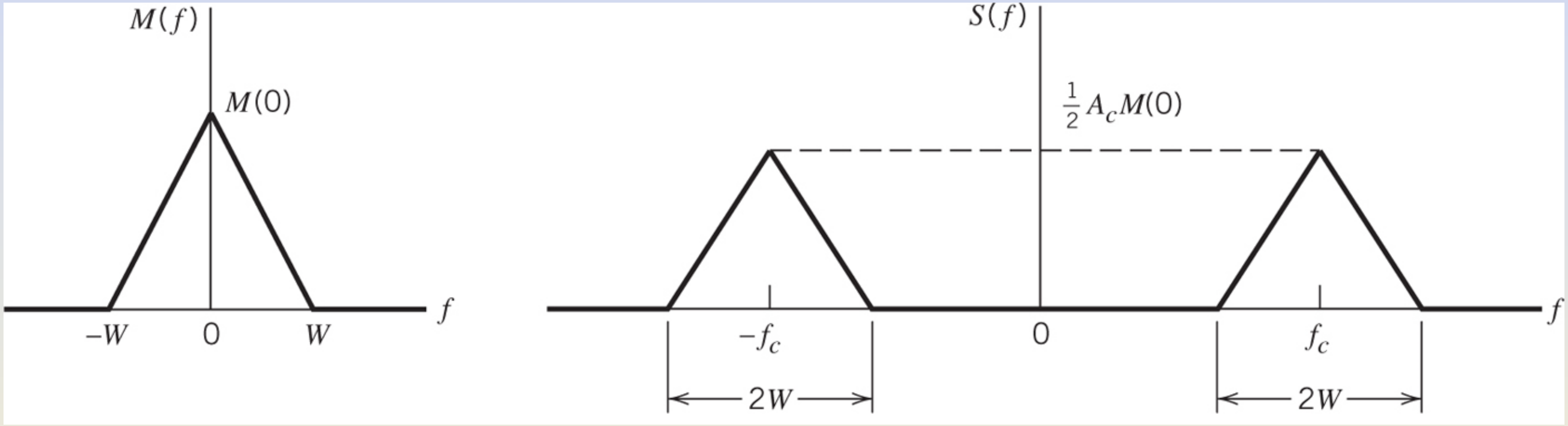
- Only one sideband (upper or lower) is required to transmit and demodulate the information (since the two sidebands carry the same information)

→ One of the two sidebands can be filtered out. This will result in a more efficiency use of the frequency spectrum (only a band of length W is used instead of 2W in the case of DSB-SC)

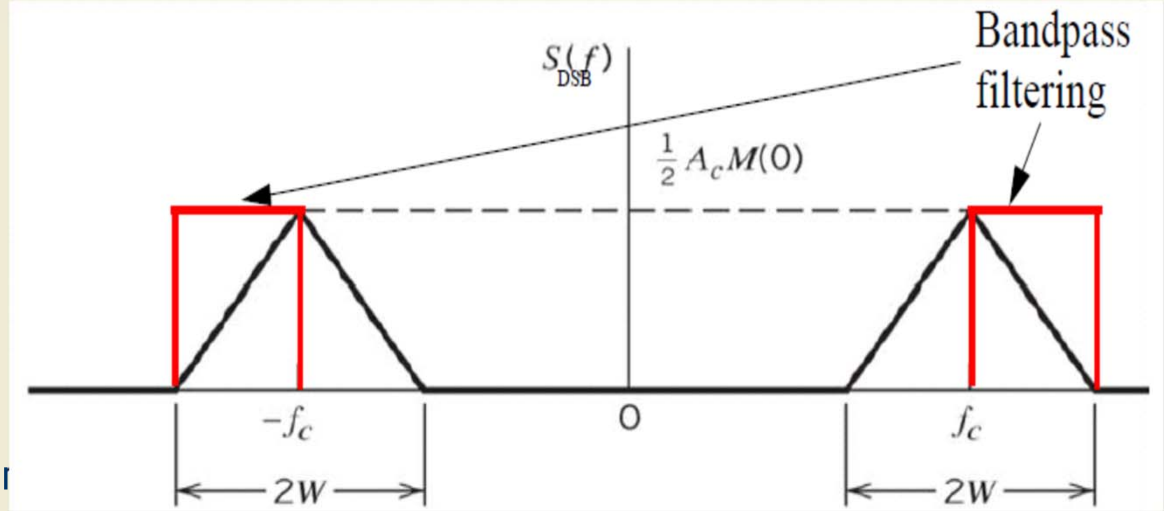
- Since the carrier is removed, and only one sideband is transmitted, this AM modulation technique has the best power efficiency
- The gain in power and spectral efficiency is obtained by trading off the design complexity, especially in the receiver

SSB–SC : Frequency Domain Analysis

- Starting from a DSB-SC spectra, we apply an ideal filter to remove one of the sidebands

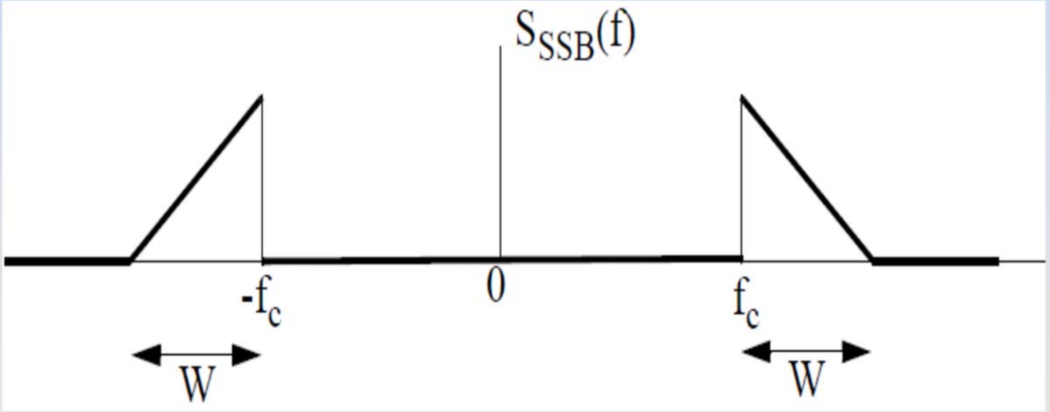


- If the upper sideband is transmitted, the transmitted spectrum will be given by:



SSB–SC : Frequency Domain Analysis

- The obtained SSB spectrum is then given by:



→ The SSB signal has a spectrum of width W

$$S_{SSB}(f) = \frac{A_c}{2} \cdot \left(M(f - f_c) + M(f + f_c) \right) \cdot H_{BP}(f)$$

where

$$H_{BP}(f) = rect\left(\frac{f - (f_c + W/2)}{W} \right) + rect\left(\frac{f + (f_c + W/2)}{W} \right)$$

SSB–SC : Advantages and Drawbacks

- Drawbacks:
  - Require more complex implementation than the conventional AM and the DSB-SC (in addition to the complex implementation of DSB-SC, an ideal filtering of one of the sidebands is required)

- Advantages:
  - Good power efficiency: No wasted power in transmitting the carrier nor in transmitting the second sideband
  - Efficient frequency band utilization requires a transmission bandwidth of 2W (half the requirement of conventional AM and DSB-SC)

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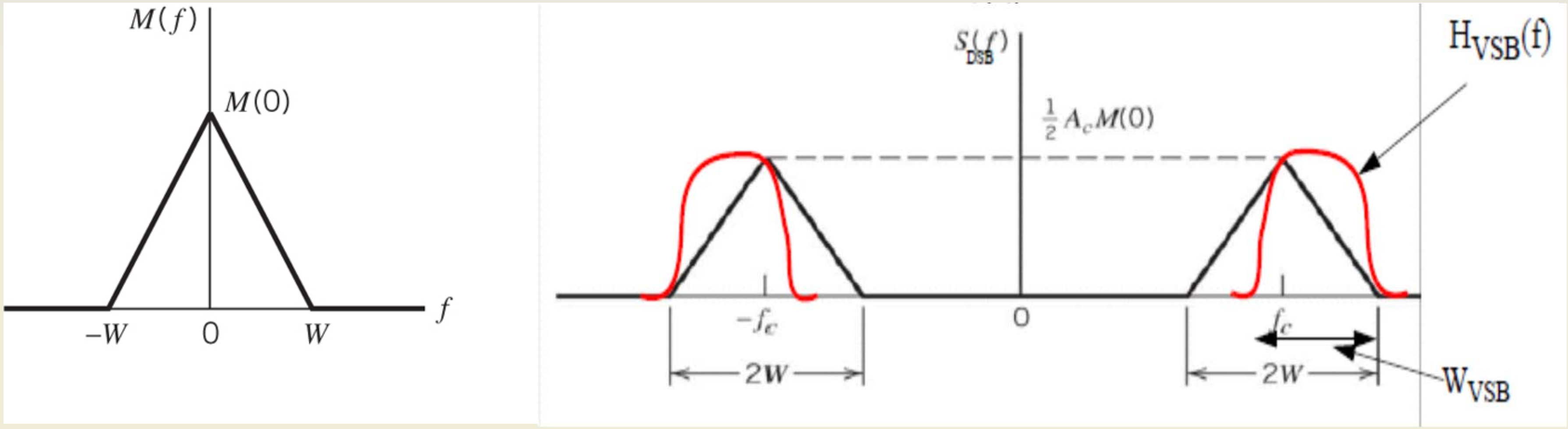
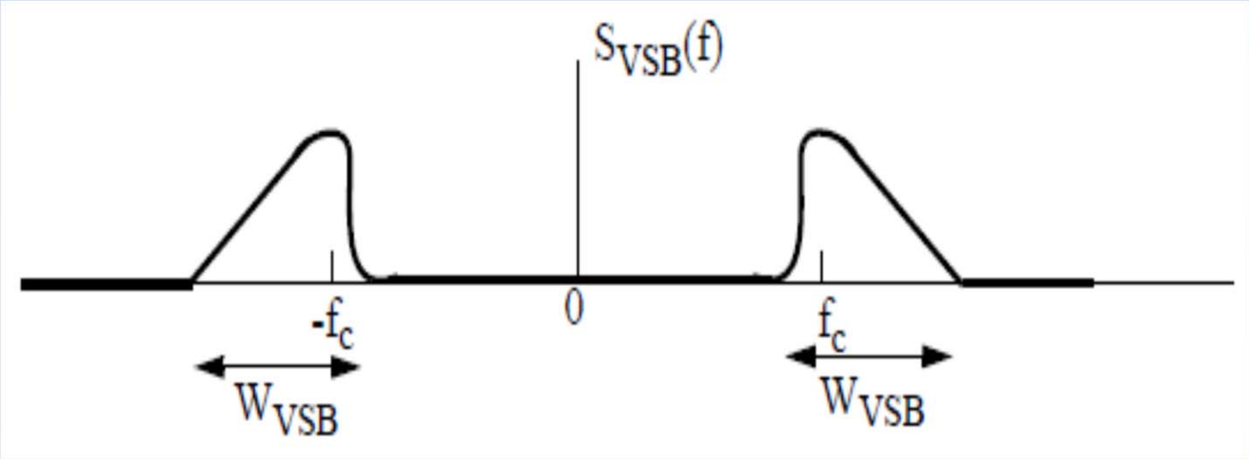
Vestigial Sideband Modulation (VSB)

- Ideal filters do not exist in practice (they are not causal)
- Filtering out one sideband completely is not possible in practice
- The vestigial sideband modulation consists of transmitting one sideband and a fraction (vestige) of the other sideband to simplify the design of sideband filters



Vestigial Sideband Modulation (VSB)



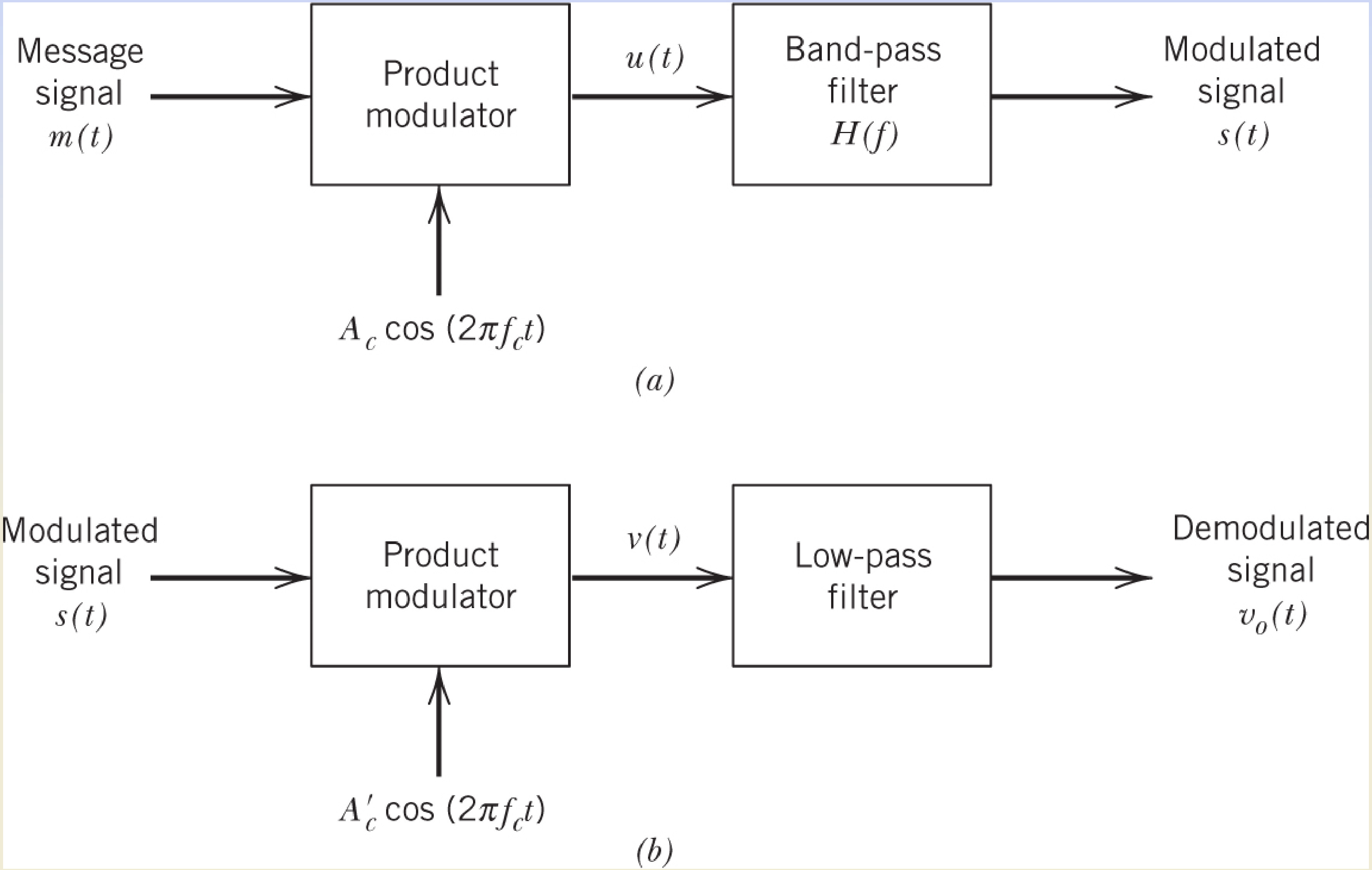
$$S_{VSB}(f) = \frac{A_c}{2} \cdot \left(M(f - f_c) + M(f + f_c) \right) \cdot H_{VSB}(f)$$

- Practical filters with a nonzero transition band can be used for sideband rejection

→ Easier design of the sideband filter but what about the signal integrity?

→ What conditions should we place on the filter to allow accurate recovery?

VSB Demodulation



VSB Demodulation

- The spectrum of the signal at the input of the demodulator is given by:

$$S(f) = \frac{A_c}{2} \cdot \left(M(f - f_c) + M(f + f_c) \right) \cdot H(f)$$

- After passing through the product modulator of the receiver, the signal $v(t)$ has the spectrum :

$$V(f) = \frac{A_c A_c'}{4} \cdot M(f) \cdot \left(H(f - f_c) + H(f + f_c) \right) +$$
$$\frac{A_c A_c'}{4} \cdot \left(M(f - 2f_c) \cdot H(f - f_c) + M(f + 2f_c) \cdot H(f + f_c) \right)$$

- After low-pass filtering, the remaining component is:

$$V_o(f) = \frac{A_c A_c'}{4} \cdot M(f) \cdot \left(H(f - f_c) + H(f + f_c) \right)$$

VSB Demodulation

- If the frequency response of the filter satisfies the following condition:

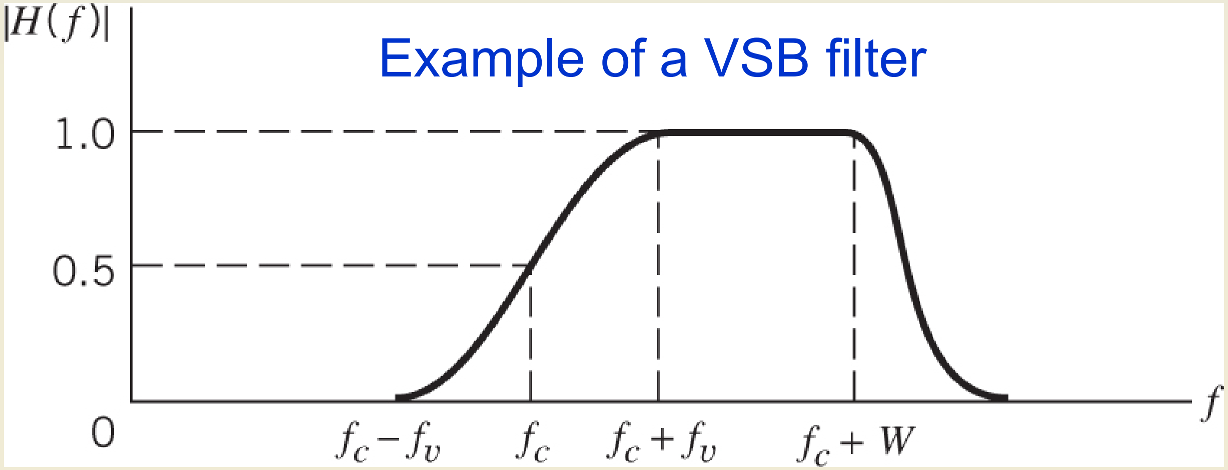
$$H(f-f_c)+H(f+f_c)=1 \qquad \text{for } -W \le f \le W$$

- The demodulated signal will have the spectrum:

$$V_o(f)=\frac{A_c A_c'}{4}\cdot M(f)$$

- The time domain representation of the demodulated signal is then given by:

$$v_o(t)=\frac{A_c A_c'}{4}\cdot m(t)$$



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Noise Definition

- Noise is undesired random time-varying wave that adds to signals and affects systems performance.

  → It is important to study its effect on the system performance and the systems effect on noise.

- It is random time-varying: can be modeled with a random process. Most often it modeled as an additive noise with Gaussian probability density function (*pdf*)

$$p_n(n) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(n-m)^2}{2\sigma^2}\right)$$

  - $m = E\left[n(t)\right]$ is the mean of the noise
  - σ is the standard deviation (σ^2 is the variance)

Noise Autocorrelation

- The autocorrelation measures the degree of dependence between two random variable (waves, signals, …):
- For the noise the expression of the autocorrelation between $n(t_1)$ and $n(t_2)$ is given by:

$$R_n\left(t_1, t_2\right) = E\left[n\left(t_1\right)n\left(t_2\right)\right]$$

- In the case of *ergodic* signals (which is the case for most signals, noises in RF), the autocorrelation is independent of time and its expression is:

$$R_n\left(\tau\right) = E\left[n\left(t\right)n\left(t+\tau\right)\right] = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} n\left(t\right)n\left(t+\tau\right)dt$$

Power Spectral Density of Noise

- The Fourier transform of the autocorrelation is the power spectral density (or frequency power spectrum) of the random variable

$$S_n(f) = \int_{-\infty}^{\infty} R_n(\tau) e^{-j2\pi f\tau} d\tau$$

$$R_n(\tau) = \int_{-\infty}^{\infty} S_n(f) e^{j2\pi f\tau} df$$

- For $\tau = 0$, we have:

$$R_n(0) = E\left[n^2\right] = \int_{-\infty}^{\infty} S_n(f) df$$

→ The total power of the noise is obtained by integrating the power spectral density over the entire frequency axis

Power Spectral Density of White Noise

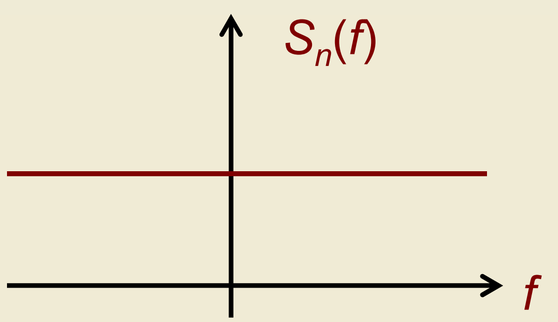
- One important particular case in communications:

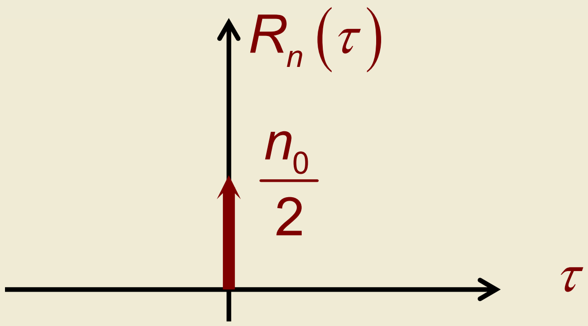
$$S_n(f) = \frac{n_0}{2}$$

→ White noise

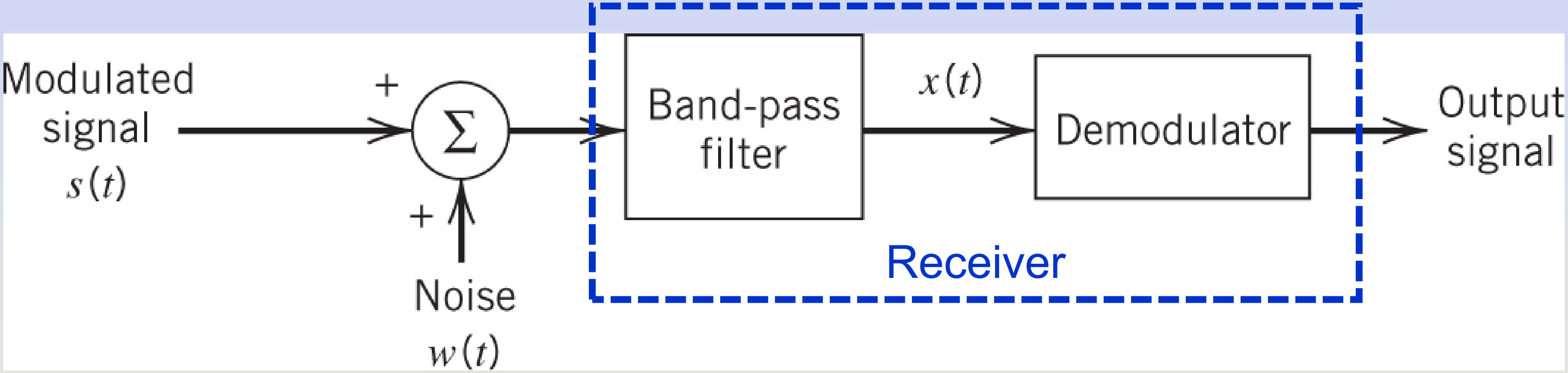
- The autocorrelation is:

$$R_n(\tau) = \frac{n_0}{2}\delta(\tau)$$





Generic Receiver Model in the Presence of Noise



- The noise process is modeled as an additive white Gaussian noise (AWGN) with zero mean and power spectral density $n_0/2$
- The band-pass filter is an ideal band-pass filter centered around tha carrier frequency f_c and with a bandwidth W equal to the bandwidth of the transmitted signal.
- AT the output of the band-pass filter, the noise process becomes a filtered noise with a limited bandwidth, W

Receiver Performance Metric: Signal-to-Noise Ratio (SNR)

- Definition of the signal-to-noise ratio: At any point in the receiver system, the SNR can be obtained by:

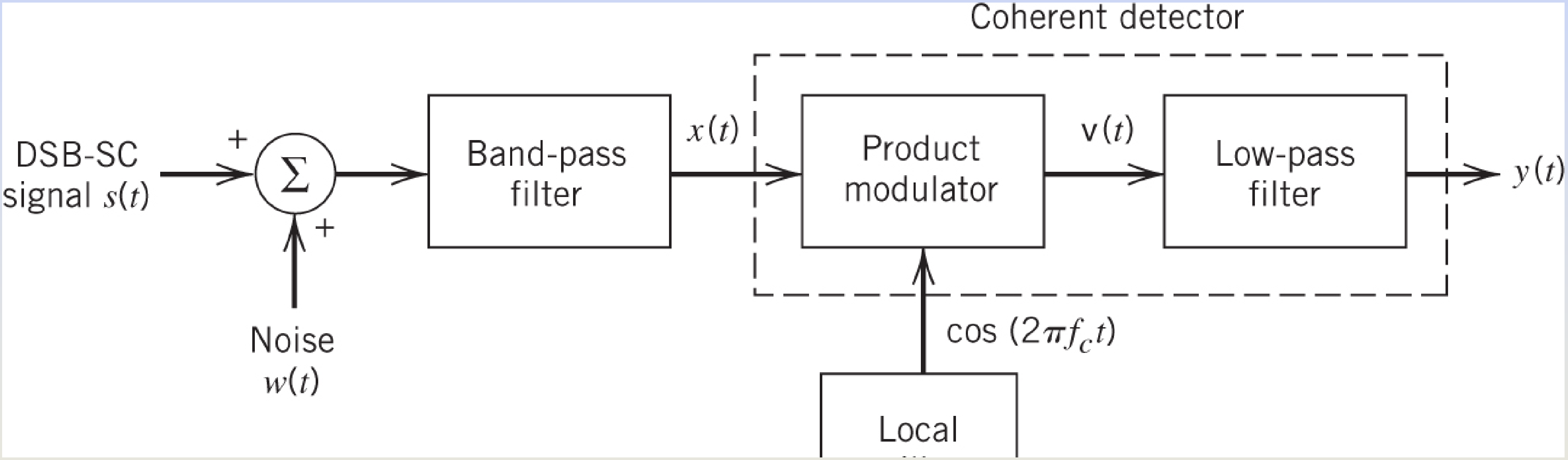
$$SNR = \frac{\text{the average power of the modulated signal}}{\text{the average power of the noise in the message bandwidth}}$$

- The contribution to noise is an important characteristic of any wireless receiver. This contribution can be quantified using the figure of merit :

$$FOM = \frac{SNR_{out}}{SNR_{in}}$$

where SNR_{in} is the SNR at the input of the receiver and SNR_{out} is the SNR at the output of the receiver.

Performance of DSB-SC with Coherent Detection in the Presence of Noise



- The DSB signal is:

$$s(t) = CA_c \cos(2\pi f_c t) m(t)$$

- If P is the average power of the message signal, the SNR at the input of the receiver is given by:

$$SNR_{in} = \frac{C^2 A_c^{\,2} P}{2Wn_0}$$

Performance of DSB-SC with Coherent Detection in the Presence of Noise

- The filtered signal is : $x(t) = s(t) + n(t)$
- At the output of the product modulator the signal is :

$$v(t) = x(t)\cos(2\pi f_c t)$$

- The SNR at the output of the DSB-SC receiver is :

$$SNR_{out} = \frac{C^2 A_c^{\,2} P}{2Wn_0}$$

- The figure of merit of the DSB-SC receiver is then:

$$FOM = \frac{SNR_{out}}{SNR_{in}} = 1$$

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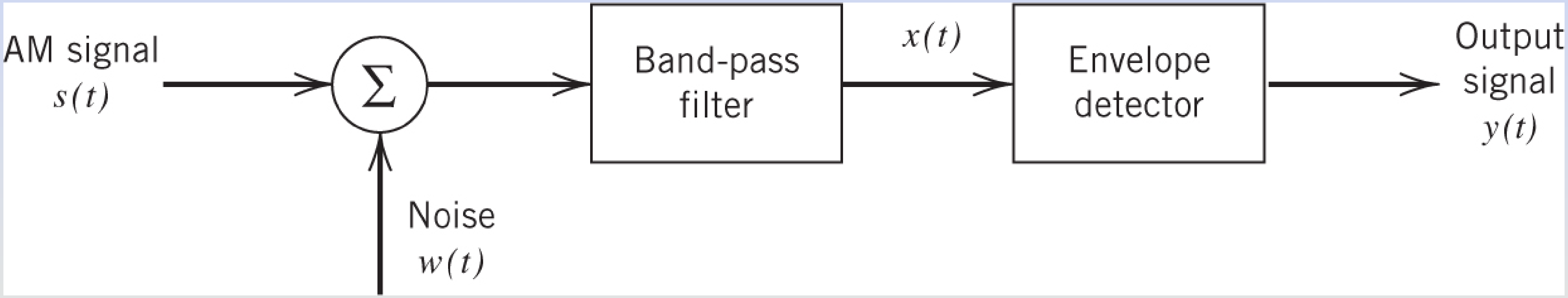
Performance of conventional AM with Envelope Detection in the Presence of Noise



- The AM signal is:

$$s(t) = A_c\left(1 + k_a m(t)\right)\cos\left(2\pi f_c t\right)$$

- If P is the average power of the message signal, the SNR at the input of the receiver is given by:

$$SNR_{in} = \frac{A_c^{\,2}\left(1 + k_a^{\,2} P\right)}{2Wn_0}$$

Performance of conventional AM with Envelope Detection in the Presence of Noise

- The filtered signal is : $x(t) = s(t) + n(t)$

- At the output of the envelope detector the signal is :

$$y(t) = \text{envelope of } \{x(t)\}$$

- The SNR at the output of the DSB-SC receiver can be approximated by :

$$SNR_{out} \approx \frac{A_c^{\ 2} k_a^{\ 2} P}{2Wn_0}$$

- The figure of merit of the DSB-SC receiver is then:

$$FOM = \frac{SNR_{out}}{SNR_{in}} \approx \frac{k_a^{\ 2} P}{1 + k_a^{\ 2} P}$$