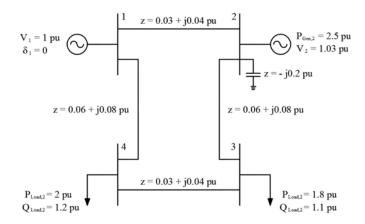
1) Consider the system below:



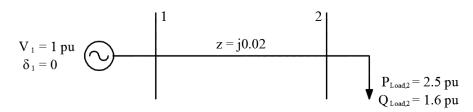
a. Fill out the following table for this system. The answer(s) should be selected from terms in the brackets. [3 marks]

Bus Number	Bus Type (Load, Gen, Slack)	Known Variable (Pgen, Qgen, V, δ, Pload, Qload)	Unknown Variable (Pgen, Qgen, V, δ, Pload, Qload)

- **b.** Create the X vector for this system (in the form of $X = [----]^T$). [4 marks]
- **c.** Create the admittance matrix Y_{bus} for system. Also, provide the B_{bus} and G_{bus} matrices. [4 marks]

d. If all the branches in the system were <u>lossless</u>, how much <u>real power</u> is injected by the slack bus? [1 mark]

2) Consider the two bus system below:



- **a.** Create the Ybus for this system. [1 mark]
- **b.** Create the X vector for this system. [1 mark]
- **c.** Write the real and reactive power flow equations for bus 2. Plug in all the known values and simplify as much as possible. [3 marks]

d. If the optimal value of δ_2 = -0.05211 radians, what is the optimal voltage at bus 2? [3 marks]

Constants/Conventions

$8.85 \text{x} 10^{-12}$ r $0.7788 \cdot r$ $D_{SL} \begin{vmatrix} \sqrt{D_S \cdot d} \\ \sqrt[3]{D_S \cdot d^2} \\ 1.091 \sqrt[4]{D_S \cdot d^3} \end{vmatrix}$ $D_{SC} \begin{vmatrix} \sqrt{r \cdot d} \\ \sqrt[3]{r \cdot d^2} \\ 1.001\sqrt[4]{r \cdot d^3} \end{vmatrix}$

General

$$\begin{array}{lll} \textbf{Single Phase $\overline{\bf S}$:} & \overline{S} = \overline{V} \cdot \overline{I}^* \\ \\ \textbf{Q for L and C:} & Q_L = \frac{V^2}{X_L} & Q_C = \frac{V^2}{X_C} \\ \\ \textbf{Y Connection:} & \overline{V_{ll}} = \sqrt{3} \angle 30^\circ \cdot \overline{V_\phi} \\ \\ \textbf{\Delta Connection:} & \overline{I_l} = \sqrt{3} \angle -30^\circ \cdot \overline{I_\phi} \\ \\ \textbf{3 Phase Power:} & \overline{S_{3\phi}} = 3 \cdot \overline{V_\phi} \cdot \overline{I_\phi^*} \\ & S = \sqrt{3} \cdot V_{ll} \cdot I_l \\ & P = S \cdot pf & S^2 = P^2 + Q^2 \end{array}$$

Per Unit

$$S_{\text{base},1\phi} = P_{\text{base},1\phi} = Q_{\text{base},1\phi}$$

$$I_{\text{base}} = \frac{S_{\text{base},1\phi}}{V_{\text{base},\text{L-N}}}$$

$$Z_{\text{base}} = R_{\text{base}} = X_{\text{base}}$$

$$Z_{\text{base}} = \frac{V_{\text{base},\text{L-N}}}{I_{\text{base}}} = \frac{V_{\text{base},\text{L-N}}^2}{S_{\text{base},1\phi}}$$

$$Three \ \text{Phase:} \qquad S_{\text{base},3\phi} = 3 \cdot S_{\text{base},1\phi}$$

$$V_{\text{base},\text{L-L}} = \sqrt{3}V_{\text{base},\text{L-N}}$$

$$I_{\text{base}} = \frac{S_{\text{base},3\phi}}{\sqrt{3}V_{\text{base},\text{L-L}}}$$

$$Z_{\text{base}} = \frac{V_{\text{base},\text{L-L}}^2}{S_{\text{base},3\phi}}$$

$$Change \ \text{of Base:} \qquad Z_{\text{pu,new}} = Z_{\text{pu,old}} \left(\frac{V_{\text{base,old}}}{V_{\text{base,new}}}\right)^2 \frac{S_{\text{base,new}}}{S_{\text{base,old}}}$$

Transmission Lines

Line Inductance:
$$L=2\mathrm{x}10^{-7}\cdot\ln\frac{D}{D_s}$$
 or
$$L=2\mathrm{x}10^{-7}\cdot\ln\frac{D_{eq}}{D_S}$$
 or
$$L=2\mathrm{x}10^{-7}\cdot\ln\frac{D_{eq}}{D_{SL}}$$

Line Capacitance:

or
$$C_{an} = \frac{2\pi\varepsilon}{\ln\frac{D}{r}}$$
 or
$$C_{an} = \frac{2\pi\varepsilon}{\ln\frac{Deq}{r}}$$
 or
$$C_{an} = \frac{2\pi\varepsilon}{\ln\frac{Deq}{D_{SC}}}$$

Line Equations: $Z_c = \sqrt{\frac{z}{y}}$

$$I(x) = I_R \cosh(\gamma x) + \frac{V_R}{Z_c} \cdot \sinh(\gamma x)$$
$$V(x) = V_R \cosh(\gamma x) + I_R Z_c \sinh(\gamma x)$$

Nominal
$$\pi$$
 Model:
$$A = D = 1 + \frac{YZ}{2}$$

$$B = Z$$

$$C = Y(1 + \frac{YZ}{4})$$
Eq π Model:
$$Z' = Z \frac{\sinh{(\gamma l)}}{(\gamma l)}$$

$$\frac{Y'}{2} = \frac{Y}{2} \frac{\tanh{\frac{(\gamma l)}{2}}}{\frac{(\gamma l)}{(\gamma l)}}$$

For:
$$x = a + jb$$
,
 $\cosh(x) = \cosh(a)\cos(b) + j\sinh(a)\sin(b)$
 $\sinh(x) = \sinh(a)\cos(b) + j\cosh(a)\sin(b)$

Power Flow

$$f_i = P_{gen,i} - P_{load,i} - \sum_{k=1}^{N} V_i V_k G[i,k] \cos(\delta_i - \delta_k)$$

$$- \sum_{k=1}^{N} V_i V_k B[i,k] \sin(\delta_i - \delta_k)$$

$$f_{N+i} = Q_{gen,i} - Q_{load,i} - \sum_{k=1}^{N} V_i V_k G[i,k] \sin(\delta_i - \delta_k)$$

$$f_{N+i} = Q_{gen,i} - Q_{load,i} - \sum_{k=1}^{N} V_i V_k G[i,k] \sin(\delta_i - \delta_k) + \sum_{k=1}^{N} V_i V_k B[i,k] \cos(\delta_i - \delta_k)$$