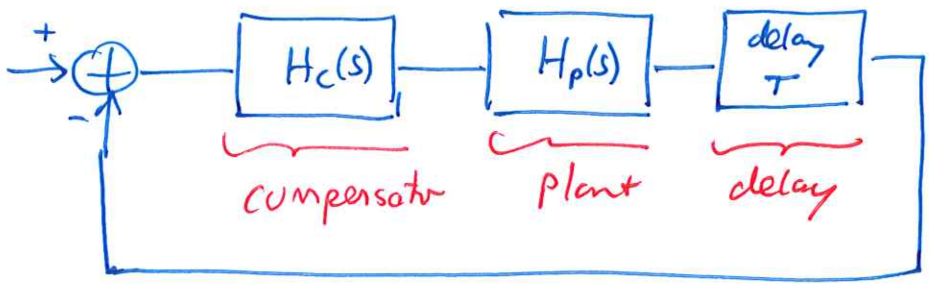
Lecture - Feedback loop compensator Design based on Bode Plots

Where we left off last week



$H_c(s) \rightarrow$ rational polynomial of s

$H_p(s) \rightarrow$ " " " "

Delay T $\quad e^{-sT}$ not a polynomial of s

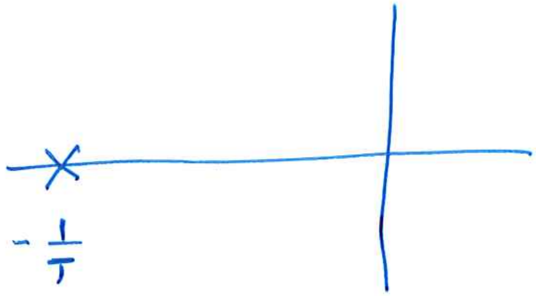
Approximation $\quad e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

for $|x| << 1$

$$e^{x} \approx 1 + x$$

write $\quad e^{-sT} = \frac{1}{e^{sT}} \approx \frac{1}{1+sT} = \frac{1/T}{\frac{1}{T} + s}$

Pole at $1/T$



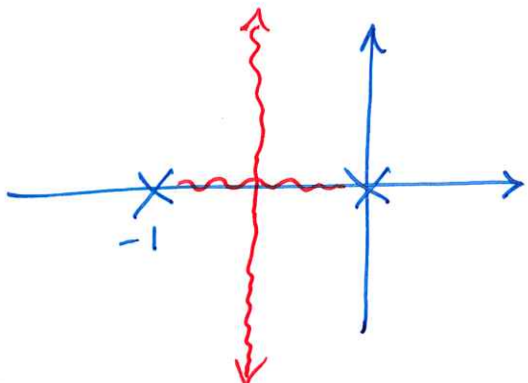
Approximate polynomial model of the delay.

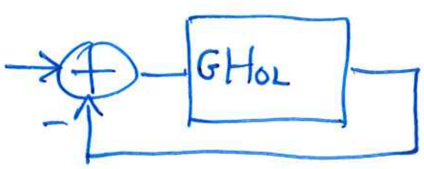
Results in a single pole.

Effect of delay pole

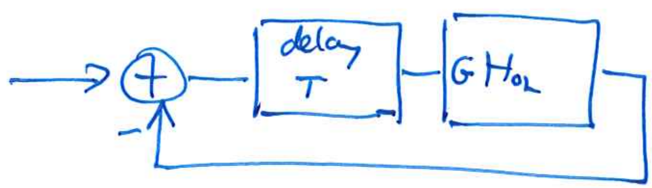
example $\quad H_{OL}(s) = \dfrac{1}{s(s+1)}$

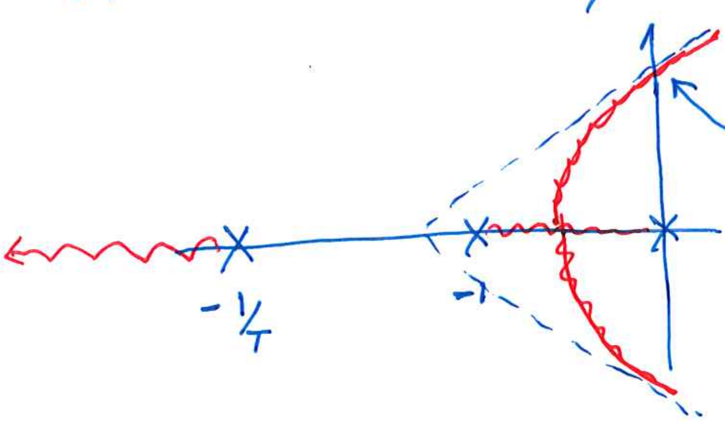
Root locus of closed loop poles





Now add delay

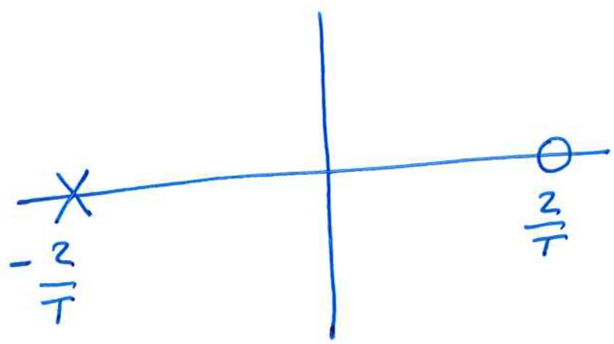




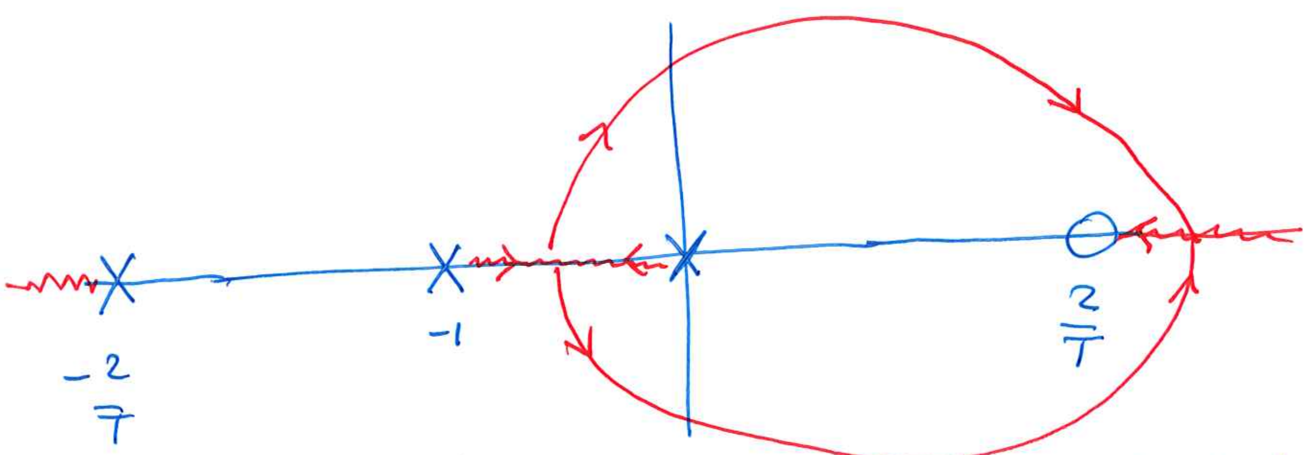
for certain value of G root locus of closed loop poles crosses into RHP and loop becomes unstable

Better approximation of e^{-sT}

$$e^{-sT} = \frac{e^{-sT/2}}{e^{sT/2}} \approx \frac{1 - sT/2}{1 + sT/2} = \frac{\frac{2}{T} - s}{\frac{2}{T} + s}$$



Consider some H_{OL} of $H_{OL}(s) = \frac{1}{s(s+1)}$



When a zero is encountered in the right hand plane then a special case for root locus is applied.

Not part of regular rule set.

Rule of looking to right on real axis does not apply here directly.

Root locus has some peculiarities when dealing with delays.

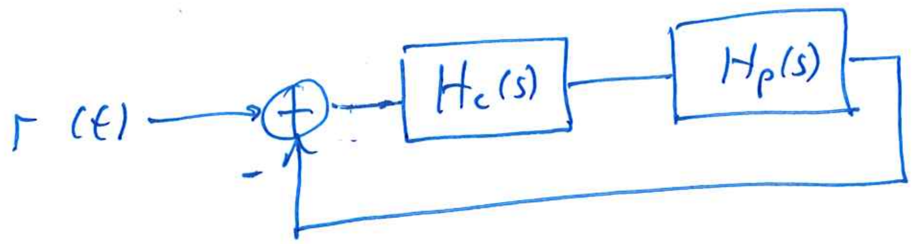
Next what if we do not have a polynomial model of $H_p(s)$?

Instead suppose we have measured data (example of arbitrary data made up)

ω	$\lvert H_p(\omega) \rvert$	$\angle H_p(\omega)$
1	1	-.01
10	0.99	-.05
50	0.9	-.1
60	0.8	-.2
70	0.9	-.35
⋮	⋮	⋮

We can use a <u>least squares curve fit</u> to get a polynomial representation of $H_p(s)$ from measurements of $H_p(\omega)$ but how many poles? zeros? to assume. Modelling is only approximate. There are issues with under fitting/over fitting data with a model.

Now we want to design a feed back loop with compensator for $H_p(\omega)$ such that $e(\infty) = 0$ for $r(t) = u(t)$.



How to do this directly? Cannot use root locus.

For example let $H_c(s) = \frac{G}{s}$

We can determine the point where the loop is marginally stable.

$$\text{if} \quad |H_c(s)|\,|H_p(s)| = 1$$

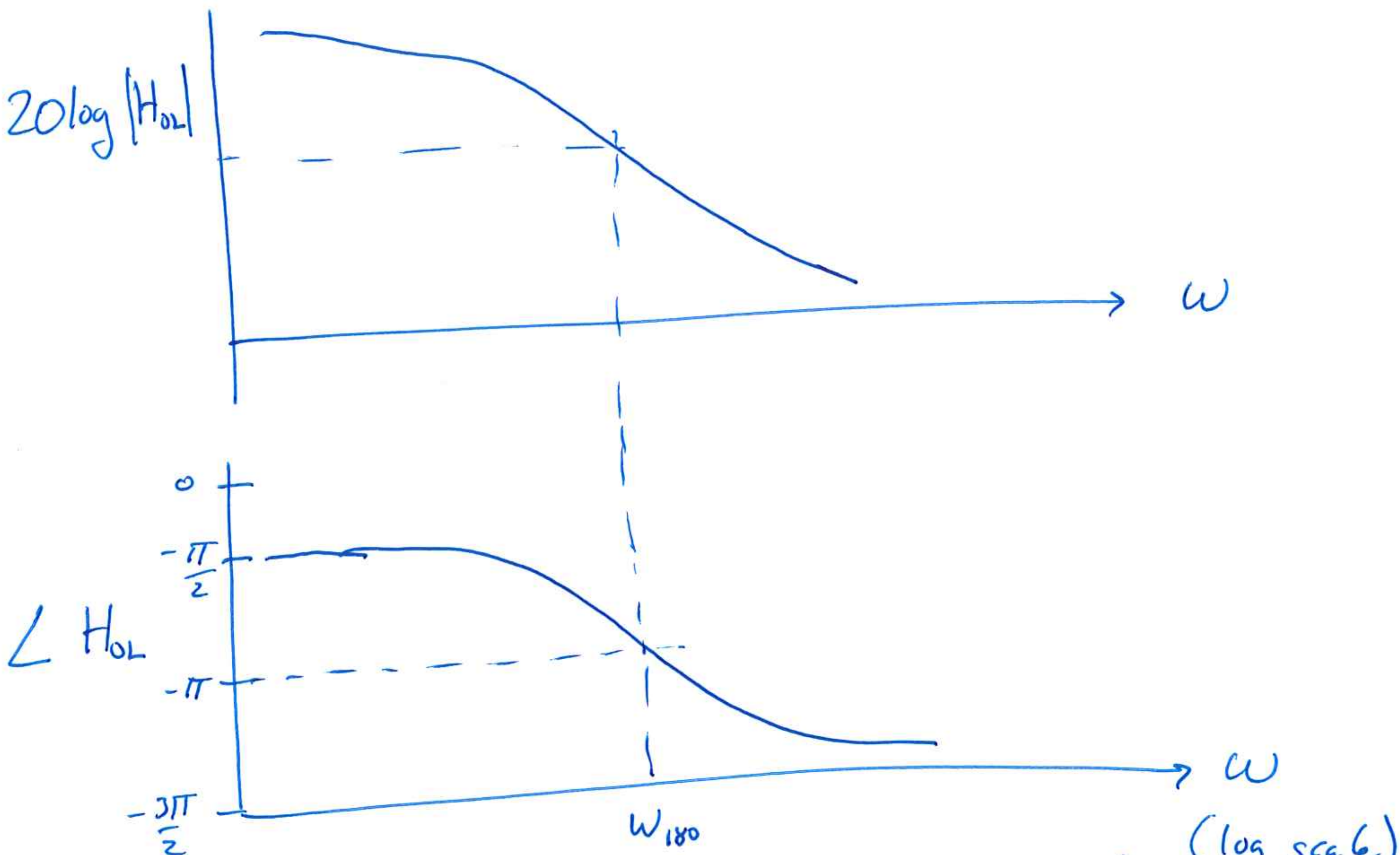
$$\text{and} \quad \angle H_c(s) + \angle H_p(s) = -180^\circ$$

then the gain around the loop is 1 and the phase shift is -360°.

At this point the loop will <u>self-oscillate</u>

That is it is marginally stable.

Hence we plot a bode plot of the open loop transfer function $H_{OL}(j\omega) = \frac{G}{j\omega} H_p(j\omega)$



Look for the frequency where $\angle H_{OL} = -180^0$

or $\angle H_{OL} = -\pi$

Call this ω_{180}

Find the corresponding value of $|H_{OL}(j\omega_{180})|$

If this is less than 1 then loop is stable

If $|H_{OL}| > 1$ then loop is unstable.

Gain margin is $-20 \log |H_{OL}(j\omega_{180})|$

Gain margin is how much more G can be increased in dB before the closed loop becomes marginally stable

If Gain margin is negative then G has to be reduced to make it stable.

We can also consider <u>Phase Margin</u>

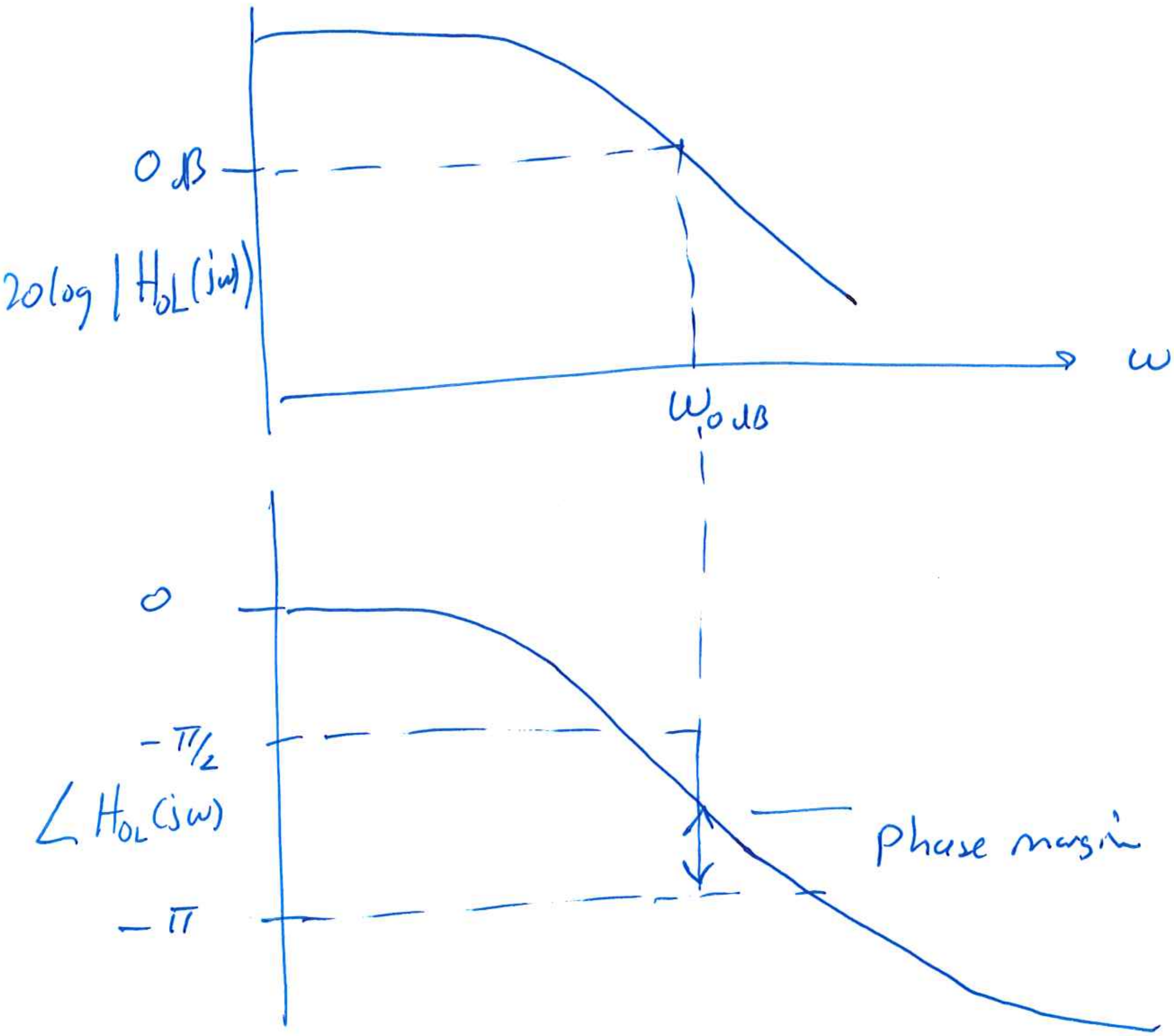
Here we find the ω where $|H_{OL}(j\omega)| = 1$

let this be ω_{0dB}

Then determine the corresponding phase.

Phase Margin is $\angle H_{OL} + \pi$

Hence if $\angle H_{OL}(\omega_{0dB}) = \pi$ then the closed loop is marginally stable. This is illustrated in the plot as follows.



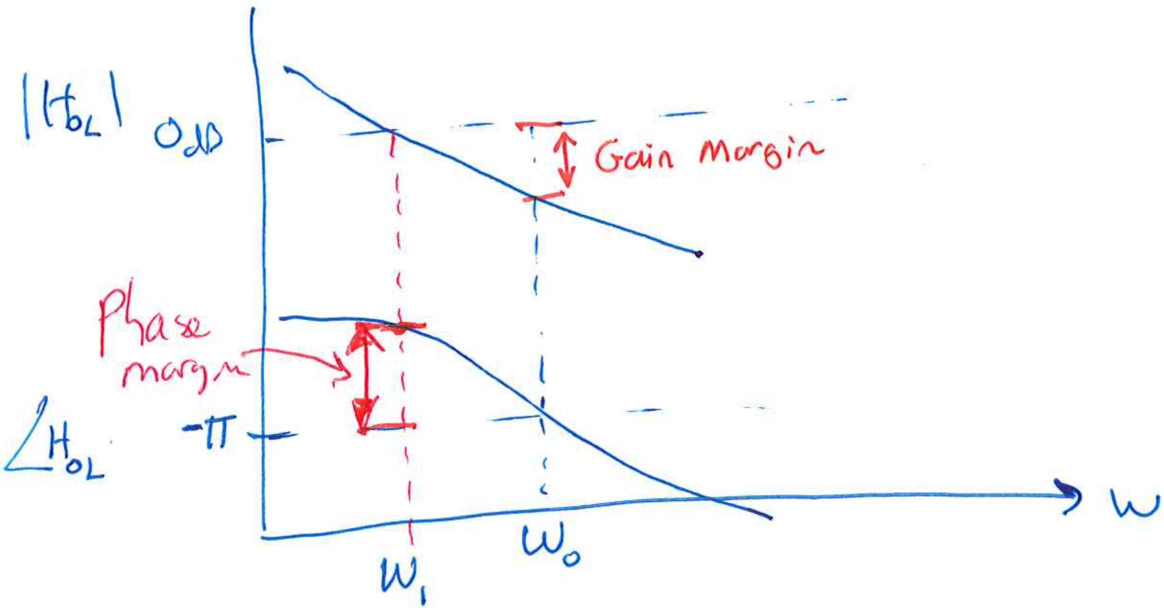
If the phase margin is positive
ie $\angle H_{OL}(i\omega) > -\pi$

Then the closed loop is stable

If phase margin is negative
ie $\angle H_{OL}(j\omega) < -\pi$

Then closed loop is unstable.

Gain Margin & Phase Margin



Gain Margin - how much the loop gain can be increased at the frequency ω_0 where $\angle H_{OL} = -\pi$ before $|H_{OL}(\omega_0)|$ gets to be 0 dB

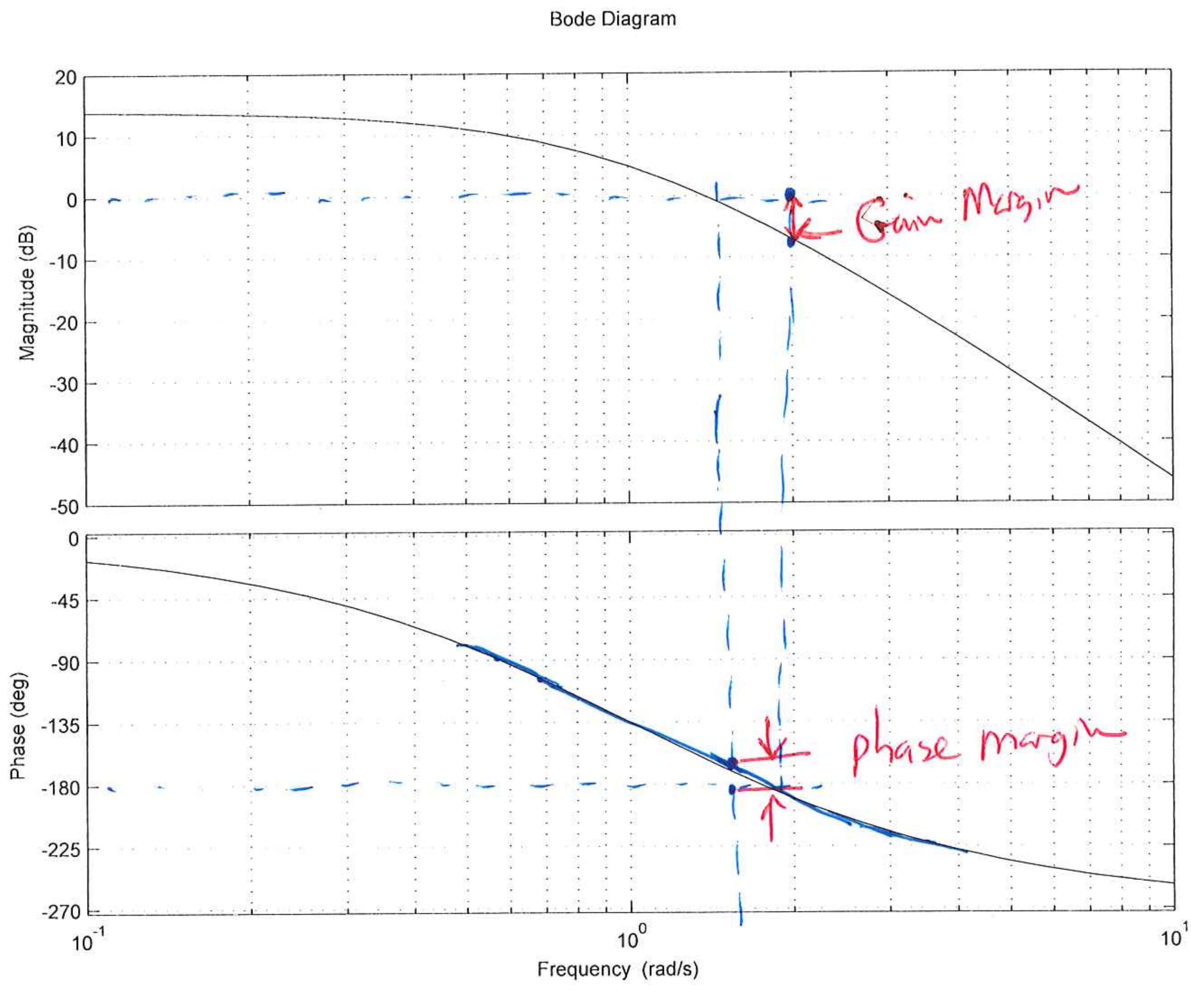
Phase Margin When $|H_{OL}(\omega_1)| \to 0$ dB

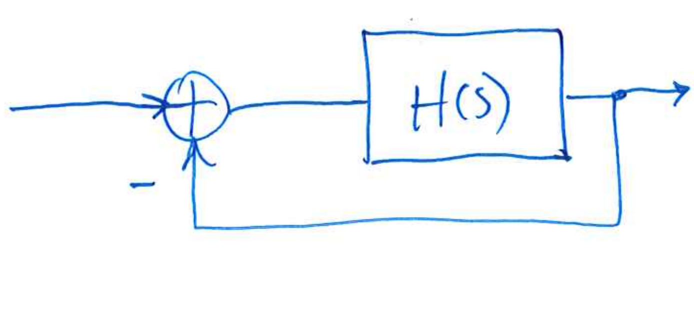
Phase margin is $\angle H_{OL} + \pi$.

Example $H_{OL}(s) = \dfrac{5}{(s+1)^3}$

Example $H(s) = \dfrac{5}{(s+1)^3}$

```
H = zpk([], [-1;-1;-1], 5)
bode(H)
```

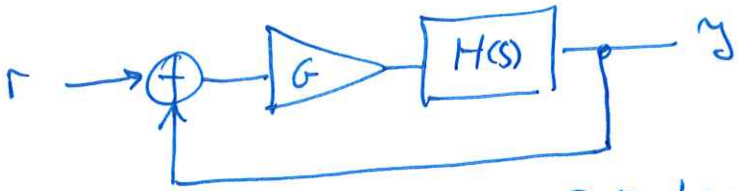




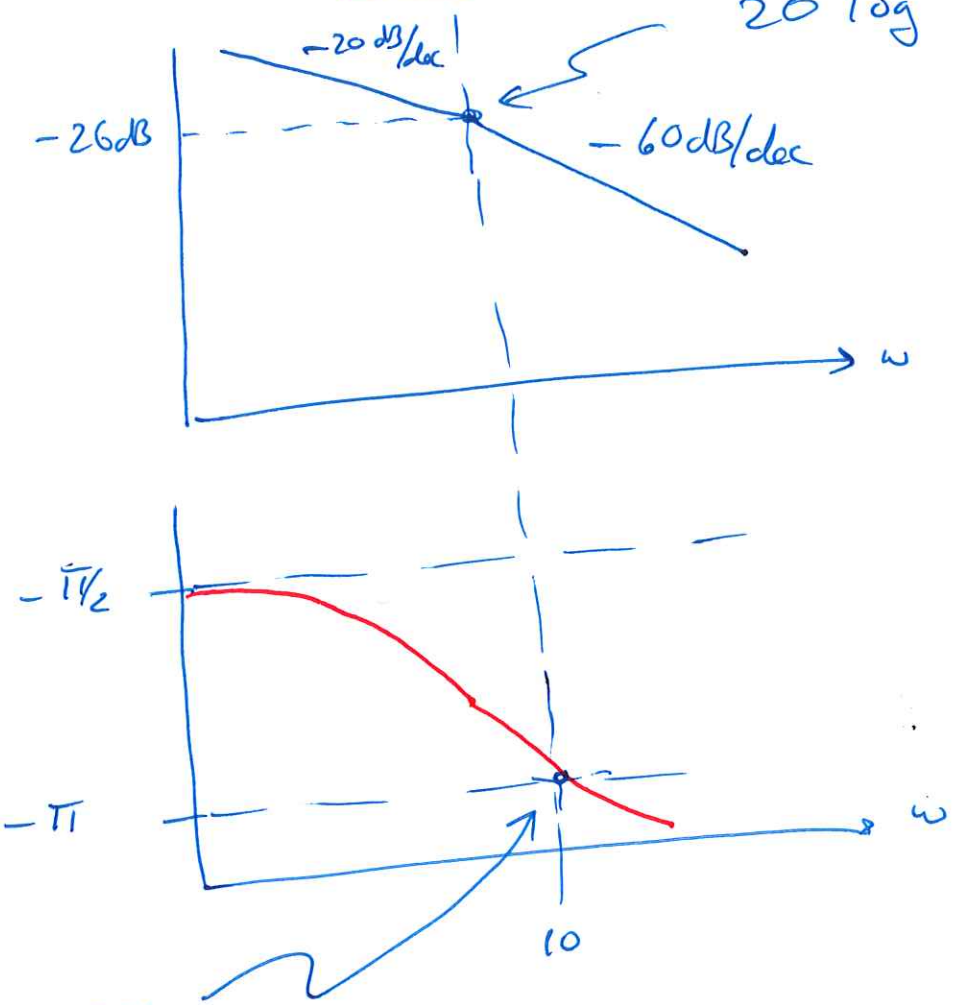
Example

Find the maximum loop gain that results in a stable response

$$H(s) = \frac{100}{(s+10)^2 s}$$



$$20 \log \left(\frac{100}{200 \cdot 10} \right) = -26\,dB$$



180° @ $\omega = 10$

G can be up to 26 dB before instability

Reading Assignment

Chapter 10 Nise

Section 10.2 review of bode plots

10.7 gain and phase margin

Section 10.12 shows the inclusion of delay in bode plots.

Bode plots lag & lead compensation

Sections 11.3 and 11.4

lead / lag circuit $H_c(s) = \frac{s+a}{s+b}$

lead $a < b$

lag $a > b$

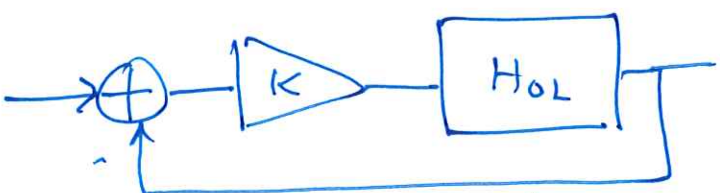
use lead if phase margin is small

use lag if phas margin is large

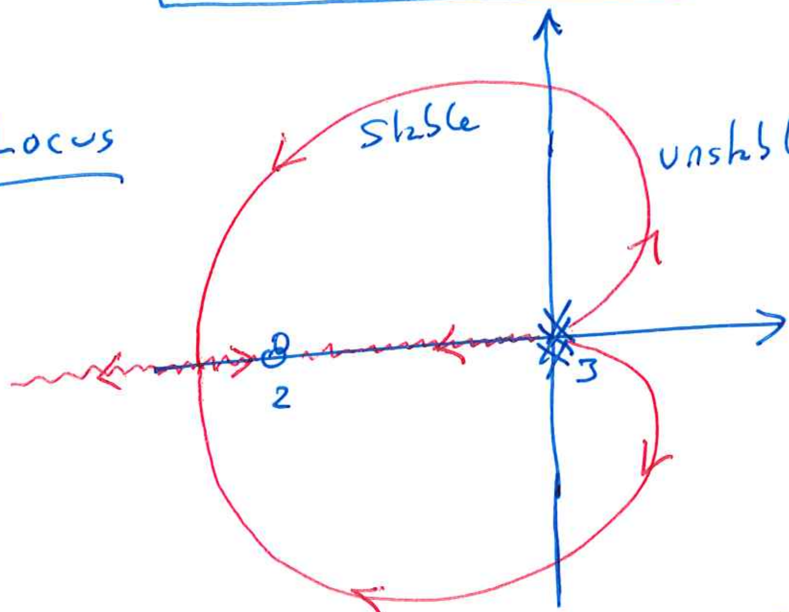
A confusion with Bode Phase and Gain margin

If phase slope is increasing at point where it crosses $-\pi$ <u>This does not indicate an instability.</u>

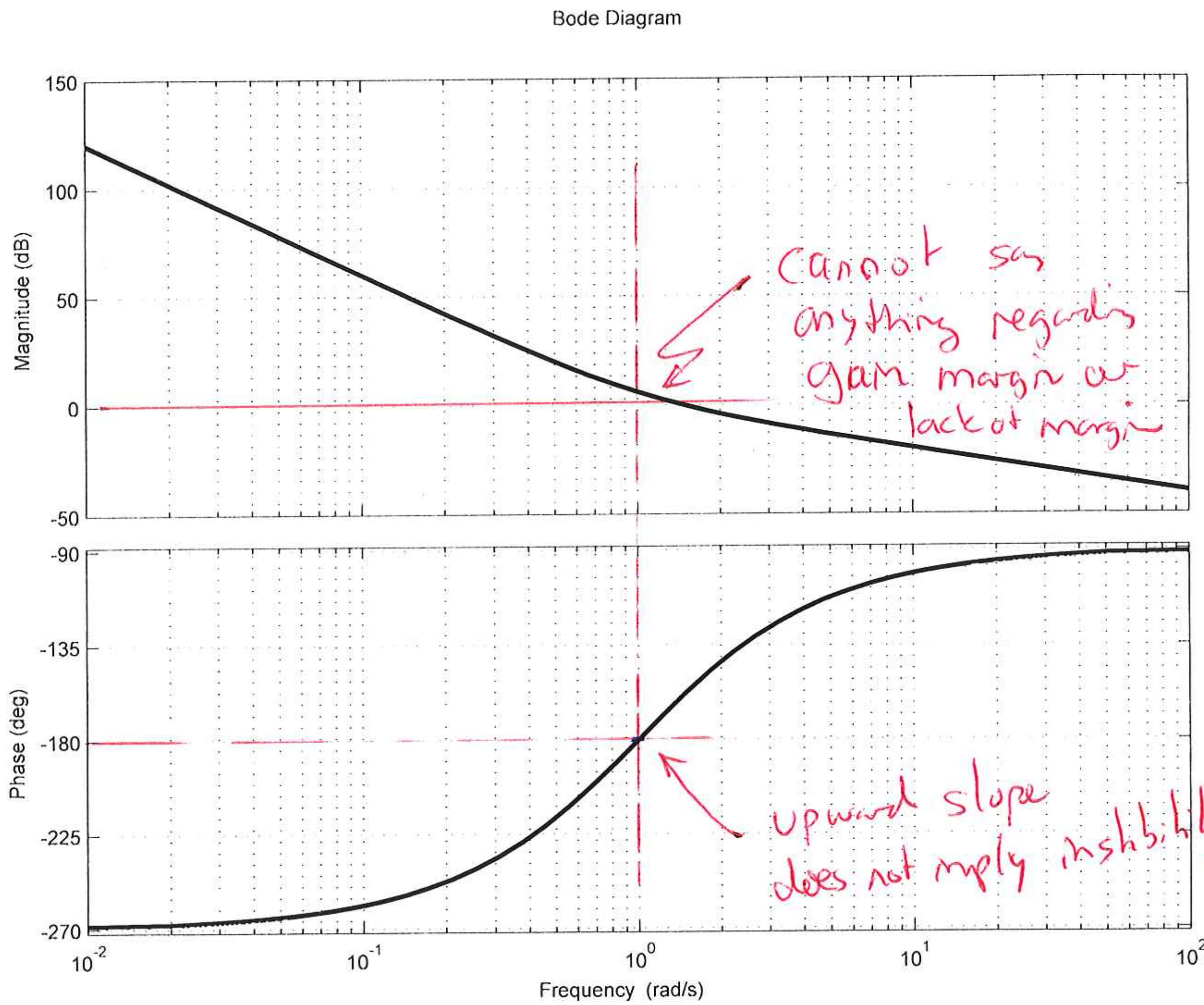
<u>Example</u> $H_{OL}(s) = \dfrac{(1+s)^2}{s^3}$



<u>Root Locus</u>



unstable for small K, stable for large K

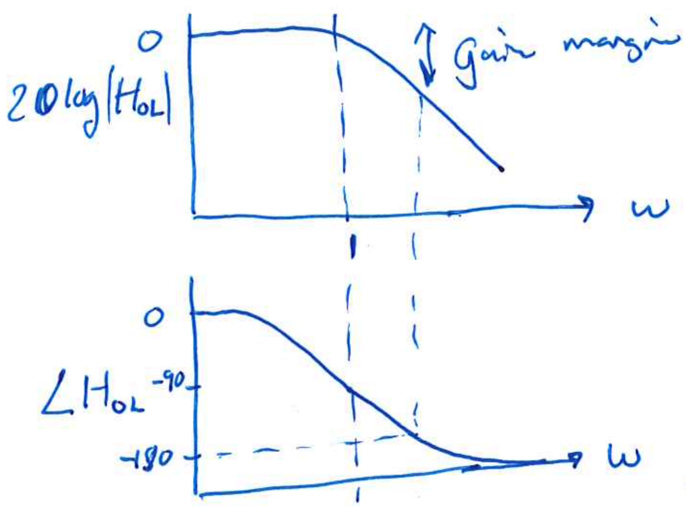


For this type of problem you have to use Root Locus.

Bode does not tell you anything!

You can use Bode for simple problems where the phase is decreasing with increasing frequency and crosses the phase of $-\pi$.

If the phase is increasing with frequency at the crossing of $-\pi$ then Bode analysis will be incorrect.



example
$$H_{OL}(s) = \frac{1}{(s+1)^3}$$

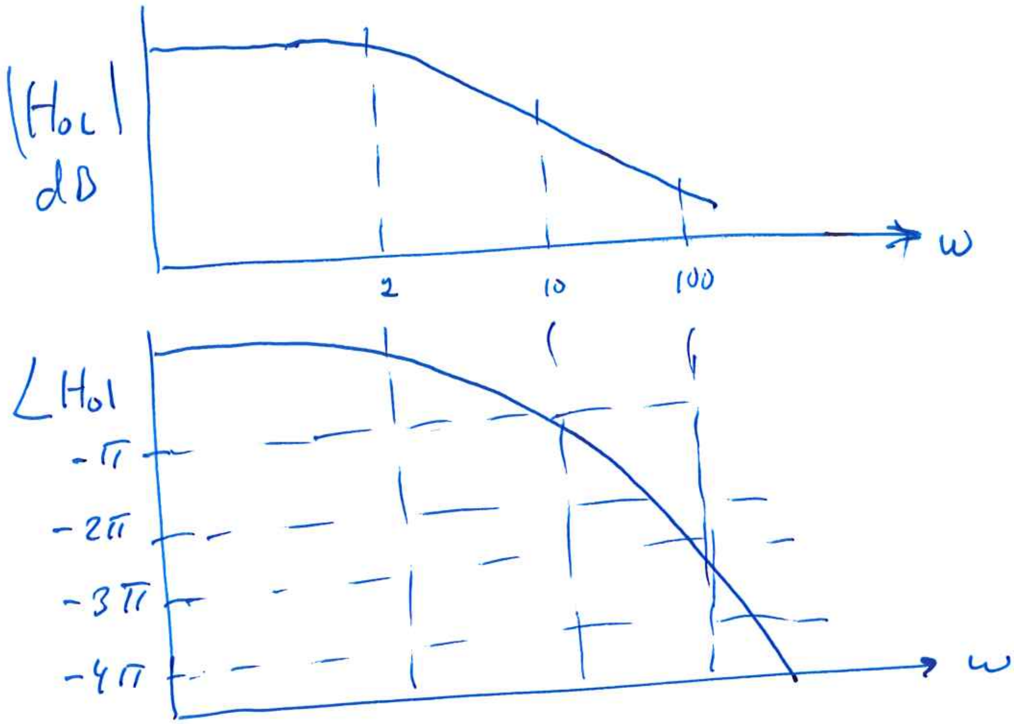
analysis of gain and phase margin is OK.

Or if we had a case where there is delay

eg. $$H_{OL}(s) = \frac{e^{-0.5s}}{(s+1)^2}$$

Enter into Matlab as

```
H = tf(1,[1,2,1],'iodelay',0.5)
bode(H)
```



Note we have said $\angle H_{OL} = -\pi$ is potential for instability (marginally stable point). However $\angle H_{OL} = -3\pi$, $\angle H_{OL} = -5\pi$, etc. are other points of potential instability.

For given measured data regarding H_{OL} you can use Bode analysis for stability if the interpolated phase is decreasing with frequency.

- Check this before making any conclusions from a Bode plot regarding closed loop stability.

Bode plots are therefore of limited use for determining stability.

<u>But</u>
Easy to use for mapping measured frequency data of the plant.

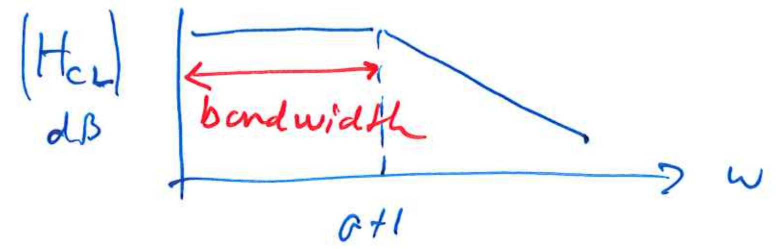
<u>Be aware</u>
Phase increasing with frequency through the point of $\angle H_{OL} = -\pi$ does not tell you anything.

However, the Bode plot of a closed loop response is useful to consider. As a simple example consider $H_{OL}(s) = \frac{1}{s+1}$ with proportional feedback of G such that

$$H_{CL}(s) = \frac{G}{s+G+1}$$

The Bode plot can show the bandwidth of the closed loop response as being $(G+1)$.

We can measure this directly and plot on a Bode plot.



The bandwidth of H_{CL} can show how fast the closed feedback loop can respond to a change in the input reference $r(t)$.