Topic 1: Power System Basics

Phasors

- Instantaneous AC voltage (or current):

phase angle (rad)

$$v(t) = \underbrace{V_m}_{\text{peak value}} \cos(\omega t + \theta_v)$$

angular frequency (rad/s)

in North America, $f = 60$ Hz $\therefore \omega = 2\pi \times 60 = 377$ rad/s

i.e. ω is constant in steady-state. $\therefore$ only need V_m & θ_v to characterize a sinusoid.

- In power systems, we use RMS (effective) value. This is the "DC equivalent" for an AC voltage or current:

$$V_{rms} \triangleq \sqrt{\frac{1}{T}\int_0^T v^2(t)\, dt}$$

for sinusoids only, $V_{rms} = V = \dfrac{V_m}{\sqrt{2}}$

i.e. we are dropping RMS subscript

- A phasor is a complex number (vector) that contains the $\underbrace{\text{magnitude (in RMS)}}_{\text{used peak values in ENGG 225}}$ & phase angle of a sinusoid

From Euler's Identity: $e^{\pm j\theta} = \cos\theta \pm j\sin\theta$

$$\therefore \quad \cos\theta = \mathrm{Re}\left[e^{j\theta}\right]$$

so, $v(t) = V_m \cos(\omega t + \theta_v)$

$$= \sqrt{2}\cdot V \ \mathrm{Re}\left[e^{j(\omega t + \theta_v)}\right]$$

$$= \sqrt{2}\ V \ \mathrm{Re}\left[e^{j\omega t}\cdot e^{j\theta_v}\right]$$

$V e^{j\theta_v} = V\angle\theta_v$ is the phasor representation of $v(t)$.

- ignored the t term here. Phasor is a snapshot of this rotating vector at $t=0$.

Time Domain		Phasor
$v(t) = \sqrt{2}\,V\cos(\omega t + \theta_v)$	$\longleftrightarrow$	$\bar{V} = \underbrace{V\angle\theta_v}$

can also use bold font to indicate a phasor

this is in polar co-ord's.

Rect. coord representation:

$$\bar{V} = V\cdot\cos\theta_v + j\,V\cdot\sin\theta_v$$

Ex: $\bar{V} = 120\angle 45^\circ$ Express in time domain?

$$v(t) = \sqrt{2}\cdot 120 \ \cos(\omega t + 45^\circ)$$

technically, should be in radians

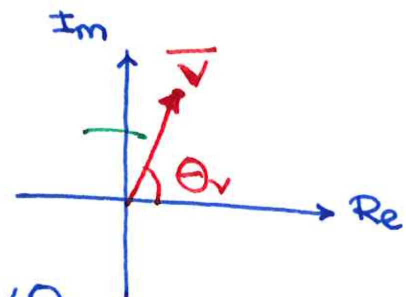
Power: Instantaneous, Real, Reactive, Complex

. Consider an element in an AC circuit (source or impedance)

if the voltage across the element is $\overline{V} = V\angle\Theta_v$,

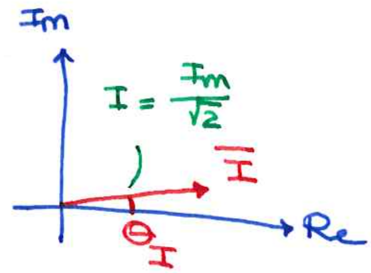
$$v(t) = V_m \cos(\omega t + \Theta_v)$$

$V = \frac{V_m}{\sqrt{2}}$



and the current through it is : $\overline{I} = I\angle\Theta_I$

$$i(t) = I_m \cos(\omega t + \Theta_I)$$



then, instantaneous power is :

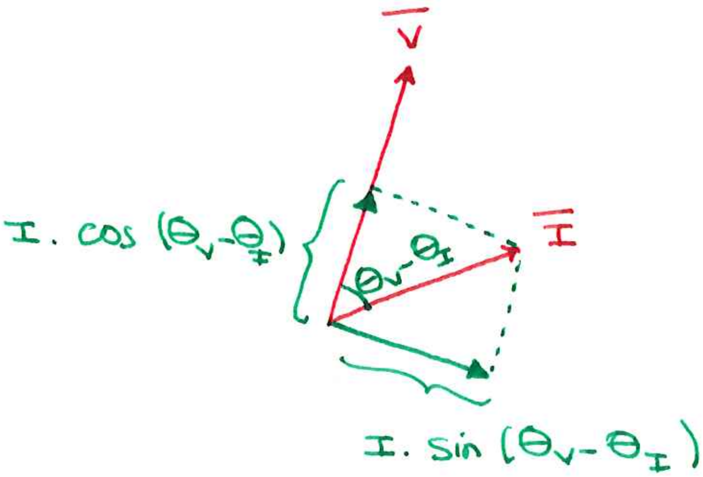
$$p(t) \triangleq v(t).\, i(t) = V_m \cos(\omega t + \Theta_v).\, I_m \cos(\omega t + \Theta_I)$$

after a lot of trig sorcery ...

$$P(t) = \underbrace{V.\,I.\,\cos(\Theta_v - \Theta_I)\left[1 + 2\cos(2\omega t + 2\Theta_v)\right]}_{\text{inst power from portion of } \overline{I} \text{ in Phase with } \overline{V} \text{ (i.e. resistive)}} +$$

$$\underbrace{V.\,I.\,\sin(\Theta_v - \Theta_I).\,\sin(2\omega t + 2\Theta_v)}_{\text{inst power from portion of } \overline{I}\ 90^\circ \text{ out of phase with } \overline{V} \text{ (i.e. reactive)}}$$

eq (i)



- Note : p(t) is not constant with time

Real Power / Resistive Power is the average of the 1st term in eq(i) :

$$\boxed{P_{avg} = P_{real} = P = V.I.\cos(\theta_V-\theta_I)}$$

unit : Watts (W)

Define power factor $\boxed{PF = \underbrace{\cos(\theta_V-\theta_I)}_{\text{power angle}}}$

$$P = \underbrace{V.I.}_{\text{apparent power, } S}(PF)$$

- product of Rms voltage & Rms current

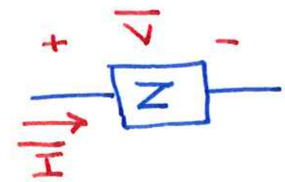
$$\boxed{S = V.I.}$$

Lagging PF : $\overline{I}$ lags $\overline{V}$: $\theta_I < \theta_V$

Leading PF : $\overline{I}$ leads $\overline{V}$: $\theta_I > \theta_V$

Power Factor for Complex Impedances

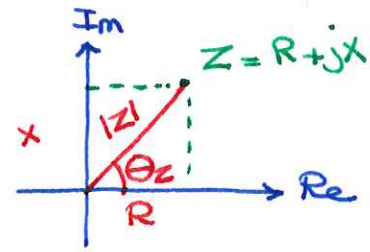
. consider a generic complex impedance Z. we can write:



$$Z = \underbrace{R}_{\text{resistance}} + j\underbrace{X}_{\text{reactance}}$$

i) for an inductive element : $X = \omega L > 0$

(reminder $Z_L = j\omega L$)

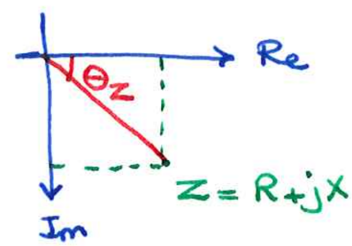


$\therefore \quad \theta_Z > 0$

θ_Z is also $\leq 90^\circ$ since $R \geq 0$

ii) for a capacitive element : $X = \dfrac{-1}{\omega C}$

(reminder $Z_C = \dfrac{1}{j\omega C} = j\,\dfrac{-1}{\omega C}$)



$\therefore \quad \theta_Z < 0$

. Now, from Ohm's Law :

$$\underbrace{\overline{V}}_{V\angle\theta_V} = \underbrace{\overline{I}}_{I\angle\theta_I} \cdot \underbrace{Z}_{|Z|\angle\theta_Z}$$

$$V\angle\theta_V = I\angle\theta_I \cdot |Z|\angle\theta_Z$$

$\therefore \quad \theta_V = \theta_I + \theta_Z \qquad \therefore \quad \boxed{\theta_Z = \theta_V - \theta_I}$

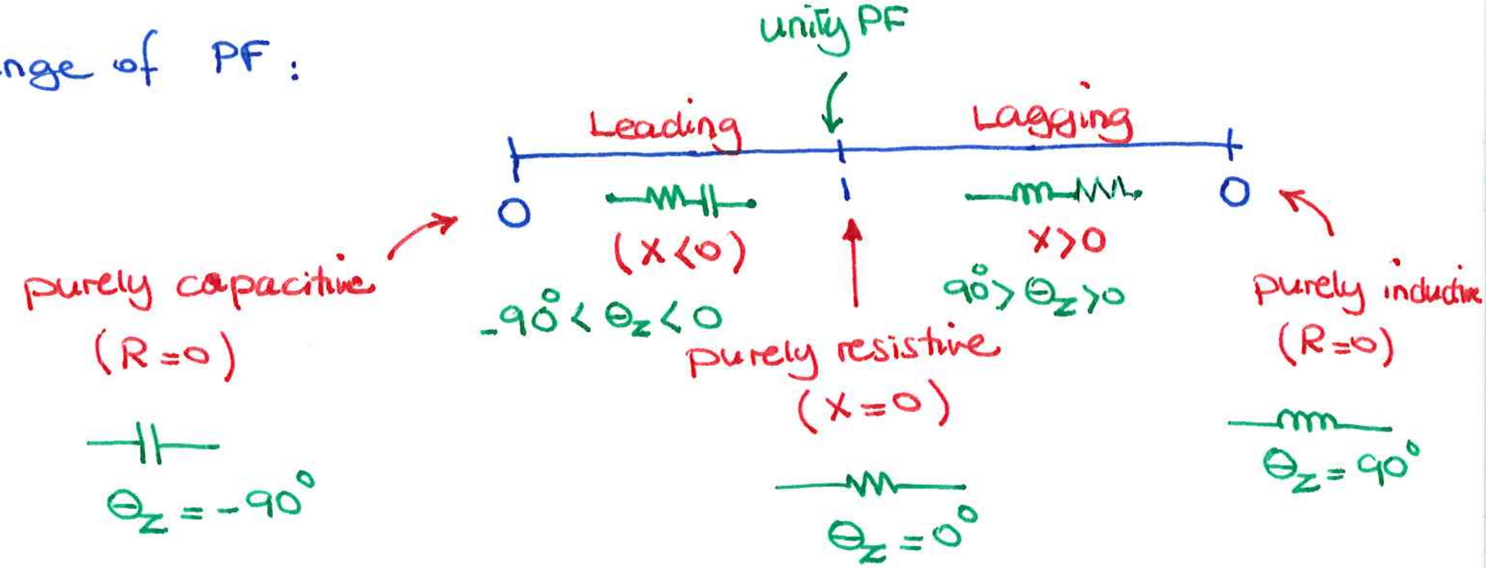
i.e. the power angle for <u>complex impedances</u> is the same as θ_Z. $\quad PF = \cos(\theta_V - \theta_I) = \cos(\theta_Z)$

i) For an inductive element, $\theta_Z > 0 \quad \therefore \theta_V - \theta_I > 0 \quad \therefore \theta_V > \theta_I$

$\therefore \bar{I}$ lags $\bar{V}$, i.e. lagging PF

ii) For a capacitive element, $\bar{I}$ leads $\bar{V}$, i.e. leading PF

· Range of PF :



Reactive Power : is the magnitude of the second term in eq(i)

The avg of second term is zero.

$$\boxed{Q = V \cdot I \cdot \sin(\theta_V - \theta_I)}$$

unit : VAr
(Volts-Amps-reactive)

It is a measure of power travelling back & forth between the source & inductors/capacitors due to creation & collapse of electric field in C & magnetic field in L.

Complex Power : $\bar{S}$ combines real power & reactive power in one quantity.

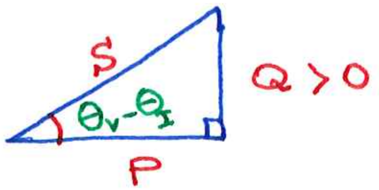
$$\bar{S} = P + jQ \quad \text{[rect. format]}$$

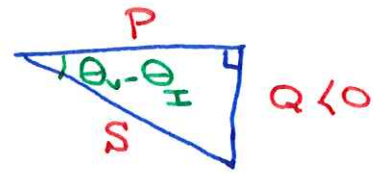
$$= S \angle \theta_V - \theta_I \quad \text{[polar format]}$$

apparent power.
Unit : VA (Volts-Amps)

magnitude of $\bar{S}$

- we can represent P, Q, and S on a power triangle:





- for $(\theta_V - \theta_I) > 0$
- for $(\theta_V - \theta_I) < 0$

- For both cases : $S^2 = P^2 + Q^2$
- Eq for complex Power: $\boxed{\overline{S} = \overline{V}\,\overline{I}^*}$ complex conjugate

from *

$$\overline{S} = (V\angle\theta_V)(I\angle\theta_I)^* = (V\angle\theta_V)\cdot(I\angle -\theta_I)$$

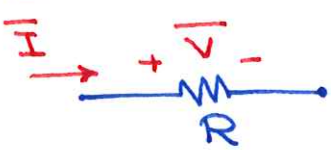
$$= VI\angle\theta_V - \theta_I$$

converting to rect. coordinates:

$$\overline{S} = \underbrace{V.I.\cos(\theta_V - \theta_I)}_{P} + j\,\underbrace{V.I.\sin(\theta_V - \theta_I)}_{Q}$$

- Note: Complex power is <u>not</u> a phasor. We cannot express it as $s(t) = V.I.\cos(\omega t + \theta_V - \theta_I)$

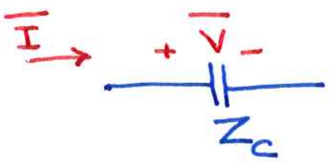
Power for impedances (R, L, C)



combining $\overline{S} = \overline{V}\,\overline{I}^*$ & $\overline{V} = \overline{I}.R$ we can show that:

$$\begin{cases} P = I^2.R = \dfrac{V^2}{R} \\ Q = 0 \end{cases}$$

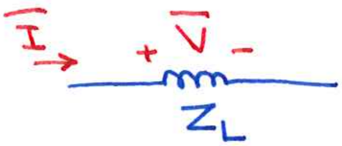
$P > 0$. Always absorbs real power



we can show that:

$$\begin{cases} P = 0 \\ Q = I^2 \cdot X_c = \dfrac{V^2}{X_c} \end{cases} \qquad \text{where } X_c = \frac{-1}{\omega C}$$

$Q < 0$. "supplies" reactive power

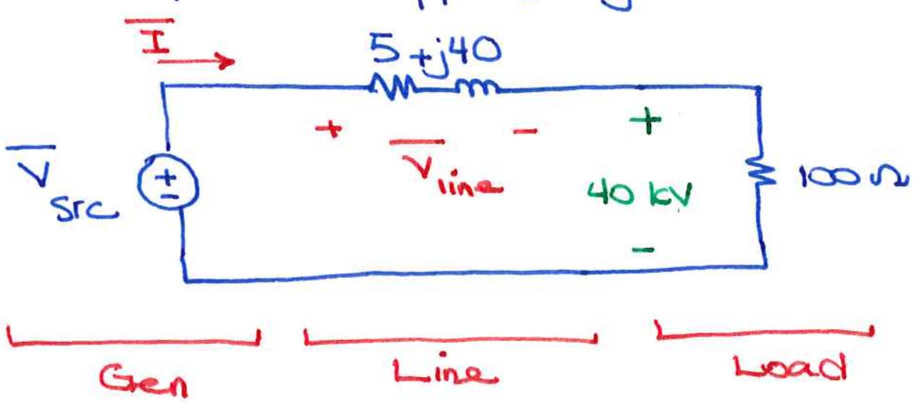


we can show that:

$$\begin{cases} P = 0 \\ Q = I^2 \cdot X_L = \dfrac{V^2}{X_L} \end{cases} \qquad \text{where } X_L = \omega L$$

$Q > 0$. "absorbs" reactive power

Ex: Find power supplied by the source:



Need to find $\overline{V}_{src}$ & $\overline{I}$ to find power from the source.

$$\overline{I} = \frac{40\text{ kV}}{100\,\Omega} = 400\text{ A}$$

From KVL: $\overline{V}_{src} = \overline{V}_{line} + 40\text{ kV} = 400\text{A}(5+j40) + 40000$

$$= 44.9\angle 20.8^\circ \text{ kV}$$

$$\overline{S}_{source} = -\overline{V}_{src}\,\overline{I}^* = -(44.9\angle 20.8^\circ)(400\angle 0)^* =$$

$$= -(17.98\angle 20.8^\circ) \text{ MVA}$$

$\overline{I}$ in dir of voltage rise

$$= \underbrace{-16.8 \text{ mW}}_{P} - j\underbrace{6.4 \text{ mVAr}}$$

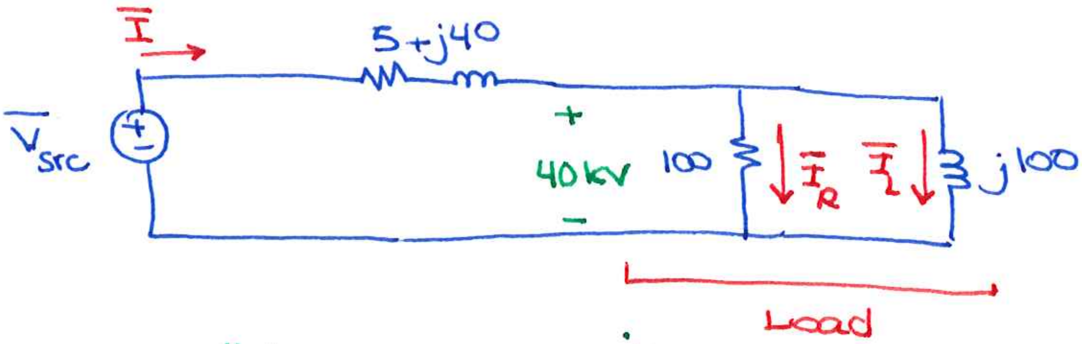
P supplies 16.8 mW

$Q = -6.4$ mVAr "supplies" reactive power

- Load power factor $= 1$ (unity PF)

$Q_{load} = 0$ since no reactive element.

- Repeat last example with an inductance added to the load (to model txfrs, motors, etc.)



$$Z_{load} = 100 \parallel j100 = \frac{100 \times j100}{100 + j100} = 70.7 \angle 45^\circ \ \Omega$$

$\theta_Z = 45^\circ$

$\therefore \theta_V - \theta_I = 45^\circ$ for combined load