Chapter V – Digital Baseband Modulation

ENEL 471 – Introduction to Communications Systems and Networks

Chapter Objectives

- At the end of this chapter, you will be able to:
  - Define the sampling theorem and be able to convert an analog signal to its digital representation
  - Analyze pulse amplitude modulated signal in the time and frequency domains
  - Analyze the performance of digital baseband systems in the presence of channel noise

Outline

- Review of Sampling Theory
  - Sampling of analog signals
  - Signal reconstruction
- Digital Baseband Modulation
  - Pulse amplitude modulation (PAM)
- Digital Baseband Demodulation
  - Message recovery from PAM
- Performance of Digital Baseband Systems in the Presence of Noise
  - Matched filter detection
  - Zero-threshold decision device
  - Bit error rate

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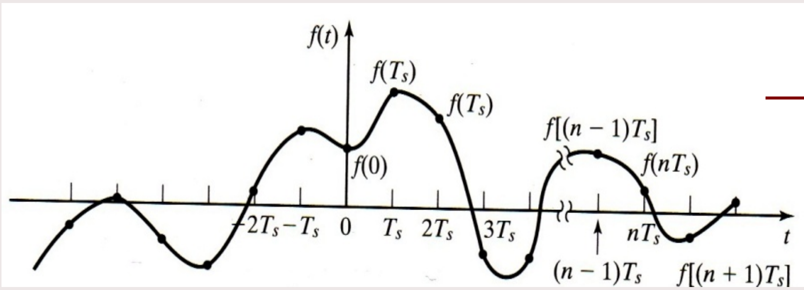
Review – Sampling of Continuous Time Signals

SAMPLING

Time Domain	Frequency Domain



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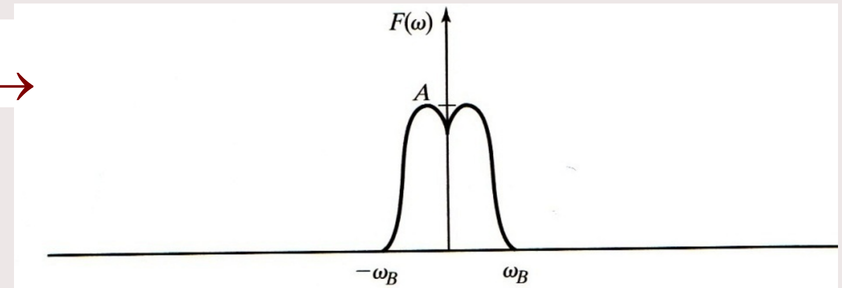
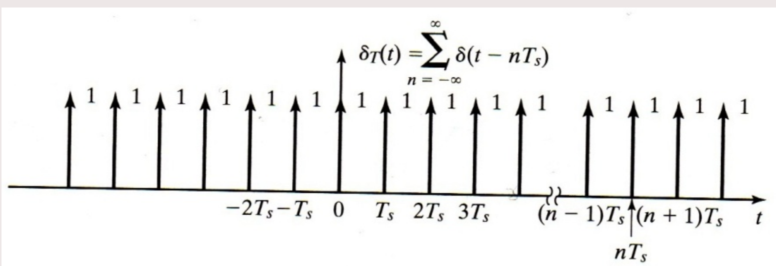


✖ Multiplication of $f(t)$ by $\delta_T(t)$ in time domain

✱ Convolution of $F(\omega)$ by $\Delta_T(\omega)$ in freq. domain



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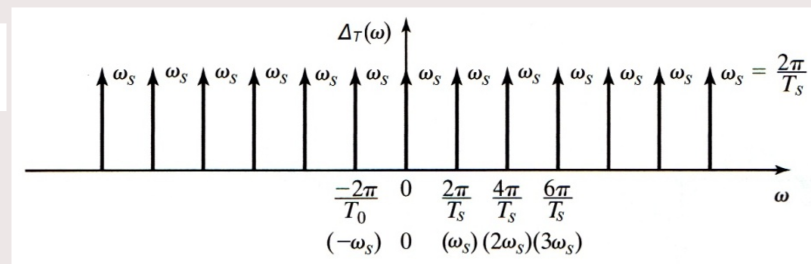

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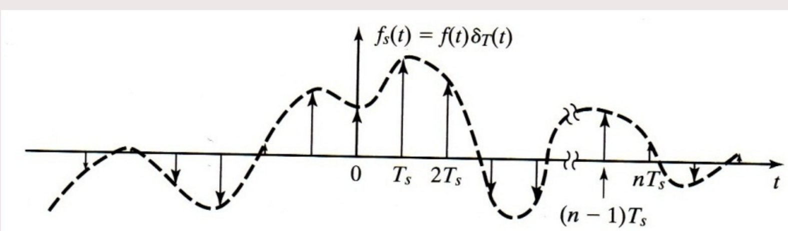

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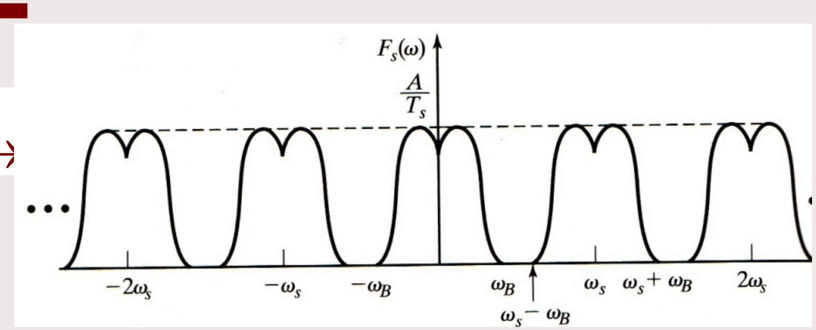

Review – Sampling of Continuous Time Signals

- In time domain, the sampled signal is obtained by multiplying the continuous-time signal with a train of impulses:

$$g_s(t) = g(t)\delta_T(t) = g(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

or: $$\boxed{g_s(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s)}$$

- Notation: $g_s(.)$ is a vector of values of $g(.)$ at times nT_s

$$g_s(nT_s) = g_s[n]$$

- The application of the Fourier transform to $g_s(t)$ gives:

$$G_s(f) = G(f) * \Delta_T(f) = G(f) * \left(f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \right)$$

or: $$\boxed{G_s(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} G(f - nf_s)}$$

→ If $f_s \geq 2W$ there is no aliasing (no loss of information). The signal $g(t)$ can be restored.

Shannon's Sampling Theorem

A function of time $f(t)$, that contains no frequency components greater than f_M hertz is determined uniquely by the values of f(t) at any set of points spaced $T_M/2$ ($T_M=1/f_M$) seconds apart

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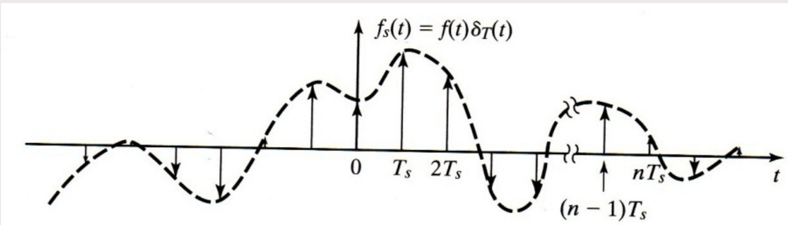
Reconstruction of Continuous Time Signals (Interpolation)

Time Domain	Frequency Domain



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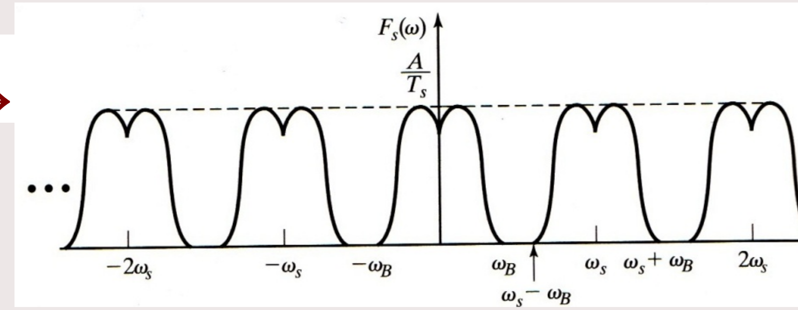
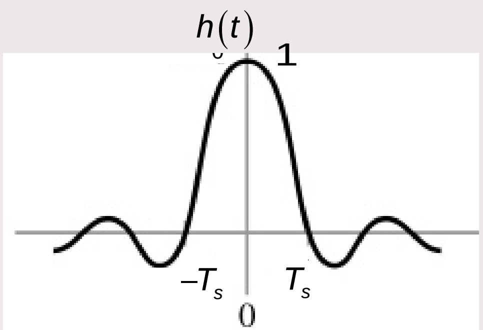


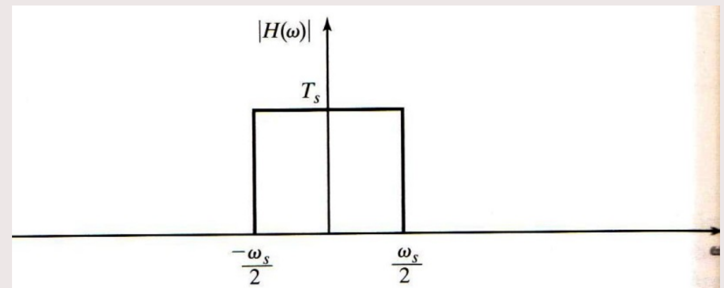
INTERPOLATION

✱ Convolution of $f_s(t)$ by $h(t)$ in time domain

✖ Multiplication of $F_s(\omega)$ by $H(\omega)$ in freq. domain



$\xleftarrow{\mathscr{F}^{-1}}$



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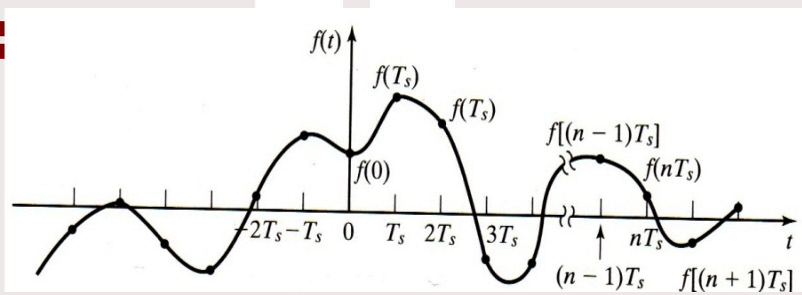

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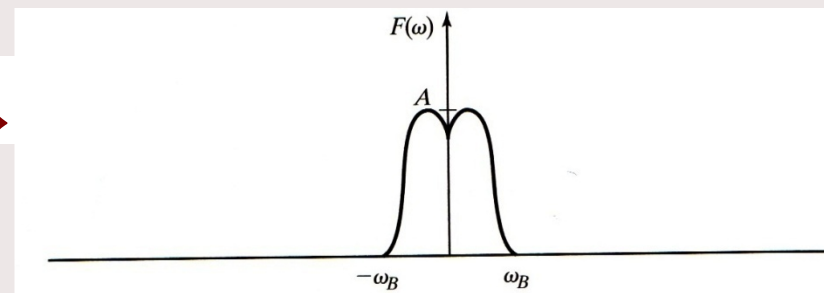


Reconstruction of Continuous Time Signals (Interpolation)

- In frequency domain, the sampled signal is filtered with an ideal low pass filter in order to reconstruct the continuous time signal (interpolation):

$$G(f) = G_s(f) \cdot T_s \operatorname{rect}\left(\frac{f}{f_s}\right)$$

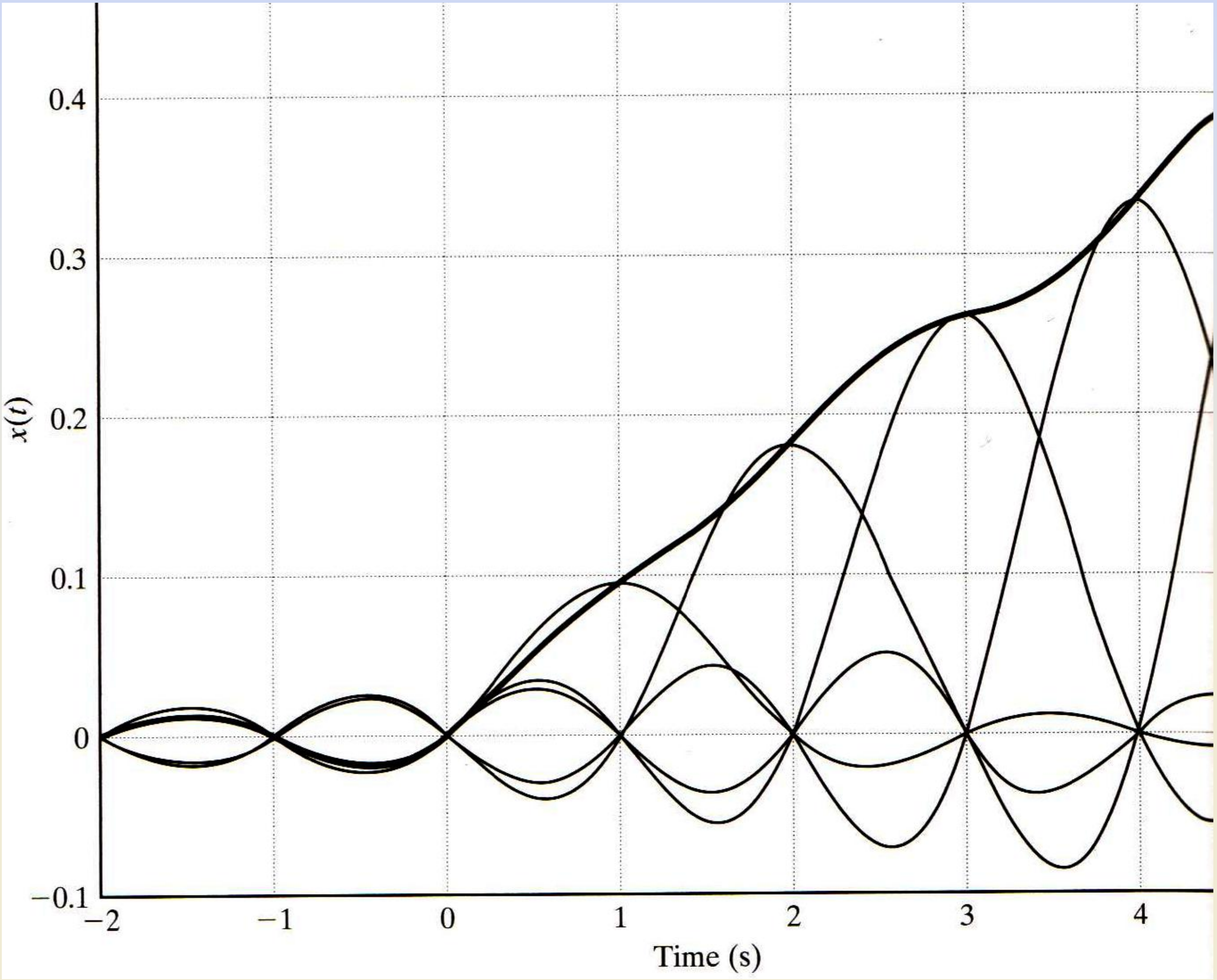
- By applying the inverse Fourier transform

$$g(t) = g_s(t) * \operatorname{sinc}\left(\frac{f_s t}{2}\right) = \left(\sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t - nT_s)\right) * \operatorname{sinc}\left(\frac{f_s t}{2}\right)$$

$$g(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\left(\delta(t - nT_s) * \operatorname{sinc}\left(\frac{f_s t}{2}\right)\right)$$

$$\boxed{g(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\operatorname{sinc}\left(\frac{f_s (t - nT_s)}{2}\right)}$$

Reconstruction of Continuous Time Signals (Interpolation)



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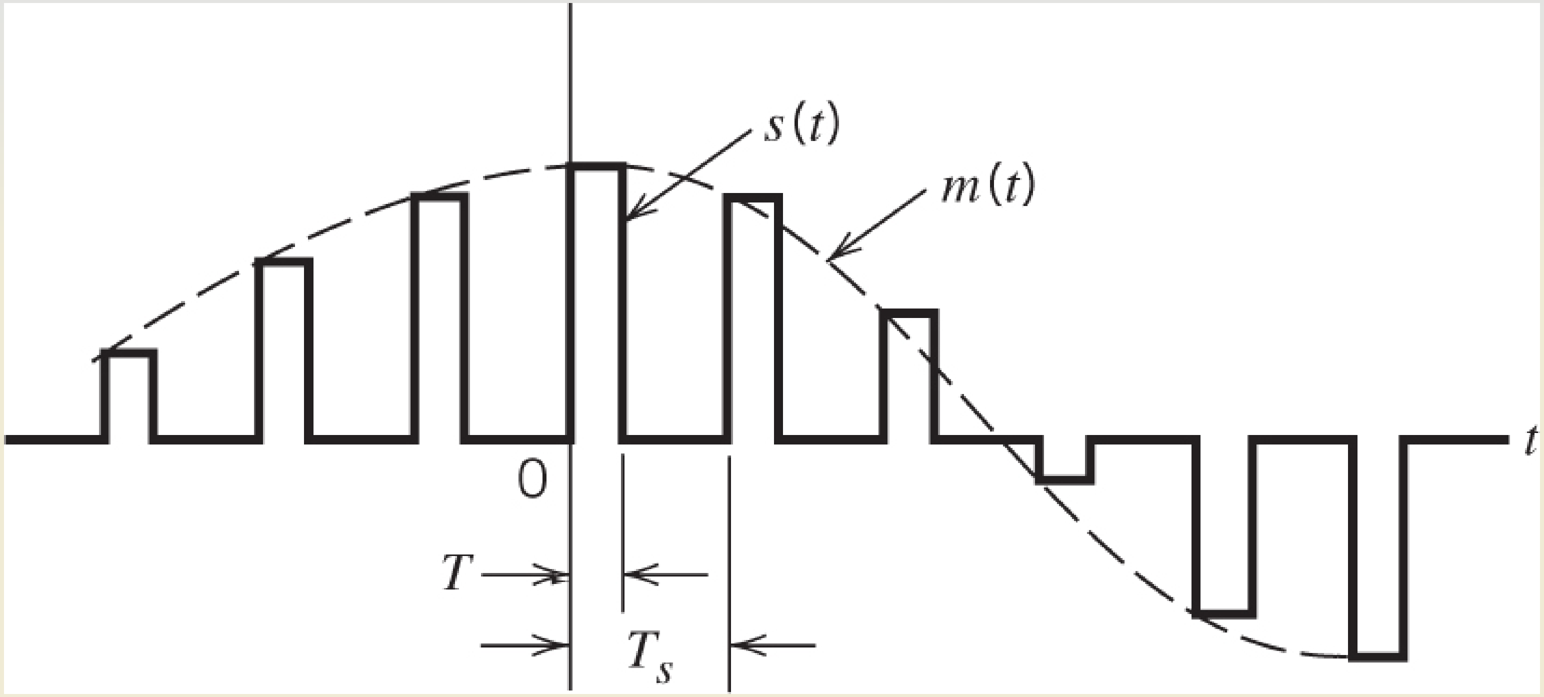
Pulse Amplitude Modulation (PAM)

- The amplitudes of regular pulses are varied in proportional to the corresponding sample values of a continuous time message signal
- It converts the sampled signal into a signal that can be produced by digital circuits
- The modulation does not change the carrier frequency of the signal. The modulated signal still has a zero Hz carrier frequency.



Time Domain Analysis of PAM

- The PAM signal can be expressed as :

$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s) \cdot h(t - nT_s)$$

where $h(t)$ is a rectangular pulse function defined by:

$$h(t) = \begin{cases} 1 & \text{for } 0 < t < T \\ 0 & \text{otherwise} \end{cases}$$

- The sampled signal is given by:

$$m_s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t - nT_s)$$

- By convolving $h(t)$ with $m_s(t)$:

$$m_s(t) * h(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t - nT_s) * h(t)$$

$$= \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s)$$

$$\boxed{m_s(t) * h(t) = s(t)}$$

Frequency Domain Analysis of PAM

- Using the Fourier transform:

$$S(f) = M_s(f) \cdot H(f)$$

where $H(f)$ is the Fourier transform of the rectangular pulse function:

$$H(f) = T\,\text{sinc}(\pi f T)\,e^{-j\pi f T}$$

$M_s(f)$ is the Fourier transform of the sampled signal:

$$M_s(f) = f_s \sum_{n=-\infty}^{\infty} M(f - nf_s)$$

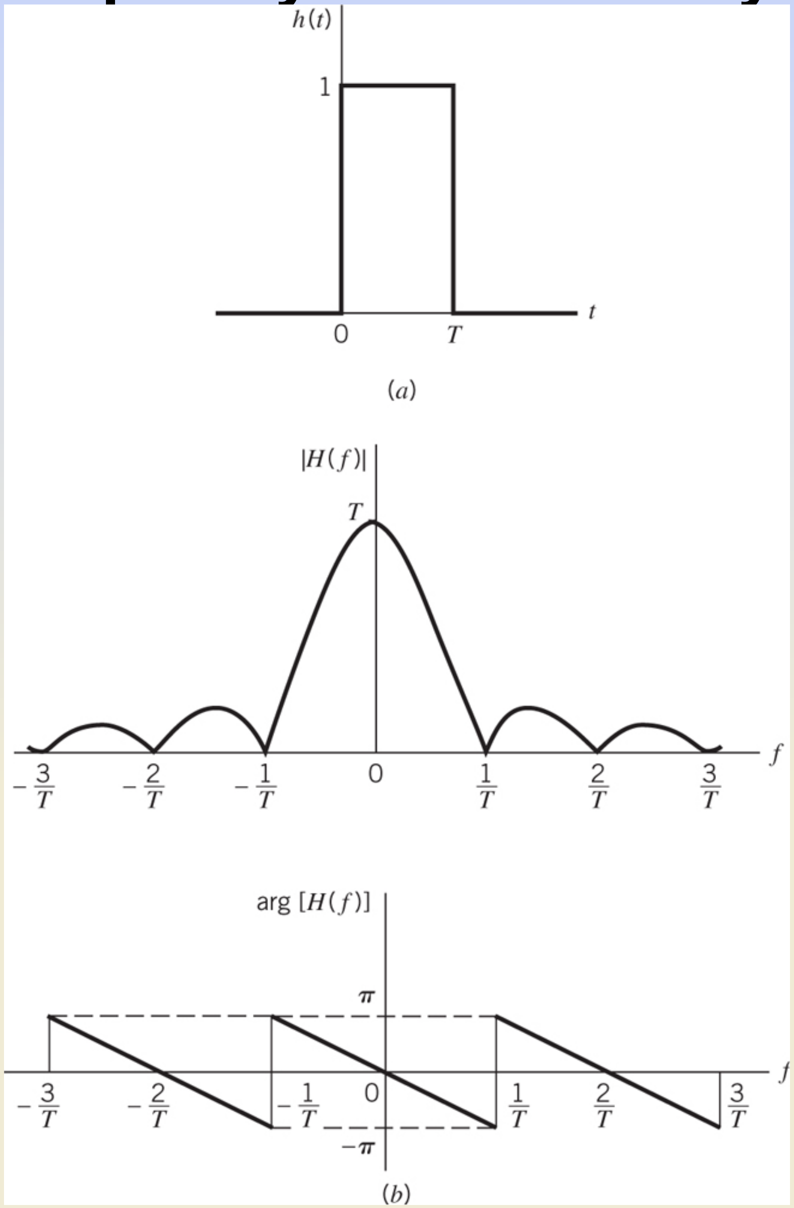
- The Fourier transform of the PAM signal is then given by:

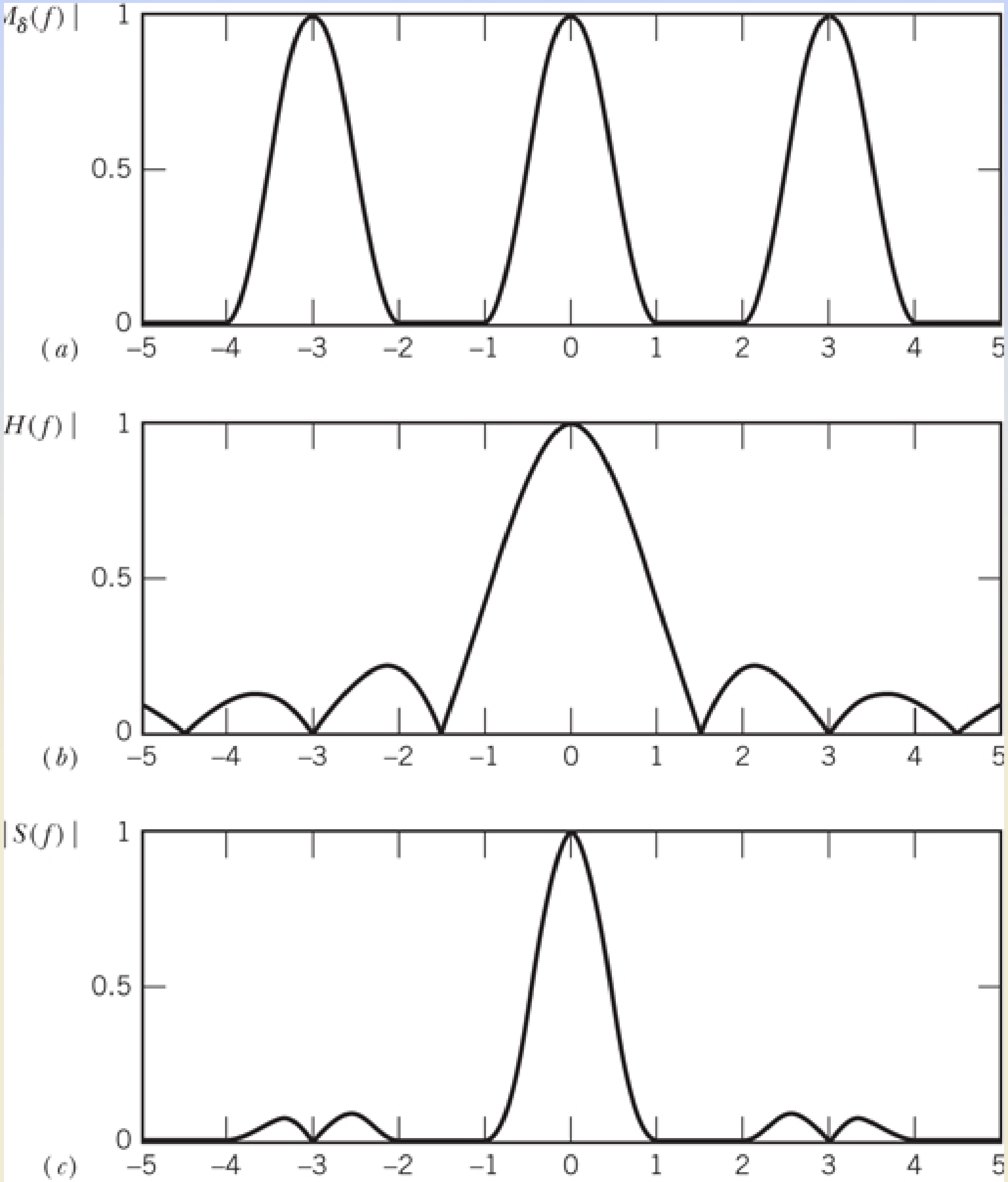
$$S(f) = f_s \sum_{n=-\infty}^{\infty} M(f - nf_s) \cdot T\,\text{sinc}(\pi f T)\,e^{-j\pi f T}$$

→ The PAM modulation introduce amplitude distortion : the message spectrum is deformed by the sinc() function. This is called **Aperture effect**

→ The PAM modulation introduces a time delay of T/2

Frequency Domain Analysis of PAM





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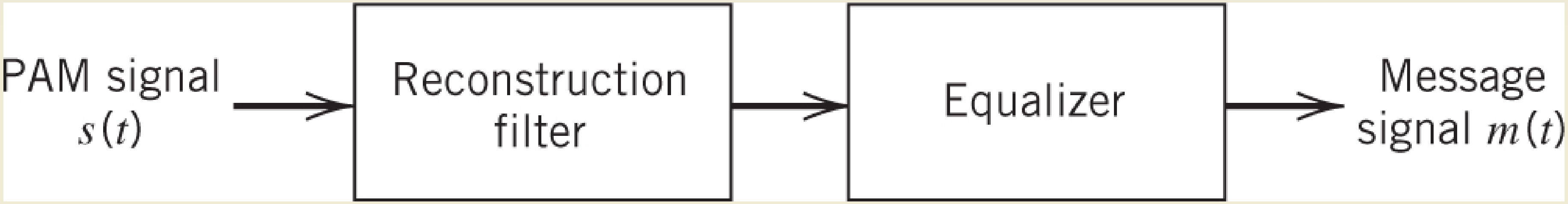
Message Recovery from PAM

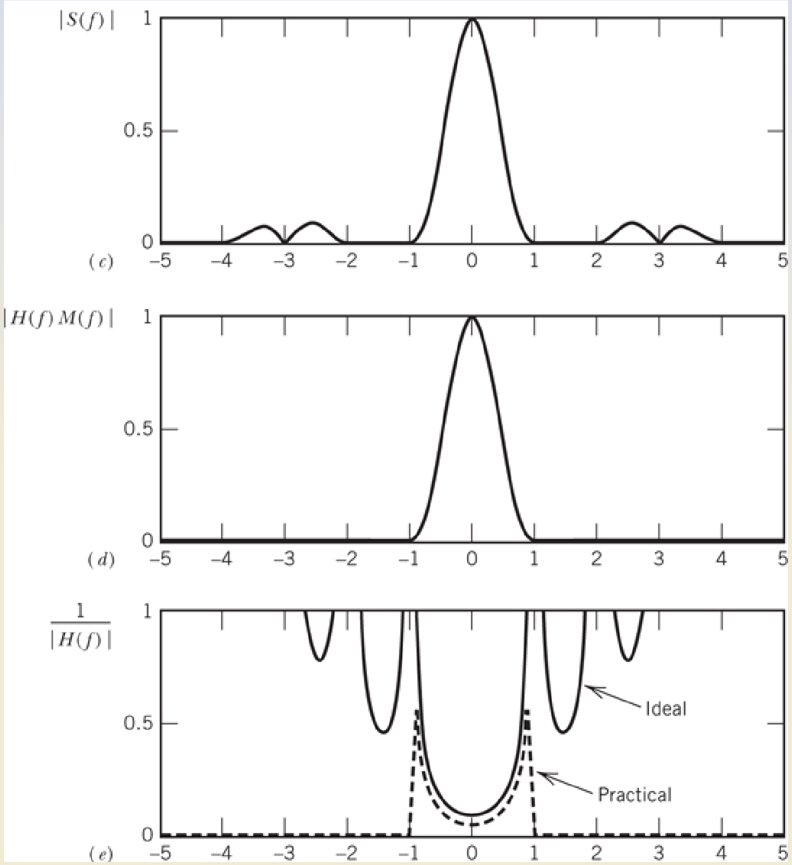
- The use of reconstruction low-pass filter removes the side lobes in the spectrum of the PAM signal but cannot compensate for the distortion due to aperture effect.
- In order to compensate for the distortion caused by the aperture effect, an equalizer has to be used in the message recovery system.
- The equalizer has to have a frequency response that is opposite to the frequency response caused by the convolution with rectangular pulses



Message Recovery from PAM

- The amplitude of the frequency response of the equalizer should then be given by:

$$\left|E(f)\right| = \frac{1}{\left|H(f)\right|} = \frac{1}{T\left|\mathrm{sinc}(\pi f T)\right|}$$



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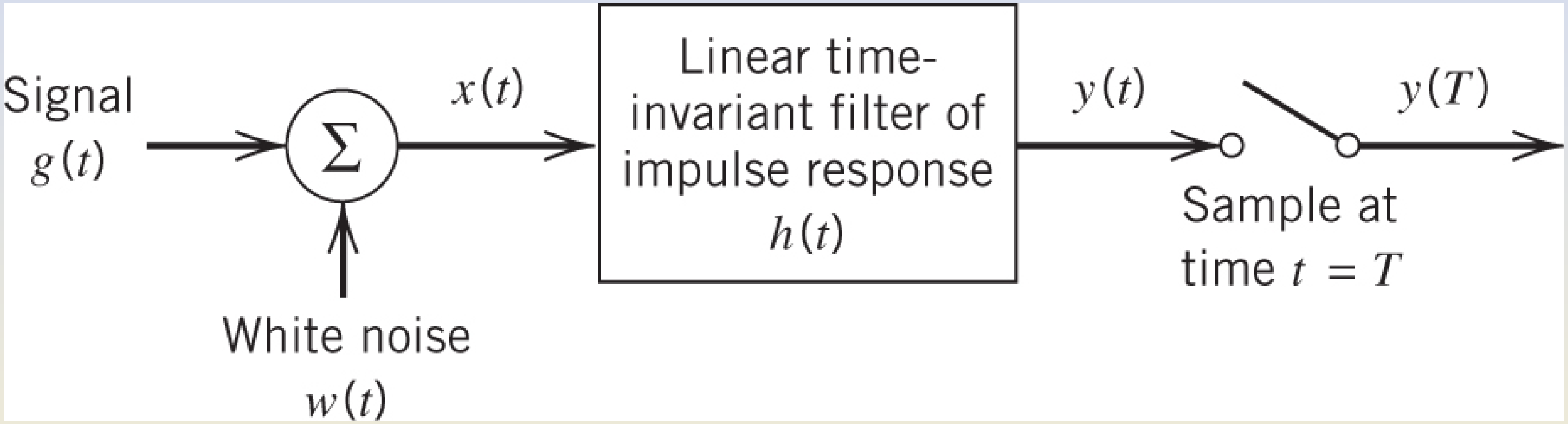
Performance of PAM in the presence of noise

- The PAM signal is a summation of rectangular pulses. Let’s do the analysis for one rectangular pulse demodulation in the presence of noise and passing through a linear receiver as shown below



- In the presence of noise, the pulse is deformed and is given by:

$$x(t) = g(t) + w(t)$$

- After passing through the linear filter the signal becomes:

$$y(t) = g_o(t) + n(t)$$

where : $g_o(t) = g(t) * h(t)$ and $n(t) = w(t) * h(t)$

Performance of PAM in the presence of noise

- The function of the receiver is to detect pulse signal g(t) in an optimum manner → <u>minimize the effect of noise on the system</u>

→ **<u>Objective</u>**: Use an optimum filter that maximizes the peak pulse signal-to-noise ratio defined as:

$$\eta = \frac{\left|g_o\left(T\right)\right|^2}{\mathrm{E}\left[n^2\left(t\right)\right]}$$

which represents the ratio of the signal power at instant T to the average noise power → <u>minimize the effect of noise on the system</u>

- It can be proven that the optimal filter should have the same pulse shape as the input signal, but time reversed and time shifted

$$h_{opt}\left(t\right) = g\left(T - t\right)$$

- This type of filter is called matched filter. It maximizes the signal-to-noise ratio at the output.

The Matched Filter

- Let the filter be a matched filter: its impulse response is given by:

$$h(t) = k \cdot g(T - t)$$

where k is a constant.

- The output *y*(*t*) is then given by: $y(t) = g_o(t) + n(t)$

where

$$g_o(t) = g(t) * \left(k \cdot g(T - t)\right) = k\int_{-\infty}^{\infty} g(\tau) \cdot g(T - t + \tau)\, d\tau$$

and therefore at time T

$$g_o(T) = k\int_{-\infty}^{\infty} g(\tau)^2\, d\tau = k \cdot E$$

E is the energy of the pulse

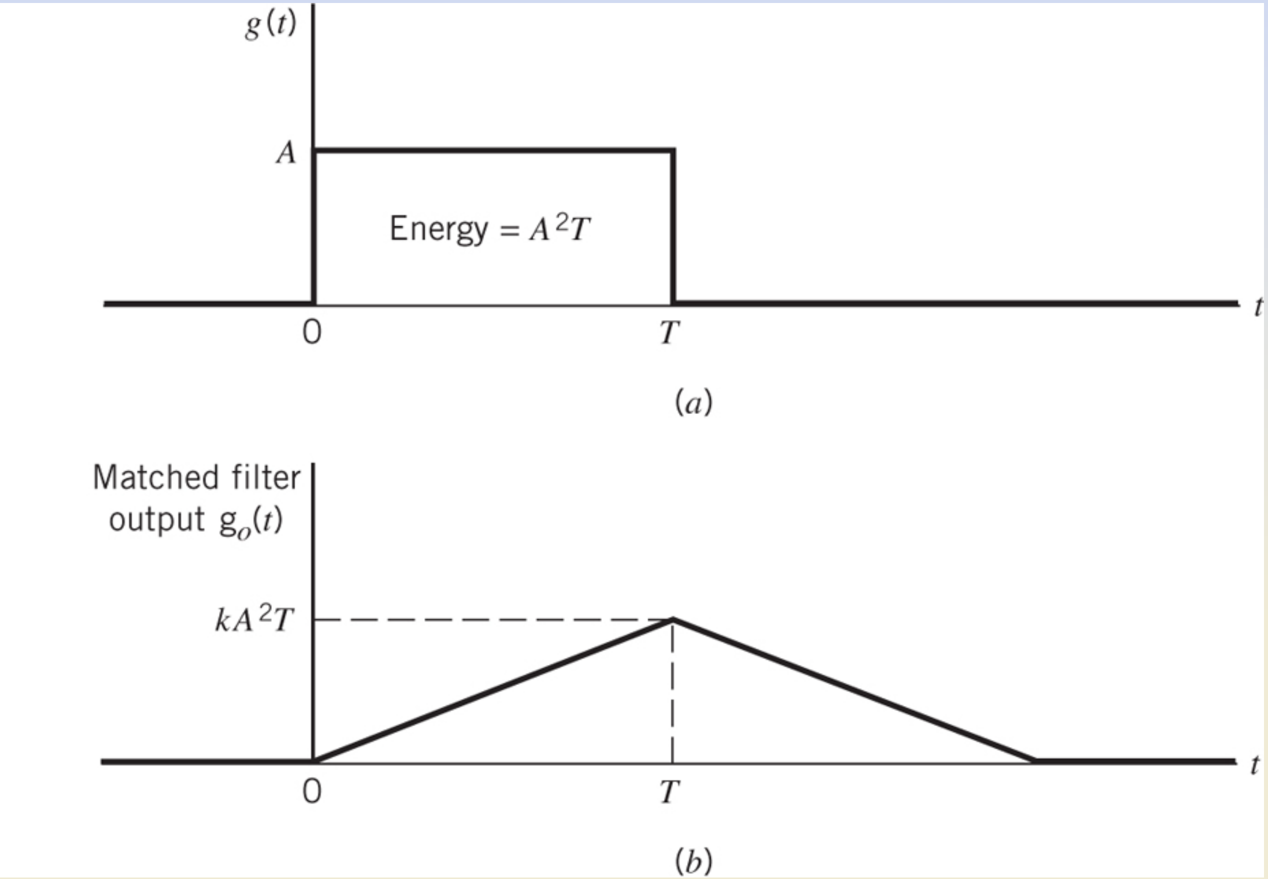
- The noise is given by: $n(t) = w(t) * \left(k \cdot g(T - t)\right)$

Its average power is therefore given by: $n(t) = N_0 k^2 E^2 / 2$

- The peak pulse signal-to-noise ratio

$$\eta = \frac{\left|g_o(T)\right|^2}{\mathrm{E}\left[n^2(t)\right]} = \frac{2E}{N_0}$$

The Matched Filter for a rectangular pulse



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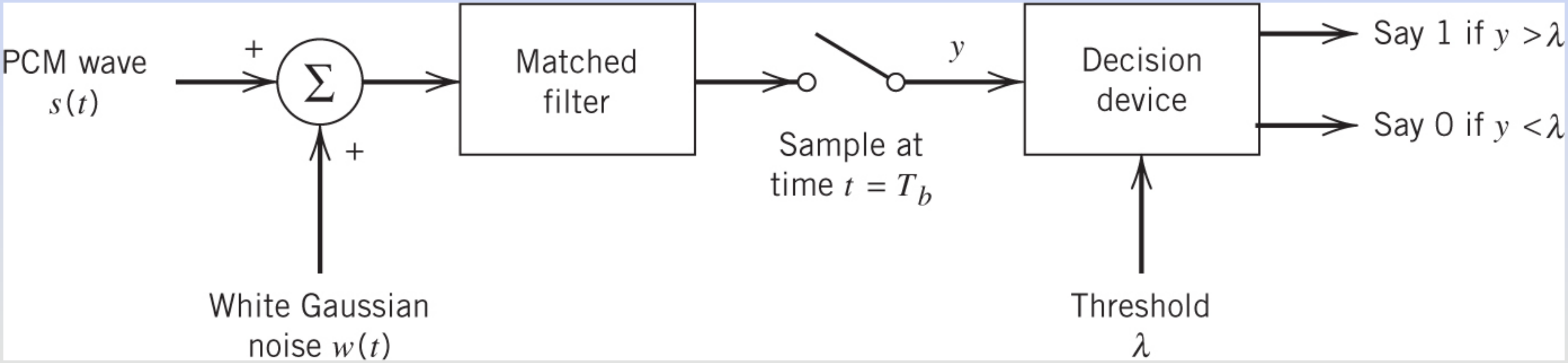
Zero Threshold Decision

- If the transmitted signal is binary (a sequence of binary numbers : ‘0’ and ‘1’), at a period of bit T_b, then, the received signal has the form:

$$x(t) = \begin{cases} A + w(t), & \text{if a '1' symbol was sent} \\ -A + w(t), & \text{if a '0' symbol was sent} \end{cases}$$

- In the receiver side, a decision has to be made on each sampled data to whether it corresponds to ‘0’ transmission or ‘1’
- The receiver includes a decision device, which decides on each sampled data by comparing it to a “zero threshold”
  - If $y(n.T_b) > 0$ then the received symbol is ‘1’
  - If $y(n.T_b) < 0$ then the received symbol is ‘0’

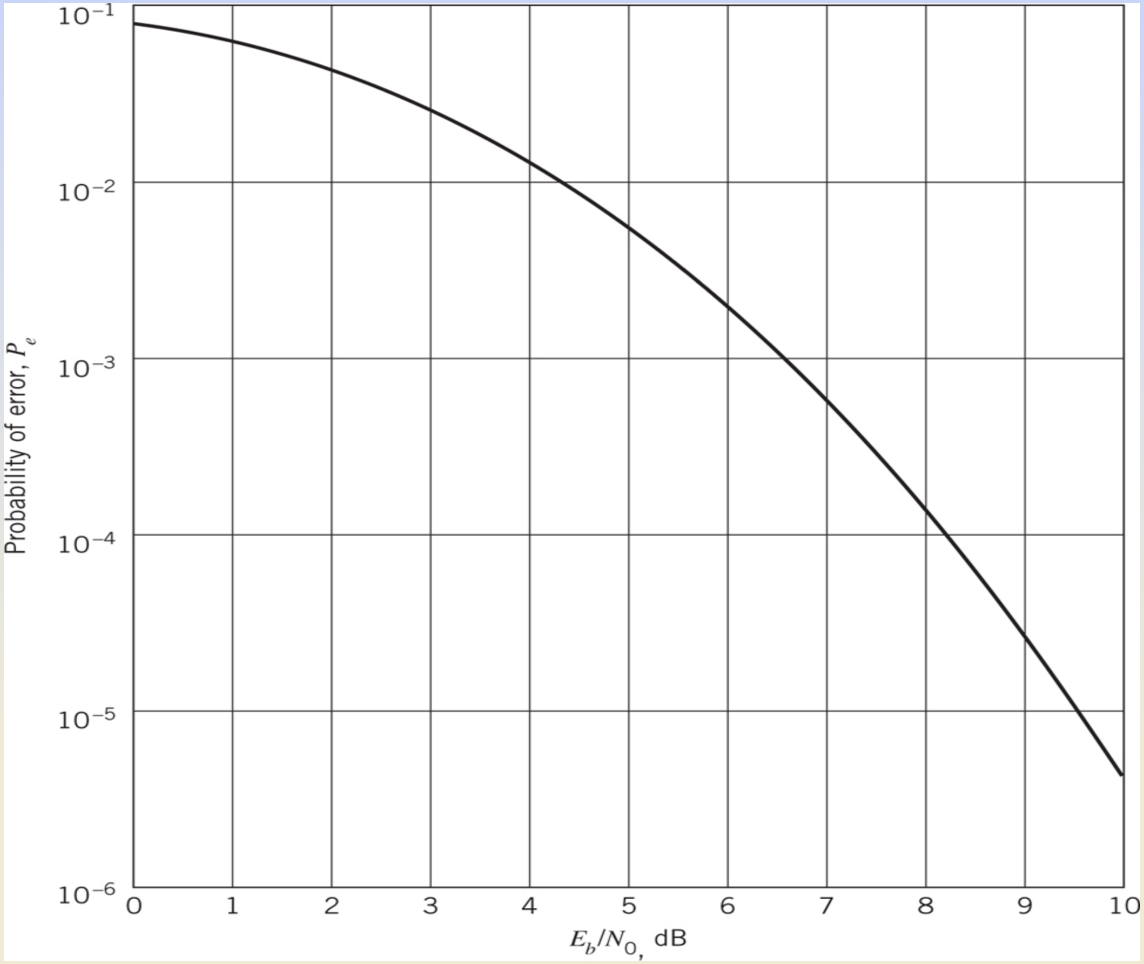
Zero Threshold Decision



- The noise will affect the decision : if the noise is large errors can occur. Two scenarios for errors are possible:
  - If a ‘0’ is transmitted, a ‘1’ is detected in the receiver
  - If a ‘1’ is transmitted, a ‘0’ is detected in the receiver
- To assess the communication system performance in the presence of noise, instead of talking about signal-to-noise ratio and figure-of-merit, the rate of erroneous detected bits is used.

→ Find the probability of error due to noise, P_e

Performance Metric : Probability of Error due to Noise



- If we increase the transmitted signal energy, *E*, the average probability of error is reduced (better transmission quality)
- Given a P_e objective, We can use the graph above to find the required E/N_0