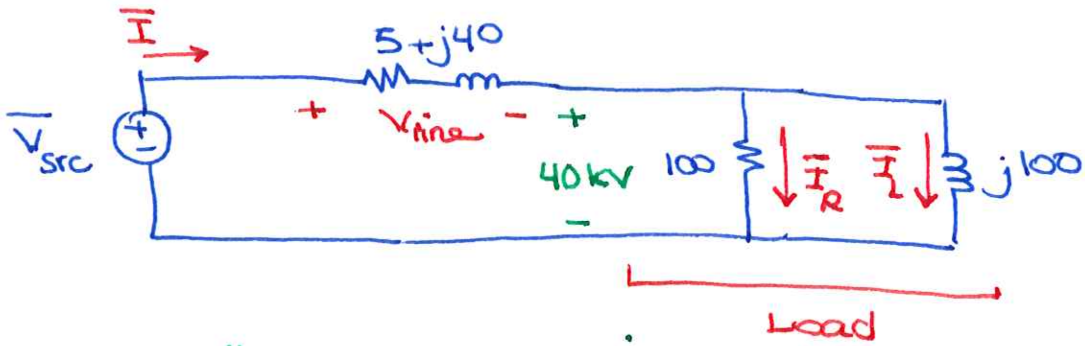
$$= \underbrace{-16.8 \text{ mW}}_{P} \quad - j\,\underbrace{6.4 \text{ mVAr}}_{Q = -6.4 \text{ mVAr}}$$

supplies 16.8 mW "supplies" reactive power

- Load power factor = 1 (unity PF)

  $Q_{load} = 0$ since no reactive element.

- Repeat last example with an inductance added to the load (to model txfrs, motors, etc.)



$$Z_{load} = 100 \parallel j100 = \frac{100 \times j100}{100 + j100} = 70.7 \underbrace{\angle 45^\circ}_{\theta_Z = 45^\circ} \ \Omega$$

$\therefore \theta_V - \theta_I = 45^\circ$ for combined load

$$\text{Load PF} = \cos(\theta_V - \theta_I) = \cos(\theta_Z) = \cos(45^\circ)$$

$$= 0.707 \text{ lagging}$$

$$\bar{I} = \frac{40 \text{ kV}}{Z_{load}} = \frac{40\,000 \angle 0^\circ}{70.7 \angle 45^\circ} = 564 \angle -45^\circ \ \text{A}$$

$$\bar{V}_{src} = \bar{V}_{line} + 40 \text{ kV} = (564 \angle -45^\circ)(5 + j40) + 40\,000$$

$$= 59.7 \angle 13.6^\circ \text{ kV}$$

$$\bar{S}_{src} = -\bar{V}_{src} \cdot \bar{I}^* = -(33.7 \angle 58.6^\circ) \text{ mVA}$$

$$= \underbrace{-17.6}_{P(\text{mW})} \quad - j\,\underbrace{28.8}_{Q = -28.8 \text{ mVAr}}$$

- Reactive power required by the load resulted in higher current which resulted in higher real power losses in the line.

  Solution: Adding power factor correction capacitors on the load side. This will bring θ_Z closer to 0° thereby reducing I in the line. i.e. Q required by inductive elements of the load now supplied by local capacitor instead of the source.

- Alternative solution: we can find P_{load} using $\dfrac{V_{load}^2}{R}$ 40kV, 100Ω

  or $I_R^2 \cdot R$, Q_{load} using $\dfrac{V_{load}^2}{X}$ 40 kV, 100 or $I_L^2 \cdot X$

  $P_{line} = I^2 \cdot R_{line}$, $Q_{line} = I^2 \cdot X_{line}$

  564 5 40

  then: $P_{src} = P_{load} + P_{line}$, $Q_{src} = Q_{load} + Q_{line}$

- when calculating P_{line}, can we use $\dfrac{V_{line}^2}{R_{line}}$?

  No, V_{line} is the voltage across the complex impedance $5+j40$. we need the <u>voltage across the 5Ω resistor only</u> to use in $\dfrac{V^2}{R}$ equation.

- **Important** : Elements in a power system operate at or near distinct, pre-defined voltage levels called rated or nominal voltage.

Example: $\underbrace{120\text{ V},\ 240\text{ V},\ 600\text{ V},\ 13.8\text{ kV},\ 25\text{ kV}, \ldots}_{\text{Gen \& distributed rated or nominal voltages}}$

and $\underbrace{69\text{ kV},\ 138\text{ kV},\ 240\text{ kV},\ 500\text{ kV}}_{\text{transmission rated or nominal voltage}}$

- Nominal voltage of the outlets in your house is 120 V (always).

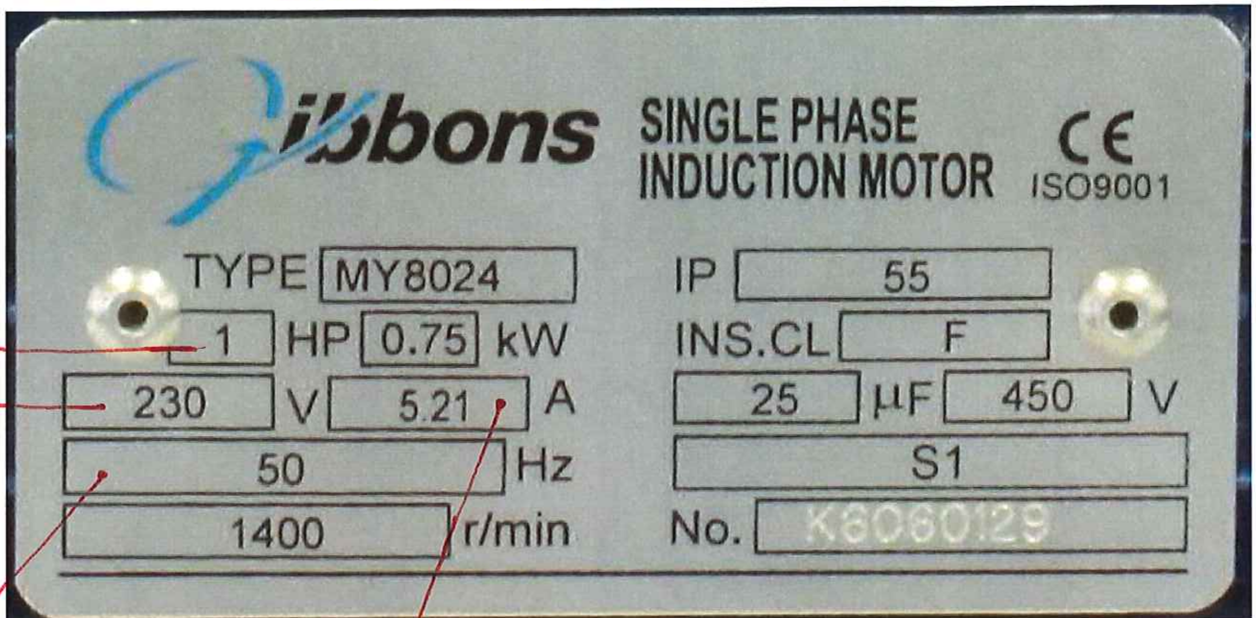
Operating voltage of the outlet changes based on operating conditions (but it should be close to the nominal voltage under normal conditions).

e.g. op. Voltage of the outlet could be 118 V, 125 V, etc.

- When solving a circuit, we must use the operating voltage
- Can't assume op. voltage = nominal (or rated) voltage unless explicitly stated.

Motor Nameplate example

shows rated values for this motor



rated output mech power, P_{mech}

rated (nominal) voltage

European!

1 HP = 746 W

rated (full load) current

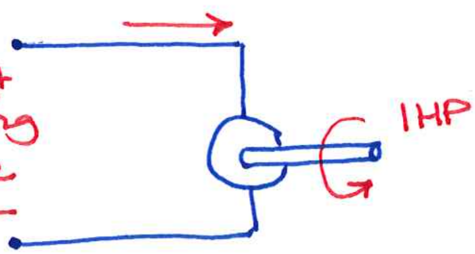
Draws 5.21 A when operating at rated conditions (delivering 1 HP with an operating voltage of 230 V)

$I = 5.21$ A if op voltage = 230 V

+

operating voltage

−

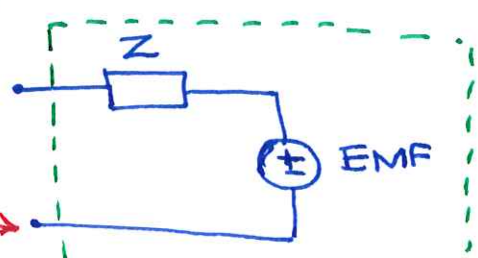


should be close to nominal voltage of 230 V

- if op voltage is 220 V (for example), the motor will draw $5.21 \times \frac{230}{220}$ to maintain the same output power.

- We will model motors (and generator) as:

motor (or gen) terminal



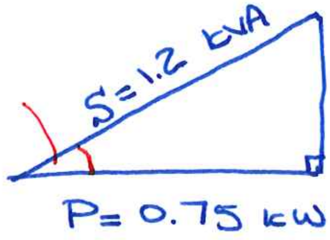
- $S_{rated} = V_{rated} \times I_{rated} = 230V \times 5.21\ A = 1.2\ kVA$

Assuming 100% efficient motor i.e. $P_{\substack{eleç \\ input}} = P_{mech} = 0.75\ kW$,

then we can calculate Q:

positive power angle

Motor is inductive

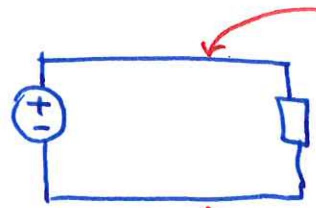


$$Q = \sqrt{1.2^2 - 0.75^2} = 0.94\ kVAr$$

Topic 2: Three Phase Systems

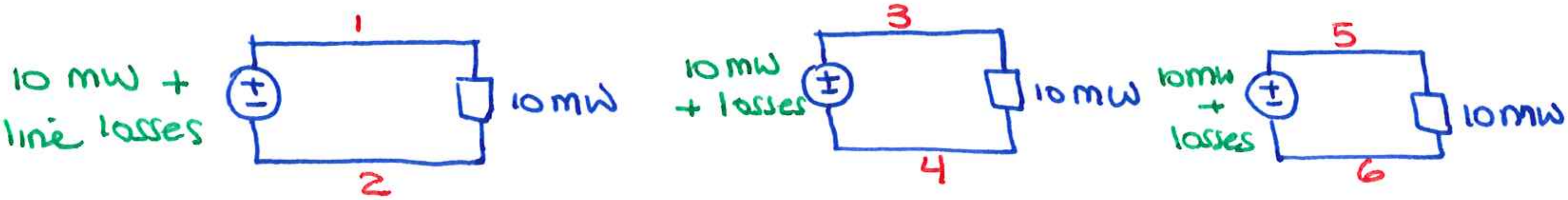
- So far, all AC circuits looked like:

"live" conductor (one phase)



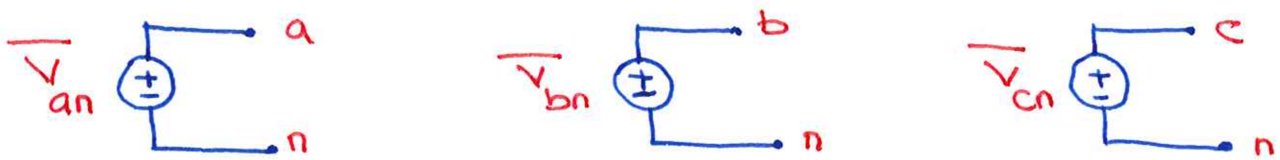
return conductor (neutral)

- Suppose we have 3 identical loads at 10 MW (for example). We can create 3 single phase systems:



  - Total of 6 conductors from gen to load. These can be 100's of km long.
  - In each gen, instantaneous elec power (p(t) from eq (i) in Topic 1) is time-varying but the input mech power is constant. This results in vibration & noise

- Consider 3 single phase sources:

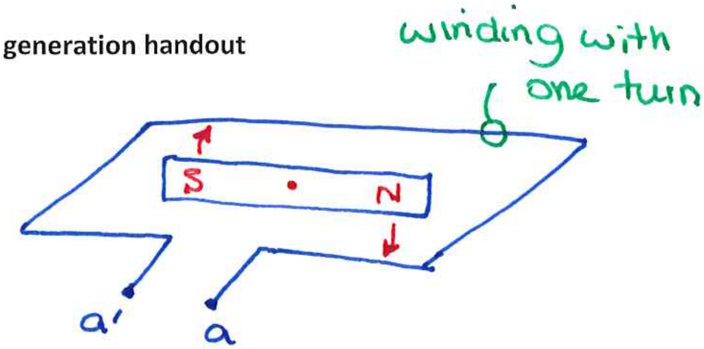


if :
- Voltage magnitudes are the same $V_{an} = V_{bn} = V_{cn}$
- voltage phases are $120°$ apart

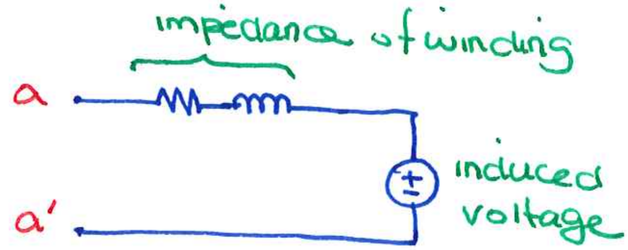
then, by connecting these 3 we will create a **balanced three phase source**. (See handout on AC generation)

Yani's cheap plastic AC generation handout

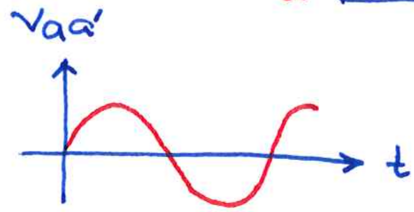
Consider this very simplified representation of an AC generator:



The rotating magnetic field induces a voltage across the aa' winding. The electrical representation of that is:

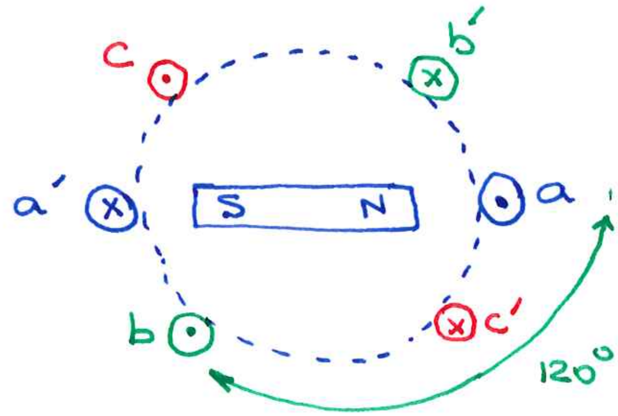


This produces sinusoidal voltage/current:

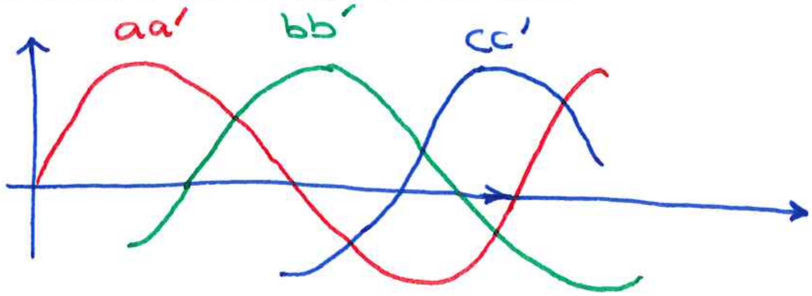


Consider the cross-sectional view of this generator:

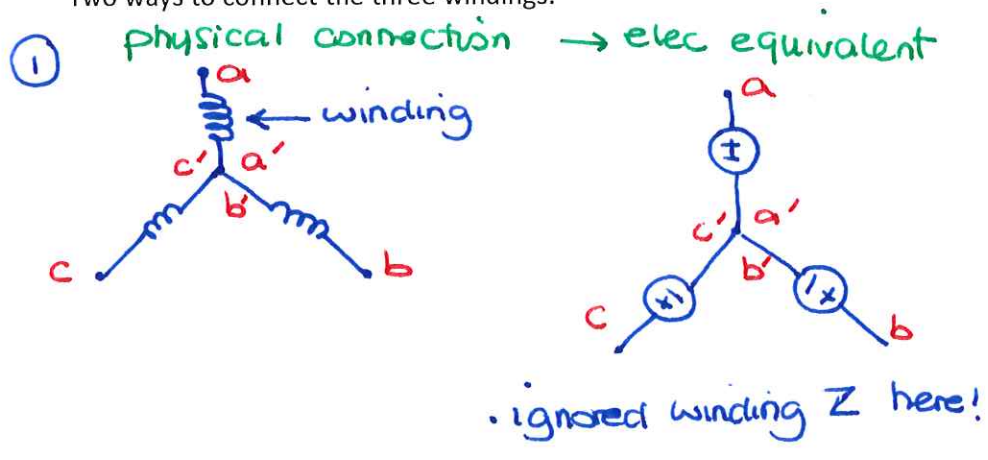
Let's add two other windings bb' and cc', all separated at 120^{0}

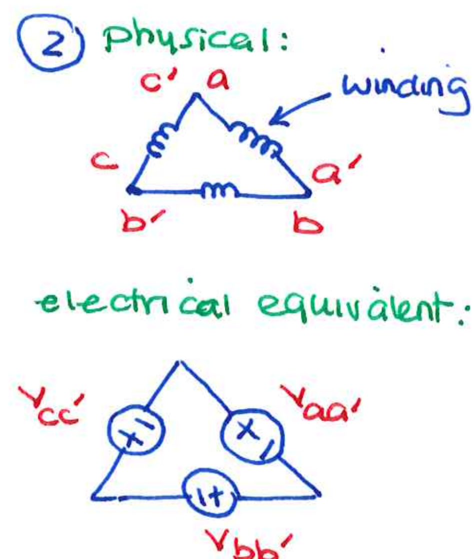


The voltages induced on these windings will be 120^{0} apart:

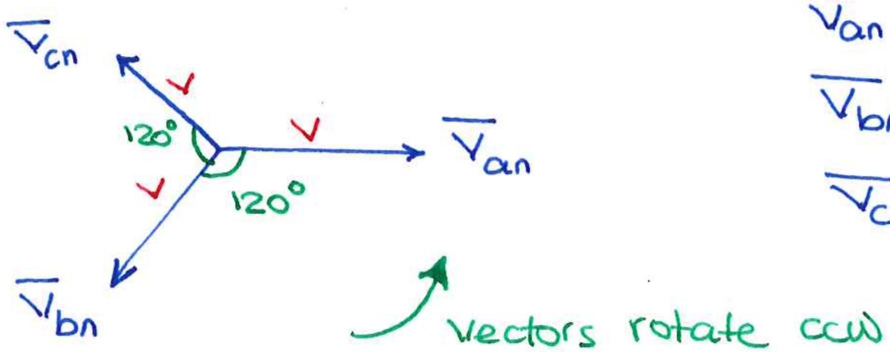


Two ways to connect the three windings:





For example, $\overline{V_{an}}$, $\overline{V_{bn}}$, and $\overline{V_{cn}}$ for a balanced three phase source can look like:



$$\overline{V_{an}} = V\angle 0°$$
$$\overline{V_{bn}} = V\angle -120°$$
$$\overline{V_{cn}} = V\angle +120°$$

this is an abc (positive) rotation; phase a followed by phase b followed by phase c.

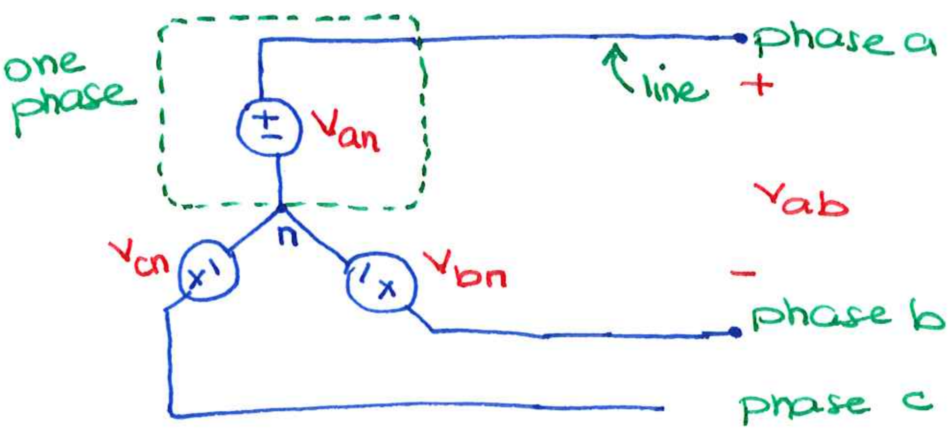
$$\overline{V_{an}} + \overline{V_{bn}} + \overline{V_{cn}} = 0$$

- Two ways to connect three-phase (3ϕ) sources & loads:

  Wye/star (Y) or Delta (Δ) * this does not apply to lines

Wye-Connected Source

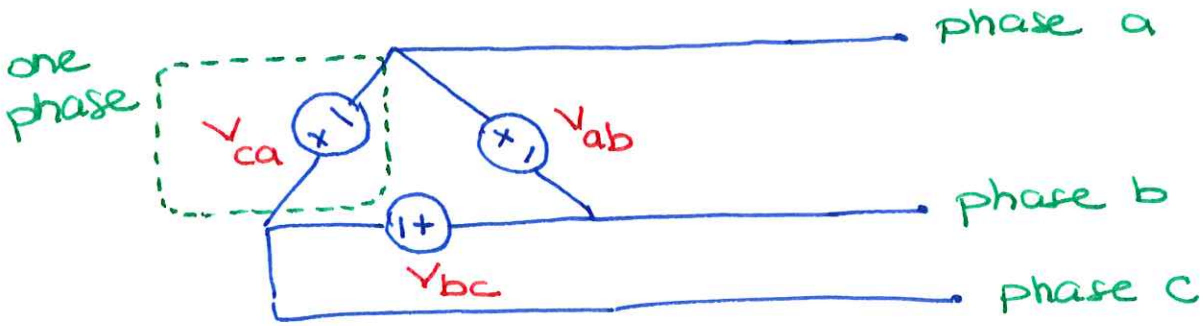
- Same terminology applies to Y-connected loads.



V_{an}, V_{bn}, V_{cn} : line-to-neutral (phase-to-neutral) voltages

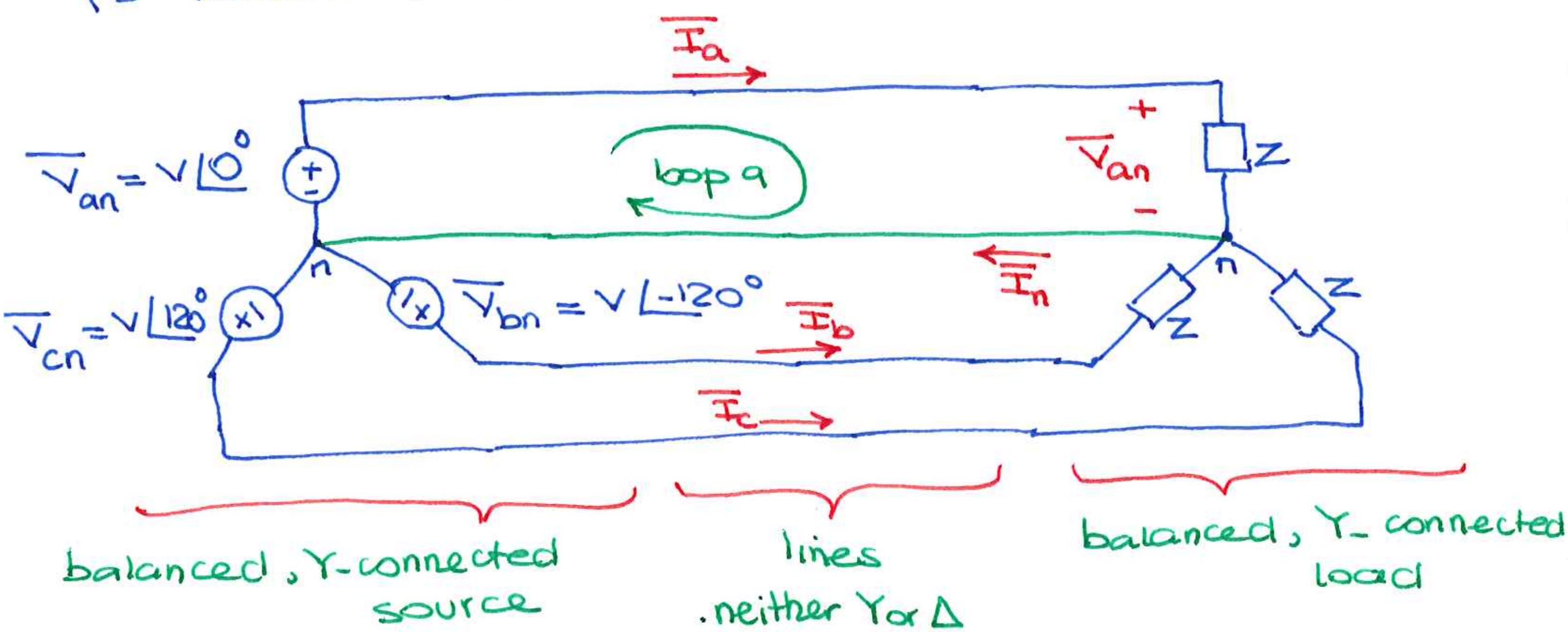
V_{ab}, V_{bc}, V_{ca} : line-to-line (phase-to-phase) voltages

Δ- connected Source



V_{ab}, V_{bc}, V_{ca} : line-to-line voltage

. we can show that the 3ϕ system delivers the same power as 3 single phase systems with ½ the conductors (3 instead of 6):



KVL around loop a: $V\angle 0 - \overline{I_a} \cdot Z = 0 \quad \therefore \quad \overline{I_a} = \frac{V}{Z}$

similarly, $\overline{I_b} = \frac{V\angle -120^\circ}{Z}$, $\overline{I_c} = \frac{V\angle +120^\circ}{Z}$

KCL at neutral point n: $\overline{I_n} = \overline{I_a} + \overline{I_b} + \overline{I_c}$

$$= \frac{V}{Z}\left(\underbrace{1\angle 0^\circ + 1\angle -120^\circ + 1\angle +120^\circ}_{0}\right)$$

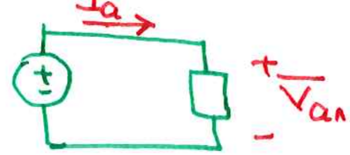
$$= 0$$

i.e. Neutral (return) conductor is not needed.

Power delivered to the 3ϕ load:

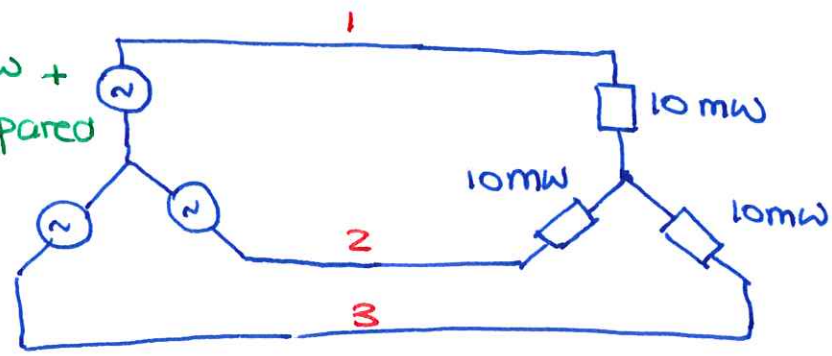
$$\overline{S}_{3\phi} = \underbrace{\overline{V}_{an}\cdot\overline{I}_a^{*}}_{V\angle 0^\circ \cdot \left(\frac{V}{Z}\angle 0^\circ\right)^{*} = \frac{V^2}{Z^*}} + \underbrace{\overline{V}_{bn}\cdot\overline{I}_b^{*}}_{\left(V\angle -120^\circ\right)\left(\frac{V}{Z}\angle -120^\circ\right)^{*} = \frac{V^2}{Z^*}} + \underbrace{\overline{V}_{cn}\cdot\overline{I}_c^{*}}_{= \frac{V^2}{Z^*}}$$

$$\therefore\ \overline{S}_{3\phi} = 3\cdot\frac{V^2}{Z^*} = 3\,\underbrace{\overline{V}_{an}\cdot\overline{I}_a^{*}}_{\text{power in } 1\phi \text{ system}} = 3\cdot\overline{S}_{1\phi}$$



· Back to the original problem:

10mw +
$\frac{1}{2}$ line losses compared
to 1ϕ option



· Not shown in the notes, but also very important:

total $p(t)$ in 3ϕ gen is constant $\therefore$ reduced vibration & noise compared to 1ϕ gen

Important: Voltages & Currents in 3ϕ Systems

- Voltage in each phase of a device (gen, load) is phase voltage
- Current in each phase of a device (gen, load) is phase current