Student ID: ______________________________ March 15, 2019 – 9:00 AM

Student name: ______________________________ Duration: 50 minutes

Problem 1 [10 pts]

The sinusoidal modulating wave: $m(t)=\frac{1}{10}\cdot\sin(8000\pi t)$ is applied to a phase modulator with phase sensitivity k_p = 2 radian per volt. The unmodulated carrier wave has frequency f_c = 1 MHz and amplitude A_c = 1 volt.

a- Determine the instantaneous frequency of this PM signal and sketch it versus time. *[2pts]*
b- Determine the time domain expression of this PM signal and sketch it versus time. *[2pts]*
c- Determine the expression of the frequency spectrum of the resulting PM signal and sketch it. **<u>Show all amplitudes and frequencies of interest</u>**. *[2pts]*
d- Construct a phasor diagram of this PM modulated signal. *[2pts]*
e- Propose of a block diagram of a system that performs this PM modulation using a FM modulator and another system. *[2pts]*

a) $\boxed{f_i(t) = f_c + \frac{k_p}{2\pi}\frac{d\,m(t)}{dt}} = f_c + \frac{1}{\pi}\,8000\pi\,\cos(8000\pi t)$

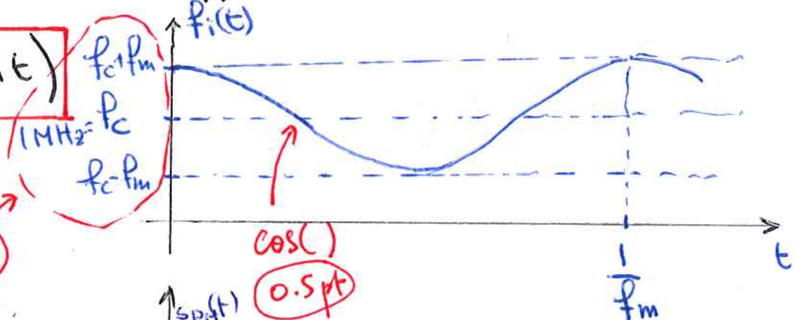
1pt or

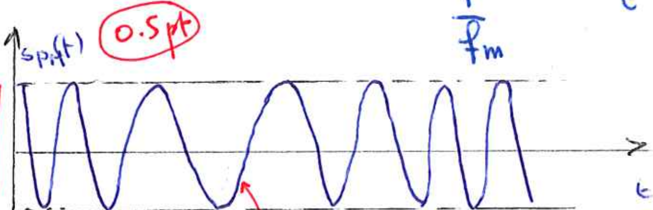
$\boxed{f_i(t) = f_c + 8000\cos(8000\pi t)}$

$f_c = 1$ MHz

& $f_m = 4$ kHz

0.5 pt





b) $\boxed{s_{PM}(t) = A_c\cos\left(2\pi f_c t + k_p\, m(t)\right)}$

1 pt or or

$\boxed{s_{PM}(t) = \cos\left(2\pi f_c t + 0.2\sin(8000\pi t)\right)}$

0.5pt ← Cos(.) with amplitude 1

0.5pt ← freq. varying according to $f_i(t)$

c) $\beta = 0.2 \ll 1$ → narrowband PM.

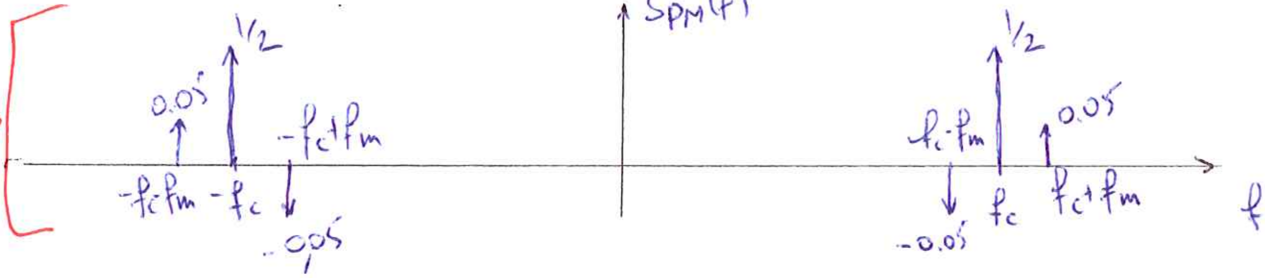
$s_{PM}(t) = \cos(2\pi f_c t) - 0.2\sin(2\pi f_c t)\,\sin(8000\pi t)$

$s_{PM}(t) = \cos(2\pi f_c t) - 0.1\cos\left(2\pi(f_c-f_m)t\right) + 0.1\cos\left(2\pi(f_c+f_m)t\right)$

1pt

$$\boxed{\begin{aligned}S_{PM}(f) = {} & \tfrac{1}{2}\,\delta(f-f_c) + \tfrac{1}{2}\,\delta(f+f_c) - 0.05\,\delta\left(f-(f_c-f_m)\right) - 0.05\,\delta\left(f+(f_c-f_m)\right) \\ & + 0.05\,\delta\left(f-(f_c+f_m)\right) + 0.05\,\delta\left(f+(f_c+f_m)\right)\end{aligned}}$$

1pt

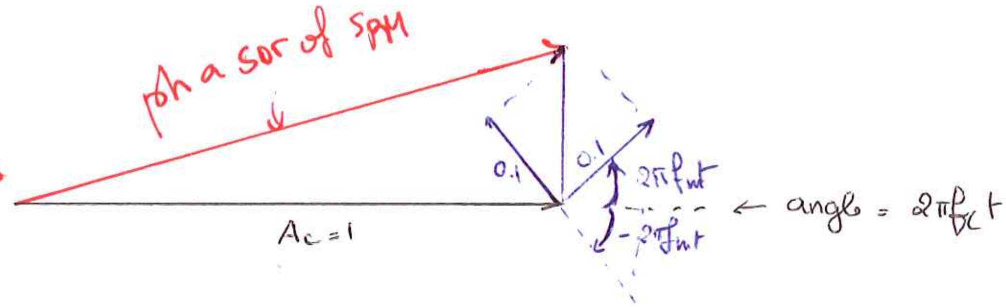


[if there is a mistake in a sign or numerical value, don't take out marks twice: [in the expression and plot]].

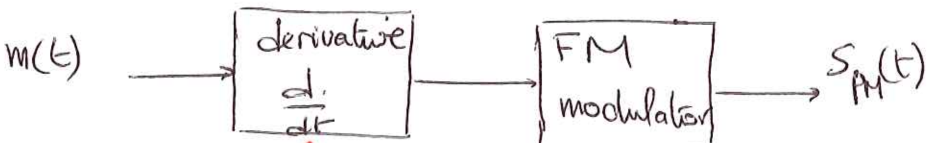
d) $s_{PM}(t) = \cos(2\pi f_c t) - 0.1 \cos(2\pi (f_c - f_m) t) + 0.1 \cos(2\pi (f_c + f_m) t)$

2pts

0.5pt on each component



e)



1pt on the derivative

1pt on the righ order of the cascade

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Problem 2 [10 pts]

An angle-modulated signal around a carrier frequency fc = 1 MHz, has the form

$$s(t) = 5\cos\left(2\pi f_c t + 2\sin(4000\pi t)\right)$$

The modulating message has a maximum amplitude $A_m = \max|m(t)| = 2$.

a. Determine the phase deviation and frequency deviation of $s(t)$. *[2pts]*

b. Determine $m(t)$ and k_f if $s(t)$ is a FM signal. *[2pts]*

c. Determine $m(t)$ and k_p if $s(t)$ is a PM signal. *[2pts]*

d. Determine the 1% bandwidth of $s(t)$. *[2pts]*

e. Sketch the frequency spectrum of the modulated signal $s(t)$. **Show only the sidebands within the approximate bandwidth calculated in d-. Indicate all the frequencies and amplitudes of interest**. (Use the table below). *[2pts]*

Values of the Bessel Functions $J_n(\beta)$

	$\beta=1$	$\beta=2$	$\beta=3$	$\beta=4$
$n=0$	0.7652	0.2239	-0.2601	-0.3971
$n=1$	0.4401	0.5767	0.3391	-0.066
$n=2$	0.1149	0.3528	0.4861	0.3641
$n=3$	0.0196	0.1289	0.3091	0.4302
$n=4$	0.0025	0.034	0.132	0.2811
$n=5$		0.007	0.043	0.1321
$n=6$		0.0012	0.0114	0.0491
$n=7$			0.0025	0.01518
$n=8$				0.004

a) phase deviation: $\boxed{\beta = 2}$ rad ← (1pt)

frequency deviation: $\boxed{\Delta f = 2 f_m = 4\,kHz}$ ← (1pt)

b) $k_f\, m(t) = \frac{1}{2\pi}\frac{d}{dt}\left(2\sin(4000\pi t)\right) = \frac{1}{2\pi} \cdot 8000\pi \cos(4000\pi t)$

(1pt) → $\boxed{m(t) = 2\cos(4000\pi t)}$

and

(1pt) → $\boxed{k_f = 2000 \ Hz/v}$

c) $k_p\, m(t) = 2\sin(4000\pi t) \longrightarrow \boxed{k_p = 1 \ rad/v}$ ← (1pt)

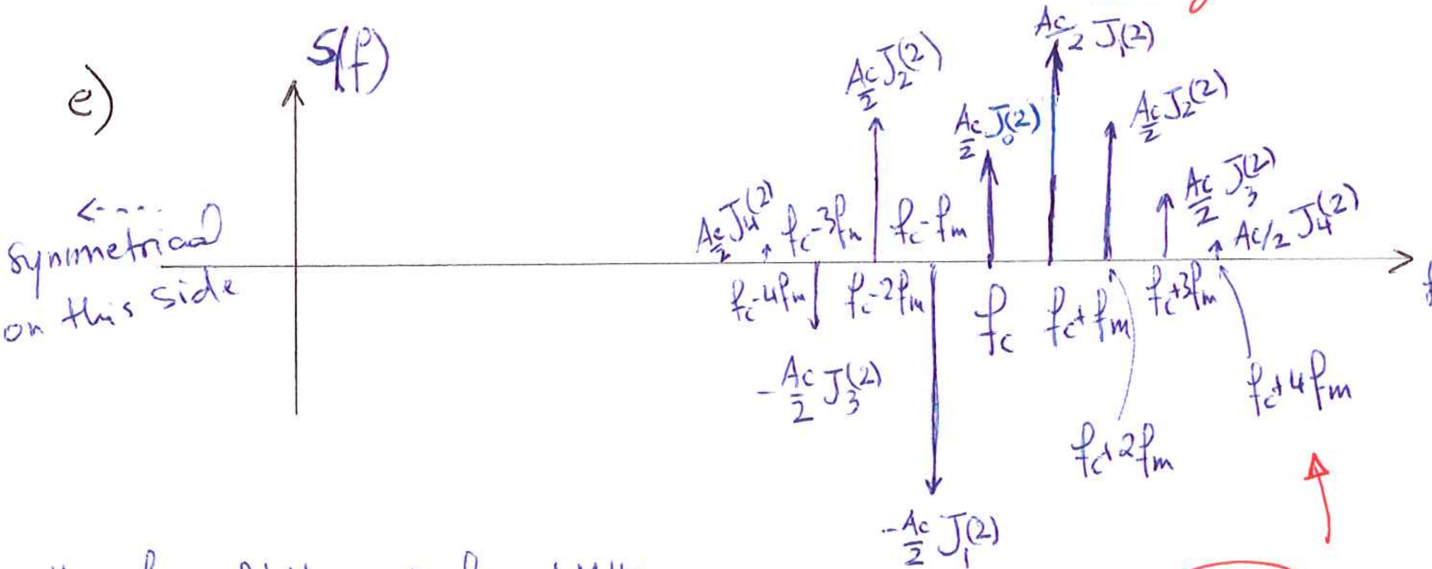
$\boxed{m(t) = 2\sin(4000\pi t)}$ ← (1pt)

d) $\beta = 2 \longrightarrow$ from the table $\boxed{n_{max} = 4} \leftarrow$ 1pt

$B_{1\%} = 2 n_{max} \cdot f_m = 8 \cdot 2kHz = \boxed{16\ kHz} \leftarrow$ 1pt

give full mark if this is correct but missing calculation details

e)



with $f_m = 2kHz$ & $f_c = 1\ MHz$

$$\frac{A_c}{2} J_0(2) = \frac{5}{2} \cdot 0.2239 = 0.55975$$

$$\frac{A_c}{2} J_1(2) = \frac{5}{2} \cdot 0.5765 = 1.44125$$

$$\frac{A_c}{2} J_2(2) = \frac{5}{2} \cdot 0.3528 = 0.882$$

$$\frac{A_c}{2} J_3(2) = \frac{5}{2} \cdot 0.1289 = 0.32225$$

$$\frac{A_c}{2} J_4(2) = \frac{5}{2} \cdot 0.034 = 0.085$$

2pts

- 0.5 on shape: set of impulses
- 0.5 on frequency separation between impulses (f_m)
- 0.5 on amplitudes $\frac{A_c}{2} J_n(\beta)$ [no need to calculate numerical value]
- 0.5 on sign of the impulses

$J_{-n}(\beta) = (-1)^n J_n(\beta)$