VI

Question 1. **(15 marks)**

An electric field is described by:

$$\vec{E}(y,t) = 12\sin(\omega t + \beta y)\vec{a}_x \quad \text{V/m}$$

The frequency is 1 GHz and the field is located in free space (ε_o, μ_o).

3

a) Express the electric field in phasor form ($\mathbf{E}_s(y)$).

$\vec{E}_s(y) = -j12e^{j\beta y}\vec{a}_x$ 1 1 1

5

b) Find the magnetic field in phasor form ($\mathbf{H}_s(y)$). You may keep the expression in terms of β.

1 expression $\nabla \times \vec{E}_s = -j\omega\mu_o\vec{H}_s$

$$\nabla \times \vec{E}_s = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{sx} & 0 & 0 \end{vmatrix}$$

$= \vec{a}_x(0) - \vec{a}_y\left(-\frac{\partial}{\partial z}E_{sx}\right)^{\nearrow 0} + \vec{a}_z\left(-\frac{\partial}{\partial y}E_{sx}\right)$

$= -\frac{\partial}{\partial y}(-j12e^{j\beta y})\vec{a}_z$

1 $= (+j12)(j\beta)e^{j\beta y}\vec{a}_z$

1 applying $\therefore -12\beta e^{j\beta y}\vec{a}_z = -j\omega\mu_o\vec{H}_s$

$\vec{H}_s = \frac{\cancel{-}12\beta e^{j\beta y}}{\cancel{-}j\omega\mu_0}\vec{a}_z$

2 $\boxed{\vec{H}_s = -j\frac{\beta(12)}{\omega\mu_0}e^{j\beta y}\vec{a}_z}$ 1 $\frac{1}{2}$ $\frac{1}{2}$

5

c) Calculate β.

1 applying curl

$$\nabla \times \vec{H}_s = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_{sz} \end{vmatrix}$$

$= \vec{a}_x\left(\frac{\partial}{\partial y}H_{sz}\right) - \vec{a}_y\left(\frac{\partial}{\partial x}H_{sz}\right)^{\nearrow 0} + \vec{a}_z(0)$

$= \frac{\partial}{\partial y}\left(\frac{-j\beta 12}{\omega\mu_0}e^{j\beta y}\right)\vec{a}_x$

$\nabla \times \vec{H}_s = \frac{-j\beta 12}{\omega\mu_0}(j\beta)e^{j\beta y}\vec{a}_x$

expression 1

$= \frac{\beta^2 12}{\omega\mu_0}e^{j\beta y}\vec{a}_x$

$\nabla \times \vec{H}_s = j\omega\varepsilon_0\vec{E}_s$

1 $\therefore \vec{E}_s = \frac{\beta^2 12}{\omega\mu_0}e^{j\beta y}\vec{a}_x$

$= j\omega\varepsilon_0(-j12e^{j\beta y})\vec{a}_x$

$\frac{\beta^2}{\omega\mu_0} = \omega\varepsilon_0$

$\beta^2 = \omega^2\mu_0\varepsilon_0$

1 $\beta = \omega\sqrt{\mu_0\varepsilon_0}$

$\beta = \frac{\omega}{c} = \frac{2\pi f}{c}$

$= 2\pi \times 10^9$

20.95

1 $\therefore \beta = \frac{20\pi}{3}$ rad

d) Write an expression for the instantaneous magnetic field ($\mathbf{H}(y,t)$).

2

$\vec{H}_s = -j\frac{\cancel{12}\left(\frac{20\pi}{3}\right)}{2\pi\times 10^9\mu_0}e^{j\beta y}\vec{a}_z$

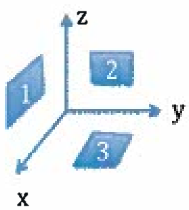
$\frac{1}{10\pi} = 0.0318$

$= j\frac{40}{10^2 \cancel{10^9} \cancel{4\pi\times 10^{-7}}}e^{j\beta y}\vec{a}_z$ 1 $\frac{1}{2}$ $\frac{1}{2}$

V1 ODD

Question 2. **(15 marks)**

Measuring an unknown magnetic field is of interest in many applications. Consider placing 3 small wire loops (1, 2 and 3) such that the surface normals of the loops are orthogonal, as shown in the figure below. Assume that the surface normals are in the $\mathbf{a}_x$, $\mathbf{a}_y$ and $\mathbf{a}_z$ directions.



9

a) Consider the magnetic field:

$$\vec{H}(t) = (3\vec{a}_x - 2\vec{a}_y + 5\vec{a}_z)\cos(\omega t) \quad \text{A/m}$$

If the area of each of the loops is A m^2, find the EMF induced in each of the three loops. Sketch each loop and indicate the direction of induced current flow during the first quarter-period of the signal.

(1) finding total flux

(1) taking time derivative

(1) $V_{emf} = -\frac{d}{dt}\phi$

(1) $d\vec{s} \Rightarrow \vec{a}_y$

$\vec{H} \cdot d\vec{s} = -2\cos\omega t$

$\phi = -2\mu A\cos\omega t$

$V_{emf} = 2A(-\sin\omega t)(\omega)$

$= -2\mu A\omega\sin\omega t$ (1)

(2) $d\vec{s} \rightarrow \vec{a}_x$

$\vec{H} \cdot d\vec{s} = 3\cos\omega t$

$\Phi_2 = 3A\mu_0\cos\omega t$

$V_{emf} = -3A\mu_0(-\sin\omega t)(\omega)$

(1) $= 3A\mu_0\omega\sin\omega t$

(1)

(3) $d\vec{s} \rightarrow \vec{a}_z$

$\vec{H} \cdot d\vec{s} = 5\cos\omega t$

$\Phi_3 = 5A\mu_0\cos\omega t$

$V_{emf} = -5A\mu_0(-\sin\omega t)(\omega)$

(1) $= 5A\mu_0\omega\sin\omega t$

(1)

(1) b) Consider the magnetic field: (1)

$$\vec{H}(t) = (-2\vec{a}_y + 5\vec{a}_z)\cos(\omega t) \quad \text{A/m}$$

3

Assuming the same loop area as in part a, what is the induced EMF in each of the loops for this field?

(1) $V_{emf} = -2A\mu_0\omega\sin\omega t$

(2) 0

(3) $V_{emf} = 5A\mu_0\omega\sin\omega t$

c) Consider the magnetic field:

$$\vec{H}(t) = (3\vec{a}_x - 2\vec{a}_y + 5\vec{a}_z) \text{ A/m}$$

1.5

Assuming the same loop area as in part a, what is the induced EMF for this field?

$\frac{1}{2}$ (1) 0

$\frac{1}{2}$ (2) 0

$\frac{1}{2}$ (3) 0

1.5

d) Based on your answers in parts a to c, are the 3 loops useful for gaining information about an unknown magnetic field?

→ yes, for time-varying fields

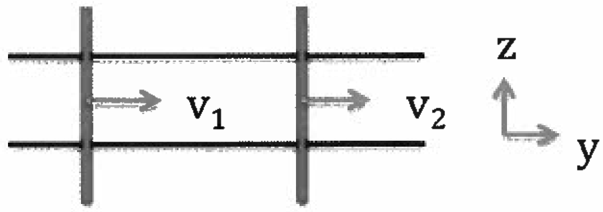
$\frac{1}{2}$ because $V_{emf} \propto$ amplitude of field component in a given direction

1

Question 3. **(10 marks)**

Select one answer for each of the following questions.

a) Two conducting bars slide over two stationary rails, as shown in the figure below.



The rails are separated by 20 cm. The rails and bars are placed in a magnetic flux density given by:

$$\vec{B}(t) = 12\vec{a}_x \quad \text{mWb/m}^2$$

i) If both bars have velocity of 10 m/s, then transformer EMF is present.

TRUE

FALSE

ii) If the bar on the left moves at 5 m/s and the bar on the right moves at 10 m/s, then motional EMF is present.

TRUE

FALSE

iii) For the second case (bars moving at 5 m/s and 10 m/s), the induced EMF is:

Zero

-6 mV

-12 mV

-18 mV

+6 mV

Other version of midterm: 20 m/s & 25 m/s with 10cm separation: −6 mV is the answer

$$\vec{v}\times\vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 0 & 5 & 2 \\ 12 & 0 & 0 \end{vmatrix}$$

$$= \vec{a}_x(0) - \vec{a}_y(0) + \vec{a}_z(-60)$$

$$= -60\vec{a}_z$$

$$V_{emf} = \oint(\vec{v}\times\vec{B})\cdot d\vec{l}$$

$$= \int_0^{0.2} -60\vec{a}_z \cdot dz\,\vec{a}_z$$

$$= -60(0.2)$$

$$= -12\,\text{mV}$$

b) An electric field in teflon (ε_r=2.5) is given by:

$$\vec{E}(y,t) = 12\sin(\omega t + \beta y)\vec{a}_x \text{ V/m}$$

The displacement current density (**J**$_d$(y,t)) is given by:

$$\vec{J}_d(y,t) = 30\varepsilon_o\omega\cos(\omega t + \beta y)\vec{a}_x$$

$$\vec{J}_d(y,t) = 12\varepsilon_o\omega\cos(\omega t + \beta y)\vec{a}_x$$

$$\vec{J}_d(y,t) = 12\varepsilon_o\omega\sin(\omega t + \beta y)\vec{a}_x$$

$$\vec{J}_d(y,t) = -30\varepsilon_o\omega\cos(\omega t + \beta y)\vec{a}_x$$

$$\vec{J}_d = \frac{d}{dt}\vec{D}$$

$$= \frac{d}{dt}(30)\varepsilon_0 \sin(\omega t + \beta y)\vec{a}_x$$

$$= 30\varepsilon_0\omega \cos(\omega t + \beta y)\vec{a}_x$$

This is answer 3 on other version of midterm.

c) A distortionless transmission line operates at 150 MHz. The impedance of the line is 75 ohms. The attenuation constant is α=0.06 Np/m. The phase velocity is v_p=2.8 x 10^8 m/s.

Which of the following combinations represents R, L, G and C?

R= 4.5 ohms/m, L=2.678 H/m, C=4.761 F/m, G=80 S/m
R= 5625 ohms/m, L=11.25 H/m, C=0.002 F/m, G=6.4 x 10^{-7} S/m
R= 4.5 ohms/m, L=2.678 x 10^{-7} H/m, C=4.761x10^{-11} F/m, G=8 x 10^{-4} S/m
Not possible to calculate.

This is answer 1 on other version of midterm.

$Z_0 = 75$ $\quad Z_0 = \sqrt{L/C}$

$\alpha = 0.06$ Np/m $\quad \alpha = \sqrt{RG}$

$V_p = 2.8 \times 10^8$ m/s $\rightarrow V_p = \omega/\beta$

$$\therefore \beta = \frac{150\times10^6 \times 2\pi}{2.8\times10^8}$$

$$R/L = G/C$$