3 RW

Name: Solution ID: 2015487

SCHULICH School of Engineering

UNIVERSITY OF CALGARY

ENEL 487 Final Examination

Wednesday, April 29, 2015

Time: 12:00 – 3:00 pm

Location: Auxiliary Gym

Instructor: Pouyan (Yani) Jazayeri

- Please note that the official University of Calgary examination regulations are printed on page 2 of this paper.
- Exam consists of 7 problems and 16 pages.
- Write answers in the space provided below each question.
- Show your work neatly in the work area. Otherwise, marks for partially correct answers cannot be given.
- Total marks for the exam is 100.
- Closed book exam. You may not refer to books or notes during the test.
- No wireless devices or earphones allowed during exam.
- Only scientific calculators without formulae storage and text display are allowed.

Question	1	2	3	4	5	6	7	Total
Mark	/15	/15	/15	/15	/15	/15	/10	/100

Problem 1:

Answer the following questions on the scantron sheet provided to you. Correct answers are rewarded 1 point.

1) For a purely capacitive element under sinusoidal steady-state excitation, the voltage and current phasors are:

   a) in phase

   b) perpendicular to each other with **V** leading **I**

   c) perpendicular to each other with **I** leading **V**

   d) None of the above

$\bar{V} = Z\bar{I}$, $(j\omega C)^{-1}$

$\bar{V} = \left(\frac{1}{\omega C}\angle -90^\circ\right)\cdot\bar{I}$

$\therefore \theta_V = \theta_I - 90^\circ$

2) The average power in a single-phase ac circuit with a purely inductive load, for sinusoidal steady-state excitation is:

   a) $(I_{rms})^2 X_L$

   b) $(V_{max})^2/X_L$

   c) Zero

   d) None of the above

$P_{avg} = 0$

3) The admittance of a -j 1/2 Ω impedance is:

   a) -j2 S

   b) j2 S

   c) -j4 S

   d) None of the above

$Y = \frac{1}{Z} = \frac{1}{-j\frac{1}{2}} = j2$

4) For a balanced Δ load supplied by a balanced source, the line currents into the load are √3 times the load currents and lag by 30 degrees.

   a) True

   b) False

5) Transmission line conductance is usually neglected in power system studies.

   a) True

   b) False

6) Bundling reduces the series reactance of the line?

   a) Yes

   b) No

7) In the Newton Raphson method, the Jacobian Matrix J(x) consists of

   a) Partial derivatives

   b) Inverses

   c) P_{branch} and Q_{branch} expressions

   d) None of the above

8) Which of the following variables is known for a load bus?

a) Real Power consumed by the load

b) Voltage angle

c) Jacobian matrix

d) Voltage magnitude

9) The Ybus of a 14 bus systems is a ________ matrix

a) 14×1

b) (14 – 1)×14

c) 14×14

d) Not enough information given

10) The bus selected as the slack bus must have a source of both real and reactive power.

a) True

b) False

11) Per Unit quantity has the same unit/dimension as that of the actual quantity.

a) True

b) False

12) The Alberta market has a demand of 5000MW at the moment. The offers received from the generators are as follows: generator A offers 1000MW at $15, generator B offers 1000MW at $25, generator C offers 4000MW at $0, generator D offers 2000MW at $20, and generator E offers 2000MW at $10. What is the market clearing price?

a) $20

b) $0

c) $15

d) None of the above

13) Which of the following is <u>not</u> a common heat dissipation technique for transformers?

a) Forced air

b) Forced oil

c) Natural air flow

d) None of the above (i.e. They are all acceptable)

14) Select the material with the best conductivity.

a) Gold

b) Aluminum

c) Copper

d) Silver

15) In design of ACSR conductors, more aluminum results in lower resistance while more steel results in higher strength.

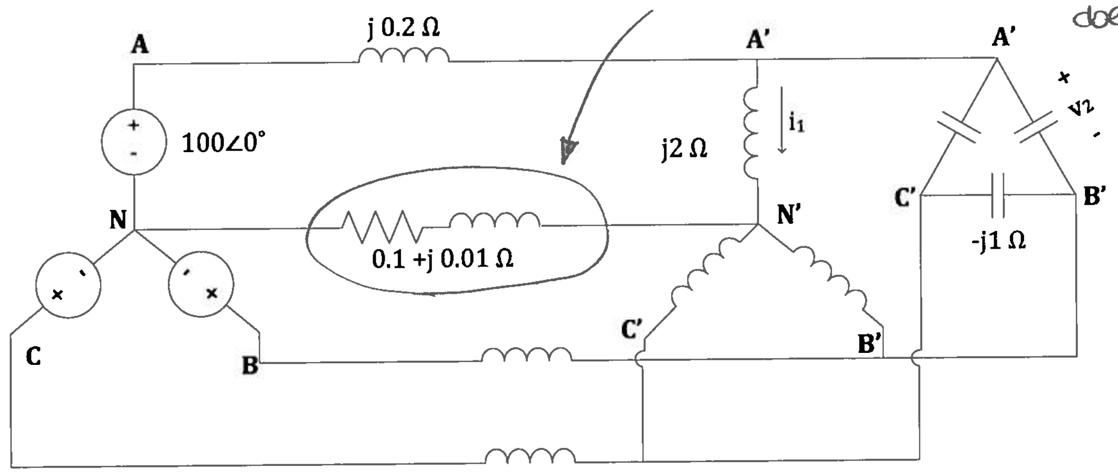
a) True

b) False

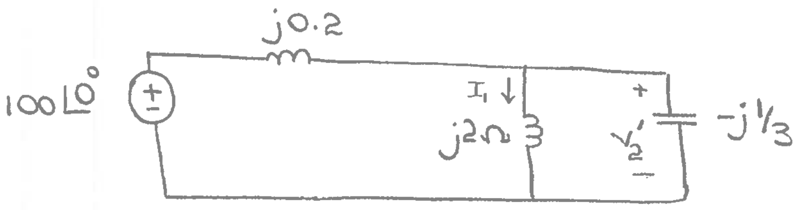
Problem 2:

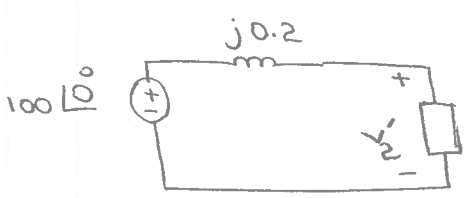
Consider the balanced three-phase system shown below. Determine $v_2(t)$ and $i_1(t)$. [15 marks]

no current in the neutral. This impedance does not show up in 1ϕ circuit



• convert Δ-connected load to Y & perform 1ϕ analysis :





$Z = j2 \parallel -j\tfrac{1}{3} = -j0.4$

$$\bar{V}_2' = 100 \frac{-j0.4}{-j0.4 + j0.2} = 200 \angle 0$$

$$\bar{V}_2 = \sqrt{3}\angle 30^\circ \cdot \bar{V}_2' = 200\sqrt{3} \angle 30^\circ \quad \text{or} \quad 346.4\angle 30^\circ$$

$$v_2(t) = \sqrt{2} \cdot 200\sqrt{3} \cos(\omega t + 30^\circ) \quad \text{or} \quad 489.9 \cos(\omega t + 30^\circ)$$

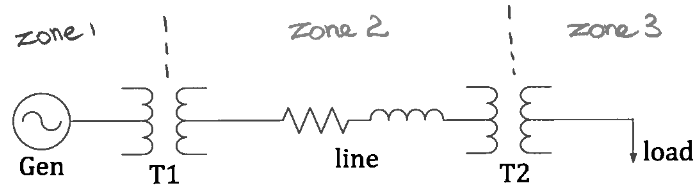
$$\bar{I}_1 = \frac{\bar{V}_2'}{j2} = \frac{200}{j2} = -j100 \quad \text{or} \quad 100\angle -90^\circ$$

$$i_1(t) = 100\sqrt{2} \cos(\omega t - 90^\circ) \quad \text{or} \quad 141.4 \cos(\omega t - 90^\circ)$$

Problem 3:

The three phase system shown below has the following ratings:

Generator:	60 MVA,	13.8kV (line-to-line),	$X_{gen} = 0.2$ pu
T1:	50 MVA,	13.2 kV/132 kV,	$X_{T1} = 0.091$ pu
T2:	50 MVA,	138 kV/13.8 kV,	$X_{T2} = 0.083$ pu
Line:	15.87 + j79.35 Ω per phase		
Load:	Z Ω per phase (i.e. unknown impedance)		



Select a power base of 60 MVA for the system and a base voltage of 13.8kV for the generator zone. Under these base values, the system was analyzed. It was calculated that the load is drawing a current of $0.7\angle{-40^\circ}$ pu, and its terminal voltage is $1\angle 0^\circ$ pu.

a) Draw the impedance diagram of the system in per unit. The load impedance can be labeled as Z_{pu}. You will need numerical values for all other impedances. [8 marks]

b) Find the load's active and reactive power in per unit. Hint: You can use $S_{pu} = V_{pu}I_{pu}^*$ [3 marks]

c) Find the actual quantity of the load's three phase active and reactive power in MW and MVAr. [1 marks]

d) What is the line current at the load terminals in Amps? [1 mark]

e) Find the total active power losses in the line in per unit and MW. [2 marks]

a) $V_{base_1} = 13.8 \text{ kV}$ (given)

$$Z_{base_2} = \frac{V_{base_2}^2}{S_{base}} = \frac{(138\text{kV})^2}{60 \text{ MVA}} = 317.4$$

$$V_{base_2} = V_{base_1} \times \frac{132}{13.2} = 138 \text{ kV}$$

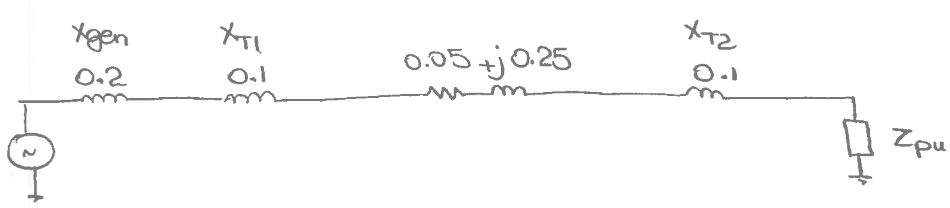
$$V_{base_3} = V_{base_2} \times \frac{13.8}{138} = 13.8 \text{ kV}$$

$X_{gen} = 0.2$ pu (does not change)

$$X_{T1} = 0.091 \times \left(\frac{13.2\text{ kV}}{13.8\text{ kV}}\right)^2 \frac{60 \text{ MVA}}{50 \text{ MVA}} \approx 0.1 \text{ pu}$$

$$X_{T2} = 0.083 \times \left(\frac{13.8\text{ kV}}{13.8\text{ kV}}\right)^2 \frac{60 \text{ MVA}}{50 \text{ MVA}} \approx 0.1 \text{ pu}$$

$$Z_{line} = \frac{Z_{actual}}{Z_{base_2}} = \frac{15.87 + j79.35}{317.4} = 0.05 + j0.25$$



b) $S_{pu} = V_{pu} \cdot I_{pu}^{*} = 1\angle 0^\circ \times (0.7\angle -40^\circ)^{*} = 0.7\angle 40^\circ$ pu

$= \underbrace{0.54}_{P_{pu}} + j\,\underbrace{0.45}_{Q_{pu}}$ pu

c) $P_{3\phi} = P_{pu} \times S_{base} = 0.54 \times 60\ MW = 32.17\ MW$

$Q_{3\phi} = Q_{pu} \times S_{base} = 0.45 \times 60\ MVAr = 27\ MVAr$

d) $I_{base3} = \dfrac{S_{base}}{\sqrt{3}\, V_{base3}} = \dfrac{60\ MVA}{\sqrt{3} \times 13.8kV} = 2510\ A$

$I_{load} = 0.7\angle -40^\circ \times I_{base3} = 1757\angle -40^\circ\ A$

e) $P_{loss,pu} = I_{pu}^{2} \times R = (0.7)^2 \times 0.05 = 0.0245$ pu

$P_{loss} = P_{loss,pu} \times S_{base} = 1.47\ MW$

Problem 4:

A 60-Hz, 200-km, three-phase overhead transmission line, constructed of ACSR conductors, has a series impedance of (0.2+ j0.8) Ω/km per phase and admittance of j10x10-6 S/km per phase. Using the nominal π circuit, compute ABCD parameters and the voltage and current at the sending end if the load at the receiving end draws 200 MVA at 0.9 lagging and at a line-to-line voltage of 230 kV. [15 marks]

$$Z = z.\ell = (0.2 + j0.8) \times 200 = 40 + j160 = 164.9 \angle 75.6^\circ \ \Omega$$

$$Y = y.\ell = (j10 \times 10^{-6}) \times 200 = j2 \times 10^{-3}$$

$$A = D = 1 + \frac{YZ}{2} = 0.84 + j0.04 = 0.841 \angle 2.73^\circ$$

$$B = Z = 164.9 \angle 75.6^\circ \ \Omega$$

$$C = Y\left(1 + \frac{ZY}{4}\right) = 0.0018 \angle 91.25^\circ \ S$$

$$V_r = \frac{230 \text{ kV}}{\sqrt{3}} = 132.8 \text{ kV} \quad \text{(line to neutral)}$$

$$|I_r| = \frac{S}{\sqrt{3}\, V_{r,\ell-\ell}} = \frac{200 \times 10^6}{\sqrt{3} \times 230 \times 10^3} = 502.04$$

$$\cos(\underset{0^\circ}{\theta_{V_r}} - \theta_{I_r}) = pf \quad \therefore \quad \theta_{I_r} = -\cos^{-1}(0.9) = -25.84$$

$$\therefore \quad I_r = 502.04 \angle -25.84$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$$

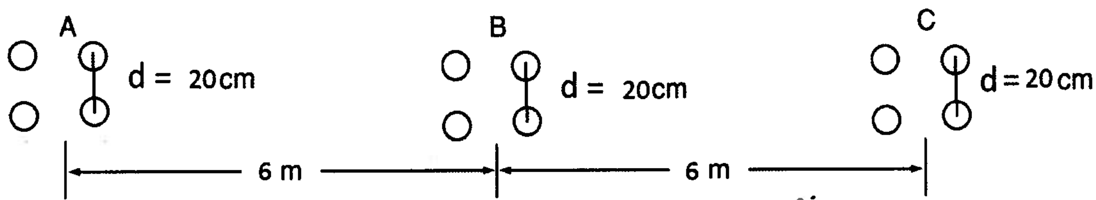
$$\therefore \quad V_s = A.V_r + B.I_r = 178.7 \angle 22.5 \ \text{kV}$$

$$|V_{s,\ell-\ell}| = \sqrt{3} \times 183.5 = 309.5 \ \text{kV}$$

$$I_s = C.V_r + D.I_r = 390.1 \angle 10.82^\circ \ \text{A}$$

Problem 5:

Figure below shows the conductor configuration of a completely transposed, three-phase overhead transmission line with bundled phase conductors. All conductors have a radius of 0.7 cm.



assume: $D_S = r'$

a) Determine the inductance per phase in mH/km. [4 marks]

b) Determine the inductive line reactance per phase in Ω/km at 60 Hz. Also, calculate the reactive power absorbed by the inductance if a 100 km transmission line with this configuration is operating at 1000A. [4 marks]

c) Determine the line-to-neutral capacitance in nF/km per phase. [3 marks]

d) Determine the capacitive reactance in Ω/km per phase. Also, calculate the reactive power supplied by the reactance if a 100 km transmission line with this configuration is operating at 200kV. [4 marks]

a) $L = 2\times10^{-7} \ln \frac{D_{eq}}{D_{SL}}$

$D_{SL} = 1.091 \sqrt[4]{D_S \cdot d^3} = 1.091 \sqrt[4]{r' \cdot d^3} = 1.091 \sqrt[4]{(0.7788 \times 0.007)(0.2)^3}$

$= 0.0887 \text{ m}$

$D_{eq} = \sqrt[3]{6\times6\times12} = 7.56 \text{ m}$

$\therefore \quad L = 2\times10^{-7} \ln \frac{7.56\text{m}}{0.0887\text{ m}} = 0.889 \text{ mH/km}$

b) $X_L = \omega L = 2\pi f \times L = 2\pi 60 \times .889 \text{ mH/km} = 0.335 \ \Omega/\text{km}$

$Q_L = 3\times(I^2 \cdot X_L) = 3\ (1000\text{A})^2\ (100 \text{ km} \times 0.335\ \Omega/\text{km}) = 100.5 \text{ mVAr}$

c) $C_{an} = \frac{2\pi\varepsilon}{\ln \frac{D_{eq}}{D_{SC}}}$ $\quad D_{eq}$ same as before; $D_{eq} = 7.56\text{m}$

$D_{SC} = 1.091 \sqrt[4]{r \cdot d^3} = 1.091 \sqrt[4]{(0.007)(0.2)^3} = 0.0944 \text{ m}$

$C_{an} = \frac{2\pi(8.85\times10^{-12})}{\ln \frac{7.56\text{ m}}{0.0944\text{ m}}} = 12.7 \text{ nF/km}$

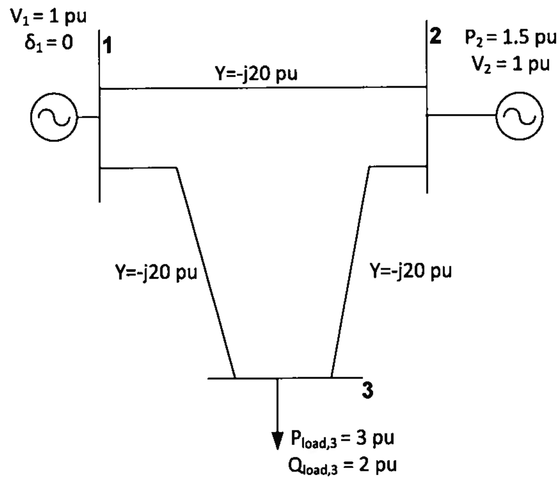
d) $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\times60\times12.7\text{ nF/km}} = 2.09\times10^5\ \Omega/\text{km}$

$Q_C = 3\times -\frac{V_{LN}^2}{X_C} = 3\times \frac{(\frac{V_{LL}}{\sqrt{3}})^2}{X_C} = -\frac{V_{LL}^2}{X_C} = \frac{-(200\text{ kV})^2}{2.09\times10^5\ \Omega/\text{km} \times 100\text{ km}}$

$= -1.91 \text{ kVAr}$

Problem 6:

Considering the system below:



a) Complete the following table. Do not include P_{load} and Q_{load} in this table. [3 marks]

Bus number	Bus type	Known variables	Unknown variables
1	Slack	V , δ	Pgen , Qgen
2	PV	V , Pgen	δ , Qgen
3	load	Pgen , Qgen	V , δ

b) Create the Y_{bus} matrix for this system [2 marks]

c) Write and simplify the active power flow equations for buses 1 and 2, and reactive power flow equation for bus 1. Plug in all the known variables, expand the equations, keep the angles in degrees, and simplify as much as possible. [6 Marks]

d) Write the expression for the element in row 3, column 3 of J_{11} and row 1, column 1 of J_{12} [4 marks]

b) $$Y_{bus} = \begin{bmatrix} -j40 & j20 & j20 \\ j20 & -j40 & j20 \\ j20 & j20 & -j40 \end{bmatrix}$$

c) $$f_1 = P_{gen,1} - \cancel{P_{load,1}} - V_1 V_1 G[\cancel{1},1] \cos(\delta_1 - \delta_1) - V_1 V_1 B[1,1] \sin(\delta_1 - \cancel{\delta_1})$$
$$- V_1 V_2 G[\cancel{1},2] \cos(\delta_1 - \delta_2) - V_1 V_2 B[1,2] \sin(\delta_1 - \delta_2)$$
$$- V_1 V_3 G[\cancel{1},3] \cos(\delta_1 - \delta_3) - V_1 V_3 B[1,3] \sin(\delta_1 - \delta_3)$$

$$= P_{gen,1} - (1)(1)(20)\sin(-\delta_2) - V_3 (20) \sin(-\delta_3)$$

$$= P_{gen,1} + 20 \sin\delta_2 + 20 V_3 \cdot \sin(\delta_3)$$

$$f_2 = P_{gen,2} - \cancel{P_{load,2}} - V_2 V_1 B[2,1] \sin(\delta_2 - \delta_1) - V_2 V_3 B[2,3] \sin(\delta_2 - \delta_3)$$

$$= 1.5 - (1)(1)(20)\sin(\delta_2) - (1) V_3 (20) \sin(\delta_2 - \delta_3)$$

$$= 1.5 - 20\sin(\delta_2) - 20 V_3 \sin(\delta_2 - \delta_3)$$

$$f_4 = Q_{gen,1} - \cancel{Q_{load,1}} + V_1 V_2 B[1,2] \cos(\delta_1 - \delta_2) + (V_1)^2 B[1,1] \cos(\delta_1 - \delta_1)$$
$$+ V_1 V_3 B[1,3] \cos(\delta_1 - \delta_3)$$

$$= Q_{gen,1} + 20\cos(-\delta_2) - 40 + 20 V_3 \cos(-\delta_3)$$

d) $J_{11}[3,3] = \dfrac{\partial f_3}{\partial x_3} = \dfrac{\partial f_3}{\partial \delta_3}$

$$f_3 = P_{gen,3} - P_{load,3} - V_3 V_1 B[3,1] \sin(\delta_3 - \delta_1) - V_3 V_2 B[3,2] \sin(\delta_3 - \delta_2)$$

$$= 0 - 3 - 20 V_3 \sin(\delta_3) - 20 V_3 \sin(\delta_3 - \delta_2)$$

$$\frac{\partial f_3}{\partial \delta_3} = -20 V_3 \cos(\delta_3) - 20 V_3 \cos(\delta_3 - \delta_2)$$

$$J_{12}[1,2] = \frac{\partial f_1}{\partial x_5} = \frac{\partial f_1}{\partial Q_{gen,2}} = 0$$

$J_{12}[1,1] = \partial f_1 / \partial x_4 = \partial P_{eq,1} / \partial Q_{gen,1} = 0$

Problem 7:

The following table shows the branch data of a 4 bus system:

Line number	Bus to bus	Branch resistance, R (p.u)	Branch Reactance, X (p.u)
1	1-2	0.564	0.981
2	2-3	0.873	0.765
3	2-4	1.092	0.45
4	3-4	0.5	1
5	4-1	0.199	0.203

a) Calculate the elements in $Y_{bus}[1,1]$ and $Y_{bus}[1,4]$? [4 marks]

b) A capacitor bank of $Z=$ j0.2 pu ~~Ω~~ is connected to bus 1. Will the Y_{bus} matrix change? If yes, calculate the modified elements in the Y_{bus}. If not, explain why. Also, state one reason for adding a capacitor bank to a bus in the power system. [6 marks]

a) $y_{12} = \frac{1}{Z_{12}} = \frac{1}{0.564 + j\,0.981} = 0.44 - j\,0.77$ pu

$y_{14} = \frac{1}{Z_{14}} = \frac{1}{0.199 + j\,0.203} = 2.46 - j\,2.51$ pu

$Y_{Bus}[1,1] = \sum$ all y connected to 1 $= y_{12} + y_{14} = 2.9 - j\,3.28$ pu

$Y_{Bus}[1,4] = -y_{14} = -2.46 + j\,2.51$ pu

b) yes, only $Y_{Bus}[1,1]$ will change. It now includes the admittance of cap bank.

$Y_{Bus}[1,1]_{new} = Y_{Bus}[1,1]_{old} + Y_{cap} = (2.9 - j3.28) + \frac{1}{-j0.2}$

$= (2.9 - j\,3.28) + (j5)$

$= 2.9 + j\,1.72$

cap bank added for power factor correction (less losses in line)

or improving the voltage