University of Calgary

Schulich School of Engineering

Department of Electrical and Computer Engineering

ENEL 476 – Electromagnetic Waves and Applications

Midterm Examination

Winter Session 2012
Wednesday March 7
6:30 – 8:00 pm

ENA 101 and ENA 103

Student Name or ID number:

Dr Fear

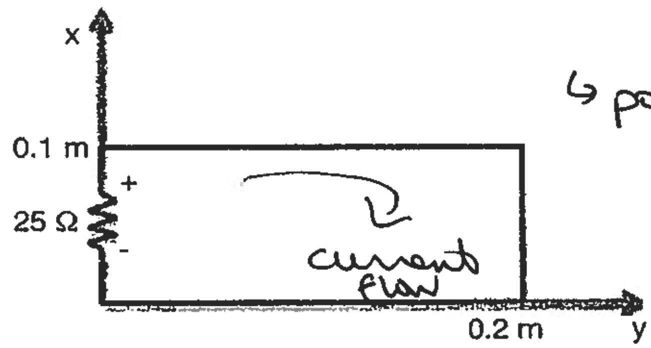
Question 1. **(2 marks each x 6 questions = 12 marks)**

Answer the following questions on the bubble sheet.

i) A conducting loop is placed in an external magnetic field. The magnetic field is described by:

$$\vec{B}(t) = 10\cos(10^6 t)\vec{a}_z \ \text{ mWb/m}$$

One side of the loop contains a resistor with R=25 Ω, as shown in the figure below.



↳ polarity: initially, $\vec{B}$ is decreasing ∴ induced flux in +z direction + current flow is as shown

∴ $V_{emf} = -200\sin(10^6 t)$

Assuming negligible resistance in the rest of the loop, the voltage across the resistor is:

A) 200 sin(10^6t) V
B) -200 sin(10^6t) V
C) 0.2 sin(10^6t) MV
D) -0.2 sin(10^6t) MV

→ Right-hand Rule ⇒ z is "into page"

→ $V_{emf} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$

$\vec{B}\cdot d\vec{s} = 10\cos(10^6 t)\,\vec{a}_z \cdot dx\,dy\,\vec{a}_z$

$= 10\cos(10^6 t)\,dx\,dy$

$\int_0^{0.2}\int_0^{0.1} 10\cos(10^6 t)\,dx\,dy$

$= (10)(0.1)(0.2)\cos(10^6 t)$

$= 0.2\cos(10^6 t)$

→ $-\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$

$= -\frac{d}{dt}(0.2\cos 10^6 t)$

$= -0.2(-\sin 10^6 t)(10^6)$

$= 2\times10^5 \sin 10^6 t$

but $\vec{B}$ has units of mW

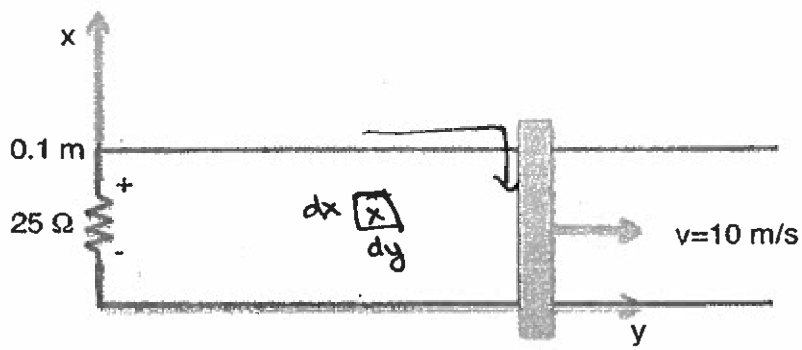
∴ $V_{emf} = 2\times10^2 \sin 10^6 t$

$= 200\sin 10^6 t$,

~~polarity: at time t=0 → t = T/4, B is decreasing ∴ current flows~~

ii) Consider the loop shown in the figure below. This is similar to part (i), however the loop has been modified to represent a rod that slides on parallel rails. The rod moves with velocity v= 10 a_y m/s. Assume that the location of the rod at t=0 is y=0. The loop is placed in an external magnetic field described by:

$$\vec{B}(t) = 10\cos(10^6 t)\vec{a}_z \text{ mWb/m}$$



$\vec{a}_z$ is into page
∴ go around loop clockwise such that fingers follow path & thumb is in direction of $d\vec{s}$ (RHR)

The total induced electromagnetic force (EMF) in the loop is:

~~A) 10^4 t sin(10^6t) + 0.01 cos(10^6t) V~~
B) 10^4 t sin(10^6t) - 0.01 cos(10^6t) V
~~C) 10^7 t sin(10^6t) +10 cos(10^6t) V~~
D) 10^7 t sin(10^6t) -10 cos(10^6t) V

$$V_{emf} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$$

$$= -\int \frac{\partial}{\partial t}\vec{B}\cdot d\vec{s} + \int(\vec{v}\times\vec{B})\cdot d\vec{l}$$

$$-\int \frac{\partial}{\partial t}\vec{B}\cdot d\vec{s} = -\int_0^y\int_0^{0.1} -(10)(10^6)\overset{\text{mWb}}{(10^{-3})}(\sin 10^6 t)\,dx\,dy$$

$$= 10^4 \sin(10^6 t)(0.1)(y)$$

$y = ?$ $\quad \frac{dy}{dt} = v \Rightarrow \frac{dy}{dt} = 10\,\vec{a}_y$

$\therefore y = 10t$

$\& \quad -\int \frac{\partial}{\partial t}\vec{B}\cdot d\vec{s} = 10^4 t \sin 10^6 t$

$\int(\vec{v}\times\vec{B})\cdot d\vec{l}$
$= ?$

$$\vec{v}\times\vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 0 & 10 & 0 \\ 0 & 0 & B_z \end{vmatrix}$$

$$= \vec{a}_x(10B_z)$$

$$\int_{0.1}^{0}(\vec{v}\times\vec{B})\cdot d\vec{l}$$

$$= \int_{0.1}^{0} 10B_z\,dx$$

$$= 100\cos(10^6 t)(-0.1)$$

3

$$= -10\cos(10^6 t)$$

but $\vec{B} \to$ mWb/m

$\therefore -0.01\cos(10^6 t)$

iii) A lossless dielectric medium has ε_r=9, μ_r=1 and σ=0 S/m. Time-varying fields are present in the dielectric and described by:

$$\vec{E} = 5\cos(\omega t + \pi z)\vec{a}_y \text{ V/m}$$

$$\vec{H} = \frac{5}{\eta}\cos(\omega t + \pi z)\vec{a}_x \text{ A/m}$$

The value of ω is:

A) $\frac{\pi^2}{9\mu_o\varepsilon_o}$

B) $\frac{\pi}{3\sqrt{\mu_o\varepsilon_o}}$

C) $\frac{3\sqrt{\mu_o\varepsilon_o}}{\pi}$

D) $\frac{1}{3\sqrt{\mu_o\varepsilon_o}}$

$\nabla \times \vec{E}_s = -j\omega\vec{B}_s$

$\vec{E}_s = 5e^{j\pi z}\vec{a}_y \; ; \; \vec{H}_s = \frac{5}{\eta}e^{j\pi z}\vec{a}_x$

$\nabla \times \vec{E}_s = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & E & 0 \end{vmatrix}$

$= \vec{a}_x\left(-\frac{\partial}{\partial z}E\right)$

$= -j5\pi e^{j\pi z}\vec{a}_x$

iv) For the fields in (iii), the value of η is:

A) $\sqrt{\frac{\mu_o}{\varepsilon_o}}$

B) $\frac{1}{3}\sqrt{\frac{\mu_o}{\varepsilon_o}}$

C) $\frac{1}{3}\sqrt{\frac{\varepsilon_o}{\mu_o}}$

D) $\frac{1}{3}\sqrt{\frac{1}{\mu_o\varepsilon_o}}$

$-j\omega\vec{B}_s = -j\omega\mu_0\frac{5}{\eta}e^{j\pi z}\vec{a}_x$

$\therefore \cancel{-}\cancel{j}\cancel{5}\pi e^{j\pi z} = \cancel{-}\cancel{j}\omega\mu_0\frac{\cancel{5}}{\eta}e^{j\pi z}$

ok, not crossed out

$\therefore$ (1) $\boxed{\pi = \frac{\omega\mu_0}{\eta}} \rightarrow \eta = \frac{\omega\mu_0}{\pi}$

(1) $\eta = \frac{\omega\mu_0}{\pi}$

(2) $\pi = 9\epsilon_0\omega\eta$

$\pi = 9\epsilon_0\omega\left(\frac{\omega\mu_0}{\pi}\right)$

$\frac{\pi^2}{9\epsilon_0\mu_0} = \omega^2 \Rightarrow \omega = \frac{\pi}{3\sqrt{\mu_0\epsilon_0}}$

$\eta = \frac{1}{3}\left(\sqrt{\frac{\mu_0}{\cdots}}\right)$

$\nabla \times \vec{H}_s = j\omega 9\epsilon_0\vec{E}_s$

$\nabla \times \vec{H}_s = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_s & 0 & 0 \end{vmatrix}$

$= \vec{a}_x(0) - \vec{a}_y\left(-\frac{\partial}{\partial z}H_s\right) + \vec{a}_z(0)$

$= \frac{5}{\eta}j\pi e^{j\pi z}$

$j\omega 9\epsilon_0\vec{E}_s =$

$j\omega(9\epsilon_0)(5e^{j\pi z})$

$\therefore \cancel{j}\pi\frac{\cancel{5}}{\eta}e^{j\pi z} = \cancel{j}\omega(9\epsilon_0)\cancel{5e}$

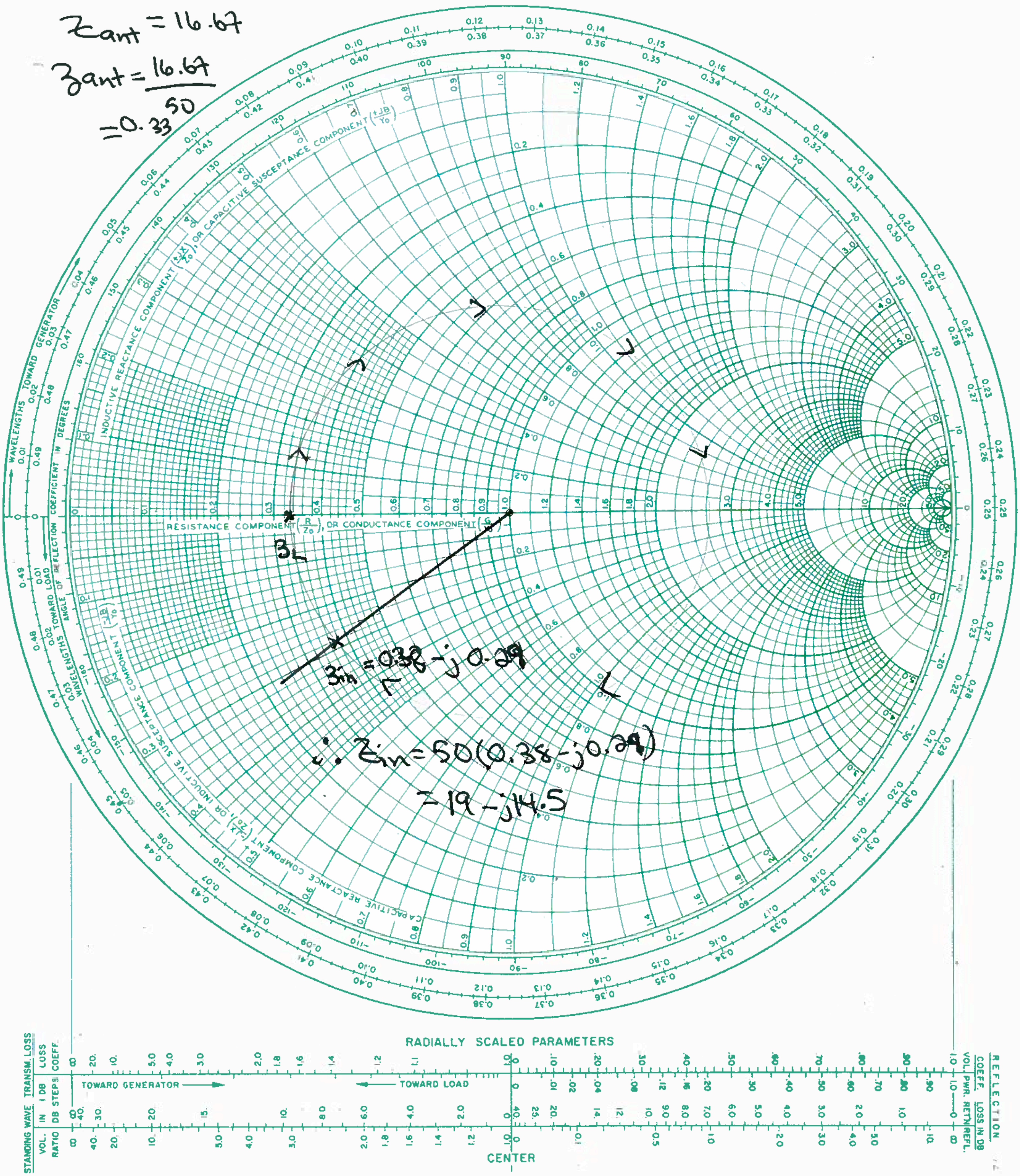
$\boxed{\frac{\pi}{\eta} = 9\omega\epsilon_0}$ (2)

Q1 vi)

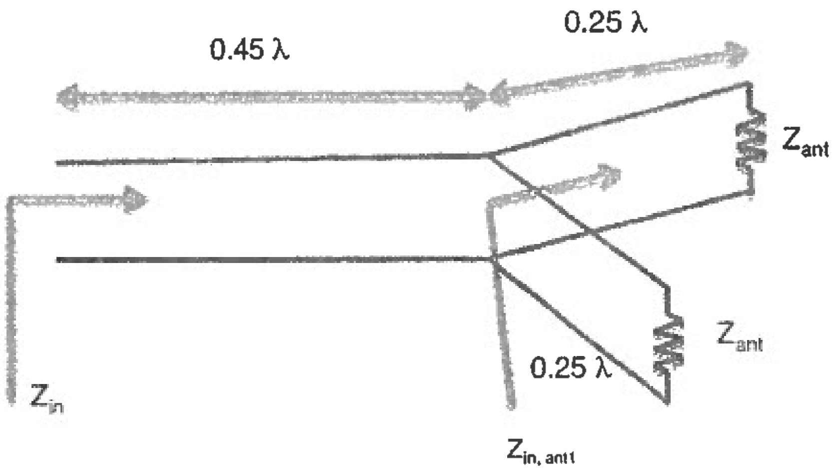
$Z_{ant} = 16.67$

$z_{ant} = \frac{16.67}{50} = 0.33$

IMPEDANCE OR ADMITTANCE COORDINATES



v) Two dipole antennas are connected in parallel with the transmission lines shown in the figure below. Each antenna has an impedance of $Z_{ant}=75\ \Omega$ and the lossless transmission lines have $Z_o=50\ \Omega$.



The figure indicates the input impedance looking down one 0.25λ line towards an antenna; only one input impedance is shown to increase the clarity of the figure (i.e. there is a second input impedance looking down the other 0.25 λ line towards the other antenna).

The impedance looking down the line at antenna 1 ($Z_{in,ant1}$) is:

A) ~~Not possible~~ to calculate.
B) 33.33 Ω
C) 112.5 Ω
D) 75 Ω

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0\tan(\pi/2)}{Z_0 + jZ_L\tan(\pi/2)}\right]$$
$$= Z_0^2/Z_L$$

vi) In the figure above, the input impedance of the antennas and transmission lines (Z_{in}) is:

$$\therefore Z_{in} = \left(\frac{50^2}{75}\right)$$
$$=$$

closest result

A) 18-j15 Ω
B) ~~15-j18 Ω~~
C) 16.65 Ω
D) 18+j15 Ω

$$Z_{antennas} = \frac{1}{\frac{1}{33.3}+\frac{1}{33.3}}$$
$$= 16.67\ \Omega$$

→ Smith chart: Rotate 0.45λ from load

↳ $\beta l = \left(\frac{2\pi}{\lambda}\right)(0.45\lambda)$
$= 0.9\pi$

Check: $Z_{in} = 50\left[\frac{16.67 + j50\tan 0.9\pi}{50 + j16.67\tan 0.9\pi}\right] = 18.2 - j14.3$

Question 2. **(13 marks)**

Consider a lossless Z_o=75 Ω transmission line terminated with load Z_L=125-j30 Ω. Using the Smith chart:

a) Plot z_L

b) Find the reflection coefficient, Γ.

c) Find the standing wave ratio, s.

d) Find the input impedance (Z_{in}) at a distance of 0.2λ from the load (and towards the generator). Next, calculate Z_{in} with the appropriate formula and explain any discrepancies between your results.

e) Find the input admittance (Y_{in}) at a distance of 0.1λ from the load.

f) Find the location of the first voltage maximum from the load.

g) Assuming that a lumped element matching network may be used to match the load to the line, sketch the appropriate circuit (you do not have to solve for the values, just sketch the configuration of elements that you would use).

g) z_L is inside r=1 circle

(1)

a) $z_L = \frac{Z_L}{Z_0}$

$= \frac{125 - j30}{75}$

(1) $= 1.67 - j0.4$ (1) plot on SC

value

b) $\Gamma = 0.3\angle -20^\circ$

(1) (1)

Check: $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$

$= 0.29\angle -22.4^\circ$

c) $s = \frac{1 + |\Gamma|}{1 - |\Gamma|}$

(1) $= 1.86$

or $s = 1.87$ from chart

d) (1) $z_{in} = 0.525 - j0.13$

$\beta l = \frac{2\pi}{\lambda}(0.2\lambda)$

$n = 39.4 - j\,9.75$

$Z_{in} = 75 \left[\frac{125 - j30 + j75\tan(0.4\pi)}{75 + j(125 - j30)\tan(0.4\pi)} \right]$

(1) $= 41.8 - j6.2$

$\Rightarrow$ accuracy of Smith chart (1) weak

e) (1) $y_{in} = 0.85 + j0.55$

(1) $Y_{in} = 0.01 + j0.007$

f) V_{max} is at 0.25λ

& load is at 0.277λ

$\therefore [0.5 - (0.277 - 0.25)]\lambda$

$= (0.5 - 0.027)\lambda$

(1) $= 0.473\lambda$

Q3

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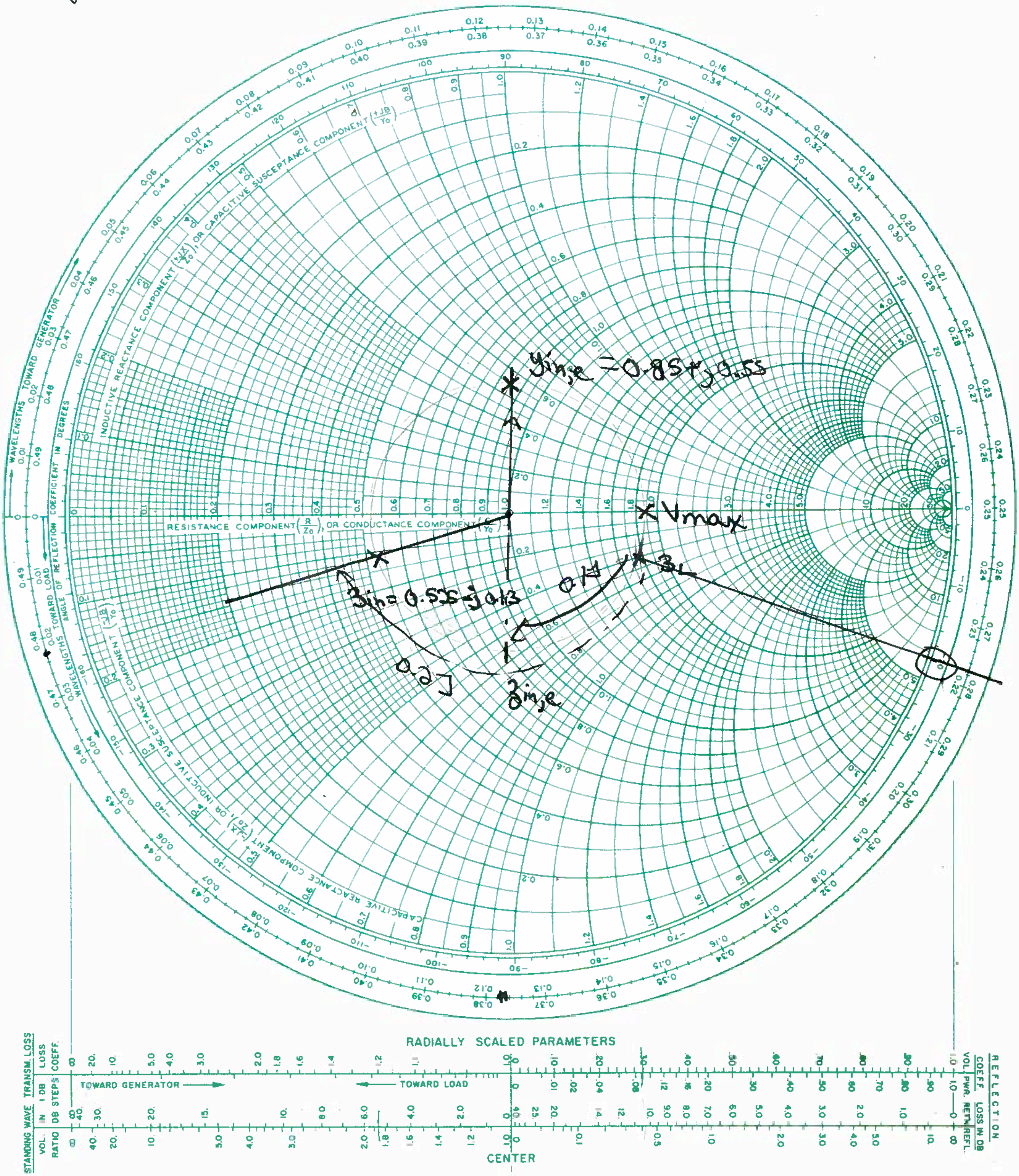

Question 3. (12 marks)

An antenna has an input impedance of Z_L=75+j10 Ω. This antenna is to be connected to a lossless transmission line with Z_o=100 Ω and phase velocity v_p=2.0 x 10^8 m/s. The frequency of operation is 5.0 GHz. You are asked to design a shunt (parallel) stub tuner to match the antenna to the line.

a) Design two different tuners using shorted stubs. Give the lengths and locations of the stubs in wavelengths and in meters.

b) Which of the two designs would you recommend and why?

a) $z_L = 0.75 + j0.1$

$f = 5\times10^9$

$v_p = 2.0\times10^8$

$v_p = \omega/\beta$

$\beta = \frac{2\pi}{\lambda}$

$v_p = \frac{\omega}{2\pi/\lambda}$

$= \frac{2\pi f \lambda}{2\pi}$

$v_p = f\lambda$

$\therefore \lambda = v_p/f$

$= \frac{2\times10^8}{5\times10^9}$

$= 2/50$ m

$= 0.04$m

1 $= 4$cm

• 6 marks on chart

• Now, plot z_L.
& then plot y_L.

• Rotate from y_L to $y_1 = 1 - j0.3$

location: $(0.362 - 0.292)\lambda$
$= 0.07\lambda$

• To cancel $-j0.3$ with shorted stub, need $+j0.3$ so rotate to stub length
length: $(0.25 + 0.046)\lambda$
$= .296\lambda$

• Sol'n two: rotate from y_L to
$y_1' = 1 + j0.3$

location: $[(0.5 - 0.292) + 0.139]\lambda$
$= 0.347\lambda$

• To cancel $+j0.3$, need shorted stub with $-j0.3$
length: $(0.454 - 0.25)\lambda$
$= 0.204\lambda$

(-1) if not in cm also

location	length
→ 0.07λ (1)	0.296λ (1)
(0.28cm)	(1.184cm)
→ 0.347λ (1)	0.204λ (1)
[illegible]	[illegible]

1 ⇒ b) choose 2ND sol'n for bandwidth

Student	
Q1	
Q2	
Q3	
Total	

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