# Solutions to selected suggested questions

**Prob. 9.1**

$$V = -\frac{\partial \psi}{\partial t} = -\frac{\partial}{\partial t}\int B \bullet dS = -\frac{\partial B}{\partial t} \bullet S$$

$$= 3770 \sin 377t \text{ x } \pi(0.2)^2 \text{ x } 10^{-3}$$

$$= \underline{\underline{0.4738 \sin 377t \text{ V}}}$$

**Prob.9.4**

Measuring the induced emf in the clockwise direction,

$$V_{emf} = \oint (u \times B) \bullet dl$$

$$= \int_0^{1.2} (5a_x \times 0.2a_z) \bullet dy a_y + \int_{1.2}^{0} (15a_x \times 0.2a_z) dy a_y$$

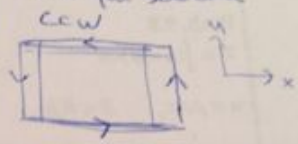
$$= -\int_0^{1.2} (1) dy - \int_{1.2}^{0} (3) dy$$

$$= -1.2 + 1.2 \times 3 = -1.2 + 3.6$$

$$= \underline{\underline{2.4 \text{ V}}}$$

alternative:

↳ if you selected CCW




$$V_{emf} = \int_0^{1.2} (15a_x \times 0.2a_z) dy\, a_y + \int_{1.2}^{0} 5a_x \times 0.2a_z\, dy\, a_y$$

$$= -3y\Big|_0^{1.2} - y\Big|_{1.2}^{0}$$

$$= -3(1.2) - (-1.2)$$

$$= -3.6 + 1.2$$

$$V_{emf} = -2.4 \text{ V}$$ → same mag, opp polarity assumed dir'n opposite to actual!

**Prob. 9.12**

$$V_{emf} = uB\ell = 410 \times 0.4 \times 10^{-6} \times 36 = \underline{\underline{5.904 \text{ mV}}}$$

**Prob. 9.18**

$$\frac{J_d}{J} = \frac{\omega \varepsilon E}{\sigma E} = \frac{\omega \varepsilon}{\sigma} = 1 \quad \longrightarrow \quad \omega = \frac{\sigma}{\varepsilon} = \frac{10^{-4}}{3 \times \frac{10^{-9}}{36\pi}} = 12\pi \times 10^5$$

$$2\pi f = 12\pi \times 10^5 \quad \longrightarrow \quad f = \underline{\underline{600 \text{ kHz}}}$$

Prob. 9.22

$$\nabla \cdot E = 0 \longrightarrow (1)$$

$$\nabla \cdot H = 0 \longrightarrow (2)$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \longrightarrow (3)$$

$$\nabla \times E = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y(x,t) & 0 \end{bmatrix}$$

$$= \frac{\partial E_y}{\partial x} \boldsymbol{a}_z = -E_o \sin x \cos t \boldsymbol{a}_z$$

$$H = -\frac{1}{\mu} \int \nabla \times E dt = \frac{E_o}{\mu_o} \sin x \sin t \boldsymbol{a}_z$$

$$\nabla \times \boldsymbol{H} = \varepsilon \frac{\partial E}{\partial t} \longrightarrow (4)$$

$$\nabla \times \boldsymbol{H} = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_z(x,t) \end{bmatrix}$$

$$= -\frac{\partial H_z}{\partial x} \boldsymbol{a}_y = -\frac{E_o}{\mu_o} \cos x \sin t \boldsymbol{a}_y$$

$$E = \frac{1}{\varepsilon} \int \nabla \times \boldsymbol{H} dt = \frac{E_o}{\mu_o \varepsilon} \cos x \cos t \boldsymbol{a}_y$$

which is off the given $\boldsymbol{E}$ by a factor. Thus, Maxwell's equations (1) to (3) are satisfied, but (4) is not. The only way (4) is satisfied is for $\mu_o \varepsilon = 1$ which is not true.

**Prob. 9.29**

(a) $J_d = \frac{\partial D}{\partial t} \longrightarrow D = \int J_d \, dt$

$$D = \frac{-60\times10^{-3}}{10^9}\cos(10^9 t - \beta z)\mathbf{a}_x = \underline{-60\times10^{-12}\cos(10^9 t - \beta z)\mathbf{a}_x \text{ C/m}^2}$$

$$\nabla\times E = -\mu\frac{\partial H}{\partial t} \longrightarrow \nabla\times\frac{D}{\varepsilon} = -\mu\frac{\partial H}{\partial t}$$

$$\nabla\times\frac{D}{\varepsilon} = \frac{1}{\varepsilon}\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ & & \\ D_x & 0 & 0 \end{vmatrix} = \frac{1}{\varepsilon}(-60)(-1)\times10^{-12}\sin(10^9 t - \beta z)\mathbf{a}_z$$

$$= \frac{60\beta}{\varepsilon}\times10^{-12}\sin(10^9 t - \beta z)\mathbf{a}_y$$

$$H = -\frac{1}{\mu}\int\nabla\times\frac{D}{\varepsilon}dt = -\frac{1}{\mu}(-1)\frac{60\beta}{\varepsilon}\times\frac{10^{-12}}{10^9}\cos(10^9 t - \beta z)\mathbf{a}_y$$

$$= \underline{\frac{60\beta}{\mu\varepsilon}\times10^{-21}\cos(10^9 t - \beta z)\mathbf{a}_y \text{ A/m}}$$

(b) $\nabla\times H = J + J_d = 0 + J_d$

$$J_d = \nabla\times H = \begin{vmatrix} \frac{\partial}{\partial_x} & \frac{\partial}{\partial_y} & \frac{\partial}{\partial_z} \\ & & \\ 0 & H_y & 0 \end{vmatrix} = \frac{(-\beta)(-1)60\beta}{\mu\varepsilon}\times(10^{-21})\sin(10^9 t - \beta z)\mathbf{a}_x$$

Equating this with the given $J_d$

$$60\times10^{-3} = \frac{60\beta^2}{\mu\varepsilon}\times10^{-21}$$

$$\beta^2 = \mu\varepsilon\cdot10^{18} = 2\times4\pi\times10^{-7}\times10\times\frac{10^{-9}}{36\pi} = \frac{2000}{9}$$

$\beta = \underline{14.907 \text{ rad/m}}$

$\beta = \omega\sqrt{\mu_0\varepsilon_0\mu_r\varepsilon_r}$

$= \frac{10^9}{3\times10^8}\sqrt{20}$

**Prob. 9.39**

(a) $A = 5\cos(2t + \pi/3 - \pi/2)\boldsymbol{a}_x + 3\cos(2t + 30^o)\boldsymbol{a}_y = \mathrm{Re}(A_s e^{j\omega t}), \omega = 2$

$$A_s = 5e^{-j30^o}\boldsymbol{a}_x + 3e^{j30^o}\boldsymbol{a}_y$$

(b) $B = \frac{100}{\rho}\cos(\omega t - 2\pi z - 90^o)\boldsymbol{a}_\rho$

$$B_s = \frac{100}{\rho}e^{-j(2\pi z + 90^o)}\boldsymbol{a}_\rho$$

(c) $C = \frac{\cos\theta}{r}\cos(\omega t - 3r - 90^o)\boldsymbol{a}_\theta$

$$C_s = \frac{\cos\theta}{r}e^{-j(3r+90^o)}\boldsymbol{a}_z$$

(d) $D_s = 10\cos(k_1 x)e^{-jk_2 z}\boldsymbol{a}_y$