Uniform plane waves in materials ①

↳ $\epsilon = \epsilon_R \epsilon_0$ ↳ source-free region

$\mu = \mu_R \mu_0$ $\rho_v = 0$

$\sigma \neq 0$

↳ $\nabla \times \vec{E}_s = -j\omega \vec{B}_s$

$$\nabla \times \vec{H}_s = \underbrace{j\omega \vec{D}_s + \boxed{\sigma \vec{E}_s}}$$

$\nabla \cdot \vec{E}_s = 0$

$\nabla \cdot \vec{B}_s = 0$

↳ $$\underbrace{\nabla \times \nabla} \times \vec{E}_s = -j\omega \boxed{\nabla \times \vec{B}_s} \rightarrow \mu \boxed{\nabla \times \vec{H}_s}$$

$$\underbrace{\nabla(\nabla \cdot \vec{E}_s)}_{0} - \nabla^2 \vec{E}_s = -j\omega\mu(j\omega \underbrace{\vec{D}_s}_{\hookrightarrow \epsilon \vec{E}_s} + \sigma \vec{E}_s)$$

$\Rightarrow$ $\nabla^2 \vec{E}_s + (\omega^2 \mu\epsilon - j\omega\mu\sigma)\vec{E}_s = 0$

$$\boxed{\nabla^2} \vec{E}_s - \gamma^2 \vec{E}_s = 0$$

$\gamma^2 = -\omega^2 \mu\epsilon + j\omega\mu\sigma$; $\gamma = \alpha + j\beta$

$\Rightarrow$ UPW assumptions → E_x, H_y, propagation in z

→ no change in x or y for $\vec{E}$ & $\vec{H}$

$$\frac{d^2 E_{s,x}}{dz^2} - \gamma^2 E_{s,x} = 0$$

$$\Rightarrow E_{s,x}(z) = \boxed{E^+} e^{-\gamma z} + \boxed{E^-} e^{\gamma z} \longrightarrow |E^-| e^{j\phi^-}$$

"initial conditions" $\Rightarrow$ $|E^+| e^{j\phi^+}$

$$E_{s,x}(z) = \underbrace{E^{+} e^{-\alpha z}}_{\text{amplitude}} e^{-j\beta z} + \underbrace{E^{-} e^{\alpha z}}_{} e^{+j\beta z}$$

$$E_x(z,t) = |E^{+}|\underbrace{e^{-\alpha z}}_{\text{attenuation}}\cos(\omega t - \boxed{\beta} z + \phi^{+}) + |E^{-}|e^{\alpha z}\cos(\omega t \boxed{+} \beta \boxed{z} + \phi^{-})$$

⇒ forward ⇒ backward

$\gamma = \alpha + j\beta$

↑ phase constant (rad/m)

↑ attenuation → Np/m ; 1Np = decrease to $\frac{1}{e}$ of original value

$$\alpha = \omega\left(\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{(1)+\left(\frac{\sigma}{\omega\epsilon}\right)^{2}} \boxed{-} 1\right]}\right)$$

$$\beta = \omega\left(\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^{2}} + 1\right]}\right)$$

↳ $\lambda = \frac{2\pi}{\beta}$

$v_p = \omega/\beta$

↳ skin depth:

$\delta = \frac{1}{\alpha}$ → depth at which wave is $\frac{1}{e}$ of initial value

$\vec{H} = ?$ $\quad \vec{E}_s = E^{+} e^{-\alpha z} e^{-j\beta z} \hat{a}_x$

$$\nabla \times \vec{E}_s = -j\omega\mu \vec{H}_s$$

$$= -(\alpha + j\beta)E^{+} e^{-\alpha z} e^{-j\beta z} \hat{a}_y$$

$$\Rightarrow \vec{H}_s = \underbrace{\left(\frac{\alpha + j\beta}{j\omega\mu}\right)}_{\text{modified amplitude}} E^{+} \underbrace{e^{-\alpha z} e^{-j\beta z}}_{\text{same attenuation + phase}} \underbrace{\hat{a}_y}_{\text{RHR}}$$

$$|\eta| = \frac{|\vec{E}_s|}{|\vec{H}_s|}$$

$$= \frac{\sqrt{\mu/\epsilon}}{\left[1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{\frac{1}{4}}} \quad ; \tan(2\theta_\eta) = \frac{\sigma}{\omega\epsilon}$$

initial

$$\vec{H}(z,t) = \frac{|E^+|}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z + \phi^+ - \theta_\eta)\vec{a}_y$$

Ex// $f = 3\text{GHz}$
$\epsilon_R = 7$
$\sigma = 0.6\ \text{S/m}$
$\mu_R = 1$

- $\vec{E}$ in y-direction, amplitude at $x=0$ is 10V/m, wave propagates in $-x$ direction

a) Find α, β, η b) Find $\vec{E}$ & $\vec{H}$

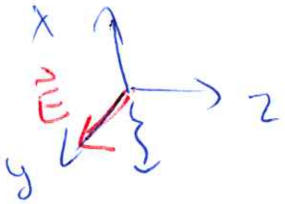
a) $$\frac{\sigma}{\omega\epsilon} = \frac{0.6}{(2\pi\times10^9\times3)(7)(8.85\times10^{-12})} \Rightarrow \alpha = 47.9\ \text{Np/m}$$

$\beta = 173.1$ rad/m

$\eta = 131.9\angle 0.27\ \Omega$

b) $$\vec{E}(x,t) = 10e^{47.9x}\cos(6\pi\times10^9 t + 173.1x)\vec{a}_y$$

$$\vec{H}(x,t) = \frac{-10}{131.9}e^{47.9x}\cos[(6\pi\times10^9 t + 173.1x) - 0.27]\vec{a}_z$$



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Ex $\vec{E}(x,t) = 50\cos(10^8 t + \beta x)\vec{a}_y$ ④

$\epsilon_r = 9$

$\mu_r = 1$

$\sigma = 0$