Chapter II – Filters

ENEL 471 – Introduction to Communications Systems and Networks

Chapter Objectives

- At the end of this chapter, you will be able to:
  - Define low-pass, high-pass, and band-pass filters
  - Analyze the time response and frequency response of low-pass filters
  - Distinguish between the characteristics of ideal and practical filters

Outline

- Filter definition and types
- Characterization of Ideal Low-pass Filters
- Practical Low-pass Filters
- Application of Filters in Communication Systems

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Definition of Filters

- Filters are systems selective in frequency:
  - Allow passage of only certain frequencies of the input signal.
  - Attenuates other unwanted frequency components
- They are very useful in communications systems. They can be used as:
  - Frequency selective devices
  - Shaping devices: modify the shape of an input signal to form a different shape for its optimum transmission
  - Cleaning devices: removing unwanted signals from an input signal to produce a clean output.

Characterization of Filters

- Filters are considered as linear and time-invariant systems
- They can be characterized by either:
  - <u>Their time domain response</u>: (impulse response) a transformation that relates the time domain representation of the input signal and the time domain representation of the output signal

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) * h(t)$$

  - <u>Their frequency response</u>: (or transfer function) a transformation that relates the spectrum of the input signal to the spectrum of the input signal and the spectrum of the output signal

$$X(f) \longrightarrow \boxed{H(f)} \longrightarrow Y(f) = X(f)\,.\,H(f)$$

Distortionless Transmission

$$x(t) \longrightarrow \boxed{\textbf{System}} \longrightarrow y(t)$$

- An input signal $x(t)$ passing through a system is not distorted by the system if:
  - The output signal $y(t)$ has the same shape as the input signal (except for a multiplicative constant, k. If k<1 : loss, if k>1 : gain)
  - The output signal can be delayed by a time constant t_d with respect to the input signal

$$y(t) = k \cdot x(t - t_d)$$

- By applying the Fourier transform, we get:

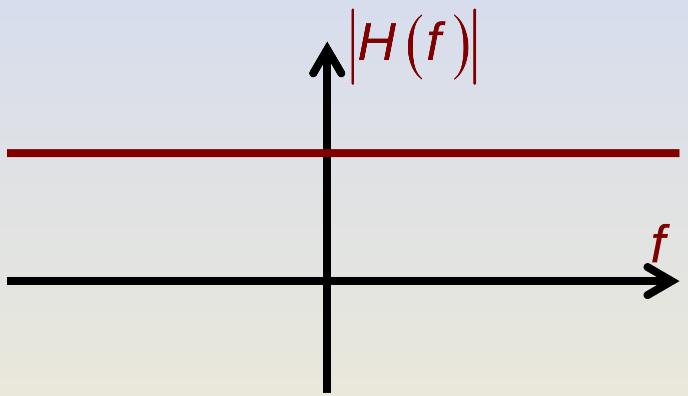
$$Y(f) = k \cdot X(f).e^{-j2\pi f t_d}$$

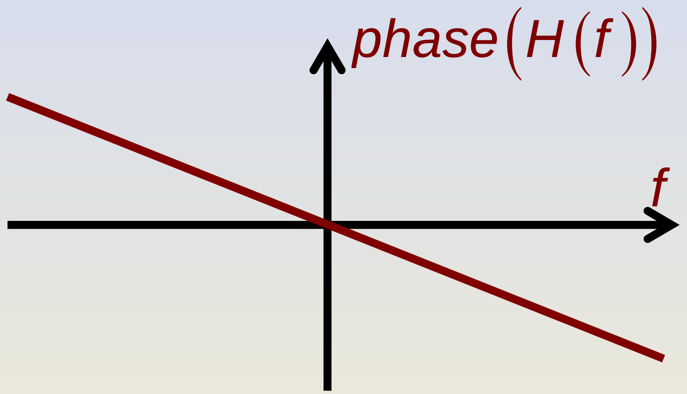
- The frequency response of the system should have the form:

$$H(f) = \frac{Y(f)}{X(f)} = k \cdot e^{-j2\pi f t_d}$$

Distortionless Transmission

- The frequency spectrum of a distortionless system is given by:



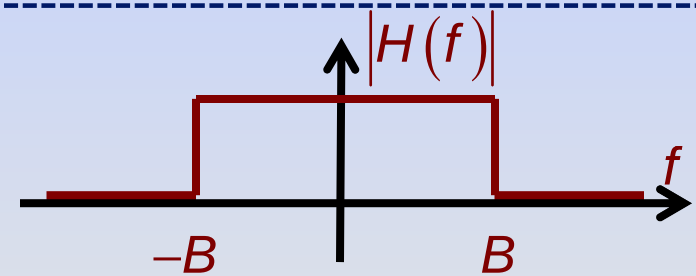


- If the system frequency spectrum is different, there may be two forms of distortions:
  - Amplitude distortion: if the amplitude spectrum ($|H(f)|$) is varying versus frequency
  - Phase distortion: if the phase spectrum is not a linear function of frequency. → all frequency components will receive different delays

Frequency Response of Ideal Filters

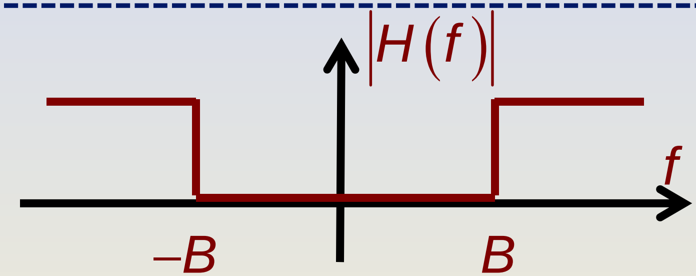
Low-pass

$$|H(f)| = rect\left(\frac{f}{2B}\right)$$



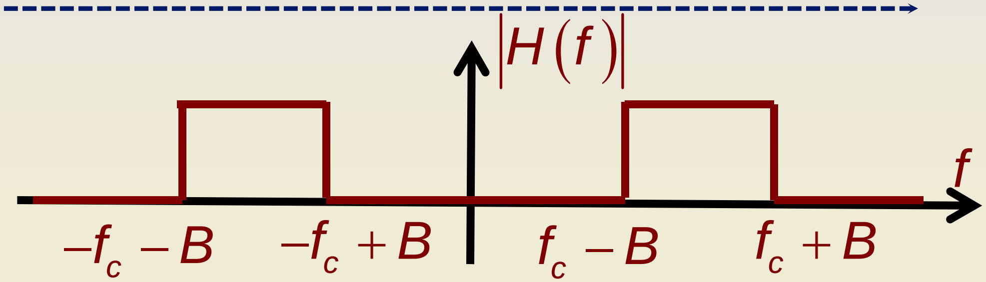
High-pass

$$|H(f)| = 1 - rect\left(\frac{f}{2B}\right)$$



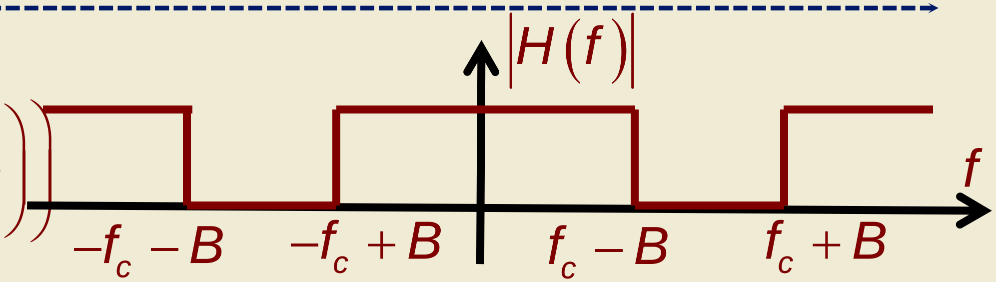
Band-pass

$$|H(f)| = rect\left(\frac{f-f_c}{2B}\right) + rect\left(\frac{f+f_c}{2B}\right)$$



Band-stop

$$|H(f)| = 1 - \left(rect\left(\frac{f-f_c}{2B}\right) + rect\left(\frac{f+f_c}{2B}\right)\right)$$



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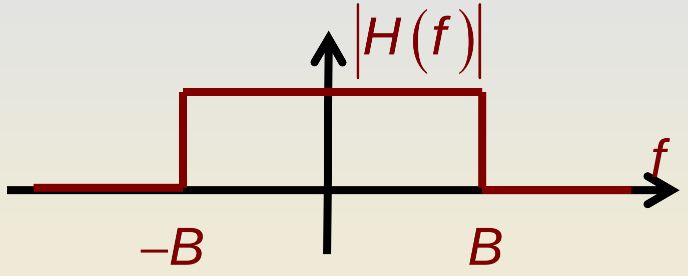
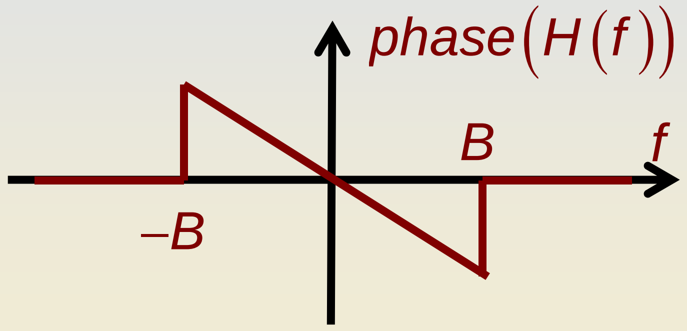
Frequency Response of Ideal Low-Pass Filter

- The general form of the frequency response of an ideal low pass filter is given by:

$$H(f) = rect\left(\frac{f}{2B}\right) \cdot e^{-j2\pi f t_d}$$





→ Satisfies the condition on distortionless transmission in the useful band. No distortion is introduced to the signal.

- Is this filter realizable?

Impulse Response of Ideal Low-Pass Filter

- The impulse response can be obtained by applying the inverse Fourier transform to *H*(*f*)

$$h(t) = \mathscr{F}^{-1}\left(H(f)\right) = \int_{-\infty}^{\infty} H(f) \cdot e^{j2\pi ft} \cdot df$$

$$h(t) = \int_{-\infty}^{\infty} rect\left(\frac{f}{2B}\right) \cdot e^{-j2\pi ft_d} \cdot e^{j2\pi ft} \cdot df$$

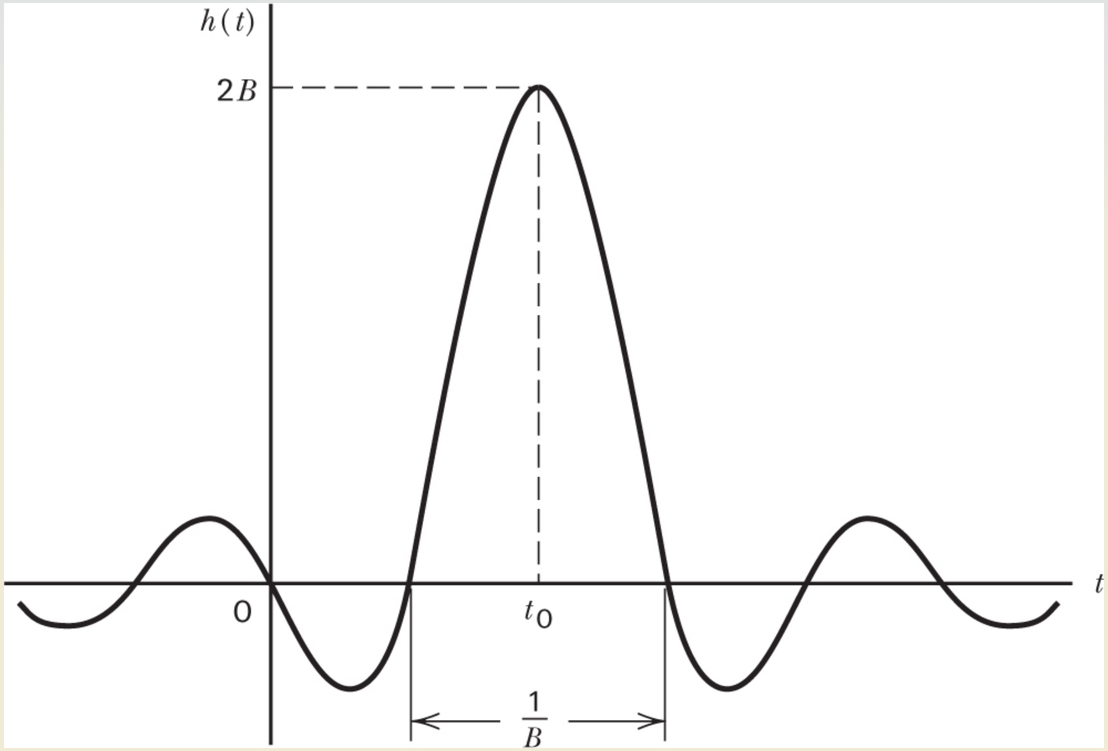
$$h(t) = 2B\text{sinc}\left(2B(t - t_d)\right)$$

- The impulse response is a delayed sinc() function

→ h(t) is not zero for negative t

→ This ideal filter is not causal

→ Not realizable



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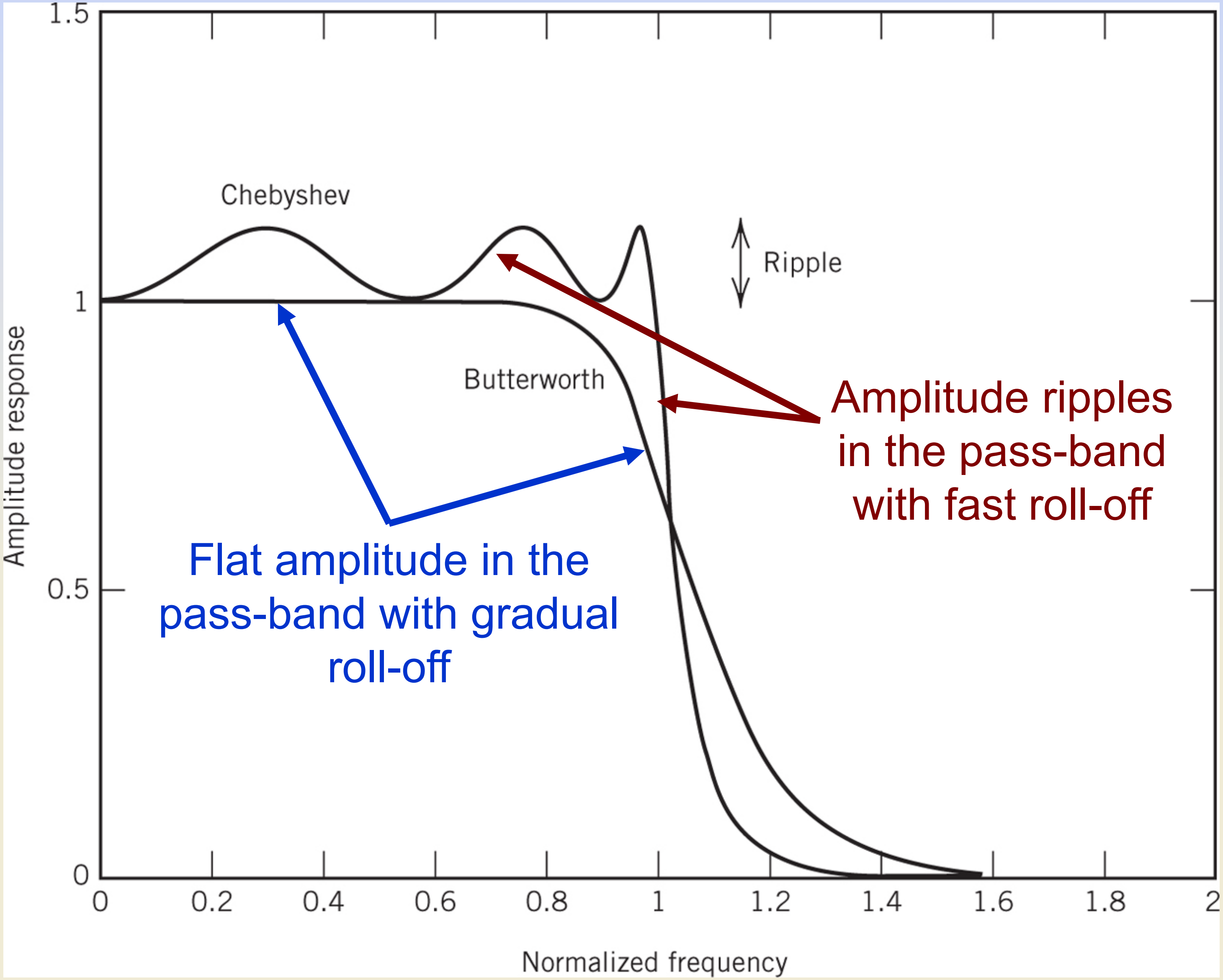
Approximations for Practical Filter Design

- The design of practical filters can be achieved in 2 steps:
- Approximation step:
  - The amplitude of the sinc() impulse response of the ideal low pass filter gets very low for values of time far from t_d
  - We can approximate this impulse response with a causal impulse response by truncating the ideal impulse response
  - In frequency domain, the transfer function is modified:
    - The frequency response is non flat
    - The rise and fall of the amplitude of the transfer function are no longer sharp
    - The phase response is not a linear function of frequency
- Realization step:
  - Implement the approximate time and frequency response by a physical device. (not the focus of this course)

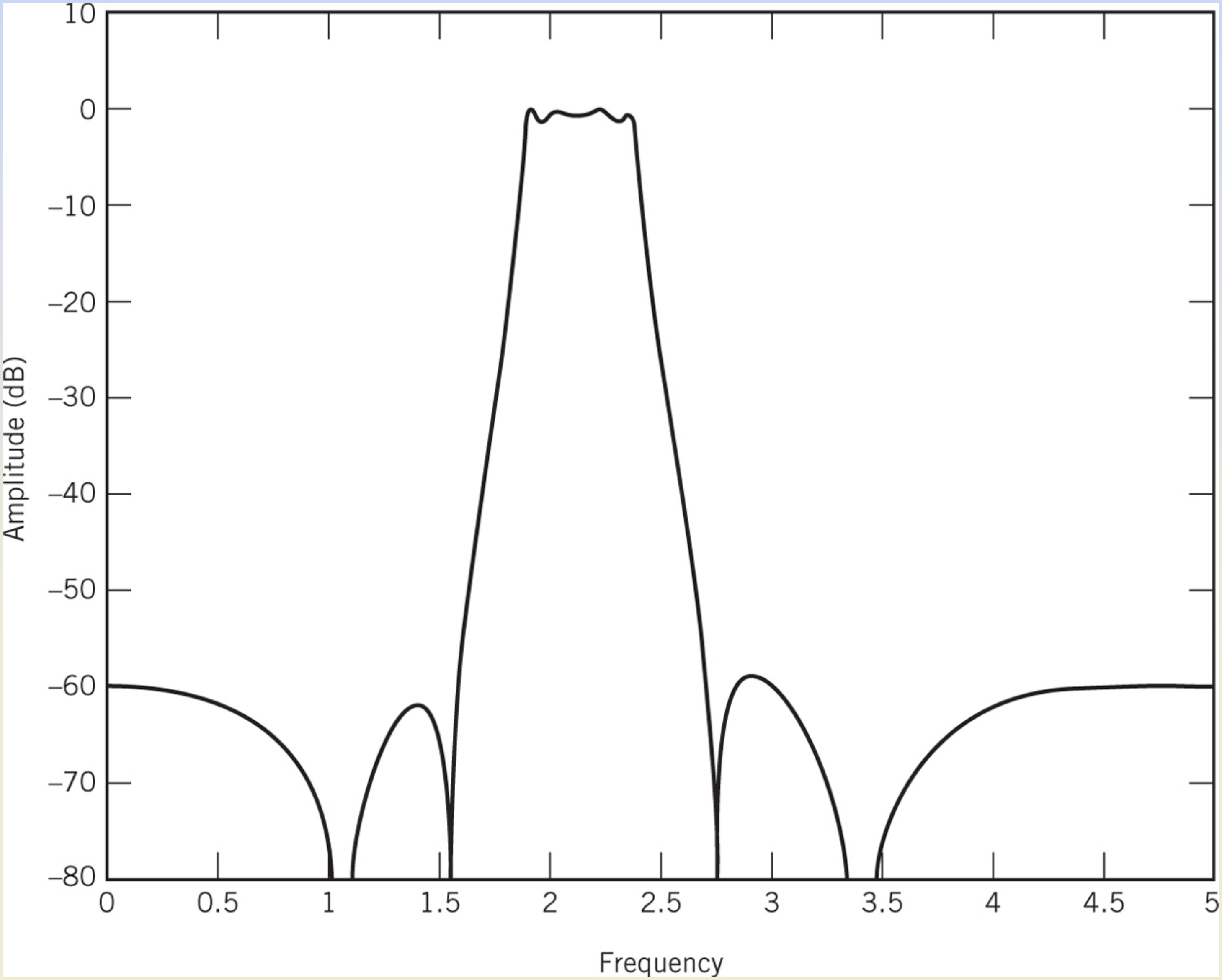
Examples of Practical Filters

- Butterworth Filters:
  - Flat amplitude response in the pass-band
  - Gradual rise-up and fall-off
  - → No distortion to the desired signal but no good rejection of unwanted signals
- Chebyshev Filters (type I):
  - Non-flat (ripples) amplitude response in the pass-band
  - Fast rise-up and fall-off
  - → Distortion to the desired signal caused by the ripples in the pass-band. But there is good rejection of unwanted signals
- Elliptic Filters:
  - Ripples in the pass-band and in the stop-band and nonlinear phase
  - Faster roll-off (faster than Chebyshev filters)
  - → Distortion to the desired signal caused by the ripples and phase nonlinearity but very good rejection of unwanted signals

Amplitude Response of Butterworth and Chebyshev Filters



Amplitude Response of Elliptic Filters



Limitations of Practical Filters

- Large delay: in order to make good approximation in the design steps, t_d is chosen to be large
- Amplitude distortion: caused by ripples in the pass-band
- Group delay distortion: cause by the nonlinearity of the phase (the phase of the frequency response is not a linear function of the frequency

→ The objective of practical filter design is to minimize these limitations

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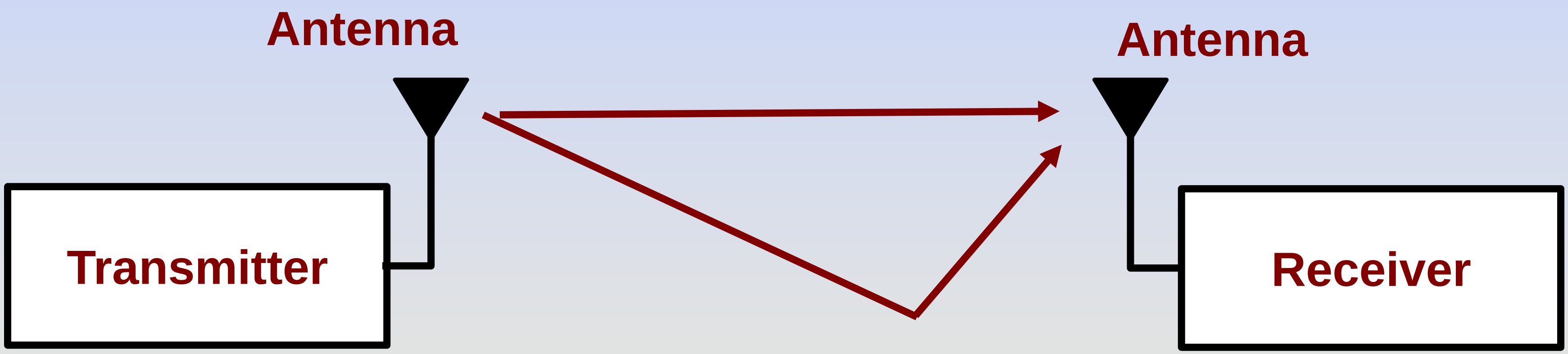
Applications of Filters in Communication Systems

- Transmitter
  - Frequency selective device (image rejection, sideband rejection, harmonics rejection)
  - Shaping device (raised cosine filter to reduce inter-symbol interference)
- Receiver
  - Cleaning device (attenuate the interference captured by the antenna in other frequency bands than the useful band)
  - Frequency selective device (image rejection, harmonic rejection)
- Channel
  - The channel is frequency selective. It can be regarded as a filter

  → Modeling the transmission channel

Communication Link Viewed as a Filter



- If the transmitted signal is received over multiple paths (for example 2 paths), the channel response to an impulse function is then given by:

$$h(t) = \delta(t) + \alpha e^{j\phi}\delta(t-\tau)$$

  - τ is the difference in propagation time between the two paths
  - $\alpha e^{j\phi}$ is the difference in complex attenuation between the two paths

Communication Link Viewed as a Filter