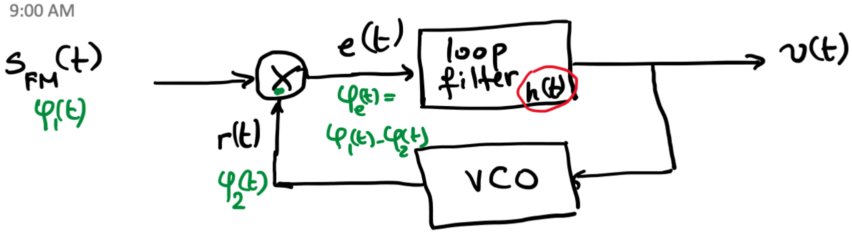
Online Lecture # 03 - FM Demodulation - PLL - Part II

Monday, March 23, 2020
9:00 AM



2 assumptions:

* $s_{FM}(t)$ and $r(t)$ have the same unmodulated carrier freq. f_c
* There is a $\pi/2$ phase shift between the unmodulated carriers of $s_{FM}(t)$ and $r(t)$

$$s_{FM}(t) = A_c \sin(2\pi f_c t + \varphi_1(t)) \quad \text{with } \boxed{\varphi_1(t) = 2\pi k_f \int_0^t m(\tau)\, d\tau}$$

$$r(t) = A_v \cos(2\pi f_c t + \varphi_2(t)) \quad \text{with } \boxed{\varphi_2(t) = 2\pi k_v \int_0^t v(\tau)\, d\tau}$$

$$e(t) = s_{FM}(t) \cdot r(t) = A_c A_v \sin(\underbrace{2\pi f_c t + \varphi_1(t)}_{a}) \cdot \cos(\underbrace{2\pi f_c t + \varphi_2(t)}_{b})$$

$e(t)$ has two freq. components:

* high freq.: $\frac{A_c A_v}{2} \sin(4\pi f_c t + \varphi_1(t) + \varphi_2(t))$ } will be completely removed by the loop filter
* low freq.: $\frac{A_c A_v}{2} \sin(\underbrace{\varphi_1(t) - \varphi_2(t)}_{=\varphi_e(t)})$ ← this will be used to calculate the output $v(t)$

$$v(t) = e(t) * h(t) = \int_{-\infty}^{\infty} e(\tau) \cdot h(t-\tau)\, d\tau$$

$$\boxed{v(t) = \frac{A_c A_v}{2} \int_{-\infty}^{\infty} \sin(\varphi_e(\tau)) \cdot h(t-\tau)\, d\tau}$$

$$\varphi_e(t) = \varphi_1(t) - \varphi_2(t) \rightarrow \frac{d\varphi_e(t)}{dt} = \frac{d\varphi_1(t)}{dt} - \frac{d\varphi_2(t)}{dt}$$

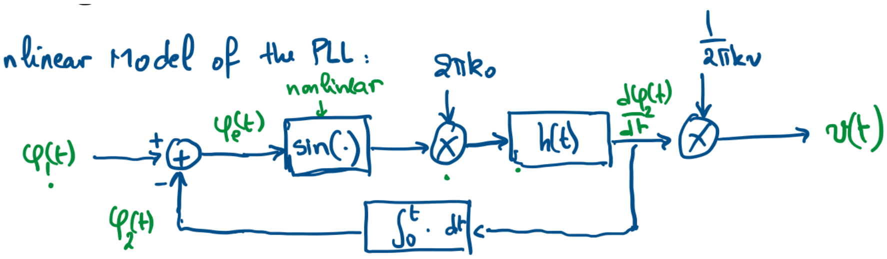
$$\frac{d\varphi_e(t)}{dt} = \frac{d\varphi_1(t)}{dt} - 2\pi k_v \cdot v(t)$$

$$\boxed{\frac{d\varphi_e(t)}{dt} = \frac{d\varphi_1(t)}{dt} - \boxed{\overbrace{2\pi k_v \cdot \frac{A_c A_v}{2}}^{K_0} \int_{-\infty}^{\infty} \sin(\varphi_e(\tau)) \cdot h(t-\tau)\, d\tau}}$$

$\frac{d\varphi_2(t)}{dt} = 2\pi k_v \cdot v(t)$

Integro-differential Equation for $\varphi_e(t)$.

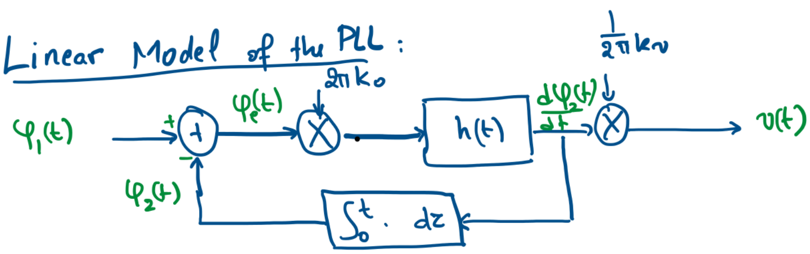
Nonlinear Model of the PLL:



Getting near the steady-state response $\varphi_2(t)$ and $\varphi_1(t)$ will be close to each other and $\varphi_e(t)$ will be small.

Using a 1st order Taylor series approximation : $\boxed{\sin(\varphi_e(t)) \simeq \varphi_e(t)}$.

Linear Model of the PLL:



→ Now we can do the analysis in Freq. domain.

$$\boxed{\frac{d\varphi_e(t)}{dt} = \frac{d\varphi_1(t)}{dt} - 2\pi k_0 \left[\varphi_e(t) * h(t)\right]}$$

Integro-differential equation for the Linear model of the PLL

In frequency domain :

$$j2\pi f \cdot \Phi_e(f) = j2\pi f \, \Phi_1(f) - k_0 \, \Phi_e(f) \cdot H(f).$$

$$\Phi_e(f)\left[1 + \frac{2\pi k_0}{j2\pi f} H(f)\right] = \Phi_1(f)$$

$$\boxed{L(f) = \frac{k_0}{jf} H(f)}$$

$$\Phi_e(f)\left[1 + \frac{k_0 H(f)}{jf}\right] = \Phi_1(f)$$

$$\boxed{\Phi_e(f) = \Phi_1(f)\left[\frac{1}{1 + L(f)}\right]}$$

$$v(t) = \frac{k_0}{k_v}\left[\varphi_e(t) * h(t)\right] \longrightarrow V(f) = \frac{k_0}{k_v} \Phi_e(f) \cdot H(f)$$