# Assignment #2

ENEL 476

Solutions

**Question 1:**

(a) $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = 0.4919 + j112.4$

(b) $\alpha = \Re\{\gamma\} = 0.4919$ Np/m

(c) $\beta = \Im\{\gamma\} = 112.4$ rad/m

(d) $\lambda = \dfrac{2\pi}{\beta} = 0.05590$ m $= 5.590$ cm

(e) $Z_0 = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}} = 44.72 + j0.1601\ \Omega$

(f) $v_p = \dfrac{\omega}{\beta} = 1.118 \times 10^8$ m/s

(g) $\frac{G}{C} = 1 \times 10^8$ and $\frac{R}{L} = 1 \times 10^7$. Therefore $\frac{G}{C} \neq \frac{R}{L}$ and the line is not distortionless.

**Question 2:**

(a) $\beta = \dfrac{2\pi}{\lambda} = 209.4$ rad/m

(b) $\Gamma(l = 0) = \dfrac{Z_L - Z_0}{Z_L + Z_0} = 0.3514 - j0.1081 = 0.3676\angle - 0.2985$ rad

(c) $SWR = \dfrac{1 + |\Gamma|}{1 - |\Gamma|} = 2.163$

(d) $\Gamma(l = 0.25\lambda) = \Gamma(l = 0)e^{-j2\beta l} = \Gamma(l = 0)e^{-j\pi} = -\Gamma(l = 0) = 0.3676\angle 2.843$ rad

(e) $\Gamma(l = 0.5\lambda) = \Gamma(l = 0)e^{-j2\pi} = \Gamma(l = 0) = 0.3676\angle - 0.2985$ rad

(f) $\Gamma(l = 0.6\lambda) = \Gamma(l = 0)e^{-j2\beta l} = 0.3676\angle - 1.555$ rad

**Question 3:**

$SWR = \dfrac{1 + |\Gamma|}{1 - |\Gamma|} = 2 \rightarrow |\Gamma| = \dfrac{SWR - 1}{SWR + 1} = \dfrac{2 - 1}{2 + 1} = \dfrac{1}{3}$

Note: Since $Z_0$ and $Z_L$ are real, $\Gamma$ is real as well. Therefore, $\Gamma = \pm 1/3$

$\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} \rightarrow Z_L = Z_0 \dfrac{\Gamma + 1}{\Gamma - 1}$

$Z_L|_{\Gamma=+1/3} = 200\ \Omega$

$Z_L|_{\Gamma=-1/3} = 50\ \Omega$

**Question 4:**

(a) $Z = \sqrt{L/C}$ and $v = 1/\sqrt{LC}$

$$\rightarrow L = Z_0/v = \frac{70\ \Omega}{2.2 \times 10^8\ \text{m/s}} = 0.3182\ \mu\text{H/m}$$

(b) Likewise: $\rightarrow C = 1/(Z_0 v) = \dfrac{1}{70\ \Omega \cdot 2.2 \times 10^8\ \text{m/s}} = 64.94\ \text{pF/m}$

**Question 5:**

$$Z_{in}(l = 0.3\lambda) = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} = 5.753 - j6.046\ \Omega$$