Unit 4 Feedback loop analysis methods

We have now completed the introduction to the fundamental components that are needed for the classical design of SISO control systems. We have seen how feedback can move the poles of the open loop response to get the desired closed loop response. Also, in the previous unit we considered the need for having higher order loops in order to provide a required steady state response. For instance, if we wanted a loop that has a zero steady state error to a parabolic input (In mechanical terms, an input of constant acceleration) then a type 3 loop is required. However, determining the required compensator coefficients is difficult. We need quantitative analysis tools to guide this design rather than the 'gut feel' and 'trial and error' approach that we have relied on till now. There are more closed loop response attributes that we need to satisfy other than a steady state response to an input.

How do we come up with a systematic design methodology for the compensator design? Of concern is:

1. Stability
2. steady state error response
3. transient behavior for input reference changes and loop disturbances
4. complexity and robustness of implementation

In this unit we will learn of a rather powerful tool for compensator design which is based on '**root locus**'.

Root locus is a graphical approach to approximating the position of the closed loop poles of a negative feedback loop as the loop gain is increased.

By approximately sketching a root locus plot we will be able to design a compensator, adding integrators, some zeros for achieving stability etc.

While root locus is a powerful method of analysis of feedback control systems which enables an effective method of compensator design, there are frequently encountered design problems where the root locus is not complete or is inconvenient to use. To use root locus we need to have the open loop transfer function given in terms of poles and zeros.

Often, we have measured plant data such as a frequency response of a transfer function. That is we could have a set of frequencies that we excite the plant with and measure the response for each frequency. In other words, we are sampling the frequency domain transfer function directly. There are estimation methods that can be used to model this frequency response as a transfer function of poles and zeros. However, this is an approximation and may be inaccurate.

Another problem is that many models necessarily include a delay. For instance, we can have a microcontroller as part of the feedback loop wherein the compensator is a digital filter. Associated with this could be a sizable delay that will have to be represented. The problem then is that the delay of T_D has a transfer function of $\exp(-sT_D)$ that can only be approximated by a rational polynomial function of s.

Often, we can overcome the limitations of the root locus by producing a Bode plot of the open loop response. With this we can take the measured frequency data and delay into account. However, Bode plots obscure important details related to loop stability. Hence in addition to Bode plots, the Nyquist locus is a plot of the open loop transfer function in the complex s-plane that is a powerful visualization of the stability and frequency response of the closed loop control system. It is a bit more tedious to sketch than the Bode plot but provides more information. Using a combination of root locus, bode plots and Nyquist plots provides a powerful complementary set of tools for analyzing a compensator design for a feedback loop. In this unit we will look at all three analysis methods. We will also introduce an effective Matlab tool for compensator design called **'sisotool'** which is primarily based on these three methods. In the next unit 5 practical implementation of compensators will be considered based on using the analysis tools developed in this unit.

You will find that Root Locus is your first analysis design tool of choice if the plant and sensor can be represented by an LTI model consisting of poles and zeros. If this is not possible then consider if the plant and sensor can be approximated as an LTI model in the frequency domain with a set of poles and zeros. If this is reasonably accurate then Root Locus can still be used. If it is not, then Nyquist and Bode frequency domain analysis can be used. More sophisticated analysis would use all three methods simultaneously. This is why SISOtool uses these three methods interactively.

The content of this unit is as follows:

1. Introduction to the root locus method. Rules for generating root locus plots

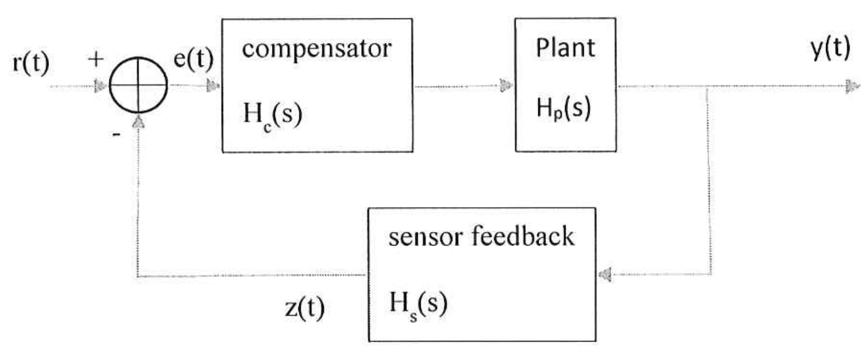
2. Application of Root locus to feedback analysis and compensator design

3. Introduction to Bode frequency domain methods

4. Introduction to Nyquist stability plots

5. Compensator analysis and design based on frequency domain methods

Throughout this unit the notation for the negative feedback control loop will be as in the diagram below:



r(t) is the reference input which is an independent function. e(t) is the error signal that is processed by the compensator of $H_c(s)$. The plant is as usual $H_c(s)$ and the sensor for the feedback is denoted by $H_s(s)$. The output is y(t).

A transfer function can be determined for any input and output point of the loop. A key observation of any SISO loop is that the poles are the same regardless of where the input and output points are. However, the zeros and the scaling will change. This simplifies the design and analysis of the loop. The poles of any transfer function related to the loop will be the same. Root locus is a powerful tool for getting the compensator to put the poles in the right place. The right place for the closed loop poles is guided by several criteria:

1. All poles in the LHP for stability

2. All poles as far into the LHP and relatively close to the real axis is ideal. However, there are limitations as the control components have to be much faster responding

3. All poles in conjugate pairs have a damping factor that is not small. Damping factors less than one lead to oscillatory transients which are problematic

4. Poles that lead to an optimum use of control energy and minimize risk of large actuator control. This topic will be covered more in advanced courses on control theory. Here we look at minimizing the control effort and having 'gentle' actuator control. As an example consider a satellite attitude control where it is imperative to conserve rocket thruster energy such that satellite lifetime can be prolonged. Of less importance is how fast the satellite recovers from a change in reference or disturbance.

The physical components of the control loop are rather ideal in ENEL441. Hence we don't get into the issue of actuator saturation, nonlinearities and power consumption of the control circuit. Such considerations are dealt with in ENEL 541 wherein loop optimization based on an arbitrary objective cost function is considered. However, in ENEL 441 you should begin to become aware of the cost of control based on closed loop pole placement. Fundamentally, the more the open loop pole is to be moved to a new closed loop position, the more feedback effort must be expended. The control signal amplitudes become large to compensated for a relatively small change in the reference or disturbance signal. Large actuator signals imply that the components must be of higher performance and therefore generally more expensive and bulky. A key to good control is to move the poles just enough for the desired transient response. Also know that there are practical limits to how far a pole can be moved and it may be that it is necessary to settle for a slower response.

The tools in this section are fundamental to visualization into the expected performance of a compensator design and provide clues as to how to realize further improvements.

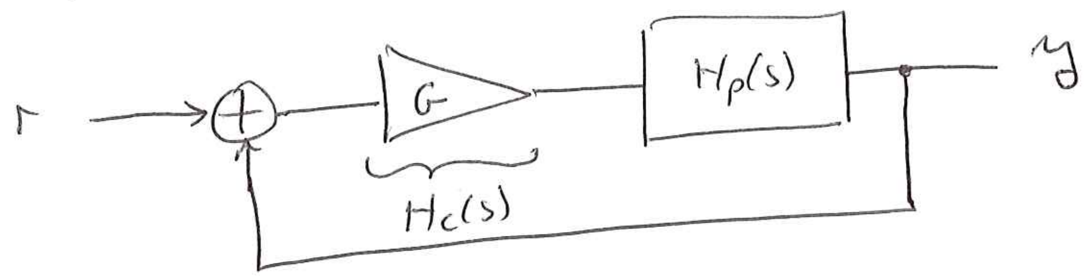
Root Locus

Start with a couple of examples to show the application of root locus, then get into the rules of drawing the locus trajectories.

Example 1

$$H_p(s) = \frac{1}{s+1}$$

change the closed loop pole to speed up response.



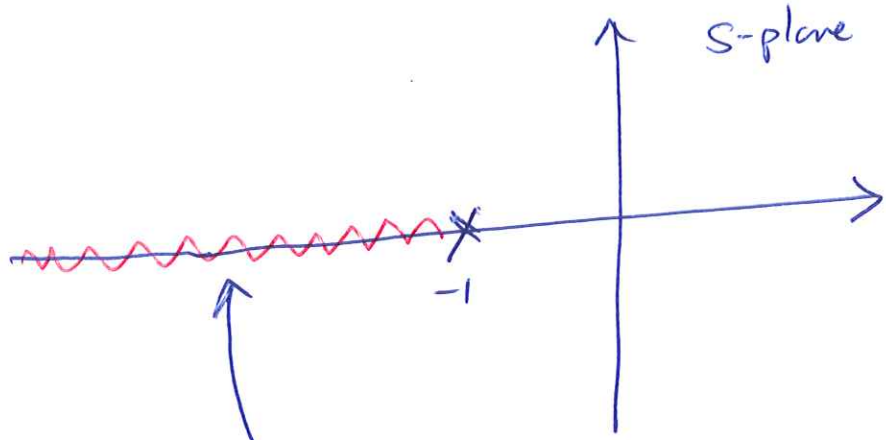
Define

$H_{OL}(s)$ open loop response $\qquad H_{OL}(s) = \frac{G}{s+1}$

$H_{CL}(s)$ closed loop response

$$H_{CL}(s) = \frac{\frac{G}{s+1}}{1 + \frac{G}{s+1}} = \frac{G}{s+1+G}$$

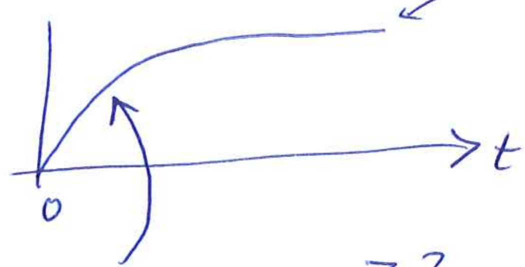
Closed loop pole is at $s = -1 - G$



trajectory of closed loop pole as G increases from zero.

Closed loop pole position indicates
- stability ⇒ in LHP?
- Robustness ⇒ how far from $j\omega$ axis
- transient behavior

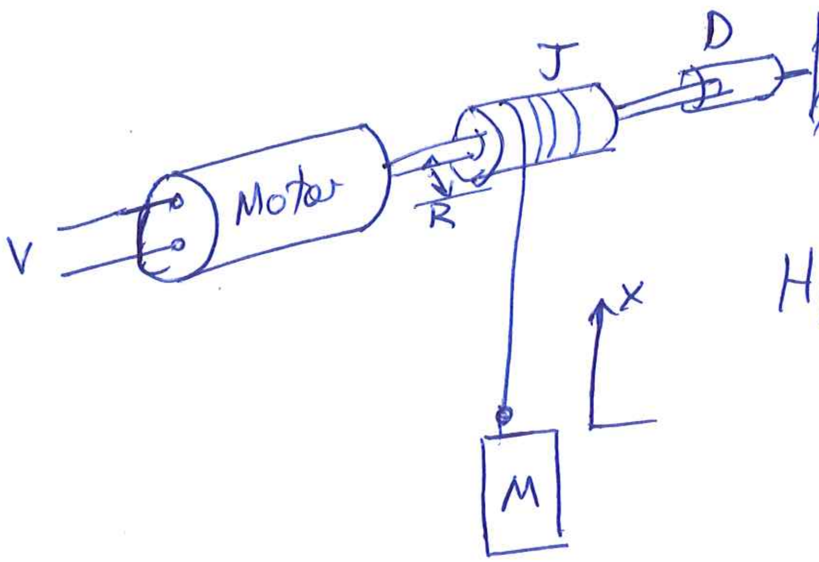
real axis pole gives exponential transient.



rise time $\frac{2.2}{1+G}$

Example

Motor weight elevator

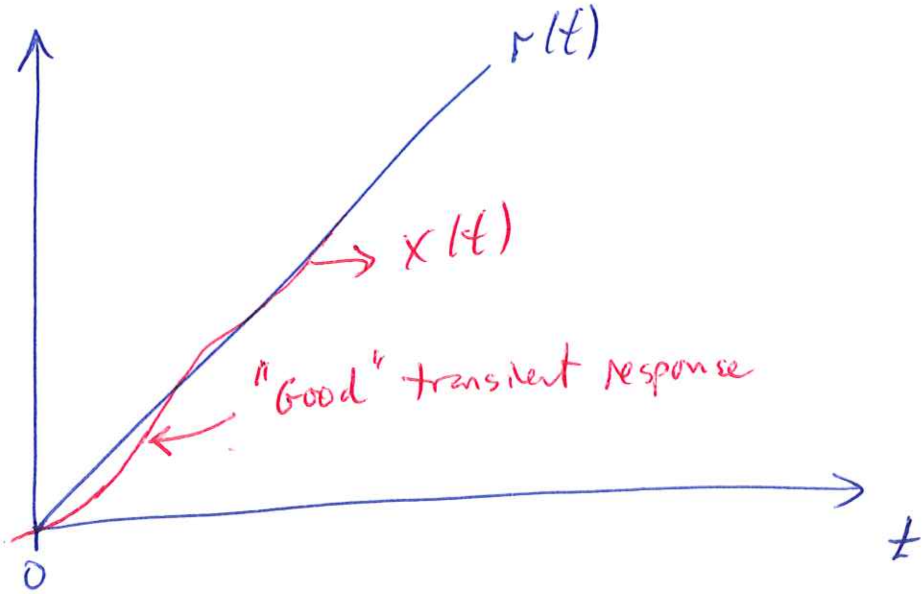


$$H_p(s) = \frac{X(s)}{V(s)} = \frac{a}{s(s+b)}$$

a, b depend on motor constants, J, D, M, R

<u>Objectives</u>

1) stable
2) track a ramp input - zero steady state error
3) fast transient, minor overshoot, ringing.



Need type-II loop → $H_{OL}(s)$ has two integrators

Integral feed back compensator

$$H_c(s) = \frac{G}{s}$$

based on root locus and $H_p(s)$ immediately tell that this will not work.

Will have to poles in RHP for any $G > 0$.

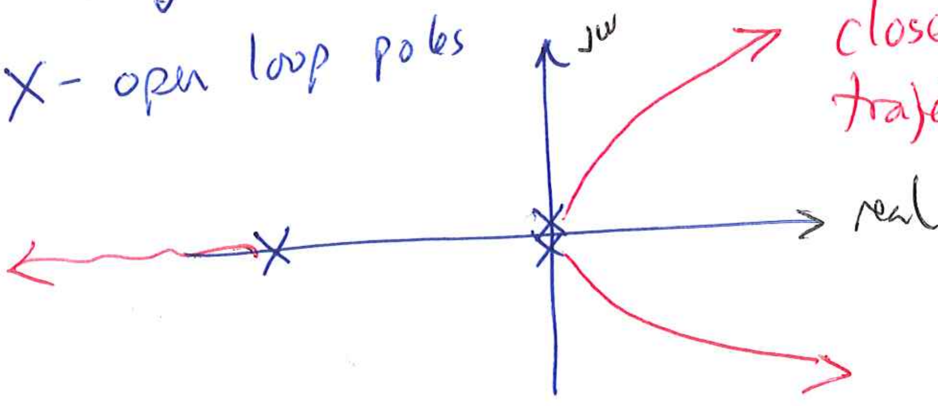
$$H_{OL}(s) = \frac{Ga}{s^2(s+a)}$$

$$H_{CL}(s) = \frac{\frac{Ga}{s^2(s+a)}}{1 + \frac{Ga}{s^2(s+a)}} = \frac{Ga}{s^3 + s^2 a + Ga}$$

Root locus allows you to sketch poles of $H_{CL}(s)$ as a function of G without having to solve cubic polynomial roots.

X - open loop poles

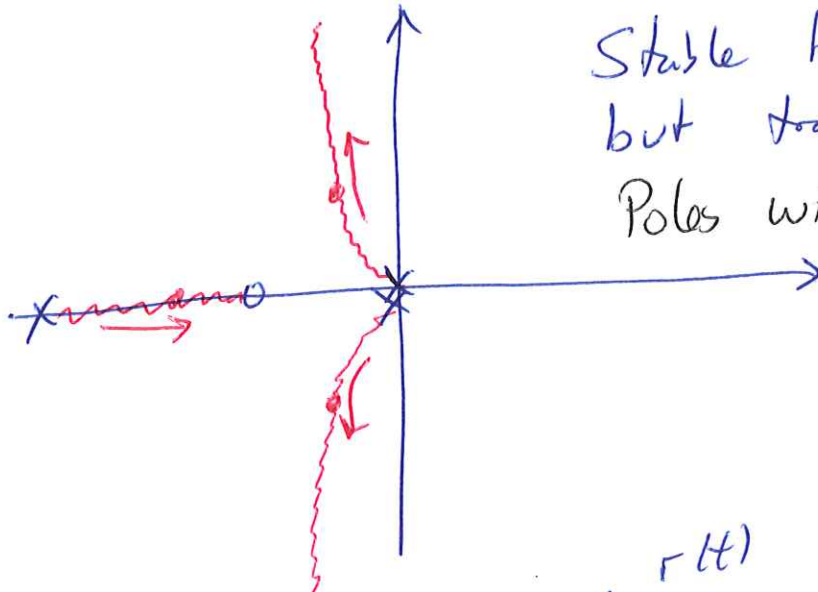
closed loop pole trajectories.



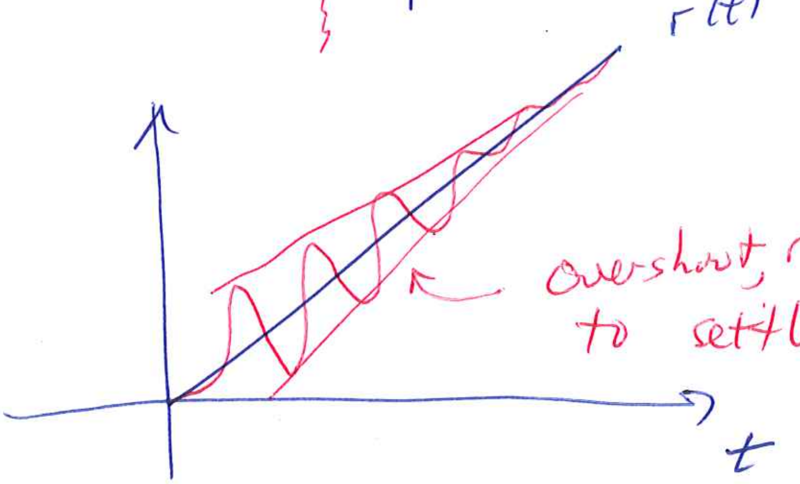
Sketch of Root locus tells us we need a zero in LHP to bring root trajectories back into LHP.

So try a "Proportional + Integral" or 'PI' compensator.

$$H_c(s) = g_1 + \frac{g_2}{s} = \frac{g_1 s + g_2}{s} = \frac{G(s+z)}{s}$$



Stable for any $G>0$
but transient not good.
Poles with relatively small D



overshoot, ringing, takes long time to settle.

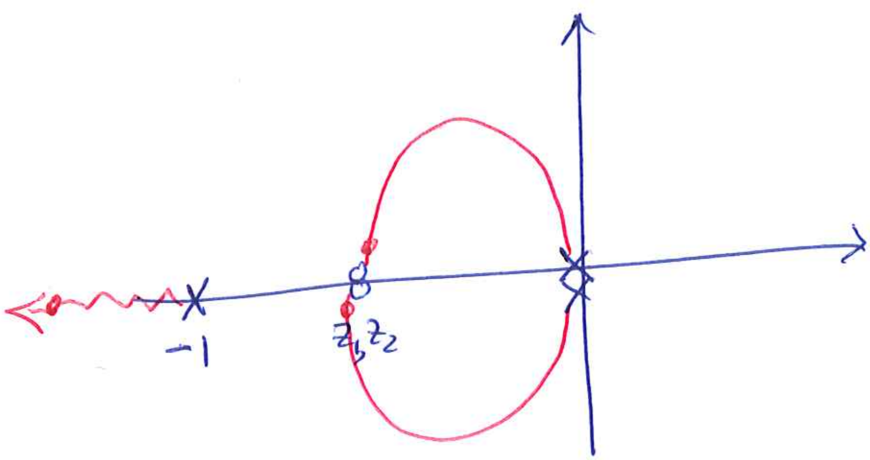
Try another zero to bring poles away from $j\omega$ axis

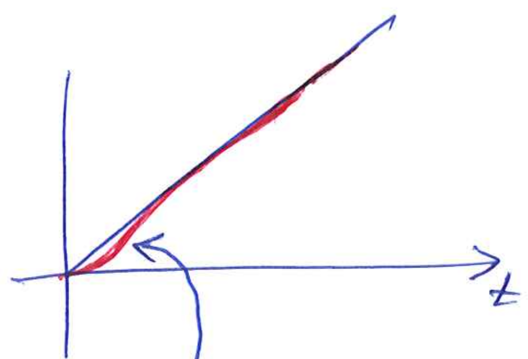
PID — proportional + integral + derivative

$$H_c(s) = g_1 + \frac{g_2}{s} + g_3 s$$

$$= \frac{g_1 s + g_2 + g_3 s^2}{s}$$

$$= \frac{G\,(s+z_1)(s+z_2)}{s}$$





But Not realizable

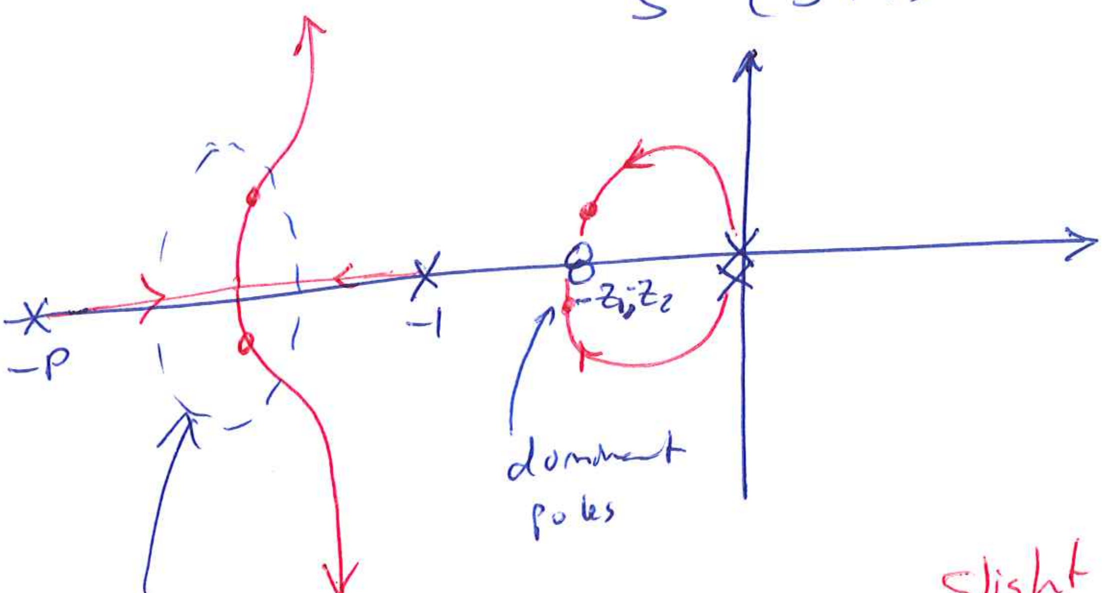
Can't make a pure derivative operator.

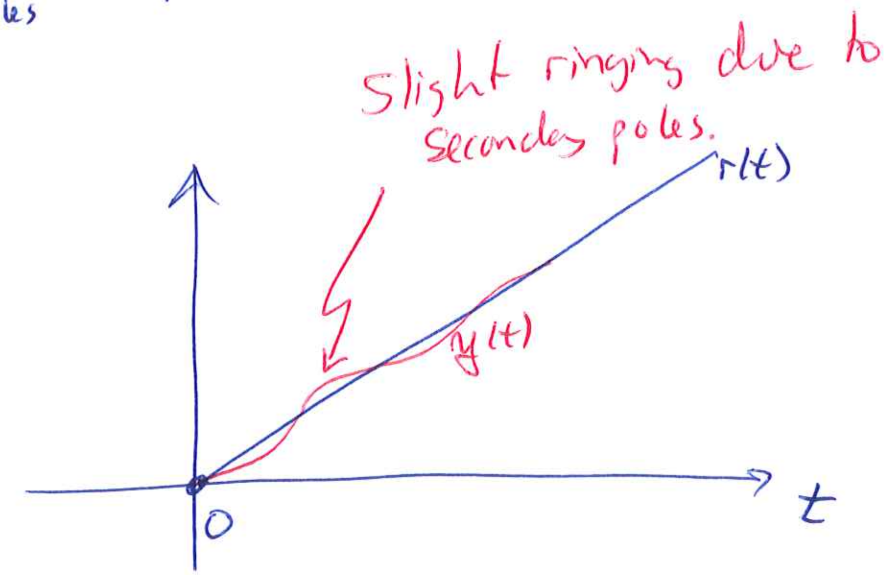
Always associated with a pole.

$$H_c(s) = g_1 + \frac{g_2}{s} + g_3 \frac{s}{s+p}$$

$$= \frac{g_1 s(s+p) + g_2(s+p) + g_3 s^2}{s(s+p)}$$

$$= G \frac{(s+z_1)(s+z_2)}{s(s+p)}$$





How to sketch the closed loop pole trajectories as a function of gain in open loop

<u>Nise Textbook</u>

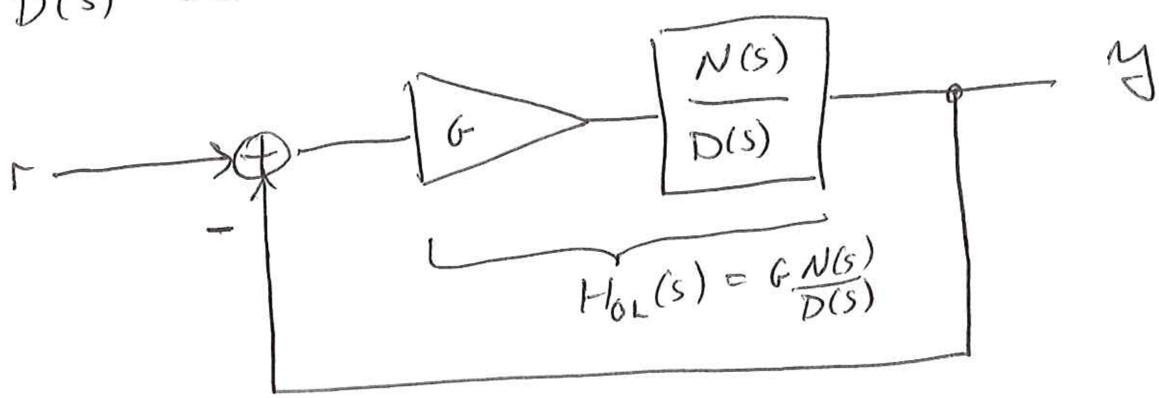
ch 8 - basic rules for sketching root locus

ch 9 - design of compensators with root locus

To apply the root locus we assume a loop with a gain G (scaling block) and a transfer function as a rational polynomial

N(s) numerator of polynomial

D(s) denominator of polynomial



$$\frac{Y}{R} = \frac{G\frac{N}{D}}{1 + G\frac{N}{D}} = \frac{GN}{D + GN}$$

closed loop poles $\rightarrow$ roots of $D + GN$

A fundamental observation:

roots of s of $\underline{D(s) + N(s)\,G = 0}$

for $G \to 0$ roots are same as roots of $D(s)$

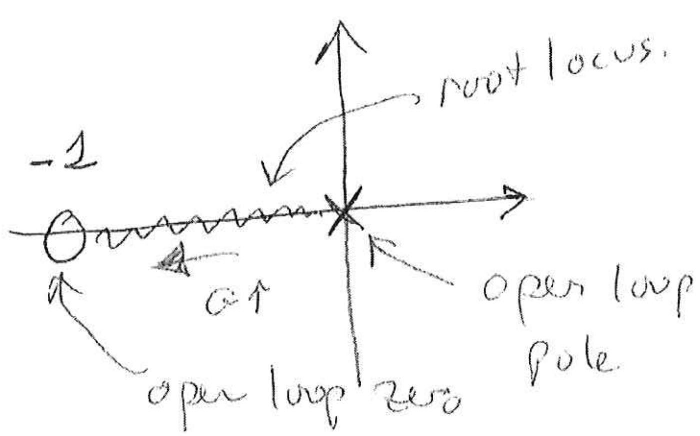
for $G \to \infty$ " " " " " " $N(s)$

In other words

The root locus starts ($G=0$) at the open loop poles (roots of $D(s)$) and ends ($G\to\infty$) at open loop zeros (roots of $N(s)$)

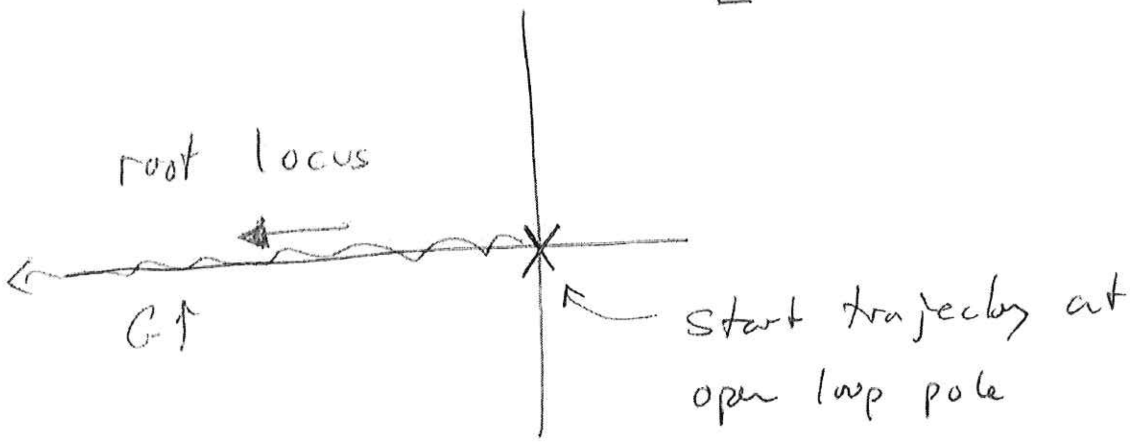
Example $H(s) = \dfrac{s+1}{s}$

$N(s) = s+1$
open loop zero $s=-1$
$D(s) = s$
open loop pole $s=0$



What if there is not a matching zero?

example $H(s) = \frac{1}{s}$ $\begin{cases} N(s) = 1 & \text{no open loop zero} \\ D(s) = s & \text{open loop pole at } s = 0 \end{cases}$



Pole of root locus (closed loop pole) goes to

$s \to -\infty$ as G increases to ∞

Missing zeros in $N(s)$ are equivalently at "infinity"

Hence refer to the "zero at infinity".

How? $\frac{1}{s} = \lim_{K \to \infty} \frac{s + K}{sK}$ $s = K \to$ zero at infinity.

Number of root trajectories Rule

$$H_{OL}(s) = G\frac{N(s)}{D(s)}$$

Root locus applies only to the cases:

$N(s)$ - polynomial of s of order n_z

$D(s)$ - polynomial of s of order n_p

such that $n_p \geqslant n_z$

ie number of open loop poles $\geqslant$ number of open loop zeros

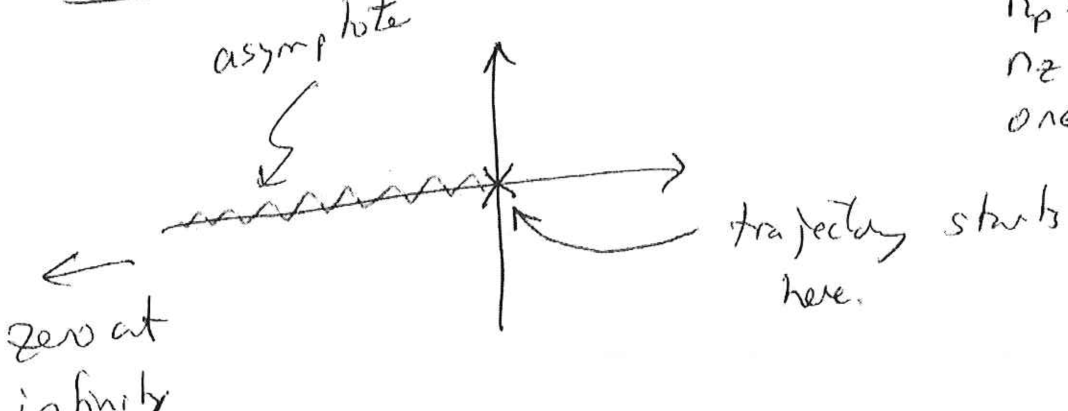
And $N(s)$, $D(s)$ must be polynomials
can not have delay terms ie e^{-as}

1. Number of Root trajectories $= n_p$
2. trajectories start at open loop poles
3. n_z trajectories terminate at the n_z zeros.

4. $n_p - n_z$ Root trajectories go to "zeros at infinity" along $(n_p - n_z)$ asymptotic lines.

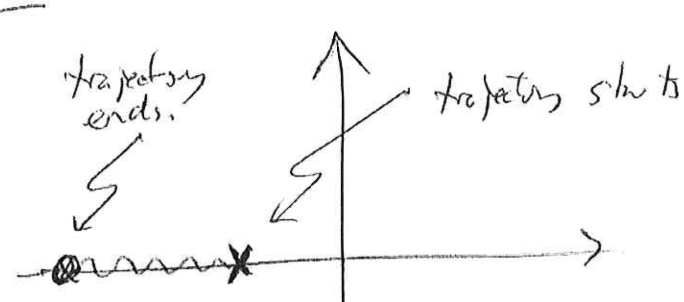
Example $\quad H_{OL}(s) = \frac{G}{s}$

$n_p = 1$
$n_z = 0$
one asymptote



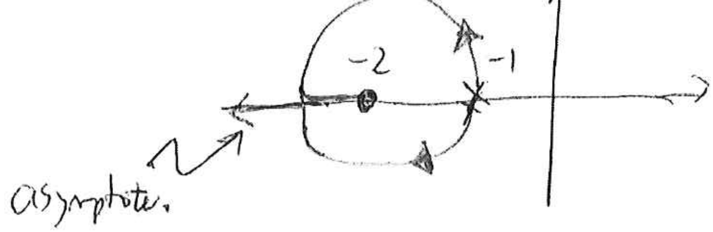
example $\quad H_{OL}(s) = G\frac{s+2}{s+1}$

$n_p = 1$
$n_z = 1$
no asymptotes



example $\quad H_{OL}(s) = G\frac{s+2}{(s+1)^2}$

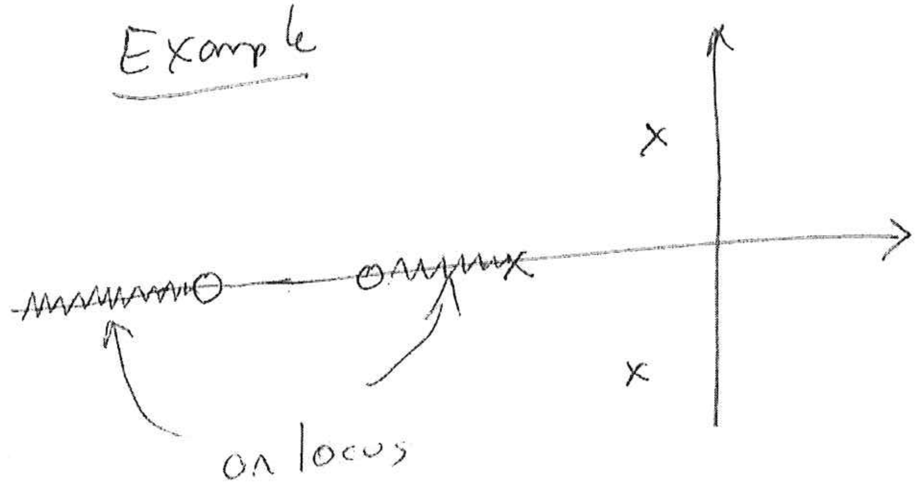
$n_p = 2$
$n_z = 1$
one asymptote



MORE ROOT LOCUS RULES

(5) A point on the real axis is on a root locus if the number of open loop poles minus the number of open loop zeros to the right of the point is odd.

Example



(6) Root trajectories are symmetric about the real axis.

→ closed loop poles always appear in conjugate pairs.

→ This is due to $H_{OL}(s)$ having all real coefficients.

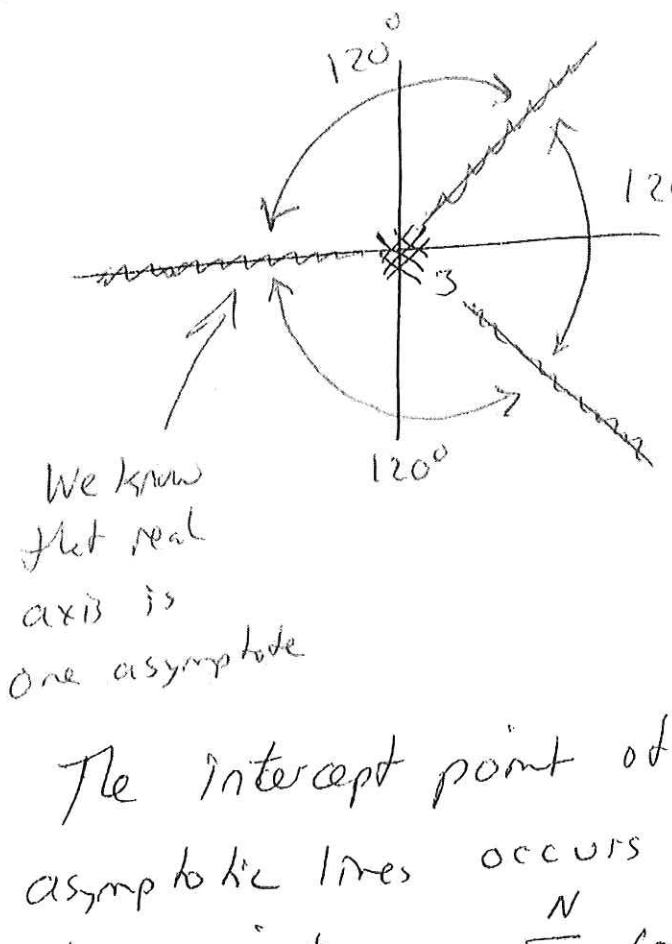
7 If there are $\begin{cases} M \text{ open loop zeros} \\ N \text{ open loop poles} \end{cases}$

Then there are $N-M$ asymptotic trajectories evenly spaced in angle

First find if the real axis as an asymptotic line, Then distribute accordingly.

Example $H(s) = \frac{1}{s^3}$ $N=3$, $M=0$

$N-M=3 \rightarrow$ 3 asymptotic lines.



8 The intercept point of the set of $N-M$ asymptotic lines occurs on the real axis at a point

$$\frac{\sum_{i=1}^{N} (\text{Pole Values}) - \sum_{j=1}^{M} (\text{zero values})}{N-M}$$

Example

$$H(s) = \frac{s+1}{s^3}$$

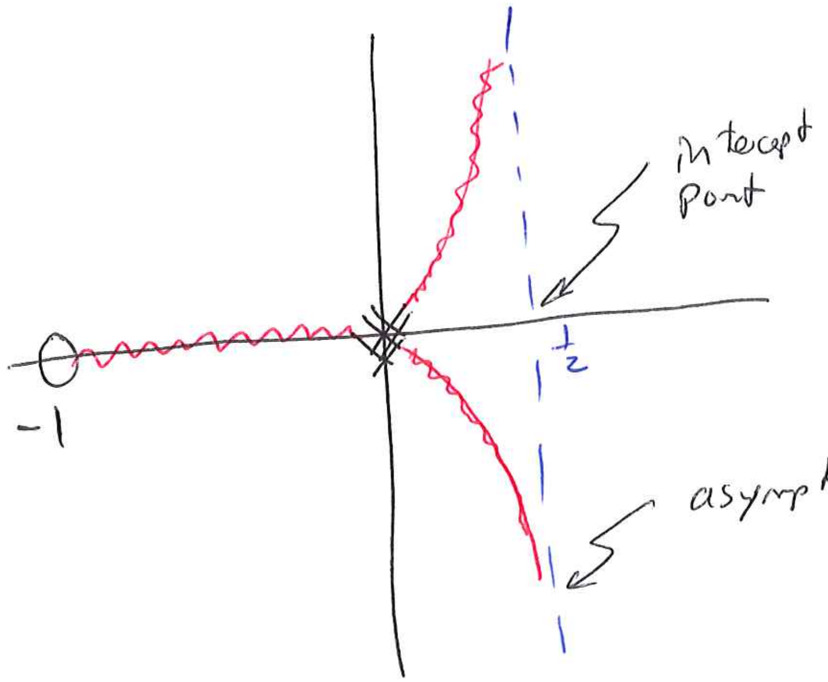
$N = 3$

$M = 1$

$N - M = 2$ asymptotes

Intercept point

$$\frac{0+0+0-(-1)}{N-M} = \frac{1}{2}$$



Summary of Rules for creating Root locus

negative feedback with open loop $H(s) = \frac{N(s)}{D(s)}$

M - number of zeros
N - number of poles

$M \le N$

R1 N trajectories

R2 N trajectories start at open loop poles

R3 M trajectories terminate on M open loop zeros

R4 N − M zeros @ infinity each with asymptote, ie N − M asymptote trajectories

R5 Point on real axis is on a root trajectory if the "number of poles to the right" minus "number of zeros to the right" is odd.

R6 Closed loop poles in conjugate pairs
Root locus plot symmetric about real axis

R7 Asymptote intercept point

$$\frac{\sum \text{pole values} - \sum \text{zero values}}{N-M}$$

R8 Asymptote lines equally spaced in angle.

Get use to using rules by going through a lot of examples.

2015 Quiz 10

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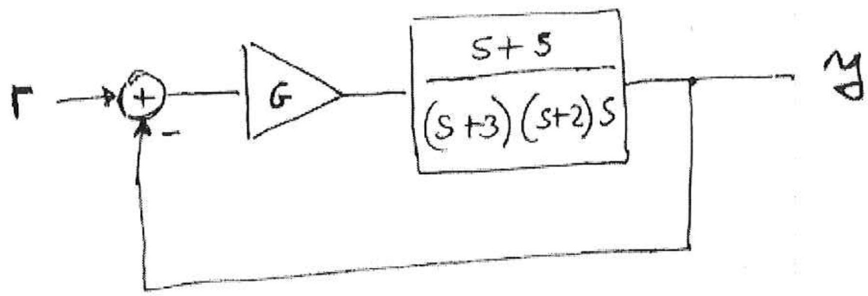
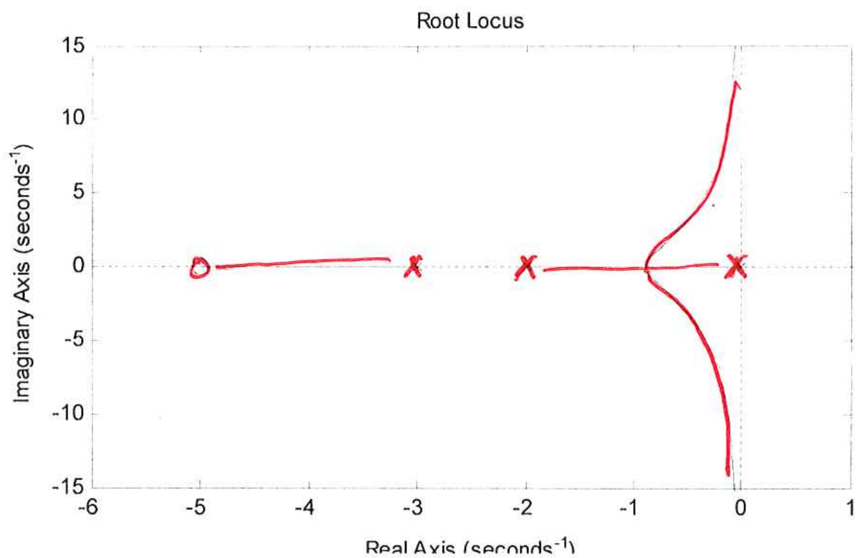
ENEL441 QUIZ 10 Name____________________________ UCID____________________________

Q1. Sketch the Root Locus diagram for the closed loop poles of the negative feedback loop as a function of gain G. Indicate any asymptotic lines for the trajectories. Calculate any intercept points on the real axis.

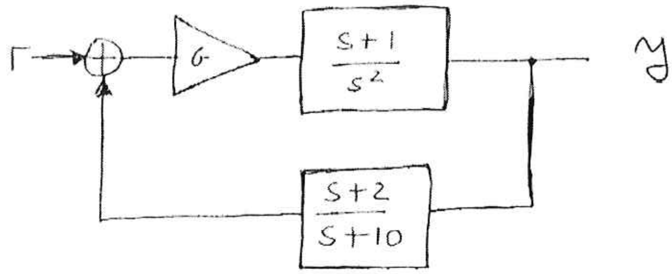


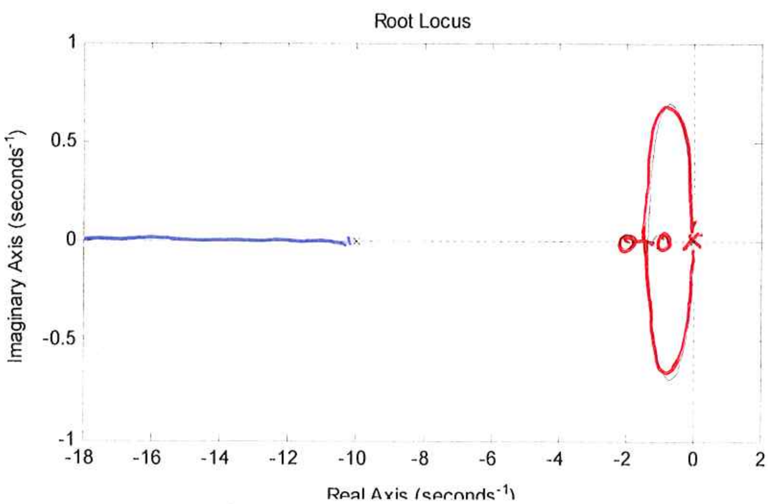


Based on the Root Locus, is the loop stable?

yes but becomes marginally stable for large G.

Q2. Sketch the Root Locus diagram for the closed loop poles of the negative feedback loop on the back side of this page. Indicate any asymptotic lines for the trajectories. Calculate any intercept points on the real axis.





Based on the Root Locus, what is the approximate rise time for a large value of G?

dominant pole at 1 so it is about 2.2 sec.

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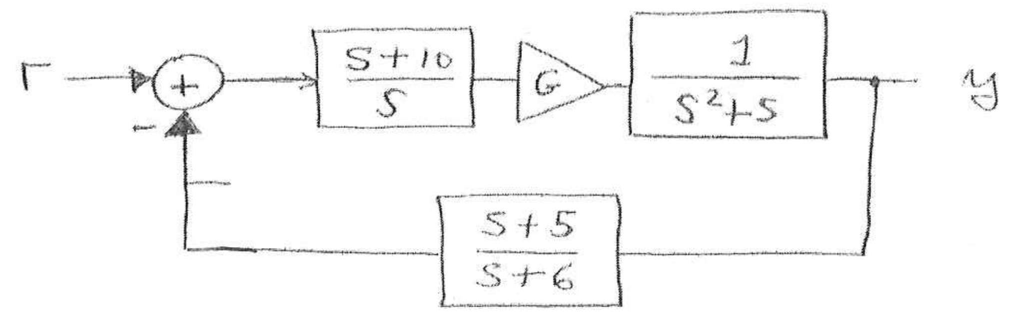
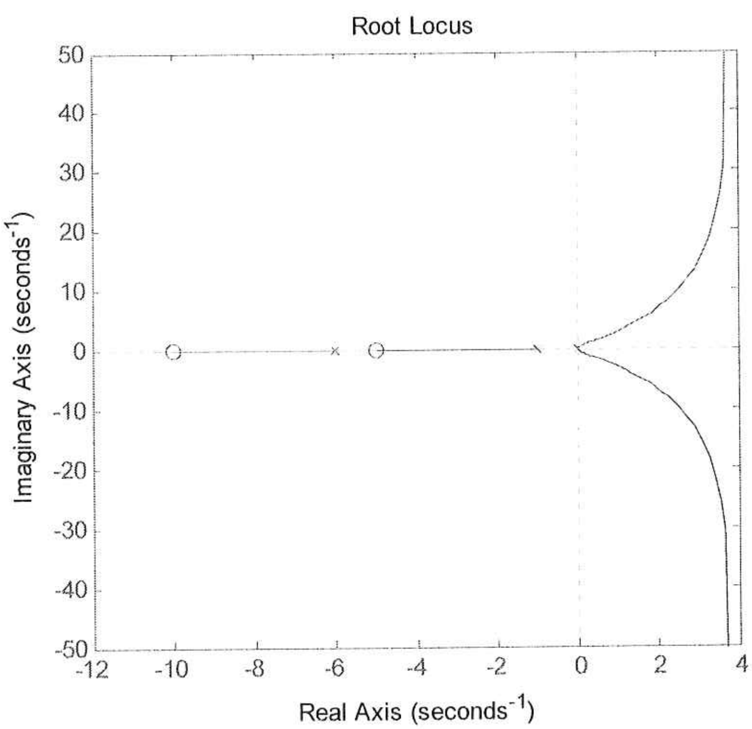
Q1.(7) Sketch the Root Locus diagram for the closed loop poles of the negative feedback loop as a function of gain G. Indicate any asymptotic lines for the trajectories. Calculate any intercept points that exist of the asymptotic lines.



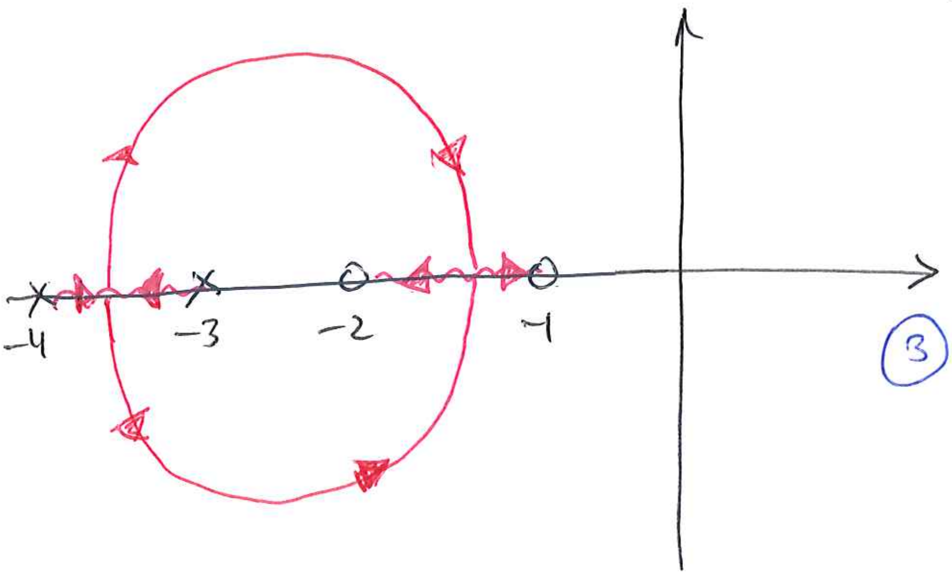


Q2)(3) Based on the root locus plot, is the loop stable? Explain your answer.

not stable for any G>0, two closed loop poles in the RHP

Practice Questions and Examples

① $H(s) = \dfrac{(s+1)(s+2)}{(s+3)(s+4)}$



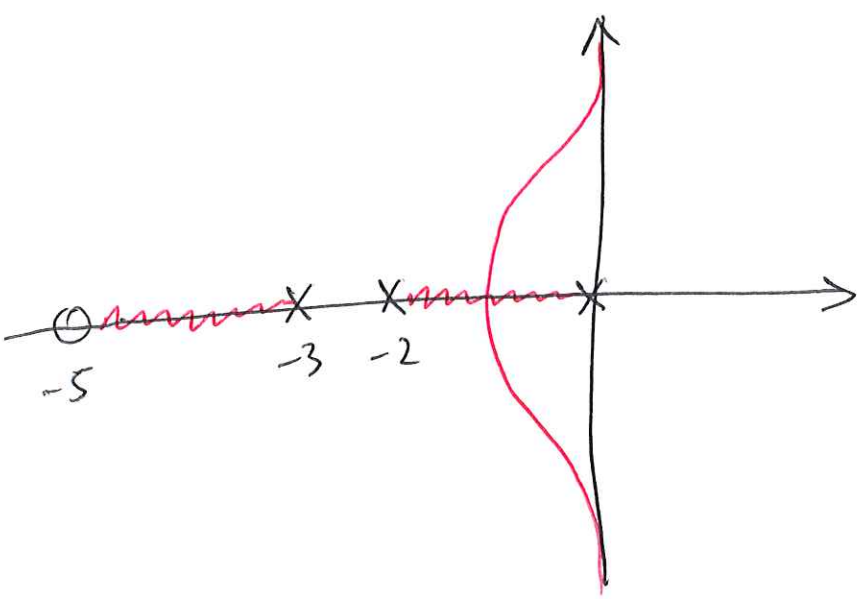
① plot pole/zero positions of H(s).

② determine that the real axis
$\{-4, -3\}$
$\{-2, -1\}$
are on the locus

③ no asymptotes
trajectories must leave poles and go to zeros

④ plot must be symmetric.

② $H(s) = \dfrac{s+5}{(s+3)(s+2)s}$



① determine that $\{-5, -3\}$, $\{-2, 0\}$ are on trajectory

② two asymptotes
intercept point

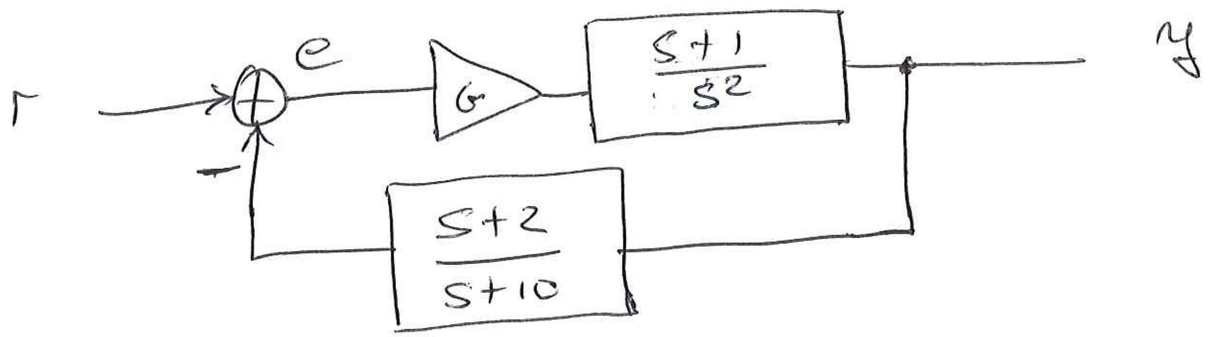
$$\frac{0 + (-2) + (-3) - (-5)}{2} = 0$$

3 Determine Root locus for

$$H(s) = \frac{(s+1)(s-10)}{s}$$

Impossible, does not have a root locus as $M=2$ is greater than $N=1$.

4 Determine the Root Locus for the following feedback loop.



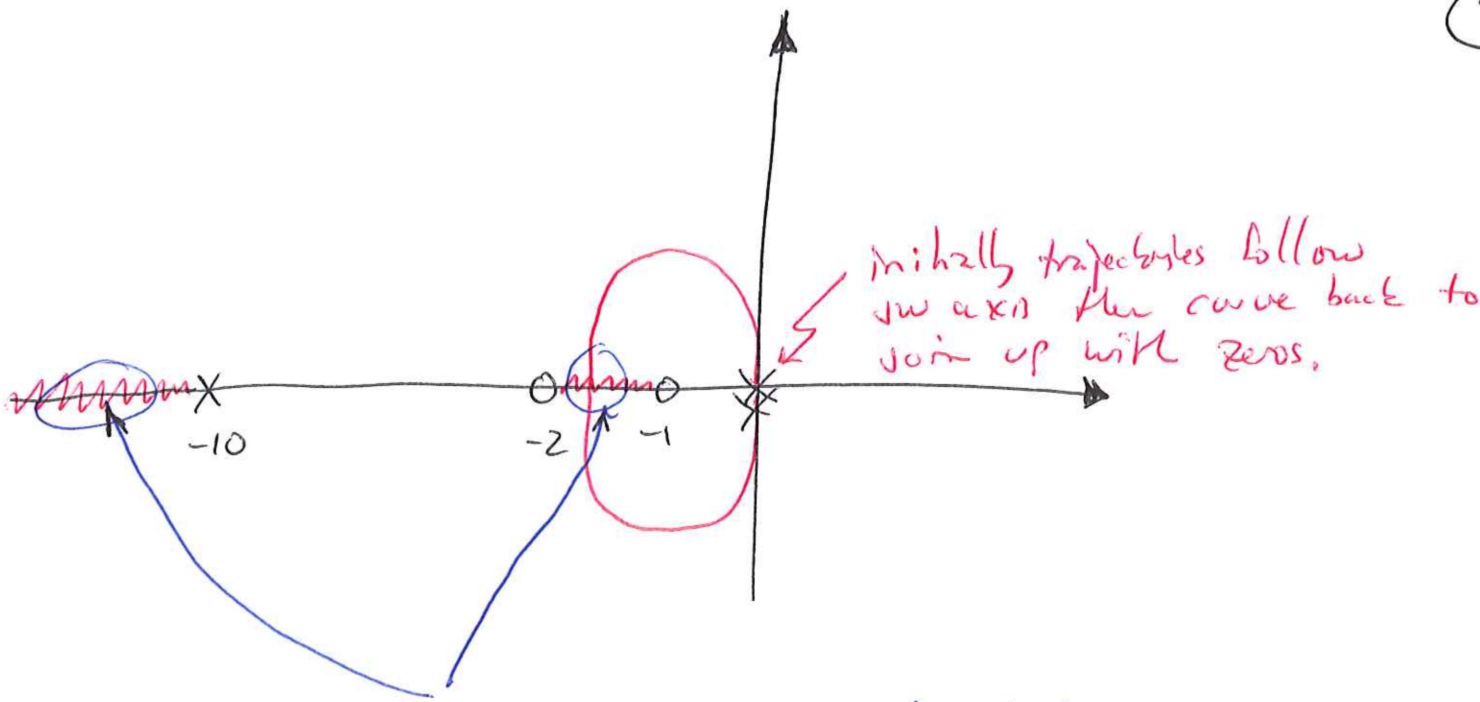
Open loop poles at $\{-10, 0, 0\} \Rightarrow N=3$

Open loop zeros at $\{-1, -2\} \Rightarrow M=2$

One asymptote line → must be real axis

Inter cept point : $\frac{-10+0+0 - (-1)-(-2)}{3-2} = -7$

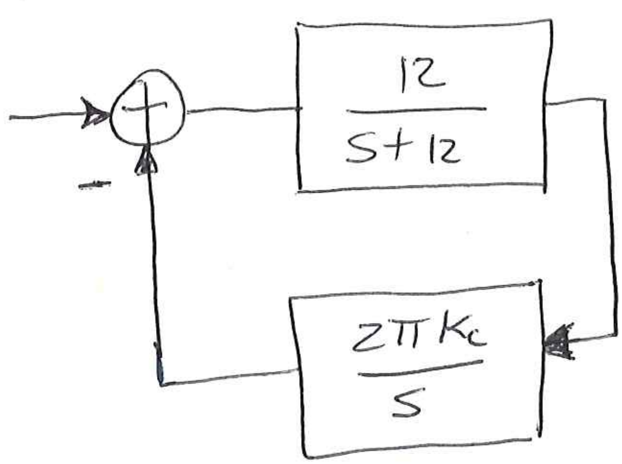
meaningless! Don't waste time calculating



G can be adjusted to place closed loop poles in this region.

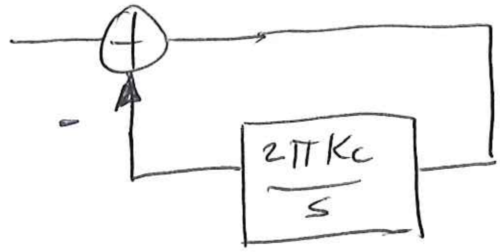
Gives good damped response.

(5) ~~In lab 3 you had a~~ PLL for the first order loop with a negative feedback as follows:

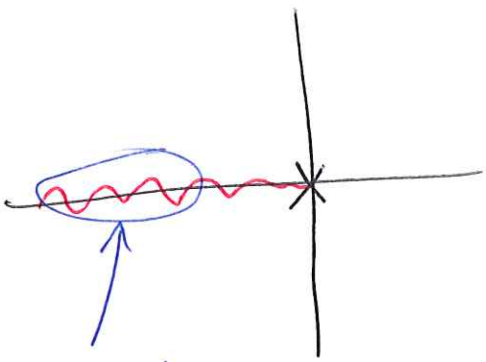


Use root locus to determine the effect of the LPF of $12/s+12$.

Without LPF:



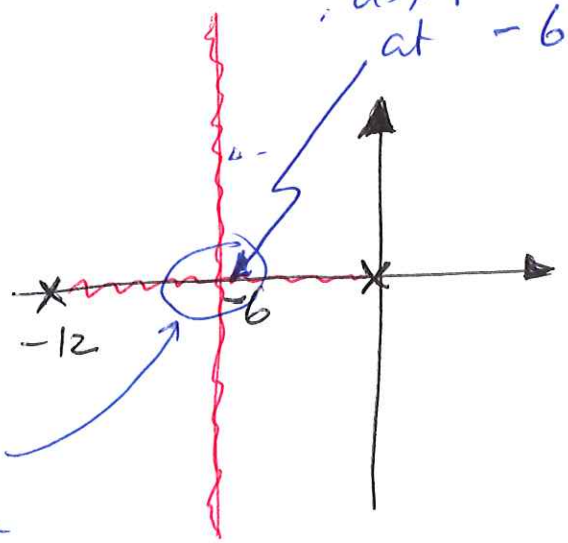
root locus



K_c set to give real axis pole

asymptote intercept at -6

With LPF

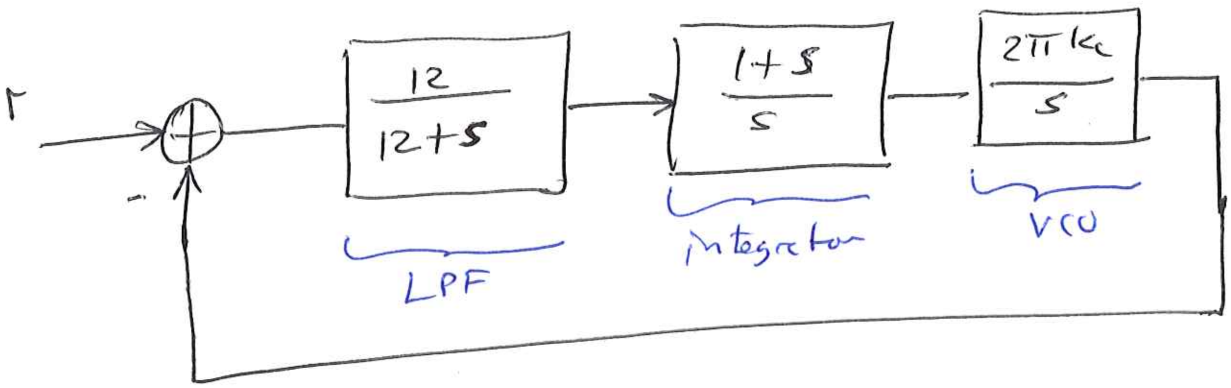


adjust K_c to give poles in this region.

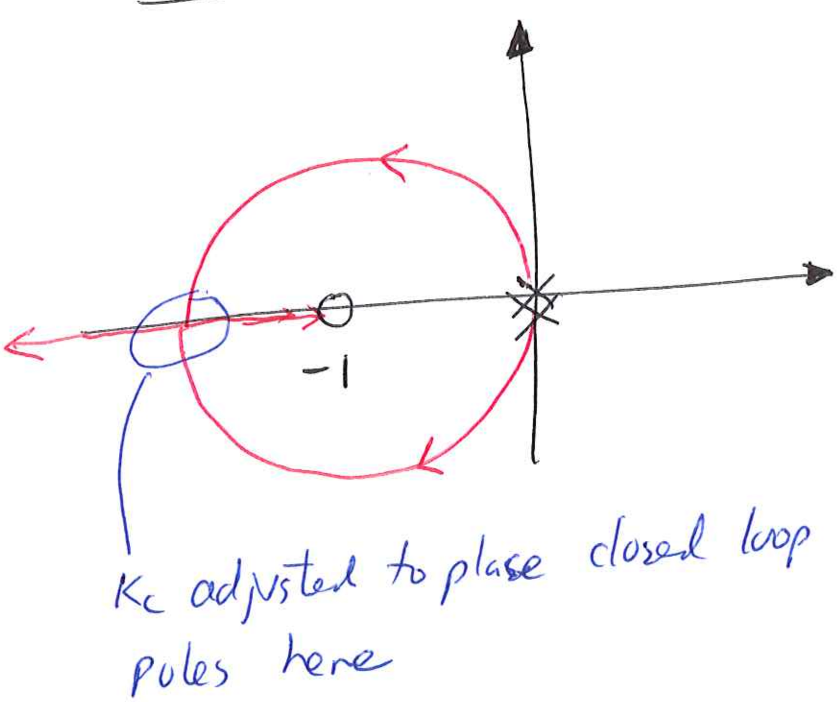
Note with LPF there is a limit to how fast the loop can respond with an increase in K_c.

6) In lab 3 you considered the second order loop. Determine what effect the LPF of $12/12+s$ of the phase detector has on the loop based on root locus analysis.

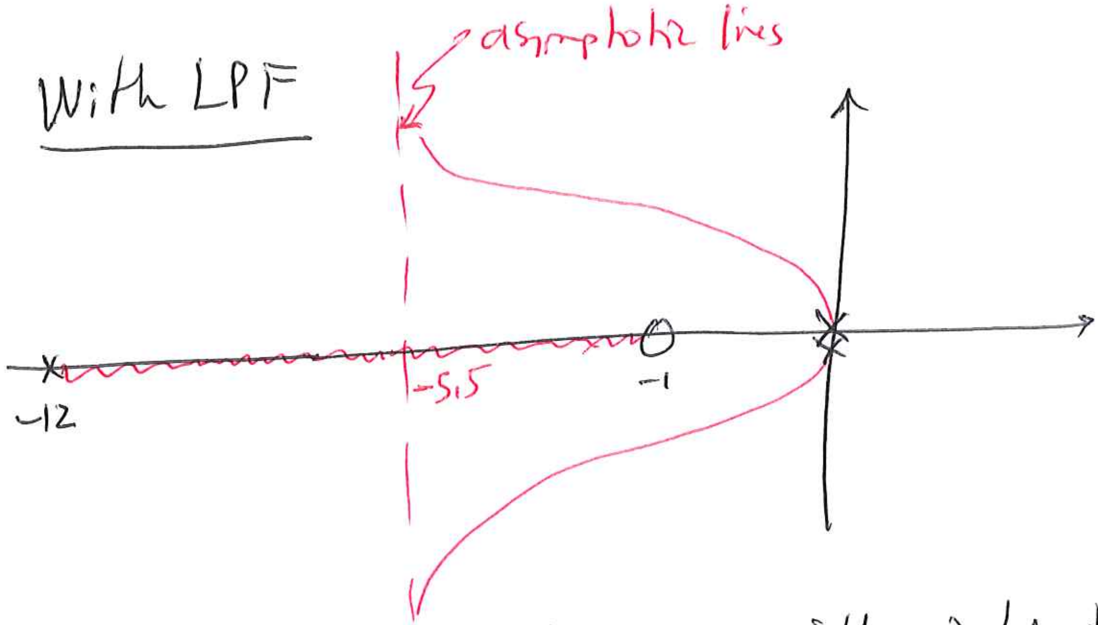
Assume the compensated integrator is $\frac{s+1}{s}$.



No LPF



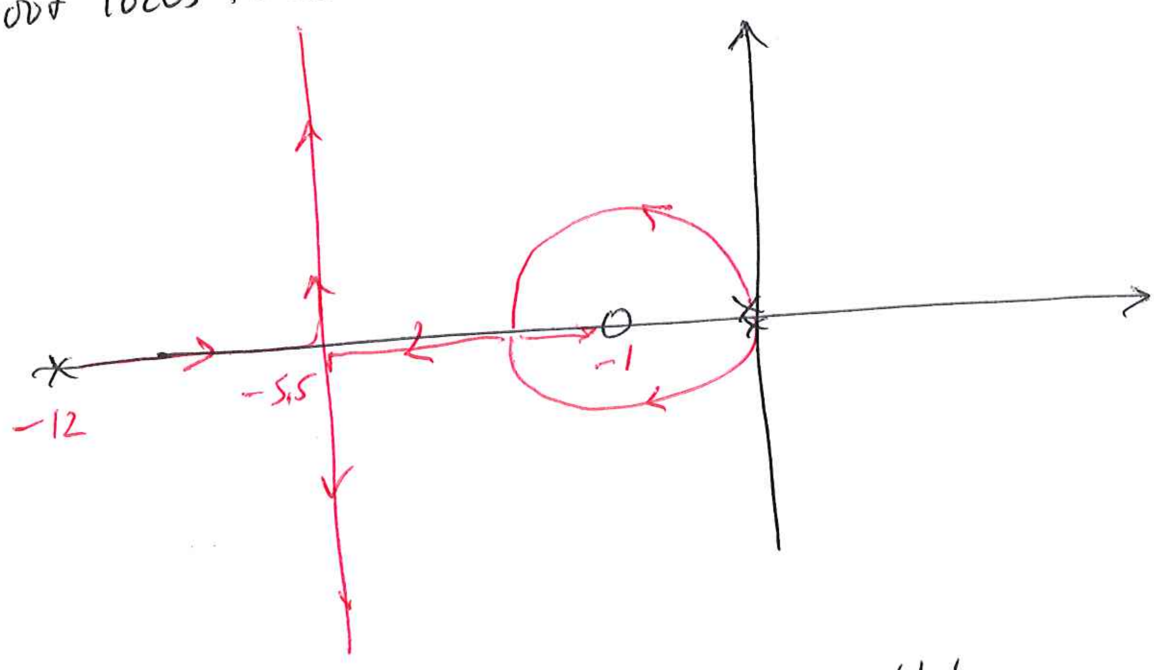
With LPF



Now two asymptote lines will intercept $\frac{-12+0+0-(-1)}{3-1}$

$= -\frac{11}{2} = -5.5$

It is not possible to tell from the simple set of root locus rules but another possible locus is as follows.



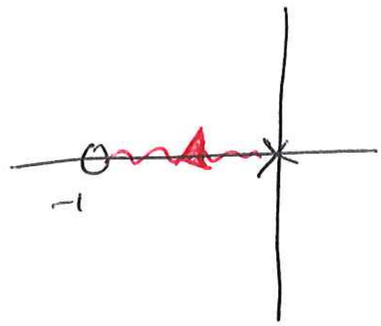
Which is it? Check with Matlab.

Regardless effect of LPF is slower response due to dominant pole that goes to the zero at -1

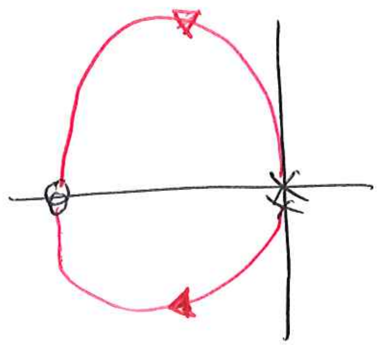
Hence as K_c increased beyond a certain point the loop actually gets slower!

7 Consider root locus for $\left(\frac{s+1}{s}\right)^K$ $K = 1, 2, \ldots 5$

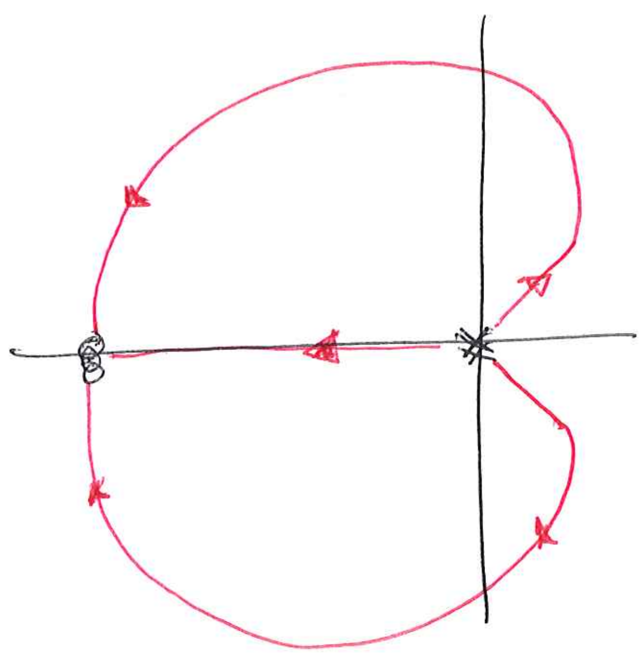
$H(s) = \frac{s+1}{s}$



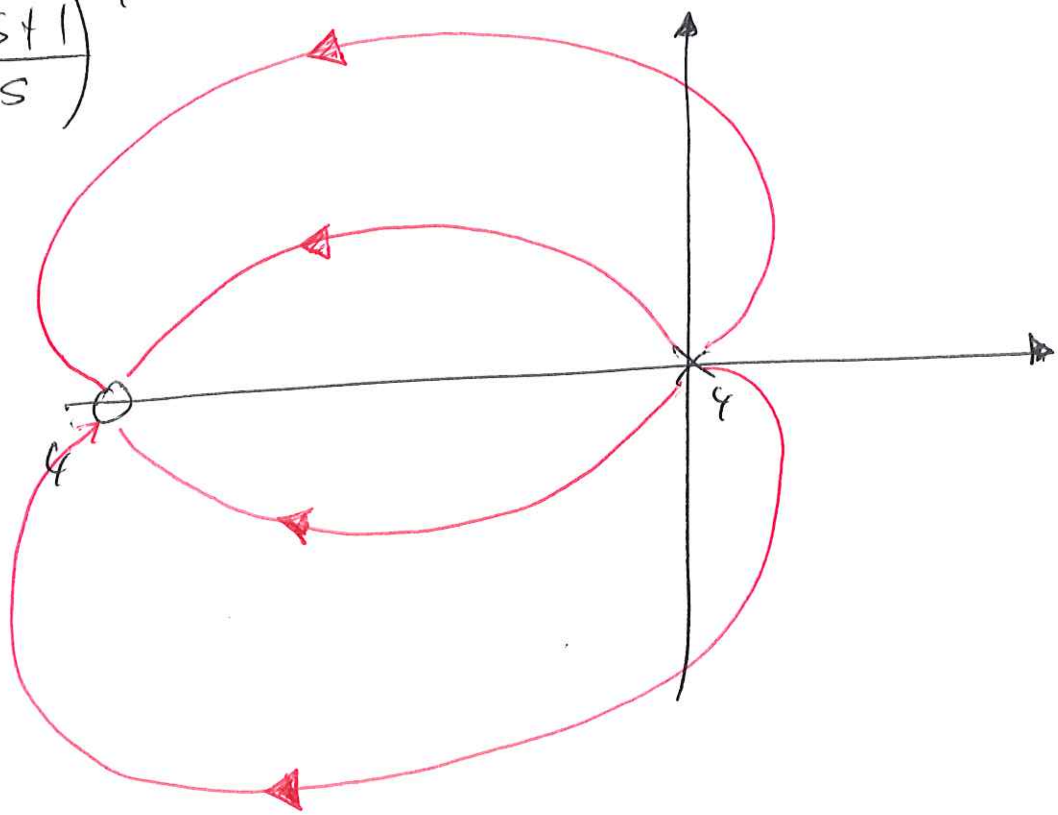
$H(s) = \left(\frac{s+1}{s}\right)^2$



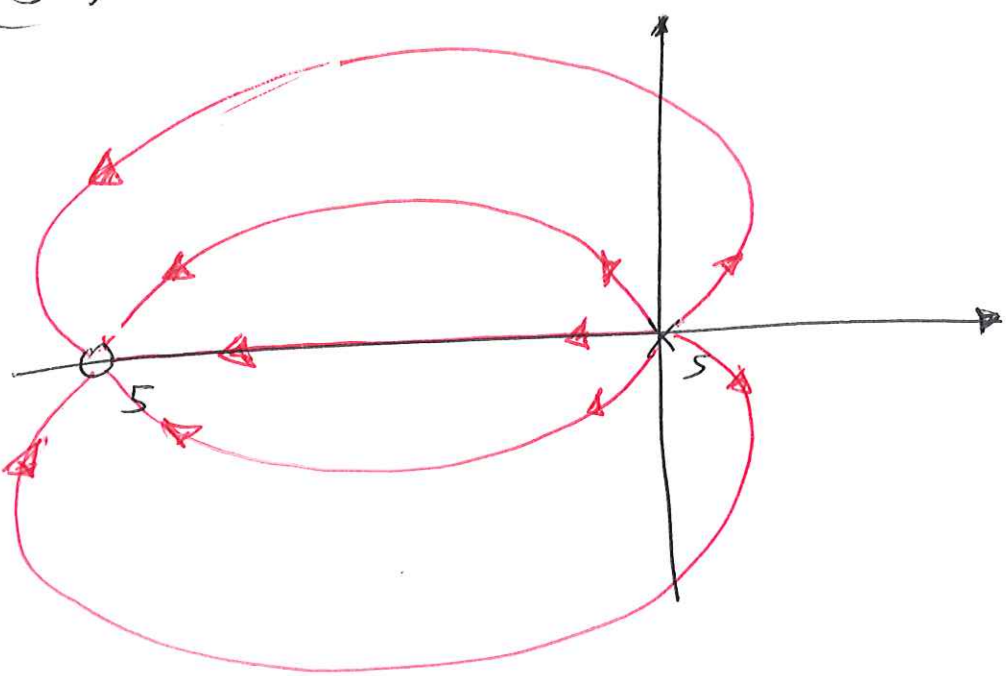
$H(s) = \left(\frac{s+1}{s}\right)^3$



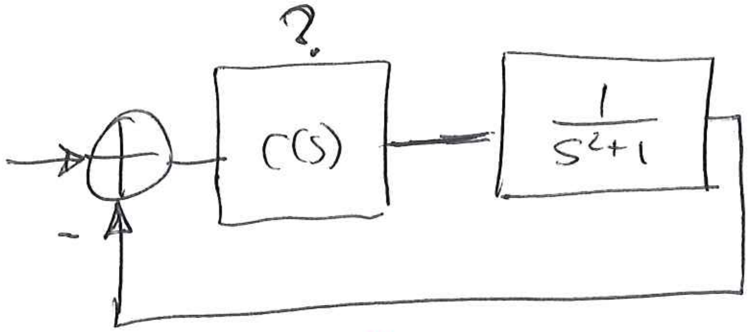
$$H(s) = \left(\frac{s+1}{s}\right)^4$$



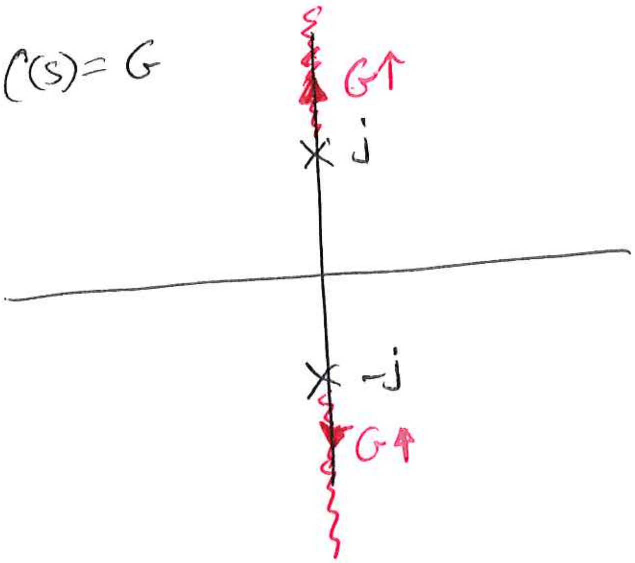
$$H(s) = \left(\frac{s+1}{s}\right)^5$$



(8) Using root locus show how to stabilize the plant with $H(s) = \frac{1}{s^2+1}$

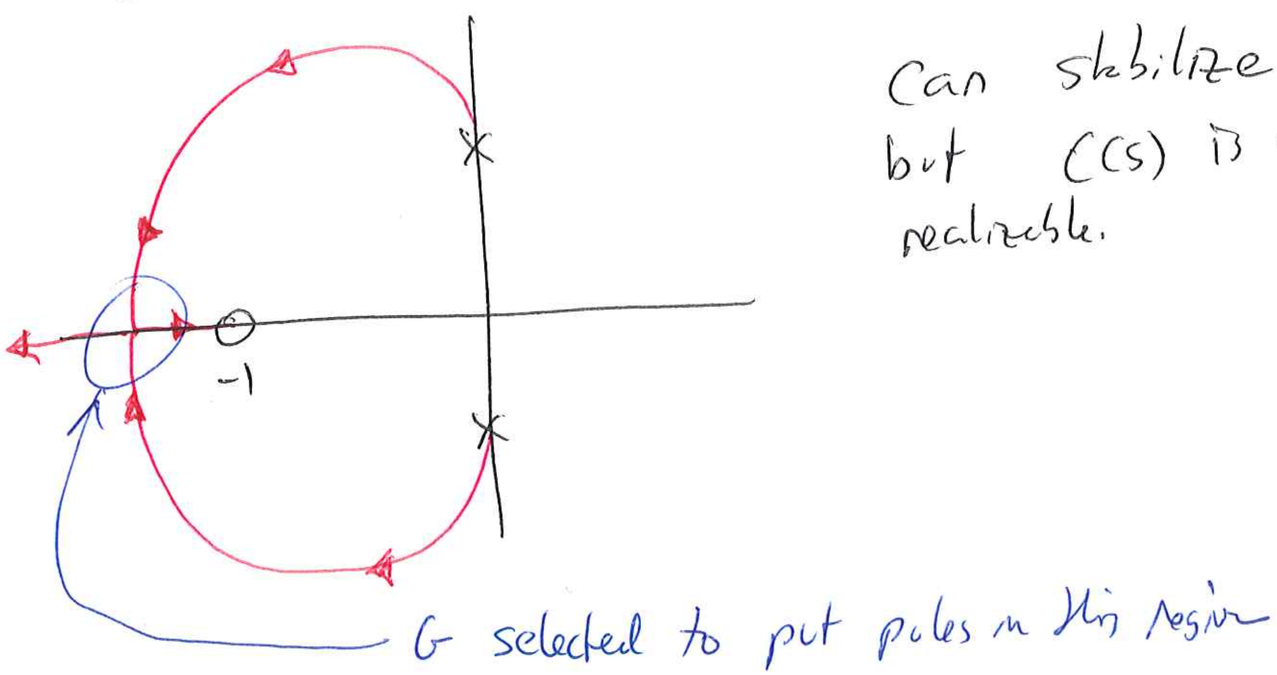


$C(s) = G$



Can't stabilize with $C(s) = G$, proportional feedback.

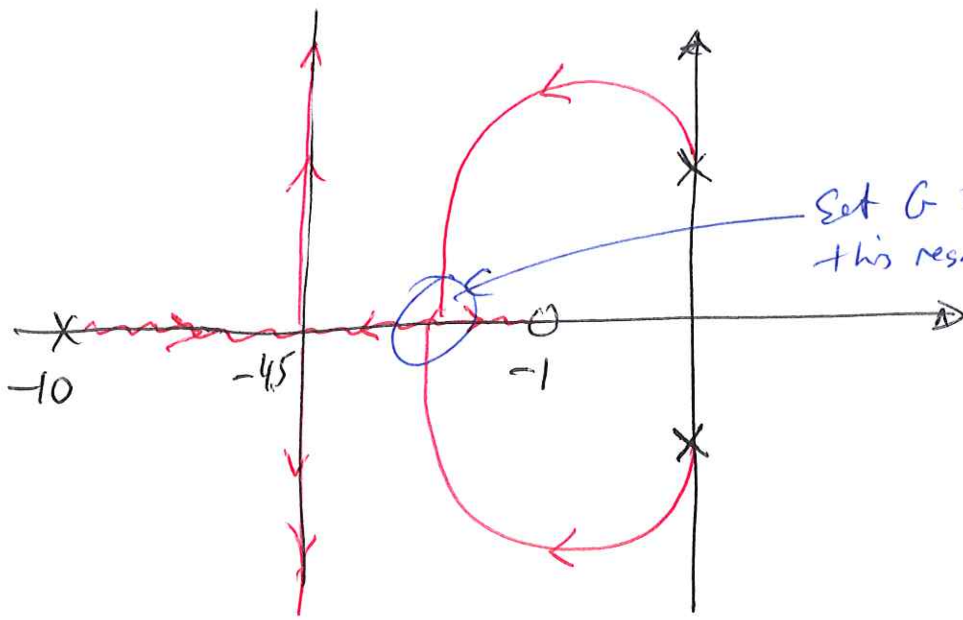
$C(s) = G(s+1)$ zero at $s = -1$



Can stabilize but $C(s)$ is not realizable.

$$C(s) = \frac{G(s+1)}{s+10}$$

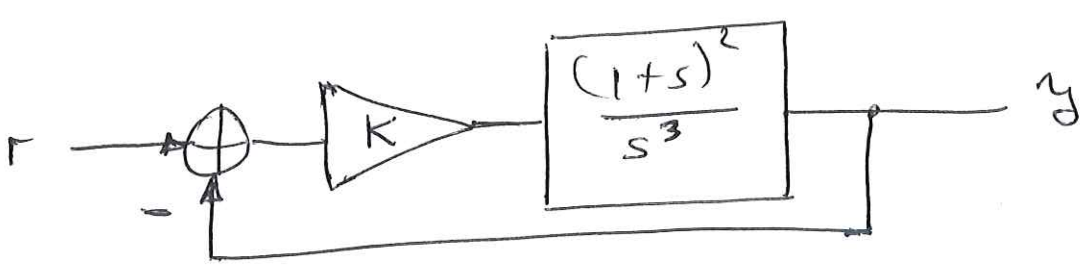
Realizable $C(s)$ with pole associated with zero



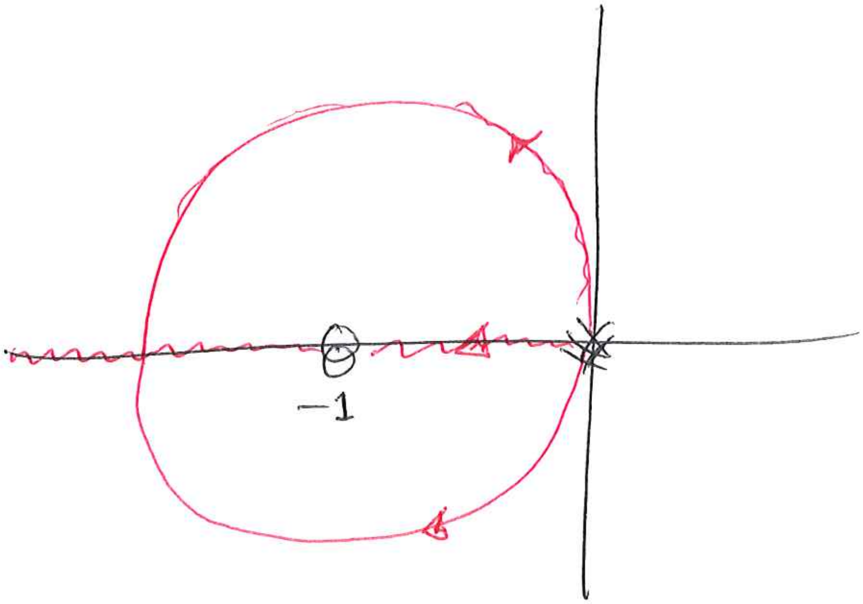
two asymptotes will intercept at

$$\frac{-10+1}{2} = -4.5$$

(9) What is the approximate step response of the following feedback loop when K gets large?

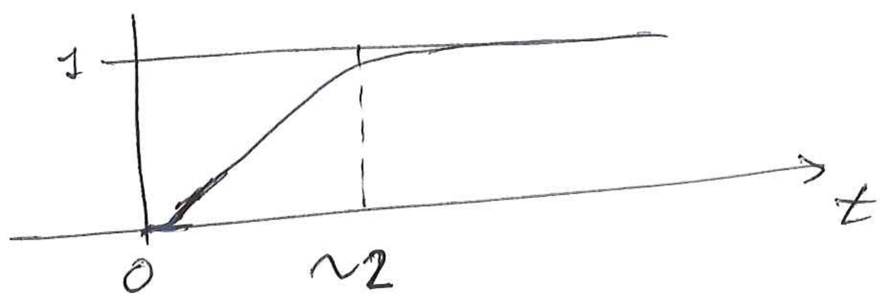


Start with consideration of the root locus



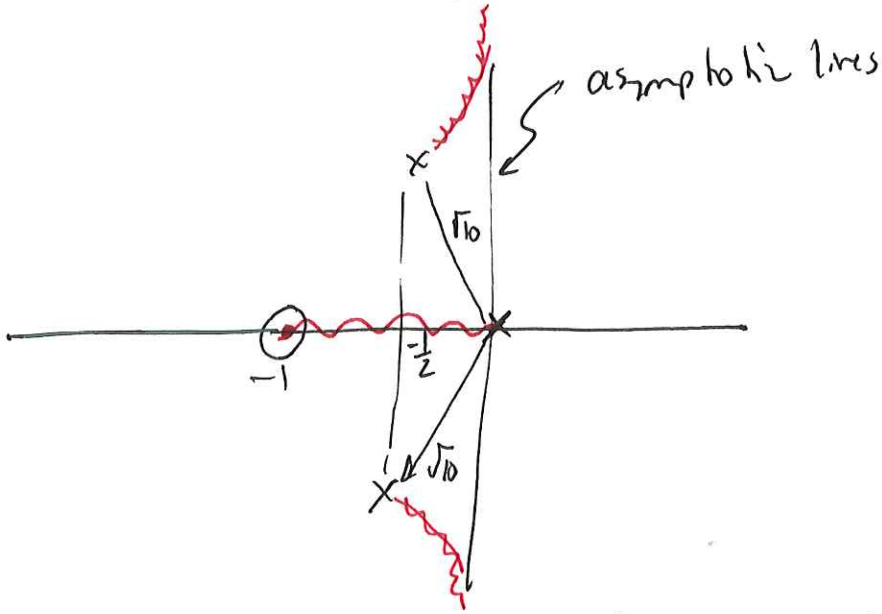
Essentially we will have two closed loop poles that are close to −1

We also know it is type-3 such that the output would asymptotically approach 1. Here a guess would be



10) Find Root locus of

$$G(s) = \frac{s+1}{(s^2+s+10)s}$$

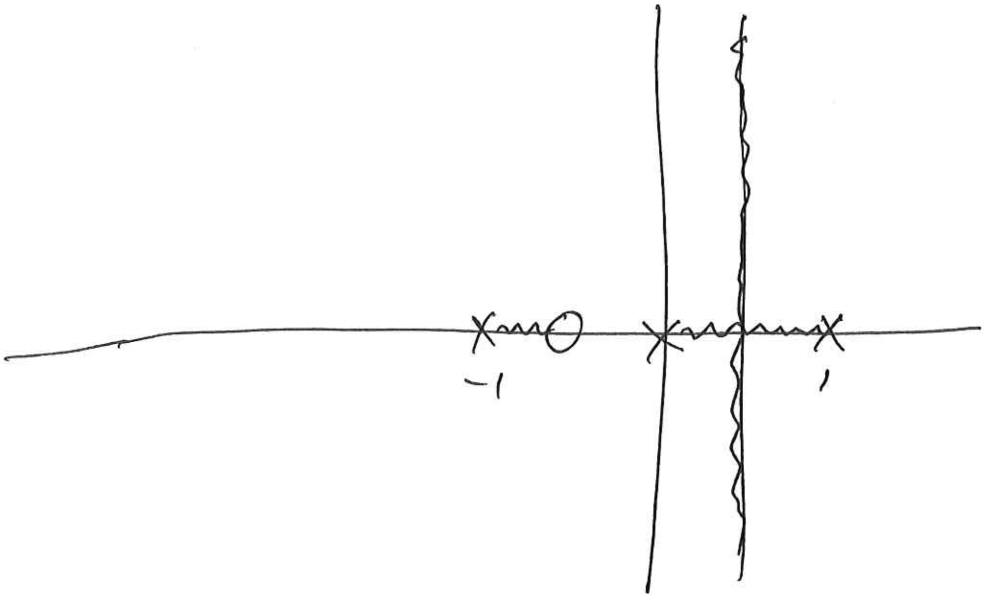


asymptote at $\dfrac{\left(-\frac{1}{2}\right)+\left(-\frac{1}{2}\right)+0-(-1)}{3-1} = \dfrac{0}{2}$

$= 0$

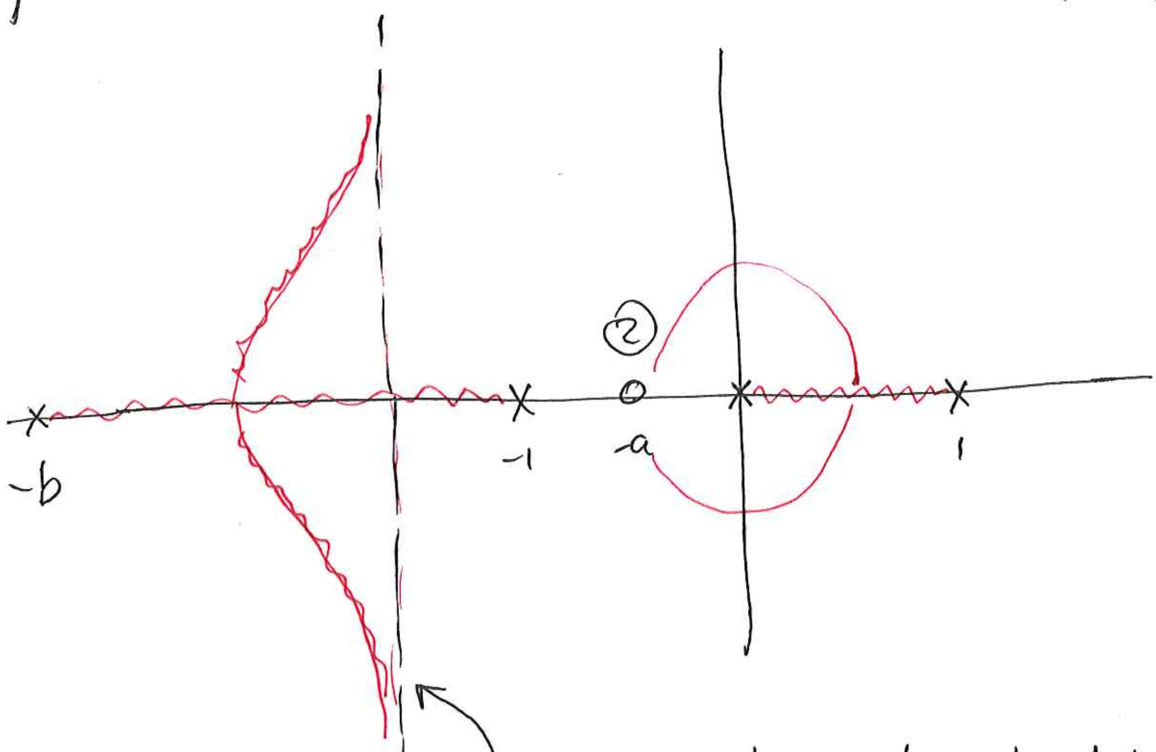
11) A Plant is unstable with a $H(s) = \dfrac{1}{s^2-1}$

It is desired to use a PDI feedback of type $C(s) = \dfrac{s+a}{s}$ to stabilize it and provide a type I loop. Is this possible?



Note a cannot be selected such that the trajectories go to the LHP. Cannot be stabilized.

12) In Q11 try with $C(s) = \left(\frac{s+a}{s}\right)\left(\frac{s+a}{s+b}\right)$



asymptotic line at $\dfrac{-b-1+0+1+a}{2}$