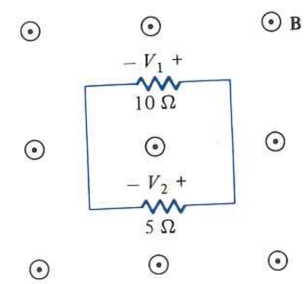

FIGURE 9.17 For Review Question 9.5.

9.4 A loop is rotating about the y-axis in a magnetic field $\mathbf{B} = B_o \sin \omega t\, \mathbf{a}_x$ Wb/m². The voltage induced in the loop is due to

(a) Motional emf
(b) Transformer emf
(c) A combination of motional and transformer emf
(d) None of the above

9.5 A rectangular loop is placed in the time-varying magnetic field $\mathbf{B} = 0.2 \cos 150\pi t\, \mathbf{a}_z$ Wb/m² as shown in Figure 9.17. V_1 is not equal to V_2.

(a) True (b) False

9.6 The concept of displacement current was a major contribution attributed to

(a) Faraday
(b) Lenz
(c) Maxwell
(d) Lorenz
(e) Your professor

9.7 Identify which of the following expressions are not Maxwell's equations for time-varying fields:

(a) $\nabla \cdot \mathbf{J} + \dfrac{\partial \rho_v}{\partial t} = 0$

(b) $\nabla \cdot \mathbf{D} = \rho_v$

(c) $\nabla \cdot \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$

(d) $\oint \mathbf{H} \cdot d\mathbf{l} = \int \left(\sigma \mathbf{E} + \varepsilon \dfrac{\partial \mathbf{E}}{\partial t}\right) \cdot d\mathbf{S}$

(e) $\oint \mathbf{B} \cdot d\mathbf{S} = 0$

9.8 An EM field is said to be nonexistent or not Maxwellian if it fails to satisfy Maxwell's equations and the wave equations derived from them. Which of the following fields in free space are not Maxwellian?

(a) $\mathbf{H} = \cos x \cos 10^6 t\, \mathbf{a}_y$
(b) $\mathbf{E} = 100 \cos \omega t\, \mathbf{a}_x$
(c) $\mathbf{D} = e^{-10y} \sin(10^5 t - 10y)\, \mathbf{a}_z$
(d) $\mathbf{B} = 0.4 \sin 10^4 t\, \mathbf{a}_z$
(e) $\mathbf{H} = 10 \cos\left(10^5 t - \dfrac{z}{10}\right) \mathbf{a}_x$
(f) $\mathbf{E} = \dfrac{\sin\theta}{r} \cos(\omega t - r\omega\sqrt{\mu_o \varepsilon_o})\, \mathbf{a}_\theta$
(g) $\mathbf{B} = (1 - \rho^2) \sin \omega t\, \mathbf{a}_z$

9.9 Which of the following statements is not true of a phasor?

(a) It may be a scalar or a vector.
(b) It is a time-dependent quantity.
(c) A phasor V_s may be represented as $V_o \angle\theta$ or $V_o e^{j\omega}$ where $V_o = |V_s|$.
(d) It is a complex quantity.

9.10 If $\mathbf{E}_s = 10\, e^{j4x}\, \mathbf{a}_y$, which of these is not a correct representation of $\mathbf{E}$?

(a) $\text{Re}(\mathbf{E}_s e^{j\omega t})$
(b) $\text{Re}(\mathbf{E}_s e^{-j\omega t})$
(c) $\text{Im}(\mathbf{E}_s e^{j\omega t})$
(d) $10 \cos(\omega t + j4x)\, \mathbf{a}_y$
(e) $10 \sin(\omega t + 4x)\, \mathbf{a}_y$

Answers: 9.1b, 9.2b, d, 9.3a, 9.4c, 9.5a, 9.6c, 9.7a,c, 9.8b, d, 9.9b, 9.10d.

PROBLEMS

Sections 9.2 and 9.3—Faraday's Law and Electromotive Forces

9.1 A conducting circular loop of radius 20 cm lies in the $z = 0$ plane in a magnetic field $\mathbf{B} = 10 \cos 377t\, \mathbf{a}_z$ mWb/m². Calculate the induced voltage in the loop.

9.2 The circuit in Figure 9.18 exists in a magnetic field $\mathbf{B} = 40 \cos(30\pi t - 3y)\mathbf{a}_z$ mWb/m². Assume that the wires connecting the resistors have negligible resistances. Find the current in the circuit.

9.3 A circuit conducting loop lies in the xy-plane as shown in Figure 9.19. The loop has a radius of 0.2 m and resistance $R = 4\ \Omega$. If $\mathbf{B} = 40 \sin 10^4\, t\mathbf{a}_z$ mWb/m², find the currrent.

9.4 Two conducting bars slide over two stationary rails, as illustrated in Figure 9.20. If $\mathbf{B} = 0.2\mathbf{a}_z$ Wb/m², determine the induced emf in the loop thus formed.

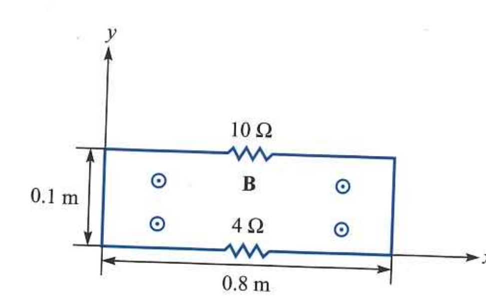

FIGURE 9.18 For Problem 9.2.

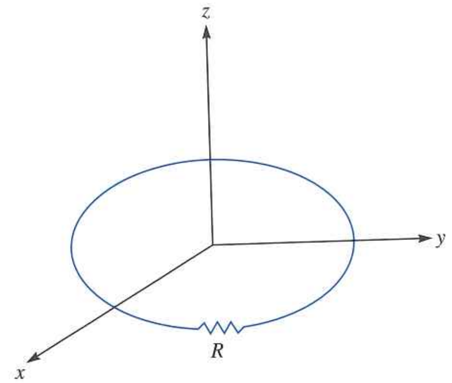

FIGURE 9.19 For Problem 9.3.

9.5 A circular loop defined by $x^2 + y^2 = 9$ is located in a magnetic field described by $\mathbf{B} = 4\sqrt{x^2 + y^2}\cos \omega t\, \mathbf{a}_z$ Wb/m². Determine the emf induced in the loop.

9.6 A square loop of side a recedes with a uniform velocity $u_o\mathbf{a}_y$ from an infinitely long filament carrying current I along $\mathbf{a}_z$ as shown in Figure 9.21. Assuming that $\rho = \rho_o$ at time $t = 0$, show that the emf induced in the loop at $t > 0$ is

$$V_{\text{emf}} = \frac{u_o a^2 \mu_o I}{2\pi\rho(\rho + a)}$$

9.7 A conducting rod moves with a constant velocity of 3 $\mathbf{a}_z$ m/s parallel to a long straight wire carrying a current of 15 A as in Figure 9.22. Calculate the emf induced in the rod and state which end is at the higher potential.

9.8 A conducting rod has one end grounded at the origin, while the other end is free to move in the $z = 0$ plane. The rod rotates at 30 rad/s in a static magnetic field $\mathbf{B} = 60\mathbf{a}_z$ mWb/m². If the rod is 8 cm long, find the voltage induced in the rod.

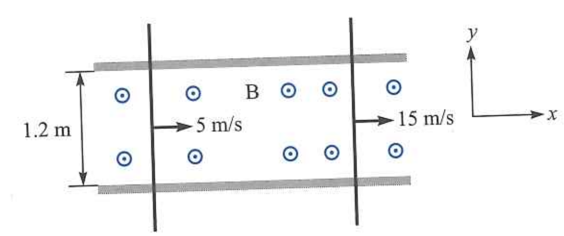

FIGURE 9.20 For Problem 9.4.

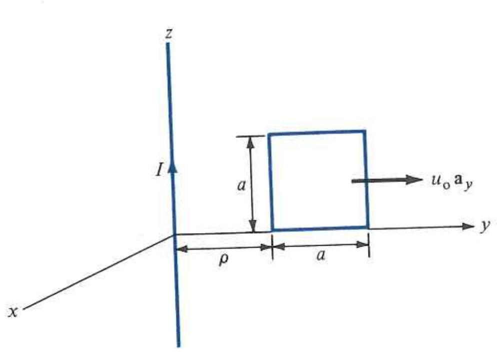

FIGURE 9.21 For Problem 9.6.

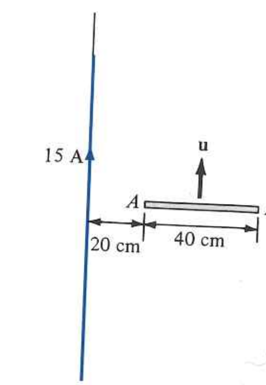

FIGURE 9.22 For Problem 9.7.

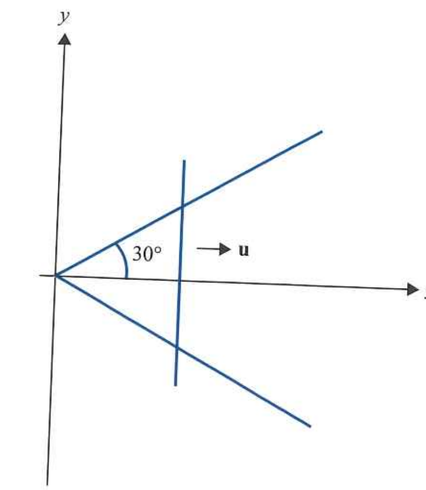

FIGURE 9.23 For Problem 9.10.

9.9 A rectangular coil has a cross-sectional area of 30 cm² and 50 turns. If the coil rotates at 60 rad/s in a magnetic field of 0.2 Wb/m² such that its axis of rotation is perpendicular to the direction of the field, determine the induced emf in the coil.

9.10 Determine the induced emf in the V-shaped loop of Figure 9.23. Take $\mathbf{B} = 0.6x\mathbf{a}_z$ Wb/m² and $\mathbf{u} = 5\mathbf{a}_x$ m/s. Assume that the sliding rod starts at the origin when $t = 0$.

9.11 A car travels at 120 km/hr. If the earth's magnetic field is 4.3×10^{-5} Wb/m², find the induced voltage in the car bumper of length 1.6 m. Assume that the angle between the earth's magnetic field and the normal to the car is 65°.

9.12 An airplane with a metallic wing of span 36 m flies at 410 m/s in a region where the vertical component of the earth's magnetic field is 0.4 μWb/m². Find the emf induced on the airplane wing.

9.13 As portrayed in Figure 9.24, a bar magnet is thrust toward the center of a coil of 10 turns and resistance 15 Ω. If the magnetic flux through the coil changes from 0.45 Wb to 0.64 Wb in 0.02 s, find the magnitude and direction (as viewed from the side near the magnet) of the induced current.

9.14 The cross section of a homopolar generator disk is shown in Figure 9.25. The disk has inner radius $\rho_1 = 2$ cm and outer radius $\rho_2 = 10$ cm and rotates in a uniform magnetic field 15 mWb/m² at a speed of 60 rad/s. Calculate the induced voltage.

Section 9.4—Displacement Current

9.15 A 50 V voltage generator at 20 MHz is connected to the plates of an air dielectric parallel-plate capacitor with a plate area of 2.8 cm² and a separation distance of 0.2 mm. Find the maximum value of displacement current density and displacement current.

9.16 A dielectric material with $\mu = \mu_o$, $\varepsilon = 9\varepsilon_o$, $\sigma = 4$ S/m is placed between the plates of a parallel-plate capacitor. Calculate the frequency at which the conduction and displacement currents are equal.

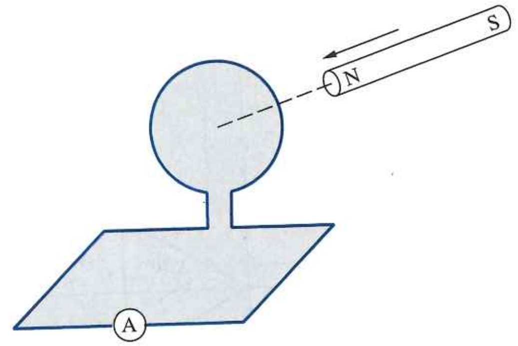

FIGURE 9.24 For Problem 9.13.

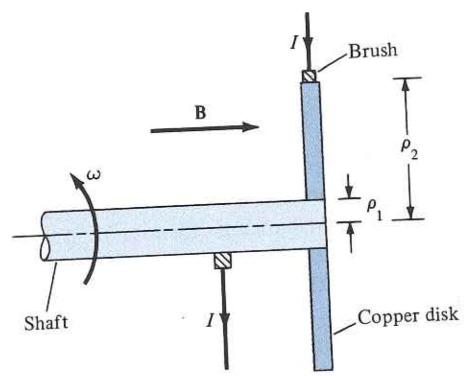

FIGURE 9.25 For Problem 9.14.

9.17 The ratio J/J_d (conduction current density to displacement current density) is very important at high frequencies. Calculate the ratio at 1 GHz for:

(a) distilled water ($\mu = \mu_o, \varepsilon = 81\varepsilon_o, \sigma = 2 \times 10^{-3}$ S/m)

(b) seawater ($\mu = \mu_o, \varepsilon = 81\varepsilon_o, \sigma = 25$ S/m)

(c) limestone ($\mu = \mu_o, \varepsilon = 5\varepsilon_o, \sigma = 2 \times 10^{-4}$ S/m)

9.18 Assume that dry soil has $\sigma = 10^{-4}$ S/m, $\varepsilon = 3\varepsilon_o$, and $\mu = \mu_o$. Determine the frequency at which the ratio of the magnitudes of the conduction current density and the displacement current density is unity.

9.19 In a dielectric ($\sigma = 10^{-4}$ S/m, $\mu_r = 1$, $\varepsilon_r = 4.5$), the conduction current density is given as $J_c = 0.4\cos(2\pi \times 10^8 t)$ A/m^2. Determine the displacement current density.

Section 9.5—Maxwell's Equations

9.20 (a) Write Maxwell's equations for a linear, homogeneous medium in terms of $\mathbf{E}_s$ and $\mathbf{H}_s$ assuming only the time factor $e^{-j\omega t}$.

(b) In Cartesian coordinates, write the point form of Maxwell's equations in Table 9.2 as eight scalar equations.

9.21 Show that in a source-free region ($\mathbf{J} = 0, \rho_v = 0$), Maxwell's equations can be reduced to two. Identify the two all-embracing equations.

9.22 Show that fields

$$\mathbf{E} = \mathrm{E}_o \cos x \cos t\mathbf{a}_y \quad \text{and} \quad \mathbf{H} = \frac{E_o}{\mu_o}\sin x \sin t\mathbf{a}_z$$

do not satisfy all of Maxwell's equations.

9.23 Assuming a source-free region, derive the diffusion equation

$$\nabla^2\mathbf{E} = \mu\sigma\frac{\partial \mathbf{E}}{\partial t}$$

9.24 In a certain region,

$$\mathbf{J} = (2y\mathbf{a}_x + xz\mathbf{a}_y + z^3\mathbf{a}_z)\sin 10^4 t \text{ A/m}$$

find ρ_v if $\rho_v(x, y, 0, t) = 0$.

9.25 Given that $\mathbf{E} = E_o\cos(\omega t - \beta z)\mathbf{a}_x$ V/m in free space, determine **D**, **H**, and **B**.

9.26 In a certain material, $\sigma = 0$, $\mu = \mu_o$, and $\varepsilon = 81\varepsilon_o$. The magnetic field intensity in this material is $\mathbf{H} = 10\cos(2\pi \times 10^9 t + \beta x)\mathbf{a}_z$ A/m. Determine **E** and β.

9.27 In free space,

$$\mathbf{E} = \frac{50}{\rho}\cos(10^8 t - kz)a_\rho \text{V/m}$$

Find k, $\mathbf{J}_d$, and **H**.

9.28 The electric field intensity of a spherical wave in free space is given by

$$\mathbf{E} = \frac{10}{r}\sin\theta\cos(\omega t - \beta r)\mathbf{a}_\theta \text{ V/m}$$

Find the corresponding magnetic field intensity **H**.

9.29 In a certain region for which $\sigma = 0$, $\mu = 2\mu_o$, and $\varepsilon = 10\varepsilon_o$

$$\mathbf{J} = 60\sin(10^9 t - \beta z)\mathbf{a}_x \text{ mA/m}^2$$

(a) Find **D** and **H**.

(b) Determine β.

9.30 Check whether the following fields are genuine EM fields (i.e., they satisfy Maxwell's equations). Assume that the fields exist in charge-free regions.

(a) $\mathbf{A} = 40\sin(\omega t + 10x)\mathbf{a}_z$

(b) $\mathbf{B} = \dfrac{10}{\rho}\cos(\omega t - 2\rho)\mathbf{a}_\phi$

(c) $\mathbf{C} = \left(3\rho^2\cot\phi\,\mathbf{a}_\rho + \dfrac{\cos\phi}{\rho}\mathbf{a}_\phi\right)\sin\omega t$

(d) $\mathbf{D} = \dfrac{1}{r}\sin\theta\sin(\omega t - 5r)\mathbf{a}_\theta$

9.31 Given the total electromagnetic energy

$$W = \frac{1}{2}\int (\mathbf{E}\cdot\mathbf{D} + \mathbf{H}\cdot\mathbf{B})\, dv$$

show from Maxwell's equations that

$$\frac{\partial W}{\partial t} = -\oint_S (\mathbf{E}\times\mathbf{H})\cdot d\mathbf{S} - \int_v \mathbf{E}\cdot\mathbf{J}\, dv$$

9.32 Given that $\mathbf{E} = E_o \cos(\omega t + \beta y - \beta z)\mathbf{a}_x$ V/m, use Maxwell's equations to find the corresponding magnetic field intensity **H**.

9.33 An antenna radiates in free space and

$$\mathbf{H} = \frac{12 \sin\theta}{r}\cos(2\pi \times 10^8 t - \beta r)\mathbf{a}_\theta \text{ mA/m}$$

Find the corresponding **E** in terms of β.

Section 9.6—Time-Varying Potentials

9.34 In free space ($\rho_v = 0, \mathbf{J} = 0$), show that

$$\mathbf{A} = \frac{\mu_0}{4\pi r}(\cos\theta\, \mathbf{a}_r - \sin\theta\, \mathbf{a}_\theta)e^{j\omega(t - r/c)}$$

satisfies the wave equation in eq. (9.52). Find the corresponding V. Take c as the speed of light in free space.

9.35 Retrieve Faraday's law in differential form from

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

9.36 In free space, the retarded potentials are given by

$$V = x(z - ct)\text{V}, \quad \mathbf{A} = x(z/c - t)\mathbf{a}_z \text{ Wb/m}$$

where $c = \dfrac{1}{\sqrt{\mu_o \varepsilon_o}}$

(a) Prove that $\nabla \cdot \mathbf{A} = \mu_o \varepsilon_o \dfrac{\partial V}{\partial t}$.

(b) Determine **E**.

9.37 Let $\mathbf{A} = A_o \sin(\omega t - \beta z)\mathbf{a}_x$ Wb/m in free space. (a) Find V and **E**. (b) Express β in terms of ω, ε_o, and μ_o.

Section 9.7—Time-Harmonic Fields

9.38 Evaluate the following complex numbers and express your answers in polar form:

(a) $(4\angle 30° - 10\angle 50°)^{1/2}$

(b) $\dfrac{1 + j2}{6 + j8 - 7\angle 15°}$

(c) $\dfrac{(3 + j4)^2}{12 - j7 + (-6 + j10)^*}$

(d) $\dfrac{(3.6\angle -200°)^{1/2}}{(2.4\angle 45°)^2(-5 + j8)^*}$

9.39 Express the following time-harmonic fields as phasors.

(a) $\mathbf{A} = 5\sin(2t + \pi/3)\mathbf{a}_x + 3\cos(2t + 30°)\mathbf{a}_y$

(b) $\mathbf{B} = \dfrac{100}{\rho}\sin(\omega t - 2\pi z)\mathbf{a}_\rho$

(c) $\mathbf{C} = \dfrac{\cos\theta}{r}\sin(\omega t - 3r)\mathbf{a}_\theta$

(d) $\mathbf{D} = 10\cos(k_1 x)\cos(\omega t - k_2 z)\mathbf{a}_y$

9.40 In a source-free vacuum region;

$$\mathbf{H} = \frac{1}{\rho}\cos(\omega t - 3z)\mathbf{a}_\phi \text{ A/m}$$

(a) Express **H** in phasor form.

(b) Find the associated **E** Field.

(c) Determine ω.

9.41 In a certain homogeneous medium, $\varepsilon = 81\varepsilon_o$, and $\mu = \mu_o$,

$$\mathbf{E}_s = 10e^{j(\omega t + \beta z)}\mathbf{a}_y \text{ V/m}$$

$$\mathbf{H}_s = H_o e^{j(\omega t + \beta z)}\mathbf{a}_x \text{ A/m}$$

If $\omega = 2\pi \times 10^9$ rad/m, find β and H_o.

9.42 Let $\mathbf{H} = 40\cos(10^9 t - \beta z)\mathbf{a}_x$ A/m in a region for which $\sigma = 0$, $\mu = \mu_o$, $\varepsilon = 4\varepsilon_o$. (a) Express **H** in phase form. (b) Find $\mathbf{J}_d$.

9.43 Given that

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + y = 2\cos 3t$$

Solve for y by using phasors.

9.44 Show that in a linear homogeneous, isotropic source-free region, both $\mathbf{E}_s$ and $\mathbf{H}_s$ must satisfy the wave equation

$$\nabla^2 \mathbf{A}_s + \gamma^2 \mathbf{A}_s = \mathbf{0}$$

where $\gamma^2 = \omega^2\mu\varepsilon - j\omega\mu\sigma$ and $\mathbf{A}_s = \mathbf{E}_s$ or $\mathbf{H}_s$.

10.9 In a good conductor, **E** and **H** are in time phase.

(a) True
(b) False

10.10 The Poynting vector physically denotes the power density leaving or entering a given volume in a time-varying field.

(a) True
(b) False

Answers: 10.1b, 10.2f, 10.3a, 10.4b,c 10.5b,e,f, 10.6c, 10.7c, 10.8b, 10.9b, 10.10a.

PROBLEMS

Section 10.2—Waves in General

10.1 An EM wave propagating in a certain medium is described by

$$\mathbf{E} = 25 \sin(2\pi \times 10^6 t - 6x)\mathbf{a}_z \text{ V/m}$$

(a) Determine the direction of wave propagation.
(b) Compute the period T, the wavelength λ, and the velocity u.
(c) Sketch the wave at $t = 0, T/8, T/4, T/2$.

10.2 Calculate the wavelength for plane waves in vacuum at the following frequencies:

(a) 60 Hz (power line)
(b) 2 MHz (AM radio)
(c) 120 MHz (FM radio)
(d) 2.4 GHz (microwave oven)

10.3 An EM wave in free space is described by

$$\mathbf{H} = 0.4 \cos(10^8 t + \beta y) \text{ A/M}$$

Determine (a) the angular frequency ω, (b) the wave number β, (c) the wavelength λ, (d) the direction of wave propagation, (e) the value of $\mathbf{H}(2, 3, 4, 10 \text{ ns})$.

10.4 (a) Show that $E(x, t) = \cos(x + \omega t) + \cos(x - \omega t)$ satisfies the scalar wave equation.
(b) Determine the velocity of wave propagation.

Section 10.3—Wave Propagation in Lossy Dielectrics

10.5 (a) Derive eqs. (10.23) and (10.24) from eqs. (10.18) and (10.20).
(b) Using eq. (10.29) in conjunction with Maxwell's equations, show that

$$\eta = \frac{j\omega\mu}{\gamma}$$

(c) From part (b), derive eqs. (10.32) and (10.33).

10.6 At 50 MHz, a lossy dielectric material is characterized by $\varepsilon = 3.6\varepsilon_o$, $\mu = 2.1\mu_o$, and $\sigma = 0.08$ S/m. If $\mathbf{E}_s = 6e^{-\gamma x}\mathbf{a}_z$ V/m, compute (a) γ, (b) λ, (c) u, (d) η, (e) $\mathbf{H}_s$.

10.7 Determine the loss tangent for each of the following nonmagnetic media at 12 MHz.

(a) wet earth ($\varepsilon = 10\varepsilon_o, \sigma = 10^{-2}$ S/m)
(b) dry earth ($\varepsilon = 4\varepsilon_o, \sigma = 10^{-4}$ S/m)
(c) seawater ($\varepsilon = 81\varepsilon_o, \sigma = 4$ S/m)

10.8 Alumina is a ceramic material used in making printed circuit boards. At 15 GHz, $\varepsilon = 9.6\varepsilon_o$, $\mu = \mu_o$, $\tan\theta = 3 \times 10^{-4}$. Calculate (a) the penetration depth, (b) the total attenuation over a thickness of 5 mm.

10.9 A medium is characterized by $\varepsilon = 4\varepsilon_o$, $\mu = 2.5\mu_o$, $\sigma = 8 \times 10^{-3}$ S/m. Calculate the phase difference between **E** and **H** at 20 MHz.

10.10 At $f = 100$ MHz, show that silver ($\sigma = 6.1 \times 10^7$ S/m, $\mu_r = 1, \varepsilon_r = 1$) is a good conductor, while rubber ($\sigma = 10^{-15}$ S/m, $\mu_r = 1, \varepsilon_r = 3.1$) is a good insulator.

10.11 Seawater plays a vital role in the study of submarine communications. Assuming that for seawater, $\sigma = 4$ S/m, $\varepsilon_r = 80$, $\mu_r = 1$, and $f = 100$ kHz, calculate (a) the phase velocity, (b) the wavelength, (c) the skin depth, (d) the intrinsic impedance.

10.12 In a certain medium with $\mu = \mu_o$, $\varepsilon = 4\varepsilon_o$,

$$\mathbf{H} = 12e^{-0.1y} \sin(\pi \times 10^8 t - \beta y)\mathbf{a}_x \text{ A/m}$$

Find (a) the wave period T, (b) the wavelength λ, (c) the electric field **E**, (d) the phase difference between **E** and **H**.

10.13 In a nonmagnetic medium,

$$\mathbf{H} = 50e^{-100x} \cos(2\pi \times 10^9 t - 200x)\mathbf{a}_y \text{ mA/m}$$

Find **E**.

10.14 A certain medium has $\sigma = 1$ S/m, $\varepsilon = 4\varepsilon_o$ and $\mu = 9\,\mu_o$ at a frequency of 1 GHz. Determine the (a) attenuation constant, (b) phase constant, (c) intrinsic impedance, and (d) wave velocity.

Sections 10.4 and 10.5—Waves in Lossless Dielectrics and Free Space

10.15 The electric field of a TV broadcast signal progagating in air is given by

$$\mathbf{E}(z,t) = 0.2 \cos(\omega t - 6.5z)\mathbf{a}_x \text{ V/m}$$

(a) Determine the wave frequency ω and the wavelength λ.
(b) Sketch $\mathbf{E}_x$ as a function of t at $z = 0$ and $z = \lambda/2$.
(c) Find the corresponding $\mathbf{H}(z, t)$.

10.16 A 60 MHz plane wave travels in a lossless medium with $\varepsilon = 3\varepsilon_o$ and $\mu = 4\mu_o$. Find the wave velocity u, its wavelength λ, and the intrinsic impedance η of the medium.

10.17 The magnetic field component of an EM wave propagating through a nonmagnetic medium ($\mu = \mu_o$) is

$$\mathbf{H} = 25 \sin(2 \times 10^8 t + 6x)\mathbf{a}_y \text{ mA/m}$$

Determine:

(a) The direction of wave propagation
(b) The permittivity of the medium
(c) The electric field intensity

10.18 A manufacturer produces a ferrite material with $\mu = 750\mu_o$, $\varepsilon = 5\varepsilon_o$, and $\sigma = 10^{-6}$ S/m at 10 MHz.

(a) Would you classify the material as lossless, lossy, or conducting?
(b) Calculate β and λ.
(c) Determine the phase difference between two points separated by 2 m.
(d) Find the intrinsic impedance.

10.19 The electric field intensity of a uniform plane wave in air is given by

$$\mathbf{E} = 50 \sin(10^8 \pi t - \beta x)\mathbf{a}_z \text{ mV/m}$$

(a) Calculate β.
(b) Determine the location(s) where **E** vanishes at $t = 50$ ns.
(c) Find **H**.

10.20 For a uniform plane wave at 4 GHz, the intrinsic impedance and phase velocity of an unknown material are measured as 105 Ω and 7.6×10^7 m/s, respectively. Find ε_r and μ_r of the material.

10.21 In a lossless medium ($\varepsilon_r = 4.5$, $\mu_r = 1$), a uniform plane wave

$$\mathbf{E} = 8 \cos(\omega t - \beta z)\mathbf{a}_x - 6 \sin(\omega t - \beta z)\mathbf{a}_y \text{ V/m}$$

propagates at 40 MHz. (a) Find **H**. (b) Determine β, λ, η, and u.

10.22 A uniform plane wave in a lossy medium has a phase constant of 1.6 rad/m at 10^7 Hz, and its magnitude is reduced by 60% for every 2 m traveled. Find the skin depth and the speed of the wave.

10.23 A uniform plane wave in a lossy nonmagnetic medium has

$$\mathbf{E}_s = (5\mathbf{a}_x + 12\mathbf{a}_y)\mathbf{e}^{-\gamma z}, \quad \gamma = (0.2 + j3.4) \text{ /m}$$

(a) Compute the magnitude of the wave at $z = 4$ m, $t = T/8$.
(b) Find the loss in decibels suffered by the wave in the interval $0 < z < 3$ m.
(c) Calculate the intrinsic impedance.

Section 10.6—Plane Waves in Good Conductors

10.24 The magnet field intensity of a uniform plane wave in a good conductor ($\varepsilon = \varepsilon_o, \mu = \mu_o$) is

$$\mathbf{H} = 20e^{-12z} \cos(2\pi \times 10^6 t + 12z)\mathbf{a}_y \text{ mA/m}$$

Find the conductivity and the corresponding **E** field.

10.25 Which of the following media may be treated as conducting at 8 MHz?

(a) Wet marshy soil ($\varepsilon = 15\varepsilon_o$, $\mu = \mu_o$, $\sigma = 10^{-2}$ S/m)
(b) Intrinsic germanium ($\varepsilon = 16\varepsilon_o$, $\mu = \mu_o$, $\sigma = 0.025$ S/m)
(c) Seawater ($\varepsilon = 81\varepsilon_o$, $\mu = \mu_o$, $\sigma = 25$ S/m)

10.26 Calculate the skin depth and the velocity of propagation for a uniform plane wave at frequency of 6 MHz traveling in polyvinyl chloride (PVC) ($\mu_r = 1$, $\varepsilon_r = 4$, $\dfrac{\sigma}{\omega t} = 7 \times 10^{-2}$).

10.27 (a) Determine the dc resistance of a round copper wire ($\sigma = 5.8 \times 10^7$ S/m, $\mu_r = 1$, $\varepsilon_r = 1$) of radius 1.2 mm and length 600 m.
(b) Find the ac resistance at 100 MHz.
(c) Calculate the approximate frequency at which dc and ac resistances are equal.

10.28 For aluminum ($\sigma = 3.5 \times 10^7$ S/m, $\varepsilon = \varepsilon_o$, $\mu = \mu_o$) at 150 MHz, find (a) the propagation constant γ, (b) the skin depth δ, (c) the wave velocity u.

10.29 For silver, $\sigma = 6.1 \times 10^7$ S/m, $\mu_r = 1$, $\varepsilon_r = 1$, determine the frequency at which the penetration depth is 2 mm.

10.30 Show that in a good conductor, the skin depth δ is approximately given by $\delta = 2\pi/\lambda$.

10.31 Brass waveguides are often silver plated to reduce losses. If the thickness of silver ($\mu = \mu_o$, $\varepsilon = \varepsilon_o$, $\sigma = 6.1 \times 10^7$ S/m) must be 5δ, find the minimum thickness required for a waveguide operating at 12 GHz.

10.32 How deep does a radar wave at 2 GHz travel in seawater before its amplitude is reduced to 10^{-5} of its amplitude just below the surface? Assume that $\mu = \mu_o$, $\varepsilon = 24\varepsilon_o$, $\sigma = 4$ S/m.

Section 10.7—Wave Polarization

10.33 The electric field intensity of a uniform plane wave in a medium ($\sigma = 0$, $\mu = \mu_o$, $\varepsilon = \varepsilon_o\varepsilon_r$) is

$$\mathbf{E} = 12 \sin(2\pi \times 10^7 t - 3y)\mathbf{a}_z \text{ V/m}$$

(a) Determine the polarization of the wave.
(b) Find the frequency.
(c) Calculate ε_r.
(d) Obtain the magnetic field intensity **H**.

10.34 Let $\mathbf{E} = 2 \sin(\omega t - \beta x)\mathbf{a}_y - 5 \sin(\omega t - \beta x)\mathbf{a}_z$ V/m. What is the wave polarization?

10.35 Determine the wave polarization of each of the following waves:

(a) $E_o \cos(\omega t + \beta y)\mathbf{a}_x + E_o \sin(\omega t + \beta y)\mathbf{a}_z$ V/m
(b) $E_o \cos(\omega t - \beta y)\mathbf{a}_x - 3E_o \sin(\omega t + \beta y)\mathbf{a}_z$ V/m

10.36 Determine the polarization of the following waves:

(a) $\mathbf{E}_s = 40e^{j10z}\mathbf{a}_x + 60e^{j10z}\mathbf{a}_y$ V/m
(b) $\mathbf{E}_s = 12e^{j\pi/3}e^{-j10x}\mathbf{a}_y + 5e^{-j\pi/3}e^{-j10x}\mathbf{a}_z$ V/m

10.37 The electric field intensity of a uniform plane wave in free space is given by

$$\mathbf{E} = 40\cos(\omega t - \beta z)\mathbf{a}_x + 60\sin(\omega t - \beta z)\mathbf{a}_y \text{ V/m}$$

(a) What is the wave polarization?
(b) Determine the magnetic field intensity.

10.38 Show that a linearly polarized plane wave of the form $\mathbf{E}_s = E_o e^{-j\beta z}\mathbf{a}_x$ can be expressed as the sum of two circularly polarized waves.

10.39 Suppose $\mathbf{E}(y,t) = E_{o1}\cos(\omega t - \beta y)\mathbf{a}_x + E_{o2}\cos(\omega t - \beta y + \phi)\mathbf{a}_z$ V/m. Determine the polarization when (a) $\phi = 0$, (b) $\phi = \pi/2$, (c) $\phi = \pi$.

Section 10.8—Power and the Poynting Vector

10.40 Show that eqs. (10.77) and (10.78) are equivalent.

10.41 The electric field intensity in a dielectric medium ($\mu = \mu_o$, $\varepsilon = \varepsilon_o\varepsilon_r$) is given by

$$\mathbf{E} = 150\cos(10^9 t + 8x)\mathbf{a}_z \text{ V/m}$$

Calculate
(a) The dielectric constant ε_r
(b) The intrinsic impedance
(c) The velocity of propagation
(d) The magnetic field intensity
(e) The Poynting vector $\mathcal{P}$

10.42 The composite fields resulting from the superposition of two uniform plane waves are given by

$$\mathbf{E} = E_{ol}\cos\alpha x\cos(\omega t - \beta z)\mathbf{a}_x + E_{o2}\sin\alpha x\sin(\omega t - \beta z)\mathbf{a}_z$$

$$\mathbf{H} = H_o\cos\alpha x\cos(\omega t - \beta z)\mathbf{a}_y$$

Determine the time-average Poynting vector.

10.43 The electric field due a short dipole antenna located in free space is

$$\mathbf{E}_s = \frac{10}{r}\sin\theta e^{-j3r}\mathbf{a}_\theta \text{ V/m}$$

Find (a) $\mathbf{H}_s$, (b) the average power crossing the surface $r = 2, 0 < \theta < \pi/6, 0 < \phi < \pi$.

10.44 The electric field component of a uniform plane wave traveling in seawater ($\sigma = 4$ S/m, $\varepsilon = 81\,\varepsilon_o, \mu = \mu_o$) is

$$\mathbf{E} = 8e^{-0.1z}\cos(\omega t - 0.3z)\mathbf{a}_x \text{ V/m}$$

(a) Determine the average power density. (b) Find the depth at which the power density is reduced by 20 dB.

10.45 In a coaxial transmission line filled with a lossless dielectric ($\varepsilon = 4.5\varepsilon_o, \mu = \mu_o$),

$$\mathbf{E} = \frac{40}{\rho}\sin(\omega t - 2z)\mathbf{a}_\rho \text{ V/m}$$

Find (a) ω and $\mathbf{H}$, (b) the Poynting vector, (c) the total time-average power crossing the surface $z = 1$ m, $2\text{ mm} < \rho < 3\text{ mm}$, $0 < \phi < 2\pi$.

10.46 An antenna is located at the origin of a spherical coordinate system. The fields produced by the antenna in free space are

$$\mathbf{E} = \frac{E_o}{r}\sin\theta\sin\omega(t - r/c)\mathbf{a}_\theta$$

$$\mathbf{H} = \frac{E_o}{\eta r}\sin\theta\sin\omega(t - r/c)\mathbf{a}_\phi$$

where $c = \dfrac{1}{\sqrt{\mu_o\varepsilon_o}}$ and $\eta = \sqrt{\dfrac{\mu_o}{\mu_o}}$. Determine the time-average power radiated by the antenna.

10.47 A plane wave in free space has

$$\mathbf{H}(x, t) = (10\mathbf{a}_y - 20\mathbf{a}_z)\sin(\omega t - 40x) \text{ A/m}$$

Find ω, $\mathbf{E}$, and $\mathbf{P}_{\text{ave}}$.

10.48 Human exposure to the electromagnetic radiation in air is regarded as safe if the power density is less than 10 mW/m^2. What is the corresponding electric field intensity?

10.49 Given that $\mathbf{E} = \cos(\omega t - \beta z)\mathbf{a}_x + \sin(\omega t - \beta z)\mathbf{a}_y$ V/m, show that the Poynting vector is constant everywhere.

10.50 At the bottom of a microwave oven, $E = 2.4$ kV/m. If this value is found uniformly over the entire area of the oven, which is 450 cm^2, determine the power delivered by the oven. Assume $\mu = \mu_o$, $\varepsilon = \varepsilon_o$.

10.51 A coaxial cable consists of two conducting cylinders of radii a and b. The electric and magnetic fields in the cable are

$$\mathbf{E} = \frac{V_o}{\rho\ln(b/a)}\sin(\omega t - \beta z)a_\rho,\ a < \rho < b$$

$$\mathbf{H} = \frac{I_o}{2\pi\rho}\sin(\omega t - \beta z)\mathbf{a}_\phi,\ a < \rho < b$$

where V_o and I_o are constants. (a) Determine the time-average Poynting vector. (b) Find the time-average power flowing through the cable.

Section 10.9—Reflection at Normal Incidence

10.52 (a) For a normal incidence upon the dielectric–dielectric interface for which $\mu_1 = \mu_2 = \mu_o$, we define R and T as the reflection and transmission coefficients for average powers, that is, $P_{r,\text{ave}} = RP_{i,\text{ave}}$ and $P_{t,\text{ave}} = TP_{i,\text{ave}}$. Prove that

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \quad \text{and} \quad T = \frac{4n_1 n_2}{(n_1 + n_2)^2}$$

where n_1 and n_2 are the refractive indices of the media.

(b) Determine the ratio n_1/n_2 so that the reflected and the transmitted waves have the same average power.

10.53 The plane wave $\mathbf{E} = 30\cos(\omega t - z)\mathbf{a}_x$ V/m in air normally hits a lossless medium ($\mu = \mu_o, \varepsilon = 4\varepsilon_o$) at $z = 0$. (a) Find Γ, τ, and s. (b) Calculate the reflected electric and magnetic fields.

10.54 A uniform plane wave $\mathbf{E}_i = 50\sin(2\pi \times 10^8 t - \beta_1 x)\mathbf{a}_z$ V/m is incident normally from air to a perfect conductor. Determine $\mathbf{E}_r$ and $\mathbf{E}_t$.

10.55 A uniform plane wave in air with

$$\mathbf{H} = 4\sin(\omega t - 5x)\mathbf{a}_y \text{ A/m}$$

is normally incident on a plastic region ($x \geq 0$) with the parameters $\mu = \mu_o, \varepsilon = 4\varepsilon_o$, and $\sigma = 0$. (a) Obtain the total electric field in air. (b) Calculate the time-average power density in the plastic region. (c) Find the standing wave ratio.

10.56 Region 1 is a lossless medium for which $y \geq 0$, $\mu = \mu_o$, $\varepsilon = 4\varepsilon_o$, whereas region 2 is free space, $y \leq 0$. If a plane wave $\mathbf{E}_i = 5\cos(10^8 t + \beta y)\mathbf{a}_z$ V/m exists in region 1, find (a) the total electric field component of the wave in region 1, (b) the time-average Poynting vector in region 1, (c) the time-average Poynting vector in region 2.

10.57 A plane wave in free space ($z \leq 0$) is incident normally on a large block of material with ($\varepsilon_r = 12, \mu_r = 3, \sigma = 0$) that occupies $z \geq 0$. If the incident electric field is

$$\mathbf{E} = 30\cos(\omega t - z)\mathbf{a}_y \text{ V/m}$$

Find (a) ω, (b) the standing wave ratio, (c) the reflected magnetic field, (d) the average power density of the transmitted wave.

10.58 A uniform plane wave in air is normally incident on an infinite lossless dielectric material occupying $z > 0$ and having $\varepsilon = 3\varepsilon_o$ and $\mu = \mu_o$. If the incident wave is $\mathbf{E}_i = 10\cos(\omega t - z)\mathbf{a}_y$ V/m, find

(a) λ and ω of the wave in air and the transmitted wave in the dielectric medium
(b) The incident $\mathbf{H}_i$ field
(c) Γ and τ
(d) The total electric field and the time-average power in both regions

10.59 A 100 MHz plane wave is normally incident from air to the sea surface, which may be assumed to be calm and smooth. If $\sigma = 4$ S/m, $\mu_r = 1$, and $\varepsilon_r = 81$ for seawater, calculate the fractions of the incident power that are transmitted and reflected.

10.60 A uniform plane wave in a certain medium ($\mu = \mu_o, \varepsilon = 4\varepsilon_o$) is given by

$$\mathbf{E} = 12\cos(\omega t - 40\pi x)\mathbf{a}_z \text{ V/m}$$

(a) Find ω.
(b) If the wave is normally incident on a dielectric ($\mu = \mu_o, \varepsilon = 3.2\varepsilon_o$), determine $\mathbf{E}_r$ and $\mathbf{E}_t$.

***10.61** A signal in air ($z \geq 0$) with the electric field component

$$\mathbf{E} = 10\sin(\omega t + 3z)\mathbf{a}_x \text{ V/m}$$

hits normally the ocean surface at $z = 0$ as in Figure 10.24. Assuming that the ocean surface is smooth and that $\varepsilon = 80\varepsilon_o$, $\mu = \mu_o$, $\sigma = 4$ S/m in ocean, determine

(a) ω
(b) The wavelength of the signal in air
(c) The loss tangent and intrinsic impedance of the ocean
(d) The reflected and transmitted **E** field

10.62 Sketch the standing wave in eq. (10.97) at $t = 0, T/8, T/4, 3T/8, T/2$, and so on, where $T = 2\pi/\omega$.

10.63 A uniform plane wave is incident at an angle $\theta_i = 45°$ on a pair of dielectric slabs joined together as shown in Figure 10.25. Determine the angles of transmission θ_{t1} and θ_{t2} in the slabs.

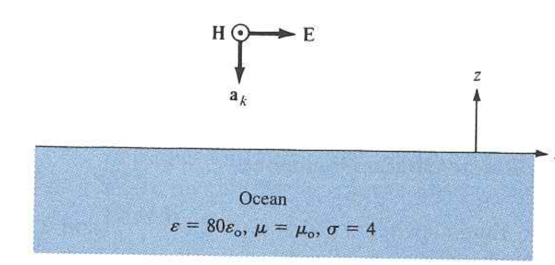

FIGURE 10.24 For Problem 10.61.

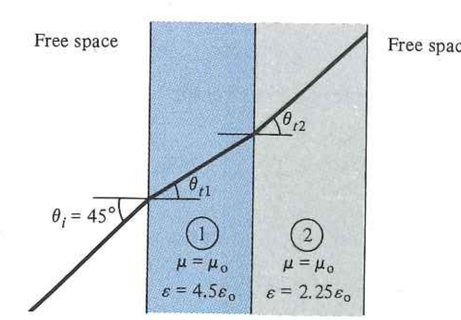

FIGURE 10.25 For Problem 10.63.

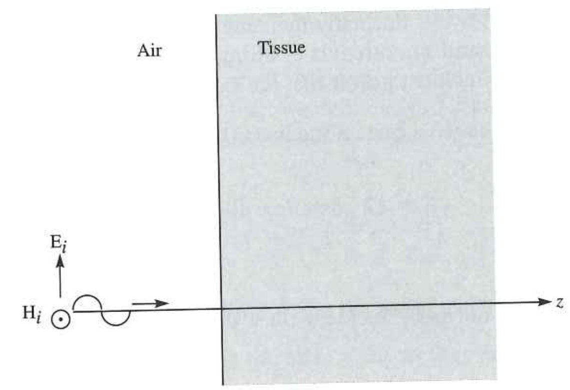

FIGURE 10.26 For Problem 10.65.

10.64 Show that the field

$$E_s = 20 \sin(k_x x) \cos(k_y y)\mathbf{a}_z$$

where $k_x^2 + k_y^2 = \omega^2\mu_o\varepsilon_o$, can be represented as the superposition of four propagating plane waves. Find the corresponding $\mathbf{H}_s$.

10.65 Electromagnetic radiation can be used to heat cancerous tumor. If a plane wave is normally incident on the tissue surface at 1.2 GHz as shown in Figure. 10.26, determine the refection coefficient. At 1.2 GHz, the electrical properties of the tissue are $\varepsilon_r = 50$, $\mu_r = 1$, $\sigma = 4$ S/m.

10.66 An EM plane wave in a lossless medium impinges normally on a lossy medium.

(a) Determine the ratio of transmitted to incident power in terms of the standing wave ratio s.

(b) Express the ratio of reflected to incident power in terms of s.

Section 10.10—Reflection at Oblique Incidence

***10.67** By assuming the time-dependent fields $\mathbf{E} = \mathbf{E}_o e^{j(\mathbf{k}\cdot\mathbf{r}-\omega t)}$ and $\mathbf{H} = \mathbf{H}_o e^{j(\mathbf{k}\cdot\mathbf{r}-\omega t)}$ where $\mathbf{k} = k_x\mathbf{a}_x + k_y\mathbf{a}_y + k_z\mathbf{a}_z$ is the wave number vector and $\mathbf{r} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$ is the radius vector, show that $\nabla \times \mathbf{E} = -\partial\mathbf{B}/\partial t$ can be expressed as $\mathbf{k} \times \mathbf{E} = \mu\omega\mathbf{H}$ and deduce $\mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_H$.

10.68 A plane wave in free space has a propagation vector

$$\mathbf{k} = 124\mathbf{a}_x + 124\mathbf{a}_y + 263\mathbf{a}_z$$

Find the wavelength, frequency, and angles $\mathbf{k}$ makes with the x-, y-, and z-axes.

10.69 In free space,

$$\mathbf{E}_s = [E_o\mathbf{a}_x + \mathbf{a}_y + (3 + j4)\mathbf{a}_z]e^{-j(3.4x-4.2y)} \text{ V/m}$$

Determine E_o, $\mathbf{H}_s$, and frequency.

10.70 Assume the same fields as in Problem 10.67 and show that Maxwell's equations in a source-free region can be written as

$$\mathbf{k}\cdot\mathbf{E} = 0$$
$$\mathbf{k}\cdot\mathbf{H} = 0$$
$$\mathbf{k}\times\mathbf{E} = \omega\mu\mathbf{H}$$
$$\mathbf{k}\times\mathbf{H} = -\omega\varepsilon\mathbf{E}$$

From these equations deduce

$$\mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_H \quad \text{and} \quad \mathbf{a}_k \times \mathbf{a}_H = -\mathbf{a}_E$$

10.71 Show that for nonmagnetic dielectric media, the reflection and transmission coefficients for oblique incidence become

$$\Gamma_\parallel = \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)}, \quad \tau_\parallel = \frac{2\cos\theta_i \sin\theta_t}{\sin(\theta_t + \theta_i)\cos(\theta_t - \theta_i)}$$

$$\Gamma_\perp = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}, \quad \tau_\perp = \frac{2\cos\theta_i \sin\theta_t}{\sin(\theta_t + \theta_i)}$$

10.72 If region 1 is in free space, while region 2 is a nonmagnetic dielectric medium ($\sigma_2 = 0$, $\varepsilon r_2 = 6.4$), compute E_{ro}/E_{io} and E_{to}/E_{io} for oblique incidence at $\theta_i = 12°$. Assume parallel polarization.

10.73 A parallel-polarized wave in air with

$$\mathbf{E} = (8\mathbf{a}_y - 6\mathbf{a}_z)\sin(\omega t - 4y - 3z) \text{ V/m}$$

impinges a dielectric half-space as shown in Figure 10.27. Find (a) the incidence angle θ_i, (b) the time-average power in air ($\mu = \mu_o$, $\varepsilon = \varepsilon_o$), (c) the reflected and transmitted $\mathbf{E}$ fields.

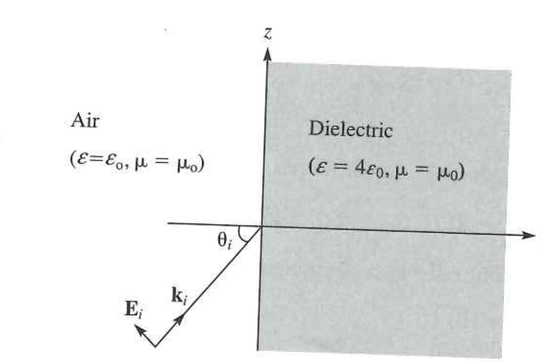

FIGURE 10.27 For Problem 10.73.

10.74 In a dielectric medium ($\varepsilon = 9\varepsilon_o, \mu = \mu_o$), a plane wave with

$$\mathbf{H} = 0.2 \cos(10^9 t - kx - k\sqrt{8}z)\mathbf{a}_y \text{ A/m}$$

is incident on an air boundary at $z = 0$. Find

(a) θ_r and θ_t

(b) k

(c) The wavelength in the dielectric and in air

(d) The incident **E**

(e) The transmitted and reflected **E**

(f) The Brewster angle

10.75 Determine the Brewster angle for an air–seawater ($\varepsilon = 81\varepsilon_o$) interface for the following cases: (a) EM plane wave passing from air to seawater, (b) EM wave passing from seawater to air.

10.76 If u is the phase velocity of an EM wave in a given medium, the index of refraction of the medium is $n = c/u$, where c is the speed of light in vacuum.

(a) Paraffin has $\mu_r = 1$, $\varepsilon_r = 2.1$. Determine n for unbounded medium of paraffin.

(b) Distilled water has $\mu_r = 1$, $\varepsilon_r = 81$. Find n.

(c) Polystyrene has $\mu_r = 1$, $\varepsilon_r = 2.7$. Calculate n.

Section 10.11—Application Note—Microwaves

10.77 Discuss briefly some applications of microwaves other than those discussed in the text.

10.78 A useful set of parameters, known as the *scattering transfer parameters*, is related to the incident and reflected waves as

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} b_2 \\ a_2 \end{bmatrix}$$

(a) Express the T-parameters in terms of the S-parameters.

(b) Find T when

$$S = \begin{bmatrix} 0.2 & 0.4 \\ 0.4 & 0.2 \end{bmatrix}$$

10.79 The S-parameters of a two-port network are:

$$S_{11} = 0.33 - j0.16, S_{12} = S_{21} = 0.56, S_{22} = 0.44 - j0.62$$

Find the input and output reflection coefficients when $Z_L = Z_o = 50\ \Omega$ and $Z_g = 2Z_o$

10.80 Why can't regular lumped circuit components such as resistors, inductors, and capacitors be used at microwave frequencies?

10.81 In free space, a microwave signal has a frequency of 8.4 GHz. Calculate the wavelength of the signal.

11.10 Two identical pulses each of magnitude 12 V and width 2 μs are incident at $t = 0$ on a lossless transmission line of length 400 m terminated with a load. If the two pulses are separated 3 μs and $u = 2 \times 10^8$ m/s, when does the contribution to $V_L(\ell, t)$ by the second pulse start overlapping that of the first?

(a) $t = 0.5\ \mu s$
(b) $t = 2\ \mu s$
(c) $t = 5\ \mu s$
(d) $t = 5.5\ \mu s$
(e) $t = 6\ \mu s$

Answers: 11.1c,d,e, 11.2b,c, 11.3c, 11.4a,c, 11.5c, 11.6 (i) D,e (ii) A, (iii) E, (iv) C, (v) B, (vi) D, (vii) B, (viii) A, 11.7a, 11.8 (a) T, (b) F, (c) F, (d) T, (e) F, (f) F, 11.9b, 11.10e.

PROBLEMS

Section 11.2—Transmission Line Parameters

11.1 An air-filled planar line with $w = 30$ cm, $d = 1.2$ cm, $t = 3$ mm has conducting plates with $\sigma_c = 7 \times 10^7$ S/m. Calculate R, L, C, and G at 500 MHz.

11.2 A coaxial cable has an inner conductor of radius $a = 0.8$ mm and an outer conductor of radius $b = 2.6$ mm. The conductors have $\sigma_c = 5.28 \times 10^7$ S/m, $\mu_c = \mu_o$, and $\epsilon_c = \epsilon_o$; they are separated by a dielectric material having $\sigma = 10^{-5}$ S/m, $\mu = \mu_o$, $\epsilon = 3.5\,\epsilon_o$. At 80 MHz, calculate the line parameters L, C, G, and R.

11.3 A coaxial cable has inner radius a and outer radius b. If the inner and outer conductors are separated by a material with conductively σ, show that the conductance per unit length is

$$G = \frac{2\pi\sigma}{\ln\dfrac{b}{a}}$$

11.4 A coaxial TV cable is designed so that $Z = 50\ \Omega$. The radius of the inner shell is $a = 6$ mm, while that of outer shell is $b = 20$ mm. Determine the dielectric constant ε_r needed to have such impedance. Assume $\mu = \mu_o$, $\sigma = 0$.

11.5 The copper leads of a diode are 16 mm in length and have a radius of 0.3 mm. They are separated by a distance of 2 mm as shown in Figure 11.46. Find the capacitance between the leads and the ac resistance at 10 MHz.

Section 11.3—Transmission Line Equations

***11.6** In Section 11.3, it was mentioned that the equivalent circuit of Figure 11.5 is not the only possible one. Show that eqs. (11.4) and (11.6) would remain the same if the Π-type and T-type equivalent circuits shown in Figure 11.47 were used.

11.7 (a) Show that at high frequencies ($R \ll \omega L$, $G \ll \omega L$),

$$\gamma \simeq \left(\frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}\right) + j\omega\sqrt{LC}$$

(b) Obtain a similar formula for Z_o.

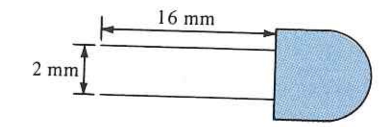

FIGURE 11.46 The diode of Problem 11.5.

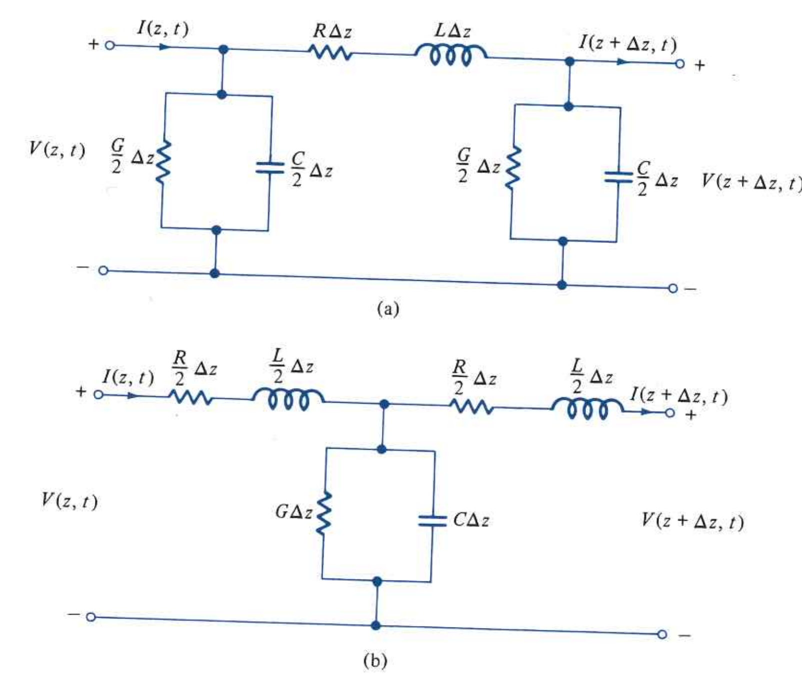

FIGURE 11.47 Equivalent circuits for Problem 11.6: **(a)** Π-type, **(b)** *T*-type.

11.8 Express the phase velocity on a lossless line in terms of C and Z_o.

11.9 At 60 MHz, the following characteristics of a lossy line are measured:

$$Z_o = 50\ \Omega, \quad \alpha = 0.04\ \text{dB/m}, \quad \beta = 2.5\ \text{rad/m}$$

Calculate R, L, C, and G of the line.

11.10 A 78 Ω lossless planar line was designed but did not meet a requirement. What fraction of the widths of the strip should be added or removed to get the characteristic impedance of 75 Ω?

11.11 A telephone line operating at 1 kHz has $R = 6.8\ \Omega$/mi, $L = 3.4$ mH/mi, $C = 8.4$ nF/mi, and $G = 0.42\ \mu$S/mi. Find (a) Z and γ, (b) phase velocity, (c) wavelength.

11.12 A TV antenna lead-in wire 10 cm long has a characteristic impedance of 250 Ω and is open-circuited at its end. If the line operates at 400 MHz, determine its input impedance.

11.13 A coaxial cable has its conductors made of copper ($\sigma_c = 5.8 \times 10^7$ S/m) and its dielectric made of polyethylene ($\varepsilon_r = 2.25$, $\mu_r = 1$). If the radius of the outer conductor is 3 mm, determine the radius of the inner conductor so that $Z_o = 75\ \Omega$.

11.14 For a lossless two-wire transmission line, show that

(a) The phase velocity $u = c = \dfrac{1}{\sqrt{LC}}$

(b) The characteristic impedance $Z_o = \dfrac{120}{\sqrt{\varepsilon_r}} \cosh^{-1} \dfrac{d}{2a}$

Is part (a) true of other lossless lines?

11.15 A twisted line, which may be approximated by a two-wire line, is very useful in the telephone industry. Consider a line comprising two copper wires of diameter 0.12 cm that have a 0.32 cm center-to-center spacing. If the wires are separated by a dielectric material with $\varepsilon = 3.5\varepsilon_o$, find L, C, and Z_o.

11.16 A distortionless cable is 4 m long and has a characteristic impedance of 60 Ω. An attenuation of 0.24 dB is observed at the receiving end. Also, a signal applied to the cable is delayed by 80 μs before is measured at the receiving end. Find R, G, L, and C for the cable.

11.17 A distortionless line operating at 120 MHz has $R = 20\ \Omega/\text{m}$, $L = 0.3\ \mu\text{H/m}$, and $C = 63$ pF/m. (a) Determine γ, u, and Z_o. (b) How far will a voltage wave travel before it is reduced to 20% of its initial magnitude? (c) How far will it travel to suffer a 45° phase shift?

11.18 On a distortionless line, the voltage wave is given by

$$V(\ell') = 120e^{0.0025\ell'}\cos(10^8 t + 2\ell') + 60e^{-0.0025\ell'}\cos(10^8 t - 2\ell')$$

where ℓ' is the distance from the load. If $Z_L = 300\ \Omega$, find (a) α, β, and u, (b) Z_o and $I(\ell')$.

11.19 The voltage on a line is given by

$$V(\ell) = 80e^{10^{-3}\ell}\cos(2\pi \times 10^4 t + 0.01\ell) + 60e^{-10^{-3}\ell}\cos(2\pi \times 10^4 t + 0.01\ell)\text{V}$$

where ℓ is the distance from the load. Calculate γ and u.

11.20 A distortionless transmission line satisfies $RC = LG$. If the line has $R = 10\ \text{m}\Omega/\text{m}$, $C = 82$ pF/m, and $L = 0.6\ \mu\text{H/m}$, calculate its characteristic impedance and propagation constant. Assume that the line operates at 80 MHz.

11.21 A coaxial line 5.6 m long has distributed parameters $R = 6.5\ \Omega/\text{m}$, $L = 3.4\ \mu\text{H/m}$, $G = 8.4$ mS/m, and $C = 21.5$ pF/m. If the line operates at 2 MHz, calculate the characteristic impedance and the end-to-end propagation time delay.

11.22 A lossy transmission line of length 2.1 m has characteristic impedance of $80 + j60\ \Omega$. When the line is short-circuited, the input impedance is $30 - j12\ \Omega$. (a) Determine α and β. (b) Find the input impedance when the short circuit is replaced by $Z_L = 40 + j30\ \Omega$.

11.23 A lossy transmission line with characteristic impedance of $75 + j60\ \Omega$ is connected to a 200 Ω load. If attenuation is 1.4 Np/m and phase constant is 2.6 rad/m, find the input impedance for $\ell = 0.5$ m.

Section 11.4—Input Impedance, Standing Wave Ratio, and Power

11.24 (a) Show that a transmission coefficient may be defined as

$$\tau_L = \frac{V_L}{V_o^+} = 1 + \Gamma_L = \frac{2Z_L}{Z_L + Z_o}$$

(b) Find τ_L when the line is terminated by (i) a load whose value is nZ_o, (ii) an open circuit, (iii) a short circuit, (iv) $Z_L = Z_o$ (matched line).

11.25 A lossy transmission line has $R = 3.5\ \Omega/\text{m}$, $L = 2\ \mu\text{H/m}$, $C = 120$ pF/m, and $G \approx 0$. At 400 MHz, determine α, β, Z_o, and u.

11.26 Find the input impedance of a short-circuited coaxial transmission line of Figure 11.48 if $Z_o = 65 + j38\ \Omega$, $\gamma = 0.7 + j2.5$/m, $\ell = 0.8$ m.

11.27 A 50 Ω transmission line of length ℓ is open-circuited. If the input impedance is $-j62\ \Omega$, determine ℓ in terms of λ.

11.28 Refer to the lossless transmission line shown in Figure 11.49. (a) Find Γ and s. (b) Determine Z_{in} at the generator.

11.29 A 60 Ω lossless line is connected to a source with $V_g = 10\angle 0°\ V_{rms}$ and $Z_g = 50 - j40\ \Omega$ and terminated with a load of $j40\ \Omega$. If the line is 100 m long and $\beta = 0.25$ rad/m, calculate Z_{in} and V at

(a) The sending end
(b) The receiving end
(c) 4 m from the load
(d) 3 m from the source

***11.30** Consider the two-port network shown in Figure 11.50(a). The relation between the input and output variables can be written in matrix form as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

For the lossy line in Figure 11.50(b), show that the $ABCD$ matrix is

$$\begin{bmatrix} \cosh \gamma\ell & Z_o \sinh \gamma\ell \\ \dfrac{1}{Z_o}\sinh \gamma\ell & \cosh \gamma\ell \end{bmatrix}$$

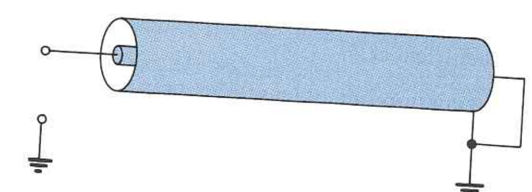

FIGURE 11.48 For Problem 11.26.

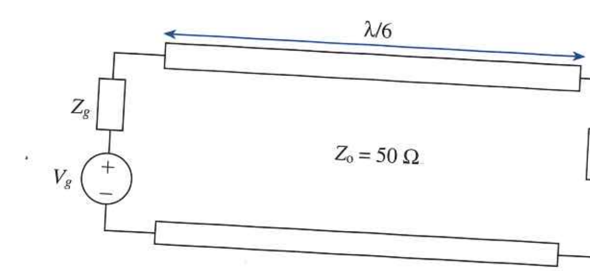

FIGURE 11.49 For Problem 11.28.

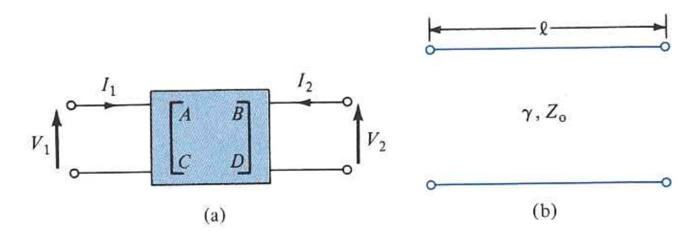

FIGURE 11.50 For Problem 11.30: **(a)** network, **(b)** lossy line.

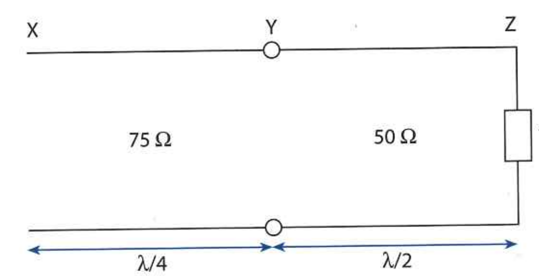

FIGURE 11.51 For Problem 11.34.

Section 11.5—The Smith Chart

11.31 Normalize the following impedance with respect to 50 Ω and locate them on the Smith chart: (a) $Z_a = 80\ \Omega$, (b) $Z_b = 60 + j40\ \Omega$, (c) $Z_c = 30 - j120\ \Omega$.

11.32 A quarter-wave lossless 100 Ω line is terminated by a load $Z_L = 210\ \Omega$. If the voltage at the receiving end is 80 V, what is the voltage at the sending end?

11.33 Determine the impedance at a point λ/4 distant from a load of impedance $(1 + j2)Z_o$.

11.34 Two lines are cascaded as shown in Figure 11.51. Determine:
(a) The input impedance
(b) The standing wave ratio for sections XY and YZ
(c) The reflection coefficient at Z

11.35 A lossless 50 Ω transmission line of length 3.2 m is terminated with an impedance of $30 - j50\ \Omega$. If the line operates at a frequency of 400 MHz, determine the input impedance.

11.36 A lossless transmission line, with characteristic impedance of 50 Ω and electrical length of $\ell = 0.27\lambda$, is terminated by a load impedance $40 - j25\ \Omega$. Determine Γ_L, s, and Z_{in}.

11.37 A 75 Ω lossless transmission line is 132 cm long, with a dielectric constant $\varepsilon_r = 3.62$. If the line operates at 400 MHz with an input impedance of $Z_{in} = 40 + j65\ \Omega$, use the Smith chart to determine the terminating load.

11.38 The distance from the load to the first minimum voltage in a 50 Ω line is 0.12λ and the standing wave ratio $s = 4$.
(a) Find the load impedance Z_L.
(b) Is the load inductive or capacitive?
(c) How far from the load is the first maximum voltage?

11.39 A lossless 50 Ω line is terminated by a load $Z_L = 75 + j60\ \Omega$. Using a Smith chart, determine (a) the reflection coefficient Γ, (b) the standing wave ratio s, (c) the input impedance at 0.2λ from the load, (d) the location of the first minimum voltage from the load, (e) the shortest distance from the load at which the input impedance is purely resistive.

11.40 A transmission line is terminated by a load with admittance $Y_L = (0.6 + j0.8)/Z_o$. Find the normalized input impedance at λ/6 from the load.

11.41 Using the Smith chart, determine the admittance of $Z = 100 + j60\ \Omega$ with respect to $Z_o = 50\ \Omega$.

11.42 A 50 Ω transmission line operates at 160 MHz and is terminated by a load of $50 + j30\ \Omega$. If its wave speed is $c/2$ and the input impedance is to be made real, calculate the minimum possible length of the line and the corresponding input impedance.

11.43 A 50 Ω coaxial cable is λ/4 long and is terminated by a load $40 - j30\ \Omega$. Use the Smith chart to find the input admittance Y_{in}.

11.44 (a) Calculate the reflection coefficient corresponding to $Z_L = (0.5 - j)Z_o$.
(b) Determine the load impedance corresponding to the reflection coefficient $0.4\angle 25°$.

11.45 An 80 Ω transmission line operating at 12 MHz is terminated by a load Z_L. At 22 m from the load, the input impedance is $100 - j120\ \Omega$. If $u = 0.8c$,
(a) Calculate Γ_L, $Z_{in,\,max}$, and $Z_{in,\,min}$.
(b) Find Z_L, s, and the input impedance at 28 m from the load.
(c) How many $Z_{in,\,max}$ and $Z_{in,\,min}$ are there between the load and the $100 - j120\ \Omega$ input impedance?

11.46 An antenna, connected to a 150 Ω lossless line, produces a standing wave ratio of 2.6. If measurements indicate that voltage maxima are 120 cm apart and that the last maximum is 40 cm from the antenna, calculate
(a) The operating frequency
(b) The antenna impedance
(c) The reflection coefficient (assume that $u = c$).

11.47 An 80 Ω lossless line has $Z_L = j60\ \Omega$ and $Z_{in} = j40\ \Omega$. (a) Determine the shortest length of the line. (b) Calculate s and Γ_L.

11.48 A 50 Ω air-filled line is terminated in a mismatched load of $40 + j25\ \Omega$. Find the shortest distance from the load at which the voltage has the smallest magnitude.

11.49 Two λ/4 transformers in tandem are to connect a 50 Ω line to a 75 Ω load as in Figure 11.52.
(a) Determine the characteristic impedance Z_{o1} if $Z_{o2} = 30\ \Omega$ and there is no reflected wave to the left of A.
(b) If the best results are obtained when

$$\left[\frac{Z_o}{Z_{o1}}\right]^2 = \frac{Z_{o1}}{Z_{o2}} = \left[\frac{Z_{o2}}{Z_L}\right]^2$$

determine Z_{o1} and Z_{o2} for this case.

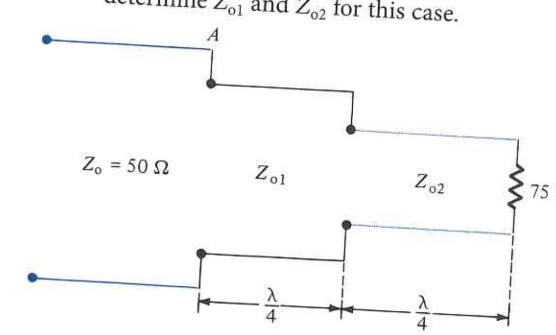

FIGURE 11.52 Double section transformer of Problem 11.49.

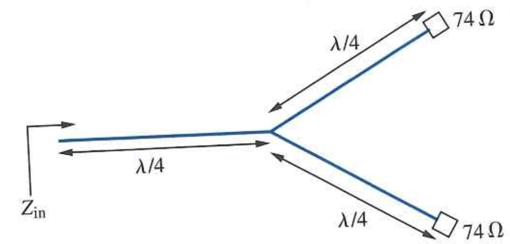

FIGURE 11.53 For Problems 11.50 and 11.51.

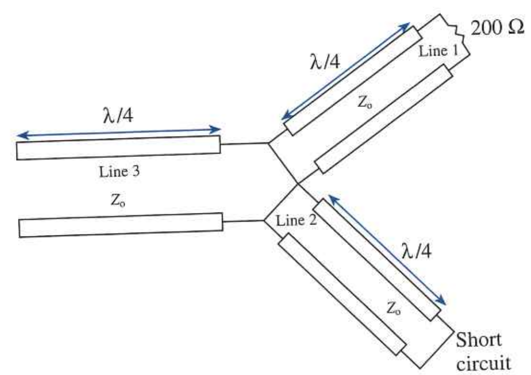

FIGURE 11.54 For Problem 11.52.

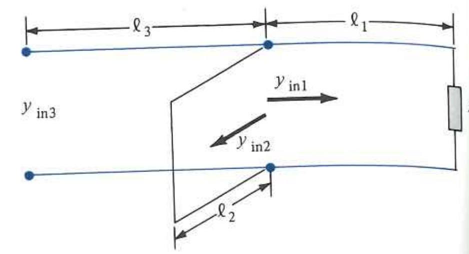

FIGURE 11.55 For Problem 11.53.

11.50 Two identical antennas, each with input impedance 74 Ω, are fed with three identical 50 Ω quarter-wave lossless transmission lines as shown in Figure 11.53. Calculate the input impedance at the source end.

11.51 If the lines in Figure 11.53 are connected to a voltage source of 120 V with an internal impedance of 80 Ω, calculate the average power delivered to either antenna.

11.52 Consider the three lossless lines in Figure 11.54. If $Z_o = 50\ \Omega$, calculate:
(a) Z_{in} looking into line 1
(b) Z_{in} looking into line 2
(c) Z_{in} looking into line 3

11.53 A section of lossless transmission line is shunted across the main line as in Figure 11.55. If $\ell_1 = \lambda/4$, $\ell_2 = \lambda/8$, and $\ell_3 = 7\lambda/8$, find y_{in_1}, y_{in_2}, and y_{in_3} given that $Z_o = 100\ \Omega$, $Z_L = 200 + j150\ \Omega$. Repeat the calculations as if the shorted section were open.

Section 11.6—Some Applications of Transmission Lines

11.54 It is desired to match a 50 Ω line to a load impedance of $60 - j50\ \Omega$. Design a 50 Ω stub that will achieve the match. Find the length of the line and calculate how far it is from the load.

11.55 A stub of length 0.12λ is used to match a 60 Ω lossless line to a load. If the stub is located at 0.3λ from the load, calculate
(a) The load impedance Z_L

(b) The length of an alternative stub and its location with respect to the load
(c) The standing wave ratio between the stub and the load

11.56 A 50 Ω lossless transmission line that is 20 m long is terminated into a $120 + j220\ \Omega$ load. To perfectly match, what should be the length and location of a short-circuited stub line? Assume an operating frequency of 10 MHz.

11.57 On a lossless line, measurements indicate $s = 4.2$ with the first maximum voltage at $\lambda/4$ from the load. Determine how far from the load a short-circuited stub should be located and calculate its length.

11.58 A 60 Ω lossless line terminated by load Z_L has a voltage wave as shown in Figure 11.56. Find s, Γ, and Z_L.

11.59 A 50 Ω air-filled slotted line is applied in measuring a load impedance. Adjacent minima are found at 14 cm and 22.5 cm from the load when the unknown load is connected, and $V_{max} = 0.95$ V and $V_{min} = 0.45$ V. When the load is replaced by a short circuit, the minima are 3.2 cm to the load. Determine s, f, Γ, and Z_L.

Section 11.7—Transients on Transmission Lines

11.60 A 50 Ω coaxial cable is connected to an 80 Ω resistive load and a dc source with zero internal resistance. Calculate the voltage reflection coefficients at the source and at the load.

11.61 The switch in Figure 11.57 is closed at $t = 0$. Sketch the voltage and current at the right side of the switch for $0 < t < 6\ell/u$. Take $Z_o = 50\ \Omega$ and $\ell/u = 2\ \mu s$. Assume a lossless transmission line.

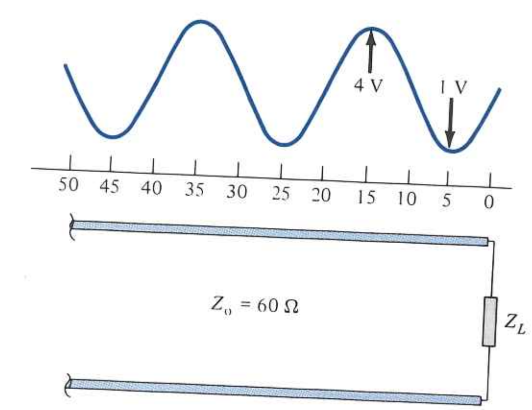

FIGURE 11.56 For Problem 11.58.

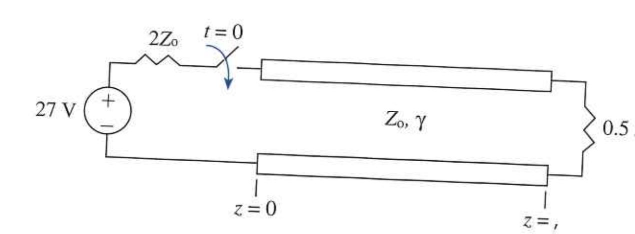

FIGURE 11.57 For Problem 11.61.