Online Lecture # 08 - Digital Baseband Modulation - Review of Sampling Theory

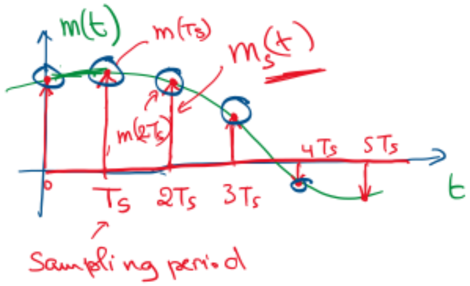
Monday, April 6, 2020
8:50 AM

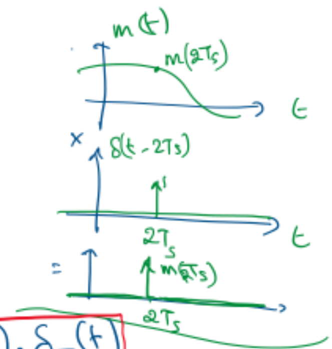
I - Review of Sampling Theory:

$m(t) \longrightarrow$ [Sampling] $\longrightarrow m_s(t)$

$m(t)$: continuous-time signal

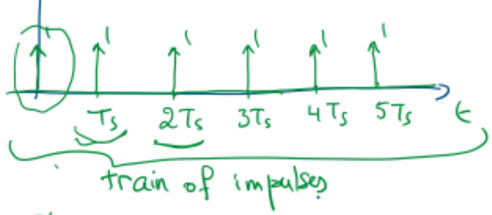
$m_s(t)$: a continuous-time signal that only keeps ON discrete values of $m(t)$





In time domain:

$$\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$



$$\boxed{m_s(t) = m(t) \cdot \delta_{T_s}(t)}$$

$$m_s(t) = m(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$m_s(t) = \sum_{n=-\infty}^{\infty} m(t) \cdot \delta(t - nT_s)$$

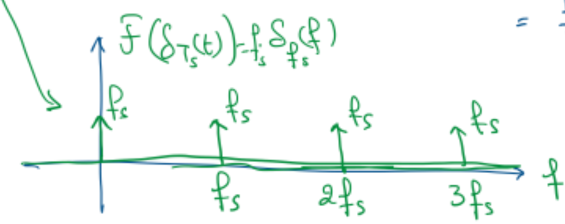
$$\boxed{m_s(t) = \sum_{n=-\infty}^{\infty} m(nT_s) \cdot \delta(t - nT_s)}$$

In frequency domain:

$$M_s(f) = M(f) * \mathcal{F}(\delta_{T_s}(t))$$

$\mathcal{F}$ We have : $\mathcal{F}(\delta_{T_s}(t)) = f_s \cdot \delta_{f_s}(f)$ with $f_s = \frac{1}{T_s}$

$$= f_s \cdot \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$

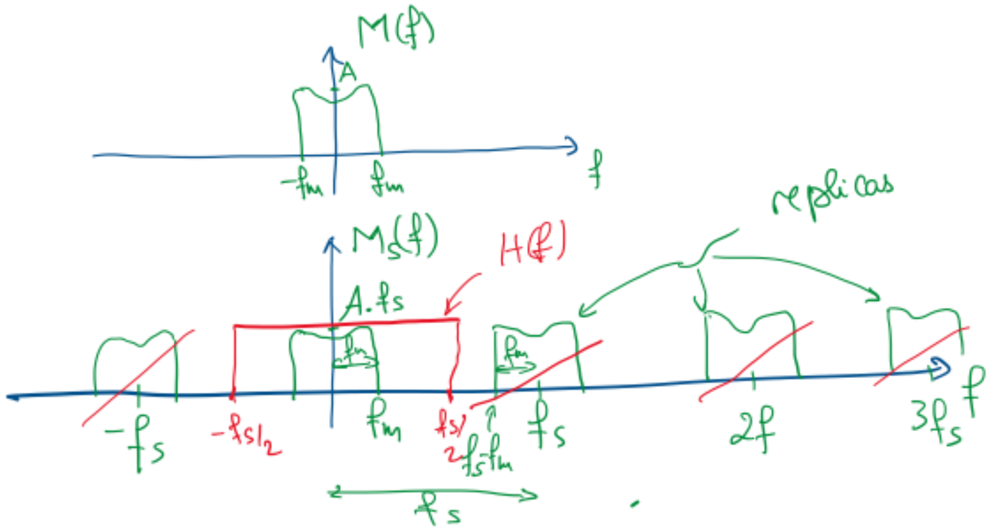


$$M_s(f) = M(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$

$$M_s(f) = f_s \sum_{n=-\infty}^{\infty} \underbrace{M(f) * \delta(f - nf_s)}_{\text{a shift of } M(f) \text{ @ } nf_s}$$

$$\boxed{M_s(f) = f_s \sum_{n=-\infty}^{\infty} M(f - nf_s)}$$

For example if:



If $f_s > 2f_m$, there is no overlap between the replicas (no aliasing).
In this case we can get back to $M(f)$ and therefore $m(t)$ using a low-pass filter.

In this case the sampling operation did not result in a loss of information.

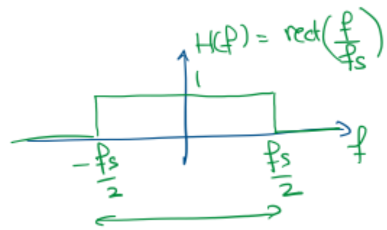
* Reconstruction of sampled signa:

if $f_s > 2f_m$:

In frequency domain :

$$\boxed{f_s \cdot M(f) = M_s(f) \cdot H(f)}$$

$$= M_s(f) \cdot \text{rect}\left(\frac{f}{f_s}\right) \xrightarrow{\mathcal{F}^{-1}} f_s \,\text{sinc}\left(\frac{f_s t}{2}\right)$$



In time domain:

$$f_s \cdot m(t) = m_s(t) * f_s \cdot \text{sinc}\left(\frac{f_s t}{2}\right)$$

$$= \sum_{n=-\infty}^{\infty} m(nT_s)\,\delta(t-nT_s) * f_s \,\text{sinc}\left(\frac{f_s t}{2}\right)$$

$$\boxed{\cancel{f_s}\, m(t) = \sum_{n=-\infty}^{\infty} \cancel{f_s} \cdot m(nT_s) \cdot \text{sinc}\left(\frac{f_s}{2}(t-nT_s)\right)}$$