Assignment #4 ENEL 476 W2014 (1)

1. $\epsilon_r = 9$ $\vec{H}(y,t) = 33\cos(\omega t - 20y)\vec{a}_z$

$\sigma = 0$

$\mu_r = 1$

a) $v_p = \omega/\beta$

$= \dfrac{\cancel{\omega}}{\cancel{\omega}\sqrt{\mu_r\mu_0\epsilon_r\epsilon_0}}$

$= \dfrac{1}{\sqrt{9\mu_0\epsilon_0}}$

$= \dfrac{3e^8}{3}$

$\boxed{v_p = 1e^8 \text{ m/s}}$

b) $f = ?$

$\beta = \omega\sqrt{\mu_r\mu_0\epsilon_r\epsilon_0}$

$20 = 2\pi f \dfrac{(\cancel{3})}{\cancel{3}e^8}$

$\dfrac{\cancel{20}e^8}{\cancel{2}\pi} = f$

$f = \dfrac{1e^9}{\pi}$

$f = 3.2\times10^8 \text{ Hz}$

$\boxed{f = 0.32 \text{ GHz}}$

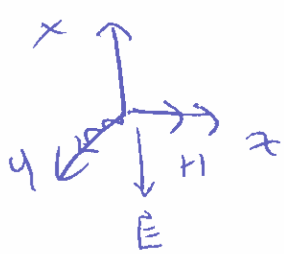
c) $\eta = \sqrt{\dfrac{\mu_r\mu_0}{\epsilon_r\epsilon_0}}$

$= \sqrt{\dfrac{\mu_0}{9\epsilon_0}}$

$= 40\pi$

$\vec{E}(y,t) = -(33)(40\pi)\cos(2\times10^9 t - 20y)\vec{a}_x$

$\boxed{\vec{E}(y,t) = -1320\pi\cos(2\times10^9 t - 20y)\vec{a}_x}$



E×H → dir'n of prop

2. $\vec{E}(z,t) = 10e^{-250z}\cos(2\pi\times10^9 t - 250z)\vec{a}_y$ mV/m

→ $\mu_r = 1$.

→ $\alpha = 250$
$\beta = 250$ } good conductor assumption

(check: $\lambda = \dfrac{2\pi}{250}$ → $\lambda = 2.51$ cm & $\delta = \dfrac{1}{\alpha}$ → $\delta = 4$ mm

2. cont'd

↳ ∴ wave attenuated by ~33% over 4mm + $\lambda = 2.51$cm

⟹ rapid attenuation + good conductor approx. ok

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

$\to \sigma = ?$

$$\alpha = \beta = \sqrt{\pi f \mu \sigma}$$

$$250 = \sqrt{(\pi)(10^9)(4\pi\times10^{-7})\sigma}$$

$$250 = \sqrt{(4\pi^2)(100)\sigma}$$

$$250 = 20\pi\sqrt{\sigma}$$

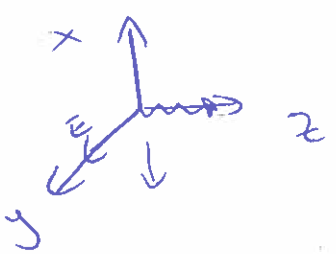
$$\boxed{\sigma = 15.8 \text{ S/m}}$$

$$|\eta| = \sqrt{\frac{(2\pi\times10^9)(4\pi\times10^{-7})}{15.8}}$$

$$= 22.35$$

$$\vec{H}(z,t) = -\frac{10}{22.35} e^{-250z} \cos(2\pi\times10^9 t - 250z - \tfrac{\pi}{4})\vec{a}_x$$

$$= -0.45 e^{-250z} \cos(2\pi\times10^9 t - 250z - \pi/4)\vec{a}_x \text{ mA/m}$$



3. $f = 800$ MHz

$\epsilon_r = 2.5$

$\mu_r = 1$

$\sigma = 0$

$|\vec{E}| = 1.0$ V/m @ inner surface of slab

+ oriented in y, propagates in z.

a) $\vec{E}(z,t) = 1\cos(\omega t - \beta z)\vec{a}_y$

$\omega = (2\pi)(8\times10^8)$

$= 1.6\pi\times10^9$

$$\beta = \omega\sqrt{\mu_r\mu_0\epsilon_r\epsilon_0}$$

$$= \frac{1.6\pi\times10^9}{3\times10^8}\sqrt{2.5}$$

$= 26.5$ rad/m

3 a) $\vec{E}(z,t) = \cos(1.6\pi\times10^9 t - 26.5z)\,\vec{a}_y$ V/m

b) $\eta = \sqrt{\dfrac{\mu_0}{2.5\epsilon_0}}$

$= 238.4$

$\vec{H}(z,t) = -4.2\cos(1.6\pi\times10^9 t - 26.5z)\,\vec{a}_x$ mA/m

c) $\vec{P}_{AV}(z) = \dfrac{|E_m^+|^2}{2\eta}\,\vec{a}_z$

$= \dfrac{1}{2(238.4)}\,\vec{a}_z$

$\vec{P}_{AV}(z) = 2.1\ \vec{a}_z$ mW/m^2

Alt

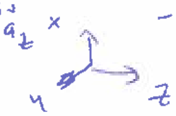
$\vec{P}_{AV} = \frac{1}{2}\,\mathrm{Re}\{\vec{E}_s \times \vec{H}_s^{*}\}$

$= \frac{1}{2}\,\mathrm{Re}\{e^{-j26.5z}\,\vec{a}_y \times -4.2e^{j26}\vec{a}_x\}$

$\times10^{-3}$

$= \frac{1}{2}\,\mathrm{Re}\{-4.2\times10^{-3}\,\underbrace{\vec{a}_y\times\vec{a}_x}_{-\vec{a}_z}\}$

$= 2.1\times10^{-3}\,\vec{a}_z$

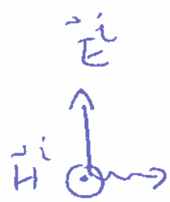


4. $\vec{E}^i(z,t) = 8\pi\cos(2\pi\times10^8 t - \beta_1 z)\,\vec{a}_x$

$\epsilon_r = 4$

$\mu_r = 1$

$\sigma = 0$



$\epsilon_r = 16$

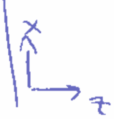
$\mu_r = 3$

$\sigma = 0.05$

$\dfrac{\sigma}{\omega\epsilon} = \dfrac{0.05}{(2\pi\times10^8)(16)(8.85\times10^{-12})}$

$= \cancel{2.25}$ $\Rightarrow$ full formulas

0.562



$\alpha_2 = 3.94$ Np/m

$\beta_2 = 15.04$ rad/m

$\eta_2 = 152.35\angle 0.256$ rad

Ω

$\eta_1 = \sqrt{\dfrac{\mu_0}{4\epsilon_0}}$

$= 60\pi$

$\beta_1 = \omega\sqrt{\mu_0\epsilon_r\epsilon_0}$

$= \dfrac{2\pi\times10^8}{3\times10^8}\sqrt{4}$

$= \dfrac{4\pi}{3}$

$\vec{H}^i(z,t) = 0.13\cos(2\pi\times10^8 t - \frac{4\pi}{3}z)\,\vec{a}_y$

4. cont'd

$\hookrightarrow \Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

$= 0.167 \angle 2.27 \text{ rad}$

$\hookrightarrow T = \dfrac{2\eta_2}{\eta_1 + \eta_2}$

$= 0.902 \angle 0.14 \text{ rad}$

b) $\vec{E}^r(z,t) = \underset{|\Gamma|}{(0.167)}(8\pi)\cos(2\pi \times 10^8 t + \frac{4\pi}{3}z + \underset{\angle\Gamma}{2.27})\vec{a}_x$

$= 4.2\cos(2\pi \times 10^8 t + \frac{4\pi}{3}z + 2.27)\vec{a}_x$

c) $\vec{H}^r(z,t) = 0.022\cos(2\pi \times 10^8 t + \frac{4\pi}{3}z + 2.27)\vec{a}_y$

d) $\vec{E}^t(z,t) = \underset{|T|}{(0.902)}(8\pi)e^{-3.94z}\cos(2\pi \times 10^8 t - 15.04z + \underset{\angle T}{0.14})\vec{a}_x$

$\vec{E}^t(z,t) = 22.67 e^{-3.94z}\cos(2\pi \times 10^8 t - 15.04z + 0.14)\vec{a}_x$

e) $\vec{H}^t(z,t) = \dfrac{22.67}{\underset{|\eta|}{152.35}} e^{-3.94z}\cos(2\pi \times 10^8 t - 15.04z + 0.14 - \underset{\angle\eta}{0.256})\vec{a}_y$

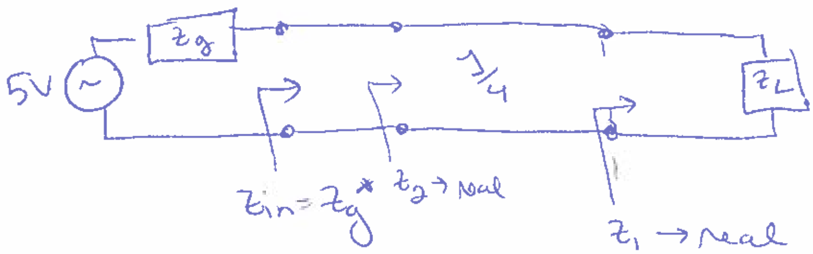
$\vec{H}^t(z,t) = 0.149 e^{-3.94z}\cos(2\pi \times 10^8 t - 15.04z - 0.116)\vec{a}_y$

5. $V_g = 5V$

$Z_g = 75 - j20\ \Omega$

$Z_0 = 50\ \Omega$

$Z_L = 20 + j70\ \Omega$



→ for maximum power transfer, we want

$Z_{in} = Z_g^*$

$\therefore\ Z_{in} = 75 + j20\ \Omega$

→ We want to use a $\lambda/4$ transformer

$\therefore$ need real values of impedance at each end

⇒ move towards load from Z_{in} to obtain real value (z_2)

⇒ move towards generator to ~~ob~~ obtain real value (z_1)

⇒ use z_1 + z_2 to design $\lambda/4$ tx.

$Z_{in} \Rightarrow \frac{75}{50} + j\frac{20}{50}$

$z_{in} = 1.5 + j0.4$

$Z_L \Rightarrow \frac{20}{50} + j\frac{70}{50}$

$z_L = 0.4 + j1.4$

5. cont'd:

$z_{in} \to 0.6 = z_2 \Rightarrow$ rotate from $(0.292 \to 0.5)\lambda$

distance $= (0.5 - 0.292)\lambda$
$= 0.208\lambda$

$z_L \to 7.5 = z_1 \Rightarrow$ rotate from $(0.156$ to $0.25)\lambda$

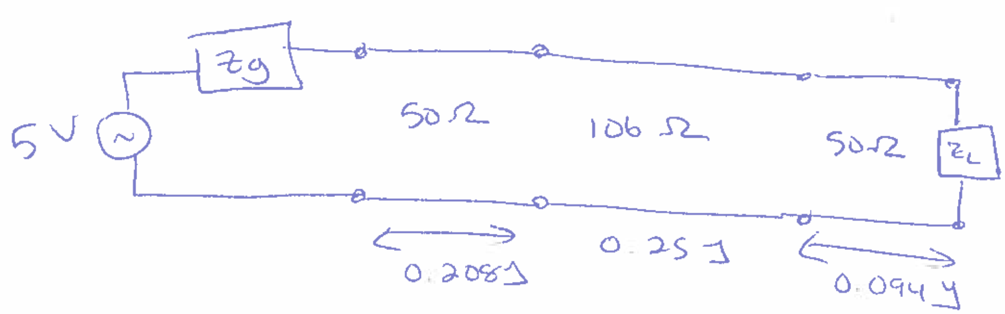
distance $= (0.25 - 0.156)\lambda$
$= 0.094\lambda$

(could also rotate $(0.094 + 0.25)\lambda$
to $z_1 = 0.13$)

$$Z_2 = (0.6)(50) = 30\ \Omega$$

$$Z_1 = 375\ \Omega$$

$$\therefore \lambda/4 \to Z_{tx} = \sqrt{Z_1 Z_2} = 106\ \Omega$$

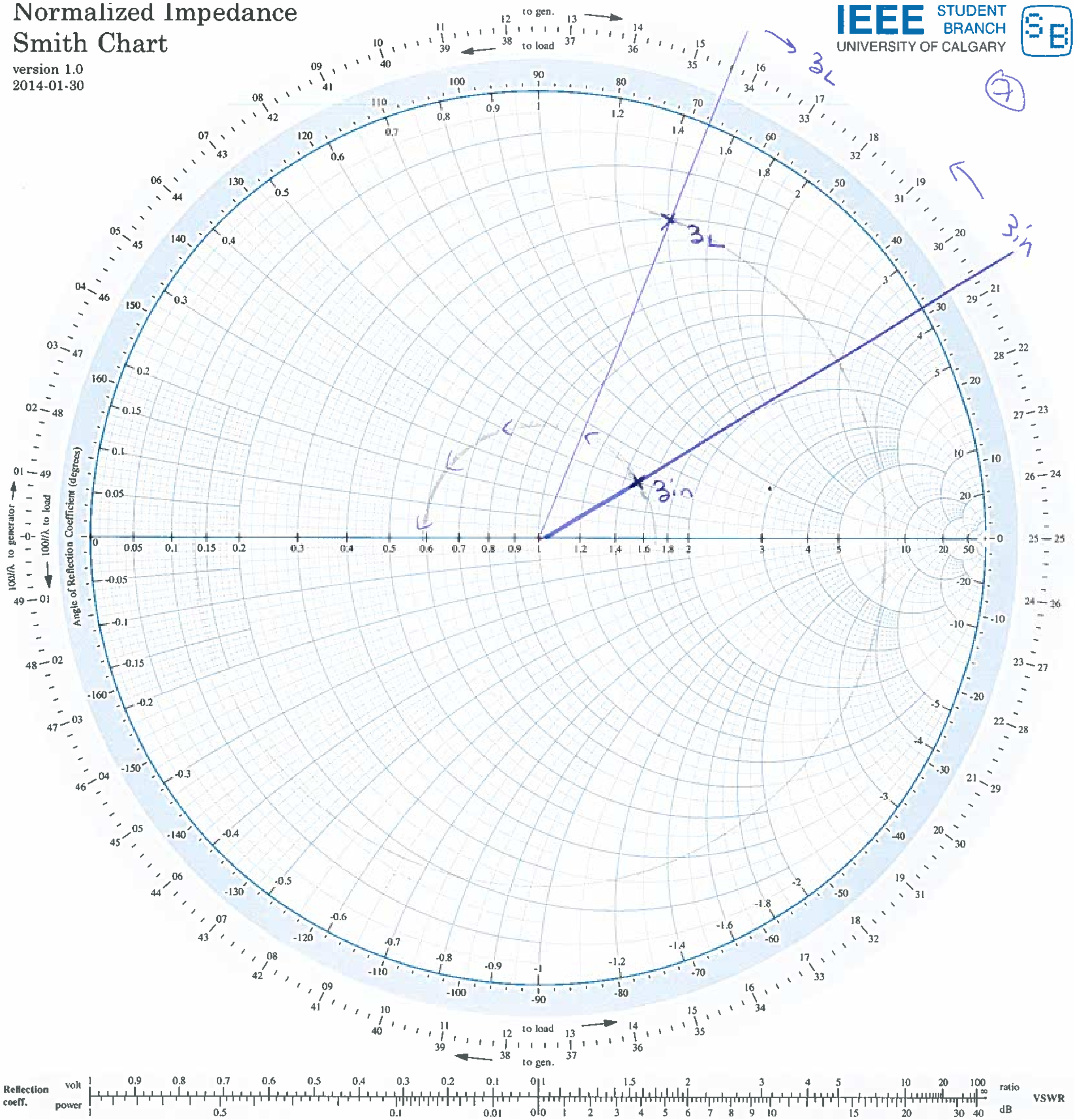


Normalized Impedance Smith Chart

version 1.0
2014-01-30

IEEE STUDENT BRANCH
UNIVERSITY OF CALGARY

7



SCHULICH School of Engineering

makedapress