January 16, 2020 12:14 PM

Faraday's Law

↳ dynamic interaction between magnetic field and charges

↳ $\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

↳ Lenz's Law

Time-varying fields

↳ 2 charged particles at rest act on each other with a force → Coulomb's law

$$\vec{F} = Q\vec{E}$$

↳ 2 charged particles with uniform velocities act on each other with magnetic force

$$\vec{F} = Q\vec{v} \times \vec{B}$$

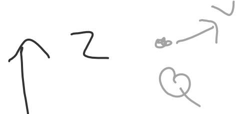
↳ accelerated particle ⇒ another force exerted on stationary or moving particles

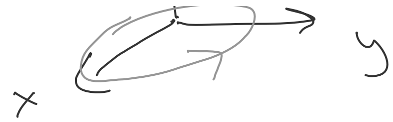
⇒ small compared to $\vec{F} = Q\vec{E}$ but many charges in conductor ⇒ measurable

⇒ same form as $\vec{F} = Q\vec{E}$ but $\vec{E}$ is different

↳ $\vec{E}_{ind}$ (induced electric field)

Consider





of referen moves with particle

→ second force acting on particle $\vec{F} = Q\vec{v}\times\vec{B}$

→ if $\vec{v} = 0$, no force

→ magnetic field, no electric field

→ particle looks stationary

→ particle experiences force ⇒ $\vec{F} = Q\vec{E}$

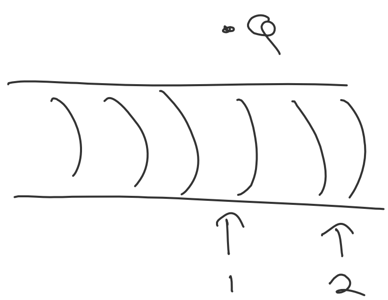
<u>but</u> $\vec{F}$ is time varying <u>and</u> magnetic field

⇒ different conclusions from same scenario with different observations

⇒ $\vec{F} = Q\vec{v}\times\vec{B}$ vs $\vec{F} = Q\vec{E}$ → time-varying magnetic field?

⇒ time-varying electric field $\vec{E}_{ind}$

Consider solenoid with charged particle:



→ observed from particle, field changes when moving from 1 → 2 ⇒ $\vec{F} = Q\vec{E}$

→ observed from solenoid, particle moves ⇒ $\vec{F} = Q\vec{v}\times\vec{B}$

⇒ regardless of cause of time-varying magnetic field, we get a time-varying electric field

Charge in static + induced field experiences:

$$\vec{F} = Q(\vec{E}_{static} + \vec{E}_{ind})$$

$\hookrightarrow \vec{E}_{ind} = \vec{v} \times \vec{B}$?

<u>but</u> time-varying currents are "sources" of $\vec{E}_{ind}$

$$\vec{E}_{ind} = -\frac{\partial}{\partial t}\left(\frac{\mu_0}{4\pi}\int \frac{\vec{J}\,dv}{r}\right) \ (V/m)$$

$\vec{J}\,dv \rightarrow$ volume

$\vec{J}\,ds \rightarrow$ surface

$i\,d\vec{l} \rightarrow$ line current

$Q\vec{v} \rightarrow$ charge

- In a region with free charge carriers (e.g. wire), $\vec{E}_{ind}$ acts on e^- with $\vec{F} = Q\vec{E}_{ind}$
- line integral of $\vec{E}_{ind}$ between 2 points : M & N

$$EMF = \int_M^N \vec{E}_{ind} \cdot d\vec{l}$$

$\Rightarrow$ loop of wire:

$$EMF = \oint \vec{E}_{ind} \cdot d\vec{l}$$

$\downarrow$

electromotive force

$$= \oint -\frac{\partial}{\partial t}\frac{\mu_0}{4\pi}\frac{Q\vec{v}}{r} \cdot d\vec{l}$$

$\vec{A} = \mu_0 \dfrac{Q\vec{v}}{4\pi r}$

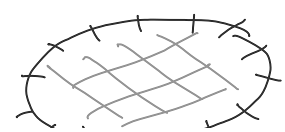
$$= -\frac{d}{dt}\oint \vec{A} \cdot d\vec{l}$$

$\vec{B} = \nabla \times \vec{A}$

$$= -\frac{d}{dt}\int (\nabla \times \vec{A}) \cdot d\vec{s}$$

$$\boxed{EMF = -\frac{d}{dt}\int \vec{B} \cdot d\vec{s}}$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{s} \longrightarrow$$



$$\nabla \times E = -\frac{\partial}{\partial t}\vec{B}$$

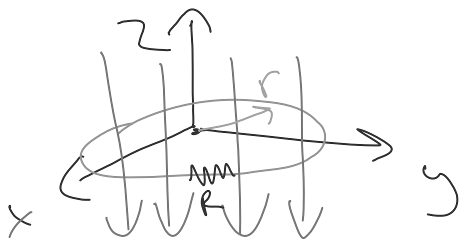
$d\vec{\ell}$

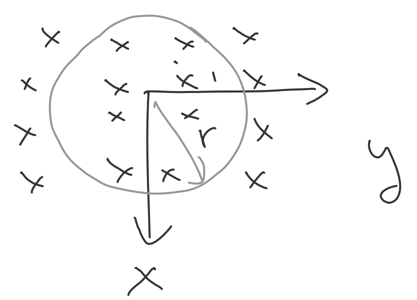
EMF → voltage that arises from conductors moving in magnetic field <u>OR</u> from changing magnetic field

→ time-varying magnetic field produces EMF that may establish current in suitable closed circuit

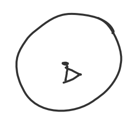
Ex Transformer EMF → changing flux
→ stationary loop

$$\vec{B} = -B_z \cos(\omega t)\vec{a}_z$$





EMF in loop?



$$EMF = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$$

$$EMF = -\frac{d}{dt}\int_0^{2\pi}\int_0^{r}(-B_z\cos(\omega t)\vec{a}_z \cdot \underbrace{\rho\, d\rho\, d\phi\, \vec{a}_z}_{d\vec{s}})$$

$$= -\frac{d}{dt}\left(-B_z\cos(\omega t)\left(\frac{r^2}{2}\right)(2\pi)\right)$$

$$= +B_z \pi r^2 \frac{d}{dt}\cos(\omega t)$$

$$= -\pi r^2 \omega B_z \sin(\omega t)$$

↳ $d\vec{s} \rightarrow \vec{a}_z$ direction

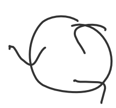
⇒ direction of integration area loop

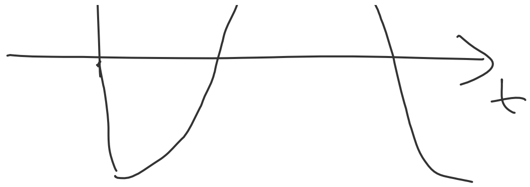
$$I = EMF/R$$

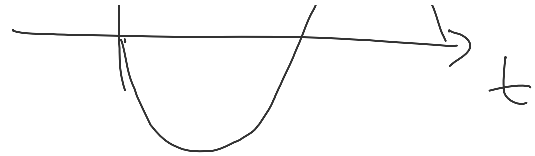
$$= -\frac{\pi r^2}{R}\omega B_z \sin(\omega t)$$

↓

current flow




t


t