## Assignment 4 Solution

**6.18** $x^3 + 8x^2 + 2x - 50 = 0$

$x(0) = 1$

$$J(i) = \left.\frac{df}{dx}\right|_{x=x(i)} = [3x^2 + 16x + 2]_{x=x(i)}$$

In the general form:

$$x(i+1) = x(i) + \frac{1}{3x(i)^2 + 16x(i) + 2} \times (-x(i)^3 - 8x(i)^2 - 2x(i) + 50)$$

Using $x(0) = 1$, we arrive at

| $i$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $x$ | 1 | 2.857 | 2.243 | 2.129 | 2.126 | 2.126 |
| $\varepsilon$ | 1.857 | 0.215 | 0.051 | 0.0018 | 2.0E-6 | |

After 5 iterations, $\varepsilon < 0.001$.

$\therefore x = 2.126$

The textbook uses $\frac{x(i+1) - x(i)}{x(i)} < \varepsilon$ as the stopping criterion. You can also use $x(i+1) - x(i) < \varepsilon$ or $f\big(x(i+1)\big) < \varepsilon$ as discussed in class.

**6.23**

$$x^0 = \begin{bmatrix}1.0\\0.5\end{bmatrix} \qquad \textit{stopping condition}: \varepsilon = 0.005$$

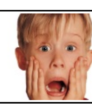
$$F(x) = \begin{bmatrix} e^{x_1x_2} - 1.2 \\ \cos(x_1+x_2) - 0.5 \end{bmatrix} \qquad J(x) = \begin{bmatrix} x_2e^{x_1x_2} & x_1e^{x_1x_2} \\ -\sin(x_1+x_2) & -\sin(x_1+x_2) \end{bmatrix}$$

Find the next value of X using:

- $J(X^{(K)}).(X^{(K+1)} - X^{(K)}) = -F(X^{(K)})$. Solve the system of linear equations to get $(X^{(K+1)} - X^{(K)})$. Then, using values of $X^{(K)}$ , you can calculate $X^{(K+1)}$
- Alternatively, you can find the inverse of the Jacobian, and use: $X^{(K+1)} = X^{(K)} - J(X^{(K)})^{-1}F(X^{(K)})$ to calculate $X^{(K+1)}$ directly.

| K | X(k) | F(X(k)) | J(X(k)) | X(k+1) | $\lVert F(X^{(k)}) \rVert$ | Another iteration? |
|---|---|---|---|---|---|---|
| 1 | $\begin{bmatrix}1\\0.5\end{bmatrix}$ | $\begin{bmatrix}0.4487\\-0.4293\end{bmatrix}$ | $\begin{bmatrix}0.8244 & 1.6487\\-0.9975 & -0.9975\end{bmatrix}$ | $\begin{bmatrix}0.6836\\0.386\end{bmatrix}$ | 0.4487 |  |
| 2 | $\begin{bmatrix}0.6836\\0.386\end{bmatrix}$ | $\begin{bmatrix}0.102\\-0.0196\end{bmatrix}$ | $\begin{bmatrix}0.5026 & 0.8901\\-0.877 & -0.877\end{bmatrix}$ | $\begin{bmatrix}0.8956\\0.1518\end{bmatrix}$ | 0.102 |  |
| 3 | $\begin{bmatrix}0.8956\\0.1518\end{bmatrix}$ | $\begin{bmatrix}-0.0544\\-0.0001\end{bmatrix}$ | $\begin{bmatrix}0.1739 & 1.026\\-0.8661 & -0.8661\end{bmatrix}$ | $\begin{bmatrix}0.8315\\0.2157\end{bmatrix}$ | 0.054 |  |
| 4 | $\begin{bmatrix}0.8315\\0.2157\end{bmatrix}$ | $\begin{bmatrix}-0.0036\\0\end{bmatrix}$ | $\begin{bmatrix}0.2580 & 0.9949\\-0.866 & -0.866\end{bmatrix}$ | $\begin{bmatrix}0.8267\\0.2205\end{bmatrix}$ | 0.0036 |  |
| 5 | $\begin{bmatrix}0.8267\\0.2205\end{bmatrix}$ | $\begin{bmatrix}0\\0\end{bmatrix}$ | $\begin{bmatrix}0.2646 & 0.992\\-0.866 & -0.866\end{bmatrix}$ | $\begin{bmatrix}0.8267\\0.2206\end{bmatrix}$ | 0.000 | Nope! |

b) Same problem but setting $x^0 = \begin{bmatrix}1.0\\2.0\end{bmatrix}$

| K | X(k) | F(X(k)) | J(X(k)) | X(k+1) | $\lVert F(X^{(k)}) \rVert$ | Another iteration? |
|---|---|---|---|---|---|---|
| 1 | $\begin{bmatrix}1\\2\end{bmatrix}$ | $\begin{bmatrix}6.1891\\-1.49\end{bmatrix}$ | $\begin{bmatrix}14.7781 & 7.3891\\-0.1411 & -0.1411\end{bmatrix}$ | $\begin{bmatrix}10.7207\\-18.2791\end{bmatrix}$ | 6.189 |  |
| 2 | $\begin{bmatrix}10.7207\\-18.2791\end{bmatrix}$ | $\begin{bmatrix}-1.2\\-0.2087\end{bmatrix}$ | $\begin{bmatrix}0 & 0\\0.9566 & 0.9566\end{bmatrix}$ |  | | |

Cannot proceed anymore since J(x) is not invertible.

## 6.28

a) $Y[1,1] = 2 - j4 + 3 - j6 = 5 - j10 = 11.18 \angle{-63.43^\circ}$

$Y[2,2] = 2 - j4 = 4.47 \angle{-63.43^\circ}$

$Y[3,3] = 3 - j6 = 6.71 \angle{-63.43^\circ}$

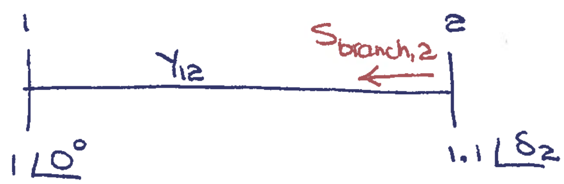
$Y[2,1] = Y[1,2] = -2 + j4 = 4.47 \angle{116.57^\circ}$

$Y[1,3] = Y[3,1] = -3 + j6 = 6.71 \angle{116.57^\circ}$

$Y[2,3] = Y[3,2] = 0$

b) at bus 2, $\quad P_{gen,2} - P_{load,2} - P_{branch,2} = 0$

$$1.5 - 0 - P_{branch,2} = 0$$




$$S_{branch,2} = \bar{V}_2 \cdot \bar{I}_{12}^{\,*}$$

$$= \bar{V}_2 \cdot \left[ (\bar{V}_2 - \bar{V}_1) \cdot Y_{12} \right]^*$$

$$= \bar{V}_2 \cdot Y_{12}^* \cdot (\bar{V}_2 - \bar{V}_1)^*$$

$$= \bar{V}_2 \cdot Y_{12}^* \cdot \bar{V}_2^{\,*} - \bar{V}_2 \cdot Y_{12}^* \, \bar{V}_1^{\,*}$$

$$= (V_2)^2 \cdot Y_{12} \angle{\theta_{12}} - V_2 \angle{\delta_2} \cdot Y_{12} \angle{-\theta_{12}} \cdot V_1 \angle{-\delta_1}$$

$$= (V_2)^2 \cdot Y_{12} \angle{-\theta_{12}} - V_1 \cdot V_2 \cdot Y_{12} \angle{\delta_2 - \delta_1 - \theta_{12}}$$

$$\therefore P_{branch,2} = (V_2)^2 (Y_{12}) \cdot \cos(-\theta_{12}) - V_1 \cdot V_2 \cdot Y_{12} \cos(\delta_2 - \delta_1 - \theta_{12})$$

This is the Pbranch equation with Y expressed in polar coordinates.

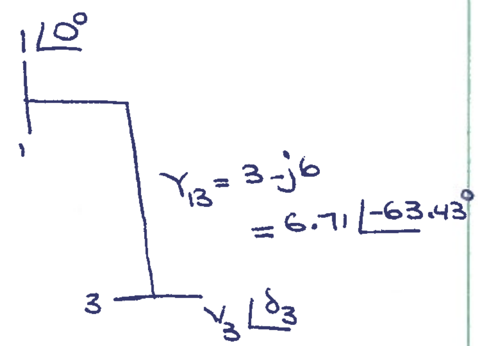
reminder $Y_{12} = 2 - j4 = 4.47 \angle{-63.43^\circ}$

Please note the difference between $Y_{12}$ and Y[1,2]

$$\therefore\ 0 = 1.5 - 0 - (1.1)^2(4.47)\cos(+63.43°) + (1)(1.1)(4.47)\cos(\delta_2 + 63.43°)$$

solve for $\delta_2$ : $\delta_2 = 15.795°$

c) Similar to part b),




$$P_{branch,3} = (V_3)^2(Y_{13})\cdot\cos(-\theta_{13}) - V_1\cdot V_3\cdot Y_{13}\cdot\cos(\delta_3 - \delta_1 - \theta_{13})$$

$$\therefore\ -1.5 - 0 - (V_3^2)(6.71)\cos(63.43°) + (1)V_3(6.71)\cos(\delta_3 + 63.43°) = 0 \quad \text{(i)}$$

$$Q_{branch,3} = (V_3)^2(Y_{13})\sin(-\theta_{13}) - V_1 V_3 Y_{13}\sin(\delta_3 - \delta_1 - \theta_{13})$$

$$\therefore\ 0.8 - 0 - (V_3)^2(6.71)\sin(63.43°) + (1)V_3(6.71)\sin(\delta_3 + 63.43°) = 0 \quad \text{(ii)}$$

Solve (i) & (ii) to get $V_3$ & $\delta_3$, painfully!

$$V_3 = 0.9723 \text{ pu}$$

$$\delta_3 = -15.1°$$

d) $$P_{branch,1} = (V_1)^2(Y_{12})\cos(-\theta_{12}) - V_1\cdot V_2\cdot Y_{12}\cos(\delta_1 - \delta_2 - \theta_{12}) + (V_1)^2(Y_{13})\cos(-\theta_{13}) - V_1\cdot V_3\cdot Y_{13}\cos(\delta_1 - \delta_3 - \theta_{13})$$

plug in all values,

$$P_{branch,1} = 0.39 \text{ pu}$$

$$P_{gen,1} - P_{load,1} - P_{branch,1} = 0$$

$$\therefore\ P_{gen,1} = 0.39 \text{ pu}$$

e) $P_{loss} = P_{gen} - P_{load}$

$= P_{gen,1} + P_{gen,2} - P_{load,3}$

$= 0.39 + 1.5 - 1.5$

$= 0.39$ pu

**6.30**

| | | | | |
|---|---|---|---|---|
| $6.25-j18.695$ | $-5.00+j15.00$ | $-1.25+j3.75$ | 0 | 0 |
| $-5.00+j15.00$ | $12.9167-j38.665$ | $-1.6667+j5.00$ | $-1.25+j3.75$ | $-5.00+j15.00$ |
| $-1.25+j3.75$ | $-1.6667+j5.00$ | $8.7990-j32.2294$ | $-5.8824+j23.5294$ | 0 |
| 0 | $-1.25+j3.75$ | $-5.8824+j23.5294$ | $9.8846-j36.4037$ | $-2.7523+j9.1743$ |
| 0 | $-5.00+j15.00$ | 0 | $-2.7523+j9.1743$ | $7.7523-j24.1443$ |

**6.31** First, we need to find the per-unit shunt admittance of the added capacitor.

$75 \text{ Mvar} = 0.75 \text{ p.u.}$

To find Y, we use the relation

$$S = V^2 Y$$

$$Y = \frac{S}{V^2} = \frac{0.75}{1^2} = 0.75 \text{ p.u.}$$

Now, we can find $Y_{44}$

$$Y_{44} = \frac{1}{0.08+j0.24} + \frac{1}{0.01+j0.04} + \frac{1}{0.03+j0.10} + j\frac{0.05+0.01+0.04}{2} + j0.75$$

$$= 9.8846 - j35.6537$$