ENEL 471 – Winter 2020
Assignment 7 – Solutions

Problem 4.7

$$s(t) = A_c \cos(\theta(t))$$
$$\theta(t) = 2\pi f_c t + k_p m(t)$$

Let $\beta = 0.3$ for $m(t) = \cos(2\pi f_m t)$.

$$\therefore\ s(t) = A_c \cos(2\pi f_c t + \beta m(t))$$
$$= A_c[\cos(2\pi f_c t)\cos(\beta\cos(2\pi f_m t)) - \sin(2\pi f_c t)\sin(\beta\cos(2\pi f_m t))]$$

for small β:

$$\cos(\beta\cos(2\pi f_m t)) \approx 1$$
$$\sin(\beta\sin(2\pi f_m t)) \approx \beta\cos(2\pi f_m t)$$

$$\therefore\ s(t) = A_c\cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t)\cos(2\pi fmt)$$
$$= A_c\cos(2\pi f_c t) - \beta\frac{A_c}{2}[\sin(2\pi(f_c + f_m)t) + \sin(2\pi(f_c - f_m)t)$$

Problems 4.8

(a) From the approximating equation of Bessel functions in Table A.3 in the Appendix:

$$J_0(\beta) = \sqrt{\frac{2}{\pi\beta}}\cos\left(\beta - \frac{\pi}{4}\right) = 0$$

Only if: $\beta - \frac{\pi}{4} = (2k+1)\frac{\pi}{2}$

Or: $\beta = k\pi + \frac{3\pi}{4}$

This give the approximate values of beta: 2.4, 5.5, 8.6, 11.8, …

(b) The modulation index is

$$\beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}$$

Therefore,

$$k_f = \frac{\beta f_m}{A_m}$$

Since $J_0(\beta) = 0$ for the first time when $\beta = 2.44$, we deduce that

$$k_f = \frac{2.44 \times 10^3}{2}$$

$$= 1.22 \times 10^3 \text{ hertz/volt}$$

Next, we note that $J_0(\beta) = 0$ for the second time when $\beta = 5.52$. Hence, the corresponding value of A_m for which the carrier component is reduced to zero is

$$A_m = \frac{\beta f_m}{k_f}$$

$$= \frac{5.52 \times 10^3}{1.22 \times 10^3}$$

$$= 4.52 \text{ volts}$$

Problems 4.9

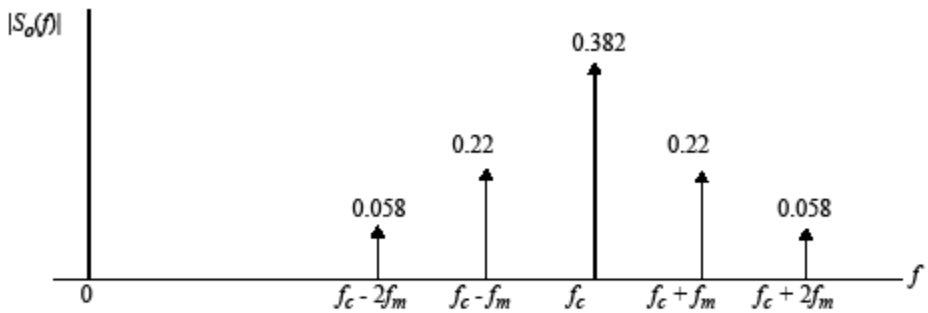
For $\beta = 1$, we have

$J_0(1) = 0.765$
$J_1(1) = 0.44$
$J_2(1) = 0.115$

Therefore, the band-pass filter output is (assuming a carrier amplitude of 1 volt)

$$s_o(t) = 0.765\cos(2\pi f_c t)$$
$$+ 0.44\{\cos[2\pi(f_c + f_m)t] - \cos[2\pi(f_c - f_m)t]\}$$
$$+ 0.115\{\cos[2\pi(f_c + f_m)t] + \cos[2\pi(f_c - 2f_m)t]\},$$

and the amplitude spectrum (for positive frequencies) is



Problems 4.10

(a) The frequency deviation is

$$\Delta f = k_f A_m = 25 \times 10^3 \times 20 = 5 \times 10^5 \text{Hz}$$

The corresponding value of the modulation index is

$$\beta = \frac{\Delta f}{f_m} = \frac{5 \times 10^5}{10^5} = 5$$

The transmission bandwidth of the FM wave, using Carson's rule, is therefore

$$B_T = 2f_m(1 + \beta) = 2 \times 100(1 + 5) = 1200\text{kHz} = 1.2\text{MHz}$$

(b) Using the universal curve of Fig. 3.36 we find that for $\beta = 5$:

$$\frac{B_T}{\Delta f} = 3$$

Therefore,

$$B_T = 3 \times 500 = 1500\text{kHz} = 1.5\text{MHz}$$

(c) If the amplitude of the modulating wave is doubled, we find that

$$\Delta f - 1\text{MHz} \quad \text{and} \quad \beta = 10$$

Thus, using Carson's rule we obtain,

$$B_T = 2 \times 100(1 + 10) = 2200\text{kHz} = 2.2\text{MHz}$$

Using the universal curve of Fig. 3.36, we get

$$\frac{B_T}{\Delta f} = 2.75$$

and $B_T = 2.75$ MHz.

(d) If f_m is doubled, $\beta = 2.5$. Then, using Carson's rule, $B_T = 1.4$ MHz. Using the universal curve, $B_T/\Delta f = 4$, and

$$B_T = 4\Delta f = 2\text{MHz}$$