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University of Calgary

Schulich School of Engineering

Department of Electrical and Computer Engineering

ENEL 476 – Electromagnetic Waves and Applications

Midterm Examination

Winter Session 2017
Tuesday February 28, 2017
12:30-1:45 pm

ENA 101

Student Name or ID number:

Dr Fear

SOLUTION

Question 1. **(18 marks; 2 per part)**

A uniform plane wave travels in a material with $\varepsilon_r=16$, $\sigma=0$ S/m and $\mu_r=1$. The direction of propagation is +x, and the electric field is oriented in +y. The maximum amplitude of the electric field is 10 V/m. The frequency is 10 MHz.

a) Find an expression for the electric field, **E**(x,t), associated with the uniform plane wave.

$$\vec{E}(x,t) = 10\cos(\underbrace{2\pi\times10^7}_{\omega} t - \beta x)\,\vec{a}_y$$

$$\beta = \omega\sqrt{\mu_0(16)\varepsilon_0}$$
$$= \frac{2\pi\times10^7}{3\times10^8}(4)$$
$$= \frac{8\pi}{30} \Rightarrow \beta = 0.84 \text{ rad/m}$$

$$\boxed{\vec{E}(x,t) = 10\cos(2\pi\times10^7 t - 0.84x)\,\vec{a}_y \text{ V/m}}$$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

b) Find an expression for the magnetic field, **H**(x,t), associated with the uniform plane wave.

$$\vec{H}(x,t) = \frac{10}{\eta}\cos(2\pi\times10^7 t - 0.84x)\,\vec{a}_z$$

$\frac{1}{2}$ $$\eta = \sqrt{\frac{\mu_0}{16\varepsilon_0}}$$
$\frac{1}{2}$ $$= 30\pi$$

$$\therefore \vec{H}(x,t) = \underset{\frac{1}{2}}{\overset{0.11}{\frac{1}{3\pi}}}\cos(2\pi\times10^7 t - 0.84x)\,\vec{a}_z \text{ A/m}$$

$\frac{1}{2}$ $\frac{1}{2}$

The uniform plane wave is normally incident on a planar interface located at x=0. The material in the region x<0 is the dielectric with $\varepsilon_r=16$, $\sigma=0$, $\mu_r=1$. The material in the region x>0 has $\varepsilon_r=4$, $\sigma=0$ and $\mu_r=2$.

c) Calculate the reflection coefficient (Γ)

(1): $\varepsilon_r=16$, $\sigma=0$, $\mu_r=1$, $\eta_1 = 30\pi$

(2): $\varepsilon_r=4$, $\sigma=0$, $\mu_r=2$

x

$$\eta_2 = \sqrt{\frac{2\mu_0}{4\varepsilon_0}} = \frac{120\pi}{\sqrt{2}}\cdot 2 \; \tfrac{1}{2} = 84.85\pi$$

$\frac{1}{2}$ $$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}$$
$$= \frac{84.85\pi - 30\pi}{84.85\pi + 30\pi}$$
$$\boxed{\Gamma = 0.478}\; 1$$

d) Calculate the transmission coefficient (T).

$$T = 1 + \Gamma \quad 1$$

$$\boxed{T = 1.478} \quad 1$$

e) Find an expression for the reflected electric field, $\mathbf{E}^r(x,t)$.

$$\vec{E}^r(x,t) = \underbrace{4.78}_{(\Gamma)(E^i)} \cos(2\pi \times 10^7 t \; \underline{+}\, 0.84x)\,\vec{a}_y \ \text{V/m}$$

$\cancel{1}$ 1 change dir'n

f) Find an expression for the reflected magnetic field, $\mathbf{H}^r(x,t)$.

$$\vec{H}^r(x,t) = -\frac{4.78}{\underbrace{120\pi}_{-0.053}} \cos(2\pi \times 10^7 t + 0.84x)\,\vec{a}_z \ \text{A/m}$$

1 1 $-\vec{a}_z$

g) Find an expression for the transmitted electric field, $\mathbf{E}^t(x,t)$.

$$\vec{E}^t(x,t) = 14.78 \cos(2\pi \times 10^7 t - 0.6x)\,\vec{a}_y \ \text{V/m}$$

1 1

$$\beta_2 = \omega\sqrt{\mu_r \mu_0 \epsilon_r \epsilon_0}$$
$$= 0.6$$

h) Find an expression for the transmitted magnetic field, $\mathbf{H}^t(x.t)$.

$$\vec{H}^t(x,t) = \frac{14.78}{\underbrace{84.85\pi}_{0.06}} \cos(2\pi \times 10^7 t - 0.6x)\,\vec{a}_z \ \text{A/m}$$

1 1

i) Show that boundary conditions at the interface are satisfied for the electric field.

$\rightarrow$ at $x = 0$,

evaluate at $x=0$ $\frac{1}{2}$

$$\vec{E}_s^i(x=0) = 10\vec{a}_y$$
$$\vec{E}_s^r(x=0) = 4.78\vec{a}_y$$
$$\vec{E}_s^t(x=0) = 14.78\vec{a}_y$$

1 $\vec{E}_{tan,1} = \vec{E}_{tan,2}$ (for this case, $\vec{E}$ is tangent to interface $\therefore \vec{D}_{IN} = 0$ & $\vec{D}_{BN} = 0$)

$$\Rightarrow \vec{E}_s^i(x=0) + \vec{E}_s^r(x=0) = \vec{E}_s^t(x=0)$$

$$10 + 4.78 = 14.78 \ \checkmark$$

$\frac{1}{2}$

Question 2 (24 marks).

Whole liquid eggs have relative permittivity of 53 and conductivity of 1.28 S/m at 1800 MHz and 20°C. The permeability is μ_o. You are interested in exploring industrial microwave heating, and decide to start with a uniform plane wave analysis. Assume that the fields in the eggs can be approximated by a uniform plane wave with maximum amplitude of 10 kV/m. The field propagates in the –z direction and the electric field is oriented in +x.

Calculate the following quantities:

a) attenuation constant (α) (3 marks)

(1) $$\frac{\sigma}{\omega\epsilon} = \frac{1.28}{(2\pi)(1.8\times10^{9})\left(\frac{1}{36\pi}\times10^{-9}\right)(53)}$$

$= 0.24 \Rightarrow$ use full formulas

(1) $$\alpha = \omega\sqrt{\frac{\mu_0\epsilon_r\epsilon_0}{2}\left[\sqrt{1+\left(\tfrac{\sigma}{\omega\epsilon}\right)^2}-1\right]}$$

$\uparrow$ 0.24

(1) $\alpha = 32.7$ Np/m

b) phase constant (β) (2 marks)

(1) $$\beta = \omega\sqrt{\frac{\mu_0\epsilon_r\epsilon_0}{2}\left[\sqrt{1+\left(\tfrac{\sigma}{\omega\epsilon}\right)^2}+1\right]}$$

(1) $\beta = 276.54$ rad/m

c) electric field in the time domain ($\mathbf{E}(z,t)$) (3 marks)

$$\vec{E}(z,t) = 10e^{32.7x}\cos(1.131\times10^{10}t + 276.54z)\,\vec{a}_x \text{ kV/m}$$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

d) electric field in phasor form ($\mathbf{E}_s(z)$) (2 marks)

$$\vec{E}_s(z) = 10e^{32.7x}\,e^{j276.54z}\,\vec{a}_x \text{ kV/m}$$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

e) intrinsic impedance of the medium (η) (2 marks)

(1) $|\eta| = 51.03$

(1) $\measuredangle \eta = 0.12$ rad

f) magnetic field in the time domain (**H**(z,t)) (2 marks)

$\vec{H}(z,t) = \frac{-10}{51.03} e^{32.7x} \cos(1.131\times10^{10}t + 276.542 - 0.12)\,\vec{a}_y$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $-\vec{a}_y$

g) velocity of propagation (v_p or u) (2 marks)

(1) $V_p = \omega/\beta \Rightarrow v_p = 1.131\times10^{10}/276.54$

(1) $= 4.09\times10^{7}$ m/s

h) wavelength (λ) (2 marks)

(1) $\lambda = \frac{2\pi}{\beta} \rightarrow \lambda = \frac{2\pi}{276.54} \Rightarrow \lambda = 2.27$ cm (1)

i) skin depth (δ) (2 marks)

(1) $\delta = \frac{1}{\alpha}$

(1) $= 3.04$ cm

j) time-averaged Poynting vector ($\mathbf{P}_{av}(z)$) (2 marks)

(1) $\vec{P}_{AV}(z) = -\frac{1}{2}\frac{|E|^2}{|\eta|} e^{2\alpha z} \cos(\theta_\eta)\,\vec{a}_z$

$= -\frac{1}{2}\frac{(10)^2}{(51)} e^{65.4z} \cos(0.12)\,\vec{a}_z$

(1) $= -0.9733 e^{65.4z}\,\vec{a}_z$ MW/m²

k) total power in a 1m² region at a depth of 5 cm into the eggs (2 marks)

(1) $\vec{P}_{AV}(z=-0.05) = -0.9733(e^{(65.4)(-0.05)})\,\vec{a}_z$

$\approx -37\,\vec{a}_z$ kW/m²

$\vec{P}_{AV} \times 1m^2 = P_{tot}$

(1) $\therefore \vec{P}_{tot} \simeq 37$ kW

Question 3 (13 marks).

a) A circular loop of wire has radius of 5 cm and contains a resistor of 10 Ω. The loop is oriented with surface normal in the +z direction and placed in an external magnetic flux density described by:

$$\mathbf{B}(t) = 10\cos(120\pi t)\,\mathbf{a}_z \text{ mWb/m}^2$$

Calculate the EMF (3 marks).

(1) $EMF = -\frac{d}{dt}\phi$

$\phi = (10\cos(120\pi t))(\pi)(0.05)^2 \times 10^{-3}$ or $\phi = \int \vec{B}\cdot d\vec{s}$

(1) $= 0.0785\cos(120\pi t) \times 10^{-3}$ $= \int_0^{2\pi}\int_0^{0.05} 10\cos(120\pi t)\,\rho\,d\rho\,d\phi$

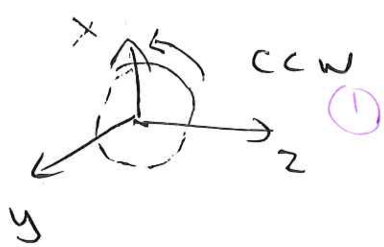
$EMF = (0.0785)(120\pi)\sin(120\pi t) \times 10^{-3}$

(1) $= 29.6\sin(120\pi t)$ mV

Calculate the current flowing in the loop (1 mark).

(1) $I = 2.96\sin(120\pi t)$ mA

Sketch the loop and indicate the direction of current flow in the first quarter period (0<t<T/4). Explain how this direction of current flow satisfies Lenz's law (3 marks).



(1)

→ Lenz's law: induced current flow has associated flux that opposes change in original flux (1)

→ $\vec{B} = 10\cos(120\pi t)\hat{a}_z$

↳ decreases from $0<t<\frac{T}{4}$ (1)

RHR gives current flow in +z

⇒ this counteracts decrease in +z noted w/ $\vec{B}$

b) A circular loop of wire has radius of 5 cm and contains a resistor of 10 Ω. The loop is oriented with surface normal in the +z direction and placed in an external magnetic flux density described by:

$$\mathbf{B}(t) = 10\cos(120\pi t)\,\mathbf{a}_z \text{ mWb/m}^2$$

The loop moves in the x direction with velocity of 0.5 m/s.

Calculate the EMF (1 mark).

(1) The flux density is uniform ∴ EMF = 29.6 sin (120πt) mV

Calculate the current flowing in the loop (1 mark).

(1) I = 2.96 sin (120πt) mA

What direction should the loop move in to maximize the EMF? (1 mark)

→ the loop would rotate as this gives a change in flux passing through the loop

(1)

→ otherwise, direction of motion in x, y, z does not result in motional EMF

c) Teflon has ε_r=2.4, σ=0 S/m, and μr=1. An electric field in the Teflon is described by:

$$\mathbf{E}(t) = 7.5\cos(2\pi \times 10^3 t)\,\mathbf{a}_x \text{ V/m}$$

Find the corresponding displacement current density (2 marks).

$$\vec{D} = \varepsilon_r \varepsilon_0 \vec{E}$$

$$= (2.4)\left(\frac{1}{36\pi}\times 10^{-9}\right)(7.5)\cos(2\pi\times 10^3 t)\,\vec{a}_x$$

(1) $$\vec{J}_D = \frac{\partial}{\partial t}\vec{D}$$

$$= \frac{(2.4)(7.5)}{36\pi \times 10^{+9}}(2\pi\times 10^3)\sin(2\pi\times 10^3 t)$$

(1) $$= -\sin(2\pi\times 10^3 t)\,\vec{a}_x \ \mu A/m^2$$

d) Time-varying fields have which of the following characteristics (1 mark):

1

the electric and magnetic fields are independent
the electric and magnetic fields are related
the electric and magnetic fields are spatially invariant.

Name	
Q1	
Q2	
Q3	
Total	