Question 1 [*8 pts*]

The message signal $m(t)$, whose spectrum is shown in Figure 1 below, is passed through the system in Figure 2 below, where $f_c = 100\,kHz$, $W = 1\,kHz$, $A_m = \max|m(t)| = 0.5$, the bandpass filter is ideal and has a bandwidth of $2W$ centered around f_c, the lowpass filter is ideal with a bandwidth of W

a- Sketch the frequency spectra of the signals $y_1(t)$ and $y_2(t)$. **Indicate all the center frequencies, bandwidths, and amplitudes of interest for all the components of these signals**. *[2 pts]*

b- What type of modulation is obtained at the output of System 1? Indicate the modulation index of this modulation. *[2 pts]*

c- Sketch the frequency spectra of the signals $y_3(t)$ and $y_4(t)$. **Indicate all the center frequencies, bandwidths, and amplitudes of interest for all the components of these signals**. *[2 pts]*

d- Propose an alternative for System 2 that does not require a coherent carrier to obtain the message signal $m(t)$ at the output $y_4(t)$. *[2 pts]*

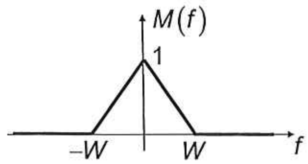


Figure 1

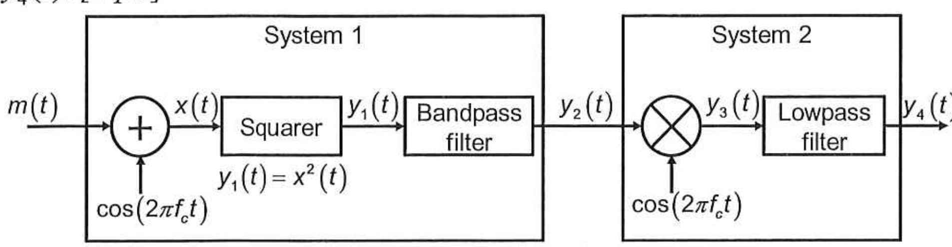


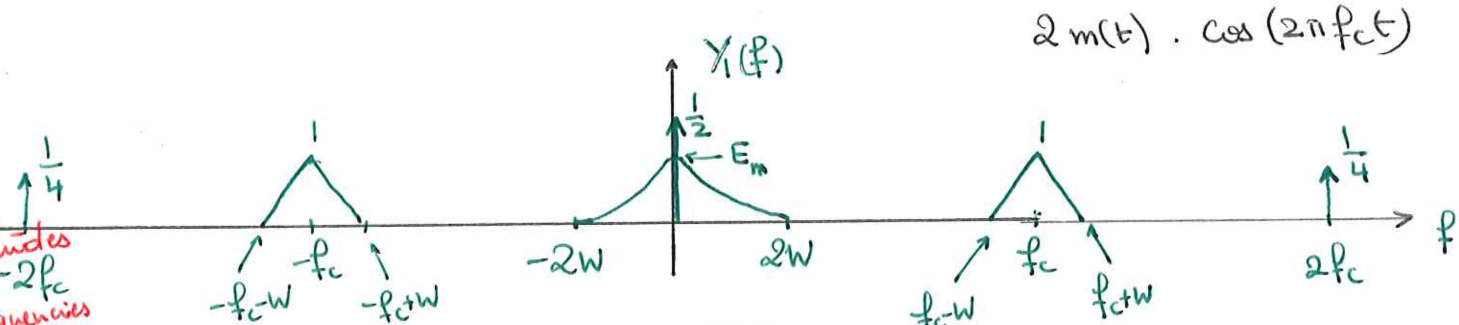
Figure 2

a) $y_1(t) = x^2(t) = \left(m(t) + \cos(2\pi f_c t)\right)^2 = m^2(t) + \frac{1}{2} + \frac{1}{2}\cos(4\pi f_c t) + 2\,m(t)\cdot\cos(2\pi f_c t)$

1 pt

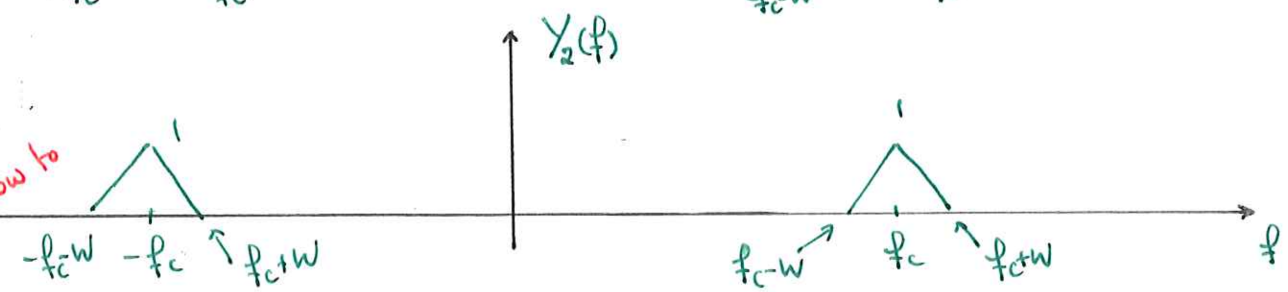
0.5: amplitudes

0.5 frequencies



1 pt

if they show that they understand how to apply a BPF



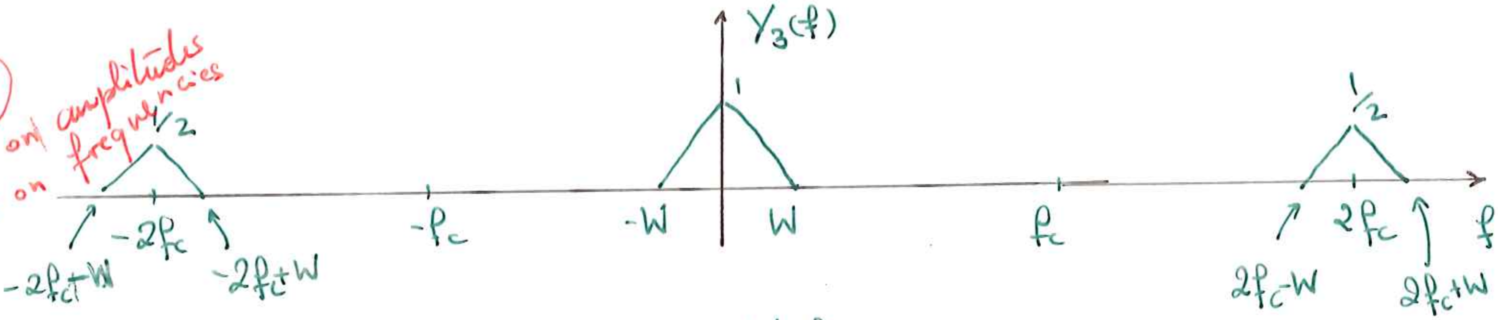
b) Modulation : DSB or DSB-SC ← 2 pts

give 1 pt if they mention only AM modulation

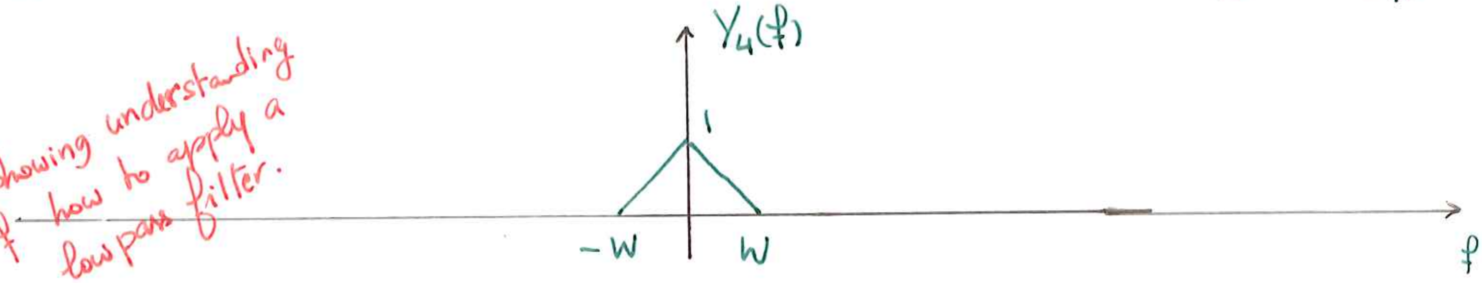
Modulation index : $\mu = 1$

c) $y_3(t) = 2\, m(t) \cos^2(2\pi f_c t) = m(t) + m(t)\cos(4\pi f_c t)$

1pt
0.5 on amplitudes
0.5 on frequencies



1pt
on showing understanding of how to apply a low pass filter.



$y_4(t) = m(t)$

d) System 2 is used to demodulate a DSB signal. A coherent carrier is required for the demodulation.

One can use a Costas receiver to perform the demodulation.

Question 2 [*10 pts*]

A conventional AM signal is given by:

$$s(t) = 5\cos(1800\pi t) + 20\cos(2000\pi t) + 5\cos(2200\pi t)$$

a- Determine the modulating signal $m(t)$ and the carrier $c(t)$. *[2 pts]*

b- Determine the modulation index and the ratio of the power in the sidebands to the power in the carrier. *[2 pts]*

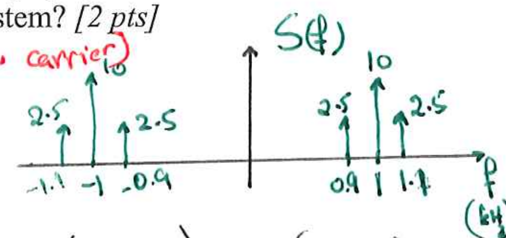
This signal is fed to a conventional AM receiver using an envelope detector. The average noise power per unit bandwidth measured at the input of the receiver front-end is 10^{-3} watt per Hertz.

c- Assuming an input resistor of $1\ \Omega$, calculate the input signal-to-noise ratio of the system. *[2 pts]*

d- Determine the output signal-to-noise ratio at the output of the receiver. *[2 pts]*

e- By how many decibels is this system inferior to a DSB modulation system? *[2 pts]*

a) $\boxed{c(t) = 20\cos(2000\pi t)}$ ← (1 pt) [identifying the carrier]



$s(t) = 20\cos(2000\pi t)\,[1 + k_a m(t)]$

$20\cos(2000\pi t)\,k_a m(t) = 5\cos(1800\pi t) + 5\cos(2200\pi t) = 10\cos(2000\pi t)\cdot\cos(200\pi t)$

$\boxed{k_a m(t) = \frac{1}{2}\cos(200\pi t)}$ ← (1 pt)

0.5 on form cos(.)
0.25 on amplitude
0.25 on freq.

b) $\mu = 0.5$ or 50 % ← (1 pt)

(1 pt)
$P_{sidebands} = 25\ W$
$P_{carrier} = 200\ W$
$\Big\} \longrightarrow \frac{P_{sideband}}{P_{carrier}} = \frac{25}{200} = 0.125$ or 12.5 %

c) $P_{n_{in}} = 2 \cdot 100 \cdot 10^{-3} = 0.2\ W$

$P_{s_{in}} = 25 + 200 = 225\ W$

$SNR_{in} = \frac{P_{s_{in}}}{P_{n_{in}}} = \frac{225}{0.2} = 1125$ or 30.5 dB

(1 pt) 0.5 on {signal, noise} power calculation
0.5 on SNR

d) $\boxed{SNR_{out} = \frac{P_{s_{out}}}{P_{n_{out}}} = \frac{50}{0.4} = 125}$

or (2 pts)

or $\boxed{SNR_{out} = \frac{\mu^2}{2+\mu^2}\,SNR_{in} = \frac{0.25}{2.25}\cdot 1125 = 125}$

1 pt on formula
1 pt on calculation

e) this is inferior to DSB modulation system by $\boxed{10\log\left(\frac{2.25}{0.25}\right) = 9.5\ dB}$ ← (2 pts)

Question 3 [*10 pts*]

An angle-modulated signal around a carrier frequency $f_c = 10\,MHz$, has the form

$$s(t) = 100\cos\left(2\pi f_c t + 4\sin\left(2000\pi t\right)\right)$$

The modulating message has a maximum amplitude $A_m = \max\left|m(t)\right| = 1$.

a- Determine the peak-phase deviation and peak-frequency deviation of $s(t)$. *[2 pts]*

b- Determine $m(t)$ and k_f if $s(t)$ is an FM signal *[2 pts]*

c- Determine $m(t)$ and k_p if $s(t)$ is a PM signal *[2 pts]*

d- Determine the approximate bandwidth of $s(t)$ using Carson's rule. *[2 pts]*

e- Sketch the spectrum of the modulated signal $s(t)$. **Show only the sidebands within the approximate bandwidth calculated in d-. Indicate all the frequencies and amplitudes of interest**. (Use the table below) *[2 pts]*

	$\beta=1$	$\beta=2$	$\beta=3$	$\beta=4$	$\beta=5$
$n=0$	0.7652	0.2239	-0.2601	-0.3971	-0.1776
$n=1$	0.4401	0.5767	0.3391	-0.066	-0.3276
$n=2$	0.1149	0.3528	0.4861	0.3641	0.04657
$n=3$	0.0196	0.1289	0.3091	0.4302	0.3648
$n=4$	0.0025	0.034	0.132	0.2811	0.3912
$n=5$		0.007	0.043	0.1321	0.2611
$n=6$		0.0012	0.0114	0.0491	0.131
$n=7$			0.0025	0.01518	0.05338
$n=8$				0.004	0.01841
$n=9$					0.0055

a) peak phase deviation : $\boxed{\beta = 4 \text{ rad}}$ ← 1pt

peak frequency deviation : $\boxed{\Delta f = \beta \cdot f_m = 4 \text{ kHz}}$ ← 1pt

b) if s(t) is an FM signal :

$\boxed{k_f = 4 \text{ kHz/V}}$ ← 1pt

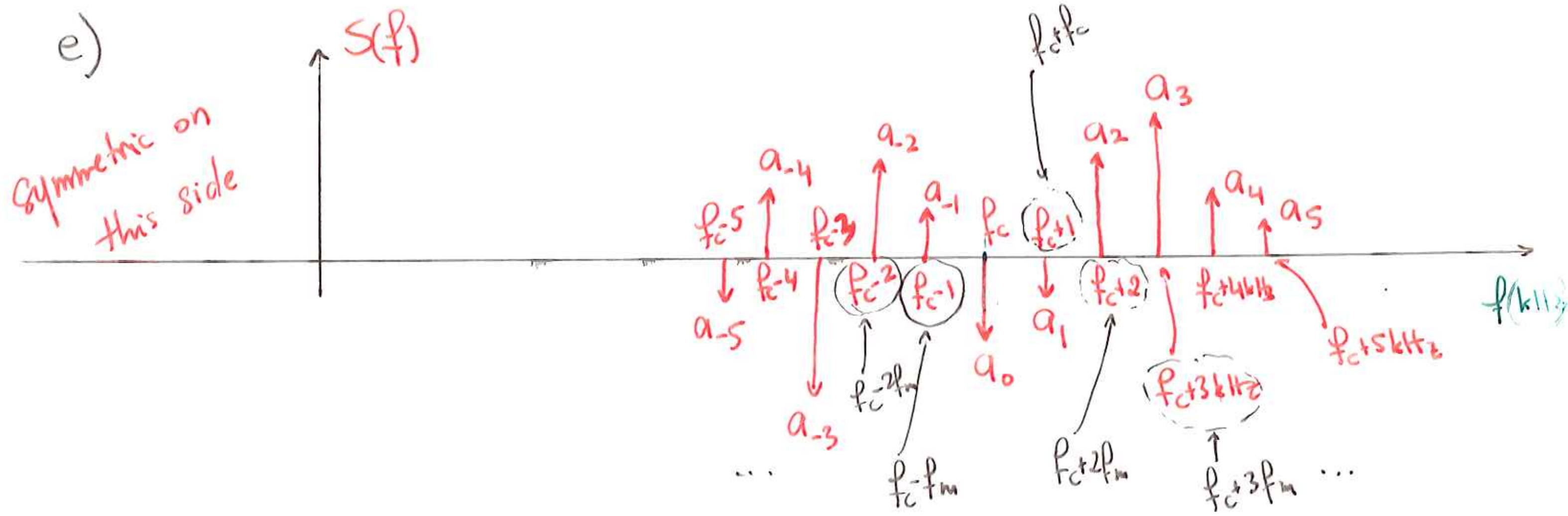
$\boxed{m(t) = \cos(2000\pi t)}$ ← 1pt

c) if s(t) is a PM signal:

$\boxed{k_p = 4 \text{ rad/V}}$ ← 1pt

$\boxed{m(t) = \sin(2000\pi t)}$ ← 1pt

d) $\boxed{B_T = (1+\beta)\cdot 2f_m = 10 \text{ kHz}}$ ← 2pts e)



$$a_0 = \frac{A_c}{2} J_0(4) = 50 \cdot (-0.3971) = \underline{\underline{-19.8}}$$

$$a_1 = \frac{A_c}{2} J_1(4) = 50 \, (-0.066) = \underline{\underline{-3.3}} \quad ; \quad a_{-1} = -a_1 = \underline{\underline{3.3}}$$

$$a_2 = a_{-2} = \frac{A_c}{2} J_2(4) = 50 \cdot (0.3641) = \underline{\underline{18.2}}$$

$$a_3 = \frac{A_c}{2} J_3(4) = 50 \cdot (0.4302) = \underline{\underline{21.5}} \quad ; \quad a_{-3} = -a_3 = \underline{\underline{-21.5}}$$

$$a_4 = a_{-4} = \frac{A_c}{2} J_4(4) = 50 (0.2811) = \underline{\underline{14.06}}$$

$$a_5 = \frac{A_c}{2} J_5(4) = 50 (0.1321) = \underline{\underline{6.6}} \qquad a_{-5} = -a_5 = \underline{\underline{-6.6}}$$

the question is on 2pts in total:

<u>0.5 pt</u> on the shape (impulses on both sides of f_c stopping at $n=5$)

<u>0.5 pt</u> on spacing between impulses (f_m or 1 kHz)

<u>0.5 pt</u> on amplitudes $\frac{A_c}{2} J_n(\beta)$

<u>0.5 pt</u> on showing $\frac{A_c}{2} J_{-n}(\beta) = (-1)^n \frac{A_c}{2} J_n(\beta)$

[antisymmetric for n odd
symmetric for n even]

Question 4 [*8 pts*]

A FM modulation system has a modulation index $\beta = 4\ rad$.

a- The SNR at the input of the FM receiver is equal to 30 dB. What is the SNR at the output of the FM receiver if no pre-emphasis and de-emphasis filters are used. *[2 pts]*

This FM modulation system uses a pair of pre-emphasis and de-emphasis filters defined by:

$$H_{pe}(f) = 1 + j\frac{f}{f_0} \quad \text{and} \quad H_{de}(f) = \frac{1}{1 + j\frac{f}{f_0}}$$

where $f_0 = 2.1\,kHz$, the message bandwidth $W = 15\,kHz$.

b- What is the value of the improvement factor I in the output signal-to-noise ratio of the FM receiver produced by using this pair of pre-emphasis and de-emphasis filters? *[2 pts]*

c- What is the SNR at the output of the FM receiver taking into account the effect of the pre-emphasis and de-emphasis filters. *[2 pts]*

d- By how many decibels this FM system is superior to a DSB system? *[2 pts]*

a) $\boxed{SNR_{out} = \left(\frac{3}{2}\beta^2\right) \cdot SNR_{in}} = \frac{3}{2}(4)^2 \cdot 1000$

1pt

or $\boxed{SNR_{out} = 24000 \text{ or } 43.8\ dB}$

1pt

give full mark if this is correct

b) $I = \dfrac{2W^3}{3\int_{-W}^{W} |H_{de}(f)|^2 f^2 df} = \dfrac{2W^3}{3\int_{-W}^{W} \frac{f^2}{1+\frac{f^2}{f_0^2}} df}$

$$\boxed{I = \frac{(W/f_0)^3}{3\left[\left(\frac{W}{f_0}\right) - \tan^{-1}\left(\frac{W}{f_0}\right)\right]}}$$

or 1pt on expression

$I = \dfrac{(7.14)^3}{3\left[7.14 - \tan^{-1}(7.14)\right]} = \boxed{21.25}$ or $\boxed{13.3\ dB}$

or 1pt on calculation

c) $SNR_{out\text{-}emphasis} = 43.8\ dB + 13.3\ dB = \boxed{57.1\ dB}$

or 2pts

or

$= 24000 \times 21.25 = \boxed{510\ 000}$

2pts

d) this system is superior to a DSB system by $(13.8 + 13.3) = \boxed{27.1\ dB}$

Question 5 [*6 pts*]

a. Plot the spectrum of a PAM wave produced by the modulating signal:

$$m(t) = 2\sin(2\pi f_m t)$$

Assuming a modulation frequency of $f_m = 1\ kHz$, sampling period $T_s = 200\ \mu s$ and pulse duration $T = 100\ \mu s$ *[2 pts]*

b. Using an ideal reconstruction filter, plot the spectrum of the filter output. *[2 pts]*

c. What should be the expression of the amplitude spectrum of the equalizer required to compensate for the aperture effect. *[2 pts]*

For the spectrum plots, indicate all the frequencies and amplitudes of interest.

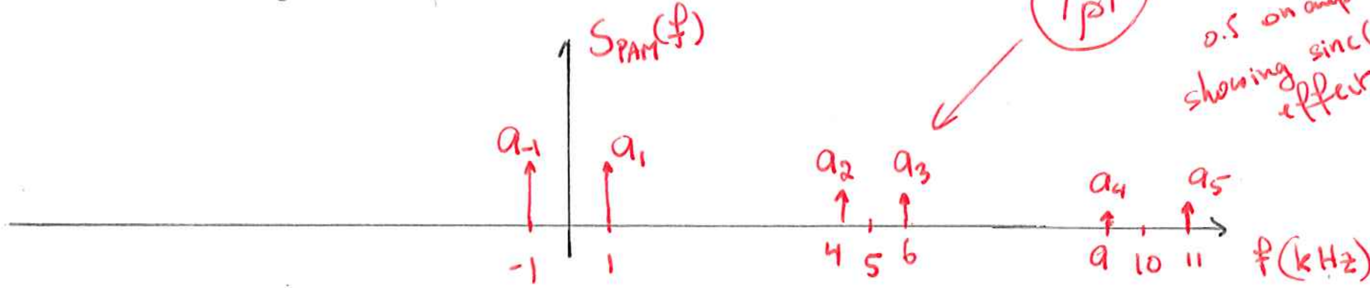
a) $$\boxed{S_{PAM}(f) = \frac{T}{T_s}\sum_{n=-\infty}^{\infty} M(f - nf_s)\ \mathrm{sinc}(\pi f T)\ e^{-j\pi f T}}$$ ← 1pt

$$M(f) = \frac{1}{j}\delta(f - f_m) - \frac{1}{j}\delta(f + f_m).$$

0.5 on shape + freq

1pt

0.5 on amplitudes showing sinc(.) effect



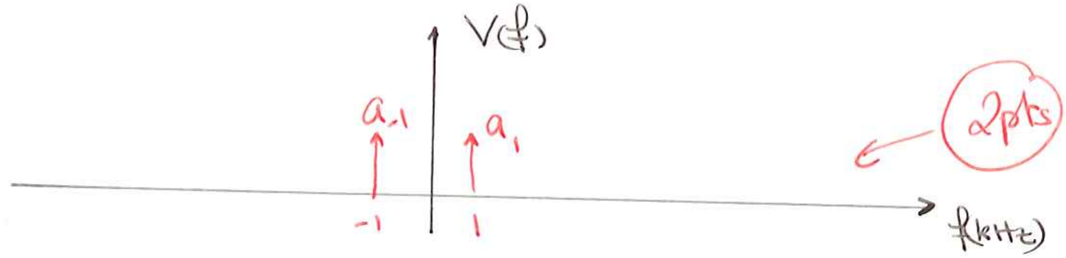
$$a_1 = \frac{1}{j}\cdot\frac{T}{T_s}\ \mathrm{Sinc}(\pi \cdot 1kHz \cdot 0.1ms)\ e^{-j\pi \cdot 1kHz \cdot 0.1ms} = \frac{1}{2j}\ \mathrm{Sinc}(0.1\pi)\ e^{-j0.1\pi}$$

$$a_{-1} = -\frac{1}{j}\cdot\frac{T}{T_s}\ \mathrm{Sinc}(\pi\ {-1}\,kHz\ .\ 0.1ms)\ e^{-j(-1kHz)\,0.1ms} = -\frac{1}{2j}\ \mathrm{Sinc}(0.1\pi)\ e^{j0.1\pi}$$

$$a_2 = -\frac{1}{2j}\ \mathrm{Sinc}(0.4\pi)\ e^{-j0.4\pi}$$

$$a_3 = \frac{1}{2j}\ \mathrm{Sinc}(0.6\pi)\ e^{-j0.6\pi}$$

b) if $v(t)$ is the signal at the output of the reconstruction filter:



$$a_1 = \frac{1}{2j} \operatorname{sinc}(0.1\pi)\, e^{-j0.1\pi}$$

$$a_{-1} = \frac{1}{2j} \operatorname{sinc}(0.1\pi)\, e^{j0.1\pi}$$

c) The frequency response of the equalizer should satisfy:

0.5

$$|E(f)| = \frac{1}{\left| T \operatorname{sinc}(\pi f T)\, e^{-j\pi f T} \right|} = \boxed{\frac{1}{T\,|\operatorname{sinc}(\pi f T)|}}$$

1.5

2pts

Question 6 [*8 pts*]

For practical considerations, a pulse $g(t)$ in a PAM signal is assumed to have the shape shown in Figure 3. The amplitude is $A=1$, and the pulse duration is $T=3\ ms$.

a. Determine the impulse response $h(t)$ of a filter matched to this signal and sketch it. *[2 pts]*

b. What is the peak value of the matched filter output? *[2 pts]*

c. What is the expression of the peak-pulse signal-to-noise ratio if a white noise with double-sided power spectral density of $N_o/2 = 10^{-4}\ W/Hz$ is added to the signal before the matched filter? *[2 pts]*

d. Compare the peak-pulse signal-to-noise ratio in this case to the ideal case where the pulse has zero rising and falling times, the same amplitude ($A=1$), and the same pulse duration ($T=3\ ms$). *[2 pts]*

$\frac{3}{4}T - \frac{T}{4}$

$\frac{2T}{4} = T/2$.

$SNR = \dfrac{k^2 T^2}{\frac{N_0}{2} k^2 T}$

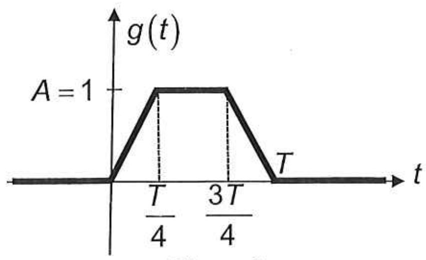
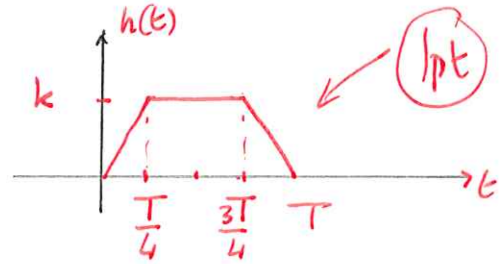


Figure 3



(1pt)

$\dfrac{|g_o(t)|^2}{E(n^2)}$

a) $\boxed{h(t) = k\ g(T-t)}$

(1pt)

$\eta = \dfrac{E_s}{E_N}$

b) $y(t) = h(t) * g(t) = k\int_0^T g(\tau)\, g(T-t+\tau)\, d\tau$

$\dfrac{E_g}{E(n^2)}$

the peak value of $y(t)$ is at $t = T$

(1pt) $\boxed{y(T) = k\int_0^T g^2(t)\, dt \quad \text{or} \quad k.E_g} = k\left[2.\int_0^{\frac{T}{4}}\left(\frac{4}{T}t\right)^2 dt + \frac{T}{2}\right]$

$y(T) = k\left[\frac{32}{T^2}\cdot\frac{1}{3}\frac{T^3}{64} + \frac{T}{2}\right] = k\left[\frac{T}{6}+\frac{T}{2}\right] = \boxed{k.\underbrace{\frac{2T}{3}}_{E_g}} = k.2 10^{-3}$

(1pt)

c) $\boxed{\eta = \dfrac{2E_g}{N_o}} = \dfrac{4T}{3N_o} = \dfrac{2.10^{-3}}{10^{-4}} = \boxed{20}$ (2pts)

d) the peak-pulse SNR in this case is $\boxed{\frac{2}{3}}$ the peak-pulse SNR in the ideal case

(2pts)

$\frac{3}{2} = 1.76$