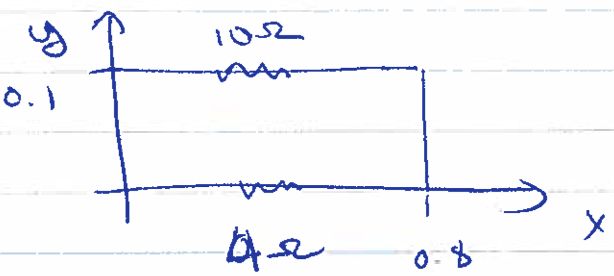
ENEL 476 Assignment #1 2015

9.2



$\vec{B} = 40\cos(30\pi t - 3y)\vec{a}_z$ mWb/m²

$$I = \frac{V_{emf}}{R} \qquad R = 10+4$$

$$V_{emf} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$$

$$= -\frac{d}{dt}\int_0^{0.1}\int_0^{0.8} 40\cos(30\pi t - 3y)\,dx\,dy \quad \times 10^{-3}$$

$$= -\frac{d}{dt}(40)(0.8)\int_0^{0.1}\cos(30\pi t - 3y)\,dy \quad \times 10^{-3}$$

$$= -\frac{d}{dt}(32)\left[-\frac{1}{3}\sin(30\pi t - 3y)\Big|_0^{0.1}\right] \quad \times 10^{-3}$$

$$= +\frac{d}{dt}\left(\frac{32}{3}\right)\left[\sin(30\pi t - 0.3) - \sin(30\pi t)\right] \quad \times 10^{-3}$$

$$= +320\pi\left[\cos(30\pi t - 2.4) - \cos(30\pi t)\right] \times 10^{-3}$$

$$= 320\pi\left[\cos(30\pi t - 2.4) - \cos(30\pi t)\right] \quad \times 10^{-3}$$

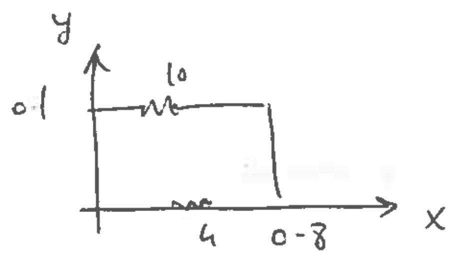
$$I = \frac{320}{14}\pi\left[\cos(30\pi t - 2.4) - \cos(30\pi t)\right] \quad \text{mA}$$

Feb 2013

Final mark for assignment 1 -

80 % Q1 Mark + 20 % Complete Assignment

Q1 (9.2) marking scheme:



$$\vec{B} = 40\cos(30\pi t - 3y)\,\hat{a}_z \ \ mWb/m^2$$

$$V_{emf} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s} \qquad 1$$

$$= -\frac{d}{dt}\int_0^{0.1}\int_0^{0.8} 40\cos(30\pi t - 3y)\,dx\,dy \times 10^{-3} \qquad 1$$

$$= -\frac{d}{dt}\ 40 \times 0.8 \times \frac{-1}{3}\sin(30\pi t - 3y)\Big|_0^{0.1} \times 10^{-3} \qquad 1$$

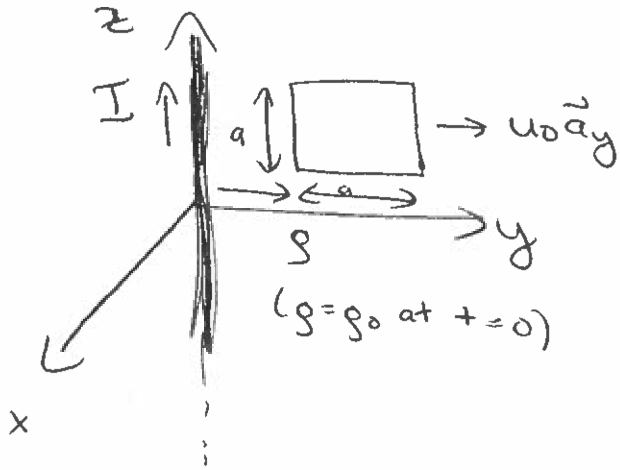
$$= \frac{32}{3}\times 10^{-3}\ \frac{d}{dt}\left[\sin(30\pi t - 0.3) - \sin(30\pi t)\right]$$

$$= 320\pi \times 10^{-3}\left[\cos(30\pi t - 0.3) - \cos(30\pi t)\right] \qquad 1$$

$$\rightarrow I = \frac{V_{emf}}{R} = \frac{320\pi}{14}\left[\cos(30\pi t - 0.3) - \cos(30\pi t)\right]\ mA \qquad 1$$

$$R = 14\ \Omega \qquad 1$$

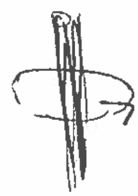
9.b



$$V_{emf} = \frac{\mu_0 I a^2 u_0}{2\pi \rho (\rho + a)}$$

$$V_{emf} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{s}$$

$\vec{B} \Rightarrow$ $\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{s}$



$$\int_0^{2\pi} H_\phi \rho \, d\phi = I$$

$$H_\phi (2\pi\rho) = I$$

$$H_\phi = \frac{I}{2\pi\rho}$$

$$\iint \vec{B} \cdot d\vec{s} = \int_0^a \int_\rho^{\rho+a} \frac{\mu_0 I}{2\pi\rho} \, d\rho \, dz$$

$$= \frac{\mu_0 I}{2\pi}(a)\left[\ln(\rho + a) - \ln(\rho)\right]$$

$$-\frac{d}{dt}\overbrace{\int \vec{B} \cdot d\vec{s}}^{\Phi} = -\left(\frac{\partial}{\partial \rho}\Phi\right)\left(\overbrace{\frac{\partial \rho}{\partial t}}^{u_0}\right)$$

$$= -\frac{\mu_0 I a}{2\pi}\left[\frac{1}{\rho + a} - \frac{1}{\rho}\right] u_0$$

$$\frac{\rho - \rho - a}{\rho(\rho + a)} = \frac{-a}{\rho(\rho + a)}$$

$$V_{emf} = \frac{+ a^2 \mu_0 I u_0}{2\pi[\rho(\rho + a)]}$$

9.16 $\mu = \mu_0$
$\epsilon = 9\epsilon_0$
$\sigma = 4$ S/m

$$\frac{\sigma}{\omega\epsilon} = 1 \Rightarrow \frac{4}{(2\pi f)(9)(\frac{1}{36\pi}\times 10^{-9})} = 1$$

$$\frac{8}{f} = 10^{-9}$$

$$f = 8 \text{ GHz}$$

9.26 $\sigma = 0, \ \mu = \mu_0, \ \epsilon = 81\epsilon_0$

$$\vec{H} = 10\cos(2\pi\times 10^9 t + \beta x)\vec{a}_z$$

$$\nabla\times\vec{H}_s = j\omega\epsilon_R\epsilon_0\vec{E}_s$$

$$\vec{H}_s = 10e^{j\beta x}\vec{a}_z$$

$$\nabla\times\vec{H}_s = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_{sz} \end{vmatrix}$$

$$= -\vec{a}_y\left(\frac{d}{dx}H_{sz}\right)$$

$$= -(10)(j\beta)e^{j\beta x}\vec{a}_y$$

$$\vec{E}_s = \frac{-10j\beta e^{j\beta x}}{j\omega\epsilon_0\epsilon_R}\vec{a}_y$$

$$\nabla\times\vec{E}_s = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_{sy} & 0 \end{vmatrix}$$

$$= \vec{a}_x\left(-\frac{d}{dz}E_{sy}\right) + \vec{a}_z\left(\frac{d}{dx}E_{sy}\right)$$

$$= \frac{-10\beta}{\omega\epsilon_R\epsilon_0}(j\beta)e^{j\beta x}\vec{a}_z$$

$$\nabla\times\vec{E}_s = -j\omega\mu_0\vec{H}_s$$

$$\frac{-j10\beta^2}{\omega\epsilon_R\epsilon_0}e^{j\beta x} = -j\omega\mu_0 10e^{j\beta x}$$

$$\beta^2 = \omega^2\mu_0\epsilon_R\epsilon_0$$

9.26 <u>cont'd</u>

$\beta = \omega\sqrt{\mu_0 \epsilon_R \epsilon_0}$

$= \frac{2\pi\times10^{9}(9)}{3\times10^{8}}$

$= 60\pi$

$\therefore\ \vec{E}_s = \frac{(-10)(60\pi)}{(2\pi\times10^{9})(\frac{1}{36\pi}\times10^{-9})(81)} e^{j\beta x}\,\vec{a}_y$

$= \frac{(-300)(4\pi)}{3\cdot 9} e^{j\beta x}\,\vec{a}_y$

$= -\frac{400\pi}{3} e^{j60\pi x}\,\vec{a}_y$

$\vec{E} = -133\pi\cos(2\pi\times10^{9}t + 60\pi x)\vec{a}_y$

Check: $\eta = \frac{120\pi}{9} = \frac{40\pi}{3}$

$|\vec{E}| = \frac{(10)(40)\pi}{3}$

$= \frac{400\pi}{3}$

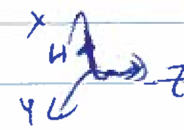
9.42 $\vec{H} = 40\cos(10^{9}t - \beta z)\vec{a}_x$ $\quad \sigma = 0,\ \mu = \mu_0,\ \epsilon = 4\epsilon_0$

a) $\vec{H}_s = 40e^{-j\beta z}\,\vec{a}_x$

b) $\beta = \frac{10^{9}}{3\times10^{8}}(2)$

$= 20/3$

$\vec{E}_s = \cancel{\frac{40}{60\pi}}\,(40)(60\pi)\cos(10^{9}t - \frac{20}{3}z)\,\vec{a}_y$



9.42

$$\vec{J}_D = \left(\frac{d}{dt}\right)\left[(4\epsilon_0)\left[-2400\pi\cos(10^9t - \frac{20}{3}z)\vec{a}_y\right]\right]$$

$$= (+2400\pi)(4\epsilon_0)(10^9)\sin(10^9t - \frac{20}{3}z)\vec{a}_y$$

$$= (\cancel{2400}^{800}\pi)\left(4 \times \frac{1}{\cancel{36\pi}_{3}} \times \cancel{10^{-9}}\right)(\cancel{10^9})\sin(10^9t - 20/3z)\vec{a}_y$$

$$= \frac{800}{3}\sin(10^9t - \frac{20}{3}z)\vec{a}_y$$

alt

$$\nabla \times \vec{H}_S = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_{Sx} & 0 & 0 \end{vmatrix}$$

$$= \vec{a}_x(0) - \vec{a}_y\left(-\frac{d}{\partial z}H_{Sx}\right) + \vec{a}_z(0)$$

$$= \frac{d}{\partial z}H_{Sx}\,\vec{a}_y$$

$$= (-40)(-j\beta)e^{-j\beta z}\vec{a}_y$$

$$= 40j\beta e^{-j\beta z}\vec{a}_y$$

$$= 40\beta e^{-j\beta z}e^{j\pi/2}\vec{a}_y$$

$$\vec{J}_{DS} = 40\beta e^{-j\beta z}e^{j\pi/2}\vec{a}_y$$

$$\vec{J}_D = (40)(\beta)\cos(10^9t - \beta z + \pi/2)\vec{a}_y$$

$$= (40)\left(\frac{20}{3}\right)\sin(10^9t - \beta z)\vec{a}_y$$