#1) $\vec{H}(x,t) = 10\cos(1.02\times10^8 t - \beta x)\,\hat{a}_y$ A/m

(a) $\vec{H}_s(x) = 10\,e^{-j\beta x}\,\hat{a}_y$ A/m

(b) Ampère's Law: $\nabla\times\vec{H} = \cancel{\vec{J}}^{\,0} + \dfrac{\partial\vec{D}}{\partial t}$

LHS: $\nabla\times\vec{H}_s = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & H_{sy} & 0 \end{vmatrix}$

$$= \left(-\cancel{\frac{\partial}{\partial z} H_{ys}}^{\,0}\right)\hat{a}_x + (0)\,\hat{a}_y + \left(\frac{\partial}{\partial x} H_{ys}\right)\hat{a}_z$$

$$= \frac{\partial}{\partial x} H_{ys}\,\hat{a}_z = -j\beta\,10\,e^{-j\beta x}\,\hat{a}_z$$

RHS: $\dfrac{\partial\vec{D}_s}{\partial t} = \varepsilon_o \dfrac{\partial\vec{E}_s}{\partial t} = j\omega\varepsilon_o\vec{E}_s$

$$\therefore\quad \vec{E}_s = \frac{-\cancel{j}\beta\,10\,e^{-j\beta x}}{\cancel{j}\omega\varepsilon_o}\hat{a}_z = \frac{-\beta\,10\,e^{-j\beta x}}{\omega\varepsilon_o}\,\hat{a}_z \quad \frac{V}{m}$$

(c) Faraday's Law $\nabla\times\vec{E}_s = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & E_{zs} \end{vmatrix}$

$$= \left(\cancel{\frac{\partial}{\partial y} E_{zs}}^{\,0}\right)\hat{a}_x + \left(-\frac{\partial}{\partial x} E_{zs}\right)\hat{a}_y + (0)\,\hat{a}_z$$

$$= -\frac{\partial}{\partial x} E_{zs}\,\hat{a}_y = -j\,\frac{\beta^2\,10\,e^{-j\beta x}}{\omega\varepsilon_o}\,\hat{a}_y$$

RHS: $-\dfrac{\partial\vec{B}_s}{\partial t} = -\mu_o\dfrac{\partial\vec{H}_s}{\partial t} = -j\omega\mu_o\vec{H}_s$

↳ where $\vec{H}_s = 10e^{-j\beta x}\hat{a}_y$ (from part a)

$$\therefore\quad -j\frac{\beta^2\,10e^{-j\beta x}}{\omega\varepsilon_o}\hat{a}_y = -j\omega\mu_o\,10\,e^{-j\beta x}\,\hat{a}_y$$

$$\beta^2 = \omega^2\mu_o\varepsilon_o \longrightarrow \beta = \omega\sqrt{\mu_o\varepsilon_o} = 1.02\cdot10^8\sqrt{4\pi\times10^{-7}\cdot 8.85\cdot10^{-12}}$$

$$\beta = 0.34$$

Note that $c = \dfrac{1}{\sqrt{\mu_o\varepsilon_o}}$ $\therefore$ $\beta = \dfrac{\omega}{c} = \dfrac{1.02\cdot10^8}{3\cdot10^8} = 0.34$

(d) $\vec{E}(x,t) = \mathrm{Re}\left\{ -\frac{10\beta}{\omega\varepsilon_0} e^{-j\beta x} e^{j\omega t} \right\} \hat{a}_z \ \frac{V}{m}$ where $\omega = 1.02 \times 10^8 \ \frac{rad}{s}$

$$= \frac{-10\beta}{\omega\varepsilon_0} \cos\left(1.02 \cdot 10^8 t - \beta x\right) \hat{a}_z \ \frac{V}{m}$$

~~(e) $\vec{J} =$~~ $= -3766 \cos\left(1.02 \cdot 10^8 t - 0.34x\right) \hat{a}_z \ \frac{V}{m}$

(e) $\vec{J}_D = \frac{\partial \vec{D}_s}{\partial t} = \varepsilon_0 \frac{\partial \vec{E}_s}{\partial t} = j\omega\varepsilon_0 \vec{E}_s = -j10\beta e^{-j\beta x} \hat{a}_z = -j3.4 e^{-j0.34x} \hat{a}_z \quad \frac{A}{m^2}$

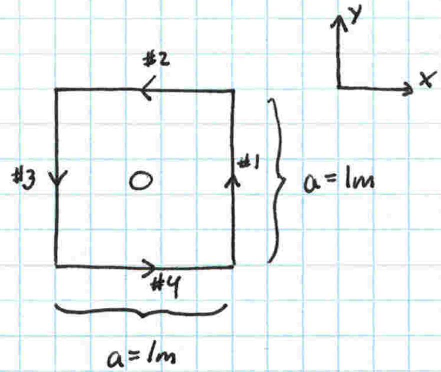
or $\vec{J}_D(x,t) = \mathrm{Re}\left\{ 3.4 \, e^{-j0.34x} e^{j\omega t} e^{-j\pi/2} \right\} \hat{a}_z \ \frac{A}{m^2}$

$$= 3.4 \cos(\omega t - 0.34x - \pi/2) \ \hat{a}_z \ A/m^2$$

$$= 3.4 \sin(\omega t - 0.34x) \ \hat{a}_z \ A/m^2$$

$$= 3.4 \sin(1.02 \cdot 10^8 t - 0.34x) \ \hat{a}_z \ A/m^2$$

#2)



Outer loop: 1m × 1m square
$I = 10\sin\omega t$ A ccw

Inner loop: circle w/ 1cm radius

What is the induced current in the inner loop?

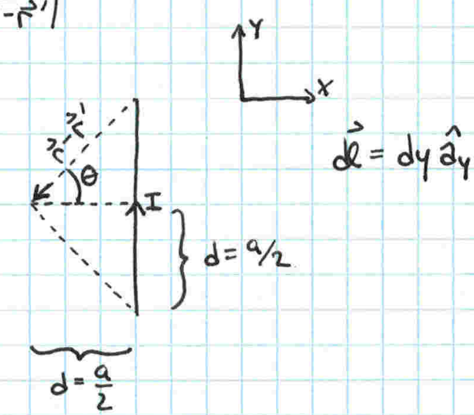
→ Need to know $\vec{B}$. $\vec{B}$ is the sum of magnetic fields from wires #1–4

$$\vec{B} = \sum_{i=1}^{4} \vec{B}_i$$

Consider wire #1:

$$\vec{a}_r = \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|}$$

$$\vec{B}_1 = \mu_0 \vec{H}_1 = \frac{\mu_0}{4\pi} I \int \frac{\vec{dl} \times \hat{a}_r}{|\vec{r}-\vec{r}\,'|^2}$$



$$\vec{dl} = dy\,\hat{a}_y$$

where: $\vec{dl} \times \vec{a}_r = \cos\theta \, dy \, \hat{a}_z$

$$|\vec{r}-\vec{r}\,'| = \frac{d}{\cos\theta}$$

$$l = d\tan\theta$$

$$dl = \frac{d}{\cos^2\theta}\, d\theta$$

$$\vec{B}_1 = \frac{\mu_0}{4\pi} I \int \frac{\cos\theta\, d\, d\theta}{\cos^2\theta \left(\frac{d}{\cos\theta}\right)^2} \hat{a}_z = \frac{\mu_0 I}{4\pi d} \int_{-\pi/4}^{\pi/4} \cos\theta \, d\theta \; \hat{a}_z$$

$$= \frac{\mu_0 I}{4\pi d}\left[\sin{\pi/4} - \sin{-\pi/4}\right] = \frac{\mu_0 I}{4\pi d}\sqrt{2}$$

→ Since all wires will cause magnetic flux densities to be in the +z directions, we can add them together

$$\vec{B} = 4\cdot\vec{B}_1 = \frac{4\mu_0 I\sqrt{2}}{4\pi d}\hat{a}_z = \frac{2\sqrt{2}\,\mu_0 I}{\cancel{4}\pi a}\hat{a}_z$$

→ $I = 10\sin\omega t$
→ $a = 1m$

$$\vec{B} = \frac{2\sqrt{2}\,\mu_0\, 10\sin\omega t}{\pi} \ \frac{Wb}{m^2}$$

$$\Phi = \int \vec{B}\cdot d\vec{S} = \frac{20\sqrt{2}\,\mu_0}{\pi} \sin\omega t \cdot \pi (0.01m)^2 \, Wb$$

$$EMF = \frac{-\partial}{\partial t}\Phi = \frac{-20\sqrt{2}\,\mu_0\omega}{\pi} \cos\omega t \, \pi(0.01m)^2$$

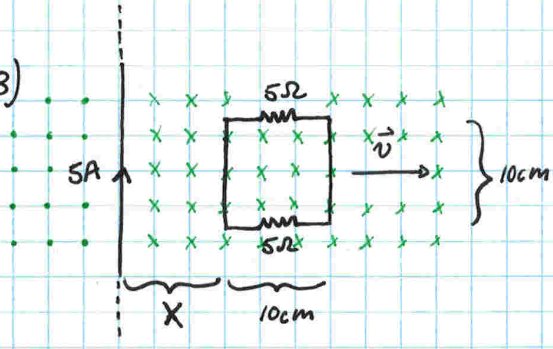
→ $I_{ind} = \frac{V_{emf}}{R} = \frac{-2\cdot 10^{-3}\sqrt{2}\,\mu_0\omega\cos\omega t}{10\,\Omega}$ A

$= -2\cdot 10^{-4}\sqrt{2}\,\mu_0\omega\cos\omega t$ A

OR

$-3.55\cdot 10^{-10}\,\omega\cos\omega t$ A

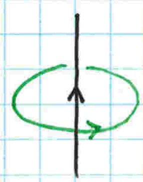
#3)



$\vec{v} = 2.5\,\hat{a}_x$ m/s

$\vec{H}$ = Green

→ Need $\vec{B}$:



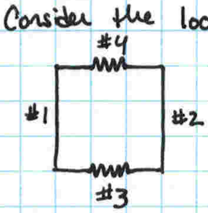
$$\oint \vec{H}\cdot d\vec{l} = I$$

$$H_\varphi\, 2\pi\rho = I$$

→ On xy-plane: (in cartesian coordinates)

$$\vec{H} = \begin{cases} \dfrac{-I}{2\pi x}\,\hat{a}_z, & x>0 \quad \text{A/m} \\[2ex] \dfrac{I}{2\pi x}\,\hat{a}_z, & x<0 \quad \text{A/m} \end{cases}$$

→ Consider the loop:



$$V_{emf} = \oint (\vec{v}\times\vec{B})\cdot d\vec{l}$$

V_{emf} will be 0 for wires #3/#4 !

$$\vec{B}_1 = \mu_0 \vec{H}_1 = \frac{-\mu_0 I}{2\pi x}\,\hat{a}_z \ \frac{\text{Wb}}{\text{m}^2} \quad \text{and} \quad \vec{B}_2 = \mu_0 \vec{H}_2 = \frac{-\mu_0 I}{2\pi (x+0.1\text{m})}\,\hat{a}_z \ \frac{\text{Wb}}{\text{m}^2}$$

→ V_{emf}:

$$V_{emf1} = \int (\vec{v}\times\vec{B})\cdot d\vec{l} = \int \left(2.5\,\hat{a}_x \times \frac{-\mu_0 I}{2\pi x}\,\hat{a}_z\right)\cdot dy\,\hat{a}_y$$

$$= \frac{2.5\mu_0 I}{2\pi x}\int dy = \frac{2.5\mu_0 I}{2\pi x}(0.1\text{m})\ \text{V} \longrightarrow \text{CW}$$

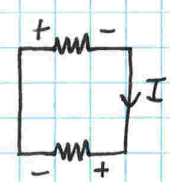
$$V_{emf2} = \frac{2.5\mu_0 I}{2\pi(x+0.1\text{m})}(0.1\text{m})\ \text{V} \longrightarrow \text{CCW}$$

$$V_{emf,total} = V_{emf1} - V_{emf2} \longrightarrow \text{CW}$$

$$= \frac{2.5\mu_0 I}{2\pi}(0.1\text{m})\left[\frac{1}{x} - \frac{1}{x+0.1\text{m}}\right]\ \text{V}$$

$$= 250\left[\frac{1}{x} - \frac{1}{x+0.1}\right]\ \text{nV} \quad \text{CW}$$

$$V_{resitor} = V_{emf,total} \div 2 = 125\left[\frac{1}{x} - \frac{1}{x+0.1}\right]\ \text{nV}$$



$$I = \frac{V_{emf}}{2R} = 25\left[\frac{1}{x} - \frac{1}{x+0.1}\right]\ \text{nA} \quad \text{CW}$$

#4) $\vec{H}(x,t) = 8\cos(2\cdot 10^6 t - 0.02x)\,\hat{a}_y$ A/m $\longrightarrow \vec{H}_s = 8\,e^{j0.02x}\,\hat{a}_y$ A/m

$$\nabla \times \vec{H}_s = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & H_{ys} & 0 \end{vmatrix} = \cancelto{0}{\left(\frac{-\partial}{\partial z} H_{ys}\right)}\hat{a}_x + (0)\,\hat{a}_y + \left(\frac{\partial}{\partial x} H_{ys}\right)\hat{a}_z$$

$$= -j0.16\,e^{-j0.02x}\,\hat{a}_z \quad \text{A/m}^2$$

$$\nabla \times \vec{H}_s = \cancelto{0}{\vec{J}} + \frac{\partial}{\partial t}\vec{D}$$

$$\frac{\partial}{\partial t}\vec{D}_s = j\omega\vec{D}_s \longrightarrow \vec{D}_s = \varepsilon_0 \vec{E}_s \longrightarrow \vec{E}_s = \frac{-\cancel{j}0.16\,e^{-j0.02x}}{\cancel{j}\omega\varepsilon_0}\,\hat{a}_z = \frac{-0.16\,e^{-j0.02x}}{\omega\varepsilon_0}\,\hat{a}_z$$

$$\vec{E}_s = -9.04\cdot 10^3\,e^{-j0.02x}$$

$$\nabla \times \vec{E}_s = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & E_{zs} \end{vmatrix} = \cancelto{0}{\left(\frac{\partial}{\partial y} E_{zs}\right)}\hat{a}_x + \left(\frac{-\partial}{\partial x} E_{zs}\right)\hat{a}_y + 0\,\hat{a}_z$$

$$= -j9.04\cdot 10^3 \cdot 0.02\,e^{-j0.02x}\,\hat{a}_y$$

$$= -j\,180.8\,e^{-j0.02x}\,\hat{a}_y$$

$$\nabla \times \vec{E}_s = \frac{-\partial}{\partial t}\vec{B}_s$$

$$\frac{-\partial}{\partial t}\vec{B}_s = -\mu_0 \frac{\partial}{\partial t}\vec{H}_s = -j\omega\mu_0\vec{H}_s = -j\omega\mu_0\mu_r\,8\,e^{-j0.02x}\,\hat{a}_y \quad \text{V/m}^2$$

$$\therefore\ \cancel{-j}\,180.8\,\cancel{e^{-j0.02x}}\,\cancel{\hat{a}_y} = \cancel{-j}\omega\mu_0\mu_r\,8\,\cancel{e^{-j0.02x}}\,\cancel{\hat{a}_y}$$

$$\mu_r = \frac{180.8}{\omega\mu_0 8} = 9$$

#5) $\vec{E} = \left(25\hat{a}_x - 15\hat{a}_y + 30\hat{a}_z\right)\cos(10^6 t)\ V/m$

(a) $\hat{a}_E = \dfrac{25\hat{a}_x - 15\hat{a}_y + 30\hat{a}_z}{\sqrt{25^2+15^2+30^2}} = 0.60\hat{a}_x - 0.36\hat{a}_y + 0.72\hat{a}_z$

→ Electric field must be perpendicular to a perfect conductor at the interface. *

$\therefore\ \hat{n} = -\hat{a}_E = -0.60\hat{a}_x + 0.36\hat{a}_y - 0.72\hat{a}_z \longrightarrow -0.60\hat{a}_x + 0.36\hat{a}_y - 0.72\hat{a}_z$

(b) For a sheet of charge

$\vec{E} = \dfrac{\rho_s}{\varepsilon}\hat{a}_n$

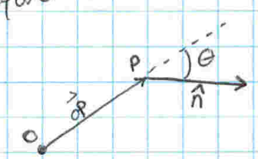
$\rightarrow \rho_s = \dfrac{\varepsilon\vec{E}}{\hat{a}_n} = -10 \cdot 8.85\times10^{-12} \cdot \sqrt{25^2+15^2+30^2}\ \cos(10^6 t)\ \dfrac{C}{m^2}$

$= -3.7\cos(10^6 t)\ \dfrac{nC}{m^2}$ or $3.7\cos(10^6 t + \pi)\ \dfrac{nC}{m^2}$

* Note: The point (0,0,0) is inside the conductor.

Therefore

$\theta < 90°$



If we choose, $\hat{n} = \hat{a}_E$

$\theta = \cos^{-1}\left(\dfrac{\vec{OP}\cdot\hat{n}}{|\vec{OP}|}\right) = 128°$

therefore, $\hat{n} = -\hat{a}_E$