The University of Calgary

Department of Electrical & Computer Engineering

ENEL 471 Introduction to Communication Systems and Networks

(Winter 2020)

Lab 1 : AM Generation

Section date	B04: Tue. Feb 4, 2020 B03: Tue. Feb. 11, 2020	B02: Wed. Feb. 5, 2020 B01: Wed. Feb. 12, 2020
Location	ENA 301	ENA 301

This document consists of 3 parts:

Part 1: Lab 1 Manual

- You must read Lab #1 Manual prior to the Lab #1 session.
- You are required to complete as much as you can on the Simulink (simulation) portion prior to the lab session.
- The hardware portion must be completed during the Lab session.

Part 2: Lab 1 Questions

- Answers to these questions must be submitted to the TAs at the end of the Lab period.
- Each group submits one set of Lab questions. Ensure names and ID numbers of the group members are written on the Lab answer sheets.

Part 3: Useful Trigonometric Identities

- Familiarize with the listed trigonometric identities.
- The listed identities are required in Labs 1, 2, 3 & 4.

Acknowledgements

The ENEL 471- Introduction to Communications and Networks Lab 1 document was originally prepared by Jennifer A. Hartwell and Dr. Mike Potter, revised (January 2013) by Warren Flaman and Dr. Abu Sesay, revised (January 2015) by Mohamed Al Masri and Dr. Mohamed Helaoui, and revised (January 2018) by Leanne Dawson and Dr. Mohamed Helaoui.

Part 1

Lab 1 Manual

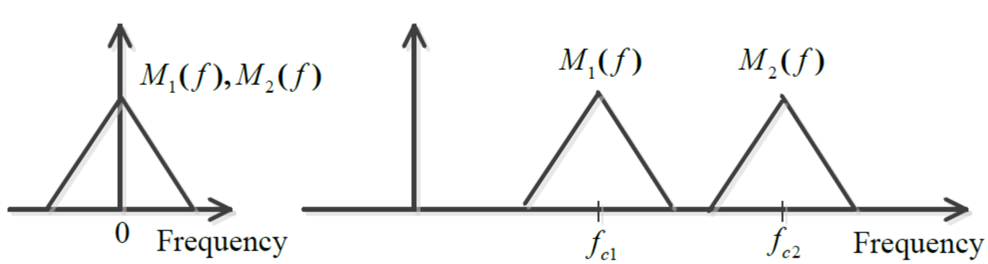
Introduction

This lab has two parts. The first part is simulation of AM signals using MATLAB's Simulink tool. The second part of the lab is generation of AM signals using an AM modulation hardware board. In both cases, the AM signal will be analyzed in time and frequency domain.

Background

Applications such as radio, television, and cell phones require that many different signals use the same physical channel (whether it be wired or wireless). To prevent the different messages from interfering with each other there are ways of dividing the channel into different sections. The two most obvious ways are frequency division and time division. Time division refers to separating the sent messages over time (i.e. only send one message during a certain slot in time, send another during a different slot).

For frequency division, each message gets translated up to a specific frequency which has been allocated for that signal only. Consider looking at the spectra of a signal; most message signals are centered around DC (0 Hz, known as baseband). If two different messages around 0 Hz are sent at the same time, they will overlap in the frequency domain and interfere with each other. If their central frequencies are changed to be far enough apart that the signals will not overlap, they can be sent over the same medium at the same time and not interfere.



The central frequencies are referred to as carrier frequencies. AM modulation is one way of translating a message to a specific carrier frequency. Basically, the carrier is a sinusoid with the desired central frequency and the amplitude of the sinusoid is altered to contain the information carried by the message. For most of this lab, the message signal will be a sinusoid (known as 'tone modulation') because the math is simple, allowing for results that are easy to verify.

1 Simulation of AM using SIMULINK

Simulink is a MATLAB tool that allows for graphical representation and simulation. There are many interesting ways to write complex functions/blocks and integrate them into the Simulink environment (even C and C++ can be compiled into Simulink blocks), and simulations developed in Simulink can now be directly converted into description files for some commercial FPGAs and DSPs. These labs will only make use of simple pre-compiled blocks (such as 'multiply' and 'add') as well as some of the filters and scopes.

1.1 AM and DSB Modulation

An AM signal can be expressed by the following formula:

$$\begin{aligned} x(t) &= [A_c + m(t)]cos(2\pi f_c t + \phi_c) \\ &= A_c[1 + \mu m_n(t)]cos(2\pi f_c t + \phi_c) \end{aligned} \tag{1}$$

where

- $m(t)$ is the message ($m_n(t)$ is normalized)
- A_c is the carrier amplitude
- μ is the modulation index
- f_c is the carrier frequency
- ϕ_c is the carrier phase

A Double Side Band (DSB) signal can be expressed by the following formula:

$$x(t) = A_c m(t) cos(2\pi f_c t + \phi_c) \tag{2}$$

DSB differs from AM by the absence of the DC term that is added to the message in AM.

1.1.1 Single Message Signals

- Open the Simulink model called AM1.slx.
- Look over the setup and verify that it will create an AM signal. Note that the frequencies are always listed as radians/sec, which explains why they always have 2π attached to their values. Double clicking on the block within the Simulink will open a dialogue box that displays some of the settings within the model. DO NOT CHANGE THESE SETTINGS unless this handout explicitly suggests it.

- Taking the values that are currently defined in the model to create the AM signal, draw the AM and DSB spectra and label the actual values for magnitude and frequency. **(Q1)**
- Run the simulation. A plot will pop up that has the magnitude frequency spectrum of the simulated signal. Double clicking on the pink scope labelled 'Signal vs. Time' will open another plot that is of the simulated signal in the time domain (which can be zoomed in using the buttons at the top of the plot window - it is best to zoom in to a time range that covers about two periods of the message signal).
- Compare the spectrum to your drawing. Check that your values are correct, noting that all Simulink spectra are plotted vs. the frequency in Hz (i.e. not in units of radians/s).
- Make a sketch of the time domain plot, be sure to mark the $A_{\max}$ and $A_{\min}$ and label the important frequencies (f_c and f_m). **(Q2/AM)**
- Double click on the block labelled 'constant'. Change the value of the constant block so that the signal generated on the screen will be a DSB signal.
- Repeat the above steps, but for the DSB signal. **(Q2/DSB)**
- Point out the main differences between the time plots for AM and DSB. **(Q3)**
- Close the AM1.slx file.

1.1.2 Multiple Message Signals

The frequency settings currently set in the AMl simulation were arbitrarily chosen to simulate and plot well in Simulink. In 'real life', the values would be much higher. For example, the commercial band for AM radio is from 540 kHz to 1605 kHz . Each station is allowed a total bandwidth of 10 kHz, meaning that the allowed carriers range from 545 kHz to 1600 kHz.

- Open the file called 'AM2.slx'. Each of the five blocks labelled 'AM Tone Modulator' outputs an AM signal like the one created in 'AMl.slx'. Double clicking on one of these blocks displays a dialogue box that allows you to enter the carrier frequency, message frequency and the modulation index. The message frequency and the modulation indices have been pre-set to values that should be left as-is. Some of the messages are 100 Hz sinusoids and some of them are 200 Hz. However, consider each AM signal as requiring 500 Hz bandwidth, and imagine that the total spectrum to be used is from the 1.25-3.75 kHz range.
- Set the values of all the carrier frequencies so that all the AM signals fit into our commercial band without interfering with each other.
- Run the simulation and verify that all the signals are where they should be and that the correct band of spectrum is being used.
- Write down all the carrier frequencies that were used. **(Q4)**
- Discuss the difference between the messages with the lower or higher modulation indices and what practical reasons you can think of for choosing higher or lower indices. **(Q5)**

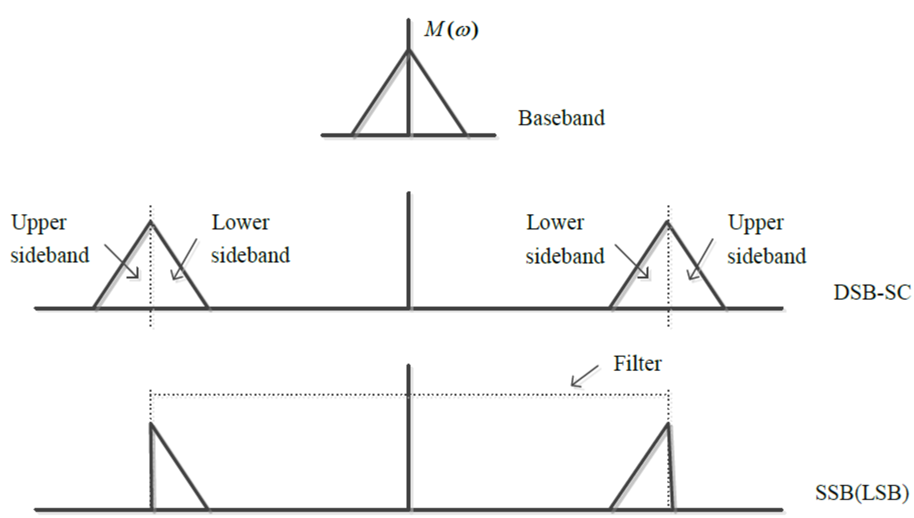
- Look at the signal vs. time plot. It is not very easy to detect the individual signals, is it? This is why the frequency domain is such an important tool!
- Change one of the message signals to have a frequency of 600 Hz.
- What effect does this have? Would this be okay in an actual system? **(Q6)**
- Take the block in the corner titled 'Random Number' and move it underneath the addition block, then drag and drop a connection from its output to the unused input on the addition block. The Random Number block creates random numbers with a Gaussian distribution, making it comparable to the Additive White Gaussian Noise (AWGN) that is seen in high frequency systems. Run the simulation and comment on the spectral effect of noise that is random in time. Based on this, what are some practical concerns regarding modulation indices? **(Q7)**
- Close the AM2.slx file.

1.2 SSB Modulation

In AM and DSB, as a consequence of the math, the message appears on either side of the carrier frequency. Each of these copies of the message is called a 'sideband'. This occurs with any message signal, regardless of the message. The overall signal, once up-converted to a higher frequency, is always symmetric, with the two sidebands being mirror images.

For a demonstration of this, open the file called 'AMsb.slx' and run the simulation. The two scopes that open will show the random signal when it is centered around DC and then the spectrum of that same signal once it is up-converted (put on a carrier). Since both sidebands have the same shape, only one of them is necessary in order to recover the message later. It is for this reason that there is Single Sideband (SSB) modulation. The signal can consist of the upper sideband (USB) or the lower (LSB). Close the AMsb.slx file.

1.2.1 Creating an SSB Signal by Filtering

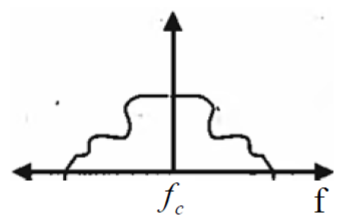


An SSB can be considered as a DSB-SC signal with one set of sidebands removed (either the upper or lower). If you consider needing to remove one of the sidebands from a DSB signal, the most intuitive way involves filtering.

- Open the file 'AMssbl.slx' and look over what is being done.
- Run it and inspect both spectra that pop up. The up-conversion is done in two steps, so the spectrum can be inspected at the Intermediate Frequency (IF) and the Radio Frequency (RF). *DO NOT concern yourself with the magnitude of the spectrum since the filters will alter them; just note the placement of the delta functions. To see what the ranges of the filters are, double-click on them and the magnitude response should be displayed. DO NOT change any of the settings in the Filter Toolbox!*
- There are four main stages to this system: multiply, filter, multiply, filter. For each stage, jot down a point form description and a crude sketch of what positive spectral components exist for each stage. Remember the trig identity that multiplying two sinusoids gives a result at sum and difference frequencies.
- What is the final **carrier** frequency? Which sideband (upper or lower) is being modulated? **(Q8)** Think about what the DSB spectra look like and how a corresponding SSB signal would look.
- What is a frequency range that the first filter could be changed to so that the opposite sideband gets modulated? **(Q9)**
- The 'constant' block is set to zero, meaning that the first signal being generated before filtering is DSB. Would it change anything if the constant block was changed so that an AM signal was generated first? Why or why not? **(Q10)** Verify your prediction.
- Why do you think the up-conversion is done in two steps? **(Q11)** HINT: When the filter toolbox is open, on the left hand side, the 'Order' of the filter is listed. The higher the order of a filter, the more complex it is. The 'tighter' range a filter has, the higher order it is. Equipment that works at higher frequencies is always more difficult to design and therefore more expensive.
- If you want to see the opposite sideband modulated switch the 'Very Narrow' Bandpass Filter # 1 with # 2.
- At the IF, look at the DSB and SSB Signals vs. Time plots. Does the SSB look correct? Would it be very easy to work out the time-domain description for a SSB signal if the message was something more complicated than a tone? **(Q12)**
- Close the AMssb1.slx file.

1.2.2 Creating an SSB Signal By a Hilbert Transform

Filtering is not the only way to create an SSB signal. Filtering appeared to work very well in part A, partly because we were only doing tone modulation. With tone modulation, each sideband is only a delta function and is not right up against the carrier frequency. However, with a more general message, the sidebands may come together at the central frequency.



In a case like this, a filter would have to cut upwards perfectly straight right at the carrier frequency (The elusive, magical 'brick-wall filter'). The 'phase shift' method is a way to completely eliminate one sideband while retaining the other.

The transfer function of the Hilbert transform is given as:

$$H(\omega) = -j\mathbf{sgn}(w) \tag{3}$$

where $\mathbf{sgn}(w)$ is the signum function, i.e. $\mathbf{sgn}(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$

The Hilbert transform of $m(t)$ and $M(t)$ is denoted by $\hat{m}(t)$ and $\hat{M}(t)$, respectively. For the case where the message is a sinusoid, it is relatively easy to write out the signal at every step and see how the end result is the SSB-USB or SSB-LSB signal. Since the Hilbert transform of a cosine is a sine, the two output signals become:

$$cos(\omega_c t)cos(\omega_m t) \mp sin(\omega_c t)sin(\omega_m t) \tag{4}$$

This is equivalent to

$$cos((\omega_c \pm \omega_m)t) \tag{5}$$

- Open the file 'AMssb2.slx'. Look over the system layout (there are two message signals to be chosen from, a cosine and a saw-tooth signal). Notice that along with the standard blocks (sinusoid carrier, multipliers) there are also a $-90°$ phase shift block and a Hilbert Transform block.
- Run the simulation AMssb2.slx with the cosine as the message and confirm that it creates both sideband messages.
- By looking at the spectrum of the DSB signal, why would the filtering method be difficult to use in this case? **(Q13)**
- Double-click the manual switch block to change the modulating signal to the saw-tooth signal.
- Run the simulation again, does the phase-shift method create SSB signals for arbitrary messages?

2 AM Modulation and Inspection using the AM Transmitter Board

This portion of the lab will use the AM transmitter w/SSB board to create AM, DSB and SSB signals, and they will be analyzed using an oscilloscope with FFT capabilities. This will be a fun and informative exercise in 'real' signals and equipment. For this portion of the lab, you are asked to get things working, make a very small number of calculations and demonstrate it.

2.1 The Transmitter (Tx) Board

At the end of Part 1 of this manual is a schematic for the board. Note that there are two versions of the board (Ver. 1.00 and Ver. 1.10, see diagrams on page 13). Ver. 1.00 boards are missing J4 and therefore J18 always outputs the 1.452 MHz range oscillator signal. The location of the Ground (TP1) is also different for the two versions. Below is a block/functional diagram of the transmitter board.

The signal selection jumper pins and the BNC connectors are labeled 'J' and the test points are labeled 'TP'. There is a lot of detail missing from the diagram; the purpose of it is to help you get an understanding of what the board does. Depending on how the Voltage Controlled Oscillators (VCOs) and the jumper pins are set up, the output (J9 or TP7) can be an AM, DSB or SSB signal.

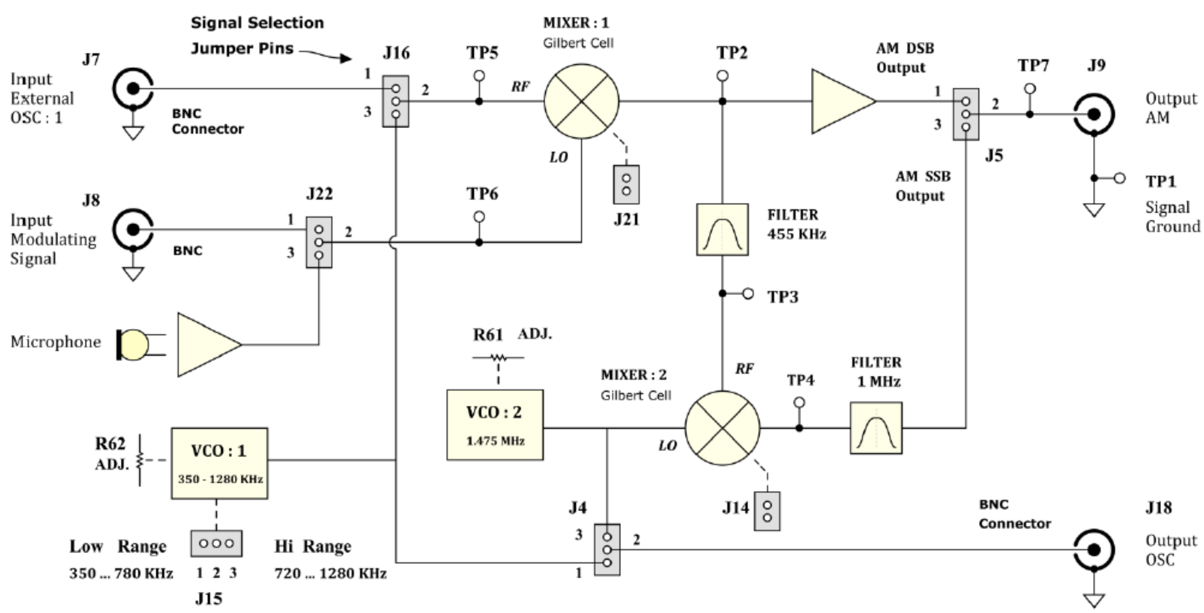


Figure 1: AM DSB/SSB Modulation Transmitter (Board Version 1.1).

The VCOs are used to create the carriers. Their frequencies can be tuned by adjusting the potentiometers R61 and R62. The Hi/Lo oscillator VCO:1 can be switched to create a signal that is either in the 1 MHz range (for AM and DSB signals) or the 452 kHz range for SSB signals.

The mixers are 'Gilbert-Cell Balanced Modulators'. A Gilbert Cell is a popular transistor configuration that implements multiplication of two signals. The mixers have a lot of carrier feed through, meaning that the DC term present in AM modulation does not have to be there and the signal will still have a strong carrier component to it. However, the carrier that is fed through can be suppressed by negative voltage, when this is done it is called a Balanced Modulator. For the Tx board, there are jumper pins that can be used to allow the carrier to feed through or suppress it (which switches the signal between AM and DSB).

Signal Selection Jumpers Pins / Switches - A Complete List:

- **J16** Pins 1 & 2 connected, selects an external carrier (J7) for the first mixer. Pins 2 & 3 connected, selects the Hi/Lo VCO:1
- **J15** Pins 1 & 2 connected, sets the Hi/Lo VCO:1 to the 452 kHz range. Pins 2 & 3 connected, sets the Hi/Lo VCO:1 to the 1MHz range.
- **J22** Pins 1 & 2 connected, selects external connector J8 to supply the modulating input signal (the message). Pins 2 & 3 connected, selects the microphone to supply the message.
- **J4** Pins 1 & 2 connected, selects the Hi/Lo VCO:1 to be accessible through J18 for testing. Pins 2 & 3 connected, selects the VCO:2 (Not an option on Ver. 1.00 boards)
- **J21** Having this jumper in place allows the carrier to feed through on the 1st mixer. If it is removed then the carrier will be suppressed.
- **J14** Having this jumper in place allows the carrier to feed through on the 2nd mixer. If it is removed then the carrier will be suppressed.
- **J5** Pins 1 & 2 connected, selects the AM signal path. Pins 2 & 3 connected, selects the SSB signal path. Both are routed to the output TP7 and J9.

2.2 AM and DSB Modulation Generation

Caution: Setup DC Power supply before you connect and power up the Transmitter board:

1. Configure the DC Power Supply: Hewlett Packard 3631A - Triple output. Turn on the power and select function Output 'On/Off' to 'Off'. This will put the power supply in standby mode (no power on the output terminals).
2. Set Power Supply voltage limits to +12 V and -12 V respectively for the two outputs:
   - Select function +25 V range button, and select button Display Limit.
   - Select the adjust Voltage/Current button if voltage adjust mode is not set.
   - Now adjust supply for +12 Volts using position arrows and Jog shuttle wheel.
   - Select function -25 V range, button and repeat adjustments for supply -12 Volts
3. Set Power Supply output current limits to 40 mA for each supply:
   - Select function +25 V range button and select button Display Limit.
   - Use the adjust Voltage/Current button to set to current adjust mode.
   - Now adjust supply output current using position arrows and Jog shuttle wheel.

- Select function -25 V range button.
- Repeat current adjustments for supply range.

4. If you are unsure if you've set the voltages correctly, check the voltage outputs using the multimeter and activate the power supply Output 'On/Off' to 'On' mode.
5. Locate connector H1 on the Transmitter board. Make sure the board is orientated so that the 'UofC' is right side up.
6. Carefully connect power supply into connector H1 on the Tx Board using banana leads and hook up wires connected to the Breadboard Station Unit.
   - Supply +12 V should be connected to the top left pin socket of connector H1.
   - The common wire (ground) goes to the top center pin socket of H1.
   - Supply -12 V should be connected the top right pin socket of connector H1.
7. Activate power to Tx Board by selecting Output 'On' from the power supply.

2.2.1 AM Generation

Objective: Create an AM signal with 1.00 MHz carrier.

Tune the carrier:

- Make sure J16 selects the Hi-Lo VCO to be the carrier.
- Set J15 to select the VCO:1 to work in the 1 MHz range.
- View the waveform at TP5 using the oscilloscope.
- Adjust potentiometer R62 using your mini-screwdriver, and tune the VCO:1 to 1.00 MHz

Create the message signal:

- Set the Arbitrary Wave Generator (AWG) to a 1 Vpp sinusoid of 10 kHz.
- Feed the message signal to J8.
- Set J22 to select J8 as the message input to Mixer:1

Look at AM signal:

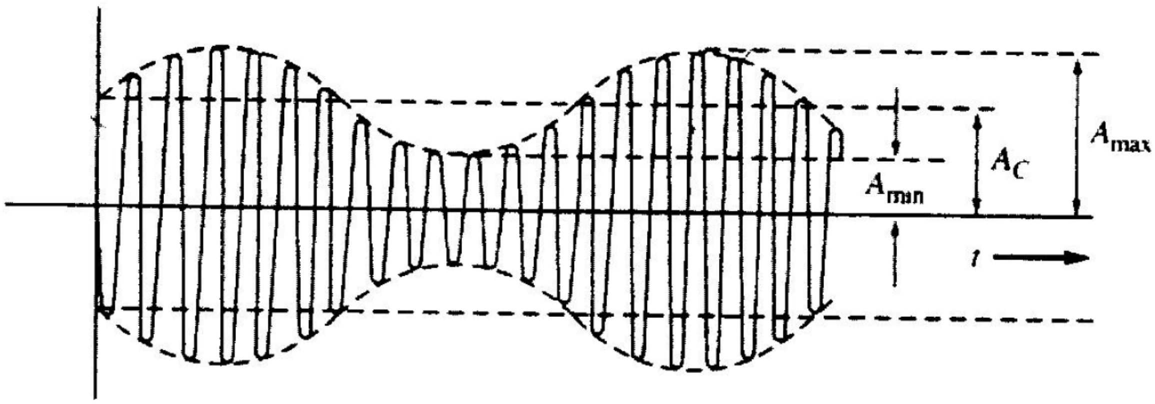
- Set J5 to select the AM output signal
- Connect either TP7 or J9 to an oscilloscope.

To see the AM signal properly:

- Select auto scale and then zoom out by turning the horizontal knob (Time/Div.) to the left until you see the envelope takes shape. It will most likely be rolling and you need to adjust the trigger level knob and if necessary, increase the holdoff time setting to make the view stationary. Holdoff time can be accessed by selecting button 'Mode Coupling' located in the scope trigger section. Then select the button underneath the scope screen menus and use the multi-purpose knob labelled with the turning arrow to set the Holdoff time.

- Check that the envelope is of the correct frequency.

You may have derived in class a simple, graphical way of determining the modulation index of an AM signal using the diagram below:



$$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} \tag{6}$$

- Measure $A_{\max}$ and $A_{\min}$ using the oscilloscope and calculate modulation μ . **(Q14(a))**
- Measure V_{pp} at TP5 and TP6. If you check the board diagram you will see that these two points are the carrier and the message before they are fed into the mixer.
- To measure more accurately with less noise, activate the oscilloscope input vertical channel, and select the BW Limit filter. The scope bandwidth is now set to 30 MHz. Note that if you activate the Auto-scale button, the BW Limit filter will turn off.
- Activate oscilloscope Cursors, select one of the vertical Y cursors and use its adjustable dial to measure. Adjust the trigger level to see the signal properly at TP6; it will be very small.
- From the two measurements, calculate μ again (divide TP6 by TP5). **(Q14(b))**
- Do the two different methods give values for μ that are at least in the same range? **(Q15)**
- Put the AM output signal back on the scope (TP7), with 1 V_{pp} input. Now increase the message's V_{pp} (by adjusting the AWG) until you can see that over-modulation has occurred. Note the AWG output value that caused this to happen. **(Q16)**

2.2.2 DSB Generation

Create a DSB signal:

- Prepare the AM signal: Have the AM signal displayed on the scope, set the message to 10 kHz, but change the message voltage to 2 V_{pp}.
- Suppress the carrier:

  - Adjust the first mixer to suppress the carrier, by removing the jumper J21
  - You may need to adjust the trigger level on the scope again.

The Balanced Modulator will not have completely eliminated the carrier in the AM signal, and other equipment factors may distort the appearance of the DSB signal. Compare what you are seeing on the scope to the simulated DSB signal. **(Q17)**

2.2.3 Spectral Analysis

***** This part requires an Agilent oscilloscope with FFT capabilities *****

Objective: View the AM and DSB spectra at TP7

- Put J21 back so that the output signal at TP7 is AM again.
- Adjust the Volt/Div on the scope so that the time signal is fairly small (spans over two divisions) and then move the waveform to top half of viewing screen. Next you will add a FFT waveform to the bottom half of the screen.
- Setup the FFT function on the scope
  - Press the 'Math' Button on the scope. Select FFT under the Operator menu, using the buttons below the scope screen that correspond to software menus displayed on the scope.
  - Using the Horizontal (Time/Div) knob at the top of the scope set the FFT sample rate to 100 MSa/s.
  - Using the buttons underneath the screen and the multi-purpose knob labelled with the lighted turning arrow, set the center frequency to 1 MHz and the Span to 100 kHz.
  - You should see 3 main spikes representing the AM spectrum. If you see more, and the center spike (the carrier) is not right in the middle (at 1 MHz) then tune R62 again until the carrier is in the right place and there are only three visible frequency components. Note: on the scope screen, a center frequency indicator triangle (color orange) is positioned on the border of the grid frame, top middle area.
- To see the DSB spectrum, remove J21.

While looking at the frequency spectrum and the time domain simultaneously, slowly increase and decrease the frequency of the message signal coming from the AWG to demonstrate that time scaling has the inverse effect in frequency (one 'spreads' while the other 'compresses'!).

Demonstrate this to a T.A. and collect a signature for every member of your group. **(Q18)**

You're done! Please put all your cords away.

*** Make sure your hand-in sheets are complete and **DON'T FORGET TO HAND THEM IN BEFORE YOU LEAVE**!!

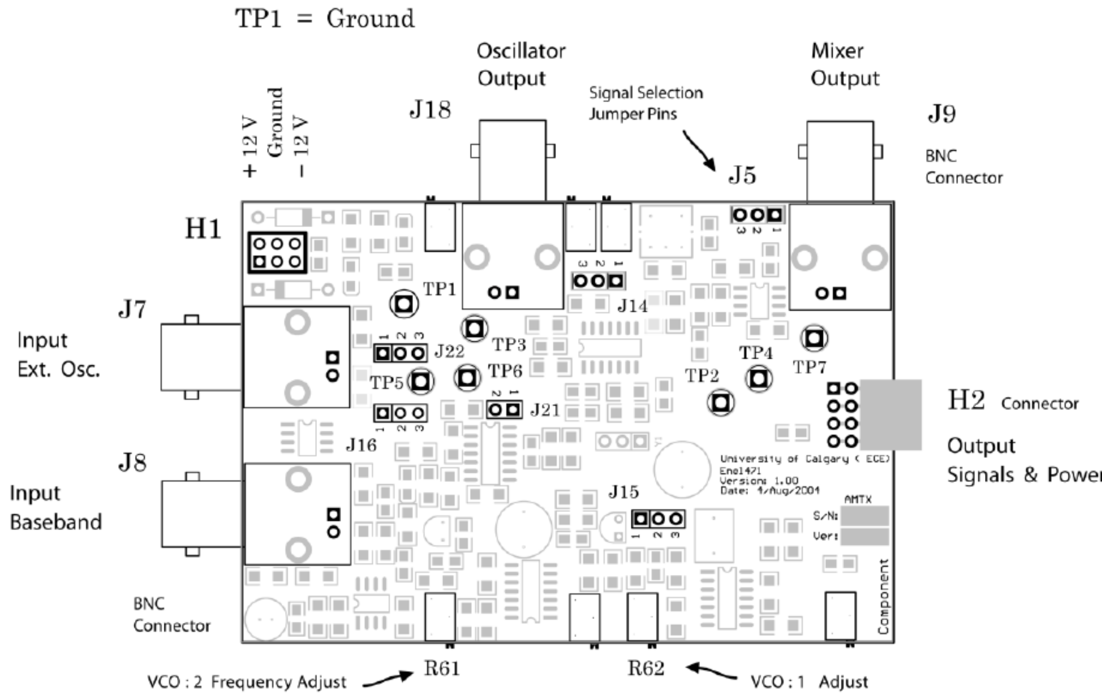


Figure 2: AM DSB/SSB Modulation Transmitter (Board Version 1.0).

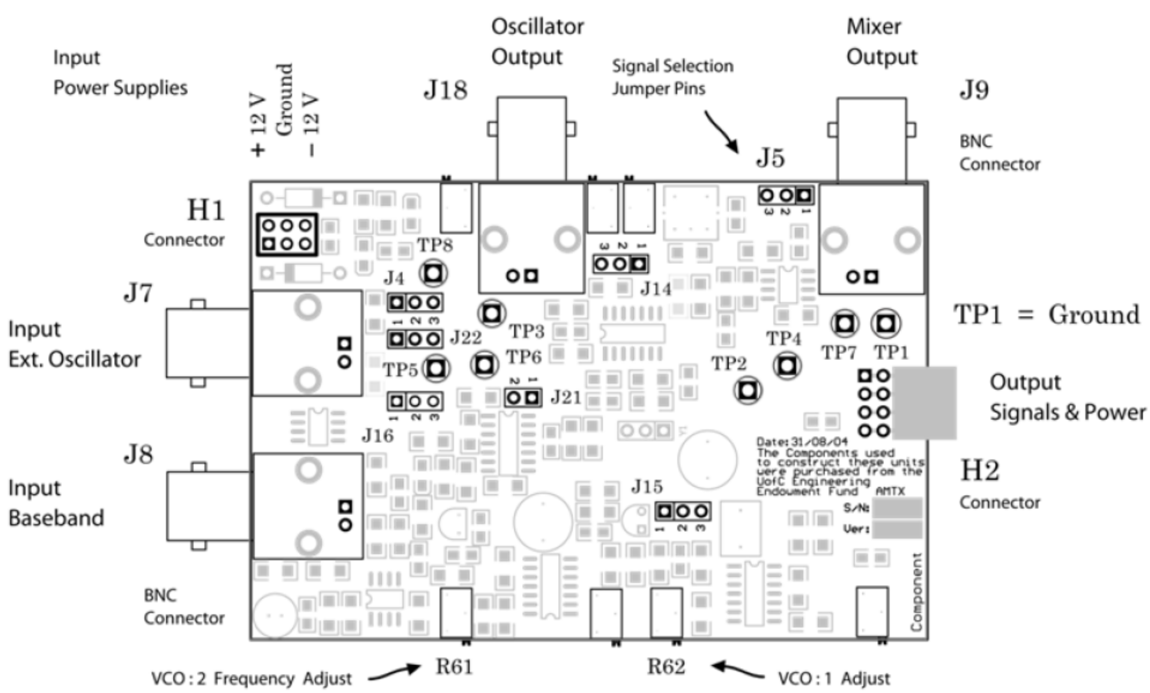


Figure 3: AM DSB/SSB Modulation Transmitter (Board Version 1.1).

Part 2

Lab 1 Questions

- Students will be required to hand in written answers (one per group) to these questions.
- Try to complete the Simulink part of the lab prior to the lab session. This is to ensure that students are out of the lab latest 5:00pm. Stations will be shut down exactly at 5:00pm

A. Simulink Part Questions (to be completed prior to the lab session): 10 marks

Q1 (AM1.slx) Draw the AM and DSB spectra and label the actual values for magnitude and frequency.

Q2 (AM1.slx) Make a sketch of the time domain plot for both AM and DSB, be sure to mark the $A_{\max}$ and $A_{\min}$ and label the important frequencies (f_c and f_m).

Q3 (AM1.slx) Point out the main difference between the time plots of the conventional AM and the Double-sideband suppressed carrier (DSB-SC).

Q4 (AM2.slx) What are the carrier frequencies used?

Q5 Discuss the difference between the messages with the lower or higher modulation indices and what practical reasons you can think of for choosing higher or lower indices.

Q6 What is the effect of changing one of the message signal frequencies to 600 Hz? Would this be okay in an actual system?

Q7 What is the effect of the additive white Gaussian noise on the spectrum?

Q8 (AMssb1) What is the final carrier frequency? Which sideband is being generated (USB or LSB)?

Q9 What is a frequency range that the first filter could be changed to so that the opposite sideband gets modulated?

Q10 The 'constant' block is set to zero, meaning that the first signal being generated before filtering is DSB. Would it change anything if the constant block was changed so that an AM signal was generated first? Why or why not?

Q11 Why do you think 'up-conversion' is implemented in two stages?

Q12 Would it be very easy to work out the time-domain description for a SSB signal if the message was something more complicated than a tone?

Q13 (AMssb2) By looking at the spectrum of the DSB signal, why do you think it would be difficult to use the filtering method?

B. Hardware Part Questions (to be completed during the lab experiments): 10 marks

Q14 Calculate the value of the modulation index using (a) A_{max} and A_{min} (off the scope) and (b) V_{pp} at TP5 and TP6.

Q15 From the two measurements, calculate μ again (divide TP6 by TP5). Do the two different methods give values for μ that are at least in the same range?

Q16 (AM) What is the value of V_{pp} of the message signal that just starts to cause over-modulation?

Q17 (DSB) How does the simulated DSB-SC signal differ from the DSB-SC signal at the balanced modulator output?

Q18 Demonstrate the effect of time-scaling on Frequency to the TA.

Write down the names and ID # of the group members and hand in the answer sheets to the TAs before you leave the lab.

Part 3

Useful Trigonometric Identities

Pythagorean Identities

- $\sin^2\varphi + \cos^2\varphi = 1$
- $1 + \tan^2\varphi = \sec^2\varphi$
- $1 + \cot^2\varphi = \csc^2\varphi$

Identities From Definitions

- $\sin\varphi = \frac{1}{\csc\varphi}$
- $\cos\varphi = \frac{1}{\sec\varphi}$
- $\tan\varphi = \frac{1}{\cot\varphi} = \frac{\sin\varphi}{\cos\varphi}$
- $\cot\varphi = \frac{1}{\tan\varphi} = \frac{\cos\varphi}{\sin\varphi}$
- $\csc\varphi = \frac{1}{\sin\varphi}$
- $\sec\varphi = \frac{1}{\cos\varphi}$

Addition Formulas

Basic Formulas

- $\sin(\varphi \pm \theta) = \sin\varphi\cos\theta \pm \cos\varphi\sin\theta$
- $\cos(\varphi \pm \theta) = \cos\varphi\cos\theta \mp \sin\varphi\sin\theta$
- $\tan(\varphi \pm \theta) = \frac{\tan\varphi \pm \tan\theta}{1 \mp \tan\varphi\tan\theta}$

Special Cases

- $\sin\left(\varphi + \frac{\pi}{2}\right) = \cos\varphi$
- $\cos\left(\varphi + \frac{\pi}{2}\right) = -\sin\varphi$

- $\sin\left(\frac{\pi}{2} - \varphi\right) = \cos\varphi$
- $\cos\left(\frac{\pi}{2} - \varphi\right) = \sin\varphi$

Double Angle Formulas

- $\sin 2\varphi = 2\sin\varphi\cos\varphi$
- $\cos 2\varphi = \cos^2\varphi - \sin^2\varphi = 2\cos^2\varphi - 1 = 1 - 2\sin^2\varphi$
- $\tan 2\varphi = \frac{2\tan\varphi}{1\ -\ \tan^2\varphi}$

Half Angle Formulas

- $\sin\frac{\varphi}{2} = \pm\sqrt{\frac{1\ -\ \cos\varphi}{2}}$ (positive if $\frac{\varphi}{2}$ in quadrants I or II, negative otherwise)
- $\cos\frac{\varphi}{2} = \pm\sqrt{\frac{1\ +\ \cos\varphi}{2}}$ (positive if $\frac{\varphi}{2}$ in quadrants I or IV, negative otherwise)
- $\tan\frac{\varphi}{2} = \frac{1\ -\ \cos\varphi}{\sin\varphi} = \frac{\sin\varphi}{1\ +\ \cos\varphi} = \pm\sqrt{\frac{1\ -\ \cos\varphi}{1\ +\ \cos\varphi}}$ (positive if $\frac{\varphi}{2}$ in quadrants I or III, negative otherwise)

Multiple Angle Formulas

- $\sin 3\varphi = 3\sin\varphi - 4\sin^3\varphi$
- $\cos 3\varphi = 4\cos^3\varphi - 3\cos\varphi$
- $\sin n\varphi = 2\sin\left((n-1)\varphi\right)\cos\varphi - \sin\left((n-2)\varphi\right)$
- $\cos n\varphi = 2\cos\left((n-1)\varphi\right)\cos\varphi - \cos\left((n-2)\varphi\right)$

Other Identities

- $\sin\varphi \pm \sin\theta = 2\sin\frac{\varphi\ \pm\ \theta}{2}\cos\frac{\varphi\ \mp\ \theta}{2}$
- $\cos\varphi + \cos\theta = 2\cos\frac{\varphi\ +\ \theta}{2}\cos\frac{\varphi\ -\ \theta}{2}$
- $\cos\varphi - \cos\theta = -2\sin\frac{\varphi\ +\ \theta}{2}\sin\frac{\varphi\ -\ \theta}{2}$
- $\sin^2\varphi = \frac{1\ -\ \cos 2\varphi}{2}$
- $\cos^2\varphi = \frac{1\ +\ \cos 2\varphi}{2}$
- $\sin^3\varphi = \frac{3\sin\varphi\ -\ \sin 3\varphi}{4}$
- $\cos^3\varphi = \frac{3\cos\varphi\ +\ \cos 3\varphi}{4}$
- $\sin\varphi\sin\theta = \frac{\cos(\varphi-\theta) -\ \cos(\varphi+\theta)}{2}$
- $\cos\varphi\cos\theta = \frac{\cos(\varphi-\theta)\ +\ \cos(\varphi+\theta)}{2}$

- $\sin\varphi\cos\theta = \frac{\sin(\varphi+\theta)\ +\ \sin(\varphi-\theta)}{2}$
- $2\cos^2\varphi\,(1+2\cos\varphi) = 1+3\cos\varphi+\cos 2\varphi+\cos 3\varphi$