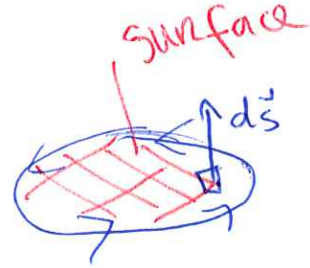
Faraday's Law

↳ $\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})$

⇒ with time varying fields,

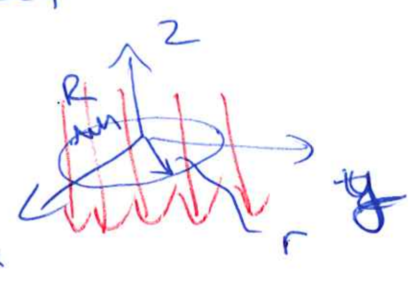
$$\vec{E}_{ind}(t) = -\frac{\mu_0}{4\pi r} Q \frac{d\vec{v}}{dt}$$



- $\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{s}$
- $V_{emf} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{s}$
- EMF

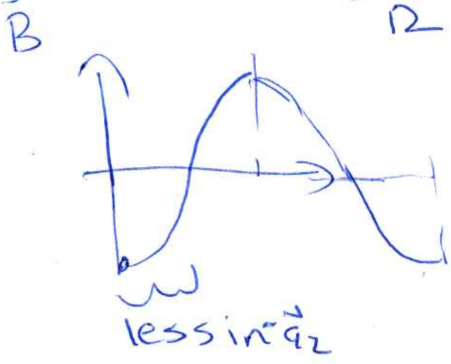
↑ Lenz's Law → induced magnetic field (produced by induced current) opposes change in original field

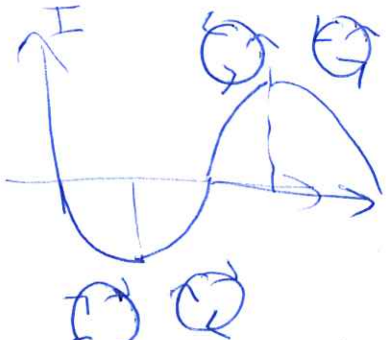
Ex $\vec{B} = -B_z \cos(\omega t)\, \vec{a}_z$



$$EMF = -\pi r^2 \omega B_z \sin(\omega t)$$

$$I = -\frac{\pi r^2}{R} \omega B_z \sin(\omega t)$$





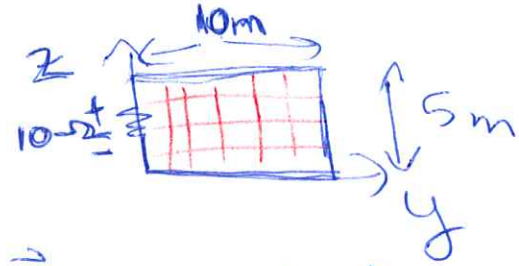
→ here, induced magnetic field opposes reduction in original $\vec{B}$ in $\vec{a}_z$ direction

counteracts changes in original $\vec{B}$

⇑

→ current flow has induced magnetic field

Ex2 2020 ②



$$\vec{B} = 6\cos(10t)\,\vec{a}_x \text{ mWb/m}^2$$

Find EMF + current through resistor.

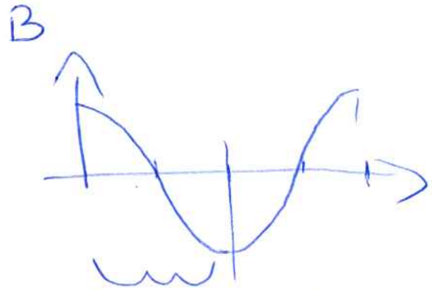
$$\text{EMF} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$$

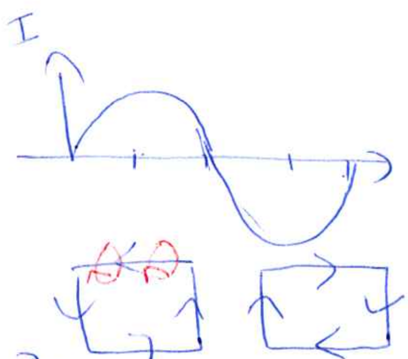
$$= -\frac{d}{dt}\int_0^5\int_0^{10} (6\cos(10t)\vec{a}_x \cdot dy\,dz\,\vec{a}_x \times 10^{-3}$$

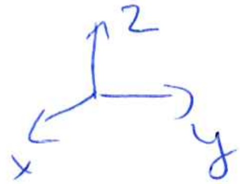
$$= -\frac{d}{dt}\left[6\cos(10t)(10)(5)\right] \times 10^{-3}$$

$$\text{EMF} = 3\sin(10t) \text{ V}$$

$$I = 0.3\sin(10t) \text{ A}$$







B decreases in $\vec{a}_x$ direction → induced current has B in $\vec{a}_x$ direction to oppose change

Transformer EMF → loop constant with time

$$\text{EMF} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$$

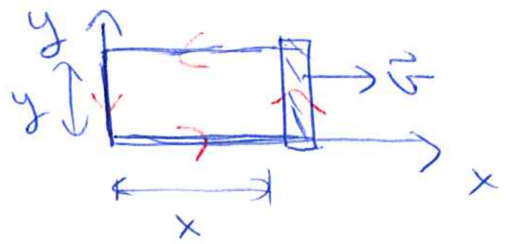
$$= -\int \frac{\partial}{\partial t}\vec{B}\cdot d\vec{s}$$

Motional EMF → loop changes with time

$$\text{EMF} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s} \quad \underline{\text{OR}} \quad \text{EMF} = \oint \vec{v}\times\vec{B}$$

2020 (3)

Ex 11



$\vec{B} = -B_z \vec{a}_z$

$$V_{emf} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s} \qquad d\vec{s} = dx\,dy\,\vec{a}_z$$

$$= -\frac{d}{dt}\left[(-B_z)(x)(y)\right]$$

$$= +B_z y \frac{d}{dt}x$$

$$= B_z y v$$

Option 2: $\vec{F} = q(\vec{v}\times\vec{B})$

$\vec{F}/q = \vec{v}\times\vec{B}$

$\vec{E}_m = \vec{v}\times\vec{B} \Rightarrow \oint \vec{E}_m\cdot d\vec{l} = \oint(\vec{v}\times\vec{B})\cdot d\vec{l}$

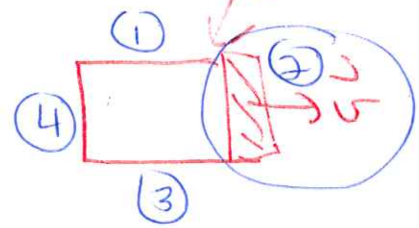
↳ motional

$\therefore EMF = \oint(\vec{v}\times\vec{B})\cdot d\vec{l}$

(1) (3), (4)

⇓

$\vec{v} = 0$



EMF ⇒ (1) $\vec{v} = 0$, $\vec{v}\times\vec{B} = 0$

(2) $\vec{v} \neq 0$ ⇒

(3) $\vec{v} = 0$, $\vec{v}\times\vec{B} = 0$

(4) $\vec{v} = 0$, $\vec{v}\times\vec{B} = 0$

$$\oint(\vec{v}\times\vec{B})\cdot d\vec{l} = \int_{(2)}(\vec{v}\times\vec{B})\cdot d\vec{l}$$

$$\vec{v}\times\vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ v_x & 0 & 0 \\ 0 & 0 & -B_z \end{vmatrix}$$

$$= \vec{a}_x(0) - \vec{a}_y(-B_z v_x) + \vec{a}_z(0)$$

$$= B_z v_x \vec{a}_y$$

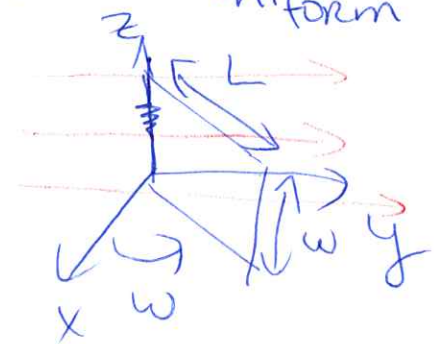
$$EMF = \int_0^y B_z v_x \vec{a}_y \cdot dy\,\vec{a}_y$$

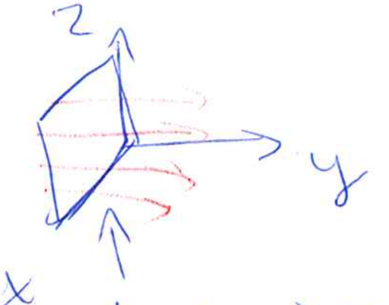
$$= B_z v_x \int_0^y dy$$

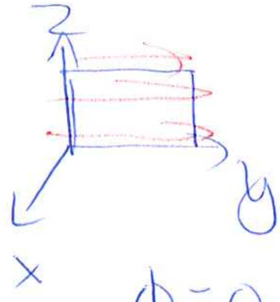
$$= B_z v_x y$$

Ex// Rectangular loop of resistance of 20 mΩ rotates at $\omega = 2$ rad/s in uniform field $\vec{B} = 10\vec{a}_y$ mWb/m^2

=> Find EMF







$\Phi = (LW)(B)$ $\quad \Phi = 0 \quad \Rightarrow \Phi = BLW\cos\theta$

$\Phi = \int \vec{B}\cdot d\vec{s} \Rightarrow EMF = -\frac{d}{dt}\Phi$

$$= BLW \sin\theta \underbrace{\frac{d\theta}{dt}}_{\omega}$$

$$= BLW\omega \sin\theta$$