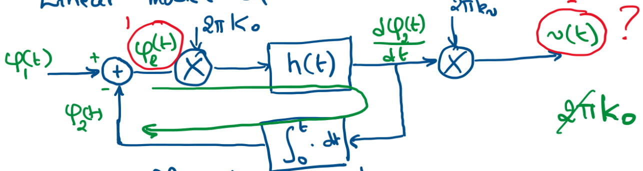
Online Lecture # 04 - FM Demodulation - PLL - Part III

Friday, March 27, 2020

8:58 AM

Linear model of the PLL:



$$\cancel{2\pi} k_o \cdot H(f) \cdot \frac{1}{j\cancel{2\pi} f} = \boxed{\frac{k_o}{jf} \cdot H(f) = L(f)}$$

Integro-differential Equation:

$$\boxed{\frac{d\varphi_e(t)}{dt} + 2\pi k_o \int_0^t \varphi_e(\tau) \cdot h(t-\tau)\, d\tau = \frac{d\varphi_1(t)}{dt}}$$

← 1- help to find $\varphi_e(t)$

$$\boxed{v(t) = 2\pi k_o \cdot \varphi_e(t) * h(t) \cdot \frac{1}{2\pi k_v}}$$

← 2- help find $v(t)$

the integro-differential equation can be written in frequency domain as:

$$j2\pi f \cdot \Phi_e(f) + 2\pi k_o \cdot \Phi_e(f) \cdot H(f) = j2\pi f\, \Phi_1(f)$$

$$\Phi_e(f) \cdot \cancel{j2\pi f} \left(1 + \frac{k_o}{jf} H(f)\right) = \cancel{j2\pi f} \cdot \Phi_1(f)$$

$$\Phi_e(f) = \frac{1}{1 + \underbrace{\frac{k_o}{jf} H(f)}_{=L(f)\,:\ \text{the loop gain.}}} \cdot \Phi_1(f) \rightarrow \boxed{\Phi_e(f) = \frac{1}{1+L(f)} \cdot \Phi_1(f)}$$

In frequency domain:

$$V(f) = \cancel{2\pi} K_o \cdot \Phi_e(f) \cdot H(f) \cdot \frac{1}{\cancel{2\pi} k_v}$$

$$V(f) = \underbrace{\frac{K_o}{k_v} H(f)}\cdot \frac{1}{1+L(f)} \cdot \Phi_1(f)$$

$$L(f) = \frac{k_o}{jf} H(f) \longrightarrow \frac{k_o}{k_v} H(f) = \frac{jf}{k_v} \cdot L(f)$$

$$\boxed{V(f) = \frac{jf}{k_v} \frac{L(f)}{1+L(f)} \cdot \Phi_1(f)}$$

If the PLL is implemented so that the Loop gain verifies:

$$\boxed{|L(f)| \gg 1}$$

then $\dfrac{L(f)}{1+L(f)} \simeq 1 \longrightarrow \boxed{V(f) \simeq \dfrac{jf}{k_v}\,\Phi_1(f)}$

$$V(f) = j2\pi f \cdot \frac{1}{2\pi k_v} \cdot \Phi_1(f)$$

Going back to time domain:

$$\boxed{v(t) = \frac{1}{2\pi k_v} \cdot \frac{d\varphi_1(t)}{dt}}$$

where $\underline{\varphi_1(t) = 2\pi k_f \int_0^t m(\tau)\,d\tau}$

$$v(t) = \frac{1}{2\pi k_v}\, 2\pi k_f \cdot m(t)$$

$$\boxed{v(t) = \frac{k_f}{k_v}\, m(t)} \longleftarrow \text{the PLL acted as an FM demodulator.}$$

Non-coherent FM Demodulation using frequency discriminators:

Frequency discriminator: a system that allows to convert an FM modulation to an AM modulation.