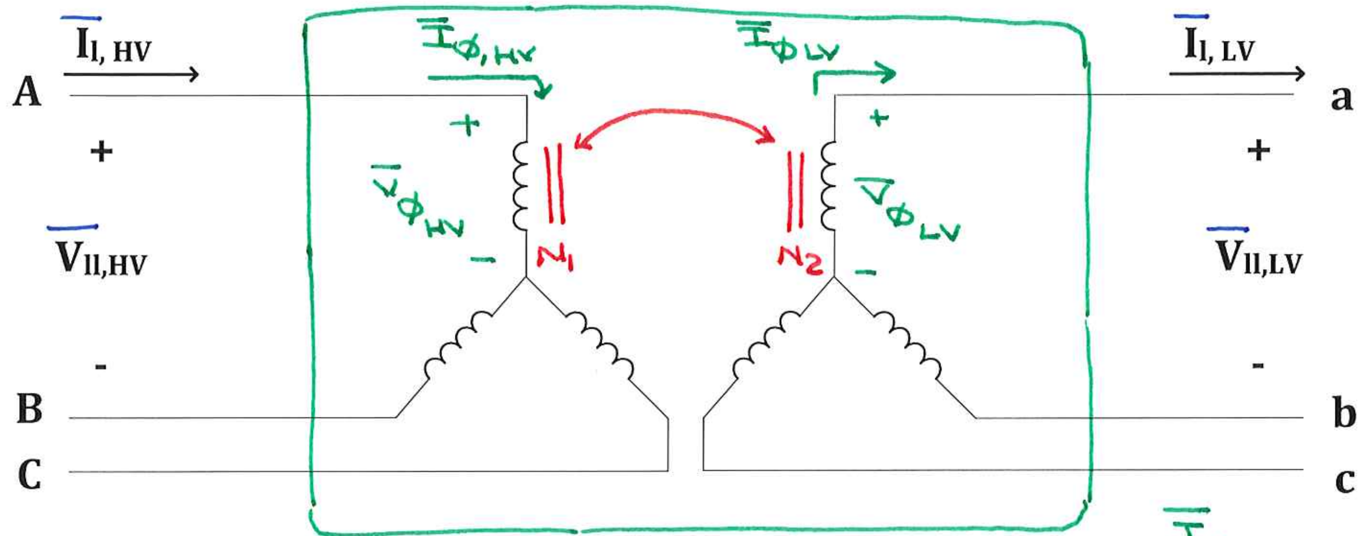
ENEL 487

Voltage and current relationships for three phase transformers

Relationships between line-to-line voltage on either side of the transformer and line current on either side of the transformer are dependent on the winding connection type.

1) Y-Y connection

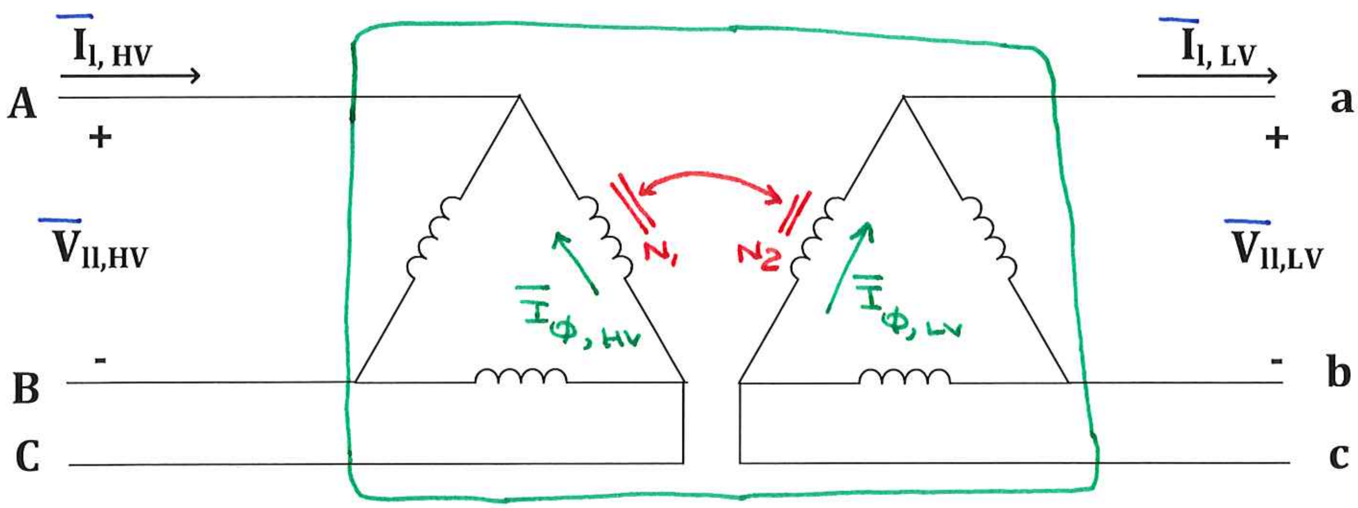


Magnetic coupling between the two winding gives: $\frac{\overline{V}_{\emptyset,HV}}{\overline{V}_{\emptyset,LV}} = \frac{N_1}{N_2}$ & $\frac{\overline{I}_{\emptyset,HV}}{\overline{I}_{\emptyset,LV}} = \frac{N_2}{N_1}$ $= \frac{\overline{I}_{l,HV}}{\overline{I}_{l,LV}}$

in Y, $\overline{I}_{\phi} = \overline{I}_{l}$

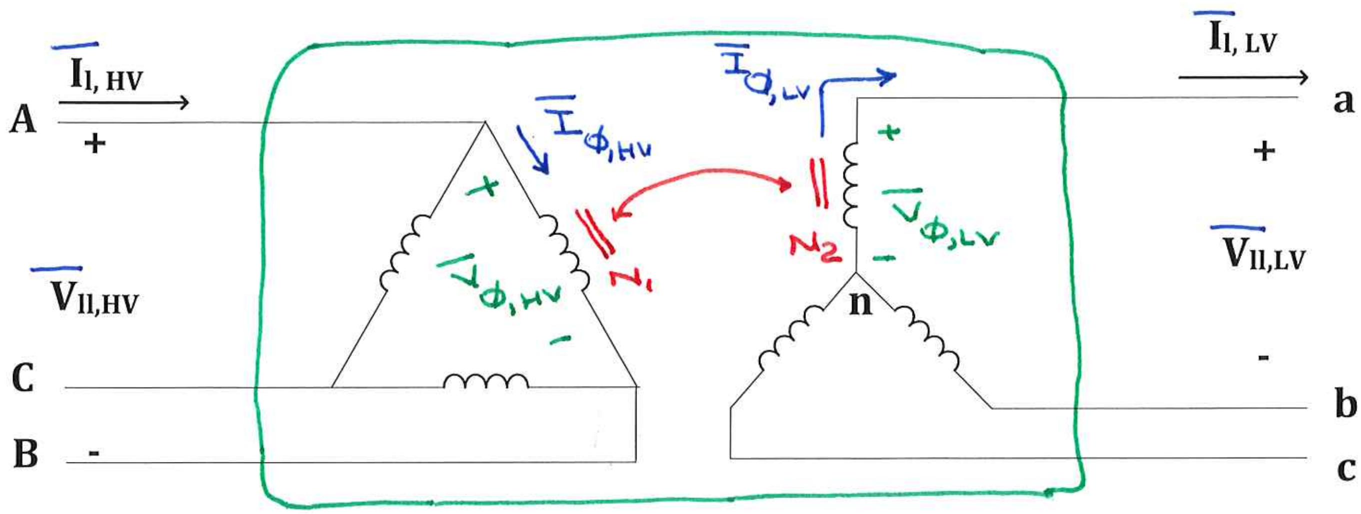
Also know that, in both sides, $\overline{V}_{ll} = \sqrt{3}\angle 30°.\overline{V}_{ln}$ therefore, $\frac{\overline{V}_{ll,HV}}{\overline{V}_{ll,LV}} = \frac{N_1}{N_2}$

2) Δ-Δ connection



Similar to Y-Y, $\frac{\overline{V}_{ll,HV}}{\overline{V}_{ll,LV}} = \frac{N_1}{N_2}$ & $\frac{\overline{I}_{l,HV}}{\overline{I}_{l,LV}} = \frac{N_2}{N_1}$

3) **Δ-Y Connection**



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Also, from Δ and Y connection properties: $\overline{\boldsymbol{V}}_{ll,LV} = \sqrt{3}\angle 30°.\overline{\boldsymbol{V}}_{\phi,LV}$ & $\overline{\boldsymbol{V}}_{ll,HV} = \overline{\boldsymbol{V}}_{\phi,HV}$

Therefore, $\frac{\overline{V}_{ll,HV}}{\overline{V}_{ll,LV}} = \frac{\overline{V}_{\phi,HV}}{\sqrt{3}\angle 30°.\overline{V}_{\phi,LV}} = \frac{N_1}{N_2.\sqrt{3}}.\angle -30°$

- The ratio of line-to-line voltage magnitudes is $\frac{N_1}{N_2.\sqrt{3}}$
- Line-to-line voltage on the Δ side lags line-to-line voltage on the Y side by 30^0

Similarly, $\frac{\overline{I}_{l,HV}}{\overline{I}_{l,LV}} = \frac{N_2.\sqrt{3}}{N_1}\angle -30°$

4) **Y-Δ Connection**

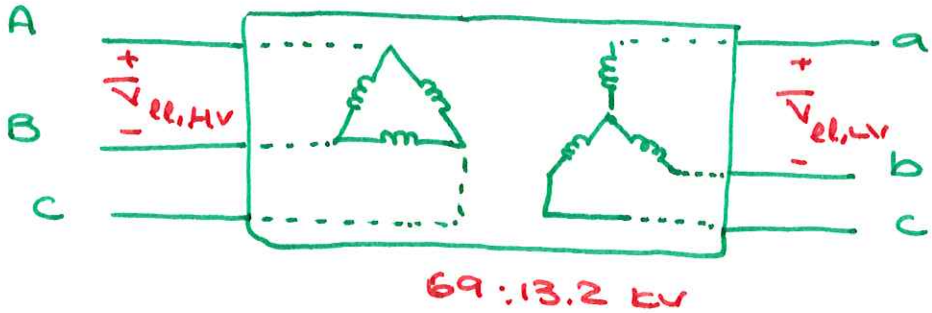
We can show that:

$$\frac{\overline{\boldsymbol{V}}_{ll,HV}}{\overline{\boldsymbol{V}}_{ll,LV}} = \frac{\sqrt{3}.N_1}{N_2}.\angle 30°$$

$$\frac{\overline{\boldsymbol{I}}_{l,HV}}{\overline{\boldsymbol{I}}_{l,LV}} = \frac{N_2.}{\sqrt{3}.N_1}\angle 30°$$

- Instead of dealing with turns ratios, we will only use the line-to-line voltage ratios for txfrs:

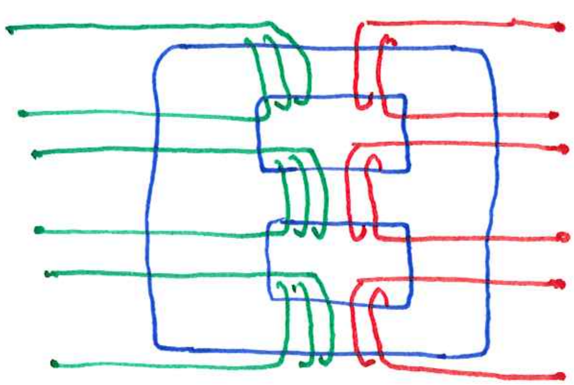
e.g: 69 : 13.2 kV ← • rated voltage of txfr
Δ Y • ratio of line-line voltages



if operating voltage on HV side $\overline{V}_{ll,HV} = 72$ kV,

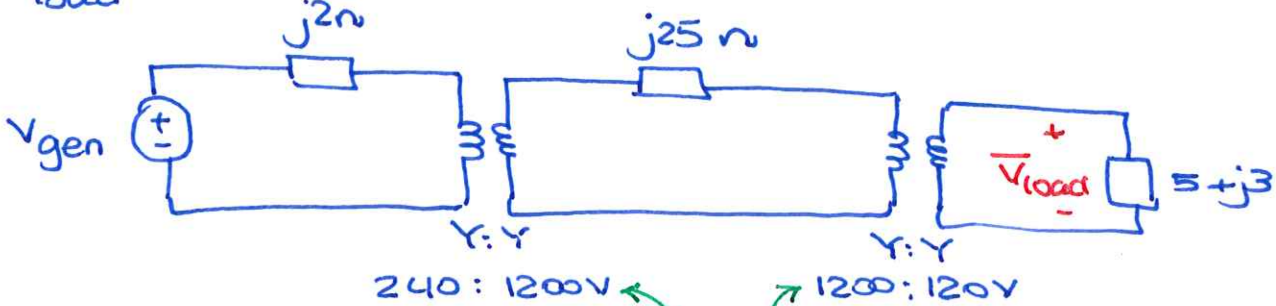
then operating voltage on LV side, $\overline{V}_{ll,LV} = 72 \times \frac{13.2}{69} = 13.8$ kV

- Typically, 3ϕ transformers are constructed such that all windings share a common core:



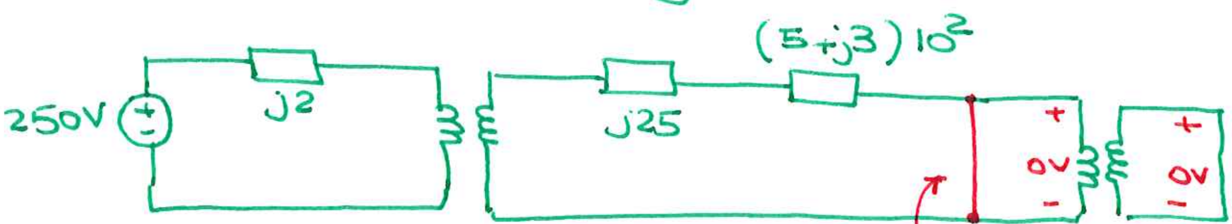
Example: Single ϕ circuit for a 3ϕ system; i.e. Source & load already converted to equivalent Y. If $V_{gen} = 250\,V$, $\bar{S}_{load} = ?$

$\bar{V}_{load} = ?$



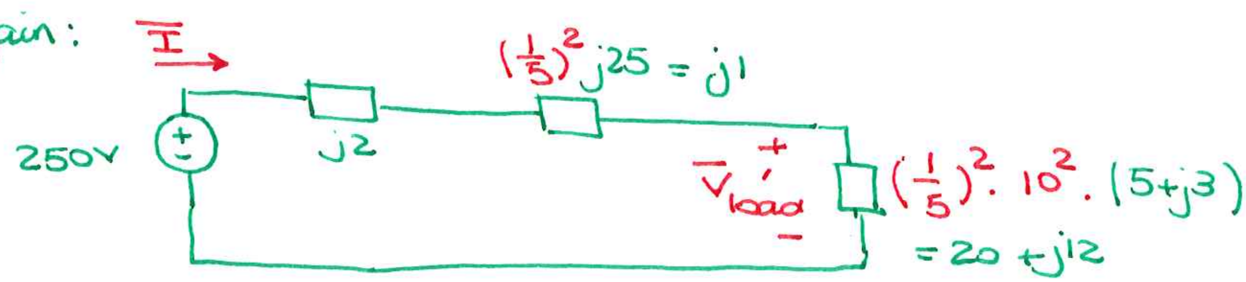
rated voltage ratios

Strategy:
- refer all impedances to one side
- Solve circuit
- Go back to orig circuit



we can replace 2nd txfr with a short

refer again:



$$\bar{I} = \frac{250}{j2 + j1 + (20 + j12)} = 10\angle{-37^\circ}\ \text{A}$$

$$\bar{V}'_{load} = \bar{I}\,(20 + j12) = 233.2\angle{-6}\ \text{V}$$

$$\bar{S}_{load} = \bar{V}'_{load} \cdot \bar{I}^* = 2332\angle{31}\ \text{VA}$$

- To find $\overline{V}_{load}$ in the orig circuit:

$$\overline{V}_{load} = \overline{V}_{load}' \times \frac{1200}{240} \times \frac{120}{1200} = 116.6 \angle{-6^\circ} \text{ V}$$

this is $V_{\ell\text{-}n}$ since this is a 1ϕ analysis

$$V_{\ell\text{-}\ell} = \sqrt{3}\, V_{\ell\text{-}n}$$

- $\overline{S}_{load}$ is the same in the orig circuit

($\overline{V}_{load}$ is $\dfrac{\overline{V}_{load}'}{2}$ but $\overline{I}_{load}$ is $\overline{I} \times 2$)

this is 1ϕ power. $\overline{S}_{load,3\phi} = 3 \times \overline{S}_{load,1\phi}$

- Yikes! there is too much work dealing with multiple voltage levels (and 1ϕ vs. 3ϕ quantities).

Help is on the way: Topic 4.

Topic 4: Per Unit Analysis

- Idea: Normalize / scale magnitudes of all variables (voltage, current, power, impedance) in the power system based on the txfr ratios so we can remove ideal txfr from the analysis.

$$\text{Quantity in per unit} = \frac{\text{actual value}}{\text{base value}}$$

- or, multiply by 100% to express as %

- The base quantities are related to each other by standard power equations:

For single phase systems:

$$I_{base} = \frac{S_{base,1\phi}}{V_{base,LN}} \quad \leftarrow \text{from } S_{1\phi} = I \cdot V_{LN}$$

all voltages are LN in 1ϕ

$$Z_{base} = \frac{V_{base,LN}}{I_{base}} \quad \leftarrow \text{from } V_{LN} = I \cdot Z$$

$$= \frac{V_{base,LN}^2}{S_{base}}$$

$$R_{base} = X_{base} = Z_{base}$$

$$Y_{base} = \frac{1}{Z_{base}}$$

$$S_{base,1\phi} = P_{base,1\phi} = Q_{base,1\phi}$$

Conversion Procedure (To analyze a 1ϕ system using P.u.)

1) Pick an S_{base} for entire system. e.g. $S_{base} = 100$ MVA

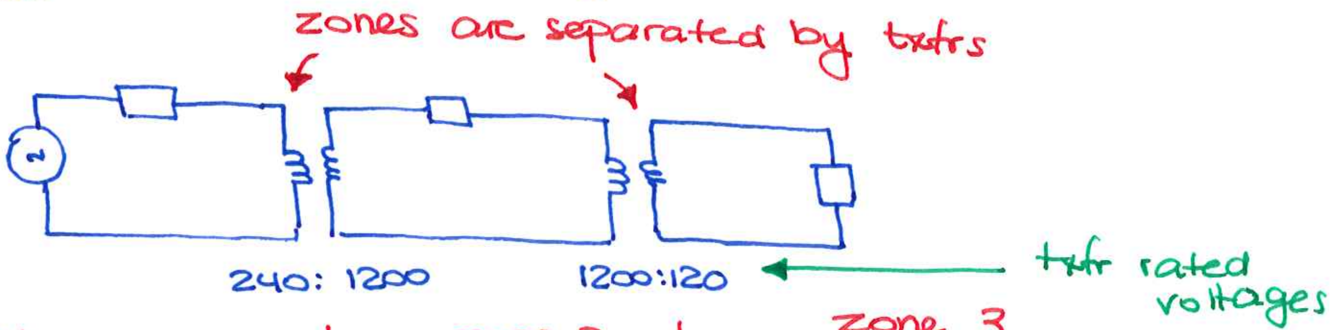
. all base values are real numbers

. P.u. only affects magnitudes, not phase angles.

2) Pick a V_{base} for one voltage level/zone.

V_{base} values are related by txfr voltage ratios

zones are separated by txfrs



Zone 1

. we can select $V_{base_1} = 240\,V$

good idea/practice to choose V_{base} to be the same as nominal/rated voltage

Zone 2

$V_{base\,2} = V_{base_1} \times \underbrace{\frac{1200}{240}}_{\text{txfr ratio}}$

$= 1200\,V$

Zone 3

$V_{base_3} = V_{base_2} \times \frac{120}{1200}$

$= 120\,V$

3) Calculate Z_{base} for each zone. $\quad Z_{base} = \frac{V_{base}^2}{S_{base}}$

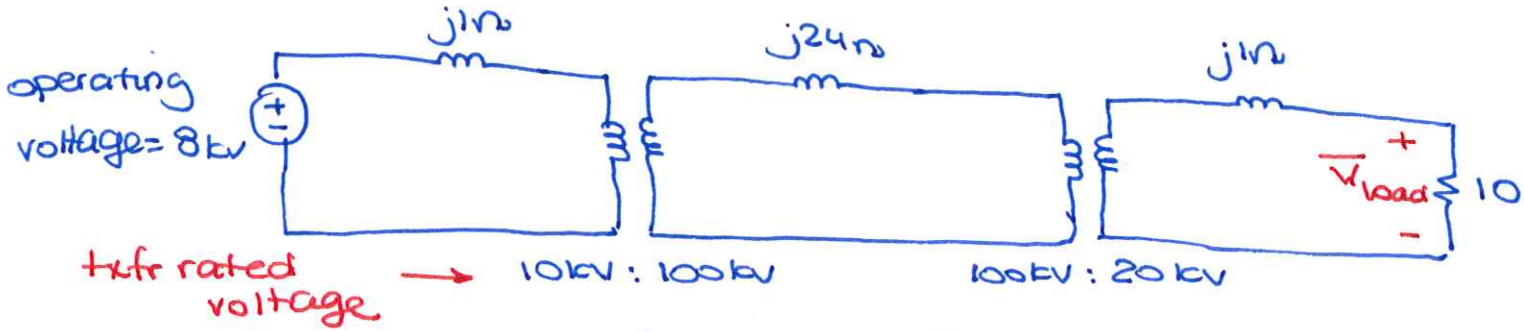
4) Calculate I_{base} for each zone. $\quad I_{base} = \frac{S_{base}}{V_{base}}$

5) Convert actual values to P.u.

. After conversion,

- Solve the P.u. circuit. obtain P.u. voltage, current, power
- convert to actual quantity by multiplying P.u values by S_{base}, V_{base}, I_{base}

Ex: Calculate load voltage, current, power. Use $S_{base} = 100$ MVA & $V_{base} = 8$ kV in gen zone.



zone 1

$V_{base_1} = 8$ kV (given)

$$Z_{base_1} = \frac{(V_{base_1})^2}{S_{base}} = 0.64\ \Omega$$

zone 2

$$V_{base_2} = V_{base_1} \times \frac{100\text{ kV}}{10\text{ kV}}$$

$= 80$ kV

$$Z_{base_2} = \frac{(V_{base_2})^2}{S_{base}}$$

$= 64\ \Omega$

$$V_{base_3} = V_{base_2} \times \frac{20\text{ kV}}{100\text{ kV}}$$

$= 16$ kV

$$Z_{base_3} = \frac{(V_{base_3})^2}{S_{base}}$$

$= 2.56\ \Omega$

$$I_{base_3} = \frac{S_{base}}{V_{base_3}}$$

$= 6.25$ kA