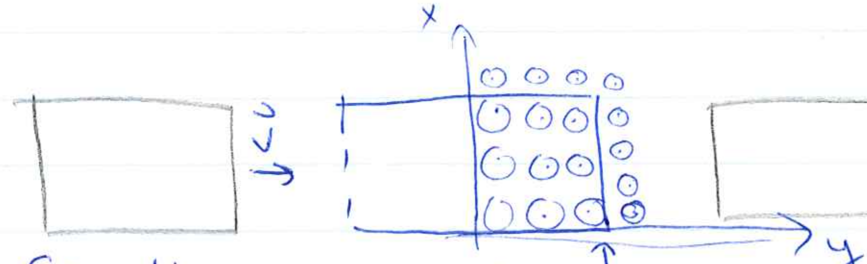
ENEL 476 Assignment #1 W2018

1. a)



Case 1: no overlap

Case 2: transition y_{loc}

Case 3: full overlap

$\vec{B} = -0.3\vec{a}_z \, , \, y > 0$

$\vec{v} = 2\vec{a}_y$ cm/s ; $t = 0$ right side of loop @ $y = 0$

Case 1: $\oint \vec{B} \cdot d\vec{s} = 0$

Case 3: $\oint \vec{B} \cdot d\vec{s} = \int_{y}^{y+0.6}\int^{0.2} (-0.3\vec{a}_z)(dx\,dy\,\vec{a}_z)$

$= (-0.3)(0.6)(0.2)$

$= -0.036$

Case 2: $\iint \vec{B} \cdot d\vec{s} = \int_0^{y_{loc}}\int_0^{0.2} (-0.3\vec{a}_z)(dx\,dy\,\vec{a}_z)$

$= -(0.3)(0.2)(y_{loc})$

$= -0.06\,y_{loc}$

$y_{loc} \rightarrow v = 2 \times 10^{-2}$

$\frac{dy}{dt} = 2 \times 10^{-2}$

$y = 2 \times 10^{-2} t + c$

$t = 0, y = 0 \therefore c = 0$

$y = 0.6, t = ?$

$0.6 = 2 \times 10^{-2} t \Rightarrow t = 30s$

So, $\Phi = \begin{cases} 0, & t \leq 0 \\ -0.0012t, & 0 < t \leq 30s \\ -0.036, & t > 30s \end{cases}$ wb

(A) $-0.06(2\times10^{-2}t)$
$= -0.0012t$

check: $t = 30s$, $-0.0012(30) = -0.036$ ✓

b) $EMF = \dfrac{-d\Phi}{dt}$

case 1: $\dfrac{-d(0)}{dt} = 0$

Case 3: $\dfrac{-d}{dt}(-0.036) = 0$

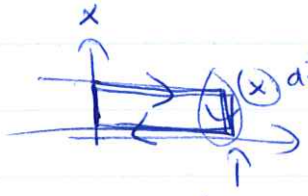
case 2: $-\dfrac{d}{dT}(-0.0012t) = -0.0012$

$\therefore$ $EMF = \begin{cases} 0, & t \leq 0 \\ +0.0012, & 0 < t \leq 30s \\ 0, & t > 30s \end{cases}$

check: $EMF = \oint(\vec{v} \times \vec{B}) \cdot d\vec{l}$

$\vec{v} \times \vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 0 & 2\times10^{-2} & 0 \\ 0 & 0 & -.3 \end{vmatrix} \Rightarrow \vec{v} \times \vec{B} = (2\times10^{-2})(-3)\vec{a}_x$
$= -6\times10^{-3}\vec{a}_x$

Now



$(x)\ d\vec{s} = dx\,dy\,\vec{a}_z$ ∴ dir'n of integration assumed as shown

integration over only this side (analogous to bar moving on rails)

$$\therefore \text{EMF} = \oint \vec{v}\times\vec{B}\cdot \underbrace{d\vec{l}}_{dx\,\vec{a}_x}$$

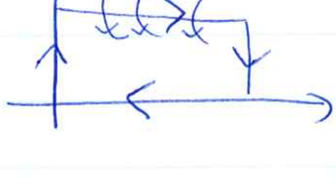
$$= \int_{0.2}^{0} -6\times10^{-3}\,dx$$

$$= (-6\times10^{-3})(-0.2)$$

$$= 0.0012$$

$$\rightarrow \text{EMF} = \begin{cases} 0.0012, & 0 \le t < 30\,s \\ 0, & \text{otherwise} \end{cases}$$

c) $$I = \begin{cases} 0, & t \le 0 \\ 0.00012, & 0 < t \le 30\,s \\ 0, & t > 30\,s \end{cases}$$



→ induced current has induced flux associated in $+\vec{a}_z$ direction

→ between 0 + 30s, the loop crossed more flux in the $-\vec{a}_z$ direction (ie loop moves into the region containing flux)

∴ the induced flux opposes the increase in flux in $-\vec{a}_z$ direction

d) $\vec{B} = -0.3\cos(2\pi\times10^3 t)\,\hat{a}_z$ Wb/m^2

Case 1: $t<0$, EMF $=0$ as loop is not in region containing flux

Case 3: $t > 30s$, $\Phi = \int_{y}^{y+0.6}\int_{0}^{0.2} (-.3\cos(2\pi\times10^3 t))\,dx\,dy$

$= (-0.3)(0.6)(0.2)\cos(2\pi\times10^3 t)$

$= -0.036\cos(2\pi\times10^3 t)$

EMF $= -\dfrac{d\Phi}{dt}$

$= 0.036\left[-2\pi\times10^3\sin(2\pi\times10^3 t)\right]$

EMF $= -72\pi\sin(2\pi\times10^3 t)$

↳ nonzero & time-varying for $t>30s$

Case 2: $0 \le t < 30$

$\iint \vec{B}\cdot d\vec{s} = \int_{0}^{y_{loc}}\int_{0}^{0.2} (-0.3)\cancel{dx\,dy}\cos(2\pi\times10^3 t)\,dx\,dy$

$= (-0.3)(0.2)\,y_{loc}\cos(2\pi\times10^3 t)$

$= (-0.06)(2\times10^{-2}t)\cos(2\pi\times10^3 t)$

$= -0.0012\,t\cos(2\pi\times10^3 t)$

EMF $= -\dfrac{d\Phi}{dt}$

$= 0.0012\left[\cos(2\pi\times10^3 t) + t(-\sin(2\pi\times10^3 t))(2\pi\times10^3)\right]$

$= 0.0012\cos(2\pi\times10^3 t) - 2.4\pi t\sin(2\pi\times10^3 t)$

→ time-varying EMF (motional + transformer)

2. a) $\vec{E}(y,t) = 40\cos(2\pi\times10^6 t - \beta y)\vec{a}_x$ mV/m

↳ $\epsilon_r = 2.4$, $\mu_r = 1$, $\sigma = 0$
↳ source - free region

$$\vec{J}_d = \frac{\partial}{\partial t}\epsilon\vec{E}$$

$$= \frac{\partial}{\partial t}\left(2.4\epsilon_0\, 40\cos(2\pi\times10^6 t - \beta y)\right)\times10^{-3}$$

$$= -96\epsilon_0\,(2\pi\times10^6 \sin(2\pi\times10^6 t - \beta y))\times10^{-3}$$

$$= -\cancel{96}^{48}\left(\frac{1}{\cancel{36\pi}_{\cancel{18}\ 9}}\times10^{-9}\right)(\cancel{2\pi}\times10^3)\sin(2\pi\times10^6 t - \beta y)$$

$\therefore\ \vec{J}_d(y,t) = -5.33\times10^3 \sin(2\pi\times10^6 t - \beta y)$ A/m^2

b) $\nabla\times\vec{E}_s = -j\omega\mu_0\vec{H}_s$

$$\vec{E}_s = 40e^{-j\beta y}\times10^{-3}\,\vec{a}_x$$

$$\nabla\times\vec{E}_s = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{d}{dy} & \frac{d}{dz} \\ E_{sx} & 0 & 0 \end{vmatrix}$$

$$= \vec{a}_x(0) - \vec{a}_y\left(-\underbrace{\frac{d}{dz}E_{sx}}_{\to 0}\right) + \vec{a}_z\left(-\frac{d}{dy}E_{sx}\right)$$

$$= \frac{-d}{dy}(40e^{-j\beta y}\times10^{-3})\vec{a}_z$$

$$= j\beta 40e^{-j\beta y}\times10^{-3}\vec{a}_z$$

$$-\cancel{j}\omega\mu_0\vec{H}_s = \cancel{j}\beta 40 e^{-j\beta y} \times 10^{-3}\,\vec{a}_z$$

$$\vec{H}_s = -\frac{\beta 40}{\omega\mu_0} e^{-j\beta y} \times 10^{-3}\,\vec{a}_z$$

$$\vec{H}(y,t) = \frac{-\beta\, 40 \cos(2\pi\times10^6 t - \beta y)}{(2\pi\times10^6)(4\pi\times10^{-7})}\,\vec{a}_z \text{ mA/m}$$

$\beta = ?$

$$\nabla\times\vec{H}_s = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 0 & 0 & H_{sz} \end{vmatrix}$$

$$= \vec{a}_x\left(\frac{d}{dy}H_{sz}\right) - \vec{a}_y\left(\underbrace{\frac{d}{dx}H_{sz}}_{0}\right) + \vec{a}_z(0)$$

$$= \frac{d}{dy}H_{sz}\,\vec{a}_x$$

$$= \frac{+\beta 40}{\omega\mu_0}(+j\beta)\, e^{-j\beta y}\,\vec{a}_x \times 10^{-3}$$

$$= \frac{j\beta^2 40}{\omega\mu_0} e^{-j\beta y}\,\vec{a}_x \times 10^{-3}$$

$$\nabla\times\vec{H}_s = j\omega\epsilon_R\epsilon_0\vec{E}_s$$

$$\therefore \vec{E}_s = \frac{\cancel{j}\beta^2 40}{(\omega\mu_0)(\cancel{j}\omega\epsilon_R\epsilon_0)} e^{-j\beta y}\,\vec{a}_x \times 10^{-3}$$

$$= \frac{\beta^2 40}{\omega^2\, 2.4\,\mu_0\epsilon_0} e^{-j\beta y}\,\vec{a}_x \times 10^{-3}$$

$\vec{E}_s = 40e^{-j\beta y}\vec{a}_x$

$\therefore \cancel{40} = \frac{\beta^2 \cancel{40}}{\omega^2 2.4 \mu_0 \epsilon_0}$

$\beta = \omega\sqrt{2.4\mu_0\epsilon_0}$

$= \frac{2\pi\times10^6\sqrt{2.4}}{3\times10^8}$

$\boxed{\beta = 0.0324 \text{ rad/m}}$

$\vec{H}(y,t) = -\frac{\cancel{2\pi\times10^6}\sqrt{2.4}\,\cancel{40}^{10}}{(3\times10^8)\cancel{(2\pi\times10^6)}(4\pi\times10^{-7})}\cos(2\pi\times10^6t - 0.0324y)\vec{a}_z$ mA/m

$= -\frac{(\sqrt{2.4})(1\cancel{0})}{3\pi\times\cancel{10}}\cos(2\pi\times10^6t - 0.0324y)\vec{a}_z$ mA/m

$= -0.164\cos(2\pi\times10^6t - 0.0324y)\vec{a}_z$ mA/m

Check: $\eta = \sqrt{\frac{\mu_0}{2.4\epsilon_0}}$

$= 243.35$

$\beta = \omega\sqrt{\mu_0\epsilon_0 2.4}$

$= \frac{2\pi\times10^6\sqrt{2.4}}{3\times10^8\ \cancel{300}\ 150}$

$= 0.0324$

$\vec{H}(y,t) = -\frac{40}{243.35}\cos(2\pi\times10^6t - 0.0324y)\vec{a}_z$

$\downarrow$

0.164