University of Calgary
Schulich School of Engineering
Department of Electrical and Computer Engineering

ENEL 476 – Electromagnetic Waves and Applications

Quiz #3
Winter Session 2016
Monday March 28, 2016
2:00-2:50 pm

ICT 102

Student Name or ID number:

Dr Fear

Question 1. (12 marks)

For a given transmission line, R=100 Ω/m, L=0.32 μH/m, G=0.03 S/m and C=96 pF/m. The frequency of operation is 100 MHz. Calculate:

a) The attenuation constant (α).

Approach 1:

$\gamma = \alpha + j\beta$

(1) $= \sqrt{(R+j\omega L)(G+j\omega C)}$

(1) $= \sqrt{\underbrace{[100+j(2\pi)(10^8)(0.32\times10^{-6})]}_{R+j\omega L = 100+j201.1 \,=\, 224.6\angle 1.11\,\text{rad}}\underbrace{[(0.03+j(2\pi)(10^8)(96\times10^{-12})]}_{G+j\omega C = 0.03+j0.06 \,=\, 0.0671\angle 1.11\,\text{rad}}}$

$= 3.882\angle 1.11$

$= 1.73 + j\,3.48$ (1) $\Rightarrow \boxed{\alpha = 1.73\ \text{Np/m}}$

b) The phase constant (β).

(1) $\boxed{\beta = 3.48\ \text{rad/m}}$

Approach 2: distortionless line $\left(\frac{R}{L} = \frac{G}{C} \text{ or } \frac{R}{G} = \frac{L}{C}\right)$

OR (1) $\alpha = \sqrt{RG}$ Np

(1) $= 1.73$ Np/m

(1) $\beta = \omega\sqrt{LC}$

$= (2\pi)(10^8)\sqrt{(0.32\times10^{-6})(96\times10^{-12})}$

(1) $= 3.48$ rad/m

c) The impedance of the line (Z_o).

Approach 1:

(1) $Z_o = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

$= \sqrt{\frac{224.6\angle 1.11}{0.0671\angle 1.11}}$

(1) $\boxed{Z_o = 57.86\ \Omega}$

Approach 2:

OR (1) $Z_o = \sqrt{L/C}$

(1) $= 57.7\ \Omega$

d) The wavelength (λ)

(1) $\lambda = \frac{2\pi}{\beta}$

(1) $\lambda = 1.81\ m$

e) The phase velocity (v_p)

(1) $v_p = \omega / \beta$

(1) $= 1.81 \times 10^8\ m/s$

f) What is the distance that the forward-traveling voltage wave must travel for its amplitude to be reduced to 90% of the initial value?

(1) $V_0^+ e^{-\alpha z} = 0.9\ V_0^+$

$e^{-\alpha z} = 0.9$

$-\alpha z = \ln(0.9)$

(1) $\boxed{z = 0.061\ m}$

Question 2. (20 marks)

Consider a 75-Ω transmission line of length 0.25λ. The line is terminated with a load of Z_L=80+j50 Ω. Find:

a) The reflection coefficient at the load (Γ_L)

(1) $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$= \dfrac{(80+j50) - 75}{80+j50+75}$

$= 0.31 \angle 1.16$ rad

(1) (1)

b) The standing wave ratio (s)

(1) $s = \dfrac{1+|\Gamma|}{1-|\Gamma|}$

(1) $= 1.892$

c) The input impedance or the impedance looking into the 0.25λ length of line terminated by the load (Z_{in}).

(1) $Z_{in} = Z_0 \dfrac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$

$\beta l = \left(\dfrac{2\pi}{\lambda}\right)(0.25\lambda)$

$= \pi/2$

$\tan(\beta l) \to \infty$

$Z_0 \tan\beta l \gg Z_L$

$\therefore Z_{in} = Z_0 \left(\dfrac{jZ_0 \tan\beta l}{jZ_L \tan\beta l}\right)$

$Z_L \tan\beta l \gg Z_0$

(1) $\to Z_{in} = \dfrac{Z_0^2}{Z_L}$

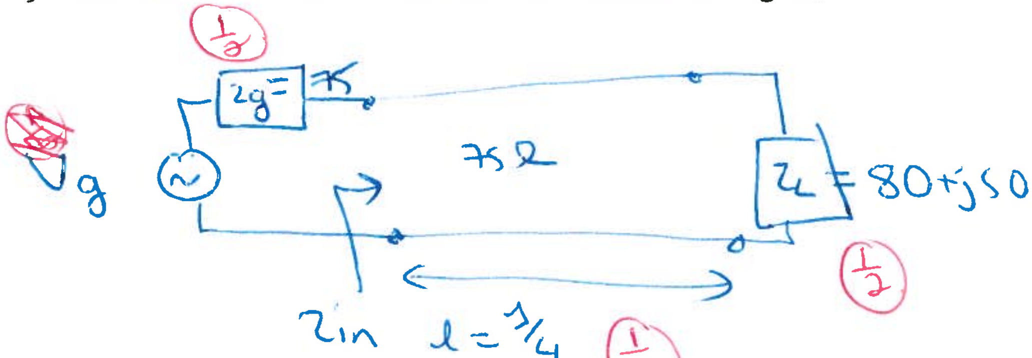
$= \dfrac{75^2}{80+j50}$

$\boxed{Z_{in} = 50.56 - j31.6\ \Omega}$

(1) (1)

The transmission line in the first part of the question is connected to a generator that delivers 10V and has internal impedance of 75Ω.

d) Draw a sketch of the source, line and load, indicating Z_{in}.



Calculate:

$\frac{1}{2}$

e) The input current to the line (I_{in})

$$I_{in} = \frac{V_g}{Z_{in} + Z_g}$$

(1)

$$= \frac{10}{75 + 50.56 - j31.62}$$

$$= 0.077\angle 0.247 \text{ rad}$$

(1) (1)

f) The power absorbed by the load (P_L)

(1) $$P_L = \frac{1}{2}|I_{in}|^2 \, \mathrm{Re}\{Z_{in}\}$$

$$= \frac{1}{2}(0.077)^2(50.56)$$

(1) $$P_L = 0.15 \text{ W}$$

g) The amplitude of the forward traveling voltage wave (V_o^+).

(1) $$V_{in} = V_0^+ e^{j\beta l} + \Gamma V_0^+ e^{-j\beta l} \Rightarrow V_{in} = V_0^+\left[e^{j\frac{\pi}{2}} + \Gamma e^{-j\frac{\pi}{2}}\right]$$

$$= V_0^+[j - j\Gamma]$$

& $$V_{in} = (V_g)\left(\frac{Z_{in}}{Z_{in} + Z_g}\right)$$

(1)

$$= 10\left(\frac{50.56 - j31.6}{50.56 - j31.6 + 75}\right)$$

$$= 4.38 - j1.41$$

$$V_0^+ = \frac{4.38 - j1.41}{j(1-\Gamma)}$$

(1)

$$V_0^+ = \frac{4.38 - j1.41}{j(1 - 0.31\angle 1.16)}$$

$$= -j5$$

(1) $\therefore V_0^+ = -5$ $\quad$ ($V_0^+ = 5$ is also fine)

$$V(z,t) = -5\cos(\omega t - \beta z + \phi) \; \Gamma 5 \cos(\omega t - \beta z + \phi + \phi_\Gamma)$$

Name	
Q1	
Q2	
Total	