Assignment 1 Solution

2.1 (a) $\overline{A}_1 = 5\angle 30° = 5\left[\cos 30° + j\sin 30°\right] = 4.33 + j\,2.5$

(b) $\overline{A}_2 = -3 + j4 = \sqrt{9+16}\;\angle \tan^{-1}\dfrac{4}{-3} = 5\angle 126.87° = 5e^{j126.87°}$

(c) $\overline{A}_3 = (4.33 + j2.5) + (-3 + j4) = 1.33 + j6.5 = 6.635\angle 78.44°$

(d) $\overline{A}_4 = (5\angle 30°)(5\angle 126.87°) = 25\angle 156.87° = -22.99 + j9.821$

(e) $\overline{A}_5 = (5\angle 30°)/(5\angle -126.87°) = 1\angle 156.87° = 1e^{j156.87°}$

2.2 (a) $\overline{I} = 400\angle -30° = 346.4 - j200$

(b) $i(t) = 5\sin(\omega t + 15°) = 5\cos(\omega t + 15° - 90°) = 5\cos(\omega t - 75°)$

$\overline{I} = \left(5/\sqrt{2}\right)\angle -75° = 3.536\angle -75° = 0.9151 - j3.415$

(c) $\overline{I} = \left(4/\sqrt{2}\right)\angle -30° + 5\angle -75° = (2.449 - j1.414) + (1.294 - j4.83)$

$= 3.743 - j6.244 = 7.28\angle -59.06°$

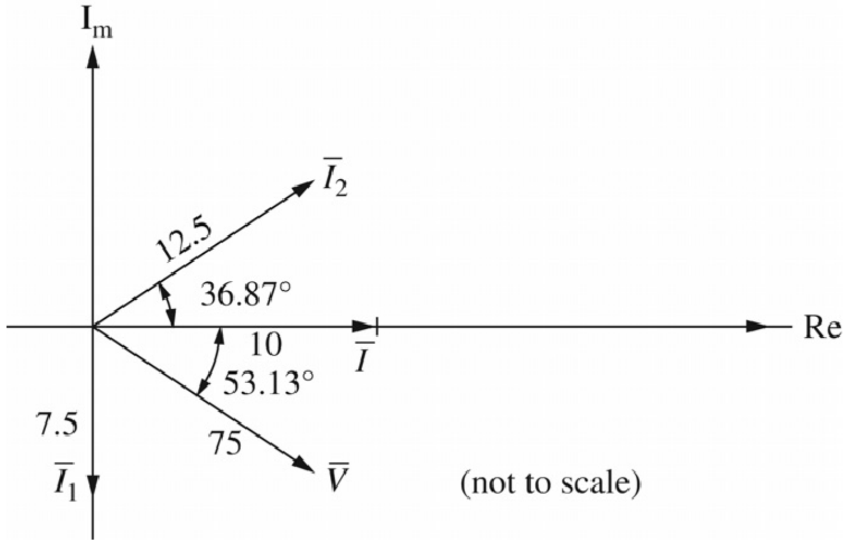
From Current division

2.4 (a) $\overline{I}_1 = 10\angle 0° \dfrac{-j6}{8 + j6 - j6} = 10\dfrac{6\angle -90°}{8} = 7.5\angle -90°\,\text{A}$

$\overline{I}_2 = \overline{I} - \overline{I}_1 = 10\angle 0° - 7.3\angle -90° = 10 + j7.5 = 12.5\angle 36.87°\,\text{A}$

$\overline{V} = \overline{I}_2(-j6) = (12.5\angle 36.87°)(6\angle -90°) = 75\angle -53.13°\,\text{V}$

(b)



2.15 (a) $\bar{I} = \left[\left(4/\sqrt{2}\right)\angle 60°\right] / \left(2\angle 30°\right) = \sqrt{2}\angle 30°\ \text{A}$

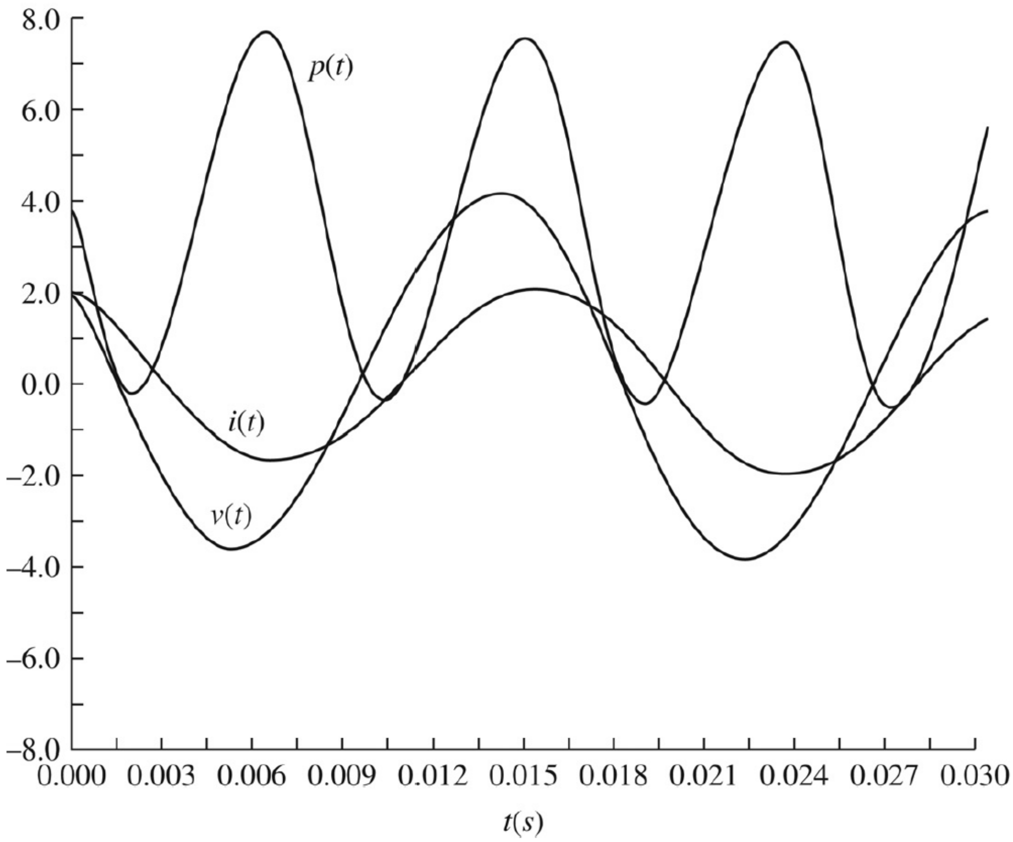
$$i(t) = 2\cos\left(\omega t + 30°\right)\text{A} \;\text{ with }\; \omega = 377\ \text{rad/s}$$

$$p(t) = \upsilon(t)i(t) = 4\left[\cos 30° + \cos\left(2\omega t + 90°\right)\right]$$

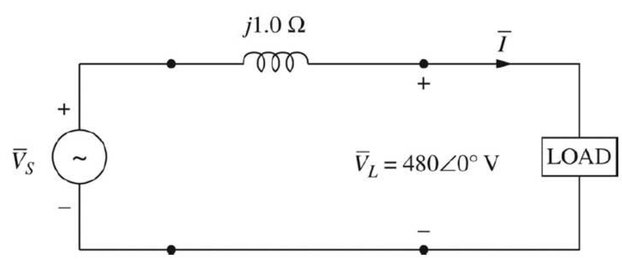
$$= 3.46 + 4\cos\left(2\omega t + 90°\right)\text{W}$$

(b) $\upsilon(t)$, $i(t)$, and $p(t)$ are plotted below: (See next page)

(c) The instantaneous power has an average value of 3.46 W, and the frequency is twice that of the voltage or current.



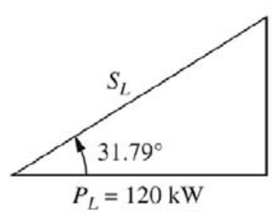
2.26 (a) The problem is modeled as shown in figure below:



$$P_L = 120\,\text{kW}$$
$$pf_L = 0.85\,\text{Lagging}$$
$$\theta_L = \cos^{-1} 0.85 = 31.79°$$

Power triangle for the load:

$$\overline{S}_L = P_L + jQ_L = 141.18\angle 31.79°\,\text{kVA}$$
$$I = S_L / V = 141{,}180/480 = 294.13\,\text{A}$$



$$Q_L = P_L \tan(31.79°)$$
$$= 74.364\,\text{kVAR}$$

Real power loss in the line is zero.

Reactive power loss in the line is $Q_{LINE} = I^2 X_{LINE} = (294.13)^2\, 1$

$$= 86.512\,\text{kVAR}$$

$$\therefore \overline{S}_S = P_S + jQ_S = 120 + j(74.364 + 86.512) = 200.7\angle 53.28^\circ\,\text{kVA}$$

The input voltage is given by $V_S = S_S / I = 682.4\,\text{V (rms)}$

The power factor at the input is $\cos 53.28^\circ = 0.6$ Lagging

(b) Applying KVL, $\overline{V}_S = 480\angle 0^\circ + j1.0(294.13\angle -31.79^\circ)$

$$= 635 + j250 = 682.4\angle 21.5^\circ\,\text{V (rms)}$$

$$(pf)_S = \cos(21.5^\circ + 31.79^\circ) = 0.6 \text{ Lagging}$$

Pf = cos (voltage angle – current angle)

2.30 (a) For load 1: $\theta_1 = \cos^{-1}(0.28) = 73.74^\circ$ Lagging

$$\overline{S}_1 = 125\angle 73.74^\circ = 35 + j120$$

$$\overline{S}_2 = 10 - j40$$

$$\overline{S}_3 = 15 + j0$$

$$\overline{S}_{TOTAL} = \overline{S}_1 + \overline{S}_2 + \overline{S}_3 = 60 + j80 = 100\angle 53.13^\circ\,\text{kVA} = P + jQ$$

$$\therefore P_{TOTAL} = 60\,\text{kW};\ Q_{TOTAL} = 80\,\text{kVAR};\ \text{kVA}_{TOTAL} = S_{TOTAL} = 100\,\text{kVA}. \leftarrow$$

$$\text{Supply } pf = \cos(53.13^\circ) = 0.6 \text{ Lagging} \leftarrow$$

(b) $$\overline{I}_{TOTAL} = \frac{\overline{S}^*}{\overline{V}^*} = \frac{100\times 10^3\angle -53.13^\circ}{1000\angle 0^\circ} = 100\angle -53.13^\circ\,\text{A}$$

At the new *pf* of 0.8 lagging, P_{TOTAL} of 60kW results in the new reactive power Q', such that

$$\theta' = \cos^{-1}(0.8) = 36.87^\circ$$

and $Q' = 60\tan\left(36.87°\right) = 45\,\text{kVAR}$

$\therefore$ The required capacitor's kVAR is $Q_C = 80 - 45 = 35\,\text{kVAR} \quad \leftarrow$

It follows then $X_C = \dfrac{V^2}{\bar{S}_C^*} = \dfrac{\left(1000\right)^2}{j35000} = -j\,28.57\,\Omega$

and $\qquad C = \dfrac{10^6}{2\pi\left(60\right)\left(28.57\right)} = 92.85\mu\text{F} \quad \leftarrow$

The new current is $I' = \dfrac{\bar{S}'^*}{\bar{V}^*} = \dfrac{60{,}000 - j45{,}000}{1000\angle 0°} = 60 - j45 = 75\angle -36.87°\,\text{A}$

The supply current, in magnitude, is reduced from 100A to 75A $\leftarrow$

Addition of capacitor bank and "correction of power factor" closer to unity resulted in lower current throught thetransmission lines, which translates to lower losses in the lines.

2.40 (a) $\bar{V}_{AN} = \dfrac{208}{\sqrt{3}}\angle 0° = 120.1\angle 0°\,\text{V}$ (Assumed as Reference)

$$\bar{V}_{AB} = 208\angle 30°\,\text{V};\ \bar{V}_{BC} = 208\angle -90°\,\text{V};\ \bar{I}_A = 10\angle -90°\,\text{A}$$

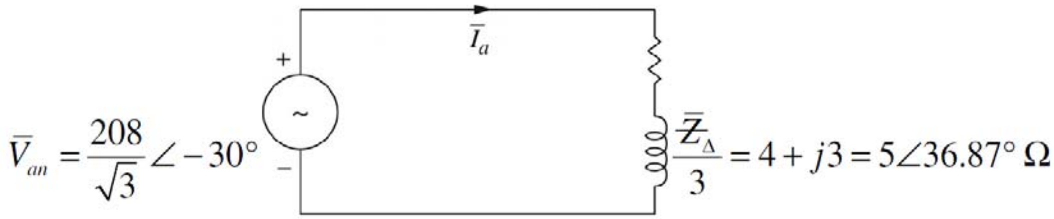
$$\bar{Z}_Y = \frac{\bar{V}_{AN}}{\bar{I}_A} = \frac{120.1\angle 0°}{10\angle -90°} = 12.01\angle +90° = \left(0 + j12.01\right)\Omega$$

(b) $\bar{I}_{AB} = \dfrac{\bar{I}_A}{\sqrt{3}}\angle 30° = \dfrac{10}{\sqrt{3}}\angle -90° + 30° = 5.774\angle -60°\text{A}$

$$\bar{Z}_\Delta = \frac{\bar{V}_{AB}}{\bar{I}_{AB}} = \frac{208\angle 30°}{5.774\angle -60°} = 36.02\angle 90° = \left(0 + j36.02\right)\Omega$$

Note: $\bar{Z}_Y = \bar{Z}_\Delta / 3$

2.42 (a) With $\overline{V}_{ab}$ as reference



$$\overline{I}_a = \frac{\overline{V}_{an}}{\left(\overline{Z}_\Delta / 3\right)} = \frac{120.1\angle -30^\circ}{5\angle 36.87^\circ} = 24.02\angle -66.87^\circ\ \text{A}$$

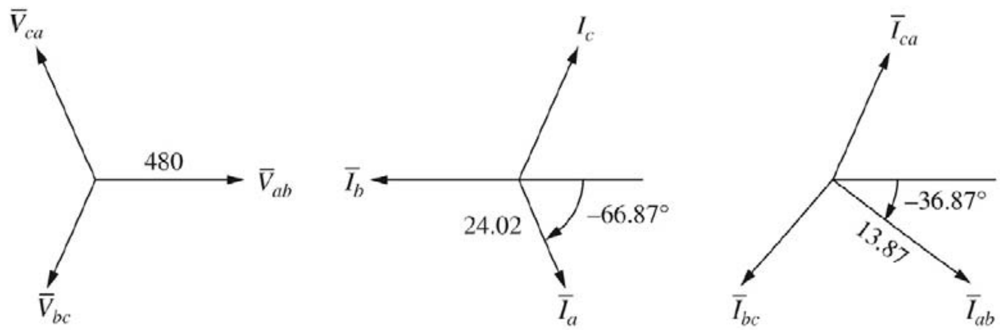
$$\overline{S}_{3\phi} = 3\overline{V}_{an}\overline{I}_a^* = 3\left(120.1\angle -30^\circ\right)\left(24.02\angle +66.87^\circ\right)$$

$$= 8654\angle 36.87^\circ = 6923 + j5192$$

$P_{3\phi} = 6923\,\text{W}; Q_{3\phi} = 5192\,\text{VAR};$ both absorbed by the load

$pf = \cos\left(36.87^\circ\right) = 0.8$ Lagging; $S_{3\phi} = \left|\overline{S}_{3\phi}\right| = 8654\,\text{VA}$

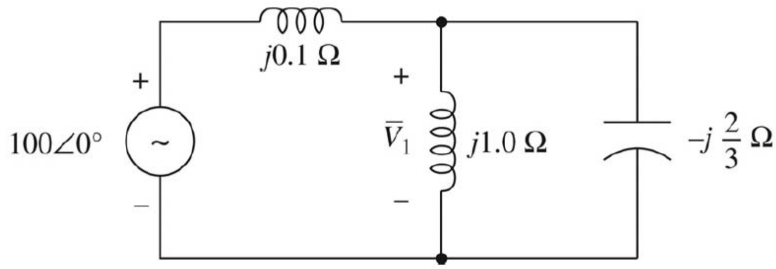
(b)



$\overline{V}_{ab} = 208\angle 0^\circ\,\text{V}$ $\quad$ $\overline{I}_a = 24.02\angle -66.87^\circ\,\text{A}$ $\quad$ $13.87\angle -36.87^\circ\,\text{A}$

2.50 Replace delta by the equivalent WYE: $\bar{Z}_Y = -j\dfrac{2}{3}\ \Omega$

Per-phase equivalent circuit is shown below:



Noting that $\left(j1.0 \,\Big\|\, -j\dfrac{2}{3}\right) = -j2$, by voltage-divider law,

$$\bar{V}_1 = \frac{-j2}{-j2 + j0.1}(100\angle 0°) = 105\angle 0°$$

$$\therefore v_1(t) = 105\sqrt{2}\cos(\omega t + 0°) = 148.5\cos\omega t \text{ V} \quad \leftarrow$$

In order to find $i_2(t)$ in the original circuit, let us calculate $\bar{V}_{A'B'}$

$$\bar{V}_{A'B'} = \bar{V}_{A'N'} - \bar{V}_{B'N'} = \sqrt{3}\,e^{j30°}\bar{V}_{A'N'} = 181.8\ \angle 30°$$

Then

$$\bar{I}_{A'B'} = \frac{181.8\ \angle 30°}{-j2} = 90.9\ \angle 120°$$

$$\therefore i_2(t) = 90.9\sqrt{2}\cos(\omega t + 120°)$$

$$= 128.6\ \cos(\omega t + 120°)\text{ A} \quad \leftarrow$$