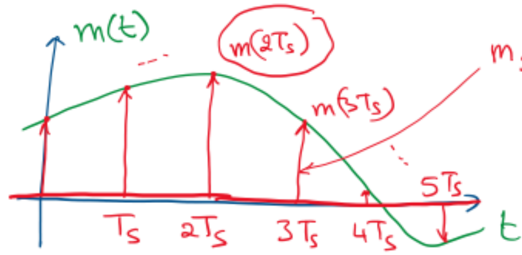
Online Lecture # 09 - Digital Baseband Modulation - Pulse Amplitude Modulation

Wednesday, April 8, 2020
8:50 AM



Ideal sampling:

$$\boxed{m_s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\,\delta(t-nT_s)}$$

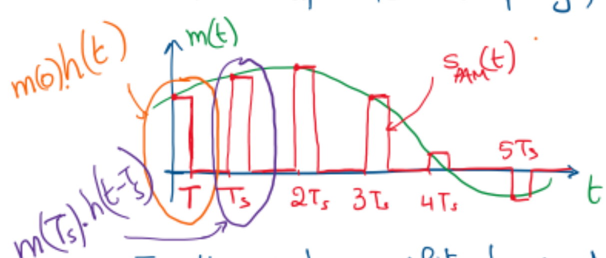
$$\delta(t-t_0)\begin{cases} = 0 & \text{for } t \neq t_0 \\ \int_{-\infty}^{\infty} \delta(t-t_0)\,dt = 1 \end{cases}$$

infinitisimally small

→ Cannot be implemented in practice.

Pulse amplitude modulation (PAM)

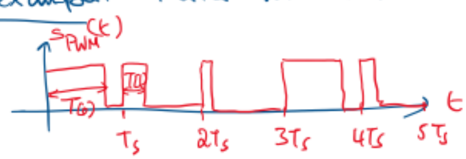
Instead of ideal sampling, we can achieve a signal as follows:



the values $m(nT_s)$ are held for a period $0 < T \leq T_s$.

In the pulse amplitude modulated signal $s_{PAM}(t)$, the information (which is the sampled points of $m(t)$: $m(nT_s)$) is encoded in the amplitude of pulses of duration T.

Another example: Pulse width modulation (PWM)



* <u>Time domain analysis of PAM signals:</u>

if we define a rectangular pulse $h(t)$ as:

$$\boxed{s_{PAM}(t) = \sum_{n=-\infty}^{\infty} m(nT_s) \cdot \underbrace{h(t-nT_s)}}$$

$$h(t) * \delta(t-nT_s) = h(t-nT_s)$$

$$\rightarrow s_{PAM}(t) = \sum_{n=-\infty}^{\infty} m(nT_s) \cdot \left(\delta(t-nT_s) * h(t)\right)$$

$$s_{PAM}(t) = \underbrace{\left[\sum_{n=-\infty}^{\infty} m(nT_s) \cdot \delta(t-nT_s)\right]}_{= m_s(t)} * h(t)$$

$$\boxed{s_{PAM}(t) = m_s(t) * h(t)}$$

* <u>Frequency domain analysis of PAM signals:</u>

$$S_{PAM}(f) = M_s(f) \cdot H(f)$$

from the previous class calculation:

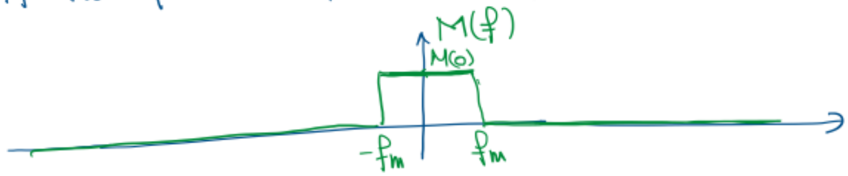
$$M_s(f) = f_s \sum_{n=-\infty}^{\infty} M(f-nf_s)$$

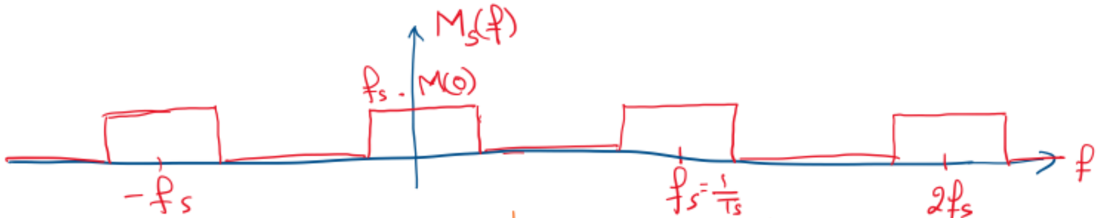
Using the Fourier Transform pairs & properties:

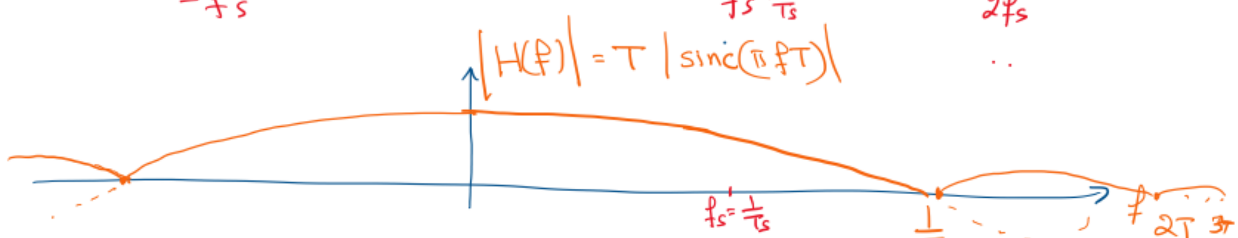
$$H(f) = T\,\mathrm{sinc}(\pi f T)\cdot e^{-j\pi f T}$$

$$\rightarrow \boxed{S_{PAM}(f) = \underbrace{f_s \sum_{n=-\infty}^{\infty} M(f-nf_s)}_{M_s(f)} \cdot \underbrace{T\,\mathrm{sinc}(\pi f T)\, e^{-j\pi f T}}_{H(f)}}$$

if the spectrum of m(t) is given by:







$H(f) = 0$ when $\pi f T = n\pi \rightarrow f = \frac{n}{T}$

$T \leq T_s \rightarrow \frac{1}{T} \geq \frac{1}{T_s} = f_s$