Set 1 - extra problems

Solutions to the following problem from the text book are provided here:
Topic 1: 2.13, 2.22, 2.24, 2.28,
Topic 2: 2.44, 2.46, 2.52

2.13 $\bar{Z} = R - jx_c = 10 - j25 = 26.93\angle -68.2°\ \Omega$

$$i(t) = (359.3/26.93)\cos(\omega t + 68.2°)$$
$$= 13.34\cos(\omega t + 68.2°)\,\text{A}$$

(a) $p_R(t) = [13.34\cos(\omega t + 68.2°)][133.4\cos(\omega t + 68.2°)]$
$$= 889.8 + 889.8\cos[2(\omega t + 68.2°)]\,\text{W}$$

(b) $p_x(t) = [13.34\cos(\omega t + 68.2°)][333.5\cos(\omega t + 68.2° - 90°)]$
$$= 2224\sin[2(\omega t + 68.2°)]\,\text{W}$$

(c) $P = I^2 R = \left(13.34/\sqrt{2}\right)^2 10 = 889.8\,\text{W}$

(d) $Q = I^2 X = \left(13.34/\sqrt{2}\right)^2 25 = 2224\,\text{VARS}$

(e) $pf = \cos\left[\tan^{-1}(Q/P)\right] = \cos\left[\tan^{-1}(2224/889.8)\right]$
$$= 0.3714 \text{ Leading}$$

2.22 (a) $\bar{Y}_1 = \dfrac{1}{\bar{Z}_1} = \dfrac{1}{(4+j5)} = \dfrac{1}{6.4\angle 51.34^\circ} = 0.16\angle -51.34^\circ$

$$= (0.1 - j0.12)\,\text{S}$$

$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{10} = 0.1\text{S}$$

$$P = V^2(G_1 + G_2) \Rightarrow V = \sqrt{\frac{P}{G_1 + G_2}} = \sqrt{\frac{1000}{(0.1+0.1)}} = 70.71\text{ V}$$

$$P_1 = V^2 G_1 = (70.71)^2\, 0.1 = 500\text{ W}$$

$$P_2 = V^2 G_2 = (70.71)^2\, 0.1 = 500\text{ W}$$

(b) $\bar{Y}_{eq} = \bar{Y}_1 + \bar{Y}_2 = (0.1 - j0.12) + 0.1 = 0.2 - j0.12$

$$= 0.233\angle -30.96^\circ\text{ S}$$

$$I_S = V\, Y_{eq} = 70.71(0.233) = 16.48\text{ A}$$

2.24 $\bar{S}_1 = P_1 + jQ_1 = 10 + j0;\ \bar{S}_2 = 10\angle\cos^{-1}0.9 = 9 + j4.359$

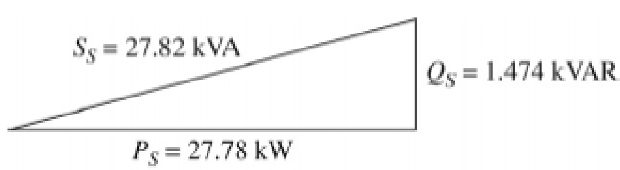
$$\bar{S}_3 = \frac{10\times 0.746}{0.85\times 0.95}\angle -\cos^{-1}0.95 = 9.238\angle -18.19^\circ = 8.776 - j2.885$$

$$\bar{S}_S = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = 27.78 + j1.474 = 27.82\angle 3.04^\circ$$

$$P_S = \text{Re}(\bar{S}_S) = 27.78\text{ kW}$$

$$Q_S = \text{Im}(\bar{S}_S) = 1.474\text{ kVAR}$$

$$S_S = |\bar{S}_S| = 27.82\text{ kVA}$$



2.28 $\overline{S}_1 = 15 + j6.667$

$\overline{S}_2 = 3(0.96) - j3\left[\sin\left(\cos^{-1} 0.96\right)\right] = 2.88 - j0.84$

$\overline{S}_3 = 15 + j0$

$\overline{S}_{TOTAL} = \overline{S}_1 + \overline{S}_2 + \overline{S}_3 = (32.88 + j5.827)\,\text{kVA}$

(i) Let $\overline{Z}$ be the impedance of a series combination of R and X

Since $\overline{S} = \overline{V}\,\overline{I}^* = \overline{V}\left(\dfrac{\overline{V}}{\overline{Z}}\right)^* = \dfrac{V^2}{\overline{Z}^*}$, it follows that

$$\overline{Z}^* = \frac{V^2}{\overline{S}} = \frac{(240)^2}{(32.88 + j5.827)10^3} = (1.698 - j0.301)\,\Omega$$

$$\therefore \overline{Z} = (1.698 + j0.301)\,\Omega \quad \leftarrow$$

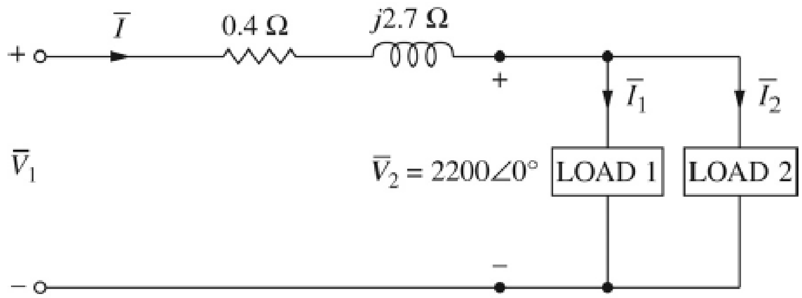
(ii) Let $\overline{Z}$ be the impedance of a parallel combination of R and X

Then $$R = \frac{(240)^2}{(32.88)10^3} = 1.7518\,\Omega$$

$$X = \frac{(240)^2}{(5.827)10^3} = 9.885\,\Omega$$

$$\therefore \overline{Z} = (1.7518 \,\|\, j9.885)\,\Omega \quad \leftarrow$$

2.44 (a) The per-phase equivalent circuit for the problem is shown below:



Phase voltage at the load terminals is $V_2 = \dfrac{2200\sqrt{3}}{\sqrt{3}} = 2200\,\text{V}$ taken as Ref.

Total complex power at the load end or receiving end is

$$\overline{S}_{R(3\phi)} = 560.1(0.707 + j0.707) + 132 = 528 + j396 = 660\angle 36.87°\,\text{kVA}$$

With phase voltage $\overline{V}_2$ as reference,

$$\overline{I} = \frac{\overline{S}^*_{R(3\phi)}}{3\overline{V}_2^*} = \frac{660{,}000\angle -36.87°}{3(2200\angle 0°)} = 100\angle -36.87°\,\text{A}$$

Phase voltage at sending end is given by

$$\overline{V}_1 = 2200\angle 0° + (0.4 + j2.7)(100\angle -36.87°) = 2401.7\angle 4.58°\,\text{V}$$

The magnitude of the line to line voltage at the sending end of the line is

$$(V_1)_{L\text{-}L} = \sqrt{3}V_1 = \sqrt{3}(2401.7) = 4160\,\text{V}$$

(b) The three-phase complex-power loss in the line is given by

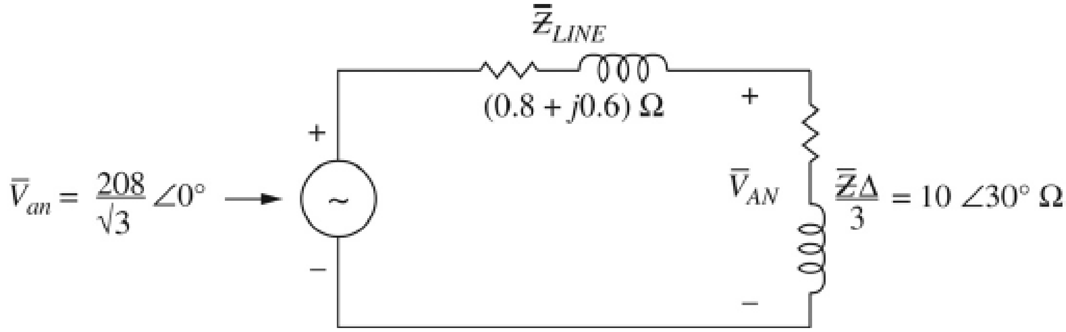
$$\overline{S}_{L(3\phi)} = 3RI^2 + j3\times I^2 = 3(0.4)(100^2) + j3(2.7)(100)^2$$
$$= 12\,\text{kW} + j81\,\text{kVAR}$$

(c) The three-phase sending power is

$$\overline{S}_{S(3\phi)} = 3\overline{V}_1\overline{I}^* = 3(2401.7\angle 4.58°)(100\angle 36.87°)$$
$$= 540\,\text{kW} + j477\,\text{kVAR}$$

Note that $\overline{S}_{S(3\phi)} = \overline{S}_{R(3\phi)} + \overline{S}_{L(3\phi)}$

2.46 (a)



Using voltage division: $\bar{V}_{AN} = \bar{V}_{an} \dfrac{\bar{Z}_\Delta / 3}{(\bar{Z}_\Delta / 3) + \bar{Z}_{LINE}}$

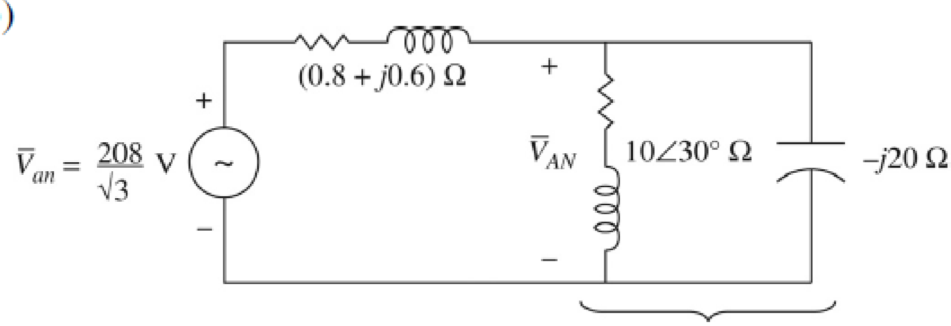
$$= \frac{208}{\sqrt{3}} \angle 0° \frac{10\angle 30°}{10\angle 30° + (0.8 + j0.6)}$$

$$= \frac{(120.09)(10\angle 30°)}{9.46 + j5.6} = \frac{1200.9\angle 30°}{10.99\angle 30.62°}$$

$$= 109.3\angle -0.62° \text{ V}$$

Load voltage = $V_{AB} = \sqrt{3}\,(109.3) = 189.3$ V Line-to-Line

(b)



$$\bar{Z}_{eq} = 10 \angle 30° \,||\, (-j20)$$

$$= 11.547 \angle 0° \Omega$$

$$\bar{V}_{AN} = \bar{V}_{an} \frac{\bar{Z}_{eq}}{\bar{Z}_{eq} + \bar{Z}_{LINE}}$$

$$= \left(208/\sqrt{3}\right) \frac{11.547}{(11.547 + 0.8 + j0.6)}$$

$$= \frac{1386.7}{12.362\angle 2.78^\circ} = 112.2\angle -2.78^\circ \text{ V}$$

Load voltage Line-to-Line $V_{AB} = \sqrt{3}\,(112.2) = 194.3$ V

2.52 (a) Let $\bar{V}_{AN}$ be the reference: $\bar{V}_{AN} = \frac{2160}{\sqrt{3}}\angle 0^\circ \simeq 2400\angle 0^\circ$ V

Total impedance per phase $\bar{Z} = (4.7 + j9) + (0.3 + j1) = (5 + j10)\,\Omega$

$\therefore$ Line Current $= \frac{2400\angle 0^\circ}{5 + j10} = 214.7\angle -63.4^\circ \text{ A} = \bar{I}_A \quad \leftarrow$

With positive A-B-C phase sequence,

$$\bar{I}_B = 214.7\angle -183.4^\circ \text{ A};\ \bar{I}_C = 214.7\angle -303.4^\circ = 214.7\angle 56.6^\circ \text{ A} \quad \leftarrow$$

(b) $\left(\bar{V}_{A'N}\right)_{LOAD} = 2400\angle 0^\circ - \left[(214.7\angle -63.4^\circ)(0.3 + j1)\right]$

$= 2400\angle 0^\circ - 224.15\angle 9.9^\circ = 2179.2 - j38.54$

$= 2179.5\angle -1.01^\circ \text{V} \quad \leftarrow$

$\left(\bar{V}_{B'N}\right)_{LOAD} = 2179.5\angle -121.01^\circ \text{V}□;\ \left(V_{C'N}\right)_{LOAD} = 2179.5\angle -241.01^\circ \text{V}□$

(c) $S/\text{Phase} = \left(V_{A'N}\right)_{LOAD} I_A = (2179.5)(214.7) = 467.94 \text{ kVA} \quad \leftarrow$

Total apparent power dissipated in all three phases in the load

$$\left[S_{3\phi}\right]_{LOAD} = 3(467.94) = 1403.82 \text{ kVA} \quad \leftarrow$$

Active power dissipated per phase in load = $\left(P_{1\phi}\right)_{LOAD}$

$$=(2179.5)(214.7)\cos(62.39°)=216.87\,\text{kW} \quad \leftarrow$$

$$\therefore \left[P_{3\phi}\right]_{LOAD}=3(216.87)=650.61\,\text{kW} \quad \leftarrow$$

Reactive power dissipated per phase in load = $\left(Q_{1\phi}\right)_{LOAD}$

$$=(2179.5)(214.7)\sin(62.39°)=414.65\,\text{kVAR} \quad \leftarrow$$

$$\therefore \left[Q_{3\phi}\right]_{LOAD}=3(414.65)=1243.95\,\text{kVAR} \quad \leftarrow$$

(d) Line losses per phase $\left(P_{1\phi}\right)_{LOSS}=(214.7)^2\,0.3=13.83\,\text{kW} \quad \leftarrow$

Total line loss $\left(P_{3\phi}\right)_{LOSS}=13.83\times 3=41.49\,\text{kW} \quad \leftarrow$