

Assignment 3 Solution

4.10

$$D = 4 \text{ ft}$$

$$L_1 = 2 \times 10^{-7} \ln \left(\frac{D}{r'} \right) \frac{\text{H}}{\text{m}}$$

$$r' = e^{\frac{-1}{4}} \left(\frac{.5}{2} \right) \left(\frac{1 \text{ ft}}{12 \text{ in}} \right)$$

$$L_1 = 2 \times 10^{-7} \ln \left(\frac{4}{1.6225 \times 10^{-2}} \right)$$

$$r' = 1.6225 \times 10^{-2} \text{ ft}$$

$$L_1 = \underline{\underline{1.101 \times 10^{-6} \frac{\text{H}}{\text{m}}}}$$

$$X_1 = \omega L_1 = (2\pi 60)(1.101 \times 10^{-6})(1000) = \underline{\underline{0.4153 \Omega / \text{km}}}$$

4.18

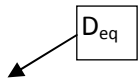
$$D_{eq} = \sqrt[3]{7 \times 7 \times 14} = 8.819 \text{ m}$$

$$\text{From Table A.4, } D_s = (0.0403 \text{ ft}) \frac{1 \text{ m}}{3.28 \text{ ft}} = 0.0123 \text{ m}$$

$$L_i = 2 \times 10^{-7} \ln(D_{eq} / D_s) = 2 \times 10^{-7} \ln \left(\frac{8.819}{0.0123} \right) = 1.315 \times 10^{-6} \text{ H/m}$$

$$X_1 = 2\pi(60)L_1 = 2\pi(60)1.315 \times 10^{-6} \frac{\Omega}{\text{m}} \times \frac{1000 \text{ m}}{1 \text{ km}} = 0.496 \Omega / \text{km}$$

4.25



$$\text{GMD} = [(41.76)(80)(41.76)]^{1/3}, \text{ using } \sqrt{40^2 + 12^2} = 41.76. \\ = 51.78 \text{ ft.}$$

[Note: from Table A.4, conductor diameter. = 1.196 in.; $r = \frac{1.196}{2} \times \frac{1}{12} = 0.0498 \text{ ft.}$] and
conductor GMR = 0.0403 ft.

$$\text{GMR for the bundle: } 1.091 \left[(0.0403)(1.667)^3 \right]^{1/4} = 0.7171 \text{ ft.}$$

Please notice the units here.
1 mile = 1609.34 m

$$\therefore X = 0.2794 \log \left(\frac{51.87}{0.7171} \right) = 0.5195 \Omega / \text{mi} \leftarrow$$

Rated current carrying capacity for each conductor in the bundle, as per Table A.4, is 1010 A; since it is a 4-conductor bundle, rated current carrying capacity of the overhead line is

$$1010 \times 4 = 4040 \text{ A} \leftarrow$$

$$4.34 \quad C_1 = \frac{2\pi\epsilon_0}{\ln\left(\frac{D}{r}\right)} = \frac{2\pi(8.854 \times 10^{-12})}{\ln\left(\frac{4}{0.25/12}\right)} = \underline{\underline{1.058 \times 10^{-11} \frac{\text{F}}{\text{m}}}}$$

$$\begin{aligned} \bar{Y}_1 &= j\omega C_1 = j(2\pi 60)(1.058 \times 10^{-11})(1000) \\ &= \underline{\underline{j3.989 \times 10^{-6} \frac{\text{S}}{\text{km}}}} \end{aligned}$$

$$4.39 \quad D_{eq} = \sqrt[3]{7 \times 7 \times 14} = 8.819 \text{ m}$$

$$\text{For Table A.4, } r = \frac{1.196}{2} \ln \left(\frac{0.0254 \text{ m}}{1 \text{ in}} \right) = 0.01519 \text{ m}$$

$$C_1 = \frac{2\pi\epsilon_0}{\ln\left(\frac{D}{r}\right)} = \frac{2\pi(8.854 \times 10^{-12})}{\ln\left(\frac{8.819}{0.01519}\right)} = 8.742 \times 10^{-12} \text{ F/m}$$

$$\bar{Y}_1 = j\omega C_1 = j2\pi(60)8.742 \times 10^{-12}(1000) = j3.296 \times 10^{-6} \text{ S/km}$$

For a 110 km line length

$$I_{chg} = Y_1 V_{LN} = (3.296 \times 10^{-6} \times 110) \left(230 / \sqrt{3} \right) = 4.814 \times 10^{-2} \text{ kA/Phase}$$

$$5.2 \quad (a) \quad \bar{A} = \bar{D} = 1 + \frac{\bar{Y} \bar{Z}}{2} = 1 + \frac{1}{2} (3.33 \times 10^{-6} \times 200 \angle 90^\circ) (0.08 + j0.48) (200) \\ = 1 + (0.0324 \angle 170.5^\circ) = 0.968 + j0.00533 = 0.968 \angle 0.315^\circ \text{ pu}$$

$$\bar{C} = \bar{Y} \left(1 + \frac{\bar{Y} \bar{Z}}{4} \right) = (6.66 \times 10^{-4} \angle 90^\circ) (1 + 0.0162 \angle 170.5^\circ) \\ = 6.553 \times 10^{-4} \angle 90.155^\circ \text{ S}$$

$$\bar{B} = \bar{Z} = 97.32 \angle 80.54^\circ \Omega$$

$$(b) \quad \bar{V}_R = \frac{220}{\sqrt{3}} \angle 0^\circ = 127 \angle 0^\circ \text{ kV}_{L-N}$$

$$\bar{I}_R = \frac{P_R \angle -\cos^{-1}(pf)}{\sqrt{3} V_{R-L-L} (pf)} = \frac{250 \angle -\cos^{-1} 0.99}{\sqrt{3} (220) (0.99)} = 0.6627 \angle -8.11^\circ \text{ kA}$$

Same as:
 $|I| = S / (\sqrt{3} V_{LL})$ & $\theta_i = \cos^{-1}(pf)$

$$\bar{V}_S = \bar{A} \bar{V}_R + \bar{B} \bar{I}_R = (0.968 \angle 0.315^\circ) (127 \angle 0^\circ) + (97.32 \angle 80.54^\circ) (0.6627 \angle -8.11^\circ) \\ = 142.4 + j62.16 = 155.4 \angle 23.58^\circ \text{ kV}_{L-N}$$

$$\bar{V}_{S-L-L} = 155.4 \sqrt{3} = 269.2 \text{ kV}$$

$$\bar{I}_S = \bar{C} \bar{V}_R + \bar{D} \bar{I}_R = (6.553 \times 10^{-4} \angle 90.155^\circ) (127) + (0.968 \angle -0.315^\circ) (0.6627 \angle -8.11^\circ) \\ = 0.6353 - j3.786 \times 10^{-3} = 0.6353 \angle -0.34^\circ \text{ kA}$$

$$5.14 \quad (a) \quad \bar{Z}_C = \sqrt{\frac{\bar{Z}}{\bar{Y}}} = \sqrt{\frac{0.03 + j0.35}{4.4 \times 10^{-6} \angle 90^\circ}} = 282.6 \angle -2.45^\circ \Omega$$

$$(b) \quad \bar{\gamma} l = \sqrt{\bar{Z} \bar{Y}} (l) = \sqrt{(0.35128 \angle 85.101^\circ) (4.4 \times 10^{-6} \angle 90^\circ)} (500) \\ = 0.622 \angle 87.55^\circ = 0.02657 + j0.6210$$

$$(c) \quad \bar{A} = \bar{D} = \cosh \bar{\gamma} l = \cosh (0.02657 + j0.6210) \\ = (\cosh 0.02657) (\cos 0.6210 \text{ radians}) + j (\sinh 0.02657) (\sin 0.6210 \text{ radians}) \\ = (1.00035) (0.8133) + j (0.02657) (0.5818) \\ = 0.8136 + j0.015458 = 0.8137 \angle 1.09^\circ \\ \sinh \bar{\gamma} l = \sinh (0.02657 + j0.6210) \\ = \sinh (0.02657) \cos (0.6210 \text{ radians}) + j (\cosh 0.02657) (\sin 0.6210 \text{ radians}) \\ = (0.02657) (0.8133) + j (1.0004) (0.5818) \\ = 0.0216 + j0.5820 = 0.5824 \angle 87.88^\circ \\ \bar{B} = \bar{Z}_C \sinh (\bar{\gamma} l) = (282.6 \angle -2.45^\circ) (0.5824 \angle 87.88^\circ) \\ = 164.6 \angle 85.43^\circ \Omega$$

$$\bar{C} = \frac{1}{\bar{Z}_C} \sinh (\bar{\gamma} l) = \frac{0.5824 \angle 87.88^\circ}{282.6 \angle -2.45^\circ} = 2.061 \times 10^{-3} \angle 90.3^\circ \text{ S}$$

$$5.26 \quad (a) \quad \bar{Z}_C = \sqrt{\frac{\bar{Z}}{\bar{Y}}} = \sqrt{\frac{j0.34}{j4.5 \times 10^{-6}}} = 274.9 \angle 0^\circ = 274.9 \, \Omega$$

$$(b) \quad \bar{\gamma}l = \sqrt{\bar{Z} \bar{Y}} (l) = \sqrt{(j0.34)(j4.5 \times 10^{-6})} (350) = j0.4330$$

$$5.41 \quad (a) \quad \bar{Z} = \bar{Z}l = (0.088 + j0.465)100 = 8.8 + j46.5 \, \Omega$$

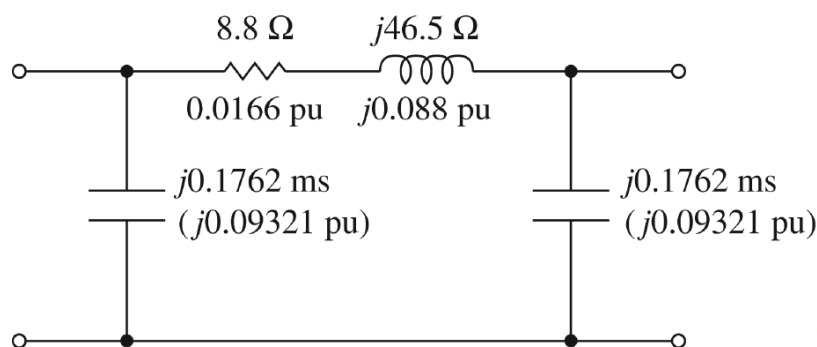
$$\frac{\bar{Y}}{2} = \frac{\bar{y}l}{2} = (j3.524 \times 10^{-6})100/2 = j0.1762 \text{ mS}$$

$$\bar{Z}_{base} = V_{L \text{ base}}^2 / S_{3\phi \text{ base}} = \frac{(230)^2}{100} = 529 \, \Omega$$

$$\therefore \bar{Z} = (8.8 + j46.5) / 529 = 0.0166 + j0.088$$

$$\frac{\bar{Y}}{2} = j0.1762 / (1/0.529) = j0.09321$$

The nominal π circuit for the medium line is shown below:



$$(b) \quad S_{3\phi \text{ rated}} = V_{L \text{ rated}} I_{L \text{ rated}} \sqrt{3} = 230(0.9)\sqrt{3} = 358.5 \text{ MVA}$$

$$(c) \quad \bar{A} = \bar{D} = 1 + \frac{\bar{Z} \bar{Y}}{2} = 1 + (8.8 + j46.5)(0.1762 \times 10^{-3}) = 0.9918 \angle 0.1^\circ$$

$$\bar{B} = \bar{Z} = 8.8 + j46.5 = 47.32 \angle 79.3^\circ \, \Omega$$

$$\bar{C} = \bar{Y} + \frac{\bar{Z} \bar{Y}^2}{4} = 0.351 \angle 90.04^\circ \text{ mS}$$