Uniform plane waves

special case of ME.
↳ TEM wave → transverse electromagnetic
- ↳ $\vec{E}$ & $\vec{H}$ are perpendicular to each other
- ↳ $\vec{E}$ & $\vec{H}$ are perpendicular to direction of propagation

special case of TEM
↳ plane wave → equiphase surface is a plane

Special case of plane wave
↳ uniform plane wave → on equiphase surface, equal amplitude of field value

UPW → assume $\vec{E} = E_x \hat{a}_x$ & travels in z

$\therefore \vec{H} = H_y \hat{a}_y$

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \Rightarrow \frac{\partial E_x}{\partial z} \vec{a}_y = -\mu_0 \frac{\partial H_y}{\partial t} \vec{a}_y \quad (1)$$

$$\nabla \times \vec{H} = \frac{\partial}{\partial t} \epsilon_0 \vec{E} \Rightarrow \frac{\partial H_y}{\partial z} \vec{a}_x = -\epsilon_0 \frac{\partial E_x}{\partial t} \vec{a}_x \quad (2)$$

$$(1) \rightarrow \frac{\partial^2 E_x}{\partial z^2} = -\mu_0 \frac{\partial^2 H_y}{\partial z \partial t}$$

$$(2) \rightarrow \frac{\partial^2 H_y}{\partial z \partial t} = -\epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

$$\Rightarrow \frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

↳ wave equation for E_x

Similarly, $\frac{\partial^2 H_y}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 H_y}{\partial t^2}$

Solution:

phase shift

$$E_x(z,t) = \underbrace{E_{x0}^+}_{\text{amplitude}} \cos(\omega t - \beta_0 z + \phi_i^+) + E_{x0}^- \cos(\omega t + \beta_0 z + \phi_i^-)$$

frequency   phase constant

$$\frac{\partial E_{\cancel{x}}}{\partial z} =$$

$$\frac{\partial^2 E_x}{\partial z} = -E_{x0}^+ \beta_0^2 \cos(\omega t - \beta_0 z + \phi_i^+)$$

$$\frac{\partial}{\partial t} E_x =$$

$$\frac{\partial^2 E_x}{\partial t^2} = -\omega^2 E_{x0}^+ \cos(\omega t - \beta_0 z + \phi_i^+)$$

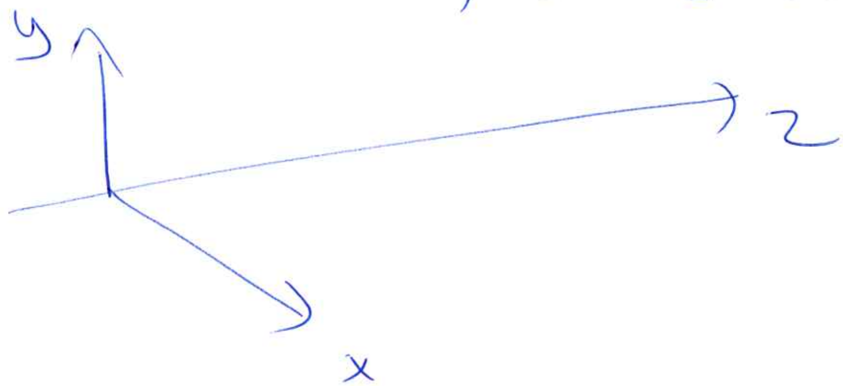
$$\frac{\partial E_x^2}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

$$\Rightarrow \cancel{-E_{x0}^+} \beta_0^2 \cancel{\cos(\omega t - \beta_0 z + \phi_i^+)} = \cancel{-}\omega^2 \mu_0 \epsilon_0 \cancel{E_{x0}^+ \cos(\omega t - \beta_0 z + \phi_i^+)}$$

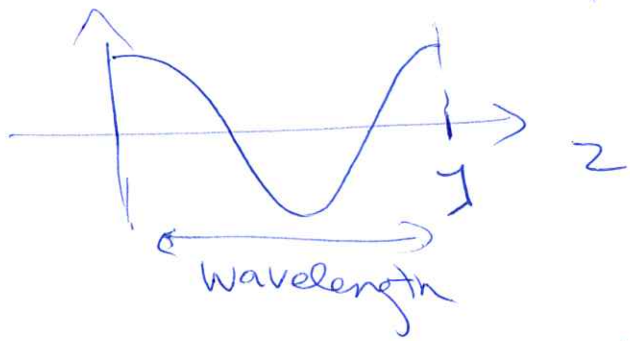
$$\Rightarrow \beta_0^2 = \omega^2 \mu_0 \epsilon_0 \Rightarrow \beta_0 = \omega \sqrt{\mu_0 \epsilon_0}$$

Consider $\vec{E} = E_0 \cos(\omega t - \beta_0 z)\,\vec{a}_x$

$\rightarrow$ set $t=0$, $\vec{E} = E_0 \cos(-\beta_0 z)\,\vec{a}_x$



$$\beta_0 = \omega\sqrt{\mu_0\epsilon_0}$$
$$= 2\pi f\sqrt{\mu_0\epsilon_0} \quad \Rightarrow \quad c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$
$$= \frac{2\pi f}{c} \qquad c/f = \lambda$$
$$= \frac{2\pi}{\lambda} \quad \underline{\text{or}} \quad \lambda = \frac{2\pi}{\beta_0}$$



$\rightarrow$ direction of propagation

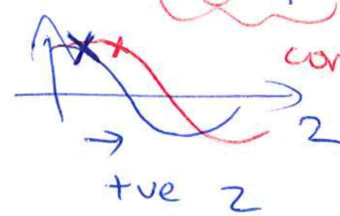
direction of propagation

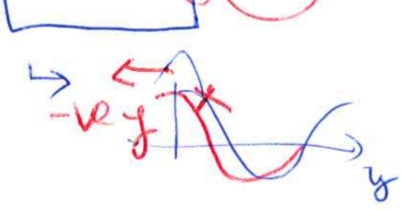
orientation of field

$\vec{E}_1 = 10\cos(\omega t - \beta_0 z)\,\hat{a}_x \Rightarrow \cos(\underset{\text{incr}}{\omega t} - \beta_0 \underset{\text{incr}}{z})$

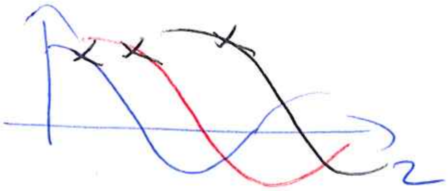
constant

$\vec{E}_2 = 20\cos(\omega t + \beta_0 y)\,\hat{a}_z$





→ phase velocity → v_p → u ⇒ speed of propagation of wave



→ $\cos(\omega t - \beta_0 z)$

$\omega t - \beta_0 z$ = constant to move with wave

$$\Rightarrow \omega - \beta_0 \frac{dz}{dt} = 0$$

$$\frac{dz}{dt} = \frac{\omega}{\beta_0}$$

$$v_p = \omega/\beta_0$$

$$= \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{\mu_0\epsilon_0}}$$

$$= \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

$$= c$$

→ magnetic field = ? $\vec{E}(x,t) = E_{x0}\cos(\omega t - \beta_0 z)\vec{a}_x$

$$\vec{E}_s = E_{x0} e^{-j\beta_0 z}\vec{a}_x$$

$$\nabla \times \vec{E}_s = -j\omega\mu_0\vec{H}_s$$

$$\nabla \times \vec{E}_s = -j\beta_0 E_{x0} e^{-j\beta_0 z}\vec{a}_y$$

↑ electric field

$$= -j\omega\mu_0\vec{H}_s \Rightarrow \vec{H}_s = \frac{\beta_0}{\omega\mu_0} E_{x0} e^{-j\beta_0 z}\vec{a}_y$$

direction

$\beta_0 = \omega\sqrt{\mu_0\epsilon_0}$ ⇒ $\sqrt{\epsilon_0/\mu_0}$

$\eta_0 = \sqrt{\mu_0 / \epsilon_0}$

$\uparrow$

eta

$= 120\pi\ \Omega$

$= 377\ \Omega$

$$\vec{H}_s = \frac{E_{x0}\, e^{-j\beta_0 z}}{\eta_0}\ \vec{a}_y$$

divide by constant

Right hand rule