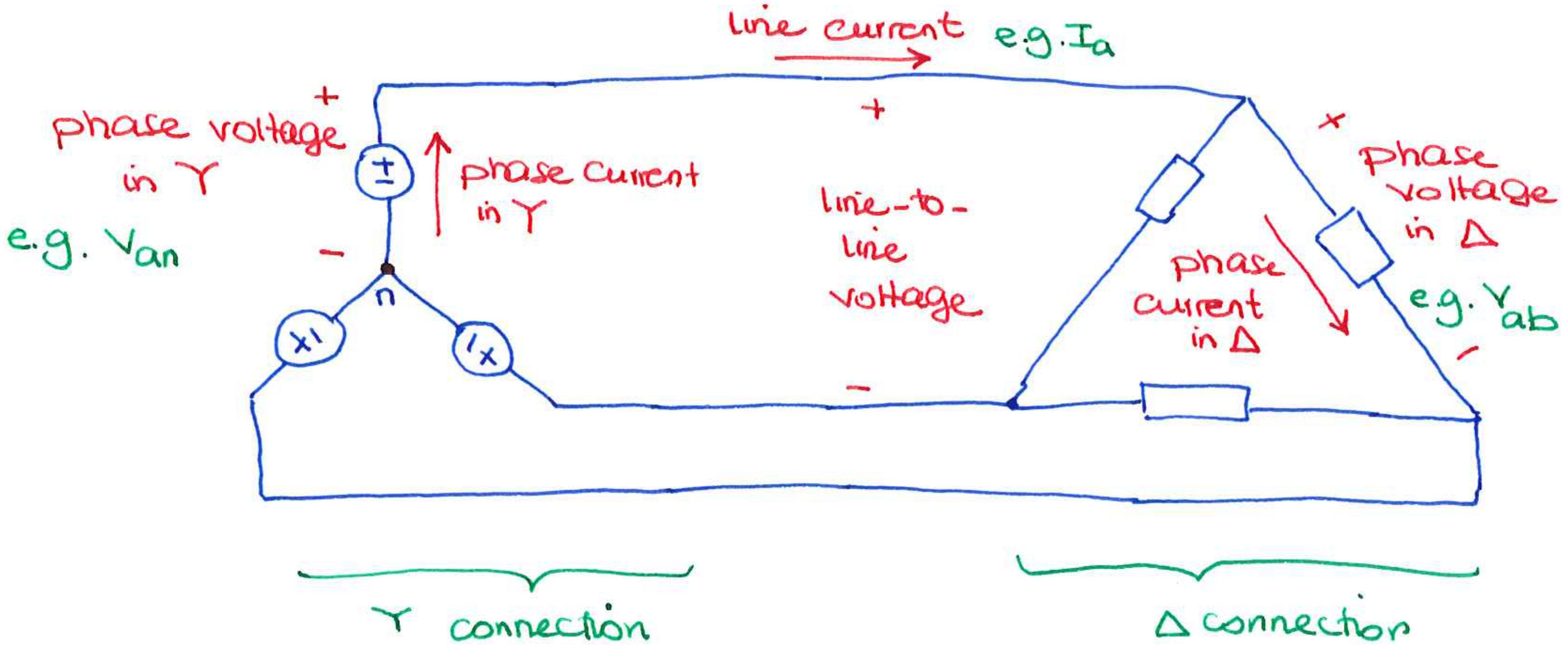
Important: Voltages & Currents in 3ϕ systems

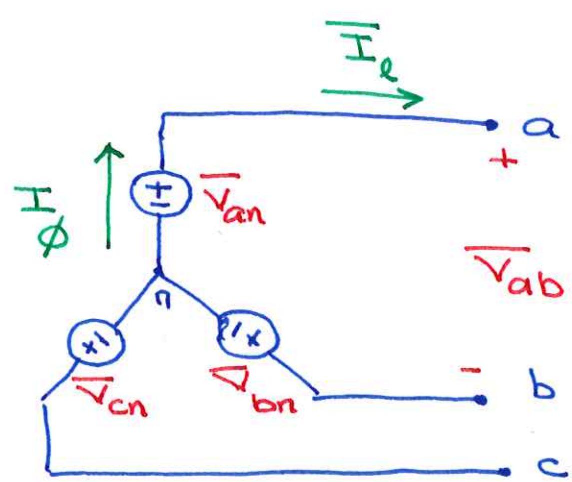
- Voltage in each phase of a device (gen, load) is <u>phase voltage</u>
- Current in each phase of a device (gen, load) is <u>phase current</u>
- Voltages between phases (lines) is <u>line-to-line</u> (phase-to-phase) voltage
- Current in a line is <u>line current</u>.



Y-connected Source / Load

- Phase current = line current

$$\boxed{\bar{I}_\ell = \bar{I}_\phi}$$



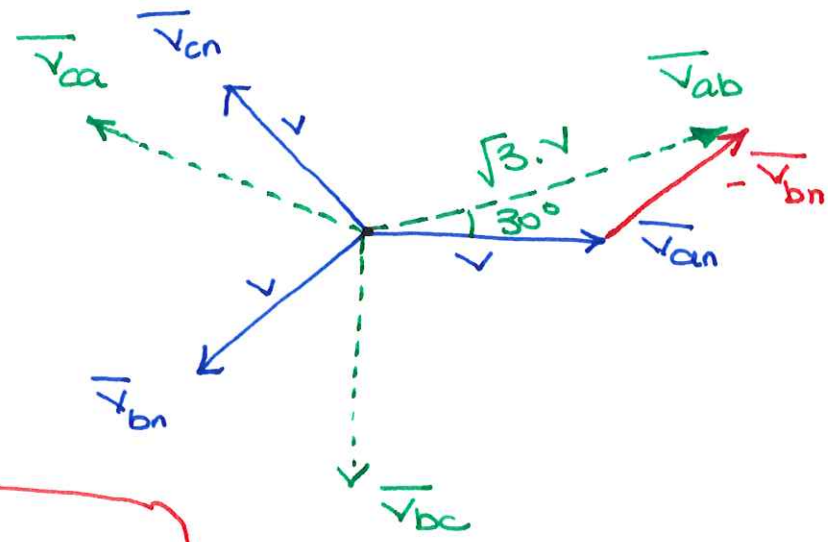
- Y-connected source shown here. Same relationships for Y load.

· Phase voltages & line-to-line voltages are related by:

$$\overline{V_{ab}} = \overline{V_{an}} - \overline{V_{bn}}$$

$$= V\angle\alpha - V\angle\alpha - 120^\circ$$

$$= \sqrt{3}.V \angle\alpha + 30^\circ$$



i.e. $V_{\ell n}$ for Y

$$\boxed{\begin{aligned} &\text{mag: } V_{\ell\text{-}\ell} = \sqrt{3}.V_{\phi} \\ &\text{phase: } \theta_{V_{\ell\text{-}\ell}} = \theta_{V_{\phi}} + 30^\circ \\ &\underline{\text{OR}} \quad \overline{V_{\ell\text{-}\ell}} = \overline{V_{\phi}}\,.\,\sqrt{3}\angle 30^\circ \end{aligned}}$$

· Line-to-line voltages lead corresponding phase (line-to-neutral) voltages by 30°. $\overline{V_{ab}}$ leads $\overline{V_{an}}$, $\overline{V_{bc}}$ leads $\overline{V_{bn}}$, $\overline{V_{ca}}$ leads $\overline{V_{cn}}$ by 30°

· Line-to-line voltages are also balanced: $\overline{V_{ab}} + \overline{V_{bc}} + \overline{V_{ca}} = 0$

· Power in 3ϕ = 3 × power in one phase

$$\left.\begin{aligned} \overline{S_{3\phi}} &= 3.\,\overline{S_{1\phi}} \\ &= 3.\,\overline{V_{\phi}}\,.\,\overline{I_{\phi}}^{*} \end{aligned}\right\} \text{ true for Y or } \Delta$$

therefore,

$$P_{3\phi} = 3.V_{\phi}.I_{\phi}.\underbrace{\cos(\theta_{V_\phi} - \theta_{I_\phi})}_{PF}$$

$$S_{3\phi} = |\overline{S_{3\phi}}| = 3.V_{\phi}.I_{\phi}$$

$$P_{3\phi} = S_{3\phi}.PF \quad , \quad Q_{3\phi} = \sqrt{S_{3\phi}^2 - P_{3\phi}^2}$$

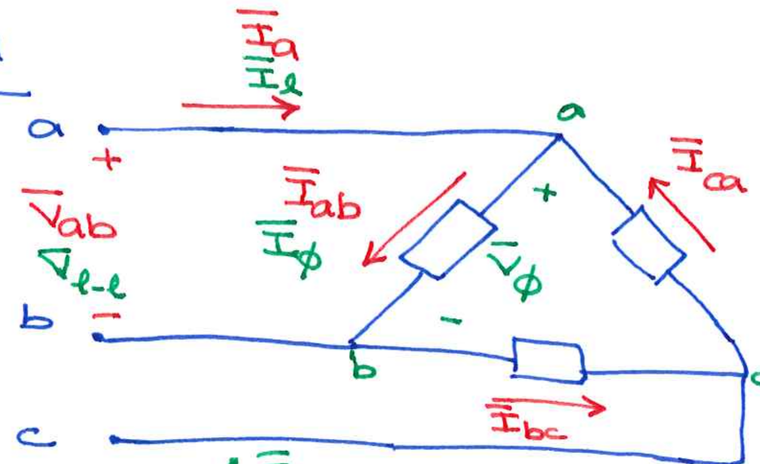
However, in Y connection : $V_\phi = \frac{V_{\ell\text{-}\ell}}{\sqrt{3}}$, $I_\phi = I_\ell$

$\therefore$ $\boxed{S_{3\phi} = \sqrt{3} \cdot V_{\ell\text{-}\ell} \cdot I_\ell}$

For all 3ϕ systems

important : Unless otherwise stated, voltages are expressed as line-to-line quantities, currents are expressed as line quantities, and power is expressed as total 3ϕ quantity.

Delta-Connected Source / Load



- Phase voltage = line-to-line voltage

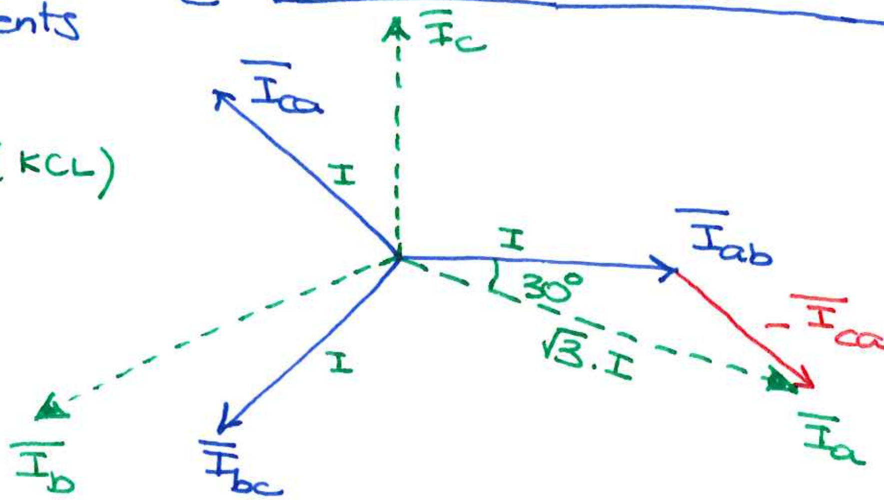
$$\boxed{\overline{V_\phi} = \overline{V_{\ell\text{-}\ell}}}$$

- Phase currents & line currents are related by:

$\overline{I_a} = \overline{I_{ab}} - \overline{I_{ca}}$ (KCL)

$= I\angle\beta - I\angle\beta+120^\circ$

$= \sqrt{3}\, I \angle\beta-30^\circ$



i.e.

mag: $I_\ell = \sqrt{3} \cdot I_\phi$ OR $\overline{I_\ell} = \overline{I_\phi} \cdot \sqrt{3}\angle-30^\circ$

phase: $\theta_{I_\ell} = \theta_{I_\phi} - 30^\circ$

- Similar to Y connection, $S_{3\phi} = 3 . V_\phi . I_\phi$

  but for Δ connection: $V_\phi = V_{\ell\ell}$ & $I_\phi = \frac{I_\ell}{\sqrt{3}}$

  $\therefore$ $\boxed{S_{3\phi} = \sqrt{3} . V_{\ell\ell} . I_\ell}$ identical to Y connection

Δ-Y Transformation :

- See handout

Per-Phase (Single Phase) Analysis

- A balanced 3ϕ system can be completely solved by analyzing one phase.

- Procedure:
  - Convert all Δ loads & sources to equivalent Y.
  - Solve one phase (phase A for example) independent of other phases. This will produce $\overline{V}_{\ell\text{-}n}$, $\overline{I}_\ell$, $\overline{S}_{1\phi}$
  - Then, we use single phase values to find desired 3ϕ values:

    e.g.: $\overline{S}_{3\phi} = 3 \times \overline{S}_{1\phi}$

    - Phase B & C voltages & currents are determined by $\pm 120^\circ$ phase shift
    - Line-to-line voltages & $\overline{I}_\phi$ for Δ connection can be determined by using relationships between $\overline{V}_{\ell\text{-}n}$ & $\overline{V}_{\ell\ell}$ and $\overline{I}_\ell$ & $\overline{I}_\phi$ for Δ

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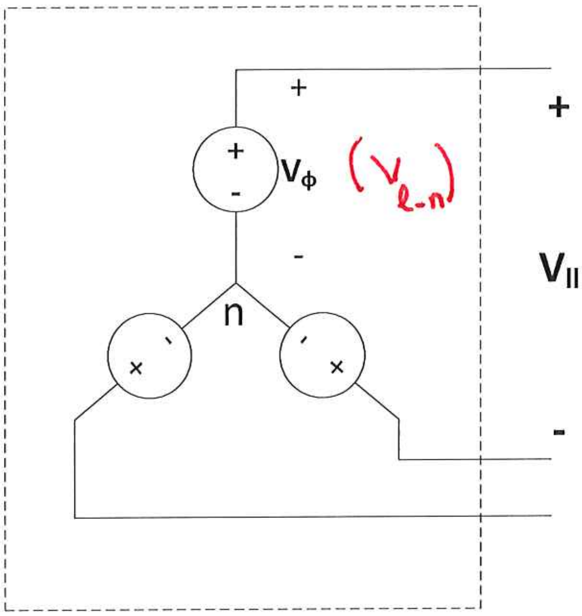
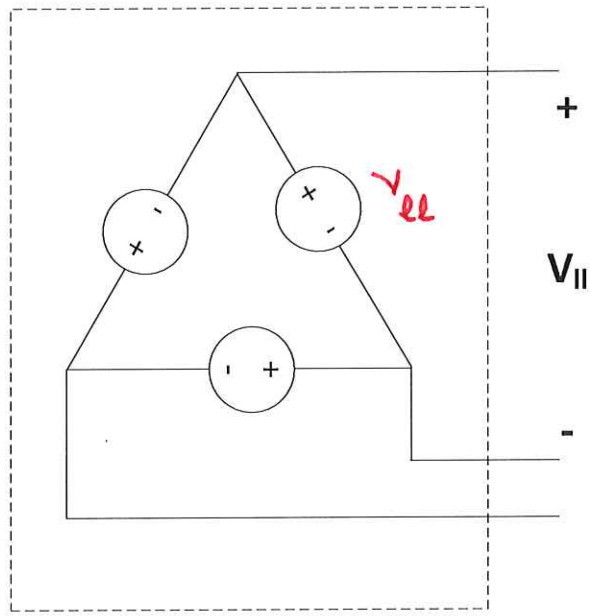
Delta-Wye transformation

__Sources:__

For sources to be equivalent, the line-to-line voltage (V_{ll}) produced by one configuration should be identical to V_{ll} produced by the other.





For the Y-connected source to be equivalent to the delta-connected source, the phase (line-to-neutral) voltage in the Y-connected source should be:

phase voltage of Y → $$\overline{V_\emptyset} = \frac{\overline{V_{ll}}}{\sqrt{3}\angle 30°}$$ ← phase voltage of equivalent Δ

For the delta-connected source to be equivalent to the Y-connected source, the phase (line-to-line) voltage in the delta connection should be the same as the line-to-line voltage in the Y connection. We can also express the phase voltage in the delta connection (line-to-line) in terms of the phase voltage in the Y connection (line-to-neutral):

$$\overline{V_{ll}} = \overline{V_\emptyset} \,.\, \sqrt{3}\angle 30°$$

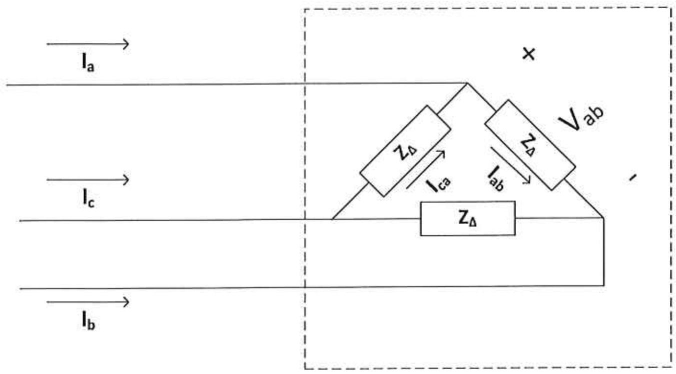
For example, Given a Δ source with $\overline{V_{ll}} = 138\angle 0°$ kV

the equivalent Y source has $\overline{V_\phi} = \overline{V_{ln}} = \dfrac{138\angle 0°}{\sqrt{3}\angle 30°}$

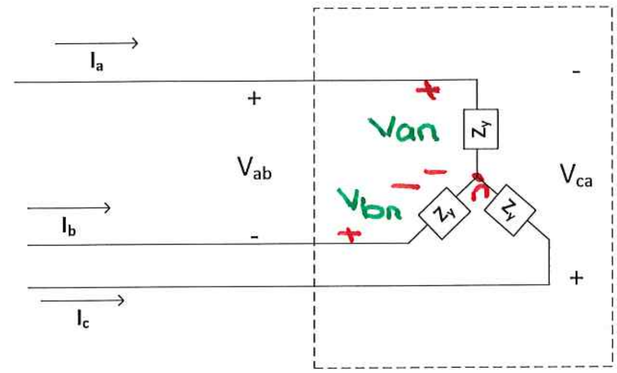
$= \dfrac{138}{\sqrt{3}}\angle -30°$ kV

Loads:

For loads to be equivalent, the line-to-line voltage (V_{ll}) at the load terminals and the line current drawn by the load (I_l) should be identical. To accomplish this, the equivalent loads should be calculated based on the following equations:



$$\boxed{Z_{\Delta} = 3.Z_y}$$



$$\boxed{Z_y = \frac{Z_\Delta}{3}}$$

Why?

$$\bar{V}_{ab} = \overbrace{Z_y.\overline{I_a}}^{\overline{V}_{an}} - \overbrace{Z_y.\overline{I_b}}^{\overline{V}_{bn}} = Z_y.(\overline{I_a} - \overline{I_b}) \qquad [1]$$

similarly,

$$\bar{V}_{ca} = Z_y.(\overline{I_c} - \overline{I_a}) \qquad [2]$$

For a balanced system,

$$\overline{I_a} + \overline{I_b} + \overline{I_c} = 0 \quad \therefore \quad \overline{I_a} = -\overline{I_b} - \overline{I_c} \qquad [3]$$

Subtract [2] from [1], then plug in [3]

$$\bar{V}_{ab} - \bar{V}_{ca} = Z_y.(\overline{I_a} \underbrace{- \overline{I_b} - \overline{I_c}}_{\overline{I_a}} + \overline{I_a})$$

$$= 3\, Z_y.\overline{I_a}$$

from [3]

$$\therefore\ 3\, Z_y = \boxed{\frac{\bar{V}_{ab} - \bar{V}_{ca}}{\overline{I_a}}}$$

KCL:

$$\overline{I_a} = \overline{I}_{ab} - \overline{I}_{ca} = \frac{\bar{V}_{ab}}{Z_\Delta} - \frac{\bar{V}_{ca}}{Z_\Delta}$$

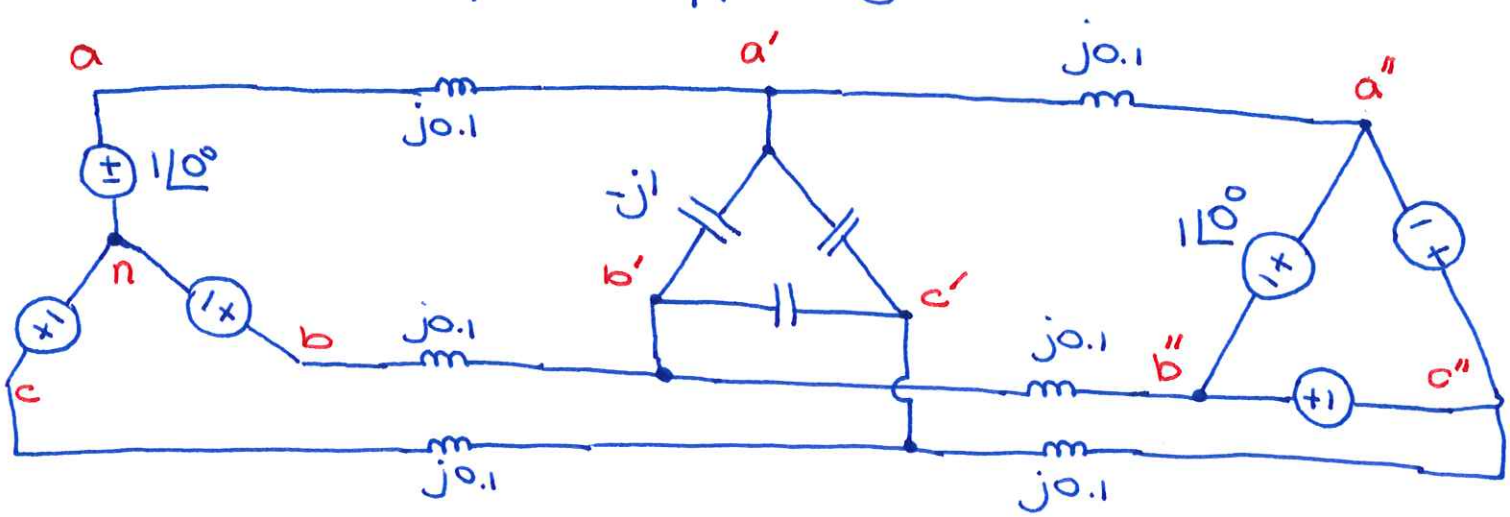
re-arrange

$$\therefore\ Z_\Delta = \boxed{\frac{\bar{V}_{ab} - \bar{V}_{ca}}{\overline{I_a}}}$$

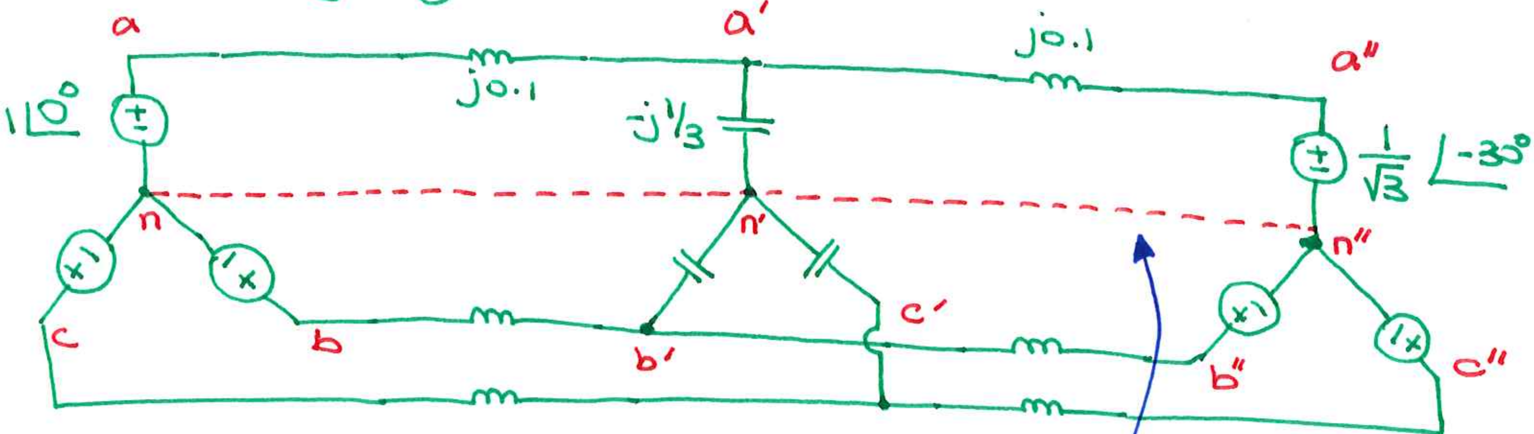
$\overline{V}_{ab}$, $\overline{V}_{ca}$, $\overline{I}_a$ should be identical for the loads to be equivalent $\therefore$ $Z_\Delta = 3.Z_Y$ or $Z_Y = \frac{Z_\Delta}{3}$

Ex: Find : • Load voltage $\overline{V}_{a'b'}$

• Total power supplied by left source

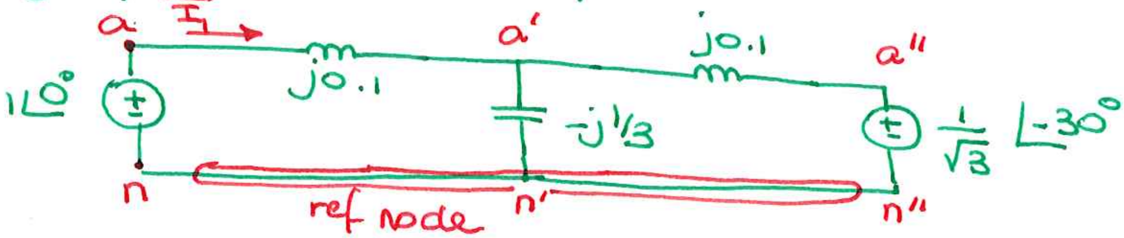


• Convert everything to Y:



added a 0Ω neutral wire to complete the circuit

Single phase circuit for phase a :



KCL (node eq) at a' : $\dfrac{\overline{V}_{a'} - 1\angle 0^\circ}{j0.1} + \dfrac{\overline{V}_{a'}}{-j1/3} + \dfrac{\overline{V}_{a'} - \frac{1}{\sqrt{3}}\angle -30^\circ}{j0.1} = 0$

$\therefore\ (-j10)(\overline{V}_{a'} - 1) + (j3)(\overline{V}_{a'}) + (-j10)\left(\overline{V}_{a'} - \frac{1}{\sqrt{3}}\angle -30^\circ\right) = 0$

$\therefore\ \overline{V}_{a'} = 0.9\angle -10.9^\circ$ ← This is $\overline{V}_{a'n'}$

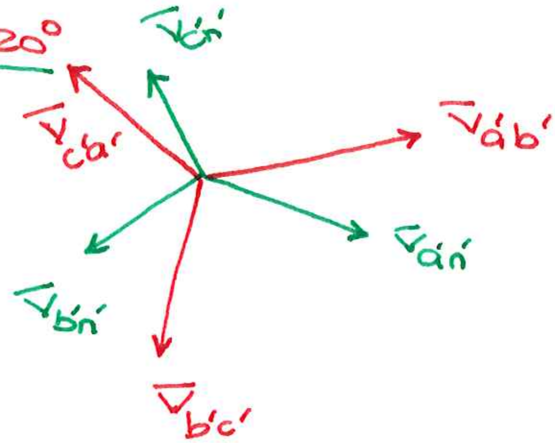
i.e. Line-to-neutral voltage for (equivalent) Y-connected capacitor

$$\overline{V_{a'b'}} = \overline{V_{a'n'}}\ \sqrt{3}\angle 30^\circ = 1.56\angle 19.1^\circ \text{ V}$$

line-to-line (phase) voltage across capacitor in the orig circuit

aside: $\overline{V_{b'c'}} = 1.56\angle 19.1^\circ - 120^\circ$

$\overline{V_{c'a'}} = 1.56\angle 19.1^\circ + 120^\circ$



To find power from the left source:

$$\overline{I_1} = \frac{\overline{V_a} - \overline{V_{a'}}}{j0.1} = \frac{1 - 0.9\angle -10.9^\circ}{j0.1} = 2.06\angle -34^\circ \text{ A}$$

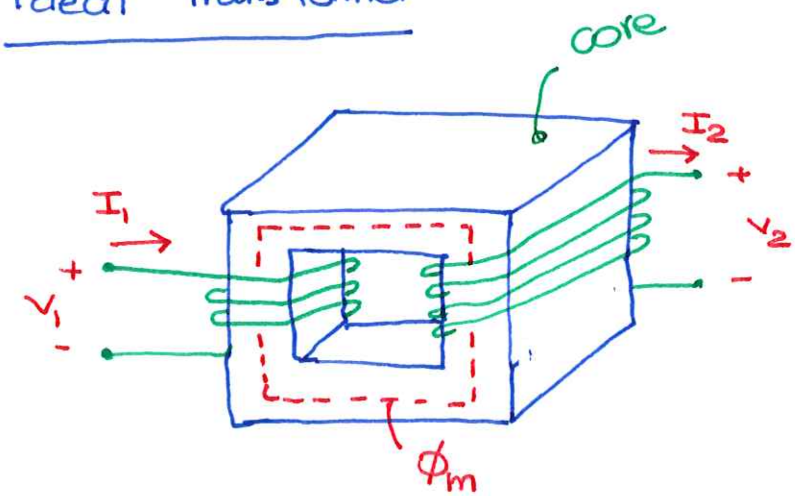
$$\overline{S}_{\substack{\text{left source}\\ 3\phi}} = 3 \cdot \underbrace{\overline{V_{an}} \cdot \overline{I_1}^*}_{\overline{S}_{1\phi}} = \underbrace{5.1}_{P_{3\phi}(\text{W})} + j\underbrace{3.5}_{Q_{3\phi}(\text{VAr})}$$

Topic 3 : (Power) Transformers

- Reminder : Power systems operate at or near a few pre-defined voltage levels, e.g: 120 V, ..., 500 kV
- Transformers (Txfr) transfer power between different AC voltage levels. We transmit power at higher voltage levels because: $S = V.I$; if $V\uparrow$ then $I\downarrow$ which results in:

  1) I^2R losses in lines $\downarrow$
  2) voltage drop across the lines $\downarrow$
  3) smaller conductors for lines

Ideal Transformer



ϕ_m: magnetic flux

μ: magnetic permeability of the core

winding 1 has N_1 turns
winding 2 has N_2 turns

assumptions :
- No real power losses in the windings $(R=0)$
- " " in the core
- μ is infinite
- no leakage flux; ϕ_m is contained in the core

From Faraday's Law : $V_1 = N_1 \frac{d\phi_m}{dt}$, $V_2 = N_2 \frac{d\phi_m}{dt}$

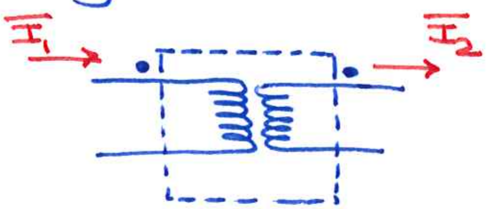
$$\therefore \frac{d\phi_m}{dt} = \frac{V_1}{N_1} = \frac{V_2}{N_2} \quad \therefore \boxed{\frac{V_1}{V_2} = \frac{N_1}{N_2} = a}$$

turns ratio

since $S_1 = V_1 I_1$ & $S_2 = V_2 I_2$ and $S_1 = S_2$

$$\therefore \boxed{\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a}}$$

- Winding direction is not always visible. Solution: dot convention

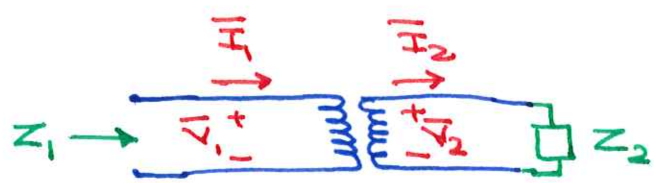


when current enters a winding at the dotted terminal, flux is in the direction of the dot

$\therefore$ when current enters dotted terminal from one side & leaves dotted terminal on other side, those currents are in phase

i.e.. $\overline{I}_1$ & $\overline{I}_2$ are in phase, just scaled up/down by $1/a$

Referring Impedances



if Z_2 is connected to winding 2 , Z_1 (impedance seen from winding 1) is :

$$Z_1 = \frac{\overline{V}_1}{\overline{I}_1} = \frac{\frac{N_1}{N_2} \cdot \overline{V}_2}{\frac{N_2}{N_1} \cdot \overline{I}_2} = \left(\frac{N_1}{N_2}\right)^2 \cdot \underbrace{\frac{\overline{V}_2}{\overline{I}_2}}_{Z_2} = \left(\frac{N_1}{N_2}\right)^2 \cdot Z_2$$