ENEL 476 Assignment #2 Winter 2015 ①

10.6, 10.26, 10.28, 10.36, 10.44, 10.56

10.6

$\epsilon_R = 3.6$

$\mu_R = 2.1$

$\sigma = 0.08$ S/m

$f = 50$ MHz

$\vec{E}_s = 6e^{-\gamma z}\,\vec{a}_z$ V/m

a) $\gamma = ?$

b) $\lambda = ?$

c) $V_p = ?$

d) $\eta = ?$

e) $\vec{H}_s = ?$

a) $\gamma = \alpha + j\beta$

$$\frac{\sigma}{\omega\epsilon} = \frac{0.08}{(2\pi)(50\times10^6)\left(\frac{1}{36\pi}\times10^{-9}\right)} = 7.99 \rightarrow \text{full formulas!}$$

$$\alpha = (2\pi\times50\times10^6)\sqrt{\frac{(2.1\times4\pi\times10^{-7})(3.6)\left(\frac{1}{36\pi}\times10^{-9}\right)}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]}$$

$= 5.41$ Np/m

$\beta = 6.13$ Rad/m

$$\therefore \boxed{\gamma = 5.41 + j6.13}$$

b) $$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6.13}$$

$$\boxed{\lambda = 1.03\text{ m}}$$

c) $V_p = \omega/\beta$

$= (2\pi)(50\times10^6)/6.13$

$= 5.13\times10^7$ m/s

d) $$|\eta| = \frac{\sqrt{\mu_r\mu_0/\epsilon_R\epsilon_0}}{\left[1+\left(\sigma/\omega\epsilon\right)^2\right]^{1/4}}$$

$= 101.4$

$\angle\eta \Rightarrow \tan 2\theta_\eta = \frac{\sigma}{\omega\epsilon}$

$\theta_\eta = 0.72$ rad

$= 41.3°$

$$\therefore \boxed{\eta = 101.4\angle 41.3°}$$

e) $$\boxed{\vec{H}_s = -\frac{6}{101.4}e^{-(5.41+j6.13)x}e^{-j0.72}\,\vec{a}_y}$$

10.26

$f = 6\,\text{MHz}$

$\mu_r = 1$

$\epsilon_r = 4$

$$\frac{\sigma}{\omega\epsilon} = 7\times10^{-2} \Rightarrow \sigma = (\omega\epsilon)(7\times10^{-2})$$
$$= 9.34\times10^{-5}\ \text{S/m}$$

$\alpha = 0.0088$ (via full equation)

$+\ \delta = \frac{1}{\alpha}$

$$\boxed{\delta = 113.7\ \text{m}}$$

$\beta = 0.2516$ rad/m

$+\ v_p = \omega/\beta$

$$= (2\pi)(6\times10^6)/0.2516$$

$$\boxed{v_p = 1.5\times10^8\ \text{m/s}}$$

10.28

$\sigma = 3.5\times10^7$ S/m

$\epsilon = \epsilon_0$

$\mu = \mu_0$

$f = 150\,\text{MHz}$

a) $\gamma = \alpha + j\beta$

b) $\delta = \frac{1}{\alpha}$

c) $v_p = \omega/\beta$

$$\frac{\sigma}{\omega\epsilon} = \frac{3.5\times10^7}{(2\pi\times150\times10^6)\left(\frac{1}{36\pi}\times10^{-9}\right)} = 4.2\times10^9 \gg 1 \quad \therefore$$

use good conductor approx.

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$
$$= 1.44\times10^5$$

$$\therefore \boxed{\gamma = 1.44\times10^5 + j\,1.44\times10^5}$$

b) $\delta = 1/\alpha$

$$\boxed{\delta = 6.95\times10^{-6}\ \text{m}}$$

c) $v_p = \omega/\beta$

$$\boxed{v_p = 6.55\times10^3\ \text{m/s}}$$

10.36 a) $\vec{E}_s = 40e^{j10z}\vec{a}_x + 60e^{j10z}\vec{a}_y$ V/m

(3)

$\Rightarrow$ linear

b) $\vec{E}_s = 12e^{j\pi/3}e^{-j10x}\vec{a}_y + 5e^{-j\pi/3}e^{-j10x}\vec{a}_z$

$\Rightarrow$ elliptical

10.44 $\sigma = 4$ S/m

$\epsilon_r = 81$

$\mu_r = 1$

$\vec{E} = 8e^{-0.1z}\cos(\omega t - 0.3z)\vec{a}_x$ V/m

a) $P_{AV}(z) = \frac{1}{2}\frac{|\vec{E}|^2}{|\eta|}e^{-2\alpha z}\cos(\theta_\eta)\vec{a}_z$

$\eta = ?$

$\alpha = 0.1$

$\beta = 0.3$

$$\frac{\alpha}{\beta} = \frac{\sqrt{\sqrt{1+(\sigma/\omega\epsilon)^2}-1}}{\sqrt{\sqrt{1+(\sigma/\omega\epsilon)^2}+1}}$$

$$\left(\frac{\alpha}{\beta}\right)^2 = \frac{\sqrt{1+(\sigma/\omega\epsilon)^2}-1}{\sqrt{1+(\sigma/\omega\epsilon)^2}+1} = \frac{1}{9}$$

$\Rightarrow 9\sqrt{1+(\sigma/\omega\epsilon)^2} - 9 = \sqrt{1+(\sigma/\omega\epsilon)^2} + 1$

$8\sqrt{1+(\sigma/\omega\epsilon)^2} = 10$

$1 + (\sigma/\omega\epsilon)^2 = \left(\frac{10}{8}\right)^2$

$$\frac{\sigma}{\omega\varepsilon} = \sqrt{(5/4)^2 - 1}$$

$$\boxed{\frac{\sigma}{\omega\varepsilon} = 0.75}$$

Now, $|\eta| = \dfrac{\sqrt{\mu_0/\varepsilon_R\varepsilon_0}}{\left(1+(0.75)^2\right)^{1/4}}$

$= 37.45$

$\theta_\eta = 0.32$ rad

$\therefore \quad P_{AV}(z) = \dfrac{1}{2}\dfrac{(8)^2}{37.45} e^{-0.2z} \cos(0.32)\,\vec{a}_z$

$= 0.81 e^{-0.2z}\,\vec{a}_z$ W/m^2

b) $20\text{ dB} = 10 \log \dfrac{P_1}{P_2} \Rightarrow \dfrac{P_1}{P_2} = 100$ $\left(\dfrac{0.81e^{-0.2z_1}}{0.81e^{-0.2(z_1+z)}}\right)$

$\dfrac{P_1}{P_2} = e^{+0.2z}$

$e^{0.2z} = 100$

$$\boxed{z = 23.03\text{m}}$$

10.56 (2) (1) (5)

free space

$\epsilon_r = 4$

$$\vec{E}_i = 5\cos(10^8 t + \beta y)\,\vec{a}_z \text{ V/m}$$

a) $\vec{E}_{tot} \Rightarrow \vec{E}_{tot} = \vec{E}_i + \vec{E}_{refl}$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}$$

$\eta_2 = 377\,\Omega$ or $120\pi\,\Omega$

$$\eta_1 = \sqrt{\frac{\mu_0}{4\epsilon_0}} = 60\pi\,\Omega$$

$$\Gamma = \frac{120\pi - 60\pi}{120\pi + 60\pi} = \frac{1}{3}$$

$$T = 1 + \Gamma = 4/3$$

$$\beta_1 = \omega\sqrt{\mu_0 \epsilon_r \epsilon_0} = \frac{10^8}{3\times10^8}(2) = 2/3$$

$$\boxed{\vec{E}_{refl} = \frac{5}{3}\cos(10^8 t - 2/3\, y)\,\vec{a}_z}$$

b) $P_{AV,tot} = P_{AV,i} + P_{AV,R}$

$$P_{AV,i} = -\frac{(5)^2}{2(60\pi)}\,\vec{a}_y$$

$$P_{AV,R} = +\frac{(5/3)^2}{2(60\pi)}\,\vec{a}_y$$

$$\therefore P_{AV,tot} = \frac{25}{18(60\pi)}\vec{a}_y - \frac{25}{2(60\pi)}\vec{a}_y$$

$$= -0.0589\,\vec{a}_y \text{ W/m}^2$$

c) $$P_{AV,2} = -\frac{\left[\left(\frac{4}{3}\right)(5)\right]^2}{2(120\pi)}\,\vec{a}_y$$

$$= -0.0589\,\vec{a}_y \text{ W/m}^2$$