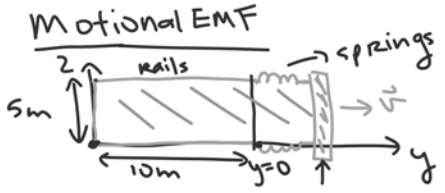
this notebook

Wednesday, January 22, 2020 12:01

Motional EMF



→ conducting bar connected to flexible leads to a pair of rails

$\vec{B} = 6\cos(10t)\vec{a}_x$ mWb/m²

$\vec{v} = 2\cos(10t)\vec{a}_y$ m/s

$y=0$ is initial position of bar

EMF = ? $\quad EMF = -\frac{d}{dt}\Phi = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$

$$\int \vec{B}\cdot d\vec{s} = \int_0^5 \int_{-10}^{y_{10c}} (6\cos(10t)\vec{a}_x \cdot dy\,dz\,\vec{a}_x \quad \times 10^{-3}$$

$$= \int_0^5 \int_{-10}^{y_{10c}} 6\cos(10t)\,dy\,dz \quad \times 10^{-3}$$

$$= (6)(5)(y_{10c}+10)\cos(10t) \quad \times 10^{-3}$$

$$-\frac{d}{dt}\int \vec{B}\cdot d\vec{s} = -30\frac{d}{dt}\left[(y_{10c}+10)\cos(10t)\right] \quad \times 10^{-3}$$

$$= -30\left[\left(\frac{dy_{10c}}{dt}\right)(\cos(10t)) + (y_{10c}+10)[-10\sin(10t)]\right] \times 10^{-3}$$

$$= +3000\sin 10t + 300\,y_{10c}\sin 10t - 30\frac{dy_{10c}}{dt}\cos 10t \quad \times 10^{-3}$$

$y_{10c} \Rightarrow \frac{dy_{10c}}{dt} = 2\cos(10t)$ & $y_{10c} = 0$ at $t=0$

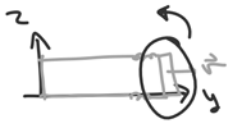
$\rightarrow y_{10c} = 0.2\sin(10t)$

$$EMF = 3000\sin 10t + 300(0.2\sin 10t)\sin 10t - 30(2\cos 10t)\times \cos 10t \times 10^{-3}$$

$$= 3\sin 10t + 0.06\sin^2 10t - 0.06\cos^2 10t$$

$$EMF = 3\sin(10t) - 0.06\cos(20t) \quad V$$

② $\oint (\vec{v}\times\vec{B})\cdot d\vec{l} \Rightarrow \vec{v}\times\vec{B} = \begin{vmatrix} a_x & a_y & a_z \\ 0 & 2\cos 10t & 0 \\ 6\cos 10t & 0 & 0 \end{vmatrix}$



$$= \vec{a}_x(0) - \vec{a}_y(0) + \vec{a}_z(-12\cos^2 10t)$$

$$= -12\cos^2 10t\,\vec{a}_z \quad \times 10^{-3}$$

$$\oint (\vec{v}\times\vec{B})\cdot d\vec{l} = \int_0^5 (-12\cos^2 10t)\vec{a}_z \cdot dz\,\vec{a}_z \quad \times 10^{-3}$$

$$= -60\cos^2 10t \quad \times 10^{-3}$$

$$= -0.06\cos^2 10t$$

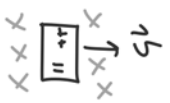
$$EMF = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$$

$$= \underbrace{\oint (\vec{v}\times\vec{B})\cdot d\vec{l}}_{\text{motional}} - \underbrace{\int \frac{\partial}{\partial t}\vec{B}\cdot d\vec{s}}_{\text{transformer}}$$

$$\Rightarrow \frac{\partial}{\partial t}\vec{B} = -60\sin 10t\,\vec{a}_x$$

$$-\int_0^5 \int_{-10}^{y_{10c}} -60\sin 10t\,dy\,dz = 3\sin 10t + 0.06\sin^2 10t$$

Motional EMF



→ $F = q\vec{v}\times\vec{B}$

→ charge separation: $F = q\vec{E}$

⇒ equilibrium: $\vec{E} = \vec{v}\times\vec{B}$

$\Rightarrow E_y = vB_x$