Wednesday, February 12, 2020 11:58

UPW in materials

↳ Ex//
- 1 MHz
- $\epsilon_r = 8$
- $\sigma = 4.8 \times 10^{-2}$ S/m
- $\mu_r = 1$
- propagation in +z
- $z=0$, $|\vec{E}| = 150$ V/m
- $\vec{E}$ in $-y$

→ $\underbrace{\vec{E}(z,t) \ \& \ \vec{H}(z,t)}$?

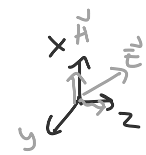
↳ $\frac{\sigma}{\omega\epsilon} = 107.9 \gg 1 \Rightarrow$ good conductor

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} \Rightarrow \alpha = \sqrt{\frac{(2\pi\times10^6)(4\pi\times10^{-7})(4.8\times10^{-2})}{2}}$$
$$= 0.435$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \measuredangle \pi/4$$
$$= 12.67 \measuredangle \pi/4$$

$$\Rightarrow \vec{E}(z,t) = -150 e^{-0.435z} \cos(2\pi\times10^6 t - 0.435z)\,\vec{a}_y$$

$$\vec{H}(z,t) = \frac{150}{12.67} e^{-0.435z} \cos(2\pi\times10^6 t - 0.435z - \pi/4)\,\vec{a}_x$$



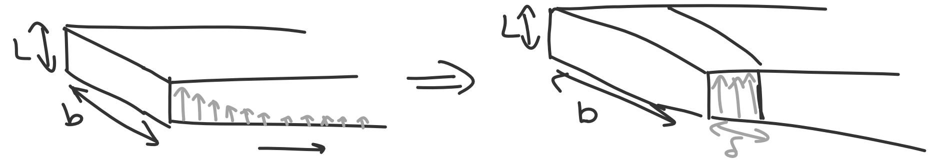
Ex// Copper at 1 GHz → $\sigma = 5.8\times10^7$ S/m, $\mu_r = 1$, $\epsilon_r = 1$

$$\delta = \frac{1}{\alpha} \Rightarrow \delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}}$$

↳ skin depth

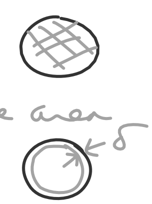
$$= 2.09\times10^{-6}\text{ m}$$

→ can show that total current in conductor of width b & infinite extent is equivalent to uniform current flow in a skin depth



Recall: $\frac{I}{A} = J \Rightarrow R_{DC} = \frac{l}{\sigma S}$

↳ surface area



$$R_{ac} = \frac{l}{\sigma(2\pi a)\delta}$$

$$\frac{R_{ac}}{R_{dc}} = \frac{\frac{l}{\sigma(2\pi a\delta)}}{\frac{l}{\sigma\pi a^2}}$$

$$= \frac{a}{2\delta} \Rightarrow R_{ac} \gg R_{DC}$$

Ex// Aluminum → $\sigma = 3.5\times10^{7}$ S/m, $\mu_r = 1$, $\epsilon_R = 1$

→ $a = 1.3$ mm

a) 2GHz $R_{ac}/R_{DC} = \frac{a}{2}\sqrt{\pi f \mu_0 \sigma} \Rightarrow \frac{1.3\times10^{-3}}{2}\sqrt{(\pi)(2\times10^{9})(4\pi\times10^{-7}\times(3.5\times10^{7})}$

$R_{ac}/R_{DC} = 341.7$

b) 10 MHz $R_{ac}/R_{DC} = 24.16$

Ex// $\vec{E}(z,t) = 10e^{-200z}\cos(2\pi\times10^{9}t - 200z)\vec{a}_y$ mV/m

↳ $\mu_r = 1$, find $\vec{H}(z,t)$

→ $\alpha = \beta \Rightarrow$ good conductor

$200 = \sqrt{\frac{\omega\mu_0\sigma}{2}}$ $|\eta| = \sqrt{\frac{\omega\mu}{\sigma}}$

$200 = \sqrt{\frac{(2\pi\times10^{9})(4\pi\times10^{-7})\sigma}{\cancel{2}}} \Rightarrow 200 = \sqrt{4\pi^2\times10^{2}\sigma}$

$\Rightarrow \cancel{200} = \cancel{2}\pi\times\cancel{10}\sqrt{\sigma}$

$10/\pi = \sqrt{\sigma}$

$\sigma = 10.13$ S/m

$|\eta| = 27.92\,\Omega$

$\vec{H}(z,t) = -\frac{10}{27.92}e^{-200z}\cos(2\pi\times10^{9}t - 200z - \pi/4)\vec{a}_x$ mA/m

Poynting vector: $\vec{P} = \vec{E}\times\vec{H}$