Wednesday, January 22, 2020 12:21

Phasors $\vec{E}(z,t) = \underbrace{E_0 e^{-\alpha z}}_{\text{amplitude}} \cos(\omega t \underbrace{- \beta z - \phi}_{\text{phase}}) \vec{a}_x$

↳ assume $\cos(\omega t)$ time variation ⇒ amplitude & phase characterize

$$\vec{E}_s(z) = E_0 e^{-\alpha z} e^{-j(\beta z + \phi)}$$

$$\vec{E}(z,t) = \mathrm{Re}\{E_0 e^{-\alpha z} e^{-j(\beta z+\phi)} e^{j\omega t}\}$$
$$= \mathrm{Re}\{E_0 e^{-\alpha z} e^{j(\omega t - \beta z - \phi)}\}$$
$$= \mathrm{Re}\{E_0 e^{-\alpha z} [\cos(\omega t - \beta z - \phi) + \cancelto{0}{j\sin(\omega t - \beta z - \phi)}]\}$$

↳ sin vs cos : $\sin(\omega t) = \cos(\omega t - \pi/2)$

$$\Downarrow$$
$$e^{-j\pi/2} = -j$$

↳ time derivative: $\frac{d}{dt} e^{j\omega t} = j\omega e^{j\omega t}$

Ex// $A\sin(\alpha)\cos(\omega t - \alpha) \Rightarrow A\sin(\alpha) e^{-j\alpha}$

$\frac{d^2}{dt^2} A\cos(\omega t - \alpha) \Rightarrow (j\omega)^2 A e^{-j\alpha} = -\omega^2 A e^{-j\alpha}$

$5e^{\alpha z}\cos(z)\cos(\omega t) \Leftarrow 5e^{\alpha z}\cos(z)\, e^{j\omega t}$

$\sin(\alpha z)\sin(\omega t + \beta z) \Leftarrow \underset{\substack{\downarrow \\ e^{-j\pi/2}}}{-j}\sin(\alpha z)\underbrace{e^{j\beta z} e^{j\omega t}}_{e^{j(\omega t + \beta z)}}$

$e^{-j\pi/2}$ or sin instead of cos

Ex// $\mu = 3\times10^{-5}$ H/m, $\epsilon = 1.2\times10^{-10}$ F/m, $\sigma = 0$; source-free region ($\rho_v = 0$ & $\vec{J} = 0$)

$\vec{H}(x,t) = 2\cos(10^{10}t - \beta x)\vec{a}_z$

a) $\vec{E}(x,t) = ?$ b) $\beta = ?$

$\nabla \times \vec{H}_s = j\omega \vec{D}_s \Rightarrow \vec{D}_s = \frac{2\beta}{\omega} e^{-j\beta x} \vec{a}_y$

$\Rightarrow \vec{D}(x,t) = \frac{2\beta}{\omega}\cos(\omega t - \beta x)\vec{a}_y \Rightarrow \vec{E}(x,t)$ & $\vec{E}_s(x)$

$\Rightarrow \beta = ? \Rightarrow \nabla \times \vec{E}_s = -j\omega \vec{B}_s$

$$\Rightarrow \vec{E}(x,t) = \frac{\vec{D}(x,t)}{\epsilon}$$
$$= \frac{2\beta\cos(10^{10}t - \beta x)\vec{a}_y}{(10^{10})(1.2\times10^{-10})}$$
$$= \underbrace{1.67\beta}\cos(10^{10}t \underbrace{- \beta x})\vec{a}_y \Rightarrow \vec{E}_s(x) = 1.67\beta e^{-j\beta x}\vec{a}_y$$

$$\vec{B}_s(x) = 6\times10^{-5} e^{-j\beta x}\vec{a}_z$$

$\nabla \times \vec{E}_s = \cancel{-j}\beta^2 1.67 \cancel{e^{-j\beta x}} \vec{a}_z$

$-j\omega\vec{B}_s = \cancel{-j}\omega\, 6\times10^{-5} \cancel{e^{-j\beta x}} \vec{a}_z$ $\Rightarrow \beta^2 1.67 = (10^{10})(6\times10^{-5})$

$\Rightarrow \beta = 600$ rad/m

Ex// $\vec{E} = \overset{\text{amp}}{50}\cos(10^8 t \overset{\text{phase}}{- kz})\vec{a}_x$

ρ

↳ in free space ($\epsilon_0, \mu_0, \sigma = 0$) & source-free region ($\rho_v = 0$ & $\vec{J} = 0$)

Find $\vec{J}_D$, $\vec{H}$ & k

a) $\vec{J}_D = \frac{\partial}{\partial t}\epsilon_0 \vec{E}$ b) $\nabla \times \vec{E}_s = -j\omega\mu_0 \vec{H}_s$ c) $\nabla \times \vec{H}_s = j\omega\epsilon_0\vec{E}_s \Downarrow k$

a) $\vec{J}_D = \frac{\partial}{\partial t}\epsilon_0 \frac{50}{\rho}\cos(10^8 t - kz)\vec{a}_\rho$

$= \frac{\epsilon_0 50}{\rho}\left(-\sin(10^8 t - kz)(10^8)\right)\vec{a}_\rho$

$= -\frac{\epsilon_0 \times 5\times 10^9}{\rho}\sin(10^8 t - kz)\vec{a}_\rho$

b) $\vec{E}_s = \frac{50}{\rho} e^{-jkz}\vec{a}_\rho \Rightarrow \nabla \times \vec{E}_s = \frac{1}{\rho}\begin{vmatrix} \vec{a}_\rho & \rho\vec{a}_\phi & \vec{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \partial/\partial z \\ E_\rho & 0 & 0 \end{vmatrix}$

$\nabla \times \vec{E}_s = \cancel{\frac{1}{\rho}}\left[-\cancel{\rho}\vec{a}_\phi\left(-\frac{\partial}{\partial z}E_\rho\right) + \vec{a}_z\left(-\cancel{\frac{\partial}{\partial\phi}E_\rho}\right)\right]$ $\to 0$

$= \frac{\partial}{\partial z}E_\rho \vec{a}_\phi$

$= -jk\frac{50}{\rho}e^{-jkz}\vec{a}_\phi$

$\cancel{-j}k\frac{50}{\rho}e^{-jkz}\vec{a}_\phi = \cancel{-j}\omega\mu_0\vec{H}_s \Rightarrow \vec{H}_s = \frac{50k}{\omega\rho\mu_0}e^{-jkz}\vec{a}_\phi$

$\Longrightarrow$

$\Downarrow$

$\nabla \times \vec{H}_s = j\omega\epsilon_0\vec{E}_s$