Student ID: ______________________________ March 14, 2018 – 9:00 AM

Student name: ______________________________ Duration: 50 minutes

Problem 1 **[10 pts]**

The sinusoidal modulating wave:

$$m(t) = 2\cos(6000\pi t)$$

is applied to a phase modulator with phase sensitivity k_p = 0.1 radian per volt. The unmodulated carrier wave has frequency f_c = 1 MHz and amplitude A_c = 1 volt.

a- Determine the instantaneous frequency of this PM signal and sketch it versus time.

b- Determine the time domain expression of this PM signal and sketch it versus time.

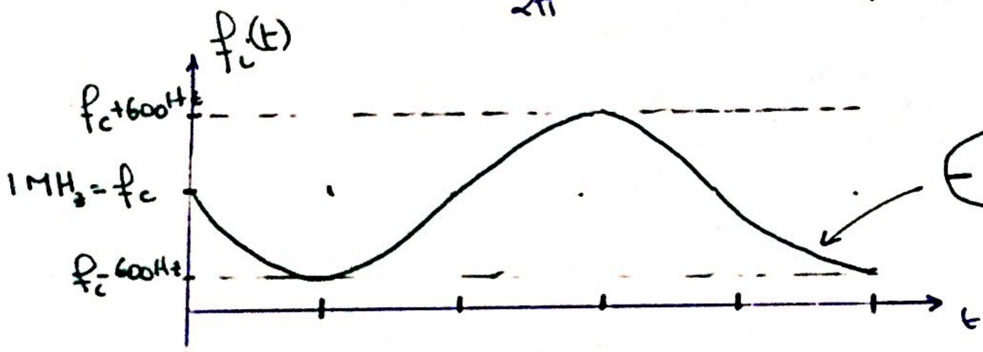
c- Determine the maximum frequency deviation Δf and the modulation index β.

d- Determine the expression of the frequency spectrum of the resulting PM signal and sketch it. **Show all amplitudes and frequencies of interest**.

e- Construct a phasor diagram of this PM modulated signal.

a) $\boxed{f_i(t) = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}}$ 1pt

$$f_i(t) = f_c - \frac{k_p}{2\pi} 12000\pi \sin(6000\pi t) = f_c - 600\sin(6000\pi t)$$



0.5 0.5

$-\sin(\cdot)$ around $f_c = 1$MHz with amplitude 600 Hz

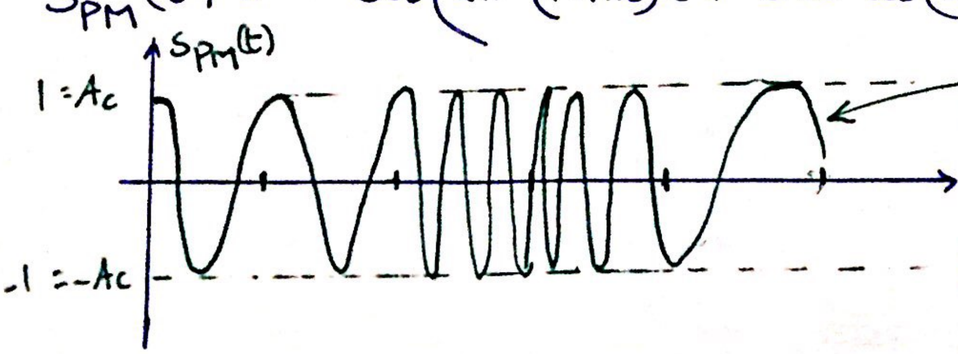
b) $\boxed{s_{PM}(t) = A_c\cos(2\pi f_c t + \beta\cos(6000\pi t))}$

1pt or

or

$\boxed{s_{PM}(t) = A_c\cos(2\pi f_c t + k_p m(t))}$

$$s_{PM}(t) = \cos(2\pi (1\text{MHz})t + 0.2\cos(6000\pi t))$$



0.5 ($\cos(\cdot)$ with amplitude $A_c = 1$

0.5 (and frequency varying according to $f_i(t)$

c) $\boxed{\Delta f = 600\ \text{Hz}}$ ← 1pt

$\boxed{\beta = 0.2\ \text{rad}}$ ← 1pt

d) $\beta << 1$ rad ⟶ narrowband PM.

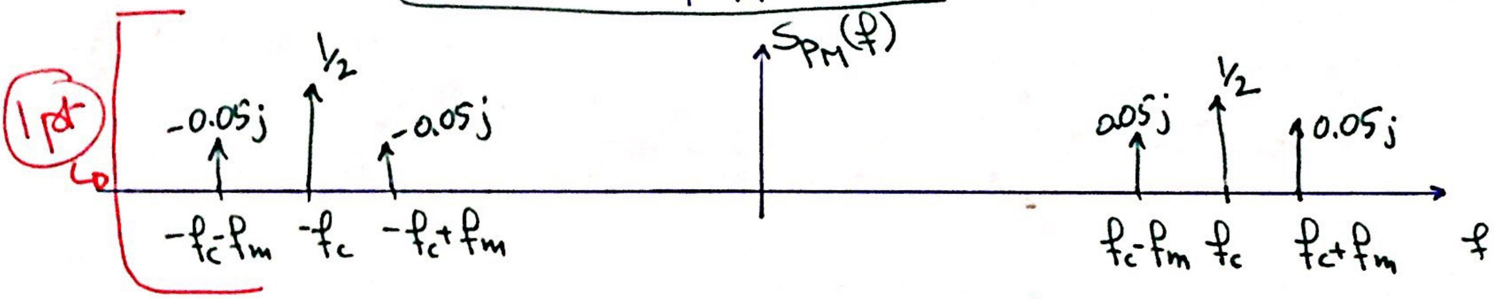
$$S_{PM}(t) = \cos\left(2\pi f_c t + 0.2\cos(6000\pi t)\right)$$
$$S_{PM}(t) = \cos(2\pi f_c t)\cos\left[0.2\cos(6000\pi t)\right] - \sin(2\pi f_c t)\sin\left(0.2\cos(6000\pi t)\right)$$
$$S_{PM}(t) \simeq \cos(2\pi f_c t) - 0.2\sin(2\pi f_c t)\cdot\cos(6000\pi t)$$

$$S_{PM}(t) \simeq \cos(2\pi f_c t) - 0.1\left[\sin\left(2\pi(f_c+3000)t\right) + \sin\left(2\pi(f_c-3000)t\right)\right]$$

1pt →
$$\boxed{\begin{aligned} S_{PM}(f) = \frac{1}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] &+ 0.05j\left[\delta(f-(f_c+f_m)) - \delta(f+(f_c+f_m))\right] \\ &+ 0.05j\left[\delta(f-(f_c-f_m)) - \delta(f+(f_c-f_m))\right] \end{aligned}}$$

with $\boxed{f_c = 1\ \text{MHz} \quad \& \quad f_m = 3\ \text{kHz}.}$

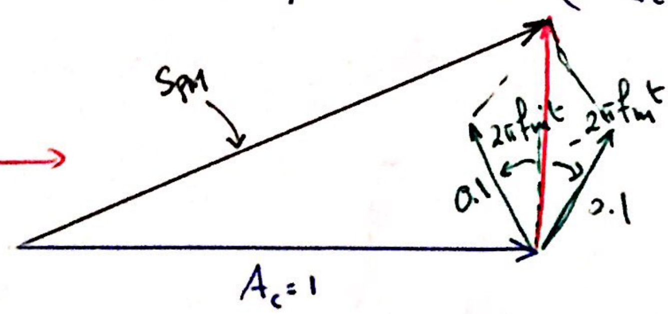
1pt



if there is a mistake in a sign or a numerical value, don't take out marks twice (in the spectrum expression and the freq. plot)

e) $$S_{PM}(t) = \cos(2\pi f_c t) + 0.1\cos\left(2\pi(f_c+f_m)t + \pi/2\right) + 0.1\cos\left(2\pi(f_c-f_m)t + \frac{\pi}{2}\right)$$

2 pts →



0.5 on each Component

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Problem 2 [10 pts]

A carrier wave of amplitude A_c = 2 volt and frequency f_c = 100 kHz is FM modulated by the message $m(t) = A_m \cos(2f_m \pi t)$, where A_m = 1 volt and f_m = 1 kHz. The frequency sensitivity of the FM modulator is k_f is 2 kHz per volt.

a- What is the time domain expression of the FM modulated signal?

b- Determine the maximum frequency deviation Δf and the modulation index β

c- Determine the transmission bandwidth of this FM signal using Caron's rule.

d- Determine the bandwidth by transmitting only those side frequencies whose amplitude exceed 1 percent of the unmodulated carrier amplitude. (Use the table below)

e- Sketch the frequency spectrum of the modulated signal. **Show only the sidebands within the bandwidth calculated in d-. Indicate all the frequencies and amplitudes of interest**. (Use the table below)

Values of the Bessel Functions $J_n(\beta)$

	$\beta=1$	$\beta=2$	$\beta=3$	$\beta=4$
$n=0$	0.7652	0.2239	-0.2601	-0.3971
$n=1$	0.4401	0.5767	0.3391	-0.066
$n=2$	0.1149	0.3528	0.4861	0.3641
$n=3$	0.0196	0.1289	0.3091	0.4302
$n_{max} \rightarrow$ $n=4$	0.0025	0.034	0.132	0.2811
$n=5$		0.007	0.043	0.1321
$n=6$		0.0012	0.0114	0.0491
$n=7$			0.0025	0.01518
$n=8$				0.004

a) $\boxed{S_{FM}(t) = A_c \cos\left(2\pi f_c t + \frac{k_f A_m}{f_m} \sin(2\pi f_m t)\right)}$

2pts

or $\boxed{S_{FM}(t) = A_c \cos\left(2\pi f_c t + \beta \sin(2\pi f_m t)\right)}$

$A_c = 2$; $f_c = 100\,kHz$; $f_m = 1\,kHz$; $A_m = 1$; $k_f = 2\,kHz/v$

$S_{FM}(t) = 2\cos\left(2\pi(100\,kHz)t + 2\sin(2000\pi t)\right)$

b) $\boxed{\Delta f = k_f \cdot A_m = 2\,kHz}$ ← 1pt

$\boxed{\beta = \frac{\Delta f}{f_m} = 2}$ ← 1pt

c) Carson's rule:

2pts

$\boxed{B_T = 2f_m(1+\beta) = 6f_m = 6\,kHz}$

or $\boxed{B_T = 2\Delta f\left(1+\frac{1}{\beta}\right) = 3\cdot\Delta f = 6\,kHz}$

d) $B_{T,1\%} = 2f_m \cdot n_{max}$

from the table : $\boxed{n_{max} = 4}$ ← 1pt

$\rightarrow \boxed{B_{T,1\%} = 8 f_m = 8\,kHz}$ ← 1pt

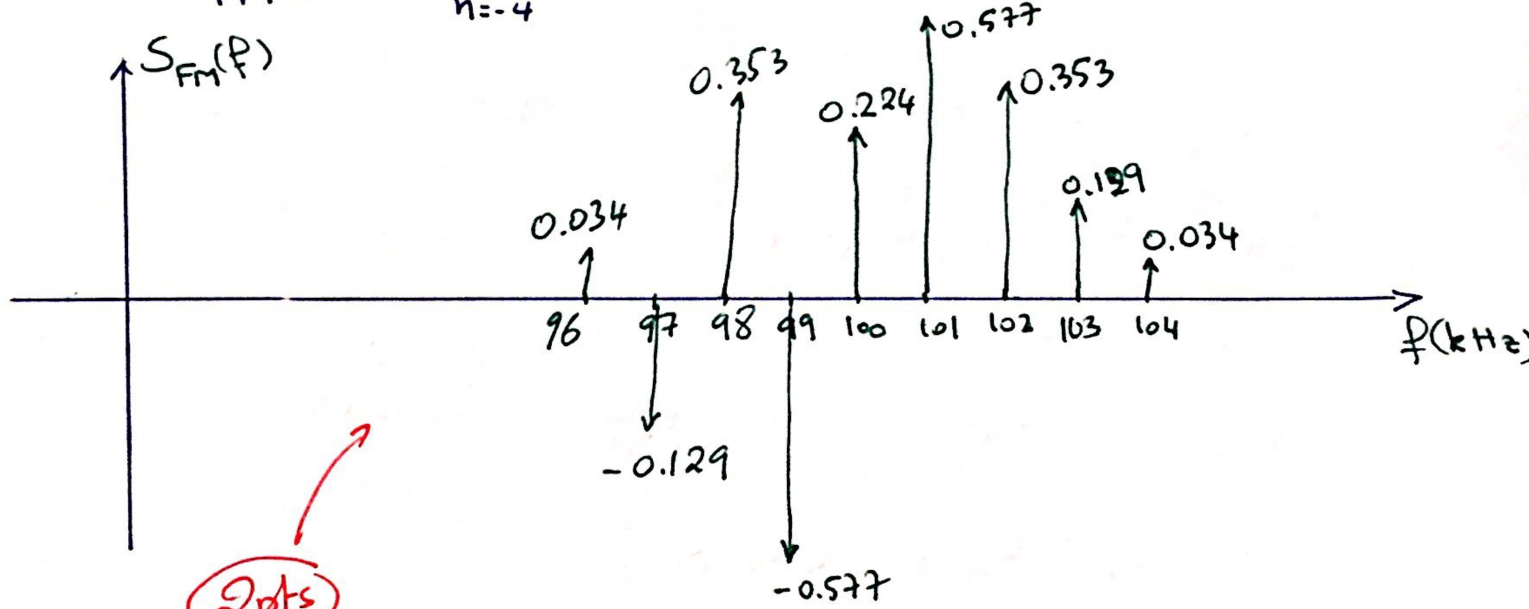
e) $\boxed{s_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(2) \cos\left(2\pi (f_c + n f_m) t\right)}$

where $f_c = 100\,kHz$; $f_m = 1\,kHz$; $A_c = 2$.

$$\boxed{S_{FM}(f) = \sum_{n=-\infty}^{\infty} J_n(2) \left[\delta\left(f - (f_c + n f_m)\right) + \delta\left(f + (f_c + n f_m)\right)\right]}$$

Using 1% Bandwidth

$$S_{FM}(f) = \sum_{n=-4}^{4} J_n(2) \left[\delta\left(f - (f_c + n f_m)\right) + \delta\left(f + (f_c + n f_m)\right)\right]$$



2pts

1pt: on showing that the amplitudes of the impulses are $J_n(2)$

1pt: on showing that the freq. of the impulses are $\begin{cases} f_c + n f_m \\ -4 \le n \le 4 \end{cases}$