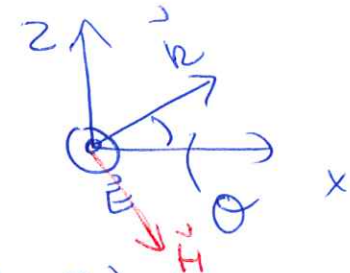
Oblique waves

$$\vec{E}_s(\vec{r}) = \vec{E}_0 \, e^{-j(\vec{k}\cdot\vec{r})}$$

$\vec{k} = k_x\vec{a}_x + k_y\vec{a}_y + k_z\vec{a}_z \Rightarrow$ propagation or wave # vector

$\vec{r} = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z$

e.g. $\vec{k} = k_x\vec{a}_x + k_z\vec{a}_z$



$\hookrightarrow \theta = \tan^{-1}(k_z/k_x)$

$\vec{E}_s(\vec{r}) = E_0 e^{-j(k_x x + k_z z)}\,\vec{a}_y$

$\hookrightarrow \lambda$ & v_p depend on direction

$\rightarrow$ in direction of $\vec{k}$: $\lambda = \dfrac{2\pi}{|\vec{k}|}$

$v_p = \omega/|\vec{k}|$

$|\vec{k}| = \omega\sqrt{\mu\epsilon}$

$\rightarrow$ in x-direction: $\lambda_x = \dfrac{2\pi}{k_x}$

$v_{px} = \omega/k_x$

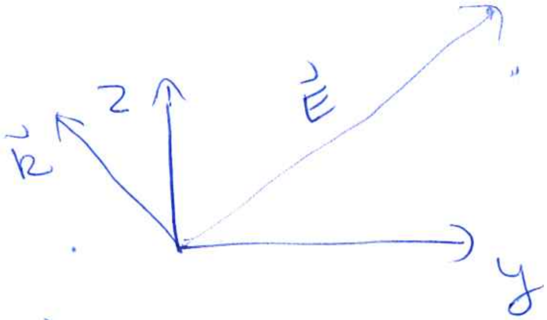
Ex// $\vec{E} = (10\vec{a}_y + 5\vec{a}_z)\cos(\omega t + 2y - 4z)$ V/m in free space.

a) Sketch $\vec{k}$ & $\vec{E}$

b) Find ω and λ.

c) Find $\vec{H}$.

a)



$\vec{k} = -2\vec{a}_y + 4\vec{a}_z$

b) $|\vec{k}| = \omega\sqrt{\mu_0\epsilon_0} \Rightarrow |\vec{k}| = \sqrt{(-2)^2+(4)^2}$
$= \sqrt{20}$

$\omega = (\sqrt{20})(3\times10^8)$
$= 1.34\times10^9$ rad/s

$\lambda = \frac{2\pi}{|\vec{k}|}$
$= \frac{2\pi}{\sqrt{20}}$
$= 1.405\text{m}$

c) $\vec{H} = ? \rightarrow \vec{E}\times\vec{H} \Rightarrow \vec{k} \Rightarrow -\vec{a}_x$ (from sketch)

$|\vec{E}| = \sqrt{(10)^2+(5)^2}$
$= \sqrt{125}$

$\eta_0 = 120\pi\ \Omega$

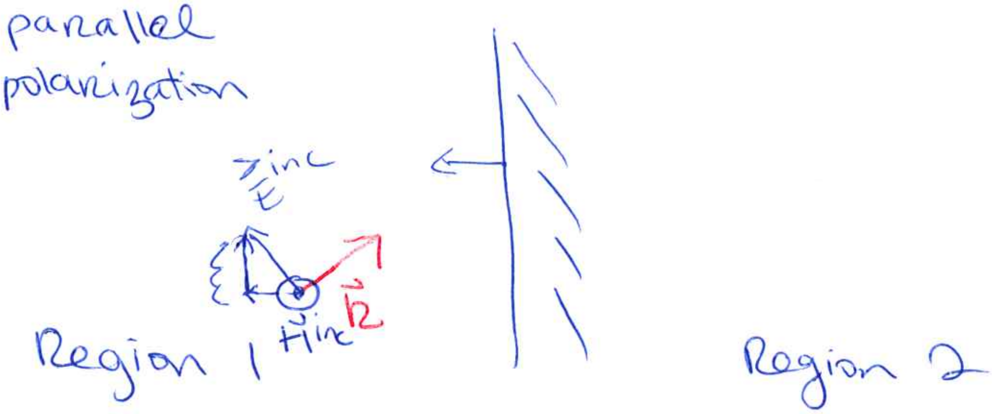
$$\vec{H} = \frac{-\sqrt{125}}{120\pi}\cos(1.34\times10^9 t + 2y - 4z)\vec{a}_x$$

$\vec{H}_s = \frac{\vec{a}_k \times \vec{E}_s}{\eta}$ normalized propagation vector

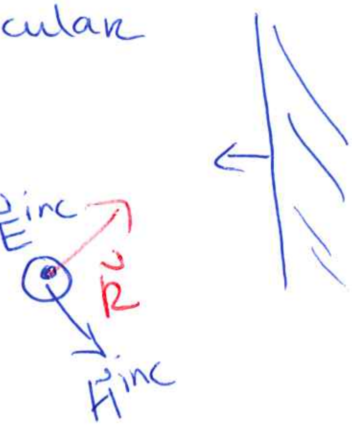
Polarization

↳ plane of incidence → defined by surface normal + $\vec{k}$

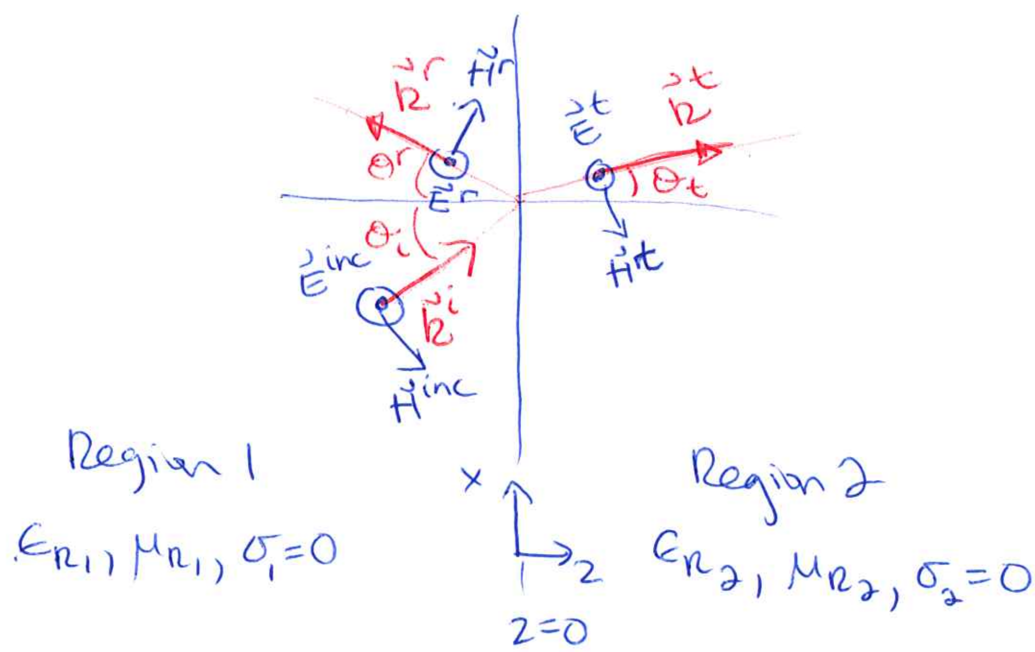
(1) parallel polarization



(2) perpendicular

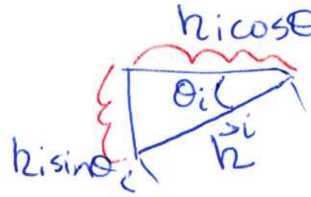


↳ consider perpendicular polarization



$$\vec{E}^i = (E^i \vec{a}_y)e^{-jk_i(x\sin\theta_i + z\cos\theta_i)}$$

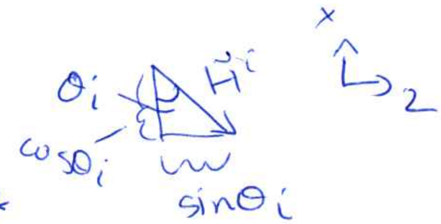
$|\vec{k}^i|$



$$\vec{k}^i = k_i\sin\theta_i\,\vec{a}_x + k_i\cos\theta_i\,\vec{a}_z$$

$$\vec{r} = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z$$

$$\vec{k}^i\cdot\vec{r} = k_i\sin\theta\, x + k_i\cos\theta_i\, z$$



$$\vec{H}^i = \frac{E^i}{\eta_1}e^{-jk_i(x\sin\theta_i + z\cos\theta_i)} *$$

$$[-\cos\theta_i\,\vec{a}_x + \sin\theta_i\,\vec{a}_z]$$

$$\vec{H}^i = \frac{E^i}{\eta_1}[-\cos\theta_i\,\vec{a}_x + \sin\theta_i\,\vec{a}_z]e^{-jk_i(x\sin\theta_i + z\cos\theta_i)}$$

$$\vec{E}^r = (\Gamma E^i\vec{a}_y)e^{-jk_i(x\sin\theta_r - z\cos\theta_r)}$$

$$\vec{H}^r = \frac{\Gamma E^i}{\eta_1}[\cos\theta_r\,\vec{a}_x + \sin\theta_r\,\vec{a}_z]e^{-jk_i(x\sin\theta - z\cos\theta_r)}$$

$$\vec{E}^t = (TE^i)(\vec{a}_y)e^{-jk_t(x\sin\theta_t + z\cos\theta_t)}$$

$$\vec{H}^t = \frac{(TE^i)}{\eta_2}[-\cos\theta_t\,\vec{a}_x + \sin\theta_t\,\vec{a}_z]e^{-jk_t(x\sin\theta_t + z\cos\theta_t)}$$

↳ apply boundary conditions $\Rightarrow z=0$

$$\vec{E}_{1,tan} = \vec{E}_{2,tan}$$

$$\cancel{E^i}\, e^{-jk_i(x\sin\theta_i)} + \Gamma \cancel{E^i}\, e^{-jk_i(x\sin\theta_r)} = T\cancel{E^i}\, e^{-jk_t x\sin\theta_t}$$

incident   reflected   transmitted

↳ $\theta_i = \theta_r \Rightarrow k_i x\sin\theta_i = k_i x\sin\theta_r$

↳ $k_i \cancel{x}\sin\theta_i = k_t \cancel{x}\sin\theta_t$

$$\Rightarrow \frac{\sin\theta_t}{\sin\theta_i} = \frac{k_1 \rightarrow k_i}{k_2 \rightarrow k_t}$$

$\Rightarrow n_1\sin\theta_i = n_2\sin\theta_t$

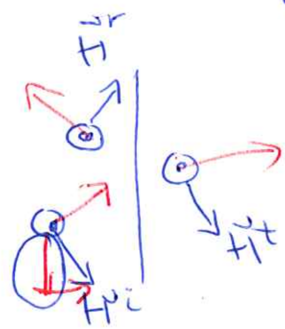
↳ Snell's law

$n_1 = c\sqrt{\mu_1\epsilon_1}$

↳ index of refraction

$\Rightarrow 1+\Gamma = T$

$\sim$ $\vec{H}_{1,tan} = \vec{H}_{2,tan}$



$$\frac{E^i}{n_1}\underbrace{[-\cos\theta_i)}_{} + \frac{\Gamma E^i}{n_1}\underbrace{[\cos\theta_r]}_{} = \frac{TE^i}{n_2}\underbrace{[-\cos\theta_t]}_{}$$

↳ $\theta_i = \theta_r$

↳ $T = 1+\Gamma \Rightarrow -n_2\cos\theta_i + n_2\Gamma\cos\theta_i = -n_1(1+\Gamma)\cos\theta_t$

$$\Gamma_{\perp} = \frac{n_2\cos\theta_i - n_1\cos\theta_t}{n_2\cos\theta_i + n_1\cos\theta_t}$$

perpendicular polarization

$$T_{\perp} = \frac{2 n_2 \cos\theta_i}{n_2 \cos\theta + n_1 \cos\theta_t}$$