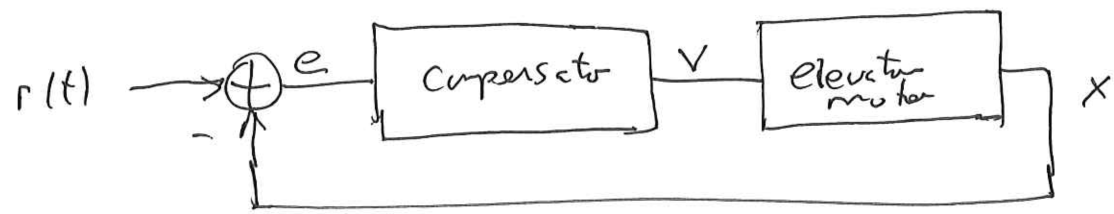
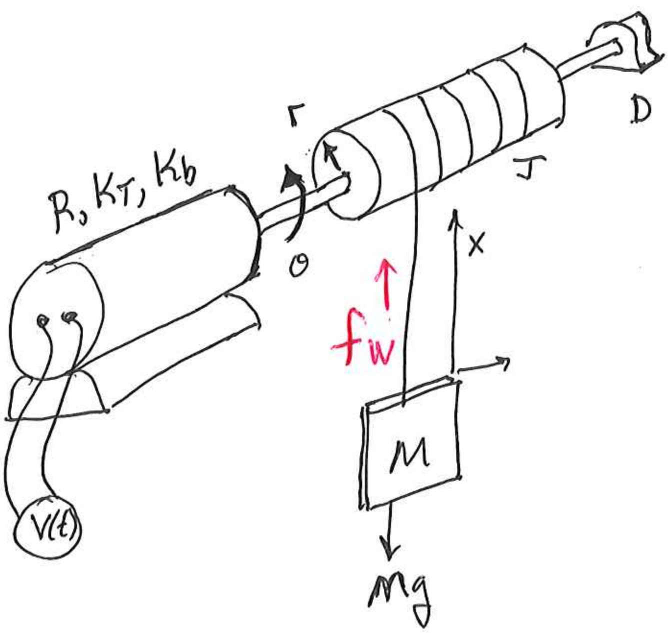
Lecture Notes March 3 2020

Problem of elevator Motor

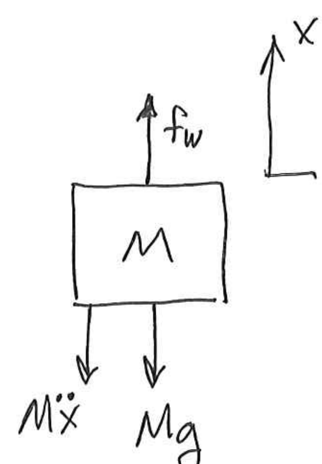
Form a control loop that sets height of elevator $x(t)$ for a given reference input $r(t)$



Start with the motor/elevator



Start with weight M



$$f_w = Mg + M\ddot{x}$$

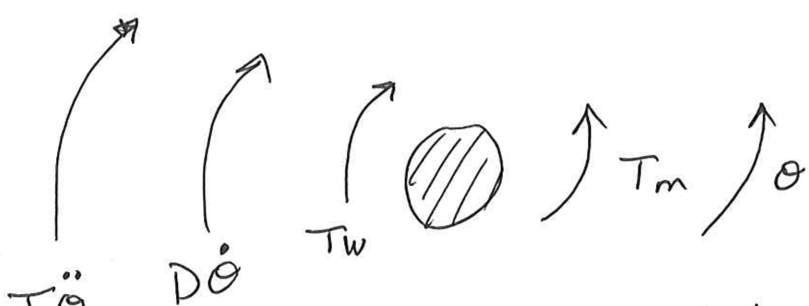
T_w = torque on cable drum from weight M

$T_w = f_w r$ (r radius of drum)

$x = \theta r$ mapping from θ to x

$$T_w = Mrg + Mr^2\ddot{\theta}$$

Next go to cable drum



T_m = torque supplied by motor

$$T_m = T_w + D\dot{\theta} + J\ddot{\theta}$$
$$= (Mr^2 + J)\ddot{\theta} + D\dot{\theta} + Mrg$$

Next go to motor

$T_m = i K_T$ $\quad$ i = current in motor

$$i = \frac{V(t) - \omega K_b}{R} = \frac{1}{R} V(t) - \dot{\theta} \frac{K_b}{R}$$

$$T_m = \frac{K_T}{R} V(t) - \frac{K_T K_b}{R} \dot{\theta}$$

Put into cable drum equation

$$\frac{K_T}{R} V(t) = (Mr^2 + J)\ddot{\theta} + \left(D + \frac{K_T K_b}{R}\right)\dot{\theta} + Mrg$$

Now assume initial conditions $V(0) = V_o$ (bias voltage)

$$\left.\begin{aligned} X(0) &= 0 \\ \dot{X}(0) &= 0 \\ \theta(0) &= 0 \\ \dot{\theta}(0) &= 0 \end{aligned}\right\} \quad \begin{aligned} \frac{K_T}{R} V_o &= Mrg \\ V_o &= \frac{MrgR}{K_T} \end{aligned}$$

This bias voltage will hold the elevator at $x=0$ and counter the gravity force of Mg on the weight.

let $V(t) = V_o + V_d(t)$

equation becomes

$$\frac{K_T}{R} V_d(t) = (Mr^2 + J)\,\ddot{\theta} + \left(D + \frac{K_T K_b}{R}\right)\dot{\theta}$$

Transfer function $H(s) = \dfrac{\Theta(s)}{V_d(s)}$

$$H(s) = \frac{K_T/R}{(Mr^2+J)\,s^2 + \left(D + \frac{K_T K_b}{R}\right)s}$$

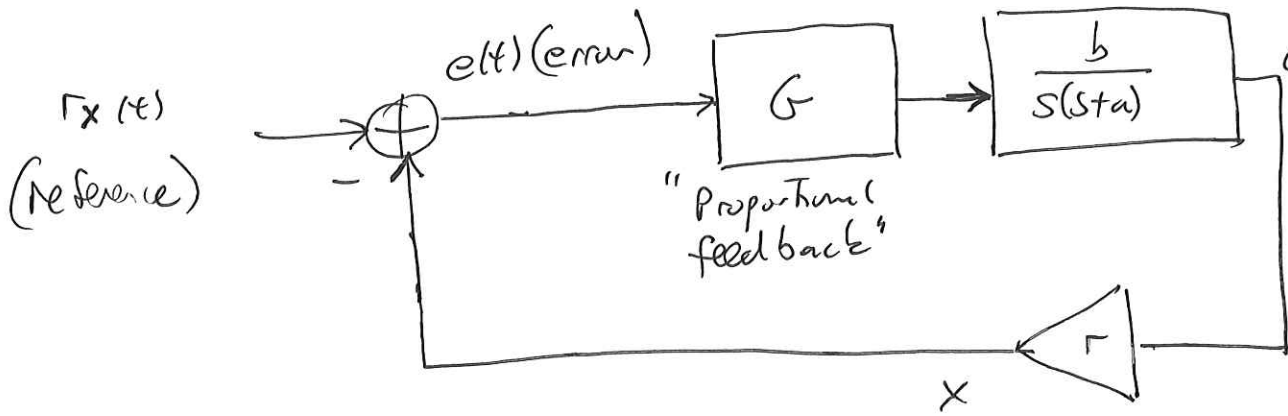
$$H(s) = \frac{\frac{K_T}{R}\,\frac{1}{Mr^2+J}}{s\left(s + \dfrac{D + \frac{K_T K_b}{R}}{Mr^2+J}\right)}$$

let $b = \dfrac{K_T}{R}\,\dfrac{1}{Mr^2+J}$

$$a = \frac{D + \frac{K_T K_b}{R}}{Mr^2 + J}$$

$$H(s) = \frac{b}{s(s+a)}$$

Positioner Control loop



r_x - input reference in x

$$\frac{X(s)}{R_x(s)} = \underbrace{H_{CL}(s)}_{\text{closed loop}} = \frac{\frac{Gbr}{s(s+a)}}{1 + \frac{Gbr}{s(s+a)}} = \frac{Gbr}{s^2 + sa + Gbr}$$

note DC gain of $H_{CL}(s)$ is 1

Also show that $x(\infty) = 1$ for $r_x(t) = u(t)$

$$x(\infty) = \underbrace{\lim_{s \to 0} s}_{\text{f.v.t.}} \underbrace{\frac{1}{s}}_{r(t)} \underbrace{\frac{Gbr}{s^2 + sa + Gbr}}_{H_{CL}(s)}$$

$$= \frac{Gbr}{Gbr} = 1$$

Poles of $H_{CL}(s)$

roots of $s^2 + sa + Gbr$

Poles @ $s = -\frac{a}{2} \pm j\sqrt{Gbr - \left(\frac{a}{2}\right)^2}$

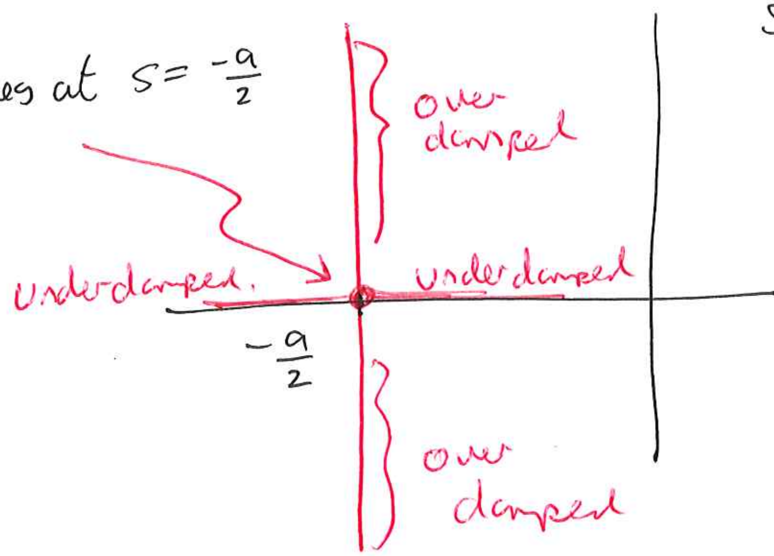
Over damped poles $G < \left(\frac{a}{2}\right)^2 \frac{1}{br}$ (not desired slow - one pole close to $j\omega$ axis)

critically damped $G = \left(\frac{a}{2}\right)^2 \frac{1}{br}$ (desired poles on real axis)

underdamped $G > \left(\frac{a}{2}\right)^2 \frac{1}{br}$ (not desired response oscillatory)

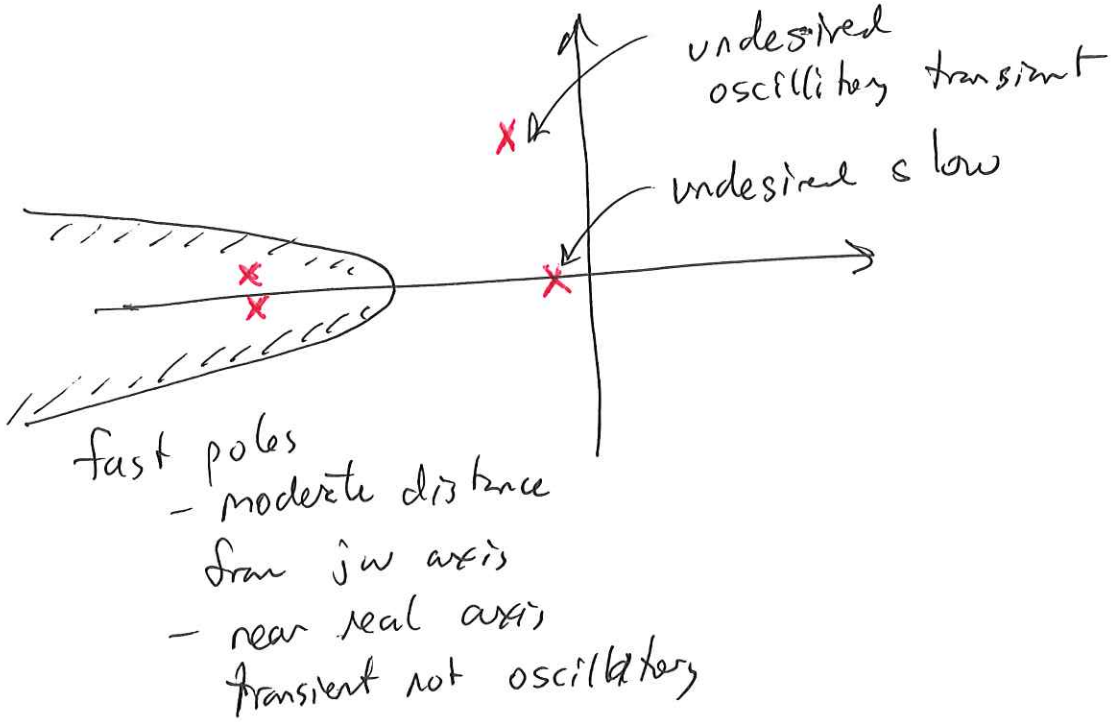
Splane

Poles at $s = -\frac{a}{2}$

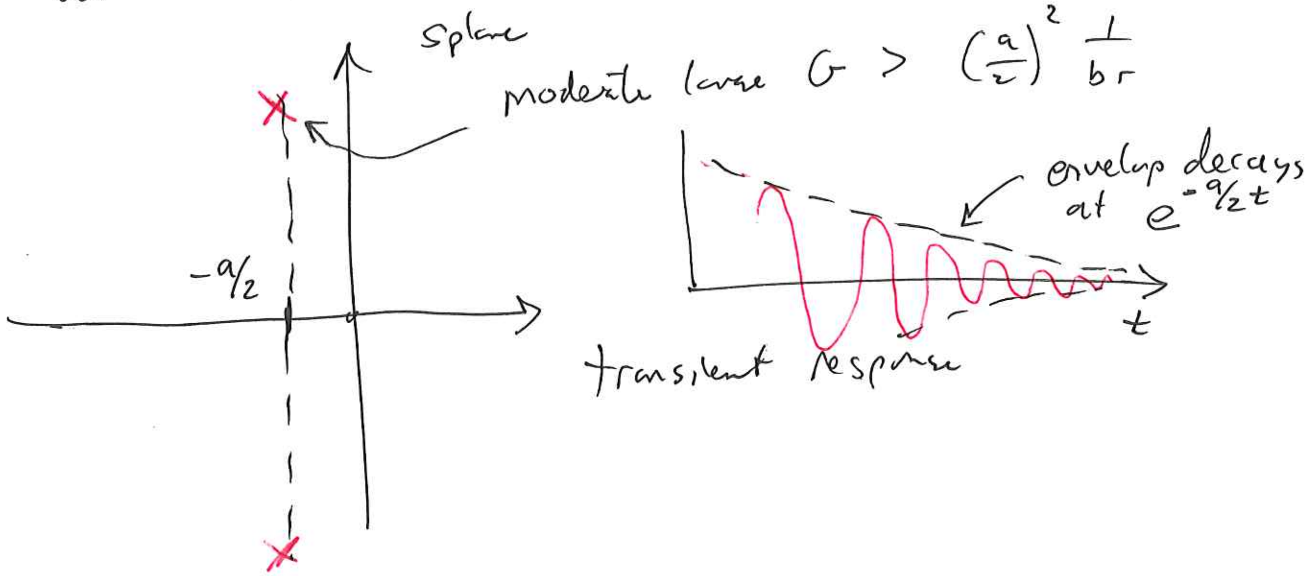


Note with proportional feed back can only get

desired location of closed loop poles



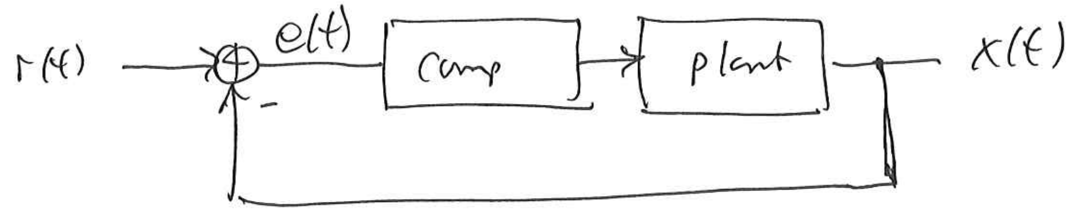
Note in present case we cannot speed up response with larger value of G



How to improve speed?

- move poles of $H_{CL}(s)$ away from $j\omega$ axis.
- How? $\Longrightarrow$ use Root Locus analysis.

Steady State Errors



zero steady state error $\Rightarrow$ $e(\infty) = 0$

finite steady state error $\Rightarrow$ $e(\infty) = \text{finite}$

find $e(\infty)$ for $r(t) = \delta(t)$ impulse

$= u(t)$ step

$= t u(t)$ ramp

$= t^2 u(t)$ parabolic

$= \ldots$ etc.

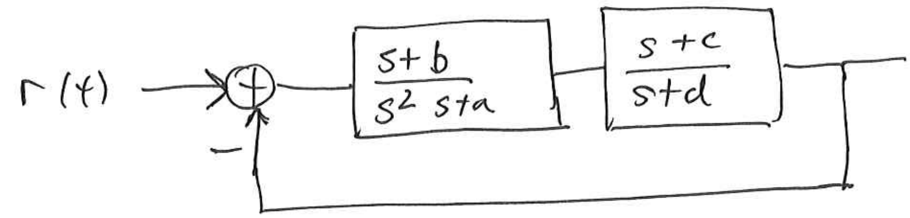
general input $r(t) = a_0 \delta(t) + a_1 u(t) + a_2 t u(t) + a_3 t^2 u(t) + \ldots$

Use linear superposition for $e(\infty)$ for arbitrary input.

Loop Type

Loop Type $=$ number of poles at $s=0$ for $H_{OL}(s)$ (open loop response)

example



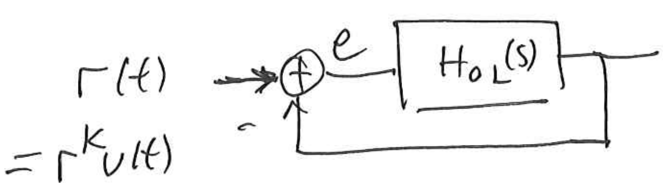
$$H_{OL}(s) = \frac{(s+b)(s+c)}{s^2(s+a)(s+d)}$$

Two poles at $s=0 \implies$ type II loop.

Relation of $r(t)$, $e(\infty)$, loop type

If $r(t) = t^K u(t)$ then
$$\begin{cases} e(\infty) = 0 & \text{if } K < \text{loop type} \\ e(\infty) = \text{finite} & K = \text{loop type} \\ |e(\infty)| = \infty & K > \text{loop type} \end{cases}$$

Consider example $H_{OL}(s) = \frac{1}{s+1}$ type $= 0$



$$e(\infty) = \underbrace{\lim_{s\to 0} s}_{\text{f.v.t.}} \; \underbrace{\frac{K!}{s^{K+1}}}_{R(s)} \; \underbrace{\frac{1}{1+\frac{1}{s+1}}}_{H_{er}(s)}$$

"r to error"

$K=0 \qquad r(t) = u(t)$

$$e(\infty) = \lim_{s\to 0} s \, \frac{1}{s} \, \frac{s+1}{s+2} = \frac{1}{2} \quad \text{finite}$$

$K = 1 \qquad r(t) = t\,u(t)$

$$e(\infty) = \lim_{s\to 0} s \, \frac{1}{s^2} \, \frac{s+1}{s+2} = \infty$$

Example $\qquad H_{OL}(s) = \dfrac{1}{s(s+1)} \qquad$ Loop type $= 1$

$$H_{er}(s) = \frac{1}{1 + \frac{1}{s(s+1)}} = \frac{s^2+s}{s^2+s+1}$$

$\underline{K=0}$

$$e(\infty) = \underbrace{\lim_{s\to 0} s}_{\text{f.v.t.}} \, \underbrace{\frac{1}{s}}_{R(s)} \, \underbrace{\frac{s^2+s}{s^2+s+1}}_{H_{er}(s)} = 0$$

$\underline{K=1}$

$$e(\infty) = \lim_{s\to 0} s \, \frac{1}{s^2} \, \frac{s^2+s}{s^2+s+1} = 1 \quad \underline{\text{finite}}$$

$\underline{K=2}$

$$e(\infty) = \lim_{s\to 0} s \, \frac{2}{s^3} \, \frac{s^2+s}{s^2+s+1} = \infty$$