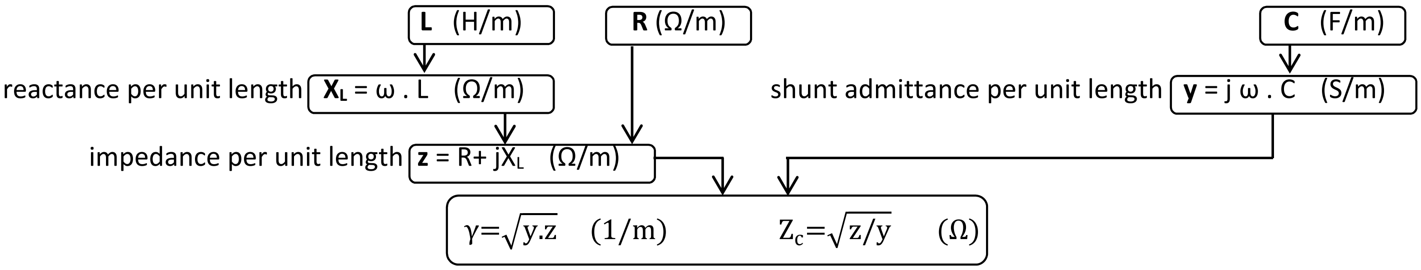
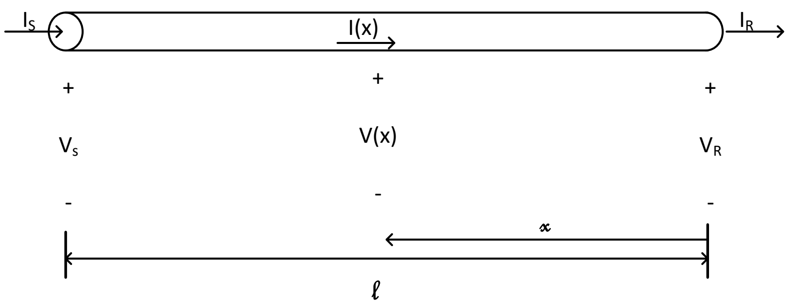
Yani's cheap plastic handout on transmission line models

In transmission line parameters section of Topic 5, we learned how to calculate distributed inductance, **L** (H/m) and distributed capacitance, **C** (F/m). Starting with these values and R (Ω/m), we can calculate the propagation constant **γ** and characteristic impedance $\mathbf{Z_c}$:



We can come up with an expression for voltage and current at any point x along the line if we know the receiving end (line to neutral) voltage V_R and receiving end current I_R.

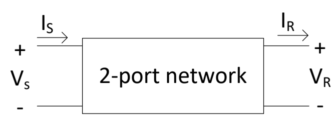
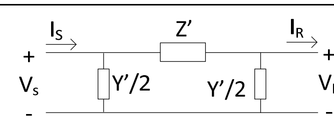


$$V(x) = V_R.\cosh(\gamma x) + I_R Z_c.\sinh(\gamma x)$$

$$I(x) = I_R.\cosh(\gamma x) + \frac{V_R}{Z_c}.\sinh(\gamma x)$$

Since we are often interested in the terminal values only (i.e. sending and receiving end voltage and current), we can plug in x= ℓ in the above equations to find out how V_s and I_s relate to V_R and I_R. (This will give us the "equivalent π model".)

We can show the relationship between the terminal values in 2 ways: a **two-port network with ABCD parameters** or an **equivalent circuit**:

	 $\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$	
Equivalent π Model (Exact parameters) Use this for long lines, ℓ > 250 km	$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_c.\sinh(\gamma l) \\ \frac{1}{Z_c}.\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix}$	$Z' = Z.\frac{\sinh(\gamma l)}{\gamma l}$ $\frac{Y'}{2} = \frac{Y}{2}.\frac{\tanh\left(\gamma l/_2\right)}{\gamma l/_2}$
Nominal π Model Use this for medium lines, 80 < ℓ <250 km	$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+\frac{YZ}{2} & Z \\ \mathrm{Y}\left(1+\frac{YZ}{4}\right) & 1+\frac{YZ}{2} \end{bmatrix}$	$Z' = Z$ $\frac{Y'}{2} = \frac{Y}{2}$
Short Line Model ℓ < 80 km	$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$	$Z' = Z$ & $\frac{Y'}{2} = 0$

Where: $Z = z.l$ & $Y = y.l$