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University of Calgary

Schulich School of Engineering

Department of Electrical and Computer Engineering

ENEL 476 – Electromagnetic Waves and Applications

Final Examination

Winter Session 2016

April 26, 2016 at 3:30 pm

ICT 102

Instructor: Elise Fear

3 hours

Closed book

Student name: ____

EXAMINATION RULES AND REGULATIONS

STUDENT IDENTIFICATION

Each candidate must sign the Seating List confirming presence at the examination. All candidates for final examinations are required to place their University of Calgary I.D. cards on their desks for the duration of the examination. (Students writing mid-term tests can also be asked to provide identity proof.) Students without an I.D. card who can produce an acceptable alternative I.D., e.g., one with a printed name and photograph, are allowed to write the examination.

A student without acceptable I.D. will be required to complete an Identification Form. The form indicates that there is no guarantee that the examination paper will be graded if any discrepancies in identification are discovered after verification with the student's file. A Student who refuses to produce identification or who refuses to complete and sign the Identification Form is not permitted to write the examination.

EXAMINATION RULES

(1) Students late in arriving will not normally be admitted after one-half hour of the examination time has passed.

(2) No candidate will be permitted to leave the examination room until one-half hour has elapsed after the opening of the examination, nor during the last 15 minutes of the examination. All candidates remaining during the last 15 minutes of the examination period must remain at their desks until their papers have been collected by an invigilator.

(3) All inquiries and requests must be addressed to supervisors only.

(4) The following is strictly prohibited:

(a) speaking to other candidates or communicating with them under any circumstances whatsoever;

(b) bringing into the examination room any textbook, notebook or memoranda not authorized by the examiner;

(c) making use of calculators, cameras, cell-phones, computers, head-sets, pagers, PDA'S or any device not authorized by the examiner;

(d) leaving answer papers exposed to view;

(e) attempting to read other student's examination papers.

The penalty for violation of these rules is suspension or expulsion or such other penalty as may be determined.

(5) Candidates are requested to write on both sides of the page, unless the examiner has asked that the left hand page be reserved for rough drafts and calculations.

(6) Discarded matter is to be struck out and not removed by mutilation of the examination answer book.

(7) Candidates are cautioned against writing in their answer book any matter extraneous to the actual answering of the question set.

(8) The candidate is to write his/her name on each answer book as directed and is to number each book.

(9) During the examination a candidate must report to a supervisor before leaving the examination room.

(10) Candidates must stop writing when the signal is given. Answer books must be handed to the supervisor-in-charge promptly. Failure to comply with this regulation will be cause for rejection of an answer paper.

(11) If during the course of an examination a student becomes ill or receives word of a domestic affliction, the student must report at once to the supervisor, hand in the unfinished paper and request that it be cancelled.

If physical and/or emotional ill health is the cause, the student must report at once to a physician/counsellor so that subsequent application for a deferred Final examination is supported by a completed Physician/Counsellor Statement form.

Students can consult professionals at SU Wellness Center during normal working hours or consult their physician/counsellor in the community.

Once an examination has been handed in for marking a student cannot request that the examination be cancelled for whatever reason. Such a request will be denied. Retroactive withdrawals will also not be considered.

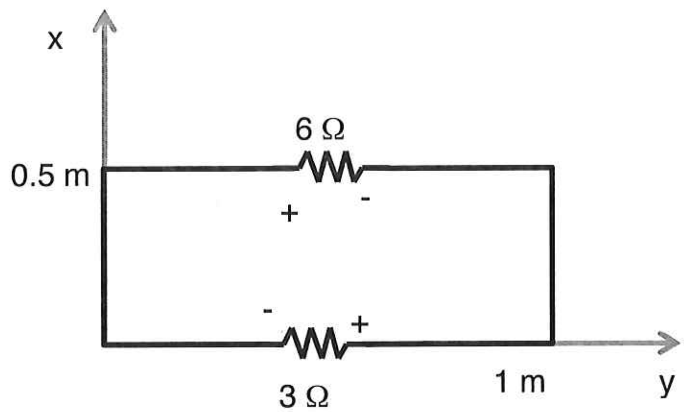
Instructions

(1) This is a closed book exam. No texts or notes are allowed.
(2) Show as much of your reasoning as time permits. Write your answers in the examination booklets.
(3) Calculators are permitted.
(4) Formulas are provided at the end of the question pages.
(5) Hand in all pages. If you detach any pages(s), write your name and UCID number on the detached page(s).
(6) If you write anything you do not want marked, put a large X through it and write "Rough work" beside it.

Question 1 (8 marks; 1 per answer) Short answer and multiple choice. Write your answers in the spaces indicated or circle the best option.

Part 1: The loop shown below is placed in an external magnetic flux density of

$$\mathbf{B} = -0.3e^{-t}\,\mathbf{a}_z \ \ \mathrm{Wb/m^2}$$



The total magnetic flux passing through the surface of the loop is: ________________

The induced current flows around the loop in the following direction (circle one):

- Clockwise
- Counterclockwise

The voltage across the 6 Ω resistor is: ________________

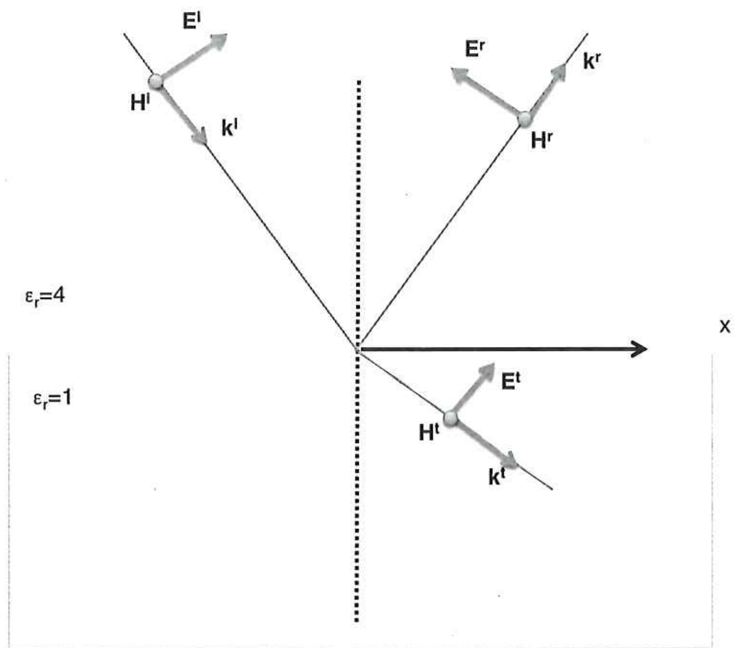
Part 2: A parallel plate capacitor has plates with areas of 9 cm^2 and separated by 1 cm. The capacitor is filled with a dielectric material with $\varepsilon_r=3$ ($\sigma=0$, $\mu_r=1$). The voltage applied to the plates is:

$$V(t)=20\cos(2\pi \times 10^3 t)\ \text{V},$$

The electric field between the plates is: ____________________

The displacement current density is: ____________________

Part 3: A uniform plane wave is obliquely incident on the interface as shown below. Assume that the planar interface is infinite in extent.



The polarization is described as:

- Perpendicular
- Parallel
- Circular

For the scenario shown in the previous figure, is it possible for the incident wave to experience total reflection?

- Yes, if the angle of incidence is 11.54°.
- Yes, if the angle of incidence is 30°.
- Yes, if the angle of incidence is greater than 30°.
- Not for this polarization.

Part 4: A uniform plane wave is normally incident from a lossless medium onto a perfect electric conductor. The total field created by the combination of incident and reflected fields is:

- A partially standing, partially traveling wave
- A standing wave
- A traveling wave
- None of these due to attenuation

Question 2 (13 marks)

Consider a uniform plane wave propagating in a material with $\varepsilon_r=4$, $\mu_r=1$, $\sigma=0$ S/m. The electric field is given by:

$$\mathbf{E}(z,t)=(10\,\mathbf{a}_x + 4\,\mathbf{a}_y)\cos(3 \times 10^6 t-\beta z)\ \text{V/m}$$

Find:

a) The phase constant, β. (2 marks)

b) The wavelength, λ. (2 marks)

c) The phase velocity, v_p or u. (2 marks)

d) The intrinsic impedance, η. (2 marks)

e) The electric field in phasor form (**E**$_s$(z)). (1 mark)

f) The polarization. (1 mark)

g) The magnetic field, **H**(z,t). (3 marks)

Question 3 (30 marks)

An implanted antenna is located 3 mm beneath the skin. The skin has ε_r=45, σ=0.9 S/m and μ_r=1 at 404 MHz. Assume that the waves propagating in the skin can be approximated with uniform plane waves and consider the skin as an infinite half space (i.e. you can consider it to be one region). Assume that the electric field inside the skin and immediately adjacent to the antenna has amplitude of 10 V/m. The wave propagates in the z-direction and the magnetic field is oriented in the x-direction.

a) Find the attenuation constant, α. (3 marks)

b) Find the phase constant, β. (2 marks)

c) Find the intrinsic impedance, η. (4 marks)

d) Find an expression for the electric field in the skin, **E**(z,t). (5 marks)

e) Find an expression for the magnetic field in the skin, **H**(z,t). (3 marks)

f) Find an expression for the time-averaged Poynting vector in the skin, $\mathbf{P}_{av}(z)$. (3 marks)

Assume that the signal is transmitted out of the body and into free space (ε_r=1, μ_r=1, σ=0 S/m).

g) Find the transmission coefficient. (3 marks)

h) Find an expression for the electric field transmitted into free space, $\mathbf{E}^t(z,t)$. (4 marks)

i) Calculate the time-averaged Poynting vector for the transmitted field. Compare the power density in free space to the power density near the antenna. (3 marks)

Question 4 (20 marks)

A distortionless transmission line has α= 40 mNp/m, Z_o= 50 Ω and waves propagate with velocity v_p=2.5 x 10^8 m/s. The line is operating at a frequency of 125 MHz.

a) Find the resistance per unit length (R), inductance per unit length (L), conductance per unit length (G), and capacitance per unit length (C). (9 marks)

A load of Z_L=70-j30 Ω is attached to the line at location z=0.

b) Find the reflection coefficient (Γ). (3 marks)

c) If the forward traveling wave has amplitude of 10 V at z=0 (at the load), find expressions for the total voltage (V(z,t)) and current (I(z,t)) on the terminated transmission line. (8 marks)

Question 5 (20 marks)

An antenna with impedance Z_L=72-j43 Ω is connected to a lossless transmission line with impedance Z_o=75Ω. Use the Smith Chart to solve the following problems.

a) Plot the normalized impedance, z_L (2 marks)

b) Find the reflection coefficient, Γ. (3 marks)

c) Find the standing wave ratio (SWR or s). (2 marks)

d) Do you recommend implementing a matching network? Why? (1 mark)

e) Find the shortest distances from the antenna to the location of the voltage maximum (V_{max}) and voltage minimum (V_{min}) on the transmission line. (4 marks)

f) The antenna is connected to transmission line of length of 0.625λ. What is Z_{in} at this location? (4 marks)

g) A generator supplies 20 V and has internal impedance of Z_g= 75 Ω. How much power is absorbed by the load connected to the generator and the 0.625λ line? (4 marks)

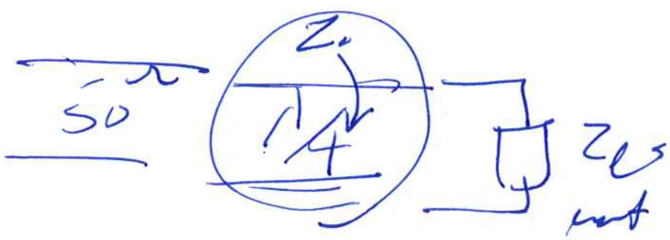
Question 6 (19 marks)

An antenna with impedance Z_a=35.5-j105 Ω is to be matched to a 50 Ω transmission line. The frequency of operation is 1 GHz and the waves propagate on the line with a velocity of 0.9c.

a) Design a series stub tuner to match the antenna to the line. Assume that the stub is terminated with an open circuit and place the stub close to the load. Give the location and length of the stub in wavelengths and cm. (7 marks)

b) Design a shunt stub tuner to match the antenna to the line. Assume that the stub is terminated with an open circuit and place the stub close to the load. Give the location and length of the stub in wavelengths and cm. (7 marks)

c) Design a quarter-wave transformer to match the antenna to the line. Give the length and location of the matching section in wavelengths and cm. Give the impedance of the quarter-wavelength line. (5 marks)



Question	Mark
1	8
2	13
3	20
4	30
5	20
6	19
Total	

$\mu_o = 4\pi \times 10^{-7}$ H/m
$1/\mu_o = 8 \times 10^5$ m/H
$\eta_o = 120\pi\ \Omega$
$\varepsilon o = 8.85\times10^{-12}$ F/m

$\mathbf{a}_r \cdot \mathbf{a}_x = \sin\theta\cos\phi$	$\mathbf{a}_\theta \cdot \mathbf{a}_x = \cos\theta\cos\phi$	$\mathbf{a}_\varphi \cdot \mathbf{a}_x = -\sin\phi$
$\mathbf{a}_r \cdot \mathbf{a}_y = \sin\theta\sin\phi$	$\mathbf{a}_\theta \cdot \mathbf{a}_y = \cos\theta\sin\phi$	$\mathbf{a}_\varphi \cdot \mathbf{a}_y = \cos\phi$
$\mathbf{a}_r \cdot \mathbf{a}_z = \cos\theta$	$\mathbf{a}_\theta \cdot \mathbf{a}_z = -\sin\theta$	$\mathbf{a}_\varphi \cdot \mathbf{a}_z = 0$

$\mathbf{a}_x \cdot \mathbf{a}_\rho = \cos\phi$	$\mathbf{a}_x \cdot \mathbf{a}\phi = -\sin\phi$	$\mathbf{a}_x \cdot \mathbf{a}_z = 0$
$\mathbf{a}_y \cdot \mathbf{a}_\rho = \sin\phi$	$\mathbf{a}_y \cdot \mathbf{a}\phi = \cos\phi$	$\mathbf{a}_y \cdot \mathbf{a}_z = 0$
$\mathbf{a}_z \cdot \mathbf{a}_\rho = 0$	$\mathbf{a}_z \cdot \mathbf{a}\phi = 0$	$\mathbf{a}_z \cdot \mathbf{a}_z = 1$

$x = \rho \cos\phi$

$y = \rho \sin\phi$

Cartesian	Cylindrical	Spherical
$dx\,\mathbf{ax} + dy\,\mathbf{ay} + dz\,\mathbf{az}$	$d\rho\,\mathbf{a}\rho + \rho d\phi\,\mathbf{a}\phi + dz\,\mathbf{az}$	$dr\,\mathbf{ar} + r\,d\theta\,\mathbf{a}\theta + r\sin\theta\,d\phi\,\mathbf{a}\phi$
$(dydz)\mathbf{ax}$	$(\rho\,d\phi\,dz)\mathbf{a}\rho$	$(r^2\sin\theta\,d\theta\,d\phi)\mathbf{ar}$
$(dxdz)\mathbf{ay}$	$(d\rho\,dz)\mathbf{a}\phi$	$(r\sin\theta\,dr\,d\phi)\mathbf{a}\theta$
$(dxdy)\mathbf{az}$	$(\rho\,d\rho\,d\phi)\mathbf{az}$	$(r\,dr\,d\theta)\mathbf{a}\phi$
$dx\,dy\,dz$	$\rho\,d\rho\,d\phi\,dz$	$r^2\sin\theta\,dr\,d\theta\,d\phi$

$r = \sqrt{x^2 + y^2 + z^2}$ $\qquad x = r\sin\theta\cos\phi$

$\theta = \cos^{-1}\dfrac{z}{\sqrt{x^2+y^2+z^2}}$ $\qquad y = r\sin\theta\sin\phi$

$\phi = \tan^{-1}\dfrac{y}{x}$ $\qquad z = r\cos\theta$

$$\nabla V = \frac{\partial V}{\partial x}\vec{a}_x + \frac{\partial V}{\partial y}\vec{a}_y + \frac{\partial V}{\partial z}\vec{a}_z$$

$$\nabla V = \frac{\partial V}{\partial \rho}\vec{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\vec{a}_\phi + \frac{\partial V}{\partial z}\vec{a}_z$$

$$\nabla V = \frac{\partial V}{\partial r}\vec{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\vec{a}_\phi$$

$$\nabla\cdot\vec{E} = \left[\frac{\partial(E_x)}{\partial x} + \frac{\partial(E_y)}{\partial y} + \frac{\partial(E_z)}{\partial z}\right]$$

$$\nabla\cdot\vec{E} = \left[\frac{1}{\rho}\frac{\partial(\rho E_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial(E_\phi)}{\partial \phi} + \frac{\partial(E_z)}{\partial z}\right]$$

$$\nabla\cdot\vec{E} = \left[\frac{1}{r^2}\frac{\partial(r^2 E_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta E_\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial(E_\phi)}{\partial \phi}\right]$$

$$\nabla^2 V = \left[\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}\right]$$

$$\nabla^2 V = \left[\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + + \frac{\partial^2 V}{\partial z^2}\right]$$

$$\nabla^2 V = \left[\begin{array}{l}\dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial V}{\partial r}\right) + \dfrac{1}{r^2\sin\theta}\dfrac{\partial}{\partial \theta}\left(\sin\theta\dfrac{\partial V}{\partial \theta}\right) \\ + \dfrac{1}{r^2\sin^2\theta}\dfrac{\partial^2 V}{\partial \phi^2}\end{array}\right]$$

$$\nabla\times\overline{\mathbf{A}} = \begin{vmatrix} a_x & a_y & a_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \mathbf{A}_x & \mathbf{A}_y & \mathbf{A}_z \end{vmatrix}$$

$$\nabla\times\overline{\mathbf{A}} = \frac{1}{\rho}\begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \dfrac{\partial}{\partial \rho} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ \mathbf{A}_\rho & \rho\mathbf{A}_\phi & \mathbf{A}_z \end{vmatrix}$$

$$\nabla\times\overline{\mathbf{A}} = \frac{1}{r^2\sin\theta}\begin{vmatrix} a_r & r a_\theta & r\sin\theta a_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ \mathbf{A}_r & r\mathbf{A}_\theta & r\sin\theta\mathbf{A}_\phi \end{vmatrix}$$

ENEL 475/6 Formula Sheet 2 Revised 10/02/09

Magnetostatics	materials
$\vec{H} = \int_L \frac{I d\vec{l} \times \vec{a}_R}{4\pi R^2}$ $W_M = \frac{1}{2}\int \vec{H}\bullet\vec{B}\,dv$ **T=m x B** $\quad L = \frac{N\Psi}{I}$ $M_{12} = \frac{N_1\Psi_{12}}{I_2} \quad \vec{F} = \oint I d\vec{l}\, x\vec{B}$	$\vec{M} = \chi_m \vec{H}$ $\nabla\times\vec{M} = \vec{J}_M$ $\mu_r=1+\chi_m$
Lorentz force	Continuity
F=q(**E**+**v**x**B**)	$\nabla\bullet\vec{J} = -\frac{\partial\rho_v}{\partial t}$
Electrostatics	Materials
$\vec{E}(\vec{r}) = \int \frac{\rho_v dv}{4\pi\varepsilon_o\varepsilon_r R^2}\vec{a}_R$ $V(\vec{r}) = \int \frac{\rho_v(\vec{r}')dv'}{4\pi\varepsilon_r\varepsilon_o \lvert\vec{r}-\vec{r}'\rvert} + C$ $V = -\int \vec{E}\bullet d\vec{l} + C$ $\vec{E} = -\nabla V$ $W_E = \frac{1}{2}\int \vec{E}\bullet\vec{D}\,dv$ $\nabla^2 V = -\frac{\rho_v}{\varepsilon} \qquad \nabla^2 V = 0$ C=Q/V	$R = l/(\sigma S)$ $\vec{P} = \chi_e\varepsilon_o\vec{E}$ $\nabla\bullet\vec{P} = -\rho_{pv}$ $\vec{P}\bullet\vec{a}_n = \rho_{ps}$ $\varepsilon_r=1+\chi_e$
Boundary conditions	
$\mathbf{D}_{1n}-\mathbf{D}_{2n}=\rho s \quad \mathbf{a}_{21}\cdot(\mathbf{D}_1-\mathbf{D}_2)=\rho s$ $\mathbf{E}_{t1}=\mathbf{E}_{t2} \quad \mathbf{a}_{21}\,\mathrm{x}\,(\mathbf{E}_1-\mathbf{E}_2)=0$ $\mathbf{B}_{1n}-\mathbf{B}_{2n}=0 \quad \mathbf{a}_{21}\cdot(\mathbf{B}_1-\mathbf{B}_2)=0$ $\mathbf{a}_{21}\,\mathrm{x}\,(\mathbf{H}_1-\mathbf{H}_2)=\mathbf{J}_s \quad \mathbf{a}_{21}\,\mathrm{x}\,(\mathbf{H}_1-\mathbf{H}_2)=\mathbf{J}_s$	$\mathbf{J}_s$ is surface current (also denoted as **K**)
Maxwell's equations	
$\oint_c \vec{E}\bullet d\vec{l} = -\frac{\partial}{\partial t}\int_s \vec{B}\bullet d\vec{s} \qquad \oint_s \vec{B}\bullet d\vec{s} = 0$ $\oint_s \varepsilon_r\varepsilon_o\vec{E}\cdot d\vec{s} = \int_v \rho_v dv$ $\oint_c \vec{H}\bullet d\vec{l} = \int_s \vec{J}\bullet d\vec{s} + \frac{\partial}{\partial t}\int_s \vec{D}\bullet d\vec{s}$ $\nabla\bullet\vec{B} = 0 \qquad \nabla\bullet\vec{D} = \rho_v$ $\nabla\times\vec{H} = \vec{J} + \frac{\partial}{\partial t}\vec{D} \qquad \nabla\times\vec{E} = -\frac{\partial}{\partial t}\vec{B}$ $V_{emf} = -\int_s \frac{\partial\vec{B}}{\partial t}\bullet d\vec{s} + \oint_L (\vec{v}x\vec{B})\bullet d\vec{l}$	Note: for static fields, time derivatives are zero. In phasor form, time derivatives become $j\omega$ terms.

Time-varying fields: UPW	
$\nabla^2\vec{E}_s - \gamma^2\vec{E}_s = 0$ $\nabla^2\vec{H}_s - \gamma^2\vec{H}_s = 0$ $\gamma = \alpha + j\beta$	Vector wave equations for time-harmonic fields in lossy medium. With lossless medium or free space, α=0.
$\lambda = \frac{2\pi}{\beta}$ $\beta = \omega\sqrt{\mu_r\mu_o\varepsilon_r\varepsilon_o}$ $T = 1/f$ $\lvert E\rvert/\lvert H\rvert = \eta = \sqrt{\frac{\mu_r\mu_o}{\varepsilon_r\varepsilon_o}}$ $v_p = \frac{\omega}{\beta}$	Uniform plane wave in lossless medium. For free space, μr=1 and εr=1.
$\vec{E}(z,t) = E_o e^{-\alpha z}\cos(\omega t - \beta z)\vec{a}_x$ $\vec{H}(z,t) = \frac{E_o}{\lvert\eta\rvert}e^{-\alpha z}\cos(\omega t - \beta z - \theta_\eta)\vec{a}_y$	One example of E and H fields in lossy medium.
$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$ $\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$ $\lvert\eta\rvert = \frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}} \qquad \tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon}$ $\delta=1/\alpha$ $\lambda = \frac{2\pi}{\beta}$	Parameters describing UPW in lossy medium.
$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} \qquad \eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^o$	good conductor: ($\sigma/\omega\varepsilon >> 1$)
$\vec{P}_{avg}(z) = \frac{1}{2}\mathrm{Re}(\vec{E}_s(z)\times\vec{H}_s^{\,*}(z))$ **P**(z,t)=**E**(z,t) x **H**(z,t)	Poynting vector
$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad T = \frac{2\eta_2}{\eta_2 + \eta_1}$	Transmission and reflection coefficients: normal incidence
$\Gamma_\parallel = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$ $T_\parallel = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$ $\Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$ $T_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$	Transmission and reflection coefficients: oblique incidence $\theta_i = \theta_r$ $\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}$

ENEL 476 Formula Sheet 3 **Revised 01/28/11**

Waves and T/R – continued

$$s = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\sin\theta_B = \sqrt{\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}}$$

$$\tan\theta_B = \sqrt{\varepsilon_2 / \varepsilon_1}$$

$$\sin\theta_c = \sqrt{\varepsilon_2 / \varepsilon_1}$$

$$R_{ac} = L/(\sigma\delta w)$$

Transmission lines:

$$V_s(z) = V^+ e^{-\gamma z} + V^- e^{\gamma z}$$

$$I_s(z) = \frac{1}{Z_o}\left[V^+ e^{-\gamma z} - V^- e^{\gamma z}\right]$$

$$Z_o = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$\gamma = \sqrt{(G+j\omega C)(R+j\omega L)} = \alpha + j\beta$$

$$P_{ave} = \frac{|V_o^+|^2}{2Z_o} e^{-2\alpha z}\cos\theta$$

Distortionless transmission lines:

R/L=G/C

$$\alpha = \sqrt{RG}$$
$$\beta = \omega\sqrt{LC}$$
$$R_o = \sqrt{\frac{R}{G}}$$

Lossless line:

$$Z_{in} = Z_o \frac{Z_L + jZ_o\tan(\beta l)}{Z_o + jZ_L\tan(\beta l)}$$

$$Z_{in\_max} = Z_o\frac{1+|\Gamma|}{1-|\Gamma|} = Z_o \cdot SWR$$

$$\Gamma_l = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_{in\_min} = \frac{Z_o}{SWR}$$

$$\Gamma(l) = \Gamma(0)e^{-j2\beta l}$$

$$SWR = \left|\frac{V_{max}}{V_{min}}\right| = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Waveguides:

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$v_p = u_p = \frac{u'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$v_g = u_g = u'\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$v_p v_g = u'^2$$

$$\alpha_d = \frac{\sigma\eta'}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\alpha_c\big|_{TE10} = \frac{2R_s}{b\eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left(0.5 + \frac{b}{a}\left(\frac{f_c}{f}\right)^2\right)$$

$$\alpha_c\big|_{TE} = \frac{2R_s}{b\eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left(\left(1+\frac{b}{a}\right)\left(\frac{f_c}{f}\right)^2 + \frac{\frac{b}{a}\left(\frac{b}{a}m^2+n^2\right)}{\frac{b^2}{a^2}m^2+n^2}\left(1-\left(\frac{f_c}{f}\right)^2\right)\right)$$

$$\alpha_c\big|_{TM} = \frac{2R_s}{b\eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left(\frac{\left(\frac{b^3}{a^3}m^2+n^2\right)}{\frac{b^2}{a^2}m^2+n^2}\right)$$