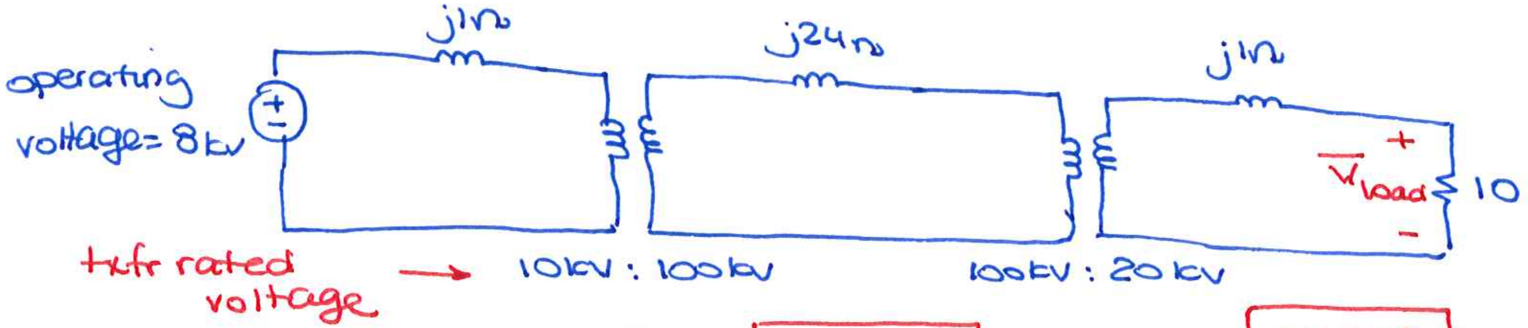
Ex: Calculate load voltage, current, power. Use $S_{base} = 100$ MVA & $V_{base} = 8$ kV in gen zone.



Zone 1

$V_{base_1} = 8$ kV (given)

$Z_{base_1} = \frac{(V_{base_1})^2}{S_{base}} = 0.64\ \Omega$

Zone 2

$V_{base_2} = V_{base_1} \times \frac{100\text{ kV}}{10\text{ kV}}$

$= 80$ kV

$Z_{base_2} = \frac{(V_{base_2})^2}{S_{base}}$

$= 64\ \Omega$

Zone 3

$V_{base_3} = V_{base_2} \times \frac{20\text{ kV}}{100\text{ kV}}$

$= 16$ kV

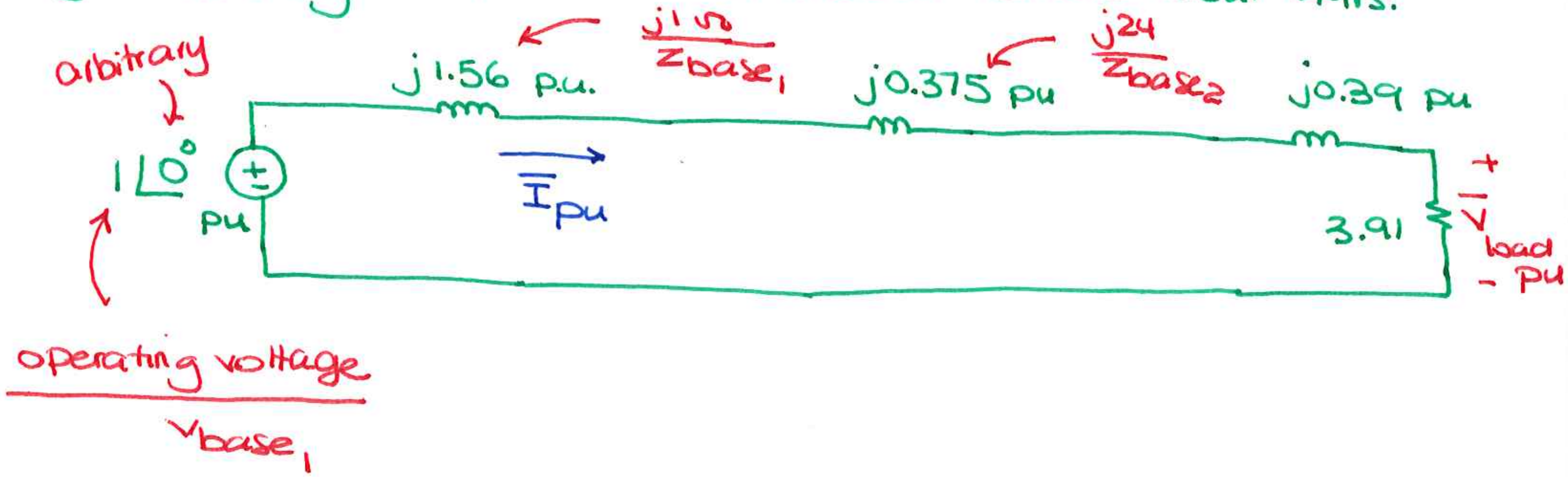
$Z_{base_3} = \frac{(V_{base_3})^2}{S_{base}}$

$= 2.56\ \Omega$

$I_{base_3} = \frac{S_{base}}{V_{base_3}}$

$= 6.25$ kA

- We can now come up with a p.u. circuit (impedance diagram) by calculating all p.u. values. We can remove ideal txfrs:



$$\overline{I}_{pu} = \frac{1\angle 0^\circ}{j1.56 + j0.375 + j0.39 + 3.91} = 0.22\angle -30.8^\circ \quad pu$$

$$\overline{V}_{load,pu} = \overline{I}_{pu} \times 3.91 = 0.859\angle -30.8^\circ \quad pu$$

$$\overline{S}_{load,pu} = \overline{V}_{load,pu} \times \overline{I}_{pu}^{*} = 0.189 \quad pu$$

To get the actual values, multiply p.u. values with their corresponding base values:

phase angles not changed in p.u.

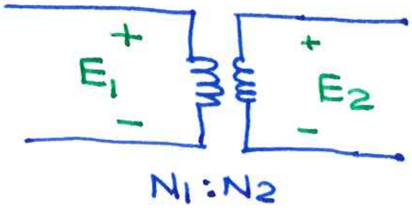
$$\overline{V}_{load} = \overline{V}_{load,pu} \times V_{base_3} = 13.7\angle -30.8^\circ \quad kV$$

$$\overline{I}_{load} = \overline{I}_{pu} \times I_{base_3} = 1375 \angle -30.8^\circ \quad A$$

$$\overline{S}_{load} = \overline{S}_{load,pu} \times S_{base} = 18.9 \quad MW$$

Transformers in PU

- Ideal txfr model:



zone 1 | zone 2

- Suppose we selected V_{base_1}

$$\therefore \quad E_{1,pu} = \frac{E_1}{V_{base_1}}$$

$$V_{base_2} = V_{base_1} \times \frac{N_2}{N_1}$$

$$E_{2,pu} = \frac{E_2}{V_{base_2}} = \frac{\cancel{\frac{N_2}{N_1}} \cdot E_1}{\cancel{\frac{N_2}{N_1}} \cdot V_{base_1}}$$

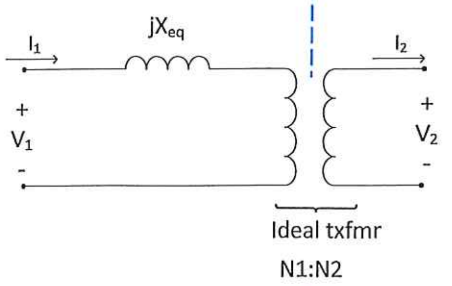
$$= E_{1,pu}$$

i.e.: pu voltages (and currents) are identical on either side of <u>ideal</u> txfr $\therefore$ we can remove ideal txfrs from pu analysis (as we did in prev. example)

ENEL 487

Non-ideal transformers in Per Unit

Let's start with the simplest non-ideal transformer model.



Zone 1

Suppose we selected $V_{base\,1}$

$$Z_{base\,1} = \frac{(V_{base\,1})^2}{S_{base}}$$

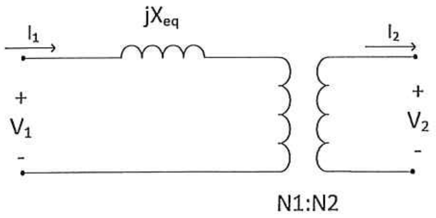
Zone 2

$$V_{base\,2} = V_{base\,1}\frac{N_2}{N_1}$$

$$Z_{base\,2} = \frac{(V_{base\,2})^2}{S_{base}} = \frac{\left(V_{base\,1}\frac{N_2}{N_1}\right)^2}{S_{base}} = \left(\frac{N_2}{N_1}\right)^2 Z_{base\,1}$$

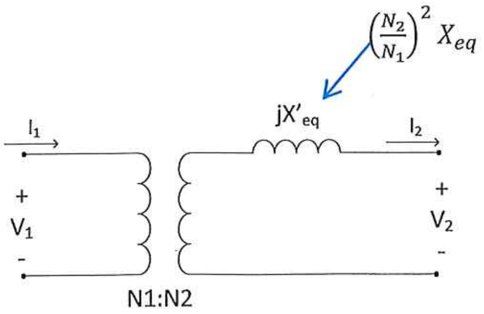
We already stablished that the ideal transformer portion can be removed in per unit analysis. The transformer impedance is the only thing left! The impedance should be divided by the correct Z_{base} in the per unit analysis.

1) With transformer impedance referred to winding 1 side:



$$X_{eq,pu} = \frac{X_{eq}}{Z_{base\,1}}$$

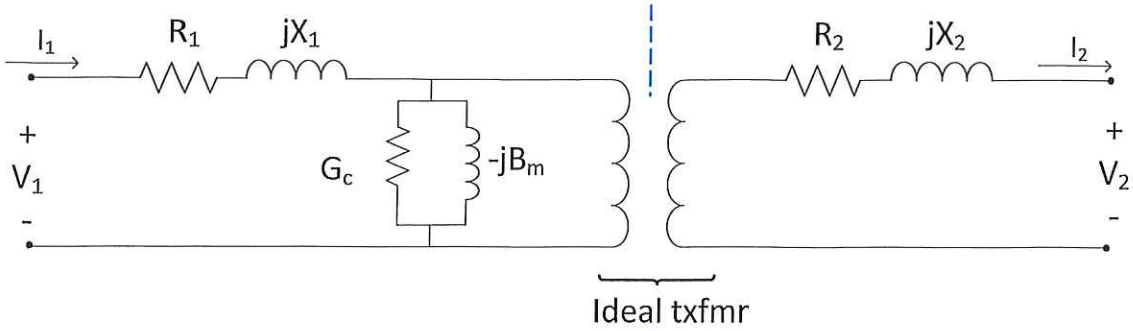
2) With transformer impedance referred to winding 2 side:



$$X'_{eq,pu} = \frac{X'_{eq}}{Z_{base\,2}} = \frac{\left(\frac{N_2}{N_1}\right)^2 X_{eq}}{\left(\frac{N_2}{N_1}\right)^2 Z_{base\,1}} = \frac{X_{eq}}{Z_{base\,1}} = X_{eq,pu}$$

Conclusion: Transformer impedance in PU is the same regardless of which side it is referred to.

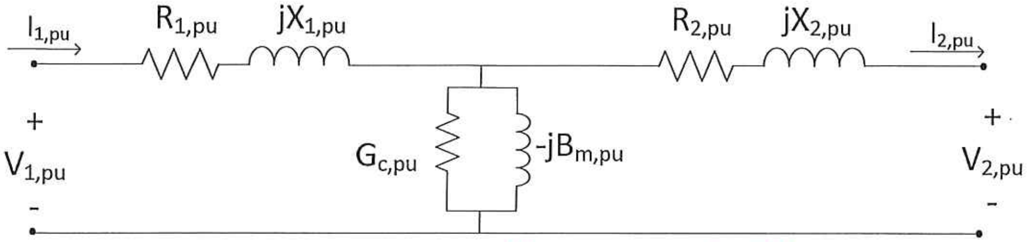
We can extend this out to the complete non-ideal transformer model:



Zone 1 with $V_{base\,1}$ and $Z_{base\,1}$ as defined before

Zone 2 with $V_{base\,2}$ and $Z_{base\,2}$ as defined before

In per Unit, this transformer model becomes:



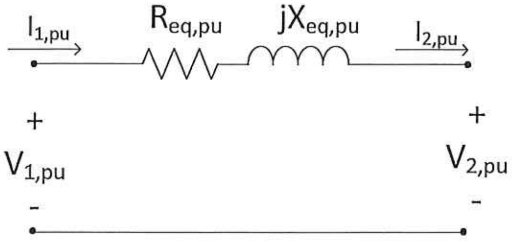
these are admittances

$$X_{1,pu} = \frac{X_1}{Z_{base\,1}} \qquad R_{1,pu} = \frac{R_1}{Z_{base\,1}} \qquad G_{c,pu} = \frac{G_c}{Y_{base\,1}} \qquad B_{m,pu} = \frac{B_m}{Y_{base\,1}} \qquad X_{2,pu} = \frac{X_2}{Z_{base\,2}} \qquad R_{2,pu} = \frac{R_2}{Z_{base\,2}}$$

base admittance

$$Y_{base} = \frac{1}{Z_{base}}$$

With the excitation branch omitted, the per unit representation of a non-ideal transformer becomes:



This is just the model from the previous page with winding resistance added.

The other conclusion: In PU analysis, non-ideal transformer becomes just another impedance in the circuit.

3ϕ Per Unit Analysis

2 options:

1) Convert 3ϕ system to per phase (i.e. convert everything to Y)
Carry per-phase P.U. analysis using 1ϕ base values as in previous example. When converting to actual 3ϕ quantities, remember $S_{3\phi} = 3 \times S_{1\phi}$, $V_{LL} = \sqrt{3} V_{LN}$

- This seems like more work than necessary! see Lab 1

2) Go directly from 3ϕ system (e.g. single line diagram) to per unit system (impedance diagram)

- Analysis is similar to 1ϕ p.u. circuit except base values are 3ϕ:
  - Use a 3ϕ S_{base}
  - V_{base} is line-to-line voltage
  - I_{base} is line current

Procedure:

i) Pick a 3ϕ power base for entire system, $S_{base,3\phi}$

ii) Pick $V_{base,LL}$ for each voltage zone

(aside: $V_{base,LL} = \sqrt{3}\ V_{base,LN}$)

iii) Calculate Z_{base}:

$$\boxed{Z_{base} = \frac{V_{base,LL}^2}{S_{base,3\phi}}}$$

↑ we can use this for all Z in the system except Δ-connected load

iv) Calculate I_{base}:

$$\boxed{I_{base} = \frac{S_{base,3\phi}}{\sqrt{3}\ V_{base,LL}}}$$

↑

reminder:
in 3ϕ systems $S_{3\phi} = \sqrt{3}\ V_{\ell\ell} \cdot I_{\ell}$

v) Convert actual values to P.u. & draw P.u. circuit (impedance diagram)

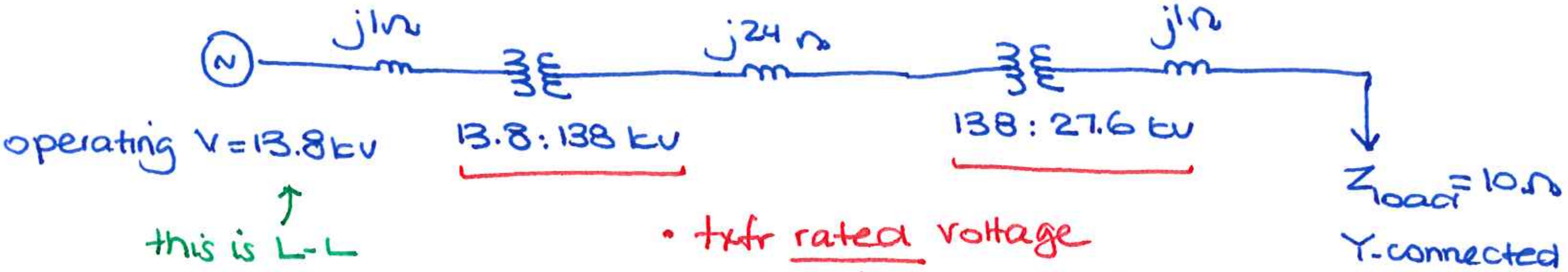
vi) Analyze P.u. circuit. Multiply P.u. quantities by corresponding base values to obtain 3ϕ quantities.

Ex: Suppose our previous example is one phase of a 3ϕ system

Find $\overline{V}_{load}$, $\overline{S}_{load}$, $\overline{I}_{load}$. Use $S_{base,3\phi} = 300$ MVA

$V_{base,LL} = 13.8$ kV in Gen zone

The single line diagram of the system is:



this is L-L

- txfr rated voltage
- ratio of L-L voltages

Zone 1

$V_{base_{1,LL}} = 13.8$ kV (given)

Vbase1

$$Z_{base_1} = \frac{(13.8\text{ kV})^2}{300\text{ MVA}}$$

Sbase

$= 0.64Ω$

Zone 2

$$V_{base_{2,LL}} = V_{base_1} \times \frac{138}{13.8}$$

$= 138$ kV

$Z_{base_2} = 64$ Ω

Zone 3

$$V_{base_{3,LL}} = V_{base_2} \times \frac{27.6}{138}$$

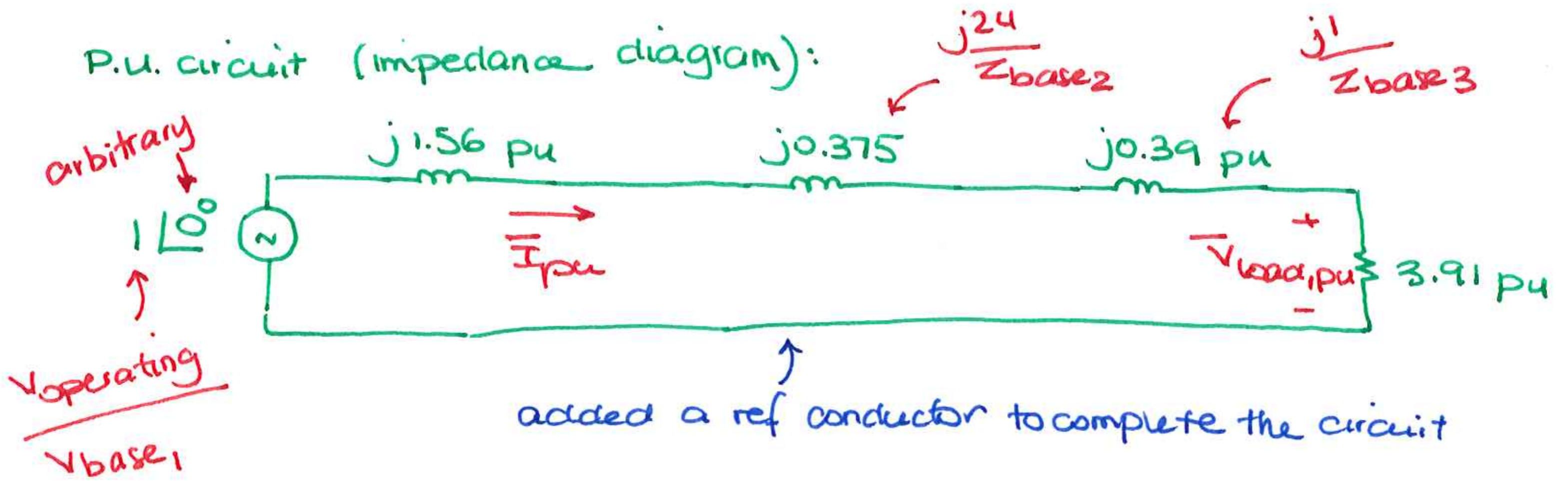
$= 27.6$ kV

$Z_{base_3} = 2.56$ Ω

$$I_{base_3} = \frac{300\text{ MVA}}{\sqrt{3} \times 27.6}$$

$= 6275.5$ A

P.u. circuit (impedance diagram):



- Notice the differences between per-phase (single phase) circuits and P.u. (impedance) diagram. While the topology is similar, the information in the circuits are different.
  - Actual values in per-phase, P.u. values in P.u diagram
  - L-N voltages in per-phase, P.u. voltages in P.u. diagram

we can solve this circuit to get : $\overline{I}_{pu} = 0.22\angle{-30.8°}$ pu

$\overline{V}_{load,pu} = 0.86\angle{-30.8°}$ pu

$\overline{S}_{load,pu} = 0.189$ pu

actual values : $\overline{V}_{load,LL} = \overline{V}_{load,pu} \times V_{base_{3,LL}} = 23.8\angle{-30.8°}$ kV

$\overline{I}_{load} = \overline{I}_{pu} \times I_{base_3} = 1.381\angle{-30.8°}$ kA

- This is the line current going into the load.
- For Y connections, this is equal to I_ϕ

$\overline{S}_{load,3\phi} = \overline{S}_{load} \times S_{base,3\phi} = 57.6$ mW

Z_{base} for Δ-connected load

- For Y-connected loads $\boxed{Z_{base} = \dfrac{V_{base,LL}^2}{S_{base,3\phi}}}$

also know that $Z_\Delta = 3 . Z_Y$

$$\therefore \quad Z_{base,\Delta} = 3 . Z_{base,Y} = \boxed{3 \frac{V_{base,LL}^2}{S_{base,3\phi}}}$$

use for Δ-connected load

use for all other Z

- regardless of load connection type, we arrive at the same pu value for a given load

Change of base

- Equipment impedances are often given in pu using the rated power & voltage as S_{base} & V_{base}. If these are different from S_{base} & V_{base} used in our analysis, we need to convert them:

$$Z_{pu,new} = \frac{Z_{actual}}{Z_{base,new}} = \frac{Z_{pu,old} \times Z_{base,old}}{Z_{base,new}}$$

$$= Z_{pu,old} \times \frac{Z_{base,old}}{Z_{base,new}}$$

$$\therefore \boxed{Z_{pu,new} = Z_{pu,old} \times \frac{V_{base,old}^2}{V_{base,new}^2} \times \frac{S_{base,new}}{S_{base,old}}}$$

Ex: $\overbrace{400\text{ MVA}}^{\text{rated power}}$, $\overbrace{144:245}^{\text{rated voltage}}$ kV transformer has leakage reactance of 13%. What is P.U impedance on $\underbrace{100\text{ MVA}}_{S_{base,new}}$ & $\underbrace{138\text{ kV}}_{V_{base,new}\text{ on LV side}}$ V_{base} on LV side?

- Pu value x 100%
- based on rated values

$$Z_{pu,new} = j0.13 \times \left(\frac{144\text{ kV}}{138\text{ kV}}\right)^2 \times \frac{100\text{ MVA}}{400\text{ MVA}} = j0.035$$

or $X_{pu,new} = 3.5\%$

- What if we used HV side values?

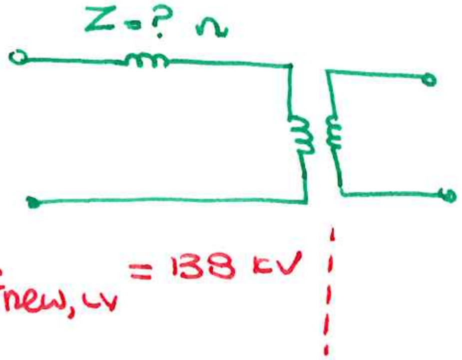
$$V_{base,new,HV} = V_{base,new,LV} \times \frac{245}{144} = 138 \times \frac{245}{144}$$

$$Z_{pu,new} = j0.13 \times \left(\frac{245\text{ kV}}{V_{base,new,HV}}\right)^2 \times \frac{100\text{ MVA}}{400\text{ MVA}} = j0.035$$

same as before!

- What is Z in Ohms referred to LV side?

$$Z = \underbrace{j0.035}_{Z_{pu,new}} \times \underbrace{\frac{(138\text{ kV})^2}{100\text{ MVA}}}_{Z_{base,new,LV}} = j6.7\,\Omega$$



$V_{base_{new,LV}} = 138$ kV

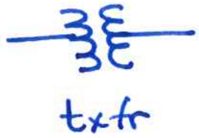
or

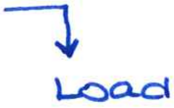
$$Z = \underbrace{j0.13}_{Z_{pu,old}} \times \underbrace{\frac{(144\text{ kV})^2}{400\text{ MVA}}}_{Z_{base,old,LV}} = j6.7\,\Omega$$

Single Line Diagram (SLD)

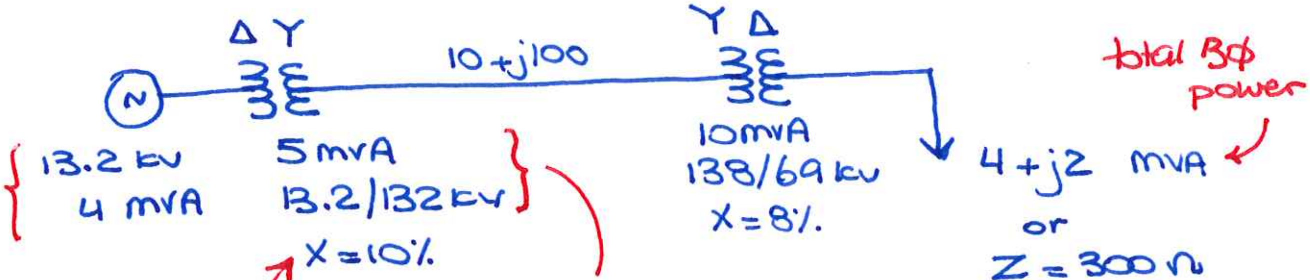
- Used for analyzing 3ϕ systems. It shows one phase of a balanced 3ϕ system. We often use symbols instead of circuit elements. Values indicated on the SLD are often 3ϕ values e.g: L-L values, 3ϕ power

- Standard symbols :





- Example :



rated voltage & power

- operating values may be different

pu reactance based on rated values

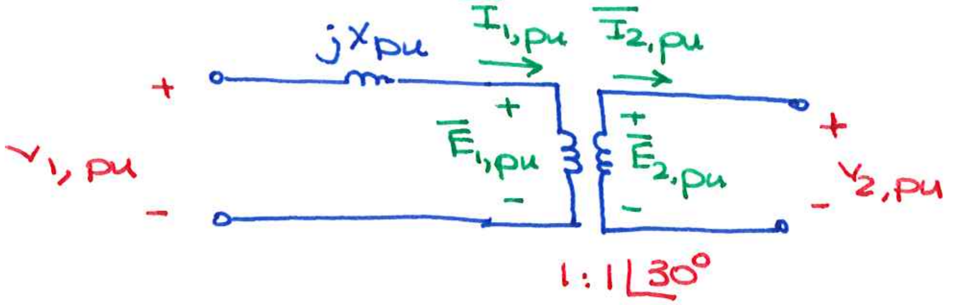
rated voltage & power

L-L voltage regardless of connection type

3ϕ Txfr in PU

1) Y-Y or Δ-Δ : line-to-line voltages on either side are in phase with each other so the txfr becomes an impedance in pu :

2) Δ-Y : 30° phase shift between 2 sides. P.u. only deals with magnitudes :



$\overline{I}_{2,pu} = \overline{I}_{1,pu} \angle 30°$

$\overline{E}_{2,pu} = \overline{E}_{1,pu} \angle 30°$

3) Y-Δ : identical to Δ-Y but with a 1:1 ∠-30° ideal txfr

- We will ignore the phase shifts for Δ-Y & Y-Δ tranrformers in this course.

Motors & Generators (machines)

- on the SLD

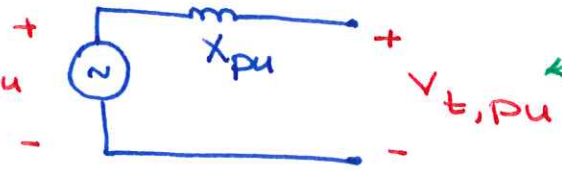
operating voltage & power may be different { 13.8 kV ← rated voltage

10 mVA ← rated power

X = 10% ← pu reactance

- In PU circuit (impedance diagram), machines are shown as an EMF behind an impedance:

operating internal EMF voltage → E_{pu}



← operating voltage at terminals

value in PU circuit

$$X_{pu} = X_{pu,old} \times \frac{V_{rated}^2}{V_{base}^2} \times \frac{S_{base}}{S_{rated}}$$

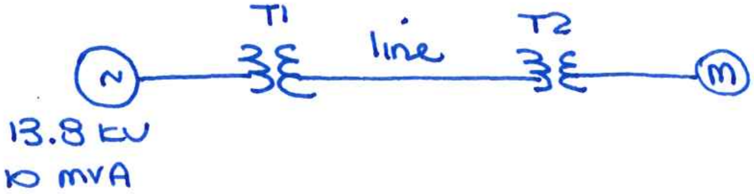
V_{rated}: $V_{base,old}$ (13.8 kV in our example)

$X_{pu,old}$: value based on rated V, S (10% in our example)

V_{base}: our choice for V_{base}

S_{rated}: $S_{base,old}$ (10 mVA in our ex.)

. SLD



. Pu Circuit

(Impedance Diagram)