Online Tutorial #03

Friday, April 3, 2020
11:40 AM

Example:

A FM receiver is operating with an FM modulated signal given by:

$$S_{FM}(t) = 2\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right)$$

where $m(t) = 10 \cos(2\pi f_m t)$; $f_m = 1\ \text{kHz}$ and $f_c = 1\ \text{MHz}$.

A_m

The average noise power per unit BW measured at the input of the receiver front-end is 10^{-5} Watt per Hertz.

N_o : single sided PSD

a) Assuming an input resistor of 1Ω, calculate the input signal-to-noise ratio of the system.

b) Determine the minimum value of the modulation index (β) that results in an output signal-to-noise ratio of the system higher than 30 dB. ($SNR_{out} \geq 30\,dB$)

c) What will be the value of the frequency sensitivity (k_f) in that case?

a) SNR_{in} or SNR_c at FM R_x input:?

$$SNR_{in} = \frac{P_{s_{in}}}{P_{n_{in}}}$$

$$P_{s_{in}} = \frac{(2)^2}{2} = 2\ \text{Watts}$$

$$P_{n_{in}} = (2 \cdot f_m) \cdot N_o = 2 \cdot 1\,\text{kHz} \cdot 10^{-5}\ W/Hz = 0.02\ \text{Watts}$$

$$SNR_{in} = \frac{P_{sin}}{P_{nin}} = 100 \quad \text{or} \quad SNR_{in\_dB} = 10\log(SNR_{in}) = \boxed{20\ dB}$$

b) β_{min}? to provide $SNR_{out} \geq 30\,dB$

$$FOM_{FM} = \frac{3}{2}\beta^2 = \frac{SNR_{out}}{SNR_{in}} \rightarrow SNR_{out} = \left(\frac{3}{2}\beta^2\right) \times SNR_{in}$$

$$SNR_{out\_dB} = 10\log_{10}\left(\frac{3}{2}\beta^2\right) + SNR_{in\_dB} \geq 30\,dB$$

$$10\log\left(\frac{3}{2}\beta^2\right) \geq 30\,dB - SNR_{in\_dB} = 30 - 20 = 10\,dB$$

$$\frac{3}{2}\beta^2 \geq 10^{\frac{10}{10}} = 10 \longrightarrow \beta^2 \geq \frac{20}{3} = 6.66$$

$\beta \geq \sqrt{6.66} = 2.58$

$$\boxed{\beta_{min} = 2.58}$$

c) k_f ?

$$\beta_{min} = \frac{A_m \overset{\Delta f}{k_f}}{f_m} \longrightarrow k_f = \frac{\beta_{min} \cdot f_m}{A_m}$$

$$k_f = \frac{2.58 \cdot 1\,kHz}{10} \Rightarrow \boxed{k_f = 0.258\ kHz/v}$$

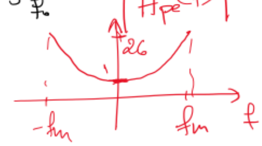
* Bandwidth of the FM signal using Carson's rule

$$B_T = (1+\beta_{min}) \cdot 2 f_m = (1+\underbrace{2.58}_{3}) \cdot 2 \cdot 1\,kHz = 4 \times 2\,kHz = \boxed{8\,kHz}$$

If the FM system uses a pre-emphasis/de-emphasis filters with the following characteristics: $\boxed{H_{pe}(f) = 1 + j\frac{f}{f_0}}$ and $H_{de}(f) = \frac{1}{1+j\frac{f}{f_0}}$

with $f_0 = 200\ Hz$.

$|H_{pe}(f)|^2 = 1 + \left(\frac{f}{f_0}\right)^2$



Calculate the new SNR_{out}.

for $H_{de}(f) = \frac{1}{1+j\frac{f}{f_0}}$; the Improvement in SNR_{out} is given by:

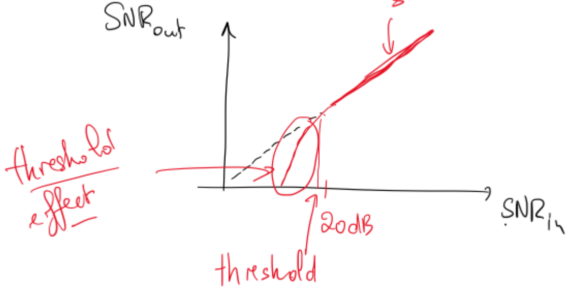
$$I = \frac{(f_m/f_0)^3}{3\left[f_m/f_0 - \tan^{-1}\left(\frac{f_m}{f_0}\right)\right]} = \frac{(5)^3}{3\left[5 - \tan^{-1}(5)\right]} = \boxed{11.5}$$

$$I = \frac{SNR_{out\_new}}{SNR_{out\_old}} \longrightarrow SNR_{out\_new} = SNR_{out\_old} \times I$$

$$SNR_{out\_new\_dB} = SNR_{out\_old\_dB} + 10\log(I) = 30\,dB + 10\log(11.5)$$

$$\boxed{SNR_{out\_new\_dB} = 40.6\ dB}$$

FM threshold effect:



$$\boxed{FOM_{FM} = \frac{SNR_{out}}{SNR_{in}}} \rightarrow SNR_{out} = \underbrace{\left(\frac{3}{2}\beta^2\right)}_{\text{Constant}} \cdot SNR_{in}$$

this is true for high values of SNR_{in}