ENEL 476 - Assignment #1 Solutions - Winter 2020

1. $\vec{E}(x,t) = 250\cos(2\pi\cdot10^8 t - 6\pi x)\hat{a}_y$ V/m $\quad \epsilon_r = 81$, $\mu_r = 1$, $\sigma = 0$

(1)

a) $\vec{J}_d(x,t)$?

$$\vec{J}_d(x,t) = \frac{\partial}{\partial t}\vec{D}(x,t)$$
$$= \frac{\partial}{\partial t}\epsilon\vec{E} = -(250)(81\epsilon_0)(2\pi\cdot10^8)\sin(2\pi\cdot10^8 t - 6\pi x)$$
$$= -112.6\sin(2\pi\cdot10^8 t - 6\pi x)\hat{a}_y \text{ A/m}^2$$

b) $\vec{H}(x,t)$?

This is a uniform plane wave so we can use wave impedance to get $\vec{H}$.

$$\vec{H}(x,t) = \frac{E_0}{|\eta|}\cos(\omega t - \beta x - \theta_n)$$

For loseless medium $\eta = \sqrt{\frac{\mu}{\epsilon}} = \eta_0\sqrt{\frac{\mu_r}{\epsilon_r}} = 120\pi\sqrt{\frac{1}{81}} = \frac{40\pi}{3}\angle 0^\circ\ \Omega$.

Direction of $\vec{H}$ must satisfy:

$$\hat{a}_y \times \hat{a}_H = \hat{a}_x$$

so

$$\hat{a}_H = \hat{a}_z$$

$$\vec{H}(x,t) = \frac{250}{\frac{40\pi}{3}}\cos(2\pi\cdot10^8 t - 6\pi x)\hat{a}_z$$
$$\vec{H}(x,t) = 6\cos(2\pi\cdot10^8 t - 6\pi x)\hat{a}_z \text{ A/m}$$

c) $\vec{E}$ inside PEC is 0.

$\vec{H}$ inside PEC is also 0 but there will be surface current $\vec{k}$ on surface to satisfy boundary conditions.

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1. $\vec{E}(x,t) = 250\cos(2\pi 10^8 t - 6\pi x)\hat{a}_y$ V/m. $\qquad \epsilon_r = 81$, $\mu_r = 1$, $\sigma = 0$

a) $\vec{J}_d(x,t)$.

$$\vec{J}_d(x,t) = \frac{\partial}{\partial t}\vec{D}(x,t)$$

$$= \frac{\partial}{\partial t}\epsilon\vec{E} = -250\cdot 81\epsilon_0\cdot 2\pi\cdot 10^8 \sin(2\pi\cdot 10^8 t - 6\pi x)\hat{a}_y$$

$$= -112.6\sin(2\pi 10^8 t - 6\pi x)\,\hat{a}_y \text{ A/m}^2$$

b) Find $\vec{H}(x,t)$

This is a uniform plane wave so we can use the wave impedance to find $\vec{H}$.

$$\vec{H}(x,t) = \frac{E_0}{|\eta|}\cos(\omega t - \beta z - \theta_n)$$

For loseless medium, $\eta = \sqrt{\frac{\mu}{\epsilon}} = \eta_0\sqrt{\frac{\mu_r}{\epsilon_r}} = 120\pi\sqrt{\frac{1}{81}} = \frac{40\pi}{3}\angle 0^\circ\ \Omega$.

Direction of $\vec{H}$ must satisfy:

$$\hat{a}_y \times \hat{a}_H = \hat{a}_x$$

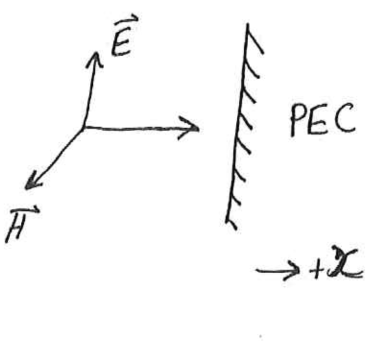
so $\quad \hat{a}_H = \hat{a}_z$

$$\vec{H}(x,t) = \frac{250}{\left|\frac{40\pi}{3}\right|}\cos(2\pi 10^8 t - 6\pi x)\,\hat{a}_z$$

$$= 6\cos(2\pi 10^8 t - 6\pi x)\,\hat{a}_z$$

(*) alternative below

c) $\vec{E}, \vec{H}$ in PEC?



$\vec{E}, \vec{H}$ are both tangential to the surface.

$\vec{E}$ inside PEC is 0.

$\vec{H}$ inside PEC is also 0, but there will be surface current $\vec{k}$ to satisfy boundary conditions.

b) option → $\nabla\times\vec{E}_s = -j\omega\mu_0\vec{H}_s$

$$\vec{E}_s = 250e^{-j6\pi x}\,\vec{a}_y$$

$$\nabla\times\vec{E}_s = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_{sy} & 0 \end{vmatrix}$$

$$= \vec{a}_x\left(-\frac{\partial}{\partial z}E_{sy}\right) - \vec{a}_y(0) + \vec{a}_z\left(\frac{\partial}{\partial x}E_{sy}\right)$$

$$\frac{\partial}{\partial x}\left(250e^{-j6\pi x}\right)\vec{a}_z = -j\omega\mu_0\vec{H}_s$$

$$\cancel{-}j6\pi(250)e^{-j6\pi x}\,\vec{a}_z = \cancel{-}j\omega\mu_0\vec{H}_s$$

$$\vec{H}_s = \frac{(250)(6\pi)e^{-j6\pi x}\,\vec{a}_z}{(2\pi\times 10^8)(4\pi\times 10^{-7})}$$

$$= \frac{750}{40\pi}e^{-j6\pi x}\,\vec{a}_z$$

$$= 6e^{-j6\pi x}\,\vec{a}_z \Rightarrow \vec{H}(x,t) \Rightarrow \text{same as above}$$

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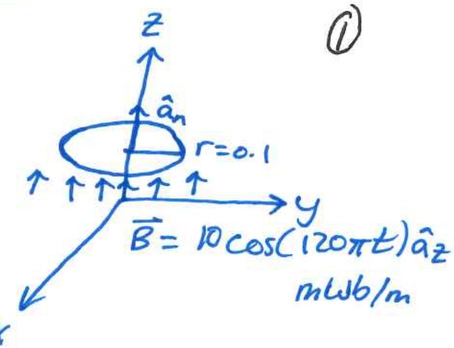
2. a) Calculate total flux through loop.



$$\Phi = \iint \vec{B} \cdot \vec{ds}$$

$$= \int_{\phi=0}^{2\pi}\int_{\rho=0}^{r} (10\cos(120\pi t)\hat{a}_z) \cdot (\rho\, d\rho\, d\phi\, \hat{a}_z)$$

$$= 10\cos(120\pi t)\int_{\phi=0}^{2\pi} d\phi \int_{\rho=0}^{r} \rho\, d\rho$$

$$= \pi r^2 \cdot 10\cos(120\pi t)$$

(Can solve by inspection as Φ = area $\cdot$ B)

$$\Phi = 0.1\pi\cos(120\pi t) \text{ mWb}$$

b) $V_{emf} = \frac{-\partial}{\partial t}\Phi$

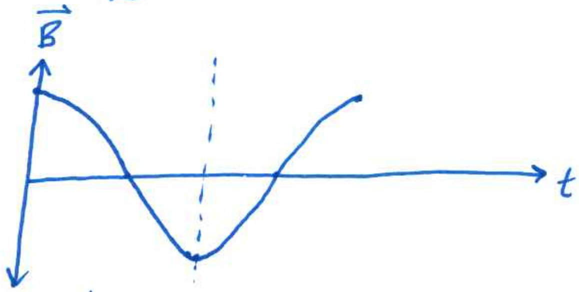
$$= (120\pi)(0.1\pi)\sin(120\pi t)$$

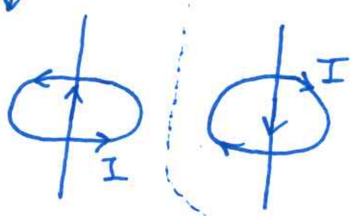
$$= 12\pi^2 \sin(120\pi t)$$

$$= 118 \sin(120\pi t) \text{ mV}$$

c) $I = \frac{V}{R} = 11.8\sin(120\pi t)$ mA

d)





Flux decreases so induced current generates counteracting magnetic flux in +z direction

Vice Versa

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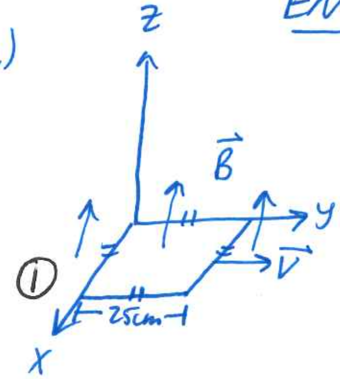
3. a)



(1) $\vec{B} = 8\cos(1.5\cdot10^8 t - 0.5x)\hat{a}_z \ \mu T$

$\rho = 125\ \Omega/m$

$\vec{v} = 50\ m/s$

b) $\Phi = \iint_S \vec{B}\cdot\vec{ds}$ (1)

$$= \int_{x=0}^{0.25}\int_{y=0}^{0.25} (8\cos(1.5\cdot10^8 t - 0.5x)\hat{a}_z)\cdot(dx\,dy\,\hat{a}_z) \quad (1)$$

$$= 8\int_{y=0}^{0.25} dy \int_{x=0}^{0.25} \cos(1.5\cdot10^8 t - 0.5x)\,dx$$

$$= 8(0.25)\left(\frac{1}{-0.5}\right)\sin(1.5\cdot10^8 t - 0.5x)\Big|_{x=0}^{0.25}$$

$$\Phi = 4\sin(1.5\cdot10^8 t) - 4\sin(1.5\cdot10^8 t - 0.125)\ \mu Wb \quad (1)$$

Note: B and Φ don't depend on y so no motional EMF (1)

c) $V_{emf} = -\frac{\partial}{\partial t}\Phi$ (1)

$$= -4(1.5\cdot10^8)(\cos(1.5\cdot10^8 t) - \cos(1.5\cdot10^8 t - 0.125))$$

$$= -600(\cos(1.5\cdot10^8 t) - \cos(1.5\cdot10^8 t - 0.125))\ V \quad (1)$$

$$I = \frac{V}{R} \overset{(1)}{=} \frac{-600}{(125\ \Omega/m)(1m)} = -4.8(\cos(1.5\cdot10^8 t) - \cos(1.5\cdot10^8 t - 0.125))\ A. \quad (1)$$

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