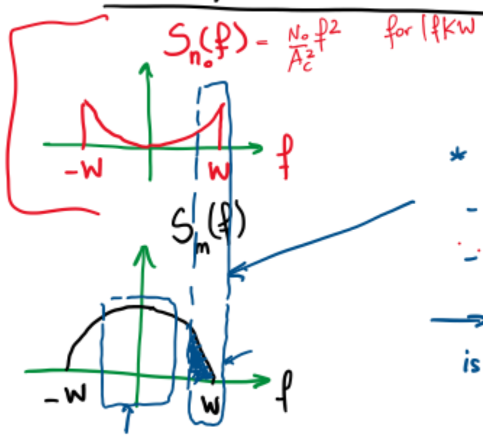
Online Lecture # 07 - Threshold Effect / Pre-emphasis and De-emphasis

Friday, April 3, 2020
8:45 AM

* Pre-emphasis and De-emphasis:



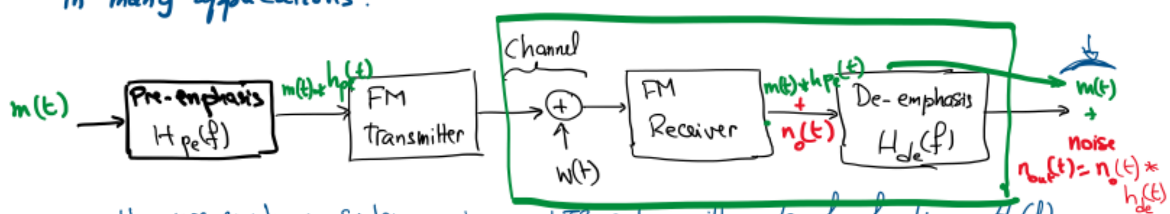
* In the outer portion of the message band
  - the signal power is low
  - the noise power is high

→ the SNR for that part of the signal is low (bad)

May result in loosing that part of the info.

most of the power in common signals is located close to 0 Hz and there is very little power at the edges of the band.

→ the loss of information at the edge of the signal band is not acceptable in many applications.



* the pre-emphasis system : is an LTI system with a transfer function = $H_{pe}(f)$

  Its role is to emphasize (amplify) the high frequency components of the message m(t).

* the de-emphasis system: perfors the opposite operation of $H_{pe}(f)$
  → Restore the original message
  → Remove/attenuate the high frequency components of the noise
  → SNR_{out} will improve. (both the overall SNR_{out} and the SNR_{out} at the edge of the message bandwidth)

for distortion-less transmission:

$$\boxed{H_{de}(f) = \frac{1}{H_{pe}(f)} \quad \text{for } -W < f < W}$$

* the improvement in the SNR_{out} due to the pre-emphasis and de-emphasis process:

$$S_{n_{out}}(f) = S_{n_o}(f) \cdot |H_{de}(f)|^2$$

$$P_{n_{out}} = \int_{-\infty}^{\infty} S_{n_o}(f) \cdot |H_{de}(f)|^2 \, df$$

$$\boxed{P_{n_{out}} = \int_{-W}^{W} \frac{N_o}{A_c^2} f^2 \cdot |H_{de}(f)|^2 \, df}$$

Without de-emphasis: $P_{n_o} = \int_{-W}^{W} \frac{N_o}{A_c^2} f^2 df = \boxed{\frac{N_o}{A_c^2} \frac{2}{3} W^3}$

and $SNR_{out} = \frac{P_{s_{out}}}{P_{n_o}}$

With de-emphasis: $SNR_{out}' = \frac{P_{s_{out}}}{P_{n_{out}}}$

the improvement on the SNR_{out} is the given by:

$$I = \frac{SNR_{out}'}{SNR_{out}} = \frac{P_{n_o}}{P_{n_{out}}}$$

$$I = \frac{\frac{N_o}{A_c^2} \frac{2}{3} W^3}{\int_{-W}^{W} \frac{N_o}{A_c^2} f^2 \, |H_{de}(f)|^2 df} \rightarrow \boxed{I = \frac{2 W^3}{3 \int_{-W}^{W} f^2 |H_{de}(f)|^2 df}}$$

Example:

An FM system is using a set of pre-emphasis and de-emphasis filters that have the following transfer functions:

$$H_{pe}(f) = 1 + j\frac{f}{f_o} \quad \text{and} \quad H_{de}(f) = \frac{1}{1+j\frac{f}{f_o}}$$

Calculate the improvement on SNR_{out} due to the pre-emph. and de. emph. operation:

$$I = \frac{2W^3}{3\int_{-W}^{W} f^2 \left| \frac{1}{1+j\frac{f}{f_0}} \right|^2 df} = \frac{2W^3}{3\int_{-W}^{W} \frac{f^2}{1+\left(\frac{f}{f_0}\right)^2} df}$$

Using the indefinite integral: $\int \frac{x^2\,dx}{a^2+b^2x^2} = \frac{x}{b^2} - \frac{a}{b^3}\tan^{-1}\left(\frac{bx}{a}\right)$

with: $a=1$ and $b = 1/f_0$

$$I = \frac{2W^3}{3\left[f \cdot f_0^2 - (f_0)^3 \tan^{-1}\left(\frac{f}{f_0}\right)\right]_{-W}^{W}} = \frac{(W/f_0)^3}{3\left[\frac{W}{f_0} - \tan^{-1}(W/f_0)\right]}$$

if $\left.\begin{array}{l} f_0 = 2.1 \text{ kHz} \\ W = 15 \text{ kHz} \end{array}\right]$ for commercial FM systems

$\rightarrow \boxed{I = 22} \rightarrow$ Improvement of $\simeq 13\,\text{dB}$.