

University of Calgary
Schulich School of Engineering
Department of Electrical and Computer Engineering

ENEL 476 – Electromagnetic Waves and Applications

Final Examination
Winter Session 2018
April 26, 2018 at 8:00 am
ENE 243
Instructor: Elise Fear

3 hours
Closed book

Student name:

Instructions

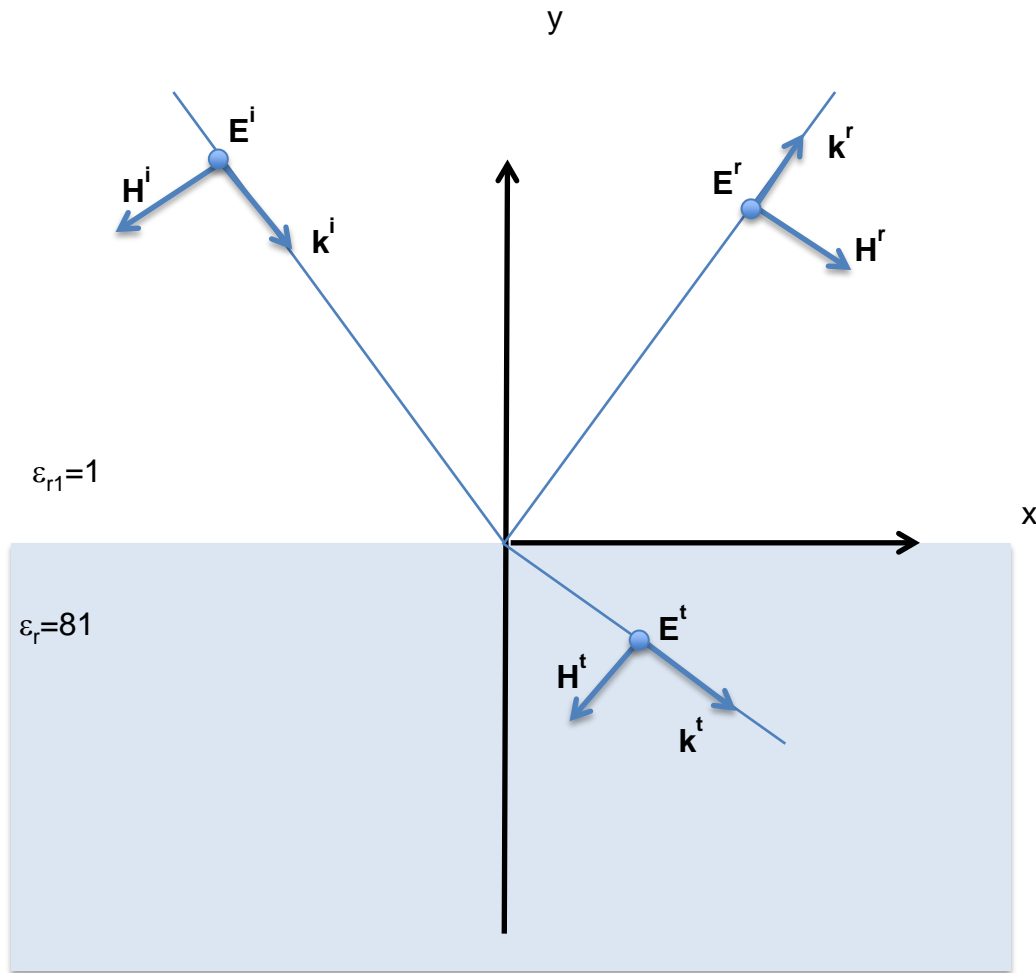
- (1) This is a closed book exam. No texts or notes are allowed.
- (2) Show as much of your reasoning as time permits. Write your answers in the examination booklets.
- (3) Calculators are permitted.
- (4) Formulas are provided at the end of the question pages.
- (5) Hand in all pages. If you detach any pages(s), write your name and UCID number on the detached page(s).
- (6) If you write anything you do not want marked, put a large X through it and write “Rough work” beside it.

Question 1 (19 marks).

A planar interface forms a boundary between air and water (assume that this interface is flat and infinite in extent). Both materials have $\mu_r=1$ and $\sigma=0$ S/m, with the relative permittivity in air of 1 ($\epsilon_{r1}=1$) and the relative permittivity in water of 81 ($\epsilon_{r2}=81$). The co-ordinate system is indicated in the figure below.

A uniform plane wave is obliquely incident the interface, as shown below. The incident electric field (\mathbf{E}^i) is given by:

$$\mathbf{E}^i(x,y,t)=20 \cos (2 \times 10^9 t - k_x x + k_y y) \mathbf{a}_z \quad \text{V/m}$$



- a) Find the propagation constant (β) in region 1 ($\epsilon_r=1$) (2 marks).

- b) If the angle of incidence is 30° , specify the phase constants (k_x and k_y) in the expression for the incident electric field (2 marks).

$$\mathbf{E}^i(x,y,t) = 20 \cos(2 \times 10^9 t - k_x x + k_y y) \mathbf{a}_z$$

- c) Calculate the angle of transmission (2 marks).
- d) Does the field exhibit perpendicular or parallel polarization? (1 mark)
- e) Find the reflection coefficient (Γ) (4 marks).
- f) Find an expression for the reflected electric field ($\mathbf{E}^r(x,y,t)$) (3 marks).

- g) For these 2 materials, is it possible to have total reflection? If so, briefly explain and calculate the critical angle (3 marks).
- h) For these 2 materials, is it possible to have total transmission? Briefly explain and, if appropriate, calculate the Brewster angle (2 marks).

- f) Find an expression for the magnetic field ($\mathbf{H}(z,t)$). (2 marks)
- g) Calculate the time-averaged Poynting vector in the tissue ($\mathbf{P}_{av}(z)$). If the wave travels 7 cm through the tissues, by how much is the power density reduced? (4 marks)
- h) With dehydration, the properties of the tissues are expected to change. A 20% decrease in permittivity and conductivity results in a 10% decrease in attenuation and phase constants. The amplitude of the intrinsic impedance increases by 12%, and the angle stays the same.

With these changes, what is the time-averaged Poynting vector in the tissue? (1 mark)

If the wave travels 7 cm through tissue with the reduced properties, what is the power density? How does this compare with the original case? (2 marks)

The sensor is embedded in a lossless dielectric with $\epsilon_r=20$ that is placed in direct contact with the tissue. This scenario is modeled by assuming a planar interface of infinite extent between the tissue and dielectric. Assume that the field in the tissue is normally incident on the interface, the wave has traveled through 7 cm of tissue, and that the interface is located at $z=0$.

- i) Find the phase constant (β) in the material (1 mark)

- j) Find the intrinsic impedance (η) of the material (1 marks).

- k) Find the transmission coefficient (T) (2 marks).

- l) Find an expression for the electric field transmitted from the tissue into the dielectric ($\mathbf{E}^t(z,t)$). (3 marks)

Question 3 (9 marks)

A coaxial cable has $a=0.5$ mm, $b=2.95$ mm, $\epsilon_r=2.25$ and $\mu_r=1$.

- a) Find the inductance per unit length (L) and capacitance per unit length (C).
Note: $C=2\pi\epsilon/\ln(b/a)$ and $L=(\mu/2\pi)*\ln(b/a)$ (2 marks)
- b) Assume that the coaxial cable is lossless. Calculate the propagation constant (β) and impedance (Z_o) at 10 GHz. (2 marks)
- c) If the cable has attenuation of $\alpha=1.06$ Np/m and assuming the conductance per unit length (G) is zero, find the resistance per unit length (R). (3 marks)
- d) Using L and C calculated in *part b* and R calculated in *part c*, find the value of G required for a distortionless line. If you did not find a value for R in *part c*, use 100 Ω/m . (2 marks)

Question 4 (20 marks)

A load of $Z_L = 40 - j20 \, \Omega$ is attached to a $60 \, \Omega$ transmission line.

a) Find reflection coefficient and VSWR using transmission line formulas. (5 marks)

b) Verify your answer to *part a* using the Smith chart. (4 marks)

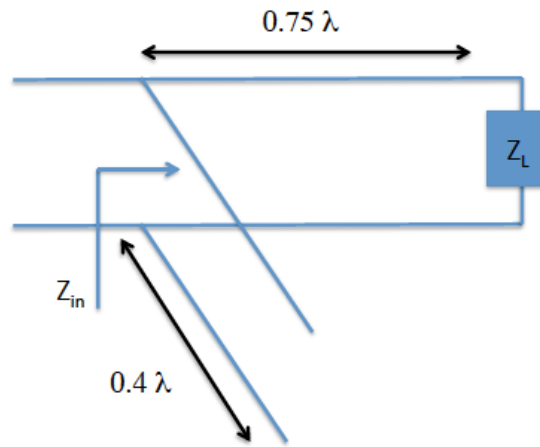
c) Find Z_{in} at a distance of 0.275λ from the load using the transmission line formulas. (3 marks)

d) Verify your answer to *part c* using the Smith chart. (2 marks)

- e) Assume that Z_L and the 0.275λ line are now connected to a transmission line with impedance of $75\ \Omega$ and length of $\lambda/4$. Using the Smith chart, find the Z_{in} looking into the combination of the quarter wavelength line, the 0.275λ line and the load. (3 marks)
- f) Is previous design a quarter wavelength transformer? If it is, explain why. If not, how would you modify the design to match to a transmission line of $60\ \Omega$? (3 marks)

Question 5 (22 marks)

A transmission line circuit is shown below. The transmission lines have impedance of $50\ \Omega$, and $Z_L = 40 - j20\ \Omega$.



- Find the normalized input admittance of the load (y_L) (2 marks).
- Find the normalized admittance 0.75λ from the load (y_{in}') (2 marks).
- Find the normalized admittance of the open-circuited stub (y_{stub}) (2 marks).
- Find the normalized admittance looking into the combination of the stub, 0.75λ of transmission line and the load (2 marks).
- Is this a stub tuner? Explain. (2 marks)

- f) Design a series stub tuner to match Z_L to a $50\ \Omega$ line. Use shorted stubs and place the stub close to the load. Specify the location of the stub relative to the load, as well as the length of the stub. (6 marks)
- g) Design a shunt stub tuner to match Z_L to a $50\ \Omega$ line. Use an open circuited stub and place the stub close to the load. Specify the location of the stub relative to the load, as well as the length of the stub. (6 marks)

Question 6 (6 marks)

- a) Calculate the displacement current density (\mathbf{J}_d) for the following electric field in distilled water ($\epsilon_r=81$): $\mathbf{E}(x,t) = 250 \cos(2\pi \times 10^8 t - 6\pi x) \mathbf{a}_y$ V/m. (2 marks)
- b) If the distilled water in *part a* is placed in a PEC container, the electric and magnetic field inside the PEC are: (1 mark)
- c) Consider a magnetic flux density of $\mathbf{B}=10 \cos(120\pi t) \mathbf{a}_z$ mW/m². Calculate the EMF induced in a loop of radius 0.1 m that has surface normal oriented in the z direction. (3 marks)

Question	Mark
1	19
2	24
3	9
4	20
5	22
6	6
Total	