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Name: Solution **ID:**__________

SCHULICH
School of Engineering
UNIVERSITY OF CALGARY

ENEL 487 Final Examination

Wednesday, April 20

Time: 8:00 – 11:00 am

Location: Red Gym

Instructor: Pouyan (Yani) Jazayeri

- Please note that the official University of Calgary examination regulations are printed on page 2 of this paper.
- Exam consists of 8 problems and 15 pages.
- Write answers in the space provided below each question.
- Show your work neatly in the work area. Otherwise, marks for partially correct answers cannot be given.
- Total marks for the exam is 90.
- Closed book exam. You may not refer to books or notes during the test.
- No wireless devices or earphones allowed during exam.
- Only scientific calculators without formulae storage and text display are allowed.

Question	1	2	3	4	5	6	7	8	Total
Mark	/15	/10	/10	/10	/10	/10	/10	/15	/90

Problem 1:

Answer the following questions on the scantron sheet provided to you. Correct answers are rewarded 1 point.

1) We can only apply per phase analysis to systems where:

   a) All loads and sources are Y connected

   b) All loads and sources are delta connected

   c) There are no restrictions to using per phase analysis with regards to connection types.

   d) All lines are Y connected

   e) A and D

2) Which statement about phase and line voltages (V) and currents (I) in Wye and Delta connections is correct?

   a) Phase I = line I in Wye, and phase V = line-line V in Delta

   b) Phase I = line I in Delta, and phase V = line-line V in Wye

   c) Phase I = line I and phase V = line-line V in Wye

   d) Phase I = line I and phase V = line-line V in Delta

   e) None of the above

3) Transformer open circuit (OC) and short circuit (SC) tests allow us to calculate certain equivalent circuit values. Which parameters are calculated from the OC test and which are from the SC test?

   a) The OC test gives the resistances and the SC test gives the reactances

   b) The OC test gives the series (leakage) impedance and the SC test gives the shunt admittance

   c) The OC test gives the shunt admittance and the SC test gives the series (leakage) impedance

   d) The OC test gives the voltages and the SC test gives currents

   e) None of the above

4) In a long line Pi model, we use series impedance Z' and shunt admittance Y'/2. In the short line model, we use:

   a) Series impedance Z' and shunt admittance Y'/2

   b) Series impedance Z' and shunt admittance Y/2

   c) Series impedance Z and shunt admittance Y/2

   d) Series impedance Z and shunt admittance 0

   e) Series impedance 0 and shunt admittance 0

5) In the power flow at the slack (swing) bus,

   a) P and V are fixed

   b) P and Q are fixed

   c) P and δ are fixed

   d) V and Q are fixed

   e) None of the above

6) If the diameter of a transmission line conductor is increased, then:

a) Both the inductance and the capacitance increase

b) The inductance increases and the capacitance decreases

c) The inductance decreases and the capacitance increases

d) Both the inductance and the capacitance decrease

e) The inductance decreases and the capacitance remains unchanged

7) Supplying reactive power locally (at the load side) leads to increased line current, thus increased line losses.

a) True

b) False

8) The total real power losses of an ideal transformer are always zero.

a) True

b) False

9) Bundled conductors can be used to decrease transmission line capacity (ampacity)

a) True

b) False

10) In a large power system, majority of busses are:

a) Slack busses

b) Generator busses

c) Load busses

d) None of the above

11) Power flow studies involve solving simultaneous__________

a) Linear algebraic equations

b) Non-linear algebraic equations

c) Linear differential equations

d) Non-linear differential equations

e) None of the above.

12) Which of the following is not true for the bus admittance matrix (Y_{bus}) in a large power system:

a) It is symmetric

b) It is a square matrix

c) It is a fully populated matrix

d) None of the diagonal elements can be zero.

e) None of the above. (All the statements are true.)

13) Which of the following is not a common heat dissipation technique for transformers?

- **a)** Forced air
- **b)** Forced oil
- **c)** Natural air flow
- **d)** Natural oil flow
- **e)** None of the above (i.e. They are all acceptable)

14) Aluminum has surpassed copper as the material for transmission lines because:

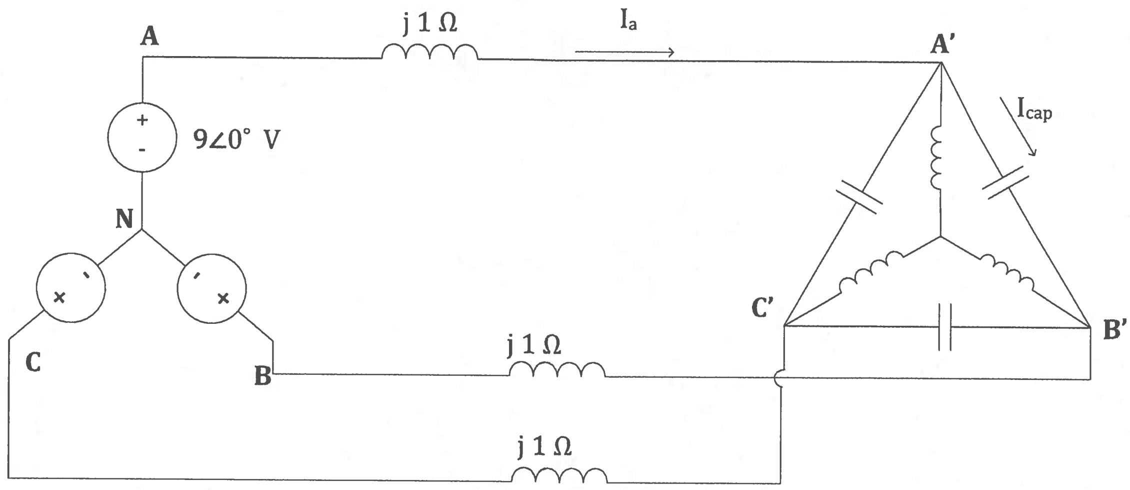
- **a)** Aluminum has better conductivity
- **b)** Aluminum is cheaper
- **c)** Aluminum is stronger
- **d)** "This probably has something to do with China! " - Donald Trump
- **e)** Both A and b

15) The Alberta market has a demand of 4500MW at the moment. The offers received from the generators are as follows: generator A offers 1000MW at $15/MWh, generator B offers 1000MW at $25/MWh, generator C offers 2000MW at $0/MWh, generator D offers 2000MW at $20/MWh, and generator E offers 1000MW at $10/MWh. Which of the following statements is true?

- **a)** Generator D receives $20/MWh and is dispatched at 2000MW
- **b)** Generator D receives less than $20/MWh and is dispatched at less than 2000MW
- **c)** Generator D receives $20/MWh and is dispatched at less than 2000MW
- **d)** Generator C receives their bid of $0/MWh and is dispatched at 2000MW
- **e)** None of the above

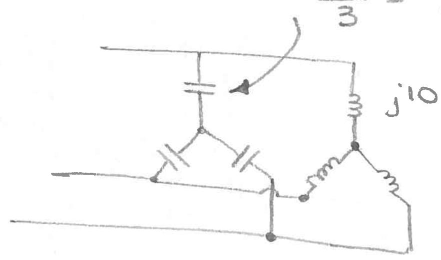
Problem 2:

Consider the balanced three-phase system shown below. Each inductor has an impedance of $Z_L = j10\Omega$ and each capacitor $Z_c = -j15\Omega$. Determine $\mathbf{I_a}$, $\mathbf{I_{cap}}$, and total three phase complex power supplied by the source.
Hint: The inductor and capacitor are simply three phase loads connected in parallel. [10 marks]

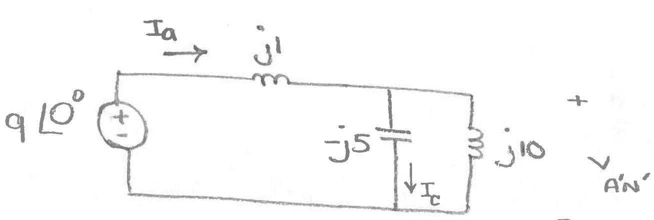


convert load to Y-connection:

$\frac{-j15}{3} = -j5$



per phase circuit:



$$Z_{load} = -j5 \parallel j10 = \frac{-j5 \times j10}{-j5 + j10} = -j10$$

$$I_a = \frac{9\angle0^\circ}{Z_{load} + j1} = \frac{9\angle0^\circ}{-j9} = 1\angle90^\circ = \boxed{j1 \text{ A}}$$

To find I_{cap}:

1

$$V_{A'N'} = 9\angle0^\circ \times \frac{Z_{load}}{Z_{load} + j1}$$
$$= 9\angle0^\circ \cdot \frac{-j10}{-j9}$$
$$= 10\angle0^\circ$$

or
$$V_{A'N'} = 9\angle0^\circ - I_a (j1)$$
$$= 9 - j1(j1)$$
$$= 10$$

2

$$I_c = I_a \cdot \frac{j10}{-j5 + j10}$$
$$= j1 \cdot \frac{j10}{j5}$$
$$= j2$$

$$I_c = \frac{V_{A'N'}}{Z_c} = \frac{10\angle0^\circ}{5\angle-90^\circ} = j2$$

$$I_{cap} = \frac{I_c}{\sqrt{3}} \cdot \angle30^\circ = \boxed{\frac{2}{\sqrt{3}}\angle120^\circ}$$

or
$$V_{A'B'} = V_{A'N'} \cdot \sqrt{3}\angle30^\circ$$
$$= 10\sqrt{3}\angle30^\circ$$

$$I_{cap} = \frac{V_{A'B'}}{Z_c} = \frac{10\sqrt{3}\angle30^\circ}{15\angle-90^\circ}$$
$$= \frac{2}{\sqrt{3}}\angle120^\circ$$

$$I_{cap} = \frac{I_c}{\sqrt{3}}\angle30^\circ$$
$$= \frac{2}{\sqrt{3}}\angle120^\circ$$

$$\overline{S}_{3\phi\,source} = 3 \cdot \overline{V_{AN}} \cdot \overline{I_a}^* = 3(9\angle0^\circ)(1\angle-90^\circ) = 27\angle-90^\circ = \boxed{-j27 \quad \text{VA}}$$

$$Q_{3\phi,source} = -27 \quad \text{VAr}$$

Problem 3:

Two balanced three phase loads are connected in parallel. Load 1 draws 10 kW at 0.8 pf lagging. Load 2 draws 20 kVA at 0.6 pf leading. The loads are supplied by a balanced 3 phase 480 V source.

a) Draw the power triangle for the combined load. [5 marks]

Load 1

$P_1 = 10$ kW

$S_1 = \frac{P_1}{pf} = \frac{10}{0.8} = 12.5$ kVA

$Q_1 = \sqrt{S_1^2 - P_1^2} = 7.5$ kVAr

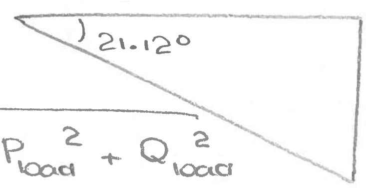
Load 2

$S_2 = 20$ kVA

$P_2 = S_2 \cdot pf = 20 \times 0.6 = 12$ kW

$Q_2 = \sqrt{S_2^2 - P_2^2} = -16$ kVAr

$P_{load} = P_1 + P_2 = 22$ kW



$Q_{load} = Q_1 + Q_2 = -8.5$ kVAr

$S_{load} = \sqrt{P_{load}^2 + Q_{load}^2}$

$= 23.59$ kVA

b) Determine the power factor of the combined load. [1 mark]

$PF = \frac{P_{load}}{S_{load}} = \boxed{0.93 \quad \text{leading}}$

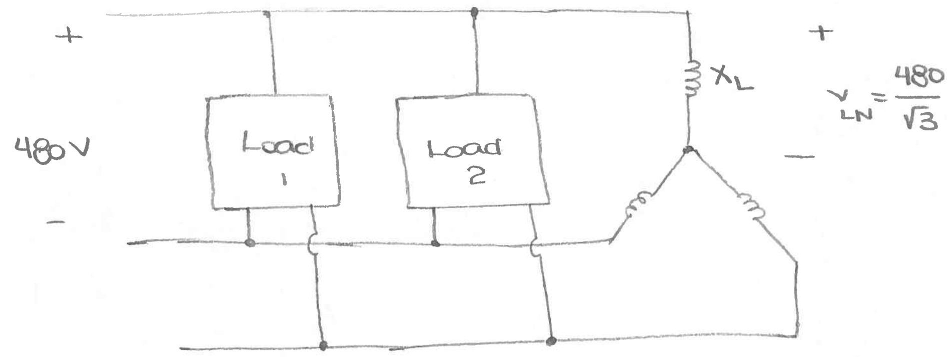
$\theta_V - \theta_I < 0$ or $Q < 0$

c) Determine the magnitude of the line current from the source. [1 mark]

$S_{3\phi} = \sqrt{3}\, V_{\ell\ell} \cdot I_\ell \quad \therefore \quad I_\ell = \frac{S_{3\phi}}{\sqrt{3}\, V_{\ell\ell}} = \frac{23.59 \text{ kVA}}{\sqrt{3} \times 480\text{V}}$

$= \boxed{28.4 \text{ A}}$

d) Y-connected inductors are now installed in parallel with the load. What value of inductive reactance is needed in each leg of the Y connection to bring the source power factor to unity? [3 marks]



3ϕ inductor bank to "consume" 8.5 kVAr

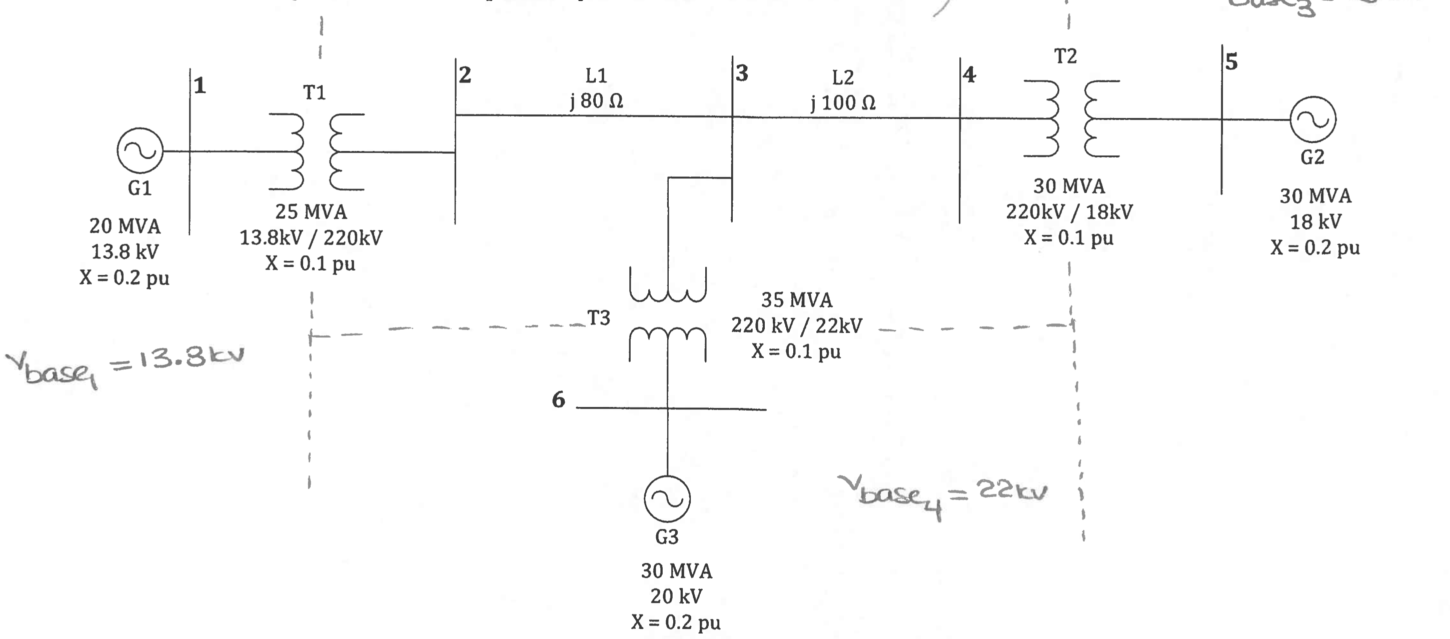
$\therefore$ each leg $Q_{1\phi} = \frac{8.5}{3}$

also $Q_{1\phi} = \frac{V_{LN}^2}{X_L}$

$\therefore \quad X_L = \frac{V_{LN}^2}{Q_{1\phi}} = \frac{\left(\frac{480}{\sqrt{3}}\right)^2}{\frac{8.5}{3}\text{ kVAr}} = \boxed{27.1\ \Omega}$

Problem 4:

The single line diagram of a three phase system is shown below.



Draw the impedance diagram for this system with all reactances marked in pu. Label the points corresponding to the busses in your impedance diagram. (i.e. there should be a point labelled 1 corresponding to bus 1, and so on). Use a power base of 50 MVA and a voltage base of 13.8kV in the generator G1 zone. [10 marks]

$$X_{G1} = 0.2 \times \frac{50 \text{ mVA}}{20 \text{ mVA}} = 0.5$$

$$X_{G2} = 0.2 \times \frac{50 \text{ mVA}}{30 \text{ mVA}} = 0.33$$

$$X_{T1} = 0.1 \times \frac{50 \text{ mVA}}{25 \text{ mVA}} = 0.2$$

$$X_{T2} = 0.1 \times \frac{50 \text{ mVA}}{30 \text{ mVA}} = 0.166$$

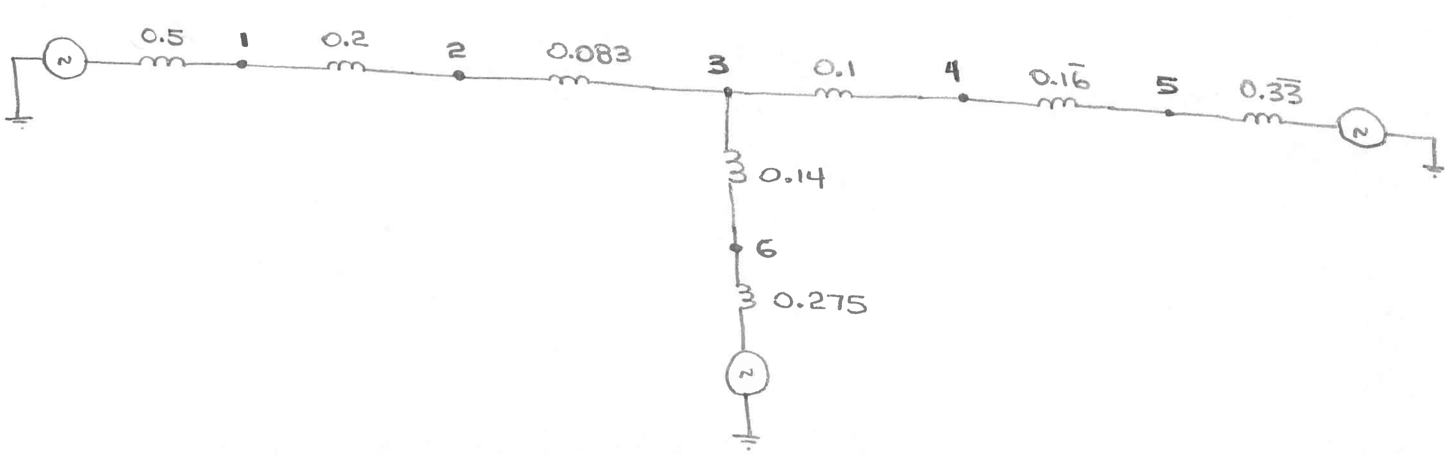
$$Z_{base_2} = \frac{(V_{base_2})^2}{S_{base}} = \frac{(220 \text{ kV})^2}{50 \text{ mVA}} = 968\ \Omega$$

$$\therefore \quad Z_{L1,pu} = \frac{j80}{Z_{base_2}} = j0.083$$

$$Z_{L2,pu} = \frac{j100}{Z_{base_2}} = j0.1$$

$$X_{G3} = 0.2 \times \frac{50 \text{ mVA}}{30 \text{ mVA}} \times \left(\frac{20 \text{ kV}}{22 \text{ kV}}\right)^2 = 0.275$$

$$X_{T3} = 0.1 \times \frac{50 \text{ mVA}}{35 \text{ mVA}} = 0.14$$



Problem 5:

The following parameters have been calculated for a 500kV line: z = 0.02 + j0.5 Ω/km, y = j7 x10^{-6} S/km. With the receiving end voltage at 500kV, the receiving end power is 100 + j500 MVA.

a) Write the expression for V(x). [7 marks]

$$\gamma = \sqrt{z \cdot y} = \sqrt{(0.02 + j0.5) \times (j7 \times 10^{-6})} = 1.87 \times 10^{-3} \angle 88.85^\circ \quad km^{-1}$$
$$= 3.7 \times 10^{-5} + j0.00187$$

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{0.02 + j0.5}{j7 \times 10^{-6}}} = 267.4 \angle -1.1^\circ \quad \Omega$$
$$= 267.4 - j5.34$$

$$V_R = \frac{500\ kV}{\sqrt{3}} = 288.7\ kV$$

$$I_R = \frac{S_{R,3\phi}}{\sqrt{3}\, V_{R,\ell\text{-}\ell}} = \frac{\sqrt{100^2 + 500^2}}{\sqrt{3} \times 500\ kV} = 588.8\ A$$

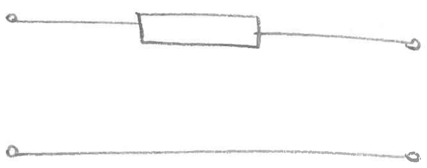
$$\theta_V - \theta_I = \tan^{-1}\left(\frac{500}{100}\right) = 78.7^\circ \quad ; \text{ assuming } \theta_{V_R} = 0 \rightarrow \theta_{I_R} = -78.7^\circ$$

$$\therefore \bar{I}_R = 588.8 \angle -78.7^\circ$$

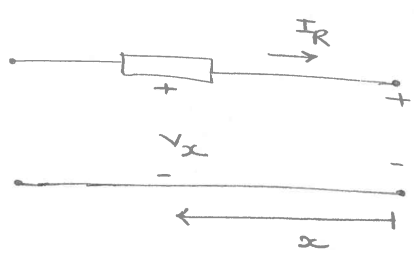
$$V(x) = \bar{V}_R \cdot \cosh(\gamma x) + \bar{I}_R \cdot Z_c \cdot \sinh(\gamma x)$$
$$= 288.7 \cosh(1.87 \times 10^{-3} \angle 88.85^\circ \cdot x) + 157.4 \angle -79.8^\circ \sinh(1.87 \times 10^{-3} \angle 88.85^\circ \cdot x)$$

b) We have decided to represent this line with a short line model. Draw this model and label all the applicable impedances and admittances. Also, write a new expression for V(x) based on this model. (i.e. An expression that provides the voltage at any point x as measured from the receiving end.) [3 marks]

$$Z = z \cdot \ell = (0.02 + j0.5)\,\ell$$



$$V(x) = V_R + I_R \cdot Z\left(\frac{x}{\ell}\right) = V_R + I_R \cdot z \cdot \cancel{\ell}\left(\frac{x}{\cancel{\ell}}\right) = V_R + I_R(0.02 + j0.5) \cdot x$$



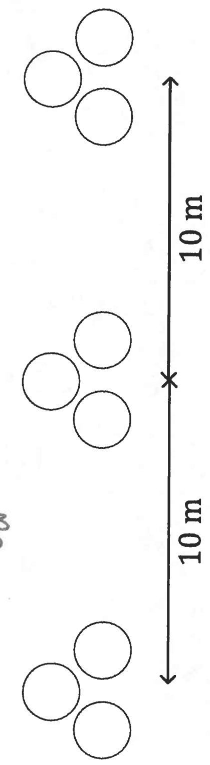
the key to the equation above is to understand that z is distributed equally along the conductor

$\therefore$ impedance from R end to point x is $(z \cdot x)$
& corresponding voltage drop is $I(z \cdot x)$

Problem 6:

A bundled, three phase, completely transposed, 60 Hz, 500kV line is composed of three ACSR Curlew 1033.5 kcmil conductor per phase with flat vertical spacing of 10 meters as shown below. The conductors have a diameter of 3.284 cm and GMR of 0.0133m. The spacing between the conductors in each bundle is 50 cm.

a) Determine the inductive line reactance per phase in Ω/km. [5 marks]



$$D_{eq} = \sqrt[3]{(10m)(10m)(20m)} = 12.6 \text{ m}$$

$$D_{SL} = \sqrt[3]{D_S \cdot d^2} = \sqrt[3]{(0.0133m)(0.5m)^2} = 0.15 \text{ m}$$

$$L = 2\times10^{-7} \ln \frac{D_{eq}}{D_{SL}} = 2\times10^{-7} \ln\left(\frac{12.6}{0.15}\right) = 8.87\times10^{-7} \text{ H/m}$$

$$X_L = \omega L = 2\pi(60)(8.87\times10^{-7} \text{ H/m}) = 3.34\times10^{-4} \ \Omega/\text{m}$$

$\times 10^3$

$$= \boxed{0.334 \ \Omega/\text{km}}$$

b) The capacitance-to-neutral in F/m and admittance-to-neutral in S/km. [5 marks]

D_{eq} same as part a).

$$D_{SC} = \sqrt[3]{r \cdot d^2} = \sqrt[3]{\frac{0.03284}{2} \times (0.5)^2} = 0.16 \text{ m}$$

$$C_{an} = \frac{2\pi\varepsilon}{\ln \frac{D_{eq}}{D_{SC}}} = \frac{2\pi\varepsilon}{\ln \frac{12.6}{0.16}} = \boxed{1.27\times10^{-11} \text{ F/m}}$$

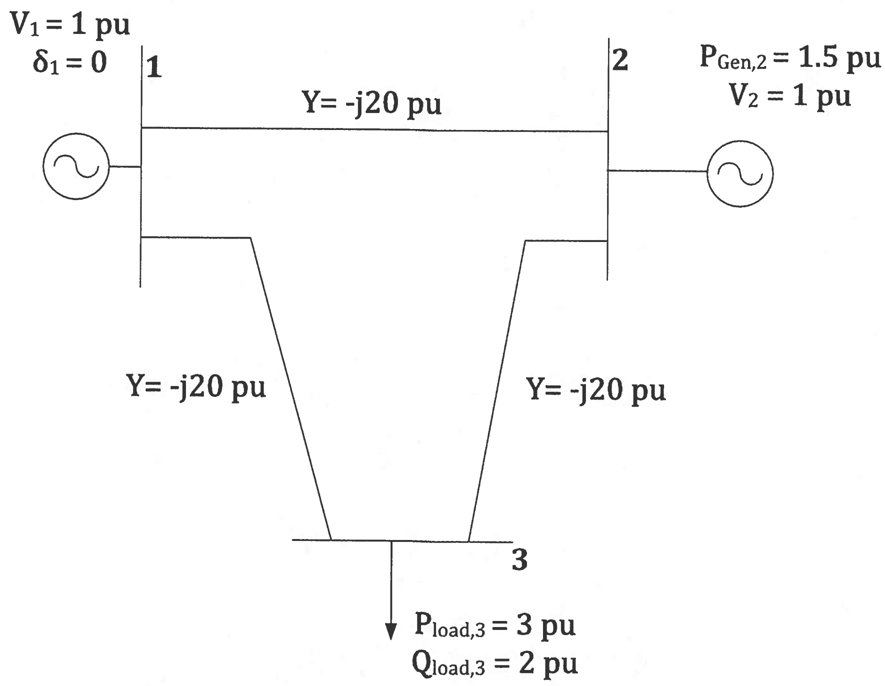
$$y_c = j\omega C = j\,2\pi(60)(1.27\times10^{-11}) = j\,4.8\times10^{-9} \text{ S/m}$$

$\times 10^3$

$$= \boxed{j\,4.8\times10^{-6} \text{ S/km}}$$

Problem 7:

Considering the system below:



Write the expression (and if possible, find the values) for the element in row 1, column 2 of J_{21} and row 2, column 2 of J_{22} [10 marks]

$$X = \begin{bmatrix} P_{gen,1} \\ \delta_2 \\ \delta_3 \\ Q_{gen,1} \\ Q_{gen,2} \\ V_3 \end{bmatrix} \qquad Y_{bus} = \begin{bmatrix} -j40 & j20 & j20 \\ j20 & -j40 & j20 \\ j20 & j20 & -j40 \end{bmatrix} \rightarrow \begin{array}{l} G_{bus} = 0 \\ \therefore B_{bus} = \begin{bmatrix} -40 & 20 & 20 \\ 20 & -40 & 20 \\ 20 & 20 & -40 \end{bmatrix} \end{array}$$

$$J_{21}[1,2] = \frac{\partial Q_{eqn,1}}{\partial \delta_2}$$

$$Q_{eqn,1} = Q_{gen,1} - Q_{load,1} - \sum_{k=1}^{3} V_1 V_k \overset{0}{\cancel{G[1,k]}} \sin(\delta_1 - \delta_k) + \sum_{k=1}^{3} V_1 V_k B[1,k] \cos(\delta_1 - \delta_k)$$

$$= Q_{gen,1} + V_1 V_1 B[1,1] \cos(\delta_1 - \delta_1) + V_1 V_2 B[1,2] \cos(\delta_1 - \delta_2) + V_1 V_3 B[1,3] \cos(\delta_1 - \delta_3)$$

$$= Q_{gen1} + (1)(1)(-40) + (1)(1)(20)\cos(-\delta_2) + (1)V_3(20)\cos(-\delta_3)$$

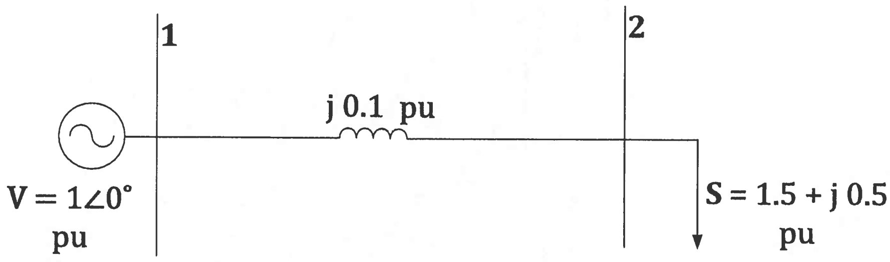
$$\therefore J_{21}[1,2] = 20\sin(-\delta_2) = \boxed{-20\sin(\delta_2)}$$

$$J_{22}[2,2] = \frac{\partial Q_{eqn,2}}{\partial Q_{gen,2}} \qquad Q_{eqn,2} = Q_{gen,2} - Q_{load,2} - \underbrace{\cdots}_{\text{terms not involving } Q_{gen,2}}$$

$$\therefore \boxed{J_{22}[2,2] = 1}$$

Problem 8:

Consider the two bus system shown below:



a) Create the Y_{bus} for this system. [2 marks]

$$y_{1,2} = y_{2,1} = \frac{1}{j\,0.1} = -j10$$

$$Y_{bus} = \begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix}$$

b) Create the X vector for this system. [2 marks]

$$X = \begin{bmatrix} P_{gen,1} \\ \delta_2 \\ Q_{gen,1} \\ V_2 \end{bmatrix}$$

c) Write the real and reactive power flow equations for bus 2. Plug in all the know values and simplify as much as possible. [4 marks]

$$P_{eqn,2} = f_2 = \overset{0}{\cancel{P_{gen,2}}} - P_{load,2} - \sum_{k=1}^{2} V_2 V_k \overset{0}{\cancel{G[2,k]}} \cos(\delta_2 - \delta_k) - \sum_{k=1}^{2} V_2 V_k B[2,k] \sin(\delta_2 - \delta_k)$$

$$= -1.5 - V_2 V_1 B[2,1] \sin(\delta_2 - \delta_1) - (V_2)^2 B[2,2] \sin\overset{0}{\cancel{(\delta_2 - \delta_2)}}$$

$$= -1.5 - V_2 (1)(10) \sin(\delta_2)$$

$$= \boxed{-1.5 - 10 V_2 \sin(\delta_2)}$$

$$Q_{eqn,2} = f_4 = \overset{0}{\cancel{Q_{gen,2}}} - Q_{load,2} - \sum_{k=1}^{2} V_2 V_k \overset{0}{\cancel{G[2,k]}} \sin(\delta_2 - \delta_k) + \sum_{k=1}^{2} V_2 V_k B[2,k] \cos(\delta_2 - \delta_k)$$

$$= -0.5 + V_2 V_1 B[2,1] \cos(\delta_2 - \delta_1) + (V_2)^2 B[2,2] \cos(\delta_2 - \delta_2)$$

$$= -0.5 + V_2 (1)(10) \cos(\delta_2) + (V_2)^2 (-10)$$

$$= \boxed{-0.5 + 10 V_2 \cos(\delta_2) - 10 (V_2)^2}$$

d) The 2 equations from part c) can form a system of non-linear equations with $[\delta_2 \quad V_2]^T$ as the unknowns. Perform one iteration of the Newtown-Raphson method in solving this system of equations (i.e. what is $X^{(1)}$?) Use an initial guess of $1\angle 30°$ for the voltage. [7 marks]

$$F(X) = \begin{bmatrix} f_2 \\ f_4 \end{bmatrix} = \begin{bmatrix} -1.5 - 10V_2 \sin(\delta_2) \\ -0.5 + 10V_2 \cos(\delta_2) - 10V_2^2 \end{bmatrix} \qquad X = \begin{bmatrix} \delta_2 \\ V_2 \end{bmatrix}$$

$$J(x) = \begin{bmatrix} \frac{\partial f_2}{\partial \delta_2} & \frac{\partial f_2}{\partial V_2} \\ \frac{\partial f_4}{\partial \delta_2} & \frac{\partial f_4}{\partial V_2} \end{bmatrix} = \begin{bmatrix} -10V_2 \cos(\delta_2) & -10 \sin(\delta_2) \\ -10V_2 \sin(\delta_2) & 10\cos(\delta_2) - 20V_2 \end{bmatrix}$$

$$x^0 = \begin{bmatrix} 30° \\ 1 \end{bmatrix}$$

$$F(x^0) = \begin{bmatrix} -1.5 - 10(1)\sin(30°) \\ -0.5 + 10(1)\cos(30°) - 10(1)^2 \end{bmatrix} = \begin{bmatrix} -6.5 \\ -1.84 \end{bmatrix}$$

$$J(x^0) = \begin{bmatrix} -10(1)\cos(30°) & -10\sin(30°) \\ -10(1)\sin(30°) & 10\cos(30°) - 20(1) \end{bmatrix} = \begin{bmatrix} -8.66 & -5 \\ -5 & -11.34 \end{bmatrix}$$

$$J(x^0) \cdot \Delta\tilde{x}^0 = -F(x^0) \quad \therefore \quad \begin{bmatrix} -8.66 & -5 \\ -5 & -11.34 \end{bmatrix} \Delta\tilde{x}^0 = \begin{bmatrix} 6.5 \\ 1.84 \end{bmatrix}$$

Solve system of linear eq : $\Delta\tilde{x}^0 = \begin{bmatrix} -0.88 \\ 0.226 \end{bmatrix}$ ← in radians! $-50.42°$

$$x^1 = x^0 + \Delta\tilde{x}^0 = \begin{bmatrix} 30° \\ 1 \end{bmatrix} + \begin{bmatrix} -50.42° \\ 0.226 \end{bmatrix} = \begin{bmatrix} -20.42° \\ 1.226 \end{bmatrix}$$