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University of Calgary

Schulich School of Engineering

Department of Electrical and Computer Engineering

ENEL 476 – Electromagnetic Waves and Applications

Midterm Examination 1

Winter Session 2014
Thursday February 6, 2014
7:00-8:30 pm

ICT 102

Student Name or ID number:

DR. FEAR

Question 1. **(15 marks)**

up to 1 mark off for missing ω substitution throughout Q1

An electric field is described by:

$$\bar{E}_s(z) = 10e^{j10\pi z}\bar{a}_y \text{ V/m}$$

The frequency (f) is 500 MHz. The field is located in a material with $\mu=\mu_o$ and $\sigma=0$. Assume a source-free region (**J**=0, ρ_v=0).

$\omega = 2\pi f = \pi \times 10^9$ rad/s

2 a) Express the electric field in the time-domain (**E**(z,t)).

$$\vec{E}(z,t) = 10\cos(\pi\times10^9 t + 10\pi z)\vec{a}_y \text{ V/m}$$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

5 b) Find the magnetic field in phasor form ($\mathbf{H}_s(z)$). You may keep the expression in terms of ε.

1 $\nabla\times\vec{E}_s = -j\omega\mu_o\vec{H}_s$

$$\nabla\times\vec{E}_s = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_{ys} & 0 \end{vmatrix}$$

1 $= \vec{a}_x\left(-\frac{\partial}{\partial z}E_{ys}\right) - \vec{a}_y(0) + \vec{a}_z\left(\frac{\partial}{\partial x}E_{ys}\right)$ → 0

$= -\frac{\partial}{\partial z}E_{ys}\,\vec{a}_x$

1 $\nabla\times\vec{E}_s = -10(j10\pi)e^{j10\pi z}\vec{a}_x$

1 $-j100\pi e^{j10\pi z}\vec{a}_x = -j\omega\mu_o\vec{H}_s$

$\vec{H}_s = \frac{100\pi}{\omega\mu_o}e^{j10\pi z}\vec{a}_x$

1 $\boxed{\vec{H}_s(z) = \frac{1}{4\pi}e^{j10\pi z}\vec{a}_x \text{ A/m}}$

c) Calculate the relative permittivity, ε_r.

1 $\nabla\times\vec{H}_s = j\omega\varepsilon_r\varepsilon_o\vec{E}_s$

$$\nabla\times\vec{H}_s = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_{xs} & 0 & 0 \end{vmatrix}$$

1 $= \vec{a}_x(0) - \vec{a}_y\left(-\frac{\partial}{\partial z}H_{xs}\right) + \vec{a}_z\left(-\frac{\partial}{\partial y}H_{xs}\right)$ → 0

$\nabla\times\vec{H}_s = \frac{\partial}{\partial z}H_{xs}\vec{a}_y$

1 $\left(\frac{1}{4\pi}\right)(j10\pi)e^{j10\pi z}\vec{a}_y = j\omega\varepsilon_r\varepsilon_o 10e^{j10\pi z}$

$\frac{5}{2} = (\pi\times10^9)(\varepsilon_r)\left(\frac{10^{-9}}{36\pi}\right)$

1 $\boxed{9 = \varepsilon_r}$

Check: $\beta = \omega\sqrt{\varepsilon_r\varepsilon_o\mu_o}$

$10\pi = \frac{\pi\times10^9}{3\times10^8}\sqrt{\varepsilon_r}$

$\sqrt{\varepsilon_r} = 3$

$\Rightarrow \varepsilon_r = 9$

d) Write an expression for the displacement current density ($\mathbf{J}_d(z,t)$).

1 $\vec{J}_d(z,t) = \frac{\partial}{\partial t}\vec{D}$

1 $\vec{D} = \varepsilon_r\varepsilon_o\vec{E}$

1 $= 9\left(\frac{1}{36\pi}\times10^{-9}\right)(10\cos(\pi\times10^9t + 10\pi z)\vec{a}_y$

→ $\vec{J}_d(z,t) = -\pi\times10^9 \times \frac{5\times10^{-9}}{2\pi}\sin(\pi\times10^9t + 10\pi z)\vec{a}_y$

1 $= -2.5\sin(\pi\times10^9t + 10\pi z)\vec{a}_y$ A/m²

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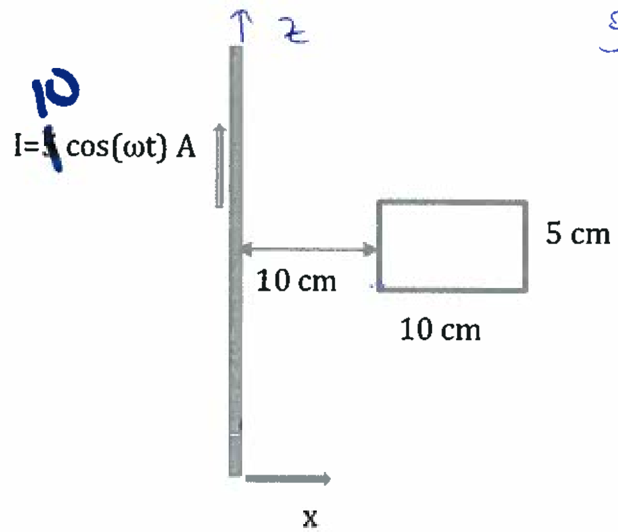
Question 2. **(15 marks)**

a) A contour is placed at a distance of 10 cm from an infinitely long wire in which a current of I=10 cos(ωt) A is flowing. The current flows in the z direction, and the contour is in the x-z plane. Assume that the current is slowly time-varying (i.e. displacement current can be neglected). The contour and wire are in free space (ε_o, μ_o, σ=0). The contour has side lengths of 10 cm and 5 cm, as shown in the figure. Find the EMF induced in the contour.

b) The contour moves with a velocity of $\mathbf{v} = 0.2\,\mathbf{a}_x$ m/s away from the wire. Find an expression for the EMF induced in the contour.

$\oint (\vec{v}\times \vec{B})\cdot d\vec{l}$ | $-\int \frac{\partial}{\partial t}\vec{B}\cdot d\vec{s}$

4.5 | 3.5



a) $EMF = -\dfrac{d\Phi}{dt}$

1 $\Phi = \int \vec{B}\cdot d\vec{s}$

$\vec{B} = \mu_o \vec{H}$

$\vec{H} \Rightarrow \oint \vec{H}\cdot d\vec{l} = I$

$d\vec{l} = \rho\, d\phi\, \vec{a}_\phi$

$\int_0^{2\pi} H_\phi \rho\, d\phi = 10\cos\omega t$

$H_\phi 2\pi\rho = 10\cos\omega t$

$H_\phi = \dfrac{10\cos\omega t}{2\pi\rho}$

For this configuration,

1 $\vec{H} = \dfrac{10\cos\omega t}{2\pi x}\vec{a}_y$

1 $\vec{B} = \dfrac{10\mu_o\cos(\omega t)}{2\pi x}\vec{a}_y$

1/2 each limits

$\int \vec{B}\cdot d\vec{s} = \dfrac{\cancel{10}^{5}\mu_o\cos(\omega t)}{\cancel{2}\pi}\int_0^{0.05}\int_{0.1}^{0.2}\dfrac{1}{x}\,dx\,dz$

$= \dfrac{5\mu_o\cos(\omega t)}{\pi}(0.05)\left(\ln\dfrac{0.2}{0.1}\right)$

1 $= \dfrac{\mu_o\cos\omega t}{4\pi}\ln 2$

$\ln 2 \sim .346$

① + $EMF = -\frac{d}{dt}\left(\mu_0 \frac{\cos\omega t}{4\pi} \ln 2\right)$

$$= -\frac{\mu_0}{4\pi}\ln 2\left(-\omega \sin \omega t\right)$$

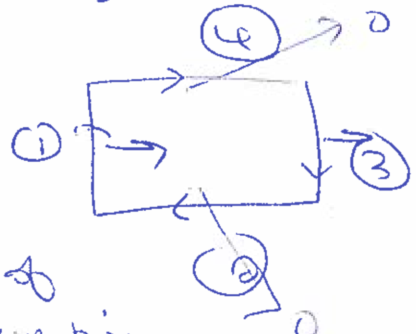
① $EMF = \frac{\mu_0 \omega}{4\pi} \ln 2 \left(\sin \omega t\right)$ V

final for b → $\boxed{EMF = \frac{0.2\mu_0 \cos(\omega t)}{4\pi}\left(\frac{1}{x} - \frac{1}{x+0.1}\right) + \frac{5\omega\mu_0}{4\pi} \sin \omega t \ln\left(\frac{x+0.1}{x}\right)}$

b) $EMF = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} + -\int \frac{\partial}{\partial t} \vec{B} \cdot d\vec{s}$ → on total flux

① ①

$$\oint (\vec{v} \times \vec{B}) \cdot d\vec{l} = \int_1 (\vec{v} \times \vec{B})_1 \cdot d\vec{l} + \int_3 (\vec{v} \times \vec{B})_3 \cdot d\vec{l}$$



$d\vec{s}$ in $\vec{a}_y$ ∴ cw dir'n of integration

$$\Rightarrow v \times \vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 0.2 & 0 & 0 \\ 0 & B_y & 0 \end{vmatrix}$$

① $= 0.2\, B_y\, \vec{a}_z$

① ①

$$\therefore \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} = \int_0^{0.05} \frac{5\mu_0 \cos(\omega t) \times 0.2}{\pi x} dz + \int_{0.05}^{0} \frac{5\mu_0 \cos(\omega t) \times 0.2}{\pi (x+0.1)} dz$$

$\frac{1}{2}$ $= \frac{\mu_0 \cos(\omega t) \times 0.2}{4\pi x} - \frac{\mu_0 \cos(\omega t) \times 0.2}{4\pi (x+0.1)}$

① $\frac{\partial}{\partial t}\vec{B} = -5 \frac{\mu_0 \omega \sin \omega t}{\pi x} \vec{a}_y$

① $-\int \frac{\partial}{\partial t} \vec{B} \cdot d\vec{s} = +5 \frac{\mu_0 \omega}{\pi} \sin \omega t \int_0^{0.05}\int_x^{x+0.1} \frac{1}{x}\, dx\, dz$

$\frac{1}{2}$ $= \frac{5\omega\mu_0}{\pi} \sin \omega t\, (0.05) \ln\left(\frac{x+0.1}{x}\right)$

Question 3. **(10 marks)**

a) A lossless transmission line has impedance of 70 Ω and β=6 rad/m. The line operates at 200 MHz. Calculate R, L, G and C.

$R=0$

$G=0$ } lossless

$Z_0 = \sqrt{L/C}$ & $\beta = \omega\sqrt{LC}$

$\sqrt{L/C} = 70$

$L/C = 70^2$

$L = 70^2 C$

$\beta = 400\pi \times 10^6 \sqrt{70^2 C^2}$

$6 = (4\pi \times 10^8)(70)C$

$C = 4\pi \times 10^9 (6/7)$

$= 68.21$ pF/m

$L = 334.2$ nH/m

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(Answers only)

R= 0 G= 0 L= 334.2 nH/m C= 68.21 pF/m

b) A coax line is characterized by R=6.5 Ω/m, L=3.4 μH/m, G=8.4 mS/m and C=21.5 pF/m. At a frequency of 1 MHz, calculate the impedance, Z_o.

$$Z_0 = \sqrt{\frac{6.5 + j(2\pi\times10^6)(3.4\times10^{-6})}{8.4\times10^{-3} + j(2\pi\times10^6)(21.5\times10^{-12})}}$$

$= 41.67 + j30.36\ \Omega$

$= 51.56 \angle 36.1°$

2

Z_o=

(Answer only, No part marks)

c) A parallel plate capacitor has plate area of 15 cm^2 and separation of 9 mm. The voltage 15 $\cos(10^3 t)$ V is applied to the plates. If the area between the plates is filled with $\varepsilon = 6\varepsilon_r$, find the displacement current density, $\mathbf{J}_d$. ← vector $\bar{6\epsilon_0}$

$V = 15\cos(10^3 t)$ $J_d = \frac{\partial D}{\partial t}$

$|E| = V/d$

$= \frac{15}{0.009}\cos(10^3 t)$ $= -10^4 \epsilon_0 (\sin 10^3 t)(10^3)$

$|D| = \frac{(6\epsilon_0)(15)(10^3)\cos(10^3 t)}{9}$ $|\vec{J}_d| = -10^7 \epsilon_0 \sin(10^3 t)$

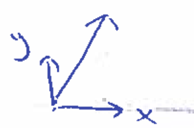
$= -\frac{10^7 \times 10^{-9}}{36\pi} \sin(10^3 t)$

$|\vec{J}_d| = -\frac{1}{3600\pi}\sin(10^3 t)$ A/

(2) $\mathbf{J}_d =$ ______________________ direction: ↓↓↓

(Answer only)

d) A material with $\varepsilon_r=6$ and $\mu_r=1.2$ forms an interface with a perfect electric conductor. The surface normal of the interface is oriented in the +x, +y direction. If the electric field in the material is given by $\mathbf{E}=(12\,\mathbf{a}_x + 6\,\mathbf{a}_y)\cos(10^9 t)$ mV/m, find the surface charge density, ρ_s.



$\vec{a}_n \cdot \vec{D} = \rho_s$

$\vec{a}_n = \frac{12\vec{a}_x + 6\vec{a}_y}{\sqrt{12^2 + 6^2}}$

144
36
180

$= 0.89\vec{a}_x + 0.45\vec{a}_y$

$\vec{a}_n \cdot \vec{D} = (0.89\vec{a}_x + 0.45\vec{a}_y)\cdot(72\epsilon_0 \vec{a}_x + 36\epsilon_0 \vec{a}_y)\cos(10^9 t)$

$= \cancel{0.71\ldots}$

(2) $\rho_s =$ ______________________

(Answer only)

$= 0.71\cos(10^9 t)$ pC/m² $\times 10^{-3}$