Wrap up

1. Understand AM Modulation. Can find $c(\mathrm{t})$ and $m(\mathrm{t})$ and k_a.
2. Understand the max and min value of the amplitude. Note that in case 3 when $k_a = 1$, $1 + 2\cos(2\pi f_m \mathrm{t})$ varies between 3 and -1. **However, the amplitude of the AM signal is the absolute value $|\mathbf{1 + 2\cos(2\pi} \boldsymbol{f_m} \mathbf{t)}|$.** So the min amplitude is 0 **NOT** -1.
3. Understand modulation factor μ. Can calculate.
4. Know the FT of cosine, sine, and other simple functions.

   $$\cos(\mathrm{at}) \Leftrightarrow \frac{\delta(f - \frac{a}{2\pi}) + \delta(f + \frac{a}{2\pi})}{2}$$

   $$\sin(\mathrm{at}) \Leftrightarrow \frac{\delta\left(f - \frac{a}{2\pi}\right) - \delta(f + \frac{a}{2\pi})}{2i}$$

   $$1 \Leftrightarrow \delta(f)$$

   $$\delta(f - \mathrm{a}) * \delta(f - \mathrm{b}) = \delta(f - \mathrm{a} - \mathrm{b})$$
5. Can draw the time domain wave and frequency domain spectrum. In time domain, horizontal axis is t. In frequency domain, horizontal axis is f. When you are asked to draw an AM signal while $\mu > 1$, you should show the phase reversal and zero crossing in the plot.

Example: Single Tone Modulation

A transmitter uses conventional AM modulation to modulate a carrier, $c(\mathrm{t}) = \cos(2\pi f_c \mathrm{t})$, with a cosine wave message, $m(\mathrm{t}) = 2 \cdot \cos(2\pi f_m \mathrm{t})$, where $f_c = 100MHz$, and $f_m = 1MHz$. For each of the following cases of amplitude sensitivity: (i) $k_a = 0.5$, (ii) $k_a = 0.1$, (iii) $k_a = 1$

a) Find the time domain representation $S_{AM}(t)$ and sketch the amplitude modulation signal.

b) What is the maximum and minimum values of the amplitude of the AM signal?

c) Calculate the modulation factor or percentage: $\mu = \max_t(|k_a m(t)|)$.

d) Calculate the frequency domain representation, $S_{AM}(f)$, and plot the frequency spectrum.

Solution:

From Eqn. 2.2 in the textbook, we have:

$$S_{AM}(t) = [1 + k_a m(t)] \cdot \mathrm{c(t)}$$

Here, $m(\mathrm{t}) = 2 \cdot \cos(2\pi f_m \mathrm{t})$ and $c(\mathrm{t}) = \cos(2\pi f_c \mathrm{t})$

Then we obtain: $S_{AM}(t) = (1 + k_a \cdot 2 \cdot \cos(2\pi f_m \mathrm{t})) \cdot \cos(2\pi f_c \mathrm{t})$

Remember that we call k_a amplitude sensitivity of the modulator.

i) $\boldsymbol{k_a = 0.5}$

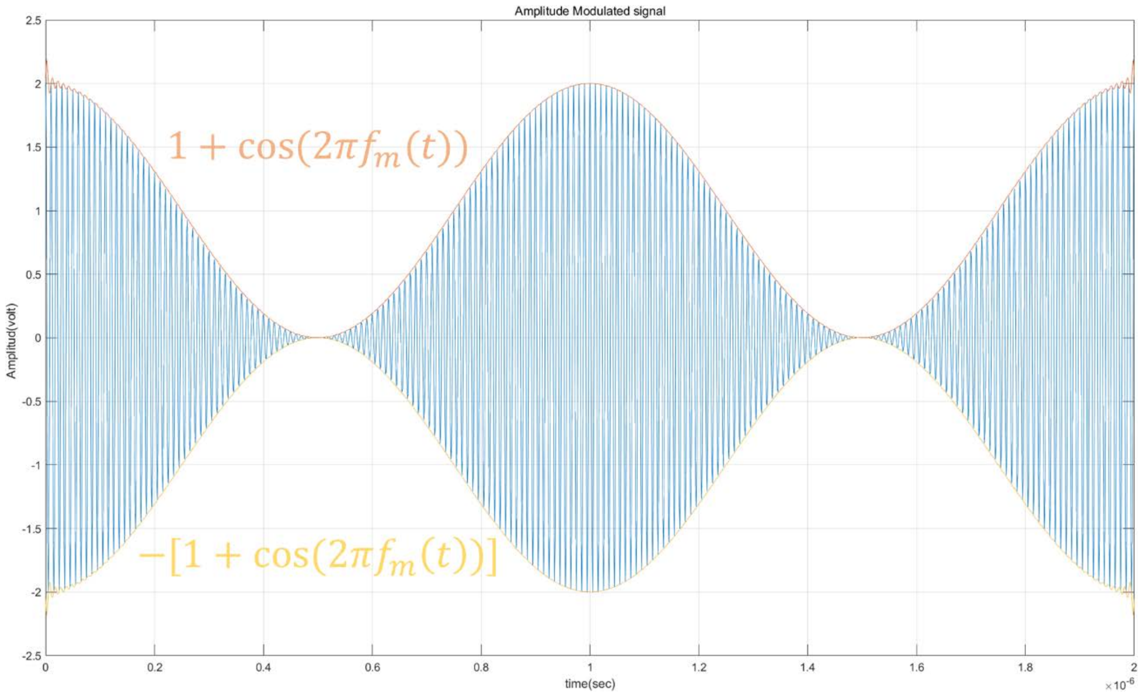
a) $S_{AM}(t) = (1 + \cos(2\pi f_m \mathrm{t})) \cdot \cos(2\pi f_c \mathrm{t})$

The wave in time domain is shown below.

Key points of this figure:

1. Since both message and carrier signals are cosine wave, at $t = 0$, both reach peak value.
2. Symmetrical to *t* axis.
3. Carrier wave (Blue) goes up and down much faster than the envelope (yellow and orange, which caused by the message). You can see around 200 cycles of the blue one, while there are just 2 cycles of the envelope.

4. No phase reversal. Min amplitude is 0.



b) The amplitude of the AM signal varies between 2 and 0. We have full swing of the amplitude without phase reversal.

c) $\mu = \max_t(|k_a m(t)|) = \max_t(|\cos(2\pi f_m t)|) = 1 = 100\%$

So we know $\mu = 100\% \leftrightarrow$ full swing.

d) Multiplication in time domain equals to convolution in frequency domain. So we can get the Fourier Transform of the cosine wave and then calculate the convolution in frequency domain.

$$S_{AM}(f) = [\delta(f) + k_a[\delta(f - f_m) + \delta(f + f_m)]] * \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$$

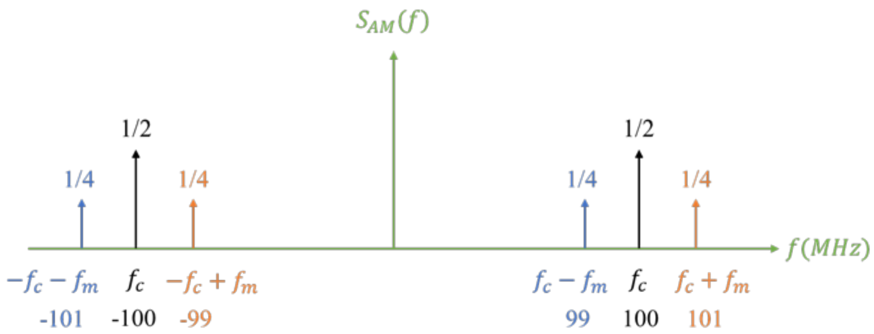
then

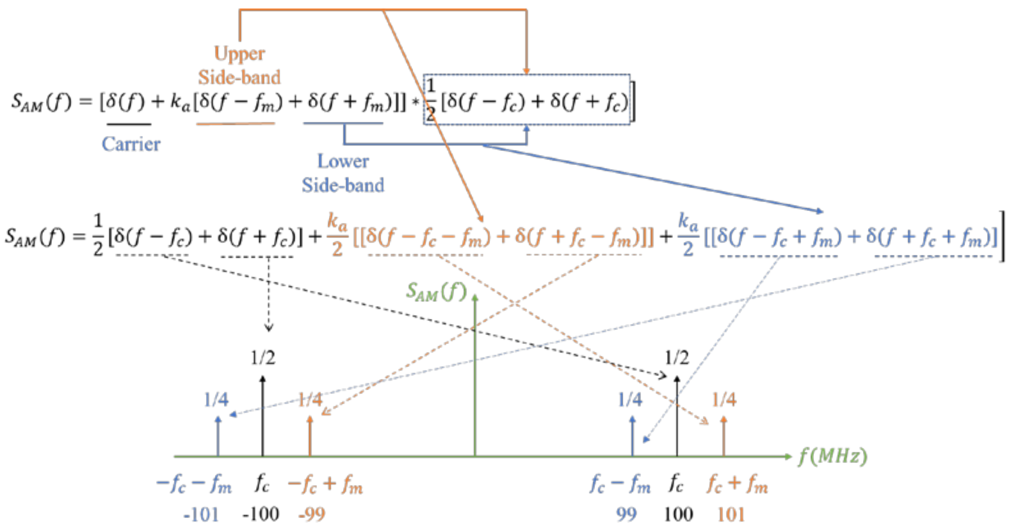
$$S_{AM}(f) = \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a}{2}[[\delta(f - f_c - f_m) + \delta(f + f_c - f_m)]] + \frac{k_a}{2}[[\delta(f - f_c + f_m) + \delta(f + f_c + f_m)]]$$

Since $k_a = 0.5$, we can get

$$S_{AM}(f) = \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{1}{4}[[\delta(f - f_c - f_m) + \delta(f + f_c - f_m)]] + \frac{1}{4}[[\delta(f - f_c + f_m) + \delta(f + f_c + f_m)]]$$

The spectrum can be plotted as follow. (The details to obtain the pulses are also shown in the next figure). Note that when we say upper side band and lower side band, they are related to the carrier f_c, not the y axis.





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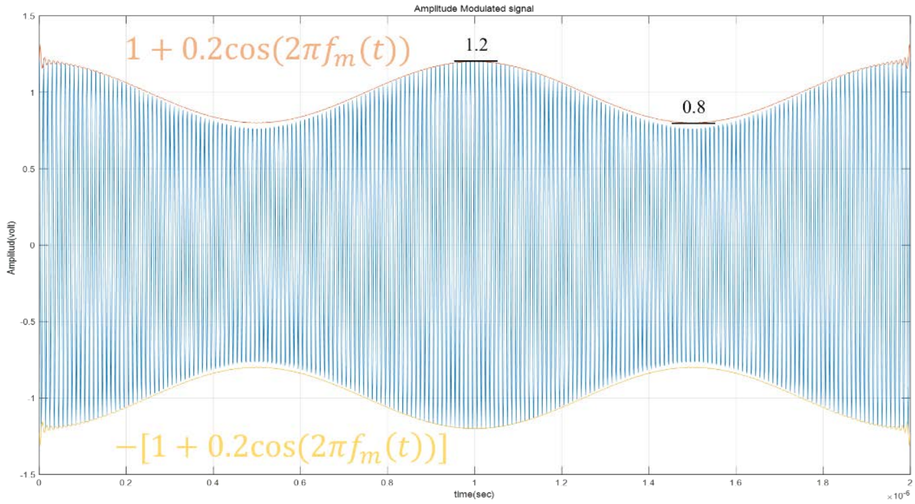
ii) $\boldsymbol{k_a = 0.1}$

a) $S_{AM}(t) = (1 + 0.2\cos(2\pi f_m t)) \cdot \cos(2\pi f_c t)$

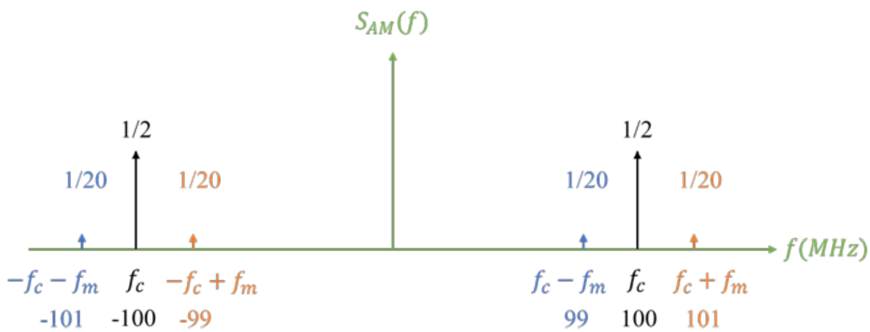


b) The amplitude of the AM signal varies between 0.8 and 1.2

c) $\mu = \max_t(|k_a m(t)|) = \max_t(|0.2\cos(2\pi f_m t)|) = 0.2 = 20\%$

Here, we have small swing (partial swing).

d) $S_{AM}(f) = \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{1}{20}[[\delta(f - f_c - f_m) + \delta(f + f_c - f_m)]] + \frac{1}{20}[[\delta(f - f_c + f_m) + \delta(f + f_c + f_m)]]$



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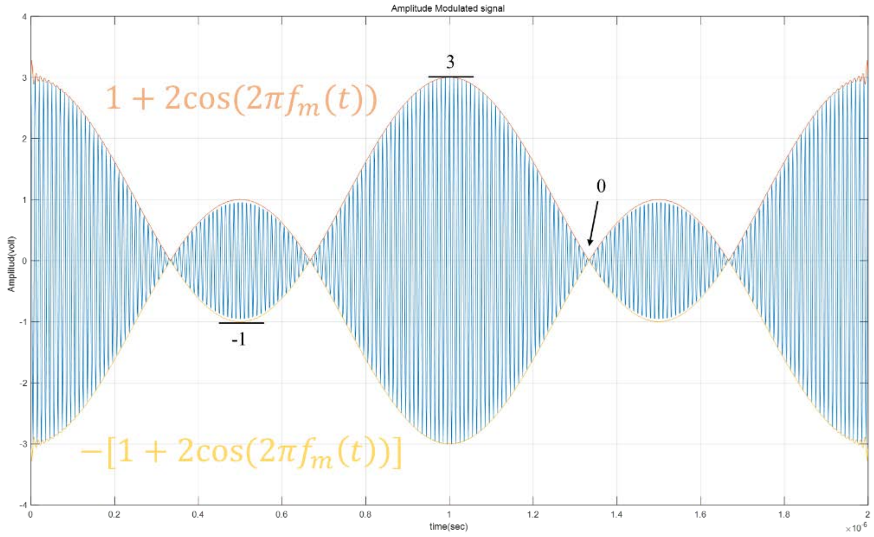
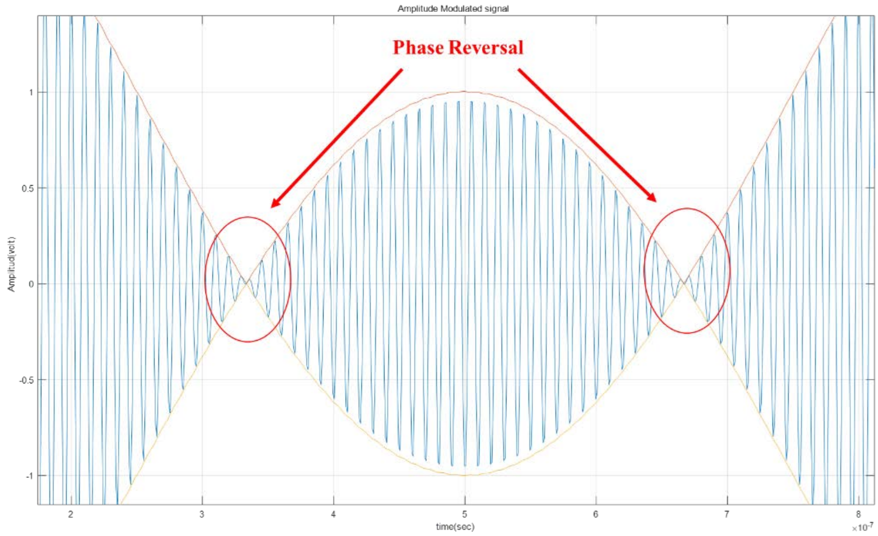
iii) $\boldsymbol{k_a = 1}$

a) $S_{AM}(t) = (1 + 2\cos(2\pi f_m t)) \cdot \cos(2\pi f_c t)$



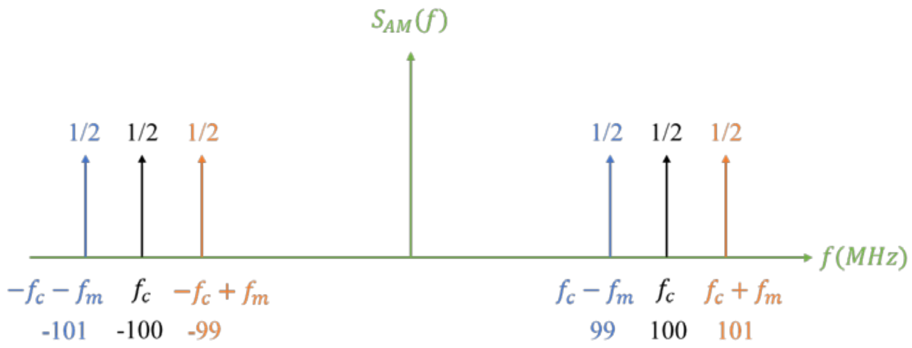


b) $1+2\cos(2\pi f_m \mathrm{t})$ varies between 3 and -1. So there is zero crossing or phase reversal. The amplitude again is $|1+2\cos(2\pi f_m \mathrm{t})|$. So its max value is 3 and min value is 0.

c) $\mu = \max_t(|k_a m(t)|) = \max_t(|2\cos(2\pi f_m \mathrm{t})|) = 2 = 200\% > 1$

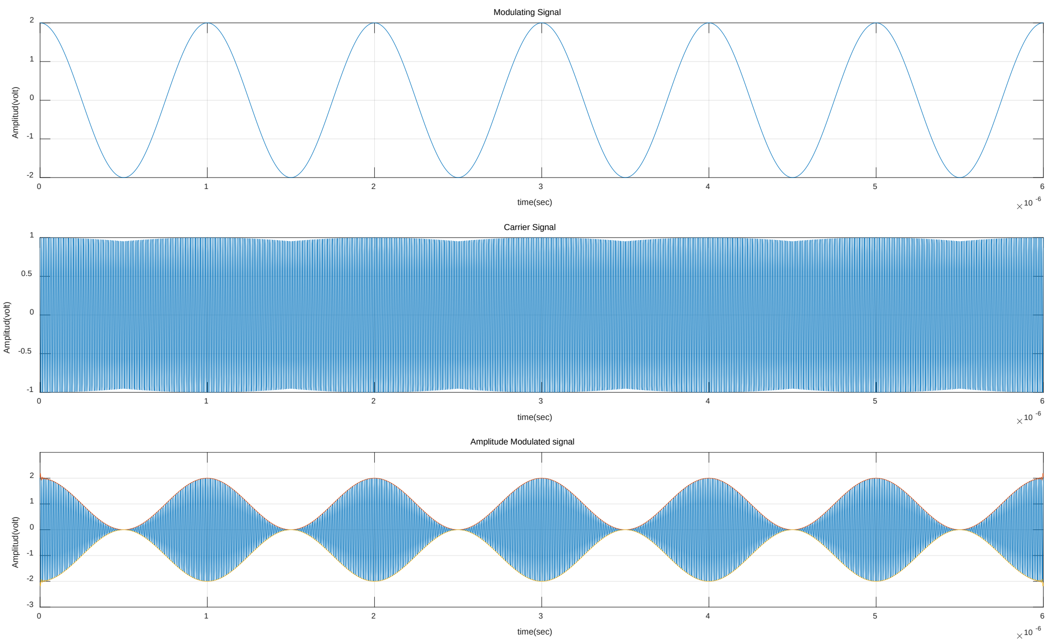
This means there is overmodulation.

d) $S_{AM}(f) = \frac{1}{2}[\delta(f-f_c)+\delta(f+f_c)] + \frac{1}{2}[[\delta(f-f_c-f_m)+\delta(f+f_c-f_m)]] + \frac{1}{2}[[\delta(f-f_c+f_m)+\delta(f+f_c+f_m)]]$

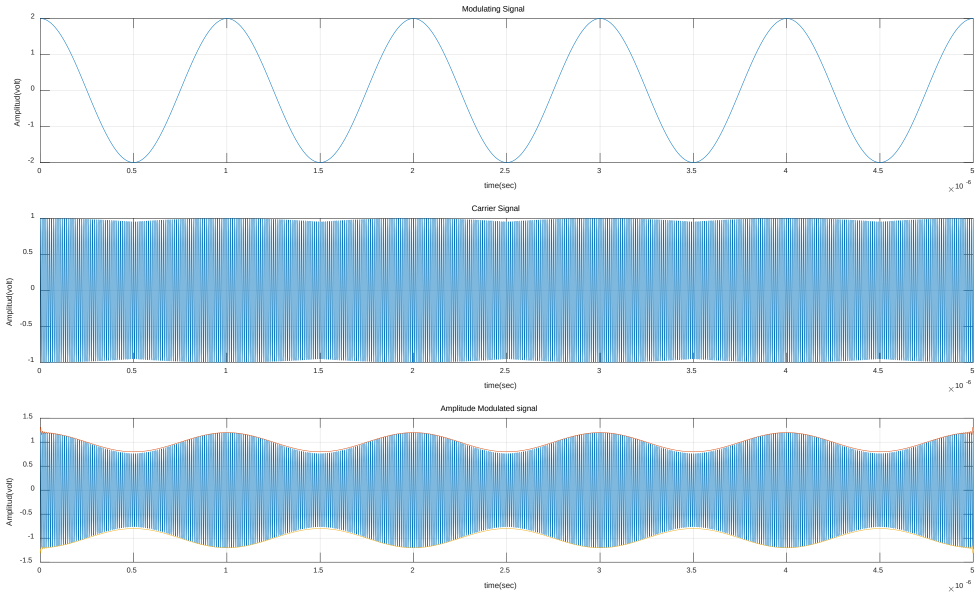


Appendix

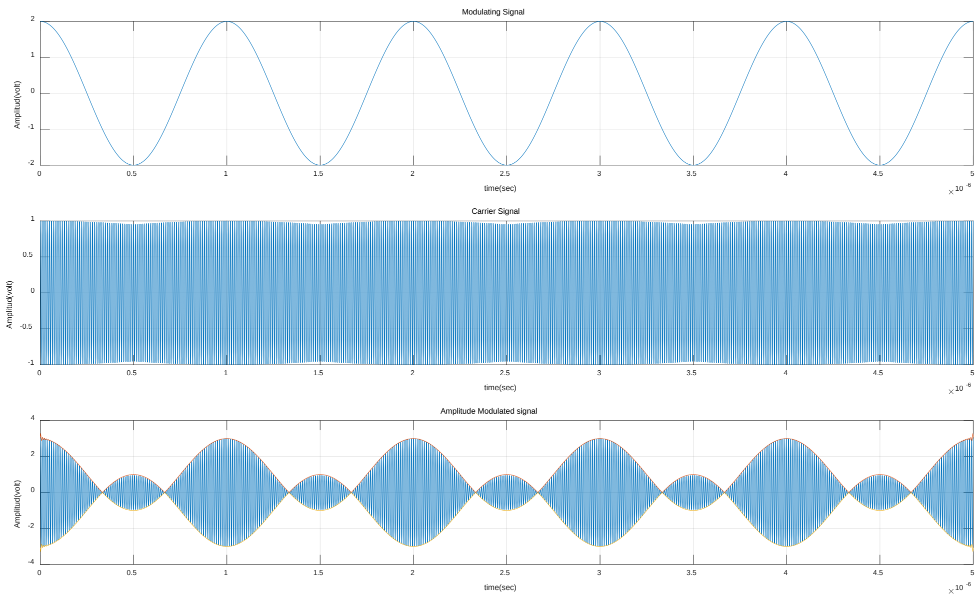
Message signal, Carrier Signal and AM Signal in 3 cases.



$$\boldsymbol{k_a = 0.5}$$


Modulating Signal
Amplitud(volt)
time(sec)
Carrier Signal
Amplitud(volt)
time(sec)
Amplitude Modulated signal
Amplitud(volt)
time(sec)

$$k_a = 0.1$$


Modulating Signal
Amplitud(volt)
time(sec)
Carrier Signal
Amplitud(volt)
time(sec)
Amplitude Modulated signal
Amplitud(volt)
time(sec)

$$k_a = 1$$

```
clc;
close all;
clear all;
%XXXXXXXXXXXXXXXXXXXXXXXXXXX Define AM modulation Index XXXXXXXXXXXXXXXXXXX
disp(' example: m=1 means 100% modulation');
%m=input(' Enter the value of modulation index (m) = ');
m=1; % for 100% modulation
if (0>m||m>1)
error('m may be less than or equal to one and geter than to zero');
end
%XXXXXXXXXXXXXXXXX modulating signal generation XXXXXXXXXXXXXXXXXXXXXXXXXX

Am=2; % Amplitude of modulating signal
ka=1;% Amplitude sensitivity
fa=1e6; % Frequency of modulating signal
Ta=1/fa; % Time period of modulating signal
t=0:Ta/999:5*Ta; % Total time for simulation
ym=Am*cos(2*pi*fa*t); % Eqation of modulating signal
figure(1)
subplot(3,1,1);
plot(t,ym), grid on;% Graphical representation of Modulating signal
title ( '  Modulating Signal   ');
xlabel ( ' time(sec) ');
ylabel (' Amplitud(volt)   ');
%XXXXXXXXXXXXXXXXXXXXX carrier signal generation XXXXXXXXXXXXXXXXXXXXXXXXXX
Ac=1;% Amplitude of carrier signal [ where, modulation Index (m)=Am/Ac ]
fc=fa*100;% Frequency of carrier signal
Tc=1/fc;% Time period of carrier signal
yc=Ac*cos(2*pi*fc*t);% Eqation of carrier signal
subplot(3,1,2);
plot(t,yc), grid on;% Graphical representation of carrier signal
title ( '  Carrier Signal   ');
xlabel ( ' time(sec) ');
ylabel (' Amplitud(volt)   ');
%XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX AM Modulation XXXXXXXXXXXXXXXXXXXXXXXXXXXXX
y=Ac*(1+ka*Am*cos(2*pi*fa*t)).*cos(2*pi*fc*t); % Equation of Amplitude
%modulated signal
subplot(3,1,3);
plot(t,y);% Graphical representation of AM signal
title ( '  Amplitude Modulated signal  ');
xlabel ( ' time(sec) ');
ylabel (' Amplitud(volt)   ');
grid on;
hold on
```

```
[up,lo]=envelope(y)
plot(t,up,t,lo)
hold off

%>>>>>>>>>>>>>>>>>>>>>> end of program <<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<
```