Topic 6: Power Flow

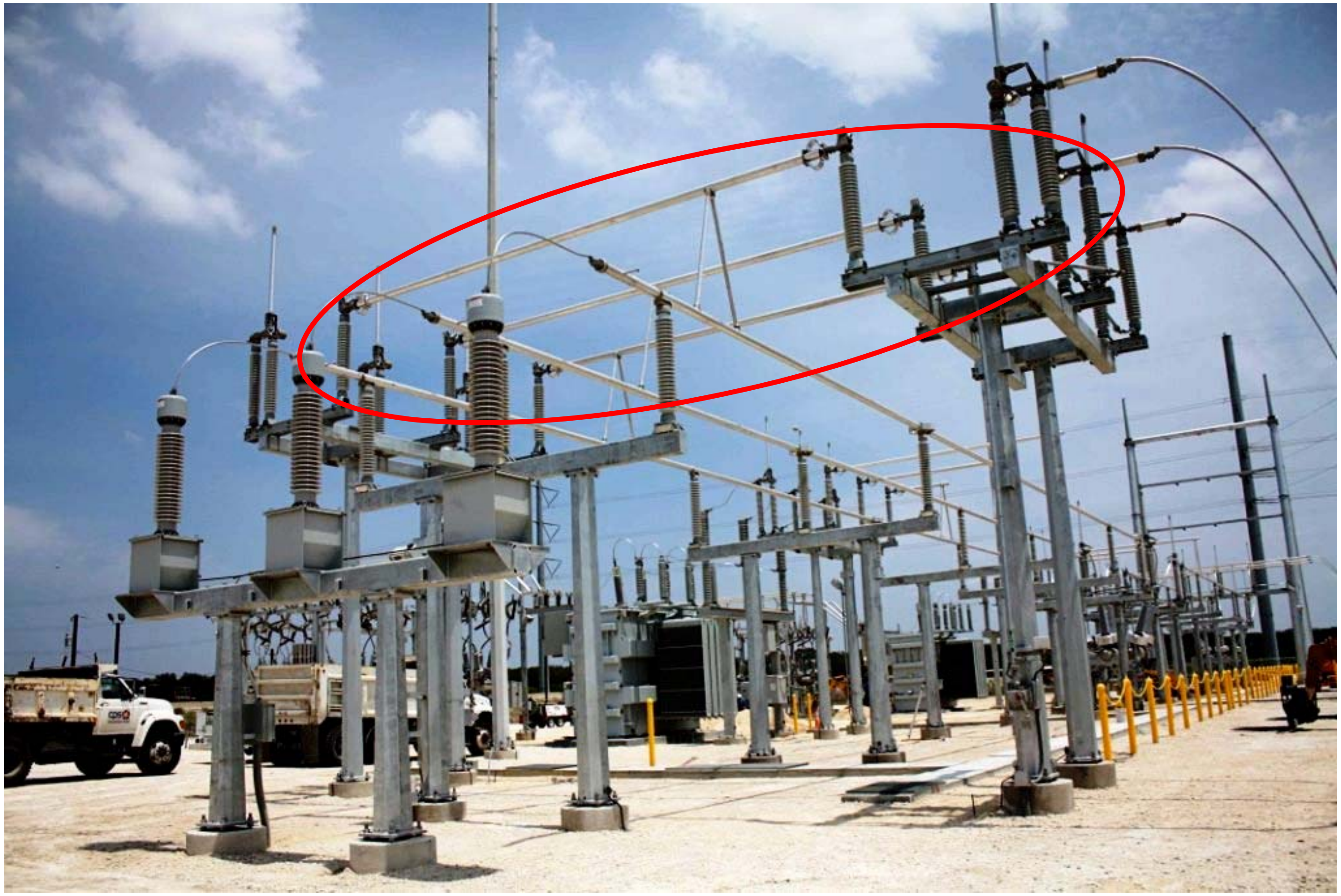
Part 1: Setting it up!

Power Flow

- Power flow (load flow):
  - Studying / determining the steady state voltages, currents, and power at different busses (nodes) in a power system.

- What's a bus?
  - A node. Where 2 or more elements (lines, transformers, load, etc.) are connected.

Rigid bus bar – outdoor substation

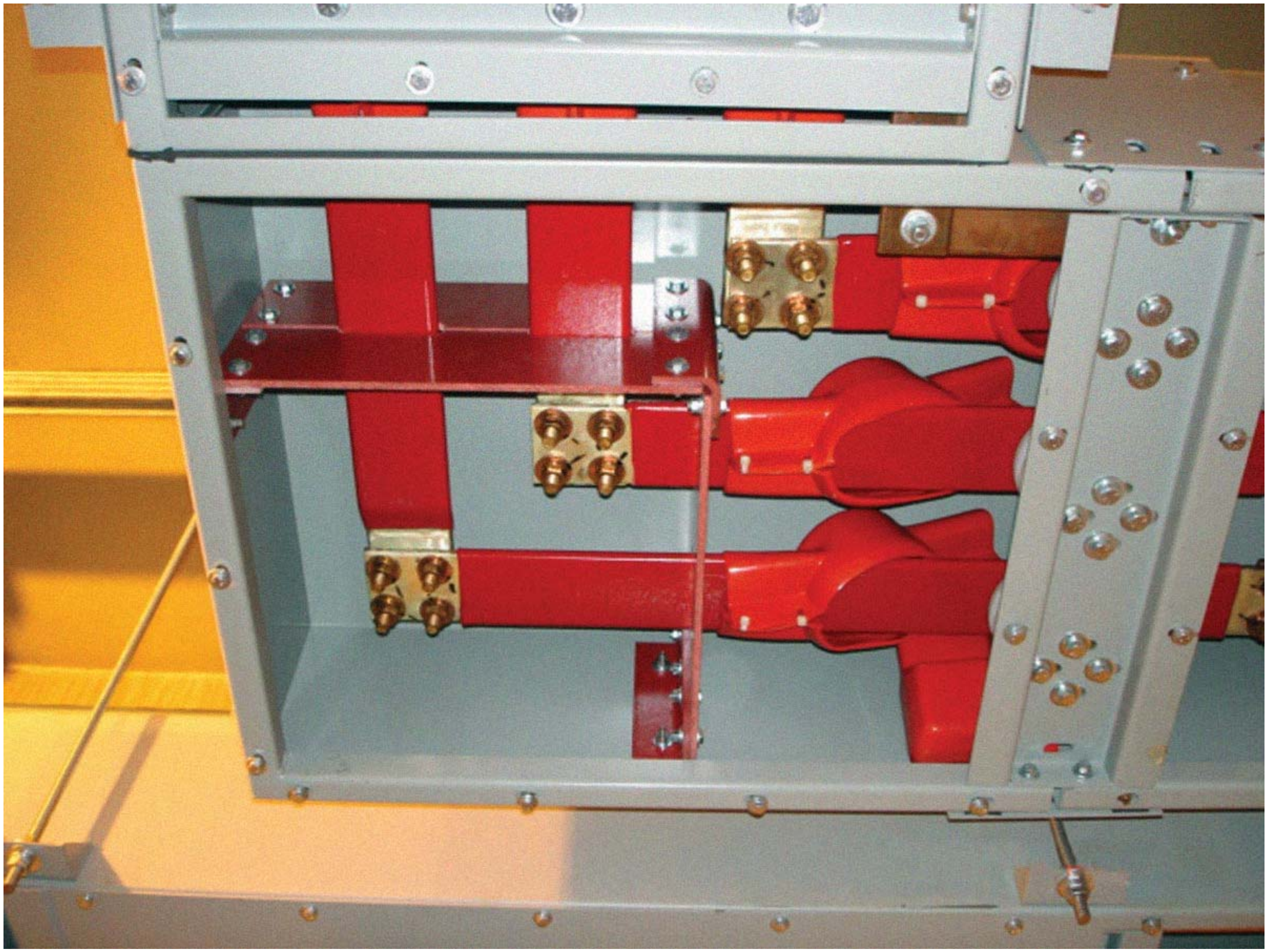


Note: One bus bar per phase

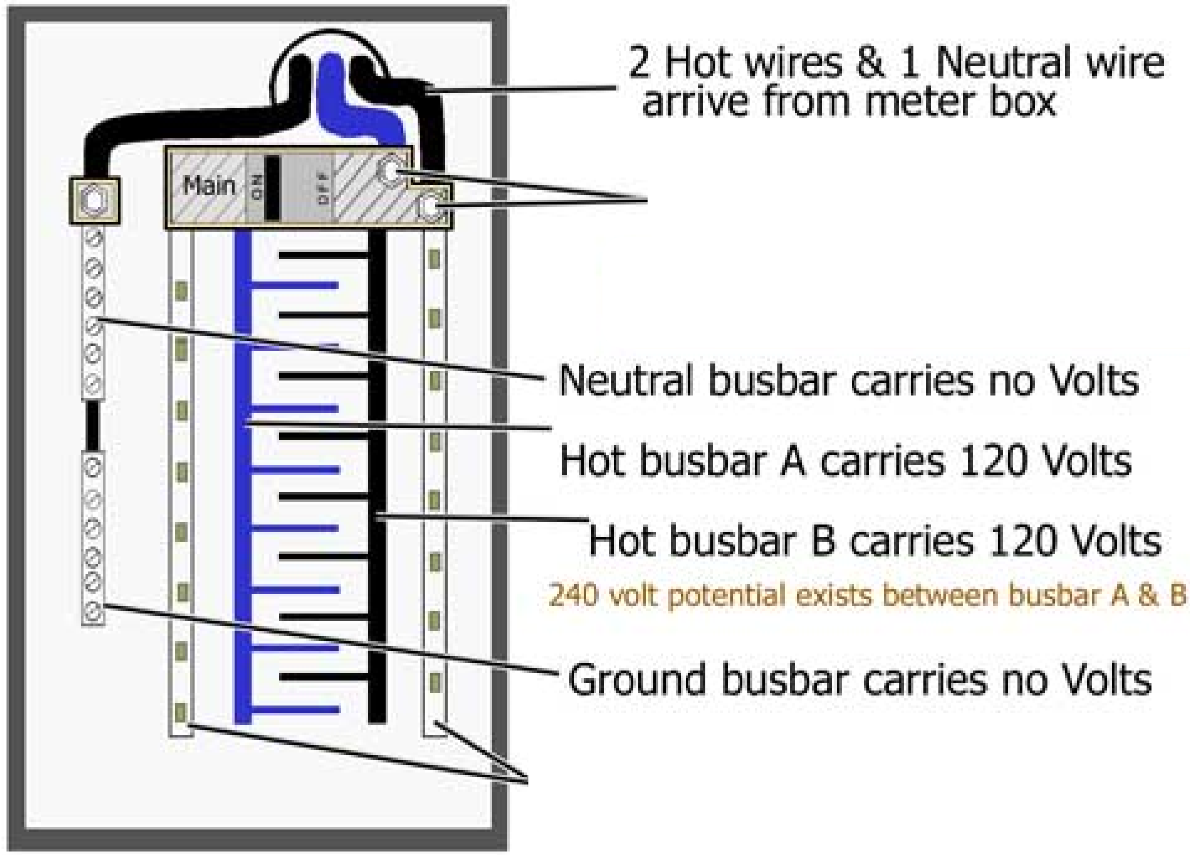
Bus in a switchgear

Bus is inside the cabinet. Let’s look inside ….

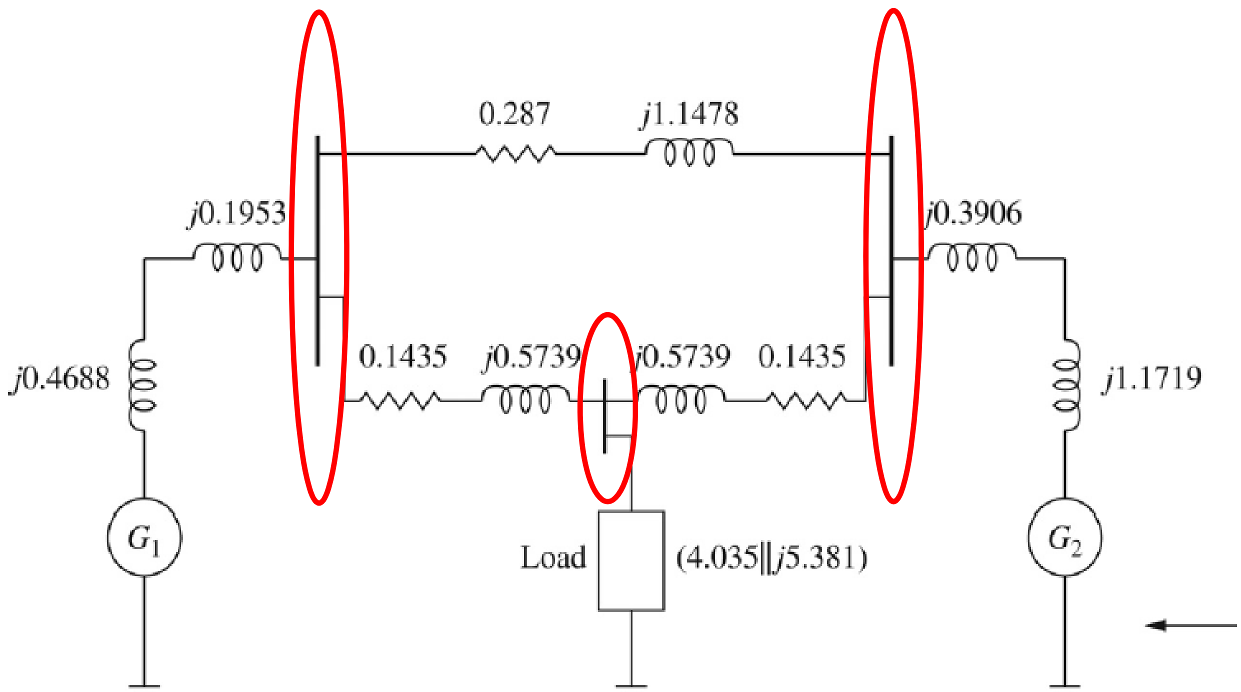
Bus in a switchgear



Bus in an electrical panel



Saw them in practice problems!



Impedance diagram of the system with pu values

Why perform power flow?

- Ensure voltages are at or near nominal/rated values.
- Ensure equipment (lines, transformer) are sized properly (i.e. can carry the required amount of power & current)
- Calculate losses
- Ensure Gen = demand + losses
- So much more!

Power Flow

- At each bus:

$$\sum \bar{S}_{gen} - \sum \bar{S}_{load} - \sum \bar{S}_{branch} = 0$$

where:

$\bar{S}_{gen}$ = local generation at the bus

$\bar{S}_{load}$ = load connected to the bus

$\bar{S}_{branch}$ = Power leaving the bus through a branch

Power Flow

- Break into real and reactive power:

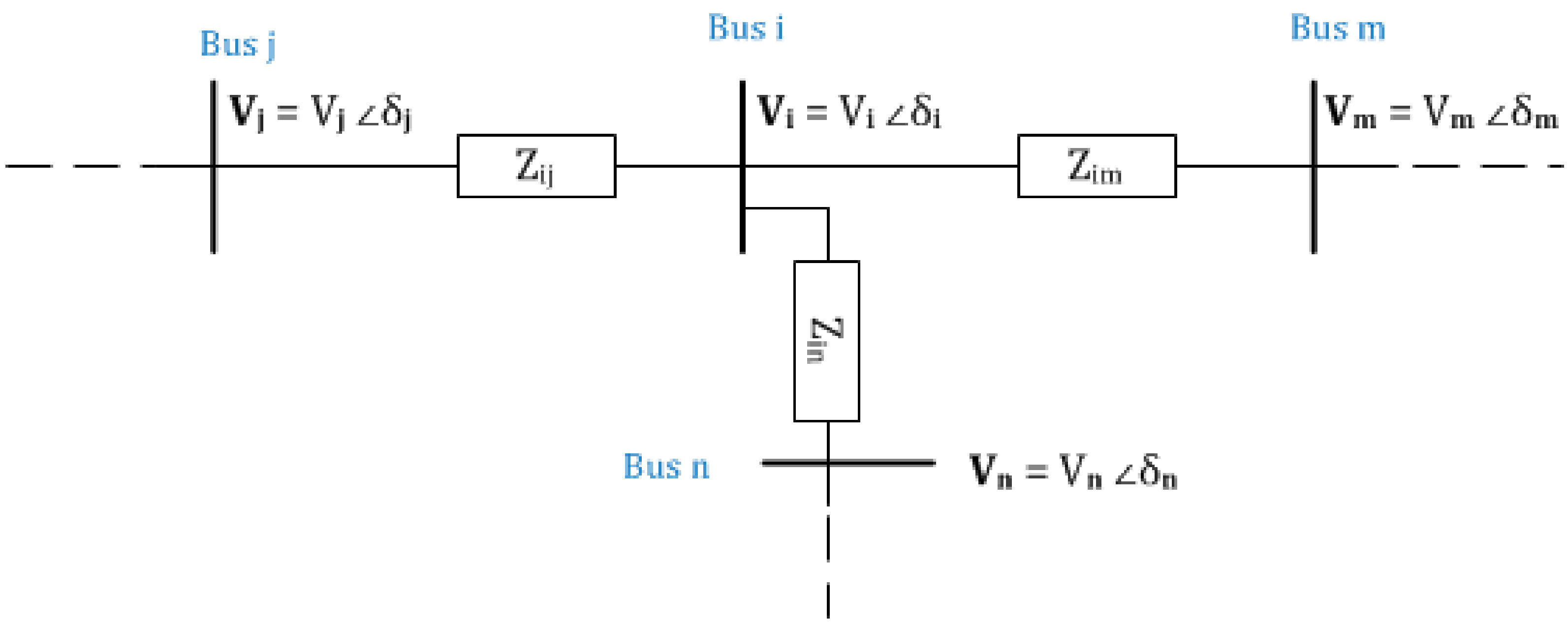
$$\sum P_{gen} - \sum P_{load} - \sum P_{branch} = 0$$

$$\sum Q_{gen} - \sum Q_{load} - \sum Q_{branch} = 0$$

- Let's try this!

S_{branch} equation

- Consider the following generic system:



S_{branch} equation

- $\bar{S}_{branch,\,i}$:
  Σ all complex power leaving bus i through branches

$$= \sum_{k \in \{bus\ connected\ to\ i\}} \bar{S}_{i,k}$$

Sending end

receiving end

- $k \in \{j, m, n\}$ in our example

- $\bar{S}_{branch,i} = \sum_{k\in\{\quad\}} \bar{V}_i . \left(\bar{I}_{i,k}\right)^*$

Topic 6: Power Flow

- Please see Intro slides

- At each bus (node): $\sum \bar{S}_{gen} - \sum \bar{S}_{load} - \sum \bar{S}_{branch} = 0$

local generation at the bus
- use (+) for power produced

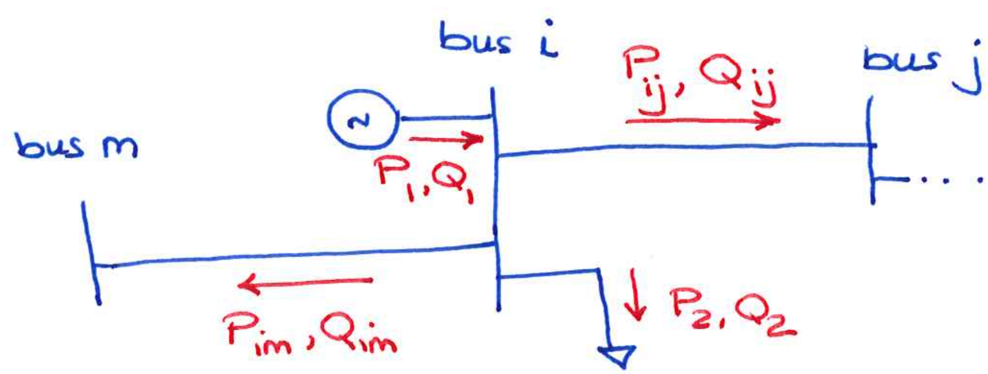
local load

power leaving the bus through a branch

break into real & reactive power:

$$\sum P_{gen} - \sum P_{load} - \sum P_{branch} = 0$$

$$\sum Q_{gen} - \sum Q_{load} - \sum Q_{branch} = 0$$



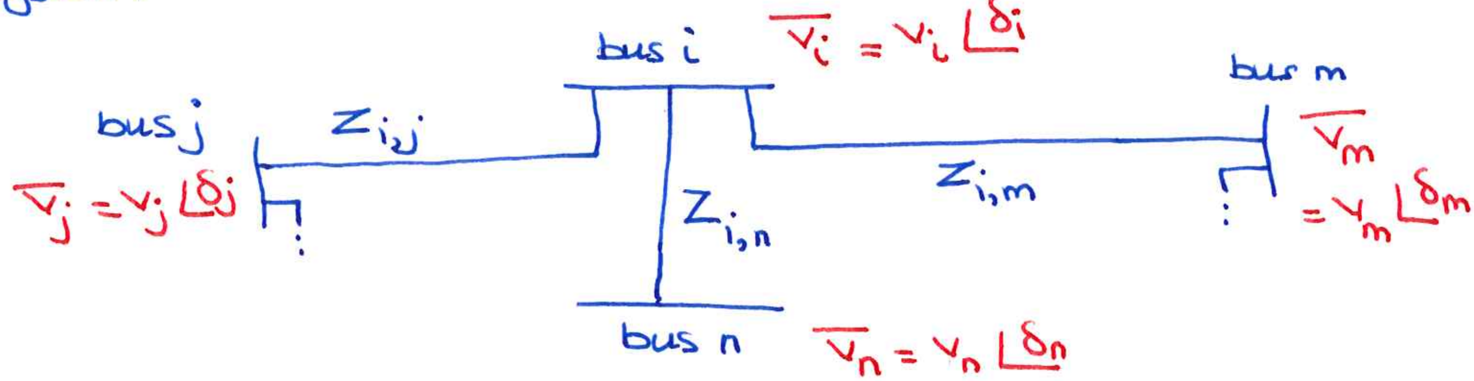
we can write 2 power flow equations at bus i:

$$P_1 - P_2 - (P_{ij} + P_{im}) = 0$$

$$Q_1 - Q_2 - (Q_{ij} + Q_{im}) = 0$$

. Values in the power flow equation could be + or −
e.g. if the load at bus i is capacitive ("supplies" reactive power) then $Q_2 < 0$.

- Power flow equations are typically written in per unit. Since transformers & lines both appear as impedances in pu, a branch could be either a line or txfr.
- The power flow equations are valid for any given point in time. P_{gen} & P_{load} (& Q_{load}) are typically known. P_{branch} (& Q_{branch}) needs more investigation.
- Let's write P_{branch} & Q_{branch} equations for a generic system:



$\overline{S}_{branch,i}$: $\sum$ all complex power leaving bus i through the branches

$$= \sum_{k \in \{\text{bus connected to } i\}} \overline{S}_{i,k}$$

i ← sending end, k ← receiving end

$k \in \{j, m, n\}$ in above system

$$= \sum_{k \in \{\text{bus connected to } i\}} \overline{V}_i \, \underbrace{\overline{I}}_{\text{current going from } i \text{ to } k}{}_{ik}^{*}$$

reminder: $\overline{S}_{pu} = \overline{V}_{pu} \cdot \overline{I}_{pu}^{*}$

$$= \sum_{k \in \{\ldots\}} \overline{V}_i \left(\frac{\overline{V}_i - \overline{V}_k}{Z_{i,k}} \right)^*$$

from Ohm's Law

$$= \sum_{k \in \{\ldots\}} \overline{V}_i \left(\overline{V}_i - \overline{V}_k \right)^* \cdot \underbrace{Y}_{\text{admittance between } i \text{ \& } k}{}_{i,k}^{*}$$

using $Y = \frac{1}{Z}$

$$\boxed{\overline{S}_{branch,i} = \sum_{k\in\{\dots\}} Y_{i,k}^{*} \cdot \overline{V}_i \cdot \overline{V}_i^{*} + \sum_{k\in\{\}} -Y_{i,k}^{*}\, \overline{V}_i\, \overline{V}_k^{*}} \quad (1)$$

Define a $1\times N$ vector $Y_i = \left[-Y_{i,1} \quad -Y_{i,2} \quad \cdots \quad \underbrace{Y_{i,i}} \quad \cdots \quad -Y_{i,N} \right]$

$\uparrow$ # of busses in system

$\uparrow$ admittance vector for bus i

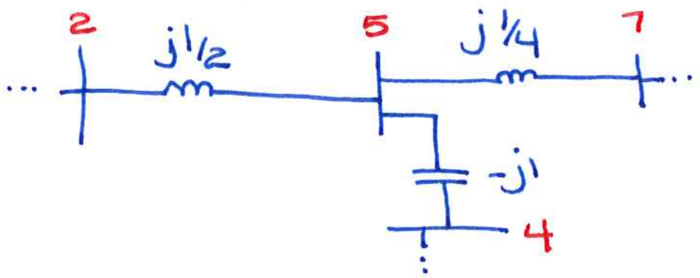
. no (-) sign

. to be defined!

where $\underset{i\neq k}{Y_{i,k}}$ = admittance between bus i & bus k

define $Y_{i,i}$: the <u>sum</u> of all admittances connected to bus i

Example: Find vector Y_5 in the following 9-bus system:



$$Y_{5,2} = \frac{1}{j\,1/2} = -j2$$

$$Y_{5,7} = \frac{1}{j\,1/4} = -j4$$

$$Y_{5,4} = \frac{1}{-j1} = j1$$

$$Y_5[5] = \sum Y_{5,k} = -j2 - j4 + j1$$

$$\downarrow$$

$$Y_5 = \left[0 \quad +j2 \quad 0 \quad -j1 \quad -j5 \quad 0 \quad +j4 \quad 0 \quad 0 \right]$$

$\uparrow$

$Y_5[2] = -Y_{5,2}$

· Re-write eq (1) using Y_i vector:

$$\overline{S}_{branch,i} = \overline{V_i} \cdot \overline{V_i}^* \left(\sum_{k \in \{..\}} Y_{i,k} \right)^* + \sum_{k \in \{..\}} \left(-Y_{i,k} \right)^* \overline{V_i} \ \overline{V_k}^*$$

how we defined $Y_i[i]$

how we defined $Y_i[k]$ $i \neq k$

$$= \overline{V_i} \ \overline{V_i}^* \ Y_i[i]^* + \sum_{\substack{k=1 \\ k \neq i}}^{N} Y_i[k]^* \ \overline{V_i} \ \overline{V_k}^*$$

we can move the 1st term into the summation & drop $k \neq i$ for $\sum$

$$\therefore \boxed{\overline{S}_{branch,i} = \sum_{k=1}^{N} Y_i[k]^* \ \overline{V_i} \ \overline{V_k}^*}$$

. Vector Y_i can be expressed in rectangular format:

$$Y_i = \underbrace{G_i}_{1 \times N \text{ vector of } Re[Y_i]} + j \overbrace{B_i}^{1 \times N \text{ vector of } Im[Y_i]}$$

1xN vector of Im $[Y_i]$

. in prev example, $B_5 = [0 \ 2 \ 0 \ -1 \ -5 \ 0 \ 4 \ 0 \ 0]$

1xN vector of Re $[Y_i]$

. in prev example, $G_5 = [0 \ \ldots \ 0]$

1x9 vector of zeros

$$Y_i^* = G_i - j B_i$$

· Finally, $\overline{S}_{branch,i} = \sum_{k=1}^{N} \left(G_i[k] - j B_i[k] \right) \underbrace{\overline{V_i} \ \overline{V_k}^*}$

$$V_i \angle \delta_i \cdot \left(V_k \angle \delta_k \right)^*$$

$$= V_i V_k \angle \delta_i - \delta_k$$

$$= V_i V_k \left(\cos(\delta_i - \delta_k) + j \sin(\delta_i - \delta_k) \right)$$

· Also, $\overline{S}_{branch,i} = P_{branch,i} + j Q_{branch,i}$

- Break $\overline{S}_{branch,i}$ expression into real & imaginary parts:

$$\boxed{\begin{aligned} P_{branch,i} &= \sum_{k=1}^{N} G_i[k] V_i V_k \cos(\delta_i - \delta_k) + \sum_{k=1}^{N} B_i[k] V_i V_k \sin(\delta_i - \delta_k) \\ Q_{branch,i} &= \sum_{k=1}^{N} G_i[k] V_i V_k \sin(\delta_i - \delta_k) - \sum_{k=1}^{N} B_i[k] V_i V_k \cos(\delta_i - \delta_k) \end{aligned}}$$

(2)

- Alternatively, we can express $Y_i[k]$ in polar format:

$Y_i[k] = |Y_{i,k}| \angle \theta_{i,k}$ and write branch power eq's as:

$$\boxed{\begin{aligned} P_{branch,i} &= \sum_{k=1}^{N} |Y_{i,k}| V_i V_k \cos(\delta_i - \delta_k - \theta_{i,k}) \\ Q_{branch,i} &= \sum_{k=1}^{N} |Y_{i,k}| V_i V_k \sin(\delta_i - \delta_k - \theta_{i,k}) \end{aligned}}$$

Y_{Bus} (Admittance Matrix)

The vector Y_i can be completed for each bus in the system:

$$Y_{bus} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{bmatrix} = \begin{bmatrix} Y_{1,1} & -Y_{1,2} & \cdots & -Y_{1,N} \\ -Y_{2,1} & Y_{2,2} & \cdots & -Y_{2,N} \\ \vdots & & & \vdots \\ -Y_{N,1} & -Y_{N,2} & \cdots & Y_{N,N} \end{bmatrix}$$

NxN matrix

- Y_{bus} is symmetric since $Y_{i,j} = Y_{j,i}$
- Also, a sparse matrix for large systems (most of the elements in the matrix $= 0$)

- $Y_{bus}[i,j] = -Y_{i,j}$ for $i \neq j$

  $Y_{bus}[i,i] = \sum$ all admittances connected to bus i

- $G_{bus} = Re[Y_{bus}]$, $B_{bus} = Im[Y_{bus}]$

Power Flow Equations

- For a system with N busses, need to solve the following $2N$ equations:

$$P_{gen,i} - P_{load,i} - \sum_{k=1}^{N} V_i V_k G[i,k] \cos(\delta_i - \delta_k) - \sum_{k=1}^{N} V_i V_k B[i,k] \sin(\delta_i - \delta_k)$$

$$= 0 \quad \text{for } i = 1, \dots, N$$

$$Q_{gen,i} - Q_{load,i} - \sum_{k=1}^{N} V_i V_k G[i,k] \sin(\delta_i - \delta_k) + \sum_{k=1}^{N} V_i V_k B[i,k] \cos(\delta_i - \delta_k)$$

$$= 0 \quad \text{for } i = 1, \dots, N$$

Solution to power flow equations: Values of unknown V, δ, P_{gen}, Q_{gen} that satisfy this system of non-linear equations

- Alternatively, we can use polar representation for Y:

textbook

$$P_{gen,i} - P_{load,i} - \sum_{k=1}^{N} |Y_{i,k}| V_i V_k \cos(\delta_i - \delta_k - \theta_{i,k}) = 0$$

$$Q_{gen,i} - Q_{load,i} - \sum_{k=1}^{N} |Y_{i,k}| V_i V_k \sin(\delta_i - \delta_k - \theta_{i,k}) = 0$$

Note: $|Y_{i,k}|$ in this expression is $|Y_{bus}[i,k]|$

$\theta_{i,k}$ is phase of $Y_{bus}[i,k]$

we want to solve $F(x)$ where:

mismatch vector

$$\underbrace{F(x)}_{\text{vector of power flow equations evaluated at every bus for a given value of } x} = \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_N(x) \\ \hline f_{N+1}(x) \\ \vdots \\ f_{2N}(x) \end{bmatrix} = \begin{bmatrix} P_{gen,1} - P_{load,1} - P_{branch,1} \\ \vdots \\ P_{gen,N} - P_{load,N} - P_{branch,N} \\ \hline Q_{gen,1} - Q_{load,1} - Q_{branch,1} \\ \vdots \\ Q_{gen,N} - Q_{load,N} - Q_{branch,N} \end{bmatrix}$$

$\sum_{k=1}^{N} v_i v_k \dots$

where x : vector of unknowns ← more on this shortly.

objective : Find $\underbrace{x^*}_{\text{optimal solution}}$ such that $F(x) \simeq 0$

or $\| F(x^*) \| < \varepsilon$ ← pre-determined small number, say 10^{-6}

Norm : largest value in the vector

Defining the elements in x

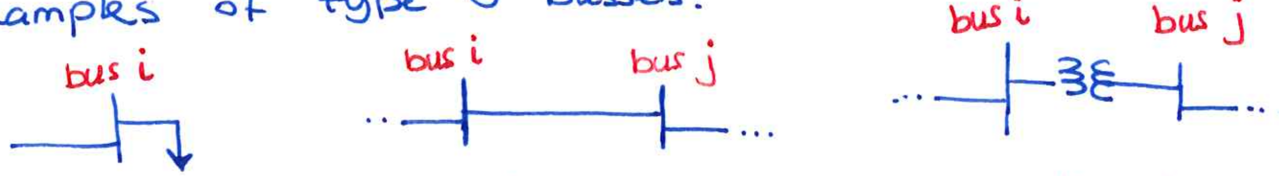
- The unknown for each bus depends on the bus type:

1) For a non-generator (load) bus [IEEE type 0 bus]
   - bus without a generator connected to it

$$P_{gen,i} = Q_{gen,i} = 0$$

v_i & δ_i are unknown → placed in x

- examples of type 0 busses:



the only req't : no gen connected to the bus

2) For a generator (PV) bus [IEEE type 2 bus]

- Generators can control voltage magnitude (using the exciter) & real power generated (using the prime mover)

$\therefore$ V_i & $P_{gen,i}$ are known

δ_i & $Q_{gen,i}$ are unknown $\longrightarrow$ placed in X

3) For a swing/slack generator [IEEE type 3 bus]

- Picks up the slack for deficient/excess P_{gen} & Q_{gen} in the whole system
- Only one slack bus in the system.

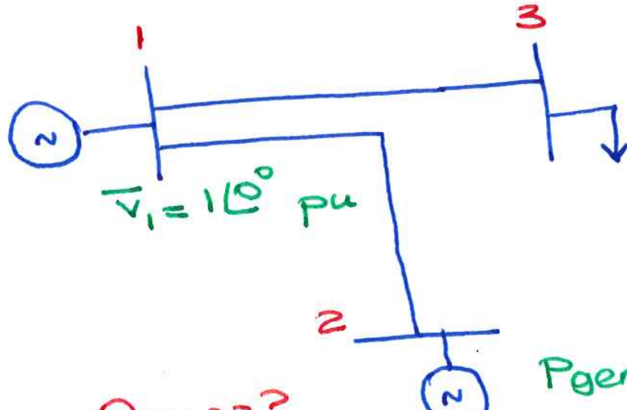
$\therefore$ $P_{gen,sb}$ & $Q_{gen,sb}$ are unknown $\rightarrow$ placed in X

$\uparrow$ bus # for slack bus

V_{sb} & δ_{sb} are known

$\uparrow$ slack bus used as phase reference i.e. $\delta_{sb} = 0^\circ$

Example: Write real power flow eq at bus 1 for the following system:



$\bar{V}_3 = V_3 \angle \delta_3 = ?$ $\begin{cases} V_3 = ? \\ \delta_3 = ? \end{cases}$

$P_{load,3} = 1$ pu

$Q_{load,3} = 1$ pu

$P_{gen,1} = ?$

$Q_{gen,1} = ?$

$P_{gen,2} = 2$ pu

$V_2 = 1.05$ pu

$Q_{gen,2} = ?$

$\delta_2 = ?$

$$f_1 = P_{eqn,1} = P_{gen,1} - \cancel{P_{load,1}}^{0}$$

$$- V_1 V_1 G[1,1] \cos(\delta_1 - \delta_1) - V_1 V_1 B[1,1] \sin(\delta_1 - \delta_1)$$

$$- V_1 V_2 G[1,2] \cos(\delta_1 - \delta_2) - V_1 V_2 B[1,2] \sin(\delta_1 - \delta_2)$$

$$- V_1 V_3 G[1,3] \cos(\delta_1 - \delta_3) - V_1 V_3 B[1,3] \sin(\delta_1 - \delta_3)$$

unknowns shown in red.

- Note: P_{load} & Q_{load} are known at every bus.
- We will organize the unknowns in vector X as follows:

$$x = [\delta_1 \;\; \delta_2 \;\; \cdots \;\; \delta_N \;\vdots\; V_1 \;\; V_2 \;\; \cdots \;\; V_N]^T \leftarrow \text{transpose}$$

this works if every bus is a non-gen bus.

modify the vector with unknowns from gen & slack bus:

$$x = [\delta_1 \;\; \delta_2 \;\; \cdots \;\; \delta_N \;\vdots\; V_1 \;\; V_2 \;\; \cdots \;\; V_N]^T$$

replace δ_{sb} with $P_{gen,sb}$

For gen & slack bus replace V_i with $Q_{gen,i}$

- In our previous example, $x = [P_{gen,1} \;\; \delta_2 \;\; \delta_3 \;\; Q_{gen,1} \;\; Q_{gen,2} \;\; V_3]^T$
- To find x^*, we will use an iterative approach:
  - Start with x^0 $\leftarrow$ iteration zero. $F(x^0) \neq 0$
  - Do some update & get x^1 $\leftarrow$ iteration one

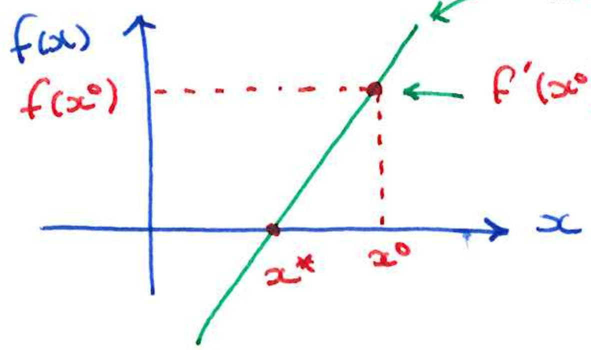
    $F(x^1) \neq 0$ & hopefully $\|F(x^1)\| < \|F(x^0)\|$
  - continue until you find x^* ; $\|F(x^*)\| < \varepsilon$

Newton-Raphson Method

- Please see handout.
- NR for scalar non-linear function:

  if $f(x)$ is a 1st order function, Δx will take us to the optimal solution:

linear $f(x)$



$$f'(x^0) = \text{slope} = \frac{f(x^0) - 0}{x^0 - x^*}$$

$$= \frac{f(x^0)}{-\Delta x}$$

$$\therefore \Delta x = - \frac{f'(x^0)}{f(x^0)}$$

Yani's cheap plastic review of Newton-Raphson (NR) Method

Idea: Using linear approximation of the function $f(x)$ or set of functions $\mathrm{F(X)}$ to solve them.

<u>Taylor Series:</u>

$$f(x) = f(x_0) + \sum_{k=1}^{\infty} \frac{f^k(x_0)}{k!}(x - x_0)^k \quad = \quad f(x_0) + f'(x_0).(x - x_0) + \frac{f''(x_0)}{2!}.(x - x_0)^2 + \cdots$$

Example: $f(x) = e^x$. Approximate e^1 and $e^{0.01}$ using Taylor Series expansion at $x_0 = 0$.

e^x is convenient since $f(x) = f'(x) = f''(x) = f^k(x) = e^x$ and they are all equal to 1 at $x = 0$ T.S. expansion gives:

$$e^x = e^0 + e^0(x - 0) + \frac{e^0}{2!}(x - 0)^2 + \cdots \quad = 1 + x + \frac{x^2}{2} + \cdots$$

If we approximate e^x using only the first two terms: $\quad e^x \approx 1 + x$

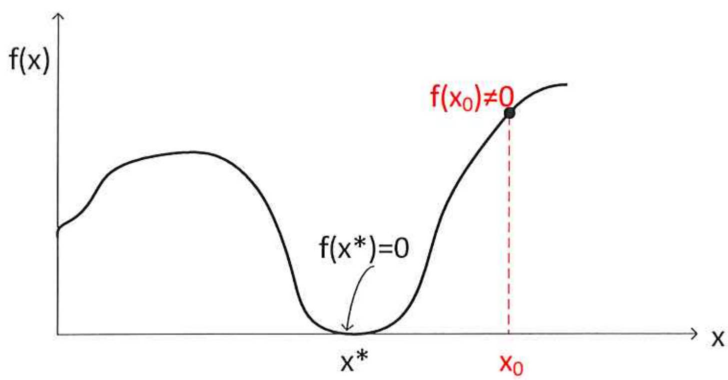
Let's calculate this for a couple of points:

x	e^x (exact value)	Taylor Series Approximation
0.01	1.0105	1.01
1	2.718	2

i.e. Taylor Series approximation is much better for x closer to x_0

<u>NR for scalar non-linear function</u>

One equation and one unknown.



Start at x_0

Want to find Δx such that $\Delta x = x^* - x_0$

Taylor Series expansion around x_0:

$$f(x) = f(x_0) + f'(x_0).(x - x_0) + H.O.T.$$

For $x = x^*$,

$$f(x^*) = f(x_0) + f'(x_0).(x^* - x_0) + \cancel{H.O.T.}$$

$$0 \approx f(x_0) + f'(x_0).\Delta x$$

Or, $\quad \Delta x \approx -\frac{f(x_0)}{f'(x_0)}$

$$\Delta\tilde{x} = -\frac{f(x_0)}{f'(x_0)}$$

- $\Delta\tilde{x}$ can <u>approximate</u> how much to change x_0 to get to x^*. We will call this the <u>update term</u> in NR.
- If x_0 and x^* are close, $\Delta\tilde{x}$ will be more accurate. (i.e. its value will be closer to the true Δx)

Steps: 1) Choose an initial value x_0 and set $k = 0$. (k keeps track of iteration #)

2) Solve for $\Delta\tilde{x}_k = -\frac{f(x_k)}{f'(x_k)}$

3) Let $x_{k+1} = x_k + \Delta\tilde{x}_k$

4) Check for convergence, i.e. $|f(x_{k+1})| < \varepsilon$ or $|\Delta\tilde{x}_k| < \varepsilon$

5) If converged, stop. Else, k++ and go to step 2.

NR for system of non-linear equations

$$\mathrm{F(X)} = \begin{bmatrix} f_1(X) \\ f_2(X) \\ \vdots \\ f_n(X) \end{bmatrix} \quad \text{where} \quad \mathrm{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Taylor Series expansion around an (N-dimensional) point X^0:

$$F(X) = F(X^0) + J(X^0).(X - X^0) + H.O.T.$$

Jacobian: n x n matrix of partial derivatives:

$$J(X) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

Same as before,

$$F(X^*) = F(X^0) + J(X^0).(X^* - X^0) + \cancel{H.O.T}$$

$$[0] \approx F(X^0) + J(X^0).(X^* - X^0)$$

$$J(X^0).\Delta\widetilde{X} = -F(X^0)$$

Steps:

- Symbolic evaluation of the Jacobian J(X) for a generic X

1) Choose an initial value X^0 and set $k = 0$.

2) Compute the Jacobian and the mismatch vector: Plug in X^k into $J(X)$ and $F(X)$

3) Solve the linear problem $J(X^k).\Delta\widetilde{X}^k = -F(X^k)$ to find $\Delta\widetilde{X}^k$

4) Let $X^{k+1} = X^k + \Delta\widetilde{X}^k$

5) Check for convergence, i.e. $\|F(X^{k+1})\| < \varepsilon$ or $\|\Delta\widetilde{X}^k\| < \varepsilon$

6) If converged, stop. Else, k++ and go to step 2.

Ex: Perform one iteration of NR to solve

$$F(X) = \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix} = \begin{bmatrix} x_1^2 + x_2^2 - 5x_1 + 1 \\ x_1^2 - x_2^2 - 3x_2 - 3 \end{bmatrix}$$

with initial guess of $X^\circ = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

$$J(x) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 2x_1 - 5 & 2x_2 \\ 2x_1 & -2x_2 - 3 \end{bmatrix}$$

1st iteration: $F(X^\circ) = \begin{bmatrix} 3^2 + 3^2 - 5\times 3 + 1 \\ 3^2 - 3^2 - 3\times 3 - 3 \end{bmatrix} = \begin{bmatrix} 4 \\ -12 \end{bmatrix}$

$$J(x^\circ) = \begin{bmatrix} 2\times 3 - 5 & 2\times 3 \\ 2\times 3 & -2\times 3 - 3 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 6 & -9 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 6 \\ 6 & -9 \end{bmatrix}}_{J(x^\circ)} \cdot \underbrace{\begin{bmatrix} \Delta\tilde{x}_1 \\ \Delta\tilde{x}_2 \end{bmatrix}}_{\Delta\tilde{X}^\circ} = - \underbrace{\begin{bmatrix} 4 \\ -12 \end{bmatrix}}_{F(x^\circ)}$$

$$\begin{cases} \Delta\tilde{x}_1 + 6\,\Delta\tilde{x}_2 = -4 \\ 6\,\Delta\tilde{x}_1 - 9\,\Delta\tilde{x}_2 = 12 \end{cases}$$

solving the above <u>linear</u> system of equations gives $\Delta\tilde{X}^\circ = \begin{bmatrix} 0.8 \\ -0.8 \end{bmatrix}$

$$X' = X^\circ + \Delta\tilde{X}^\circ = \begin{bmatrix} 3 \\ 3 \end{bmatrix} + \begin{bmatrix} 0.8 \\ -0.8 \end{bmatrix} = \begin{bmatrix} 3.8 \\ 2.2 \end{bmatrix}$$

check: $\| F(x') \| = \left\| \begin{bmatrix} 1.28 \\ 0 \end{bmatrix} \right\| = 1.28$

we can repeat the above steps for X' in the next iteration

- we now try to apply Newton-Raphson to solve the power flow problem

Jacobian for Power Flow Equations

$$\sum_{k=1}^{N} V_i V_k G[i,k] \cos(\delta_i - \delta_k) \dots$$

Find $J(x)$ for $F(x) = \begin{bmatrix} f_1(x) \\ \vdots \\ f_{2N}(x) \end{bmatrix} = \begin{bmatrix} P_{gen,i} - P_{load,i} - P_{branch,i} \\ \hline Q_{gen,i} - Q_{load,i} - Q_{branch,i} \end{bmatrix} \begin{matrix} \} P_{eq,i} \\ \} Q_{eq,i} \end{matrix}$

Let's form the Jacobian assuming all busses are type 0 (non-gen)

i.e. $x = \left[\delta_1 \ \dots \ \delta_N \,\middle|\, V_1 \ \dots \ V_N \right]^T$

we will then go back & correct this for other types of busses (PV & slack)

- Let's break up the Jacobian into 4 sub-matrices:

$$\underbrace{J(x)}_{2N \times 2N \text{ matrix}} = \left[\begin{array}{c|c} J_{11} & J_{12} \\ \hline J_{21} & J_{22} \end{array}\right]$$

$J_{11}, J_{12}, J_{21}, J_{22}$ are all $N \times N$ matrices

assuming all non-gen busses, then

$$J_{11} = \frac{\partial P_{eq}}{\partial \delta} \qquad J_{12} = \frac{\partial P_{eq}}{\partial V}$$

$$J_{21} = \frac{\partial Q_{eq}}{\partial \delta} \qquad J_{22} = \frac{\partial Q_{eq}}{\partial V}$$

. Let's build the generic equations for J_{11}.

$$P_{eq,i} = P_{gen,i} - P_{load,i} - \sum_{k=1}^{N} V_i V_k G[i,k] \cos(\delta_i - \delta_k) - \sum_{k=1}^{N} V_i V_k B[i,k] \sin(\delta_i - \delta_k)$$

. off-diagonal elements of J_{11}:

$$\boxed{\frac{\partial P_{eq,i}}{\partial \delta_k} = - V_i V_k G[i,k] \sin(\delta_i - \delta_k) + V_i V_k B[i,k] \cos(\delta_i - \delta_k)}$$

for $i \neq k$

Diagonal elements of J_{11}:

$$\boxed{\frac{\partial P_{eq,i}}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^{N} V_i V_k G[i,k] \sin(\delta_i - \delta_k) - \sum_{\substack{k=1 \\ k \neq i}} V_i V_k B[i,k] \cos(\delta_i - \delta_k)}$$

. Let's re-visit our 3-bus example system:

$$f_1 = P_{eqn,1} = P_{gen,1} - P_{load,1} - \sum_{k=1}^{3} V_1 V_k G[1,k] \cos(\delta_1 - \delta_k) - \sum_{k=1}^{3} V_1 V_k B[1,k] \sin(\delta_1 - \delta_k)$$

if all 3 busses are non-gen busses:

$$\text{row 1 of } J_{11} = \left[\underbrace{\frac{\partial f_1}{\partial \delta_1}}_{\text{diagonal}} \quad \frac{\partial f_1}{\partial \delta_2} \quad \frac{\partial f_1}{\partial \delta_3} \right]$$

off-diagonal

$$J_{11}[1,1] = \sum_{k=2}^{3} V_1 V_k G[1,k] \sin(\delta_1 - \delta_k) - \sum_{k=2}^{3} V_1 V_k B[1,k] \cos(\delta_1 - \delta_k)$$

$$J_{11}[1,2] = - V_1 V_2 G[1,2] \sin(\delta_1 - \delta_2) + V_1 V_2 B[1,2] \cos(\delta_1 - \delta_2)$$

- Note: in a large system, most of the off-diagonal terms in the Jacobian $= 0$ since $B[i,k] = G[i,k] = 0$ for $i \neq k$

Each bus is only directly connected to a few other busses.

Incorporating Gen & Slack busses into the Jacobian

- In J_{11}: partial derivatives of $P_{eqn,i}$ w.r.t the first N terms in x (top half of x). This set includes δ for all busses except the slack bus

$$x = \left[\delta_1 \quad \cdots \quad \delta_N \;\vdots\; V_1 \quad \cdots \quad V_N \right]^T$$

for slack bus, replace δ_{sb} with $P_{gen,sb}$

- Already formed J_{11} assuming all non-gen busses, i.e. took the $\frac{\partial P_{eqn}}{\partial \delta}$ partial derivative

Now, correct J_{11} to account for $P_{gen,sb}$ being in top half of x:

$$x = \left[\delta_1 \cdots \quad P_{gen,sb} \cdots \quad \delta_N \;\vdots\; \cdots \right]$$

$$J_{11} = \begin{bmatrix} \frac{\partial f_1}{\partial \delta_1} & \cdots & \frac{\partial f_1}{\partial P_{gen,sb}} & \cdots & \frac{\partial f_1}{\partial \delta_N} \\ \frac{\partial f_2}{\partial \delta_1} & \cdots & \frac{\partial f_2}{\partial P_{gen,sb}} & \cdots & \frac{\partial f_2}{\partial \delta_N} \\ \vdots & & & & \vdots \\ \frac{\partial f_N}{\partial \delta_1} & \cdots & \underbrace{\frac{\partial f_N}{\partial P_{gen,sb}}}_{} & \cdots & \frac{\partial f_N}{\partial \delta_N} \end{bmatrix}$$

this column replaces $\frac{\partial f_i}{\partial \delta_{sb}}$

- If slack bus is bus #7 for example, we will have to correct column 7 in J_{11} since it was formed by using $\frac{\partial f_i}{\partial \delta_7}$ but δ_7 is not an unknown; we need $\frac{\partial f_i}{\partial P_{gen,7}}$

Let's divide f_i into 2 sets:

$$f_i = P_{gen,i} - P_{load,i} - P_{branch,i} \qquad \text{for } i = 1, \dots, N,\ i \neq sb$$

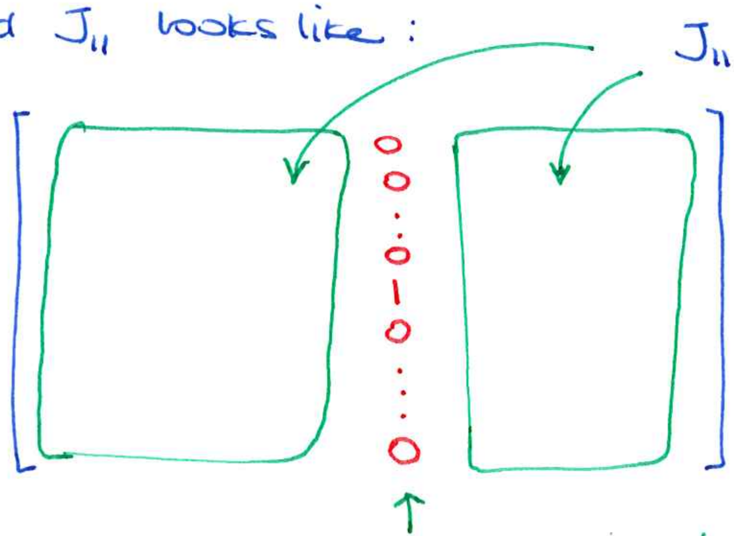
then $\frac{\partial f_i}{\partial P_{gen,sb}} = 0$

$$f_{sb} = P_{gen,sb} - P_{load,sb} - P_{branch,sb}$$

then $\frac{\partial f_{sb}}{\partial P_{gen,sb}} = 1$

- Corrected J_{11} looks like:

J_{11} developed according to the equations derived previously



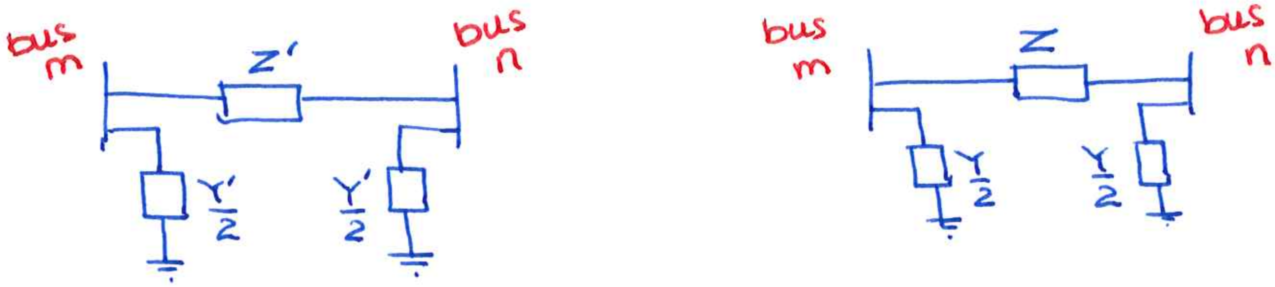
column corresponding to slack bus #

$$J_{11}[sb, sb] = 1$$

Homework: Repeat this for J_{12}, J_{21}, J_{22}.

Incorporating shunt branches into power flow

- Recall nominal π or equivalent π models:



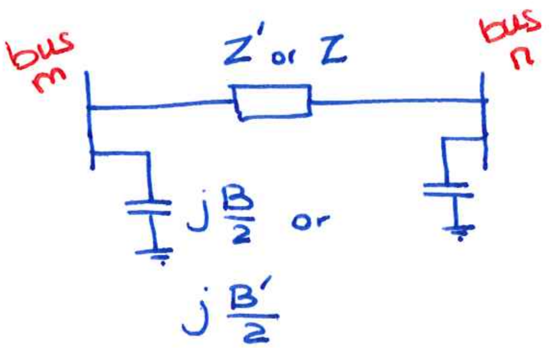
Equivalent π   Nominal π

- For both $\frac{Y}{2}$ and $\frac{Y'}{2}$, we ignored G

$$\frac{Y}{2} \text{ or } \frac{Y'}{2} = \cancel{G} + j\frac{B}{2} \text{ or } j\frac{B'}{2}$$

usually neglected in transmission lines

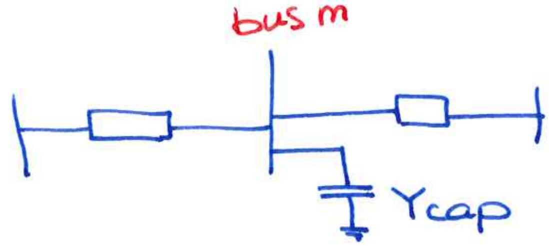
to update Y_{bus} to account for shunt branches:



Add $\frac{B}{2}$ or $\frac{B'}{2}$ to diagonal element $B_{bus}[m,m]$ & $B_{bus}[n,n]$

Reminder: Diagonal elements of Y_{bus} are defined as Σ of all admittances connected to that bus.

- Similarly, for a shunt capacitor connected to a bus (e.g for power factor correction):



Add Y_{cap} to $Y_{bus}[m,m]$

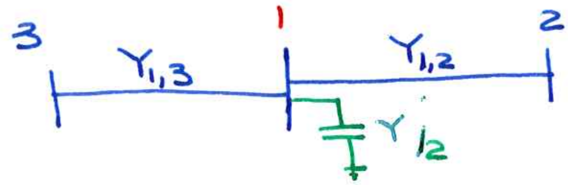
. why can we do this?

we derived the expression for $\overline{S}_{branch,i} = \sum_{k=1}^{n} Y^{*}_{bus}[i,k] \cdot \overline{V}_i \overline{V}_k^{*}$

where $Y_{bus}[i,k] \triangleq -Y_{i,k}$ for $i \neq k$

$Y_{bus}[i,i] \triangleq \sum$ all admittances connected to bus i

let's apply this to a 3-bus system:



$$\overline{S}_{branch,1} = \overline{V}_1 (\overline{V}_1 - \overline{V}_2)^{*} Y_{1,2}^{*} + \overline{V}_1 (\overline{V}_1 - \overline{V}_3)^{*} \cdot Y_{1,3}^{*} + \overline{V}_1 (\overline{V}_1 - 0)^{*} \left(\frac{Y}{2}\right)^{*}$$

$$= \underbrace{\left(Y_{1,2} + Y_{1,3} + \frac{Y}{2}\right)}^{*} \cdot \overline{V}_1 \overline{V}_1^{*}$$

this should be $Y_{bus}[1,1]$

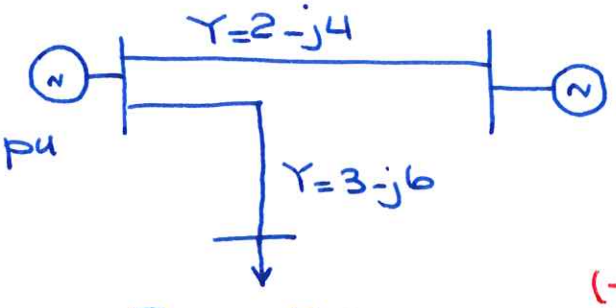
$$+ (-Y_{1,2})^{*} \overline{V}_1 \overline{V}_2^{*}$$

i.e justifies adding $\frac{Y}{2}$ to diagonal element of Y_{bus}

$$+ (-Y_{1,3})^{*} \overline{V}_1 \overline{V}_3^{*}$$

this is the expanded format of $\overline{S}_{branch}$ equation from above

Problem 6.28



$\overline{V}_1 = 1\angle 0^\circ$ pu

$P_2 = 1.5$ pu

$V_2 = 1.1$ pu

(+) indicates $P_{gen,2} = 1.5$ pu

(−) indicates $P_{load,3} = 1.5$ pu $\rightarrow P_3 = -1.5$ pu

$\rightarrow Q_3 = +0.8$ pu

(+) indicates $Q_{load,3} = -0.8$ pu (ie. capacitive load)

- A note about P & Q values on the SLD: textbook use positive P & Q values if real/reactive power is generated at the bus, negative values if consumed.

$$P_{textbook} = P_{gen} - P_{load}, \quad Q_{textbook} = Q_{gen} - Q_{load}$$

Solution:

Bus 1: slack/swing

Bus 2: Gen/PV

Bus 3: Non-gen/load

$$Y_{bus} = \begin{bmatrix} 5-j10 & -2+j4 & -3+j6 \\ -2+j4 & 2-j4 & 0 \\ -3+j6 & 0 & 3-j6 \end{bmatrix}$$

$$f_2 = P_{eqn,2} = P_{gen,2} - P_{load,2} - \sum_{k=1}^{3} V_2 V_k G[2,k] \cos(\delta_2 - \delta_k)$$
$$- \sum_{k=1}^{3} V_2 V_k B[2,k] \sin(\delta_2 - \delta_k)$$

$$= 1.5 - 0 - \left(V_2 V_1 G[2,1] \cos(\delta_2 - \delta_1) + V_2 \cdot V_2 G[2,2] \cos(\delta_2 - \delta_2) \right)$$
$$- \left(V_2 V_1 B[2,1] \sin(\delta_2 - \delta_1) + V_2 \cdot V_2 B[2,2] \sin(\delta_2 - \delta_2) \right)$$

$$= 1.5 - (1.1)(1)(-2)\cos(\delta_2 - 0) - (1.1)^2(2)\cos(0)$$
$$- (1.1)(1)(4)\sin(\delta_2 - 0) - (1.1)^2(-4)\cancel{\sin(0)}^{\,0}$$

$$= 1.5 + 2.2\cos(\delta_2) - 2.42 - 4.4\sin(\delta_2)$$

it would be easier to solve for δ_2 if we used polar format. See posted solution on D2L for Set 4.

- The posted solution derives an $\overline{S}_{branch}$ equation involving $Y_{1,2}$ expressed in polar coordinates.
- How does this match with the expression from the notes:

$$P_{gen,i} - P_{load,i} - \sum_{k=1}^{N} \underbrace{|Y_{i,k}|}_{|Y_{bus}[i,k]|} V_i V_k \cos(\delta_i - \delta_k - \underbrace{\theta_{i,k}}_{\text{phase of } Y_{bus}[i,k]})$$

$$Y_{bus} = \begin{bmatrix} \cdots & & \\ -2+j4 & 2-j4 & 0 \\ \cdots & & \end{bmatrix}$$

$-2+j4 = 4.47\angle 116.57^\circ$ $\qquad$ $2-j4 = 4.47\angle -63.43^\circ$

$$f_2 = P_{gen,2} - P_{load,2} - 4.47\, V_2 \cdot V_1 \cos(\delta_2 - \delta_1 - 116.57^\circ) - 4.47\, V_2 \cdot V_2 \cdot \cos(\delta_2 - \delta_2 - (-63.43^\circ))$$

$$= 1.5 - \underbrace{4.47(1.1)(1)\cos(\delta_2 - 0 - 116.57^\circ)} - 4.47(1.1)^2\cos(63.43^\circ)$$

we can use trig with this term:

$$+\,4.47(1.1)(1)\cos(\delta_2 - 116.57^\circ + 180^\circ)$$

which now matches the posted solution

- Let's form the X vector & a few Jacobian terms for system in problem 6.28:

$$x = \left[P_{gen,1} \quad \delta_2 \quad \delta_3 \;\vdots\; Q_{gen,1} \quad Q_{gen,2} \quad V_3 \right]^T$$

- what is the expression for $\underbrace{J_{11}}[2,2]$?

partial derivatives of 1st half of mismatch vector w.r.t. 1st half of x

i.e. $\dfrac{\partial \, P_{ean}}{\partial \text{ top half of } x}$

$$\therefore \; J_{11}[2,2] = \frac{\partial P_{eqn,2}}{\partial \delta_2} = -2.2\sin(\delta_2) - 4.4\cos(\delta_2)$$

2nd element in top half of x

- what is $J_{11}[2,3]$?

$$J_{11}[2,3] = \frac{\partial P_{ean,2}}{\partial \delta_3} = 0$$

3rd element in top half of x

- what is $J_{22}[2,3]$?

partial derivatives of 2nd half of mismatch vector (i.e. Q_{ean})

w.r.t 2nd half of x vector

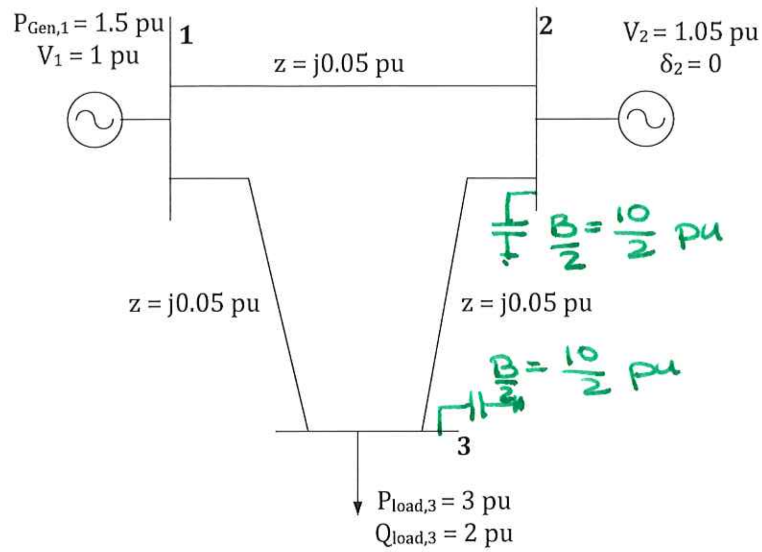
$$J_{22}[2,3] = \frac{\partial Q_{ean,2}}{\partial V_3}$$

3rd element in bottom half of x.

ENEL 487 | Winter 2018 | Quiz 3

Name: ID:

1) Consider the system below:



a. Fill out the following table for this system. The answer(s) should be selected from the terms in the brackets. [3 marks]

Bus Number	Bus Type (Load, Gen, Slack)	Known Variables (P_{gen}, Q_{gen}, V, δ, P_{load}, Q_{load})	Unknown variables (P_{gen}, Q_{gen}, V, δ, P_{load}, Q_{load})
1	Gen		
2	Slack		
3	Non-gen		

b. Create the X vector for this system [3 marks] $[\delta_1 \quad P_{gen,2} \quad \delta_3 \quad Q_{gen,1} \quad Q_{gen2} \quad V_3]$

c. For the transmission line between bus 2 and bus 3, a nominal π model will be used with a line susceptance of B = 10 pu. Create the admittance matrix Y_{bus} for this system. Also, provide B_{bus} and G_{bus} matrices. [4 marks]

d. Write and simplify the active (real) power flow equation at bus 1. Use the power flow equation with Y_{bus} expressed in rectangular coordinates. Plug in all the known variables, expand the equations, keep the angles in degrees, and simplify as much as possible. [3 marks] $1.5 - 21\sin(\delta_1) - 20 V_3 \sin(\delta_1 - \delta_3)$

e. If during one iteration of the Newton-Raphson method, $\delta_1 = 90^\circ$, $\delta_3 = 0^\circ$, and $V_3 = 1.1$ pu. Calculate the value of the first element in the mismatch vector. [2 marks]

$f_1(x) = -41.5$

$$Y_{bus} = \begin{bmatrix} -j40 & j20 & j20 \\ j20 & -j35 & j20 \\ j20 & j20 & -j35 \end{bmatrix}$$

2018 Quiz 3 Problem 2

update term

mismatch vector

$$J(x^0) \cdot \Delta \tilde{x} = -F(x^0)$$

2) For the system of equations:

$f_1(\mathbf{X}) = 10\, x_1 \sin x_2 + 2$

$f_2(\mathbf{X}) = 10\, (x_1)^2 - 10\, x_1 \cos x_2 + 1$

Using the Newton-Raphson method, determine the values of x_1 and x_2 after the first iteration. Use $\underbrace{x_1 = 1, x_2 = 0}$ as an initial guess. [5 marks]

$$x^0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

k

2018 Quiz 3

2) $$J(x) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 10\sin(x_2) & 10x_1 \cdot \cos(x_2) \\ 20x_1 - 10\cos(x_2) & 10x_1 \cdot \sin(x_2) \end{bmatrix}$$

$$F(x^0) = \begin{bmatrix} f_1(x^0) \\ f_2(x^0) \end{bmatrix} = \begin{bmatrix} 10(1)\cdot\sin(0) + 2 \\ 10(1)^2 - 10(1)\cos(0) + 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$J(x^0) = \begin{bmatrix} 10\sin(0) & 10(1)\cdot\cos(0) \\ 20(1) - 10\cos(0) & 10(1)\cdot\sin(0) \end{bmatrix} = \begin{bmatrix} 0 & 10 \\ 10 & 0 \end{bmatrix}$$

$$J(x^0)\cdot\Delta\tilde{x} = -F(x^0) \therefore \begin{bmatrix} 0 & 10 \\ 10 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} \Delta\tilde{x}_1 \\ \Delta\tilde{x}_2 \end{bmatrix}}_{\Delta\tilde{x}} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

$$\begin{cases} 10\cdot\Delta\tilde{x}_2 = -2 \\ 10\cdot\Delta\tilde{x}_1 = -1 \end{cases} \qquad \therefore \Delta\tilde{x} = \begin{bmatrix} -0.1 \\ -0.2 \end{bmatrix}$$

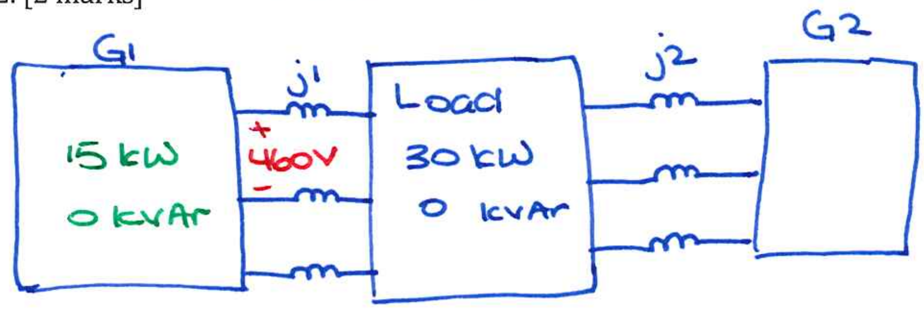
$$x^1 = x^0 + \Delta\tilde{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.1 \\ -0.2 \end{bmatrix} = \begin{bmatrix} 0.9 \\ -0.2 \end{bmatrix}$$

Problem 3:

Two three-phase generators supply a three-phase load through separate three-phase lines. The load absorbs a total of 30 kW at unity (1.0) power factor. The line impedance is 0+j1 Ω per phase between generator G1 and the load, and 0+j2 Ω per phase between generator G2 and the load. If generator G1 supplies 15 kW (three-phase) at unity power factor with a terminal voltage of $\underbrace{460\text{ V}}_{}$ → Line to line, Determine:

a) The real power supplied by generator G2. [2 marks]

Load requires 30 kW of real power. No real power loss in lines



$$P_{G2} = P_{load} - P_{G1} = 15 \text{ kW}$$

b) Magnitude of the line-to-line voltage at the load terminals. [3 marks]

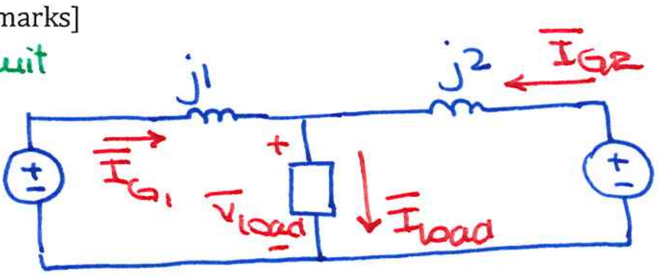
per-phase circuit

$$\bar{S}_{G1,1\phi} = 5 + j0 \text{ kVA}$$

$\dfrac{460}{\sqrt{3}} \angle 0°$

L-N voltage required for 460 V L-L

assigned



$$\bar{S}_{G1,1\phi} = \bar{V}_{G1,LN} \cdot \bar{I}_{G1}^{*} \quad \therefore \quad \bar{I}_{G1} = \frac{5000 \angle 0°}{\frac{460}{\sqrt{3}} \angle 0°} = 18.82 \angle 0° \text{ A}$$

$$\bar{V}_{load} = \bar{V}_{G1,LN} - (\bar{I}_{G1} \cdot j1) = 266.2 \angle -4.05° \text{ V}$$

$$|V_{load,LL}| = |\bar{V}_{load}| \times \sqrt{3} = 461.1 \text{ V}$$

c) The reactive power supplied by generator G2. [5 mark]

use $\bar{S}_{load,1\phi} = \bar{V}_{load} \cdot \bar{I}_{load}^{*}$ to find $\bar{I}_{load}$

$$\bar{I}_{G2} = \bar{I}_{load} - \bar{I}_{G1}$$

$$Q_{G2} = Q_{loss} \text{ on branches} = \underbrace{I_{G1}^2 \cdot (1) + I_{G2}^2 \cdot (2)}_{Q = I^2 \cdot X}$$

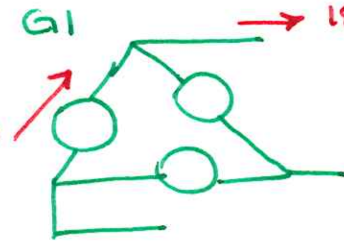
Notes about Problem 3, 2017 Final

- we can use per unit here but there is no real advantage as there are no transformers.

- **Phase** current of G1?
  - Y-connected G1 : $\overline{I}_{\phi G1} = 18.82 \angle 0^\circ$
  - Δ-connected G1 : $\overline{I}_{\phi G1} = \dfrac{18.82 \angle +30^\circ}{\sqrt{3}}$

$\overline{I}_{G1}$ is the line current coming out of G1

$\overline{I}_{\phi}$ $\rightarrow$ 18.82



- **phase** voltage of load
  - Y-connected : $\overline{V}_{\phi load} = 266.2 \angle -4.05^\circ$
  - Δ-connected : $\overline{V}_{\phi load} = \overline{V}_{load}\left(\sqrt{3} \angle 30^\circ\right)$

$\overline{V}_{load}$ is L-N

L-L for Δ connection