ENEL 476 W2018 A#2 (1)

- free space
  - ↳ $\epsilon_r = 1$
  - ↳ $\mu_r = 1$
  - ↳ $\sigma = 0$

$$\vec{E}(x,t) = 10\cos(3\times10^8 t + \beta_0 x)\vec{a}_y$$

a) $\beta_0 = \omega\sqrt{\mu_0\epsilon_0}$

$$= \frac{3\times10^8}{3\times10^8}$$

$$\boxed{\beta_0 = 1 \text{ rad/m}}$$

b) $\lambda_0 = 2\pi/\beta_0$

$$= 2\pi/1$$

$$\boxed{\lambda_0 = 2\pi \text{ m}}$$

c) $v_p = \omega/\beta_0$

$$= \frac{3\times10^8}{1}$$

$$\boxed{v_p = 3\times10^8 \text{ m/s}}$$

d) $\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$

$$\boxed{\eta_0 = 120\pi\ \Omega}$$

e) $\vec{E}$ is in $\vec{a}_y$ ⇒ varies between $+10\vec{a}_y$ & $-10\vec{a}_y$ ⇒ $\boxed{\text{LINEAR}}$

f) $\vec{H}_s(x) \Rightarrow \vec{E}_s(x) = 10e^{jx}\,\vec{a}_y$

$$\vec{H}_s(x) = \frac{-10}{120\pi}e^{jx}\,\vec{a}_z$$

$$\boxed{\vec{H}_s(x) = -\frac{1}{12\pi}e^{jx}\,\vec{a}_z \text{ A/m}}$$

↳ shale

$\epsilon_r = 25$

$\mu_r = 1$

$\sigma = 0.25$ S/m

g) $\beta = ?$

$$\frac{\sigma}{\omega\epsilon} = \frac{0.25}{(\cancel{3}\times\cancel{10^8})(\cancel{25})\left(\frac{1}{\cancel{36}\pi}\times\cancel{10^{-9}}\right)}$$

$$= \frac{12\pi}{10}$$

$$= 1.2\pi$$

1 $= 3.77 \Rightarrow$ use full formulas

h) $$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+(\sigma/\omega\epsilon)^2}-1\right]}$$

$$= 3\times10^8\sqrt{\frac{\mu_0 25\epsilon_0}{2}\left[\sqrt{1+(3.77)^2}-1\right]}$$

$$= \frac{3\times10^8}{3\times10^8}\sqrt{\frac{25}{2}\left[\sqrt{15.21}-1\right]}$$

1 $\boxed{\alpha = 6.02 \text{ Np/m}}$

g) $\beta = \omega \sqrt{\left(\frac{\mu\epsilon}{2}\right)\left[\sqrt{1+(\sigma/\omega\epsilon)^2}+1\right]}$

$= \frac{3\times10^8}{3\times10^8}\sqrt{(25/2)\left[\sqrt{15.21}+1\right]}$

(1) $\boxed{\beta = 7.83 \text{ rad/m}}$

i). $\delta = \frac{1}{\alpha}$

(1) $= 0.166 \text{ m}$

j) $|\eta| = \frac{\sqrt{\mu/\epsilon}}{\left[1+(\sigma/\omega\epsilon)^2\right]^{1/4}}$

$= \frac{\sqrt{\mu_0/25\epsilon_0}}{\left[1+(3.77)^2\right]^{1/4}}$

$= \frac{\frac{120\pi}{5}}{(15.21)^{1/4}}$

(1) $\boxed{|\eta| = 38.18\ \Omega}$

$\measuredangle\eta = \frac{1}{2}\tan^{-1}(\sigma/\omega\epsilon)$

$= \frac{1}{2}\tan^{-1}(3.77)$

(1) $\boxed{\measuredangle\eta = 0.656 \text{ rad}}$

k) $\dfrac{\sigma}{\omega\epsilon} > 10$ (1)

$$\frac{0.25}{\omega(25)\epsilon_0} > 10$$

$$\frac{0.25}{\omega} > 250\epsilon_0$$

$$\frac{1}{\omega} > \frac{\overset{1000}{250\epsilon_0}}{\cancel{0.25}}$$

$$\omega < \frac{1}{1000\epsilon_0}$$

$$\omega < 1.13\times10^{8}$$

(1) $\therefore$ $\underline{f < 1.8\times10^{7}\ Hz}$ for a good conductor

l) $\vec{E}(x,t) = 1.86e^{6.02x}\cos(3\times10^{8}t + 7.83x)\vec{a}_y$ V/m

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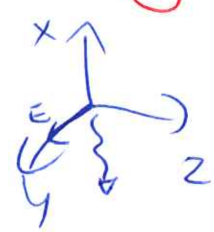
m) $\vec{H}(x,t) = -\dfrac{1.86}{38.18}e^{6.02x}\cos(3\times10^{8}t + 7.83x - 0.656)\vec{a}_z$ A/m

$-\vec{a}_z$ (1) (1) (1)



n) $\vec{P}_{AV}(x) = -\dfrac{|\vec{E}|^2}{2|\eta|}e^{2\alpha x}\cos(\theta_\eta)\vec{a}_x$

$$= -\frac{(1.86)^2}{2(38.18)}e^{2(6.02)x}\cos(0.656)\vec{a}_x$$

$\vec{P}_{AV}(x) = -0.036e^{+12.04x}\vec{a}_x$ W/m²

(1) (1) (1) $-\vec{a}_x$

(1) decreases by 8.08%

$\vec{P}_{AV}(x=0) = -0.036\vec{a}_x$

$\vec{P}_{AV}(x = 7mm) = -0.036(e^{12.04(0.007)})\vec{a}_x \Rightarrow (0.036)(0.919)$