University of Calgary

Schulich School of Engineering

Department of Electrical and Computer Engineering

ENEL 476 – Electromagnetic Waves and Applications

Midterm Examination

Winter Session 2018
Tuesday February 27, 2017
12:30-1:45 pm

ENE 243

Student Name or ID number:

Dr. Fear

c) $\Phi = \int \vec{B} \cdot d\vec{s}$

(1) $= \cancel{\vec{B} \int d\vec{s}} = \int_0^{0.2}\int_0^{0.1} (B_z)\, \vec{a}_z \cdot dx\,dy\, \vec{a}_z$

$= (\vec{B}_z) \int_0^{0.1}\int_0^{0.2} dx\,dy$ ↳ constant w.r.t. space

$0.1 \cos 10^3 t \times 10^{-3} = B(0.1)(0.2)$

(1) divide by area $|\vec{B}| = \frac{\cancel{0.1}}{(\cancel{0.1})(0.2)} \times 10^{-3} \cos 10^3 t$

$= 5 \cos 10^3 t$ mWb/m²

↳ direction ⇒ $\vec{a}_z$ (same as $d\vec{s}$)

(1) $\vec{B} = 5 \cos 10^3 t\, \vec{a}_z$ mWb/m²

(½ if not including direction)

d) No, $\vec{B} \cdot d\vec{s}$ ⇒ could have components of $\vec{B}$ in x or y direction (1)

(1) OR constant value

(e.g. $5\cos 10^3 t + 10$)

⇒ this would give same resulting induced current

e) Lenz's law: induced flux associated with induced current opposes change in original flux (1)

↳ for $\vec{B} = 5\cos(10^3 t)\, \vec{a}_z$ mWb/m², the flux density

(1) is decreasing in the 1ST $T/4$ $(0 < t < T/4)$.

↳ the induced current flows in the counter-clockwise direction & has an associated induced flux in

(1) the +z direction

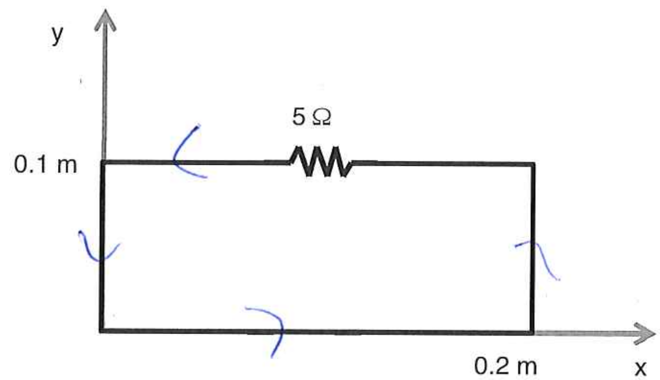
⇒ this opposes the decrease in the original flux

Question 1. **(12 marks)**

Consider the rectangular loop of wire shown in the figure below. The loop contains a 5 Ω resistor. The loop is placed in an external magnetic flux density (**B**) that changes with time but does not vary with location (i.e. is constant in space).

The induced current flows counter-clockwise in the loop and is described as:

$$I(t) = 0.02 \sin(10^3 t) \text{ A}$$



a) Find the corresponding EMF (1 mark).

b) Find the total flux (Φ) through the loop (3 marks).

c) Find an expression for the magnetic flux density (**B**) (3 marks).

d) Is this expression for magnetic flux density uniquely related to the induced current (i.e. is this the only magnetic flux density capable of generating the induced current specified above)? (2 marks)

e) State Lenz's law. Consider the first quarter period (0<t<T/4). Explain how Lenz's law is upheld for your magnetic flux density and the induced current. (3 marks)

a) $V = IR \Rightarrow \text{EMF} = (0.02\sin(10^3 t))(5)$

$= 0.1 \sin 10^3 t \text{ V}$ (1)

b) $\phi \Rightarrow \text{EMF} = -\frac{d\phi}{dt}$ (1)

$\phi = \int -\text{EMF}\, dt$

$= \frac{0.1}{10^3} \cos 10^3 t$ (1) integrate w.r.t. time

$\phi = 0.1 \cos 10^3 t \text{ mWb}$ (1)

Question 2. (21 marks)

Ground penetrating radar (GPR) may be used to assist with archeological exploration. Consider modeling GPR exploration with a uniform plane wave propagating in dry and sandy soil. This soil has ε_r=2.5, σ=0.5 mS/m and μ_r=1. The direction of propagation of the wave is -z, and the electric field is oriented in +y. The frequency is 300 MHz.

a) Calculate the attenuation constant (α) (3 marks)

b) Calculate the phase constant (β) (2 marks)

c) Calculate the wavelength (λ) (2 marks)

d) Calculate the skin depth (δ) (2 marks)

e) Calculate the intrinsic impedance of the medium (η) (3 marks)

f) If the electric field has amplitude of 1 kV/m at z=0 m, find an expression for the electric field in the time domain (**E**(z,t)) (3 marks)

g) Find an expression for the magnetic field in the time domain (**H**(z,t)) (2 marks)

h) Calculate the time-averaged Poynting vector ($\mathbf{P}_{av}(z)$) (2 marks)

i) How much does the time-averaged power density decrease after the wave propagates a distance of 1 m in soil? (2 marks)

a) $\dfrac{\sigma}{\omega\varepsilon} = \dfrac{0.5\times10^{-3}}{(2\pi\times 3\times10^{8})(2.5)(\frac{1}{36\pi}\times10^{-9})}$

$= \dfrac{60\times10^{-3}}{5}$

(1) $= 0.012$ ⇒ not a good conductor ∴ use full formulas

(1) formula

$\alpha = 2\pi\times3\times10^{8}\sqrt{\dfrac{\mu_0\varepsilon_0 2.5}{2}\left(\sqrt{1+(0.012)^2}-1\right)}$

$= \dfrac{2\pi\times3\times10^{8}}{3\times10^{8}}\sqrt{\left(\dfrac{2.5}{2}\right)\left(\sqrt{1+(0.012)^2}-1\right)}$

(1) $= 0.0596$ Np/m

(1) formula

b) $\beta = 2\pi\sqrt{\left(\dfrac{2.5}{2}\right)\left(\sqrt{1+(0.012)^2}+1\right)}$

$= 9.95$ rad/m

(1)

c) $\dfrac{2\pi}{\beta} = \lambda$

(1) $\lambda = \dfrac{2\pi}{9.95}$

(1) $= 0.63$ m

d) $\delta = \dfrac{1}{\alpha}$

(1)

(1) $= 1.85$m

16.78 m

e) $\eta = \dfrac{\sqrt{\mu_0/2.5\varepsilon_0}}{\left[1+(0.012)^2\right]^{1/4}}$

(1/2) formula

(1) $= 238.43\,\Omega$

(1/2) formula $\tan 2\theta_n = \sigma/\omega\varepsilon$

(1) $\theta_n = 0.343°$

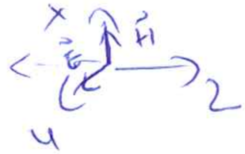
$= 0.006$ Rad

f) $\vec{E}(z,t) = 1000 e^{0.0596z} \cos(6\pi\times10^{8}t + 9.95z)\vec{a}_y$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

g) $\vec{H}(z,t) = \dfrac{1000}{238.43} e^{0.0596z} \cos(6\pi\times10^{8}t + 9.95z - 0.006)\vec{a}_x$

$\frac{1}{2}$ $\frac{1}{2}$ 1



h) $\vec{P}_{AV}(z) = -\dfrac{|E|^2}{2|\eta|} e^{2\alpha z} \cos(\theta_n)\vec{a}_z$

2097.05 $= \left(-\dfrac{(1000)^2}{2(238.43)}\right) e^{0.1192z} \cos(0.006)\vec{a}_z$

1 $\frac{1}{2}$ $\frac{1}{2}$

$\vec{P}_{av}(z) = -2097.01 e^{0.1192z} a_z$ amplitude attenuation $-\vec{a}_z$ dir'n

i) $\vec{P}_{AV}(z=-1) = \vec{P}_{AV}(z=0) e^{-0.1192}$

1 evaluate $\vec{P}_{AV}(z=-1)$ $= 0.8876 \vec{P}_{AV}(z=0)$

$\therefore$ power decreases by 11.24%

1

3

Question 3. (8 marks).

a) Consider a planar interface located at z=0. The region z<0 is free space (ε_r=1, σ=0, μ_r=1), while the region z>0 is a perfect dielectric with ε_r=25, σ=0, μ_r=1.

i) Calculate the reflection coefficient (Γ) (2 marks)

assume: ii) Calculate the transmission coefficient (T) (2 marks)

① $\varepsilon_r=1$, $\sigma=0$, $\mu_r=1$ $z=0$ ② $\varepsilon_r=25$, $\sigma=0$, $\mu_r=1$

(1) regions defined

i) $\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}$ ($\frac{1}{2}$) $\eta_1 = 120\pi$ ($\frac{1}{2}$) $\eta_2 = \sqrt{\frac{\mu_0}{25\varepsilon_0}} = 24\pi$

$= \frac{24\pi - 120\pi}{24\pi + 120\pi}$

ii) (1) $= -0.6667$

($\frac{1}{2}$) $T = 1+\Gamma$

($\frac{1}{2}$) $= 0.33$

b) An electric field is given by:

$$\mathbf{E}(z,t) = 5\sin(3\text{x}10^9 t - 20z)\mathbf{a}_x + 10\cos(3\text{x}10^9 t - 20z)\mathbf{a}_y$$

iii) What is the type of polarization of this field? (1 mark)

iv) What is the corresponding displacement current density? Assume that the material is lossless (σ=0) and has μ_r=1. (3 marks)

iii) elliptical polarization

iv) $\vec{D} = \varepsilon_r\varepsilon_0 \frac{d\vec{E}}{dt}$ ($\frac{1}{2}$)

$\varepsilon_r = ? \Rightarrow \beta = 20$

($\frac{1}{2}$) $= \omega\sqrt{\mu_0\varepsilon_r\varepsilon_0}$

$20 = \frac{\overset{10}{3\text{x}10^9}}{3\text{x}10^8}\sqrt{\varepsilon_r}$

$2 = \sqrt{\varepsilon_r}$

$\varepsilon_r = 4$ ($\frac{1}{2}$)

($\frac{1}{2}$) $\frac{d}{\partial t}$

$\vec{\dot{D}} = 4\varepsilon_0 \, 3\text{x}10^9 \left[5\cos(3\text{x}10^9 t - 20z)\vec{a}_x - 10\sin(3\text{x}10^9 t - 20z)\vec{a}_y\right]$

$= (12\text{x}10^9)(\frac{1}{36\pi}\text{x}10^{-9})\left[5\cos(3\text{x}10^9 t - 20z)\vec{a}_x - 10\sin(3\text{x}10^9 t - 20z)\vec{a}_y\right]$

$= \frac{5}{3\pi}\cos(3\text{x}10^9 t - 20z)\vec{a}_x - \frac{10}{3\pi}\sin(3\text{x}10^9 t - 20z)\vec{a}_y$

($\frac{1}{2}$)   ($\frac{1}{2}$)