Time-varying fields

Ex 11 $\vec{E} = \frac{50}{\boxed{\rho}} \cos(10^8 t \boxed{-kz})\vec{a}_\rho$

$\nabla \times \vec{E}_s = -j\omega\mu \vec{H}_s$

↑ cylindrical co-ords

$\vec{H}_s = \frac{50k}{10^8 \mu_0 \rho} e^{-jkz} \vec{a}_\phi$

$\nabla \times \vec{H}_s = j\omega\epsilon_0 \vec{E}_s \Rightarrow \nabla \times \vec{H}_s = \frac{1}{\rho}\left(-\frac{\partial}{\partial z}\rho H_\phi \vec{a}_\rho + \frac{\partial}{\partial \rho}\rho H_\phi \vec{a}_z\right)$

$\rightarrow 0$

$\nabla \times \vec{H}_s = \frac{\cancel{j}k^2 50}{10^8 \mu_0 \cancel{\rho}} \cancel{e^{-jkz}} \vec{a}_\rho$

$j\omega\epsilon_0 \vec{E}_s = \cancel{j}\omega\epsilon_0 \left(\frac{50}{\cancel{\rho}} \cancel{e^{-jkz}}\right)\vec{a}_\rho$

$\Rightarrow \frac{k^2}{10^8 \mu_0} = \omega\epsilon_0$

$k^2 = (10^8)^2 \epsilon_0 \mu_0$

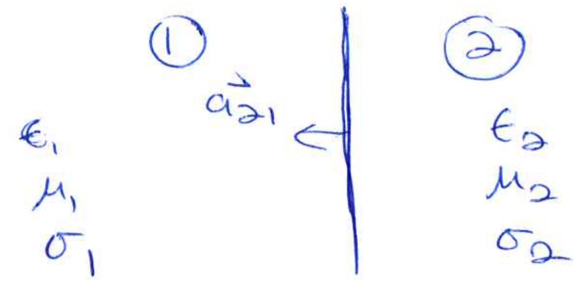
$\Downarrow$

$\frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3\times10^8$

$k^2 = \left(\frac{10^8}{3\times10^8}\right)^2$

$k = \frac{1}{3}$

Boundary conditions



$\vec{a}_{21} \times (\vec{E}_1 - \vec{E}_2) = 0$

$\vec{a}_{21} \times (\vec{H}_1 - \vec{H}_2) = \vec{K}$

$\vec{a}_{21} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$

$\vec{a}_{21} \cdot (\vec{B}_1 - \vec{B}_2) = 0$

→ perfect electric conductor ($\sigma \to \infty$); assuming $\vec{J}$ is finite

(1) | (2) PEC → $\vec{E}_2 = 0$ in conductor

+ $\vec{H}_2 = 0$ in conductor

⇒ current flows in extremely thin layer on surface ⇒ $\vec{E}_{1t} = 0$ $\quad \vec{D}_{1N} = \rho_s$

⇒ $\vec{B}_{1N} = 0$ $\quad \vec{a}_{21} \times \vec{H}_1 = \vec{K}$

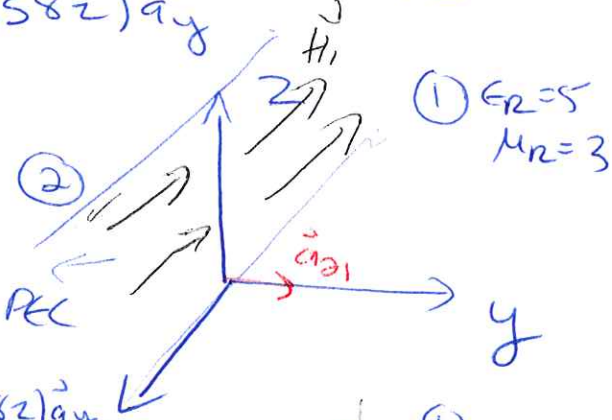
$(\vec{a}_{21} \cdot \vec{B}_1 = 0)$

Ex// $y = 0$ is a PEC (2) and $y > 0$ has $\epsilon_R = 5$, $\mu_R = 3$, $\sigma = 0$

• $\vec{E}_1 = 20\cos(2\times10^8 t - 2.58z)\vec{a}_y$

• $\vec{E}_{1N}$, $\vec{E}_{1t}$, ρ_s, $\vec{H}_1$, $\vec{K}$

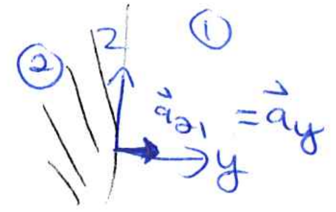
$\vec{E}_{1N} \to \vec{E}_1$, $\quad \vec{E}_{1t} \to 0$



$\rho_s = \vec{a}_{21} \cdot \vec{D}_1$

$= \underbrace{\vec{a}_y}_{} \cdot (5\epsilon_0)(20\cos(2\times10^8 t - 2.58z)\underbrace{\vec{a}_y}_{\times})$

$= 8.85\times10^{-10}\cos(2\times10^8 t - 2.58z)$



$$\nabla \times \vec{E}_s = -j\omega\mu \vec{H}_s$$

$$\vec{H}_1 = -6.84 \times 10^{-2} \cos(2\times10^8 t - 2.582)\vec{a}_x$$

$$\vec{a}_{21} \times \vec{H}_1 = \vec{K}$$

$$\Rightarrow \vec{K} = 6.84\times10^{-2}\cos(2\times10^8 t - 2.582)\vec{a}_z$$

Material characterization:

↳ ratio between conduction + displacement currents

$$\vec{J} = \sigma\vec{E} \Rightarrow \vec{J}_s = \sigma \vec{E}_s$$

$$\vec{J}_D = \frac{\partial}{\partial t}\epsilon\vec{E} \Rightarrow \vec{J}_{Ds} = j\omega\epsilon\vec{E}_s$$

$$\Rightarrow \left|\frac{\vec{J}_s}{J_{Ds}}\right| = \frac{\sigma}{\omega\epsilon}$$

e.g. muscle $f = 496$ MHz $\epsilon_R = 56.5$

$\sigma = 0.821$ S/m

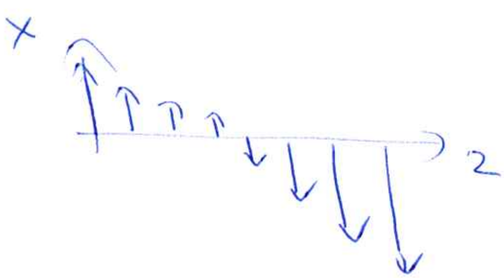
$$\frac{\sigma}{\omega\epsilon_R\epsilon_0} = \frac{0.821}{(2\pi)(496\times10^6)(56.5)(8.85\times10^{-12})}$$

$$= 0.527$$

Uniform plane waves

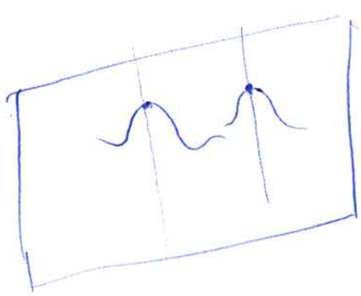
↳ $\nabla \times \vec{H} = \epsilon \frac{\partial}{\partial t}\vec{E}$ → $\vec{E}$ changes with time
→ relates to curl of $\vec{H}$
↳ $\vec{H}$ varies spatially perpendicular to orientation $\vec{H}$

↳ $\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$ → $\vec{E}$ vary spatially perdicular to orientation of $\vec{E}$

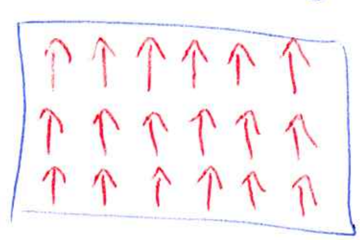


↳ TEM → transverse electromagnetic
↳ $\vec{E}$ is perpendicular to $\vec{H}$
↳ $\vec{E}$ + $\vec{H}$ are perpendicular to direction of propagation

↳ plane wave → equiphase surface is a plane



→ uniform plane wave ⇒ on the equiphase surface, equal amplitudes of field values



- assume $\vec{E} = E_x \vec{a}_x$ & travels in z

$\Rightarrow \vec{H} = H_y \vec{a}_y$ & travels in z

↳ free space ($\epsilon = \epsilon_0$, $\mu = \mu_0$, $\sigma = 0$)

↳ source-free region ($\rho_v = 0$, $\vec{J} = 0$)

$$\nabla \times \vec{E} = \frac{\partial E_x}{\partial z} \vec{a}_y \quad \& \quad -\mu_0 \frac{\partial}{\partial t} \vec{H} = -\mu_0 \frac{\partial H_y}{\partial t} \vec{a}_y$$

↳ $\nabla \times \vec{E} = -\mu_0 \dfrac{\partial \vec{H}}{\partial t}$

$\Downarrow$

$$\frac{\partial E_x}{\partial z} = -\mu_0 \frac{\partial H_y}{\partial t}$$