3.9 (a) 
$$a = N_1 / N_2 = 2000 / 500 = 4$$
  
 $R_{eq1} = 2 + 0.125 (4)^2 = 4 \Omega; X_{eq1} = 8 + (0.5) 4^2 = 16 \Omega$   
 $\overline{Z}'_2 = 12 (4)^2 = 192 \Omega$ 

The equivalent circuit referred to primary is shown below:

(b) 
$$V_{2,NL} = V_1/a = 1000/4 = 250 \text{ V}$$
  
Voltage Regulation =  $\frac{250 - 243.8}{243.8} \times 100 = 2.54\% \leftarrow$ 

3.10 Rated current magnitude on the 66-kV side is given by

$$I_{1} = \frac{13,000}{66} = 197.0 \,\mathrm{A}$$

$$I_{1}^{2} R_{eq1} = (197.0)^{2} R_{eq1} = 100 \times 10^{3}$$

$$\therefore R_{eq1} = 2.58 \,\Omega \quad \leftarrow$$

$$\frac{Z_{eq1}}{197.0} = \frac{5.5 \times 10^{3}}{197.0} = 27.9 \,\Omega$$
Then  $X_{eq1} = \sqrt{\frac{Z_{eq1}^{2} - R_{eq1}^{2}}{197.0}} = \sqrt{(27.9)^{2} - (2.58)^{2}} = 27.8 \,\Omega \quad \leftarrow$ 

## **3.11** Turns Ratio = $a = N_1/N_2 = 66/11.5 = 5.74$

With high-voltage side designated as 1, and L-V side as 2,

Just a long-winded way of saying:  

$$G_c = P_{oc} / (V_{rated, HV})^2$$

$$(11.5 \times 10^3)^2 a^2 G_{C1} = 65 \times 10^3$$
, based on O.C test.

Note: To transfer shunt admittance from H-V side to L-V side, we need to multiply by  $a^2$ .

$$\therefore G_{C1} = \frac{65 \times 10^{3}}{\left(11.5 \times 10^{3}\right)^{2} \left(5.74\right)^{2}} = 14.9 \times 10^{-6} \,\mathrm{S} \quad \leftarrow$$

$$Y_{1} = \frac{I_{2}}{V_{2}} \times \frac{1}{a^{2}} = \frac{30}{11.5 \times 10^{3}} \times \frac{1}{\left(5.74\right)^{2}} = 79.2 \times 10^{-6} \,\mathrm{S} \quad \longleftrightarrow \quad \mathrm{Identical\ to:\ |Y| = I_{HV,OC} / V_{rated,\ HV}}$$

$$\therefore B_{m1} = \sqrt{Y_{1}^{2} - G_{C1}^{2}} = 10^{-6} \sqrt{\left(79.2\right)^{2} - \left(14.9\right)^{2}}$$

$$= 77.79 \times 10^{-6} \,\mathrm{S} \quad \leftarrow$$

In the given operating conditions (10 MW of load at 0.8 PF and rated voltage):

$$\begin{split} S_{load} &= \frac{P_{load}}{pf} = \frac{10}{0.8} = 12.5 \, MVA \\ I_{load}(referred \ to \ HV \ side) &= \frac{S_{load}}{V_{load,ref \ to \ HV}} = \frac{12.5 \times 10^6}{66 \times 10^3} = 189.4 \, A \\ Losses \ in \ winding \ resistance &= I_{load}(referred \ to \ HV \ side)^2 \times R_{eq,1} = 189.4^2 \times 2.58 = 92.5 \, KW \end{split}$$

(In the above calculation, we neglected the excitation branch current and assumed that all of  $I_{load}$  referred to HV side will go through  $R_{eq}$ . You can calculate the excitation branch current =  $66kV/(G_c+jB_m)$  and subtract this from  $I_{load}$  referred to HV side to get the exact current through  $R_{eq}$ .)

Losses in 
$$G_c=65kW$$
 (from the OC test. Can you see why?) 
$$\eta=\frac{10~MW}{10~MW+92.5~KW+65~KW}=98.4\%$$

3.23

$$\begin{split} G: X &= 0.18 \left(\frac{100}{90}\right) = 0.2; T_1: X = 0.1 \left(\frac{100}{50}\right) = 0.2 \\ T_2: X &= 0.06 \left(\frac{100}{40}\right) = 0.15; T_2: X = 0.06 \left(\frac{100}{40}\right) = 0.15 \\ T_3: X &= 0.064 \left(\frac{100}{40}\right) = 0.16; T_4: X = 0.08 \left(\frac{100}{40}\right) = 0.2 \\ M: X &= 0.185 \left(\frac{100}{66.5}\right) \left(\frac{10.45}{11}\right)^2 = 0.25 \end{split}$$

For Line 1, 
$$\frac{Z_{BASE}}{100} = \frac{(220)^2}{100} = 484 \Omega$$
 and  $X = \frac{48.4}{484} = 0.1$ 

For Line 2, 
$$\frac{Z_{BASE}}{100} = \frac{(110)^2}{100} = 121 \Omega$$
 and  $X = \frac{65.43}{121} = 0.54$ 

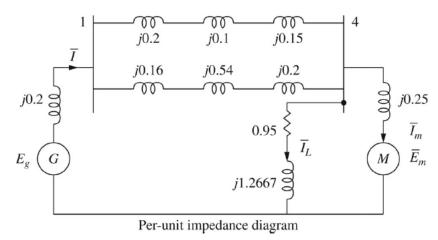
The load complex power at 0.6 Lagging pf is  $\overline{S}_{L(3\phi)} = 57 \angle 53.13^{\circ} \text{MVA}$ 

∴ The load impedance in OHMS is 
$$\overline{Z}_{L} = \frac{(10.45)^{2}}{57 \angle -53.13^{\circ}} = \frac{V_{LL}^{2}}{\overline{S}_{L(3\phi)}^{*}}$$
  
= 1.1495 + j1.53267 \Omega

The base impedance for the load is  $(11)^2/100=1.21 \Omega$ 

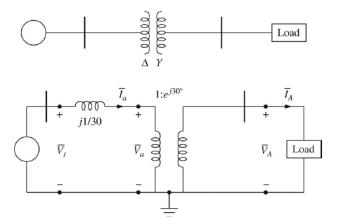
:. Load Impedance in pu = 
$$\frac{1.1495 + j1.53267}{1.21} = 0.95 + j1.2667$$

The per-unit equivalent circuit is shown below:



You are not required to know how to model motors in 487, yet!
For (steady-state) circuit analysis, motors are modelled identical to generators, i.e. they are an EMF behind an impedance. The only difference is generators produce power while motors consume it.

**3.38** (a) The single-line diagram and the per-phase equivalent circuit, with all parameters in per unit, are given below:



Current supplied to the load is  $\frac{240 \times 10^3}{\sqrt{3} \times 230} = 602.45 \,\text{A}$ 

Base current at the load is  $100,000/(\sqrt{3} \times 230) = 251.02 \text{ A}$ 

Reminder: Pf =  $\cos (\theta_v - \theta_i)$ Since we're choosing  $\theta_v = 0$ ,  $\theta_i = -\cos^{-1}(Pf)$ 

The power-factor angle of the load current is  $\theta = \cos^{-1} 0.9 = 25.84^{\circ}$  Lag . With  $\overline{V}_{\!\scriptscriptstyle A} = 1.0 \angle 0^{\circ}$  as reference, the line currents drawn by the load are

$$I_A = \frac{602.45}{251.02} \angle -25.84^\circ = 2.4 \angle -25.84^\circ \text{ per unit}$$

$$\overline{I}_B = 2.4 \angle -25.84^\circ -120^\circ = 2.4 \angle -145.84^\circ \text{ per unit}$$

$$\overline{I}_C = 2.4 \angle -25.84^\circ +120^\circ = 2.4 \angle 94.16^\circ \text{ per unit}$$

(b) Low-voltage side currents further lag by 30° because of phase shift

$$\overline{I}_a = 2.4 \angle -55.84^{\circ}; \overline{I}_b = 2.4 \angle 175.84^{\circ}; \overline{I}_c = 2.4 \angle 64.16^{\circ}$$

(c) The transformer reactance modified for the chosen base is

$$X = 0.11 \times (100/330) = \frac{1}{30} \text{ pu}$$

The terminal voltage of the generator is then given by

$$\overline{V}_t = \overline{V}_A \angle -30^\circ + jX\overline{I}_a$$
  
= 1.0\angle -30^\circ + j(1/30)(2.4\angle -55.34^\circ)  
= 0.9322 - j0.4551=1.0374\angle -26.02^\circ pu

Terminal voltage of the generator is  $23 \times 1.0374 = 23.86 \text{ kV}$ The real power supplied by the generator is

$$\text{Re}\left[\overline{V}_{t}\overline{I}_{a}^{*}\right] = 1.0374 \times 2.4\cos(-26.02^{\circ} + 55.84^{\circ}) = 2.16 \text{ pu}$$

which corresponds to 216 MW absorbed by the load, since there are no  $I^2R$  losses.

- (d) By omitting the phase shift of the transformer altogether, recalculating  $\overline{V}_t$  with the reactance  $j\bigg(\frac{1}{30}\bigg)$  on the high-voltage side, the student will find the same value for  $V_t$  i.e.  $\left|\overline{V}_t\right|$ .
- 3.49 Base kV in transmission-line circuit = 132 kVBase kV in the generator  $G_1$  circuit =  $132 \times \frac{13.2}{165} = 10.56 \text{ kV}$ Base kV in the generator  $G_2$  circuit =  $132 \times \frac{13.8}{165} = 11.04 \text{ kV}$

On the common base of 100 MVA for the entire system,

$$\begin{split} G_1: \overline{\overline{Z}} &= j0.15 \times \frac{100}{50} \times \left(\frac{13.2}{10.56}\right)^2 = j0.4688 \, \mathrm{pu} \\ G_2: \overline{\overline{Z}} &= j0.15 \times \frac{100}{20} \times \left(\frac{13.8}{11.04}\right)^2 = j1.1719 \, \mathrm{pu} \\ T_1: \overline{\overline{Z}} &= j0.1 \times \frac{100}{80} \times \left(\frac{13.2}{10.56}\right)^2 = j0.1953 \, \mathrm{pu} \\ T_2: \overline{\overline{Z}} &= j0.1 \times \frac{100}{40} \times \left(\frac{13.8}{11.04}\right)^2 = j0.3906 \, \mathrm{pu} \end{split}$$

Base impedance in transmission-line circuit is

$$\frac{\left(132\right)^{2}}{100} = 174.24\Omega$$

$$\overline{Z}_{TR.LINE 1} = \frac{50 \times j200}{174.24} = 0.287 + j1.1478 \,\text{pu}$$

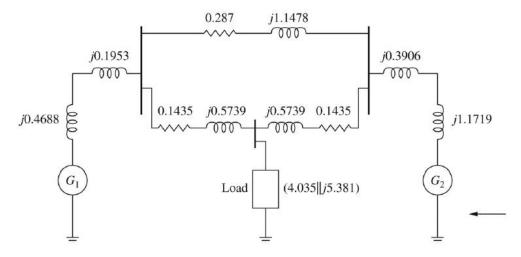
$$\overline{Z}_{TR.LINE 2} = \frac{25 \times j100}{174.24} = 0.1435 + j0.5739 \,\text{pu}$$

$$LOAD: 40\left(0.8 + j0.6\right) = \left(32 + j24\right) \,\text{MVA}$$

$$R_{LOAD} = \frac{\left(150\right)^{2}}{32} = 703.1\Omega = \frac{703.1}{174.24} \,\text{pu} = 4.035 \,\text{pu}$$

$$X_{LOAD} = \frac{\left(150\right)^{2}}{24} = 937.5\Omega = \frac{937.5}{174.24} \,\text{pu} = 5.381 \,\text{pu}$$

$$\overline{Z}_{LOAD} = \left(R_{LOAD} \parallel jX_{LOAD}\right)$$



Impedance diagram of the system with pu values