Charging Current

The current drawn by the line capacitance is called charging current. This charging current can introduce problems when a transmission line is in operation.

For example, in underground cables, the charging current is significant due to the fact that the distance between conductors in underground cables is very small.

Charging current can be calculated as follows:

$$\boxed{\bar{I}_{chg} = \frac{\bar{V}_{an}}{Z_{cap}} = \frac{\bar{V}_{an}}{\frac{1}{j\omega C_{an}}} = j\omega C_{an} \bar{V}_{an}}$$

The reactive power associated with the charging current is given as

$$\boxed{Q_{chg} = \frac{V_{an}^2}{X_{cap}} = \frac{V_{an}}{-\frac{1}{\omega C_{an}}} = -\omega C_{an} V_{an}^2}$$

Note: The negative sign shows that the line is supplying reactive power to the line.

Representation of Transmission Lines

In this lecture, we will leverage the four parameters of a transmission line (resistance, Inductance, capacitance, & Conductance) to model transmission lines.

Transmission lines can be divided into three types (namely long, medium, and short lines) based on their length.

For short and medium lines, we consider the parameters of transmission lines we calculated earlier to be lumped at a point along the transmission line. This simplification does not introduce significant errors in calculations. However, for long lines, we do not ~~used~~ use the lump strategy. Rather we consider the line parameters to be distributed along the line. The table below gives details about short, medium, and long transmission line models.

Table: Transmission line models

Type of Line	Length (l)	Remarks
Short	$l < 80$ Km	R & L used. C omitted because it is negligible.
Medium	$80 < l < 250$ Km	R, L & C used
Long	$l \geq 250$ Km	R, L & C assumed to be distributed

Note that for a linear passive, bilateral two-port network, the following equation is always satisfied:

$$\boxed{AD - BC = 1}$$

Also, the units of the constants are as follows.

$A = D$ = [dimensionless (no units)]

B = [ohms]

C = [Mhos or Siemens].

Voltage Regulation (V.R)

It can be defined as the percentage change in the magnitude of the receiving end voltage (expressed as percent of full-load voltage) when the load is varied from no-load to full-load.

Mathematically,

$$\boxed{\text{Percent V.R} = \frac{|V_{R(NL)}| - |V_{R(FL)}|}{|V_{R(FL)}|} \times 100}$$

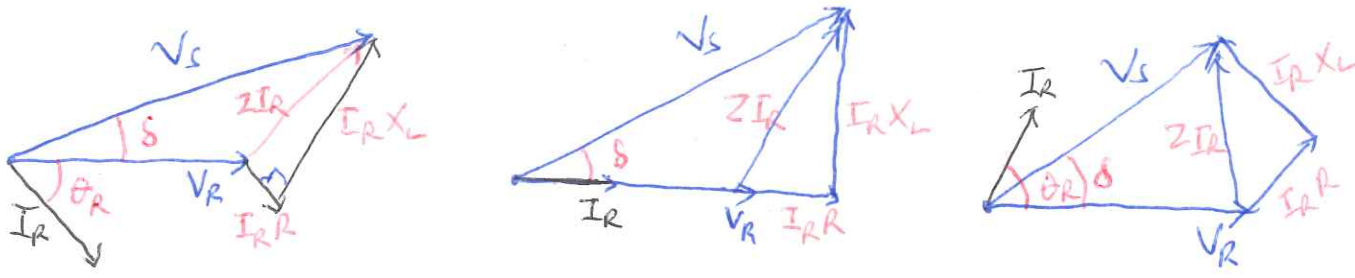
where $V_{R(NL)}$ = receiving end voltage at no-load; $V_{R(FL)} = V_R$ at full load.

We are interested in calculating percent V.R because we would like to keep the power quality at the consumers' end constant irrespective of load variations. One way of satisfying the customer is by keeping the supply voltage constant over a wide range of load conditions.

However, depending on load conditions & the impedance of the transmission line, the required sending-end

Voltage (V_s) to satisfy the receiving-end voltage Constraint will change.

Effect of power factor on Sending end voltage



(a) Lagging power factor

(b) Unity Power factor

(c) Leading power factor

Fig Nos	Difference	Remarks
(a)	I_R is lagging V_R	Requires larger V_s Compared to (b)
(b)	I_R is in-phase with V_R $\therefore$ $\theta_R = 0°$	Baseline Case
(c)	I_R is leading V_R	Requires lower V_s compared to (b)

Calculating Voltage Regulation for a Short-line

At no-load, $\bar{I}_R = 0$;

From ABCD Constants based equation for a Short line,

$$V_{R(NL)} = \frac{V_s}{A}$$

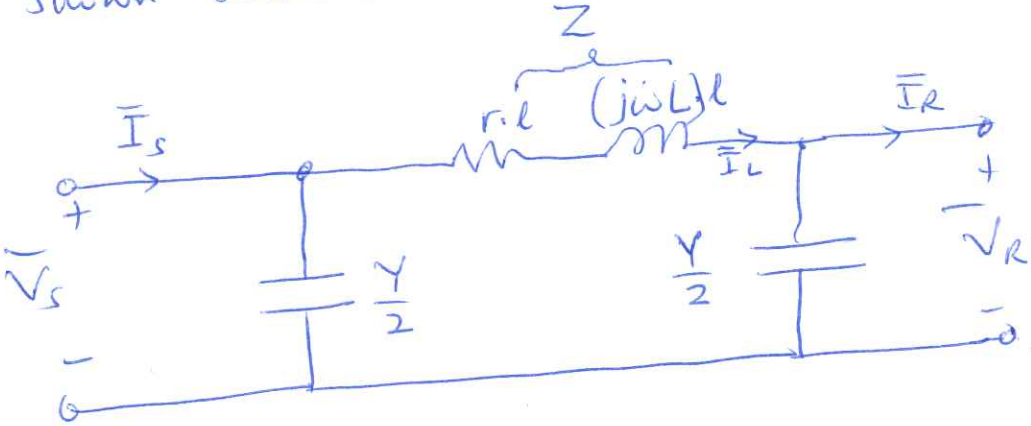
But $A = 1$ for short line

$$\therefore \boxed{\text{Percent V.R} = \frac{|V_s| - |V_{R(FL)}|}{|V_{R(FL)}|} \times 100}$$

(b) Medium length transmission lines

When the length of the line exceeds 80km, the line charging current becomes appreciable, thus the shunt capacitance cannot be neglected.

For medium lines, we consider half of the shunt capacitance to be lumped at each end of the line as shown below:



Nominal π circuit

where $Y = (g + j\omega C)l$; $g = 0$

↓ Conductance

C = Capacitance per length

ω = fundamental frequency in rad/s.

l = length of the line.

Applying KCL at the sending end and at the receiving end, we get

$$\boxed{\begin{aligned} \bar{I}_s &= \bar{I}_L + \frac{Y}{2}\bar{V}_s \\ \bar{I}_L &= \bar{I}_R + \frac{Y}{2}\bar{V}_R \end{aligned}}$$

Applying KVL yields

$$\cancel{\bar{V}_s = \bar{V}_R \bar{I}_R} \qquad \boxed{\bar{V}_s = Z I_L + \bar{V}_R}$$

Solving for $\bar{V}_S$ and $\bar{I}_S$ yield

$$\boxed{\begin{bmatrix} \bar{V}_S \\ \bar{I}_S \end{bmatrix} = \begin{bmatrix} 1+\frac{YZ}{2} & Z \\ Y\left(1+\frac{YZ}{4}\right) & 1+\frac{YZ}{2} \end{bmatrix} \cdot \begin{bmatrix} \bar{V}_R \\ \bar{I}_R \end{bmatrix}}$$

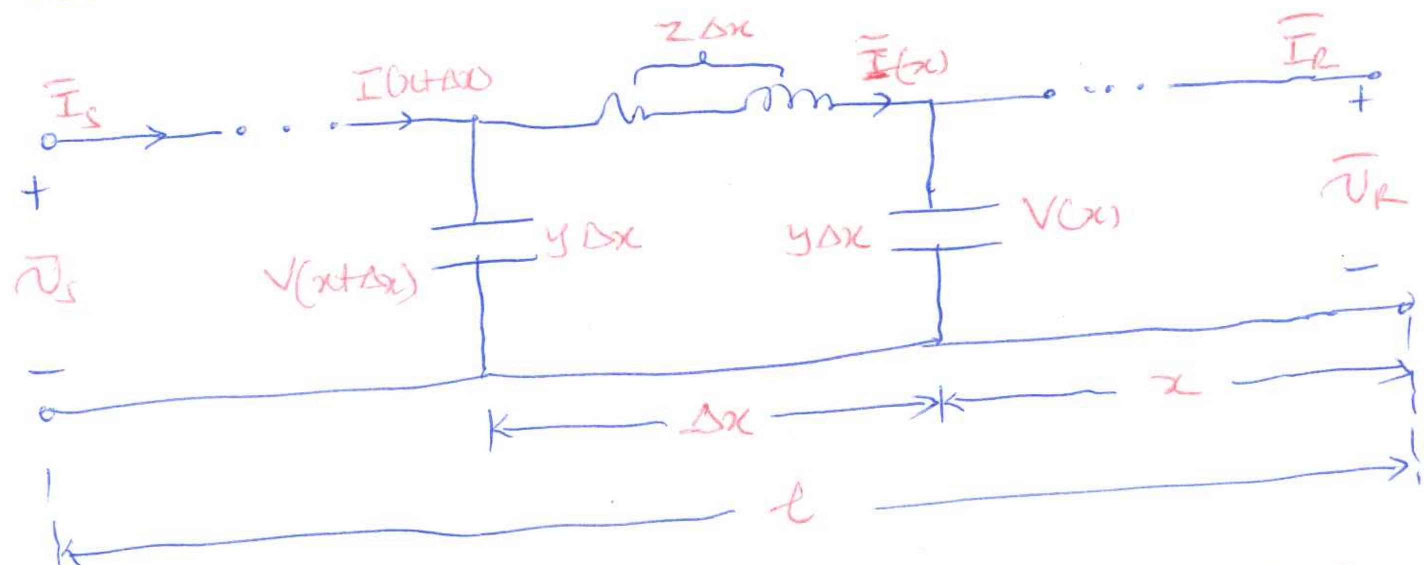
where $A = 1 + \frac{YZ}{2}$; $B = Z$

$C = Y\left(1+\frac{YZ}{4}\right)$; $D = 1 + \frac{YZ}{2}$

Quick question: Check if the ABCD Constants for a medium line satisfies the equation: $AD - BC = 1$.

(c) The long transmission line

As stated earlier in the Introduction, for long transmission lines, we will consider the line parameters to be distributed across the line. With the long transmission line model, we can calculate the voltage, current and power at any point on the transmission line provided we know the voltage, current and power at one point along the line. Consider the diagram below.



where x : distance from the receiving end of the line

dx : differential element of length.

$z\,dx$: series impedance of the infinitely small section of length dx.

$y\,dx$: shunt admittance of the infinitely small section of length dx.

V : Voltage phasor that varies with x.

I : Current phasor that varies with x.

From KVL,

$$\frac{V(x+\Delta x) - V(x)}{\Delta x} = z\,I(x) \quad \text{-----} \ *$$

Taking limit as $\Delta x \to 0$,

$$\frac{d\,V(x)}{dx} = z\,I(x)$$

From KCL,

$$\frac{I(x+\Delta x) - I(x)}{\Delta x} = y\,V(x+\Delta x)$$

Taking limit as $\Delta x \to 0$,

$$\frac{d\,I(x)}{dx} = y\,V(x) \quad \text{-----} \ **$$

Differentiating (*) and substituting from (**), yields

$$\boxed{\frac{d^2 V(x)}{dx^2} = z\,d\,I(x) = zy\,V(x)}$$

Let $\gamma^2 = zy$ where γ is the propagation constant in ($1/m$).

Then $$\boxed{\frac{d^2 V(x)}{dx^2} - \gamma^2 V(x) = 0}$$

The solution of the above differential equation will take the form of

$$\boxed{V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x}}$$

After manipulating the equation, we get

$$\boxed{\begin{aligned}\bar{V}(x) &= \bar{V}_R \cdot \cosh(\gamma x) + \bar{I}_R Z_c \sinh(\gamma x) \\ \bar{I}(x) &= \frac{\bar{V}_R}{Z_c} \cdot \sinh(\gamma x) + \bar{I}_R \cdot \cosh(\gamma x)\end{aligned}}$$

Where $Z_c = \sqrt{\frac{z}{y}}$ and it is Known as the characteristic impedance

$\bar{V}_R = \bar{V}(x=0)$

$\bar{I}_R = \bar{I}(x=0)$

At the sending end, $x = l$ $\therefore$

$$\boxed{\begin{aligned}\bar{V}(x=l) &= \bar{V}_s = \bar{V}_R \cosh(\gamma l) + \bar{I}_R Z_c \sinh(\gamma l) \\ \bar{I}(x=l) &= \bar{I}_s = \frac{\bar{V}_R}{Z_c} \cdot \sinh(\gamma l) + \bar{I}_R \cdot \cosh(\gamma l)\end{aligned}}$$

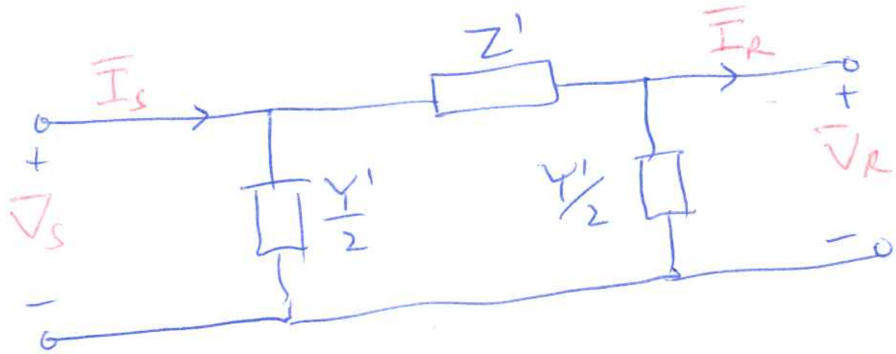
where $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_c \cdot \sinh(\gamma l) \\ \frac{1}{Z_c} \sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix}$

Quick question: Verify if $A = D$ and $AD - BC = 1$ in the long transmission line equation presented above.

Equivalent π – model

For circuit modeling and analysis purposes, it is more convenient to represent the long transmission line model using the nominal π – representation as it was done for the medium transmission line.

Consider the figure below



The prime in the series impedance and shunt admittance denote the quantities are equivalent of the ones obtained for the medium length lines.

For the equivalent π -model, we can write

$$\boxed{\begin{aligned} \bar{V}_s &= \left(1 + \frac{Z'Y'}{2}\right)\bar{V}_R + Z'\bar{I}_R \\ \bar{I}_s &= Y'\left(1 + \frac{Z'Y'}{4}\right)V_R + \left(1 + \frac{Z'Y'}{2}\right)\bar{I}_R \end{aligned}}$$

Comparing the ABCD constants in the equation above with the ABCD constants obtained for the long line, we get

$$\boxed{Z' = Z \cdot \frac{\sinh(\gamma l)}{\gamma l}}$$

where $Z = z \cdot l$

Total impedance

distributed impedance

$$\boxed{\frac{Y'}{2} = \frac{Y}{2}\,\frac{\tanh(\gamma l/2)}{\frac{\gamma l}{2}}}$$

where $Y = y l$

Y — Total admittance; y — distributed admittance

Thus, we can redraw the long transmission line model to be