ENEL 476

Winter 2012

Quiz #2 **March 28, 2012** **EDC 179, 10:00-10:50 am**

Name or ID VDR Fear

Instructions

- Closed book and closed notes.
- Formula sheet is provided.
- Complete the quiz on the quiz paper. If you run out of paper, request additional blank paper from the invigilators.
- Only the solution of the multiple-choice questions will be evaluated. ~~Clearly circle only one choice.~~
- Programmable calculators may be used.

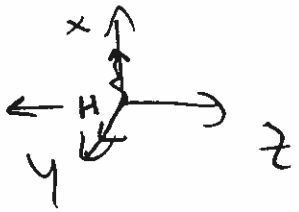
Question 1. (2 marks)

Consider the following expression for the magnetic field associated with a uniform plane wave in the region x>0:

$$\mathbf{H}(x,t)=0.5e^{-0.21x}\cos(10^8t-18.7x)\,\mathbf{a}_y$$

Circle the statements that are true:

- the wave propagates in the –x direction → x propagates in +x
- the electric field is oriented in the –z direction
- the wavelength is 3 m
- the electric and magnetic field do not have a phase shift between them.



$\beta = 18.7$

$\lambda = \frac{2\pi}{18.7}$

$= 0.34$

$\alpha \neq 0 \therefore$ n is complex

Question 2 (2 marks)

An electric field in free space is given by:

$$\mathbf{E}_s(z)=10(e^{j\pi}\mathbf{a}_x+e^{j\pi/2}\mathbf{a}_y)e^{j1.5\pi z}$$

Circle the statements that are true:

- The wave is elliptically polarized. → circularly polarized
- The frequency is 0.225 GHz.
- The magnetic field is in the z-direction.
- The time-averaged Poynting vector varies with z.

↓

no, free space

∴ $P_{Avg}(z)$ = constant!

no, magnetic field is in xy plane

$\beta = 1.5\pi$

$\beta = \omega/c$

$= 2\pi f / 3\times10^8$

$(3\times10^8)(1.5\pi) = 2\pi f$

$f = 2.25\times10^8$

$= 0.225\text{GHz}$

Question 3. (2 marks)

Consider the following expression for the electric field associated with a uniform plane wave:

$$E(y,t)=25\cos(10^6t-3.5\pi y)a_x$$

The wave propagates in a region with $\varepsilon_r=9$, $\sigma=0$ S/m and $\mu_r=1$.

Circle the statements that are true:

- the wavelength is 0.57 m.
- the wave travels faster than a wave described by the same expression would travel in free space
- an expression for the magnetic field is: $H(y,t)=0.625\pi\cos(10^6t-3.5\pi y)a_x$
- if a wave described by the same expression traveled in free space, the wavelength would be smaller than the wavelength in the dielectric.

doesn't work to do this (if ω is the same, β changes!)

$v_p = \omega/\beta \rightarrow \beta > \sqrt{\varepsilon_R}$ ∴ wave is slower than free space (at same freq)

if $\beta_1 = \beta_0$, same speed ∴ not faster

$\beta = 3.5\pi$

$\lambda = \frac{2\pi}{\beta}$

$= \frac{2\pi}{3.5\pi}$

$= \frac{4}{7}$

$= 0.57$ m

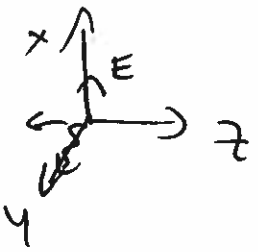
$\lambda = \frac{2\pi}{\beta}$

$\rightarrow \beta > \sqrt{\varepsilon_R}$

∴ wavelength is smaller in dielectric for wave at same freq

$\rightarrow$ if $\beta_1 = \beta_0$, then $\lambda_1 = \lambda_0$

$\tilde{H}(y,t) = \frac{25}{120\pi}\cos(10^6t - 3.5\pi y)\frac{J}{a_z}$



Question 4. (4 marks)

The electric field of a uniform plane wave traveling in free space is given by:

$$\mathbf{E}_s(z)=10e^{j\pi/4}e^{j1.5\pi z}\,\mathbf{a}_y$$

Find:

- (½) the direction of propagation: $-z$
- (½) the phase constant (β): $\beta = 1.5\pi$ rad/m
- (½) wavelength (λ): $\lambda = 2\pi/\beta = 2/1.5 = 4/3$ $\lambda = 1.33$ m
- (½) frequency (f): $\beta = \omega\sqrt{\mu_0\epsilon_0}$, $f = \frac{\beta}{2\pi\sqrt{\mu_0\epsilon_0}} = \frac{1.5\pi}{2\pi}\times 3\times10^8 = 2.25\times10^8$

check: $\lambda = c/f$, $f = c/\lambda = \frac{3\times10^8\,(3)}{4} = \frac{9}{4}\times10^8$, $f = 2.25\times10^8$ Hz

- an expression for $\mathbf{H}_s(z)$ (½): $\vec{H}_s(z) = \frac{10}{120\pi}e^{j\pi/4}e^{j1.5\pi z}\,\vec{a}_x$ (½)
- an expression for $\mathbf{H}(z,t)$ (½):

$$\vec{H}(z,t) = -\frac{1}{12\pi}\cos(4.5\pi\times10^8 t + 1.5\pi z - \pi/4)\,\vec{a}_x \text{ A/m}$$ (½)

- the time-averaged Poynting vector, $\mathbf{P}_{avg}(z)$ (½):

$$\vec{P}_{AVG}(z) = -\frac{|E_m^+|^2}{2\eta_0}\vec{a}_z = -\frac{(10)^2}{2(120\pi)}\vec{a}_z = -\frac{10}{24\pi}\vec{a}_x \text{ W/m}^2$$