Question 1

A coaxial line has R = 6 Ω/m, L = 3 μH/m, G = 8 mS/m and C = 4 nF/m. The frequency of operation is 1 MHz. Find Z_0, α and β.

Solution 1 - Short Method

For this question, we can either use the general formulas or evaluate to see if the line is distortion less. A distortionless line must satisfy the condition:

$$G' = \frac{RC}{L} \text{ (1.1.1)}$$

where G' should equal G (8 mS/m) if the line is distortion less. Substituting are known line parameters into equation 1.1.1 we obtain:

$$G' = \frac{6\ \Omega/m * 4\ nF/m}{3\ \mu H/m}$$

$$G' = 8\ mS/m = G$$

proving that the line is distortion less. Using the distortionless line equations we can find Z_0 from the following equation:

$$Z_0 = \frac{\sqrt{L}}{\sqrt{C}} \text{ (1.1.2) (+0.5 marks)}$$

substituting are known line parameters into equation 1.1.2 we obtain:

$$Z_0 = \frac{\sqrt{3\ \mu H/m}}{\sqrt{4\ nF/m}}$$

$$Z_0 = 27\ \Omega \text{ (+0.5 marks)}$$

The attenuation coefficient (α) can then be calculated using the following equation:

$$\alpha = \frac{R}{Z_0} \text{ (1.1.3) (+0.5 marks)}$$

substituting are known line parameters into equation 1.1.3 we obtain:

$$\alpha = \frac{6\ \Omega/m}{27\ \Omega}$$

$$\alpha = 0.222\ Nepers/m \text{ (+0.5 marks)}$$

Finally we solve for β using the following equation:

$$\beta = \omega\sqrt{LC} \text{ (1.1.4) (+0.5 marks)}$$

substituting are known line parameters into equation 1.1.4 we obtain:

$$\beta = 2 * \pi * 1\ MHz * \sqrt{3\ \mu H/m * 4\ nF/m}$$

$$\beta = 0.688\ rads/m \text{ (+0.5 marks)}$$

If the transmission line is not shown to be distortion less then -1.5 marks. If any units missing for answers, then -0.5 marks (only once for entire question).

Total mark assigned out of 3.

Solution 2 - Long Method

Using the general formulas for a lossy transmission line, Z_0 is given by:

$$Z_0 = \frac{\sqrt{R + j\omega L}}{\sqrt{G + j\omega C}} \text{ (1.2.1) (+0.5 marks)}$$

substituting are known line parameters into equation (1.2.1) we obtain:

$$Z_0 = \frac{\sqrt{6\ \Omega/m + j * 2\pi * 1\ MHz * 3\ \mu H/m}}{\sqrt{8mS/m + j * 2\pi * 1\ MHz * 4nF/m}}$$

$$Z_0 \approx 27\ \Omega \text{ (+0.5 marks)}$$

γ is given by:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \text{ (1.2.2) (+0.5 marks)}$$

substituting are known line parameters into equation (1.2.2) we obtain:

$$\gamma = \sqrt{(6\ \Omega/m + j * 2\pi * 1\ MHz * 3\ \mu H/m)(8\ mS/m + j * 2\pi * 1\ MHz * 4nF/m)}$$

$$\gamma = 0.219\ Nepers/m + j0.688\ rad/m \text{ (+1.5 marks)}$$

the α and β are the real and imaginary parts of this respectively.

$$\alpha = \text{RE}(\gamma) \text{ (+0.5 marks)}$$

$$\beta = \text{IM}(\beta) \text{ (+0.5 marks)}$$

If any units missing for answers, then -0.5 marks (only once for entire question).

Total mark assigned out of 3.

Question 2

Consider a lossless transmission line with capacitance per unit length of C = 15 pF/m and a characteristic impedance of Z_0 = 300 Ω. The angular frequency of operation is $\omega = 10^8$ rad/s. A load is then attached to the end of 20 m of this line. The impedance of the load is Z_l = 440 - j265 Ω.

Solution

a) First we find the inductance per unit length(L). Rearranging equation 1.1.2 we arrive at the following:

$$L = Z_0^2 C \text{ (2.1) (+0.5 marks)}$$

substituting are known line parameters into equation 2.1 we obtain:

$$L = (300)^2 (4\ pF/m)$$

$$L = 1.35\ \mu H/m \text{ (+0.5 marks)}$$

b) Next we find β. Substituting are known line parameters into equation 1.1.4 we obtain:

$$\beta = 10^8 \sqrt{1.35\ \mu H/m * 4\ pF/m}$$

$$\beta = 0.45\ rads/m \text{ (+0.5 marks)}$$

c) Next we find γ. Using the relationship $\beta = 2\pi/\lambda$ we obtain:

$$\lambda = \frac{2\pi}{\beta} \text{ (2.2)}$$

substituting are known line parameters into equation 2.2 we obtain:

$$\lambda = \frac{2\pi}{0.45\ rads/m}$$

$$\lambda = 13.96\ m \text{ (+0.5 marks)}$$

20 m would therefore represent 1.43 λ (+0.5 marks).

d) Yes, the line is distortionless as <u>all losseless lines are distortionless</u> (0.5 marks, must give a valid reason)

e) The reflection coefficient at the load can be calculated by the impedance mismatch as follows:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (2.3)$$

substituting in our known impedances

$$\Gamma_L = \frac{(440 - j265\ \Omega) - (300\ \Omega)}{(440 - j265\ \Omega) + (300\ \Omega)}$$

$$\Gamma_L = 0.281 - j0.257 = 0.381\angle{-42.4^o} \ (+0.5 \text{ marks})$$

f) The VSWR is given by:

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad (2.4)$$

substituting in our known Γ_L:

$$VSWR = \frac{1 + |0.381\angle 42.4^o|}{1 - |0.381\angle 42.4^o|}$$

$$VSWR = 2.23 \ (+0.5 \text{ marks})$$

g) The Z_{in} is given by:

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \quad (2.5) \ (+0.5 \text{ marks})$$

substituting in our known impedances, β, and l:

$$Z_{in} = (300\ \Omega) \frac{(440 - j265\ \Omega) + j(300\ \Omega)\tan(0.45\ rads/m * 20\ m)}{(300\ \Omega) + j(440 - j265\ \Omega)\tan(0.45\ rads/m * 20\ m)}$$

$$Z_{in} = 662 + j64\ \Omega = 665\angle 5.5^o\ \Omega \ (+0.5 \text{ marks})$$

If any units missing for answers, then -0.5 marks (only once for entire question).

Total mark assigned out of 5.

Question 3

A transmission line with $Z_0 = 50\ \Omega$ is terminated with a load of $Z_L = 170$ - j20 Ω. The frequency of operation of f = 2 GHz and phase velocity on the line is $v_p = 0.8c$ where c is the speed of light in free space. Use the Smith chart to solve the following questions.

Solution

a) *See the smith chart for question 3 and look for the area marked a).* First we find the z_L and then we will use the smith chart. z_L is given by:

$$z_L = \frac{Z_L}{Z_0} \quad (3.1)$$

substituting in the known impedances we obtain:

$$z_L = \frac{170 - j20\ \Omega}{50\ \Omega}$$

$$z_L = 3.4 - j0.4$$

This is then plotted on the smith chart (+0.5 marks).

b) *See the smith chart for question 3 and look for the two areas marked b).* The $|\Gamma|$ can be found by using the magnitude ruler on the bottom of the smith chart with the radius of the z_l, which gives $|\Gamma| = 0.55$ (+0.5 marks). The angle of Γ can be found from the angle metric on the outside of the smith chart which give 6^o (+0.5 marks).

c) *See the smith chart for question 3 on the next page and look for the area marked c).*The VSWR can be found by using the magnitude ruler on the bottom of the smith chart with the radius of the z_l, which gives VSWR = 3.5 (+0.5 marks).

d) *See the smith chart for question 3 on the next page and look for the area marked d).*The correct VSWR is drawn on the smith chart (+0.5 marks).

e) *See the smith chart for question 3 and look for the two areas marked e).* The highest voltage (V_{Max}) will occur when the resistance is the highest. This corresponds to the intersection of the unity axis (imaginary component of $z_{in} = 0$) with the VSWR circle on the right of the smith chart. Another way to look at this is to consider that the V_{Max} will occur where the line is closest to an open circuit on the smith chart. The first V_{Max} will occur at when we rotate by 0.492 λ, toward the generator (+0.5 marks). We must rotate toward the generator since we are at the load. Calculating the wavelength using the formula:

$$\lambda = \frac{v_p}{f} \ (3.2)$$

substituting in the known v_p and f we obtain:

$$\lambda = \frac{0.8 * 3E8}{2E9\ Hz} \ (3.2)$$

$$\lambda = 12\ cm$$

The first $VMax$ occurs at 5.9 cm (0.5 marks).

f) *See the smith chart for question 3 on the next page and look for the two areas marked f).* The short is located on the left edge of the smith chart on the unity axis (r = 0) (+0.5 marks) and the open is located on the right edge of the smith chart on the unity axis (r $->\infty$) (+0.5 marks).

A smith chart must be included and readable. Otherwise either a mark of zero can be assigned or -2 marks for improper annotation on the smith chart.If any units missing for answers, then -0.5 marks (only once for entire question).

Total mark assigned out of 5.

Question 3

$\lambda = \frac{c}{f} = \frac{0.86}{2E9Hz} = 12cm$

Rotate toward generator by $0.5\lambda = (0.78\lambda - 0.25\lambda) = 0.492\lambda = 5.9cm$

e)

The Smith Chart

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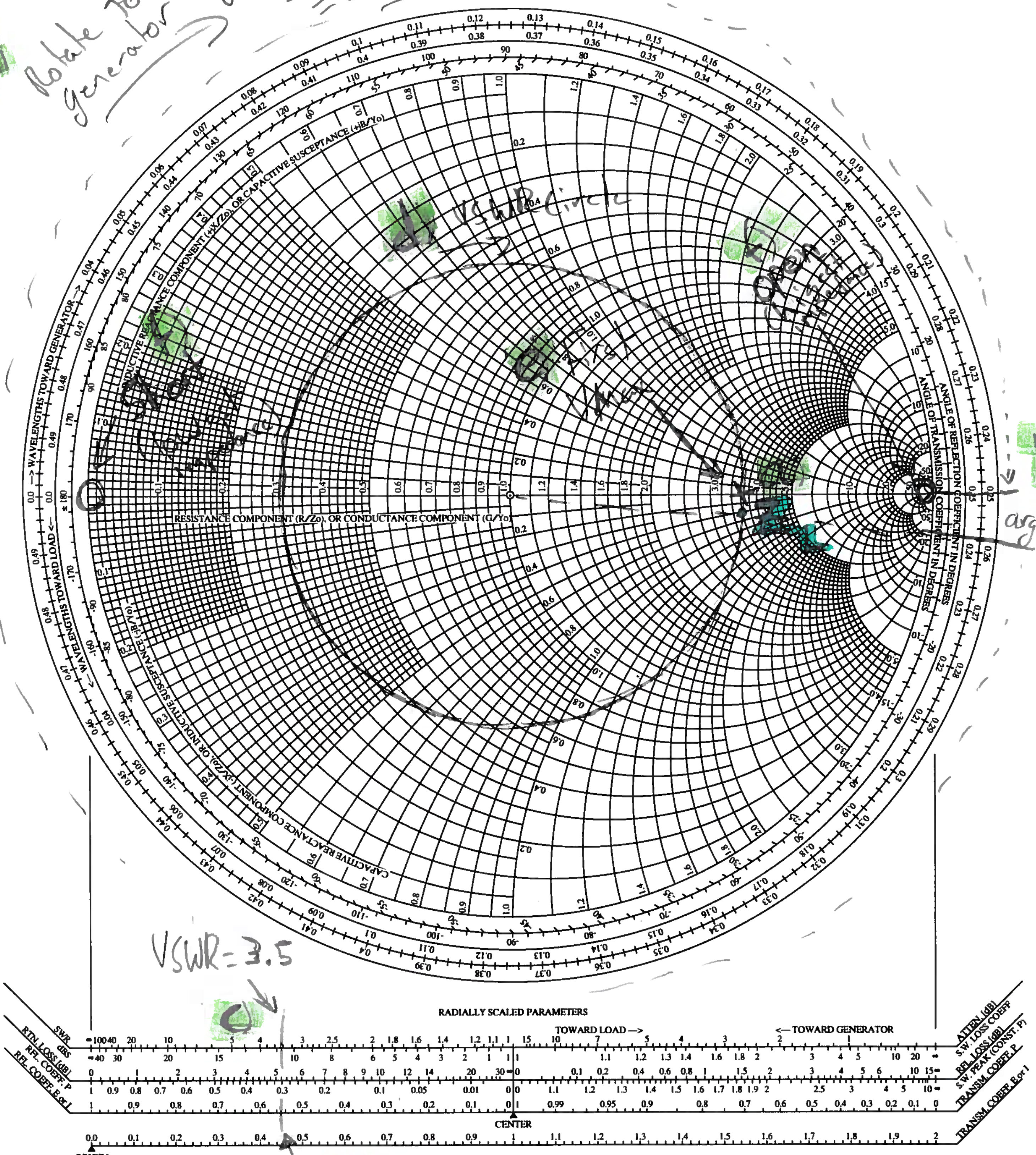

b)

$|\Gamma| = 0.55$

Question 4

A transmission line with $Z_0 = 75\ \Omega$ is terminated with a load of $Z_L = 30$ - j60 Ω. The frequency of operation is f = 5 GHz and $\lambda = 6$ cm. A length of 5 cm line is then attached to the antenna. Solve parts a)-c) on both the smith chart and using equations. Solve parts d)-e) using the smith chart only.

Solution

a) *See the smith chart for question 4 and look for the two areas marked a).*

First we find the z_L and then we will use the smith chart to find Γ_L. Substituting in our known impedances into equation 3.1 we obtain:

$$z_L = \frac{30 - j60\ \Omega}{75\ \Omega}$$
$$z_L = 0.4 - j0.8$$

This is then plotted on the smith chart.

The $|\Gamma_L|$ can be found by using the magnitude ruler on the bottom of the smith chart with the radius of the z_l, which gives $|\Gamma_L| = 0.62$ (+0.5 marks). The angle of Γ_L can be found from the angle metric on the outside of the smith chart which give 97.5^o (+0.5 marks).

We can check this answer by using the equation 2.3 and substituting in our known impedances:

$$\Gamma_L = \frac{(30 - j60\ \Omega) - (75\ \Omega)}{(30 - j60\ \Omega) + (75\ \Omega)}$$
$$\Gamma_L = -0.077 - j0.615\ \Omega = 0.62\angle 97.1^o \text{ (+0.5 marks)}$$

The methods agree. Some error can be expected with the smith chart method as it is graphical however.

b) *See the smith chart for question 4 and look for the area marked b).*

The VSWR can be found by using the magnitude ruler on the bottom of the smith chart using the radius of the z_L for the length, which gives VSWR = 4.3 (+0.5 marks).

We can check this answer using equation 2.4 and substituting in Γ_L

$$VSWR = \frac{1 + |0.62\angle 97.1^o|}{1 - |0.62\angle 97.1^o|}$$

$$VSWR = 4.27 \text{ (+0.5 marks)}$$

The methods agree.

c) *See the smith chart for question 4 and look for the two areas marked c).*

Using the smith chart, finding Z_{in} can be much faster than using the equations. First find the position of the z_{in} on the smith chart in terms of wavelengths toward the load metric. This was found to be 0.382λ. Next calculate the length of the line in terms of wavelength.

$$L = \frac{5cm}{6cm} = 0.833\lambda$$

Now impedance is periodic every $\lambda/2$ so the line has one full rotation and a 0.333λ offset relative to z_L (+0.5 marks).

Finally, we calculate the actual location in terms of the wavelengths toward the generator metric on the smith chart.

$$Location = 0.333\lambda - (0.5\lambda - 0.386\lambda)$$

$$Location = 0.219\lambda \text{ (+0.5 marks)}$$

Rotating around to this point on the smith chart and finding the intersection with the VSWR circle we find

$$z_{in} = 2.6 + j2.0 \text{ (+0.5 marks)}$$

and denormalizing this value we find

$$Z_{in} = 195 + j150\ \Omega = 246\angle 37.5^o\ \Omega\text{(+0.5 marks).}$$

We can check this using equation 2.5. First we need to find β using the relationship $\beta = 2\pi/\lambda$.

$$\beta = 2\pi/\lambda = 105\ rads/m \text{ (+0.5 marks)}$$

Substituting in our known impedances, β,l:

$$Z_{in} = (75\ \Omega)\ \frac{(210 - j150\ \Omega) + j(75\ \Omega)\tan(105\ rads/m * 0.06\ m)}{(75\ \Omega) + j(210 - j150\ \Omega)\tan(105\ rads/m * 0.06\ m)}$$

$$Z_{in} = 191 + j149\ \Omega = 242\angle 37.9^o\ \Omega \text{ (+0.5 marks)}$$

The methods agree.

d) *See the smith chart for question 4and look for the two areas marked d).*

The lowest voltage (V_{min}) will occur when the resistance is the lowest. This corresponds to the intersection of the unity axis (imaginary component of $z_{in} = 0$) with the VSWR circle on the left of the smith chart. Another way to look at this is to consider that the V_{min} will occur where the line is closest to an short circuit on the smith chart. The first V_{min} will occur at when we rotate by

$$d_{min} = (0.5 - 0.382)\lambda = 0.118\lambda = 0.684\ cm$$

(+0.5 mark for answer in λ, +0.5 mark for answer in meters)

e) *See the smith chart for question 4 and look for the two areas marked e).*

The shortest distance to a resistive load is the first intersection with the unity axis of the smith chart with the VSWR circle, rotating toward the generator. This is in fact the same position as the first V_{min}.

$$d_{resitive} = d_{V_{min}} = 0.118\lambda = 0.684cm \text{ (+0.5 marks for either unit)}$$

A smith chart must be included and readable. Otherwise either a mark of -4.5 marks (all of the smith chart component) can be assigned or -2 marks for improper annotation on the smith chart. If any units missing for answers, then -0.5 marks (only once for entire question).

Total mark assigned out of 7.

Question 4)

The Smith Chart

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c) $l = 5cm = 0.833\lambda$

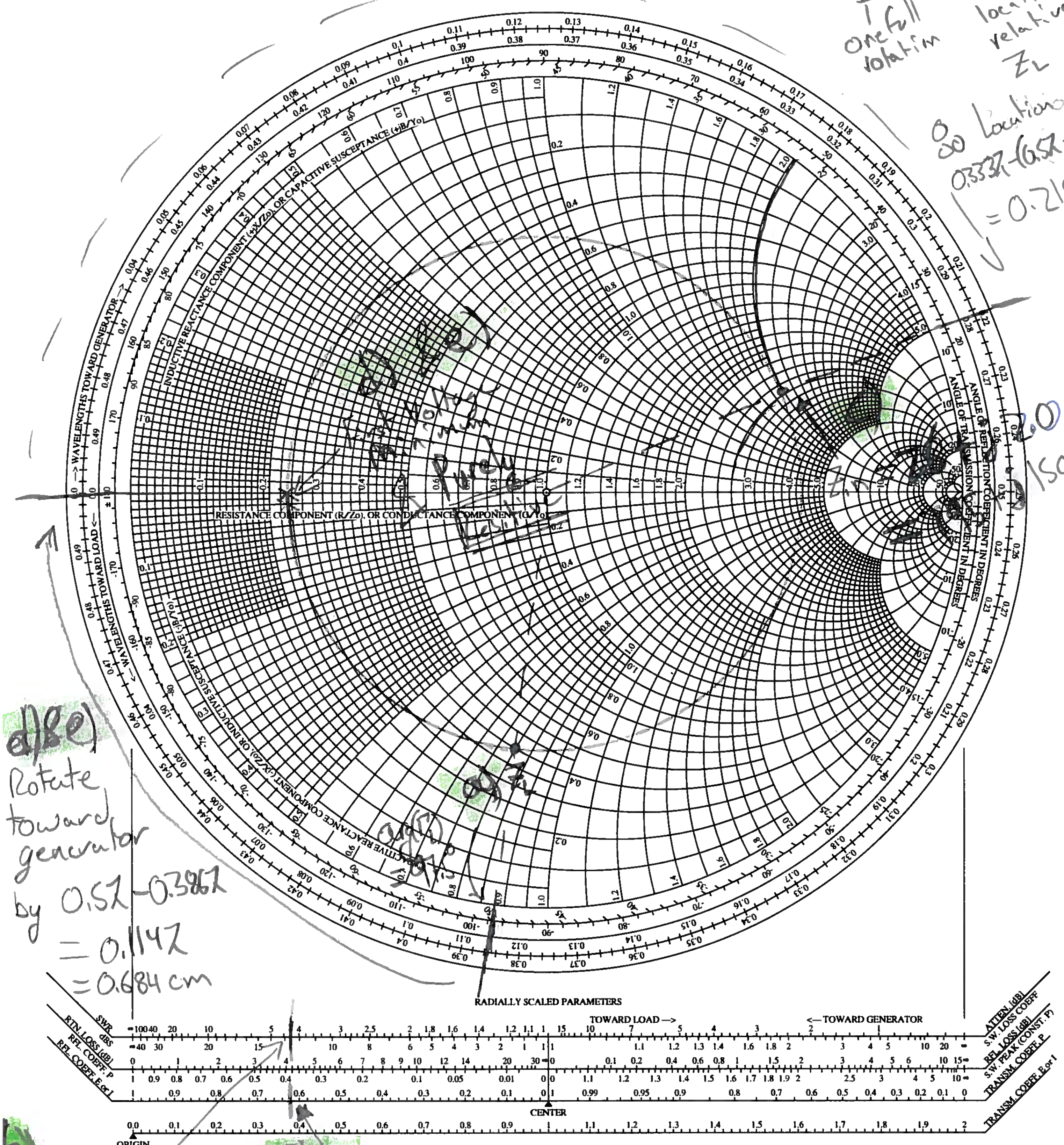
$= 0.5\lambda + 0.333\lambda$

one full rotation

location relative to Z_L

So location $= 0.333\lambda - (0.5\lambda - 0.386)$

$= 0.219\lambda$



d)&e) Rotate toward generator by $0.5\lambda - 0.386\lambda$

$= 0.114\lambda$

$= 0.684\ cm$

b)

c) $|\Gamma| = 0.62$

VSWR = 4.3