

**ENEL 476 Winter 2018**

**Assignment #2**

**Due on Friday Feb 16, 2018 at 5 pm**

You are designing a microwave imaging system to investigate the brain. The system design involves transmitting a wave from an antenna towards the head. The signal propagates through free space towards the head, so you start by modeling this wave.

The electric and magnetic fields of this wave are represented as a uniform plane wave propagating in free space ( $\epsilon_r=1$ ,  $\mu_r=1$ ,  $\sigma=0$  S/m). You can assume that only free space is present. The electric field is given by:

$$\mathbf{E}(x,t)=10\cos(3 \times 10^8 t + \beta x) \mathbf{a}_y \text{ V/m}$$

Find:

- a) The phase constant,  $\beta$ .
- b) The wavelength,  $\lambda$ .
- c) The phase velocity,  $v_p$  or  $u$ .
- d) The intrinsic impedance,  $\eta$ .
- e) The polarization.
- f) The magnetic field in phasor form,  $\mathbf{H}_s(x)$ .

In the head, the wave propagates through various tissues. You start by investigating wave propagation in skull, which has  $\epsilon_r=25$ ,  $\mu_r=1$ ,  $\sigma=0.25$  S/m. Consider a uniform plane wave propagating in an infinite region filled with this material (i.e. you don't need to consider transmission and reflection).

- g) Find the phase constant ( $\beta$ ) in the material.
- h) Find the attenuation constant ( $\alpha$ ) in the material.
- i) Find the skin depth in the material.
- j) Find the intrinsic impedance ( $\eta$ ) of the material.
- k) Assuming that the properties of the materials don't change with frequency, over what frequency range might this material be considered a good conductor?

The electric field amplitude in the skull at  $x=0$  is given by 1.86 V/m. The wave propagates in the  $-x$  direction and the electric field is oriented in the  $y$  direction. The frequency is the same as the wave in free space.

- l) Find an expression for the electric field ( $\mathbf{E}(x,t)$ ) in the skull.
- m) Find an expression for the magnetic field ( $\mathbf{H}(x,t)$ ) in the skull.
- n) Find an expression for the time-averaged Poynting vector ( $\mathbf{P}(x)$ ) in the skull. How much does this power density decrease after the wave travels a distance of 7 mm?