ENEL 471 – Winter 2020

Assignment 1 - Solutions

Problem 2.16

The transfer function of the summing block is:

$$H_1(f) = \left[1 - \exp(-j2\pi fT)\right].$$

The transfer function of the integrator is:

$$H_2(f) = \frac{1}{j2\pi f}$$

These elements are cascaded. The equivalent transfer function of the cascade is:

$$\begin{aligned} H(f) &= \left(H_1(f)H_2(f)\right)\cdot\left(H_1(f)H_2(f)\right) \\ &= -\frac{1}{\left(2\pi f\right)^2}\left[1 - \exp(-j2\pi fT)\right]^2 \\ &= -\frac{1}{\left(2\pi f\right)^2}\left[1 - 2\exp(-j2\pi fT) + \exp(-j4\pi fT)\right] \end{aligned}$$

Problem 2.21

$$H(f) = X(-f)\exp(j2\pi fT)$$

$$\begin{aligned} X(f) &= \frac{A}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] * T\operatorname{sinc}(fT)\exp(-j2\pi f\frac{T}{2}) \\ &= \frac{AT}{2}\left[\operatorname{sinc}(T(f - f_c)) + \operatorname{sinc}(T(f + f_c))\right]\exp(-j\pi fT)\exp(j\pi N) \end{aligned}$$

Let $f_c = \frac{N}{T}$ for N large

$$
\begin{aligned}
Y(f) &= H(f)X(f) \\
&= X(-f)\exp(j2\pi fT)\exp(-j\pi fT)\exp(j\pi N)\frac{AT}{2}\left[\operatorname{sinc}\left(T(f-f_c)\right)+\operatorname{sinc}\left(T(f+f_c)\right)\right] \\
&= \exp(j2\pi fT)\frac{A^2T^2}{4}\left[\operatorname{sinc}\left(T(f-f_c)\right)+\operatorname{sinc}\left(T(f+f_c)\right)\right]\left[\operatorname{sinc}\left(T(-f-f_c)\right)+\operatorname{sinc}\left(T(-f+f_c)\right)\right] \\
&= \exp(j2\pi fT)\frac{A^2T^2}{4}\left[\operatorname{sinc}(-fT-N)+\operatorname{sinc}(-fT+N)\right]\left[\operatorname{sinc}(fT-N)+\operatorname{sinc}(fT+N)\right]
\end{aligned}
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But sinc(x)=sinc(-x)

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\therefore Y(f) = \exp(j2\pi fT)\frac{A^2T^2}{4}\left[\operatorname{sinc}(fT-N)+\operatorname{sinc}(fT+N)\right]^2
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