BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY

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Project Report on,

Signal Sampling & Reconstruction Via Proteus

Submitted to:

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1. Introduction

Digital computers can process discrete time signals using extremely flexible and powerful algorithms. However, most signals of interest are continuous time signals, which is how data almost always appears in nature. This report introduces the concepts & practical circuits behind converting continuous time signals into discrete time signals through a process called sampling. The sampling process produces a discrete time signal from a continuous time signal by examining the value of the continuous time signal at equally spaced points in time. Reconstruction, also known as interpolation, attempts to perform an opposite process that produces a continuous time signal coinciding with the points of the discrete time signal. Because the sampling process for general sets of signals is not invertible, there are numerous possible reconstructions from a given discrete time signal, each of which would sample to that signal at the appropriate sampling rate. This report will also highlight on reconstruction of the input signals. Perfect reconstruction of a bandlimited continuous time signal from its sampled version is possible using the Whittaker-Shannon reconstruction formula, which makes use of the ideal lowpass filter and its sinc function impulse response, if the sampling rate is sufficiently high.

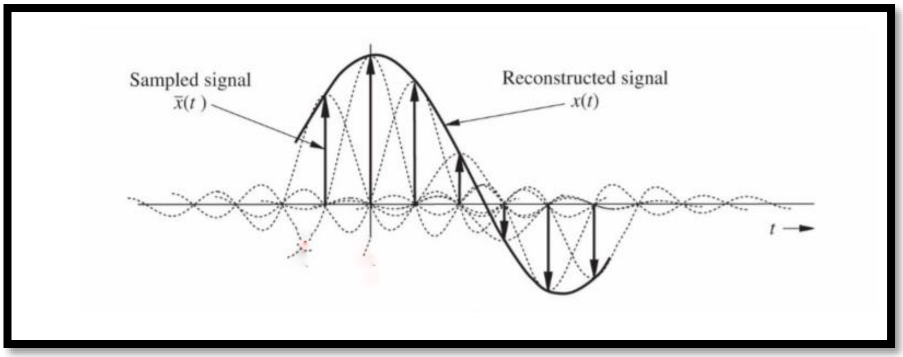

Figure 1 Signal Sampling & Reconstruction Illustration

2. Background Theory of Circuital Implementation

2.1 Sampling Brief

Sampling a continuous time signal produces a discrete time signal by selecting the values of the continuous time signal at equally spaced points in time. Basically, there are three types of sampling techniques such as:

(i) Instantaneous sampling

(ii) Natural sampling

(iii) Flat top sampling

Out of these three, instantaneous sampling is called ideal sampling whereas natural sampling and flat-top sampling are called practical sampling methods.

2.1.1 Instantaneous Sampling

In this type of sampling, the sampling function is a train of impulses.

x(t) is the input signal (i.e., signal to be sampled) as shown in fig.2(a).

Fig.2(c) shows a circuit to produce instantaneous or ideal sampling. This circuit is known as the switching sampler.

The working principle of this circuit is quite easy. The circuit simply consists of a switch. Now if we assume that the closing time ‘t’ of the switch approaches zero, then the output g(t) of this circuit will contain only instantaneous value of the input signal x(t).

Since the width of the pulse approaches zero, the instantaneous sampling gives a train of impulses of height equal to the instantaneous value of the input signal x(t) at the sampling instant. We know that the train of impulses may be represented as,

$$\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nTs)$$

This is known as sampling function and its waveform is shown in fig.2(b).

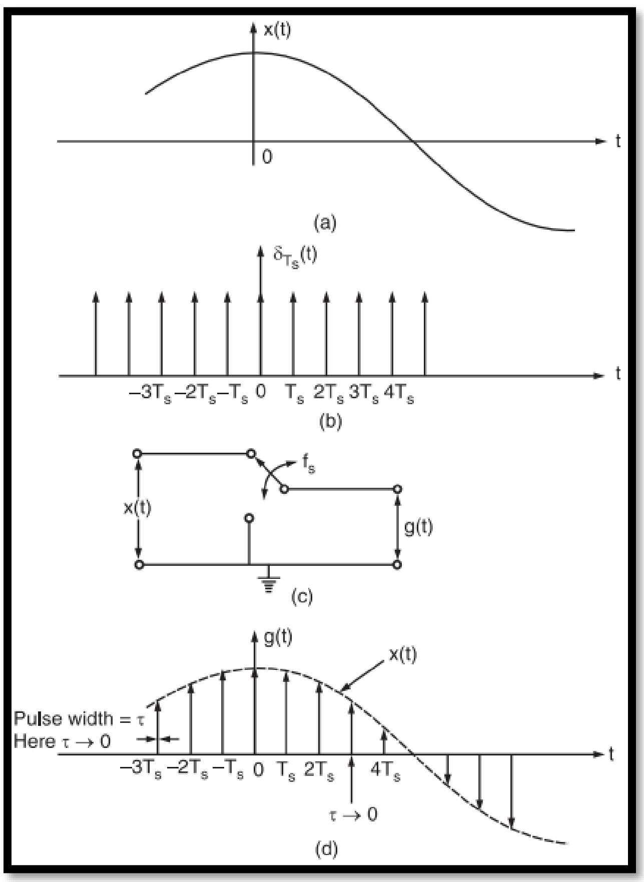

Figure 2(a) Baseband signal, (b) impulse train, (c) functional diagram of a switching sampler, (d) sampled signal

The sampled signal g(t) is expressed as the multiplication of x(t) and $\delta T_s(t)$.

Thus,

$$g(t) = x(t) \cdot \delta_{T_s}(t)$$

$$g(t) = x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

Or

$$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \delta(t - nT_s)$$

2.1.2 Natural Sampling

As we have already discussed, the instantaneous sampling results in the samples whose width τ approaches zero. Due to this, the power content in the instantaneously sampled pulse is negligible. Thus, this method is not suitable for transmission purpose.

Natural sampling is a practical method. In this type of sampling, the pulse has a finite width equal to τ.

Let us consider an analog continuous-time signal x(t) to be sampled at the rate of f_s Hertz.

Here it is assumed that f_s is higher than Nyquist rate such that sampling theorem is satisfied.

Again, let us consider a sampling function c(t) which is a train of periodic pulses of width τ and frequency equal to f_s Hz.

Fig.3 shows a functional diagram of a natural sampler.

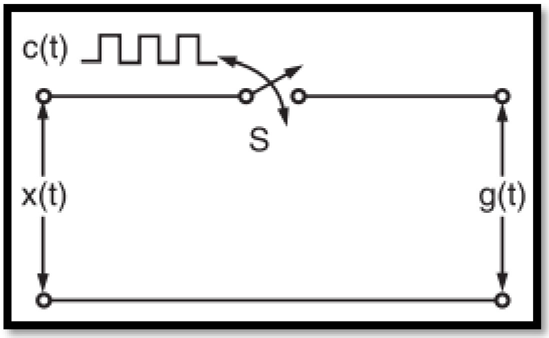

Figure 3 A functional diagram of a natural sampler

With the help of this natural sampler, a sampled signal g(t) is obtained by multiplication of sampling function c(t) and input signal x(t).

Now, according to fig.3, we have when c(t) goes high, the switch ‘S’ is closed. Therefore,

g(t) = x(t) when c(t) = A

g(t) = 0 when c(t) = 0

where A is the amplitude of c(t).

The waveforms of signals x(t), c(t) and g(t) have been illustrated in fig.4(a), (b) and (c) respectively.

Now, the sampled signal g(t) may also be described mathematically as

g(t) = c(t). x(t)

Here, c(t) is the periodic train of pulse of width t and frequency f_s.
We know that the exponential Fourier series for any periodic waveform is expressed as,

$$c(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi nt/T_0}$$

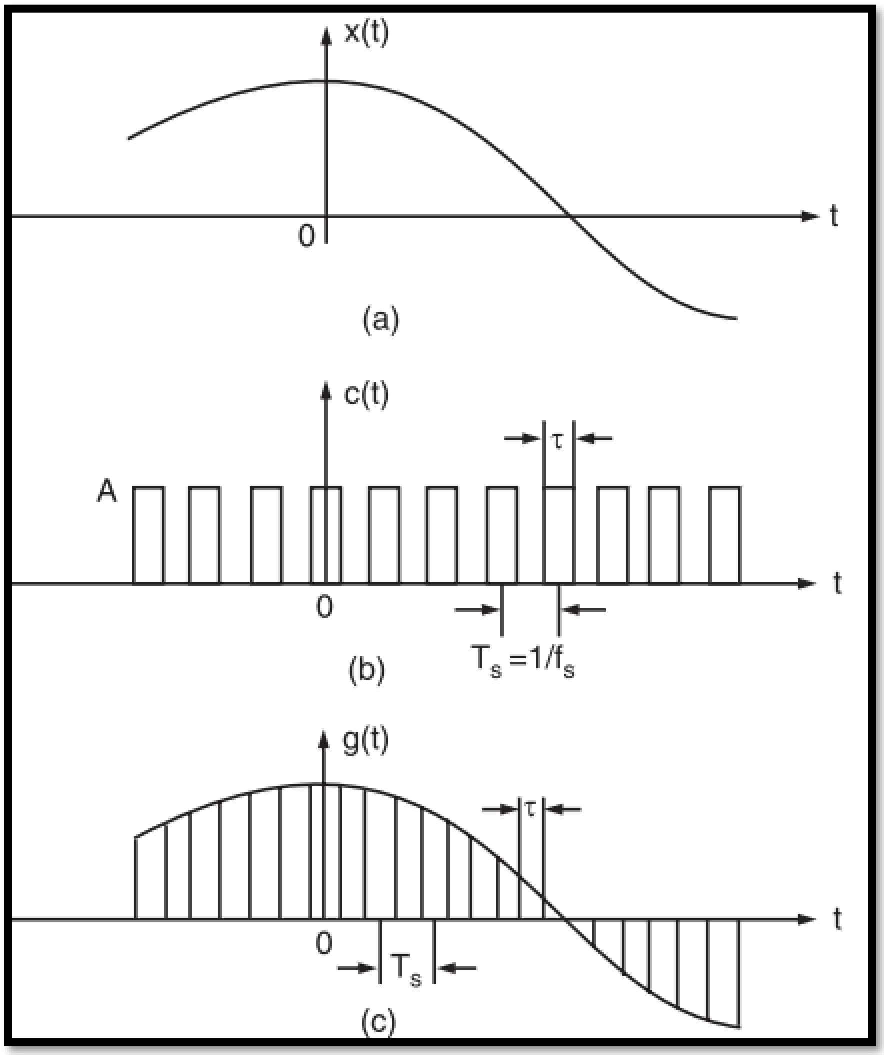

Figure 4 (a) Continuous time signal x(t), (b) Sampling function waveform i.e., periodic pulse train, (c) Naturally sampled signal waveform g(t)

Also, for the periodic pulse train of c(t), we have,

$$T_0 = T_s = \frac{1}{f_s} = \text{period of } c(t)$$

or $$f_0 = f_s = \frac{1}{T_0} = \frac{1}{T_s} = \text{frequency of } c(t)$$

So, we have

$$c(t) = \sum_{n=-\infty}^{\infty} C_n . e^{j2\pi f_s nt} \quad \text{with} \quad \frac{1}{T_0} = f_s$$

Now, it may be noted that since c(t) is a rectangular pulse train, therefore C_n for this waveform will be expressed as,

$$C_n = \frac{TA}{T_0} \sin c\,(f_n.T)$$

here T = pulse width = τ

and f_n = harmonic frequency

But here, $f_n = nf_s$

or,

$$f_s = \frac{n}{T_0} = nf_0$$

Hence,

$$C_n = \frac{\tau.A}{T_s} \cdot \sin c(f_{n.}\tau)$$

Therefore, the Fourier series representation for c(t) will be given as,

$$c(t) = \sum_{n=-\infty}^{\infty} \frac{\tau.A}{T_s} \cdot \sin c(f_{n.}\tau)\, e^{j2\pi f_s \cdot nt}$$

Now, substituting the value of c(t) in the equation of g(t), we get,

$$g(t) = \frac{\tau A}{T_s} \cdot \sum_{n=-\infty}^{\infty} \sin c(f_{n.}\tau).\, e^{j2\pi f_s nt} .x(t)$$

This is required time-domain representation for naturally sampled signal g(t).

2.1.3 Flat Top Sampling

Flat top sampling like natural sampling is also a practically possible sampling method. But natural sampling is little complex whereas flat top sampling is quite easy.

In flat-top sampling or rectangular pulse sampling, the top of the samples remains constant and is equal to the instantaneous value of the baseband signal x(t) at the start of sampling.

The duration or width of each sample is τ and sampling rate is equal to $f_s = 1 / T_S$. Fig.5(a) shows the functional diagram of a sample and hold circuit which is used to generate the flat top samples.

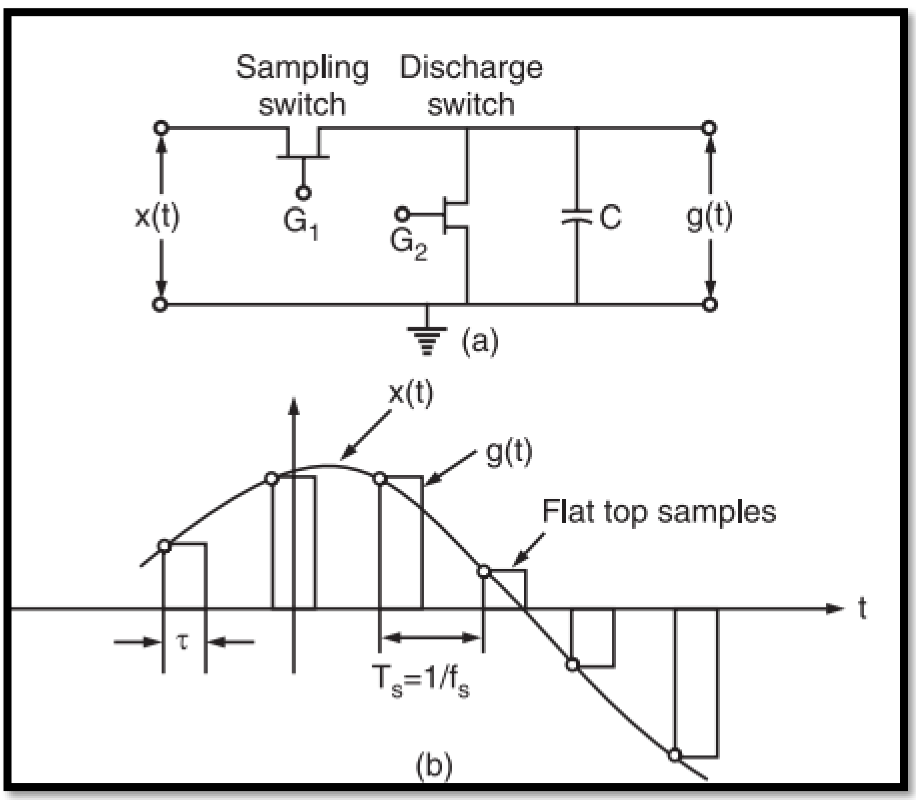

Figure 5:(a) A sample and hold circuit to generate flat top samples (b) A general waveform of flat top sampling

Fig. 5(b) shows the general waveform of flat top samples. From fig.5(b), it may be noted that only starting edge of the pulse represents instantaneous value of the baseband signal x(t).

Also, the flat top pulse of g(t) is mathematically equivalent to the convolution of instantaneous sample and a pulse h(t) as depicted in fig.6.

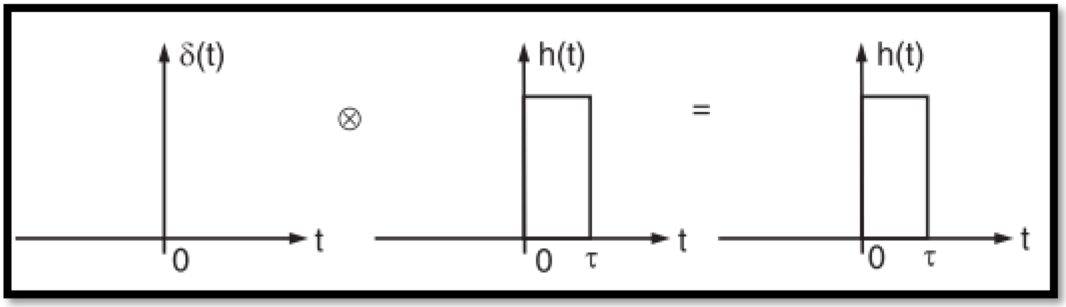

Figure 6 Convolution of any function with delta function is equal to that function

This means that the width of the pulse in g(t) is determined by the width of h(t) and the sampling instant is determined by delta function.

In fig. 5(b), the starting edge of the pulse represents the point where baseband signal is sampled and width is determined by function h(t). Therefore, g(t) will be expressed as,

$$g(t) = s(t) \otimes h(t)$$

This equation has been explained in fig.6.

Now, from the property of delta function, we know that for any function f(t)

$$f(t) \otimes \delta(t) = f(t)$$

This property is used to obtain flat top samples. It may be noted that to obtain flat top sampling, we are not applying the above equation directly here i.e., we are applying a modified form of the above equation.

Thus, in this modified equation, we are taking s(t) in place of delta function δ(t).

Observe that δ(t) is a constant amplitude delta function whereas s(t) is a varying amplitude train of impulses. This means that we are taking s(t) which is an instantaneously sampled signal and this is convolved with function h(t).

Therefore, on convolution of s(t) and h(t), we get a pulse whose duration is equal to h(t) only but amplitude is defined by s(t).

Now, we know that the train of impulses may be represented mathematically as,

$$\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

The signal s(t) is obtained by multiplication of baseband signal x(t) and $\delta T_s(t)$. Thus,

$$s(t) = x(t) \cdot \delta_{T_s}(t)$$

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \delta(t - nT_s)$$

Now, sampled signal g(t) is given as

$$g(t) = s(t) \otimes h(t)$$

or $$g(t) = \int_{-\infty}^{\infty} s(\tau)h(t-\tau)\, d\tau$$

or $$g(t) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(nT_s)\, \delta(\tau - nT_s)h\,(t-\tau)\, d\tau$$

or $$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \int_{-\infty}^{\infty} \delta(\tau - nT_s)h\,(t-\tau)\, d\tau$$

According to shifting property of delta function, we know that

$$\int_{-\infty}^{\infty} f(t)\, \delta(t-t_0) = f(t_0)$$

Hence,

$$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, h(t-nT_s)$$

This equation represents value of g(t) in terms of sampled value $x(nT_s)$ and function $h(t - nT_s)$ for flat top sampled signal.

In figure 7, the entire scheme of this flat top sampling or also popularly known as rectangular pulse sampling scheme has been shown at a glance.

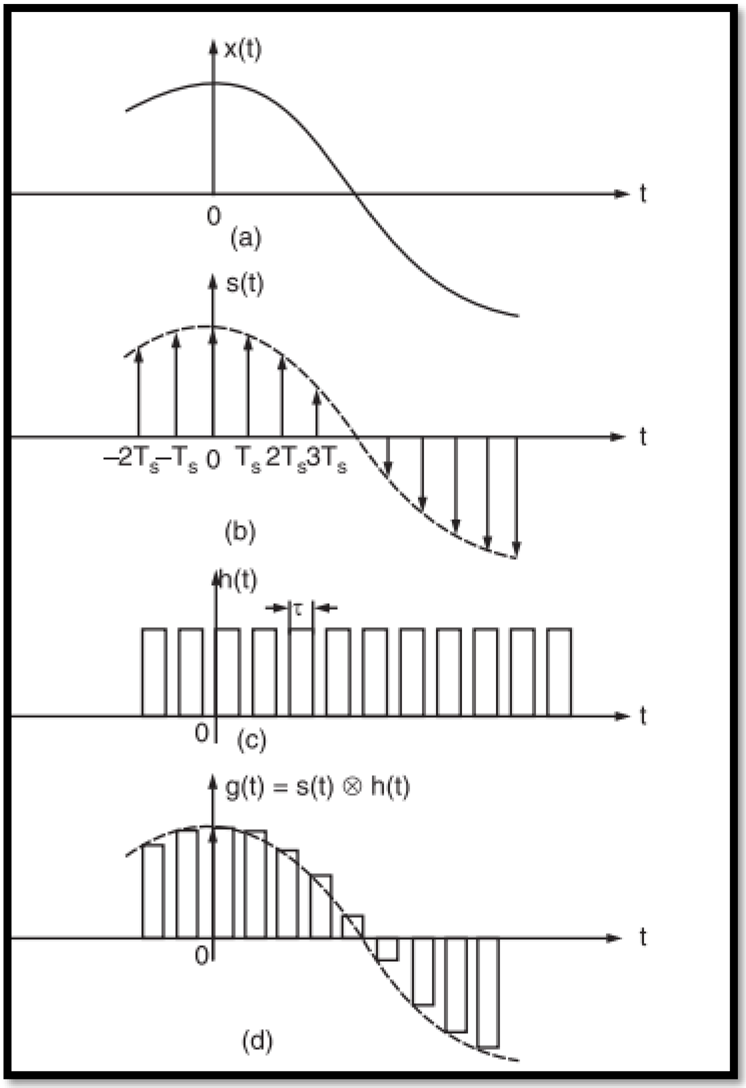

Figure 7 (a) Baseband signal x(t), (b) Instantaneously sample signal s(t), (c) Constant pulse width function h(t), (d) Flat top sampled signal g(t) obtained through convolution of h(t) and s(t)

2.2 Reconstruction Brief

In all the sampling cases which had outputs later to be reconstructed via low pass filters, the shape or width of the roll-off also called the "transition band", for a simple first-order filter may be too long or wide and so active filters designed with more than one "order" are required.

The complexity or filter type is defined by the filters "order", and which is dependent upon the number of reactive components such as capacitors or inductors within its design. We also know that the rate of roll-off and therefore the width of the transition band, depends upon the order number of the filter and that for a simple first-order filter it has a standard roll-off rate of 20dB/decade or 6dB/octave.

Then, for a filter that has an n-th number order, it will have a subsequent roll-off rate of 20n dB/decade or 6n dB/octave. So, a first-order filter has a roll-off rate of 20dB/decade (6dB/octave), a second-order filter has a roll-off rate of 40dB/decade (12dB/octave), and a fourth-order filter has a roll-off rate of 80dB/decade (24dB/octave), etc, etc.

2.2.1 Low Pass Butterworth Filter

The frequency response of the Butterworth Filter approximation function is also often referred to as "maximally flat" (no ripples) response because the pass band is designed to have a frequency response which is as flat as mathematically possible from 0Hz (DC) until the cut-off frequency at -3dB with no ripples. Higher frequencies beyond the cut-off point rolls-off down to zero in the stop band at 20dB/decade or 6dB/octave. This is because it has a "quality factor", "Q" of just 0.707.

Butterworth filters are not designed to keep a constant phase angle at the cutoff frequency. A basic low pass filter of -20 dB/decade has a phase angle of -45^o at ω_c. A -40 dB/decade Butterworth filter has a phase angle of -90^o at ω_c, and a -80 dB/decade filter has a phase angle of -180^o at ω_c. Therefore, for each increment of -20 dB/decade, the phase angle will increase by -45^o at ω_c.

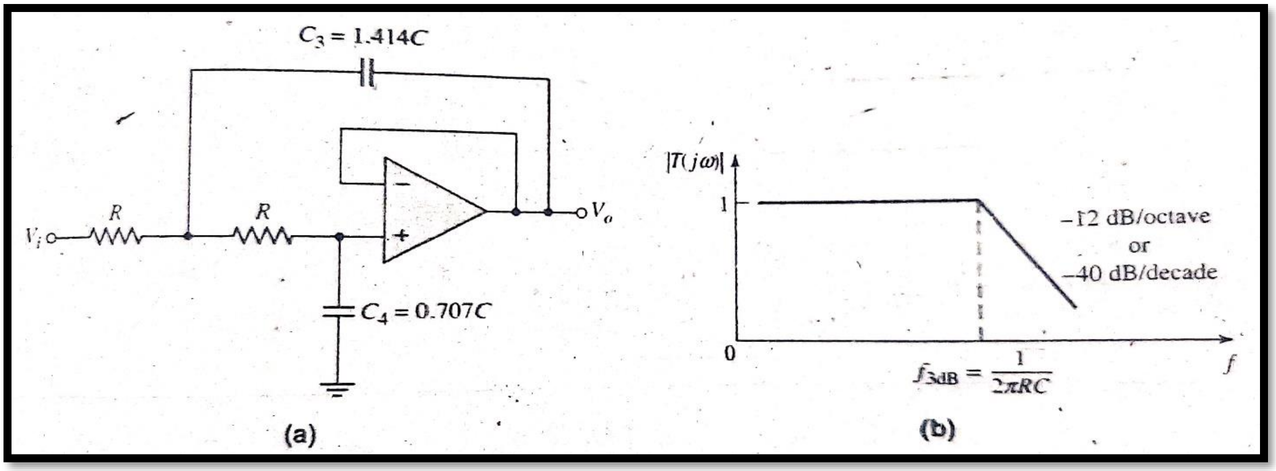

Figure 8 (a)Two-pole Low Pass Butterworth filter (b)Bode plot, transfer function magnitude

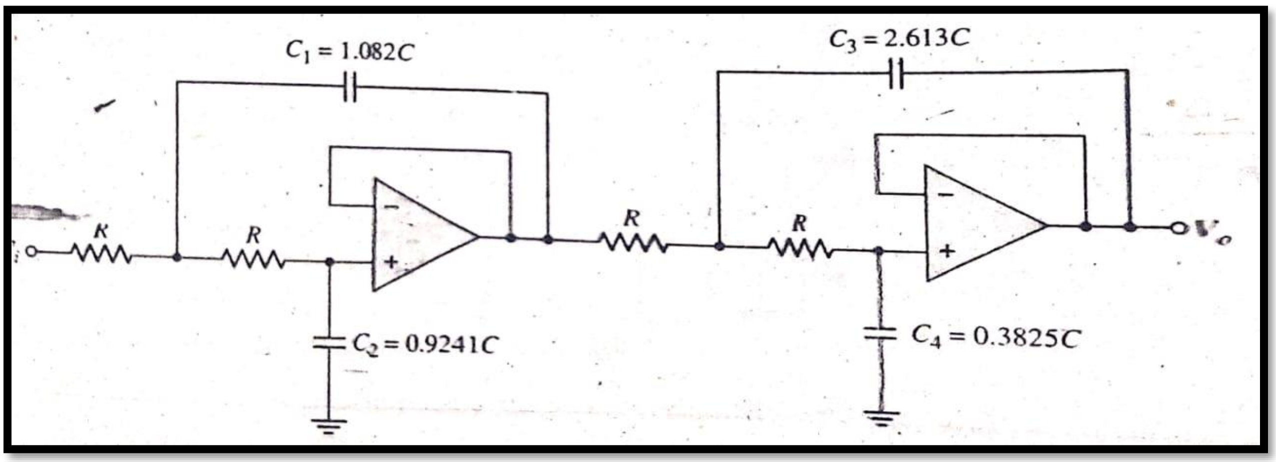

Figure 9 Four-pole low-pass Butterworth Filter

The two-pole LPF was used in this project for the detection of the single input signals & the higher order four-pole LPF was used for the multiple signals of different frequencies. For the higher order filter, the maximally flat response is not obtained simply by cascading two two-pole filters. The relationship between the capacitors is found through the first three derivatives of the transfer function.

3. Circuit Implementation: Single Input

3.1 Ideal Sampling & Reconstruction

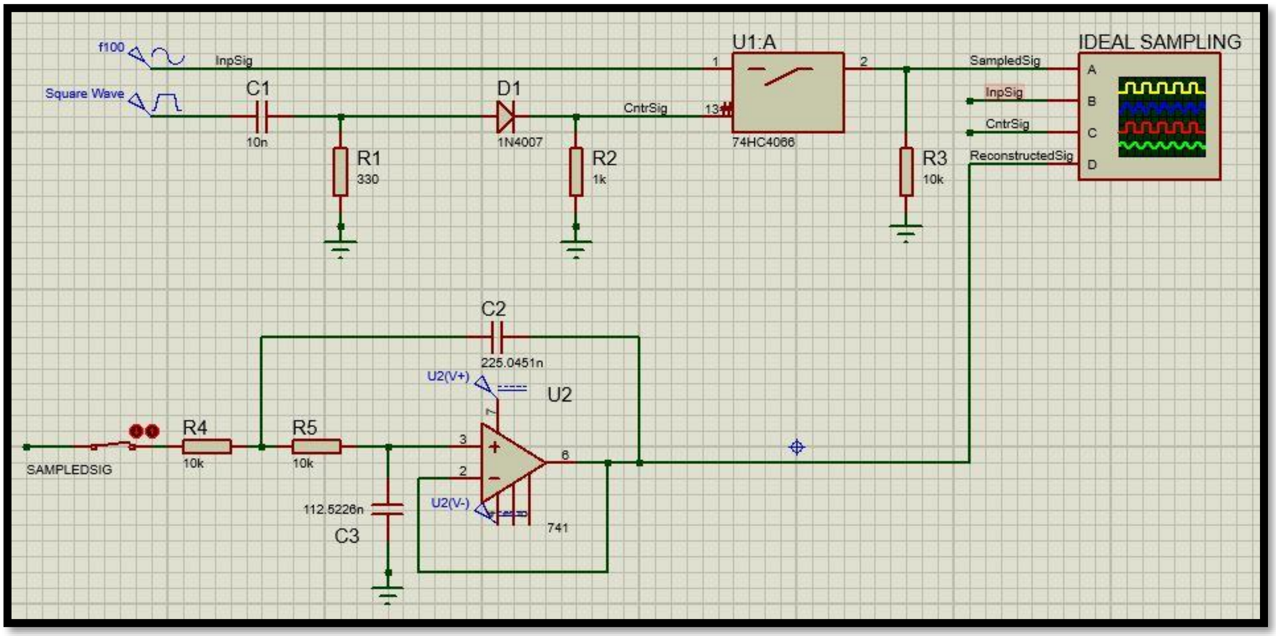

Figure 10 Instantaneous sampling & reconstruction circuit

- A 100Hz sinusoid was used as the input.
- 74HC4066 switch was used as a sampling switch. This is a high-speed CMOS Quad Logic Chip, which contains silicon gate CMOS technology. Widely used as analog switches. They can turn ON or OFF with external logic signals. As the closing time of the switch approaches zero, then the output of this circuit will contain only instantaneous value of the input signal.
- As the logic/control signal, an impulse signal was generated by using a square wave(1000Hz) initially & then having it differentiated & rectified.Since the width of the pulse approaches zero, the instantaneous sampling gives a train of impulses of height equal to the instantaneous value of the input signal at the sampling instant.

- After sampling the signal, the output is fed to a 100Hz two-pole LP Butterworth filter which reconstructed the signal at a 45^{o} lag.

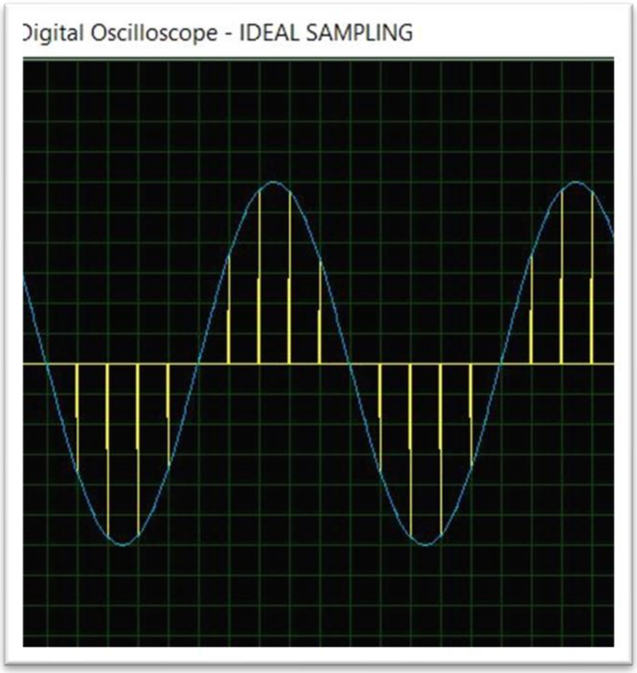

Figure 11 Sampled Signal (Yellow)

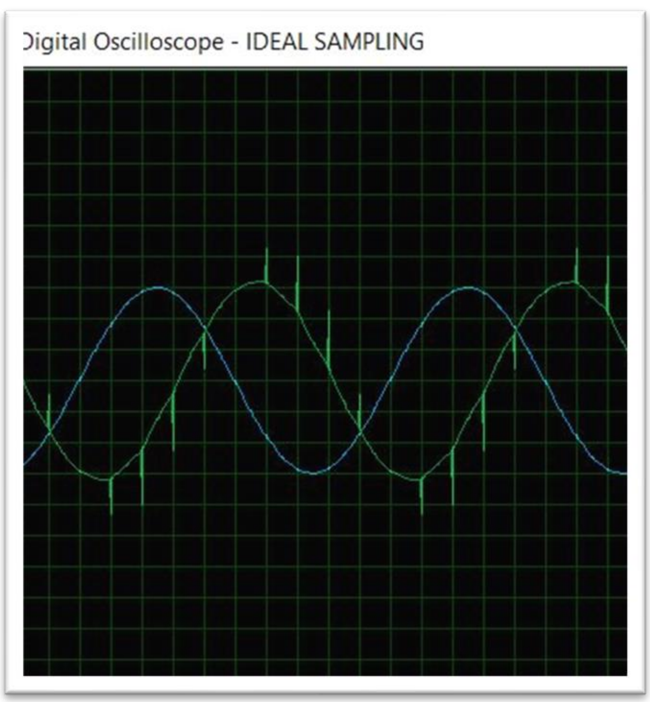

Figure 12 Reconstructed Signal (Green)

3.2 Natural Sampling & Reconstruction

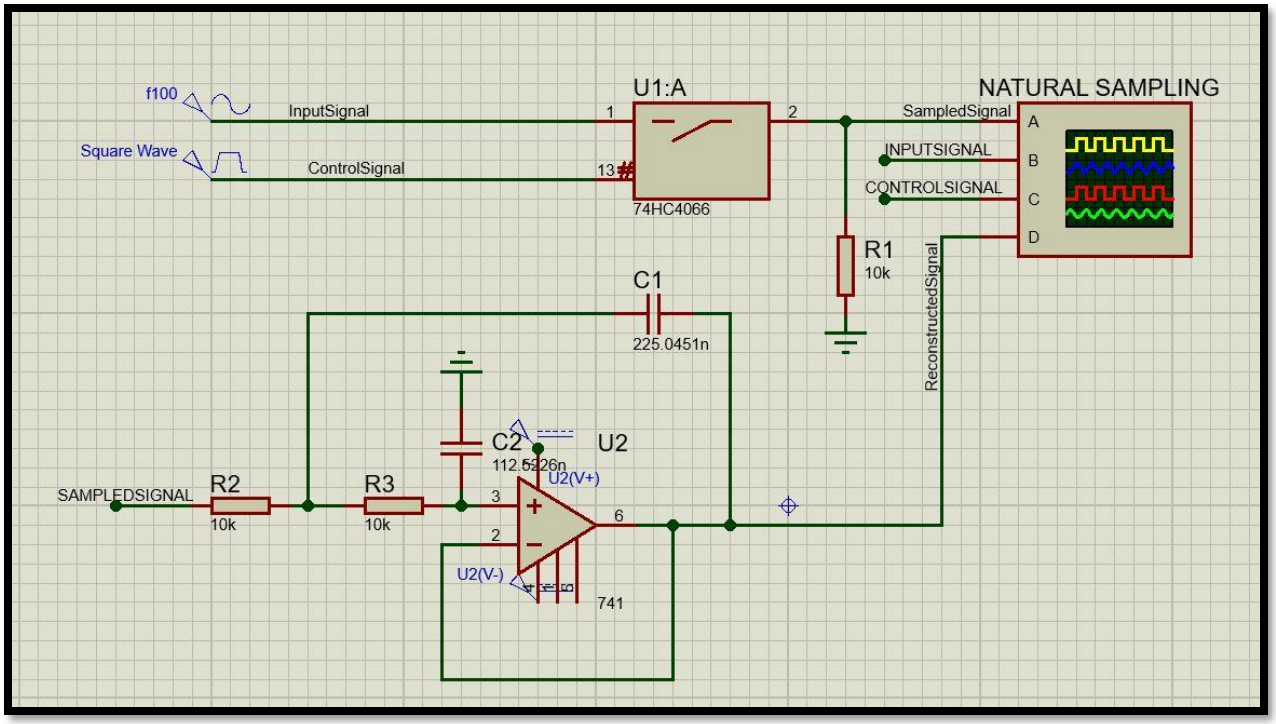

Figure 13 Natural Sampling & Reconstruction Circuit

- A 100Hz sinusoid was used as the input
- 74HC4066 switch was used as a sampling switch. The is a high-speed CMOS Quad Logic Chip, which contains silicon gate CMOS technology. Widely used as analog switches. They can turn ON or OFF with external logic signals.
- As the logic/control signal, a square wave was used of 1000Hz.
- The circuit works on the chopping principle. As the closing time of the switch approaches zero, then the output of this circuit will contain only the value of the multiplication of those signals at that moment.
- After sampling the signal, the output was fed to a 100Hz two-pole LP Butterworth filter which reconstructed the signal at a 45^{o} lag.

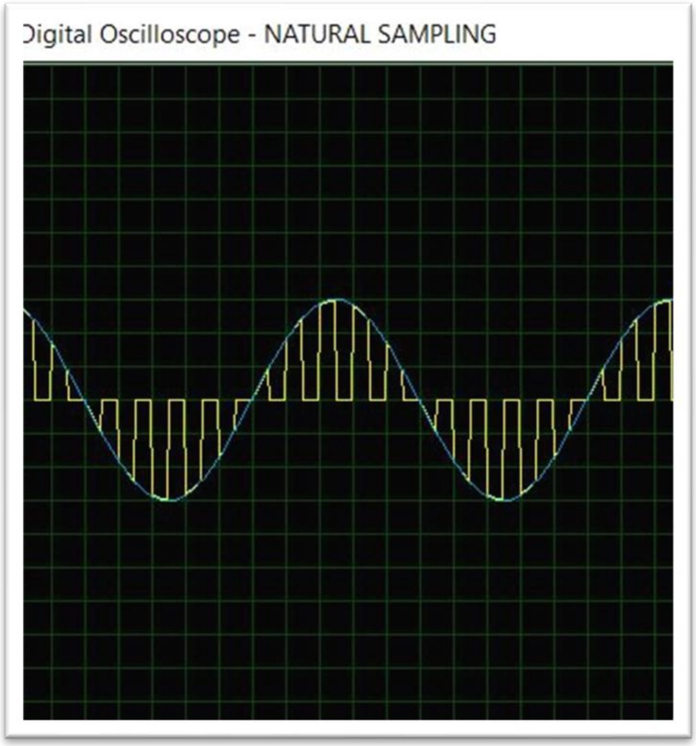

Figure 14 Sampled Signal (Yellow)

Figure 15 Reconstructed Signal (Green)

3.3 Flat Top Sampling & Reconstruction

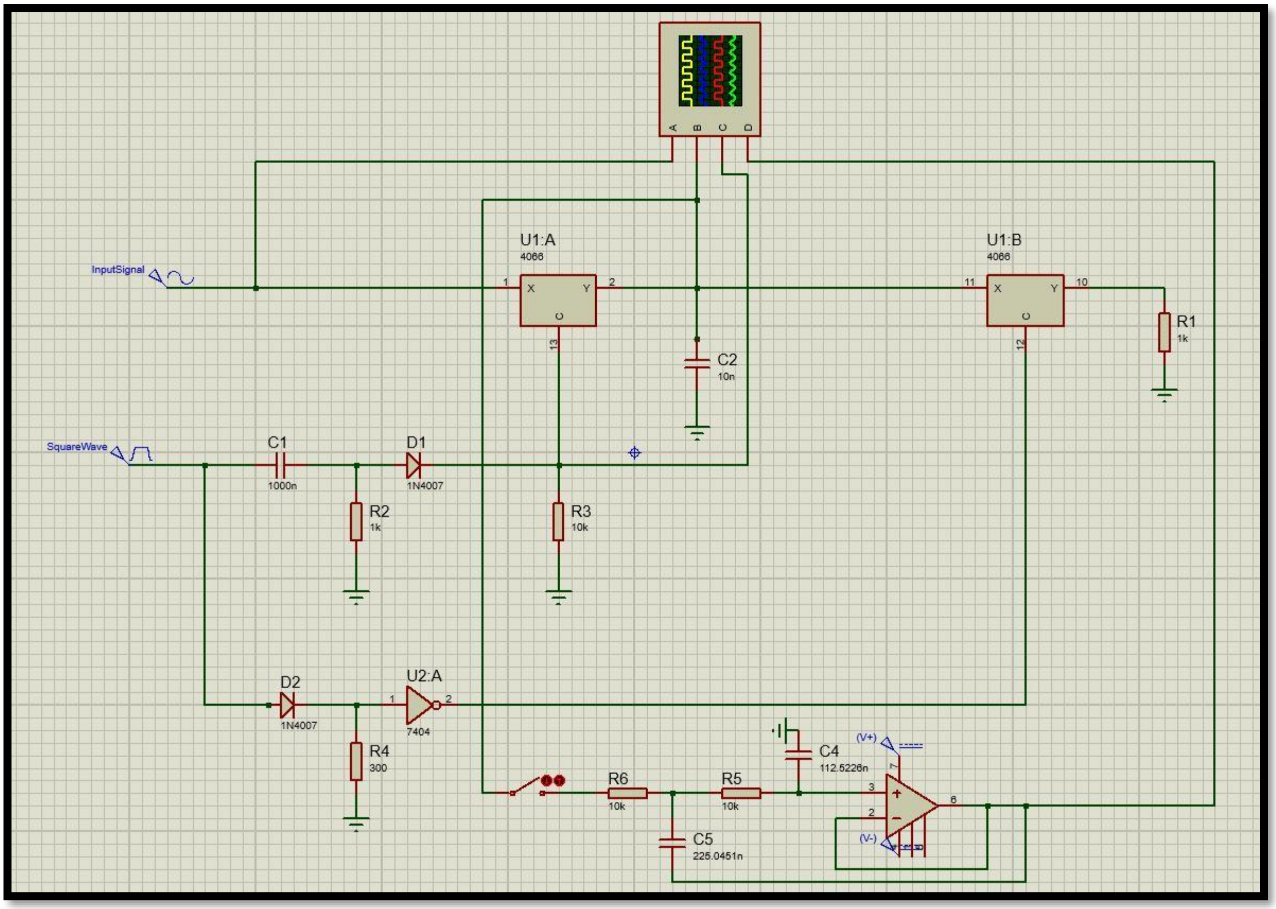

Figure 16 Flat Top Sampling & Reconstruction Circuit

- A 100Hz sinusoid was used as the input.
- 4066 switches were used one as a sampling switch and the other as a discharging switch. They are high-speed CMOS Quad Logic Chip, which contains silicon gate CMOS technology. Widely used as analog switches. They can turn ON or OFF with external logic signals.
- The sampling switch samples the input & the discharging switch holds the sampled value accordingly to the control signal.

- A square wave of 1000Hz was initially introduced. Afterwards, for the sampling switch, the wave was made into an impulse signal to use as the control signal. These made instantaneous samples in the output of the switch of the input, storing in a capacitor.
- After that, the same initial square wave was rectified & inverted to use as the control signal for the discharging switch. Inverting principle was done sothat for the cycle when the signal gets instantaneously sampled at the sampling switch, the discharging switch was not in operation. After that in the next cycle when the sampling switch gets opened, the discharging switch starts its operation & the previously charged capacitor gets discharges whilst generating our desired flat-top sampling.
- Thus, the top of the samples remains constant and is equal to the instantaneous value of the baseband signal at the start of sampling, and later utilizing the sample & hold configuration via discharging, generates the desired flat top sampling.
- The entire output has been taken across the middle capacitor as it is the one which gets charged & discharged simultaneously with a view to producing the sampled output.
- After sampling the signal, the output is fed to a 100Hz two-pole LP Butterworth filter which reconstructed the signal at a 45^o lag.

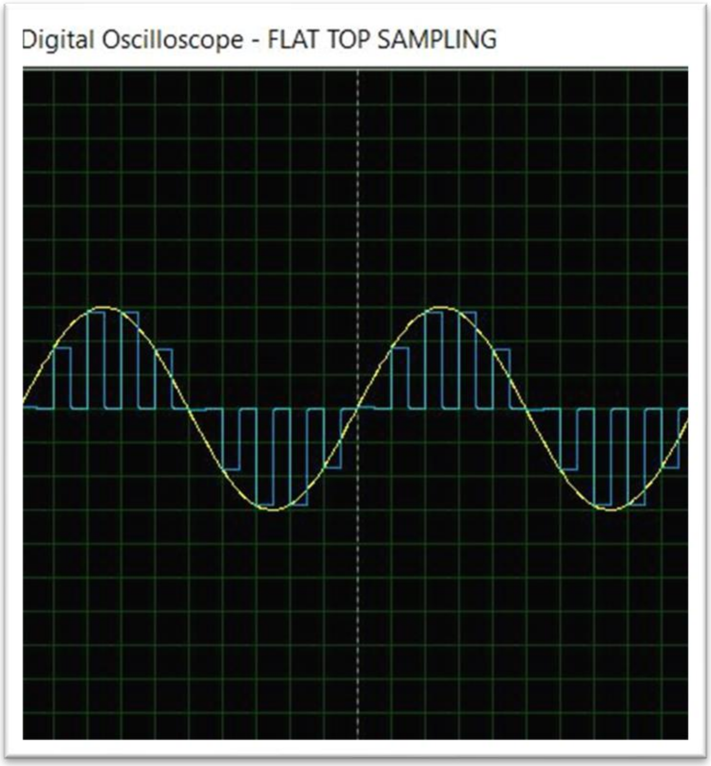

Figure 17 Sampled Signal (Blue)

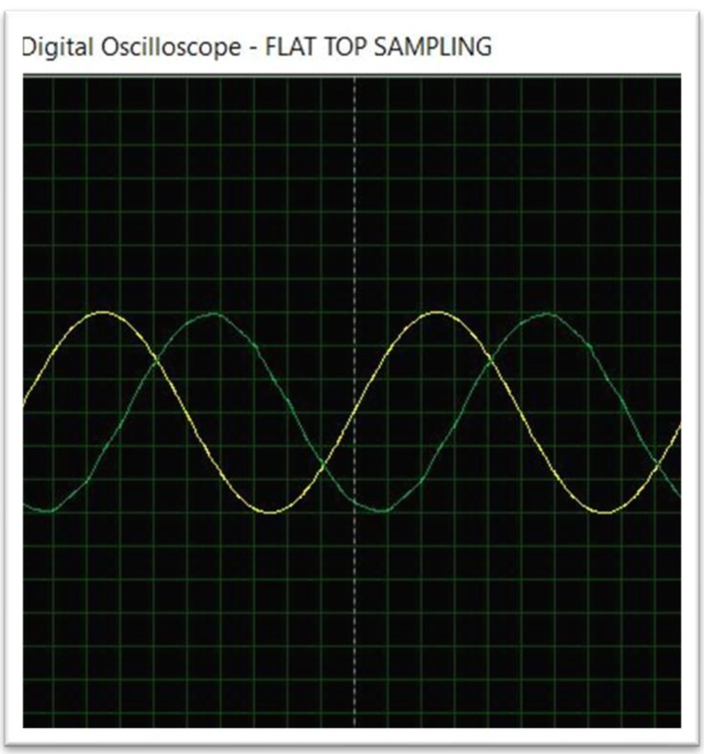

Figure 18 Reconstructed Signal (Green)

4. Circuit Implementation: Multiple Input Signals with Different Frequencies

In the below final circuit, all the parameters were adjusted to work both with & without the Noise addition to the sampled signal. Necessary switches have been set with distinct tags to operate the circuit for required modes.

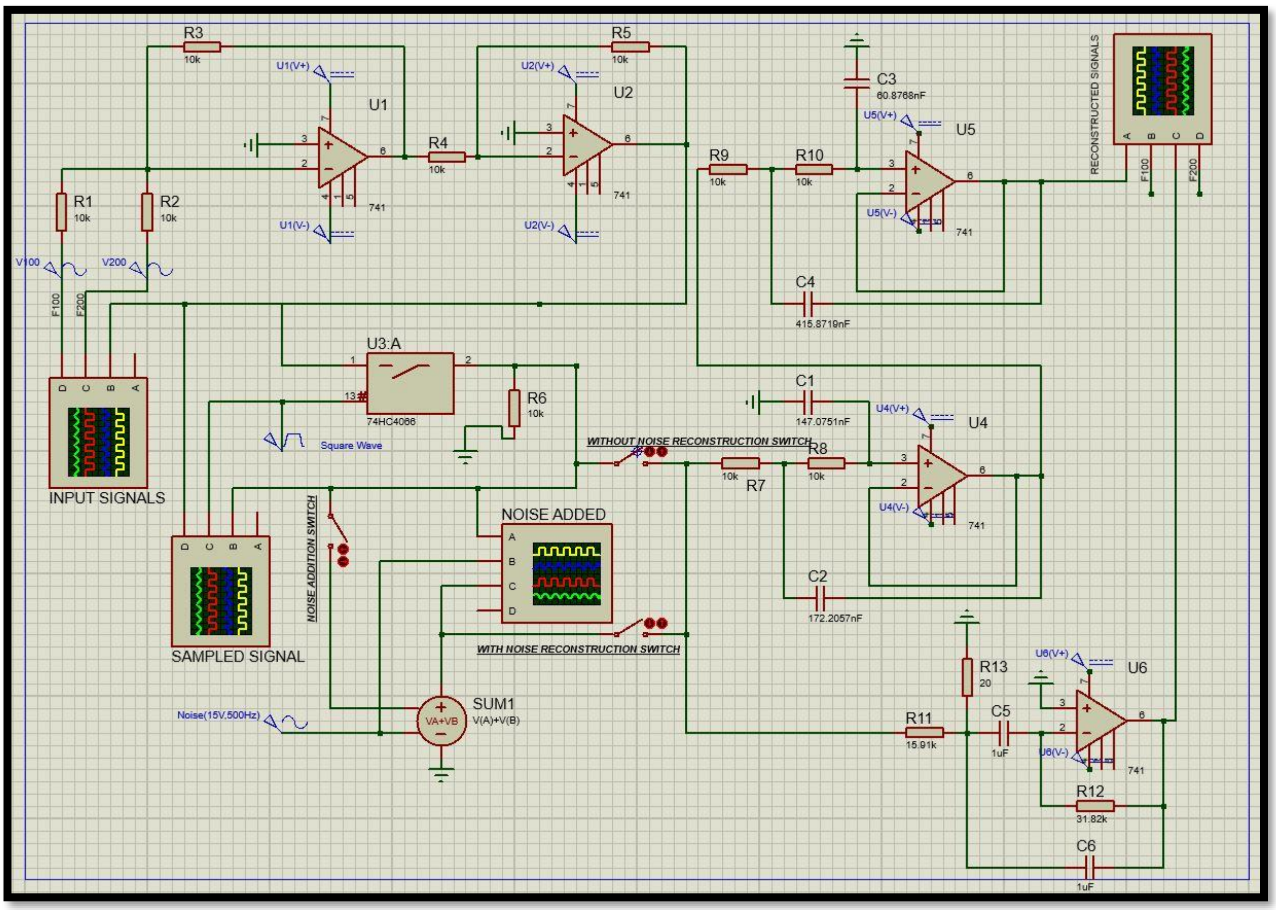

Figure 19 Multiple Input Signals with Different Frequencies Sampling & Reconstruction Circuit Both With & Without Noise Addition

4.1 Circuit Blocks Introduction

- 02 sine wave input generators having parameters:

  15V Amp, 100 Hz Freq (F100)

  15V Amp, 200 Hz Freq (F200)

- Non-inverting Adder (to sum the inputs)
- 01 square wave control generator of 1000Hz frequency
- 74HC4066 switch for Natural Sampling
- A noise generator & summer block to add noise in the transmission line
- A 4-pole Butterworth LPF to reconstruct the 100Hz signal & a Narrowband Bandpass Filter to reconstruct the 200Hz signal
- 04 Digital Oscilloscopes to show required signals at various terminals

4.2 Compound Signal Sampling

- The input signals were added using the adder
- Afterwards the compound signal was sampled using the natural sampling method because Ideal sampling is impractical & Flat Top sampling has very high noise interference.

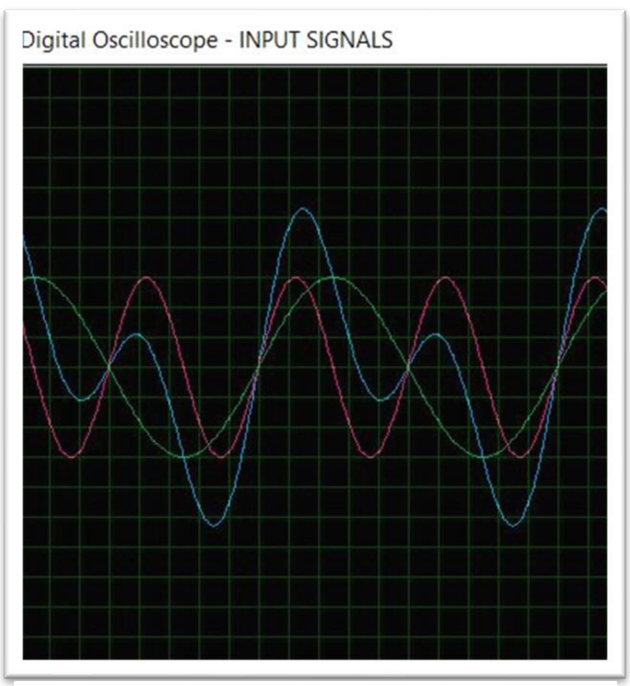

Figure 21 The Blue lined signal is the compound signal made from the 100Hz (Green) & 200H z(Pink) signals

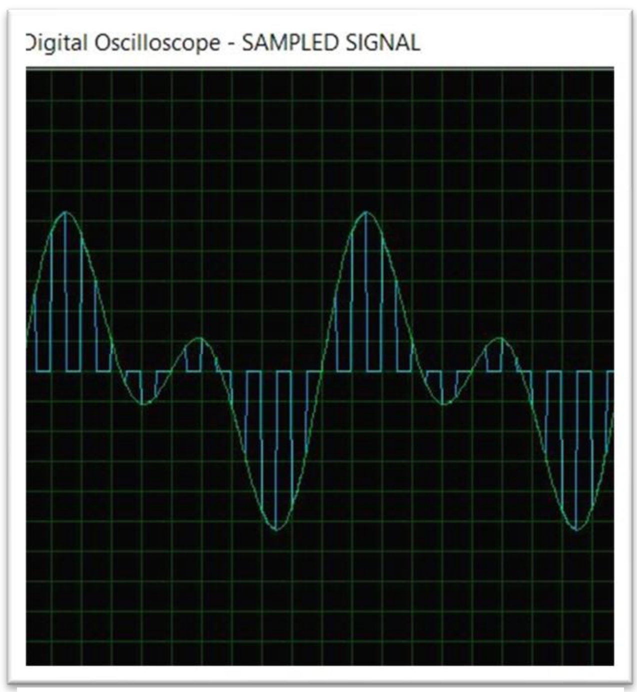

Figure 20 The compound signal (Green) is afterwards naturally sampled (Blue)

4.3 Filter Establishment

4.3.1 Detection of 100Hz Signal

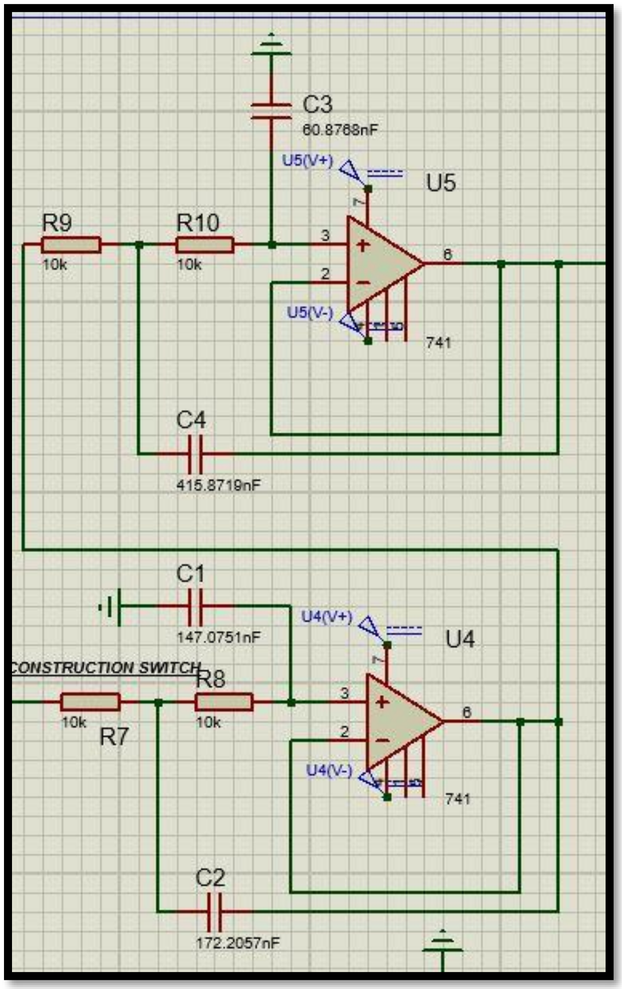

Figure 22 Four-pole low pass Butterworth Filter

- According to the filter characteristics & parameters-based figure in section 2.2.1, this filter has a cut-off frequency of 100Hz.
- For fc = 100Hz & taking the resistors R=10k ohms, we find the general C as,

$$C=\frac{1}{2*\pi*fc*R}=159.1549nF$$

- From the first three derivatives of the transfer function we get,

Capacitor	Formula	Value
C1	0.9241C	147.0751nF

C2	1.082C	172.2057nF
C3	0.3825C	60.8768nF
C4	2.613C	415.8719nF

- The bias voltages of the op-amp were set to 100V & -100V respectably for the positive bias & negative bias.
- At cut-off frequency, the filter gives a (-45*4)^o = -180^o phase increment thus at reconstruction oscilloscope, output is inverted to viewers ease.

4.3.2 Detection of 200Hz Signal

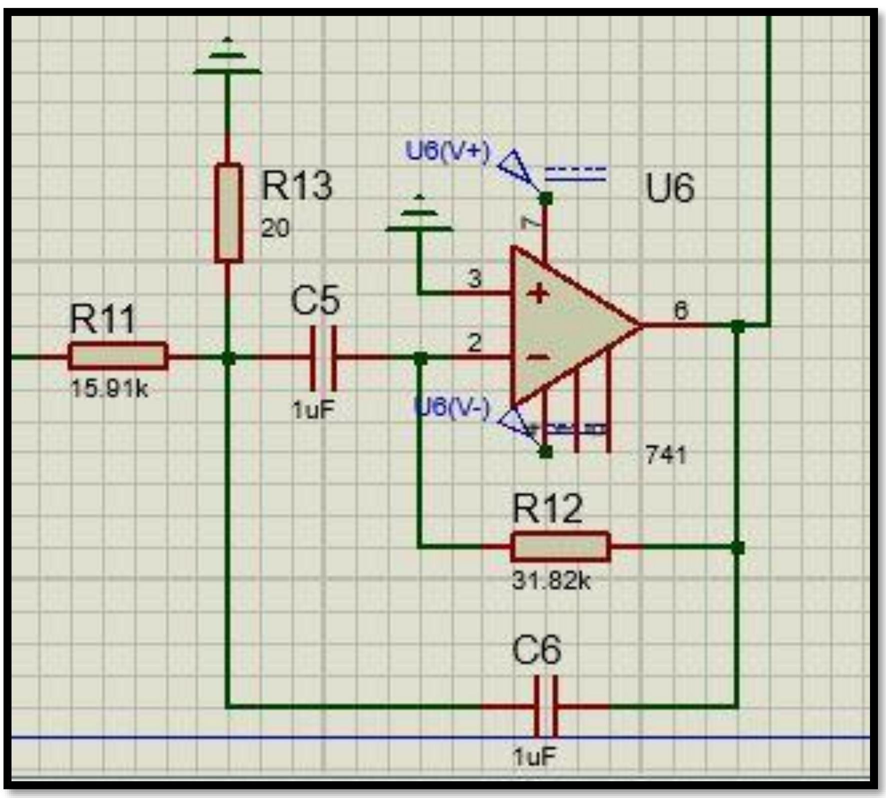

Figure 23 Unity Gain Narrowband Bandpass Filter

- The follow parameters where selected accordingly to use this filtration in order to detect the 200Hz input signal:

Parameter	Value
Lower cut-off frequency, f_l	195 Hz
Upper cut-off frequency, f_h	205 Hz
Resonant Frequency, f_r	$\sqrt{195 * 205}$ = 200 Hz
Bandwidth, B	(205-195) = 10Hz
Quality Factor, Q	200/10 = 20

- Following are the calculated resistive & capacitive values:

Parameter	Formula	Value
C5	Chosen analytically	1 uF (C)
C6	Chosen analytically	1 uF
R11	$\frac{0.1591}{B * C}$	15.91k Ohms (R)
R12	2*R	31.82k Ohms
R13	$\frac{R}{2 * Q^2 - 1}$	20 Ohms

- The filter takes input in it’s inverting side thus provides an inverting output. For the ease of viewers, the reconstructed signal in the oscilloscope is inverted.

4.4 Signal Reconstruction: Without Noise Addition

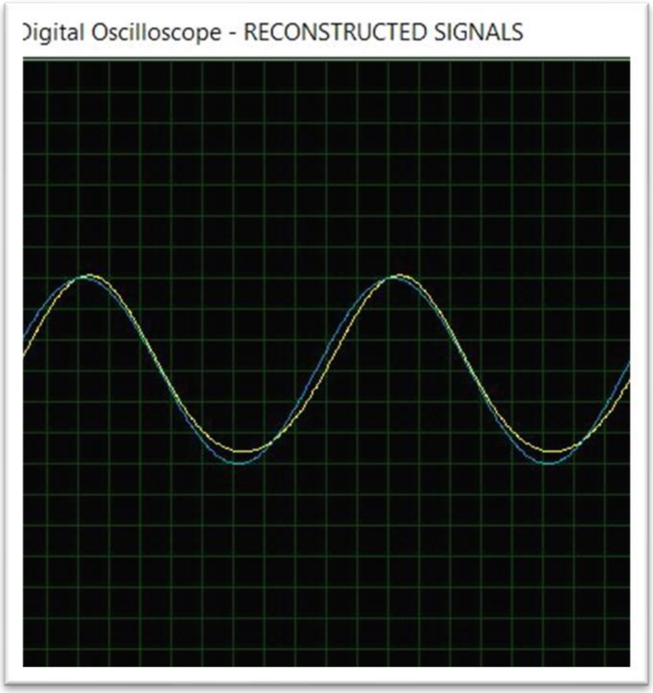

Figure 24 the Reconstructed signal(yellow) for the input of 100Hz(blue)

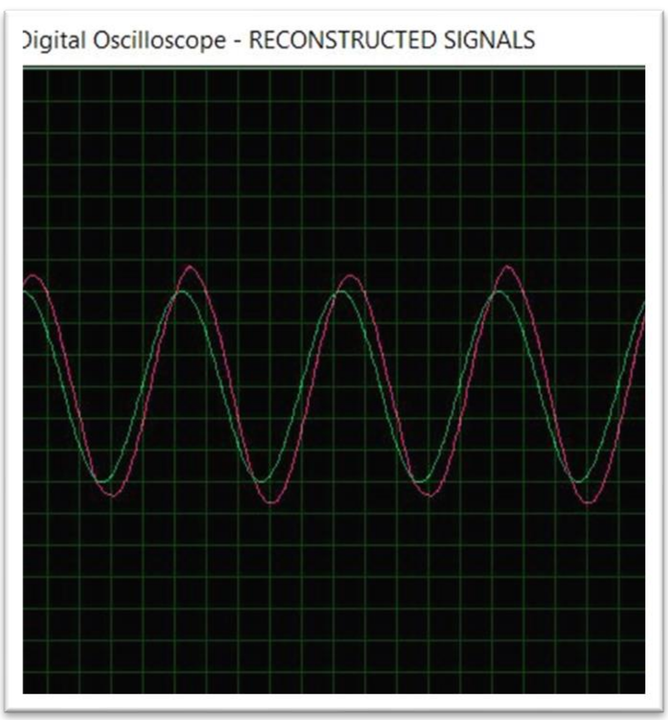

Figure 25 the Reconstructed signal(pink) for the input of 200Hz(green)

4.5 Noise Addition to The Sampled Signal

- A 15V sinusoid was added with the sampled signal as a noise
- The noise frequency was varied for two cases & the signals were successfully reconstructed from them

4.5.1 Low Frequency Noise

- Frequency of Noise = 500 Hz

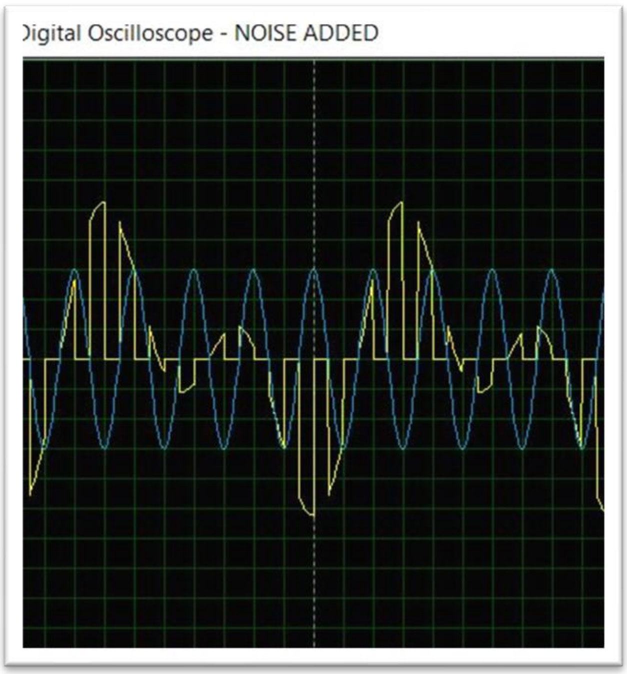

Figure 27 A 15V 500Hz Sinusoid was added as a noise (Blue) with the compound sampled signal(yellow)

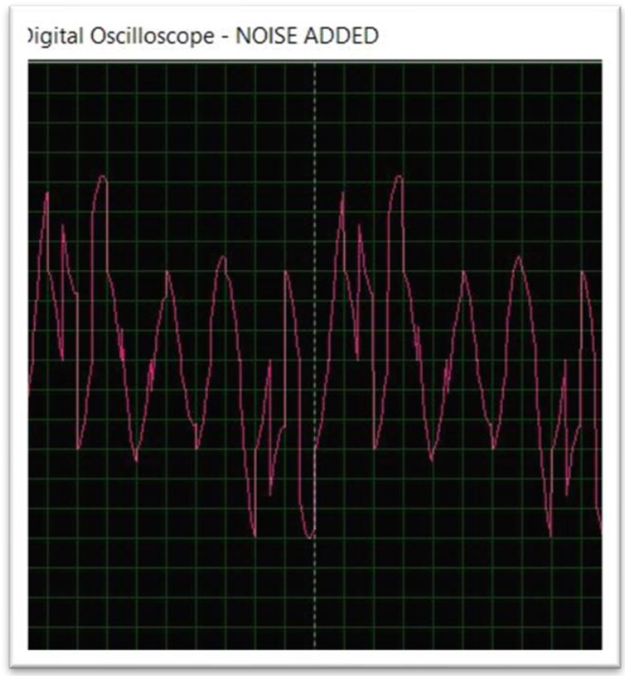

Figure 26 The output sampled signal after the addition of noise

4.5.2 High Frequency Noise

- Frequency of Noise = 5000 Hz

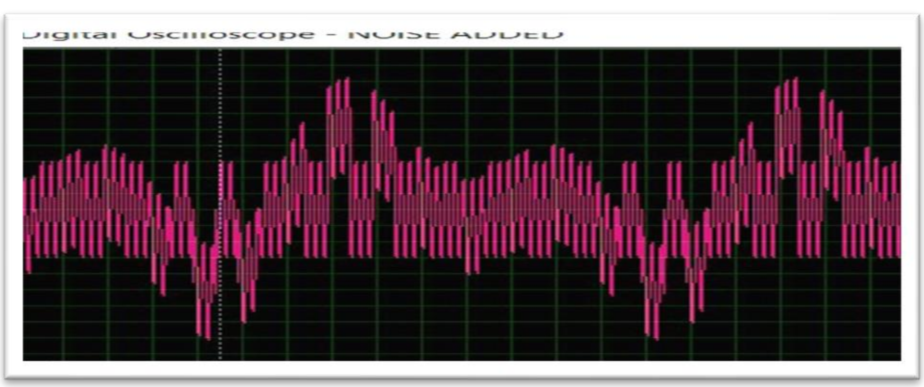

Figure 28 The output sampled signal after the addition of 5k frequency noise

4.6 Signal Reconstruction: With Noise Addition

4.6.1 Low Frequency Noise: 500Hz

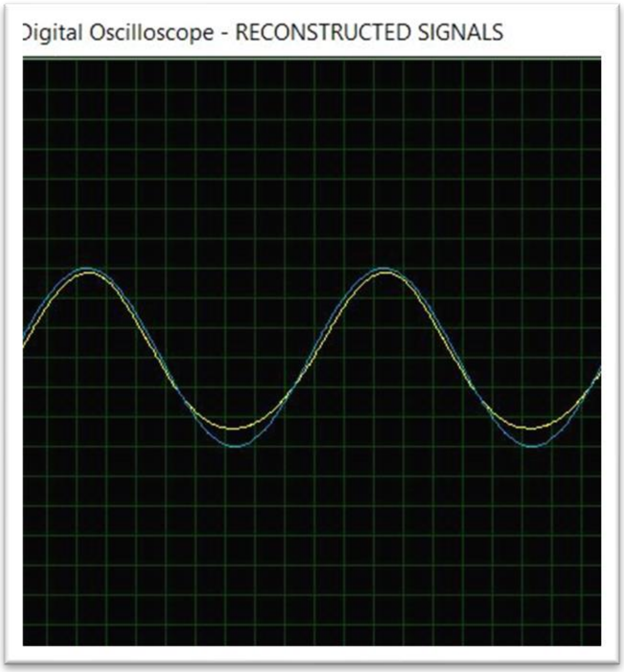

Figure 29 the reconstructed signal(yellow) for the input of 100Hz(blue)

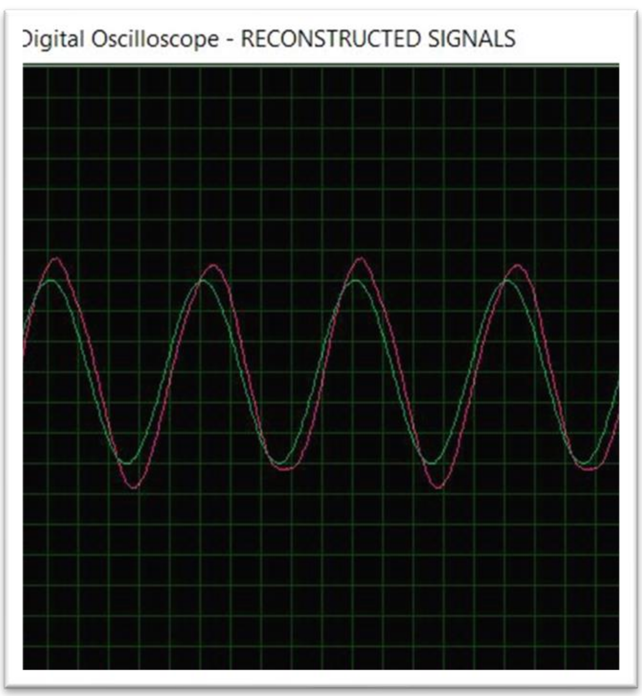

Figure 30 the reconstructed signal(pink) for the input of 200Hz(green)

4.6.2 High Frequency Noise: 5000Hz

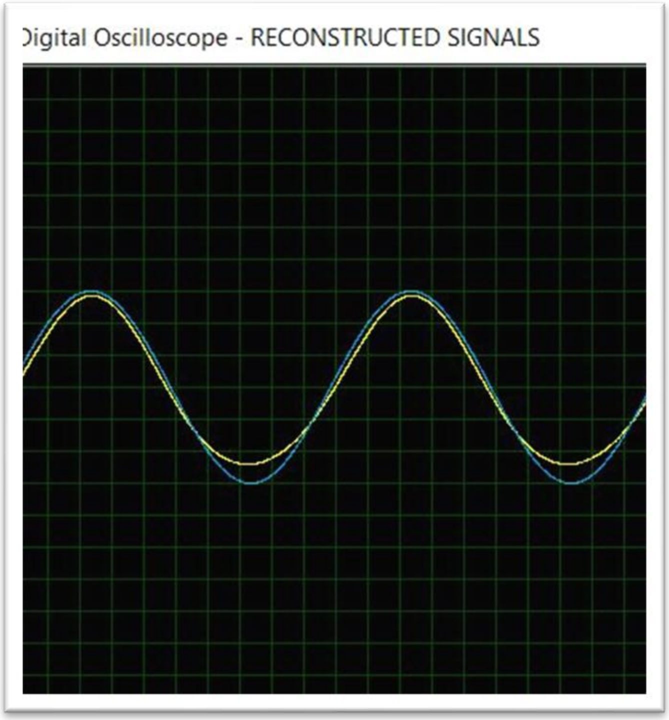

Figure 31the reconstructed signal(yellow) for the input of 100Hz(blue)

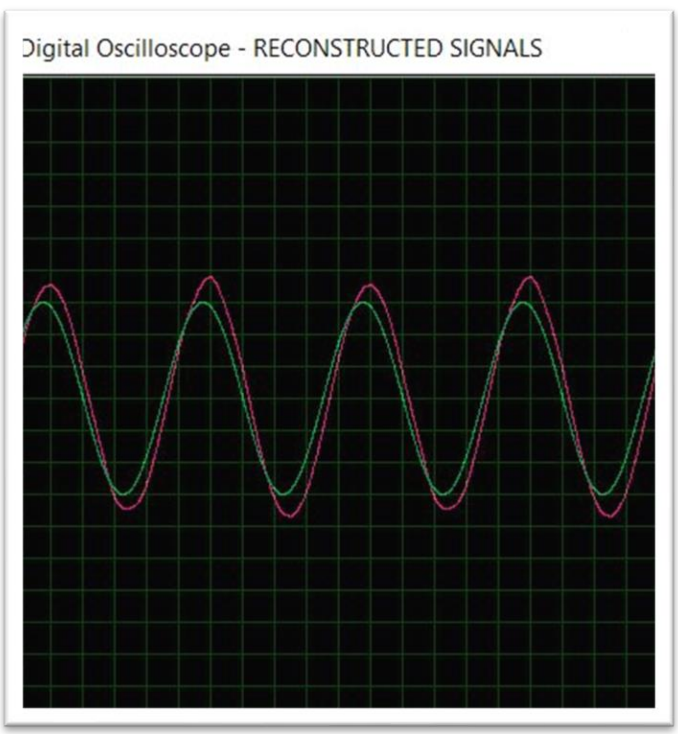

Figure 32 the reconstructed signal(pink) for the input of 200Hz(green)

5. Conclusion

Digital hardware, including computers, take actions in discrete steps. So, they can deal with discrete-time signals, but they cannot directly handle the continuous-time signals that are prevalent in the physical world. This project basically upholds the interface between these two worlds, one continuous, the other discrete. A discrete-time signal is constructed by sampling a continuous-time signal, and a continuous-time signal is reconstructed by interpolating a discrete-time signal – this simple wording has been implemented via various circuits in Proteus whilst staying in the time-domain & also showed how better selection and calculation of certain components can tackle even the noises that might appear in the circuit channels.

6. References

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