Investments in Cryptocurrencies

WANG Jiancheng 1155107822
Jhumka Abdoulaye 1155116361

Supervisor: Prof. HE Xuedong

November 2020

Abstract

This research project applied classical portfolio selection models (CAPM and 1/N) for cryptocurrency investment and compared their performances to determine the best portfolio model. To evaluate the performances of these models, Sharpe Ratio and Maximum Utility are adopted, both of which have become the most widely used methods for portfolio analysis. The results of this research conclude that the CAPM model which allows short-selling, with constituents based on their market capitalization, has the highest return of 176% for a holding period of five months (from June 1st 2020 to November 3rd 2020).

Contents

1 Introduction

1.1 Background and motivation

At the start of this semester, on the 7th of September, the market capitalization of cryptocurrencies was approximately 323 billion U.S. dollars, as obtained from https://coinmarketcap.com. At the time of writing this report, on the 25th of November, the market capitalization of cryptocurrencies is approximately 580 billion U.S. dollars. During one semester, the cryptocurrency market increased by more than 250 billion U.S. dollars, an increase of nearly 80%.

The straightforward reason for such an exponential increase in the market size of cryptocurrencies is the presence of the pandemic caused by COVID-19. However, the pandemic had a negative impact on stock markets worldwide, when they reported, on 28 February 2020, their largest single-week declines since the 2008 financial crisis, the same crisis that led to the birth of Bitcoin. Satoshi Nakamoto, the creator of Bitcoin, must be proud now given that his creation and its predecessors bloomed during this current financial crisis, proving that decentralized digital currencies are more appealing to the public when the economy is undergoing a recession.

Moreover, there are many reasons why investing in cryptocurrencies is more beneficial than investing in stocks. Some reasons are listed as follows:

- 24/7 access to investment: to invest in the stock market, investors need to first find a broker. Only when they completed the administrative procedures can they start trading. Moreover, investors cannot trade all day long. Stock exchanges have trading sessions: timeframes when traders can place orders. As observed, access to the stock market is regulated. Therefore, it makes planning hard. Unlike trading stocks and commodities, the cryptocurrency market is not strictly regulated. Setting up an account to trade is relatively easy and there is no need for a broker. Moreover, it is accessible 24/7 across a growing number of exchanges, allowing for swift trade movements.

- Low transaction fees: in addition to strict regulations, there are also expensive fees. A considerable portion of an investment is lost in the process of buying stocks. The costs accumulate as the investment traverse the stock exchange: brokers charge a fee or commission, banks will charge for the payments and the profits are taxed. In contrast, trading on cryptocurrency exchanges incurs relatively fewer costs. The costs associated with transacting on the blockchain are minuscule, consisting only of any mining fees. Exchanges themselves thus incur lower costs when buying and selling cryptocurrencies than do brokers for stock exchanges.

- Global access and Transparency: stocks are usually traded in the country in which they are registered. Cryptocurrencies, led by Bitcoin, are accepted globally (more than 100 countries accept Bitcoin). Despite being globally available, there is no inter-border friction when transferring assets, and transaction fees are still lower than stock exchanges. Moreover, the transaction would be clean, swift, transparent, and irreversible.

- Not only assets: cryptocurrencies can be used as payment for goods and services gives them a great advantage over stocks.

Despite the potential of high returns, investors are still skeptical about investing in the cryptocurrency market due to high volatility. Early empirical results find Bitcoin to be partially 'inefficient' in Fama's sense of efficient markets and predictability of returns (Urquhart, 2016)[3], whereas a more recent work finds cryptocurrency asserts to be efficient (Nadarajah and Chu, 2017)[4]. In a recent paper, Brauneis and Mestel (2018)[1] concluded that efficiency of a single cryptocurrency is closely and positively related to its liquidity. Added to a portfolio, a sufficient amount of CC will have a market capitalization of billions of dollars, and paired with the fact that they usually provide lower transaction costs than the most efficient financial markets (Kim, 2017)[5], it indicates ample liquidity.

Taken as a whole, these empirical findings, such as extreme volatility, bubble-behavior, and inefficiencies in the price discovery process of individual cryptocurrency, give rise to considering diversified cryptocurrency investments as a means of mitigating risk exposure in cryptocurrency markets.

1.2 Goal

As stated above, there are plenty of cryptocurrencies being traded in the market and their influence on the financial world is enormous and unpredictable. The objective of this project is to analyze the mechanism and market of cryptocurrencies, and the related regulation, to download the historical data of cryptocurrencies, to build up and apply classical portfolio selection models to cryptocurrency investment, and to test their performances.

1.3 Summary of contribution

This project was completed by Jhumka Abdoulaye (a third-year FTEC student) and WANG Jiancheng (a fourth-year FTEC student). Jhumka Abdoulaye constructed portfolio investment strategies and performed result analysis of six portfolio models; WANG Jiancheng researched optimization algorithm and built portfolio models. Jhumka Abdoulaye and WANG Jiancheng both worked on the literature part and helped each other in the coding part.

2 Existing Literature

2.1 Cryptocurrency-portfolios in a mean-variance framework, by Alexander Brauneis and Roland Mestelb[2]

The aim of this research is to construct different portfolios, with varying parameters, using classical portfolio selection models such as CAPM and to compare their performances. First, they calculated the pairwise correlations among 500 cryptocurrencies. They observed that 98 percent of the values fall within the range from 0.10 to 0.20, exposing the weak correlation between these assets. Therefore, according to the Modern Portfolio Theory, these assets can be added to a diversified portfolio and the portfolio risk will decrease, without sacrificing return.

The parameters they tuned are:

- The number of constituents in a portfolio, K: The cryptocurrencies were chosen from a basket of 500, and the K most liquid assets were chosen, according to the mean dollar trading volume over a pre-determined period.
- The holding period of the portfolio, h: The portfolios are formed and held for h days, subsequently, at time (t + h), cryptocurrencies holdings are rebalanced using the various selection models.
- The volatility of individual cryptocurrencies: They defined as a cutoff point historical daily volatility of 0.15 percent, in other words, all CC exceeding this volatility level over f past days will not be considered for the selection of one type of portfolio. The rationale behind this is to rule out potential biases stemming from extreme risk CC.

They chose 8 Portfolios, some of them are:

- The naively diversified portfolio (1/N), which was used as a benchmark.
- The optimal portfolio based on CAPM with risk-free rate equal to zero.
- The minimum variance portfolio on the efficient frontier.
- The maximum returns portfolio consisting of only one CC, which had the highest returns.
- three equally spaced target-return portfolios in between the minimum variance on the efficient frontier and the maximum returns CC.

To compare the portfolios, they used Sharpe Ratio and Certainty Equivalent Return, risk aversion set to one.

Other important details about this experiment are:

- The risk-free rate is zero, implying that at any time all funds are entirely invested in CC.
- No short selling was allowed.
- They included a fixed transaction cost.

The conclusions they reached are:

- Despite higher transaction costs, frequent rebalancing within a given portfolio strategy results in higher mean daily returns without leading to higher standard deviations.
- Irrespective of the examined holding period, the 1/N portfolios yield the highest returns.
- A broader investment universe (K = 30, 50, 100) alters results towards a higher mean return at no additional risk but linked to substantially reduced liquidity of constituents' CC on the downside.
- An equally weighted portfolio is the best choice of investment since the minimum Sharpe ratio of all 1/N portfolios exceeds the Sharpe ratios of more than 75 percent of the optimized portfolios and 1/N portfolios on average outperform at least 75 percent of mean-variance optimal portfolios.

2.2 Portfolio diversification across cryptocurrencies, by Weiyi Liu[7]

In this paper, they analyzed the investability and role of diversification in the cryptocurrency market by applying six classical portfolio selection models through an out-of-sample evaluation method. They compared the performances of the different models using various evaluation criteria to understand which model performs the best in one or more specific aspects.

The 10 cryptocurrencies analyzed in this article are Bitcoin, Ethereum, Ripple, Litecoin, Stellar, Monero, Dash, Tether, NEM and Verge.

The portfolio models considered in this article are: 1/N equally weighted rule, minimum variance, risk parity, Markowitz, maximum Sharpe ratio, and maximum utility. No short selling is allowed.

The conclusions they reached are:

- The minimum variance model achieves the smallest out-of-sample volatility and maximum drawdown but performs less attractive in Sharpe ratio and utility.
- The maximum Sharpe model does not maximize the out-of-sample Sharpe ratio.
- The maximum utility model attains the highest out-of-sample return and utility.
- None of the sophisticated models beat the naïve 1/N portfolio in the criterion of Sharpe ratio.
- Risk parity performs analogously to the 1/N model, but none of the models are consistently better than the naïve 1/N portfolio in Sharpe ratio.
- If the utility as the evaluation criterion is considered, then maximum utility stably dominates the performances.

As the reader can observe, the research done on investment diversification in cryptocurrencies is limited, so our goal is to contribute to this topic.

3 Solution Approach

A list of the top ten cryptocurrencies depending on their Market Capitalization as of 31 May 2020 will be created. The choice of cryptocurrency also depends on the availability of its data from Yahoo Finance API as from 1 Jan 2019. The list contains Bitcoin, Ethereum, XRP, BitcoinCash, Chainlink, BinanceCoin, EOS, Cardano, Litecoin, and Stellar.

Moreover, another list will be created depending on their Liquidity, based on the mean dollar trading volume. The list contains Bitcoin, Ethereum, XRP, BitcoinCash, EOS, TRON, EthereumClassic, Cardano, Litecoin, and Stellar.

The portfolio models used for this semester are the 1/N model, and the CAPM model with the risk-free rate equal to 0. No risk-free asset was considered, implying that at any time all funds are entirely invested in the 10 cryptocurrencies. This research is not going to build a weighted cryptocurrency model based on market capitalization because it is inappropriate as Bitcoin has a market share of 60%, thus the results will be biased towards the returns of Bitcoin. Although the two previous research did not consider short-selling of assets, the constraint is considered in this research project.

For the CAPM model:

- The fixed parameters: the portfolio holding period is 1 week, the investment period is from 1/6/20 to 3/11/20 (five months) and the number of constituents of the portfolio is ten.
- The parameters tuned are the constituents of the portfolios, based on their Market Capitalization or Liquidity, and their corresponding weights based on the Capital Allocation Line.
- The data used to decide the weights of the portfolios' constituents is as from 1 Jan 2019.

4 Mathematical and Financial Methods

4.1 Portfolio models

In this project, there are two main portfolio models which are the capital asset price model (CAPM) and the 1/N model being used, both of which are able to generate the optimal portfolio. First, in an attempt to address and quantify portfolio effects in the cryptocurrency investment universe, CAPM model combined with the traditional mean-variance portfolio selection framework as proposed by Markowitz can reveal preliminary evidence on portfolio effects, i.e. properties of multiple cryptocurrency investments. Second, the reason why 1/N is also adopted in this project is that "1/N-portfolio outperformed single cryptocurrencies and more than 75% of mean-variance optimal portfolios".[2]

4.1.1 CAPM model

In the CAPM model, given a specific level of rate of return, there will be various portfolios with different variances, and only the portfolio with minimal variance is the optimal one. Then the model will adjust the rate of return into different values ranging from the minimal return of those selected cryptocurrencies to the maximal one of them. Consequently, an efficient frontier is constructed based on a set of optimal portfolios that offer the lowest risk for a given level of expected return. Thus, the optimal portfolio of the CAPM model consists of a risk-free asset and an optimal risky asset portfolio, and the optimal risky asset portfolio is at the point where the capital allocation line (CAL) is tangent to the efficient frontier. This portfolio is optimal because the slope of CAL is the highest, which means the model can achieve the highest returns per additional unit of risk. Therefore, the optimal weights of cryptocurrencies can be obtained from the optimal portfolio, which is used to construct future investment strategies.

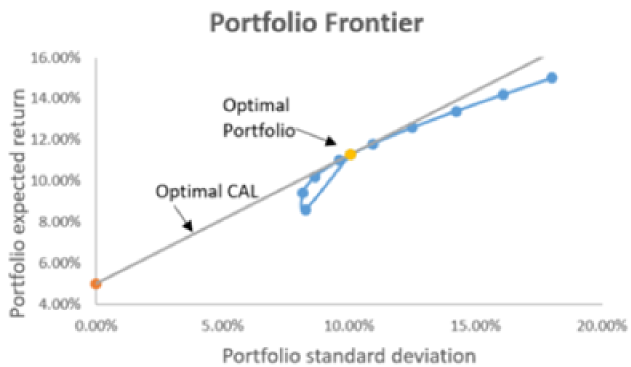

Figure 1: Optimal Portfolio on Efficient Frontier

4.1.2 1/N model

The 1/N model is used to assign each cryptocurrency with equal weight, which may be seen as a benchmark for data-driven portfolios. The reason why the naively diversified 1/N equal-weighted portfolio is adopted is that price changes in each cryptocurrency are unpredictable, which results in the inaccuracy of calculated portfolios. Therefore, the 1/N model can also be used to compare with other portfolio models.

4.2 Optimization algorithms

This project mainly focuses on two optimization methods to solve the optimization problems in the CAPM model. In specific, the portfolio risk of return is quantified by σ_p^2 and in mean-variance analysis, only the first two moments (σ_p^2 and μ_p) are considered in the portfolio model. To minimize the portfolio risk no matter whether the portfolio model allows short-selling or not, the first method is to apply the closed form combined with quadratic programming; the second one is to use a function named "minimize" from the package "scipy.optimize" in Python.

4.2.1 Closed form and quadratic programming

The first approach is to apply the closed form to get the optimal portfolio with short-selling and quadratic programming to get the optimal one without short-selling since there is no closed-form solution in the inequality constrained optimization problem.

Firstly, for closed from, the solution can be calculated in the equality constrained optimization problem. To solve the problem, Markowitz's mean-variance analysis should be used and its mathematical formulation of given by:

$$\min \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(w_i w_j \sigma_{ij})$$

subject to $\sum_{i=1}^{N} w_i R_i = \mu_p$ and $\sum_{i=1}^{N} w_i = 1$. Given the target expected rate of return of portfolio μ_p, find the portfolio strategy that minimizes σ_p^2. To form the Lagrangian:

$$L = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(w_i w_j \sigma_{ij}) - \lambda_1(\sum_{i=1}^{N} w_i - 1) - \lambda_2(\sum_{i=1}^{N} w_i R_i - \mu_p)$$

where λ_1 and λ_2 are Lagrangian multipliers. Then differentiate L with respect to w_i and Lagrangian multipliers, and set the derivative to zero.

$$\frac{\partial L}{\partial w_i} = \sum_{j=1}^{N} \sigma_{ij} w_j - \lambda_1 - \lambda_2 R_i = 0, i = 1, 2, ..., N$$

$$\frac{\partial L}{\partial \lambda_1} = \sum_{i=1}^{N} w_i - 1 = 0$$

$$\frac{\partial L}{\partial \lambda_2} = \sum_{i=1}^{N} w_i R_i - \mu_p = 0$$

Let Ω denote the covariance matrix of cryptocurrencies of the portfolio so that $a = 1^T\Omega^{-1}1$, $b = 1^T\Omega^{-1}\mu$, $c = \mu^T\Omega^{-1}\mu$. Solving for λ_1 and λ_2: $\lambda_1 = \frac{c-b\mu_p}{\Delta}$ and $\lambda_2 = \frac{a\mu_p-b}{\Delta}$, where $\Delta = ac - b^2$. Thus, the optimal weight is given by:

$$w^* = \Omega^{-1}(\lambda_1 1 + \lambda_2 \mu)$$

Secondly, for quadratic programming, Markowitz's algorithm with no short selling requires the weights of all cryptocurrencies to be positive or zero. With inequality constraints, the Lagrange multiplier method no longer works because it imposes equality in the constraint. This optimization problem must be solved numerically, e.g. using the cvxopt.solver.qp in Python or the function solve.QP() in the R package quadprog. Quadratic programming problems are of the form:

$$\min \frac{1}{2}x^T D x - d^T x \ s.t.$$

$$A_{neq}^T x \geq b_{neq}$$

$$A_{eq}^T x = b_{eq}$$

where D is an $n \times n$ matrix, x and d are $n \times 1$ vectors, A_{neq}^T is an $m \times n$ matrix, b_{neq} is an $m \times 1$ vector, A_{eq}^T is an $l \times n$ matrix, b_{eq} is an $l \times 1$ vector. Now, consider the portfolio optimization problem:

$$\min_w \sigma^2 = w^T \Omega w \ s.t.$$

$$w^T R = \mu_p$$

$$w^T 1 = 1$$

$$w_i \geq 0 \ i = 1, 2, ..., N$$

Then combine the inequality constraints and equality constraints into matrices separately and use Python or R to solve the inequality constrained optimization problem. Thus, the optimal weight will be computed.

4.2.2 "scipy.optimize.minimize"

The second approach is to use Python package "scipy.optimize" to solve the minimization problem regardless of whether the optimal portfolio enables short-selling or not. The default method of "scipy.optimize.minimize" is BFGS to solve both unconstrained minimization problems and constrained minimization problems. The package provides L-BFGS-B algorithm (L-BFGS-B), a modification of Powell's method (Powell), and a truncated Newton algorithm (TNC) for bound-constrained minimization, and constrained optimization BY linear approximation method (COBYLA), sequential least squares programming (SLSQP), and trust-region algorithm (trust-constr) for constrained minimization.

4.3 Performance evaluation of portfolio models

In order to test the performances of portfolio models, this project adopts two criteria which are Sharpe Ratio and Maximum Utility, both of which have become the most widely used methods for risk-adjusted return calculation and portfolio analysis.

4.3.1 Sharpe Ratio

The Sharpe ratio is the average return earned in excess of the risk-free rate per unit of volatility or total risk. Generally, the greater the value of the Sharpe ratio is, the more attractive the portfolio is.

$$Sharpe\ Ratio\ = \frac{R_p - r_f}{\sigma_p}$$

where R_p is the return of portfolio, r_f is the risk-free rate, and σ_p is the standard deviation of the portfolio' excess return.

4.3.2 Maximum Utility

The maximum utility is the average return minus the portfolio's adjusted risk, which is a utility function to test whether the investment money spent yield a reasonable marginal utility.

$$Maximum\ Utility\ = R_p - r_f - \frac{\gamma}{2}\sigma_p$$

where R_p is the return of portfolio, r_f is the risk-free rate,γ reflects risk aversion which is set to one (DeMiguel et al., 2009)[6], and σ_p is the standard deviation of the portfolio.

5 Results

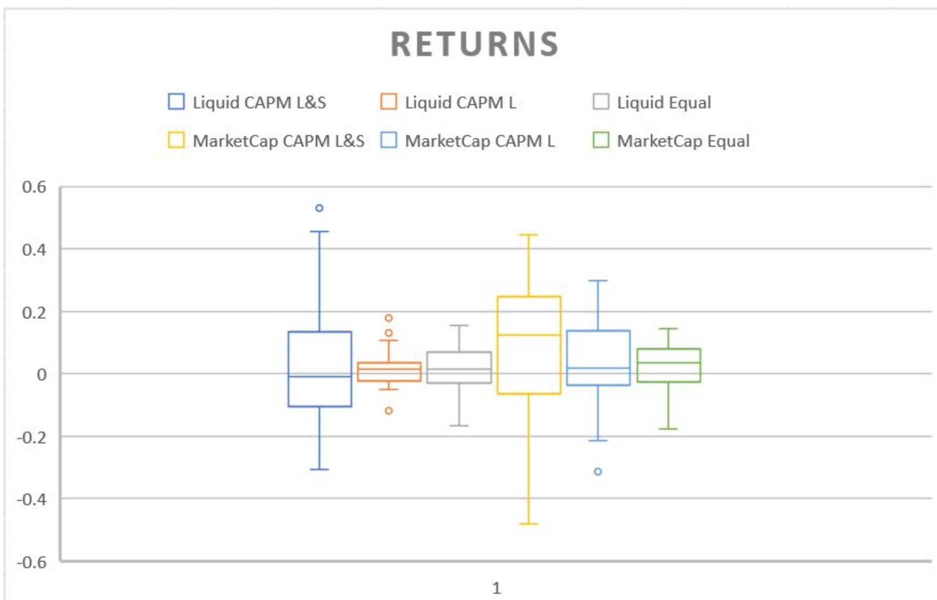

Figure 2: Box and Whisker Diagram for Weekly Returns

This is the Box and Whisker diagram of the real portfolio returns for the 22 weeks of investment, which are calculated as the optimal weight of each portfolio given the first N weeks' data times the real return of the next week (the $(N+1)_{th}$ week). These real returns can be seen as the evaluation of these portfolios' performances. Each portfolio name indicates the type of portfolio used. Refer to the list below to understand each portfolio type and the name assigned to it.

- Liquid CAPM L&S: The constituents of the portfolio used are from the set of cryptocurrencies based on their liquidity, the model used to assign the weights is the CAPM model and short-selling constraint is allowed.

- Liquid Equal: The constituents of the portfolio used are from the set of cryptocurrencies based on their liquidity, the model used to assign the weights is the 1/N model.

- Market CAPM L: The constituents of the portfolio used are from the set of cryptocurrencies based on their market capitalization, the model used to assign the weights is the CAPM model and short-selling constraint is not allowed.

- Liquid CAPM L: The constituents of the portfolio used are from the set of cryptocurrencies based on their liquidity, the model used to assign the weights is the CAPM model and short-selling constraint is not allowed.

- MarketCap CAPM L&S: The constituents of the portfolio used are from the set of cryptocurrencies based on their market capitalization, the model used to assign the weights is the CAPM model and short-selling constraint is allowed.

- MarketCap Equal: The constituents of the portfolio used are from the set of cryptocurrencies based on their market capitalization, the model used to assign the weights is the 1/N model.

As observed from the graph above, the only portfolio with median returns less than 0 is the Liquid CAPM L&S portfolio, while also having the maximum week return, which is an outlier. It indicates that most of its weekly returns are losses and its maximum return is not considered normal. However, the spread of its values above the 0 line is greater than its spread below the 0 line, which indicates, along with its only outlier, that its mean will be positive. While the MarketCap CAPM L&S portfolio has the highest median, it also has the minimum week return among all the portfolios, which is not an outlier. It has the largest spread of return values and all the values in its upper quartile range are highly positive. It will suit the investors who want to take high risks and reap high profits in the long term.

The MarketCap Equal portfolio and the Liquid Equal portfolio are the other 2 portfolios with no outliers, and they have nearly the same spread of values. However, the median of the MarketCap Equal portfolio is higher, making it the better choice. The returns of these 2 portfolios are stable since they have a low spread of values and no outliers. They may be best for new investors since both portfolios use the 1/N model, they are easy to invest into due to no advanced calculation needed, and their stability will suit those who are risk-averse.

	Liquid CAPM L&S	Liquid CAPM L	Liquid Equal	MarketCap CAPM L&S	MarketCap CAPM L	MarketCap Equal
Mean	0.022	0.016	0.013	0.08	0.038	0.02

Figure 3: Mean Real Returns

It can be observed from Figure 3 that, in terms of mean real returns, the portfolios which considered the short-selling constraint produce better results; the CAPM model is always better than the Naïve Equal Model; the portfolios with constituents based on their market capitalization have greater returns than those based on liquidity.

	Liquid CAPM L&S	Liquid CAPM L	Liquid Equal	MarketCap CAPM L&S	MarketCap CAPM L	MarketCap Equal
Mean	0.026	0.006	-0.003	0.051	0.018	0.002

Figure 4: Maximum Utility

	Liquid CAPM L&S	Liquid CAPM L	Liquid Equal	MarketCap CAPM L&S	MarketCap CAPM L	MarketCap Equal
Mean	0.233	0.112	0.026	0.338	0.193	0.072

Figure 5: Sharpe Ratio

The 2 tables above correspond to the mean of the Maximum Utility and Sharpe Ratio of each portfolio at the end of each week.

From the 2 tables above, it can be inferred that the Market CAPM L&S is the best portfolio to invest in since it has the highest Maximum Utility and Sharpe Ratio. With its mean Maximum Utility value being twice as large as the second-best portfolio, it is clearly the best choice. The equal weight liquid portfolio performed the poorest, which contradicts the conclusion reached in the research by Brauneis and Mestel (2019)[2]. It is believed that the market of cryptocurrency is ever-changing, thus investment models that worked in different past periods may not always produce the same expected results.

6 Discussion and Future Plan

When the portfolio model doesn't enable short-selling, i.e. when the portfolio can only long the cryptocurrencies, the weights allocated to each cryptocurrency is not distributed appropriately; the portfolios constructed by such models usually only focus on three to four cryptocurrencies, and do not consider the rest. One possible explanation for this could be that some cryptocurrencies dominantly outperforms the others, providing higher returns while having a lower risk level. For future research, more cryptocurrencies could be included, and inefficient ones should be eliminated.

Although the CAPM model performs better than the 1/N model, for further research, the Fama French three-factor model and the Fama French five-factor model will be introduced for comparison of their performances. In the meantime, more evaluation tools will be applied to test performances of these models.

After all the above four models are constructed, the Machine Learning algorithms such as Neural Networks (NNs) and Polynomial Regression will be used to assign parameters to different models based on their performances and finally integrate them into one comprehensive model. This final model is supposed to be the optimal one and can achieve a relatively high return given a specific level of risk.

7 Conclusion

This research project introduced CAPM and 1/N portfolio models to generate six optimal portfolios based on three criteria (whether short-selling is allowed; whether the chosen cryptocurrencies are market-capitalized or liquidized; and whether CAPM or 1/N model is adopted). The best portfolio model considering Return, Sharpe Ratio, and Maximum Utility is the CAPM portfolio model, which allows short-selling, with constituents based on their market capitalization.

References

[1] Brauneis A. and Mestel R. Price discovery of cryptocurrencies: Bitcoin and beyond. *Economics Letters*, 165:58 – 61, 2018.

[2] Brauneis A. and Mestel R. Cryptocurrency-portfolios in a mean-variance framework. *Finance Research Letters*, 28:259 – 264, 2019.

[3] Urquhart A. The inefficiency of bitcoin. *Economics Letters*, 148:80 – 82, 2016.

[4] Nadarajah S. and Chu J. On the inefficiency of bitcoin. *Economics Letters*, 150:6 – 9, 2017.

[5] Kim T. On the transaction cost of bitcoin. *Finance Research Letters*, 23:300 – 305, 2017.

[6] DeMiguel V., Garlappi L., and Uppal R. Optimal Versus Naive Diversification: How Inefficient is the 1/N Portfolio Strategy? *The Review of Financial Studies*, 22(5):1915–1953, 12 2007.

[7] Liu W. Portfolio diversification across cryptocurrencies. *Finance Research Letters*, 29:200 – 205, 2019.