We have

$$\dot{x}(t) = \nu w(t) + \xi(t)$$

Moments of w and ξ are known $\Rightarrow$ moments of $\dot{x}(t)$ are known, we can write down the relevant correlators

i.e. let $\mathbb{E}_x$ denote expectation over the x-ensemble. Then

$\mathbb{E}_x[\dot{x}(t)] = 0,$ no mention of w or ξ

$\mathbb{E}_x[\dot{x}(t_1)\dot{x}(t_2)] = f_1(\alpha_+, \alpha_-, \nu)$

$\mathbb{E}_x[\dot{x}(t_1)\dot{x}(t_2)\dot{x}(t_3)] = f_2(\alpha_+, \alpha_-, \nu)$ etc.

Also $\dot{x} - \nu w$ is a white noise process so

$$P(\{\dot{x}\} \mid \{w\}) \propto \exp\left(-\frac{1}{4D}\int_0^t (\dot{x} - \nu w)^2 \, ds\right)$$

So,

$$P(\{\dot{x}\}) = \mathbb{E}_w\left[\exp\left(-\frac{1}{4D}\int_0^t (\dot{x} - \nu w)^2 \, ds\right)\right]$$

w ensemble average

$\Rightarrow$

$$\log\left(\frac{P(\{\dot{x}\})}{P(\{\dot{x}\}^R)}\right)$$

At this point in the current manuscript, we average over ξ, then again over w, so we calculate

$$\mathbb{E}_w\left[\mathbb{E}_\xi \log\left(\frac{\mathbb{E}_w[\exp \cdots]}{\mathbb{E}_w[\exp \cdots]}\right)\right]$$

Instead, average over x at this point, i.e.

$$\mathbb{E}_x\left[\log\left(\frac{P(\{x\})}{P(\{\hat{x}\}^R)}\right)\right] = \mathbb{E}_x\left[\log\left(\frac{\mathbb{E}_w[\exp\cdot\text{--}]}{\mathbb{E}_w[\exp\cdot\text{--}]}\right)\right]$$

Not averaging over the same process anymore! To average over x use the moments calculated above, i.e. $f_1(\alpha_+, \alpha_-, 2l)$, $f_2(\alpha_+, \alpha_-, 2s)$ etc.

It's just a matter of changing the order of the operations we perform.