$$K = \frac{1}{2} m(\dot{x}^2 + (l_0 + x)^2 \dot{\theta}^2) = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} m(l_0 + x)^2 \dot{\theta}^2) = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} m(l_0^2 + 2l_0 x + x^2)\dot{\theta}^2$$

$$P = -mg(l_0 + x) * \cos\theta + \frac{1}{2} Kx^2$$

$$L = \frac{1}{2} m\dot{x}^2 + (\frac{1}{2} m l_0^2 + \frac{1}{2} m * 2 l_0 x + \frac{1}{2} m x^2)\dot{\theta}^2 - P = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} m l_0^2 \dot{\theta}^2 + m l_0 x \dot{\theta}^2 + \frac{1}{2} m x^2 \dot{\theta}^2$$

$$+ mgl_0 \cos\theta + mgx\cos\theta - \frac{1}{2} Kx^2$$

$$\frac{\partial L}{\partial \dot{q}_1} \Rightarrow \begin{matrix} q_1 = x \\ \dot{q}_1 = \dot{x} \end{matrix} \Rightarrow m\dot{x} = \frac{\partial L}{\partial \dot{q}_1} \; ; \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_1}\right) = m\ddot{x}$$

$$\frac{\partial L}{\partial q_1} = m l_0 \dot{\theta}^2 + m x \dot{\theta}^2 + mg\cos\theta - Kx$$

now for $\begin{matrix} q_2 = \theta \\ \dot{q}_2 = \dot{\theta} \end{matrix} \Rightarrow \frac{\partial L}{\partial \dot{q}_2} = m l_0^2 \dot{\theta} + 2 m l_0 x \dot{\theta} + m x^2 \dot{\theta}$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_2}\right) = m l_0^2 \ddot{\theta} + 2 m l_0 \dot{x}\dot{\theta} + 2 m l_0 x \ddot{\theta} + 2 m \dot{x} x \dot{\theta} + m x^2 \ddot{\theta}$$

$$\frac{\partial L}{\partial q_2} = -mgl_0 \sin\theta - mgx\sin\theta$$

Creating full lagrangian for the first variable:

$$\tau_1 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_1}\right) - \frac{\partial L}{\partial q_1} = m\ddot{x} - m l_0 \dot{\theta}^2 - m x \dot{\theta}^2 + mg\cos\theta + Kx$$

Creating full Lagrangian for the second variable:

$$\tau_2 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_2}\right) - \frac{\partial L}{\partial q_2} = m l_0^2 \ddot{\theta} + 2 m l_0 \dot{x}\dot{\theta} + 2 m l_0 x \ddot{\theta} + 2m\dot{x} x \dot{\theta} + m x^2 \ddot{\theta} + mgl_0 \sin\theta + mgx\sin\theta$$

$$\rightarrow m\ddot{x} = + m l \dot{\theta}^2 + m x \dot{\theta}^2 + mg\cos\theta - Kx$$

$$\Rightarrow \ddot{x} = + l_0 \dot{\theta}^2 + x\dot{\theta}^2 + g\cos\theta - \frac{K}{m} x$$

$$\rightarrow \ddot{\theta}(m l_0^2 + 2 m l_0 x + m x^2) = -2 m l_0 \dot{x}\dot{\theta} - 2 m x \dot{x} \dot{\theta} + mgl_0 \sin\theta + mgx\sin\theta$$

$$\Rightarrow \ddot{\theta} = \frac{-2 m l_0 \dot{x}\dot{\theta} - 2 m x \dot{x} \dot{\theta} - mgl_0 \sin\theta - mgx\sin\theta}{m l_0^2 + 2 m l_0 x + m x^2}$$

$$X = \begin{bmatrix} X \\ \dot{X} \\ \theta \\ \dot{\theta} \end{bmatrix}, \dot{X} = \begin{bmatrix} \dot{X} \\ \ddot{X} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} X_2 \\ l_o X_4^2 + X_1 X_4^2 + g \cos X_3 - K/m\, X_1 \\ X_4 \\ \dfrac{-2\, l_o X_2 X_4 - 2\, X_1 X_2 X_4 - g l_o \sin X_3 - g X_1 \sin X_3}{l_o^2 + 2\, l_o X_1 + X_1^2} \end{bmatrix}$$

$$\ddot{X} = l_o X_4^2 + X_1 X_4^2 + g \cos X_3 - \frac{K}{m} X_1$$

$$\ddot{\theta} = \frac{-2\, l_o X_2 X_4 - 2\, X_1 X_2 X_4 - g l_o \sin X_3 - g X_1 \sin X_3}{l_o^2 + 2\, l_o X_1 + X_1^2}$$

For the equilibrium to exist the speed should be zero:

$$\dot{X} = 0,\ f(x,u) = 0 \Rightarrow \dot{X} = \begin{bmatrix} \dot{X} \\ \ddot{X} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = 0 \Rightarrow \begin{matrix} X_2 = 0 \\ X_4 = 0 \\ \tau_1, \tau_2 = 0 \end{matrix} \Rightarrow g \cos X_3 - \frac{K}{m} X_1 = 0$$

$$g \cos X_3 = \frac{K}{m} X_1$$

$$\cos X_3 = \frac{K X_1}{mg}$$

$$X_3 = \cos^{-1}\left(\frac{K X_1}{mg}\right)$$

$$-mg\, l_o \sin X_3 - mg X_1 \sin X_3 = 0$$

$$\boxed{-l_o = X_1}$$

$$\Rightarrow \quad = \begin{bmatrix} 0 \\ g \cos X_3 - K/m\, X_1 = 0 \\ 0 \\ -mg\, l_o \sin X_3 - mg X_1 \sin X_3 = 0 \end{bmatrix}$$

yes, the equilibrium is stable since X_1 at equilibrium becomes a function of a constant (initial length of the spring). Also, X_3 which is θ, is only a function of X_1 (change in length), which it self is a constant. So, the whole system is stable @ equilibrium point.

Problem 2: $\ddot{x} - 5\dot{x} + 10x = 0$ $\xrightarrow{\text{writing down in SS}}$ $\ddot{x} = 5\dot{x} - 10x$

$\ddot{x} = 5x_2 - 10x_1$

1. $x = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$, $\dot{x} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} x_2 \\ 5x_2 - 10x_1 \end{bmatrix}$

2. For the equilibrium the velocity state should be zero. this means that all the related (dependent) term should also be zero.

$\dot{x} = \begin{bmatrix} x_2 \\ 5x_2 - 10x_1 \end{bmatrix} = 0 \Rightarrow$ $x_2 = 0$

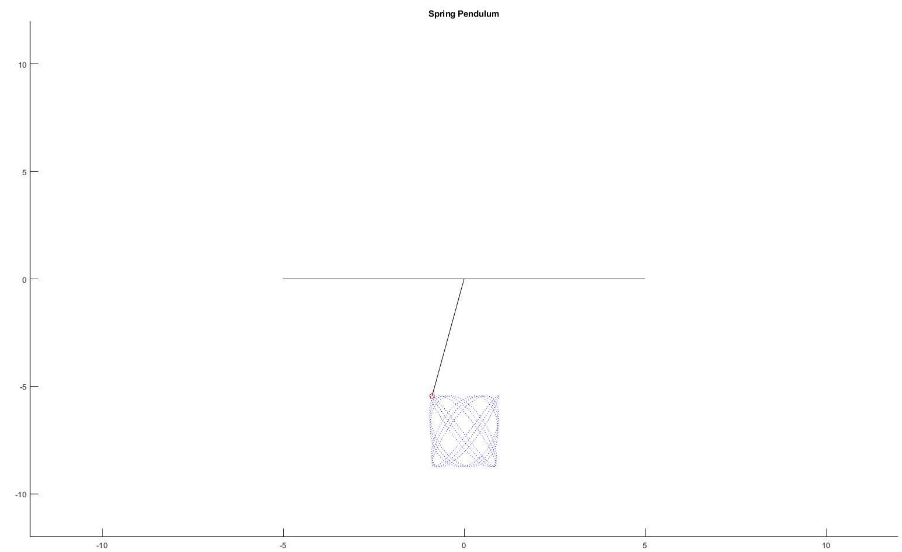
$5x_2 - 10x_1 = 0 \Rightarrow x_2 = 2x_1$

solving the differential equation: $x = e^{2t + c_1}$

Farid Tavakkolmoghaddam

HW1 Problem 1

```
clear all;
k=10; %stifness coefficient
l=2; % initial length
m=5; % mass
% m2=5;
x=3.5; % initial deflection of the spring
p=10; % initial deflection angle of the spring
g=9.81; % gravitational acceleration
y10=[p/360*2*pi 0 x 0]; % Initial Conditions X=[ x1 x1' theta theta']

tspann=[linspace(0,40,201)];% Duration
 f=@(t,y)[y(2) ; ...
         (-2*m*(l+y(3))*y(4)*y(2)-m*g*(l+y(3))*sin(y(1)))/(m*(l
+y(3))^2); ...
        y(4) ; ...
        (l+y(3)*m*y(2)^2+m*g*cos(y(1))-k*y(3))/m];

[t,y]=ode45(f,tspann,y10);

x2=(l+y(:,3)).*sin(y(:,1));
y2=(l+y(:,3)).*cos(y(:,1));


for k=1:1:1
    figure(1)
for i=1:1:length(t)
    hold on;
    axis([-12 12 -12 12]);

    title('Spring Pendulum');
    plot( [0 x2(i) ],-[0 y2(i)] ,'k -');

    plot(x2(1:i),-y2(1:i),'b :');

    plot(x2(i),-y2(i),'r o');% massenpunkt m2
    plot([-5 5],[0 0],'k -');
    plot(x2(i),-y2(i),'r o');
    if i==length(t)
        break
    end
    drawnow;
    clf;
end
hold off
end
```



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