(1) State space form:

$$x_1 = z(t)$$
$$x_2 = \dot{z}(t)$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = (-kz - r\dot{z}) \cdot \frac{1}{m} = -\frac{k}{M}z - \frac{r}{m}\dot{z}$$
$$= -\frac{k}{m}x_1 - \frac{r}{m}x_2$$

(2) with input: $m\ddot{z} = -r\dot{z} - kz + F$

state space form: $u = F$.

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{r}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F$$

(3) $A = \begin{bmatrix} 0 & 1 \\ -\frac{2}{5} & -\frac{1}{5} \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ \frac{1}{5} \end{bmatrix}$

$[B \quad AB] = \begin{bmatrix} 0 & \frac{1}{5} \\ \frac{1}{5} & -\frac{1}{25} \end{bmatrix}$ is full rank.

∴ The system is controllable.

(4) set point tracking. $y_r = 5$

use feedback controller

$$u = -Kx + K_r \cdot y_r$$

substitute in the linear system

$$\dot{x} = Ax + Bu$$
$$= Ax - BKx + BK_r y_r$$

The equilibrium of the new system is

$$Ax_e - BKx_e + BK_r y_r = 0$$

$$x_e = -(A-BK)^{-1} B K_r y_r$$

Because at the equilibrium. $y_e = Cx_e = y_r$

$$\Rightarrow \underbrace{-[1 \quad 0](A-BK)^{-1} B k_r}_{=1} y_r = y_r$$

① Select K to make $A-BK$ stable

(so that the system is stable at the equilibrium)

pole-placement: select two stable poles for $\mathrm{eig}(A-BK)$ to be

$$\text{Poles} = \begin{bmatrix} -1 \\ -1.5 \end{bmatrix}$$

using matlab: place $[A, B, \text{Poles}]$.

obtain $K = [5.5 \quad 11.5]$

② select K_r so that

$$-[1 \quad 0](A-BK)^{-1} B K_r = 1$$

$$K_r = 7.5$$

(Note: you may choose different gains.)