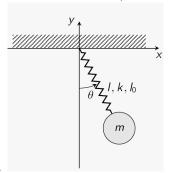Total : 50pt.

If you use matlab, please attach the published pdf from matlab (see how to: https://blogs.mathworks.com/community/2009/11/16/publish-to-pdf/) as well as the original matlab file.

Problem 1

35pt.

Consider the Elastic Pendulum, where the mass is connected to the wall via an elastic string, with elastic

constant k.

- (10pt) Use the Euler-Lagrangian method, to derive the dynamic model of the system. Note that the length l varies over time too. It is assumed that the spring is massless.

- (5pt) Define a set of state variables and express the system using state-space form.

- (5pt) Under no external input, what is the equilibrium of the system? Please give the reasoning.

- (5pt) Is the equilibrium stable or unstable? Justify your answer.

- (10pt) Use matlab ode45, simulate the dynamic behavior of the pendulum with some initial configuration (up to your own choice), with $l_0 = 2$, $m = 5$ and $k = 10$.

- (not graded) Feel free to change the spring constant and observe how the behavior changes under different k using the simulator. If you like, you can share your observation in the submission.

Problem 2

(15 pt) Consider the following system:
$$\ddot{x} - 5\dot{x} + 10x = 0$$

1. (5pt) Write down the state space form of the dynamical system.

2. (5pt) Determine the equilibrium of the system.

3. (5pt) Is the equilibrium stable or unstable? Justify your answer.