1) $m\ddot{z} + \gamma\dot{z} + Kz = 0$

write down the state space form:

$X = \begin{bmatrix} z \\ \dot{z} \end{bmatrix}$, $\dot{X} = \begin{bmatrix} \dot{z} \\ \ddot{z} \end{bmatrix}$; $\dot{X} = AX + Bu \Rightarrow \dot{X} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & -\frac{\gamma}{m} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix} u}_{\text{no input}}$

$\dot{X} = \begin{bmatrix} X_2 \\ -\frac{\gamma}{m}X_2 - \frac{K}{m}X_1 \end{bmatrix}$, $m\dot{X}_2 = -\gamma\dot{z} - Kz \rightarrow \ddot{z} = \dot{X}_2 = -\frac{\gamma}{m}X_2 - \frac{K}{m}X_1$

$A = \begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & -\frac{\gamma}{m} \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

2) write down the equation of motion & put it in a state space form:

equation of motion: $m\ddot{z} + \gamma\dot{z} + Kz = F$

state space form: $X = \begin{bmatrix} z \\ \dot{z} \end{bmatrix}$, $\dot{X} = \begin{bmatrix} \dot{z} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} X_2 \\ \frac{F}{m} - \frac{\gamma}{m}X_2 - \frac{K}{m}X_1 \end{bmatrix}$

$m\ddot{z} = F - \gamma\dot{z} - Kz$

$m\dot{X}_2 = F - \gamma X_2 - KX_1 \xRightarrow{\div m} \ddot{z} = \dot{X}_2 = \frac{F}{m} - \frac{\gamma}{m}X_2 - \frac{K}{m}X_1$

$\dot{X} = AX + Bu \Rightarrow \begin{bmatrix} X_2 \\ \frac{F}{m} - \frac{\gamma}{m}X_2 - \frac{K}{m}X_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & -\frac{\gamma}{m} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F$

$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + [0]u$

Question 3: Given, $K=2$, $m=5$, $\gamma=1$ is the system Controllable?

Substituting the Constants in the A & B matrices $\Rightarrow A = \begin{bmatrix} 0 & 1 \\ -\frac{2}{5} & -\frac{1}{5} \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \frac{1}{5} \end{bmatrix}$

$M_c = [B \quad AB]$

$AB = \begin{bmatrix} 0 & 1 \\ -\frac{2}{5} & -\frac{1}{5} \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{5} \end{bmatrix} = \begin{bmatrix} \frac{1}{5} \\ -\frac{1}{25} \end{bmatrix} = \begin{bmatrix} 0.2 \\ -0.04 \end{bmatrix}$; $M_c = \begin{bmatrix} 0 & 0.2 \\ 0.2 & -0.04 \end{bmatrix}$ M_c is full rank

There fore is Controlable. ✓

4) Define the output, $y = CX$ with $C = [1 \quad 0]$, that is the output is the position of the mass. let the origin $\vec{X} = 0$ be the equilibrium of the system when no external force $F = 0$ is applied. Design a set point controller so that the system stabilizes to $y_r = 5$. with zero velocity.

For gain matrix K we have to place the poles of the closed loop system at a location that ensures stability. This means that the eigen values of closed loop system must have negative real parts. We assume that we would want our poles to be located at $(-1, -2.5)$. Thus, we have: $(\lambda+1)(\lambda+2.5) \rightarrow$ desired pole locations.

$$A - BK = \begin{bmatrix} 0 & 1 \\ -0.4 & -0.2 \end{bmatrix} - \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} [K_1 \quad K_2] = \begin{bmatrix} 0 & 1 \\ -\frac{K_1}{5} - \frac{4}{10} & -\frac{2K_2}{10} - \frac{2}{10} \end{bmatrix}$$

$\text{EigenV}(A - BK) = (\lambda+1)(\lambda+2.5) \rightarrow$ finding the values for K_1, K_2

$\rightarrow K_1 = 10.5,\ K_2 = 16.5 \rightarrow K = [10.5 \quad 16.5]$ now we have the desired gain which places the closed loop poles @ $-1, -2.5$.

now we calculate K_r using the following formula:

$$K_r = -(C(A - BK)^{-1}B)^{-1} =$$

$$A - BK = \begin{bmatrix} 0 & 1 \\ -\frac{4}{10} & -\frac{2}{10} \end{bmatrix} - \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} [10.5 \quad 16.5] = \begin{bmatrix} 0 & 1 \\ -2.5 & -3.5 \end{bmatrix} \rightarrow (A - BK)^{-1} = \begin{bmatrix} -1.4 & -0.4 \\ 1 & 0 \end{bmatrix}$$

$$K_r = -\left([1 \quad 0] \begin{bmatrix} -1.4 & -0.4 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}\right)^{-1} = \underline{12.5}$$

$$u = K_r r - KX \rightarrow u = 12.5 * 5 - [10.5 \quad 16.5] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} =$$

$$u = 62.5 - 10.5 X_1 - 16.5 X_2$$

$$\dot{X} = AX + Bu \rightarrow (A - BK)X + BK_r r \rightarrow \begin{bmatrix} 0 & 1 \\ -2.5 & -3.5 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} * 12.5 * 5 = \dot{X}$$

$$y_r = [5; 0]' = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

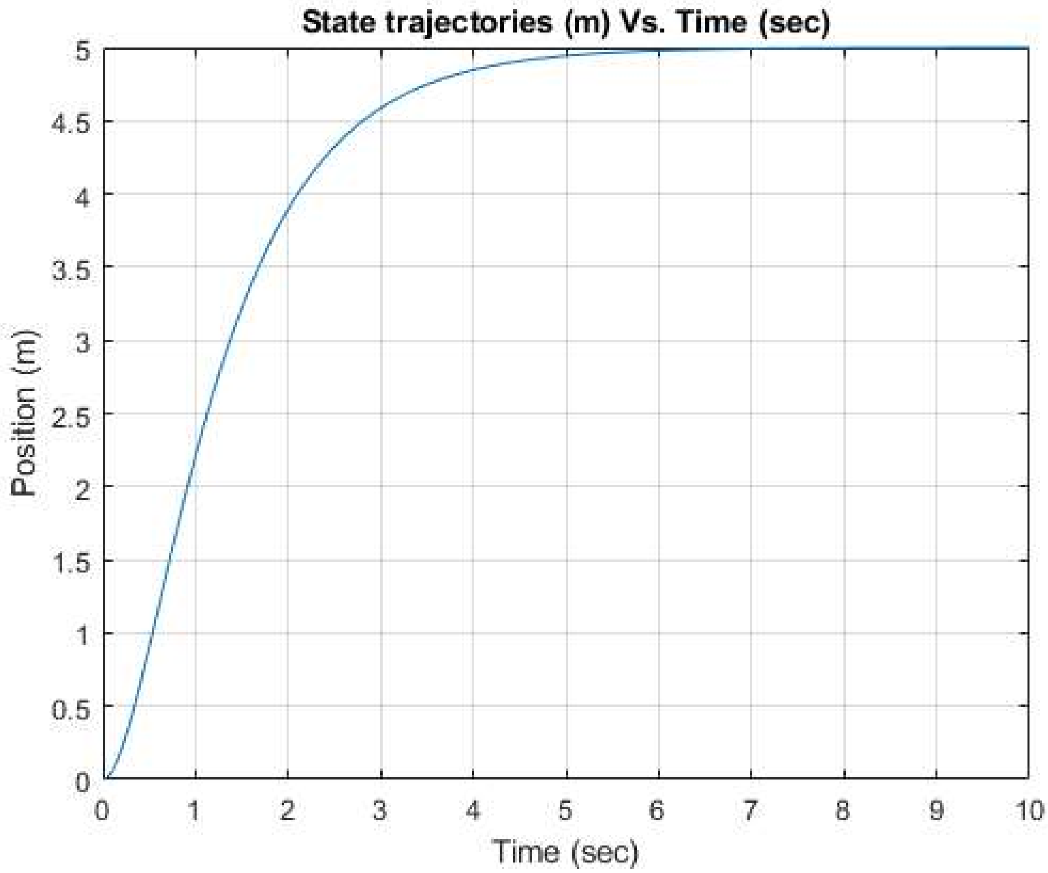
Contents

HW2 Farid Tavakkolmoghaddam

```
clc; clear;
A=[0 1;-0.4 -0.2]; %System dynamics
B=[0;0.2];
C=[1 0];
D=0;
x0 = [0 0]; % initial position and velocity at the equilibrium point
sys_open_loop=ss(A,B,C,D) % open_loop transfer function
% desired poles at the  -1 -2.5 finding the gain (K)
P=[-1 -2.5];% desired poles
disp('desired K that places the poles at the -1 , -2.5 :')
K=place(A,B,P)%desired gain
disp('desired Kr that places the poles at the -1 , -2.5 :')
Kr=-inv(C*inv((A-B*K))*B)% desired Kr
sys_closed_loop=ss(A-B*K,B*Kr,C,D) % closed loop system


t = 0:0.01:10;
u = 5*ones(size(t)); % input with the reference input of 5
% plotting the trajectory of the system
[y,t,x] = lsim(sys_closed_loop,u,t,x0);
figure('Name','With the equilibrium initial condition')
plot(t,y)
title('State trajectories (m) Vs. Time (sec)')
xlabel('Time (sec)')
ylabel('Position (m)')
grid
```

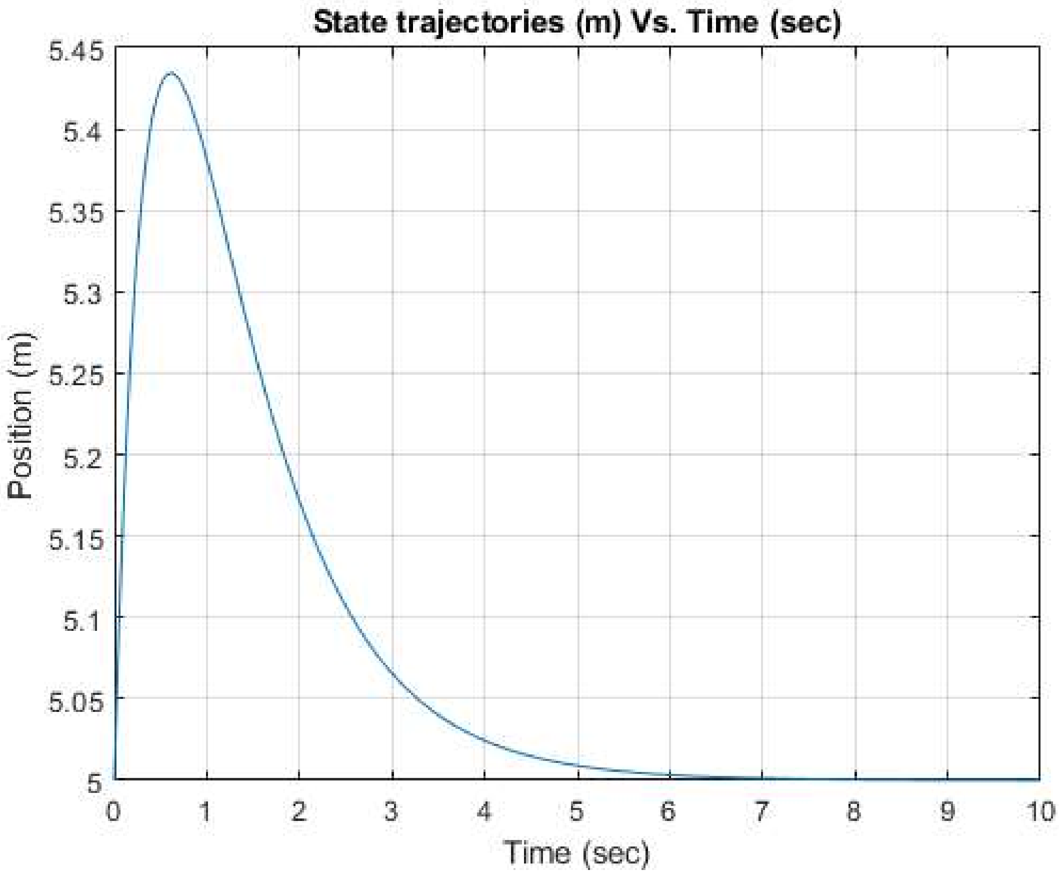
```
sys_open_loop =

  A = 
         x1    x2
   x1     0     1
   x2  -0.4  -0.2

  B = 
        u1
   x1    0
   x2  0.2

  C = 
       x1  x2
   y1   1   0

  D = 
       u1
   y1   0
```

```
Continuous-time state-space model.

desired K that places the poles at the -1 , -2.5 :

K =

   10.5000   16.5000

desired Kr that places the poles at the -1 , -2.5 :

Kr =

   12.5000


sys_closed_loop =

  A =
         x1    x2
   x1     0     1
   x2  -2.5  -3.5

  B =
        u1
   x1    0
   x2  2.5

  C =
       x1  x2
   y1   1   0

  D =
       u1
   y1   0

Continuous-time state-space model.
```



Trying with the different initial position and velocity

```
x0 = [5 2]; % initial position is 5 m and the initial velocit is
% plotting the trajectory of the system
[y,t,x] = lsim(sys_closed_loop,u,t,x0);
figure('Name','With different initial condition')
plot(t,y)
title('State trajectories (m) Vs. Time (sec)')
xlabel('Time (sec)')
ylabel('Position (m)')
grid
```


State trajectories (m) Vs. Time (sec)
Position (m)
Time (sec)
5.45
5.4
5.35
5.3
5.25
5.2
5.15
5.1
5.05
5
0
1
2
3
4
5
6
7
8
9
10