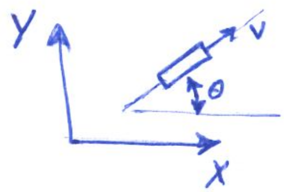
1) $\dot{x} = v\cos\theta$

$\dot{y} = v\sin\theta$

$\dot{\theta} = \omega$

$$\rightarrow \dot{X} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \\ \omega \end{bmatrix}$$



dynamical wrt deviation

$$e_x = \begin{bmatrix} x - x_d \\ y - y_d \\ \theta - \theta_d \end{bmatrix}, \quad e_u = \begin{bmatrix} v - v_d \\ \omega - \omega_d \end{bmatrix}$$

$v_d = v_r \qquad v_r = 10 \qquad v_d = 10$

$x_d = v_r t \qquad y_r = 2 \qquad x_d = 10t$

$y_d = y_r \qquad\qquad y_d = 2$

$$\begin{bmatrix} \theta_d = 0 \\ \omega_d = 0 \end{bmatrix} \qquad \theta_d = 0 \qquad \omega_d = 0$$

$$X_d = \begin{bmatrix} v_r t \\ y_r \\ 0 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

$$u_d = \begin{bmatrix} v_r \\ \omega_d \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$u = \begin{bmatrix} v \\ \omega \end{bmatrix}$$

$$\dot{e}_x = \dot{X} - \dot{X}_d = f(e_x + X_d, e_u + u_d) - f(X_d, u_d) \approx f(X_d, u_d) + \frac{\partial f}{\partial X}\Big|_{\substack{X_d \\ u_d}} e_x + \frac{\partial f}{\partial u}\Big|_{\substack{X_d \\ u_d}} e_u - \cdots - f(X_d, u_d)$$

$$\dot{e}_x \approx \underbrace{\frac{\partial f}{\partial x}\Big|_{X_d, u_d}}_{A(t)} e_x + \underbrace{\frac{\partial f}{\partial u}\Big|_{X_d, u_d}}_{B(t)} e_u$$

$$\dot{X} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \\ \omega \end{bmatrix} = f(X, u)$$

$$\frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -v\sin\theta \\ 0 & 0 & +v\cos\theta \\ 0 & 0 & 0 \end{bmatrix}\Bigg|_{X_d, u_d} = \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 10 \\ 0 & 0 & 0 \end{bmatrix}}_{A(t)}$$

$$\frac{\partial f}{\partial u} = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} \\ \frac{\partial f_3}{\partial u_1} & \frac{\partial f_3}{\partial u_2} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix}\Bigg|_{u_d, X_d} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}}_{B(t)}$$

$$\dot{e}_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 10 \\ 0 & 0 & 0 \end{bmatrix} e_x + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} e_u, \text{ where, } e_x = X - X_d, \; e_u = u - u_d$$

$$e_x = \begin{bmatrix} X - 10t \\ y - 2 \\ \theta - 0 \end{bmatrix}, \quad e_u = \begin{bmatrix} v - 10 \\ \omega - 0 \end{bmatrix}$$

using Pole placement techique we find the desired K (gain) which places the poles in a stable LHS of the imaginary axis. K is calculated as : K=[5000, 200, 300; 1000, 200, 300]

Also, by using the reachability matrix we see that the system is full rank and Controllable.

$$\text{rank}([B \quad AB]) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \text{rank} = 3$$

$u = u_d + e_u, \; e_u = -Ke_x$

$$u = -K e_x + u_d \rightarrow u = -\begin{bmatrix} 5000 & 200 & 300 \\ 1000 & 200 & 300 \end{bmatrix} \begin{bmatrix} X - X_d \\ Y - Y_d \\ \theta - \theta_d \end{bmatrix} + \begin{bmatrix} v_d \\ \omega_d \end{bmatrix}$$

$$u = \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\underbrace{u = -\begin{bmatrix} 5000(X - 10t) + 200(Y - 2) + 300(\theta) \\ 1000(X - 10t) + 200(Y - 2) + 300(\theta) \end{bmatrix} + \begin{bmatrix} 10 \\ 0 \end{bmatrix}}_{\text{Feedback Controller}}$$