(1) $e_x = \begin{bmatrix} x - x_d \\ y - y_d \\ \theta - \theta_d \end{bmatrix}$ $\quad e_u = \begin{bmatrix} e_v \\ e_w \end{bmatrix} = \begin{bmatrix} v - v_r \\ w - w_d \end{bmatrix}$

$$\dot{e}_x = \begin{bmatrix} \dot{x} - \dot{x}_d \\ \dot{y} - \dot{y}_d \\ \dot{\theta} - \dot{\theta}_d \end{bmatrix}$$

$$= \begin{bmatrix} v\cos\theta - v_d\cos\theta_d \\ v\sin\theta - v_d\sin\theta_d \\ w - w_d \end{bmatrix}$$

$$\theta_d = 0$$
$$w_d = 0$$

$$\dot{e}_x = \begin{bmatrix} v\cos\theta - v_r \\ v\sin\theta - 0 \\ \omega \end{bmatrix}$$

$$v = e_v + v_d$$
$$w = e_w + w_d$$
$$\theta = e_\theta + \theta_d$$

$$\dot{e}_x = \begin{bmatrix} (e_v + v_r)\cos(e_\theta) \\ (e_v + v_r)\sin e_\theta \\ e_w \end{bmatrix}$$

Because: $\theta_d = 0, \Rightarrow e_\theta = \theta$

$w_d = 0 \Rightarrow e_w = w.$

(2) Jacobian linearization.

The vector $f(e_x, e_u) = \begin{bmatrix} (e_v + v_r)\cos e_\theta \\ (e_v + v_r)\sin e_\theta \\ e_w \end{bmatrix}$

$$\frac{\partial f}{\partial e_x} = \begin{bmatrix} 0 & 0 & -(e_v + V_r)\sin e_\theta \\ 0 & 0 & (e_v + V_r)\cos e_\theta \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial f}{\partial e_u} = \begin{bmatrix} \frac{\partial f}{\partial e_v} & \frac{\partial f}{\partial e_w} \end{bmatrix}$$

$$= \begin{bmatrix} \cos e_\theta & 0 \\ \sin e_\theta & 0 \\ 0 & 1 \end{bmatrix}$$

Evaluate at the desired trajectory.

$$\left.\frac{\partial f}{\partial e_x}\right|_{\substack{e_x=0 \\ e_u=0}} = \begin{bmatrix} 0 & 0 & -V_r \sin 0 \\ 0 & 0 & V_r \cos 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & V_r \\ 0 & 0 & 0 \end{bmatrix}$$

$$\left.\frac{\partial f}{\partial e_u}\right|_{\substack{e_x=0 \\ e_u=0}} = \begin{bmatrix} \cos 0 & 0 \\ \sin 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Thus: the linear approximation gives.

$$\dot{e}_x \approx \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & V_r \\ 0 & 0 & 0 \end{bmatrix} e_x + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} e_u$$

Explicitly: $\dot{e}_x = e_v$ subsystem 1

$\dot{e}_y = V_r e_\theta$

$\dot{e}_\theta = e_w$ subsystem 2

For subsystem 1: $e_v = -k e_x$, $k > 0$.

$\dot{e}_x = -k e_x$ is stable

input velocity: $\boxed{v} = e_v + V_r = -k(x - x_d) + V_r$

$= -k(x - V_r t) + V_r$

For subsystem 2. $\underset{A}{\begin{bmatrix} 0 & V_r \\ 0 & 0 \end{bmatrix}}$ $\underset{B}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}$

design $e_w = -k_1 e_y - k_2 e_\theta$

Such that the closed loop is

$$\begin{bmatrix} 0 & V_r \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -k_1 & -k_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & V_r \\ k_1 & k_2 \end{bmatrix}$$

pole placement.

$$e_w = -k_1 e_y - k_2 e_\theta$$

$$w - w_d = -k_1 (y - y_d) - k_2 (\theta - \theta_d)$$

$$\boxed{w = -k_1 (y - y_r) - k_2 \theta}$$

Steering control.