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Hybrid force/position control scheme for flexible joint robot with friction between and the end-effector and the environment

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ABSTRACT

In this paper a novel adaptive-sliding mode control design is developed for trajectory tracking of a class of flexible robots. This control design uses a force/position controller for robots with flexible joints where the friction between the end effector and the environment is taken into account. For systematic reasons the controller is designed considering the rigid links subsystem and the flexible joints. The proposed control system, satisfies the stability of the two subsystems and cope with the uncertainty of the robot dynamics. The feedback for the flexible subsystem is based on the joint torque signal and estimation of the torque derivative which is provided by a nonlinear sliding observer. For the stability of the observer, it is assumed that the uncertainty of the observed system is bounded. A MRAC algorithm is used for the estimation of the friction forces at the contact point between the end effector and environment. Finally simulation results are given to demonstrate the effectiveness of the proposed controller.

1. Introduction

The control of flexible robots has been an important research topic and many approaches have been proposed [1,2]. Feedback linearization based adaptive control is suitable for the control of nonlinear systems with accurate nominal models or linearly parametrizable dynamical models. However, due to modeling errors, these control methods may not be very effective without proper compensation to overcome the modeling error effects. The design of robust control for uncertain nonlinear systems is still a challenging task. In the analysis of the robot dynamics, frequently the rigidity of the links and joints is assumed as a prerequisite. Also friction forces at the end point of the robot are not taken into account. These assumptions facilitate the implementation of the control laws but some times are too far from the reality.

In the present paper, a new sliding-control-based approach is presented to overcome the above modeling problems based on sliding control. It was tried to make the least assumptions and build a controller that can encounter successfully the real world problems.

1.1. Related work

The hybrid robot force control literature reports different approaches for the control of robots executing constrained motion [3,4]. A variety of dynamic/model-based hybrid controllers [5] which take into account the full rigid body dynamical model of the robot and guarantee asymptotically exact force trajectory tracking have since been proposed for the hard contact case.

Due to the high ratio between the stiffness of the links and joints, the compliance of commercial robots is concentrated mainly in the joints. Therefore the compensation of the joint flexibility is a very interesting task in the control design of the

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industrial robots. Several control approaches have been proposed to compensate the low joint stiffness such as the adaptive sliding control [6], feedback linearization and the singular perturbation methods [7]. Some researchers have concentrated on the combination of sliding and adaptive algorithms in order to control the manipulators with compliant joints [8–10].

In [11] an adaptive algorithm is presented for the joint control of flexible joint robots. An adaptive law estimates the unknown parameters of the system such us the inertia of links, the stiffness of joints, etc. and avoids the singularities in the estimation of the unknown parameters. The control law requires a double differentiation of the inertial and centrifugal matrices of the rigid subsystem and a triple differentiation of the actuators angular position. This increases the computational complexities, while the influence of the triple differentiation on the measured data due to the existence of noise needs further investigation.

A smooth sliding-mode controller for friction compensation was introduced in [12] under the assumption that the friction model is exponential. The friction force is bounded by a hyperbolic function and the static friction force is estimated adaptively. The asymptotic stability of the controller is proved via a Lyapunov function.

A new general scheme for modeling of friction forces is presented in [13], which is based on a linear differential form. This formulation facilitates the design of an adaptive algorithm for the prediction of friction forces. This adaptive algorithm is used in the present paper in order to achieve the compensation of the friction forces.

A challenge in the research of the flexible joint robot control is the minimization of number and cost of the sensors required for the control algorithms' implementation. Several controllers have been introduced including observers or filters that estimate some of the state variables to reduce the number of required sensors. A critical point in the design of the observers is that a minimum knowledge of the physical system is necessary. Therefore, a balance between the knowledge about the physical system and the number of the system states that can be reconstructed by an observer has to be achieved. In [14,15] different trends concerning this balance are presented.

In [16] a new observer design is introduced to reconstruct the state space of a flexible joint robot based only on the links global position. In this configuration, it is assumed that the manipulator characteristics are exactly known. The proposed method is valid for full rigid manipulators too.

1.2. Our approach

In the present paper, a sliding-adaptive control algorithm for a class of flexible joint manipulators is presented. The robot joints are assumed to act as torsional springs. The controller is designed in two steps. In the first step the desired torque of the motors is determined, in order to obtain the desired response of the rigid subsystem. In the second step, the control input for the motors is defined in order to meet the desired motor torque. The compensation of the friction at the endpoint of the robot is included in the desired torque for the rigid subsystem. The actuators are controlled to obtain the desired torque for the rigid subsystem. The joints torque is used in the control law because it is a signal of high gain and is more convenient to know the torque which is needed to move the links, rather than the displacements of flexible joints. In the control law, the time derivative of the joints torque is needed, but is not recommended to measure this signal directly. Therefore an observer is used to obtain an accurate estimation of the torque time derivative. In [17], the authors first introduced the basic structure of this observer. In the present paper, this type of the observer is extended for the case of parametric uncertainty.

While most previously reported hybrid (or force) controllers for flexible-joint robots guarantee stability, none has considered both the rigid links and flexible joints (two subsystems) at the same time with friction between the end-effector and the environment. The controller provably guarantees stability for force/position trajectory tracking with minimized error. To the best of our knowledge, no provably stable hybrid force/position trajectory tracking controller has been reported in the literature which treats both the rigid and flexible subsystems considering parametric uncertainties.

The design of observer presented in [17] is also extended for the case of parametric uncertainty.

The development of the control algorithm is based on the following assumptions:

- The displacement, velocity and the acceleration of the links are available for measurement.
- The length of each link is assumed known, but the inertia of the links is assumed unknown.
- The inertia and the stiffness of the motor rotors of the joints are assumed unknown, while its bounds and the maximum motors torque, are assumed known.
- The Jacobian matrix has full rank over the configuration space.
- The desired trajectory of the robot is at least four times differentiable with respect to time and the desired forces are at least twice.
- The environment is assumed to be rigid.
- Friction force is developed between the environment and the end effector. The used friction model includes the exponential decay part, the Coulomb and viscous friction. The unknown parameters of the developed friction force are the coefficients of Coulomb and viscous friction.

2. The robot model

In this section the model of a robot with flexible joints applying force to the rigid environment is presented. A revolute flexible joint direct-drive robot is described by the following equations:

$$
\begin{aligned}
&H_r(q)\ddot{x}(r) + C_r(q,\dot{q})\dot{x}(r) + g_r(q) = J^{-T}\tau_j - F_s \\
&H_m\ddot{\theta} + \tau_j = u \\
&\tau_j = K_j(\theta - q) \\
&x_r = J\dot{q}
\end{aligned} \tag{1}
$$

where H_r, $C_r \in R^3 \times R^3$ are the inertia and the coriolis matrices of the rigid subsystem in the C-Frame respectively. $G_r \in R^3$represents the gravity forces for the rigid subsystem. H_m, $K_j \in R^n \times R^n$ are the inertia matrix of the actuator and the joint stiffness matrix respectively. The vectors q and θ are the angular displacements of the links and actuators respectively. The vectors u and $\tau_j \in R^n$ are the torque input and the joint torque respectively. $\mathbf{x}_r \in R^3$ is the position vector of the robot end effector. The vector $F_s \in R^3$ is the generalized force measured by the force sensor mounted on the wrist. J represents the Jacobian matrix of the rigid subsystem and n is the number of robot joints. Furthermore, the coupling inertial term is assumed to be negligible as compared to the inertia of the drive system.

The control system has to obtain the following desired response of the robot:

$$
\begin{aligned}
&\Sigma x_r = x_d, \quad \Sigma\dot{x}_r = \dot{x}_d \\
&F_s^n = F_d
\end{aligned} \tag{2}
$$

where $x_d \in R^3, \dot{x}_d \in R^3$ are the desired position and velocity for the robot end effector respectively, $F_d \in R^3$ is the desired force applied by the robot to the environment, $\Sigma \in R^3 \times R^3$ is a selection matrix. The force vector F_s can be analyzed in two components; the $F_s^n \in R^3$ normal to the environment surface and F_s^t tangent to the contact surface. The component F_s^n is equal to the normal force applied by the robot to the environment. The component F_s^t is equal to the friction force between the robot end-point and the environment surface.

The vector F_s and its components satisfy the following equations:

$$
\begin{aligned}
&F_s = F_s^n + F_s^t \\
&F_s^n = TF_s = TK_s(x_r - x_d) \\
&F_s^t = \Sigma F_s = \Sigma K_s(x_r - x_d)
\end{aligned} \tag{3}
$$

where $K_s \in R^3 \times R^3$ is the force sensor stiffness matrix. The diagonal matrices T and Σ satisfy the following equation:

$$\Sigma + T = I \tag{4}$$

where I is the identity matrix. The friction model used in this paper is analyzed in [18]. Based on this approach the friction coefficient is obtained by the following differential equation:

$$
\begin{aligned}
&\frac{d\mu}{dt} + k\cdot\mu\cdot\frac{dv}{dt}\cdot v = \lambda\cdot v\cdot\frac{dv}{dt} + \rho\cdot v\frac{dv}{dt} \\
&\mu_s = \mu(0) + \frac{\lambda}{k} - \frac{\rho}{k^2} \\
&\mu_c = \frac{\lambda}{k} - \frac{\rho}{k^2} \\
&\mu_r = \frac{\rho}{k\cdot w} \\
&w = \|F_s^n\|_2 \\
&v = \frac{abs(\dot{x}_r^T\Sigma^T)}{\|\dot{x}_r^T\Sigma^T\|}\cdot\Sigma\cdot\dot{x}_r
\end{aligned} \tag{5}
$$

where μ_s is the static friction coefficient, μ_c is the Coulomb friction coefficient, μ_r is the Reyleigh friction coefficient, v is the signed magnitude of the velocity of the robot tool at the contact point. The variables λ, ρ, κ are unknown parameters of the friction model. In the following the estimated values of the variables are noted by the $\hat{}$ symbol above them and the difference between the real and the estimated values by the $\sim$ symbol.

3. Control strategy and stability study

The developed control strategy is divided into two steps for systematic reasons. In the first step, the desired torque of the joints is determined in order to obtain the desired response of the rigid subsystem. In the second step, the control input for the motors is determined in order to meet the desired torque determined in the first step. In order to calculate the desired joint torque, the following sliding surface is defined:

$$s_r = \Sigma\cdot[(\dot{x}_r - \dot{x}_d) + \Lambda_r(\dot{x}_r - \dot{x}_d)] + [\dot{F}_s^n + \Lambda_r(F_s^n - F_d)] \tag{6}$$

where $\Lambda \in R^3 \times R^3$ is a positive definite constant diagonal matrix.

Since force measurement signals tend to be noisy and the noise is amplified by the differentiation of the force error. Therefore, using (3), the following form of the sliding surface is used by the controller:

$$s_{\mathrm{r}} = [(\Sigma + TK_{\mathrm{s}}) \cdot (\dot{x}_{\mathrm{r}} - \dot{x}_{\mathrm{d}}) + \Sigma \Lambda_{\mathrm{r}}(\dot{x}_{\mathrm{r}} - \dot{x}_{\mathrm{d}})] + \Lambda_{\mathrm{r}}(F_{\mathrm{s}}^{n} - F_{\mathrm{d}}) \tag{7}$$

The final form of the sliding error that is directly related to the position and velocity error of the end point is given as

$$s_{\mathrm{r}} = (\Sigma + TK_{\mathrm{s}})[(\dot{x}_{\mathrm{r}} - \dot{x}_{\mathrm{d}}) + \Lambda_{\mathrm{r}}(\dot{x}_{\mathrm{r}} - \dot{x}_{\mathrm{d}})] \tag{8}$$

As it can be seen the hybrid sliding error is converted to a position sliding error. Eq. (8) can be written in the following form:

$$\begin{aligned} s'_{\mathrm{r}} &= (\dot{x}_{\mathrm{r}} - \dot{x}_{\mathrm{d}}) + \Lambda_{\mathrm{r}}(\dot{x}_{\mathrm{r}} - \dot{x}_{\mathrm{d}}) \\ s_{\mathrm{r}} &= (\Sigma + TK_{\mathrm{s}})s'_{\mathrm{r}} \end{aligned} \tag{9}$$

The reference velocity is given by the following equation:

$$\begin{aligned} v'_{\mathrm{r}} &= [\ddot{x}_{\mathrm{d}} - \Lambda_{\mathrm{r}}(\dot{x}_{\mathrm{r}} - \dot{x}_{\mathrm{d}})] \\ v_{\mathrm{r}} &= (\Sigma + TK_{\mathrm{s}})v'_{\mathrm{r}} \end{aligned} \tag{10}$$

Based on the definition of the sliding error of the rigid subsystem, the following desired torque is proposed:

$$\tau_{\mathrm{md}} = J^{\mathrm{T}} \cdot \left[Y_{v} \cdot \hat{p}_{\mathrm{r}} - R \cdot (\Sigma + TK_{\mathrm{s}})^{-1} s_{\mathrm{r}} + F_{\mathrm{s}}^{n} + \widehat{F}_{\mathrm{s}}^{t} \right] \tag{11}$$

where $R \in R^3 \times R^3$ is a positive definite matrix, $\hat{p}_{\mathrm{r}} R^5$ is the vector of the rigid subsystem estimated parameters. The regression matrix $Y_v \in R^3 \times R^5$ and the estimated parameters of the rigid subsystem are satisfying the following linearity property [19]:

$$Y_{v} \cdot \widehat{P}_{\mathrm{r}} = \widehat{H}_{\mathrm{r}}(q) \cdot \dot{v}_{\mathrm{r}} + \widehat{C}_{\mathrm{r}}(q \cdot \dot{q}) \cdot v_{\mathrm{r}} + \hat{g}_{\mathrm{r}}(q) \tag{12}$$

The estimated parameters vector $\widehat{P}_{\mathrm{r}}$ is calculated by the following update law:

$$\widehat{P}_{\mathrm{r}} = -\Gamma_{\mathrm{r}}^{\mathrm{T}} \cdot Y_{v}^{\mathrm{T}} \cdot (\Sigma + TK_{\mathrm{s}})^{-1} \cdot s_{\mathrm{r}} \tag{13}$$

where $\Gamma_{\mathrm{r}} \in R^3 \times R^5$ is a diagonal positive definite matrix.

For the estimation of the friction coefficients a MRAC (Model reference adaptive control) model is used [18] and the proposed adaptive laws are the following:

$$\begin{aligned} &\frac{\mathrm{d}\hat{\mu}}{\mathrm{d}t} + \hat{k} \cdot v \cdot \frac{\mathrm{d}v}{\mathrm{d}t}\hat{\mu} = v \cdot \frac{\mathrm{d}v}{\mathrm{d}t}\hat{\lambda} + \hat{\rho} \cdot v\frac{\mathrm{d}v}{\mathrm{d}t} - G \cdot (\tilde{\mu}) \\ &\dot{\hat{\lambda}} = -\frac{1}{\rho_f} \cdot \frac{\mathrm{d}v}{\mathrm{d}t} \cdot v \cdot \tilde{\mu} \\ &\dot{\hat{k}} = \frac{1}{\rho_f} \cdot \mu \cdot \frac{\mathrm{d}v}{\mathrm{d}t} \cdot v \cdot \tilde{\mu} \\ &\dot{\hat{\rho}} = -\frac{1}{\rho_f} \cdot v \cdot \frac{\mathrm{d}v}{\mathrm{d}t} \cdot \tilde{u} \\ &g = \left| \eta - \hat{k} \cdot v \cdot \frac{\mathrm{d}v}{\mathrm{d}t} \right| |\tilde{\mu}| + \left| s_{\mathrm{r}}^{\prime\mathrm{T}} \cdot \tilde{\mu} \cdot F_{\mathrm{s}}^{n} \cdot \Sigma \cdot \frac{\dot{x}_{\mathrm{r}}}{\|\dot{x}_{\mathrm{r}}\|} \right| \end{aligned} \tag{14}$$

where ρ_f is a positive constant, and η is a positive constant. As it can be seen the second term of g in (14) is the internal product of the vector s_{r} and the error of the friction force estimation. This term represents the contribution of the friction force error to the sliding errors_{r}.

3.1. Stability analysis for the rigid subsystem

For the stability study of the system along the desired trajectory the following Lyapunov candidate function is defined:

$$V(t) = \frac{1}{2}s_{\mathrm{r}}^{\prime\mathrm{T}} H_{\mathrm{r}} s'_{\mathrm{r}} + \frac{1}{2}\tilde{p}_{\mathrm{r}}^{\mathrm{T}} \Gamma_{\mathrm{r}}^{-1} \tilde{p}_{\mathrm{r}} + V_{\mathrm{fr}}(t) \tag{15}$$

The positive function $V_{\mathrm{fr}}(t)$ is referred to friction estimator stability. The Lyapunov function candidate $V(t)$ represents a measure of the robot rigid system sliding error, the unknown parameters estimation error and the $V_{\mathrm{fr}}(t)$. The candidate Lyapunov function is globally positive definite, continuous and differentiable with respect to time. The time derivative of the Lyapunov function candidate is the following:

$$\dot{V}(t) = s_{\mathrm{r}}^{\prime\mathrm{T}} H_{\mathrm{r}} s'_{\mathrm{r}} + \frac{1}{2}s_{\mathrm{r}}^{\prime\mathrm{T}} \dot{H}_{\mathrm{r}} s'_{\mathrm{r}} + \dot{\tilde{p}}_{\mathrm{r}}^{\mathrm{T}} \Gamma_{\mathrm{r}}^{-1} \tilde{p}_{\mathrm{r}} + \dot{V}_{\mathrm{fr}}(t) \tag{16}$$

Replacing (1) and (11) in (16) the following expression is obtained:

$$\dot{V}(t) = s_{\mathrm{r}}^{\prime\mathrm{T}}[Y_{v}\tilde{p}_{\mathrm{r}} - Rs'_{\mathrm{r}}] + \dot{\tilde{p}}_{\mathrm{r}}^{\mathrm{T}} \Gamma_{\mathrm{r}}^{-1} \tilde{p}_{\mathrm{r}} + s_{\mathrm{r}}^{\prime\mathrm{T}} \tilde{\mu} F_{\mathrm{s}}^{n} \cdot \Sigma \frac{\dot{x}_{\mathrm{r}}}{\|\dot{x}_{\mathrm{r}}\|} + \dot{V}_{\mathrm{fr}}(t) \tag{17}$$

For the clarity of the calculations, (17) is separated into the following parts:

$$\dot{V}_1(t) = s_{\mathrm{r}}^{\prime\mathrm{T}}[Y_v\tilde{p}_{\mathrm{r}} - Rs_{\mathrm{r}}'] + \dot{\tilde{p}}_{\mathrm{r}}^{\mathrm{T}}\Gamma \tag{18}$$

$$\dot{V}_2(t) = s_{\mathrm{r}}^{\prime\mathrm{T}}\mu F_{\mathrm{s}}^n \cdot \Sigma\frac{\dot{x}_{\mathrm{r}}}{\|\dot{x}_{\mathrm{r}}\|} + \dot{V}_{\mathrm{fr}}(t) \tag{19}$$

If the following inequalities are satisfied then the stability of the system can be achieved:

$$\dot{V}_1(t) \leqslant 0 \tag{20}$$

$$\dot{V}_2(t) \leqslant 0 \tag{21}$$

The first inequality is referred to the rigid subsystem of the robot, the second to the friction compensation. Eq. (20) is proved by replacing $\dot{\tilde{p}}_{\mathrm{r}}$ given by (13) in (18) and the resultant inequality is the following:

$$\dot{V}_1(t) \leqslant s_{\mathrm{r}}^{\prime\mathrm{T}}Rs_{\mathrm{r}}' \tag{22}$$

What remains to be proved is the satisfaction of inequality (21) which is related with the friction compensation system.

3.1.1. Stability analysis of the friction compensation

In this section the stability analysis of the friction compensation system is presented. The friction model is given by (5) where the constants λ and ρ are the parameters of the friction coefficient function versus velocity. If these and the initial condition for the (5) are known then the friction force is known in the whole region of velocities v.

For the stability study of the friction estimator the following Lyapunov candidate function is defined:

$$V_{\mathrm{fr}}(t) = \frac{1}{2}\tilde{\mu}^2 + \frac{1}{2}\rho_f(\tilde{k}^2 + \tilde{\lambda}^2 + \tilde{\rho}^2) \tag{23}$$

The time derivative of the candidate Lyapunov function is the following:

$$\dot{V}_{\mathrm{fr}}(t) = \tilde{\mu}\dot{\tilde{\mu}} + \rho_f(\tilde{k}\dot{\tilde{k}} + \tilde{\lambda}\dot{\tilde{\lambda}} + \tilde{\rho}\dot{\tilde{\rho}}) \tag{24}$$

Based on (23) the (19) is transformed to the following form:

$$\dot{V}_2(t) = s_{\mathrm{r}}^{\prime\mathrm{T}}\mu F_{\mathrm{s}}^n \cdot \Sigma\frac{\dot{x}_{\mathrm{r}}}{\|\dot{x}_{\mathrm{r}}\|} + \tilde{\mu}\dot{\tilde{\mu}} + \rho_f(\tilde{k}\dot{\tilde{k}} + \tilde{\lambda}\dot{\tilde{\lambda}} + \tilde{\rho}\dot{\tilde{\rho}}) \tag{25}$$

Replacing (14) and (5) in (25) the following inequality is obtained:

$$\dot{V}_2(t) \leqslant -\eta\tilde{\mu}^2 \tag{26}$$

From (22) and (26) it can be concluded that the time derivative of the Lyapunov function $V(t)$ given by (16) is negative semi-definite, therefore the rigid subsystem is stable if the input torque is given by (11). In the next sub-section the stability of the flexible subsystem is proved.

3.1.2. Stability analysis of the flexible subsystem

For the study of the flexible subsystem stability the following sliding error is formed:

$$s = (\dot{\tau}_J - \dot{\tau}_{\mathrm{md}}) + \Lambda_J(\tau_J - \tau_{\mathrm{md}}) \tag{27}$$

where $\Lambda_{\mathrm{m}} \in R^n \times R^n$ is a positive definite and diagonal matrix. The candidate Lyapunov function formed to investigate the stability of the flexible subsystem is the following:

$$V_{\mathrm{fr}}(t) = \frac{1}{2}s^2 H_{\mathrm{m}}K_J^{-1}s \tag{28}$$

The time derivative of the candidate Lyapunov function is given by

$$\dot{V}_{\mathrm{fr}}(t) = s^{\mathrm{T}}H_{\mathrm{m}}K_J^{-1}\{[K_J H_{\mathrm{m}}^{-1}K_J^{-1}(u - \tau_J - H_{\mathrm{m}}\ddot{q}) - \ddot{\tau}_{\mathrm{md}}] + \Lambda_J(\dot{\tau}_J - \dot{\tau}_{\mathrm{md}})\} \tag{29}$$

Where u is the control input to the robot joints which is given by the following expression:

$$\begin{aligned} u &= -P_J s + \tau_J + \widehat{H}_{\mathrm{m}}\ddot{q} - \widetilde{A}\Lambda_J(\dot{\tau}_J - \dot{\tau}_{\mathrm{md}}) - G_J\mathrm{sign}(s) \\ A &= H_{\mathrm{m}}K_J^{-1} \end{aligned} \tag{30}$$

where $P_J \in R^n \times R^n$, $G_J \in R^n \times R^n$ are positive definite and diagonal matrices. It is assumed that the bounds of the uncertainty for the matrices H_{m} and A are known and the vector of the maximum torques of the motors T_{m} is known too. The bound matrices are the following:

$$D_{\widetilde{A}} \in R^n \times R^n, \text{ with } \{D_{\widetilde{A}}\}_{i,j} \geqslant |\{\widetilde{A}\}_{i,j}| \tag{31}$$

$$D_{\widetilde{H}_{\mathrm{m}}} \in R^n \times R^n, \text{ with } \{D_{\widetilde{H}_{\mathrm{m}}}\}_{i,j} \geqslant |\{\widetilde{H}_{\mathrm{m}}\}_{i,j}| \tag{32}$$

The diagonal elements of the matrix G_J are selected to be given by the following equation:

$$\{G_J\}_{i,i} = \{D_{\widetilde{H}_m}\}_{i,i} \cdot |\ddot{q}_i| + \{D_{\widetilde{A}}\}_{i,i} \cdot \{\Lambda_J\}_{i,i} \cdot |\{\dot{\tau}_J\}_i - \{\dot{\tau}_{md}\}_i| + \{T_m\}_i + \{N_J\}_{i,i} \tag{33}$$

where $N_J \in R^n \times R^n$ is a positive definite and diagonal matrix and i is the index of rows. By substitution of (30) and (33) in (29) the following inequality is proved:

$$\dot{V}_{fl}(t) \leqslant -s^T N_J \mathrm{sign}(s) \tag{34}$$

3.1.3. A sliding observer for the estimation of the torque time derivative

In the control law for the robot motors given by (30), the time derivative of joint torques is used. As it is known, the quality of the first derivative of force or torque signals is low due to the induced noise therefore a sliding observer is employed in the control design in order to avoid these difficulties. The main structure of this observer is based on a model presented in [17], extended to be valid in systems with uncertainties. The dynamics of the flexible robot subsystem given by (1) is written in the following form:

$$\begin{aligned} x_1 &= \tau_J \\ x_2 &= \dot{\tau}_J \\ \dot{x}_1 &= x_2 \\ \dot{x}_2 &= A^{-1}(u - x_1) - K_J\ddot{q} \end{aligned} \tag{35}$$

It is assumed that the observer is described by the following equations:

$$\begin{aligned} \dot{\hat{x}}_1 &= -\Gamma_1 \cdot \tilde{x}_1 - \Lambda_1 \mathrm{sgn}(\tilde{x}_1) + \tilde{x}_2 \\ \dot{\hat{x}}_2 &= -\Gamma_2 \cdot \tilde{x}_1 - \Lambda_2 \mathrm{sgn}(\tilde{x}_1) + \widehat{A}^{-1}(u - x_1) - \widehat{K}_J\ddot{q} \end{aligned} \tag{36}$$

where $\Gamma_1 \in R^n \times R^n$, $\Gamma_2 \in R^n \times R^n$, $\Lambda_1 \in R^n \times R^n$, $\Lambda_2 \in R^n \times R^n$, are positive definite and diagonal matrices. In the general case the matrices Λ_1 and Λ_2 could be constant. In the following we will discus this issue further and we will investigate the possibility to have time variable matrix.

In the structure of the observer, it is assumed that knowledge uncertainty of the inertia and stiffness matrices of the flexible robot subsystem appears. This point makes the difference between this observer and the one introduced in [17]. In order to improve the computation time and to focus on the uncertainty of the system the following two expressions are introduced in the structure of the observer:

$$\begin{aligned} Y_1\tilde{r}_1 &= \widetilde{A}^{-1} \cdot (U - x_1) \\ Y_2\tilde{r}_2 &= -K_J^{-1}\ddot{q} \end{aligned} \tag{37}$$

where r_1 and r_2 are the vectors with the unknown quantities and Y_1 and Y_2 are the regression matrices. By substituting (37) in (36) and subtracting (35) the following form for the observer error is obtained:

$$\begin{aligned} \dot{\tilde{x}}_1 &= -\Gamma_1 \cdot \tilde{x}_1 - \Lambda_1 \mathrm{sgn}(\tilde{x}_1) + \tilde{x}_2 \\ \dot{\tilde{x}}_2 &= -\Gamma_2 \cdot \tilde{x}_1 - \Lambda_2 \mathrm{sgn}(\tilde{x}_1) + Y_1\tilde{r}_1 + Y_2\tilde{r}_2 \end{aligned} \tag{38}$$

The stability of the observer is based on the convergence of the error $\tilde{x}_1$, which is a measurable quantity, since x_1 is measured using the joint torque sensors. For the study of the convergence of $\tilde{x}_1$, the following candidate function is defined:

$$V_{01}(t) = \frac{1}{2}\tilde{x}_1^T x_1 \tag{39}$$

Using Eqs. (36) and (39) the time derivative of $V_{01}(t)$ is given by the following expression:

$$\dot{V}_{01}(t) = \tilde{x}_1^T[-\Gamma_1\tilde{x}_1 - \Lambda_1 \mathrm{sign}(\tilde{x}_1) + \tilde{x}_2] \tag{40}$$

If the elements of the matrix Λ_1 satisfy the following inequality then the stability of $\tilde{x}_1$ around 0 is succeeded and it is exponential [19]:

$$\{\Lambda_1\}_{i,i} \geqslant |\{\tilde{x}_2\}_i| \tag{41}$$

Based on expression (41) we will investigate the stability of convergence of $\tilde{x}_1$ around 0 and the next step will be the definition of matrices Λ_1 and Λ_2. The time derivative of the Lyapunov function satisfies the following inequality:

$$\begin{aligned} &\dot{V}_{01}(t) \leqslant -\tilde{x}_1^T \cdot \Gamma_1 \cdot \tilde{x}_1 \Rightarrow \dot{V}_{01}(t) \leqslant -\rho_1 \cdot \|\tilde{x}_1\|_2^2 \\ &\rho_1 = \min\{\Gamma_1\}_{i,i}, i = 1, 2, \ldots, n \end{aligned} \tag{42}$$

This inequality insures the global exponential stability of $\|\tilde{x}_1\|_2$ around 0. In the next step the convergence of the error $\|\tilde{x}_2\|_2$ is investigated, which is not technically a measurable quantity. While the convergence of $\|\tilde{x}_1\|_2$ is succeeded, the vector $\tilde{x}_1$ is oscillating on the surface $\tilde{x}_1 = 0$, and the invariance of this surface is given by the equation $\dot{\tilde{x}}_1 = 0$. This means that the vector $\dot{\tilde{x}}_1$ is tangent to the surface $\tilde{x}_1 = 0$. However this tangential geometry does not appear due to the chattering effect but it can be concluded that the average dynamics satisfy the invariance defined by the condition $\dot{\tilde{x}} = 0$. According to Fillipov's mathematical construction on variable systems [20], the following equation for the system dynamics is written:

$$\begin{aligned} &\dot{\tilde{x}}_1 = Qf^+ + (I-Q)f^- \\ &f = -\Gamma_1\tilde{x}_1 - \Lambda_1\mathrm{sgn}(\tilde{x}_1) + \tilde{x}_2 \end{aligned} \tag{43}$$

where f^+ and f^- are the dynamic's actions due to the sign part of (36). Therefore (38) is written in the following form:

$$\dot{\tilde{x}}_1 = Q\cdot(-\Gamma_1\cdot\tilde{x}_1 - \Lambda_1\cdot G + \tilde{x}_2) + (I-Q)\cdot(-\Gamma_1\cdot\tilde{x}_1 - \Lambda_1\cdot G + \tilde{x}_2) \tag{44}$$

where $G \in R^n$,$\{G_i\}$ = 1 and $Q \in R^n \times R^n$ is a diagonal matrix. Taking into account the invariance defined by condition $\dot{\tilde{x}}_1 = 0$ the following equation is obtained:

$$\{Q\}_{i,i} = \frac{1}{2\{\Lambda_i\}_{i,i}}(\{\tilde{x}_2\}_i + \{\Lambda_1\}_{i,i} - \{\Gamma_1\}_{i,i}\{x_1\}_i) \tag{45}$$

Substituting (44) in (38) results the following equality:

$$\dot{\tilde{x}}_2 = (\Lambda_1^{-1}\cdot\Lambda_2\cdot\Gamma_1 - \Gamma_2)\cdot\tilde{x}_1 - \Lambda_1^{-1}\cdot\Lambda_2\cdot\tilde{x}_2 + Y_1\tilde{r}_1 + Y_2\tilde{r}_2 \tag{46}$$

In order to study the convergence of $\tilde{x}_2$ the following Lyapunov candidate function is formed:

$$V_{02}(t) = \frac{1}{2}\tilde{x}_2^{\mathrm{T}}x_2 \tag{47}$$

The elements of the matrices Γ_2 and Λ_2 are selected to be given by the following equations:

$$\begin{aligned} &\{\Lambda_2\}_{i,i} = \phi\left[\{\Lambda_1\}_{i,i}\{\Delta\}_{i,i} + \frac{\{\Lambda_1\}_{i,i}}{\delta}(\{Y_1\}_{i,i}\{d_1\}_i + \{Y_2\}_{i,i}\{d_2\}_i)\right] \\ &\Gamma_2 = \Lambda_1^{-1}\Lambda_2\Gamma_1 \end{aligned} \tag{48}$$

where $\phi \in R$, $\phi \geqslant 1$ and $\Delta \in R^n \times R^n$ is a diagonal positive definite matrix, $d_1 \in R^n$, $d_1 \geqslant 0$ and $d_2 \in R^n$, $d_2 \geqslant 0$ are the known bounds of the uncertainty of $\tilde{r}_1$ and$\tilde{r}_2$. Finally $\delta \geqslant 0$ is the desired accuracy for the convergence of the error $\tilde{x}_2$. Using (48) it is proved that the time derivative of $V_{02}(t)$ satisfies the following inequality:

$$\dot{V}_{02}(t) \leqslant \tilde{x}_2^{\mathrm{T}}\Delta\tilde{x}_2 \tag{49}$$

This condition ensures the exponential convergence of error $\|\tilde{x}_2\|_2$ to the surface of the sphere $B(0,\sqrt{n\delta})$ where n is the dimension of observer vectors. The convergence continues further but not in an exponential way until the inequality of (49) becomes equality, where the Euclidean norm of the error $\tilde{x}_2$ satisfies the following equality:

$$\dot{V}_{02}(t) = 0 \tag{50}$$

After that the time derivative of the function $\dot{V}_{02}(t)$ becomes positive and the error $\tilde{x}_2$ diverges until it passes the surface of the sphere $B(0{\cdot}\rho_l)$ where ρ_l is the radius of the sphere that refers to the condition of (50). Then the convergence starts again. In other words the Euclidean norm of $\tilde{x}_2$ oscillates around the shell of sphere $B(0{\cdot}\rho_l)$. To guarantee that this oscillation will not drive $\|\tilde{x}_2\|_2$ out of $B(0,\sqrt{n\delta})$ is the continuity of $\dot{V}_2(t)$. So the condition to attain required accuracy δ is always satisfied. While $\tilde{x}_1$ belongs to the surface $\|\tilde{x}_1\|_2 = 0$ and the condition of (41) is always satisfied, the measure $\|\tilde{x}_2\|_2$ converges to $B(0{\cdot}\rho_l)$.

Until now we have done all the above analyses assuming that the elements of matrix $\Lambda 1$ are constant. However matrix $\Lambda 1$ can have time varying elements which will be reduced during time with a rate lower than the rate of the $\{\tilde{x}_2\}_i$ decay. Assume that the matrix Λ_1 is given by the following equation (see (41) and [17]):

$$\begin{aligned} &\Lambda_1 = \Lambda_0 e^{-k_1 t} + K_2 l \\ &\{\Lambda_0\}_{i,i} \geqslant \sum_{i=1}^{n}|\{\tilde{x}_2(0)\}_i| \\ &0 \leqslant k_1 \leqslant \frac{\rho_2}{2} \\ &\rho_2 - \min\{\{\Delta\}_{i,i}, i = 1,2,\ldots,n\} \\ &k_2 \geqslant \delta \end{aligned} \tag{51}$$

where $\Lambda_0 \in R^n \times R^n$ is a diagonal matrix. It will be proved now that the matrix Λ_1 satisfies (41). For $t = 0$ the following equality satisfies (41):

$$\lim_{t\to 0}\Lambda_1 = \Lambda_0 + k_2 I \tag{52}$$

For t tends to infinity the following equation results:

$$\lim_{\to\infty}\Lambda_1 = k_2 I \tag{53}$$

It is obvious that (52) satisfies the inequality (41). It remains to be proved that Λ_1 satisfies (41) for $t \in (0,\infty)$. The following inequality can be proved easily from (51):

$$\{\Lambda_0\}_{i,i}^2 e^{-2k_1 t} \geqslant \|\tilde{x}_2(0)\|^2 e^{-\rho_2 t} \tag{54}$$

From this inequality the following inequality can be obtained:

$$\{\Lambda_0\}_{i,i} e^{-k_1 t} \geqslant |\{\tilde{x}_2\}_i| \quad \forall t \in (0,\infty) \tag{55}$$

The last relation is enough to satisfy (41)

After these the stability of the observer is proved and the vector $\tilde{x}_2$ can replace $\dot{\tau}_J$ in the control law given by (30) if the rate of convergence of $\tilde{x}_2$ is much higher than that of s.

4. Results

A simulation of a 2 DOF robot manipulator with flexible joints demonstrates the performance of the proposed robot controller. The parameters used in the simulation results were of a six degree-of-freedom PUMA robot developed at Simulation

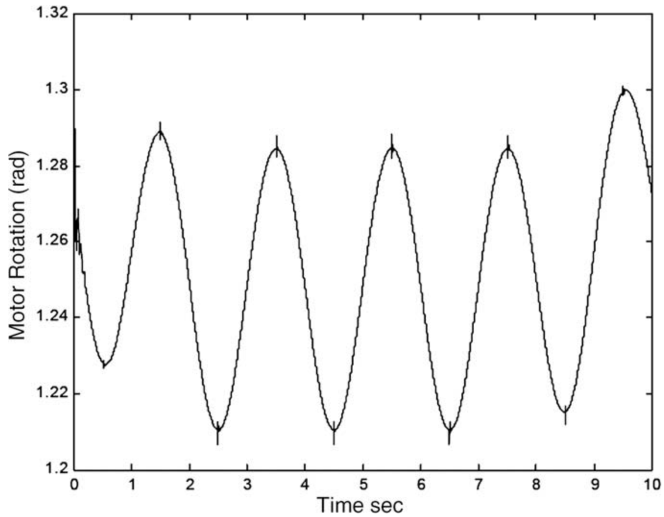

Fig. 1. Robot's 1st joint motor rotation.

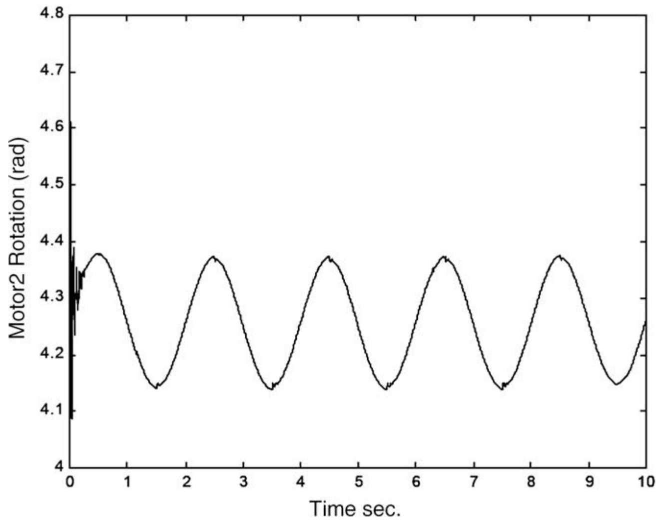

Fig. 2. Robot's 2nd joint motor rotation.

and machine Control Laboratory (S&MC) Nanjing University of Aeronautics & Astronautics [21]. The authors assumed that all the joints of the robot, except shoulder and elbow, were immobilized. The characteristics of the robot are the following:

$$m1 = 26.3\ \text{kg}, \quad m2 = 15\ \text{kg}, \quad l1 = 0.44\ \text{m}, \quad l2 = 0.38\ \text{m}$$

where $m1$, $m2$ are the masses and $l1$, $l2$ are the lengths of the links. It is assumed that the masses $m1$ and $m2$ are concentrated at the distal end of each link. The following matrices give the characteristics of the flexible subsystem:

$$H_\text{m} = \text{diag}(1.0, 1.0)\ \text{kg m}^2, \quad K_J = \text{diag}(100, 100)\,\frac{\text{N m}}{\text{rad}}$$

The stiffness matrix of the force sensor is

$$K_\text{s} = \text{diag}(100, 100)\,\frac{\text{N m}}{\text{rad}}$$

Both stiffness values are too low in order to demonstrate that the proposed controller behaves quite well in robots with soft joints and sensors. The characteristics of the friction coefficient according to (5) are the following:

$$\mu_\text{s} = 0.2, \quad \mu_\text{c} = 0.15, \quad \rho = 1$$

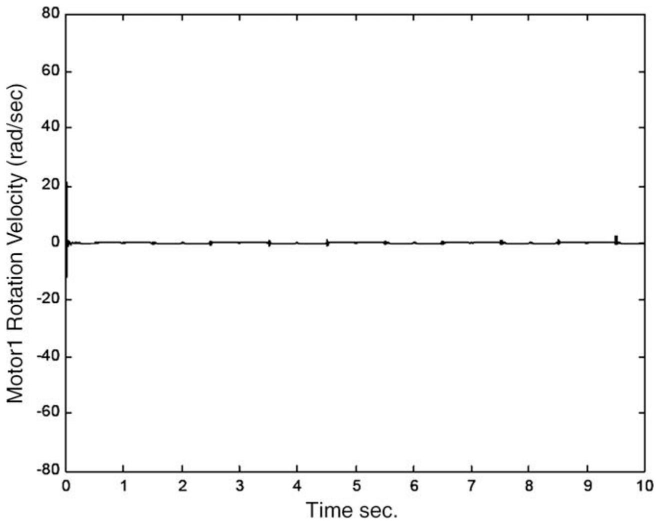

Fig. 3. Robot's joint 1 motor velocity.

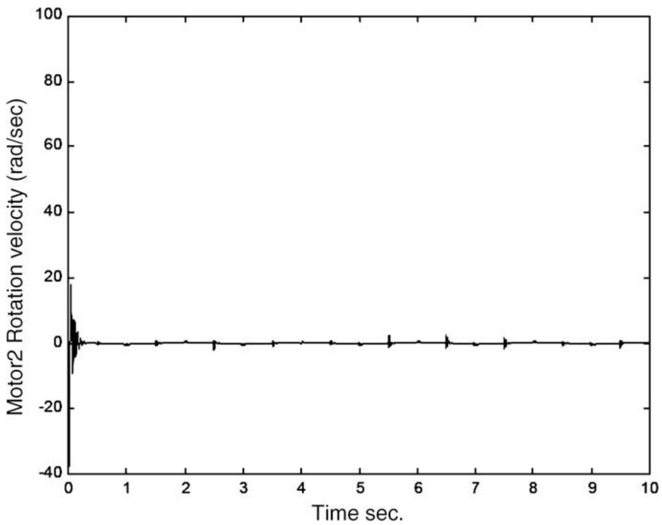

Fig. 4. Robot's joint 2 motor velocity.

The force applied by the end effector is given by the following formula:

$$F(t) = \begin{cases} a_f \sin\left(\frac{2\pi}{T_x} t\right), & t \leqslant t_1 \\ a_f, & t_1 \leqslant t \leqslant t_2 \\ a_f \sin\left(\frac{2\pi}{T_x} t\right), & t \leqslant t_2 \end{cases}$$

The parameters of the desired profile of motion and force are the following:

$$a_f = -2.0\ \mathrm{N}, \quad T_f = 10\ \mathrm{s}, \quad t_1 = -2.5\ \mathrm{s}, \quad t_2 = 7.5\ \mathrm{s}, \quad T_x = 1.5\ \mathrm{s}$$

The parameters and the gains for the control system of the rigid subsystem are the following:

$$\Lambda_\mathrm{r} = \mathrm{diag}(250, 250), \quad \Gamma_\mathrm{r} = \mathrm{diag}(0.1, 0.1, 0.1, 0.1, 0.1)$$
$$R = \mathrm{diag}(50, 50)$$

The parameters and the gains for the control of the flexible subsystem are the following:

$$P_J = \mathrm{diag}(250, 250), \quad \Lambda_\mathrm{j} = \mathrm{diag}(200, 200), \quad T_\mathrm{m} = [10, 10]^\mathrm{T}\mathrm{N\ m}$$
$$N_J = \mathrm{Diag}(200, 200), \widehat{H}_\mathrm{m} = \mathrm{Diag}(1.2, 1.2)\ \mathrm{kg\ m^2/rad}, \quad \widehat{A} = \mathrm{Diag}(0.012, 0.012)\ \mathrm{s}^2$$
$$D_{\underset{Hm}{\sim}} = \mathrm{Diag}(0.22, 0.22)\ \mathrm{kg\ m^2/rad}, D_{\underset{A}{\sim}} = \mathrm{Diag}(2.2\mathrm{e}^{-3}, 2.2\mathrm{e}^{-3})\ \mathrm{s}^2$$

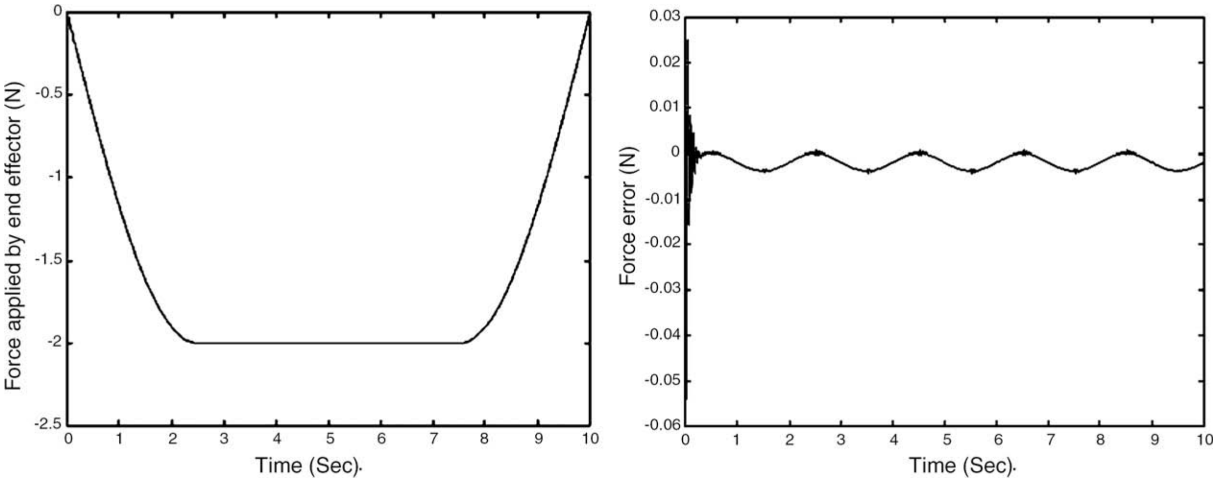

Fig. 5. Force applied by robot end effector.

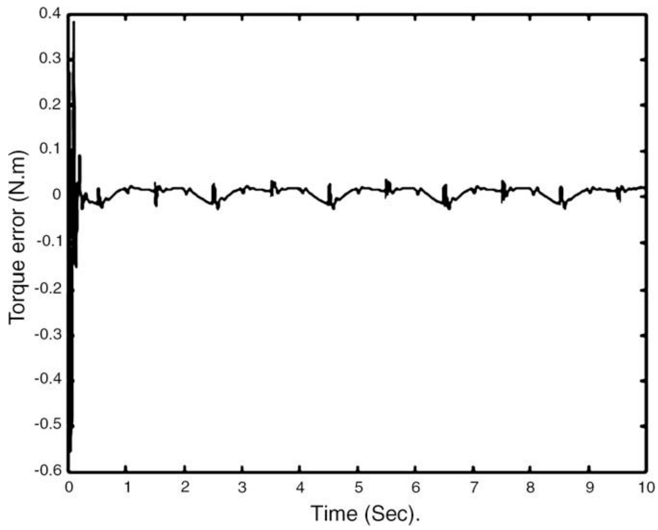

Fig. 6. Torque estimation error of 1st joint.

The parameters and the gains for the identification of the friction coefficient are: $\rho_f = 50$, $\eta = 100$ .The parameters and the gains for the observer of the torque time derivative are the following:

$$\Gamma_1 = \mathrm{Diag}(20, 20), \quad \Lambda_0 = \mathrm{Diag}(40, 40), \quad \Delta = \mathrm{Diag}(210, 210)$$
$$k_1 = 2, \quad k_2 = 100, \quad \delta = 0.1, \quad \Lambda_0 = \mathrm{Diag}(40, 40)$$

The bounds of the uncertainties are identical with the ones used in the flexible subsystem control law.

The general idea for the selection of the control scheme parameters is that the flexible subsystem should be faster than the rigid subsystem in order to provide the desired torque in time. Under this consideration the gains for the flexible subsystem are higher than the gains of the rigid subsystem. However, for the selection of observer parameters, the main caution is focused on the choice of the desired accuracy δ.

In Figs. 1–4, the response of the flexible subsystem in the Cartesian space is shown. The results show that the errors of robot end-point motion are under 0.5% and the desired path is successfully followed. The error peaks are observed at the points where velocity changes sign. The velocities error along the direction of the motion of the robot end point is reduced very fast, while spikes are observed at the points where velocity changes sign. It should be mentioned that at these points the friction force takes the static value and this causes the peaks of the error at these points. The joint velocities are high due to the low joint stiffness and the fast desired response. We noticed that robot motors show some irregular short peaks at the points where the end point velocity changes sign. The magnitude of this over- action depends on the stiffness of joints.

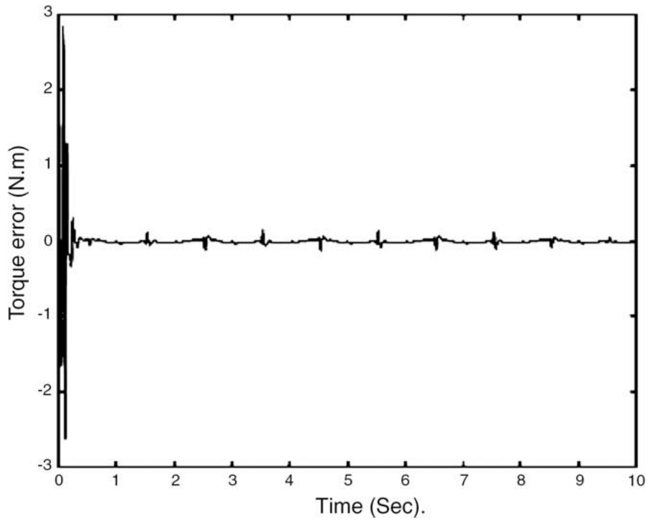

Fig. 7. Torque estimation error of 2nd joint.

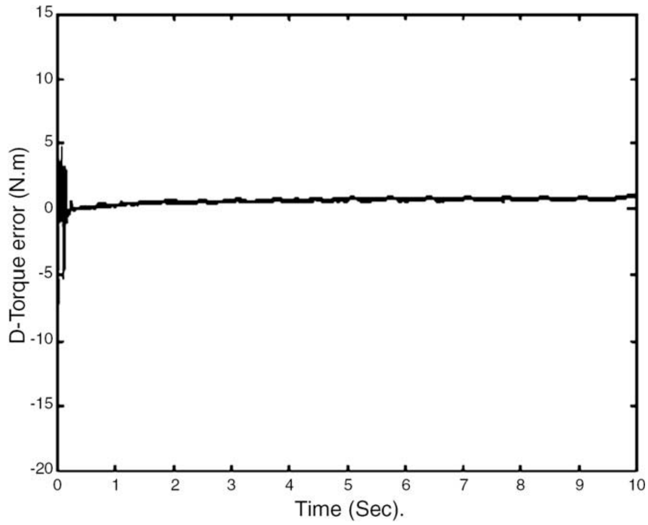

Fig. 8. D-Torque error of 1st joint.

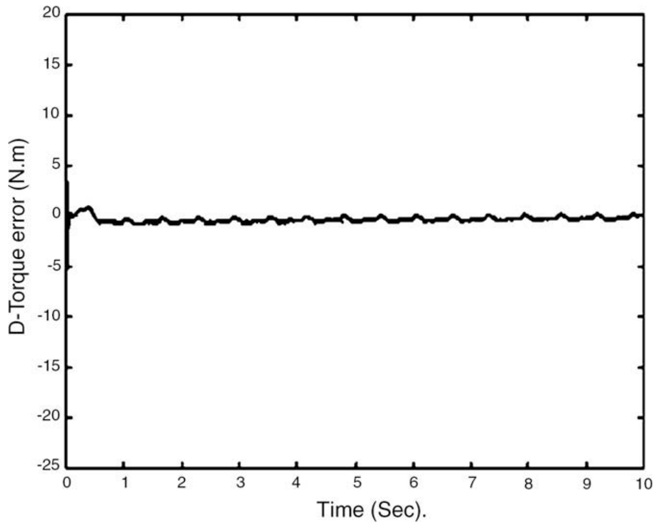

Fig. 9. D-Torque error of 2nd joint.

In Fig. 5 the profile of the applied force by the robot and the force error are shown respectively. The desired profile is followed quite well and the relevant error is less than 0.1%. The motion of the robot end point along the *x*-axis causes the variation of the force error.

In Figs. 6–9 the results of the observer response and the estimation error of the joint torque time-derivative are shown. The torques error is converging very fast to zero while the error of their time derivatives is less than the predefined accuracy. This shows that an observer can take into account the system uncertainty and it can work even though the global stability is not established. From the above analysis is proved that the error of the observer is uniformly bounded.

5. Conclusions

In the present paper a hybrid force/position controller for flexible joint robot is introduced. The controller is based on joint torque feedback and it takes into account the uncertainty of the robot structure. For the estimation of the torque time derivative a nonlinear sliding observer is used.

In the observer, the structure of the observed system is not described exactly due to its bounded uncertainty. Friction forces appear between the end point of the robot and the environment. For their estimation and the avoidance of stick-slip, an MRAC algorithm is used.

The simulation results show good tracking performance of the system and error in friction estimation are shown. The observer operates without instabilities and the time derivative of the joint torques is estimated accurately. The results of the current paper shows that joint torque sensors can be used successfully for the control of robots when the general design of the control system takes into account the changes in the architecture of the whole structure. For instance the flexibility of the joints should be modeled accurately and the derivatives of the torques should be calculated explicitly and not via numerical differentiation.

In the case of flexible robot the required information for the robot structure is improved, this means that any kind of uncertainty in this information is more critical than the case of rigid robot. The robust and adaptive algorithms proposed in this paper encountered the uncertainties without any instabilities or inaccuracies.

Further work can be done regarding the extension of the stability of the observer for the case of systems with parametric uncertainty. The current control algorithm should be implemented in a real robot in order to investigate its behavior in the real world environment. The overall design of the controller including these new approximations will be a considerable contribution to the real world problem of controlling direct drive robots.

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References

[1] E.J. Park, J.K. Mills, Dynamic modeling of flexible payloads manipulated by a smart gripper in robotic assembly, International Journal of Robotics and Automation 11 (1) (2006) 50–62.

[2] M. C Chien, A. C Huang, Adaptive control for flexible-joint electrically driven robot with time-varying uncertainties, IEEE Transactions on Industrial Electronics 54 (2) (1992) 1032–1038.
[3] C.C. Cheah, S. Kawamua, S. Arimoto, Stability of hybrid position and force control for robotic manipulator with kinemetic and dynamic uncertainties, Automatica 39 (5) (2003) 847–855.
[4] Y. Touati, K. Djouani, Y. Amirat, Neuro-fuzzy based approach for hybrid force/position robot control, Integrated Computer-Aided Engineering 11 (1) (2004) 85–98.
[5] C.K. Loo, R. Mandava, M.V.C. Rao, A hybrid intelligent active force controller for articulated robot arms using dynamic structure neural network, Journal of Intelligent and Robotic Systems: Theory and Applications 40 (2) (2004) 113–145.
[6] T. Mrad, A. Shaheen, Adaptive control of flexible joint robots using position and velocity feedback, International Journal of Control 55 (5) (1992) 1255–1277.
[7] C.A. caundas de Wit, B. Siciliano, G. Bastin, Theory of Robot Control, second ed., Springer, 1997.
[8] G. Yugheng, W. Peng-Yun, Adaptive fuzzy sliding mode control for robotic manipulators, in: Proceedings of the IEEE International Conference on Decision and Control, 2003, pp. 2174–2179.
[9] S. Elangovan, W. Peng-Yun, Adaptive fuzzy sliding control for a three-link passive robotic manipulator, Robotica 23 (5) (2005) 635–644.
[10] G. Song, L. Cai, A new approach to robust force/position control of flexible joint manipulators, Journal of Robotic Systems 13 (7) (1996) 429–444.
[11] L. Rogelio, B. Bernard, Adaptive control of robotic manipulators with flexible joints, IEEE Transactions on Automatic Control 37 (2) (1992) 174–181.
[12] S.Y. Ling, D.M. Dawson, H. Jun, M.S. de Queiroz, An adaptive link position tracking controller for rigid-link flexible joint robots without velocity measurements, IEEE Transactions on Systems, Man and Cybernetics 27 (3) (1997) 412–427.
[13] C.J. Tsaprounis, N.A. Aspragathos, A linear differential formulation of friction forces for adaptive estimator algorithms, Robotica 19 (4) (2001) 407–421.
[14] L. Alessandro, D. Schroder, M. Thummel, An acceleration-based state observer for robot manipulators with elastic joints, in: Proceedings of the IEEE International Conference on Robotics and Automation, 2007, pp. 3817–3823.
[15] Zhang, J. Furusho, Control of robot arms using joint torque sensors, IEEE Control Systems Magazine 18 (1) (1997) 48–55.
[16] N. Lechevin, P. Sicard, Observer design of flexible joint manipulators with parameters uncertainties, in: Proceedings of the IEEE International Conference on Robotics and Automation, 1997, pp. 2547–2552.
[17] C.C. de Witt, J.J. Slotine, Sliding observers robot manipulator, Automatica 27 (5) (1991) 859–864.
[18] A. Gurcel, K. Sadettin, B. Sedat, A robust motion technique for flexible-jointed manipulation systems, Robotica 24 (1) (2006) 95–103.
[19] J.J. Slotine, L. Wiping, Applied Non-Linear Control, Prentice Hall, 1991.
[20] H. Yan-Ru, V. George, Position and force control of flexible joint robots during constrained motion tasks, Mechanism and Machine Theory 35 (10) (2001) 853–871.
[21] M. Farooq, D.B. Wang, Implementation of a new PC based controller for a PUMA robot, Journal of Zhejiang University Science A 8 (12) (2007) 1962–1970.