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Independent Hybrid Force/Motion Control of Constrained Six-Degrees-of-Freedom Manipulators

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Abstract. Controlling the dynamics of a constrained manipulator includes position tracking as well as stabilization of the contact wrench. In this paper we derive a control scheme, that makes it possible to treat position and force control independently. The approach is based on a mass-orthogonal splitting of the space of joint torques, allowing independent actuation and therefore independent specification of control laws. An appropriate definition of the reference wrench makes it possible to achieve independent stability of the position and force loop.

Key words: force/motion control, constrained manipulators, mass-orthogonality, feedback linearization, augmented PD control.

1. Introduction

There are many applications in robotics, where the end-effector has to operate on the environment, as for example in the case of machining a workpiece. To complete a task of this kind successfully, the actuation of the manipulator should meet the following conditions. (i) The end-effector is steered along the desired trajectory, (ii) the required contact wrench is applied, and (iii) permanent contact between end-effector and environment is maintained. The corresponding control problem can be classified depending on the modeling of the system manipulator-environment.

First, the environment can be modeled as a rigid or as a deformable object. In the former case the contact can be described by algebraic equations, implicating that the dynamics of the constrained manipulator are given by a differential-algebraic system. Then position and contact wrench are not directly related to each other and have to be tracked independently. The control problem that has to be solved for this setup is usually named ‘Hybrid Force/Motion Control’. In the latter case the environment is normally modeled as a mass interacting with the manipulator by a spring-damper system, and position and contact wrench are directly related to each other. The equations of motion are then given by a set of ordinary differential equations, describing the dynamics of the free manipulator, charged with a mechanical impedance. The most widely used control scheme in this case is called 'Impedance Control', where position and contact wrench are tracked by controlling the impedance.

Second, it must be decided, whether joints and/or links have to be modeled as flexible elements. If so, the model of the manipulator contains more than six components such that the system is underactuated, meaning that the number of actuators is less than the degree of freedom. Control strategies would have to deal with the zero dynamics of the system. Only if joints and links are modeled as nondeformable rigid elements, the system would be fully actuated.

Finally, it must be determined, what modeling inaccuracies the controller has to deal with. It is reasonable to classify them into structural and parametric uncertainties. For example, the former include non-modeled friction or elastic components and require robust control schemes in any case. The latter arise from inaccurate calculation or measurement of the dynamic and/or geometric parameters of the manipulator and must be controlled by either robust or adaptive control schemes. In a theoretical case where the modeling is assumed to be exact and coincides with the real system, a control strategy is merely required, in order to deal with uncertainties in the initial conditions of the state.

For practical applications it is of course necessary to design controllers which are robust against structural and parametric uncertainties. Still the 'exact/rigid' case is very important, because it enlightens the basic difference between controlling force and position, and provides the basis for more advanced problems.

In this paper we will analyse hybrid force/motion control for rigid six-degrees-of-freedom manipulators with an open-chain structure, assuming that the model is exact. The contact between end-effector and environment is supposed to be holonomic and unilateral. It is further assumed that all joint angles and joint velocities can be measured, and that actuation is realized by direct drives in each joint.

Numerous approaches to this problem have been proposed. The works by Khatib [4] and McClamroch and Wang [6], for example, are based on a local bijection of a neighborhood of the constraint manifold to a linear space, where the force/motion problem is decoupled with respect to the Euclidean metric. A decoupling which is independent of such a parameterization, based on the concept of mass-orthogonality, has been realized in [3, 9].

There are mainly two points, however, which have not been clarified in any of the cited references. First, it has not been pointed out that force and position can be controlled by different control strategies, which can be designed independently from each other. This, however, makes it possible to stabilize the position loop by any conventional control strategy developed for unconstrained manipulators, while the force loop can be controlled by any appropriate control scheme independent of the position control scheme. Second, independent stability of the contact wrench from position tracking in the general case has not been achieved, due to the definition of the reference wrench. These points will be discussed in this paper.

As an example we have analysed a six-degrees-of-freedom manipulator, applied in order to polish a sphere. The simulation results will exhibit the basic differences between force and motion control.

2. Joint Space and Workspace*

The joint space Q is defined as the set of all possible joint positions, and is equal to the configuration space of the unconstrained manipulator. The workspace is defined as the configuration space of the end-effector, which is a subset of $SE(3)$.

If a coordinate frame K_E is defined on the end-effector, its position can be described by $g := (a, R) \in SE(3)$, containing the translation a and rotation R of K_E with respect to some inertial system K_0. The kinematic relation between joint angles and end-effector position can then be expressed by a map called forward kinematics.

$$h : Q \to SE(3), \quad q \mapsto h(q) = g. \tag{1}$$

The velocity of the end-effector is denoted by $V =: (v, \omega) \in \mathbb{R}^6$, where $v := R^T \dot{a}$ is the translational and $\omega := (R^T \dot{R})^\vee$ is the angular velocity both described with respect to K_E. The kinematic relation between joint velocities and the end-effector velocity at $q \in Q$ can then be expressed by a tangent map called geometric Jacobian.**

$$J_q : T_q Q \to \mathbb{R}^6, \quad \dot{q} \mapsto J_q \dot{q} = V. \tag{2}$$

A wrench that is exerted from the environment onto the end-effector is denoted by $F =: (\rho, l) \in \mathbb{R}^6$ containing a force ρ and a torque l, both described with respect to K_E. The correct transformation of F at $h(q) \in SE(3)$ to a vector of joint torques F_q is derived from required equivalence of work $F_q^T \dot{q} = F^T V, \forall V = J_q \dot{q}$ and is given by

$$J_q^T : \mathbb{R}^6 \to T_q^* Q, \quad D \mapsto J_q^T F = F_q. \tag{3}$$

3. Constraints

The holonomic contact between environment and end-effector can be modeled by m algebraic equations

$$\Psi(g) = 0, \tag{4}$$

where $\Psi : SE(3) \to \mathbb{R}^m$, defining the set of admissible end-effector positions

$$\Sigma := \{g \in SE(3) \mid \Psi(g) = 0\}. \tag{5}$$

* For background on $SE(3)$ theory and the notation used here, we refer the reader to [2, 8, 10].
** Throughout the paper it is assumed that J_q has rank 6.

From (4) a map $E_g : \mathbb{R}^6 \to \mathbb{R}^m$ can be derived, such that the nullspace of E_g* describes the set of admissible end-effector velocities at $g \in \Sigma$

$$\sigma_g := \{V \in \mathbb{R}^6 \mid E_g V = 0\}. \tag{6}$$

By d'Alemberts Principal the space of contact wrenches is equal to the annihilator of σ_g

$$\sigma_g^\circ := \{D \in \mathbb{R}^6 \mid D^T V = 0, \forall V \in \sigma_g\}, \tag{7}$$

which can be parametrized by $E_g^T : \mathbb{R}^m \to \mathbb{R}^6$, $\lambda \mapsto E_g^T \lambda = D$. Using the kinematical relations introduced in the last section these definitions can be transformed to joint space. One finds the set of admissible joint positions

$$\Sigma_q := \{q \in Q \mid \Psi_q(q) = 0\} \tag{8}$$

with $\Psi_q := \Psi \circ h$, the set of admissible joint velocities at $q \in \Sigma_q$

$$\sigma_q := \{\dot{q} \in T_q Q \mid E_q \dot{q} = 0\} \tag{9}$$

with $E_q := E_g J_q$ and likewise

$$\sigma_q^\circ := \{D_q \in T_q^* Q \mid D_q^T \dot{q} = 0, \forall \dot{q} \in \sigma_q\}, \tag{10}$$

which can be parameterized by $E_q^T : \mathbb{R}^6 \to T_q^* Q$, $\lambda \mapsto E_q^T \lambda = D_q$.

A joint space trajectory $q(t)$ of constrained motion must therefore fulfill

$$E_q \dot{q} = 0 \tag{11}$$

as well as

$$E_q \ddot{q} + \dot{E}_q \dot{q} = 0 \tag{12}$$

with $\dot{E}_q = \dot{E}_g J_q + E_g \dot{J}_q$.

4. Dynamics of the Constrained System

A redundant formulation of the dynamical equations of the constrained manipulator is given by the following differential-algebraic system

$$M\ddot{q} + C\dot{q} + N - D_q = \tau, \tag{13}$$

$$\Psi_q = 0, \tag{14}$$

where $M(q)$, $C(q, \dot{q})\dot{q}$,** $N(q)$ denote inertia matrix, coriolis vector and gravity vector of the unconstrained system, τ is the vector of joint torques. $D_q = E_q^T \lambda$ can be obtained as a function of q and $\dot{q}$ by multiplying (13) by the projector

* Throughout the paper it is assumed that E_g has rank m on Σ.

** As the Coriolis vector is a quadratic form in $\dot{q}$, one can find a symmetric bilinear form $C(q, \cdot)\cdot$ such that $C(q, \dot{q})\dot{q}$ is equal to the coriolis vector. This notation is important for various control concepts as the one in Section 8.2.

$$\beta_q^T := E_q^T M_\lambda E_q M^{-1} \tag{15}$$

and using (12)

$$D_q = \beta_q^T (C\dot{q} + N) - E_q^T M_\lambda \dot{E}_q \dot{q} - \beta_q^T \tau, \tag{16}$$

where $M_\lambda := (E_q M^{-1} E_q^T)^{-1}$. A possible set of underlying ordinary differential equations of (13–14) is obtained by multiplying (13) by*

$$\alpha_q^T := I - \beta_q^T \tag{17}$$

and using (12)

$$M\ddot{q} + \alpha_q^T (C\dot{q} + N) + E_q^T M_\lambda \dot{E}_q \dot{q} = \alpha_q^T \tau. \tag{18}$$

5. The Hybrid Force/Motion Control Problem

For given reference position and reference wrench

$$g_d : \quad [t_0, t_1] \to \Sigma, \tag{19}$$

$$D_d : \quad \Sigma \to \Sigma \times \sigma_g^\circ, \tag{20}$$

design a control law

$$c : (t, q, \dot{q}) \to c(t, q, \dot{q}) = \tau, \tag{21}$$

such that actuation by τ leads to independent exponentially stable tracking of position and contact wrench on the constraint manifold Σ

$$g(t) - g_d(t) \to 0, \tag{22}$$

$$D(t) - D_d(g(t)) \to 0, \quad \text{for growing time } t, \tag{23}$$

$$\Psi(g(t)) = 0, \quad \forall t \in [t_0, t_1]. \tag{24}$$

6. Mass-Orthogonal Splitting

The space of joint torques $T_q^* Q$ can be split into the subspaces σ_q° and its mass-orthogonal complement $\sigma_q^{\circ\perp}$, meaning that $T_q^* Q = \sigma_q^\circ \oplus \sigma_q^{\circ\perp}$. The physical interpretations of σ_q° and $\sigma_q^{\circ\perp}$, which is relevant for the design of a force/motion control scheme is stated in the following notes.

* Throughout the paper I denotes the identity matrix. Its dimension should be clear respectively from the context.

NOTE 6.1. *Actuation by* $\tau \in \sigma_q^\circ$ *does not change the state of motion at* $q \in \Sigma$.

Proof. If $\tau \in \sigma_q^\circ$ there exists $\lambda \in \mathbb{R}^m$ with $\tau = E_q^T \lambda$ and therefore $\alpha_q^T \tau = 0$. By (18) it follows that the acceleration is not changed. □

The mass-orthogonal complement of σ_q° is defined by

$$\sigma_q^{\circ\perp} = \{F_q \in T_q^* Q \mid D_q^T M^{-1} F_q = 0,\ \forall D_q \in \sigma_q^\circ\}. \tag{25}$$

NOTE 6.2. *Actuation by* $\tau \in \sigma_q^{\circ\perp}$ *does not change the contact wrench at* $q \in \Sigma$.

Proof. If $\tau \in \sigma_q^{\circ\perp}$ it follows by (25) that $\lambda^T E_q M^{-1} \tau = 0$, $\forall \lambda \in \mathbb{R}^m$, or $E_q M^{-1} \tau = 0$ and therefore $\beta_q^T \tau = 0$. Equation (16) completes the proof. □

7. Decoupling of the Control Problem

Equations (16) and (18) describe the decoupled control loops, however, in a redundant form, where the number of variables to be controlled is bigger than the number of actuation variables. Therefore these equations have to be transformed to the corresponding parameter spaces. By multiplying (16) by $M_\lambda E_q M^{-1}$ and setting $D_q = E_q^T \lambda$ we have

$$-\lambda + C_\lambda \dot{q} + N_\lambda = M_\lambda E_q M^{-1} \tau, \tag{26}$$

where

$$\begin{aligned} M_\lambda &:= (E_q M^{-1} E_q^T)^{-1}, \\ C_\lambda &:= M_\lambda E_q M^{-1} C - M_\lambda \dot{E}_q, \\ N_\lambda &:= M_\lambda E_q M^{-1} N. \end{aligned} \tag{27}$$

On the other hand we assume that $B_u : \mathbb{R}^f \to T_q Q$, $\dot{u} \mapsto B_u \dot{u} = \dot{q}$, with $f = 6 - m$, is an explicit parameterization of σ_q, thus

$$E_q B_u = 0. \tag{28}$$

By multiplying (18) by B_u^T and setting $\dot{q} = B_u \dot{u}$ we have

$$M_u \ddot{u} + C_u \dot{u} + N_u = B_u^T \tau, \tag{29}$$

where

$$\begin{aligned} M_u &:= B_u^T M B_u, \\ C_u &:= B_u^T C B_u + B_u^T M \dot{B}_u, \\ N_u &:= B_u^T N. \end{aligned} \tag{30}$$

In order to make it possible to design control laws for position and contact wrench independently, the joint torques have to be split within the meaning of Notes 6.1 and 6.2 into motion control torques $\tau_{\text{motion}} \in \sigma_q^{\circ\perp}$ and force control torques $\tau_{\text{force}} \in \sigma_q^{\circ}$

$$\tau = \tau_{\text{motion}} + \tau_{\text{force}}. \tag{31}$$

From (25) and (28) it can be derived that $MB_uM_u^{-1} : \mathbb{R}^f \to T_q^*Q$ is an explicit parameterization of $\sigma_q^{\circ\perp}$, while it has already been mentionned that E_q^T is an explicit parameterization of σ_q°. We might thus set

$$\tau_{\text{motion}} = MB_uM_u^{-1}\tau_u, \tag{32}$$

$$\tau_{\text{force}} = E_q^T\tau_\lambda \tag{33}$$

and therefore by (31)

$$\tau = MB_uM_u^{-1}\tau_u + E_q^T\tau_\lambda, \tag{34}$$

where $\tau_u \in \mathbb{R}^f$ and $\tau_\lambda \in \mathbb{R}^m$ are still free to choose. Inserting (34) into (26) and (29) and using (28) leads to

$$M_u\ddot{u} + C_u\dot{u} + N_u = \tau_u, \tag{35}$$

$$-\lambda + C_\lambda\dot{q} + N_\lambda = \tau_\lambda. \tag{36}$$

Equations (35) and (36) describe the decoupled control loops, which can be controlled by designing τ_u and τ_λ independently. The total joint torques are then given by (34).

8. Control Schemes and Stability Analysis

For fully actuated systems there are three basic approaches in nonlinear control theory. The Computed Torque Method (inverse dynamics) consists of a nonlinear feedback that compensates the nonlinearities coupled with a linear PD controller. Due to the linear structure of the closed loop, it belongs to the class of feedback linearization methods. Another conventional control strategy is called Augmented PD Control, which takes advantage of the passivity structure of Lagrangian systems. It actually is an extension of a PD controller, within the meaning that the two methods are equivalent when an equilibrium point of the system has to be stabilized. Finally, the method called Feedforward Decoupling consists of a nonlinear feedforward, steering the system along the given reference trajectory and a linear PD controller. This method affords a linearization of the dynamical equations along the reference trajectory, in order to design the control gains. All strategies can be applied in the setting of joint- or workspace control, which only differ by the definition of the error that is fed back to the controller. Because they are all model-based, their performance depends on the model accuracy. In this paper we will discuss feedback linearization and augmented PD control based on an error measure defined in joint space and assuming an exact model.

8.1. FEEDBACK LINEARIZATION

Applying computed torque control to the position loop (35) as well as to the force loop (36) gives

$$\tau_{u,fb} = M_u(\ddot{u}_d - K_D\dot{e}_u - K_P e_u) + C_u\dot{u} + N_u, \tag{37}$$

$$\tau_{\lambda,fb} = -\lambda_d + C_\lambda\dot{q} + N_\lambda, \tag{38}$$

where $e_u := u - u_d$ and $e_\lambda := \lambda - \lambda_d$ denote position and force error. The closed loop is given by

$$\ddot{e}_u + K_D\dot{e}_u + K_P e_u = 0, \tag{39}$$

$$e_\lambda = 0. \tag{40}$$

Hence $g(t) \to g_d(t)$ for $K_D, K_P > 0$ and $D(t) = D_d(g(t))$ because $D(t) = E_{g(t)}\lambda(t)$ and $D_d(g(t)) = E_{g(t)}\lambda_d(g(t))$ (see Section 9). We can conclude, that the control law

$$c_{fb,fb} : (t, q, \dot{q}) \to c_{fb,fb}(t, q, \dot{q}) = MB_u M_u^{-1}\tau_{u,fb} + E_q^T\tau_{\lambda,fb} \tag{41}$$

solves the force/motion control problem (19–24).

8.2. AUGMENTED PD CONTROL

Alternatively, the position loop could also be controlled by augmented PD control

$$\tau_{u,aPD} = M_u\ddot{u}_d + C_u\dot{u}_d + N_u - K_D\dot{e}_u - K_P e_u. \tag{42}$$

The closed loop is then given by

$$M_u\ddot{e}_u + C_u\dot{e}_u + K_D\dot{e}_u + K_P e_u = 0. \tag{43}$$

The proof that augmented PD control leads to asymptotically stable tracking in the case of unconstrained manipulators is standard in control literature (see, for example, [14]). It is actually based on the fact that $\dot{M} - 2C$ is skew-symmetric, which is characteristic for Lagrangian systems. In order to show that (43) can be made stable it therefore suffices to show that $\dot{M}_u - 2C_u$ is skew-symmetric. By (30) $\dot{M}_u = 2B_u^T M\dot{B}_u + B_u^T\dot{M}B_u$ and $C_u = B_u^T CB_u + B_u^T M\dot{B}_u$. Thus $\dot{M}_u - 2C_u = B_u^T(\dot{M} - 2C)B_u$ is also skew-symmetric. (In fact, (35) are just Lagrangian equations.) We can conclude, that the control law

$$c_{aPD,fb} : (t, q, \dot{q}) \to c_{aPD,fb}(t, q, \dot{q}) = MB_u M_u^{-1}\tau_{u,aPD} + E_q^T\tau_{\lambda,fb} \tag{44}$$

solves the force/motion control problem (19–24).

Remarks. When $g_d(t) = g_d(t_0)$ the position control law would be

$$\tau_{u,PnP} := N_u - K_D\dot{e}_u - K_P e_u. \tag{45}$$

Together with $\tau_{\lambda,fb}$ this leads to a solution of a pick and place problem on the constraint manifold.

9. Discussion

In this section we will make some comments on the numerical calculation of the terms that the controller has to evaluate and on the stability analysis of the force loop.

9.1. NUMERICAL EVALUATION OF THE CONTROL LAW

M, $C\dot{q}$ and N as well as J_q and $\dot{J}_q\dot{q}$ can be calculated efficiently using the recursive Newton–Euler algorithm. E_g and $\dot{E}_g V$ have to be determined by hand.

For motion control we further need B_u and $\dot{B}_u$ as well as u_d, $\dot{u}_d$, $\ddot{u}_d$ from the reference data (19) and u, $\dot{u}$ from the measured data q, $\dot{q}$. Here it is appropriate to use coordinate partitioning, which is based on the following ideas. Let A be a constant $f \times 6$ matrix, such that $\left[\begin{smallmatrix} A \\ E_q \end{smallmatrix}\right]$ is regular. If we use A to select the parameters u by $u = Aq$ we find

$$A\dot{q} = \dot{u}, \tag{46}$$

$$E_q\dot{q} = 0, \tag{47}$$

and therefore

$$\dot{q} = \begin{bmatrix} A \\ E_q \end{bmatrix}^{-1} \begin{bmatrix} I \\ 0 \end{bmatrix} \dot{u} =: B_u\dot{u}. \tag{48}$$

Differentiating B_u with respect to time yields

$$\dot{B}_u\dot{u} = -\begin{bmatrix} A \\ E_q \end{bmatrix}^{-1} \begin{pmatrix} 0 \\ \dot{E}_q\dot{q} \end{pmatrix}. \tag{49}$$

u, $\dot{u}$, u_d, $\dot{u}_d$ and $\ddot{u}_d$ will then be determined by applying A to q, $\dot{q}$, q_d, $\dot{q}_d$ and $\ddot{q}_d$, the latter being calculated from g_d by solving the inverse kinematics.

Remarks. (i) Note that an explicit parameterization of the constraint manifold is not needed for the evaluation of the control schemes. This means that the kinematics of the constrained system does not have to be solved on position level. (ii) For the calculation of A one might use a singular value decomposition of E_q

at $q(t_0)$, which generates a matrix whose rows span the nullspace of E_q and are, therefore, linearly independent from the rows of E_q – a good choice for A. It is possible, however, that A has to be changed during the simulation.

For force control we finally need to determine λ_d from the reference wrench (20), such that $E_g^T \lambda_d = D_d$. Let

$$\lambda_d(q) := M_\lambda E_q M^{-1} D_{qd}(q) \tag{50}$$

with $D_{qd}(q) = J_q^T D_d(h(q))$. Because β_q^T is a projector onto σ_q° we have $E_q^T \lambda_d = E_q^T M_\lambda E_q M^{-1} D_{qd}(q) = \beta_q^T D_{qd} = D_{qd}$ and therefore $E_g^T \lambda_d = D_d$ as must be.

9.2. STABILITY OF THE FORCE LOOP

In conventional approaches the reference wrench is defined as a function of time $D_d : t \in [t_0, t_1] \to g_d(t) \times \sigma^\circ_{g_d(t)}$ - thus defined along $g_d(t)$ only. This has two adverse consequences. First, it seems to make no sense to track $D_d(t)$ when $g(t) \neq g_d(t)$. Second, $\lambda(t) = \lambda_d(t)$ does not induce $D(t) = D_d(t)$ if $E_{g(t)} \neq E_{g_d(t)}$, because $D_d(t) = E^T_{g_d(t)} \lambda_d(t)$ and $D(t) = E^T_{g(t)} \lambda(t)$. Therefore, the stability of the contact wrench is not independent of the stability of position. By (20) however, the reference wrench is defined at each point on the constraint manifold and $D_d(g(t)) = E^T_{g(t)} \lambda_d(q(t))$ at $g(t) = h(q(t))$. But then $D(t) = D_d(g(t))$ follows directly from $\lambda(t) = \lambda_d(q(t))$, leading to stability of the contact wrench which is independent of position tracking, and finally assuring that the end-effector stays in permanent contact with the constraint manifold.

10. Example

As an example, we analyse the problem of polishing a sphere. The end-effector is modeled as a small rigid annular disc, attached to the last joint of the wrist. Motion control aims at tracking a curve on the sphere, as well as stabilizing the discs angular velocity at constant speed. Force control is required in order to stabilize the normal force exerted on the sphere, while keeping the torques between disc and sphere equal to zero, in order to achieve a uniform distribution of the contact force along the circular contact line.

10.1. DESCRIPTION OF THE CONSTRAINTS

At the base of the manipulator an inertial coordinate system $K_0 := \{0, e_x, e_y, e_z\}$ is defined. In order to describe the configuration of the disc, a coordinate system K_E is attached to it, such that the z-axis coincides with the disc's axis of rotation and the origin touches the sphere. The contact between disc and sphere is characterized by one constraint of the origin of K_E and two constraints on the discs orientation

$$\Psi(g) = \begin{pmatrix} \psi(a) \\ \nabla_a^T \psi R e_1 \\ \nabla_a^T \psi R e_2 \end{pmatrix} = 0, \tag{51}$$

where $\psi(a) = (a - a_0)^2 - r^2$, $\nabla_a \psi$ denotes the gradient of ψ at a, a_0 the midpoint, r the radius of the sphere and e_i the i-th unit vector. From (51) one can derive

$$E_g = \begin{bmatrix} e_3^T & 0 \\ e_1^T & -r e_2^T \\ e_2^T & r e_1^T \end{bmatrix}. \tag{52}$$

10.2. REFERENCE POSITION AND REFERENCE WRENCH

The reference trace on the sphere is specified by two orthogonal meridians connected by the equator and by the requirement that each part has to be covered within 1 second. Because this trace is non-differentiable, the velocity has to vanish at the edges of the trace. One might choose

$$a_d(t) = a_0 + \begin{cases} r(-\sin s(t), 0, \cos s(t)), & t \in [0, 1], \\ r(-\cos(s(t) - \frac{\pi}{2}), -\sin(s(t) - \frac{\pi}{2}), 0), & t \in [1, 2], \\ r(0, -\cos(s(t) - \pi), \sin(s(t) - \pi)), & t \in [2, 3], \end{cases} \tag{53}$$

with

$$s(t) := \frac{\pi}{2} t - \frac{1}{4} \sin 2\pi t.$$

The reference orientation is then determined by specifying the rotational speed $\omega_{3,d}$ of the annular disc, which we set 10 Hz. By (52) it is $\omega_{1,d} = -v_{2,d}/r$ and $\omega_{2,d} = v_{1,d}/r$, thus

$$\omega_d(t) = \begin{pmatrix} -\dot{a}_d^T R_d e_2 \\ \dot{a}_d^T R_d e_1 \\ \omega_{3,d} \end{pmatrix}. \tag{54}$$

$R_d(t)$ can then be calculated from solving the differential equation

$$\dot{R}_d = R_d \hat{\omega}_d, \quad R_d(0) = I. \tag{55}$$

The reference wrench shall be specified by $\rho_{3,d} = 20$ N and $l_{1,d} = l_{2,d} = 0$ Nm, expressing that the reference angular torques along the x- resp. y-axis of K_E have to vanish. It follows that

$$D_d(g) = \begin{pmatrix} 20 e_3 \\ 0 \end{pmatrix}. \tag{56}$$

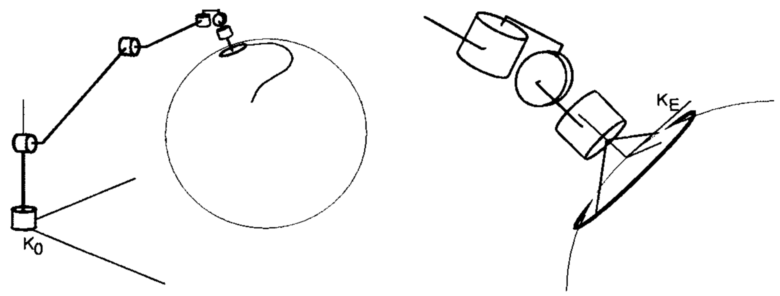

Figure 1. Six-degrees-of-freedom manipulator with an annular disc on a sphere.

10.3. MODELING

The first three links have been modeled as hollow cylinders with inner radius $r_i = 0.005$ m, outer radius $r_a = 0.01$ m and lengths $L_1 = 0.2$ m, $L_2 = 0.4726$ m and $L_3 = 0.7638$ m. For the last three links we set $L_4 = L_5 = L_6 = 0$. The positioning of the joints is shown in Figure 1. The density of the first three links is $\varrho = 7000$ kg/m^3, the masses for the last three links have been neglected. The mass of the disc is $m_d = 2$ kg. The sphere is specified by $a_0 = (1$ m, 0, 0.2 m) and $r = 0.4$ m.

11. Simulation

The dynamical equations have been programmed in MATLAB as a first order ODE in the redundant form (18) and numerically stabilized by the method developed in [1], with parameter $\gamma = 5$. For numerical integration the solver *ode*113 (see [12]). was chosen with $abstol = 10^{-6}$ and $reltol = 10^{-6}$.

The controller is based on computed torque for the position loop as well as for the force loop as presented in Section 8.1. K_P and K_D are diagonal matrices with eigenvalues 10 and $2\sqrt{10}$, such that the position loop is critically damped. Matrix A from (46) has been calculated in a pre-processing at discrete positions by evaluating the singular value decomposition of E_q along the reference trajectory $q_d(t)$ each 0.3 seconds. The initial conditions coincide with the reference values at $t_0 = 0$ except for the angular speed of the disc which is zero in the beginning. The aim of the simulation is to exemplify the basic differences of motion and force tracking, regarding convergence behaviour of the closed loops and independency concerning stability as well as actuation.

Because the dynamical equations have been solved as a redundant ODE it is necessary to analyse the numerical drift-off, before evaluating the time-dependent state evolution. Figure 2 makes plain that the numerical stabilization works very well, meaning that the constraints are fulfilled during the simulation. Motion tracking is illustrated in Figure 3. The convergence behaviour of the angular velocity ω_3 is that of a critically damped state representing the dynamical behaviour of the

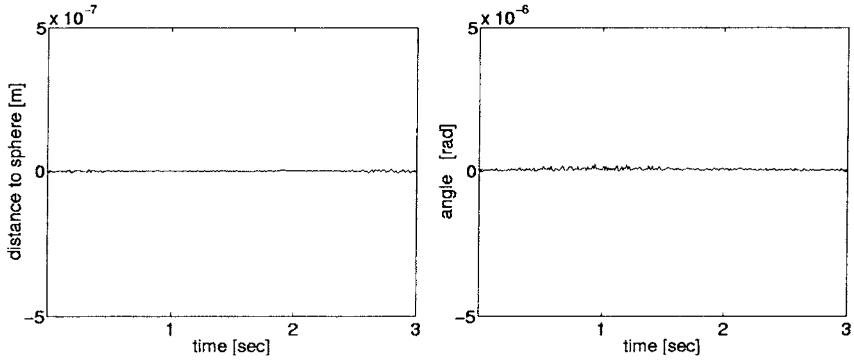

Figure 2. Numerical drift-off: (a) position constraint: distance from the discs origin to the sphere; (b) orientation constraint: angle between the discs axis and the normal to the sphere.

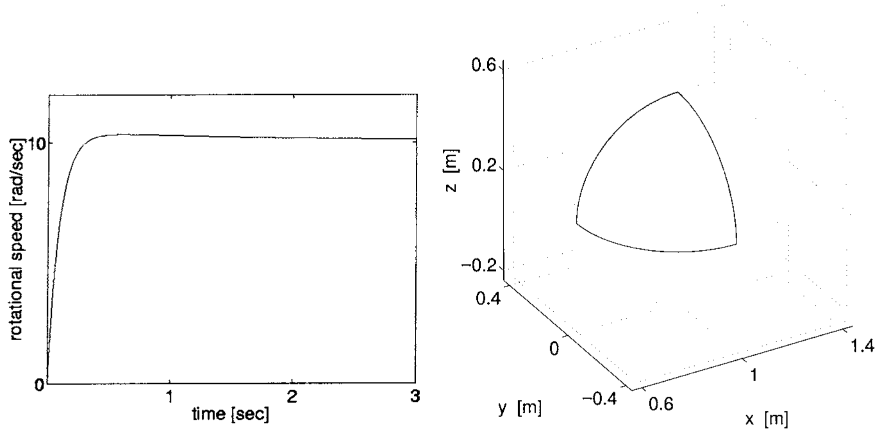

Figure 3. Motion tracking: (a) angular velocity ω_3 of the disc; (b) position on the sphere.

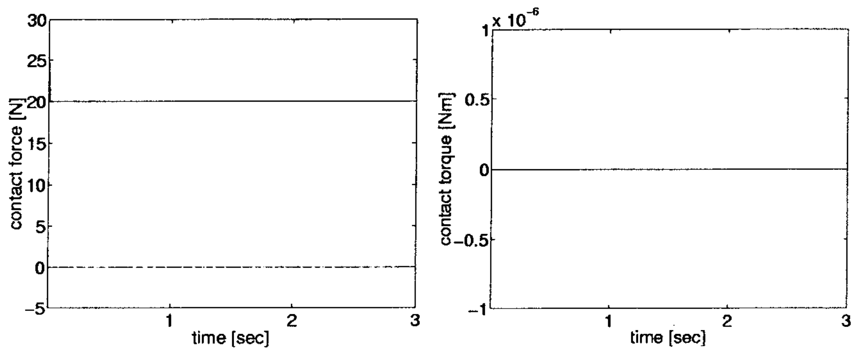

Figure 4. Force tracking: (a) contact force ρ (- - - ρ_1, -.- ρ_2, — ρ_3), (b) contact torque l (- - - l_1, -.- l_2, — l_3).

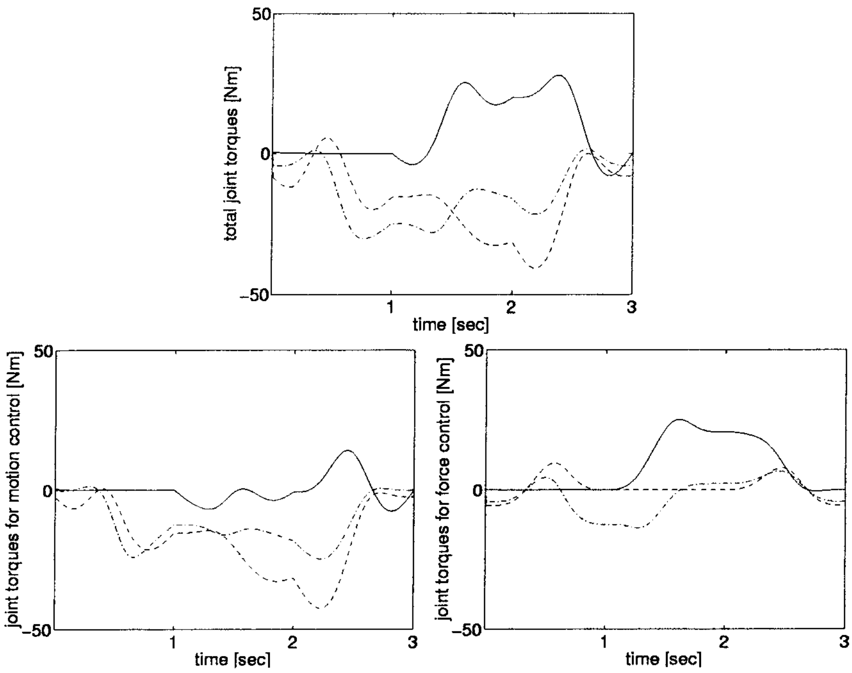

Figure 5. Actuation of joints 1-3 (— 1, - - - 2, -.- 3): (a) total joint torques τ; (b) joint torques for motion control τ_{motion}; (c) joint torques for force control τ_{force}.

position loop. Figure 4 points out that the reference wrench is identically tracked, although this is not the case for the configuration of the end-effector, indicating that force tracking is fully independent of motion tracking. Figures 5b and 5c point out that the joint torques for motion control are non-differentiable due to the edges in the reference trace, while the joint torques for force control are differentiable, indicating that they are fully independent of the reference trace. Furthermore, it can be guessed from Figures 5a–5c and Figures 6a–6c that the total joint torques τ are given by a superposition of the joint torques for motion control τ_{motion} and force control τ_{force} respectively. The peaks in Figures 6a and 6b are due to the required acceleration of the discs angular velocity in the beginning.

12. Conclusions

We have analysed the problem of hybrid force/motion control for rigid six-degrees-of-freedom manipulators with an open-chain structure. The reference position has been defined as a curve, while the reference wrench has been defined as a field on the constraint manifold. Based on a mass-orthogonal splitting of the space of joint torques, the control problem has been decoupled into a position loop and a force loop. It has been shown that both loops can be controlled by independent control laws and that the stability of both loops are independent from each other.

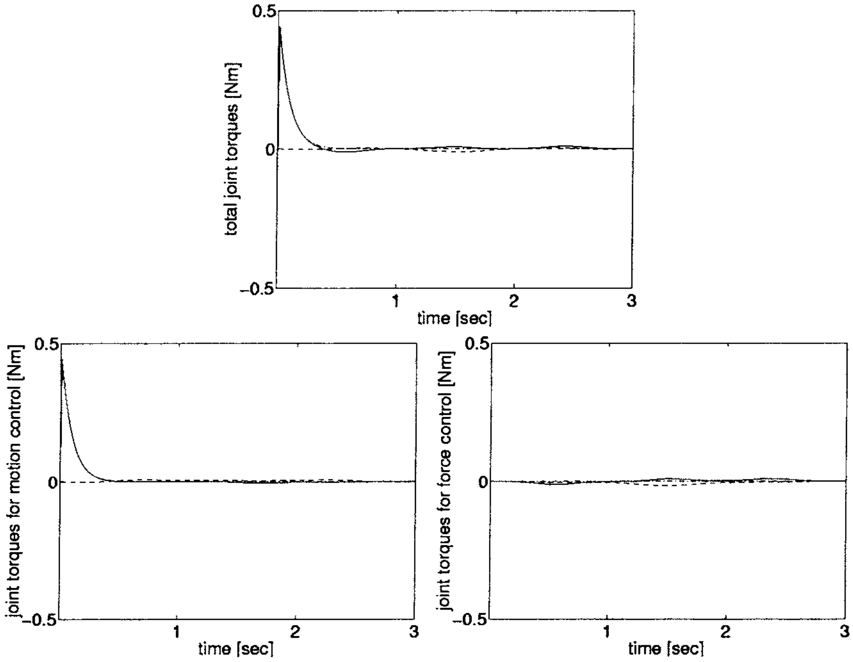

Figure 6. Actuation of joints 4–6 (— 4, - - - 5, -.- 6): (a) total joint torques τ; (b) joint torques for motion control τ_{motion}; (c) joint torques for force control τ_{force}.

The resulting control law that steers the joint actuators could then be calculated by a linear superposition of the control laws for motion control and force control respectively.

References

1. Ascher, U.M., Chin, H., Petzold, L.R. and Reich, S., 'Stabilization of constrained mechanical systems with DAEs and invariant manifolds', *Mechanics of Structures and Machines* **23**(2), 1995, 135–157.
2. Brockett, R.W., Stokes, A. and Park, F.C., 'A geometrical formulation of the dynamical equations describing kinematic chains', in *Proceedings IEEE Conference on Robotics and Automation*, Atlanta, May, J.T. Butler et al. (eds), IEEE Computer Society Press, Los Alamitos, 1993, 637–641.
3. Jankowski, K.P. and ElMaraghy, H.A., 'Dynamic decoupling for hybrid control of rigid-/flexible-joint robots interacting with the environment', *IEEE Transactions on Robotics and Automation* **8**(5), 1992, 519–534.
4. Khatib, O., 'A unified approach for motion and force control of robot manipulators: The operational space formulation', *IEEE Journal of Robotics and Automation* **RA-3**(1), 1987, 43–53.
5. Krishnan, H. and McClamroch, N.H., 'Tracking in nonlinear differential-algebraic control systems with applications to constrained robot systems', *Automatica* **30**(12), 1994, 1885–1897.
6. McClamroch, N.H. and Wang, D., 'Feedback stabilization and tracking of constrained robots', *IEEE Transactions on Automatic Control* **33**(5), 1988, 419–426.

7. Mills, J.K. and Goldenberg, A.A., ‘Force and position control of manipulators during constrained motion tasks’, *IEEE Transactions on Robotics and Automation* **5**(1), 1989, 30–46,
8. Murray, R.M., Li, Z. and Sastry, S.S., *A Mathematical Introduction to Robotic Manipulation*, CRC Press LLC, Boca Raton, FL, 1994.
9. Narikiyo, T., Schmidt, G. and Akuta, T., ‘Theoretical aspect of hybrid position/force control’, in *Proceedings IFAC Robot Control*, Capri, Italy, L. Sciavicco, C. Bonivento and F. Nicolò (eds), Pergamon, Oxford, 1995, 407–412.
10. Park, F.C., Bobrow, J.E. and Ploen, S.R., ‘A Lie group formulation of robot dynamics’, *The International Journal of Robotics Research* **14**(6), 1995, 609–618.
11. Raibert, M.H. and Craig, J.J., ‘Hybrid position/force control of manipulators’, *ASME Journal of Dynamic Systems, Measurement and Control* **102**, 1981, 126–133.
12. Shampine, L.F. and Reichelt, M.W., ‘The Matlab ODE suite’, *SIAM Journal on Scientific Computing* **18**(1), 1997, 1–22.
13. Tahboub, K., *Modeling and Control of Constrained Robots*, VDI Fortschritt-Berichte No. 361, VDI, Düsseldorf, 1993.
14. de Wit, C.C., Siciliano, B. and Bastin, G. (eds), *Theory of Robot Control*, Springer-Verlag, Berlin, 1996.
15. Yoshikawa, T., ‘Dynamic hybrid position/force control of robot manipulators – Description of hand constraints and calculation of joint driving forces’, *IEEE Journal of Robotics and Automation* **RA-3**(5), 1987, 386–392.