

Part (a): Kruskal's Algorithm - Steps

Kruskal (G, V, E):

1. Initialize an empty set T to store the MST edges.
2. Sort all edges in E in non-decreasing order by their weights $w(e)$.
3. Initialize a disjoint set (Union-Find) for the vertices in V :
 - a. $\text{MakeSet}(v)$ for each vertex v in V . // Each vertex is its own component.
4. For each edge (u, v) in the sorted edge list:
 - a. If $\text{Find}(u) \neq \text{Find}(v)$: // Check if u and v are in different components
 - i. Add edge (u, v) to T .
 - ii. $\text{Union}(u, v)$. // Merge the components of u and v
 - b. If $|T| = |V| - 1$: // Stop if MST contains $(|V| - 1)$ edges
Break.
5. Return T and the total weight of the MST.

Part (b): Analysis of Kruskal's Algorithm

Time Complexity:

1. **Sorting the edges:** Sorting takes $O(E \log E)$, where E is the number of edges.
2. **Find Operation:** Each find operation is $O(\alpha(V))$, where α is the inverse Ackermann function (which grows very slowly and is effectively constant for all practical input sizes).
3. **Union Operation:** The union operation is $O(\alpha(V))$.
4. Since we process all edges, the overall time complexity is dominated by the sorting step, making it $O(E \log E)$.

Thus, the time complexity of Kruskal's algorithm is $O(E \log E)$.