# IEEE SSCS International Student Circuit Contest 2019: Problem & Solution

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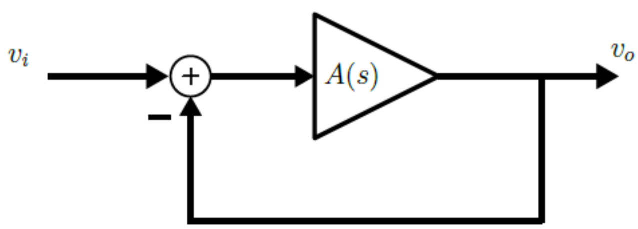
## I. Problem Statement

For the feedback amplifier shown below:




The open loop amplifier gain, A(s) is given by:

$$A(s) = \frac{10(s+1)(s+1)}{(s+0.1)(s+0.1)(s+0.1)} = \frac{10(s+1)^2}{(s+0.1)^3} \qquad (1)$$

If we investigate the Bode plot of the loop gain, we can confirm that at $\omega_{180}$ (that is the angular frequency at which the phase of the loop gain reaches -180 degrees), the magnitude of the loop gain is much greater than 1 (in fact it is 36dB). We may naturally conclude that this feedback amplifier is unstable, simply because a signal at $\omega_{180}$ must grow out of bound as it goes around the loop. Yet, when we use the Nyquist plot, we find that this system is indeed stable.

As a participant, I have to provide an intuitive explanation for why the intuition given by above (using Bode Plot and loop gain) is incorrect!

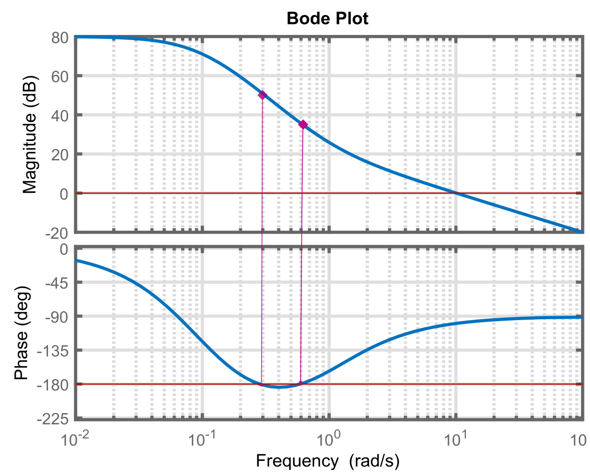



**Fig. 1:** Bode plot of equation (1). At the first -180°phase crossover, gain>40dB while at the second -180°phase crossover, gain = 36dB. Magnitude of the loop gain is thus 36dB.

## II. Solution

For the unity gain feedback system, if I analyze the open loop transfer function as described by equation (1) , there is an order 3 pole at -0.1 and an order 2 zero at -1. Since all the poles are at left-half plane (LHP) , the open loop transfer function is stable. Bode plots are used in analysing closed loop stability for stable open loop systems only. That condition is fulfilled.

Now from the bode plot in Figure 1 , it is clear that at $\omega_{180}$, the magnitude of the loop gain is much greater than 1 and it may be concluded that this feedback amplifier is unstable, simply because a signal at $\omega_{180}$ must grow out of bound as it goes around the loop. However, there is a catch.

A common statement about Bode Stability Criterion is that it cannot be used if the loop gain frequency response shows **non-monotonic** phase angle or gain at frequencies higher than the first phase crossing of -180°[1]. As the figure exhibits, at the first -180°phase crossover frequency, both the gain and phase angle are downward. However, at the second -180°phase crossover frequency which is at a higher value than the first one , the phase angle curve is upward while gain curve is still downward. Hence, at a frequency higher than the first phase crossing of -180°, gain is monotonic but phase angle is non-monotonic. Bode stability criterion cannot be used in cases like this. However, even with monotonicity of both gain and phase angle, bode stability criterion fails for a system having two negative 180°phase crossover points in [2]. In general, Bode stability criterion is a sufficient condition, but not necessary [2,3] as this criterion fails to correctly predict the stability of a closed loop system in a number of cases . It is suggested that a system should only be analyzed for stability using Bode criterion if it has at most one phase crossover frequency [2]. Some writers even add that for multiple -180°phase crossings, one must use the Nyquist criterion [3].

## III. Final Verdict

The feedback amplifier shown in the problem statement is stable. Due to having nonmonotonic phase angle at loop gain frequency response, the closed loop system stability cannot be evaluated using Bode criterion.

## References


[1] T. Marlin, *Process Control: Designing Processes and Control Systems for Dynamic Performance*, NY: McGraw-Hill, 2000.

[2] J. Hahn, T. Edison, and T. Edgar. ”A note on stability analysis using Bode plots”, *Chemical Engineering Education*, vol.35, no.3 , pp. 208-211, 2001

[3] B. Ogunnaike and W. Ray, *Process Dynamics, Modelling, and Control*, NY: Oxford University Press, 1994.