Segment and test approach on transport for hamiltonian systems

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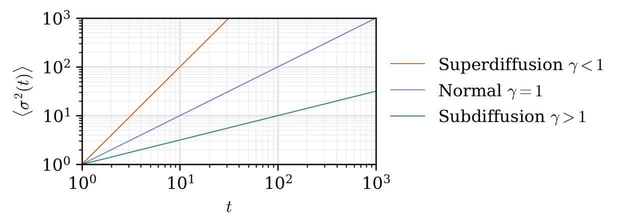
What is transport?

- How things, such as heat and mass, spread over time:

normal, such as heat in a metal bar

anomalous, as it is in some plasmas

One tool to characterize transport is with the behavior of a power law of the mean square displacement: $\sigma(t)^2 = Ct^\gamma$



Segment and test - Why and How

Why do it?

- Evaluation of γ is expensive, requiring thousands of initial condition iterated over long times
- Ballistic modes are sufficient condition for anomalous transport

What is it?

- Fast method to identify anomalous transport trough the existence of ballistic modes
- Tunable parameters
- Around 10 times, cheaper compared to usual methods

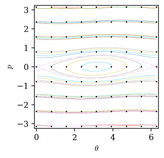
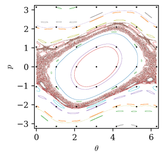
How do it?

1. Pick some initial conditions ≈ 100
2. Create the Poincaré section
3. Separate regions with image processing
4. Test the regime in each region

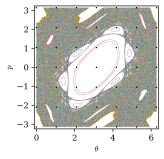
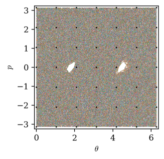
Segment and test - Example

We can test the method on the standard map:

$$p_{n+1} = p_n + K\sin(\theta_n)\ ,\ \theta_{n+1} = \theta_n + p_{n+1} \bmod(2\pi)$$

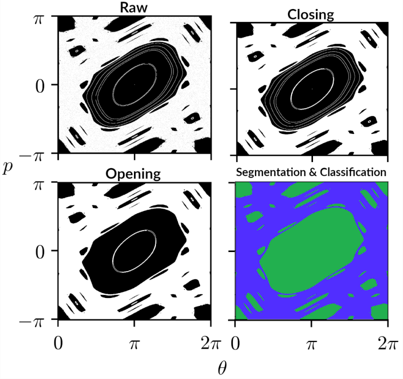



$K = 0.1;\ \gamma = 0$ $K = 0.9;\ \gamma = 0$

$K = 1.5;\ \gamma = 1$ $K = 6.5;\ \gamma > 1$

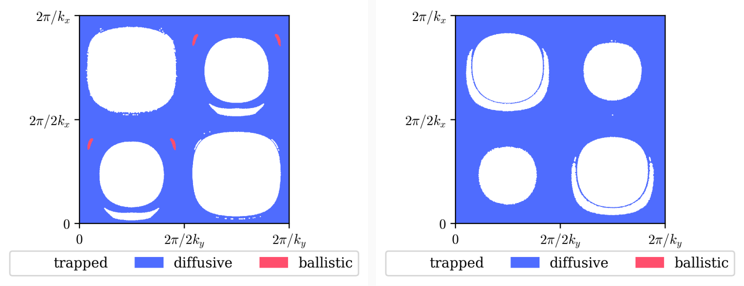
How the map changes with K



Example of the method

Continuous time system - The actual problem

$$H(x, y, t) = \phi_o - v_1 x + A_1 \sin(k_{x1} x) \cos(k_{y1} y) + A_2 \sin(k_{x2} x + \theta_x) \cos(k_{y2}(y - vt)).$$



(a) $A_2 = 0.16$ **(b)** $A_2 = 0.2$

Figure 2: Categorization on the two wave system. $\theta_x = \frac{\pi}{4}$.

Thank you
See you at the poster

Extras - Morphology

(a) Referene **(b)** Erosion **(c)** Dilation

Figure 3: Basic Operations

(a) Closing **(b)** Opening

Figure 4: Filtering operations

Extras - Where standard map appears

- On mechanics: periodically kicked rotor
- On particle wave interaction: accelerators
- Numerical integration of the a pendulum

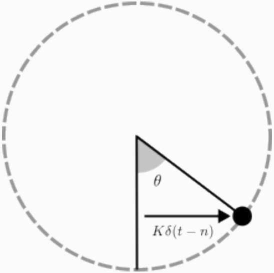


Figure 5: Kicked rotor

Extras - What is a Poincaré section

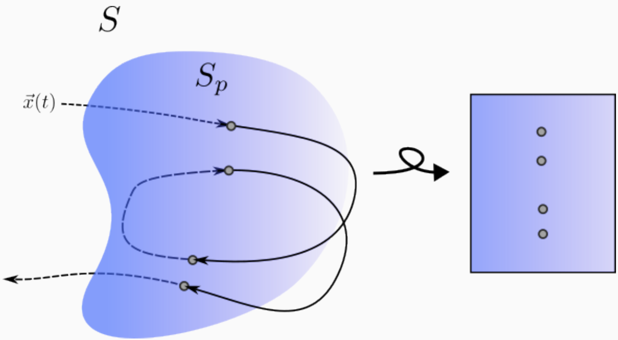


Figure 6: Visual representation of a Poincaré section

Extras - Regimes standard map

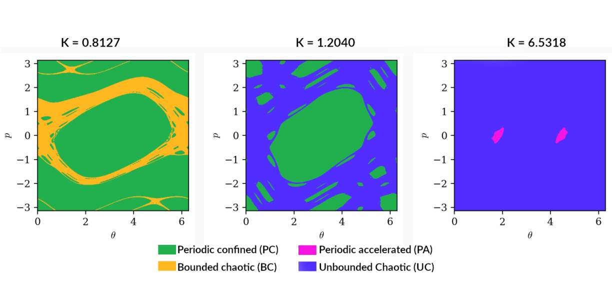


Figure 7: The different regimes of the standard map

Extras - Methods comparison

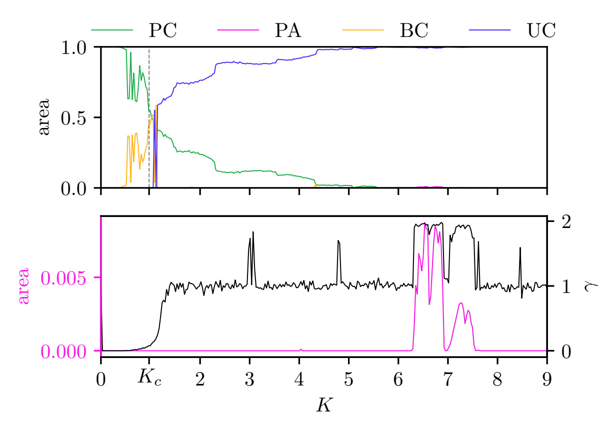
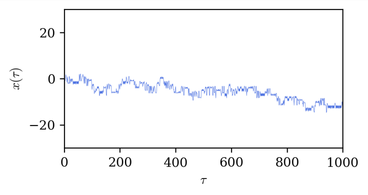


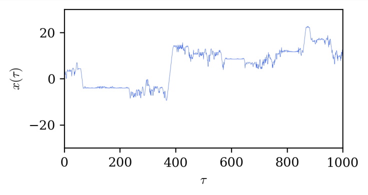
Figure 8: Existence of anomalous transport when accelerator modes are present

Extras - continuous time system - flights

$$H(x,y,t) = \phi_o - v_1 x + A_1 \sin(k_{x1} x)\cos(k_{y1} y) + A_2 \sin(k_{x2} x + \theta_x)\cos(k_{y2}(y - vt)). \quad (1)$$



(a) Trajectory with normal transport



(b) Duperdiffusive Trajectory

Figure 9: General Trajectory behavior

Extras - continuous time system - phase space

$$H(x,y,t) = \phi_o - v_1 x + A_1 \sin(k_{x1}x)\cos(k_{y1}y) + \underset{\uparrow}{A_2}\sin(k_{x2}x + \underset{\uparrow}{\theta_x})\cos(k_{y2}(y - vt)). \tag{2}$$

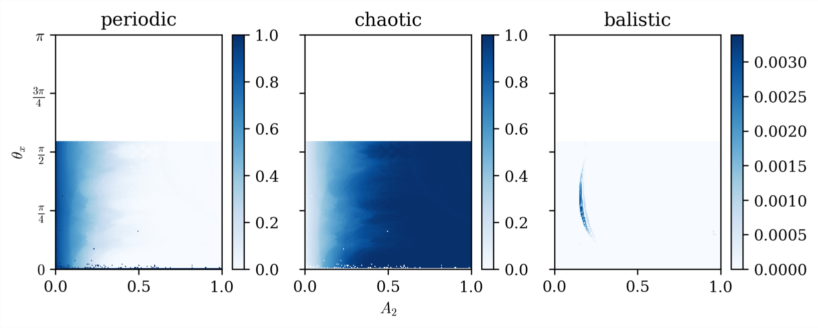


Figure 10: Categorized regions; Estimated total wall clock simulation time: 20 days with 64 core parallelization on Intel Xeon Gold 5118 at 2.3 GHz

Operations required for different methods

	Detailed displacement	γ evaluation	Segment and test
Parameters	$N_x \times N_y \times N_{it}$	$N \times N_{it}$	$N \times N_{it}$
Order	$10^3 \times 10^3 \times 10^2$	$10^3 \times 10^4$	$10^2 \times 10^4$
Order	10^8	10^7	10^6

Table 1: Approximated iterations of some methods to identify anomalous transport