2 NUCLEAR REACTIONS AND APPLICATIONS

Introduction

The study of nuclear radioactive decay provides us only a little knowledge about the nucleus. This is because only few types of radioactive processes occur in nature. In these processes only few isotopes are formed and also only few excited states of nuclei can be studied. However in nuclear reaction experiments we have the freedom to select any nuclear species and any excited states of the species. Thus we obtain too many informations regarding the nucleus.

In this chapter we shall discuss some of the different nuclear reactions that can occur and we study the properties of those reactions. Two important reactions are of particular importance. They are fusion and fission reaction processes. Then we will see how these reactions are useful as sources of energy. Finally we go for applications of nuclear reactions.

Types of nuclear reactions

When a nuclei comes in contact with another nuclei, they interact with each other. In this context we say that nuclear reaction takes place. A typical nuclear reaction experiment is conducted in a laboratory is as follows. A beam of particles of type x is incident (projectile) on a target nuclei of type X. The nuclear reaction occurs. After the reaction an outgoing particle y is observed in the laboratory, leaving a residual nucleus Y. Symbolically this reaction is written as

$$x + X \rightarrow y + X$$

For example,

$$^{4}_{2}He + ^{14}_{7}N \rightarrow ^{1}_{1}H + ^{17}_{8}O$$

Like chemical reaction, a nuclear reaction must be balanced. That is the total number of protons must be the same before and after the reaction and the total number of neutrons must be also the same before and after the reaction. In the example given above there are (2 + 7) a protons on each side and 9 neutrons on each side. In this reaction there will be no beta decays. This is because in beta decays neutron can be converted into proton and vice. This will up set the balancing condition. In order to have beta decay the time scale of interaction between projectile and the target must be of the order of 10^{-10} s. But in nuclear reactions the interaction time is only 10^{-20} s. Since there is not enough time for interaction, there will be no beta decay. It may also be noted that the protons and neutrons can be rearranged among the reacting nuclei, but their balancing should not upset.

In nuclear reactions there are only internal forces of interaction between the projectiles and the targets. As there are no external forces that come into play, nuclear reactions conserve their energy, linear momentum and angular momentum.

In most nuclear reactions we observe only the outgoing light particle y. They heavy nucleus Y usually loses all its kinetic energy, by collisions with atoms, and they stay within the target at rest.

In nuclear reactions we assume that the target particle (X) is at rest and the projectile (x) is coming with a kinetic energy k_x. The product particles share this kinetic energy plus or minus any additional energy from the rest energy difference of the initial and final nuclei.

Projectiles (x) can be either charged particles supplied by a suitable nuclear accelerators or neutrons from nuclear reactor sources. There are two types of charged particle accelerators in use. They are Cyclotron and Vandegraaf generator. Cyclotron can impart kinetic energy to charged particles of the order of 10 to 20 MeV per unit charge. The Vandegraaff accelerator can produce a potential of 25MV. The kinetic energy of the particle is about 25MeV per unit charge.

In nuclear reaction experiments we usually measure the energy of the particle y and its probability to emerge at a certain angle with certain energy.

Measuring the particle energy

We found that in nuclear reaction the kinetic energy (K_X) of the projectile (x) is shared by the product particles Y and y. If the product particles have no excited states, we can easily calculate the energy of the particle y using law of conservation of energy and momentum. If the residual nucleus is left in an excited state then the energy of the particle y is reduced by the excitation energy of Y. When the residual nucleus goes to higher and higher excited states then the energy of the particle y becomes lower and lower. This shows that measurement of the energy of the particle y gives us information about the excited state of the residual nucleus. To get the exact values energies of excited states of Y, conduct an experiment in which we measure different energies of the particle y. From this we can deduce the energies of various excited states of Y. A typical set of experimental results are plotted in a graph and shown below.

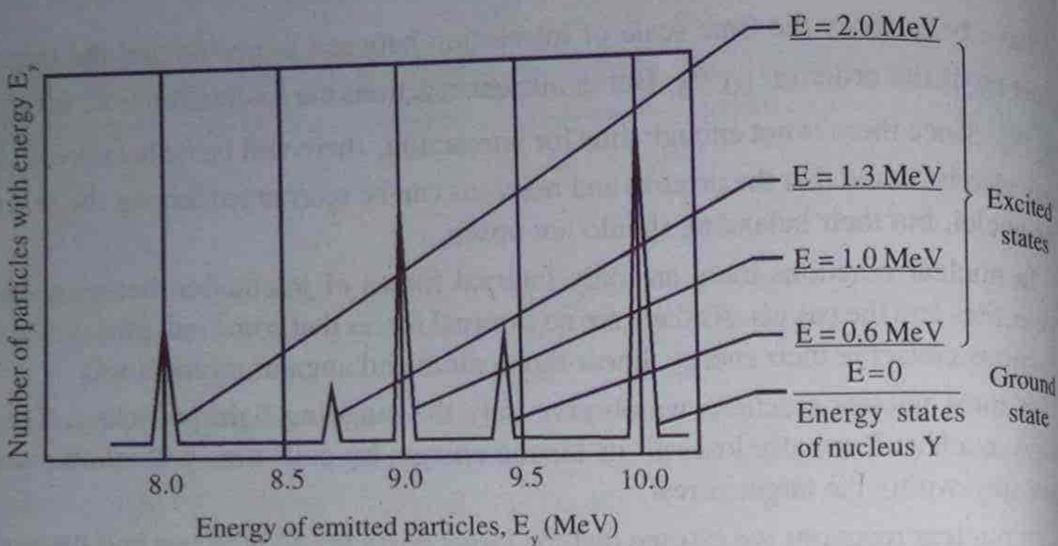


Figure 2.1: A spectrum of energies of the outgoing particles and the corresponding excited states

Each peak in the figure indicates the energy of the particle E_y. For example whe[n] $E_y = 9.0\,\text{MeV}$ and the corresponding energy of excited state of Y is 1.0 MeV an[d] so on.

Measuring the reaction probability

From the figure we can see that there are different peaks having different heights. The height of the peak tells us about which is more probable. Comparing the heights we can infer about the relative probabilities. For example height of the peak corre-sponding to E_y= 9 MeV is twice the height of the peak corresponding to E_y = 9.4 MeV. The excited state energy corresponds to $E_y = 9\,\text{MeV}$ is $E_y = 1.0\,\text{MeV}$ and that corresponds to E_y = 9.4 MeV is E_y = 0.6 MeV. This means that in this nuclear reaction the probability of leaving Y in its excites is twice the probability of leaving Y in its first excited state (E_y = 0.6 MeV). To calculate the reaction probability actually we have to solve the Schrodinger equation. Unfortunately this is not pos-sible since this is a many body problem. So what we do is starting from the experi-mental results and work backward together some informations regarding nuclear force.

The reaction cross section

The concept of cross section plays a vital role in nuclear physics. Most of the informations about atomic nuclei are derived from nuclear reactions experiments. In nuclear reaction experiments high energetic particles are bombarded with a stationary target nuclei. The target nuclei present a certain area, called its cross section, to the incident particles. Any incident particle that is directed at this area interacts with the target particle. Hence grater the cross section, the greater the likelihood of interaction. In other words interaction cross section (σ) is a measure of the effective size of the nucleus for a particular scattering. It can be thought of as an effective area of nucleus at right angles to the direction of motion of bombaring particles. A reaction (interaction) will occur only if the incident particle is in the area σ. Hence probability that a reaction will occur is proportional to σ. In other words cross section measures the likelihood of a particular reaction. The reaction cross section of a target particle varies with the nature of process involved and with the energy of the incident particle. It may be greater or lesser than the geometrical crossection. Cross sections have the dimensions of area and its unit is m^2. Since the size of the cross section is very very small and comparable to cross section of a nucleus another unit named barn is used.

$$1 \text{ barn} = 1\text{b} = 10^{-28}\text{m}^2.$$

The reaction cross sections of iodine and Xe non involved in reactions is given below as an example.

$I + n \rightarrow n + I$ (inelastic scattering), $\sigma = 4b$

$Xe + n \rightarrow Xe + n$ (inelastic scattering), $\sigma = 4b$

$I + n \rightarrow I + \gamma$ (neutron capture), $\sigma = 7b$

$Xe + n \rightarrow Xe + \gamma$ (neutron capture) $\sigma = 10^6$ b.

Xenon ($^{54}_{54}Xe$) and iodine ($_{53}I$) are isotopes of neighbouring elements. In inelastic scattering both have same cross sections ($\sigma = 4$ b). But in neutron capture reactions Xe and I have very different cross sections telling us about the unusual properties of the nucleus Xenon.

Expression for cross section

Suppose a beam of partiacles is incident on a thin target of area S which contains a total of N nuclei. The effective area of each nucleus is the cross section σ, and so the total effective area of all the nuclei in the target is σN. The fraction of the target area that this represents is $\frac{\sigma N}{S}$. This fraction is the probability for the reaction to occur.

Suppose the incident particles strike the target at a rate of I_0 particles per second. The number of particles per second is called beam intensity. After reaction the outgoing particles y are emitted at a rate of R per second. This is also the rate at which the product nucleus is formed. Then the reaction probability can also be expressed as the rate of y divided by the rate of x i.e.,

$$\text{The reaction probability } = \frac{R}{I_0}$$

$$\text{The reaction probability already obtained } = \frac{\sigma N}{S}$$

Equating the two, we get

$$\frac{R}{I_0} = \frac{\sigma N}{S}$$

or
$$R = \frac{\sigma N}{S} I_0 \qquad \text{..... (1)}$$

This is the relation between the rate of emission of y (R) and the reaction cross section (σ).

In a reactor, the beam intensity of neutrons is defined as the number of neutrons cross unit area perpendicular to the beam. This is also called neutron flux ϕ (neutrons per cm^2 per second). The cross section is σ (It is cm^2 per nucleus per incident neutron). The rate of emission of y also depends upon the number of target nuclei. If M is the molar mass of the target. This means that M grams containing one Avogadro number (N_A) of atoms.

$\therefore$ 1 gram contains $\dfrac{N_A}{M}$ atoms.

If m (gram) is the mass of the target. Then the number of atoms in the target

$$N = \frac{mN_A}{M}$$

Equation (1) can be rewritten as $R = \sigma N \left(\dfrac{I_0}{S}\right)$

$\frac{I_0}{S}$ is the neutron intensity (ϕ)

$$R = \sigma N \phi$$

substituting for N $\quad R = \sigma\phi \frac{mN_A}{M}$ ..... (2)

Example 1

A beam of 20.0 μ A of protons is incident on 2.0 cm^2 of a target of ^{107}Ag of thickness 4.5 μm producing the reaction.

$$p + {}^{107}Ag \rightarrow {}^{105}cd + 3n$$

Neutrons are observed at a rate of 8.5×10^6 per second. What is the cross section for this reaction at this proton energy. Density of Ag = 10.5 gcm^{-3}.

Solution

We know that, current $= \frac{\text{Total charge}}{\text{time}}$

$$20\ \mu A = \frac{Ne}{t}$$

$\therefore$ Number of protons incident per second

$$\frac{N}{t} = \frac{20 \mu A}{e} = \frac{20 \times 10^{-6}}{1.6 \times 10^{-19}} = 1.25 \times 10^{14}$$

i.e., Intensity of the proton beam

$$I_0 = 1.25 \times 10^{14} \text{ protons s}^{-1}$$

Using $\quad R = \frac{\sigma N\, I_0}{S}$

But $\quad \frac{N}{S} = \frac{(mN_A / M)}{S} = \frac{V\rho N_A}{SM}$

$$\frac{N}{S} = \left(\frac{V}{S}\right) \frac{\rho N_A}{M} = \frac{t\, \rho N_A}{M}$$

Where t is the thickness of the target

$$\therefore \qquad R = \frac{\sigma I_0\, t\, \rho N_A}{M}$$

or

$$\sigma = \frac{RM}{I_0 t \rho N_A}$$

$$R = \frac{1}{3}(8.5 \times 10^6)\ s^{-1} \ \text{(given)}$$

$$M = 107 \text{gmole}^{-1}, \ I_0 = 1.25 \times 10^{14}$$

$$\rho = 10.5\text{g cm}^{-3}, \quad t = 4.5 \times 10^{-4}\text{cm}$$

$$N_A = 6.022 \times 10^{23} \text{atoms / mole}$$

$$\therefore \qquad \sigma = \frac{8.5 \times 10^6}{3} \times \frac{107}{1.25 \times 10^{14} \times 4.5 \times 10^{-4} \times 10.5 \times 6.022 \times 10^{23}}$$

$$\sigma = 8.523 \times 10^{-28}\ \text{cm}^2 = 8.523 \text{barn}$$

Example 2

A beam of alpha particles is incident on a target of ^{63}Cu, resulting in the reaction $\alpha + {}^{63}Cu \rightarrow {}^{66}Ga + n$. Assume that cross section for the particular alpha energy to be 1.25b. The target is in the form of a foil 2.5μm thick. The beam has a circular cross section of diameter 0.50 cm and a current of 7.5μA. Find the rate of neutron emission.

Density of copper = 8.96 gcm^{-3}

Solution

$$\sigma = 1.25\text{b} = 1.25 \times 10^{-24}\text{cm}^2, \quad t = 2.5 \times 10^{-4}\text{cm}$$

$$d = 0.50\text{cm}, \quad \text{current} = 7.5\mu\text{A}$$

$$M = 63\text{g and } \rho = 8.96 \ \text{gcm}^{-3}$$

Using $R = \dfrac{\sigma I_0 N}{S}$

$$\text{Current} = \frac{\text{Total charge}}{\text{time}} = \frac{N.2e}{t}$$

$$\therefore \quad \frac{N}{t} = I_0 = \frac{\text{Current}}{2e} = \frac{7.5 \times 10^{-6}}{2 \times 1.6 \times 10^{-19}}$$

$$I_0 = 2.344 \times 10^{13} \text{ particles s}^{-1}$$

$$\frac{N}{S} = \frac{mN_A}{SM} = \frac{V\rho N_A}{\pi r^2 M} = \frac{t\rho N_A}{M}$$

$$\frac{N}{S} = \frac{2.5 \times 10^{-4} \times 8.96 \times 6.022 \times 10^{23}}{63}$$

$$\frac{N}{S} = 2.141 \times 10^{19}$$

$$\therefore \quad R = 1.25 \times 10^{-24} \times 2.141 \times 10^{19} \times 2.344 \times 10^{13}$$

$$R = 6.273 \times 10^{8} \text{ neutrons per second.}$$

Radioisotope production in nuclear reactions

Nuclear reactions are also used to produce radioisotopes. In this process of reaction a stable nonradioactive isotope X is irradiated with a particle x to form the radioactive isotope Y. The outgoing particle is y. Here the outgoing particle is not our interest so we don't observe them. Our interest is to observe the number of radio active isotopes formed that remains at rest within the target after reaction. Since the particle Y is radioactive it will undergo decay.

Here our aim is to calculate the activity of the isotope Y for a certain time t. Let R be the constant rate at which Y is produced. This is related to the intensity (I_0) of the particle x (see equation 1). In a time interval dt, the number of Y nuclei produced is Rdt. Since Y is radioactive it decays. In time dt the number of Y nuclei decayed is $\lambda N dt$, where λ is the decay constant and N is the number of Y nuclei present.

Therefore the net change in the number (dN) of Y nuclei is

$$dN = Rdt - \lambda N dt$$

or

$$\frac{dN}{dt} = R - \lambda N$$

$$\frac{dN}{dt} + \lambda N = R$$

Multiplying throughout by the integrating factor $e^{\lambda t}$, we get

$$e^{\lambda t}\frac{dN}{dt} + e^{\lambda t}\lambda N = Re^{\lambda t}$$

or $$\frac{d}{dt}(Ne^{\lambda t}) = Re^{\lambda t}$$

Integrating we get

$$Ne^{\lambda t} = \frac{Re^{\lambda t}}{\lambda} + c \qquad \text{..... (4)}$$

Where c is the constant of integration

when $t = 0$, $N = 0$

$\therefore$ $$0 = \frac{R}{\lambda} + c$$

i.e., $$c = -\frac{R}{\lambda}$$

Put this in equation 4, we get

$$Ne^{\lambda t} = \frac{Re^{\lambda t}}{\lambda} - \frac{R}{\lambda}$$

$$N = \frac{R}{\lambda} - \frac{R}{\lambda}e^{-\lambda t}$$

$$N = \frac{R}{\lambda}(1 - e^{-\lambda t}) \qquad \text{..... (5)}$$

or $$\lambda N = R(1 - e^{-\lambda t})$$

λN is called activity denoted by $a(t)$

Thus, activity

$$a(t) = R(1 - e^{-\lambda t}) \qquad \text{..... (6)}$$

When $t = 0$, $a(t) = 0$. This shows that at $t = 0$, there is no radioactive isotope of y hence activity is zero. Equation 6 tell us that as the irradiation time (t) passes, the activity a(t) of the isotope increases exponentially. See figure shown below.

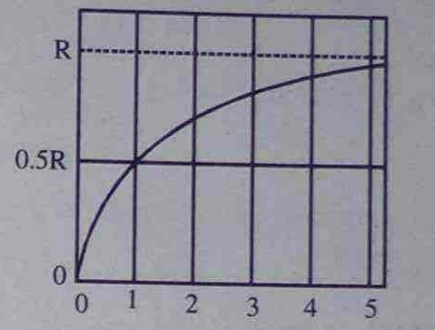


Figure 2.2: Formation of activity in a nuclear reaction

For large irradiation times $t \gg T_{1/2}$,

$$a(t) = R \qquad \text{..... (7)}$$

When $t \ll T_{1/2}$, $\lambda t = \dfrac{0.693}{T_{1/2}} t$ is very very small. Thus $e^{-\lambda t} \simeq 1 - \lambda t$

$$\therefore \quad a(t) = R(1-e^{-\lambda t}) = R(1-(1-\lambda t)$$

$$a(t) = R\lambda t \qquad \text{..... (8)}$$

Example 3

A radioactive isotope o half life $T_{1/2}$ is produced in a nuclear reaction. What fraction of the maximum possible activity is produced in an irradiation time of

a) $T_{1/2}$ b) $2\,T_{1/2}$ c) $4\,T_{1/2}$

Solution

Maximum activity of the isotope

$$a_{max} = R$$

The fraction of maximum activity,

$$f = \frac{a(t)}{R} = \frac{R(1-e^{-\lambda t})}{R} = 1 - e^{-\lambda t}$$

a) $f = 1-e^{-\lambda t} = 1-e^{\frac{-0.693}{T_{1/2}} \cdot T_{1/2}} = 1-e^{-0.693}$

$f = 1-0.50 = 0.50$

b) $f = 1-e^{\frac{-0.693}{T_{1/2}} \cdot 2T_{1/2}} = 1-e^{-0.693\times2}$

$f = 1-0.5^2 = 0.75$

c) $f = 1-e^{\frac{-0.693}{T_{1/2}} \cdot 4T_{1/2}} = 1-e^{-0.693\times4}$

$f = 1-(0.5)^4 = 1-0.0625 = 0.9375$

Note: $e^{-0.693} \simeq 0.5$

Example 4

Neutron capture in sodium occurs with a cross section of 0.53b and leads to radioactive $^{24}Na\ (T_{1/2} = 15h)$. What is the activity that results when 1.00 μg of Na is placed in a neutron flux 2.5×10^{13} neutron $cm^{-2}s^{-1}$ for 4.00 hours.

Solution

$$\sigma = 0.536\,b = 0.53\times10^{-24}\,cm^2$$

$$T_{1/2} = 15h$$

$$m = 1.00\mu g = 1.00\times10^{-6}\,g$$

$$\phi = 2.5\times10^{13}\ \text{neutrons}\ cm^{-2}s^{-1}$$

$$t = 4.00h \quad \text{and}\ M = 23$$

Using $R = \dfrac{\sigma\phi m N_A}{M}$

$$R = \frac{0.53\times10^{-24}\times2.5\times10^{13}\times10^{-6}\times6.02\times10^{23}}{23}$$

$$R = 3.47\times10^5\ s^{-1}$$

$\therefore$ Activity, $a(t) = R\,(1-e^{-\lambda t})$

$$a(t) = 3.47\times10^5 \left(1-e^{-\frac{0.693}{15}\times4}\right)$$

$$a(t) = 3.47\times10^5 \ (1-0.831)$$

$$a(t) = 3.47\times10^5 \times0.169 = 5.86\times 10^4 \, s^{-1}$$

Low energy reaction kinematics

Consider a projectile x moving with momentum $\vec{p}_x$ and kinetic energy K_x. The target is at rest and the reaction products (y and Y) have momenta $\vec{p}_y$ and $\vec{P}_y$ and kinetic energies K_y and K_Y. The particles y and Y are emitted at angles θ_y and θ_Y with respect to the initial direction of the projectile. This is illustrated in the figure given below. We assume that the product nucleus Y is not observed in the laboratory. If it is a heavy nucleus it moves with slow speed or generally comes to rest within the target. One more assumption that we make is, the velocities of the nuclear particles are sufficiently small that we can use non-relativistic kinematics.

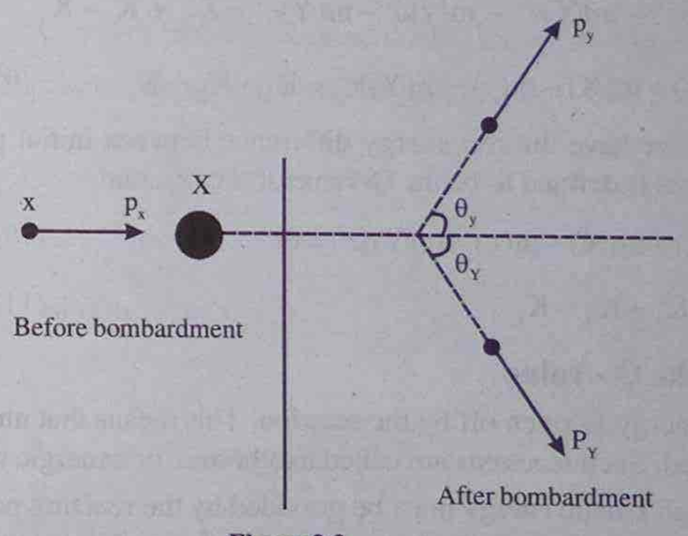


Figure 2.3

Our aim here is to find out the Q-value of the reaction.

Using law the conservation of energy:

Energy before reaction = Energy after reaction i.e., $m_x c^2 + K_x + m_X c^2 = m_y c^2 + K_y + m_Y c^2 + K_Y$

All masses are nuclear masses.

We found that in nuclear reactions number of protons are balanced.

i.e., $$z_x + Z_X = z_y + Z_Y$$

We can therefore add equal number of electrons on both sides. We get

$$m_x c^2 + z_x m_e c^2 + K_x + m_X c^2 + Z_X m_e c^2$$

$$= m_y c^2 + z_y m_e c^2 + K_y + m_Y c^2 + Z_Y m_e c^2 + K_Y$$

i.e, $$[m_x + z_x m_e]\, c^2 + K_x + [m_X + Z_X m_e]\, c^2$$

$$= [m_y + z_y m_e]\, c^2 + K_y + [m_Y + z_Y m_e]\, c^2 + K_Y$$

The terms inside the square brackets are atomic masses of respective nuclei.

i.e, $$m(x)c^2 + K_x + m(x)c^2 = m(y)c^2 + K_y + m(Y)c^2 + K_Y$$

or $$m(x)c^2 + m(X)c^2 - m(y)c^2 - m(Y)c^2 = K_Y + K_y - K_x$$

$$[m(x) + m(X) - m(y) - m(Y)]c^2 = K_y + K_Y - K_x \qquad ..... (9)$$

On the L.H.S we have the rest energy difference between initial particles and final particles. This is defined to be the Q-value of the reaction.

$$[m(x) + m(X) - m(y) - m(Y)]c^2 = Q \qquad ..... (10)$$

Thus $$Q = K_y + K_Y - K_x \qquad ..... (11)$$

Discussion on the Q - value

If $Q > 0$, the energy is given off by the reaction. This means that nuclear energy has been converted. Such reactions are called exothermic or exoergic reactions.

If $Q < 0$, enough kinetic energy must be provided by the reacting particles. This means that these reactions require input energy in the form of the kinetic energy of x. Such reactions are called endothermic or endoergic reactions.

In endoergic reaction, we must supply enough kinetic energy to provide the additional rest energy of the reaction products. There is thus some minimum or threshold kinetic energy of x below which the reaction will not take place. This threshold kinetic energy not only must supply the additional rest energy but also some kinetic energy to the products. This is because after reaction either one product particle or two product particles are in motion. For this motion they require kinetic energy. If both particles are at rest after reaction this would violate law of conservation of momentum. This problem can easily analyse in the centre of mass reference frame.

Lab frame and centre of mass frame - A review

Consider a particle of mass m(x) moving with speed u_x approaches a particle of mass m(X) at rest. The frame of reference with origin at m(X) defines the lab frame.

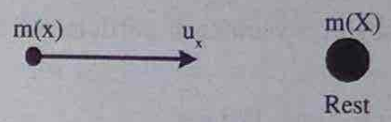


Figure 2.4

The centre of mass of m(x) and m(X) is defined to be

$$R_{cm} = \frac{m(x)\, r_x + m(X)\, r_X}{m(x) + m(X)}$$

Where r_x and r_X are the position vectors of m(x) and m(X) respectively.

Differentiating the above equation with respect to time

$$\frac{dR_{cm}}{dt} = \frac{m(x)\,\frac{dr_x}{dt} + m(x)\,\frac{dr_X}{dt}}{m(x) + m(X)}$$

i.e. $$V_{cm} = \frac{m(x)u_x + m(X)u_X}{m(x) + m(X)}$$

But $u_X = 0$ $\quad \therefore \; V_{cm} = \dfrac{m(x)u_x}{m(x) + m(X)}$

In the lab frame kinetic energy is that of the incident particle only.

i.e, $$K_{lab} = \frac{1}{2} m(x) v_X^2$$

Centre of mass frame

Consider two particles of masses m(x) and m(X) moving with velocities u'_x and u'_X. Here the origin of the co-ordinate system is at the centre of mass we have

$$R'_{cm} = \frac{m(x)r'_x + m(X)\, r'_X}{m(x) + m(X)} = 0$$

i.e, $$m(x)r'_x = -m(X)\, r'_X$$

Where r'_x and r'_X are the position vectors of particles of masses m(x) and m(X) respectively.

Differentiating with respect to time, We get

$$m(x)\frac{dr'_x}{dt} = -m(X)\frac{d\,r'_X}{dt}$$

i.e. $$m(x)u'_x = -m(X)u'_X$$

This means that in C.M frame the direction of the two particles will be opposite to each other.

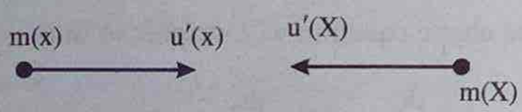


Figure 2.5

Relation between velocities in lab frame and that in C.M frame

From the figure we have

$$\vec{r}_x = \vec{r}'_x + \vec{v}_{cm}t$$

Differentiating with respect to time, we get

$$\vec{u}_x = \vec{u}'_x + v_{cm}$$

$$\therefore \quad u'_x = u_x - v_{cm}$$

and $$u'_X = u_X - v_{cm} \quad \text{But } u_X = 0$$

$$\therefore \quad u'_X = -v_{cm}$$

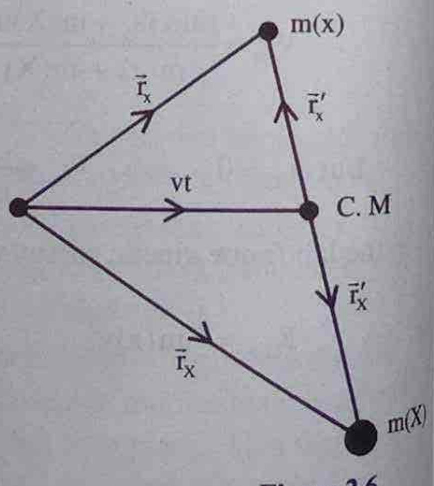


Figure 2.6

or $$u'_x = u_x - \frac{m(x)u_x}{m(x)+m(X)}$$

$$\therefore \quad u'_x = \frac{m(X)u_x}{m(x)+m(X)} \qquad \text{..... (12)}$$

and $$u'_X = -\frac{m(x)u_x}{m(x)+m(X)} \qquad \text{..... (13)}$$

When you go from Lab frame to C.M frame u_x [the speed of m(x)] must be replaced by u'_x and the speed of $u_x (u_X = 0)$ must be replaced by u'_X.

Expression for threshold kinetic energy

In the centre of mass reference frame the projectile x of mass m(x) is moving with a speed u'_x and the target particle (X) moving with a spead u'_X in opposite directions. After nuclear reaction the product particles y and Y would be at rest. Since we find the threshold kinetic energy.

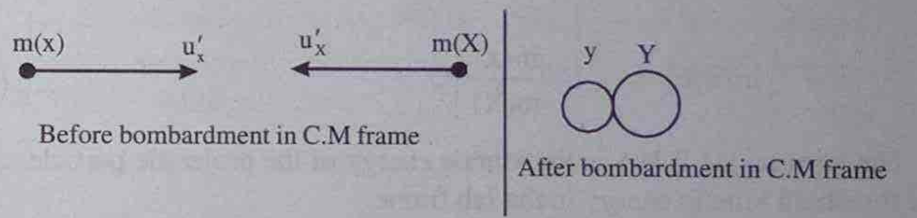


Figure 2.7

Using law of conservation of energy, we have

Total energy before reaction = Total energy after reaction

$$m(x)c^2 + \frac{1}{2}m(x)u'^2_x + m(X)c^2 + \frac{1}{2}m(X)u'^2_X = m(y)c^2 + m(Y)c^2$$

or $$\frac{1}{2}m(x)u'^2_x + \frac{1}{2}m(X)u'^2_X = m(y)c^2 + m(Y)c^2 - m(x)c^2 - m(X)c^2$$

Substituting for u'_x from eq. (12) and u'_x from eq. (13) in the above, we get

$$\frac{1}{2}m(x)\left[\frac{m(X)u_x}{m(x)+m(X)}\right]^2+\frac{1}{2}m(X)\left[-\frac{m(x)u_X}{m(x)+m(X)}\right]^2$$

$$= m(y)c^2+m(Y)c^2-m(x)c^2-m(X)c^2$$

$$\frac{\frac{1}{2}m(x)m^2(X)u_x^2}{[m(x)+m(X)]^2}+\frac{\frac{1}{2}m(X)m^2(x)u_X^2}{[m(x)+m(X)]^2}=-[m(x)+m(X)-m(y)-m(Y)]c^2$$

$$\frac{1}{2}m(x)u_x^2\ m(X)\frac{[m(X)+m(x)]}{[m(x)+m(X)]^2}=-Q$$

On the R.H.S we have the Q-value of the reaction

$$\therefore \qquad \frac{1}{2}m(x)u_x^2\ \frac{m(X)}{m(x)+m(X)}=-Q$$

or
$$\frac{1}{2}m(x)u_x^2=-\frac{m(x)+m(X)}{m(X)}Q$$

$$\frac{1}{2}m(x)u_x^2=-\left[1+\frac{m(x)}{m(X)}\right]Q \qquad \text{..... (14)}$$

The term on the L.H.S is the kinetic energy of the projectile particle x, which is the threshold kinetic energy in the lab frame.

i.e.
$$K_{Th}=-\left[1+\frac{m(x)}{m(X)}\right]Q \qquad \text{..... (15)}$$

This is the expression for the threshold kinetic energy.

Note: While deriving the expression for K_{Th}, we used law of conservation of energy. The rest energy is mc^2, which is relativistic but kinetic energy is non-relativistic. This is admissible since $u_x << c$.

Example 5

Find the Q-value of the reaction

$$^{3}He + {}^{40}Ar \rightarrow {}^{41}K + {}^{2}H$$

Given: $m(He) = 3.016029$ u, $m(Ar) = 39.962383$ u
$m(K) = 40.961826$ u and $m(H) = 2.014102$ u.

Solution

Q- value of the reaction is :

$$Q = [m(He) + m(Ar) - m(K) - m(H)]c^2$$

$$Q = [3.016029\,u + 39.962383\,u - 40.961826\,u - 2.014102\,u]c^2$$

$$Q = 0.2484\,u\,c^2$$

Using $1u = 931.5\dfrac{MeV}{c^2}$

$$Q = 0.2484 \times 931.5 \text{ MeV}$$

$$Q = 2.313846 \text{ MeV}$$

Example 6

In the reaction ${}^2H + {}^3He \rightarrow p + {}^4He$, deutrons of energy 5.000 MeV are incident on ${}^3He$ at rest. Both the proton and the alpha particle are observed to travel along the same direction as the incident deutron. Find the sum of kinetic energies of the proton and the alpha particle.

Given: $m({}^2H) = 2.014102\,u, m({}^3He) = 3.016029\,u$

$m({}^1H) = 1.007825\,u$ and $m({}^4He) = 4.002603\,u$

Solution

The Q-value of the reaction is

$$Q = \left[m({}^2H) + n({}^3He) - m({}^1H) - m({}^4He)\right]c^2$$

$$Q = [2.014102u + 3.016029\,u - 1.007825\,u - 4.002603\,u]c^2$$

$$Q = 0.019703\,u\,c^2$$

$$Q = 0.019703 \times 931.5\,MeV$$

$$Q = 18.353 \text{ MeV}$$

But $\quad Q = K_y + K_Y - K_x$ in general

Here $Q = K_p + K_{^4He} - K_{^2H}$

$$18.353 = K_p + K_{^4He} - 5.000$$

$$K_p + K_{^4He} = 23.353 \text{ MeV}$$

Example 7

What is the Q-value of the reaction $p + {}^4He \rightarrow {}^2H + {}^3He$

a) What is the threshold energy for protons incident on ${}^4He$ at rest.

b) What is the threshold energy if ${}^4He$ are incident on proton at rest m(H) =1.007825u, $m({}^4He) = 4.002603u$, $m({}^2H) = 2.014102u$ and $m({}^3He) =$ 3.016029 u.

Solution

$$Q = \left[m({}^1H) + m({}^4He) - m({}^2H) - m({}^3He)\right]c^2$$

$$Q = [1.007825u + 4.002603u - 2.014102u - 3.016029u]c^2$$

$$Q = -0.019703uc^2$$

$$Q = -0.019703 \times 931.5 \text{MeV}$$

$$Q = -18.353$$

a) Threshold kinetic energy

$$K_{Th} = -Q\left[1 + \frac{m(x)}{m(X)}\right]$$

$$K_{Th} = -Q\left[1 + \frac{m({}^1H)}{m({}^1He)}\right]$$

$$K_{Th} = 18.353\left[1 + \frac{1.007825}{4.002603}\right]$$

$$K_{Th} = 22.974 \text{ MeV}$$

b) $$K_{Th} = -Q\left[1+\frac{m(^4He)}{m(^1H)}\right]$$

$$K_{Th} = --18.353\left[1+\frac{4.002603}{1.007825}\right]$$

$$K_{Th} = 91.242 \text{ MeV}$$

Nuclear Fission

Nuclear fission is the phenomenon of splitting a heavy nucleus (A>230) into two lighter nuclei with the release of energy. These are of two types. (1) Spontaneous nuclear fission and (2) induced nuclear fission.

Spontaneous nuclear fission

By a suitable selection of a projectile and a target we can conduct collisions in laboratories to produce a heavy nucleus which is radioactive. For example the massive nucleus californium $^{254}_{98}Cf$ can be produced in accelerators by collision. This heavy nucleus is radio active, decaying with a half life 60.5 days. The Californium decays into xenon and ruthenium and with the release of four neutrons.

i.e., $$^{254}_{98}Cf \rightarrow {}^{140}_{54}Xe + {}^{110}_{44}Ru + 4n$$

If we calculate the Q-value for positive and negative beta decay, they are found to be negative. But the Q-value of alpha decay is positive. Thus alpha is energetically possible for californuim. But this alpha particles cannot overcome the high Coulomb barrier of californium. So alpha decay is also not possible. In effect californium does not undergo nuclear decays (α or $\beta^{\pm}$) but disintegrate into smaller nuclei. This mode of decay is known as nuclear fission.

When a heavy nucleus is bombarded with neutrons, nucleus becomes excited. This excited nucleus is unstable and disintegrates into two lighter nuclei. This is known as induced nuclear fission. For example

$$n + {}^{235}_{92}U \rightarrow {}^{93}_{37}Rb + {}^{141}_{55}Cs + 2n$$

Note: The Californium nucleus was particularly selected owing to some special interest. This is because it is also produced in supernova explosions and knowledge of its properties provides a key to understanding the formation of elements in stars.

Mechanism of fission

A nucleus consists of protons and neutrons moving about under the mutual attraction of their nuclear forces and Coulombian repulsive force. As a result of these interactions many nuclei assume spherical in shape and behave like liquid drops according to liquid drop model. When this nucleus experiences some external force (arising from external energy) the equilibrium spherical shape will be distorted and becomes ellipsoidal in shape. Such a nucleus undergoes rapid oscillations about their equilibrium shape. In other words nucleus behaves like a stretched spring when released. see figure (2.8).

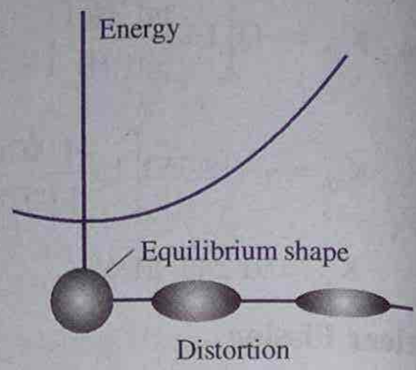


Figure 2.8: The energy of a nucleus with a spherical equilibrium shape increases as the distortion increases

Other nuclei which are already in ellipsoidal shape at equilibrium, when such a nucleus experiences an external force (arising from external energy) it undergoes rapid oscillations and finally comes back to their ellipsoidal shape. If the external force is large nucleus may not return to its equilibrium but instead may split into two. See figure (2.9).

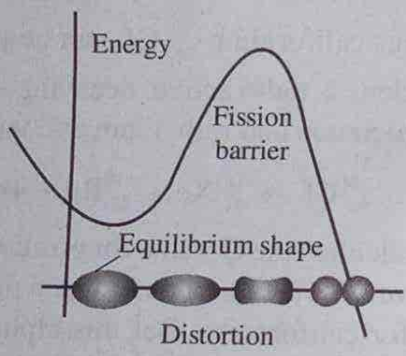


Figure 2.9: The energy of a nucleus with a nonspherical equilibrium shape. If enough energy is added, the nucleus can tunnel through the fission barrier and split into two pieces

For example, consider the collision between a neutron and a heavy nucleus $^{236}_{92}U$. They combine to form a compound nucleus which is highly energetic. Its extra energy is partly the kinetic energy of the neutron but largely the added binding energy of the incident neutron. This energy appears to initiate a series of rapid oscillations in the drop which at times assumes the shape B shown in the figure below. The restoring force of the nucleus arises from the short range inter nucleon forces. If the oscillations become so violent that the stage D is reached and each half is now positively charged, the final fission into stage E is inevitable. Thus this is a threshold energy or a critical energy required to produce stage D after which the nucleus cannot return to A, because of Coulombian repulsion of the two parts.

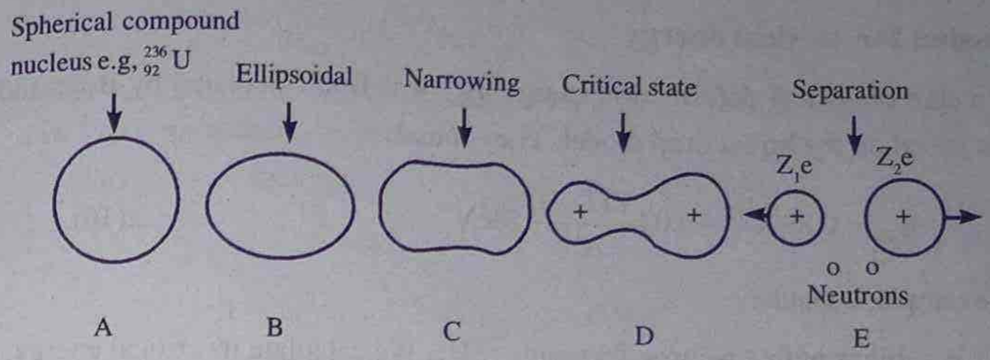


Figure 2.10: Mechanism of fission in liquid-drop model of nucleus

The critical energy which must be supplied with the neutron, is best illustrated in figure below which is a potential energy diagram. In this diagram we see how the energy E_{crit} must be added to the system to enable the energy of the nucleus to become greater than the stability barrier energy E_b. Once the maximum barrier height has been overcome the system descends to the state of lowest potential energy and the fragments separate. When the mass of the compound nucleus is greater than the masses of the total fission fragments, fission is possible and the mass difference is released as energy according to Einstein's relation $\Delta E = \Delta mc^2$.

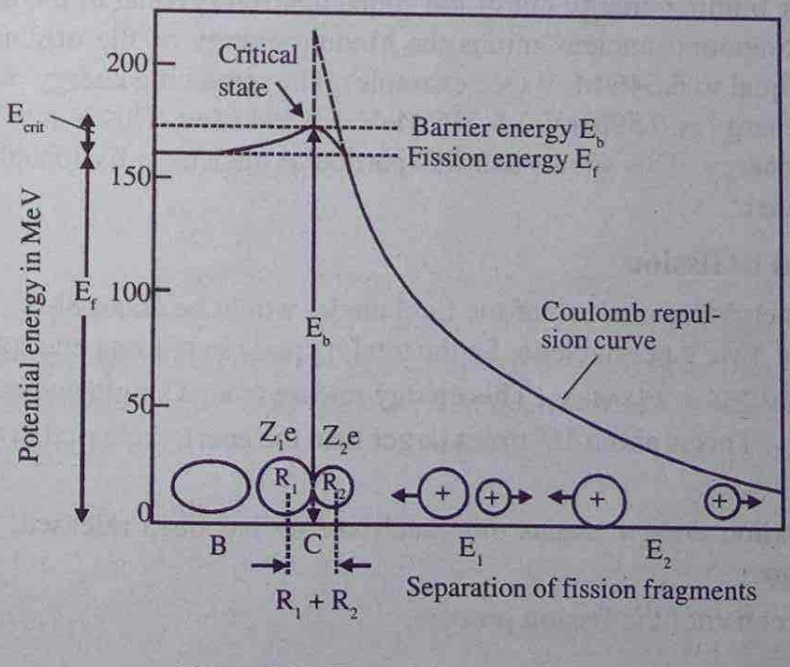


Figure 2.11: Potential energy curve for fission

Expression for critical energy

The value of critical deformation energy E_{cri} was first calculated by Bohr and wheeler based on the liquid drop model. They found

$$E_{cri} = 0.89A^{2/3} - 0.02\frac{Z(Z-1)}{A^{1/2}}\text{MeV} \qquad .....(16)$$

For example, consider

$^{235}_{92}U$ combines with a neutron forming $^{236}_{92}U$. We calculate its critical energy.

$$E_{cri} = 0.89A^{2/3} - 0.02\frac{Z(Z-1)}{A^{1/3}}\text{MeV}$$

$$E_{cri} \text{ of } ^{236}_{92}U = 0.89\times(236)^{2/3} - 0.02\times\frac{92\times(92-1)}{(236)^{1/3}}\text{MeV}$$

$$= 0.89\times 38.19 - 29.10$$
$$= 33.99 - 29.10$$
$$= 6.89 \text{ MeV}$$

This energy must be added as the kinetic energy and binding energy of the incident neutron. The binding energy out of the added neutron is equal to the binding energy of 236 compound nucleus minus the binding energy of the original 235 nucleus. This is equal to 6.546 MeV (see example). The remaining energy required to attain critical energy is 6.89MeV – 6.546 MeV = 0.344 Mev. This is supplied as neutrons kinetic energy. This shows that this particular nucleus is fissionable with low energy neutrons.

Energy released in fission

The binding energy per nucleon of the final nuclei would be about 8MeV. Thus there is increase of 1MeV per nucleon. So the total increase in binding energy of the nucleus is about $1\times 254 = 254\text{MeV}$. This energy release from a single nucleus is an enormous quantity. This is about 10^8 times larger than the energy released in chemical processes.

Increase in binding energy means this much energy has been released. Where does this energy go?

To clarify this consider the fission process

$$^{254}_{98}Cf \rightarrow {}^{127}_{49}In + {}^{127}_{49}In$$

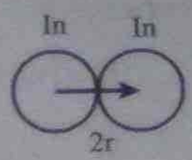


Figure 2.12

when they are about to split two positively charged indium nuclei, just touching at their surfaces

The radius of each nuclei is

$$r = r_0\ A^{1/3}$$

$$r = 1.2\ (127)^{1/3} = 6.03 fm$$

$\therefore$ The separation between the two nuclei = 2r = 12.06 fm

The Coulombian repulsion between them is

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 (2r)} = \frac{(Z_1 e)(Z_2 e)}{4\pi\varepsilon_0 \cdot 2r}$$

$$U = \frac{9\times10^9\ (49e)(49e)}{12.06\times10^{-15}}\ J$$

$$U = \frac{9\times10^9 \times (49)^2 (1.6\times10^{-19})^2}{12.06\times10^{-15}}$$

$$U = \frac{9\times10^9\ (49)^2 \times (1.6\times10^{-19})^2}{12.06\times10^{-15}\times1.6\times10^{-13}}\ MeV$$

$$U = \frac{34574.4\times10^{-1}}{12.06}$$

$$U = 286.68\ MeV$$

This is very much close to our estimate of the binding energy released. These two charged objects repel one another, so that Coulomb energy quickly becomes kinetic energy. This energy is given to the fission products, the decay products (betas and gammas) and other particles like neutrons.

Example 8

Find the Q-value of the fission decay

$$^{254}Cf \rightarrow {}^{127}In + {}^{127}In$$

m(In) = 126.917353 u and m (Cf) = 254.087323 u

Solution

$$Q = [m(Cf) - 2m(In)]c^2$$

$$Q = [254.087323u - 2\times 126.917353u]c^2$$

$$Q = 0.252617uc^2$$

$$Q = 0.252617\times 931.5MeV = 235.312 MeV$$

Example 9

Find the energy released in the fission of 1.00kg of uranium that has been enriched to 3.0% in the radioisotope of ^{235}U. Each fission releases about 200 MeV.

Solution

The mass of isotope that contains in 1.00kg = $1.00\times\frac{30}{100} = 0.03kg = 30g$

$\therefore$ The number of atoms in

$$^{235}U \text{ isotope} = \frac{m\,N_A}{M} = \frac{30\times 6.022\times 10^{23}}{235}$$

$$= 7.689\times 10^{22}\,\text{atoms}$$

Since fission releases about 200 MeV, the total energy released,

$$E = 7.689\times 10^{22}\times 200$$

$$E = 15.378\times 10^{24}\,MeV$$

or

$$E = 15.378\times 10^{24}\times 1.6\times 10^{-13}\,J$$

$$E = 24.6\times 10^{11} = 2.46\times 10^{12}\,J$$

Induced fission

We found that radio active decay of californium is an example of spontaneous fission. There are other examples of fissionable nuclei that may occur naturally or are produced artificially. These nuclei can be made to fission by the addition of some energy, which might be in the form of absorption of photons or neutrons. In these cases the energy input is very small compared with the energy released in the fission process.

An example of such fission process is

$$^{235}_{92}U + n \rightarrow {}^{236}_{92}U \rightarrow {}^{144}_{56}Be + {}^{89}_{36}Kr + 3n + Q.$$

The Q-value in this reaction is about 200 MeV.

In a bulk sample of uranium each of the neutrons emitted in fission can be absorbed by another ^{235}U and thus induce another fission process, resulting in the emission of still more neutrons. This will be followed by more fissions and so forth. As long as the average number of neutrons available to produce new fissions is greater than 1per nucleon, the number of fissions grows with time. the avalanche or chain reaction of fission occurs each with a release of about 200 MeV energy. This occurs in a rapid and uncontrolled conditions as in nuclear weapons. The energy output can be taken out under slower and carefully controlled conditions as in a nuclear reactor.

Electrical power from fission

Nuclear energy originates from the splitting of uranium nuclei by a process called nuclear fission. The thermal energy so produced is used to boil water producing steam. This steam is used by a turbine generator to generate electricity. But there is a problem, ordinary uranium by itself cannot undergo fission even if it is bombarded by a neutron. There are mainly three reasons for this. They are called (1) enrichment (2) moderation and (3) control.

Enrichment

Natural uranium consists of 0.7% of ^{235}U and 99.3% of ^{238}U. ^{235}U is fissionable but ^{238}U is not fissional. To produce fission in the ^{235}U, we require one neutron and one fission produces two neutrons. If there are two neutrons absorbed by ^{238}U, no further fission occurs. If by chance one neutron is free among the two neutrons produced which can initiate another fission of ^{235}U. So we cannot expect a sustained production of energy. To maintain a steady energy production from fission reactions, we should have one free neutron from each fission produce another fission. To overcome this problem it is necessary to use enriched uranium, which contains 3-5% of ^{235}U. Enrichment is a difficult process because ^{235}U and ^{238}U are chemically identical. By forcing gaseous uranium through a porus barrier in which the more massive ^{238}U atoms diffuse more slowly there by we get more ^{235}U in the output.

Moderation

The neutrons released in fission have high kinetic energies of the order of a few

MeV. Such neutrons cannot trigger the fission of ^{235}U further. This is because the fission cross section generally decreases with increasing neutron energy. At the same time there is a high chance of absorption of neutrons by ^{238}U, which is not fissionable. it is therefore necessary to slow down, or moderate, these neutrons in order to increase their chances of initiating further fission process. the fissionable material is surrounded by a moderator, and the neutrons lose energy in collisions with the atoms of the moderator. When a neutron is scattered from a heavy nucleus like uranium, the loss of energy of neutron is negligibly small or nil. But when neutron collides with a light nuclei, the neutron can lose its substational amount of energy.

The main considerations for the choice of a moderator are (a) Mass comparable with that of neutrons and (b) low capture of neutrons. Graphite (Carbon) is therefore the best moderator which has a relatively small neutron absorption cross section. Enrico Fermi and his co-workers built the first nuclear reactor in 1942 at the University of Chicago. This reactor used carbon in the form of graphite blocks as moderator.

Ordinary water is frequently used as a moderator, because collisions with the protons are very effective in slowing the neutrons. However, neutrons have a relatively high probability of being absorbed by the water according to the reaction $p + n \rightarrow {}_1^2H_1 + \gamma$. ${}_1^2H$ is called deuterium. In water hydrogen is replaced by deuterium, it is called heavy water. so heavy water is more useful as a moderator. it has virtually no neutron absorptions cross section. Many reactors use heavy water as moderator.

Control

To have continuous nuclear fission in reactors, the average number of neutrons in each fission react on that is available to produce next set of fission reactions must be exactly equal to one. If it is even slightly greater than one, the reaction rate will grow exponentially and out of control. Control of the reaction rate is accomplished by inserting the control rods made of cadmium. The cadmium has a very large cross section for absorbing neutrons and thus removing them from the fission process. Control rods are also used to shut down the reactor in case of an emergency. However, small fluctuations in the reaction rate occur much too rapidly so it is not possible to move control rods in and out to control the emitted neutrons instantly.

Fortunately, nature has provided us a solution to this difficulty. About 10% of the neutrons emitted in fission are delayed neutrons, produced not at the instant of fission but some what later, following radioactive decays of fission fragments. For example Rubidium ^{93}Rb, which might be produced in the fission ^{235}U, beta decays with a half life of 6s to strontium (^{93}Sr), which is occasionally (in about 1% of decays) produced in a very excited state that is unstable to neutron emission. The neutron appears to emerge with the 6s half life of the beta decay. This short delay time is enough to allow the control rods to be adjusted to maintain a constant reaction rate.

The process is as follows. A uranium nucleus ^{235}U captures a neutron and splits into two heavy fragments and two prompt neutrons. One of the fragments emits a delayed neutron. The three neutrons are slowed by passage through the moderator. Two of the neutrons cause new fissions and the third is captured by ^{238}U, eventually to form fissionable plutonium ^{239}Pu, which can be recovered from the fuel by chemical means. See the figure below.

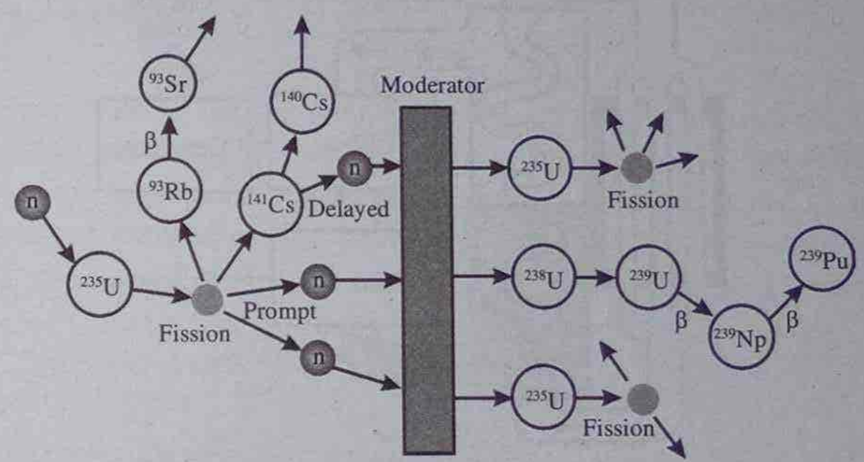


Figure 2.13: A typical sequence of processes in fission. A nucleus of ^{235}U absorbs a neutron and fissions; two prompt neutrons and one delayed neutron are emitted. Following moderation, two neutrons cause new fissions and the third is captured by ^{238}U, resulting finally in ^{239}Pu.

Fission reactors

Nuclear reactor is a source of intense heat. This heat is extracted to generate electric power. A fission reactor consists of mainly 3 parts.

1) Reactor core 2) Coolant system and 3) Shield

Reactor core

It contains fissionable material. Commonly used materials are ^{238}U enriched in ^{235}U and plutonium (^{239}Pu). The fissionable materials are made in the form rods about 1cm in diameter. To control the rate of fusion and also to shutdown the reactor in the case of emergency, cadmium control rods are inserted at the middle of the reactor core. Water circulates inside the reactor core which act as the moderator.

Coolant system

A large amount of heat is produced in the reactor. This has to be extracted from the reactor core. This can be accomplished pressurised water reactor as shown in figure below. The heat is extracted in two step process. Water circulates through the core under great pressure. This will prevent turning boiled water into steam. This hot water then allowed to pass through another water system kept normal pressure. This will deliver steam to the turbine. Since steam never enters reactor core, it does not become radioactive, thus there is no radioactive material in the vicinity of the turbine.

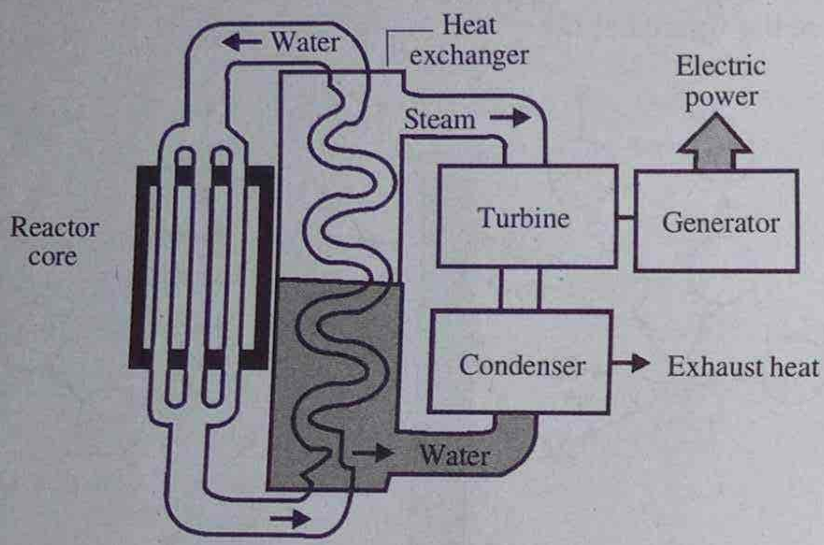


Figure 2.14: The components of a pressurized-water reactor

Shield

Various radiations emitted from the reactor which are injurious to the persons working near it. For their protection, the reactor core is surrounded by a radiation shield in the form of a thick concrete wall.

Almost all countries of the world use nuclear power plants. But there are some technological issues associated with it that are the subjects of active debate and investigation. Some of the radioactive isotopes among the fission fragments have very long half lives of the order of many years. The radio active waste from reactors must be stored in a manner that prevents leakage of radioactive material into the environment. Another botheration is about the resistance of reactors to natural disasters such as earth quakes, flooding or to acts of terrorism or sabotage.

There are two major disastrous and catastrophic incidents occurred in the history of nuclear reactors. In 1906, a graphite moderated power reactor of Chernobyl in the former U.S.S.R suffered a serious accident due to the disabling of the cooling system, which is designed to extract the intense heat generated in the reactor core. The resulting temperature rise ignited the graphite moderator and caused an explosion of the reactor containment vessel. It released radioactive fission products and exposing the inhabitants of the region to life threatening radiation loses. The water moderated reactors cannot suffer this kind of accident.

Another incident occurred in Japan. In 2011, the earth quake triggered tsunami that struck Japan's Fukushima reactor complex. The flooding of the reactor building caused the pumps supplying cooling water to fail. As a result, the reactor core over heated due to the radioactive decay of the fission products, and a partial meltdown of the fuel rods occurred. The release of radioactivity contaminated a wide region of the Japanese country side.

Fusion

Nuclear fusion is the phenomenon of two or more lighter nuclei to form to a single heavy nucleus. The mass of the product nucleus is less than the sum of the masses of the lighter nuclei fusing together. The difference in mass Δm results in the release of tremendous amount of energy according to Einstein's mass energy relation $\Delta E = \Delta mc^2$. In other words we can say that the energy released in this process is the excess binding energy of the heavy nucleus compared with the lighter nuclei. This process can release energy as long as the final nucleus is less massive than about $A = 60$.

For example consider the reaction

$$ {}_1^2H + {}_1^2H \rightarrow {}_1^3H + {}_1^1H $$

The Q-value of the reaction can be estimated to be 4 MeV. (See example 10) so this reaction liberates about 1MeV per nucleon, roughly the same as the fission reaction. This reaction can occur when a beam of deuterons is accelerated onto a deuterium target. In order to take place the fusion, the incident and the target deuterons must be close enough such that nuclear force between them must overcome the electrostatic repulsion between them. The Coulombian repulsion energy can be estimated to be 0.5 MeV (see example 11). This means that a deuteron with 0.5 MeV of kinetic energy can initiate nuclear fusion and it can release an energy (4 MeV + 0.5 MeV) of 4.5 MeV.

Doing this reaction in an accelerator, in which the beam currents are in the micro ampere range, would produce only a small amount of energy. To obtain significant amounts of energy it is necessary to work with much larger quantities of deuterium. For example the fusion energy from the deuterium in a litre of water (which contains 0.015% D_2O) would be equivalent to the chemical energy obtained from burning about 300 litres of gasoline.

The nuclear fusion can also take place under the conditions of high temperature and pressure. In this the deuterium gas is heated to a high temperature so that each atom of deuterium has about 0.25MeV of thermal kinetic energy. Then in the collision between two deuterium atoms, the total of 0.5 MeV of kinetic energy would be sufficient to overcome the Coulomb repulsion. Hence this is called thermo nuclear fusion.

The temperature corresponding to 0.25 MeV energy is

$$\frac{1}{2}kT = 0.25 \text{ MeV}$$

$$\frac{1}{2}kT = 0.25 \times 1.6 \times 10^{-13}$$

$$\therefore \quad T = \frac{2 \times 0.25 \times 1.6 \times 10^{-13}}{1.38 \times 10^{-23}} = 0.58 \times 10^{9}\,\text{K}.$$

It is very difficult to produce temperature of the order of 10^9 K. However this temperature would exist in the interior of stars where energy is produced due to nuclear fusion. Scientists and engineers are in the hot pursuit of producing nuclear fusion energy in the laboratories for generating electric power small scale production of fusion energy for a small scale of time is attained in laboratories. We are waiting for a sustained and controllable fusion reactor that can deliver constant output. It is not yet a reality. A great deal of effort is currently underway to resolve various difficulties in the development of a successful device. Nevertheless controlled fusion is regarded as the ultimate energy source because of the abundant availability of its main fuel: water.

Example 10

Calculate the Q – value of the reaction: ${}^2_1H + {}^2_1H \rightarrow {}^3_1H + {}^1_1H$

$$m({}^2H) = 2.014102\,u, \quad m({}^3H) = 3.016029\,u \text{ and } m({}^1H) = 1.007825\,u$$

Solution

$$Q = [2m(^2H) - m(^3H) - m(^1H)]c^2$$

$$Q = [2 \times 2.014102\,u - 3.016029\,u - 1.007825\,u]c^2$$

$$Q = 0.0435uc^2$$

or $$Q = 0.0435 \times 931.5 \text{ MeV} = 4.04 \text{ MeV}$$

Example 11

Calculate the Coulomb energy between two deuterons (^2_1H), when they just touch each other.

Solution

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 R}$$

$$U = \frac{2e2e}{4\pi\varepsilon_0 R}$$

Radius of deutron $r = r_0 A^{1/3} = 1.2(2)^{1/3}\,\text{fm}$

$$r = 1.512 \text{ fm}$$

$$\therefore \quad R = 2r = 2 \times 1.512\text{fm} = 3.024 \times 10^{-15}\text{m}$$

$$\therefore \quad U = \frac{9 \times 10^9 \times (2 \times 1.6 \times 10^{-19})^2}{3.024 \times 10^{-15}}\,\text{J}$$

$$U = \frac{9 \times 10^9 \times (2 \times 1.6 \times 10^{-19})^2}{3.024 \times 10^{-15} \times 1.6 \times 10^{-13}}\,\text{MeV}$$

$$U = \frac{9 \times 4 \times 1.6 \times 10^{-1}}{3.024} = 0.629 \text{ MeV}$$

Fusion processes in stars

Stars are composed of ordinary hydrogen rather than deuterium. Nuclear fusion to take place, it is necessary to produce deuterium. This is done according to the reaction.

$${}_1^1H + {}_1^1H \rightarrow {}_1^2H + e^+ + \upsilon$$

i.e., proton combines with proton giving one deuterium, a positron and a neutrino. This is analogues to β^+ decay in which one proton is converted to a neutron, a positron and a neutrino.

Next reaction that occurs is the deuterium combines with another proton resulting in ${}_2^3He$ and a gamma ray according to:

$${}_1^2H + {}_1^1H \rightarrow {}_2^3He + \gamma$$

For the next reaction to occur we need one more ${}_2^3He$. For this first two reactions must occur twice. Finally ${}_2^3He$ combines with ${}_2^3He$ forming ${}_2^4He$ and two protons. Accordingly we have

$${}_2^3He + {}_2^3He \rightarrow {}_2^4He + 2{}_1^1H$$

In the overall processes, four protons are involved. So the net process can be written as

$$4{}_1^1H \rightarrow {}_2^4He + 2e^+ + 2\upsilon + 2\gamma$$

To calculate the Q-value, all nuclear masses must be converted into atomic masses. For this add $4m_e$ on both sides. On the L.H.S four protons added with 4 electrons giving four neutral hydrogen atom. On the R.H.S two electrons added with ${}_3^4He$ nuclei giving a neutral helium atom. The remaining two electrons combines with two positrons giving additional gamma rays.

$$e^- + e^+ \rightarrow 2\gamma$$

This will give additional energy from the reaction.

According to definition, the Q-value is

$$Q = (m_i - m_f)c^2 = [4m({}_1^1H) - m({}_2^4He)]c^2$$

Substituting the values of atomic masses of hydrogen and helium, we get

$$Q = [4 \times 1.007825\,u - 4.002603\,u]c^2$$

$$Q = 0.028697uc^2$$

Using $1u = 931.5\,\frac{MeV}{c^2}$

$$Q = 0.028697 \times 931.5\,\text{Mev}$$

$$Q = 26.73\,\text{MeV}$$

This shows that each fusion liberates about 26.73 MeV of energy.

Now we will see at what rate these reactions must occur in the Sun? Earth receives 1400 joules of energy in every second from the sun by an area 1m^2. This is called the solar constant. So the total energy received by overall area $4\pi r^2$ is $4\pi r^2 \times 1400$. This comes about 4×10^{26} joules (where $r = 1.5 \times 10^{11}\,\text{m}$, the distance between the sun and the earth). In other words the power output of the sun is 4×10^{26} W. Converting this into MeV, we get $\frac{4 \times 10^{26}}{1.6 \times 1^{-13}} \frac{\text{MeV}}{\text{s}}$. This is about $2.5 \times 10^{39}\,\text{MeV/s}$. Each fusion reaction liberates about 26 MeV and thus there must be $\frac{2.5 \times 10^{39}}{26} \simeq 10^{38}$ fusion reactions per second. This will consume about 4×10^{38} protons per second. The sun's mass is about $2 \times 10^{30}\,\text{kg}$ which corresponds to about 10^{57} protons. This is enough to burn for the next few billion years.

The sequence of reactions described above is called proton-proton cycle. However it is probably not the primary source of fusion energy in many stars. This is because the first reaction which is similar to β^+ decay takes place only on a very long time scale and is therefore unlikely to occur. The carbon cycle is believed to be the more likely sequence of reactions.

Carbon cycle reactions

The cycle proceeds as follows:

$$^{1}_{1}H + ^{12}_{6}C \rightarrow ^{13}_{7}N + \gamma$$

$$^{13}_{7}N \rightarrow ^{13}_{6}C + ^{0}_{1}e^{+} + \upsilon$$

$$^{1}_{1}H + ^{13}_{6}C \rightarrow ^{14}_{7}N + \gamma$$

$$^{1}_{1}H + ^{14}_{7}N \rightarrow ^{15}_{8}O + \gamma$$

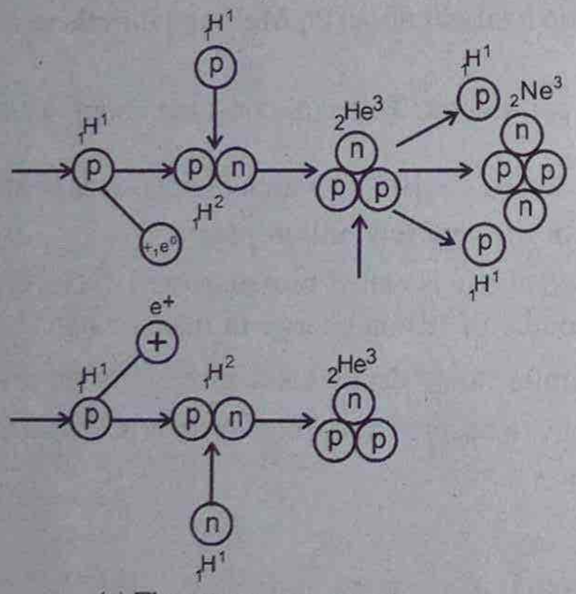
$$^{15}_{8}O \rightarrow ^{15}_{7}N + ^{0}_{1}e^{+} + \upsilon$$

$$^{1}_{1}H + ^{15}_{7}N \rightarrow ^{12}_{6}C + ^{4}_{2}He$$

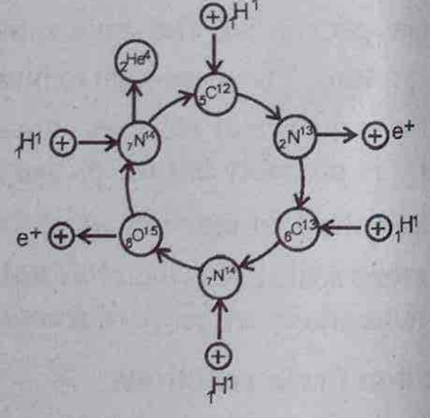
The overall result is the combination of four hydrogen nuclei to form a helium nucleus plus two positrons in the same way as in the proton-proton cycle. This reaction neither consumes nor produce any ${}^{12}_{6}C$. ${}^{12}_{6}C$ plays the role as a catalyst. The value is the same as before (26.73MeV). (See example 12)

The Coulomb repulsion between ${}^{1}_{1}H$ and ${}^{12}_{6}C$ is larger than the Coulomb repulsion between ${}^{1}_{1}H$ and ${}^{1}_{1}H$, so more thermal energy and a correspondingly higher temperature are needed for the carbon cycle. The carbon cycle is important at a temperature about 20×10^6 K, while the Sun's interior temperature is only about 15×10^6 K.

The symbolic diagram of proton-proton cycle and carbon cycle are given below.



(a) The proton-proton cycle



(b) The carbon cycle

Figure 2.15

Fusion reactors

We know that both the reactors of nuclear fission and fusion yield much more energy than other conventional nuclear reactions. They can, therefore, be put to use to generate energy. But fission based reactors are already being used to generate electricity, fusion based controlled reactors have so far not become a practical possibility. They are, however, reactors of the future because of their many advantages over the fission reactions. Some of these advantages are.

1. The basic fuel for nuclear fusion - hydrogen and deuterium is available in unlimited quantities in sea water. The most important thing is that deuterium is present in sea water is cheap to extract. The concentration of deuterium in sea water is only $33 g/m^3$ and this add upto a total of about 10^{15} tonnes of deuterium in the worlds ocean. The deuterium in a gallon of sea water can yield as much energy through fusion as 600 gallons of gasoline can through combustian.
2. The energy yield per unit mass of the fuel is much more in fusion than in fission.
3. There are no radioactive pollutants arising as a by product of the fusion reactions. On the other hand, fission results in some radio active isotopes that are quite long lived and, therefore, almost a permanent source of radioactive pollution.

Because of these advantages, scientists are keenly exploring possibility of a fusion reactor. It is clear that fusion holds the promise of meeting the energy requirements challenge of the future. Hence fusion reactors are indeed the reactors of future.

There are mainly two types of nuclear fusion reactions. They are deutron-deutron (D-D) reaction and deutron - triton (D-T) reaction.

D-D reactions

In this two deuterons fuse to form a triton and a proton and energy. ${}_1^2H + {}_1^2H \rightarrow {}_1^3H + {}_1^1H + 4MeV$ or two deutrons combine to form a ${}_2^3He$ nucleus and a neutron and energy.

${}_1^2H + {}_1^2H \rightarrow {}_2^3He + {}_0^1n + 3.3MeV$. Both D-D reactions have about equal probabilities.

D-T reaction

In this a deutron and a triton fuse to form a helium nucleus and a neutron with the release of energy.

$${}_1^3H + {}_1^2H \rightarrow {}_2^4He + {}_0^1n + 17.6MeV$$

D-T reaction releases much more energy than D-D reaction. But the availability of tritium is difficult and sea water contains too little tritium to be extracted economically. However it can be produced by the neutron bombardment of the two isotopes of natural lithium.

$${}_3^6Li + {}_0^1n \rightarrow {}_1^3H + {}_2^4He$$

$${}_3^7Li + {}_0^1n \rightarrow {}_1^3H + {}_0^1n$$

Basic requirements of fusion reactor

1. During fusion processes very strong coulomb repulsion exist between the two fusing nuclei. Hence very large kinetic energies, of the order of 1MeV, are needed so that the nuclei get close enough for the attractive nuclear forces to become effective and cause fusion. These large kinetic energies can be obtained in an accelerator.
2. To obtain energy from fusion, the particles must be heated to a temperature great enough for the fusion reaction to occur as the result of random thermal collisions. Because of thermal energy some particles can overcome the coulomb barrier. A temperature T corresponding to $kT = 10keV$ is required for a reasonable number of fusion reactions to occur. The temperature corresponding $kT = 10eV$ is of the order of $10^8 K$. Such temperatures occur in the interiors of stars. At these temperatures, a gas consists of positive ions and negative electrons called plasma. The main problem lies in confining the plasma long enough for the reactions to take place. The enormous gravitational field of the sun confines plasma in the interior of the sun.
3. The energy required to heat a plasma is proportional to the density of its ions n. The fusion rate is proportional to square of density n^2. The output energy is proportional to $n^2\tau$, where τ is the confinement time. If the output energy is to exceed the input energy, we must have

$$C_1 n^2 \tau > C_2 n$$

where C_1 and C_2 are constants. Lawson derived the following relation between density and confinement time, known as Lawson's criterion

$$n\tau > 10^{20}\ sm^{-3}$$

The product $n\tau$ is called the confinement quality parameter. When the above criterion is met the energy released by a fusion reactor will just equal to the energy input, hence called break even condition. For this to happen the thermal energy of the ions is great enough. In the case of D-T plasma this is about 10keV. For a feasible reactor much more energy must be released.

Confinement methods

We have found that for nuclear fusion to take place plasma must be confined.

Magnetic Confinement

A magnetic field can confine a plasma because the charged particles spiral around the magnetic field lines. Figure below shows a toroidal magnetic confinement. There are two contributions to the magnetic field: one is along the toroid axis another is around the axis. The combination of these two gives a helical field along the toroid axis, and the charged particles are confined as they spiral about the field lines. This type of device is called a tokamak (from the Russian acronym for toroidal magnetic chamber. A current passed through the plasma serves both ot heat the plasma and to create one of the magnetic components.

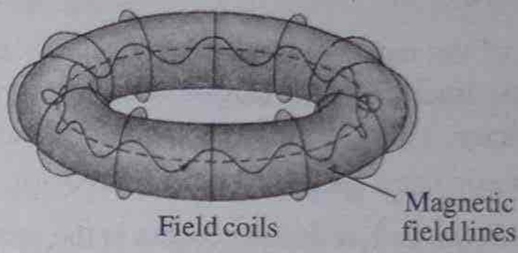


Figure 2.16: The toroidal geometry of plasma confinement. The ionized atoms circulate around the ring, trapped by the magnetic field lines. The coils produce a magnetic field along the axis of the toroid (dashed line). Another field component is produced by a current along the axis in the plasma. The two components of the field produce the helical field lines shown.

The Tokamak Fusion Test Reactor (TFTR) at Princeton University, which operated from 1982 to 1997 achieved an ion temperature of $5.1\times10^{8}\,K$ and a fusion power level of 10.7 MW. This came very close to reaching Lawson's criterion with a plasma density of $n = 10^{20}$ particles per m^3 and a confinement time of $\tau = 0.2s$

Inertial confinement

In this confinement process a small pellet of D.T fuel is compressed to high densities for very short confinement times by striking the pellet from many directions with intense laser beams. This will first vapourise the pellet and convert it to a plasma, and then heat and compress it to the point at which fusion can occur. The laser pulses last only about 1 ns. Thus according to Lawson's criteria the density must exceed 10^{29} particles per m^3. ($n\tau \geq 10^{20}$ and $\tau = 10^{-9}s$). However, because of inefficiencies of the lasers and other losses a self sustaining laser fusion reactor must exceed this minimum by about 2-3 orders of magnitude. In 2010 the first inertial confined system was operated at the Lawrence Livermore National Laboratory (LLNL). It is designed so that a 2mm diameter pellet of D.T is struck simultaneously by 192 laser beams that deliver an energy of 1MJ in a pulse lasting a few nanosecond, which is expected to compress the pelllet to a central density that is 100 times that of lead. In June 13, 2018 an experimental campaign conducted at LLNL, achieved a total fusion energy output of 54kJ.

The main difficulty in the D-T fusion reaction is the recovery of the energy from the neutrons and its conversion into electrical power. One possibility for fusion reactor design is that the reaction area is surrounded by lithium, which captures neutrons by the reaction

$$^{6}_{3}\mathrm{Li} + \mathrm{n} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{3}_{1}\mathrm{H}$$

The kinetic energies of the reaction products are rapidly dissipated as heat, and the thermal energy of the liquid lithium can be used to convert water to steam in order to generate electricity. This reaction has the added advantage of producing tritium ($^{3}\mathrm{H}$) which is needed as a fuel for the fusion reactor.

Another difficulty with the D-T reaction process is the production of large number of neutrons. Although fusion reactors will not produce the radio active wastes that fission reactors do, the neutrons make the immediate area surrounding the reactor radioactive. This results in the damage of reactor vessel. Here once again the lithium is helpful, because a 1m thickness of lithium should be sufficient to stop essentially all of the neutrons.

Fusion energy is the subject of vigorous research all over the world. All technological problems are being attacked with a variety of methods. Researchers are hopeful that solutions can be found out soon, so that fusion can help to supply our electrical power requirements.

Example 12

Calculate the Q-value of the carbon cycle reaction process:

$$4\,^{1}_{1}\mathrm{H} + {}^{12}_{6}\mathrm{C} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{12}_{6}\mathrm{C} + 2e^{+} + 2\upsilon + 3\gamma$$

$$m(^{1}_{1}\mathrm{H}) = 1.007825\,\mathrm{u}, \quad m(^{4}\mathrm{He}) = 4.002603\,\mathrm{u}$$

Solution

$$Q = [4m(^{1}_{1}\mathrm{H}) - m(^{4}_{2}\mathrm{He})]c^2$$

$$Q = (4 \times 1.007825\mathrm{u} - 4.002603\mathrm{u})c^2$$

$$Q = 0.028697\mathrm{u}c^2$$

$$Q = 0.028697 \times 931.5\mathrm{MeV}$$

$$Q = 26.73\ \mathrm{MeV}.$$

This is same as the Q-value of proton-proton cycle.

Example 13

Show that the D-T fusion reaction releases 17.6 MeV of energy.

$$m(^2H) = 2.014102u,\ m(^3H) = 3.016049u,$$

$$m(^4He) = 4.002603u \text{ and } m_n = 1.008665u$$

Solution

D-T Fusion reaction is

$$^2_1H + ^3_1H \rightarrow ^4_2He + n$$

The Q-value is:

$$Q = [m(^2H) + m(^3H) - m(^4He) - m_n]c^2$$

$$Q = [2.014102u + 3.016049u - 4.002603u - 1.008665u]c^2$$

$$Q = 0.018883uc^2$$

$$Q = 0.018883 \times 931.5\text{MeV}$$

$$Q = 17.5895 \text{ MeV}$$

$$Q = 17.6 \text{ MeV}$$

Example 14

If a tokamak fusion reactor were able to achieve a confinement time of 0.60s, what minimum particle density is required.

Solution

a) $\tau = 0.60$ s

We have $n\tau \geq 10^{20}$

$$\therefore \quad n \geq \frac{10^{20}}{0.6} = 1.67 \times 10^{20}\ m^{-3}$$

$$n = 1.67 \times 10^{21}\ m^{-3}$$

Applications of nuclear physics

So far we found only two main applications one is the estimation of the age of the rocks and woods, the other one is the production of electric power. Besides these there are somany applications of nuclear physics. Here we discuss only three among them.

Neutron activation analysis

Neutron Activation Analysis (NAA) is a method to determine the concentrations of elements in a given substance.

The technique involves irradiating the material to be analysed with a neutron beam. The irradiation will result in the absorption of neutrons by various nuclei of the elements in the material yielding many radio isotopes. The energies and intensities of the gamma-ray emissions of the radio isotopes produced are then measured with a sensitive semiconductor detector and the elements in the sample as well as their concentrations are determined by a correlating the gamma ray energy lines and their intensities with standards irradiated simultaneously with the unknown material.

For example ^{59}Co is placed in a flux of neutrons, neutron absorption results in the production of radioactive isotope ^{60}Co. ^{60}Co undergoes beta decay with a half life of 5.27 years. Following the beta decay, nickel ^{60}Ni emits two gamma -rays of energies 1.17 MeV and 1.33 MeV and of equal intensity. If an unknown material is placed in a flux of neutrons and observe the spectrum. If the spectrum contains 1.17 MeV and 1.33 MeV, it is certain the unknown material contains cobalt.

NAA has high level of sensitivity for the trace elements (< 0.01%) and accuracy it is commonly employed in many fields of study including the biological sciences, agriculture, industry food and nutrition. It is also commonly used in forensic science because of characteristic signatures that the gamma spectra will display after neutron irradium of samples.

There are two main drawbacks to the use of NAA. Even though the technique is essentially non destructive, the irradiated sample will remain radioactive for many years after initial analysis. This requires careful handling and disposal of this radioactive material.

An example of a neutron activation analysis study of a sample of hair is shown in figure.

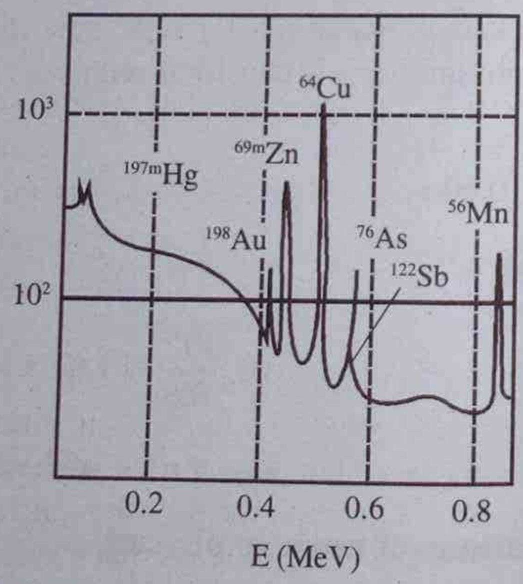


Figure 2.17: Gamma-ray spectrum following neutron activation of a sample of human hair. The sample shows traces of mercury, gold, zinc, copper, arsenic, antimony, and manganese.

Medical radiation physics

One of the most important applications of nuclear physics is in the medical field for diagnostic and therapeutic purposes. The use of X-rays for producing images for medical diagnosis is known to everybody. Though X-rays show distinct and detailed images of bones, they are unable to take photographs of soft issues inside the human body. To take the photographs of soft tissues radioactive isotopes are introduced into the human body in chemical forms that have an affinity for certain organs, such as bone or thyroid gland. A sensitive detector (gamma ray camera) can observe the radiations from the isotopes that are concentrated in the organ and can produce an image that shows how the activity is distributed in the patient. These detectors are capable of determining where each gamma -ray photon originates in the patient.

Another technique used in medical diagnosis is called positron emission tomography (P E T). This reveals a wealth of information. Positron emission tomography is an imaging technique that uses radioactive substances to visualise and identifying changes at the cellular level.

PET uses small amounts of radioactive materials called radio tracers, a special camera and a computer to evaluate organ and tissue functions. By identifying changes at the cellular level, PET may detect the early onset of disease before other imaging tests can.

Functioning of PET

Radio tracers (Positron emitting isotope) are either swallowed, inhale or injected into the vein depending on what part of the body is being examined. Certain organs and tissues then absorb the tracer. The tracer will collect in areas of higher chemical activity which is helpful. Because certain tissues of the body and certain diseases have a higher level of activity. These areas of disease will show up as bright spot in the PET scan.

Usually radio tracers used are ^{15}O ($T_{1/2}$ = 2 minutes) ^{13}N ($T_{1/2}$= 10 minutes) ^{11}C ($T_{1/2}$= 20 minutes) and ^{18}F ($T_{1/2}$=110 minutes). Since these isotopes have short half-lives this must be produced at the site of the diagnostic facility by a cyclotron. When a positron emitter decays, the positron quickly annihilates with an electron and produces two 511 keV gamma -rays that travel in opposite directions. By surrounding the patient with a ring of detectors, it is possible to determine exactly where the decay occurred and from a large number of such events the physician can produce an image that reconstructs the distribution of radioisotope in the patient. One advantage of PET is that it can produce a dynamic image-changes in the patient during the measuring time can be observed.

In radiation therapy uses the effect of radiations in destroying unwanted tissue in the body such as a cancerous growth or an overactive thyroid gland. When radiation is passed through matter it ionises the atoms. The ionised atoms then participate in chemical reactions that lead to their incorporations into molecules and subsequent alteration of their biological function, possibly the destruction of a cell or the modification of its genetic material. For example, an overactive thyroid gland is often treated by giving the patient radioactive iodine (^{131}I), which collects in the thyroid. The beta emissions from the isotope damage the thyroid cells and ultimately lead to their destruction. Certain cancers are treated by implanting needles or wires containing radium or other radioactive substances. The decays of these radioisotopes cause localised damage to the cancerous cells.

Other cancers can be treated using beams of particles that cause nuclear reactions within the body at the location of the tumor. Pions and neutrons are used for this purpose. The absorption of a pion or a neutron by a nucleus causes a nuclear reaction, and the subsequent emission of particles or the reaction products again causes local damage that is concentrated at the site of the tumor, inflicting maximum damage to the tumor and minimum damage to the surrounding healthy tissue.

Alpha scattering application

Radioactive sources emitting alpha particles have been used in a variety of ways. Alpha particles emit at a fixed rate in any location. This property is made use of in most of the applications.

1) Alpha particle emission can be used to produce electric power. The energy of the alpha particle absorbed can be converted into electric power through thermo electric power. The power is of the order of one watt per gram material (see example 15). These powers are sufficient to operate many devices such as cardiac pace makers, voyager space craft which photographed Jupiter, Saturn and Uranus etc.

2) Alpha particles are used in ionisation type smoke detectors. Alpha particles from the decay of Americium (^{241}Am) are scattered by the ionised atoms that comes from smoke. When the smoke detector senses a decrease in the rate at which alphas are counted, the alarm is triggered.

3) In Rutherford scattering alpha particle emission enables us to calculate the change in kinetic energy of the scattered alpha particles that scatters through 180° using

$$\Delta K = K \frac{4m/M}{\left(1+\frac{m}{M}\right)^2}$$

This can be used for material analysis (See example 16). The survey or space craft that landed on the Moon and the Viking landed on Mars carried Rutherford back scattering experiments to analyse the chemical composition of the surfaces of those planets.

Example 15

A radioactive source is to be used to produce electric power from the alpha decay of plutonium $^{238}Pu(T_{1/2} = 88y)$. What is the Q-value for the decay? (b) Assuming 100% conversion efficiency, how much power could be obtained from the decay of 1.0g of ^{238}Pu. $m(Pu) = 238.049560\,u \cdot m(U) = 234.040952$ and $m(He) = 4.002603u$

Solution

$$^{230}Pu \rightarrow {}^{234}U + \alpha$$

a) $Q = [m(Pu) - m(U) - m(^4He)]c^2$

$$Q = [238.049560u - 234.040952u - 4.002603u]c^2$$

$$Q = 0.06005u\,c^2$$

$$Q = 0.06005 \times 931.5 \text{ MeV}$$

$$Q = 5.593 \text{ MeV}$$

b) $1.0g = \frac{1}{238}\text{mole} = \frac{1}{238} \times 6.22 \times 10^{23}$ atoms

$$= 2.53 \times 10^{21} \text{ atoms.}$$

$$\therefore \text{Activity, } a = \lambda N = \frac{0.693}{T_{1/2}} N$$

$$a = \frac{0.693 \times 2.53 \times 10^{21}}{88 \times 365 \times 24 \times 60 \times 60}$$

$$a = 6.32 \times 10^{11}\ s^{-1}$$

$$\therefore \text{Power, } P = aQ = = 6.32 \times 10^{11} \times 5.593 \times 1.6 \times 10^{-13} \frac{J}{s}$$

$$P = 0.566W$$

Example 16

An alpha particle of mass m makes an elastic head on collision with an atom of mass M at rest. Show that the loss in kinetic energy

$$\Delta K = K\left[\frac{4m/M}{\left(1+\frac{m}{M}\right)^2}\right]$$

Solution

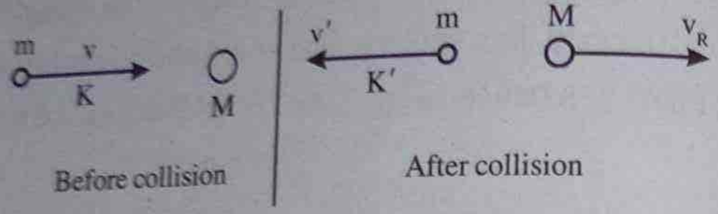


Figure 2.18

Momentum conservation gives

$$mv = -mv' + Mv_R \qquad \text{..... (1)}$$

Energy conservation gives

$$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + \frac{1}{2}Mv_R^2 \qquad \text{..... (2)}$$

or
$$K = K' + \frac{1}{2}Mv_R^2$$

From eq. (1), we get

$$m(v+v') = mv_R \qquad \text{..... (3)}$$

From eq. 2, we have

$$m(v^2 - v'^2) = Mv_R^2 \qquad \text{..... (4)}$$

$\frac{Eq4}{Eq3}$ gives
$$v - v' = v_R \qquad \text{..... (5)}$$

solving eqs 3 and 5 we get

$$v' = v\,\frac{(1-m/M)}{(1+m/M)}$$

$\therefore$
$$\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv'^2$$

$$\Delta K = \frac{1}{2}mv^2\left(1-\frac{v'^2}{v^2}\right)$$

$$\Delta K = \frac{1}{2}mv^2\left[\frac{\left(1-\frac{m}{M}\right)^2}{\left(1+\frac{m}{M}\right)^2}\right] = \frac{4m/M}{\left(1+\frac{m}{M}\right)^2}$$

Synthetic elements

Synthetic elements are chemical elements that do not occur naturally on earth. There are totally 24 synthetic elements that have been created. They are created in nuclear reactors, particle accelerators or in the explosion of atom bombs. Since they are artificially (or man made) created they are called as synthetic elements.

The mechanism for the creation of synthetic elements is to force protons or neutrons onto the nucleus of an element with atomic number less than 95. All synthetic elements are unstable but they decay at widely varying rate. Their half-lives range from 15.6 million years to a few hundred micro seconds. The names of the synthetic elements are given, in the increasing order of atomic number, below in the table 2.1

Table 2.1: Synthetic elements

No.	Atomic number	Name of the element	No	Atomic number	Name of the element
1	43	Technetium	13	106	Seaborgium
2	61	Promethium	14	107	Bohrium
3	85	Astatine	15	108	Hassium
4	87	Francium	16	109	Meitnerium
5	95	Americium	17	110	Darmstadtium
6	99	Einsteinium	18	111	Roentgenium
7	100	Fermium	19	112	Copernicium
8	101	Mendelevium	20	113	Nihonium
9	102	Nobelium	21	114	Flerovium
10	103	Lawrencium	22	115	Moscovium
11	104	Rutherfordium	23	116	Livermorium
12	105	Dubnium	24	118	Oganesson

The known atoms beyond uranium ($Z = 92$) are all radioactive, with half-lives compared with the age of the earth. Therefore they are not present on earth, but they can be produced in the laboratory. The chemical elements with atomic number greater than 92 are called transuranium elements. The atomic number starting at 93 and ending at 118 are transuranium elements. Sometimes chemical elements with atomic number greater than 103 are called transactinides.

The process of neutron capture followed by beta decay is used to produce elements with $Z = 93$ to $Z = 100$. Beyond this accelerators, reactors, atom bomb (mini) explosions are required. All elements upto $Z = 118$ have been observed and named.

Elements with atomic number greater 99 have no use other than scientific research. Since they have extremely short half-lives and thus never been produced in large quantities.

IMPORTANT FORMULAE

1. Reaction probability:

a) $R = \dfrac{\sigma N I_0}{S}$ b) $Q = \sigma\phi\dfrac{mN_A}{M}$

2. Production of activity in reaction:

$$a(t) = \lambda N = R(1 - e^{-\lambda t})$$

Where $\lambda = \dfrac{0.693}{T_{1/2}}$

3. Fraction of the maximum possible activity:

$$f = \frac{a(t)}{R} = 1 - e^{-\lambda t}$$

4. Reaction Q-value:

$$x + X \rightarrow y + Y$$

$$Q = (m_i - m_f)c^2$$

$$Q = [(m(x) + m(X) - m(y) - m(Y)]c^2$$

$$Q = K_y + K_Y - K_x$$

5. Expression for threshold kinetic energy $K_{Th} = -Q\left[1 + \dfrac{m(x)}{m(X)}\right]$

6. Spontaneous nuclear fission decay

   Example $\quad {}^{254}_{98}Cf \rightarrow {}^{140}_{54}Xe + {}^{110}_{44}Ru + 4n$

7. Induced nuclear fission decay

   Example $\quad n + {}^{235}_{92}U \rightarrow {}^{93}_{37}Rb + {}^{141}_{55}Cs + 2n$

8. Expression for critical energy

$$E_{cri} = 0.89\,A^{2/3} - 0.02\,\frac{Z(Z-1)}{A^{1/3}}\ \text{MeV}$$

9. D-T fusion reaction

   ${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + n$

10. Lawsons criterion

   $n\tau \geq 10^{20}\,m^{-3}s$

11. Change in kinetic energy in alpha back scattering:

$$\Delta K = K\left[\frac{4\dfrac{m}{M}}{\left(1+\dfrac{m}{M}\right)^2}\right]$$

UNIVERSITY MODEL QUESTIONS

Section A

(*Answer questions in about* **two** *or* **three** *sentences*)

Short answer type questions

1. How the study of nuclear reaction is more important than the study of nuclear decays?
2. What is a nuclear reaction?
3. A nuclear reaction doesn't involve beta decay. Justify?
4. What are the observations that we make in a nuclear reaction experiment?
5. How nuclear reaction experiment is conducted in laboratories?
6. What is reaction cross section? What is its unit?
7. How do we produce radioisotopes in nuclear reactions?
8. Write down an expression for the activity of a radioisotope produced in nuclear reaction and explain the symbols?
9. Write down an expression for the Q-value of a nuclear reaction and explain the symbols?
10. What is meant by threshold kinetic of a projectile involved in nuclear reaction?

11. Write down an expression for threshold kinetic energy and explain the symbols?
12. What is nuclear reaction? Give an example?
13. What is meant by critical energy of nuclear fission?
14. Write down an expression for critical energy of nuclear fission and explain the symbols used?
15. What is induced nuclear fission?
16. Ordinary uranium is not fissionable. Then what are main things required to make it fissionable?
17. What is the use of cadmium rods in nuclear reactors?
18. What is the function of moderators in nuclear reactions?
19. Write down two names of moderators?
20. Why ^{238}U is not suitable for nuclear fission?
21. Which are the main parts of a nuclear reactor?
22. What is the function of shield in a nuclear reactor?
23. What was the cause of Chernobyl accident of nuclear reactor?
24. What is nuclear fusion? Where does the energy come from?
25. What are the conditions under which nuclear fusion occurs?
26. "Controlled nuclear fusion is the ultimate energy source of mankind", Justify?
27. What is tokamak?
28. What is Lawson's criteria?
29. What is neutron activation analysis?
30. Give any three uses of neutron activation analysis?
31. What does PET stand for?
32. What are radio tracers? Give two examples?
33. Write down two uses of medical radiation physics?
34. How does alpha particle emission use to produce electric power?
35. What are synthetic elements?
36. Give three names of synthetic element?
37. What are the properties of synthetic elements?
38. What are transuranic elements?
39. What are transactinoids?
40. How does synthetic element whose mass number 99 is prepared in laboratory?

Section B

(*Answer questions in a* **paragraph** *of about half a page to one page*)

Paragraph / Problem type questions

1. How will you measure the outgoing particle energy in a nuclear reaction experiment?
2. How will you evaluate the reaction probability in a nuclear reaction experiment?
3. Derive an expression for reaction cross section?
4. Derive an expression for activity of a radioactive isotope in the process of its production?
5. What are exothermic and endothermic reaction?
6. Explain what is meant by spontaneous nuclear fission?
7. Explain what is meant by artificial nuclear fission?
8. Explain the mechanism behind nuclear fission?
9. The critical energy for the ^{239}U compound nucleus is 7.3 MeV. Justify?
10. What is meant by enrichment of natural uranium?
11. What is meant by moderation in reference to nuclear fission?
12. How do we control nuclear fission processes?
13. What are the main problems associated with nuclear reactors?
14. What are the essential components of reactor core?
15. What is the function of a coolant system in a nuclear reactor?
16. Explain the fusion process in stars?
17. Calculate the number of fusion reactions that occurs in the sun in one second?
18. Explain the proton-proton cycle reaction with an example?
19. Explain the carbon-nitrogen cycle reaction with an example?
20. Explain D-D reaction with an example?
21. Explain D-T reaction with an example?
22. Write down three basic requirements of fusion reactor?
23. What are the advantages of nuclear fusion reactors over nuclear fission reactors?
24. Explain the magnetic confinement of plasma?
25. Explain the inertial confinement of plasma?
26. Write a short note on the future energy source of reactors?
27. Explain the technique of neutron activation analysis?
28. Explain the technique of taking photograph of soft tissues inside a human body?
29. How does positron emission tomography function?
30. How does radiation effect on cancerous tumor?
31. How does an overactive thyroid is treated with radiation?
32. How alpha particle emission is used in smoke detectors?

33. Write brief note on synthetic elements?

34. Fill up the missing particle in the reactions given below.

a) $^{4}He + {}^{14}N \rightarrow {}^{17}O + -...$ c) $^{27}Al + {}^{4}He \rightarrow n + ...$

b) $^{9}Be + {}^{4}He \rightarrow {}^{12}C + -...$ d) $^{12}C + ... \rightarrow {}^{13}N + n$

[a) $^{1}_{1}H$ b) $^{1}_{0}n$ c) $^{30}_{15}P$ d) $^{2}_{1}H$]

35. In a certain nuclear reaction, outgoing protons are observed with energies 16.2 MeV, 14.8 MeV, 11.6 MeV, 8.9 MeV and 6.7 MeV. No energies higher than 16.2 MeV are observed. Construct a level scheme of the product nuclear?

36. For a certain incident proton energy the reaction $P + {}^{56}Fe \rightarrow n + {}^{56}Co$ has a cross section of 0.40b. If we bombard a target in the form of a 1.00cm^2, 1.00 μm thick iron foil with a beam of protons equivalent to a current of 3.00 μA, and if the beam is spread uniformly over the entire surface of the target. At what rate the neutrons are produced. Density of iron 7.90 gcm^{-3} (6.37×10^{7} particles per second)

37. In order to determine the cross section for neutron capture, you are irradiating a thin gold foil in the form of a circular disc of diameter 3.00mm and thickness 1.81 μm; with neutrons to produce the reaction $n + {}^{197}Au \rightarrow {}^{195}Au + \gamma$. By observing the outgoing gamma-ray photons in a detector, you determined that the gold decays at a rate of 5.37×10^{6} per second. From an independent measurement, you have determined the neutron flux to be 7.25×10^{10} neutrons cm^{-2}s^{-1}. What value you deduce for the cross section for this reaction? Density of gold 19.30 gcm^{-3} [9.82b]

38. The radioisotope $^{15}O(T_{1/2} = 122s)$ is used to measure respiratory function. Patients inhale the gas by irradiating nitrogen gas with deuterons (^{2}H). Consider a cubical cell measuring 1.24 cm on each edge which holds nitrogen gas at a pressure of 2.25 atm and a temperature of 293K. One face of the cube is uniformly irradiated with a deuteron beam having current of 2.05A. At the chosen deutron energy, the reaction cross section is 0.21b. a) At what rate is ^{15}O produced in the cell? b) After an irradiation lasting for 60.0s, what is the activity of ^{15}O in the cell? [a) $4.03 \times 10^{8}s^{-1}$ b) 3.14m Ci]

39. 30 milligrams of gold are exposed to a neutron flux of 3.0×10^{12} neutrons cm^{-2}s^{-1} for 1.0 minute. The neutron capture cross section of gold is 99b. Find the resultant activity of ^{198}Au? (130μCi)

40. Find the Q-value of the reaction

$$p + {}^{55}Mn \rightarrow {}^{54}Fe + 2n$$

$$m({}^{1}H) = 1.007825u, \ m(Mn) = 54.938045u,$$

$$m(Fe) = 53.939614u, \ m_n = 1.008665u$$

(−10.313MeV)

41. Find the Q-value of the reaction

$$^{7}\text{Li} + {}^{2}\text{H} \rightarrow {}^{8}\text{Be} + \text{n}$$

$$\text{m(Li)} = 7.016005\text{u}, \ \text{m}(^{2}\text{H}) = 2.014102\text{u},$$

$$\text{m(Be)} = 8.005305\text{u}, \text{and } \text{m}_{\text{n}} = 1.008665\text{u}$$ (15.032 MeV)

42. Calculate the threshold kinetic energy for the reaction $p + {}^{3}H \rightarrow {}^{2}H + {}^{2}H$
   a) If protons are incident on ^{3}H at rest
   b) If ^{3}H (tritons) are incident on protons at rest. $m(^{1}H) = 1.007825u$, $m(^{3}H) = 3.016049u$ and $m(^{2}H) = 2.014102u$

[a) 5.381 MeV b) 16.10 MeV]

43. Find the Q-value and therefore the energy released in the fission reaction $^{235}U + n \rightarrow {}^{93}Rb + {}^{141}Cs + 2n$. Use $m(U) = 235.043930u$, $m_n = 1.008665u$, $m(Rb) = 92.922402u$ and $m(Cs) = 140.9200464$. (179.94 MeV)
44. Find the energy difference between ^{235}U and ^{236}U. We can regard this as the excitation energy of ^{236}U. Repeat this for $^{238}U + n$ and ^{239}U. Comparing the two results and justify ^{235}U is fissionable with slow neutrons and ^{238}U requires fast neutrons.
45. In the D-T reaction, the kinetic energies of ^{2}H and ^{3}H are small compared with typical nuclear binding energies. Find the kinetic energy of the emitted neutron. Q-value is 17.6 MeV and $\frac{m_n}{m_{He}} = 0.25$ (14.1 MeV)

Section C

(*Answer questions in about* **one** *or* **two** *pages*)

Long answer type questions (Essays)

1. Derive an expression for threshold kinetic energy of nuclear reaction?
2. Explain the construction and working of a nuclear fission reactor?

Hints to problems

34. Use conservation number of protons and neutrons on both sides of the reaction.
   a) $^{4}_{2}He + {}^{14}_{7}N \rightarrow {}^{17}_{8}O + {}^{1}_{1}H$
   b) $^{9}_{4}Be + {}^{4}_{2}He \rightarrow {}^{12}_{6}C + {}^{1}_{0}n$
   c) $^{27}_{3}Al + {}^{4}_{2}He \rightarrow {}^{1}_{0}n + {}^{30}_{15}P$
   d) $^{12}_{6}C + {}^{2}_{1}H \rightarrow {}^{13}_{7}N + {}^{1}_{0}n$

35. The energy levels are:

$$16.2 - 14.8 = 1.4\,\text{MeV}$$
$$16.2 - 11.6 = 4.6\,\text{MeV}$$
$$16.2 - 8.9 = 7.3\,\text{MeV}$$
$$16.2 - 6.7 = 9.5\,\text{MeV}$$

So the energy levels are

______________ 9.5

______________ 7.3

______________ 4.6

______________ 1.4

______________ 0.0 E(MeV)

36. Using $R = \dfrac{N\sigma I_0}{S} = \dfrac{mN_A}{M}\dfrac{\sigma I_0}{S}$

but $m = St\rho \quad \therefore \quad R = \dfrac{N_A I_0 \sigma t\rho}{M}$

$$I_0 = \frac{3.0 \times 10^{-6}}{1.6 \times 10^{-19}} = 1.875 \times 10^{13} \text{ protons per second}$$

$$R = \frac{6.022 \times 10^{23} \times 1.875 \times 10^{13} \times 0.4 \times 10^{-24} \times 10^{-4} \times 7.9}{56}$$

$R = 6.37 \times 10^{7}$ particles per second.

37. Using $R = \dfrac{\sigma\phi m N_A}{M}$

$$\sigma = \frac{MR}{\phi m N_A} = \frac{197 \times 5.37 \times 10^{6}}{7.25 \times 10^{10} \times m \times 6.022 \times 10^{23}}$$

Where $m = V\rho = \pi r^2 t\rho = 3.14 \times (0.15)^2 \times 1.81 \times 10^{-4} \times 19.3$

$$= 2.468 \times 10^{-4}$$

$$\sigma = 9.82 \text{ barn}$$

38. The number of nitrogen molecules in the cell is:

$$N = \frac{PV}{kT} = \frac{2.25 \times 1.01 \times 10^{5} \times (0.0124)^3}{1.38 \times 10^{-23} \times 293}$$

$$N = 1.07 \times 10^{20}$$

The number nitrogen nuclei = 2N.

$$I_0 = \frac{2.05 \times 10^{-6}}{1.6 \times 10^{-19}} = 1.28 \times 10^{13}$$

$$R = \frac{\sigma N I_0}{S}$$

$$\sigma = 0.21 \times 10^{-28}\,\text{m}^2 \quad N = 2.14 \times 10^{26} \quad \text{and } s = 0.0124^2$$

Thus $\quad R = 4.03 \times 10^{8}\,\text{s}^{-1}$

Activity $\quad a(t) = R = (1 - e^{-\lambda t}) = 1.16 \times 10^{8}\,\text{s}^{-1} = 3.14\text{ mCi}$

39. $R = \phi\,\sigma \dfrac{m}{M} N_A \quad = \dfrac{3 \times 10^{12} \times 99 \times 10^{-24} \times 0.03 \times 6.02 \times 10^{23}}{197}$

$R = 2.7 \times 10^{10}\ \text{s}^{-1}$

In the case $t \ll T_{1/2}$

$\therefore\ a(t) = R\lambda t = 4.8 \times 10^{6}\,\text{s}^{-1} = 130\mu\text{Ci}$

40. $Q = [m(^1H) + m(M_n) - m(F_e) - 2m_n]c^2$

$1u = 931.5\dfrac{\text{MeV}}{c^2}$

$Q = -10.313\text{MeV}$

41. $Q = \left[m(\text{Li}) + m(^2H) - m(\text{Be}) - m(n)\right]c^2$

$Q = -15.0316\text{MeV}$

42. $Q = \left[m^1H + m(^3H) - 2m(^2H)\right]c^2 \quad = -4.0335\text{ MeV}$

a) $\quad K_{Th} = -Q\left[1 + \dfrac{m(^1H)}{m(^3H)}\right] = 3.381\text{MeV}$

b) $\quad K_{Th} = -Q\left[1 + \dfrac{m(^3H)}{m(^1H)}\right] = 16.10\text{MeV}$

43. $Q = [m(U) + m_n(n) - m(Rb) - 2m_n]c^2$

$$= 179.94 \text{ MeV}$$

44. $^{235}U + n \rightarrow {}^{236}U$

$$\Delta E_9 = [m(^{235}U) + m_n - m(^{236}U)]c^2 = 6.546 \text{ MeV}$$

$$^{238}U + n \rightarrow {}^{239}U$$

$$\Delta E_2 = [m(^{238}U) + m_n - m(^{239}U)]c^2$$

$$\Delta E_2 = 4.807 \text{ MeV}$$

The excitation energy of ^{235}U is greater than the excitation energy of ^{238}U. The second one needs an additional energy of $(6.546 - 4.807) = 1.739$ MeV. This should come from the kinetic energy of neutrons. So ^{238}U can be fissioned only by fast neutrons.

45. $^2H + {}^3H \rightarrow {}^4He + n$

$$Q = K_{He} + K_n = 17.6\text{MeV} \text{ given.}$$

$$Q = \frac{P_{He}^2}{2m_{He}} + \frac{p_n^2}{2m_n}$$

Neglecting initial kinetic energies of ^2H and ^3H. Then conservation of momentum gives

$$O = P_{He} + P_n$$

or $\quad P_H = -P_n$

$$\therefore \quad Q = \frac{p_n^2}{2m_{He}} + \frac{p_n^2}{2m_n}$$

$$Q = \frac{p_n^2}{2m_n}\left(1 + \frac{m_n}{2m_{He}}\right)$$

$$\therefore \quad Q = \frac{p_n^2}{2m_n} = \frac{Q}{1 + \frac{m_n}{m_{He}}} = \frac{17.6}{1 + 0.25}$$

$$K_n = 14.1 \text{ MeV}$$