UNIT ONE

1 NUCLEAR STRUCTURE AND RADIOACTIVITY

Introduction

Nucleus is the central core of every atom which occupies a very small volume. Within the nucleus there are Z positive charges. These positive charges repel due to electrostatic forces between them. So to hold these positive charges inside the nucleus, there must be some nuclear attractive force inside the nucleus to overcome the electrostatic force of repulsion. This nuclear force is the strongest force among the four known force. The nuclear force provides nuclear binding energies that are millions of times stronger than atomic binding energies.

There are many similarities between atomic and nuclear structure, which will make our study of the properties nucleus simple.

Similarities between nuclei and atoms

Both are governed by quantum mechanical laws. Both have ground state and excited states and emit radiations when they go from excited states to lower states. Both are labelled by their angular momentum quantum number.

Dissimilarities between nuclei and atoms

There are two major differences between the study of nuclei and atoms.

First one is that in atomic physics, the electrons in atoms experience the force provided by an external agent whereas in nuclear physics, nuclei can never experience an external force. In atomic physics we consider the interaction between nucleus and the electrons (single body problem) and the interaction between the electrons is often considered as a perturbation to the single body problem. In nuclear physics mutual interaction between the nuclear constituents is considered which provides the nuclear force. This interaction is a many body problem which cannot be treated as a single body problem.

The second difference between atomic and nuclear physics is that we cannot write the nuclear force in a simple form like the Coulomb force. There is no analytical expression describing the mutual interactions of constituents of nuclei.

Inspite of these difficulties we can learn about the properties of nuclei by studying the interactions between different nuclei, the radioactive decay of nuclei and properties of nuclear constituents. In this chapter we discuss the properties of nuclei.

This study is of great importance because of several reasons. One is that the very existence of the various elements is due to the ability of nuclei to possess multiple electric charges. The second one is that energy involved in almost processes are due to nuclear reactions and transformations. Above all the liberation of nuclear energy in reactors and weapons play a crucial role in our everyday life.

Nuclear constituents

The work of Rutherford, Bohr and etal revealed that the entire positive charge of atom is concentrated in a small of space (10^{-15} m) called nucleus. The nucleus in an atom of atomic number Z has charge of Ze. The mass of these charges provides 99.9% of its atomic mass. At that time it was also known that the masses of the atoms were integers within an accuracy of 0.1%, we call this integer A the mass number. It was therefore reasonable to suppose that nuclei are composed of a mass number A. At that time it was known that the mass of the proton is nearly 1u (atomic mass unit). If the mass is Au, we can very well say that the nucleus contains A proton.

Such a nucleus would have a nuclear charge of Ae rather than Ze, because $A > Z$ for all atoms heavier than hydrogen. This model gives two much positive charge to the nucleus. This difficulty was removed by bringing another model called proton-electron model. In this model they postulated that the nucleus also contained (A-Z) electrons called nuclear electrons. Under these assumptions the nuclear mass would be A times the mass of the proton, because the mass of the electrons is negligible. Then the nuclear charge would be $Ae + (A - Z)(-e) = Ze$. This is in agreement with the experimental results. The postulate of nuclear electrons was supported by the fact that certain radioactive nuclear spontaneously emit electrons, a phenomenon called beta decay. However there are strong evidences against the idea of nuclear electrons. They are

1. Nuclear size

When an electron is confined to a box of nuclear dimensions it possesses an energy more than 19 MeV according to uncertainty principle whereas electrons emitted during beta decay have energies 2-3 MeV.

According to uncertainty principle

$$\Delta x \Delta p \approx \hbar, \text{ here } \Delta x = 10^{-14}\,\text{m}$$

$$\therefore \quad \Delta p \approx \frac{\hbar}{\Delta x} \simeq \frac{10^{-34}}{10^{-15}} = 10^{-20}$$

Using $E = \sqrt{p^2c^2 + m_0^2c^4} = \sqrt{(10^{-20})^2 \times (3\times10^8)^2 + (9\times10^{-31})^2(3\times10^8)^4}$

$$= 3\times10^{-12}\,J = 18.75\ MeV$$

2. Nuclear spin

Protons and electrons are fermions with spins $\frac{1}{2}$. Thus nucleus with even number of protons plus electrons should have zero or integral spin, those with an odd number of protons plus electrons should have half integral spins. But it has been experimentally found that it is not true. For example a deuterium nucleus consists of two protons and an electron, its nuclear spin should be either $\frac{1}{2}$ or $\frac{3}{2}$. But the spin observed is one.

3. Magnetic moment

It has been found that the magnetic moment of proton is only about 0.15 present that of electrons. If nuclear electrons exist, the magnetic moment of a nucleus is of the order of magnitude of that of the electron. But the observed magnetic moment of the nucleus is found to be of the order of magnitude of protons.

Because of these and other reasons the idea of nuclear electrons was abandoned. The resolution of this dilemma came in 1932 with the discovery of neutron by James Chadwick. The discovery of neutron with atomic mass unit roughly the same as the proton (actually 0.14% more massive than proton) without electric charge led to the assumption that every atomic nucleus consists of protons and neutrons. This hypothesis was used for the first time as the basis of a detailed theory of the nucleus by Heisenberg in 1932. According to proton-neutron model, the total number of elementary particles in the nucleus, protons and neutrons together, is equal to the mass number A of the nucleus. The number of protons is given by Z and the number of neutrons is A-Z.

The proton and neutron together are called as nucleus. Some properties of nucleons are given below.

Table 1.1: Properties of nucleons

Name	Symbol	Charge	Mass	Rest energy	Spin
Proton	p	+e	1.007276 u	938.28 MeV	$\frac{1}{2}$
Neutron	n	0	1.008665 u	939.57 MeV	$\frac{1}{2}$

Note: Recall that one atomic mass unit, $1u = 931.5\ MeV/c^2$.

i.e. to convert one atomic mass unit into MeV multiply by 931.5

A nuclide is a specific nucleus of an atom which is characterised by its atomic number Z and mass number A. It is represented as $_Z X^A$ or $^A_Z X$, where X is the chemical symbol of the species.

Each nuclear species with a given Z and A is called a nuclide. Each Z characterises a chemical element with symbol X. For example, Al for aluminium (Z = 13), Ca for calcium (Z = 20). Nuclear mass is roughly the sum of its constituents proton and neutron masses (the slight difference being the binding energy of the nucleus). The nuclear charge is +e times the number (Z) of protons ($1e = 1.6 \times 10^{-19}$ C).

The chemical properties of an atom are determined by its electron configuration because the number of electrons and protons are equal in neutral atom. The chemical properties are essentially determined by Z. The dependence of the chemical properties on N is negligible.

Nuclei with the same Z but different A are called isotopes. For example ^1_1H (ordinary hydrogen), ^2_1H (deuterium) and ^3_1H (tritium) are isotopes of hydrogen. Another example is the four isotopes of carbon: $^{11}_6C$, $^{12}_6C$, $^{13}_6C$ and $^{14}_6C$.

Nuclides with the same neutron number are called isotones

For example: $^{14}_6C$, $^{15}_7C$, $^{16}_8C$ and $^{17}_9C$.

Nuclide with the same mass number are called isobars.

For example: $^{16}_6C$, $^{16}_7N$, $^{16}_8O$ and $^{16}_9F$.

Example 1

Give the proper isotopic symbols for (a) the isotope of fluorine with mass number 19 (b) an isotope of gold with 120 neutrons (c) an isotope of mass number 107 with 60 neutrons.

Solution

(a) Fluorine has $Z = 9$, so the symbol is ${}^{19}_{9}F$.

(b) Gold has $Z = 79$. So $N + Z = 120 + 79 = 199$. Thus the symbol is ${}^{199}_{79}Au$.

(c) Here $A = 107, N = 60 \quad \therefore \quad Z = A - N = 107 - 60 = 47$. $Z = 47$ is silver. Thus the symbol is ${}^{107}_{47}Ag$.

Nuclear sizes and shapes

Atoms have hard surfaces thereby they have well defined radii even though they are probabilistic. But nuclei do not have a hard surface hence no easily definable radius. At the same time different types of experiments often reveal different values of the radius for the same nucleus.

Several experiments had been conducted to find the variation of nuclear density with the radius of the nuclei. This is given in figure below.

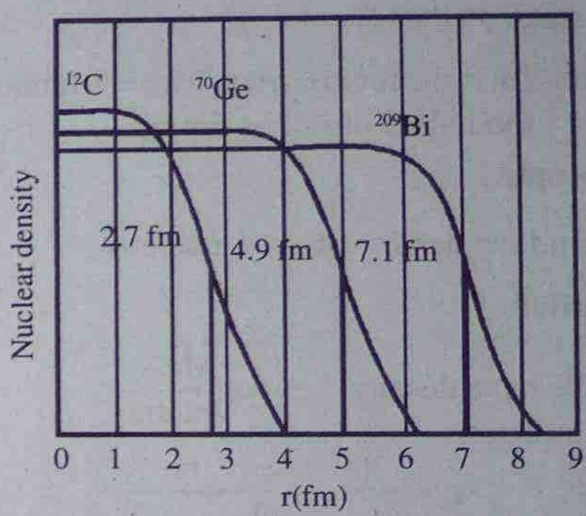

Figure 1.1: The radial dependence of the nuclear charge density

From the graph two things can be inferred.

1. Nuclear density is independent of mass number A.
2. Nuclear density is almost uniform for all nuclei.

From this important clue, we can arrive at the relation between the radius and mass number of nuclei.

$$\text{density } (\rho) = \frac{\text{Mass}}{\text{Volume}} = \frac{A}{\frac{4}{3}\pi R^3} = \text{constant}$$

Assumed that nucleus is a sphere of radius R.

i.e., $$A = \text{constant} \cdot \tfrac{4}{3}\pi R^3$$

i.e., $$A \propto R^3$$

This relationship is expressed in inverse form as

$$R \propto A^{1/3}$$

or

$$R = R_0 A^{1/3} \qquad \text{..... (1)}$$

where R_0 is a constant whose value can be easily calculated from the above graph.

For example, for ^{12}C radius is 2.7 fm from the graph and A = 12, use this in equation (1), we get

$$R_0 = \frac{R}{A^{1/3}} = \frac{2.7\text{fm}}{(12)^{1/3}} \simeq 1.2\text{fm}$$

As A is different for different atoms atomic nuclei of different atoms have different sizes.

Note: fm is the femtometer. 1 fm = 1 femtometer is also called as fermi in honour of the Italian physicist Fermi.

Example 2

Find the density of $_6C^{12}$ nucleus.

Solution

We have density, $\rho = \dfrac{\text{Mass}}{\text{Volume}}$

i.e.,

$$\rho = \frac{m}{\frac{4}{3}\pi R^3} = \frac{m}{\frac{4}{3}\pi(R_0 A^{\frac{1}{3}})^3}$$

$$m = 12u = 12 \times 1.66 \times 10^{-27}\,\text{kg}$$

from fig (1.1)

$$R_0 A^{\frac{1}{3}} = 1.2 \times 10^{-15} \times (12)^{\frac{1}{3}} = 2.7 \times 10^{-15}\,\text{m}$$ – these values define the mean radius the point at which the density falls to half the central value.

$$\therefore \quad \rho = \frac{12 \times 1.66 \times 10^{-27}}{\frac{4}{3} \cdot \pi \times (2.7 \times 10^{-15})^3} = 2.4 \times 10^{-17}\,\text{kgm}^{-3}$$

Example 3

Find the nuclear radius of (a) ^{197}Au (b) ^{20}Ne and (c) ^{4}He

Solution

(a) $A = 197,\ R_0 = 1.2\,\text{fm}$

Using $R = R_0 A^{1/3}$

$$R = 1.2 \times (197)^{1/3} = 6.98\,\text{fm}$$

(b) $A = 20,\ R = 1.2$ fm

$$R = 1.2 \times (20)^{1/3} = 3.26\,\text{fm}$$

(c) $A = 4,\ R\ \ R_0 = 1.2\,\text{fm}$

$$R = 1.2 \times (4)^{1/3} = 1.90\,\text{fm}$$

Nuclear masses and binding energies

Consider a proton and an electron at rest separated by a large distance. The total energy of this system (proton and electron) is the total rest energy of the two particles.

i.e., total energy of the system = $m_p c^2 + m_e c^2$ ..... (2)

Now suppose that these two particles bring together to form a hydrogen atom in its ground state. In the process of bringing energy is liberated in the form of photons, say it is 13.6 eV. Now the total energy of the system is rest energy of the hydrogen atom $m(H)c^2$ plus the energy liberated (13.6 eV).

i.e., total energy of the system = $m(H)c^2 + 13.6$ eV ..... (3)

According to law of conservation of energy, for an isolated system total energy is conserved. i.e., eq (2) = eq (3)

Thus we get

$$m_p c^2 + m_e c^2 = m(H)c^2 + 13.6\ \text{eV}$$

or

$$m_p c^2 + m_e c^2 - m(H)c^2 = 13.6\ \text{eV} \qquad \text{..... (4)}$$

This equation says that the rest energy of the combined system $m(H)c^2$ is less than the rest energy of its constituents by an amount 13.6 eV. This energy difference is called the binding energy (E_b) of the atom. Thus binding energy can be defined as follows.

Binding energy is the extra energy that we obtain when we assemble an atom from its constituents or it is the energy that we must supply to dissemble the atom into its constituents.

Equation (4) can be written as

$$[m_p + m_e - m(H)]c^2 = 13.6 \text{ eV}$$

The term inside the square bracket is the difference mass of an atom and its constituents is called the mass defect denoted by Δm.

i.e., $$\Delta mc^2 = 13.6 \text{ eV}$$

In general $$\Delta mc^2 = E_b \qquad \text{..... (5)}$$

Thus binding energy E_b is the energy equivalent of the mass defect of the atom. In a similar way we can calculate nuclear binding energies.

Example: Consider the nucleus of deutirium ${}^2_1H$, which composed of one proton and one neutron. The nuclear binding energy of deuterium is the difference between the total rest energy of the constituents and the rest energy of their combination:

$$E_b = m_n c^2 + m_p c^2 - m_D c^2 \qquad \text{..... (6)}$$

where m_D is the mass of the deuterium nucleus.

Usually we do calculations by using standard tables of atomic masses. Thus m_p is replaced by mass of the hydrogen atom $m({}^1_1H)$ and m_D is replaced by deutron mass m(D)

$$m({}^1H_1)c^2 = m_p c^2 + m_e c^2$$

or $$m_p c^2 = m({}^1_1H)c^2 - m_e c^2$$

$$m(D)c^2 = m_D c^2 + m_e c^2$$

or $$m_D c^2 = m(D)c^2 - m_e c^2$$

Replacing $m_p c^2$ and $m_D c^2$ from equation (6) by the above relations we get

$$E_b = m_n c^2 + m({}^1_1H)c^2 - m_e c^2 - (m(D))c^2 - m_e c^2)$$

$$E_b = m_n c^2 + m({}^1_1H)c^2 - m(D)c^2$$

or $$E_b = [m_n + m({}^1_1H)c^2 - m(D)]c^2 \qquad \text{..... (7)}$$

Nuclear masses are expressed in atomic mass units. Substituting the values of m_n, m(H) and m(D), we get

$$E_b = [1.008665\,u + 1.007825\,u - 2.014102\,u]c^2$$

$$E_b = 0.002388\,u\,c^2$$

Using $1\,u = 931.5\,\frac{MeV}{c^2}$

$$E_b = 0.002388 \times 931.5\,\frac{MeV}{c^2}c^2$$

$$E_b = 0.002388 \times 931.5\ MeV$$

$$E_b = 2.224422\ MeV$$

Example: In the case of $^{16}_{8}O$, we have 8 neutrons and 8 protons.

$$\text{Mass of 8 neutrons} = 8\ m_n$$

$$= 8 \times 1.008665\ u$$

$$= 8.06932\ u$$

$$\text{Mass of 8 protons} = 8m(^1_1H) = 8 \times 1.007825\ u$$

$$= 8.0626\ u$$

$\therefore$ Total mass of constituents $= 8.06932u + 8.0626\,u$

$$= 16.13192\,u$$

Mass of $^{16}_{8}O$ atom $= 16\,u$

$\therefore$ The mass defect, $\Delta m = 16.13192\,u - 16\,u$

$$\Delta m = 0.13192\,u$$

$\therefore$ Thus binding energy $E_b = \Delta mc^2 = 0.13192\,uc^2$

Using $1u = 931.5\,\frac{MeV}{c^2}$

$$E_b = 0.13192 \times 931.5\ MeV$$

$$E_b = 122.88348\ MeV$$

i.e., the binding energy of oxygen nucleus is 122.88348 MeV.

Now we can generalise this process to calculate the binding energy of a nucleus X of mass number A with Z protons and N neutrons. Let m_X represent the mass of this nucleus.

The binding energy of a nucleus ${}^{A}_{Z}X$ is given by

$$E_b = Nm_n c^2 + Zm_p c^2 - m_X c^2 \qquad \text{..... (8)}$$

We do this calculation by using standard tabulated atomic masses. For this we replace the nuclear mass m_X with its corresponding atomic mass $m({}^{A}_{Z}X)$.

$$m({}^{A}_{Z}X)c^2 = m_X c^2 + Zm_e c^2 - e_b$$

where e_b represents the total binding energy of all electrons in this atom. Usually $m_X c^2$ is of the order of 10^9 to 10^{11} eV, $Zm_e c^2$ is of the order of 10^6 to 10^8 eV and e_b is of the order of 1 to 10^5 eV. Thus e_b is very small compared with other two energies we can very well neglect e_b.

Thus we have

$$m({}^{A}_{Z}X)c^2 = m_X c^2 + Zm_e c^2$$

or

$$m_X c^2 = m({}^{A}_{Z}X)c^2 - Zm_e c^2 \qquad \text{..... (9)}$$

Now replace proton mass m_p with atomic mass of hydrogen $m({}^{1}_{1}H)$.

$$m({}^{1}_{1}H)c^2 = m_p c^2 + m_e c^2$$

or

$$m_p c^2 = m({}^{1}_{1}H)c^2 - m_e c^2 \qquad \text{..... (10)}$$

substituting equations 9 and 10 in equation 8 we get

$$E_b = Nm_n c^2 + Z(m({}^{1}_{1}H)c^2 - m_e c^2) - (m({}^{A}_{Z}X)c^2 - Zm_e c^2)$$

$$E_b = Nm_n c^2 + Z\,m({}^{1}_{1}H)c^2 - m({}^{A}_{Z}X)c^2$$

$$E_b = [Nm_n + Z\,m({}^{1}_{1}H) - m({}^{A}_{Z}X)]c^2 \qquad \text{..... (11)}$$

This is the general expression for the binding energy of a nucleus ${}^{A}_{Z}X$

Therefore the binding energy per nucleon for a given nucleus is $\dfrac{E_b}{A} = \bar{E}_b$

i.e., $$E_b = \frac{[Nm_n + Zm(^1H_1) + m(^A_ZX)]c^2}{A} \quad ..... (12)$$

It may be noted that masses are expressed in atomic masses and automatically electron masses cancel out.

Example 4

Calculate the binding energy of tin nucleus ($^{120}_{50}Sn$). Given: atomic mass of $^{120}Sn = 119.\ 9022$ u, mass of hydrogen atom $m(^1_1H) = 1.007825u$, mass of neutron $m_n = 1.008665$ u.

Solution

Binding energy of nucleus, $E_b = [Nm_n + Zm(^1_1H) - m(^A_ZX)]c^2$

Tin($^{120}_{50}Sn$) has 70 neutrons and 50 protons.

Thus, $$E_b = [70 \times 1.008665\,u + 50 \times 1.007825\,u - 119.9022\,u]c^2$$

$$E_b = [70.60655\,u + 50.39125\ u - 119.9022\ u]c^2$$

$$E_b = 1.0956\,u\,c^2$$

Using $1\,u = 931.5 \dfrac{MeV}{c^2}$

$$\therefore \quad E_b = 1.0956 \times 931.5\ MeV$$

$$E_b = 1020.5514\ MeV$$

Example 5

Find the total binding energy and the binding energy per nucleon for

a) $^{208}_{82}Pb$ and b) $^{90}_{40}Zr$

$m_n = 1.008665\,u$, $m(^1_1H) = 1.007825\,u$, $m(Pb) = 207.976652\,u$ and $m(Zr) = 89.904704\,u$.

Solution

Binding energy, $E_b = [Nm_n + Zm(^1_1H) - m(^A_ZX)]c^2$

a) For lead $E_b = [Nm_n + Zm({}^1_1H) - m(Pb)]c^2$

Lead nuclei containing 126 neutrons and 82 protons

$$\therefore \quad E_b = [126 \times 1.008665\,u + 82 \times 1.007825\,u - 207.976652\,u]c^2$$

$$E_b = [127.09179\,u + 82.64165\,u - 207.976652\,u]c^2$$

$$E_b = 1.756788\,u\,c^2$$

$$E_b = 1.756788 \times 931.5 \text{ MeV}$$

$$E_b = 1636.448022 \text{ MeV}$$

$\therefore$ Binding energy per nucleon $\bar{E}_b = \dfrac{E_b}{A} = \dfrac{1636.448022}{208}$

$$\bar{E}_b = 7.868 \text{ MeV per nucleon.}$$

b) For Zirconium nucleus

$$E_b = [Nm_n + Z\,m({}^1_1H) - m(Zr)]c^2$$

Zirconium nucleus has 50 neutrons and 40 protons.

$$E_b = [50 \times 1.008665\,u + 40 \times 1.007825\,u - 89.904704\,u]c^2$$

$$E_b = [50.43325\,u + 40.313\,u - 89.904704\,u]c^2$$

$$E_b = 0.841546\,uc^2$$

$$E_b = 0.841546 \times 931.5\,\text{MeV}$$

$$E_b = 783.9\,\text{MeV}$$

$\therefore$ Binding energy per nucleon,

$$\bar{E}_b = \frac{E_b}{A} = \frac{783.9}{90} = 0.71\,\text{MeV} \text{ per nucleon.}$$

Binding energy curve

A plot of binding energy per nucleon $\left(\bar{B} = \dfrac{BE}{A}\right)$ as a function of mass number A for various stable nucleus is called a binding energy curve. Figure given above shows the binding energy curve. Following points may be noted from the curve.

(i) The greater the binding energy per nucleon the more stable is the nucleus. The graph has its maximum value of 8.8 MeV/nucleon for $^{56}_{26}Fe$ i.e., number of nucleon is 56. This is the most stable nucleus, since maximum energy is needed to pull a nucleon away from it.

(ii) If each nucleon interacted with every nucleon, there would be (A-1) interactions for each nucleon, and the binding energy would be proportional to (A-1) rather than constant. Instead, there is a fixed number of interactions per nucleon, because each nucleon is attracted to its nearest neighbours. Such a situation also leads to a constant nuclear density. If the binding energy per nucleon were instead proportional to the number of nucleons, then the radius would be approximately constant, as in the case of atoms.

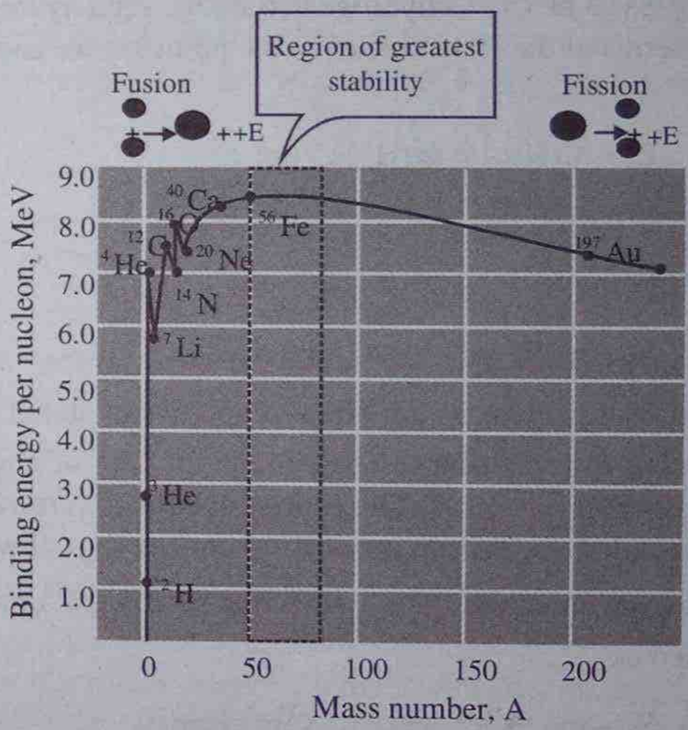

Figure 1.2

(iii) If we split a heavy nucleus into two medium sized ones, each of the new nuclei will have more binding energy per nucleon than the original nucleus did. The extra energy will be given off.

Binding energy curve droops at high mass number which tells us that nucleons are more tightly bound when they are assembled into two middle-mass nuclei rather than into a single high- mass nucleus. Energy is released in the nuclear fission of a single massive nucleus into two smaller fragments.

(iv) The drooping of the binding energy curve at low mass number tells us that energy will be released if two nuclei of small mass number combine to form a single middle mass nucleus. New nucleus has more binding energy per nucleon.

For instance, if two $^{2}_{1}H$ deuterium nuclei combine to form $^{4}_{2}He$ helium nucleus, over 23 MeV is released. Such process is called **nuclear fusion**.

Proton and neutron separation energies

Firstly we recall the definition of ionisation energy (E_i) of an atom. **It is the amount of energy need to remove an electron from an atom**. For example, the ionisation energy of hydrogen atom in the ground state is +13.6 eV. If we add the ionisation energy E_i(13.6eV) to a hydrogen atom, we get a hydrogen ion (H^+) and a free electron. In terms of the rest energies of the particles, we can write this process as

$$E_i + m(H)c^2 = m(H^+)c^2 + m_e c^2$$

In general, for any atom X

$$E_i + m(X)c^2 = m(X^+)c^2 + m_e c^2$$

or

$$E_i = m(X^+)c^2 + m_e c^2 - m(X)c^2$$

The ionisation energy gives us important properties of atoms.

A process similar to ionisation can be done in the case of nucleus: Removing a proton or a neutron from a nucleus. **The energy needed to remove the least tightly bound proton is called the proton separation energy (S_P)**. If we add energy S_p to a nucleus $^{A}_{Z}X$, we get a nucleus $^{A-1}_{Z-1}X'$ and a free proton (p). In mathematical form this can be written as

$$S_P + m(^{A}_{Z}X)c^2 = m\left(^{A-1}_{Z-1}X'\right)c^2 + m_p c^2$$

or

$$S_P = m\left(^{A-1}_{Z-1}X'\right)c^2 + m_p c^2 - m(^{A}_{Z}X)c^2$$

In terms of atomic mass units, this can be written as

$$S_P = m\left(^{A-1}_{Z-1}X'\right)c^2 + m(^{1}_{1}H)c^2 - m(^{A}_{Z}X)c^2$$

or $$S_P = \left[m\left({}^{A-1}_{Z-1}X' \right) + m({}^{1}_{1}H) - m({}^{A}_{Z}X) \right] c^2 \qquad ..... (13)$$

This is the expression for proton separation energy. (see also example 6).

Similarly if we add the neutron separation energy (S_n) to a nucleus ${}^{A}_{Z}X$, we get the nucleus ${}^{A-1}_{Z-1}X$ and free neutron.

i.e. $$S_n + m({}^{A}_{Z}X)c^2 = m\left({}^{A-1}_{Z-1}X \right)c^2 + m_n c^2$$

or $$S_n = \left[m\left({}^{A-1}_{Z-1}X \right) + m_n - m({}^{A}_{Z}X) \right] c^2 \qquad ..... (14)$$

This is the expression for neutron separation energy.

Proton and neutron energies are in the range of 5 – 10 MeV, this is the same as the average binding energy per nucleon. (see also example 7).

The proton and neutron separation energies play a role in nuclei similar to ionisation energy in atoms. Figure below shows a plot of the neutron separation energies of nuclei with a valence neutron from Z = 36 to Z = 62. Following points may be noted from the graph

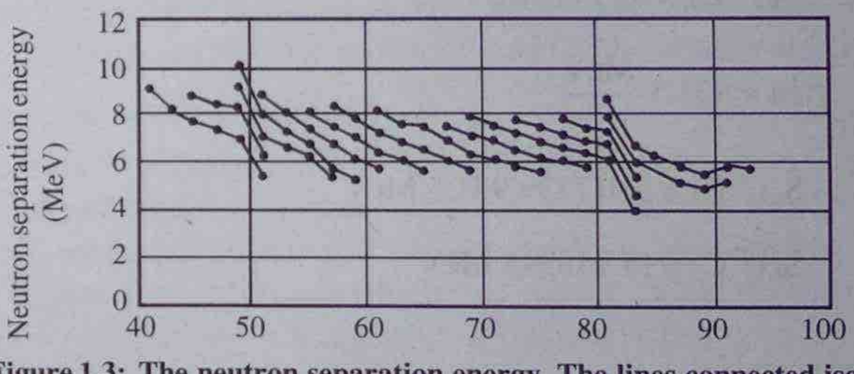

Figure 1.3: The neutron separation energy. The lines connected isotopes of the same element that have an odd neutron starting on the left at Z = 36 and ending on the right at Z = 62

As we add neutrons the neutron separation energy decreases smoothly except near N = 50 and N = 82, where there are more sudden decreases in the separation energy. In analogy with atomic physics these sudden decreases are associated with the filling of shells. The motions of neutrons and protons in the nucleus are described in terms shell structure that is similar to that of atomic shells and when a neutron or proton is placed into a new shell it is less tightly bound and its separation energy decreases. The neutron separation data indicate that there are closed neutron shells at N = 50 and N = 82. The figure above thus provides an important information about the shell structure of the nuclei.

Example 6

Find the proton separation energy of ^{12}C. Given that $m(^1_1H) = 1.007825\,u$, $m(^{11}B) = 11.009305\,u$ and $m(^{12}C) = 12.000000\,u$.

Solution

If we add proton separation energy to a nucleus ^A_ZX it becomes $^{A-1}_{Z-1}X' + p$.

In the case of carbon it becomes boron (B)

Separation energy of proton in general is given by

$$S_p = \left[m\left(^{A-1}_{Z-1}X'\right) + m(^1_1H) - m(^A_ZX)\right]c^2$$

$$S_p(^{12}C) = \left[m(^{11}B) + m(^1_1H) - m(^{12}C)\right]c^2$$

Substituting the m values given, we get

$$S_p(^{12}C) = [11.009305\,u + 1.007825\,u - 12.000000\,u]c^2$$

$$S_p(^{12}C) = 0.01713\,u\,c^2$$

Using $\quad 1u = 931.5\,\dfrac{MeV}{c^2}$

$$S_p(^{12}C) = 0.01713 \times 931.5\ MeV$$

$$S_p(^{12}C) = 15.956595\ MeV$$

Example 7

Find the neutron separation energy of ^{57}Fe. Given that $m_n = 1.008665\,u$ $m(^{56}Fe) = 55.934937\,u$ and $m(^{57}Fe) = 56.935394\,u$

Solution

The neutron separation energy is given by

$$S_n = \left[m\left(^{A-1}_{\ \ Z}X\right) + m_n - m\left(^A_ZX\right)\right]c^2$$

$$S_n(^{57}Fe) = \left[m(^{56}Fe) + m_n - m(^{57}Fe)\right]c^2$$

Substituting the values of m, we get

$$S_n(^{57}Fe) = [55.934937\,u + 1.008665\,u - 56.935394\,u]c^2$$

$$S_n(^{57}Fe) = 0.08208\,u\;c^2$$

Using $$1\,u = 931.5\frac{MeV}{c^2}$$

$$S_n(^{57}Fe) = 0.08208 \times 931.5\ MeV$$

$$S_n(^{57}Fe) = 7.645752\ MeV$$

Nuclear models

To explain the behaviour and properties of nuclei several models have been proposed so far. Unlike the case of the atom, physicists however, still donot have a very clear understanding of the details of nuclear forces or the way the nucleons interact with each other. This lack of understanding has made them propose very many different models each designed to explain a particular category of nuclear phenomena or nuclear properties. These models even tend to contradict each other. This has led to considerable confusion and attempts are constant on to clear this confusion and get a model that incorporates the essential details of these very many models.

The nuclear models proposed so far can be categorized into two broad types. They are (1) the strong interaction models and (2) the independent particle models.

The strong interaction models

These models are based on the assumption that the nucleons, in a nucleus, are strongly coupled together. The one model of this category is liquid drop model. This model explains the phenomenon of nuclear fission. It has also been used to arrive at the semi empirical mass formula which enables us to calculate the nuclear masses in a satisfactory way (from the knowledge of binding energy).

The independent particle models

These models are based on the assumption that there exists a common nuclear potential within the nucleus and all the nucleons move nearly independently within this common nuclear potential. The most successful model of this category is the shell model. This model helps us to understand the periodicity observed in many properties of the nuclei.

Here we shall deal with the liquid drop model and the shell model.

The liquid drop model

The liquid drop model of the nucleus was first suggested by Bohr. He along with wheeler used it to explain the essential details of the phenomenon of nuclear fission.

This explanation has been the most significant achievement of this model but it failed to understand other nuclear properties. It is now a more or less obsolete model.

Because of some striking similarities between a liquid drop and a nucleus, this model was proposed. We give some of the similarities below.

1. Small drops of a liquid are known to acquire a spherical shape. We know that this is because of the symmetrical surface tension forces acting towards the centre of the drop. The gravitational forces, acting vertically downwards, tend to flatten out the drop. Ultimately, it is the balance between these two forces which decides the shape of the drop.

A nucleus also has two kinds of opposing forces acting within it. On the one hand there are the (short ranged and very strong) attractive nuclear forces that try to keep the nucleus intact. On the other hand, there are the repulsive electrostatic forces between the protons contained in the nuclei. It is again a balance between these two opposing forces which decides the stability level of a nucleus. For light stable nuclei, the attractive nuclear forces dominate the other repulsive (disruptive) forces. Such nuclei, therefore, tend to be spherical in shape.

2. For a spherical drop, the density is known to be independent of its volume. This also implies that the radius must vary as the cube root of its mass. It is known that the density of nuclear matter is (nearly) constant for all nuclei and nuclear radii vary as the cube root of their mass numbers. We know that, unlike different nuclei, the densities of different liquids are different. However, there is still sufficient reason to think of a nucleus as if it were akin to a spherical drop.

3. The molecules of a liquid are known to be in continuous random thermal motion. We use this to explain the phenomenon of continuous evaporation of a liquid and the increase in the rate of this evaporation with an increase in the temperature of the liquid. If we were to think of a similar random motion for the nucleons inside a nucleus, we could think of phenomenon like spontaneous emission of α-particles as essentially similar to the evaporation of molecules from the surface of a liquid.

According to this model the nucleus is regarded as a collection of neutrons and protons forming a droplet of incompressible liquid which behaves in some ways like a liquid drop.

The liquid drop model is based on two properties of nuclei.

(i) The interior mass densities are same

(ii) Their total binding energies are approximately proportional to their masses. i.e.,

$$\frac{\Delta E_b}{A} = \text{a constant (except for small A).}$$

The semiempirical mass formula

This formula helps us to calculate the mass of a given nucleus from a knowledge of its mass number A and its atomic number (Z). This was derived by Von Weizsacker in 1935.

His arguments were based on the assumption of a liquid drop type model for a nucleus.

By simple logic, we expect the mass, M of a nucleus of atomic number Z and mass number A to be

$$M = Zm_p + (A - Z)m_n$$

Here m_p and m_n stand for the masses of the proton and neutron respectively. Precise measurements of nuclear masses however indicated that the observed nuclear masses are always less than their ideally expected mass given by the above formula. We now believe that this lost mass gets converted into energy as per the Einstein mass energy relation $(E = mc^2)$. This energy equivalent of this lost mass (mass defect) when divided by the number A of the nucleus is referred to as the binding energy per nucleon for the given nucleus. If a given nucleus has a total binding energy (E_b), its mass ${}^A_Z M$ can be correctly expressed through the relation.

$${}^A_Z M = Zm_p + (A - Z)m_n - \frac{E_b}{c^2}$$

Von Weizsacker used the essential ideas of the liquid drop model to calculate the binding energy E_b in terms of number of terms such as volume energy, surface energy, Coulomb energy. In otherwords the binding energy of a nucleus is due to volume energy, surface energy and Coulomb energy according to liquid drop model.

1. Volume energy

We know that the binding energy per nucleon has an approximately constant value over a wide range of mass numbers. Hence as a first approximation, we can regard the binding energy of a nucleus to be proportional to the total number of nucleons (A) in it. Therefore it is proportional to nuclear volume, hence called volume energy (E_v).

$\therefore$ $$E_v \propto A$$

or $$E_v = a_v A \qquad \text{..... (1)}$$

The constant of proportionality a_v is the binding energy per nucleon.

2. Surface energy

In writing the above expression for volume energy, we have implicitly assumed that all the nucleons are attracted by all the other nucleons. However, this is not quite true. The surface nucleons have fewer nearer neighbours than the nucleons which are deep within the nuclear volume. This gives rise to a slight decrease in the total binding energy of the nucleus. This decrease is proportional to the number of nucleons on the surface. Hence it is proportional to the surface area of the nucleus. If r is the radius of the nucleus then we know that

$$r = r_0 \ A^{\frac{1}{3}}$$

Since the surface area is proportional to r^2, it must be proportional to $A^{\frac{2}{3}}$. Thus the contribution of the surface effect to the binding energy of the nucleus can be written as

$$E_s \propto A^{\frac{2}{3}}$$

or

$$E_s = -a_s \ A^{\frac{2}{3}} \qquad \text{..... (2)}$$

where a_s is an empirical constant. –ve sign shows that the binding energy gets reduced. This effect is similar to that in a liquid drop where the surface nucleons are less tightly bound and therefore evaporate out more easily from the liquid.

Coulomb energy

The electrostatic repulsion between each pair of protons in a nucleus also contributes toward decreasing its binding energy. The Coulomb energy (E_c) of a nucleus is the work that must be done to bring together Z protons from infinity into a spherical aggregate the size of the nucleus. The potential energy of a pair of protons r apart is equal to

$$V = -\frac{e^2}{4\pi\varepsilon_0 r}$$

Since there are $\frac{Z(Z-1)}{2}$ pairs of protons

$$E_c = \frac{Z(Z-1)}{2} V \ = -\frac{Z(Z-1)}{8\pi\varepsilon_0}\left(\frac{1}{r}\right)_{av}$$

where $\left(\frac{1}{r}\right)_{av}$ is the value of $\frac{1}{r}$ averaged over all proton pairs. If the protons are uniformly distributed throughout a nucleus of radius R, $\left(\frac{1}{r}\right)_{av}$ is proportional to $\frac{1}{R}$ and hence to $\frac{1}{A^{\frac{1}{3}}}$.

$$\therefore \qquad E_c = -a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}}$$

The Coulomb energy is negative because it arises from an effect that opposes nuclear stability.

Thus the total binding energy is the sum of its volume, surface and Coulomb energies.

$$E_b = E_v + E_s + E_c$$

$$E_b = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}}$$

$\therefore$ The binding energy per nucleon is

$$\frac{E_b}{A} = a_v - a_s A^{-\frac{1}{3}} - a_c Z(Z-1) A^{-\frac{4}{3}}.$$

A graph is drawn between $\frac{E_b}{A}$ and A by appropriately choosing the coefficients so as to resemble as closely as possible to the empirical binding energy curve. This shows that liquid drop model has some validity.

Corrections to the formula

The binding energy formula derived in the last section is modified by taking two more factors which donot come under liquid drop model. This makes sense in terms of a model that provides for nuclear energy levels (shell model). The factors which contribute to binding energy are asymmetry energy and pairing energy.

Asymmetry energy

We found that among different isotopes of nuclei, the ones having equal number of protons and neutrons will be most stable and will therefore possess maximum binding energy. Thus we can say that an excess of neutrons over protons leads to a decrease in binding energy and causes instability. This is called asymmetry energy contribution to binding energy.

We know that like energy levels in atoms, nuclear energy levels also exist. Nucleons which have spin $\frac{1}{2}\hbar$ obey Paulis exclusion principle. Hence each nuclear energy level can contain two neutrons of opposite spins and two protons of opposite spins. Energy levels in nuclei are filled in sequence to achieve configuration of minimum energy and maximum stability. When nucleus contains equal number of neutrons and protons all lower energy levels are filled hence it is stable. When the neutrons in a nucleus outnumber the protons, the higher energy levels have to be filled (see figure below).

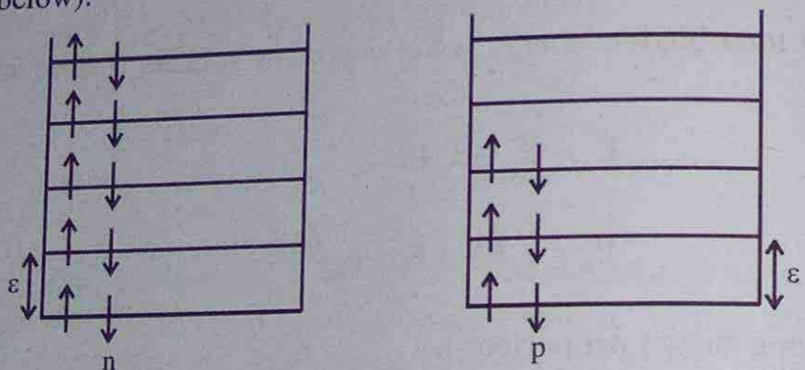

Figure 1.4

Let us suppose that the uppermost neutron and proton energy levels have the same spacing ε. Inorder to produce a neutron excess say $N-Z$ without changing

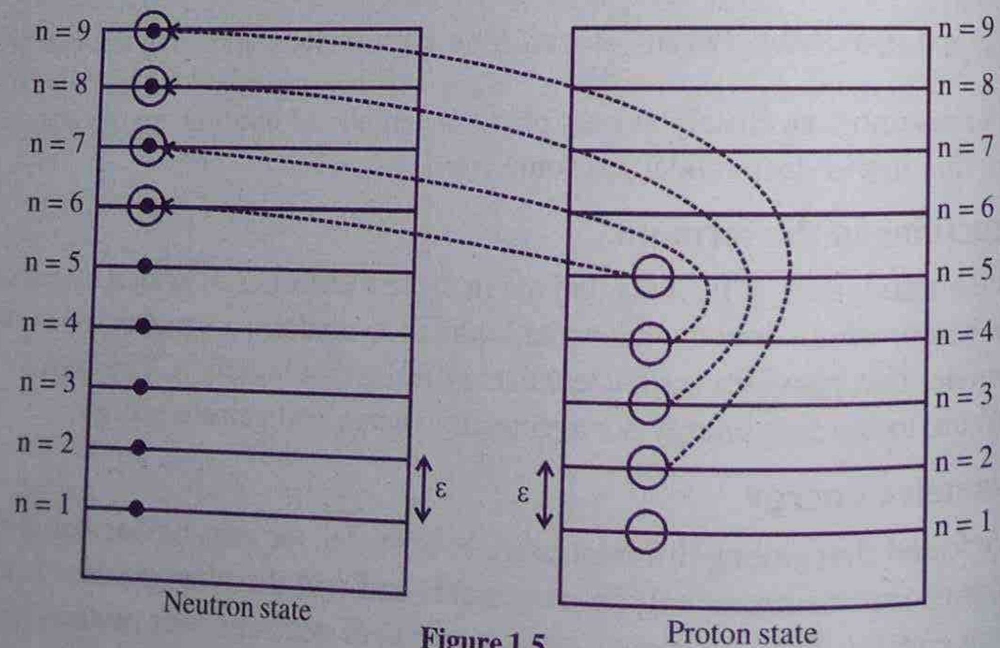

Figure 1.5

A, $\frac{1}{2}(N-Z)$ neutrons would have to replace protons in an original nucleus in which $N = Z$.

[This is because when we add one neutron, one proton has be removed to keep A constant. i.e., for each neutron replacement two neutrons are increased. So to have $N-Z$ excess neutrons only $\frac{N-Z}{2}$ neutrons will have to replace protons in the original nucleus in which $N = Z$]

Now will calculate the total energy change when $\frac{N-Z}{2}$ neutrons are added by replacing protons. There are ways of doing this. If we remove a proton from level 5 and add a neutron to level 6 the work required to be done for this is ε.

Now remove the proton from level 4 and add a neutron to level 7, the work required to be done for this is $3\varepsilon\,(E_7 - E_4 = 3\varepsilon)$.

To add 2 neutrons total work done = $\varepsilon + 3\varepsilon = 4\varepsilon$

Now remove the proton from levels 3 and add a neutron to level 8, the work required to be done for this is $5\varepsilon (E_8 - E_3 = 5\varepsilon)$.

$\therefore$ To add 3 neutrons total work done = $\varepsilon + 3\varepsilon + 5\varepsilon = 9\varepsilon$

Now remove the proton form level 2 and add a neutron to level 9, the work required to be done for this is $7\varepsilon (E_9 - E_2 = 7\varepsilon)$

$\therefore$ To add 4 neutrons total work done $= \varepsilon + 3\varepsilon + 5\varepsilon + 7\varepsilon = 16\varepsilon$

Now we summarise

Work done to add one neutron = ε

Work done to add two neutrons $= 4\varepsilon = 2^2\,\varepsilon$

Work done to add three neutrons $= 9\varepsilon = 3^2\,\varepsilon$

Work done to add four neutrons $= 16\varepsilon = 4^2\,\varepsilon$

In general, work done to add $\left(\frac{N-Z}{2}\right)$ neutrons

$$= \left(\frac{N-Z}{2}\right)^2 \varepsilon$$

$\therefore$ The total energy change $= \left(\frac{N-Z}{2}\right)^2 \varepsilon$.

This energy change is called asymmetry energy which contributes to the binding energy (E_a)

i.e., $$E_a = -\left(\frac{N-Z}{2}\right)^2 \varepsilon$$

Negative sign shows that this contribution decreases the binding energy

Using $N = A - Z$

$$E_a = -\left(\frac{A-2Z}{2}\right)^2 \varepsilon$$

But the spacing of energy level decreases with A, according to $\varepsilon \propto \frac{1}{A}$.

Thus the asymmetry energy contribution can be written as

$$E_a = -a_a \frac{(A-2Z)^2}{A}$$

Pairing energy

This correction term arises from the following observations of nuclides. We find that

(i) All nuclides having an even number of protons and even number of neutrons are generally the most stable of all.

(ii) All nuclides having an odd number of protons and an odd number of neutrons are the least stable of all.

(iii) Nuclides belonging to even-odd (even Z, odd N) or odd-even (odd Z, even N) category have an intermediate level of stability. Of the two categories of these even - odd nuclides are relatively more stable than odd-even nuclides.

Depending upon the pairing, stability of nuclides vary so also binding energy. For even-even nuclides all the protons as well as neutrons are paired off and this makes them the most stable. For example nuclei such as ${}^{4}_{2}He$, ${}^{12}_{6}C$, ${}^{16}_{8}O$ are most stable and appears as peak on the empirical curve of binding energy per nucleon. For odd-odd nuclides there is at least one unpaired proton and one unpaired neutron and this makes them least stable and have relatively low binding energies. For the even - odd or odd- even case we have one unpaired proton or one unpaired neutron and this makes them an intermediate stability. To take account of this pairing effect, pairing energy term is is produced. The pairing energy E_p is positive for even - even nuclei, for odd-even and even-odd nuclei and negative for odd-odd nuclei. On the basis of more detailed analysis, it is seen that, the pairing energy varies with A as $A^{-\frac{3}{4}}$.

i.e., $$E_p = a_p A^{-\frac{3}{4}}$$

where a_p is called the pairing energy constant.

Hence we have

$$E_p = a_p A^{-\frac{3}{4}} \qquad \text{for even-even}$$

$$E_p = -a_p A^{-\frac{3}{4}} \qquad \text{for odd-odd.}$$

$$E_p = 0 \qquad \text{for even-odd, and odd-even.}$$

Now taking all contributions to binding energy, we get

$$E_b = E_v + E_s + E_c + E_a + E_p$$

i.e., $$E_b = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_a \frac{(A-2Z)^2}{A} + E_p$$

This is called the semiempirical binding energy formula obtained by Von. Weizscaker in 1935. This is also called as Weizsacker formula. From this semi empirical mass formula can be written down.

The mass of nucleus is given by

$$^{A}_{Z}M = Zm_p + (A-Z)m_n - \frac{E_b}{c^2}$$

Substituting for E_b, we get

$$^{A}_{Z}M = Zm_p + (A-Z)m_n - \frac{1}{c^2}\left[a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_a \frac{(A-2Z)^2}{A} + E_p\right]$$

This is called the semiempirical mass formula. Experimental results give the following values of the constants occurring in the semi empirical mass formula.

$$a_v = 15.753 \text{ MeV}$$

$$a_s = 17.80 \text{ MeV}$$
$$a_c = 0.713 \text{ MeV}$$
$$a_a = 23.6925 \text{ MeV}$$
$$a_p = 33.6 \text{ MeV}.$$

However it must be kept in mind that no single unique set of values satisfies the equations for all known nuclides. Other sets of values of the constants may be found.

The semi empirical formula can account for the stability of nuclei against α and β decay and also it estimates the masses of wide range of nuclides. This shows that the liquid drop model is a good approximate model.

But liquid model fails to explain high stability of nuclei with magic numbers. This model also doesnot explain the measured spins and magnetic moments of the nuclei.

Example 8

Isobars are nuclides that have the same mass number A. Derive a formula for the atomic number of the most stable isobar of a given A and use it to find the most a stable isobar of A = 25, $a_c = 0.595$ and $a_a = 19$.

Solution

We have

$$E_b = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_c \frac{(A-2Z)^2}{A} + E_p$$

For stability E_b must be maximum.

i.e.,
$$\frac{dE_b}{dZ} = 0.$$

Different outing the above equation with respect to Z, we get

$$\frac{dE_b}{dZ} = -a_c A^{-1/3}(2Z-1) + \frac{4a_c}{A}(A-2Z) = 0$$

or
$$\frac{4a_c}{A}(A-2Z) = a_c A^{-1/3}(2Z-1)$$

$$4a_c A - 8a_a Z = a_c A^{2/3} 2Z - a_c A^{2/3}$$

$$4a_a A + a_c A^{2/3} = Z(2a_c A^{2/3} + 8a_a)$$

$$\therefore \quad Z = \frac{4a_a A + a_c A^{2/3}}{2a_c A^{2/3} + 8a_a}$$

For A = 25

$$Z = \frac{4 \times 19 \times 25 + 0.595 \times (25)^{2/3}}{2 \times 0.595 \times (25)^{2/3} + 8 \times 19}$$

$$Z = \frac{1900 + 5.087}{10.174 + 152} = \frac{1905.087}{162.174}$$

$$Z = 11.747 \simeq 12$$

so the nuclide is $^{25}_{12}Mg$ which is obviously a stable isobar.

Shell model

The nuclear shell model is one of the most important and useful models of nucleus structure. According to this model the nucleons (protons and neutrons) have been interpreted as forming closed shells of neutrons and protons in analogy with the filling of electrons shells in atoms and the neutron and proton shells appear to be independent of each other. This model is based on the assumption that there exists a common nuclear potential within the nucleus and all the nucleons move nearly independent within this common nuclear potential.

The emergence of shell model

According to atom model the electrons in an atom are occupying positions in shells designated by the various quantum numbers. Certain important aspects of atoms behaviour is determined by the number of electrons filled in the outer most shell. For example, atoms with 2, 10, 18, 36, 54 and 86 electrons have all their electron shells completely filled. Such electron structures have high binding energies and exceptionally stable, which accounts for the chemical inertness of the rare gases.

The same kind of effect is observed in nuclei. Nuclei that have 2, 8, 20, 28, 50, 82 and 126 neutrons or protons are stable. Stability is related to high binding energy and also to high natural abundance. This is experimentally found to be true for all numbers 2, 8, 20, 28, 50, 82 and 126. These numbers are commonly referred to as magic numbers. The above stated similarity between atoms and nuclei motivated scientist to propose shell model to nuclear structure.

Another interesting property exhibited by nuclei with magic N or Z is that with regard to electric quadrupole moments. The magnitude of the quadrupole moment is a measure of the deviation of a nucleus from spherical shape. i.e., A spherical nucleus has no quadrupole moment, while one shaped like football has positive moment and one shaped like a pumpkin has a negative moment. Nuclei of magic N and Z are found to have zero quadrupole moment and hence are spherical, while the other nuclei are distorted in shape.

1. Hydrogen $^{4}_{2}H$ (Z = 2, N = 2)
2. Oxygen $^{16}_{8}O$ (Z = 8, N = 8)
3. Sulphur $^{36}_{16}S$ (Z = 16, N = 20)
4. Calcium $^{40}_{20}C_{a}$ (Z = 20, N = 20)
5. Molybdenum $^{93}_{42}M_{o}$ (Z = 42, N = 50)
6. Lead $^{208}_{82}pb$ (Z = 82, N = 126)

Nuclei for which both Z and N are magic numbers are called doubly magic $^{4}_{2}H$, $^{16}_{8}O$, $^{40}_{20}C_{a}$ are doubly magic and are particularly tightly bound.

Nuclear energy levels from the shell model

According to shell model each nucleon is moving independently in a nuclear potential. But the exact form of the potential energy function is not known unlike the case of an atom. Hence a suitable function has to be assumed. A reasonable guess is a square well potential with rounded corners. Having thought of this nuclear potential, we can solve the Schrodinger equation for the motion of this nucleon. The results obtained are similar to those for electrons orbiting around the force field of the nucleus. We find that there exists a set of discreate allowed energy levels where each of these energy levels has a specific value of the principle quantum number n associated with it. We also find that we must also associate an orbital quantum number l with a given nucleon. This quantum number tells us about the discrete set of values of the orbital angular momentum. In essence the energy of a given nucleon is given by the quantum numbers n and l. Further like in the case of electrons we have to think of the nucleons having spin quantum numbers $\left(\pm\frac{1}{2}\hbar\right)$. This implies that both proton and the neutron are fermions and hence obey Paulis exclusion principle.

The similarity between these pictures of the electrons and nucleons implies that

(i) An energy state with a given value of l can accommodate a total of $2(2l+1)$ nucleons. For each value of l there are $2l+1$ sub states. The exclusion principle limits the number of neutrons or protons occupying a level to $2(2l+1)$.

(ii) We can designate the nuclear energy states as s, p, d, g, f for orbital angular momentum quantum number $l = 0, 1, 2, 3, 4$.

Explanation of magic numbers

We found that a nuclear energy level with a given value of l can accommodate $(2l+1)$ nucleons. So the nuclear closed shells can contain either 2 or $2+6=8$, or $2+6+10=18$, or $2+6+10+2=20$, or $2+6+10+2+14=34$ and so on. The magic numbers 2, 8, 20 are thus understood easily. The other magic numbers 28, 50, 82, 126 etc. however, cannot be explained on this basis. To circumvent this difficulty a new concept of strong interaction between the orbital angular momentum $\vec{L}$ and the spin angular momentum $\vec{S}$. There are two ways of interaction between $\vec{L}$ and $\vec{S}$. One is called LS coupling and the other is called J J coupling. The shell model assumes that LS coupling holds only for lightest nuclei and JJ coupling holds for heavier nuclear. In the case of LS coupling the spin angular momentum S_i of the particles (protons and neutrons separately) are coupled together into a total spin angular momentum S. The orbital angular momentum L_i are separately coupled together into a total orbital angular momentum $\vec{L}$. Then $\vec{L}$ and

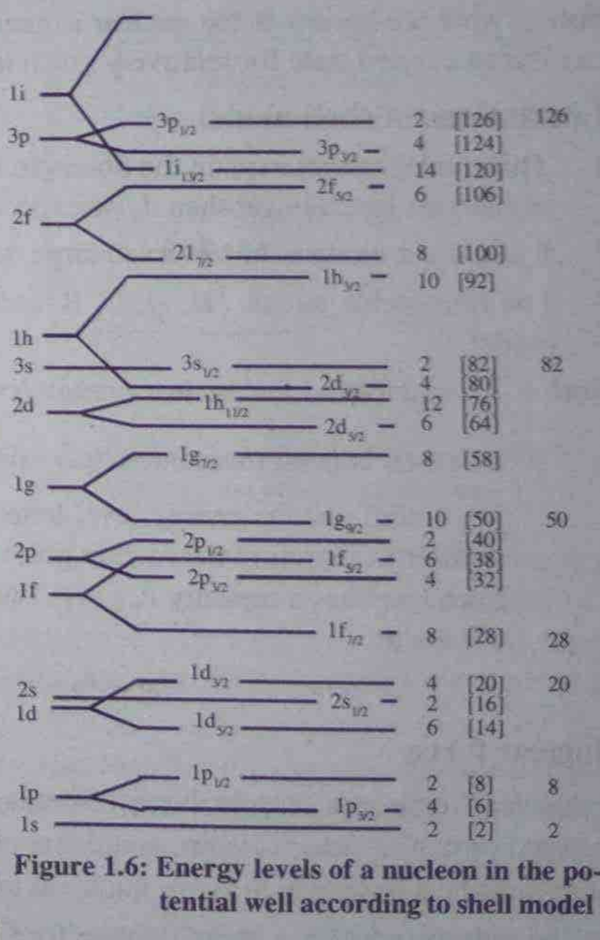

Figure 1.6: Energy levels of a nucleon in the potential well according to shell model

$\bar{S}$ are coupled to form a total angular momentum $\bar{J}$ with magnitude $\sqrt{J(J+1)}\,\hbar$. In the case of JJ coupling the S_i and L_i of each particle are first coupled to form a j_i for that particle of magnitude $\sqrt{j(j+1)}\,\hbar$. The various j_i are coupled to form the total angular momentum J. The modified picture then helps us to understand the whole magic numbers. See the nuclear energy level diagram (figure 1.6).

The merits of the shell model

The shell model has proved useful and successful in explaining and accounting for the observed

(i) Angular momentum.

(ii) Magnetic moment.

(iii) Electric quadrupole moment.

Besides these, the shell model also helps us to understand the observed distribution of what are known as the nuclear isomers. Nuclear isomers are nuclei that can exist in an excited state for relatively much longer times (half life $\geq$ 1s).

Limitations of shell model

1. This model cannot explain the observed first excited states in even-even nuclei at energies much lower than those expected from single particle excitation.
2. It could not explain the observed large quadrupole moments of odd nuclei.
3. The four stable nuclei ${}_1^2H$, ${}_3^6L_i$, ${}_5^{10}B$ and ${}_7^{14}N$ donot fit in the scheme of this model.

Note : To understand the nuclear energy level (figure above) note the following. The total angular momentum has values $j = l + \frac{1}{2}$ and $j = l - \frac{1}{2}$. Empirically it is found that the energy level with higher value of j lies below that with smaller j. Therefore the former gives a more tightly bound nucleonic state. Each level has a capacity $(2j+1)$. The ordering of l for all the unsplit levels of l are as

spdsfpgdshfpig

Nuclear force

Nuclear forces are responsible for keeping nucleons together. Repulsive force between protons inside nucleons would tear nucleons apart. Since nucleus is stable, there must be a force which keeps nucleons together is called nuclear force.

The only way to learn about nuclear force is from experiments. The scattering experiments of neutrons and protons and other varieties of experiments, the following characteristics of nuclear force have been noted.

Properties of nuclear force

1. The nuclear force is the strongest of the four known forces, hence called strong force. For two adjacent protons in a nucleus, the nuclear interaction is 10 - 100 times stronger than the electromagnetic interaction.
2. The nuclear force is short range.

   The distance over which the nuclear force acts is limited to about 10^{-15}m. This comes from the fact that central density of nuclear matter is constant. As we add nucleons to be nucleus, each added nucleon feels a force only from its neighbours and not from all other nucleons in the nucleons. In this respect, a nucleus behaves somewhat like a crystal, in which each atom interacts primarily with its neighbours and addition of atoms make the crystal larger but don't change its density. Another piece of evidence comes from the relation between nuclear binding energy and the separation distance of nucleons. It has been found that nuclear binding energy is a constant for separation distance is less than about 10^{-15}m and it is zero for separation distances greater than 10^{-15}m. (see figure).

   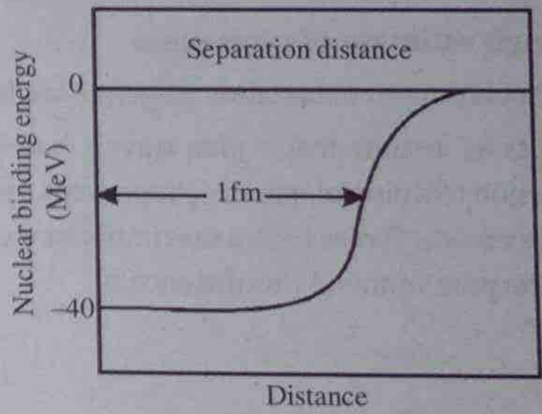

   Figure 1.7: Dependence of nuclear binding energy on separation distance

   From this we can conclude that for short range strong force, the binding energy is proportional to A because binding energy per nucleon is constant. For a force with long range (gravitational and electrostatic forces having infinite range) the binding energy is roughly proportional to the square of the number of interacting particles $(E_b \propto Z^2)$. This is because each of the Z protons in a nucleus feels the repulsion of the other $Z-1$ protons. Thus the total electrostatic energy of the nucleus is proportional to $Z(Z-1)$. For large Z it is Z^2.
3. The nuclear force between any two nucleons does not depend on whether the nucleons are protons or neutrons. The n-p nuclear force is the same as the n-n nuclear force which is in turn same as the p-p nuclear force.
4. The nuclear force is not completely central force. It depends on the orientation of the spins relative to the line joining the two nucleons. This property has been

deduced by noting that even in the simplest nucleus (deuterium), the orbital angular momentum of the two nucleons relative to the centre of mass is not constant whereas for central forces angular momentum is conserved.

A successful model for the origin of this short range force was put for ward by Japanese physicist Yukawa. According to this model some particles are responsible for nuclear force. These particles are called pions. Pions may be charge π^+, π^- or neutral π^0. According to this model every nucleon continuously emits and reabsorbs pions. The transfer of momentum associated with the shift of pion is equivalent to the action of a force. Thus nuclear force is the result of the exchange of pions between the nucleons. That is nuclear force is an exchange force.

Two nucleons experience an attractive force at small distance because of the virtual exchange of pions between them. The situation, qualitatively, is analogous to two dogs grappling for control over a bone. One dog grabs the bone from the other and then, the second snatching it back again. The continual exchange of the bone results in each dog pulled towards the other.

Rough estimate of pion mass

According to uncertainty principle we have $\Delta E\ \Delta t \sim \hbar$

Let us assume that a pion travels between nucleons at a speed $v \sim c$, that the emission of a pion of mass m_π represents a temporary energy uncertainty $\Delta E \sim m_\pi c^2$. Since nuclear forces have a maximum range r of about 1.7fm and the time Δt needed for the pion to travel this distance is

$$\Delta t = \frac{r}{v} \sim \frac{r}{c}$$

$$\therefore \qquad \Delta E\ \Delta t \sim \hbar$$

or $$m_\pi = \frac{\hbar}{rc} = \frac{1.05\times10^{-34}}{1.7\times10^{-15}\times3\times10^{8}} = 2\times10^{-28}\,\text{kg}$$

$$m_\pi = \frac{2\times10^{-28}\,m_e}{7.7\times10^{-31}} \simeq 220 m_e.$$

After 12 years of the prediction of pions, they were experimentally detected. The rest mass of the charged pions were found to be $273m_e$ and that of neutral pions to be $264m_e$. There were two reasons for the belated discovery of pion. One is that particles with kinetic energies of several hundred MeV energy was required to produce pions. This much energy producing accelerators were not available at that time. The second reason is the pions instability. The mean life time of the charged pion is only 2.6×10^{-8}s and that of the neutral pion is 8.4×10^{-17}s.

Radioactive decay

We found that for the lighter stable nuclei the proton and neutron numbers are roughly equal. However, for the heavy stable nuclei, the factor $Z(Z-1)$ in the Coulomb repulsion energy grows rapidly, so extra neutrons are required to supply the additional binding energy needed for stability. For this reason, all heavy stable nuclei have $N > Z$. Whenever the stability condition is upset they transform themselves into more stable nuclei by changing their Z and N through alpha decay or beta decay. Nuclei are unstable in excited states and they undergo gamma decay to achieve stability.

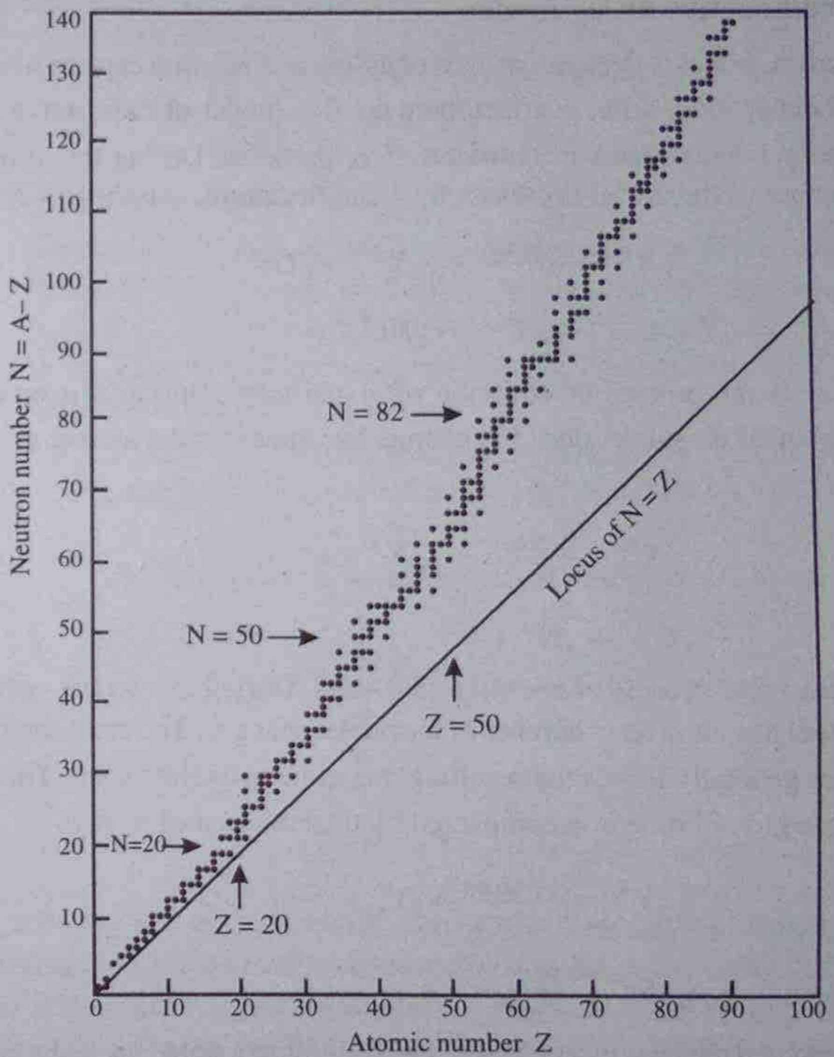

Figure 1.8: N-Z plot of stable nuclides

Which type of decay is occurring can be easily understood from the proton number versus neutron number plot. A graph is plotted between proton number (Z) on the horizontal axis and neutron number on the vertical axis by taking all known stable nuclides. We get a graph shown above. The dotted points represent the stable nuclides called stability curve.

Nuclei to the right of the stability line where the protons dominate over neutrons, they undergo β_+ decay. Nuclei to the right of the stability line where the neutrons dominate over protons, they undergo β_- decay. When the proton number is greater than $82 (Z > 82)$, they undergo alpha decay.

Different radioactive decay modes

Apart from α, β and γ decays, positron emission and electron capture were added to the list of decay modes. i.e. in effect there are five modes of radioactive decay.

1. Alpha decay is the process of emission of α particles. During this process the mass number of the nuclei decreases by 4 and the atomic number by 2.

   i.e. $${}_{Z}X^{A} \xrightarrow{\alpha\text{-decay}} {}_{Z-2}Y^{A-4} + {}_{2}He^{4}$$

   Example: $${}_{92}U^{238} \rightarrow {}_{92}Th^{234} + {}_{2}He^{4}$$

2. Beta decay is the process of emission of β-particles. During this process the mass number of the nuclei does not change but increases the atomic number by one.

   i.e. $${}_{Z}X^{A} \xrightarrow{\beta\text{-decay}} {}_{Z+1}Y^{A} + \bar{e}$$

   Example: $${}_{6}C^{14} \rightarrow {}_{7}N^{14} + \bar{e}$$

3. Gama decay is the process of emission of γ-rays. During this process neither the mass number nor the atomic number of the nuclei changes. The emission of α and β-particles generally leaves the resulting nuclei in an excited state. Then return to the stable ground state is accompanied by the emission of γ-rays.

   $$\left({}_{Z}Y^{A}\right)^{*} \xrightarrow{\gamma\text{-decay}} {}_{Z}Y^{A} + \gamma$$

   $$\left({}_{38}Sr^{87}\right)^{*} \rightarrow {}_{38}Sr^{87} + \gamma$$

4. When the excited nuclei is proton rich it often returns to a stable nuclei by capturing an orbital (K-shell) electron. This process is called electrons capture or K-capture.

The vacancy caused in the K.shell is filled by transition of electrons from higher orbits with subsequent emission of characteristic x-rays.

$$\left({}_{Z}X^{A}\right)^{*} + \bar{e} \xrightarrow[\text{capture}]{\text{electrons}} {}_{Z-1}Y^{A}$$

It is regarded as another form of β-decay since during this process the mass number remains the same and the atomic number decreases by one.

Example: $\quad {}_{29}Cu^{64} + \bar{e} \rightarrow {}_{28}Ni^{64}$

5. Positron emission is the process of emission positrons. During this process the mass number remains unchanged but atomic number decreases by one.

$${}_{Z}X^{A} \xrightarrow[\text{emission}]{\text{Positrons}} {}_{Z-1}Y^{A} + e^{+}$$

Example: $\quad {}_{29}Cu^{64} \rightarrow {}_{28}Ni^{64} + e^{+}$

Law of radioactive decay

The disintegration of radioactive nucleus was studied by Rutherford and Soddy in 1902. Whatever be the nature of decay, it is essentially a statistical process. In a given sample of a radioactive substance, the various identical nuclei take different times to decay. There is absolutely no way of predicting the moment of decay of any given nucleus in the sample.

In general, the statistical character of the decay process can be expressed as follows. If a radioactive sample contains N nuclei at a given instant the ratio of the rate of decay $\left(-\frac{dN}{dt}\right)$ to the number of nuclei present at that instant is a constant

Thus
$$\frac{-\frac{dN}{dt}}{N} = \lambda \text{ (constant)}$$

or
$$\frac{dN}{dt} = -\lambda N \qquad \text{..... (15)}$$

The constant λ is called the decay constant or disintegration constant. Its value is characteristics of the radioactive substance and signifies the decay probability of the sample. The negative sign indicates that N is decreasing with time. Equation (15) expresses the law of radioactive decay. The following interesting characteristics of the radioactive substance can be obtained from this law.

(i) Exponential decay

Equation (15) can be written in the form

$$\frac{dN}{N} = -\lambda\, dt$$

Integrating both sides, we get

$$\ln N = -\lambda t + C \qquad .....(16)$$

at $t = 0$, $N = N_0$

$\therefore \quad \ln N_0 = -\lambda \times 0 + C$

or $\quad C = \ln N_0$

$\therefore$ Eq (16) becomes

$$\ln N = -\lambda t + \ln N_0$$

or $\quad \ln N - \ln N_0 = -\lambda t$

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

i.e. $\quad \dfrac{N}{N_0} = e^{-\lambda t}$

$$\therefore \quad N = N_0 e^{-\lambda t} \qquad .....(17)$$

Half Life

The time interval during which half of the atoms of the given radioactive sample decay is called half life. It is denoted by T

i.e., when $t = T$, $N = \dfrac{N_0}{2}$

using $\quad N = N_0 e^{-\lambda t}$

$$\frac{N_0}{2} = N_0 e^{-\lambda t}$$

$$\frac{1}{2} = e^{-\lambda t}$$

$$2 = e^{\lambda T}$$

i.e., $\quad \ln 2 = \lambda T$

$$2.303 \times \log 2 = \lambda T$$

$$2.303 \times 0.3010 = \lambda T$$

$$0.693 = \lambda T$$

Hence $$T = \frac{0.693}{\lambda} \quad .....(18)$$

Thus half life of a radioactive substance is a characteristic property and is inversely proportional to the decay constant. It cannot be changed by any chemical or physical means. For example half life of radium is 1620 years, that of ${}_{92}U^{238}$ is 4.51×10^9 years and ${}_{84}Po^{214}$ is 10^{-6} s. In numerical problems, instead of specifying decay constant, usually half life of a nucleus is specified. The equation (18) is a convenient expression for relating half life to the decay constant. Note that after one half life, $N_0/2$ radioactive nuclei remain; after two half lives $N_0/4$ radioactive nuclei are left; after three half lives, $N_0/8$ are left; and so on. In general, the number of nuclei remaining after n half lives is $N_0/2^n$. The decay of a radioactive nucleus is a statistical process. If we take a radioactive sample of 1mg with a half life of 1h, about 50% of the 1 mg sample will decay in 1h ; during the second hour probability of decay is still 50%, for each remaining nucleus. The total probability that a given nucleus did not decay is $0.5 \times 0.5 = 0.25$ or 25%. The probability of decay is 75%, which is a fraction of the original nucleus expected to be disintegrated in 2 h.

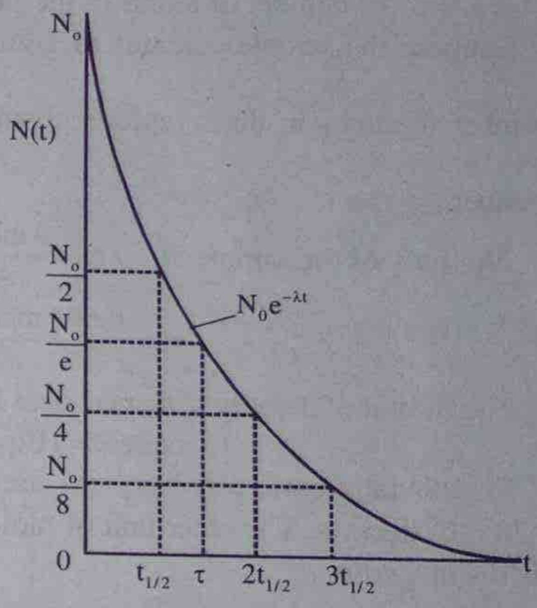

Figure 1.9

Note : After n half lives $$N = N_0\left(\frac{1}{2}\right)^n$$

we have $$N = N_0e^{-\lambda t}$$

If $$\lambda = \frac{1}{t}, N = N_0e^{-1}$$

or $$N = \frac{N_0}{e} = \frac{N_0}{2.718} = 0.368N_0 = 36.8\%N_0$$

Thus decay constant may be defined as the inverse of the time during which the number of the atoms in the radioactive sample is reduced to 36.8% of their original value.

Activity of the given sample

The number of disintegrations per second of the radioactive sample is called activity. It is denoted by R.

i.e.,
$$R = \frac{dN}{dt} = \lambda N$$
where N is the number of atoms of the sample

Suppose the sample contains m grams of the radio active element $_{Z}X^{A}$. The number of atoms in the sample is given by $N = \frac{mNa}{A}$ where Na is the Avagadro number,

$\therefore$ Activity of the sample $R = \lambda N = \frac{\lambda mNa}{A}$

$$R = \frac{0.693}{T}\frac{mNa}{A}$$

The SI unit of activity is named after Becquerel.

1becquerel=1Bq=1decay/s

Traditionally curie (ci) has been used as the unit of activity, 1curie = 1Ci = 3.70×10^{10} decay/s. The other unit of radioactivity is rutherford (Rd), 1rutherford = 10^{6} disintegration /s.

The mean life time: Mean life (average life) τ is defined as the average time the nucleus survives before it decays. Mean life can be determined by calculating total life time of all the nuclei initially present (N_0) and dividing it by total number of nuclei. Let the number of nuclei decaying in time intervals t and t+dt be dN, and the life time of these nuclei be t. Then the total life time of these nuclei is t dN.

The total life time of all the nuclei $\int_{N=N_0}^{N} tdN$

From equation (3), $dN = N_0 \lambda e^{-\lambda t} dt$

Now recognising the fact that N = 0 for $t \to \infty$ and $N = N_0$ for $t \to 0$, total life time of all the nuclei is

$$= \int_{N=N_0}^{N} tdN = \int_0^{\infty} tN_0 \lambda e^{-\lambda t} dt = N_0 \lambda \int_0^{\infty} te^{-\lambda t} dt = \frac{N_0}{\lambda}$$

(we can carry out integration by parts to get the result). Thus the mean can be obtained by dividing the total life of all the nuclei to decay by the total number of nuclei. Thus,

$$\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693}$$

Radioactive carbon dating : Radioactive ^{14}C is produced in our atmosphere by the bombardment of ^{14}N by cosmic rays.

$$n + {}^{14}N \rightarrow {}^{14}C + p$$

The ratio of ^{14}C to ^{12}C in the carbon dioxide molecules of our atmosphere has a constant value of approximately 1.3×10^{-19}. All the living organisms exchange carbon dioxide from atmosphere; hence a constant ratio of ^{14}C to ^{12}C is maintained in them. When an organism dies, it does not absorb ^{14}C from atmosphere, hence the ratio of ^{14}C to ^{12}C decreases as a result of beta decay of ^{14}C, which has a half life of 5730 year. It is therefore possible to measure the age of a material by measuring its activity caused by radioactive ^{14}C.

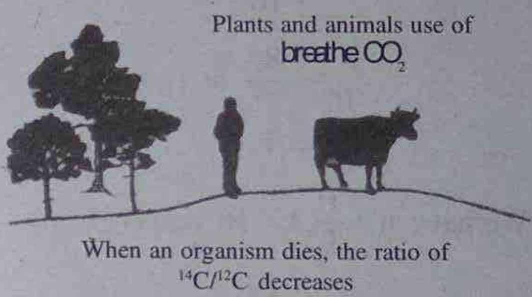

Figure 1.10

Example 9

The half life of Radon is 3.8 days. Calculate how much of 15 milligram of Radon will remain after 38 days.

Solution

T = 3.8 days, t = 38 days

Number of half lives in 38 days is

$$n = \frac{38}{3.8} = 10$$

$$N = N_o \left(\frac{1}{2}\right)^n$$

$$= 15 \times \left(\frac{1}{2}\right)^{10} = 0.014\text{mg}$$

Example 10

Find the decay constant of a sample of the radio active sample whose activity becomes $\frac{1}{16}$th in 10 years

Solution

Since activity is proportional to the number of atoms in the sample. Therefore activity after n half lives is given by

$R = \frac{R_o}{2^n}$, where R_o is the activity in the beginning

Here $R = \frac{R_o}{16}$

$\therefore \quad \frac{R_o}{16} = \frac{R_o}{2^n}$ or $16 = 2^n$

i.e., $n = 4$

We have $n = \frac{t}{T}$, t = 10 years (given)

$$4 = \frac{10}{T}$$

$$T = 2.5 \text{ years}$$

$\therefore$ decay constant $\lambda = \frac{0.693}{2.5} = 0.277/\text{year}$

Example 11

The half life of a radio active sample is 4 days. What fraction of 1 gram sample will remain after 20 days.

Solution

Amount of substance left after n half lives is given by

$$M = \frac{M_n}{2^n}$$

$$M_o = 1g,\ n = \frac{20}{4} = 5$$

Hence $M = \frac{M_o}{2^n} = \frac{1}{2^5} = 0.03125g$

Example 12

A radioactive substance has a half life period of 30 days. Calculate the time taken for 3/4 of original number of atoms to distintegrate.

Solution

$$T = 30\text{days}$$

$$N = \frac{N_o}{4}$$

Number of half lives $n = \frac{t}{T} = \frac{t}{30}$

Using $N = \frac{N_o}{2^n}$

$$\frac{N_0}{4} = \frac{N_0}{2^n}$$

$$4 = 2^n$$

$$n = 2$$

or $\frac{t}{30} = 2$

$$t = 60\text{days}$$

Example 13

Determine the amount of ${}_{84}Po^{210}$ having activity equal to 5 millicurie. The half life of Po is 138 days.

Solution

Activity of the sample

$$R = \frac{0.693}{T}\left(\frac{mNa}{A}\right)$$

$R = 5\text{mci} = 5 \times 10^{-3} = 18.5 \times 10^{7}\, s^{-1}$

$(1\text{ci} = 3.7 \times 10^{10} s^{-1})$

$m = ?\quad Na = 6.025 \times 10^{23}, A = 210$

$$T = 138 \times 24 \times 60 \times 60\text{s}$$

$$m = \frac{T \times R \times A}{0.693 \times Na}$$

$$= \frac{138 \times 24 \times 60 \times 60 \times 18.5 \times 10^7 \times 210}{0.693 \times 6.025 \times 10^{23}}$$

$$= 1.1 \times 10^{-6} \text{ g}$$

Example 14

The disintegration rate of a certain radioactive sample at any instant is 4750 disintegration per minute. Five minute later the rate of disintegration becomes 2700 disintegrations per minute. Calculate the half life of the sample.

Solution

$$\frac{dN_o}{dt} = -\lambda N$$

At $t = 0$ $\quad \frac{dN_o}{dt} = -\lambda N_0$

After 5 minutes $\frac{dN}{dt} = -\lambda N$

Hence $\quad \frac{N_0}{N} = \frac{(dN_0/dt)}{(dN/dt)} = \frac{4750}{2700} = 1.759$

Using $\quad N = N_0 e^{-\lambda t}$

or $\quad \lambda = \frac{1}{t} \times 2.303 \log\left(\frac{N_0}{N}\right) = \frac{1}{5} \times 2.303 \log(1.759)$

$$= 0.1130/\text{minutes}$$

$$T = \frac{0.693}{\lambda} = \frac{0.693}{0.1130} = 6.13/\text{minutes}$$

Example 15

A piece of burnt wood of mass 20g is found to have a C^{14} activity of 4 decays/s. How long has the tree that this wood belonged to been dead. Given $T_{1/2}$ of $C^{14} = 5730$ year.

Solution

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{5730 \times 3.17 \times 10^7} = 3.83 \times 10^{-12} \text{ s}^{-1}$$

$\because \quad 1 \text{ year} = 3.17 \times 10^7 \text{ s}$

To find the number of C^{14} nuclei in 20g of burnt wood, we first calculate the number of C^{12} nuclei in 20g of carbon (burnt wood)

Thus $$N(C^{12}) = \frac{6.02\times10^{23}}{12}\times 20 \approx 10^{24}$$

Now assuming that the ratio of C^{14} to C^{12} is 1.3×10^{-12}, the number of C^{14} nuclei in 20 g before decay is,

$$N_0(C^{14}) = (1.3\times10^{-12})(10^{24}) = 1.3\times10^{12}$$

We thus have for the initial activity of the sample

$$R_0 = N_0\lambda = (1.3\times10^{12})\times(3.83\times10^{-12})$$

$$= 4.979\text{decay/s} \approx 5\text{decay/s}$$

The age of the sample can now be calculated from the releation,

$$R = R_0 e^{-\lambda t}$$

or $$e^{\lambda t} = \frac{R_0}{R}$$

or $$t = \frac{1}{\lambda}\ln\left(\frac{R_0}{R}\right)$$

It is given that R = 4 decay/s and we have calculated $R_0 = 5\text{decay/s}$.

Thus $$t = \frac{1}{\lambda}\ln\left(\frac{5}{4}\right) = \frac{\ln(1.25)}{3.83\times10^{-12}}\text{s}$$

$$= \frac{0.223}{3.83\times10^{-12}}\text{s} = 0.58\times10^{11}\text{s} = 1842\text{year}$$

Example 16

Find the activity of 1mg of radon whose atomic mass is 222u. $T_{1/2} = 3.8$ days.

Solution

$$T_{1/2} = 3.8 \text{ days} = 3.8\times86400\text{s}$$

$$\therefore \quad \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{3.8\times86400} = 2.11\times10^{-6}\text{s}^{-1}$$

The number of atoms in 1mg of radon.

$$N = \frac{1\times10^{-3}}{222u} = \frac{10^{-3}}{222\times1.66\times10^{-27}} = 2.71\times10^{18}$$

$$\therefore \quad \text{Activity } R = \lambda N = 2.11\times10^{-6}\times2.71\times10^{18}$$

$$= 5.72\times10^{12}\,\text{decay s}^{-1}$$

$$= \frac{5.72\times10^{12}}{3.7\times10^{10}}\ \text{Ci}$$

$$= 155\ \text{Ci}$$

Example 17

Find the probability that a particular nucleus of ^{38}Cl will undergo beta decay in any 1 second period. The half life of ^{38}Cl is 37.2 minutes.

Solution

$$T_{1/2} = 37.2\ \text{minutes} = 37.2\times60\text{s}$$

$$\therefore \quad \lambda = \frac{0.693}{37.2\times60} = 3.10\times10^{-4}\,\text{s}^{-1}$$

$$\therefore \quad \text{Probability} = 3.10\times10^{-4}$$

Example 18

The half life of the alpha emitter ^{210}Po is 138 days. What mass of polonium (210) is needed for a 10mCi source.

Solution

$$T_{1/2} = 138\ \text{days} = 138\times86400\text{s}$$

$$\therefore \quad \lambda = \frac{0.693}{138\times86400} = 5.812\times10^{-8}\ \text{decay s}^{-1}$$

we have $R = \lambda N \qquad R = 10\text{m Ci (given)}$

$$R = 10\text{m Ci} = 10^{-2}\,\text{Ci} = 10^{-2}\times3.7\times10^{10}\ \text{decay s}^{-1}$$

$$= 3.7\times10^{8}\ \text{decay s}^{-1}$$

$$\therefore \quad N = \frac{R}{\lambda} = \frac{3.7\times10^{8}}{5.812\times10^{8}} = 6.366\times10^{15}\ \text{atoms}$$

$$\therefore \quad \text{Mass of } 6.366\times10^{15} \text{ atoms} = \frac{6.366\times10^{15}\times210}{6.02\times10^{23}}$$

$$= 22.2\times10^{-7}\,\text{g}$$

$$= 22.2\times10^{-10}\,\text{kg}$$

Conservation laws in radioactive decay

Our study of radioactive decays and nuclear reactions reveals that these processes occurring in nature are not arbitrarily but according to certain laws. The laws are called conservation laws. These laws put some limitations on the outcome of the processes. At the same time these laws give us important insight into the fundamental workings of nature. There are five conservation laws that can be applied to radioactive decays.

1. Conservation of energy

This states that energy before decay is equal to energy after decay. This enables us to calculate rest energies or kinetic energies of decay products. Consider a nucleus X decay into lighter nuclei X' with the emission of one or more particles we call it collectively as x. Let m_X be the rest mass of the nucleus X, $m_{X'}$ be the rest mass of the nucleus after decay and m_x be the rest energy of the emitted particles.

According to the law of conservation of energy.

$$m_X c^2 = m_{X'} c^2 + m_x c^2 + \text{ Energy released}$$

Energy released in the form of kinetic energy of decay products. This is usually called as the Q value.

Thus

$$m_X c^2 = m_{X'} c^2 + m_x c^2 + Q \qquad \text{..... (19)}$$

or

$$Q = [m_X - (m_{X'} + m_x)]c^2 \qquad \text{..... (20)}$$

The decay is possible only Q values is positive. For this to be possible only if the rest energy of the nucleus X is greater than the total rest energy of the decay products X' and x

i.e. $\quad m_X > m_{X'} + m_x$

If the Q value appears as kinetic energy of the decay products

$$Q = K_{X'} + K_x \qquad \text{..... (21)}$$

2. Conservation of linear momentum

The law states that the momentum of decaying nucleus before is equal to the momentum of the decay products. If the decaying nucleus is at rest initially it momentum is zero. After the decay the total momentum is $\vec{p}_{X'} + p_x$.

According to the law

Momentum before decay = Momentum after decay

i.e., $$0 = \vec{p}_{X'} + \vec{p}_x \qquad \text{..... (22)}$$

Usually the emitted particles x are less massive than the residual nucleus X′. Thus the recoil momentum $p_{X'}$ yields a very small kinetic energy $K_{X'}$. This is because

$$K_{X'} = \frac{p_{X'}^2}{2m_{X'}}$$

and $$K_x = \frac{p_x^2}{2m_x}$$

$$\therefore \quad \frac{K_{X'}}{K_x} = \frac{p_{X'}^2}{2m_{X'}} \frac{2m_x}{p_x^2}$$

$$\frac{K_{X'}}{K_x} = \frac{m_x}{m_{X'}} \qquad (\because \quad p_{X'} = -p_x)$$

since $m_x < m_{X'}$, we get $K_{X'} < K_x$.

If there is only one emitted particle, solving equations 21 and 22 we get the values $K_{X'}$ and K_x. If two or more particles are emitted we get no unique solution. In this case a range of values from some minimum to some maximum is permitted for the decay products.

3. Conservation of angular momentum

This law states that the total spin angular momentum of the initial particle before the decay must be equal to the total angular momentum of all of the product particles after decay.

For example

$$n \rightarrow p + e^-$$

$$\frac{1}{2} \rightarrow \frac{1}{2} \pm \frac{1}{2}$$

Here total angular momentum is not conserved. Adding integer units of orbital angular momentum to the electron does not restore conservation.

4. Conservation of electric charge

This law states the total charge before decay must be equal to the total charge of decay products.

5. Conservation of nucleon number

This law states that the total nucleon number A does not change in decay processes. In this protons can be transformed into neutrons or vice versa but N + Z must be remain invariant.

Alpha decay

We found that in alpha decay an unstable nucleus decays into a lighter nucleus and an alpha particle. Thus we have

$${}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}X' + \alpha \qquad (\alpha = {}^{4}_{2}He)$$

This decay process releases energy in the form of kinetic energy of the decay products. The Q value of the process is

$$Q = [m_X - (m_{X'} + m_\alpha)]c^2$$

Here all m's are nuclear masses

and $$Q = K_{X'} + K_\alpha \qquad \text{..... (24)}$$

From the law of conservation of linear momentum, we have

$$O = \vec{p}_{X'} + \vec{p}_\alpha$$

$$O = m_{X'} v_{X'} + m_\alpha v_\alpha$$

or $$m_{X'} v_{X'} = -m_\alpha v_\alpha$$

But $$K_{X'} = \frac{1}{2} m_{X'} v_{X'}^2 = \frac{1}{2} m_{X'} \left(\frac{-m_\alpha v_\alpha}{m_{X'}} \right)^2$$

$$K_{X'} = \frac{1}{2} \frac{m_\alpha^2 v_\alpha^2}{m_{X'}}$$

or
$$m_{X'}K_{X'} = \frac{1}{2}m_\alpha^2 v_\alpha^2 = \frac{1}{2}m_\alpha v_\alpha^2 \cdot m_\alpha$$

$$m_{X'}K_{X'} = K_\alpha m_\alpha \qquad \left(\because\; K_\alpha = \frac{1}{2}m_\alpha v_\alpha^2\right)$$

Using $m_\alpha = 4$ and $m_X \simeq A-4$, we get

$$(A-4)K_{X'} = 4K_\alpha$$

$$\therefore \qquad K_{X'} \simeq \frac{4K_\alpha}{A-4} \qquad \text{..... (25)}$$

Putting the value of $K_{X'}$ in equation 24, we get

$$Q \simeq \frac{4K_\alpha}{A-4} + K_\alpha$$

$$Q \simeq \frac{AK_\alpha}{A-4}$$

$$\therefore \qquad K_\alpha \simeq \frac{A-4}{A}Q \qquad \text{..... (26)}$$

Putting the value of K_α in equation 25, we get

$$K_{X'} \simeq \frac{4}{A}Q$$

For heavy nuclei, A is large and $A - 4 \simeq A$

Then $K_\alpha \simeq Q$ and $K_{X'} = 0$

This shows that α particles emitted carry off practically all of the disintegration energy available in the form of kinetic energy.

Note: It may be noted that all masses appearing in Q value are nuclear masses but while performing calculation we use standard values of atomic masses. So all nuclear masses must be replaced by atomic masses. In this manipulation electron masses will automatically cancel out.

Thus $\qquad Q = [m(X) - m(X') - m(\alpha)]c^2.$

Example 19

Find the kinetic energy of alpha particle emitted in the decay of ^{234}U.

$m(U) = 234.040952\,u$, $m(Th) = 230.033134\,u$ and $m(\alpha) = 4.002603\,u$.

Solution

Kinetic energy of alpha particle emitted, $K_\alpha = \frac{A-4}{A}Q$.

The decay process is

$$^{234}U \longrightarrow {}^{230}Th + {}^{4}_{2}He$$

$$\therefore \quad Q = [m(U) - m(Th) - m(\alpha)]c^2$$

$$Q = [234.040952\,u - 230.033134\,u - 4.002603\,u]c^2$$

$$Q = 0.005125\,u\,c^2$$

Using $\quad 1u = 931.5\frac{MeV}{c^2}$

$$Q = 0.005125 \times 931.5\text{ MeV} = 4.858\text{ MeV}$$

$$\therefore \quad K_\alpha = \frac{A-4}{A}Q = \frac{(234-4)}{234} \times 4.858\text{ MeV}$$

$$K_\alpha = 4.775\text{ MeV}$$

Example 20

Check whether ^{203}Hg undergoes alpha decay. Given $m(Hg) = 202.972872\,u$, $m(Pt) = 198.970593\,u$ and $m(\alpha) = 4.002603\,u$.

Solution

The decay process is

$$^{203}Hg \longrightarrow {}^{199}Pt + {}^{4}_{2}He$$

The value is

$$Q = [m(Hg) - m(Pt) - m(\alpha)]c^2$$

$$Q = [202.972872\,u - 198.970593\,u - 4.002603\,u]c^2$$

$$Q = -0.000324\,u\,c^2$$

$$Q = -0.000324 \times 931.5\ M\,c^2$$

$$Q = -0.3018\,MeV.$$

Since Q value is negative, this decay is not permitted.

Quantum theory of alpha decay

In alpha decay of heavy nuclei, two protons and two neutrons come together to form an α particle. As long as α particle is inside the nucleus it is acted upon by short range nuclear force which dominates over coulombian repulsion. Once α particle is ejected out of nucleus it is acted upon by the coulomb force due to daughter nucleus. The variation of potential energy (V) of α particle with distance from the centre of the atom is shown in figure below.

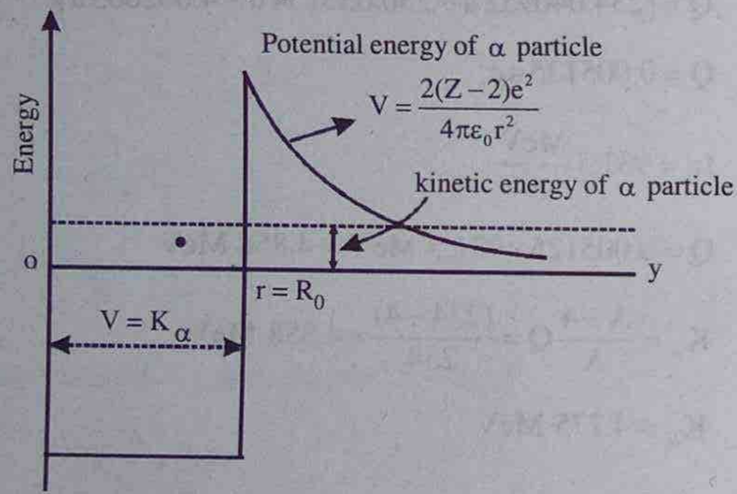

Figure 1.11

Outside the nucleus i.e. $r > R_0$, the potential energy V of α particle is due to coloumb force between nucleus of daughter atom and α particle. If Z is the atomic number of parent atom.

$$V = \frac{1}{4\pi\varepsilon_0} 2 \frac{(Z-2)e^2}{r} \qquad \text{..... (27)}$$

where ε_0 is the permittivity of free space. The change from the attractive nuclear force to coulomb force takes place at $r = R_0$, the radius of the nucleus.

Now consider α decay of $^{228}_{88}Rn$. According to the equation

$$^{228}_{88}Rn \rightarrow\ ^{224}_{86}X +\ ^{4}_{2}He + Q$$

The potential energy on the surface of nucleus

$$V_S = \frac{2(Z-2)e^2}{4\pi\varepsilon_0 r} = \frac{9\times10^9\times2\times(88-2)\times(1.6\times10^{-19})^2}{10^{-14}}$$

$$\approx 25\text{MeV}.$$

But $\quad Q = (m_{228} - m_{224} - m_{\alpha})c^2$

$$Q \approx 5\text{MeV}.$$

This is the energy of the α-particle inside the nucleus. Here Q is much less than V_s. Classically this means that α-particle cannot escape from the nucleus. To escape from the nucleus α-particle must have an energy greater than 25 MeV. But quantum mechanics says that there is a probability of penetration of α-particle through the potential barrier V_s. This is known as the tunnelling effect in quantum mechanics. This idea was firstly put forward by Gamow in 1928 and is known as Gamow's theory of α-decay.

To develop the theory, three assumptions are made. They are

1. An α-particle may exist as an entity within a heavy nucleus.
2. α-particle is in constant motion and is held in the nucleus by a potential barrier.
3. There is a small but definite probability that the particle may tunnel through the barrier (despite its height) each time a collision with it occurs.

According to WKB perturbation theory of barrier penetration, the decay probability per unit time (λ) is given by

$$\lambda = \upsilon T \qquad \text{..... (28)}$$

where υ is the number of times per second an alpha particle strikes the potential barrier and T is the transmission probability of the α - particle.

If we assume that at any moment only one alpha particle exists as such in a nucleus and that it moves back and forth along a nuclear diameter.

Thus $\quad \upsilon = \frac{1}{t} = \frac{v}{x} = \frac{v}{2R_0}$

where v is the velocity of the α - particle when eventually leaves the nucleus and R_0 is the radius of the nucleus.

$$\therefore \qquad \lambda = \frac{v}{2R_0} T \qquad \text{..... (29)}$$

From the theory of tunnel effect, we have

$$T = e^{-2k_2L} \qquad \text{..... (30)}$$

with $k = \sqrt{\frac{2m(V-E)}{\hbar^2}}$ and L is the width of the barrier.

Thus $$\lambda = \frac{v}{2R_0} e^{-2kL} \qquad \text{..... (31)}$$

By making suitable rough estimates for the thickness (L) and height of the barrier (V), we can calculate the decay probability λ such that it tallies with the observed λ of isotopes of nuclei.

An exact calculation of the decay probability was first done in 1928 by George Gamow and was one of the first successful applications of the quantum theory.

Beta decay

Beta decay is the process by means of which a nucleus can alter its composition to become more stable. In beta decay, a neutron in the nucleus changes into a proton (or a proton into a neutron); Z and N each change by one unit, but A does not change. The emitted particles, which were called beta particles when first observed in 1898, were soon identified as electrons. There are three types of β-decays observed in nature. These are

1. The negatron emission or β^- decay
2. The positron emission or β^+ decay
3. Orbital electron capture.

We have already discussed these decays in brief. Here we shall discuss some more details of these.

Negatron emission or β^- decay

Consider the decay of a neutron into a proton and an electron.

$$n \rightarrow p + e^-$$

This decay appears to violate the law of conservation of angular momentum that we already discussed. Experiments have shown that the energy emitted by electrons varies continuously from zero to a maximum value. This implies apparent violation of law of conservation of energy, because all electrons should emerge from the decay $n \rightarrow p + e^-$ with precisely the same energy. Instead, all electrons emerge with less energy but in varying amounts.

To overcome these difficulties in 1930 Wolfgang Pauli suggested that there is a third particle emitted in the decay process. Since the charge is already conserved the third particle emitted must be chargeless. In order to preserve the conservation of angular momentum, the third particle must have spin $\frac{1}{2}$. The missing energy will be carried by the third particle. The new particle is named neutrino (little neutral one in Italian). Every particle has an antiparticle, the antiparticle of neutrino (υ) is antineutrino ($\overline{\upsilon}$). It is in fact an antineutrino is emitted in the decay process. The complete decay process is

$$n \rightarrow p + e^- + \overline{\upsilon}$$

Since the emitted electron has negative charge, this electron is called negatron and the process is called negatron emission or simply β^--decay. Consider a nucleus ${}^{A}_{Z}X$ undergoes a β^- decay giving a new nucleus ${}^{A}_{Z+1}X'$:

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}X' + e^- + \overline{\upsilon}$$

Using law of conservation of energy

$$m_X c^2 = m_{X'} c^2 + m_e c^2 + K_{X'} + K_e + K_{\overline{\upsilon}}$$

Here m_X and $m_{X'}$ are nuclear masses and $K_{X'} + K_e$ are kinetic energies of new nucleus X' and emitted electron respectively. $K_{\overline{\upsilon}}$ is the kinetic energy of the antineutrino. Usually the recoil kinetic energy ($K_{X'}$) of the nucleus X' is negligibly small. So the Q value is:

$$Q = K_e + K_{\overline{\upsilon}}$$

Thus the above equation becomes

$$m_X c^2 = m_{X'} c^2 + m_e c^2 + Q$$

or

$$Q = (m_X - m_{X'} - m_e)c^2 \qquad \text{..... (32)}$$

This the expression for the Q-value of β^--decay process.

To do the calculations we convert nuclear masses into atomic masses.

$$m_X = m(X) - Zm_e \quad \text{and} \quad m_{X'} = m(X') - (Z+1)m_e$$

where $m(X)$ and $m(X')$ are atomic masses.

Putting the values of m_X and $m_{X'}$ in equation 32, we get

$$Q = \left[m(X) - Zm_e - m(X') + (Z+1)m_e - m_e\right]c^2$$

or $$Q = \left[m(X) - m(X')\right]c^2 \qquad \text{..... (33)}$$

This is the expression for the Q-value of β^--decay process in terms of atomic masses.

Example 20

Consider β^--decay represented by ${}^{15}_{6}C \longrightarrow {}^{15}_{7}N + e^- + \bar{\upsilon}$

Given $m(C) = 15.010599\,u$, $m(N) = 15.000109\,u$. Calculate the Q-value.

Solution

The Q-value of the process is

$$Q = [m(C) - m(N)]c^2$$

$$Q = [15.010599\,u - 15.000109\,u]c^2$$

$$Q = 0.01049 \times u\,c^2$$

$$Q = 0.01049 \times 931.5\,\text{MeV}$$

$$Q = 9.771435\,\text{MeV}$$

Since Q is positive this process is possible.

Positron emission or β^+ decay

In this process a proton decays into a neutron, positive electron (positron) and a neutrino. The process is given by

$$p \longrightarrow n + e^+ + \upsilon$$

Since it emits a positron, the process is called positron emission or β^+-decay. In this process the proton number decreases by one but A remains unchanged.

Consider a nucleus ${}^{A}_{Z}X$ undergoes a β^+-decay giving a new nucleus ${}^{A}_{Z-1}X'$:

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}X' + e^+ + \upsilon$$

Using law of conservation of energy

$$m_X c^2 = m_{X'} c^2 + m_e c^2 + K_{X'} + K_{e^+} + K_\upsilon$$

where m_X and $m_{X'}$ are nuclear masses. $K_{X'}$, K_{e^+} and K_υ are the kinetic energies of the nucleus after decay, positron and neutrino respectively. Usually the recoil kinetic energy $(K_{X'})$ of the nucleus (X') is negligibly small. So the Q-value is:

$$Q = K_{e^+} + K_\upsilon$$

Thus the above equation becomes

$$m_X c^2 = m_{X'} c^2 + m_e c^2 + Q$$

$$\therefore \quad Q = (m_X - m_{X'} - m_e)c^2 \quad \text{..... (34)}$$

This is the expression for the Q-value of β^+-decay process.

Now we convert nuclear masses into atomic masses.

$$m_X = m(X) - Zm_e$$

and

$$m_{X'} = m(X') - (Z-1)m_e$$

where $m(X)$ and $m(X')$ are the atomic masses. Putting the values of m_X and $m_{X'}$ in equation (34), we get

$$Q = [m(X) - Zm_e - m(X') + (Z-1)m_e - m_e]c^2$$

$$Q = [m(X) - m(X') - 2m_e]c^2 \quad \text{..... (35)}$$

This is the expression for Q-value of β^+-decay process in terms of atomic masses.

Example 21

Consider the β^+-decay represented by

$${}^{107}_{49}In \longrightarrow {}^{107}_{48}Cd + e^+ + \upsilon$$

Given that $m(In) = 106.918263\,u$

$m(Cd) = 106.906618\,u$ $m_e = 0.0005486\,u$. Calculate the Q-value

Solution

The Q-value of the process is

$$Q = [m(In) - m(Cd) - 2m_e]c^2$$

$$Q = [106.918263\,u - 106.906618\,u - 2 \times 0.0005486\,u]c^2$$

$$Q = 9.825276 \text{ MeV}$$

Since Q is positive, this process is possible.

Electron capture

In this process the nucleus absorbs one of the orbital electrons of the atom. The electrons nearest to the nucleus are in K-shell. Therefore the nucleus is most likely to absorb an electron in K-shell. For this reason this capture is known as K-capture. The probability of a L or M shell electron to be absorbed by nucleus is much less than the probability of K-shell capture. The basic electron capture process is

$$p + e^- \longrightarrow n + \upsilon$$

In a nucleus K-capture can be represented as

$$^{A}_{Z}X + e^- \longrightarrow {}^{A}_{Z-1}X' + \upsilon$$

In this process the proton number decreases by one and A remains the same. Using law of conservation of energy, we have

$$m_X c^2 + m_e c^2 + K_e = m_{X'} c^2 + K_{X'} + K_\upsilon$$

Neglecting initial kinetic energy of the electron and the recoil kinetic energy of the nucleus, the Q-values energy of the nucleus, the Q-values becomes:

$$Q = K_\upsilon$$

Thus we have

$$m_X c^2 + m_e c^2 = m_{X'} c^2 + Q$$

$$\therefore \quad Q = [m_X + m_e - m_{X'}]c^2 \qquad \text{..... (36)}$$

This is the expression for Q-value of the electron capture process.

Now convert nuclear masses into atomic masses

$$m_X = m(X) - Zm_e$$

$$m_{X'} = m(X') - (Z-1)m_e$$

Putting the value of m_X and $m_{X'}$ in equation 36, we get

$$Q = [m(X) - Zm_e + m_e - m(X') + (Z-1)m_e]c^2$$

$$Q = [m(X) - m(X')]c^2 \qquad \text{..... (37)}$$

This is the expression for the Q-value of electron capture process in terms of atomic masses.

In contrast to beta-decay processes, a mono energetic neutrino is emitted in electron capture. It may also be noted that if $m(X) > m(X')$ electron capture will occur. If $m(X) > m(X') + 2m_e$, β^+-decay process occurs.

Example 22

^{75}Se decays by electron capture to ^{75}As. Find the energy of the emitted neutrino. $m(Se) = 74.922523\,u$, $m(As) = 74.921596\,u$.

Solution

The Q-value of electron capture = energy of emitted neutrino

$$Q = [m(Se) - m(As)]c^2$$

$$Q = 74.922523\,u - 74.921596\,u]c^2$$

$$Q = 0.8635\,\text{MeV}.$$

Gamma decay

Like atoms, nuclei also have excited states. When a nucleus jumps from an excited state to ground state, it emits a photon (γ-ray) whose energy is equal to the energy difference between the excited state and the ground state. The energies of emitted gamma rays are typically in the range of 100 KeV to a few MeV.

When a nucleus emits an alpha particle or beta particle, the final nucleus may be left in an excited state. Most excited nuclei have very short half lives in the order of 10^{-9}s to 10^{-12}s. After this the excited nucleus comes back to the ground state after emitting one or two photon known as nuclear gamma rays.

There are few cases of excited states with half lives of hour, days or even years.

Along lived excited state is called an isomer of the same nucleus in its ground state. The excited nucleus $\left({}^{87}_{38}\mathrm{Sr}\right)^*$ has a half life of 2.8 hours. Thus Sr^* is an isomer of Sr. Similarly $\left({}^{79}_{35}\mathrm{Br}\right)^*$ and $\left({}^{81}_{25}\mathrm{Br}\right)^*$ are isomeric states of ${}^{80}_{35}\mathrm{Br}$.

Consider a nucleus jumps from an initial state with energy E_i to final state of energy E_f emitting a γ-ray of energy E_γ. According to law of conservation of energy, we have

$$E_i = E_f + E_\gamma + K_R$$

where K_R is the recoil kinetic energy of the nucleus. Usually K_R is negligibly small.

Thus $$E_i = E_f + E_\gamma$$

or $$E_\gamma = E_i - E_f$$

This shows that the gamma-ray energy is equal to the difference between the initial and final energy states.

Assume that before the emission, nucleus is at rest so its momentum zero. When its a γ-ray of momentum p_γ normally nucleus will recoil to conserve momentum. Let p_R be the recoil momentum of the nucleus. Conservation of momentum gives

$$0 = \vec{p}_R + \vec{p}_\gamma$$

or $$\vec{p}_R = -\vec{p}_\gamma$$

The recoil energy $$K_R = \frac{p_R^2}{2m}$$

where m is the mass of the nucleus

or $$K_R = \frac{p_\gamma^2}{2m} \qquad (\because \quad p_R = -p_\gamma)$$

or $$K_R = \frac{p_\gamma^2 c^2}{2mc^2} = \frac{(p_\gamma c)^2}{2mc^2} = \frac{E_\gamma^2}{2mc^2}$$

If $E_\gamma = 1\text{MeV}$ and $m = 100\,u$

$$K_R = \frac{1(\text{MeV})^2}{2\times 100\,u\,c^2}$$

$$1\,u = 931.5\frac{\text{MeV}}{c^2}$$

$$\therefore \quad K_R = \frac{(\text{MeV})^2}{2\times 100\times 931.5\ \text{MeV}}$$

$$K_R = \frac{\text{MeV}}{200\times 931.5} = \frac{10^6\,\text{eV}}{200\times 931.5} = 5.368\,\text{eV}$$

This is obviously negligibly small.

Example 23

The nucleus ^{198}Hg has excited states at 0.412 and 1.088 MeV. Following the beta decay of ^{198}Au to ^{198}Hg, three gamma-rays are emitted. Find the energies of these three gamma rays.

Solution

The energies of the emitted gamma rays are

E_3	______	1.088 MeV
E_2	______	0.412 MeV
E_1	______	0 MeV

$$\Delta E_{21} = E_2 - E_1 = 0.412 - 0 = 0.412\ \text{MeV}$$

$$\Delta E_{31} = E_3 - E_1 = 1.088 - 0 = 1.088\ \text{MeV}$$

$$\Delta E_{32} = E_3 - E_2 = 1.088 - 0.412 = 0.676\ \text{MeV}$$

Here we neglected the small recoil energy.

Example 24

Compare the recoil energy of a nucleus of mass 200 that emits (a) a 5.0 MeV alpha particle and (b) a 5.0 MeV gamma ray.

Solution

(a) When a nucleus emits an alpha particle its Q-value is:

$$Q = K_{X'} + K_\alpha$$

where $K_{X'}$ is the recoil energy of the nucleus

$$\therefore \quad K_{X'} = Q - K_\alpha$$

Using
$$Q = \frac{A}{A-4} K_\alpha$$

$$K_{X'} = \frac{A}{A-4} K_\alpha - K_\alpha = \frac{4}{A-4} K_\alpha$$

$$K_{X'} = \frac{4 \times 5}{200-4} = \frac{20}{196} = 0.102 \text{ MeV.}$$

(b) The gamma-ray recoil energy is given by:

$$K_R = \frac{E_\gamma^2}{2mc^2} = \frac{(5.0 \text{ MeV})^2}{2 \times 200 \, \text{u} \, c^2}$$

$1 \, \text{u} = 931.5 \text{ MeV}/c^2$

$$K_R = \frac{(5.0 \text{ MeV})^2}{400 \times 931.5 \dfrac{\text{MeV}}{c^2} \cdot c^2}$$

$$K_R = \frac{25}{400 \times 931.5} \text{ MeV}$$

$$K_R = 67.096 \text{ eV.}$$

Natural radioactivity

The radioactivity exhibited by naturally occurring elements is called natural radioactivity. It takes place spontaneously without any external causation i.e. no cause effect relationship (causality) is involved in the decay process. It is exhibited by the nuclei with $Z > 82$. Some examples are radium, actinium, polonium, etc. The ultimate product of the natural radioactivity is lead with $Z = 82$.

Certain nuclei which are stable can be converted into unstable isotopes by bombarding them with fast moving particles. The unstable isotope so formed undergo radioactive decay. Thus, the phenomenon of radioactive decay by artificial means is called artificial radioactivity. The half life of the artificially produced isotope may be short and the decay may stop as soon as the bombardment with fast moving particles ceases. The phenomenon of artificial radioactivity that persists long after the bombardment with fast moving particles ceases is called induced radioactivity.

Cause of radioactivity

Unstability of nucleus is the main cause of radioactivity. We know that the nuclei with higher atomic number have low binding energy per nucleon. Also due to the presence of large number of protons in the nucleus, the Coulombian repulsion is also stronger. Owing to these three reasons nuclei with $Z > 82$ spontaneously decay emitting radiations so as to achieve stability.

Cause of natural radioactivity surrounds us

All of the elements beyond the very lightest (hydrogen and helium) were produced by nuclear reactions in the interiors of stars. These reactions not only produce stable elements but also radioactive ones. Most radioactive elements have half-lives that are much smaller than the age of the earth (about 4.5×10^9 y), so those radioactive elements that may have been present when the earth was formed have decayed to stable elements. However, a few of the radioactive elements created long ago have half-lives that are greater than the age of the earth. These elements can still be observed to undergo radioactive decay and account for part of the back ground of natural radioactivity that surrounds.

Radioactive series

When a radioactive substance (parent) emits an α or β particle an entirely new substance (product) is left which is still radioactive and sooner or later emit another particle to become a still different atom. The product has different chemical and physical properties from the parent. This process continues through a series of elements ending up finally with an atom which is stable and not radioactive. This is called radioactive series.

It has been experimentally observed that most of the radioactive substances found in nature are members of four series namely thorium series, neptunium series, uranium series and actinium series. The mass numbers of above four known series could be represented by the following set of numbers. 4n for the thorium series, 4n + 1 for the neptunium series, 4n + 2 for the uranium series and 4n + 3 for the actinium series, where n is an integer, see the table below. It may also be noted that since the half life of neptunium is so small compared with the age of the solar system that members of the series are not found on the earth today.

Four Radioactive Series

Mass Numbers	Series	Parent	Half-Life Years	Stable end Product
4n	Thorium	${}^{232}_{90}Th$	1.39×10^{10}	${}^{208}_{82}Pb$
4n + 1	Neptunium	${}^{237}_{93}Np$	2.25×10^{6}	${}^{209}_{83}Bi$
4n + 2	Uranium	${}^{238}_{92}U$	4.47×10^{9}	${}^{206}_{82}Pb$
4n + 3	Actinium	${}^{235}_{92}U$	7.07×10^{8}	${}^{207}_{82}Pb$

It may be noted that the neptunium series $(4n+1)$ begins with ${}^{237}NP$, which has a half life of only $2.25\times10^{6}y$ which is less than the age of the earth $4.5\times10^{9}y$. Thus all of the ${}^{237}Np$ might have decayed to ${}^{209}_{83}Bi$.

One important use of study of natural radioactivity is to find the ages of the rocks there by that of earth. If we examine a sample of uranium bearing rock, we can find the ratio of ${}^{238}U$ atoms to ${}^{206}Pb$ atoms. From this we can estimate the age of the rock. See example given below.

Example 25

In a rock the ratio of uranium to lead is found to be one. Estimate the age of the rock. $t_{1/2}=4.5\times10^{9}y$

Solution

Let N_0 be the original number of uranium atoms present. $N(N_0e^{-\lambda t})$ be the number of uranium atoms present now. It means that N_0-N atoms have been decayed into Pb and presently observed as ${}^{206}Pb$. Thus the ratio (R) of uranium to lead is

$$R=\frac{\text{Number of } {}^{238}U \text{ present now}}{\text{Number of } {}^{206}Pb \text{ present now}}$$

i.e.

$$R=\frac{N_0e^{-\lambda t}}{N_0-N_0e^{-\lambda t}}$$

or

$$R=\frac{1}{e^{\lambda t}-1}$$

$$e^{\lambda t}-1=\frac{1}{R}$$

$$e^{\lambda t} = \frac{1}{R} + 1$$

Taking log on both sides, we get

$$\lambda t = \ln\left(1 + \frac{1}{R}\right)$$

$$t = \frac{1}{\lambda}\ln\left(1 + \frac{1}{R}\right)$$

Using $$\lambda = \frac{0.693}{T_{1/2}}$$

$$t = \frac{T_{1/2}}{0.693}\ln\left(\frac{1}{R} + 1\right)$$

$\therefore$ $$t = \frac{4.5\times10^{9}}{0.693}\ln(1+1) = \frac{4.5\times10^{9}\times\ln 2}{0.693}$$

$$t = 4.5\times10^{9} \text{ years.}$$

Thus the age of the rock is 4.5×10^{9} years.

Example 26

The radioactive decay of ^{232}Th leads eventually to ^{208}Pb. A certain rock is examined and found to contain 3.65 grams of ^{232}Th and 0.75 grams of ^{208}Pb. Assuming all of the Pb was produced in the decay of Th, what is the age of the rock. $T_{1/2}$ of thorium is 1.41×10^{10} y.

Solution

Number of atoms contained in 3.65 g of thorium,

$$N_{Th} = \frac{3.65}{232}\times N_A$$

Number of atoms contained in 0.75 g of ^{208}Pb,

$$N_{Pb} = \frac{0.75}{208}\times N_A$$

$\therefore$ $$R = \frac{N_{Th}}{N_{Pb}} = \frac{3.65\times208}{232\times0.75} = \frac{759.2}{174}$$

$$R = 4.363.$$

Using $$t = \frac{T_{1/2}}{0.693} \ln\left(\frac{1}{R} + 1\right)$$

$$t = \frac{1.41 \times 10^{10}}{0.693} \ln\left(\frac{1}{4.363} + 1\right)$$

$$t = 4.198 \times 10^{9} \text{ years.}$$

Mossbauer effect

Mossbauer effect is the phenomenon which involves the resonant and recoil free emission and absorption of gamma-rays by atomic nuclei bound in a solid.

This phenomenon was discovered by German physicist Rudolf Mossbauer in the year 1958. For this Mossbauer was awarded the 1961 Nobel Prize in physics.

One way of studying atomic systems is to do resonance experiments. Resonance is the phenomenon of overlapping of the emission and absorption spectrum. In resonant experiments what we do is radiation from a collection of atoms in an excited state is allowed to incident on a collection of identical atoms in their ground state. The ground state atoms can absorb the emitted photons by the former one and jump to the corresponding excited state. In principle resonant absorption will not occur. This is because when atom emits a photon, it is due to the recoil of the atom, the photons energy is less than the transition energy by the recoil kinetic energy. The absorbing atom also recoil. Hence photons energy is less than the transition by $2K_R$. So normally expect that absorption will not occur. However, the absorption is still possible. This is because the excited states don't have exact energies. According to uncertainty principle $\Delta E \Delta t \approx \hbar$, the state lives for time Δt and during that time we can't determine its energy to an accuracy less than ΔE. That is the energy spectrum spreads over a width ΔE.

Since K_R is much less than the width ΔE the emission spectrum and the absorption spectrum can overlap there by occurring absorption process. See figure (a) given below.

In the case of nuclear gamma rays the situation is different. A typical life is 10^{-10} s, so width

$$\Delta E = \frac{\hbar}{10^{-10}} \simeq 10^{-5} \text{ eV}.$$

i.e. the width of energy spread is very narrow. For γ-rays

$$E_\gamma = E_i - E_f + K_R$$

If $E_\gamma = 10^5 eV$ we found that $K_R = 1eV$ so $2K_R = 2eV$

This shows that, since $2K_R(2eV)$ is much larger than $\Delta E(10^{-5} eV)$ no overlapping is possible. So resonance absorption cannot occur. See figure (b) below.

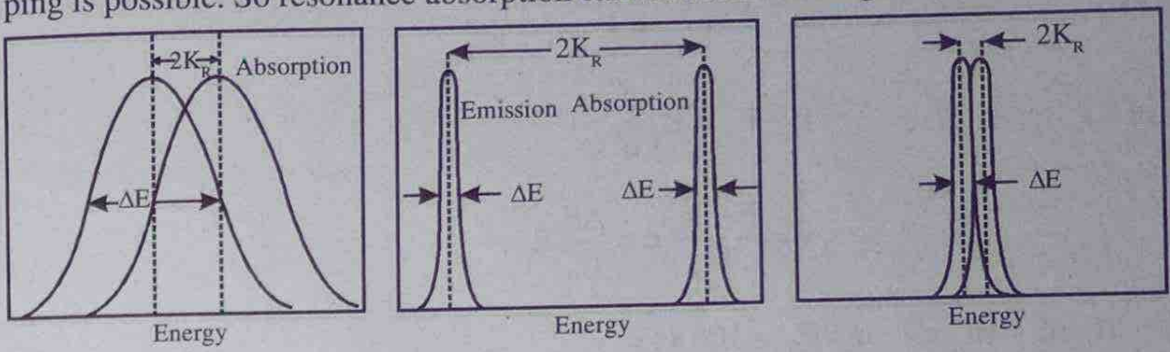

Figure 1.12(a): Emission and absorption energies in an atomic system

Figure 1.12(b): Emission and absorption energies in a nuclear system

Figure 1.12(c): Absorption energies for nuclei bound in crystal lattice

For the overlapping to occur we have to make recoil kinetic energy as small as possible. The recoil kinetic energy of gamma ray is given by $K_\gamma = \frac{E_\gamma^2}{2mc^2}$. To make K_γ smaller m has to be made larger. This is what Mossbauer did. Mossbauer placed the radioactive nuclei and the absorbing nuclei in crystals. The crystalline binding energies are large compared with K_R, so the individual atoms are held tightly to their position in the crystal lattice and are not free to recoil. When gamma-ray radiation occurs in the crystal, the whole crystal that recoils. This makes m in the expression for K_γ larger, may be 10^{20} times larger than the atomic mass. When the recoil energy is very small resonant absorption occurs. That is emission spectrum and absorption spectrum overlaps. See figure (c) above.

To obtain a complete overlap we have to shift either the emission or absorption energies. This can be done by Doppler shift. According to Doppler effect

$$\upsilon = \upsilon_0 \left(1 + \frac{v}{c}\right)$$

when a source of frequency υ_0 moving towards the observer with a speed v. The above equation can be written as

$$h\upsilon = h\upsilon_0\left(1+\frac{v}{c}\right)$$

$$E = E_0\left(1+\frac{v}{c}\right)$$

or
$$E - E_0 = E_0\frac{v}{c}$$

$$v = \frac{E-E_0}{E_0}c = \frac{\Delta E}{E_0}E$$

If $\Delta E = 10^{-5}\,\text{eV}$ and $E_0 = 100\,\text{keV}$

we get
$$v = \frac{10^{-5}}{10^5}\times 3\times 10^8 = 3\,\text{cm/s}.$$

This shows that when a gamma-ray emitting source is moving towards a gamma ray absorbing source with a speed 3 cm/s, the spreads will overlap.

The Mossbauer effect can be used as a probe to observe interactions between a nucleus and its electrons. This is because gamma-rays have very narrow line width. This means that this is very sensitive to small changes in energies of nuclear transitions. Thus the Mossbauer effect is an extremely precise method for measuring small changes in the energies of photons.

IMPORTANT FORMULAE

1. Nuclear radius, $R = R_0A^{1/3}$, where $R_0 = 1.2$ fm
2. Nuclear binding energy:

$$E_b = [Nm_n + Zm({}^1_1H) - m(X)]c^2$$

where $m({}^1_1H)$ and $m(X)$ are atomic masses.

3. Proton separation energy:

$$S_p = \left[m\left({}^{A-1}_{Z-1}X'\right) + m\left({}^1_1H\right) - m\left({}^A_ZX\right)\right]c^2$$

4. Neutron separation energy:

$$S_n = \left[m\left({}^{A-1}_{Z-1}X\right) + m_n - m\left({}^A_ZX\right)\right]c^2$$

5. Semiempirical binding energy formula:

$$E_b = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_a \frac{(A-2Z)^2}{A} + E_p$$

6. Most stable isobar:

$$Z = \frac{4a_a A + a_c A^{2/3}}{2a_c A^{2/3} + 8a_a}$$

7. Radioactive decay law:

$$N = N_0 e^{-\lambda t}, \text{ where } \lambda = \frac{0.693}{T_{1/2}}$$

8. The number of nuclei after n half lives:

$$N = \frac{N_0}{2^n}$$

9. Activity of a radioactive sample:

$$R = \frac{dN}{dt} = \lambda N$$

or

$$R = \frac{0.693}{T_{1/2}} \frac{mN_a}{A}$$

10. Expression for mean life time

$$\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693}$$

11. Age of the sample (wood):

$$t = \frac{1}{\lambda} \ln\left(\frac{R_0}{R}\right)$$

R_0 is the initial activity and R is the present activity.

12. Q-value of alpha decay:

$$Q = [m(X) - m(X') - m(\alpha)]c^2$$

where

$$Q = K_{X'} + K_\alpha$$

$$K_\alpha \simeq \frac{A-4}{A} Q.$$

13. Q-values of beta decays:

a) β^--decay

$$Q = [m(X) - m(X')]c^2$$

where $$Q = K_{X'} + K_e + K_{\bar{\upsilon}}$$

b) β^+-decay

$$Q = [m(X) - m(X') - 2m_e]c^2$$

where $$Q = K_{X'} + K_{e^+} + K_{\upsilon}$$

14. Q-value of electron capture:

$$Q = [m(X) - m(X')]c^2$$

where $Q = K_{\upsilon}$ neglecting K_e and $K_{X'}$

15. Recoil energy of gamma-decay:

$$K_R = \frac{E_\gamma^2}{2mc^2}$$

16. Expression for the age of the rock:

$$t = \frac{T_{1/2}}{0.693} \ln\left(\frac{1}{R} + 1\right)$$

where R is the ratio number of decaying atoms at present and the number of decayed atom.

UNIVERSITY MODEL QUESTIONS

Section A

(*Answer questions in* **two** *or* **three** *sentences*)

Short answer type questions

1. What is a nucleus? Give three of its properties.
2. What is the origin of nuclear force?
3. What are the similarities between atomic and nuclear structure.
4. Write down two major differences between the study of nuclei and atoms.
5. Why the study of nuclear structure is important?
6. What are isotopes? Give an example.
7. What are isotones? Give an example.
8. What are isobars? Give an example.

9. What are nuclear electrons?
10. What is proton-electron model of nucleus?
11. What is proton-neutron hypothesis?
12. Draw the graphical variation of nuclear density with radius of nuclei.
13. Define binding energy of a nucleus.
14. What is a binding energy curve?
15. What is meant by proton separation energy?
16. What is meant by neutron separation energy?
17. Give the name of three nuclear models.
18. Distinguish between strong interaction model and independent particle model.
19. Write down the semiempirical mass formula and explain the symbols used.
20. What is volume energy of a nucleus?
21. What is surface energy of a nucleus?
22. What is Coulomb energy of a nucleus?
23. What is shell model?
24. What are magic numbers?
25. What is the cause of magic numbers?
26. What is nuclear force?
27. Define mass defect and binding energy.
28. Why is the binding energy curve steep for lighter nuclei and falling off slowly for the heavy nuclei?
29. Account for the unstability of heavy nuclei.
30. Why the nuclei, that are off the line of stability, unstable?
31. What is the density of the nucleus to that of water?
32. Why is N approximately equal to Z for stable nuclei?
33. Why N is greater than Z for heavy nuclei?
34. Why there are no stable isotopes with $Z > 83$?
35. What is meant by radioactivity?
36. State the law of radioactive disintegration.
37. Distinguish between natural and artificial radioactivity.
38. What is the cause of radioactivity?
39. Define radioactive decay constant.
40. What is alpha decay? Give an example.
41. What is beta decay? Give an example.

42. What is gamma decay? Give an example.
43. What is meant by electron capture?
44. What is meant by positron emission?
45. What is meant by activity of a radioactive sample? What is its unit?
46. Define Q-value of a decay process.
47. Write down expression for Q-value of the alpha decay process.
48. Write down an expression for Q-value of the β^--decay process.
49. Write down an expression for Q-value of the β^+ decay.
50. Write down an expression for Q-value of the electron capture process.
51. What is meant by inverse β-decay?
52. What is gamma-decay?
53. What is an isomer?
54. What is meant by natural radioactivity?
55. Distinguish between natural and artificial radioactivity.
56. What is meant by induced radioactivity?
57. What is the cause of radioactivity?
58. What is radioactive series? Give their names.
59. What is Mossbauer effect?
60. What is the use of Mossbauer effect?

Section B

(*Answer questions in a* **paragraph** *of about half a page to one page*)

Paragraph / Problem type questions

1. Give three evidences against the idea of nuclear electrons.
2. How did proton-electron hypothesis fail?
3. Justify that electrons cannot exist inside a nucleus using the uncertainty principle.
4. Give the symbol for the following.
   a) The isotope of helium with mass number 4.
   b) The isotope of tin with 66 neutrons.
   c) An isotope with mass number 235 that contains 143 neutrons.

   [a) ${}^{4}_{2}He$, b) ${}^{116}_{50}Sn$ b) ${}^{235}_{92}Sn$]
5. How can you estimate the radius of a nucleus?
6. Arrive at an expression for binding energy of a nucleus.

7. What are the informations that we can obtain from a binding energy curve?
8. Arrive at an expression for proton separation energy.
9. Arrive at an expression for neutron separation energy.
10. Explain the liquid drop model.
11. What are the factors on which the binding energy of a nucleus depend according to liquid drop model?
12. Explain the assymmetry energy of a nucleus.
13. Explain the pairing energy of a nucleus.
14. What are the assumptions on which shell model is based?
15. What are the merits and demerits of liquid drop model?
16. What are the merits and demerits of shell model?
17. Give four properties of nuclear force.
18. Briefly explain nuclear force model proposed by Yukawa.
19. Roughly estimate the mass of the pion using uncertainty principle.
20. Why do we say that a nucleus behaves like a drop of a liquid?
21. What is meant by half life? Derive an expression for it.
22. What is meant by meanlife. Derive an expression for it.
23. Explain radioactive carbon dating.
24. Explain the tunnel theory of alpha decay in brief.
25. What are the assumptions made in the theoryof alpha decay.
26. Explain how does β-decay apparently violate law of conservation of energy.
27. Explain the apparent violation of conservation of linear momentum in β-decay.
28. Explain the apparent violation of law of conservation of angular momentum in β-decay.
29. In β-decay, law of conservation of energy, linear momentum, angular momentum are apparently violated. How it is overcome?
30. What is neutrino hypothesis?
31. Derive expression for the recoil energy of a nucleus which emitted a gamma-ray.
32. How will you estimate the age of a rock by using radioactivity?
33. What is meant by radioactive carbon dating?
34. Give a brief account of Mossbauer effect.
35. How to achieve complete overlap of emission spectrum and absorption spectrum?
36. Compute the density of a typical nucleus and find the resultant mass if we could produce a nucleus with a radius of 1 cm. $[2\times10^{17}\,\mathrm{kgm^{-3}},\ 8\times10^{11}\,\mathrm{kg}]$

37. Compute the Coulomb repulsion energy between two nuclei of ^{16}O that just touch at their surfaces. [15.26 MeV]

38. Find the total binding energy and the binding energy per nucleon for $^{133}_{55}Cs$. Given that $m_n = 1.008665\,u$, $m(^1_1H) = 1.007825\,u$ and $m(Cs) = 132.905452\,u$

[1118.53868 MeV, 8.410065 MeV per nucleon]

39. Find the proton separation energy of ^{40}Ca. Given that $m(^1_1H) = 1.007825\,u$, $m(^{39}K) = 38.963707\,u$, $m(^{39}Ca) = 39.962591\,u$ [8.3285415 MeV]

40. Find the neutron separation energy of ^7Li. Given: $m_n = 1.008665\,u$, $m(^6Li) = 6.015123\,u$ $m(^7Li) = 7.016005\,u$. [7.249865 MeV]

41. Which isobar of A = 75 does the liquid drop model suggest is most stable $a_a = 19$ and $a_c = 0.595$. $\left[{}^{75}_{33}As\right]$

42. Tritium ^3_1H has halflife of 12.5 years against β-decay. What fraction of the sample of pure tritium will remain unchanged after 25 years $\left[\frac{1}{4}\right]$

43. The half life of $^{238}_{92}U$ against alpha decay is 4.5×10^9 years. Calculate the time taken by uranium to reduce 80% of the initial amount [1.45×10^9 years]

44. A radioactive isotope X has a half life of 3 second. Initially a sample of this isotope contains 8000 atoms. calculate
a) Its decay constant
b) The time t_1 when 1000 atoms of the isotope X remain in the sample.
c) The number of decay per second in the sample at $t = t_1$

$$\begin{bmatrix} \text{a) } 0.231\ s^{-1} \\ \text{b) } 9s \\ \text{c) } 231\ s^{-1} \end{bmatrix}$$

45. The mean lives of a radioactive substances are 1620 year and 405 year for alpha emission and beta emission respectively. Find the time during which three fourth of a sample will decay if it is decaying both by alpha and beta emission simultaneously

[449 years]

46. Calculate the mass in gram of a radioactive sample ^{214}Pb having an activity of one microcuri and a half life of 26.8 minute. [3.04×10^{-14}g]

47. An animal bone fragment found in an archeological site has a carbon mass of 200g. It registers an activity of 16 decays.What is the age of the bone $\lambda = 3.83\times10^{-12}\ s^{-1}$

[9400 year]

48. A bone suspected to have originated during the period of Asoka the great was found in Bihar. Accelarator technique gives $\frac{^{14}C}{^{12}C} = 1.1\times10^{-12}$. Is the bone old enough to have belonged to that period. [No, 719 year]

49. The atomic ratio between the uranium isotopes ^{238}U and ^{234}U in a mineral sample is found to be 1.8×10^4. The half life of ^{234}U is $T_{1/2}(234) = 2.5\times10^5 y$. Find the half life of ^{238}U. [4.5×10^9 y]

50. A laboratory has 1.49μg of $^{13}_{7}N$, which has a half life of 10 minutes. (a) How many nuclei are present initially (b) What is the activity initially (c) What is the activity after 1 hour. After how long will the activity drop to less that one per second

[a) 6.9×10^{16}, b) $8\times10^{13}s^{-1}$, c) $1.23\times10^{12}s^{-1}$, d) $2.76\times10^4 s$]

51. A rock sample contains 1mg of ^{206}Pb and 4mg of ^{238}U whose half life is $4.47\times10^9 y$. How long ago the rock formed. [1.64×10^9 y]

52. The activity R of a sample of a unknown radio nuclide is measured at hourly intervals. The results in MBq are 80.5 and 36.2. Find the half life of the radio nuclide

[52 minutes]

53. Calculate the approximate mass of uranium which must undergo fission to produce the same energy as 100 tonnes of coal. Heat combustian of coal 8000 cal/g. 1 calorie = 4J. Fission of uranium releases 200 MeV [39.2g]

54. Find the kinetic energy of the alpha particle emitted in the decay process

$$^{226}Ra \longrightarrow {}^{222}Ra + {}^{4}_{2}He$$

$m(Ra) = 226.025410\,u$, $m(Rn) = 222.0175778\,u$ and $m(\alpha) = 4.002603\,u$

[4.7846 MeV]

55. Check whether the decay processes allowed or not

$$^{211}At \longrightarrow {}^{207}Bi + \alpha$$

$m(At) = 210.987496\,u$, $m(Bi) = 206.978471\,u$ $m(\alpha) = 4.002603\,u$ [allowed]

56. Find the maximum kinetic energy of the electrons emitted in the negative β-decay of ^{11}Be. Given: $m(Be) = 11.021658\,u$ $m(\beta) = 11.009305\,u$ [11.506 MeV]

57. ^{23}Ne decays to ^{23}Na by negative β-emission what is the maximum kinetic energy of the emitted electrons. Given: m(Ne) = 22.994467 u, m(Na) = 22989769 u [4.376 MeV]

58. ^{40}K isotope decays by electron capture. Find the Q-value. Given: m(K) = 39.964998u m(Ar) = 39.962383 u. [1.504 MeV]

59. ^{12}N beta decays to an excited state of ^{12}C, which subsequently decays to the ground state with the emission of a 4.43 MeV gamma-ray. What is the maximum kinetic energy of the emitted beta particle. Given: m(C) = 12.000000 u, m(N) = 12.018613 u $m_e = 0.000549\,u$. [11.89 MeV]

60. The 4n radioactive decay series begins with $^{232}_{90}Th$ and ends with $^{248}_{82}Pb$.
   a) How many alpha decays are in the series
   b) How many beta decays
   c) How much energy is released in the complete series. $m(Th) = 232.038050\,u$, $m(Pb) = 207.976636\,u$ and $m_\alpha = 4.002603\,u$ [a) 6 b) 4 c) 42.658974 MeV]

Section C

(*Answer questions in about* **two** *pages*)

Long answer type questions (Essays)

1. Explain the postulates of a liquid drop model. Derive Weizsacker semi empirical mass formula.
2. Disscuss the various factors which contribute to the binding energy of nuclei and derive a formula for atomic mass M of a nucleus based on these considerations.
3. Give salient features of nuclear shell model and point out its success and failures.

Hints to problems

36. $\rho = \frac{m}{V} = \frac{Am_p}{\frac{4}{3}\pi R^3}$, $R = 1.2 \times A^{1/3}\,fm$

$m_p = 1.67 \times 10^{-27}\,kg$, get ρ.

$$m = \rho V = 2 \times 10^{17} \times \frac{4}{3}\pi(0.01)^3 = 8 \times 10^{11}\,kg$$

37. Radius of oxygen nucleus

$$R = R_0 A^{1/3} = 1.2 \times (16)^{1/3} = 3.02\,fm$$

Coulomb energy, $U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$

$q_1 = 8e, q_2 = 8e, r = 2R = 6.04\,fm$

$$U = \frac{9\times10^{9}\times64\times e^{2}}{(6.04\times10^{-15})}$$

$$U = \frac{9\times10^{9}\times64\times(1.6\times10^{-19})^{2}}{(6.04\times10^{-15})} J$$

or $$U = \frac{9\times10^{9}\times64\times(1.6\times10^{-19})^{2}}{6.04\times10^{-15}\times1.6\times10^{-13}} MeV$$

$$U = 15.26 MeV$$

38. See example 4 or 5
39. See example 6
40. See example 7
41. See example 8
42. In 12.5 years half of the sample will be decayed, in the next 12.5 years again half will be decayed. Hence $\frac{1}{4}$ of the tritium will remain undecayed after 25 years.
43. $N = N_0 e^{-\lambda t}, \lambda = \frac{0.693}{T_{\frac{1}{2}}}$
44. a) $\lambda = \frac{0.693}{T_{\frac{1}{2}}}$ b) $N = N_0 e^{-\lambda t}, N_0 = 8000, N = 1000$ c) $\left[\frac{dN}{dt}\right]_{t=t_1} = \lambda N$
45. Total decay constant $\lambda = \lambda_\alpha + \lambda_\beta$

    $\lambda_\alpha = \frac{1}{1620}, \lambda_\beta = \frac{1}{405},$

    Using $N = N_0 e^{-\lambda t}$, $N = \frac{N_0}{4}$
46. $R = \lambda N$, $R = 1\mu ci = 3.7 \times 10^4$

    $\therefore$ $N = 8.58\times10^{7}$

    214 g of Pb contains 6.02×10^{23} atoms

    $\therefore$ Mass of 8.58×10^{7} atoms $= \frac{8.58\times10^{7}\times214}{6.02\times10^{23}}$
47. $N_0 = \frac{6.02\times10^{23}\times200\times1.3\times10^{-12}}{12} = 1.3\times10^{13}$

$$\left(\frac{dN}{dt}\right)_0 = \lambda N_0 = 50s^{-1}$$

Using $\frac{dN}{dt} = \lambda N_0 e^{-\lambda t}$, $\frac{dN}{dt} = 16s^{-1}$ calculate t.

48. Initial ratio of $\frac{^{14}C}{^{12}C}$ at the time of death was, $R_0 = 1.2 \times 10^{-12}$

using $N = N_0 e^{-\lambda t}$

$$R = \frac{N(^{14}C)}{N(^{12}C)} = \frac{N_0(^{14}C)e^{-\lambda t}}{N(^{12}C)}$$

$$= R_0 e^{-\lambda t}$$

or $$t = \frac{\ln\left(\frac{R_0}{R}\right)}{\ln 2} T_{\frac{1}{2}}$$ find t

$$R = 1.1 \times 10^{-12},\ T_{\frac{1}{2}} = 5730 \text{ years}$$

49. We have $\lambda_1 N_1 = \lambda_2 N_2$

$$\frac{N(238)}{T_{\frac{1}{2}}(238)} = \frac{N(234)}{T_{\frac{1}{2}}(234)}$$

$$\therefore \quad T_{\frac{1}{2}}(238) = \frac{N(238)}{N(234)} T_{\frac{1}{2}}(234)$$

50. a) 13 g of N contains 6.02×10^{23} nuclei

$\therefore$ 1 g contains $= \frac{6.02 \times 10^{23}}{13}$

1.49 mg contains $= \frac{6.02 \times 10^{23}}{13} \times 1.49 \times 10^{-6} = No$

b) $R_0 = \lambda N_0$, $\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{600} = 1.16 \times 10^{-3}$

c) After one hour $R = \lambda N \quad R = \lambda N_0 e^{-\lambda t} = Roe^{-\lambda t}$

d) $R = 1s^{-1}$, $R = \lambda N_0 e^{-\lambda t}$. Find t.

51. Try yourself
52. $\lambda N_1 = 80.5, \lambda N_2 = 36.2$

$$\frac{N_2}{N_1} = e^{-\lambda t}, \ t = 1 \text{ hour, find } \lambda \text{ and use } T_{\frac{1}{2}} = \frac{0.693}{\lambda}.$$

53. Heat produced on combustian $= 10^5 \times 8000 \times 10^3 \times 4\ \text{J} = 22 \times 10^{11}\ \text{J}$

Energy released per fission $= 200 \times 10^6 \times 1.6 \times 10^{-19}\ \text{J} = 3.2 \times 10^{-11}$

$\therefore$ Number of atoms required to produce

$$32 \times 10^{11}\ \text{J of energy} = \frac{32 \times 10^{17}}{3.2 \times 10^{-17}} = 10^{23}$$

$\therefore$ Mass of uranium $\dfrac{235 \times 10^{23}}{6.02 \times 10^{23}} = 39.2\text{g}$

54. See example 19
55. See example 20
56. $Q = [m(Be) - m(B)]c^2 = 11.506\,\text{MeV}$

$Q = K_B + K_e + K_\upsilon$

The electrons have their maximum kinetic energy when $K_B \simeq 0$ and $K_\upsilon \simeq 0$.

57. $Q = [m(Ne) - m(Na)]c^2 = 4.376$ MeV.

$Q = K_{Na} + K_e + K_\upsilon$ when $K_{Na} \simeq 0$ and $K_\upsilon \simeq 0$ $(K_e)_{max} = Q$.

58. $Q = [m(K) - m(Ar)]c^2 = 1.540\,\text{MeV}$

$40K + e^- = {}^{40}Ar + \upsilon \cdot$

59. The Q-value in β^+-decay is

$Q = [m(N) - m(C^*) - 2m_e]c^2$

$$m(C^*) = 12.000000\,u + \frac{4.43\,\text{MeV}}{931.5\ \text{MeV/u}}$$

60. a) $N_\alpha = \dfrac{232 - 208}{4} = 6$

b) Balancing the charges $90 = 82 + 2N_\alpha - N_e$ $\quad\therefore\quad N_e = 4$

c) $Q = [m(Th) - m(Pb) - 6m_\alpha]c^2 = 42.658974$ MeV.