UNIT - I

1 FERMAT'S PRINCIPLE

Introduction

Fermat's principle is the most important fundamental principle in optics. The rectilinear propagation of light, the laws of reflection and refraction which form the very basis of the whole of geometrical optics are remarkably unified in a general principle first enunciated by pierre de Fermat (1601-1665) a French mathematician in 1658. All laws of geometrical optics such as principle of reversibility, lens makers formula, thin lens formula etc. can be deducted from Fermat's principle.

Optical path

The distance S that a light ray travels in any medium is the product of velocity (v) and time (t),

i.e. $$S = vt$$

From the definition of refractive index $\mu = \frac{c}{v}$ or $v = \frac{c}{\mu}$

$\therefore$ $$S = \frac{c}{\mu}t \quad \text{or} \quad \mu S = ct$$

For a given time R.H.S is a constant hence L.H.S μS is a constant. This product μS is called optical path or

Optical path is defined as the product of geometrical distance and the refractive index. Optical path is the equivalent air path. Since the value of μ for air doesnot appreciably differ from that of vacuum, it *represents the distance in vacuum that light would travel in the same time in which it travels the distance S in the medium.* i.e. In a given time, light travels the same optical path in different media. For example if light travels a distance S_1 in a medium of refractive index μ_1, a distance S_2 in a medium of refractive index μ_2 and so on. Then the optical path,

$$\mu_1 S_1 = \mu_2 S_2 = ...$$

when a ray travels distances S_1, S_2, S_3 etc in media of refractive indices μ_1, μ_2, μ_3 etc., then the optical path is given by

$$S = \mu_1 S_1 + \mu_2 S_2 + \mu_3 S_3 + ...$$

$$= \sum_{i=1} \mu_i S_i$$

For a medium of continuously varying optical density, the optical path of a ray going from P to Q is given by

$$S = \int_P^Q \mu \, dS$$

Fermat's principle of least time

According to this principle "*when a ray travels from one point to another point through a set of media, out of all possible paths it always follows that path along which the time taken is minimum.*

Suppose a ray of light travels from P to Q. There are several possible paths like PAQ, PBQ, PCQ etc. According to Fermats principle of least time, the ray will choose the path in which the time taken is minimum.

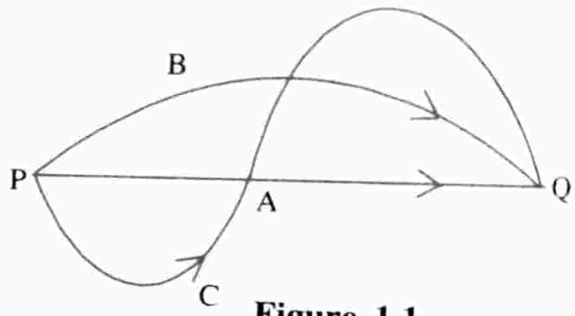


Figure 1.1

Let dt be the time in which light ray travels a distance dS in the medium. Then

$dt = \frac{dS}{v}$ where v is the velocity of light in that medium.

using $\mu = \frac{c}{v}$ or $v = \frac{c}{\mu}$

We get $dt = \frac{\mu}{c} dS$

$\therefore$ The time required to travel from P to Q is

$$t = \int_P^Q \frac{\mu \, dS}{c} = \frac{1}{c} \int_P^Q \mu \, dS$$

where $\int_P^Q \mu \, dS$ is called optical path

If we consider different paths from P to Q, according to Fermat's principle of least time the ray will take the path for which time taken is minimum.

i.e. $$\delta t = 0 \quad \text{or} \quad \delta t = \delta \int_P^Q \frac{\mu}{c} dS = 0$$

when time is minimum optical path length is minimum since c is a constant. We therefore restate the Fermats principle as follows. Out of all possible paths light travels along a path having the minimum path length.

i.e. $$\delta \int_P^Q \mu \, dS = 0.$$

Fermat's principle of extremum path

The Fermat's principle of least time discussed above is not complete and slighty incorrect. This is because there are a number of cases where the optical path is maximum (light reflects from a spherical surface) or else neither a maximum nor a minimum but stationary in the case of formation of images by a lens in which all rays proceeding from an object point traverse the same constant path to the corresponding image point. Thus Fermat's principle of least time is modified to Fermat's principle of extremum path.

When a ray travels from one point to another point through a set of media, out of all possible paths it always follows that path along which the time taken is extremum. (Minimum or maximum or stationary). This is called Fermat's principle of extremum path or Fermat's principle of stationary time. The fundamental laws of rectilinear propagation, reflection and refraction can be deduced from Fermats principle.

Rectilinear propagation of light

According to Fermat's principle, we have

$$\delta \int_P^Q \mu \, dS = 0$$

For homogeneous and isotropic medium, μ being constant and taken outside the integral. Thus we have

$$\delta \left[\mu \int_P^Q dS \right] = 0 \quad \text{or} \quad \delta \int_P^Q dS = 0$$

which means that between two points a ray will travel in a straight line and not any other path. Thus in such a medium rectilinear propagation of light is explained.

Laws of reflection and refraction from Fermat's principle

Laws of reflection

First law of reflection

Consider a plane mirror $M_1 M_2 M_3 M_4$. Let A and B be two points above the mirror and located in a plane ABCD normal to the plane of the mirror. Light coming from point A is reflected towards B. Suppose the light ray passes through a point P. It means that the path of the ray is along APB.

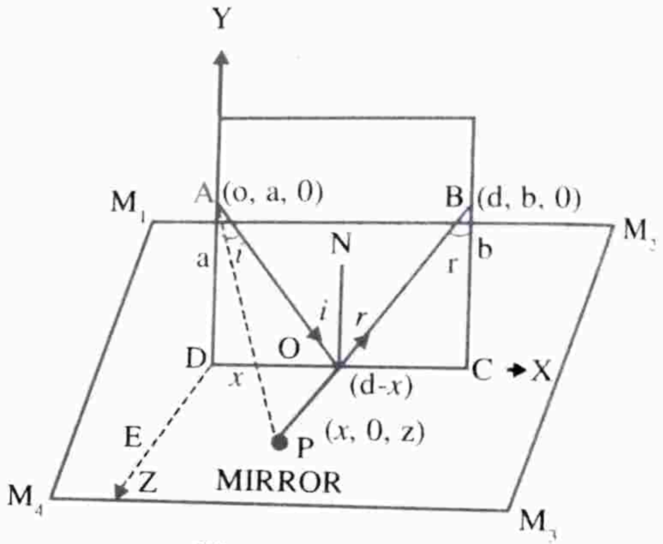


Figure 1.2

Draw a plane ABCD normal to the mirror plane $M_1 M_2 M_3 M_4$. Take the origin of coordinates at D. Let DC, DA DE be the X, Y and Z axes. Let DA = a, CB = b and DC = d. The point P has the coordinates (x, o, z)

To check whether the path APB (AP + PB) is the actual path taken by the ray, we have to apply Fermat's principle. Since μ is constant all along the path, we have to minimise only the path length APB = AP + PB instead of optical path length μ APB.

The path length APB, L = AP + PB

$$L = \left[(x-o)^2 + (o-a)^2 + (z-o)^2\right]^{1/2} + \left[(x-d)^2 + (o-b)^2 + (z-o)^2\right]^{1/2}$$

or
$$L = (x^2 + a^2 + z^2)^{1/2} + \left[(x-d)^2 + b^2 + z^2\right]^{1/2} \qquad \text{..... (1)}$$

We now apply Fermat's principle to get actual path length. The path APB can be varied by varying x and z. We obtain the minimum value of L, that is the shortest path by taking the derivative of L with respect to z and setting the derivative equal to zero. Thus,

$$\left(\frac{\partial L}{\partial z}\right)_x = \frac{1}{2}\frac{1}{(x^2 + a^2 + z^2)^{1/2}} \cdot 2z + \frac{1}{2}\frac{1}{\left[(x-d)^2 + b^2 + z^2\right]^{1/2}} 2z = 0.$$

or $$z\left\{\frac{1}{(x^2+a^2+z^2)^{1/2}}+\frac{1}{\left[(x-d)^2+b^2+z^2\right]^{1/2}}\right\}=0.$$

As the factor within the brackets cannot be zero, z must be zero.

The above result means that P must lie in the plane ABCD, which is normal to the mirror. O is such a position for P. When P coincide with O, it is obvious that the incident ray AO, the normal ON and the reflected ray OB lie in the same plane. This is the first law of reflection.

Second law of reflection

Using $z = 0$ in equation (1), we get

$$L=(x^2+a^2)^{1/2}+\left[(x-d)^2+b^2\right]^{1/2}$$

Now taking the derivative of L with respect to x, and setting the derivative equal to zero.

$$\frac{dL}{dx}=\frac{2x}{2(x^2+a^2)^{1/2}}+\frac{2\cdot(x-d)}{2\left[(x-d)^2+b^2\right]^{1/2}}=0.$$

or $$\frac{x}{(x^2+a^2)^{1/2}}=\frac{d-x}{\left[(x-d)^2+b^2\right]^{1/2}}.$$

From the $\Delta\, AOD$, we have $\frac{x}{(x^2+a^2)^{1/2}}=\sin i$ and from the ΔBOC, we have

$$\frac{d-x}{[(x-d^2)+b^2]^{1/2}}=\sin r$$

$\therefore\quad \sin i=\sin r \quad \text{or}\ i=r$

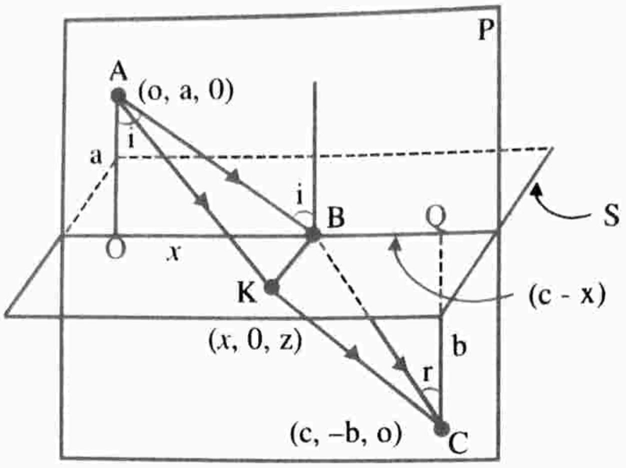
This is the second law of reflection.

Laws of refraction

First law of refraction

Consider a plane surface S separating two media of refractive indices μ_1 and μ_2. Let A and C be two points lying in the two different media. A light ray starts from A and reaches at C. The ray can choose different paths such as ABC or AKC. According to Fermats principle this path will be such that the optical path is minimum. Suppose the ray takes the path AKC. To check whether the optical path AKC

$(\mu_1 AK + \mu_2 KC)$ the actual path we have to apply Fermat's principle. Let the various lines have the indicated magnitudes in the figure below.



OA = a
OQ = c
OB = x
BQ = c − x
QC = b

Figure 1.3

The optical path length through the point $K(x, o, z)$ is given by

$L = \mu_1 AK + \mu_2 KC$

$$L = \mu_1 (x^2 + a^2 + z^2)^{1/2} + \mu_2 \left[(x-c)^2 + b^2 + z^2\right]^{1/2} \quad \text{..... (1)}$$

We now apply Fermat's principle to get actual path length. The path can be varied by varying x and z. We obtain the minimum value of L that is the shortest optical path by taking the derivative of L with respect to z and setting the derivative equal to zero, thus

$$\left(\frac{\partial L}{\partial z}\right)_x = \frac{\mu_1 \cdot 2z}{2(x^2+a^2+z^2)^{1/2}} + \frac{\mu_2 \cdot 2z}{2\left[(x-c)^2+b^2+z^2\right]^{1/2}} = 0$$

$$z\left\{\frac{\mu_1}{(x^2+a^2+z^2)^{1/2}} + \frac{\mu_2}{2\left[(x-c)^2+b^2+z^2\right]^{1/2}}\right\} = 0.$$

As the factor within the bracket cannot be zero, z must be zero. It means that K coincides with B. It means that the incident ray, the refracted ray and the normal lie in the same plane. This is the first law of refraction.

Second law of refraction

Using $z = 0$ in equation (1), we get

$$L = \mu_1 (x^2 + a^2)^{1/2} + \mu_2 \left[(x-c)^2 + b^2\right]^{1/2}$$

Now taking the derivative of L with respect to x and setting the derivative equal to zero.

$$\frac{dL}{dx} = \frac{\mu_1 2x}{2(x^2 + a^2)^{1/2}} + \frac{\mu_2 \cdot 2(x - c)}{2\left[(x - c)^2 + b^2\right]^{1/2}} = 0$$

or $$\frac{\mu_1 x}{(x^2 + a^2)^{1/2}} = \frac{\mu_2 (c - x)}{\left[(x - c)^2 + b^2\right]^{1/2}}$$

From the Δ AOB, $\frac{x}{(x^2 + a^2)^{1/2}} = \sin i$

From the Δ BCQ, $\frac{c - x}{\left[(x - c)^2 + b^2\right]^{1/2}} = \sin r$

$\therefore \quad \mu_1 \sin i = \mu_2 \sin r$

or $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$ This is Snell's law of refraction.

REFRACTION AND REFLECTION BY SPHERICAL SURFACES

Introduction

In this section we will discuss the formation of images by simple optical systems. In general an optical system consists of a succession of elements which includes lenses, mirrors, light sources, detectors, projection screens, reflecting prisms, dispersing devices, filters and thin films and fibre-optics bundles. Here we deal only with simple optical systems. Our optical system is assumed to be made up of a number of refracting surfaces like combination of lenses or a combination of a number of reflecting surfaces like mirrors. To form images by these optical systems (by refraction) it is necessary to apply Snell's laws of refraction at each refracting surface. They are

(i) The incident ray, the refracted ray and the normal to the surface lie in the same plane.

(ii) If i and r represent the angles of incidence and refraction respectively then

$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$, where μ_1 and μ_2 are the refractive indices of two media.

Refraction at a single spherical surface

Consider a spherical refracting surface XY of refractive index μ_2 placed in a medium of refractive index μ_1 (where $\mu_1 < \mu_2$). Suppose the surface is convex towards the rarer medium (μ_1). Let P be the pole, C be the centre of curvature and $PC = r$ be the radius of curvature.

Consider a point object O lying on the principal axis of the surface. The point object O emitting rays in all directions. We will use Snell's laws of refraction to determine the image of the point O. It may be noted that not all rays emanating from O converge to a point. However, if we consider only those rays which make small angles with the principal axis do converge to a single point I. This is known as paraxial approximation and according to Fermats principle, all paraxial rays take the same amount of time to travel from O to I.

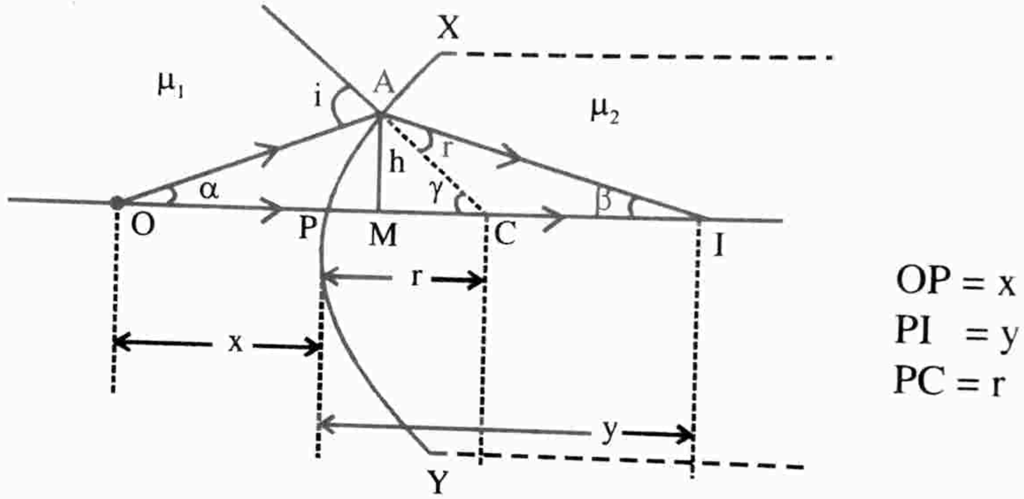


Figure 1.4(a)

From the ΔOAC

$$i = \alpha + \gamma \qquad \text{..... (1)}$$

(Exterior angle = sum of the interior opposite angles)

From the $\Delta\, ACI$,

$$\gamma = r + \beta$$

or

$$r = \gamma - \beta \qquad \text{..... (2)}$$

Using paraxial approximation

$i \simeq \sin i \simeq \tan i$, similarly for all angles.

so equation (1) can be written as

$$\sin i = \tan\alpha + \tan\gamma$$

$$\sin i = \frac{AM}{OM} + \frac{AM}{MC}$$

$AM = h$, $OM \simeq x$ and $MC = PC = r$

$$\therefore \quad \sin i = \frac{h}{x} + \frac{h}{r} \qquad \text{..... (3)}$$

similarly equatoin (2) can be written as

$$\sin r = \tan\gamma - \tan\beta$$

$$\sin r = \frac{AM}{MC} - \frac{AM}{MI}$$

$$\sin r = \frac{h}{r} - \frac{h}{y} \qquad \text{..... (4)}$$

$$MI \simeq PI = y$$

$\dfrac{\text{eq } 3}{\text{eq } 4}$ gives

$$\frac{\sin i}{\sin r} = \frac{\dfrac{h}{x} + \dfrac{h}{r}}{\dfrac{h}{r} - \dfrac{h}{y}}$$

According to Snell's law $\dfrac{\sin i}{\sin r} = \dfrac{\mu_2}{\mu_1}$

$$\therefore \quad \frac{\mu_2}{\mu_1} = \frac{\dfrac{1}{x} + \dfrac{1}{r}}{\dfrac{1}{r} - \dfrac{1}{y}}$$

or $$\mu_2\left(\frac{1}{r} - \frac{1}{y}\right) = \mu_1\left(\frac{1}{x} + \frac{1}{r}\right)$$

$$\frac{\mu_2}{r} - \frac{\mu_2}{y} = \frac{\mu_1}{x} + \frac{\mu_1}{r}$$

$$\frac{\mu_2}{r} - \frac{\mu_1}{y} = \frac{\mu_1}{x} + \frac{\mu_2}{y}$$

$$\frac{(\mu_2 - \mu_1)}{r} = \frac{\mu_1}{x} + \frac{\mu_2}{y}$$

i.e., $$\frac{\mu_1}{x} + \frac{\mu_2}{y} = \frac{(\mu_2 - \mu_1)}{r} \qquad \text{..... (5)}$$

This is the required relation between object distance (x), the image distance (y) and the radius of curvature.

When the refracting surface is concave towards the rarer medium (μ_1), the ray diagram is shown below .

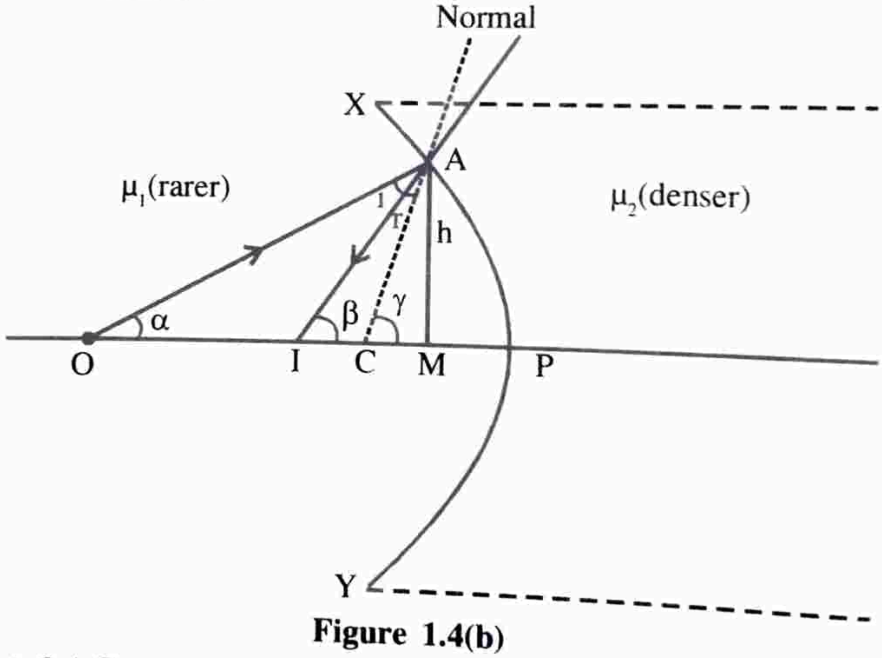


Figure 1.4(b)

From the ΔOAC

$$\gamma = i + \alpha$$

or $$i = \gamma - \alpha$$

$\therefore$ $$\sin i = \tan\gamma - \tan\alpha$$

$$\sin i = \frac{AM}{CM} - \frac{AM}{OM}$$

AM = h, CM $\simeq$ CP = r and OM $\simeq$ OP = x

$$\sin i = \frac{h}{r} - \frac{h}{x} \qquad \text{..... (6)}$$

From the Δ IAC

$$\gamma = \beta + r$$

$$r = \gamma - \beta$$

$$\therefore \quad \sin r = \tan\gamma - \tan\beta \qquad \tan\beta = \frac{AM}{IM}$$

$$\sin r = \frac{h}{r} - \frac{h}{y} \qquad IM \simeq IP = y \qquad \text{..... (7)}$$

$\dfrac{\text{eq 6}}{\text{eq 7}}$ gives

$$\frac{\sin i}{\sin r} = \frac{\frac{h}{r} - \frac{h}{x}}{\frac{h}{r} - \frac{h}{y}}$$

According to Snell's law $\dfrac{\sin i}{\sin r} = \dfrac{\mu_2}{\mu_1}$

$$\frac{\mu_2}{\mu_1} = \frac{\left(\frac{1}{r} - \frac{1}{x}\right)}{\left(\frac{1}{r} - \frac{1}{y}\right)}$$

$$\mu_2\left(\frac{1}{r} - \frac{1}{y}\right) = \mu_1\left(\frac{1}{r} - \frac{1}{x}\right)$$

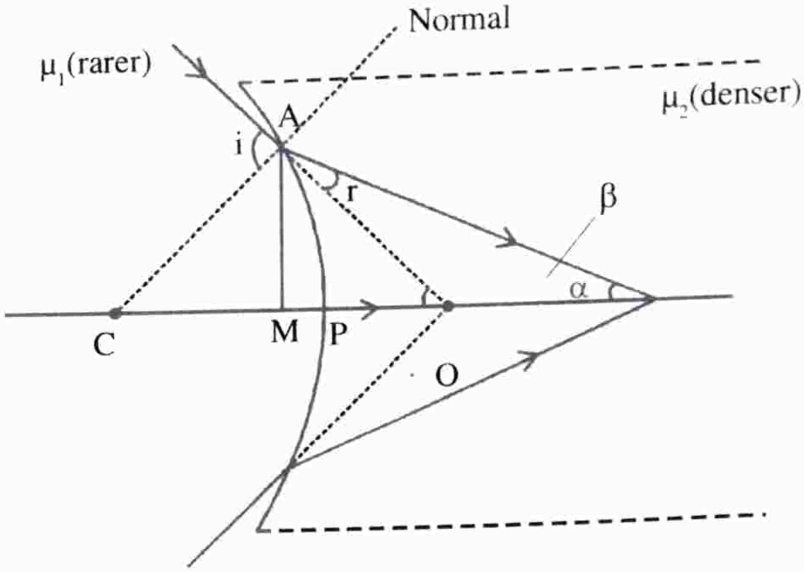
$$\frac{\mu_2}{r} - \frac{\mu_1}{r} = -\frac{\mu_1}{x} + \frac{\mu_2}{y}$$

i.e.,

$$\frac{-\mu_1}{x} + \frac{\mu_2}{y} = \frac{(\mu_2 - \mu_1)}{r} \qquad \text{..... (8)}$$

We can see that equations 5 and 8 are different.

When the refracting surface is concave towards the rarer medium (μ_1) and the point object is kept in the denser medium (μ_2), the ray diagram is given below.



OP = x
PI = y
PC = r

Figure 1.4(c)

Proceed as before we get another formula.

Sign convention

We found that while dealing with refraction or reflection of spherical surfaces, depending upon the positions of the object and nature of the spherical surfaces the object-image relation will be different. Then it will be very difficult to memorise all the formulae. To overcome this difficulty we introduce a sign convention. if we apply this sign convention, always we have only one formula connecting the object and the image distance.

Here we shall adopt the following systems of signs called new Cartesian sign conventions. For this assume that the pole (P) of spherical surface is the origin of the coordinate system and distances are measured from P. Following are the sign convention.

1. The rays are always incident from the left on the refracting (or reflecting surface)
2. All distances measured to the right of P (right of the origin of the coordinate) are positive and distances measured to the left are negative. For example in figure 1.4(a) the object distance $x = -u$ and the image distance $y = +v$. The radius of curvature $r = +R$, as the curvature is towards right of P.

3. The angle is taken to be positive if the axis has to be rotated in the anticlockwise direction through the acute angle to coincide with the ray and conversely. For example in figure 1.4(a), α is positive and β is negative.
4. The angle that a ray makes with the normal to the surface is taken as positive if the normal has to be rotated in the anticlockwise direction through the acute angle to concide with the ray and conversely. For example in figure 1.4(a) i and r are positive.
5. All distances measured from the axis upward are taken positive and all distances measured in the downward direction are negative.

When we apply the sign conventions to different cases we discussed all the formulae become one and the same. For example, in figure 1.4(a), we have

$$\frac{\mu_1}{x}+\frac{\mu_2}{y}=\frac{\mu_2-\mu_1}{r}$$

applying sign convention $x=-u,\ y=+v$ and $r=R$

$$\frac{-\mu_2}{u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R} \qquad \text{..... (9)}$$

From figure 1.4(b), we have

$$\frac{-\mu_1}{x}+\frac{\mu_2}{y}=\frac{(\mu_2-\mu_1)}{r}$$

Applying sign convention, here $x=-u,\ y=-v$ and $r=-R$

We get $\dfrac{-\mu_1}{-u}+\dfrac{\mu_2}{-v}=\dfrac{(\mu_2-\mu_1)}{-R}$

cancelling $-$ve sign throughout, we get

$$\frac{-\mu_1}{u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R} \qquad \text{..... (10)}$$

We can see that equations 9 and 10 are one and the same.

So remember only one formula.

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R} \qquad \text{..... (11)}$$

The equation (8) is known as Gaussian formula for a single surface

Note : 1. While using this formula always remember that this formula is fake so apply sign convention once again, to get back the original one, especially doing problems.

2. When the refraction occurs from denser to rarer the formula becomes

$$\frac{\mu_1}{v}-\frac{\mu_2}{u}=\frac{\mu_1-\mu_2}{R}$$

Example 1

Light from a point source in air falls on a spherical glass surface ($\mu = 1.5$ and $R = 20\,\text{cm}$). The distance of the light source from the glass surface is 100 cm. At what position does the image form.

Solution

$$\mu_1 = 1 \text{ (air) } \mu_2 = 1.5$$

$$R = 20\,\text{cm} = +0.2\text{m}$$

$$u = -100\,\text{cm} = -1\,\text{m}$$

$$v = ?$$

Using $$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

$$\frac{1.5}{v}-\frac{1}{-1}=\frac{1.5-1}{0.2}$$

$$\frac{1.5}{v}+1=2.5$$

$$\frac{1.5}{v}=1.5$$

$\therefore$ $$v = 1\text{m}.$$

Here we assumed that convex surface is towards the point source.

Example 2

A mark on the surface of a glass sphere is viewed through the glass from a position directly opposite. If the diameter of the sphere is 10 cm and refractive index of glass is 1.5, find the position of the image.

Solution

$$u = -10\,\text{cm},\ R = -5\,\text{cm},\ \mu_1 = 1,\ \mu_2 = 1.5$$

Here the refraction occurs from denser to rarer medium. Using

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

$$\frac{1}{v} - \frac{1.5}{-10} = \frac{1-1.5}{-5}$$

$$\frac{1}{v} + \frac{1.5}{10} = \frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1.5}{10} = \frac{-0.5}{10} = \frac{-1}{20}$$

$$\therefore \qquad v = -20\,\text{cm}.$$

Example 3

A concave spherical surface of radius curvature 100 cm separates two media of refractive indices 1.5 and $\frac{4}{3}$. An object is kept in the first medium at a distance of 30 cm from the surface. Find the position of the image.

Solution

$$u = -30\,\text{cm},\ R = -100\,\text{cm}$$

Here the refraction is from denser to rarer, then we have

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

$$\frac{\frac{4}{3}}{v} - \frac{1.5}{-30} = \frac{\frac{4}{3} - 1.5}{-100}$$

$$\frac{\frac{4}{3}}{v} + \frac{1}{20} = \frac{1}{600}$$

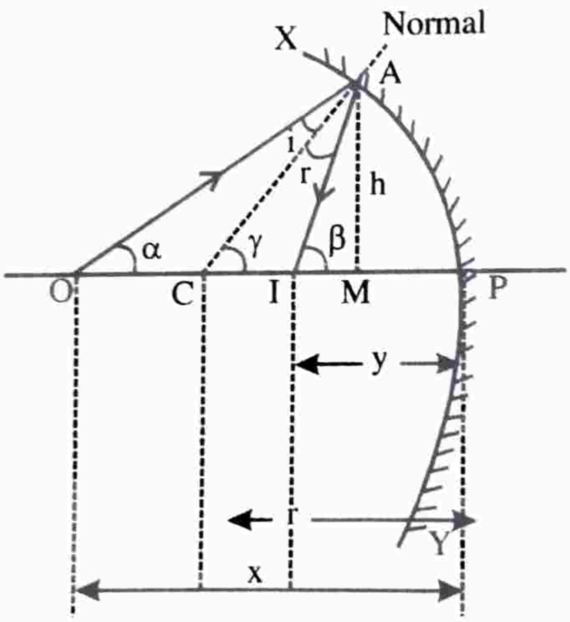
$$\frac{\frac{4}{3}}{v} = \frac{1}{600} - \frac{1}{20} = \frac{1-30}{600} = \frac{-29}{600}$$

$$v = \frac{-4}{3} \times \frac{600}{29} = -27.58\,\text{cm}.$$

Reflection by a single spherical surface

He we construct an image of a point object by a spherical mirror and derive the relation between the object and image distance.

Consider a spherical mirror XY whose concave surface is towards the point object O kept at a distance x from the pole P. The ray diagram is shown in figure below. i b the incident angle of incidence (angle between incident ray and the normal) and r be the angle of reflection (the angle between the reflected ray and the normal). The image is formed at I, where $PI = y$. The point C represents the centre of curvature of the mirror with PC = r.



$OM \simeq OP = x$
$CM \simeq CP = r$
$IM \simeq IP = y$

Figure 1.5

Here we consider paraxial approximation

From the ΔOAC,

$$\gamma = \alpha + i$$

$$i = \alpha - \gamma = \tan\alpha - \tan\gamma$$

$$i = \frac{AM}{OM} - \frac{AM}{CM} = \frac{h}{x} - \frac{h}{r}$$

From the ΔACI

$$\beta = r + \gamma$$

$$r = \beta - \gamma = \tan\beta - \tan\gamma$$

$$r = \frac{AM}{IP} - \frac{AM}{CM} = \frac{h}{y} - \frac{h}{r}$$

From the law of reflection $i = r$

$$\therefore \quad \frac{h}{x} - \frac{h}{r} = \frac{h}{y} - \frac{h}{r}$$

or $$\frac{1}{x} + \frac{1}{y} = \frac{2}{r} \qquad \text{..... (12)}$$

Applying the sign convention

$$x = -u,\ y = -v\ r = -R$$

we get $$\frac{1}{u} + \frac{1}{v} = \frac{2}{R} \qquad \text{..... (13)}$$

Note : According to the sign convention i is negative and r is positive.

According to Snell's law of refraction

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

In the case of reflection $i = -r$, thus we get

$$\therefore \quad -1 = \frac{\mu_2}{\mu_1}$$

or $$\mu_2 = -\mu_1$$

This shows that Snell's law of refraction becomes law of reflection if $\mu_2 = -\mu_1$. That is if we put $\mu_2 = -\mu_1$ in the refraction formula, it will become the reflection formula.

We have

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Put $\mu_2 - \mu_1$, we get

$$\frac{-\mu_1}{v} - \frac{\mu_1}{u} = \frac{-\mu_1 - \mu_1}{R}$$

i.e., $$\frac{1}{v} + \frac{1}{u} = \frac{2}{R}.$$

Example 4

Consider a spherical mirror of focal length f. If the object is kept at a distance x_1 from the focus and the image is obtained at a distance x_2 from the focus. Show that $x_1 x_2 = f^2$.

Solution

$$u = f + x_1 \text{ and } v = f + x_2 \text{ given}$$

Using $$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{f + x_1} + \frac{1}{f + x_2} = \frac{1}{f}$$

which on simplification gives $x_1 x_2 = f^2$

Example 5

A plane mirror is placed 22.5 cm in front of a concave mirror of focal length 10cm. Find where an object can be plaed between the two mirrors, so the first image in both the mirrors coincides.

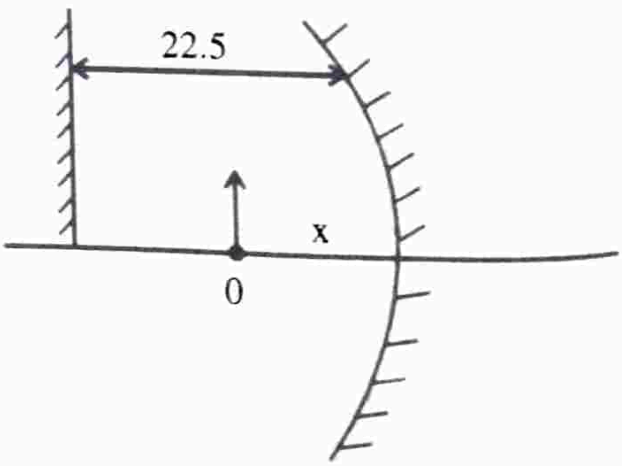


Figure 1.6

Solution

Let the object be placed at a distance x from the mirror. The image of this object will be at a distance of 22.5–x behind the plane mirror. For the concave mirror we have

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{-x}+\frac{1}{v}=\frac{1}{-10} \qquad \text{..... (1)}$$

According to the problem the two images must coincide. For this to happen

$$v=-(22.5+22.5-x)=-(45-x)$$

Put this in eq (1), we get $x=15\,\text{cm}$ or $30\,\text{cm}$. $x=30\,\text{cm}$ is not admissible, so $x=15\,\text{cm}$.

The thin lens

A medium bounded by two spherical refracting surfaces is referred to as a spherical lens. If the thickness of such a lens is very small compared to object and image distances and to the radii of curvature of the refracting surfaces then the lens is referred to as a thin spherical lens. In general a lens may have non spherical surfaces such as cylindrical, parabaloidal etc. However most lenses employed in optical systems are spherical.

Lenses are of two types (i) convex lens or converging lens and (ii) concave lens or diverging lens.

Convex lens

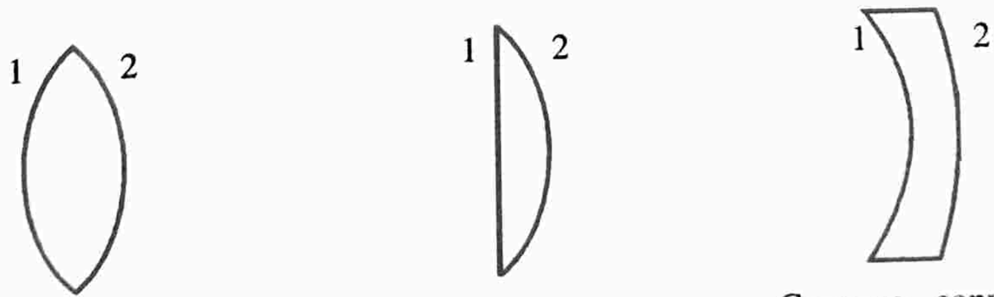
A lens which is thicker in the middle and thinner at the edges These are of three types

(1) Double convex lens

(2) Plano - convex lens

(3) Concavo - convex lens

These lenses are shown below with appropriate sign to radii of curvature.



Double convex $R_1 > 0,\ R_2 < 0$

Plano - convex $R_1 = \infty,\ R_2 < 0$

Concavo - convex $R_1 < 0,\ R_2 < 0$

Figure 1.7

Concave lens

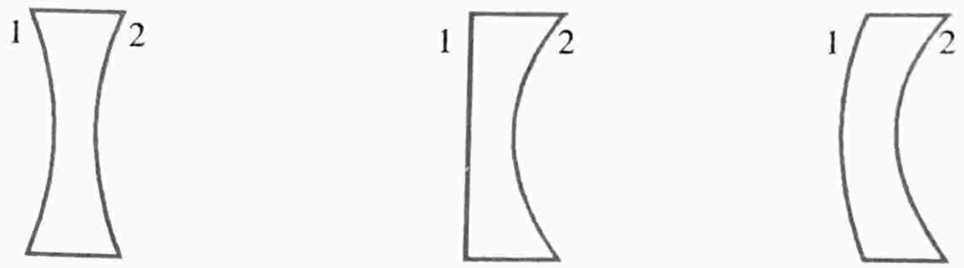
A lens which is thinner in the middle and thicker at the edges. These are also of three types.

(1) Double concave lens

(2) Plano - concave lens

(3) Convexo - concave lens

These lenses are shown below with appropriate sign to radii of curvature.



Double concave $R_1 < 0, R_2 > 0$

Plano - concave $R_1 = \infty\ R_2 > 0$

Convexo - concave $R_1 > 0, R_2 > 0$

Figure 1.8

Thin lens formula

Here we discuss the paraxial image formation by a thin lens.

Consider a convex lens ABDE made up of two refracting surfaces ABD and AED. Let μ_2 be the refractive index of the material of the lens and μ_1 be the refractive index of rarer medium in which lens is placed. C_1 and C_2 be the centres of curvature of two surfaces with optic centre C.

Consider a point object O lying on the principal axis of the lens. A ray of light starting from O and incident normally on the surface ABD along OC passes straight. Another ray incident on ABD along OA is refracted along AI_1. The two refracted rays meet at I_1 forming an image at I_1.

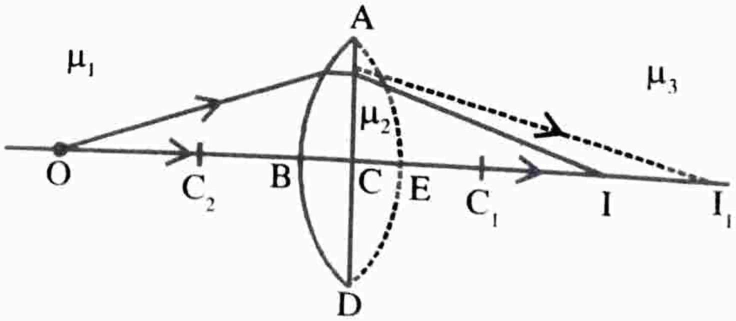


Figure 1.9

$OC = u$, $CI_1 = v'$ and $CC_1 = R_1$ radius of curvature of ABD.

Using refraction formula for the single surface ABD

$$\frac{\mu_2}{v'} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

where $v' = CI'$

$$\frac{\mu_2}{v'} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \qquad \text{..... (14)}$$

Now consider the second surface AED. For this surface I_1 acts as an object and due to refraction at this surface final image is formed at I.

For the surface AED, we have

$CI_1 = v'$, $CI = v$, $CC_2 = R_2$ Radius of curvautre of AED.

As refraction is from denser to rarer medium, using refraction formula

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

we have

$$\frac{\mu_1}{v} - \frac{\mu_2}{v'} = \frac{\mu_1 - \mu_2}{R_2} \qquad \text{..... (15)}$$

Adding equations 14 and 15, we get

$$\frac{\mu_1}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or
$$\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{..... (16)}$$

Put $\frac{\mu_2}{\mu_1} = \mu$, the refractive index of the material of the lens with respect to the surrounding medium.

$$\therefore \qquad \frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad \text{.....(17)}$$

Equation 17 is known as the thin lens formula.

If the object is at infinity, the image is formed at the principal focus of the lens.

i.e., when $u=\infty \quad v=f$, equation 17 becomes

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad \text{.....(18)}$$

This is known as the lens makers formula.

Comparing eqns 17 and 18 we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \qquad \text{.....(19)}$$

This is known as the Gauss' formula for a lens.

We derived the thin lens formula for a particular lens (convex) and for a particular object distance 'u' and assumed first image position v'. So depending upon the object distance and nature of the lens we obtain different formulae. To overcome this difficulty we apply the sign convention then irrespective of the object distances and the nature of the lenses we have only one formula. It is given by

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \qquad \text{.....(20)}$$

where $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ .....(21)

If should be remembered that while using this formula sign convention has to be applied.

The lens makers formula tells us that depending upon μ, R_1 and R_2, f may be positive or negative. f positive means a converging lens and f negative means a diverging lens.

For lens placed in air $\mu=\frac{\mu_2}{\mu_1}>1$. If the lens is convex, R_1 is +ve and R_2 is –ve

$\therefore \quad \frac{1}{R_1}-\frac{1}{R_2}$ is negative. It implies that f is +ve and the lens acts like a converging lens.

If $\mu > 1$ and the lens is concave. R_1 is –ve R_2 is +ve $\therefore \frac{1}{R_1} - \frac{1}{R_2}$ is –ve. It implies that f is –ve and the lens acts like a diverging lens. If a double convex lens of refractive index μ_2 is placed in a medium of refractive index μ_1 with $\mu_1 > \mu_2$, then

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\therefore \mu_1 > \mu_2$, $\frac{\mu_2}{\mu_1} - 1$ is negative, for a convex lens $\frac{1}{R_1} - \frac{1}{R_2}$ is +ve.

$\therefore$ f is –ve. i.e., convex lens acts like a diverging lens.

If a double concave lens is placed a medium whose refractive index is greater than that of the material of the lens $(\mu_1 > \mu_2)$. Then $\frac{\mu_2}{\mu_1} - 1$ is negative. $\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ is also negative, so $\frac{1}{f}$ is positive. i.e., concave lens acts like a converging lens.

The principal foci and focal lengths of a lens

Every lens has two surfaces. Each surface has its own focal points and planes and the lens as a whole has its own pair of focal points and focal planes. The focal points and focal planes of the lens are known as principal focal points and principal focal planes. Our aim is to locate principal focal points and focal planes.

First principal focal point

If a point object is placed on the principal axis is such that the rays refracted by the lens are parallel to the axis, then the position of the point object is called first principal focus (F_1) of the lens.

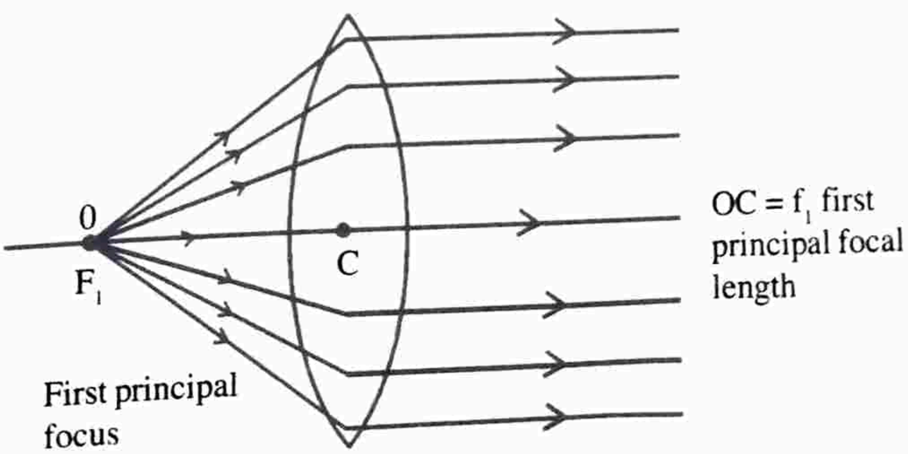


Figure 1.10

See figure 1.10. The distance between the first principal focus (F_1) and the optic centre of the lens (C) is called the first principal focal length f_1.

From thin lens formula, we have

$$\frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Here $u=f_1$, $v=\infty$

$$\therefore \qquad \frac{1}{\infty}-\frac{1}{f_1}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{-1}{f_1}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad \text{..... (22)}$$

This is the expression for first principal focus.

The plane perpendicular to the axis and passing through the first focal point is known as first principle focal plane.

Second principal focal point

If the object is situated at infinity, the position of the image on the axis is known as the second principal focus F_2. See figure 1.11. The distance between the second principal focus F_2 and the optic centre of the lens (C) is known as the second principal focal length f_2.

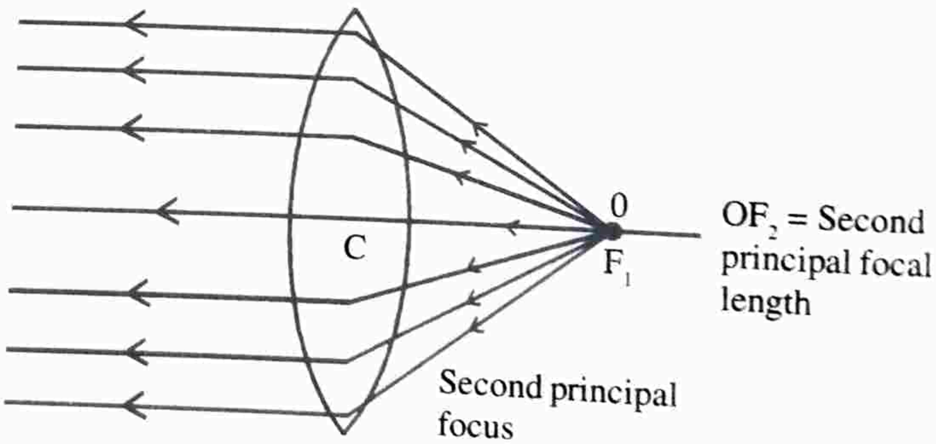


Figure 1.11

Using $u=\infty$ and $v=f_2$ in thin lens formula we get

$$\frac{1}{f_2}-\frac{1}{\infty}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f_2} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{..... (23)}$$

The plane perpendicular to the axis and passing through the second focal point is known as the second focal plane.

Comparing equations 22 and 23, we get $f_1 = -f_2$.

It may be noted that for a converging lens f_1 is negative and f_2 is positive where as for a diverging lens f_1 is positive and f_2 is negative.

The above discussion shows that the two focal lengths of a lens can be evaluated. Once f_1 and f_2 are known the paraxial images can be constructed by set of rules we already familiar.

Rules to draw ray diagram

(i) A ray passing through the first principal focus will after refraction emerge parallel to the axis.

(ii) A ray parallel to the axis after refraction either pass through or apper to come from the second principal focus.

(iii) A ray passing through the optic centre pass undeviated.

The Newton formula

In thin lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, distances u and v are measured from the optic centre. Instead Newton measured object and image distances from the focal points F_1 and F_2 so that we can accomodate the first and second principal focal lengths f_1 and f_2 in the equation.

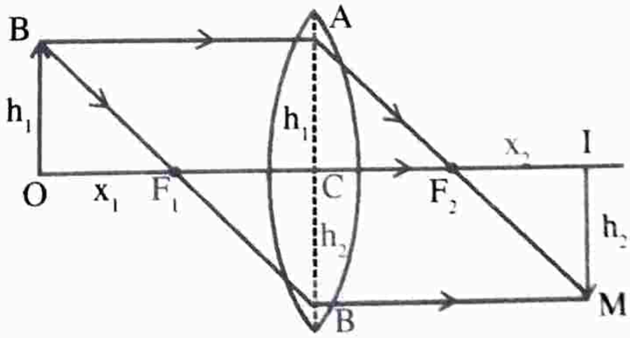


Figure 1.12

Let x_1 be the distance of the object from the first principal focus F_1 and let x_1 be the distance of the image from the second principal focus F_2 as shown in figure 1.12.

Let h_1 and h_2 be the heights of the object and image respectively. From the figure the Δ's OBF_1 and BCF_1 are similar

$$\therefore \qquad \frac{OB}{BC} = \frac{OF_1}{CF_1}$$

$$\frac{h_1}{h_2} = \frac{x_1}{f_1} \qquad \text{..... (24)}$$

Similarly the Δ's ACF_2 and IMF_2 are similar,

$$\therefore \qquad \frac{AC}{IM} = \frac{CF_2}{IF_2}$$

$$\frac{h_1}{h_2} = \frac{f_2}{x_2} \qquad \text{..... (25)}$$

Equating equations 24 and 25 we get

$$\frac{x_1}{f_1} = \frac{f_2}{x_2}$$

$$x_1 \ x_2 = f_1 f_2 \qquad \text{..... (26)}$$

This is known as Newton's law. When the thin lens has the same medium on both sides $-f_1 = f_2 = f$,

$$x_1 \ x_2 = -f^2$$

Negative sign shows that x_1 and x_2 must be of opposite sign. Thus if the object lies on the left side of the first principal focus then the image will be on the right of the second principal focus and versa.

Note : Though the Newton's formula is derived for convex lens, in general it is true.

Lateral magnification

The lateral magnification (m) is the ratio of the height of the image to that of the object. i.e., $m = \frac{h_2}{h_1}$

From equation 24, we have

$$\frac{h_2}{h_1} = \frac{x_1}{f_1}$$

here h_2 is –ve and h_1 is +ve.

$$\because \qquad \frac{h_2}{h_1} = -m$$

$$\therefore \qquad m = \frac{-x_1}{f_1} \qquad \text{..... (27)}$$

From equation 25, we get

$$m = \frac{-f_2}{x_2} \qquad \text{..... (28)}$$

If m_1 and m_2 represent the magnifications produced by two refracting surface, the total magnification.

$$m = m_1 m_2.$$

Another form of magnification

Referring to the last figure

we have

$$\frac{h_1}{OC} = \frac{h_2}{CI}$$

$$\therefore \qquad \frac{h_2}{h_1} = \frac{CI}{OC} = \frac{v}{u}.$$

$$m = \frac{v}{u}.$$

If m is positive, the image is errect and if m is negative the image is inverted.

Example 6

A thin lens having refracting index μ is placed in a medium such that the refractive indices on both sides are equal. Find the first and second principal focal lengths.

Solution

We have

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$v = \infty,\ u = -f_1 \quad \text{first principal focus}$$

$$\therefore \qquad \frac{-1}{f_1} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For the second principal focus

$$u = -\infty,\ v = f_2$$

$$\therefore \qquad \frac{1}{f_2} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Example 7

A thin biconvex lens made of a material whose refractive index is 1.5. The radii of curvature of the first and second surfaces are $+100\,\text{cm}$ and $-60\,\text{cm}$ respectively. The lens is placed in air. For an object at a distance of 100 cm from the lens, determine the position and linear magnification of the paraxial image. Also calculate x_1 and x_2 and verify Newton's formula.

Solution

$$\mu = 1.5,\ R_1 = +100\,\text{cm},\ R_2 = -60\,\text{cm}$$

$$u = 100\,\text{cm},\ v = ?\ m = ?$$

Using $$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{v} - \frac{1}{-100} = (1.5 - 1)\left(\frac{1}{100} - \frac{1}{-60}\right)$$

$$\frac{1}{v} + \frac{1}{100} = 0.5\left(\frac{1}{100} + \frac{1}{60}\right) = \left(\frac{1}{200} + \frac{1}{120}\right) = \frac{1}{75}$$

$$\frac{1}{v} = \frac{1}{75} - \frac{1}{100}$$

$$\therefore \qquad v = +300\,\text{cm}$$

$$|x_1| = u - f = 100 - 75 = 25$$

$$|x_2| = v - f = 300 - 75 = 225$$

$$x_1 = -25$$

$$x_2 = 225$$

$$\therefore \quad x_1 x_2 = -f^2$$

Example 8

An object of height 1 cm is placed at a distance of 24 cm from a convex lens of focal length 15 cm. A concave of focal length – 20 cm is placed beyond the convex lens at a distance of 25 cm. Determine the position and size of the final image.

Solution

For the convex lens

$$u = -24, \; f = 25\,\text{cm} \quad v = ?$$

Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

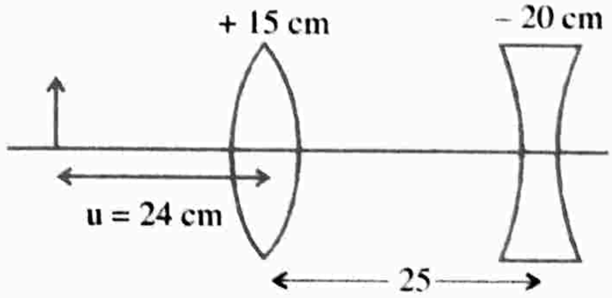


Figure 1.13

$$\frac{1}{v} - \frac{1}{-24} = \frac{1}{15}$$

$$\frac{1}{v} = \frac{1}{15} - \frac{1}{24} = \frac{1}{40}$$

$$v = +40\,\text{cm}.$$

This shows that convex lens forms an image at a distance 40 cm on the R.H.S. This image acts like an object for the concave lens. For the concave lens

$$u = 40 - 25 = 15\,\text{cm}. \quad f = -20\,\text{cm},\; v = ?$$

Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} - \frac{1}{15} = \frac{1}{-20}$$

$$\frac{1}{v}=\frac{1}{20}+\frac{1}{15}=\frac{1}{60}$$

$\therefore$ $v = 60\,\text{cm}.$

i.e., final image will be formed at a distance of 60 cm on R.H.S. of the concave lens.

Using $\frac{v}{u}=\frac{60}{-24}=-2.5$

Final image will be 2.5 times larger than the object.

Example 9

Consider a thin lens (μ_2) having different media on both sides. Let μ_1 and μ_3 be the refractive indices of the media derive the Gauss' formula then find first and second principal focal lengths.

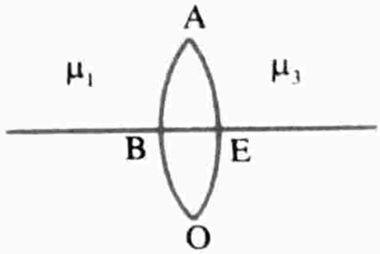


Figure 1.14

Solution

For the first surface we have

$$\frac{-\mu_1}{u}+\frac{\mu_2}{v'}=\frac{(\mu_2-\mu_1)}{R_2} \qquad \text{..... (1)}$$

For the second surface v' acts as an object, we have

$$\frac{-\mu_2}{v'}+\frac{\mu_3}{v}=\frac{(\mu_3-\mu_2)}{R_2} \qquad \text{..... (2)}$$

Adding eqns (1) and (2)

$$\frac{\mu_3}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R_1}+\frac{(\mu_3-\mu_2)}{R_2}$$

For the first principal focus

$$v=\infty \quad u=-f_1$$

$$\therefore \qquad \frac{\mu_1}{f_1}=\frac{(\mu_2-\mu_1)}{R_1}+\frac{(\mu_3-\mu_2)}{R_2}$$

$$\frac{1}{f_1}=\frac{1}{\mu_1}\left[\frac{(\mu_2-\mu_1)}{R_1}+\frac{(\mu_3-\mu_2)}{R_2}\right]$$

For the second principal focus

$$u = -\infty,\ v = f_2$$

$$\therefore \quad \frac{\mu_3}{f_2} = \frac{(\mu_2 - \mu_1)}{R_1} + \frac{(\mu_3 - \mu_2)}{R_2}$$

$$\frac{1}{f_2} = \frac{1}{\mu_3}\left[\frac{(\mu_2 - \mu_1)}{R_1} + \frac{(\mu_3 - \mu_2)}{R_2}\right]$$

Example 10

An object of 2 cm high is placed in front of a double convex lens of focal length 12.5 cm. On the other side of the lens a concave mirror of focal length 10 cm is placed at a distance of 45 cm form the lens. If the separation between the object and the mirror is 70 cm, calculate the location, nature and magnification of the image.

Solution

For the convex lens

$$u = -25\,\text{cm},\quad f = 12.5\,\text{cm}$$

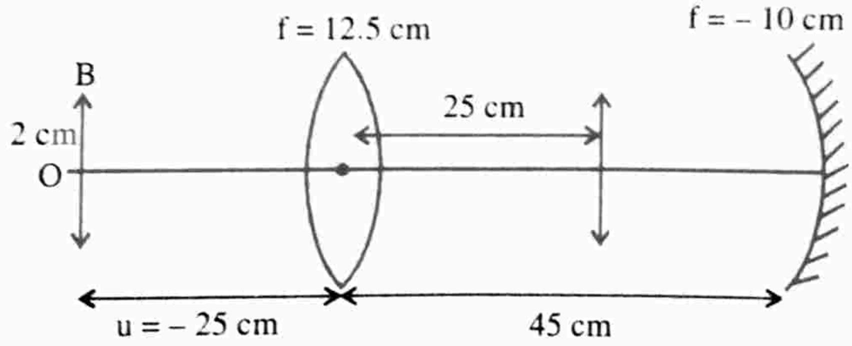


Figure 1.15

Using $\quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} + \frac{1}{25} = \frac{2}{25}$$

$$\frac{1}{v} = \frac{2}{25} - \frac{1}{25} = \frac{1}{25}$$

$$v = 25\,\text{cm}.$$

magnification, $m_1 = \frac{v}{u} = \frac{25}{-25} = -1$

The image of the lens acts as an object for the mirror. For the convex mirror

$$u = -(45 - 25) = -20$$

$$f = -10\,\text{cm}$$

Using $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$$\frac{1}{-20} + \frac{1}{v} = \frac{-1}{10}$$

$$\frac{1}{v} = \frac{-1}{10} + \frac{1}{20} = \frac{-1}{20}$$

$$v = -20\,\text{cm}.$$

magnification, $m_2 = \frac{-v}{u} = -\frac{-20}{-20} = -1$

This image will again acts an object for the convex lens

where $u = -25\,\text{cm}$

$$f = 12.5\,\text{cm}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-25} = \frac{1}{12.5}$$

$$\frac{1}{v} = \frac{-1}{25} + \frac{2}{25} = \frac{1}{25}$$

$$v = 25\,\text{cm}$$

magnification $m_3 = \frac{v}{u} = \frac{25}{-25} = -1$

$\therefore$ Total magnification, $m = m_1 \times m_2 \times m_3$

$$-1 \times -1 \times -1 = -1$$

Thus final image is inverted and real.

UNIVERSITY MODEL QUESTIONS

Section A

(*Answer questions in* **two** *or* **three** *sentences*)

Short answer type questions

1. What is the importance of Fermat's principle in optics?
2. Define optical path length.
3. State Fermat's principle of least time.
4. State Fermat's princple of extremum path.
5. When optical path length is minimum and when it is maximum?
6. State two uses of Fermat's principle.
7. Give three important laws that can be deducted from Fermat's principle.
8. State the laws of reflection.
9. State the laws of refraction.
10. State the Snell's law of refraction.
11. What is an optical system?
12. What is meant by paraxial approximation of image formation?
13. Why sign convention is necessary in optics?
14. What is a spherical lens? When it is called a thin lens?
15. Write down the thin lens formula and explain the symbols used.
16. Define the first principal focus and the second principal focus.
17. Write down Newton's formula and explain the symbols used.
18. Define lateral magnification and write it in terms of object and image focal distance.
19. What is the meaning of magnification $m = +1$ and $m = -1$?
20. Draw the diagrams of formation first principle focus and second principal focus of a concave lens.

Section B

(*Answer questions in a* **paragraph** *of about half a page to one page*)

Paragraph / Problem type questions

1. Define optical path length. Set up an expression for it.
2. State and explain Fermat's principle of least time.
3. State and explain Fermat's principle of extremum path.
4. Give examples for which the optical path length, the light prefers to take maximum, minimum and stationary.
5. Explain the rectilinear propagation of light using Fermat's principle of least time.

6. State and prove Fermat's principle of least time.
7. Show that the product of geometrical distance and the refractive index is a constant.
8. Write down the rules of sign convention.
9. Why sign convention is necessary while making use of mirror formula or thin lens formula?
10. Deduce expressions for first and second focal distances from thin lens formula.
11. Derive Newton's lens formula.
12. Write down three rules for drawing ray diagrams?
13. Discuss the convergence and divergence of lens placed in different media.
14. What is the refractive index of material of a plano convex lens, if the radius of curvature of the convex surface is 10 cm and focal length of the lens is 30 cm? $\left[\mu = \frac{4}{3}\right]$
15. Diameter of a plano-convex lens is 6 cm and its thickness at the centre is 3mm. What is the focal length of the lens if the speed of light in the material of lens is $2 \times 10^8 \, \text{ms}^{-1}$? $[f = 30 \, \text{cm}]$
16. A convex lens $(\mu = 3/2)$ has a focal length of 10 cm. Find the focal length of the lens if it is immersed in water. [40 cm]
17. Two lenses of focal lengths f_1 and f_2 are put in contact what is the effective focal length. $\left[F = \frac{f_1 f_2}{f_1 + f_2}\right]$
18. Two plano-convex lenses each of $\mu = 1.5$ have radii of curvature of 20 cm and $30 \, \text{cm}$. They are placed in contact with curved surfaces towards each other and the space between them is filled with a liquid of $\mu = \frac{4}{3}$, Find the focal length of the system. [– 72 cm]
19. The radius of curvature of the convex face of a palno-convex lens is 12 cm and $\mu = 1.5$. The plane surface of the lens is silvered. A point object is placed on the axis 20 cm from the lens. Find the position of the image.
20. A convex lens of focal length 20 cm and a concave lens of focal length 10 cm are separated by 8 cm. An object of height 1 cm is placed at a distance 40 cm from the convex lens. Calculate the position and size of the image.

$$\begin{bmatrix} v = -14.5 \text{ cm} \\ m = 0.453 \end{bmatrix}$$

Section C

(*Answer questions in about* **one** *or* **two** *pages*)

Long answer type questions (Essays)

1. Using Fermat's principle prove the laws of reflection and refraction.
2. State and explain Fermat's principle of stationary time. Derive the laws of refraction using this principle.
3. Derive the relation between u, v and R due to refraction at a single spherical surface.
4. Derive the relation between u, v and R due to reflection at a single spherical surface.
5. Derive thin lens formula and deduce lens makers formula.

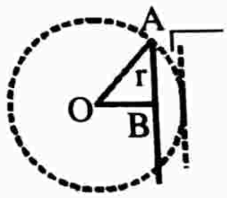
Hint to problems

14. $$\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$f=+30,\ \mu_1=1,\ R_1=+10,\ R_2=\infty$ calculate μ_2.

15. $$\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$\mu_2=\frac{c}{v}=1.5,\ \mu_1=1,\ R_1=\infty,\ R_2=-R$



y = 3mm = 0.3 cm
OB = R–y
OA = R

To find R_2

Using pythagorus theorem

$$R^2=r^2+(R-y)^2$$

$$R=\frac{r^2}{2y}=\frac{(6/2)^2}{2\times 0.3}=15\text{cm}\text{, proceed}$$

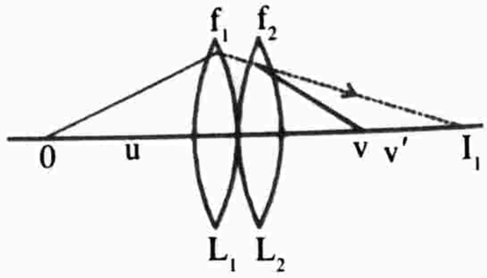
16. $$\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f_{air}}=\left(\frac{3/2}{1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad .....(1)$$

$$\frac{1}{f_{water}}=\left(\frac{3/2}{4/3}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad .....(2)$$

$\frac{eq\,1}{eq\,2}$ gives the answer.

17.



Using $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

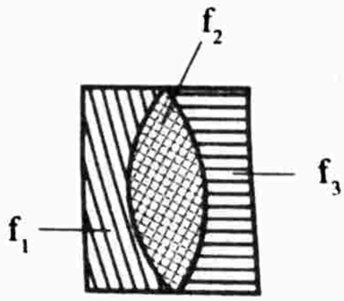
For the lens L_1

$$\frac{1}{v'}-\frac{1}{u}=\frac{1}{f_1} \qquad \text{..... (1)}$$

For the length L_2

$$\frac{1}{v}-\frac{1}{v'}=\frac{1}{f_2} \qquad \text{..... (2)}$$

Adding eqs 1 and 2 we get the result.



18. Using $\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$$\frac{1}{f_1}=\left(\frac{3/2}{1}-1\right)\left(\frac{1}{-\infty}-\frac{1}{20}\right)=\frac{-1}{40}$$

Similarly $\frac{1}{f_2}=\frac{5}{180}$

and $\frac{1}{f_3}=\frac{-1}{60}$

Finally use $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}$

19.  f of the lens before silvering. $\frac{1}{f_L} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$f_L = 24\,\text{cm}.$

On silvering, the second surface acts like a plane mirror. The light ray from the object passing through the lens will be incident on the mirror which will reflect it back through the lens again. The effective focal length of the system $\frac{1}{F} = \frac{1}{f_L} + \frac{1}{f_M} + \frac{1}{f_L}$

$f_M = \frac{R}{2} = \infty$. $\frac{1}{F} = \frac{2}{f_L}$ gives $F = -12\,\text{cm}$. Since the system behaves like a concave mirror. Using $\frac{1}{u} + \frac{1}{v} = \frac{1}{F}$, $u = -20, v = ?$ $F = -12$, we get $v = -30\,\text{cm}$.

20. For convex lens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, $u = -40, f = 20$

$\therefore \quad v = +40.$

For the concave lens $u = 40 - 8 = +32, f = -10$

$\therefore \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ gives $v = -14.5$

$$m_1 = \frac{v}{u} = \frac{40}{-40} = -1, \quad m_2 = \frac{v}{u} = \frac{14.5}{32} = -0.453$$

$\therefore \quad m_1 m_2 = -1 \times -0.45 = 0.453$

IMPORTANT FORMULAE

1. Optical path $= \mu S,\ S = vt$

   Optical path $= \int_P^Q \mu dS$

   (For a medium of continuously varying optical density, travels from P to Q.

2. Fermats principle $\delta t = \int_P^Q \frac{\mu}{c} dS = 0$

   or $\int_P^Q \mu dS = 0$

3. Law of reflection $i = r$

Law of (Snell's) refraction $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$

when $\mu_2 = -\mu_1$, law of refraction becomes law of reflection

4. The relation between u, v and R due to refraction at a single spherical surface.

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \quad \text{– rarer to denser}$$

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R} \quad \text{– denser to rarer}$$

5. The relation between u, v and R due to reflection at a single spherical surface

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

when $\mu_2 = -\mu_1$, the law governing refraction becomes that of reflection.

6. Thin lens formula

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \mu = \frac{\mu_2}{\mu_1}$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{– Lens makers formula}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

7. First and second principal focal lengths

$$\frac{1}{f_1} = -(\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_2} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

8. Newton's formula

$$x_1 x_2 = f_1 f_2$$

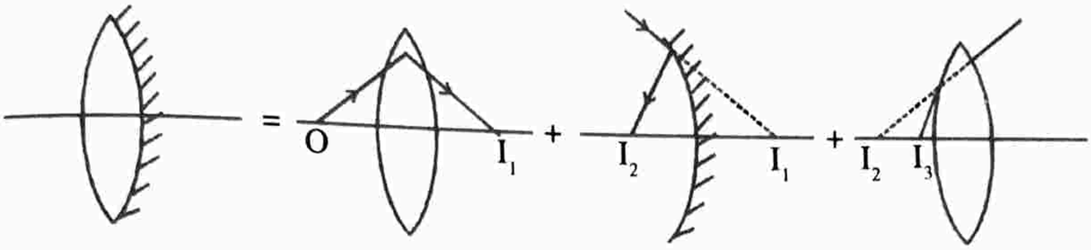
If the lens has same media on both sides $x_1 x_2 = -f^2$

9. Lateral magnification $m = \frac{v}{u}$ – Lens

or $m = \frac{-x_1}{f_1}$ or $m = \frac{-f_2}{x_2}$

10. Silvering of surfaces:

(i) If the back surface of a lens is silvered and object is placed in front of the lens.



$$\frac{1}{F} = \frac{1}{f_L} + \frac{1}{f_M} + \frac{1}{f_L}$$

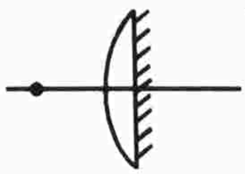
$$\frac{1}{f_L} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \quad \frac{1}{f_m} = \frac{R_2}{2}$$

$$\frac{1}{F} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{2}{R_2} + (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\frac{1}{F} = \frac{2(\mu-1)}{R}$, Apply sign convention $F = \frac{-2(\mu-1)}{R}$. The system acts like a curved mirror.

(ii) Silvered plano-convex lens

(a) When the plane surface is silvered and the object is in front of the curved surface



$$\frac{1}{f_2} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{(\mu - 1)}{R}$$

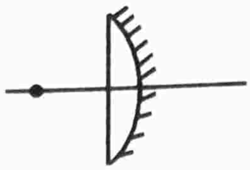
$$f_M = \frac{\infty}{2} = \infty$$

$$\frac{1}{F} = \frac{1}{f_L} + \frac{1}{f_M} + \frac{1}{f_L} = \frac{2}{f_L} = \frac{2(\mu - 1)}{R}$$

$$F = \frac{2(\mu - 1)}{R}, \text{ Apply sign convention } F = \frac{-2(\mu - 1)}{R}$$

The system acts like a concave mirror.

(b) When the curved surface is silvered and the object is in front of the plane surface.



$$\frac{1}{f_L} = (\mu - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = \frac{(\mu - 1)}{R}$$

$$f_M = \frac{R}{2}$$

$$\frac{1}{F} = \frac{1}{f_L} + \frac{1}{f_M} + \frac{1}{f_L} = \frac{2(\mu - 1)}{R} + \frac{2}{R}$$

$$\frac{1}{F} = \frac{2\mu}{R}, \quad F = \frac{R}{2\mu}. \text{ Apply sign convention } F = \frac{-R}{2\mu}$$

The system behaves like a converging mirror of focal length $\frac{R}{2\mu}$.