5 FRESNEL DIFFRACTION

Introduction

In chapter 4 we found that the phenomenon of diffraction can be broadly classified under two categories namely Fraunhofer and Fresnel. The last chapter dealt with Fraunhofer class of diffraction in detail. This chapter deals with Fresnel class of diffraction. The principle of Fresnel class of diffraction is the Huygens principle of secondary wavelets and the principle of interference. According to this each point on a wavefront is a source of secondary disturbance and secondary wavelets emanating from different points mutually interfere. In the case of Fresnel diffraction the source of light and the screen are at finite distances from the obstacle. The observation of Fresnel diffraction phenomenon does not require any lenses. The incident wave is not plane. As a result the phase of secondary wavelets is not the same at all points in the plane of the obstacle. Experimental observation of Fresnel diffraction is simple but the analysis is very complex.

Fresnel diffraction

According to Huygens wave theory of light, each progressive wave produces secondary waves, the envelop of which forms the secondary wave front. S is a source of monochromatic light and AB is a small aperture. XY is the screen placed in the path of light. CD is the illuminated portion of the screen and above C and below D is the region of the geometrical shadow. Considering AB as the primary wave front. According to Huygens construction, if secondary wave fronts are drawn, one would expect encroachment of light in the geometrical shadow. This bending of light around the edges of an obstacle or the encroachment of light within the geometrical shadow is known as diffraction.

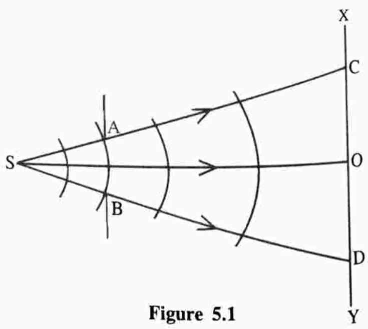


Figure 5.1

To explain the diffraction pattern Fresnel combined Huygen's wavelets principle

with the principle of interference and could satisfactorily explain the phenomenon of diffraction. For this he made the following assumptions. They are

(i) Every point of the wavefront sends secondary waves continuously.

(ii) The wave front can be divided into large number of strips or zones called Fresnel zones.

(iii) The resultant effect at any point on the screen is due to the combined effect of all the secondary waves coming from various zones.

(iv) The effect at any point on the screen due to any particular zone depends on the distance of the point from the zone.

(v) The effect at any point on the screen also depends on the inclination of the point with reference to the zone under consideration.

Note : It is to be noted that the resultant effect at any point is directly proportional to $(1+\cos\theta)$ called inclination factor. When $\theta = 0$, the inclination factor is maximum thus effect is maximum.

When $\theta = 180^\circ$, the inclination factor is zero. Thus effect is also zero. This explains why there is no backward propagation of energy.

Fresnel's half period zones

Let ABCD be a plane wave front of a monochromatic light of wavelength λ, travelling in the positive direction. P be a point at a distance b perpendicular to ABCD. To find the resultant intensity at P due to the wave front ABCD, divide the wave front into small areas called half period zones (HPZ). The half period zones

Figure 5.2

are drawn as follows. With P as centre and radii equal to $b+\frac{\lambda}{2}$, $b+\frac{2\lambda}{2}$, $b+\frac{3\lambda}{2}$ etc. construct series of spheres. The spheres cut the wave front into annular strips or zones. The section of these spheres on the plane wave front are concentric circles with O as their centre and radii equal to OM_1, OM_2, OM_3 etc. The area of the circle whose radius OM_1 is called first half period zone (HPZ). The annular ring between the circles of radii OM_2 and OM_1 is called second half period zone. Similarly the annular ring between n^{th} and $(n-1)^{th}$ circles is called n^{th} half period zone. These areas are called half period zones because the secondary wave reaching from successive zones reach at P with a path difference of $\frac{\lambda}{2}$, i.e. time difference of half period.

From the figure, the radius of the first half period zone,

$$OM_1 = \sqrt{PM_1^2 - PO^2}$$

i.e.

$$OM_1 = \sqrt{\left(b+\frac{\lambda}{2}\right)^2 - b^2}$$

or

$$OM_1 = \sqrt{b\lambda + \frac{\lambda^2}{4}}$$

Since λ is a small quantity $\frac{\lambda^2}{4}$ is negligible

$$\therefore \quad OM_1 = \sqrt{b\lambda} \qquad \text{......(1)}$$

$\therefore$ The area of the first half period zone

$$= \pi OM_1^2 = \pi b\lambda \qquad \text{......(2)}$$

The radius of the second half period zone,

$$OM_2 = \sqrt{PM_2^2 - PO^2}$$

$$= \sqrt{\left(b+\frac{2\lambda}{2}\right)^2 - b^2}$$

or

$$OM_2 = \sqrt{2b\lambda + \lambda^2}$$

Again neglecting λ^2, we get

$$OM_2 = \sqrt{2b\lambda} \qquad \text{......(3)}$$

$\therefore$ The area of the first two half period zones

$$= \pi\, OM_2^2 = 2\pi b\lambda$$

$\therefore$ The area of the second half period zone

$$= \pi\, OM_2^2 - \pi\, OM_1^2$$

$$= 2\pi b\lambda - \pi b\lambda = \pi b\lambda \qquad \text{......(4)}$$

Similarly, the radius of the $(n-1)^{th}$ half period zone

$$OM_{n-1} = \sqrt{PM_{n-1}^2 - PO^2} = \sqrt{\left[b+(n-1)\frac{\lambda}{2}\right]^2 - b^2}$$

$$OM_{n-1} = \sqrt{(n-1)b\lambda} \qquad \text{......(5)}$$

The radius of the n^{th} half period zone

$$OM_n = \sqrt{PM_n^2 - PO^2}$$

$$OM_n = \sqrt{nb\lambda} \qquad \text{......(6)}$$

The area of the n^{th} half period zone

$$= \text{area of the first n zones} - \text{area of the first (n−1) zones}$$

$$= \pi OM_n^2 - \pi OM_{n-1}^2 = \pi[nb\lambda - (n-1)b\lambda]$$

$$= \pi b\lambda \qquad \text{......(7)}$$

Thus we find from equations 1, 3, 5, and 6 that radii of the half period zones are directly proportional to the square root of the order of the zone. Equations 2, 4, 6 and 7 show that the areas of zones are approximately equal and independent of the order of the zone.

Amplitude at P due to individual half period zones

To find out the resultant effect of the whole wavefront at P now reduces to find out the resultant of a large number of secondary waves originating from the zones into which the whole wavefront is divided. The amplitude due to a zone reaching the point P depends on the following factors.

(i) It is directly proportional to the area of the zone.

(ii) It is inversely proportional to the mean distance of the zone from the point P and

(iii) It is directly proportional to the inclination factor $(1+\cos\theta)$, where θ is the angle between the normal to the zone and the line joining the zone to P.

$\therefore$ Amplitude due to n^{th} zone,

$$R_n \propto \frac{\text{Area of the } n^{th} \text{ zone} \times (1+\cos\theta_n)}{\text{mean distance of the } n^{th} \text{ zone}}$$

$$R_n \propto \frac{(\pi OM_n^2 - \pi OM_{n-1}^2)\,(1+\cos\theta_n)}{\frac{1}{2}\left\{\left(b+n\frac{\lambda}{2}\right)+\left[b+(n-1)\frac{\lambda}{2}\right]\right\}}$$

$$\pi OM_n^2 - \pi OM_{n-1}^2 = \pi\left[\left(b+n\frac{\lambda}{2}\right)^2 - b^2\right] - \pi\left\{\left[b+(n-1)\frac{\lambda}{2}\right]^2 - b^2\right\}$$

$$= \pi\left[nb\lambda + \frac{n^2\lambda^2}{4} - b(n-1)\lambda - \frac{(n-1)^2\lambda^2}{4}\right]$$

$$= \pi\left[nb\lambda + \frac{n^2\lambda^2}{4} - bn\lambda + b\lambda - \frac{n^2\lambda^2}{4} + \frac{2n\lambda}{4} - \frac{\lambda^2}{4}\right]$$

$$= \pi\lambda\left[b+(2n-1)\frac{\lambda}{4}\right]$$

$$\therefore \quad R_n \propto \frac{\pi\lambda\left[b+(2n-1)\frac{\lambda}{4}\right](1+\cos\theta_n)}{\left[b+(2n-1)\frac{\lambda}{4}\right]}$$

or

$$R_n \propto \pi\lambda(1+\cos\theta_n) \qquad \text{...... (8)}$$

This equation shows that as the order of the zone n increases, θ_n increases and $\cos\theta_n$ decreases therefore the amplitude of the wave at P due to a zone decreases as the order of the zone increases.

Resultant amplitude and intensity at p due to the whole wave front

Let $R_1, R_2, R_3 \ldots\ldots R_n$ be the amplitudes of the wave at P due to first, second, third ...... n^{th} zones respectively. These amplitudes are in slightly decreasing order because of the inclination. If the phase of the wavelets starting from the point O is taken to be zero and that of the wavelet from the circumference of M_1 is π. Hence the phase of the wave coming out of first half period zone will be the average of these two phases i.e. $\frac{0+\pi}{2}=\frac{\pi}{2}$. Similarly the phase of the wavelet starting from second half period zone will be the mean of π and 2π. i.e. $\frac{\pi+2\pi}{2}=\frac{3\pi}{2}$. It shows that the phases of wavelets coming from successive outer zones are $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$, etc. which means that wavelets arriving from two consecutive zones will be out of phase. Hence $R_1, R_2, R_3 \ldots\ldots R_n$ are alternately positive and negative. Therefore the resultant amplitude at P due to entire wave front is

$$R = R_1 - R_2 + R_3 - R_4 + \cdots\cdots \pm R_n \qquad \text{...... (9)}$$

Since R_1, R_2, R_3 are in continuously decreasing order, to a close approximation we can take

$$R_2 = \frac{R_1+R_3}{2}$$

$$R_4 = \frac{R_3+R_5}{2} \quad \text{etc.}$$

Thus

$$R = \frac{R_1}{2} + \frac{R_1}{2} - \frac{R_1}{2} - \frac{R_3}{2} + \frac{R_3}{2} + \frac{R_3}{2} - \frac{R_3}{2} - \frac{R_5}{2} + \cdots\cdots \frac{R_{n-1}}{2} - R_n \text{ or } \frac{R_n}{2}$$

Last factor is $\frac{R_n}{2}$, if n is odd and it is $\frac{R_{n-1}}{2} - R_n$ or nearly $-\frac{R_n}{2}$ when n is even

$$\therefore \quad R = \frac{R_1}{2} \pm \frac{R_n}{2}$$

If the wavefront is large, n is large the term R_n is negligible because of inclination

Hence

$$R = \frac{R_1}{2} \qquad \text{...... (10)}$$

i.e. the resultant amplitude at point P is only half of the amplitude of the wavelet from first half period zone.

$\therefore$ Intensity at P, $I \propto (\text{amplitude})^2 \propto \left(\frac{R}{2}\right)^2 \propto \frac{R_1^2}{4}$. Thus the intensity at P is only one fourth of that due to the first half period zone alone. Here, only half the area of the first half period zone is effective in producing the illumination at the point P.

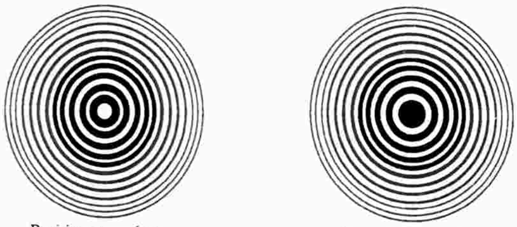
A small obstacle of the size of half the area of the first half period zone placed at

O will screen the effect of the whole wave front and the intensity at P due to the rest of the wave front will be zero. While considering the rectilinear propagation of light, the size of the obstacle used is far greater than the area of the first half period zone and hence the bending effect of light around corners cannot be observed. If the size of the obstacles placed in the path of light is comparable to the wavelength of light, then it is possible to observe illumination in the region of the geometrical shadow also. Thus, rectilinear propagation of light is only approximately true.

Zone plate

A zone plate is a specially constructed plate having alternate transparent and opaque zones.

To construct a zone plate concentric circles are drawn on a white paper such that the radii are proportional to square root of natural numbers. First circle represents the first zone and the annular ring between the first and second circle represents second zone and so on. Now if the odd numbered zones (1^{st}, 3^{rd}, 5^{th} etc.) are covered with black ink and a reduced photograph on a plane glass plate is taken. The resulting photographic transparency (negative film) is called positive zone plate.



Positive zone plate   Negative zone plate

Figure 5.3

i.e. *A positive zone plate is a plate in which the odd numbered zones are transparent to light and even numbered zones will cut off the light.*

But if we *make a plate such that even numbered zones are transparent and odd numbered zones are opaque then the plate is called a negative zone plate.*

For a positive zone plate, the resultant amplitude at the image point P becomes

$$R = R_1 + R_3 + R_5 + \cdots\cdots$$

This is because for a positive zone plate, odd numbers zones are transparent.

For a negative plate, the resultant amplitude at the image point P becomes

$$R = -(R_2 + R_4 + R_6 + \cdots\cdots)$$

It is to be noted that in either case the amplitude and hence intensity is enormously increased.

Theory of zone plate

Consider a monochromatic source of light S, emitting spherical waves of wavelength λ. Let AB represent the cross section of the zone plate. P be a point on the screen at which we have to find the intensity. Both S and P are on the axis of the zone plate at distances a and b respectively.

Let $r_1, r_2, r_3, \ldots\ldots, r_n$ be the radii of first, second, third ...... n^{th} zone respectively.

From the figure we have

$$SO + OP = a + b$$

$$SM_1 + M_1P = a + b + \frac{\lambda}{2}$$

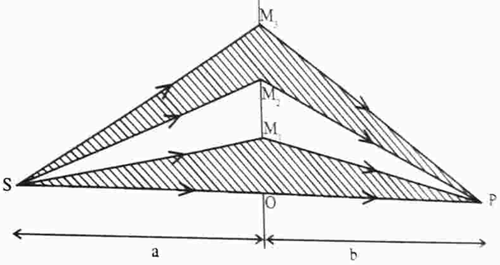


Figure 5.4

This is because we have found that the waves reaching at P from successive zones differ in path by $\frac{\lambda}{2}$.

Similarly $$SM_2 + M_2P = a + b + 2\frac{\lambda}{2}$$

$\therefore$ $$SM_n + M_nP = a + b + n\frac{\lambda}{2} \quad \ldots\ldots(11)$$

But $$SM_n = (SO^2 + OM_n^2)^{\frac{1}{2}}$$

$$SM_n = (a^2 + r_n^2)^{\frac{1}{2}} \quad \ldots\ldots(12)$$

and
$$M_nP = (PO^2 + OM_n^2)^{\frac{1}{2}}$$

$$M_nP = (b^2 + r_n^2)^{\frac{1}{2}} \qquad \text{...... (13)}$$

Using equations 12 and 13, equation 11 becomes

$$(a^2 + r_n^2)^{\frac{1}{2}} + (b^2 + r_n^2)^{\frac{1}{2}} = a + b + n\frac{\lambda}{2}$$

$$a\left(1 + \frac{r_n^2}{a^2}\right)^{\frac{1}{2}} + b\left(1 + \frac{r_n^2}{b^2}\right)^{\frac{1}{2}} = a + b + n\frac{\lambda}{2}$$

$$a\left(1 + \frac{r_n^2}{2a^2}\right) + b\left(1 + \frac{r_n^2}{2b^2}\right) = a + b + n\frac{\lambda}{2}$$

[In the last step Binomial approximation is used i.e. $(1+x)^n \approx 1 + nx$ if $x \ll 1$]

or
$$a + \frac{r_n^2}{2a} + b + \frac{r_n^2}{2b} = a + b + \frac{n\lambda}{2}$$

$$r_n^2\left(\frac{1}{a} + \frac{1}{b}\right) = n\lambda$$

or
$$\frac{1}{a} + \frac{1}{b} = \frac{n\lambda}{r_n^2} \qquad \text{...... (14)}$$

Applying new Cartesian sign convention a is –ve, b is +ve, we get

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{\frac{r_n^2}{n\lambda}} \qquad \text{...... (15)}$$

This is very much similar to the lens equation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad \text{...... (16)}$$

comparing two equations, we get

$$\frac{1}{f} = \frac{1}{r_n^2/n\lambda}$$

i.e.
$$f = \frac{r_n^2}{n\lambda} \qquad \text{...... (17)}$$

Here f is called the principal focal length of the zone plate. Thus a zone plate acts as a converging lens. For different half period zones n will be different hence f will also be different. f will also be different for different wave lengths. Thus the zone plate has number of foci depending on the number of zone and wavelengths.

Multiple foci of the zone plate

Suppose a plane wavefront is incident normally on the zone plate i.e. the point source is at infinity or $a = \infty$. The zone plate focuses the wavefront at any point (say p) whose distance is given by

$$\frac{1}{b} - \frac{1}{a} = \frac{n\lambda}{r_n^2} = \frac{1}{f}$$

i.e.
$$\frac{1}{b} = \frac{n\lambda}{r_n^2} \qquad (\because \ a = \infty)$$

or
$$f = b = \frac{r_n^2}{n\lambda}$$

This distance is called the principal focal length of the zone plate.

In this case the area of each zone is $\pi b\lambda$. Now consider another point (say p_3) on the axis of the zone plate at a distance $\frac{b}{3}$ from the zone plate. Then the area of each half period zone with respect to the point p_3 will be $\frac{1}{3}\pi b\lambda$. And each zone plate will contain three half period zones. Hence the resultant amplitude at p_3 will be

$$R = (R_1' - R_2' + R_3') + (R_7' - R_8' + R_9')$$

Here R_1', R_2' ...... will be hearly equal to $\frac{R_1}{3}, \frac{R_2}{3}$ ...... respectively. And R_4', R_5', R_6' are cut off due to opaque zone.

Therefore
$$R = \frac{1}{2}(R_1' + R_3' + R_7' + R_9' \cdots\cdots)$$

Thus point p_3 will also be sufficiently bright with respect to other points. And it represents the second focus of the zone plate. The focal length of the second focus is given by

$$f_3 = \frac{r_n^2}{3n\lambda}$$

Similarly the intensity of light is sufficiently large at the distances $\frac{b}{5}, \frac{b}{7}$ etc.
Hence the third and fourth focal lengths are given by

$$f_5 = \frac{r_n^2}{5n\lambda}, \quad f_7 = \frac{r_n^2}{7n\lambda}$$

Thus we see that a zone plate has multiple foci. But the intensity at the principal focus is maximum and goes on decreasing for the successive foci.

Comparison of zone plate and convex lens

Similarities

(i) Focal length of zone plate and convex lens depend upon wavelength and hence both show chromatic aberration.

For a zone plate $f \,\alpha \frac{1}{\lambda}$.

Thus $f_r < f_v \quad \because \quad \lambda_r > \lambda_v$

For a convex lens $f \,\alpha\, \lambda$

Thus $f_r > f_v$

(ii) Formulae connecting conjugate distance in the case of zone plate and convex lens are similar.

For the lens we have $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

For the zone plate, we have $\frac{1}{b} - \frac{1}{a} = \frac{1}{f}$ where $\frac{1}{f} = \frac{n\lambda}{r_n^2}$

(iii) Zone plate and convex lens both form real image of an object on opposite side of the object.

(iv) The formula for linear magnification for a zone plate is similar to that of a lens.

Dissimilarities

(i) The zone plate, unlike convex lens has multiple foci but convex lens has only one.

(ii) For a convex all the rays reaching the image point have the same optical path while in case of a zone plate the path difference between the waves reaching the image point from two successive transparent zones is λ.

(iii) The intensity of the image formed by a zone plate is less than that formed by the lens.

Phase reversal zone plates

If the alternate zones of a zone plate instead of being blocked are coated with thin film of some transparent material such that it introduces an additional path difference of $\frac{\lambda}{2}$ between the waves emananting from successive zones. Hence amplitudes from successive zone will add each other. Thus the intensity of the image would be four fold. Such a zone plate is called phase reversal zone plate.

Example 1

What is the radius of sixth zone in a zone plate of focal length 10cm for a light of wavelength $\lambda = 6000\text{Å}$

Solution

$$f = 10\text{cm} = 10 \times 10^{-2}\text{m}$$

$$\lambda = 6000\text{Å} = 6000 \times 10^{-10}\text{m}$$

Using $f = \frac{r_n^2}{n\lambda}$ or $r_n = \sqrt{nf\lambda}$

$$r_6 = \sqrt{6 \times f \times \lambda}$$

$$r_6 = \sqrt{6 \times 10 \times 10^{-2} \times 6000 \times 10^{-10}}$$

$$r_6 = 6 \times 10^{-4}\text{m}$$

Example 2

The radius of the first zone on the zone plate is 0.05cm. If a plane wave front of light of wavelength $\lambda = 5000\text{Å}$ is incident on it. Find the distance of the screen from the zone plate so that light is focussed to bright spot.

Solution

$$r_n = 0.05\text{cm} = 5 \times 10^{-4}\text{m}$$

$$\lambda = 5000\text{Å} = 5000 \times 10^{-10}\text{m} = 5 \times 10^{-7}\text{m}$$

Using $f = \frac{r_n^2}{n\lambda}$

Maximum brightness is obtained when n = 1

$$\therefore \quad f = \frac{r_1^2}{\lambda} = \frac{(5 \times 10^{-4})}{5 \times 10^{-7}} = 0.5\text{m}$$

Thus, the zone plate has to be kept at a distance of 0.5m from the screen.

Example 3

A point source of light of wavelength 5000Å placed at a distance of 1.5m on the axis of a zone plate of diameter 1cm produces the remotest image at a distance of 3 cm from the plate. Calculate the number of zones marked on the plate.

Solution

$$\lambda = 5000\text{Å} = 5000 \times 10^{-10}\,\text{m} = 5 \times 10^{-7}\,\text{m}$$

$$a = -1.5\text{m} = \frac{-3}{2}\,\text{m}$$

$$b = 3\text{m}$$

$$r_n = 0.5 \times 10^{-2}\,\text{m} \qquad (\because \; d = 1\text{cm} = 10^{-2}\,\text{m})$$

Using $\frac{1}{b} - \frac{1}{a} = \frac{n\lambda}{r_n^2}$

$$\frac{1}{3} + \frac{2}{3} = \frac{n \times 5 \times 10^{-7}}{(0.5 \times 10^{-2})^2}$$

$$1 = \frac{n \times 5 \times 10^{-7}}{0.25 \times 10^{-4}} = \frac{n \times 10^{-1}}{5}$$

$$\therefore \quad n = 50$$

Example 4

Calculate the size of the circular opening in an opaque screen which will transmit 10 Fresnel zones to a point 1m away. Given $\lambda = 6000\text{Å}$

Solution

$$b = 1\text{m}$$

$$\lambda = 6000\text{Å} = 6 \times 10^{-7}\,\text{m}$$

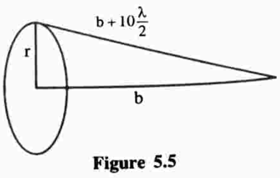


Figure 5.5

From the figure $\left(b + 10\frac{\lambda}{2}\right)^2 = b^2 + r^2$

$$b^2 + 10b\lambda + 25\lambda^2 = b^2 + r^2$$

or $\quad r^2 = 10b\lambda + 25\lambda^2$

Since λ is small, λ^2 can be neglected

$$\therefore \quad r = \sqrt{10b\lambda}$$

$$r = \sqrt{10 \times 1 \times 6 \times 10^{-7}} = 10^{-3}\sqrt{6}$$

$$r = 2.449 \times 10^{-3}\,\text{m}$$

Note: Or use directly $\pi r^2 = n\pi b\lambda$

Area of circular disc = area of n Fresnel half period zones.

Example 5

A zone plate with radii $r_n = 0.1\sqrt{n}$ cm. For $\lambda = 5 \times 10^{-5}$cm, calculate the positions of various foci.

Solution

$$r_n = 0.1\sqrt{n}, \quad \lambda = 5 \times 10^{-5}\,\text{cm}$$

We have $\quad f = \frac{r_n^2}{n\lambda}$

For the primary image $n = 1$

$$\therefore \quad f_1 = \frac{r_1^2}{\lambda} = \frac{10^{-2}}{5 \times 10^{-5}} = 200\text{cm}$$

For the secondary image $n = 3$

$$\therefore \quad f_2 = \frac{r_1^2}{\lambda} = \frac{200}{3}\,\text{cm}$$

Similarly $\quad f_3 = \frac{r_1^2}{5\lambda} = \frac{200}{5}\,\text{cm}$ and so on.

Example 6

Consider a plane wave of wavelength 6×10^{-7}cm incident normally on a circular aperture of radius 0.01cm. Calculate the positions of the brightest and the darkest points on the axis.

Solution

For the brightest point, the aperture should contain only the first zone.

Thus we have $\quad r_1 = \sqrt{b\lambda} \quad$ (see eqn 1)

or $\quad r_1^2 = b\lambda \;\; (\pi r_1^2 = \pi b\lambda)$

$$r_1 = 0.01\text{cm} = 10^{-2}\,\text{cm}$$

$$\therefore \qquad 10^{-4} = b \times 6 \times 10^{-7}$$

$$\text{or} \qquad b = \frac{10^{-4}}{6\times10^{-5}} = \frac{10}{6} = 1.67\text{cm}$$

For the darkest point we have

$$r_1^2 = 2\frac{b_1\lambda}{2}$$

$$\therefore \qquad b_1 = \frac{r_1^2}{2\lambda} = \frac{1.67}{2} = 0.835\text{cm}$$

Example 7

If a zone plate has to have a principle focal length of 50 cm corresponding to $\lambda = 6\times10^{-7}$m, obtain an expression for the radii of different zones. What would be its principal focal length for $\lambda = 5\times10^{-7}$m.

Solution

Using $f = \frac{r_n^2}{n\lambda}$ $\quad f = 0.5\text{m}, \quad \lambda = 6\times10^{-7}\text{m}$

$$0.5 = \frac{r_n^2}{n\times6\times10^{-7}}$$

$$\therefore \qquad r_n = \sqrt{0.3\times10^{-6}n} = \sqrt{0.3\times n}\ \text{mm}$$

When $\lambda = 5\times10^{-7}$m,

$$f \propto \frac{1}{\lambda}$$

$$0.5 \propto \frac{1}{6\times10^{-7}} \qquad \text{......(1)}$$

$$f \propto \frac{1}{5\times10^{-7}} \qquad \text{......(2)}$$

$$\frac{\text{eqn 2}}{\text{eqn 1}} \text{ gives} \qquad \frac{f}{0.5} = \frac{6}{5}$$

$$\therefore \qquad f = 0.6\text{m} = 60\text{cm}$$

Example 8

A plane wavefront of wavelength 6×10^{-7}m is intercepted by a circular aperture of radius 0.3cm. Calculate the number of half period zones in the aperture with respect to an axial point at distance (i) 100 cm (ii) 500 cm.

Solution

$\lambda = 6\times10^{-7}$m, $a = \infty$, $b = 100\text{cm} = 1\text{m}$

$r_n = 0.3\times10^{-2}$m

For the zone plate we have

$$\frac{1}{b} - \frac{1}{a} = \frac{n\lambda}{r_n^2} \qquad \text{(1)}$$

$$1 = \frac{n\times6\times10^{-7}}{(0.3\times10^{-2})^2}$$

$$\therefore \qquad n = \frac{(0.3\times10^{-2})^2}{6\times10^{-7}} = \frac{0.09\times10^{-4}}{6\times10^{-7}} = \frac{90\times10}{6\times10^{-7}}$$

$$n = 15$$

(ii) Fromeqn (1) $\quad \frac{1}{b} \propto n$

i.e. $\quad 1 \propto 15 \quad$ in part (i)

$\frac{1}{5} \propto n \quad$ in part (ii) b = 500 cm = 5 m

$$\therefore \qquad 5 = \frac{15}{n}$$

$$\text{or} \qquad n = 3$$

Example 9

A circular aperture of 1.4×10^{-3}m diameter is illuminated by plane wave of monochromatic light. The diffracted light is received on a distant screen which is slowly moved towards the aperture. The centre of circular patch of light first becomes dark when the screen is at 0.49m from the aperture. Calculate the wavelength of light.

Solution

The centre is dark when the aperture is equal to two half period zones

i.e.

$$2\pi b\lambda = \pi r^2$$

$$\therefore \qquad \lambda = \frac{r^2}{2b} = \frac{(0.7\times10^{-3})}{2\times0.49} = 5\times10^{-7}\text{m}$$

Example 10

A point source of monochromatic light of wavelength $\lambda = 5.5\times10^{-7}$ m is kept at a distance of 0.5 m from an aperture of radius 5×10^{-4} m. Calculate the farthest point along the axis where the intensity is zero. Also calculate the radius of the dark central disc formed on the screen placed there.

Solution

$\lambda = 5.5\times10^{-7}$ m, $a = -0.5$m, $r = 5\times10^{-4}$ m

$b = ?$ $n = 2$ since the centre to be dark.

Using
$$\frac{1}{b}-\frac{1}{a}=\frac{n\lambda}{r^2}$$

$$\frac{1}{b}+\frac{1}{0.5}=\frac{2\times5.5\times10^{-7}}{(5\times10^{-4})^2}=\frac{110}{25}=\frac{22}{5}$$

or
$$\frac{1}{b}=\frac{22}{5}-\frac{10}{5}=\frac{12}{5}$$

$$b=\frac{5}{12}=0.4167\text{ m}$$

∴ The radius of the nth dark ring.

$$r_n=\frac{nb\lambda}{2r}$$

∴ The radius of the central dark disc

$$r_1=\frac{b\lambda}{2r}=\frac{0.4167\times5.5\times10^{-7}}{2\times5\times10^{-4}}=2.3\times10^{-4}\text{m}$$

UNIVERSITY MODEL QUESTIONS

Section A

(*Answer questions in* **two** *or* **three** *sentences*)

Short answer type questions

1. What is diffraction according to Fresnel?
2. What are the peculiarities of Fresnel diffraction with reference to Fraunhofer diffraction?
3. What are the assumptions made by Fresnel to explain the diffraction pattern?
4. What are Fresnels half period zones? Why are they called so?
5. What is a zone plate? What are the two types of it?

6. Compare a zone plate and a convex lens.
7. What are the factors on which the amplitude due to n^{th} zone depend?
8. What are the dissimilarities between a zone plate and a convex lens?
9. What is meant by phase reversal zone plate?
10. What happens to intensity of the image in a phase reversal zone plate?
11. What is the principle of Fresnel diffraction?
12. Define the principal focal length of a zone plate. Write down an expression for it
13. Discuss the nature of fringes produced if white light is incident on a straight edge
    [We have $y_1 \propto \sqrt{\lambda}$]
    If white light is incident on the straight edge, then in the position of first bright fringe we observe colours in order of violet to red. In the second bright fringe due to overlapping of colours the pattern becomes uniform.
14. In a zone plate the focal length of red colour is less that of violet. How?
15. When white light is incident on a straight edge, how does the colour of the centre of the diffraction pattern become violet?

Section B

(*Answer questions in a* **paragraph** *of about half a page to one page*)

Paragraph / Problem type questions

1. Show that areas of half period zones are equal.
2. Show that areas of half period strips are in decreasing order.
3. What would happen when the circular aperture in Fresnel diffraction pattern is replaced by a circular disc of the same radius.
   [In the diffraction pattern due to circular disc the centre of pattern is always bright, and it is surrounded by dark and bright rings, within the geometrical shadow. And there are brighter and broader rings beyond the geometrical shadow. The intensity of brighter rings goes on decreasing while that of the dark rings goes on increasing as we move outside the geometrical shadow. These rings are unequally spaced.]
4. Show that a zone plate has multiple focii.
5. Derive an expression for the position of n^{th} bright band due to a straight edge diffraction.
6. Derive an expression for the position of n^{th} dark band due to a straight edge diffraction.
7. The diameter of the first ring of a zone plate is 1.1mm. If plane waves fall on $(\lambda = 6000\text{Å})$ the plate, where should the screen be placed so that light is focused to a bright spot. [0.5 m]
8. Find the radii of the first three transparent zone of zone plate whose first focal length is 1m. $\lambda = 5893\text{Å}$. [0.77mm, 1.3mm, 1.7mm]

9. The radius of central zone of the zone plate is 1×10^{-3} m. A point source emitting light of wavelength 6000Å is placed at a distance of 5m from the zone plate. Calculate the position of primary and secondary images [0.4 m, 0.625 m]
10. Assume a plane wave ($\lambda = 5 \times 10^{-5}$cm) to be incident on a circular aperture of radius 0.5 mm. Calculate the positions of the brightest and darkest points on the axis. [50 cm, 25 cm]
11. Consider a circular aperture of diameter 2mm illuminated by a plane wave. The most intense point on the axis is at a distance of 200 cm from the aperture. Calculate the wavelength. [5×10^{-7} m]
12. A zone plate gives a series of images of a point source on its axis. If the strongest and second strongest images are at distances of 30 cm and 6 cm respectively from the zone plate both on the same side away from the source. Calculate the distance of the source from the zone plate [a = 0.3 m]

Section C

(*Answer questions in about* **one** *or* **two** *pages*)

Long answer type questions (Essays)

1. Explain the rectilinear propagation of light on the basis of Fresnels half period zones.
2. What is a zone plate. How it forms the image of an object and derive an expression for its focal length.

Hints to problems

1 to 6 see book work

7. See example 2

8. Using $f = \frac{r_n^2}{n\lambda}$, $r_n = \sqrt{nf\lambda}$

Find r_n for $n = 1, 3$ and 5

9. $f = \frac{r_n^2}{n\lambda}$

For primary image $n = 1$

$$\therefore \quad f_1 = \frac{r_1^2}{\lambda} = \frac{5}{3}\text{m}$$

Using $\frac{1}{b} - \frac{1}{a} = \frac{1}{f_1}$ find b where $a = -5$m .

For the secondary image $n = 3$

$$\therefore \quad f_2 = \frac{r_1^2}{3\lambda} = \frac{5}{9}\text{m}$$

Using $\frac{1}{b} - \frac{1}{a} = \frac{1}{f_2}$, find b

10. See example 6.

11. $r_1 = 1\text{mm} = 10^{-3}\text{m}$, $b = 2\text{m}$

Using $r_1^2 = b\lambda$

$$\therefore \quad \lambda = \frac{r_1^2}{b} = \frac{10^{-6}}{2} = 5 \times 10^{-7}\text{m}$$

12. For the first image $f_1 = \frac{r_n^2}{n\lambda}$

$$\therefore \quad \frac{1}{b} - \frac{1}{a} = \frac{n\lambda}{r_n^2} \quad .....(1)$$

b = 0.3m

For the second image $f_3 = \frac{r_n^2}{3n\lambda}$

$$\therefore \quad \frac{1}{b} - \frac{1}{a} = \frac{3n\lambda}{r_n^2} \quad .....(2)$$

solving eqns 1 and 2 we get a

IMPORTANT FORMULAE

1. Radius of the first half period zone: $r = \sqrt{b\lambda + \frac{\lambda^2}{4}}$

$$r = \sqrt{b\lambda}$$

Radius of the nth half period zone: $r_n = \sqrt{nb\lambda}$

2. Area of the nth half period zone: $= \pi b\lambda$
3. Amplitude due to nth zone: $R_n \propto \pi\lambda(1 + \cos\theta_n)$
4. Principal focal length of the zone plate: $f = \frac{r_n^2}{n\lambda}$

where $\frac{1}{b} - \frac{1}{a} = \frac{1}{f}$