

Z

RUTHERFORD-BOHR MODEL OF THE ATOM

Introduction

The structure of matter which has been a subject of study since long, that shapes the world around us. Dalton was the first person who postulated that matter is made up of atoms which are indivisible. However with the discovery of electron this hypothesis proved wrong. Therefore physicists started thinking about the structure of the atom. J.J. Thomson was the first to propose a model for the structure of the atom. It was modified by Rutherford and Bohr. In this chapter we shall discuss these models and experimental studies of atoms to understand some of the details of atomic structure.

Here we discuss two experiments that are important in the development of our theory of atomic structure. They are

1. Scattering of charged particles by atoms. This gives us information about the distribution of electric charges in atoms.
2. The emission and absorption of radiations by atoms. This gives us information about their excited states.

These two informations help us to develop an atomic model. From this we can understand and explain the properties of atoms. The informations that we obtained may not be sufficient to explain all the properties since a model is an over simplified picture of a more complex system.

Basic properties of atoms

Before explaining the experiments we write down the basic properties of atoms.

1. Atoms are very small about 10^{-10} m in radius. (see footnote)
2. Atoms are stable

They do not spontaneously break apart into pieces. From this we can infer that there must be a force in the atom to counter balance the internal forces that hold the atom together.

3. Atoms are neutral with negatively charged electrons.

The idea that atoms contain negatively charged electrons come from the experiments of photo electric effect and Compton effect. Since bulk matter which is made up of atoms is neutral we can very well assume that atoms are also neutral. From these experimental fact we can deduce that an atom with Z negatively charged electrons must contain net positive charge Ze to make atom.

4. Atoms emit and absorb electromagnetic radiation in all wavelengths. i.e., visible, X-rays, ultraviolet, infrared etc. This idea is extracted from the experimental observations of emission spectrum and absorption spectrum.

Note : Consider 1cc of iron in cubical form. The molar mass of iron is 56g. This one mole of iron (56g) contains one Avagadro number of atoms.

$$\therefore 1 \text{ gram of iron contains } \frac{56}{6 \times 10^{23}} \text{ atoms}$$

$$\text{i.e., } 1\text{cc of iron contains } \frac{56}{8 \times 6 \times 10^{23}} \text{ atom.}$$

Where $d = 8\text{gcm}^{-3}$ is the density of iron.

or 1 atom has volume $= 10^{-23}\text{cc}$

$$\text{i.e., } \frac{4}{3} \pi r^3 = 10^{-23}$$

$$\text{or } 2r = \left(\frac{3 \times 2}{4\pi} 10^{-23} \right)^{\frac{1}{3}} = 2 \times 10^{-8} \text{ cm}$$

$$= 2 \times 10^{-10} \text{ m.}$$

Though this is only a rough and crude estimate it suffices to account property (1).

Scattering experiments and the Thomson model

J.J. Thomson in 1911 proposed a model for the structure of the atom. The Thomson model incorporates many of the known properties of atoms such as size, mass, number of electrons and electrical neutrality. According to Thomson model every atom is positively charged sphere of radius 10^{-10} m in which positive charge of the atom are uniformly distributed. Inside this sphere electrons are embedded like raisins in a plum pudding (hence sometimes called plum-pudding model). The number of electrons is such that their negative charge is equal to the positive charge of the atom. Thus the atom is electrically neutral. But we will see that Thomson's model doesn't agree with the experimental observations.

To verify Thomson's model we have to conduct experiments to see the distribution of electric charges inside the atom. For this we can go for scattering experiments. In scattering experiment we bombard the atom under study with charged particles and observing the angle by which particles are deflected from their original direction. Ideally we do the experiment with a single atom as shown in figure 2.1.

The scattering angle θ depends on the impact parameter, which measures from the atom that a projectile would pass if it were not deflected.

Each different value of the impact parameter results in a different value of the scattering angle.

Consider positively charged projectile bombarded with the atom. The projectile experiences two force. One is the repulsive force due to the positive charge of the atom and the second one is the attractive force due to the negatively charged electrons. We assume that the mass of the projectile is much greater than the mass of the electron and much less than the mass of the atom. In this circumstance during collision the deflection of the projectile is negligible due to electrons and the cause of deflection of projectile is due to positive charges insides the atom.

In actual experiment we cannot do the experiment with a single atom and a single projectile. So what we do is bombard a thin foil containing large number of atoms with a beam of projectile particles. The scattering angle θ that we measure in the experiment is the result of scattering by many atoms with impact parameters that we do not know and cannot control. Let us assume that for a single atom the average scattering angle is θ_{av} . θ_{av} is the average over all possible impact parameters from zero to the atom radius R .

Theoretically the total scattering angle θ can calculated from statistical considerations. The relation between scattering angle θ and the average individual scattering angle θ_{av} is given by

$$\theta \approx \sqrt{N} \theta_{av} \quad \dots (1)$$

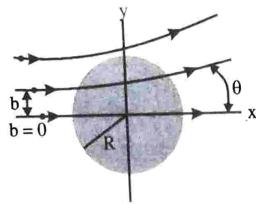


Figure 2.1

A positively charged particle is deflected by an angle θ as it passes through a positively charged sphere, representing a Thomson model atom. The scattering angle depends on the value of the impact parameter b , which varies from 0 to R .

Where N is the number of scatterings (number of atoms). This θ can be measured experimentally.

Theoretical calculation of θ_{av}

Consider a projectile of positive charge ze is incident on an atom of radius R . According to Thomson model Z electrons are embedded in a uniform sphere of charge Ze and radius R as shown in figure 2.2.

The force on the projectile when it is at a distance r from the centre of the atom is given by

$$F = \frac{zZe^2}{4\pi\epsilon_0 R^3} r \quad \dots (2)$$

(see footnote)

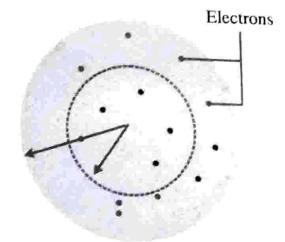


Figure 2.2

The Thomson model of the atom. Z electrons are imbedded in a uniform sphere of positive charge Ze and radius R . An imaginary spherical surface of radius r contains a fraction r^3/R^3 of the positive charge.

Suppose initially the projectile is in the x -direction but atom exerts a force in they-direction (see figure 2.1). This produces a momentum p_y . Using Newton's second law, we can evaluate p_y

$$p_y = \int F_y dt \quad \left(F_y = \frac{dp_y}{dt} \right)$$

Approximately

$$p_y \approx F_y \Delta t$$

$$\therefore p_y \approx \frac{zZe^2 r}{4\pi\epsilon_0 R^3 v} R$$

where $r = b = \frac{R}{2}$ average value of impact parameter

$$p_y \approx \frac{zZe^2}{8\pi\epsilon_0 R v}$$

But

$$\tan \theta_{av} = \frac{p_y}{p_x}$$

$$\theta_{av} = \frac{p_y}{p_x}$$

$$\theta_{av} = \frac{z Ze^2}{8\pi \epsilon_0 R v m v}$$

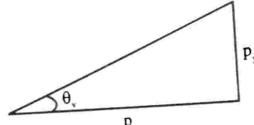


Figure 2.3

Using $\frac{1}{2}mv^2 = K$ (kinetic energy of the projectile).

$$\theta_{av} = \frac{z Ze^2}{16\pi \epsilon_0 K R} \quad \dots \dots (3)$$

This is the average scattering angle according to Thomson model. Take the bombarding particles as α particles and gold foil.

Then $z = 2$, $Z = 79$ (for gold)

$K = 3 \text{ MeV}$ say

$R = 0.179 \text{ nm}$ (for gold nuclei)

Substituting these parameters in equation (3), we get

$$\theta_{av} = 0.01^\circ$$

Note : The electric field at a distance r from the centre of the atom is $E = \frac{Zer}{4\pi\epsilon_0 R^3}$

From Gauss's law,

$$F = qE = zeE = \frac{z Ze^2 r}{4\pi\epsilon_0 R^3}$$

According to Thomson model, the average scattering angle for a single atom is evaluated to be of the order of 0.01° . Thus for a foil of thickness 10^{-8} m ($\frac{10^{-4}}{10^{-8}} = 10^4 \text{ atoms}$), the net scattering angle is about 1° . This is consistent with experimental observations.

To observe projectiles scattered through a total angle greater than 90° we must have about 10^4 successive scatterings. This scattering results in either larger or smaller angles. Since these two probabilities are equal, the probability of scattering towards larger angle is only $\left(\frac{1}{2}\right)^{10000}$. This can be written as

$$y = \left(\frac{1}{2}\right)^{10000}$$

Take log on both sides

$$\log y = 10000 \ln \frac{1}{2} = -10000 \times \ln 2$$

$$\log y = -10000 \times 0.3010 = -3000$$

Taking antilog, we get

$$y = (10)^{-3000}$$

$$\text{i.e., } \left(\frac{1}{2}\right)^{10000} = (10)^{-3000}$$

An experiment was conducted by Geiger and Marsden in the laboratory of Rutherford at Manchester university in 1910. They used α -particles (nuclei of helium with charge $2e$ emitted in radioactivity) as the projectile and gold foil as the target. Their result showed that the probability of an alpha particle scattering at angles greater than 90° was about 10^{-4} . Thus Thomson model failed.

There is one more reason for the failure of Thomson model. The electric field inside the atom was found to be $E = \frac{Zer}{4\pi\epsilon_0 R^3}$

\therefore The force experienced by the electron in the atom at a distance r is

$$F = eE = \frac{-Ze^2}{4\pi\epsilon_0 R^3} r$$

This can be written as $F = -kr$

with $k = \frac{Ze^2}{4\pi\epsilon_0 R^3}$.

This restoring force permits electron to oscillate about its equilibrium position.

$$\text{Thus } m \frac{d^2r}{dt^2} = -\frac{Ze^2}{4\pi\epsilon_0 R^3} r$$

$$\text{or } \frac{d^2r}{dt^2} = -\frac{Ze^2}{4\pi\epsilon_0 m R^3} r$$

Comparing this equation with standard equation

$$\frac{d^2r}{dt^2} = -\omega^2 r$$

$$\text{we get } \omega = \sqrt{\frac{Ze^2}{4\pi\epsilon_0 m R^3}} \quad \dots\dots (4)$$

$$\text{or } v = \sqrt{\frac{Ze^2}{16\pi^3\epsilon_0 m R^3}} \quad (\omega = 2\pi v)$$

This is the characteristic frequency of oscillation, of Thomson model. This did not agree with experimental observations. Moreover the oscillating electrons inside the atom should radiate electromagnetic radiation which makes atom unstable. This was also not found to be true.

Drawbacks of Thomson model

1. It could not explain large angle scattering of α particles from thin metal foil.
2. Electrostatically this model cannot be stable, slightest disturbance of the atom will set the electrons into oscillations and as a result energy should be radiated out of the atom as electromagnetic waves.
3. The frequencies observed for radiation emitted by atoms is not in accordance with frequency predicted by the model.

Example 1

Compute the oscillation frequency of the electron and the expected absorption

and emission expected wavelength in a Thomson model hydrogen atom. Use $R = 0.053\text{nm}$. compare with the observed wavelength in hydrogen 122nm .

Solution

$$\text{We have } v = \sqrt{\frac{Ze^2}{16\pi^3\epsilon_0 m R^3}}$$

$$v = \sqrt{\frac{1 \times (1.6 \times 10^{-19})^2}{16 \times 3.14^3 \times 8.85 \times 10^{-12} \times 9.1 \times 10^{-31} \times (0.053 \times 10^{-9})^3}}$$

$$v = 45.7 \text{ nm}.$$

This is about 3 times smaller than the observed wavelength.

Example 2

Consider the Thomson model for an atom with 2 electrons. Let the electrons be located along a diameter on opposite sides of the centre of the sphere, each a distance x from the centre. Show that the configuration is stable if

$$x = \frac{R}{2}$$

Solution

The force on one electron due to positively charged supposed to be at the centre of the atom

$$F_+ = \frac{z Ze^2 r}{4\pi\epsilon_0 R^3}$$

$$\text{Here } z = 1, Z = 1, r = x$$

$$\therefore F_+ = \frac{e^2 x}{4\pi\epsilon_0 R^3} \quad \dots\dots (1)$$

The force exerted by the electron by the other electron

$$F_- = \frac{e \cdot e}{4\pi\epsilon_0 (2x)^2}$$

$$F_- = \frac{e^2}{16\pi\epsilon_0 x^2} \quad \dots\dots (2)$$

At equilibrium

$$\frac{e^2 x}{4\pi\epsilon_0 R^3} = \frac{e^2}{16\pi\epsilon_0 x^2}$$

$$\text{or } x = \frac{R}{2}$$

Rutherford atom model

In order to investigate the structure of the atom Ernest Rutherford (1871-1937) along with his two associates Geiger and Marsden studied (in 1911) the scattering of α -particles from thin gold foils.

The experimental set up consists of a source of α -particles which is kept in a lead container having fine cavity. The collimated beam of α particles is incident on a thin gold foil of thickness $2 \times 10^{-7} \text{ m}$. The scattered α -particles in different directions are observed with the help of rotatable detector consisting of a zinc sulphide screen and a microscope. The α -particles produce bright flashes or scintillations on the ZnS screen. These are observed in the microscope and counted at different angles from the direction of incidence of the beam.

Observations

A graph is plotted between the scattering angle θ and the number of α -particles scattered. From the graph we observe that (i) Most of the α -particles pass through the gold foil without appreciable scattering. (ii) Very few α -particles scatter through angles more than 90° (iii) Rarely, an α -particle (about 1 in 8000) rebounces i.e., it is scattered through an angle 180° .

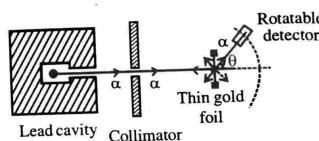


Figure 2.4

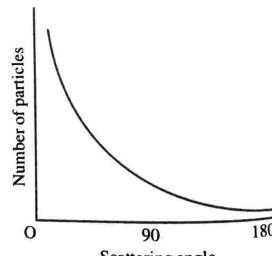


Figure 2.5

Explanation

The scattering of α -particles is due to Coulombian interaction of α -particles with positive charges and electrons in the every atom of the gold foil. If positive charges and electrons were distributed uniformly in an atom the scattering angle θ would be very small.

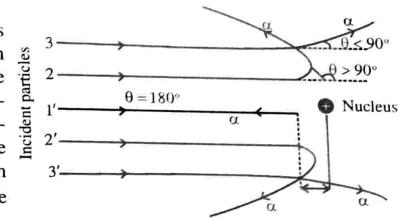


Figure 2.6

An α -particle is over 7000 times more massive than an electron and in this experiment, α -particle is travelling at a high speed. Therefore very strong forces alone could have deflected them through large angles. This led Rutherford to postulate that the entire positive charge of the atom must be concentrated in a tiny central core of each atom called atomic nucleus.

An α -particle (1) tending to collide head on with the nucleus, slows down due to repulsive force of the nucleus finally stops and is then repelled back. This α -particle is therefore retraces its path, scattering through 180° .

α -particle (2, 2') tending to hit the nucleus at its periphery experiences small repulsive forces and get scattered through large angles ($\theta > 90^\circ$).

The α -particle (3, 3') which pass at a distance from the nucleus experience small repulsive forces get scattered through small angles. The α -particles which pass still larger distances from the nucleus go almost undeviated.

We divide our study of Rutherford scattering formula into three parts.

1. Derivation of Rutherford's scattering formula and its experimental verification.
2. Calculation of fraction of projectiles scattered at angles greater than some angle θ and
3. The closest approach of a projectile to the nucleus.

To derive Rutherford's scattering formula we require the relation between impact parameter and the scattering angle.

Relation between impact parameter and scattering angle

This relation plays an important role in all scattering experiments.

Consider a projectile (positively charged particle with charge ze) moving towards a target (atom with initial charge Ze concentrated at its centre) with initial momentum $\vec{p}_i = mv$. Let b be the impact parameter. When the projectile reaches within the influence of positive charge Ze at the centre, it experiences coulombian force of repulsion given by

$$F = \frac{ze Ze}{4\pi\epsilon_0 r^2} = \frac{z Ze^2}{4\pi\epsilon_0 r^2}$$

acting along the direction shown in figure.

As a result projectile scattered by an angle θ and moves with momentum \vec{p}_f . The change in momentum due to collision

$$\Delta\vec{p} = \vec{p}_i - \vec{p}_f$$

$$\text{i.e., } \Delta p = \sqrt{\vec{p}_i^2 + \vec{p}_f^2 - 2\vec{p}_i \cdot \vec{p}_f \cos\theta}$$

Since the momentum is conserved

$$|\vec{p}_i| = |\vec{p}_f| = p$$

$$\text{thus } \Delta p = \sqrt{p^2 + p^2 - 2p^2 \cos\theta}$$

$$\Delta p = \sqrt{2p^2(1-\cos\theta)}$$

$$\Delta p = \sqrt{2p^2 \sin^2 \frac{\theta}{2}} = 2p \sin \frac{\theta}{2}$$

$$\Delta p = 2mv \sin \frac{\theta}{2} \quad \dots\dots (5)$$

The direction of change in momentum is given by $-\phi$.

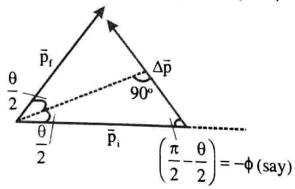


Figure 2.8

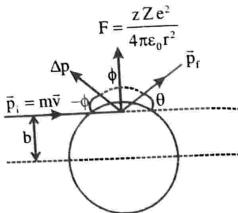


Figure 2.7

The change in momentum Δp can also be calculated from Newton's second law.

$$\bar{F} = \frac{d\vec{p}}{dt}$$

$$\text{or } \Delta\vec{p} = \int \bar{F} dt$$

$$\text{Here } \Delta p = \int \bar{F} \cos\phi dt$$

$$\text{or } \Delta p = \int \frac{zZe^2}{4\pi\epsilon_0 r^2} \cos\phi dt$$

the angle ϕ can vary from $-\phi$ to ϕ

$$\Delta p = \int_{-\phi}^{\phi} \frac{zZe^2}{4\pi\epsilon_0 r^2} \cos\phi d\phi \frac{dt}{d\phi}$$

$$\frac{d\phi}{dt} = \omega$$

Using conservation of angular momentum

$$mvb = mr^2\omega$$

$$\omega = \frac{vb}{r^2}$$

$$\therefore \Delta p = \int_{-\phi}^{\phi} \frac{zZe^2}{4\pi\epsilon_0 r^2} \cos\phi d\phi \frac{r^2}{vb}$$

$$\text{or } \Delta p = \frac{zZe^2}{4\pi\epsilon_0 vb} \int_{-\phi}^{\phi} \cos\phi d\phi$$

$$\Delta p = \frac{zZe^2}{4\pi\epsilon_0 vb} 2 \sin\phi$$

$$\text{or } \Delta p = \frac{zZe^2}{4\pi\epsilon_0 vb} 2 \sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$

$$\Delta p = \frac{zZe^2}{4\pi\epsilon_0 vb} 2 \cos \frac{\theta}{2} \quad \dots (6)$$

Equating equation (5) and (6), we get

$$2mv \sin \frac{\theta}{2} = \frac{zZe^2}{4\pi\epsilon_0 vb} 2 \cos \frac{\theta}{2}$$

$$b = \frac{zZe^2}{4\pi\epsilon_0 mv^2} \cot \frac{\theta}{2} = \frac{zZe^2}{8\pi\epsilon_0 K} \cot \frac{\theta}{2} \quad \dots (7)$$

This is the relation between impact parameter and scattering angle.

Note : In Rutherford experiment projectile was α particle so $z = 2$

$$b = \frac{Ze^2}{4\pi\epsilon_0 \frac{1}{2} mv^2} \cot \frac{\theta}{2} = \frac{Ze^2 \cot \frac{\theta}{2}}{4\pi\epsilon_0 K}$$

$$\text{The value of } \frac{e^2}{4\pi\epsilon_0} = 1.44 \text{ MeVfm}$$

Rutherford scattering formula and its experimental verification

Rutherford scattering formula gives the expression for total scattering cross section of the experiment. It gives valuable information regarding the target particle. The actual time rate of loss of particles due to scattering is characterised by the effective loss of area normal to the direction of the beam that originally contained these scattered particles. This effective loss of total effective area from the incident beam cross section is called cross section of scattering. It is denoted by σ_T .

$$\sigma_T = \frac{\text{Total number of particles scattered per unit time}}{\text{Total number of particles incident per unit area per unit time}} \quad \dots (8)$$

The total number of particles incident per unit area per unit time is called incident intensity. We also introduce one more term called differential cross section $\sigma(\Omega)$. It is defined as the ratio between the total number of particles scattered per unit time into the solid angle $d\Omega$ to the incident intensity.

$$\sigma(\Omega)d\Omega = \frac{\text{Total number of particles scattered per unit time into solid angle}}{\text{Incident intensity}} \quad \dots (9)$$

$\sigma(\Omega)d\Omega$ gives us the effective loss of normal area from the incident beam into the solid angle $d\Omega$.

Integrating $\sigma(\Omega)d\Omega$ we get the total scattering cross section

$$\text{i.e., } \sigma_T = \int \sigma(\Omega)d\Omega$$

$$\text{or } \frac{d\sigma_T}{d\Omega} = \sigma(\Omega)$$

$\sigma(\Omega)$ and σ_T have dimensions of area. So its unit is m^2 . Usually at the microscopic level it is expressed in barn. 1 barn = 10^{-28}m^2

Derivation of Rutherford scattering formula

Consider a beam of particles (projectiles) encountering a target particle. Let I be the intensity of incident beam. Now we consider particles within the impact parameters b and $b + db$ shown in figure 2.9 below. These particles scattered through angles θ and $\theta - d\theta$.

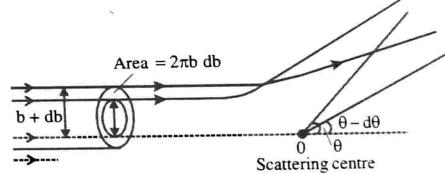


Figure 2.9 : Scattering of particles with impact parameters between b and $b + db$ are scattering through angles between θ and $\theta - d\theta$

The number of particles incident in time with imparameter lying between b and $b + db = I \cdot 2\pi b db$ (10)

The number of particles scattered into solid angle $d\Omega$

$$= \sigma(\theta) d\Omega I \quad \text{From equation (8).}$$

$$= \sigma(\theta) 2\pi \sin \theta d\theta I \quad \dots (11)$$

where $d\Omega = 2\pi \sin \theta d\theta$

The number of particles scattered in unit time into the solid angle $d\Omega$ must be equal to the number of particles in unit time with impact parameter between b and $b+db$. Hence

$$\sigma(\theta) 2\pi \sin \theta d\theta I = I 2\pi b db \quad \dots(12)$$

$$\therefore \sigma(\theta) = \frac{bdb}{\sin \theta d\theta}$$

$$\text{But we have } b = \frac{zZe^2 \cot \frac{\theta}{2}}{8\pi\epsilon_0 K} \quad (\text{see equation 7})$$

Taking differential on both sides, we get

$$db = \frac{zZe^2}{8\pi\epsilon_0 K} \csc^2 \frac{\theta}{2} \cdot \frac{1}{2} d\theta \quad \dots(14)$$

Substituting equation 13 and 14 in equation 12, we get

$$\sigma(\theta) = \left(\frac{zZe^2 \cot \frac{\theta}{2}}{8\pi\epsilon_0 K} \right) \cdot \frac{zZe^2 \csc^2 \frac{\theta}{2} \frac{1}{2} d\theta}{8\pi\epsilon_0 K \sin \theta d\theta}$$

$$\sigma(\theta) = \left(\frac{zZ}{2K} \right)^2 \left(\frac{e^2}{4\pi\epsilon_0} \right)^2 \frac{\cot \frac{\theta}{2} \cdot \csc^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}$$

$$\text{or} \quad \sigma(\theta) = \frac{1}{16} \left(\frac{zZe^2}{K4\epsilon_0} \right)^2 \csc^4 \frac{\theta}{2} \quad \dots(15)$$

This is the Rutherford's scattering formula.

According to Rutherford's scattering formula

$$\sigma(\theta) \propto z^2$$

$$\sigma(\theta) \propto Z^2$$

$$\sigma(\theta) \propto K^{-2}$$

$$\sigma(\theta) \propto \csc^4 \frac{\theta}{2}$$

Experimental verification

In Rutherford's laboratory Geiger and Marsden tested the predictions of the formula in a remarkable series of experiments involving the scattering of alpha particles ($z = 2$) from a variety of thin metal foils. In all, four predictions are found to be experimentally true.

The closest approach of a projectile to the nucleus

The distance of closest approach (size of nucleus) is the smallest distance between the nucleus and the projectile particle fired for head on collision towards the nucleus is called distance of closest approach. Suppose a projectile with positive charge ze is directed towards the nucleus with initial kinetic energy K . On account of coulombian repulsive force between nucleus and the projectile kinetic energy of projectile goes on decreasing and in turn electrical potential energy of the particle goes on increasing. At a certain minimum distance r_{min} from the nucleus the speed of the projectile is v_{min} .

Initial total energy of the projectile

$$E = K + U = K + 0 = K = \frac{1}{2} mv^2$$

Total energy of the projectile at a distance r_{min}

$$E = K + U$$

$$E = \frac{1}{2} mv_{min}^2 + \frac{zZe^2}{4\pi\epsilon_0 r_{min}}$$

According to law of conservation of energy, total energy is conserved, thus we have

$$\frac{1}{2} mv^2 = \frac{1}{2} mv_{min}^2 + \frac{zZe^2}{4\pi\epsilon_0 r_{min}} \quad \dots(16)$$

Initially angular momentum of the projectile = mvb , where b is the impact parameter.

See figure below. When the projectile is at r_{min} , angular momentum = $mv_{min}r_{min}$ since angular momentum is conserved

$$mv_{min}r_{min} = mvb$$

or $v_{\min} = \frac{vb}{r_{\min}}$

Putting this in equation 16, we get

$$\frac{1}{2}mv^2 = \frac{1}{2}\frac{mv^2b^2}{r_{\min}^2} + \frac{zZe^2}{4\pi\epsilon_0 r_{\min}} \quad \dots(17)$$

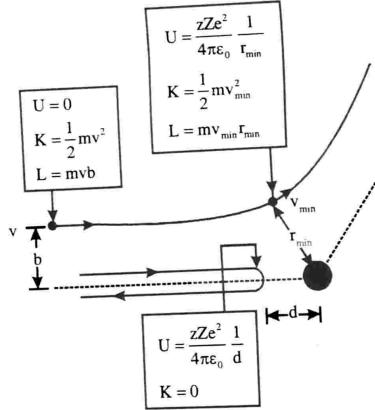


Figure 2.10 : Closet approach of the projectile to the nucleus

The value of r_{\min} can be calculated from equation 17 knowing all other parameters.

When the projectile loses all its kinetic energy, thus get closest to the nucleus. At this point its distance from the nucleus is d , called the distance of closest approach (see figure above).

i.e., when $b=0$ $r_{\min}=d$, Thus we get

$$d = \frac{zZe^2}{4\pi\epsilon_0 K} \quad \dots(18)$$

$$K = \frac{1}{2}mv^2$$

This is the expression for the distance of closest approach.

In Rutherford experiment if the energy of the α - particle is 5.5MeV

$$\text{We get } d = \frac{2 \times 79 \times (1.6 \times 10^{-19})^2 \times 9 \times 10^9}{5.5 \times 10^6 \times 1.6 \times 10^{-19}}$$

$$d = 4.13 \times 10^{-14} \text{ m.}$$

This gives an approximate size of the nucleus.

Example 3

Alpha particles of kinetic energy 5.00 MeV are scattered at 90° by a gold foil. (a) What is the impact parameter (b) What is the minimum distance between alpha particles and gold nucleus. (c) Find the kinetic and potential energies at that distance.

Solution

a) We have $b = \frac{zZe^2}{4\pi\epsilon_0 2K} \cot \frac{\theta}{2}$

Here $z = 2, Z = 79, \frac{e^2}{4\pi\epsilon_0} = 1.44 \text{ MeV fm}$

$\theta = 90^\circ$ and $K = 5.00 \text{ MeV}$

$$\therefore b = \frac{2 \times 79 \times 1.44}{2 \times 5} \cot 45^\circ$$

$$b = 22.75 \text{ fm}$$

b) We have the relation

$$\frac{1}{2}mv^2 = \frac{1}{2}\frac{mv_{\min}^2}{r_{\min}^2} + \frac{zZe^2}{4\pi\epsilon_0 r_{\min}}$$

or $\frac{1}{2}mv^2 = \frac{1}{2}\frac{mv^2b^2}{r_{\min}^2} + \frac{zZe^2}{4\pi\epsilon_0 r_{\min}}$

Multiply through out by r_{\min}^2

$$\frac{1}{2}mv^2 r_{\min}^2 = \frac{1}{2}mv^2 b^2 + \frac{zZe^2}{4\pi\epsilon_0} r_{\min}$$

Put $\frac{1}{2}mv^2 = K$ and $\frac{z(Ze^2)}{4\pi\epsilon_0} = c$ we get

$$Kr_{\min}^2 - cr_{\min} - Kb^2 = 0$$

This is a quadratic equation in r_{\min}

$$r_{\min} = \frac{c \pm \sqrt{c^2 - 4K^2b^2}}{2K}$$

$$c = 2 \times 79 \times 1.44 \text{ MeV fm} = 227.5$$

$$K = 5 \text{ MeV} \text{ and } b = 22.75 \text{ fm}$$

$$\therefore r_{\min} = 55.00 \text{ fm or } -9.5 \text{ fm}$$

r_{\min} negative is meaningless

$$\therefore r_{\min} = 55 \text{ fm}$$

$$c) U = \frac{zZe^2}{4\pi\epsilon_0 r_{\min}}$$

$$U = \frac{2 \times 79 \times 1.44}{55} \text{ MeV}$$

$$U = 4.137 \text{ MeV}$$

$$K = E - U = 5 - 4.137 = 0.863 \text{ MeV}$$

Example 4

How much kinetic energy must an alpha particle have before its distance of closest approach to a gold nucleus is equal to the nuclear radius $7.0 \times 10^{-15} \text{ m}$.

Solution

$$\text{We have } d = \frac{zZe^2}{4\pi\epsilon_0 K}$$

$$\text{or } K = \frac{zZe^2}{4\pi\epsilon_0 d} = \frac{2 \times 79 \times 1.44}{7}$$

$$K = 32.5 \text{ MeV.}$$

Example 5

What is the distance of closest approach when α - particles of kinetic energy 6.0 MeV are scattered by a thin copper foil.

Solution

$$\text{We have } d = \frac{zZe^2}{4\pi\epsilon_0 K}$$

Here $z = 2$, $Z = 29$ for copper, $K = 6 \text{ MeV}$

$$d = \frac{2 \times 29 \times 1.44}{6} \text{ fm}$$

$$d = 13.92 \text{ fm}$$

Rutherford atom model

On the basis of his study of scattering of α - particles Rutherford postulated the following model of the atom.

- Every atom consists of a tiny central core, called the atomic nucleus in which the entire positive charge and almost entire mass of the atom are concentrated.
- The size of the nucleus is of the order of 10^{-15} m , which is very small compared to the size of the atom which is of the order of 10^{-10} m .
- The space around the nucleus is virtually empty with electrons revolving round the nucleus in the same way as the planets revolve around the sun.
- The electrostatic attraction of the nucleus provides centripetal force to the orbiting electrons.
- Total positive charge in the nucleus is equal to the total negative charge of the orbiting electrons.

This model could explain large angle scattering of α - particles through thin foils and justified the classification of elements in the periodic table on the basis of their atomic number.

Limitations

- An electron revolving around the nucleus has acceleration, therefore, according to classical electromagnetic theory, it must radiate energy continuously in the form of electromagnetic radiations. As the energy of revolving electron decreases continuously, the radius of its orbit would decrease continuously. It

- would follow a spiral path and ultimately fall into the nucleus. But this is not so. Thus Rutherford's model of atom cannot account for stability of the atom.
- (ii) According to the Rutherford's model the electrons can revolve around the nucleus in all possible orbits. Hence, the atom should emit radiations of all wavelengths. That is, energy spectrum of the radiations emitted by the atom should be continuous. However in practice the atoms are found to have line spectrum or discrete spectrum. Thus the observed spectra of elements cannot be accounted for by Rutherford's atom model.

Line spectra

The radiations emitted by atoms are classified into two. They are

1. continuous spectra and
2. discrete spectra (line spectra)

Continuous spectra

The radiation emitted by atoms containing all wavelengths from some minimum (say 0) to some maximum (say $\rightarrow \infty$) is called continuous spectra.

For example the radiation from a hot, glowing body. White light is a mixture of all of the different colours of visible light. An object that glows white hot is emitting light at all wavelengths of the visible spectrum.

Discrete spectra

The radiation emitted by some isolated atoms which contains only bright or dark lines at very distinct and sharply defined wavelengths is called discrete spectra.

There are two types of discrete spectra. They are

- (1) emission (bright line) spectra and
- (2) absorption (dark line) spectra.

Emission spectra

When an electric discharge is passed through a tube containing vapour of some element (mercury, sodium or neon) light is emitted at a few discrete wavelengths called emission spectra.

An experimental arrangement to view emission spectrum is shown below.

Note : The prism shown in the arrangement is to disperse different wavelengths. The white lines are emission spectrum.

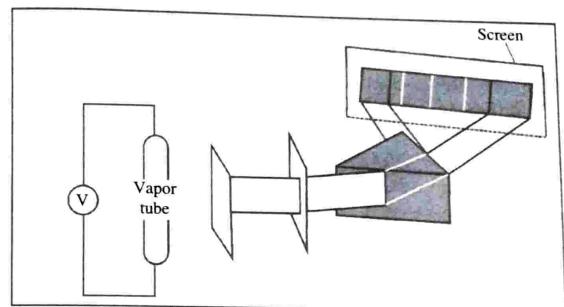


Figure 2.11: Apparatus for observing emission spectra

Absorption spectrum

When a beam of white light, containing all wavelengths, is passed through a tube containing vapour of some gas, we find that certain wavelengths have been absorbed from the light. The spectrum so obtained contains dark lines (wavelengths absorbed) called absorption spectrum.

An experimental setup to view absorption spectrum is shown below.

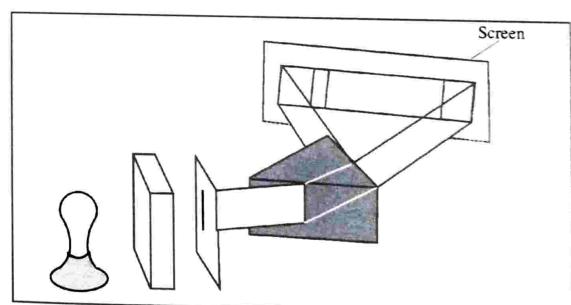


Figure 2.12: Apparatus for observing absorption spectra

In general interpretation of line spectra of complex atoms is very difficult. So we deal with spectra of simple atom hydrogen. The emission and absorption spectral series of hydrogen is given below.

- Several informations are obtained from the spectra.
1. The regularities in the spacing of spectral lines appear in both the emission and absorption spectra.
 2. Every series has a limiting wavelength. The spectra lines get closer together as the limit of each series is approached. (dashed line is the limit of each series)

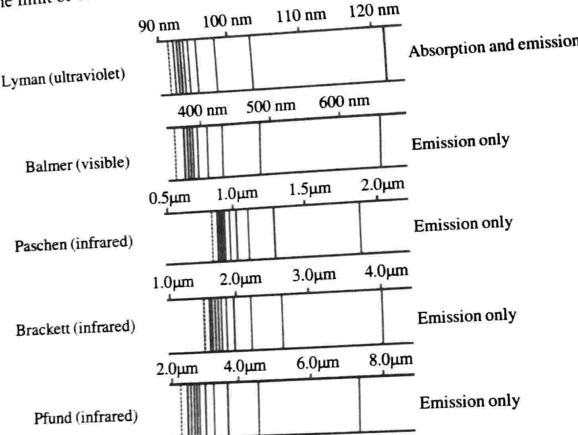


Figure 2.13 : Emission and absorption spectral series of hydrogen

3. All series are present in the emission spectrum whereas Lyman series appears in the absorption spectrum.
4. The wavelengths of the group of emission lines in the visible region could be calculated by the formula $\lambda = (364.5 \text{ nm}) \frac{n^2}{n^2 - 4}$ ($n = 3, 4, 5 \dots$)(19)

For example $n = 3$

$$\therefore \lambda = 364.5 \times \frac{9}{9 - 4} = 656.1 \text{ nm.}$$

Equation 19 is called Balmer formula and the series of lines that it fits is called the Balmer series.

In general the above formula can be written as

$$\lambda = \lambda_{\min} \frac{n^2}{n^2 - n_0^2} \quad (n = n_0 + 1, n_0 + 2 \dots) \quad \dots\dots (20)$$

where λ_{\min} is the wavelength of the particular series limit.

For Lyman series $n_0 = 1$

Balmer series $n_0 = 2$

Paschen series $n_0 = 3$

Brackett series $n_0 = 4$

Pfund series $n_0 = 5$

5. Ritz combination principle

It is another interesting property exhibited by hydrogen spectrum. If we convert the hydrogen wavelengths to frequencies. It is found that certain pairs of frequencies added together give other frequencies that appear in the spectrum.

We can conclude by saying that any successful model of the hydrogen atom must be able to explain the above said observations.

Example 6

The shortest wavelength of the Lyman series 91.13 nm. Find the three longest wavelength in this series.

Solution

We have $\lambda = \lambda_{\min} \frac{n^2}{n^2 - n_0^2}$

For the Lyman series $n_0 = 1$

$$\lambda_{\min} = 91.13 \text{ nm} \text{ (given)}$$

$$\lambda = 91.13 \times \frac{n^2}{n^2 - 1}$$

$$\text{Put } n = 2 \quad \lambda = 91.13 \times \frac{4}{3} = 121.51 \text{ nm}$$

$$\text{Put } n = 3 \quad \lambda = 91.13 \times \frac{9}{8} = 102.52 \text{ nm}$$

$$\text{Put } n = 4 \quad \lambda = 91.13 \times \frac{16}{15} = 97.21 \text{ nm}$$

Example 7

One of the lines in the Brackett series (series limit = 1458 nm) has a wavelength of 1944 nm. Find the next higher and next lower wavelengths in this series.

Solution

$$\text{We have } \lambda = \lambda_{\min} \frac{n^2}{n^2 - n_0^2}$$

For the Brackett series $n_0 = 4$

$$1944 = 1458 \frac{n^2}{n^2 - 16}$$

$$\frac{n^2}{n^2 - 16} = \frac{1944}{1458} = \frac{4}{3}$$

$$\text{or } 3n^2 = 4n^2 - 64$$

$$n^2 = 64$$

$$n = 8$$

Next higher wavelength corresponds to $n = 7$

$$\therefore \lambda = 1458 \frac{7^2}{7^2 - 4^2} = \frac{1458 \times 49}{33}$$

$$\lambda = 2164.91 \text{ nm}$$

Next lower wavelength corresponds to $n = 9$

$$\therefore \lambda = 1458 \frac{9^2}{9^2 - 4^2} = \frac{1458 \times 81}{65}$$

$$\lambda = 1816.89 \text{ nm}$$

Example 8

The longest wavelength in the Pfund series is 7459 nm. Find the series limit.

Solution

$$\text{We have } \lambda = \lambda_{\min} \frac{n^2}{n^2 - n_0^2}$$

For the Pfund series $n_0 = 5$. The longest wavelength corresponds to $n = 6$

$$\therefore 7459 = \lambda_{\min} \frac{6^2}{6^2 - 5^2} = \lambda_{\min} \frac{36}{11}$$

$$\lambda_{\min} = \frac{7459 \times 11}{36}$$

$$\lambda_{\min} = 2279.14 \text{ nm}$$

Bohr's model of the atom

To account for the drawbacks of the Rutherford's model atom, Danish physicist Niels Henrick David Bohr (1885-1962) modified the Rutherford's model of the atom as follows. In this model Bohr retained the following features of Rutherford's model.

- (i) Atom consists of positively charged nucleus around which electrons revolve in circular orbits.
- (ii) Majority of the mass and whole of the positive charge of the atom is concentrated in the nucleus.
- (iii) Size of the nucleus is very small compared to the size of the atom.

Bohr's postulates

Bohr added following postulates to the Rutherford's model of the atom.

1. The electrons revolve around the nucleus only in certain permitted orbits in which the angular momentum of the electron is an integral multiple of $\frac{h}{2\pi}$ where h is the Planck's constant.
2. The electron does not radiate energy while revolving in the permitted orbits. That is, the permitted orbits are stationary, non radiating orbits.
3. The energy is radiated only when the electron jumps from an outer permitted orbit to some inner permitted orbit.
4. If the energy of the electron in n^{th} and m^{th} orbits be E_n and E_m respectively then, when the electron jumps from n^{th} to m^{th} orbit, the radiation of frequency v is emitted, such that

$$E_n - E_m = hv$$

This is called Bohr's frequency equation.

Bohr's theory of hydrogen atom

By using the above mentioned postulates Bohr calculated the radius, orbital velocity and the energy of the orbital electron in the hydrogen atom as explained below.

A hydrogen atom consists of a proton (positive charge $+e$) in the nucleus and has one electron with charge $-e$ going round the nucleus. The permitted orbits are numbered as $n = 1, 2, 3, \dots$ etc. with the innermost as $n = 1$. Let v be the speed of the electron in the n^{th} orbit and r be the radius of the n^{th} orbit. Let m be the mass of the electron.

The centripetal force $\left(\frac{mv^2}{r}\right)$ required for the electron to keep electron in circular orbit is provided by the electrostatic attraction between the electron and the nucleus.

$$\therefore \frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

$$\text{i.e., } mv^2 = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \quad \dots \dots (1)$$

\therefore The kinetic energy of the electron

$$E_k = \frac{1}{2} mv^2 = \frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \quad \dots \dots (2)$$

The electrostatic potential energy of the electron

$$E_p = \frac{-1}{4\pi\epsilon_0} \frac{e^2}{r} \quad \dots \dots (3)$$

\therefore The total energy of the electron,

$$E_n = E_k + E_p = \frac{-1}{2} \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \right) \quad \dots \dots (4)$$

$$\text{i.e., } E_k = -E_n \text{ and } E_p = 2E_n$$

Radius of the orbit

According to Bohr's postulates, the angular momentum of the

$$\text{electron} = \frac{nh}{2\pi}$$

$$\text{i.e., } mvr = \frac{nh}{2\pi}$$

$$\text{or } m^2 v^2 r^2 = \frac{n^2 h^2}{4\pi^2}$$

$$mv^2 = \frac{n^2 h^2}{4\pi^2 m r^2}$$

Substituting this in eq (1), we get

$$\frac{1}{4\pi\epsilon_0} \frac{e^2}{r} = \frac{n^2 h^2}{4\pi^2 m r^2}$$

$$\therefore r = \left(\frac{\epsilon_0 h^2}{\pi m e^2} \right) n^2 \quad \dots \dots (5)$$

This shows that $r \propto n^2$

Velocity of the orbital electron

The orbital radius is given by

$$r_n = \left(\frac{\epsilon_0 h^2}{\pi m e^2} \right) n^2$$

Also we have

$$mvr = \frac{nh}{2\pi} \text{ or } v = \frac{nh}{2\pi mr}$$

Substituting the value of r in the above equation, we get

$$v = \frac{nh}{2\pi m} \left(\frac{\pi m e^2}{\epsilon_0 h^2 n^2} \right) = \frac{e^2}{2\epsilon_0 hn} = \frac{1}{4\pi\epsilon_0} \frac{2\pi e^2}{hn} \quad \dots \dots (6)$$

This can be written as

$$v_n = \frac{c}{n} \left[\frac{1}{4\pi\epsilon_0} \frac{2\pi e^2}{ch} \right]$$

Here $\frac{1}{4\pi\epsilon_0} \frac{2\pi e^2}{ch}$ is a dimensionless quantity called the fine structure constant and is denoted by α .

.....(7)

$$v_n = \frac{c}{n} \alpha$$

The value of α comes out to be $\frac{1}{137} = 0.0073$. Thus in the first orbit ($n=1$) of

hydrogen atom orbital velocity of electron is $\frac{1}{137}$ of the velocity of light in vacuum. Further the orbital velocity of electron in outer orbits is smaller as compared to its value in the inner orbits.

Wavelength of the radiations emitted

Total energy of the electron in the n^{th} orbit is

$$E_n = -\frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \quad (\text{see eq 4})$$

substituting the value of r from eq 5, we get

$$E_n = -\left(\frac{1}{4\pi\epsilon_0}\right)^2 \left(\frac{2\pi^2 me^4}{h^2}\right) \frac{1}{n^2}$$

If the electron jumps from the outer orbit n_2 to the inner orbit n_1 , then the energy released is given by

$$E = E_2 - E_1 = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \left(\frac{2\pi^2 me^4}{h^2}\right) \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

If λ be the wavelength of the radiation emitted then $E = \frac{hc}{\lambda}$

$$\frac{hc}{\lambda} = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \left(\frac{2\pi^2 me^4}{h^2}\right) \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\therefore \frac{1}{\lambda} = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \left(\frac{2\pi^2 me^4}{h^3 c}\right) \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$\frac{1}{\lambda} = \bar{v}$, called the wave number of radiation emitted i.e., number of complete waves in unit length.

$$\left(\frac{1}{4\pi\epsilon_0}\right)^2 \left(\frac{2\pi^2 me^4}{h^2}\right) = R, \text{ a constant called Rydberg constant.}$$

Putting the values of the terms in above relation we get the value of Rydberg constant

$$R = 1.097 \times 10^7 \text{ m}^{-1}$$

$$\therefore v = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right). \quad \dots \dots (8)$$

This is called Rydberg formula for the spectrum of hydrogen atom.

$$\text{From equation 8, } v = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$v = R \frac{(n_2^2 - n_1^2)}{n_1^2 n_2^2}$$

$$\text{or } \lambda = \frac{n_1^2 n_2^2}{R(n_2^2 - n_1^2)}$$

For the Balmer series $n_1 = 2, n_2 = 3, 4, \dots$

$$\text{Thus } \lambda = \frac{1}{1.09737 \times 10^7} \frac{4 \times n_2^2}{(n_2^2 - 4)}$$

$$\lambda = 364.5 \text{ nm} \times \frac{n_2^2}{n_2^2 - 4}$$

This is exactly same as our Balmer formula.

Potential and kinetic energy of the orbital electron.

We already have

$$E_n = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \left(\frac{2\pi^2 me^4}{h^2}\right) \frac{1}{n^2}$$

substituting the value and converting into eV, we get

$$E_n = -\frac{13.6 \text{ eV}}{n^2}$$

This shows that the total energy of electron in a stationary orbit is negative, which means the electron is bound to the nucleus and is not free to leave. Further as n increases, negative energy decreases i.e., in outer orbits, total energy of electron is more than inner orbits.

Spectral series of hydrogen atom

Long before to Bohr gave his theory of hydrogen atom, various scientists had discovered experimentally the spectral series of hydrogen atom. Bohr offered a theoretical explanation of these spectral series as follows.

According to Bohr's theory the wavelength of radiations emitted from hydrogen atom is given by

$$\frac{1}{\lambda} = R \left(\frac{1}{n_i^2} - \frac{1}{n_j^2} \right)$$

The different groups or the series of spectral lines depend upon the value of n was described below.

1. Lyman series

If the electron jumps from any one of the outer orbits to the inner most orbit, we obtain the Lyman series i.e., for Lyman series $n_i = 1$ and $n_j = 2, 3, 4, \dots$

The wavelength of the radiation is given by

$$\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{n_j^2} \right)$$

2. Balmer series

If the electron jumps from 3rd or higher orbit to the 2nd orbit, we obtain the Balmer series, i.e., for Balmer series $n_i = 2, n_j = 3, 4, 5, \dots$

The wavelength of the radiation is given by

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n_j^2} \right)$$

They lie in the visible region

3. Paschen series

If the electron jumps from fourth or higher orbit to the third orbit, we obtain

Paschen series i.e., when $n_i = 3$ and $n_j = 4, 5, 6, \dots$. The wavelength of the radiation is given by

$$\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{n_j^2} \right)$$

They lie in the infra red region.

4. Brackett series

If the electron jumps from fifth or higher orbit to the fourth orbit we obtain Brackett series i.e., when $n_i = 4$ and $n_j = 5, 6, 7, \dots$

The wavelength of the radiation is given by

$$\frac{1}{\lambda} = R \left(\frac{1}{4^2} - \frac{1}{n_j^2} \right)$$

They lie in the infra red region.

5. Pfund series

If the electron jumps from sixth or higher orbit to the fifth orbit we obtain pfund series i.e., when $n_i = 5$ and $n_j = 6, 7, 8, \dots$

The wavelength of the radiation is given by

$$\frac{1}{\lambda} = R \left(\frac{1}{5^2} - \frac{1}{n_j^2} \right)$$

They also lie in the infra red spectrum.

The various spectral series of hydrogen atom are shown in figure below.

Energy level diagram

A diagram which represents the total energies of electron in different stationary orbits of an atom is called the energy level diagram of that atom.

In this diagram total energies of electron in various stationary orbits are represented by parallel horizontal lines drawn according to some suitable energy scale.

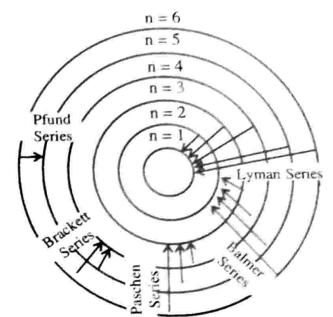


Figure 2.14

The vertical line connecting any two states represents the transition of the electron from one to the other of these states. Difference of energies of these states gives the amount of energy emitted or absorbed according as the electron goes from higher to lower state or from lower to higher energy level in the atom.

Total energy of electron in n^{th} orbit of hydrogen atom is

$$E_n = \frac{-13.6\text{eV}}{n^2}$$

Putting $n = 1, 2, 3, \dots$ we get the energies of electrons in various stationary orbits as

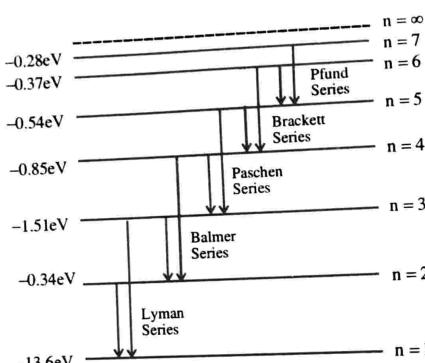


Figure 2.15

$$E_1 = \frac{-13.6\text{eV}}{1^2} = -13.6\text{eV}$$

$$E_2 = \frac{-13.6\text{eV}}{2^2} = -3.4\text{eV}$$

$$E_3 = \frac{-13.6\text{eV}}{3^2} = -1.51\text{eV}$$

$$E_4 = \frac{-13.6\text{eV}}{4^2} = -0.85\text{eV}$$

$$E_5 = \frac{-13.6\text{eV}}{5^2} = -0.54\text{eV}$$

$$E_6 = \frac{-13.6\text{eV}}{6^2} = -0.37\text{eV}$$

As n increases, E_n becomes less negative until at $n = \infty$, $E_n = 0$

Drawbacks of Bohr's theory

- Bohr's theory can explain the radiation spectrum of only hydrogen or hydrogen like atoms such as He^+ , Li^{++} , $\text{Be}^{+++} \dots$ etc. i.e., it fails to explain the spectrum of multi-electron atoms.
- It does not explain the fine structure of spectral lines even in hydrogen atom.
- It does not say anything about the relative intensities of spectral lines.
- It does not take into account the wave nature of electrons.
- It does not explain why orbits of electrons are taken as circular while elliptical orbits are also possible.

Hydrogen like atoms

The ions such as He^+ , Li^{++} , $\text{Be}^{+++} \dots$ etc. possess only one electron orbiting around the nucleus. They are called hydrogen like atoms. Bohr's atomic theory can be applied to them keeping in view that the charge in their nucleus is Ze , where Z is the atomic number. Below we give simplified expressions for the various parameters of hydrogen like atoms.

The energy of the orbital electron in the hydrogen like atom is given by

$$E_n = \left(\frac{1}{4\pi\epsilon_0} \right)^2 \left(\frac{2\pi me^4}{h^2} \right) \frac{Z^2}{n^2} = -hcR \frac{Z^2}{n^2}$$

orbital radius is given by

$$r_n = \frac{\epsilon_0 h^2}{\pi m e^2} \frac{n^2}{Z} \text{ and}$$

orbital velocity is given by

$$v_n = \frac{1}{4\pi\epsilon_0} \frac{2\pi e^2}{h} \frac{Z}{n}$$

from the above relations we can write

$$(i) E_n \propto \frac{Z^2}{n^2} \text{ or } E_n = -13.6 \frac{Z^2}{n^2} \text{ eV}$$

$$(ii) r_n \propto \frac{n^2}{Z} \text{ or } r_n = a_0 \frac{n^2}{Z}$$

$$(iii) v_n \propto \frac{Z}{n} \text{ or } v_n = c \alpha \frac{Z}{n}$$

Excitation and ionisation energy

When an electron revolving in a stationary orbit of an atom, electron can absorb energy. During absorption two things may occur.

- By absorbing energy electron may jump over to an outer orbit of higher energy. This process in which absorption of energy by an electron takes the electron to some outer orbit of higher energy is called excitation. Then the atom is said to be in the excited state. The energy required for this is called excitation energy.

It is given by $E_e = hcR \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$. The excitation energy is defined as the

minimum energy required to take an electron from a lower orbit to any higher orbit.

For example, the energy of the orbital electron in the ground state ($n = 1$) is -13.6 eV and that in the first excited state ($n = 2$) is -3.4 eV

\therefore The excitation energy for the first excited state

$$= -3.4 \text{ eV} - (-13.6 \text{ eV}) = 10.2 \text{ eV}$$

Similarly, energy required to raise an electron from ground state ($n = 1$) to second excited state ($n = 3$).

$$= -1.51 - (-13.6) = 12.09 \text{ eV}$$

The potential corresponding to excitation energy is called excitation potential. For example, when an electron jumps from $n = 1$ to $n = 2$ the excitation potential is 10.2 volt .

- If the energy absorbed by the electron is very large so that it can jump from the present orbit to the infinite orbit. This process is called ionisation. Thus, ionisation is the phenomenon of removal of an electron from an atom and the energy required for this process is called ionisation energy.

$$\text{It is given by } E_i = hcR \frac{1}{n^2}$$

For example, total energy of electron in ground state of hydrogen atom $= -13.6 \text{ eV}$. To remove this electron from the hydrogen atom $+13.6 \text{ eV}$ energy is required.

\therefore Ionisation energy of hydrogen atom $= 13.6 \text{ eV}$

Ionisation potential

The potential through which an electron need to be accelerated so that it acquires energy equal to the ionisation energy is called ionisation potential. For example the ionisation potential of hydrogen atom in the ground state is 13.6 volts .

Example 9

In the $n = 3$ state of hydrogen, find electrons velocity, kinetic energy and potential energy.

Solution

$$\text{Velocity } v_n = \frac{c}{n} \alpha, \text{ here } n = 3$$

$$v_3 = \frac{3 \times 10^8}{3} \cdot \frac{1}{137} = \frac{100 \times 10^6}{137}$$

$$v_3 = 7.299 \times 10^5 \text{ ms}^{-1}$$

$$\text{Kinetic energy } K = \frac{1}{2} mv_3^2$$

$$K = \frac{1}{2} \times 9.1 \times 10^{-31} \times (7.299 \times 10^5)^2$$

$$K = 4.55 \times 10^{-31} \times 7.299^2 \times 10^{10}$$

$$K = 2.42 \times 10^{-19} \text{ J}$$

$$K = \frac{2.42 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.51 \text{ eV}$$

$$\text{Potential energy } = -2K = -2 \times 1.51$$

$$= -3.02 \text{ eV}$$

Example 10

Use the Bohr theory to find the series wave length limits of the Lyman and Paschen series of hydrogen

Solution

$$\text{We have } \lambda = \frac{1}{R} \frac{n_1^2 n_2^2}{n_2^2 - n_1^2}$$

$R = 1.09737 \times 10^7$, For the Lyman series $n_1 = 1$, For the series limit $n_2 = \infty$

$$\lambda = \frac{1}{R} \frac{n_1^2}{1 - \frac{n_1^2}{n_2^2}} = \frac{1}{R} \cdot n_1^2 = \frac{1}{R}$$

$$\lambda = \frac{1}{1.09737 \times 10^7} = 91.13 \text{ nm}$$

For the Paschen series $n_1 = 3$ and $n_2 = \infty$

$$\lambda = \frac{1}{R} \frac{n_1^2}{1 - \frac{n_1^2}{n_2^2}} = \frac{n_1^2}{R} = \frac{9}{R}$$

$$\lambda = \frac{9}{1.09737 \times 10^7} = 820.1 \text{ nm}$$

Example 11

An electron is in the $n = 5$ state of hydrogen. To what states can the electron make transitions and what are the energies of emitted radiation.

Solution

Since electron is in the 5th state it can make transitions by jumping to any lower state ($n = 4, 3, 2, 1$).

$$\text{We have } E_n = -\frac{13.6}{n^2} \text{ eV}$$

$$\text{In the 5th state, } E_5 = -\frac{13.6}{25} = -0.544 \text{ eV}$$

$$\text{In the 4th state, } E_4 = -\frac{-13.6}{16} = -0.85 \text{ eV}$$

∴ The transition energy is $(5 \rightarrow 4)$

$$\begin{aligned} \Delta E &= E_5 - E_4 = -0.54 - (-0.85) \\ &= 0.306 \text{ eV} \end{aligned}$$

Similarly

$$5 \rightarrow 3, \Delta E = E_5 - E_3 = 0.97 \text{ eV}$$

$$5 \rightarrow 2, \Delta E = E_5 - E_2 = 2.86 \text{ eV}$$

$$5 \rightarrow 1, \Delta E = E_5 - E_1 = 13.1 \text{ eV}$$

Example 12

Calculate the excitation energy for $n = 3$ of He^+ atom.

Solution

For H_2 atom we have

$$E_i = -\frac{13.6}{n_i^2} \text{ eV}$$

$$E_j = -\frac{13.6}{n_j^2} \text{ eV}$$

Excitation energy $E_j - E_i$

$$\begin{aligned} &= -\frac{13.6}{n_j^2} + \frac{13.6}{n_i^2} \\ &= 13.6 \left(\frac{1}{n_i^2} - \frac{1}{n_j^2} \right) \text{ eV} \end{aligned}$$

For any atom, Excitation energy

$$= 13.6 Z^2 \left(\frac{1}{n_i^2} - \frac{1}{n_j^2} \right) \text{ eV}$$

For He^+ atom $Z = 2$ and $n_i = 19$ and $n_j = 3$

$$= 13.6 \times 4 \left(\frac{1}{3^2} \right)$$

$$= 13.6 \times 4 \times \frac{8}{9} = 48.35 \text{ eV}$$

Example 13

What is the shortest wavelength emitted by Balmer series in hydrogen spectrum?

Solution

We have

$$\frac{1}{\lambda} = R \left(\frac{1}{n_j^2} - \frac{1}{n_i^2} \right) = 1.097 \times 10^7 \left(\frac{1}{n_j^2} - \frac{1}{n_i^2} \right)$$

For Balmer series $n_j = 2$. To have shortest wavelength $n_i = \infty$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{n_j^2} - \frac{1}{\infty} \right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \times \frac{1}{4}$$

$$\therefore \lambda = \frac{4}{1.097 \times 10^7} = 365 \times 10^{-9} \text{ m}$$

Example 14

Calculate the wavelength of radiation emitted when He^+ makes a transition from the state $n_i = 3$ to the state $n_j = 2$.

Solution

We have $\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_j^2} - \frac{1}{n_i^2} \right)$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \times 2^2 \left(\frac{1}{2^2} - \frac{1}{3^2} \right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \times 4 \left(\frac{1}{4} - \frac{1}{9} \right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \times 4 \left(\frac{(9-4)}{36} \right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \times \frac{4 \times 5}{36}$$

or $\lambda = \frac{36}{1.097 \times 10^7 \times 20} = 164 \text{ nm}$

Example 15

A particular hydrogen like ion emits radiation of frequency $2.647 \times 10^{15} \text{ Hz}$ when it makes transition from $n_i = 2$ to $n_j = 1$. What will be the frequency of the radiation emitted in a transition from $n_i = 3$ to $n_j = 1$.

Solution

We have $\frac{1}{\lambda} = R \left(\frac{1}{n_j^2} - \frac{1}{n_i^2} \right)$

or $\frac{c}{\lambda} = cR \left(\frac{1}{n_j^2} - \frac{1}{n_i^2} \right)$

or $v = cR \left(\frac{1}{n_j^2} - \frac{1}{n_i^2} \right)$

When $n_i = 2$, $n_j = 1$ $v = 2.647 \times 10^{15}$

$$2.647 \times 10^{15} = cR \left(\frac{1}{1^2} - \frac{1}{2^2} \right)$$

or $2.647 \times 10^{15} = cR \cdot \frac{3}{4}$ (1)

When $n_i = 3$, $n_j = 1$ $v = ?$

$$v = cR \cdot \left(\frac{1}{1^2} - \frac{1}{3^2} \right)$$

$$v = cR \cdot \frac{8}{9}$$

$$\text{eq(2)} \rightarrow \frac{v}{2.647 \times 10^{15}} = \frac{8}{9} \times \frac{4}{3} = \frac{32}{27}$$

$$v = 2.647 \times 10^{15} \times \frac{32}{27}$$

$$v = 3.13 \times 10^{15} \text{ Hz}$$

..... (2)

Example 16

A proton and an electron both at rest initially combine to form a hydrogen atom in the ground state. A single photon is emitted in this process. What is its wavelength.

Solution

The energy of the photon emitted

= Binding energy of H₂ atom

$$\text{i.e., } hv = 13.6 \text{ eV}$$

$$\text{or } \frac{hc}{\lambda} = 13.6 \times 1.6 \times 10^{-19} \text{ J}$$

$$\therefore \lambda = \frac{hc}{13.6 \times 1.6 \times 10^{-19}}$$

$$\lambda = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{13.6 \times 1.6 \times 10^{-19}} = 912 \text{ Å}$$

Example 17

How much energy is required to remove an electron in n = 2 state from a hydrogen atom.

Solution

$$\text{We have } E_n = -\frac{13.6}{n^2} \text{ eV}$$

$$\text{For } n = 2, E_n = -\frac{13.6}{4} \text{ eV} = -3.4 \text{ eV}$$

This is the energy possessed. So to remove it an energy of 3.4eV is required.

Example 18

The longest wavelength in the Lyman series is 121.5nm and the shortest wavelength in a Balmer series is 364.6nm. Use the figures to find the longest wavelength of light that could ionise hydrogen.

Solution

$$\text{We have } \frac{1}{\lambda} = R \left(\frac{1}{n_i^2} - \frac{1}{n_j^2} \right)$$

For Lyman series n_i = 1, for longest wavelength n_j = 2

$$\therefore \left(\frac{1}{\lambda_{\max}} \right)_L = R \left(1 - \frac{1}{2^2} \right) = \frac{3}{4} R \quad \dots\dots (1)$$

For Balmer series n_i = 2, for shortest wavelength n_j = ∞

$$\therefore \left(\frac{1}{\lambda_{\min}} \right)_B = R \left(\frac{1}{2^2} - \frac{1}{\infty} \right) = \frac{R}{4} \quad \dots\dots (2)$$

Adding eqns (1) and (2), we get

$$\left(\frac{1}{\lambda_{\max}} \right)_L + \left(\frac{1}{\lambda_{\min}} \right)_B = R \quad \dots\dots (3)$$

The ionisation energy of hydrogen atom is

$$E_i = hcR \cdot \frac{1}{n^2} \text{ or } hv = \frac{hcR}{n^2}$$

$$\frac{v}{c} = \frac{R}{n^2}$$

$$\frac{1}{\lambda} = \frac{R}{n^2}$$

$$\left(\frac{1}{\lambda_{\max}} \right)_H = R \quad \dots\dots (4)$$

$$\therefore \text{We have } \left(\frac{1}{\lambda_{\max}} \right)_H = \left(\frac{1}{\lambda_{\max}} \right)_L + \left(\frac{1}{\lambda_{\min}} \right)_B$$

$$\left(\frac{1}{\lambda_{\max}} \right)_H = \frac{1}{121.5} + \frac{1}{364.6} = \frac{4861}{369.6 \times 121.5}$$

$$\therefore (\lambda_{\max})_H = \frac{121.5 \times 364.6}{4861} = 91.1 \text{ nm.}$$

Franck-Hertz experiment

In 1914 James Franck and Gustav Hertz (nephew of Heinrich Hertz) performed a series of experiments demonstrating conclusively the quantised nature of the energy levels of the atoms postulated by Neils Bohr in 1913. For this Franck and Hertz were awarded Nobel prize in physics in 1925.

Mechanism

In this experiment mercury vapour was bombarded with electrons of the known kinetic energy. When this kinetic energy is less than the energy of the first excited state of atomic mercury, the only energy which the colliding electron loses is the small amount of kinetic energy (about 1 part in 10^5) that it can transfer to the massive mercury atom by an elastic collision. However, when the kinetic energy of the incident electron just exceeds the energy of the first excited state of mercury atom, then the electrons give virtually all of its kinetic energy to the mercury atom in an inelastic collision. Obviously by comparing the kinetic energy of the electron before and after collision one can determine how much energy is transferred to the target atoms.

Experimental set up

The experimental arrangement consists of a mercury tube containing a cathode, plate and accelerating grid which is located near the plate. (see figure). Both the plate and the grid were maintained at positive potential with respect to the cathode but the plate potential was slightly lower than the grid potential. This small retarding potential on the plate prevents contributions to the plate current from electrons having negligible kinetic energy. Hence, if electrons to lose most of their kinetic energy in inelastic collisions with mercury atoms, the retarding potential would prevent them from reaching the plate and a drop in plate current would result. Such a drop in plate current was found to occur at 4.9 volts. Further more, Franck and

Hertz noted that at this voltage, the 2536 Å spectral line of mercury appeared in the emission spectrum of the vapour. A simple calculation reveals that the photon energy of the 2536 Å line corresponds to 4.9 eV.

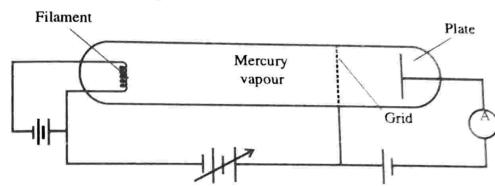


Figure 2.16

$$\text{i.e., we have } E = hv = \frac{hc}{\lambda}$$

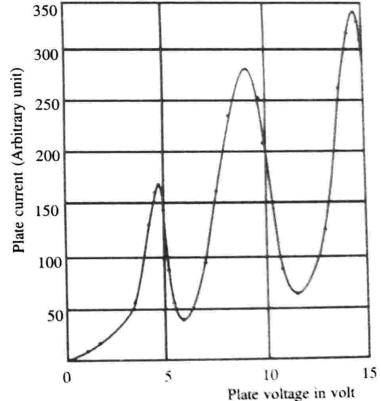
$$\therefore \lambda = \frac{hc}{E} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{4.9 \times 1.6 \times 10^{-19}}$$

$$= 2536 \text{ Å.}$$

At slightly higher voltages a large drop in the plate current occurs and new lines appear in the emission spectrum of the mercury vapour. This behaviour is repeated at multiples of 4.9 volts as shown in figure. Thus we must conclude that the Franck-Hertz experiment confirms the discrete energy states of the mercury atom there by the postulate of Bohr. The experiment was conducted for several other atoms also.

Example 19

The first excited state of sodium decays to the ground state by emitting a photon of wavelength 590nm. If sodium vapour is used for the Frank-Hertz experiment, at what voltage will the first current drop be recorded.



The variation of the plate current with voltage V as measured in the original experiment of Franck and Hertz

Figure 2.17

Solution

$$\text{We have } E = \frac{hc}{\lambda}$$

$$E = \frac{1240 \text{ eV nm}}{590 \text{ nm}} = 2.10 \text{ eV.}$$

At $V = 2.10$ volts, we expect a first decrease in current.

Correspondence principle

This principle was enunciated by Neils Bohr. According to Bohr's theory of hydrogen atom, the frequency of radiation emitted or absorbed is determined by the difference in energy of two allowed orbital state. This frequency is not equal to frequency of rotation in the stationary orbits. A correlation between the frequency of rotation (f) and the frequency of radiation (v) can be achieved is known as Bohr's correspondence principle. According to quantum mechanics energy levels are quantised and classical mechanics says that energy is continuous. When we go to higher and higher energy levels (for large n) the energy levels are so close that we can take it that energies are continuous. In other words for large quantum numbers quantum mechanics predicts the same as what classical physics predicts. In this context the frequency of rotation (f) and the frequency of radiation (v) emitted are equal.

The frequency of rotation of the electron in a hydrogen atom can be easily calculated. We know that

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

If the time taken is period (T) of one revolution then obviously the distance travelled is $2\pi r$.

$$\text{i.e., } v = \frac{2\pi r}{T}$$

$$\text{or } \frac{1}{T} = \frac{v}{2\pi r}$$

$$\text{or } f = \frac{v}{2\pi r} \quad \left(\because \frac{1}{T} = f \right)$$

Substituting for v from eqn (6), we get

$$f = \frac{e}{2\pi\sqrt{4\pi\varepsilon_0 mr}} = \frac{e}{2\pi\sqrt{4\pi\varepsilon_0 mr^3}}$$

Now substituting from eq (5), we get

$$f = \frac{e}{2\pi\sqrt{4\pi\varepsilon_0 m}} \frac{\pi^{1/2} m^{3/2}}{n^3 h^3 \varepsilon_0^{3/2}} e^3$$

$$\text{or } f = \frac{me^4}{4\varepsilon_0^2 h^3} \cdot \frac{1}{n^3} = \frac{6.58 \times 10^{15}}{n^3} \text{ Hz} \quad \dots\dots (1)$$

The frequency of radiation emitted is given by

$$v = \frac{me^4}{8\varepsilon_0^2 h^3} \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) \quad \dots\dots (2)$$

Take $n_f = n$ and $n_i = n - p$ where where $p = 1, 2, 3, \dots$

$$v = \frac{me^4}{8\varepsilon_0^2 h^3} \left(\frac{1}{(n-p)^2} - \frac{1}{n^2} \right)$$

$$v = \frac{me^4}{8\varepsilon_0^2 h^3 c} \left[\frac{n^2 - (n-p)^2}{(n-p)^2 n^2} \right]$$

$$v = \frac{me^4}{8\varepsilon_0^2 h^3} \left[\frac{(n^2 - n^2 + 2np - p^2)}{(n-p)^2 n^2} \right]$$

$$v = \frac{me^4}{8\varepsilon_0^2 h^3} \frac{(2np - p^2)}{n^2(n-p)^2}$$

When n is very large, obviously $n \gg p$

$$\text{So } 2np - p^2 \approx 2np$$

$$(n-p)^2 \approx n^2$$

$$v = \frac{me^4}{8\varepsilon_0^2 h^3} \frac{2np}{n^2 \cdot n^2}$$

$$\text{or } v = \frac{me^4}{8\epsilon_0^2 h^3} \left(\frac{2p}{n^3} \right)$$

when $p = 1$

$$v = \frac{me^4}{4\epsilon_0^2 h^3} \left(\frac{1}{n^3} \right)$$

Comparing eqn 1 and 3, we can see that

$$f = v$$

It shows that for large quantum number the classical and quantum predictions are same. This is called Bohr's correspondence principle.

Note : It may also be noted that when n is very small $n = 2$ or 3 , the predictions strongly differ.

Example 20

Compare the frequency of revolution of an electron with the frequency of photons emitted in transitions from n to $n-1$ for

- a) $n = 10$, b) $n = 100$, c) $n = 1000$, d) $n = 10000$.

Solution

The frequency of revolution is given by

$$f_n = \frac{me^4}{4\epsilon_0^2 h^3} \cdot \frac{1}{n^3} = \frac{6.58 \times 10^{15}}{n^3} \text{ Hz}$$

Frequency of radiation emitted is given by

$$v = \frac{me^4}{8\epsilon_0^2 h^3} \frac{(2np - p^2)}{n^2(n-1)^2}$$

Here $p = 1$ (given)

$$v_n = \frac{me^4}{8\epsilon_0^2 h^3} \frac{(2n-1)}{n^2(n-1)^2}$$

$$v_n = \frac{6.58 \times 10^5 (2n-1)}{2n^2(n-1)^2} \text{ Hz}$$

a) For $n = 10$, $f_{10} = 6.58 \times 10^{12} \text{ Hz}$ and $v_{10} = 7.72 \times 10^{12} \text{ Hz}$

b) For $n = 100$, $f_{100} = 6.58 \times 10^8 \text{ Hz}$ and $v_{100} = 6.68 \times 10^8 \text{ Hz}$

c) For $n = 1000$, $f_{1000} = 6.58 \times 10^6 \text{ Hz}$ and $v_{1000} = 6.59 \times 10^6 \text{ Hz}$

d) For $n = 10000$, $f_{10000} = 6.58 \times 10^3 \text{ Hz}$ and $v_{10000} = 6.58 \times 10^3 \text{ Hz}$

This shows that n becomes larger f and v coincide in accordance with correspondence principle.

Deficiencies of the Bohr model

Here we discuss some of the apparently seen deficiencies of Bohr model.

- According to Bohr model electron orbiting around the nucleus containing proton. But actually electron and proton both orbit about their centre of mass. Thus kinetic energy of atom should include the kinetic energy of proton. In other words it becomes a two body problem. This can be converted into a one body by introducing the concept of reduced mass μ . It is not the mass of electron alone contributes to the energy but also the mass of proton (m_p).

$$\text{The reduced mass } \mu = \frac{m_e m_p}{m_e + m_p}$$

In order to refine the theory of Bohr model mass of the electron must be replaced by reduced mass μ . As a result of this the calculated energy will be slightly decreased so also predicted frequency. The increase in wavelength is calculated about 0.05%. Hence in Rydberg formula Rydberg constant modifies to

$$R' = R \left(\frac{1 + m_e}{m_p} \right)$$

- Another discrepancy found in Bohr model was in the conversion of frequency to wavelength. Wavelength measurements are done in air. So wavelength

$$\lambda = \frac{c_{\text{in air}}}{v_{\text{in air}}}$$

$c_{\text{in air}}$ is slightly less than the c we used. This results in slight decrease in wavelength λ . This comes about 0.03%.

3. A serious failure of Bohr model is the prediction of angular momentum.
According to Bohr model angular momentum $L = n\hbar$.
For the ground state of the hydrogen atom $n = 1 \therefore L = \hbar$.
But experiment shows that $L = 0$ for the ground state.
4. Another deficiency of Bohr model is that it violates the uncertainty principle.
According to uncertainty principle we have $\Delta z \Delta p_z \geq \hbar$.
Although Bohr model has so many defects, it introduced so many concepts like stationary states, quantization of angular momentum, correspondence principle etc. which are found to be true in the realm of wave mechanics. Moreover Bohr model provided a stepping stone to develop more refined theories.

IMPORTANT FORMULAE

1. According to Thomson model the electric field at a distance r from the centre of the atom is

$$E = \frac{Ze}{4\pi\epsilon_0 R^3} r$$

2. Characteristic frequency of oscillation of electrons of Thomson model

$$\omega = \sqrt{\frac{Ze^2}{4\pi\epsilon_0 m R^3}}$$

3. Relation between impact parameter (b) and scattering angle

$$b = \frac{zZe^2}{4\pi\epsilon_0 K} \cot \frac{\theta}{2}$$

4. Rutherford's scattering formula

$$\sigma(\theta) = \frac{1}{16} \left(\frac{zZe^2}{K4\pi\epsilon_0} \right)^2 \csc^4 \frac{\theta}{2}$$

5. Distance of closest approach

$$d = \frac{zZe^2}{4\pi\epsilon_0 K}$$

6. Balmer formula

$$\lambda = (364.5 \text{ nm}) \frac{n^2}{n^2 - 4} (n = 3, 4, 5, \dots)$$

7. In general $\lambda = \lambda_{\min} \frac{n^2}{n^2 - n_0^2}$, λ_{\min} is the wavelength of the series limit
For Lyman series $n_0 = 1$
Balmer series $n_0 = 2$
Paschen series $n_0 = 3$
Brackett series $n_0 = 4$
Pfund series $n_0 = 5$
8. Bohr frequency equation when an electron jumps from the n th to m th orbit, the radiation of frequency ' v ' emitted is

$$v = \frac{E_n - E_m}{h}$$

9. Expression for the radius of the orbit

$$r_n = \left(\frac{\epsilon_0 h^2}{\pi m e^2} \right) n^2$$

$$r_n = a_0 n^2 \quad a_0 = 0.0529 \text{ nm} - \text{Bohr radius}$$

10. Velocity of the orbital electron

$$V_n = \frac{ca}{n} \quad \alpha = \frac{1}{137}$$

11. Energies of Bohr orbits in hydrogen atom.

$$\text{Potential energy } U = \frac{-e^2}{4\pi\epsilon_0 r_n}$$

$$\text{Kinetic energy } K = \frac{1}{2} \frac{e^2}{4\pi\epsilon_0} \frac{1}{r_n}$$

$$\text{Total energy } E_n = \frac{-1}{2} \frac{e^2}{4\pi\epsilon_0} \frac{1}{r_n}$$

$$\text{with } r_n = a_0 n^2$$

$$\text{or } E_n = \frac{-13.6 \text{ eV}}{n^2}$$

12. For hydrogen like atoms

$$r_n = \frac{e_0 h^2}{\pi m_e^2} \frac{n^2}{Z} = \frac{a_0 n^2}{Z}$$

$$v_n = \frac{c \alpha \cdot Z}{n} \quad \alpha = \frac{1}{137}$$

$$E_n = \frac{-13.6 Z^2}{n^2} \text{ eV}$$

13. Excitation energy

$$E_e = h c R \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$$

where $R = 1.09737 \times 10^7 \text{ m}$

14. Ionization energy

$$E_i = h c R \frac{1}{n^2}$$

15. Hydrogen wavelengths in Bohr model

$$\lambda = \frac{1}{R} \frac{n_1^2 n_2^2}{n_2^2 - n_1^2}$$

For $n_2 = \infty$ gives λ series limit.

16. The frequency of revolution in Bohr orbit

$$f_n = \frac{6.5 \times 10^{15}}{n^3}$$

17. Frequency of radiation emitted

$$v_n = \frac{6.58 \times 10^{15} \cdot (2np - p^2)}{2n^2(n-1)^2}$$

18. Modified Rydberg constant due to reduced mass μ

$$R' = R \left(1 + \frac{m_e}{m_p} \right), \quad \mu = \frac{m_e m_p}{m_e + m_p}$$

UNIVERSITY MODEL QUESTIONS**Section A**

(Answer questions in two or three sentences)

Short answer type questions

1. Mention two experimental studies that are important in the development of the theory of atomic structure.
2. What information would be obtained from the experimental study of scattering of charged particles by atoms?
3. Mention the name of the experiment that gives us information about the excited states of atoms.
4. Write down three basic properties of atoms.
5. Atoms are neutral with negatively charged particles. Where does this statement come from?
6. What is Thomson's atomic model?
7. What is scattering experiment?
8. Write down the formula for the angular frequency of electron predicted by Thomson's model.
9. Mention any two drawbacks of Thomson's model.
10. According to Rutherford what is the model of an atom.
11. Define impact parameter.
12. Write down the relation between impact parameter and scattering angle and explain the symbols used.
13. Define differential cross section of scattering.
14. Define intensity in scattering experiment.
15. Write down the Rutherford scattering formula and explain the symbols used.
16. What is meant by closest distance of approach?
17. Write down the expression for closest distance of approach and explain the symbols used.
18. What are the advantages of Rutherford's atom model over Thomson's model.
19. Mention two limitations of Rutherford's atom model.
20. Mention three classifications of spectra.
21. What is a line spectra?
22. What is continuous spectra?
23. What is discrete spectra?
24. What is emission spectra?

25. What is absorption spectra?
26. Write down Balmer formula and explain the symbols used.
27. What is Balmer series?
28. What is Ritz combination principle?
29. Define ground state and excited state of an atom.
30. What is the origin of line spectra?
31. Write down the names of the spectral lines emitted by hydrogen atom.
32. Write down the expression for the wavelength emitted by hydrogen atom and explain the symbols used.
33. What is Bohr's correspondence principle?
34. What is the mechanism of Hertz experiment?
35. Draw a diagram of Frank-Hertz experimental setup.
36. What was the aim of Frank-Hertz experiment?
37. What was the conclusion of Frank-Hertz experiment?
38. Explain the function of grid in the Frank-Hertz experiment.
39. Explain the function of plate in the Frank-Hertz experiment.
40. Mention two deficiencies of Bohr atom model.
41. What are the postulate of Bohr atom model?
42. Give two drawbacks of Bohr's atom model.
43. What is excitation energy?
44. What is ionisation potential?
45. How does Rydberg constant change when reduced mass is taken into account?

Section B

(Answer questions in about half a page to one page)

Paragraph / Problem type questions

1. Derive an expression for the characteristic frequency of oscillation of Thomson model.
2. Set up an expression for the distance of closest approach.
3. Derive an expression for the radius of the Bohr orbit.
4. Derive an expression for the velocity of the orbital electron according to Bohr model.
5. Write down Bohr's postulates and what are the drawbacks of Bohr's model.
6. What are the deficiencies of Bohr model?
7. Calculate the ionisation energy of the ground state of Li^{++} atom [1224eV]
8. An electron collides with a hydrogen atom in its ground state and excites it to a state of $n = 3$. How much energy was given to the hydrogen atom in this elastic collision? [12,08eV]

9. Calculate longest wavelength present in the Balmer series of hydrogen corresponding to the H_α line. [656 nm]
10. Calculate the shortest and longest wavelength limits of Lyman series $R = 1.097 \times 10^7 \text{ m}^{-1}$. [911.5Å, 1215Å]
11. An excited hydrogen atom emits a photon of wavelength λ in returning to the ground state. Derive a formula that gives the quantum number of initial state in terms of λ and R .

$$n = \sqrt{\frac{\lambda R}{\lambda R - 1}}$$
12. When an excited atom emits a photon, the linear momentum of the photon must be balanced by the recoil momentum of the atom. As a result some of the excitation energy of the atom goes into the kinetic energy of the recoil. Modify $E_i - E_f = h\nu$ to include this effect.

$$\left[h\nu \left(1 + \frac{h\nu}{2Mc^2} \right) \right]$$
13. Find the wavelength of the spectral line that corresponds to a transition in hydrogen from the $n = 10$ state to the ground state. In what part of the spectrum is this. [9.21nm, ultra violet]
14. Compare the ionisation energy in positronium with that in hydrogen. [6.8eV]
15. The maximum kinetic energy that an alpha particle can transmit to an electron occurs during a head on collision. Compute the kinetic energy lost by an alpha particle of kinetic energy 8.0MeV in a head on collision with an electron at rest. [0.0044MeV]
16. Alpha particles of energy 9.6MeV are incident on a silver foil of thickness 7.0 μm. For a certain value of the impact parameter, the alpha particle loses exactly half their incident kinetic energy when they reach their minimum separation from the nucleus. Find the minimum separation, the impact parameter and the scattering angle.

$$[28.2 \text{ fm}, 19.9 \text{ fm}, \theta = 38.9^\circ]$$
17. Find the shortest and the longest wavelengths of the Lyman series of singly ionised helium [30.4 nm 22.8 nm]
18. A collection of hydrogen atoms in the ground state is illuminated with ultraviolet light of wavelength 59.0 nm. Find the kinetic energy of the emitted electrons. [7.4eV]
19. Find the ionisation energy of
 - a) the $n = 3$ level of hydrogen [1.51eV]
 - b) the $n = 2$ level of He^+ [13.6eV]
 - c) the $n = 4$ level of L_i^{++} [7.65eV]
20. What is the difference in wavelength between the first line of the Balmer series in ordinary hydrogen ($M = 1.007825\mu$) and in heavy hydrogen ($M = 2.014102\mu$)? [0.178 nm]

Section C

(Answer questions in about two pages)

Long answer type questions (Essays)

1. Derive the relation between impact parameter and scattering angle.
2. Derive Rutherford scattering formula.
3. State and prove correspondence principle.
4. What is Bohr model of atom. Derive an expression for the energy levels of hydrogen atom and draw the energy level diagram.
5. Describe the Frank-Hertz experiment in detail.

Hints to problems

7. $E_i = 13.6 \frac{Z^2}{n^2}$ $Z = 3$ and $n = 1$

8. $E_f - E_i = -13.6 \left(\frac{1}{n_j^2} - \frac{1}{n_i^2} \right) \text{eV}$

9. $\frac{1}{\lambda} = R \left(\frac{1}{n_i^2} - \frac{1}{n_j^2} \right)$ For Balmer $n_i = 2$, to have longest wavelength $n_j = 3$

10. We have $\frac{1}{\lambda} = R \left(\frac{1}{n_i^2} - \frac{1}{n_j^2} \right)$

For Lyman series $n_i = 1$ For shortest wavelength $n_j = \infty$ For longest wavelength $n_j = 2$

11. $\frac{1}{\lambda} = R \left(\frac{1}{n_i^2} - \frac{1}{n_j^2} \right)$

For ground state $n_i = 1$ take $n_j = n$

$\therefore \frac{1}{\lambda} = R \left(1 - \frac{1}{n^2} \right)$

$$\frac{1}{\lambda R} = 1 - \frac{1}{n^2} \quad \text{or} \quad 1 - \frac{1}{\lambda R} = \frac{1}{n^2}$$

$$\frac{\lambda R - 1}{\lambda R} = \frac{1}{n^2} \quad \text{or} \quad n = \sqrt{\frac{\lambda R}{\lambda R - 1}}$$

12. Photon momentum = Recoil momentum of the atom

$$\frac{hv}{c} = Mv \quad \therefore K.E = \frac{\left(\frac{hv}{c}\right)^2}{2M}$$

$$(E_i - E_f) = hv + \frac{\left(\frac{hv}{c}\right)^2}{2M}$$

13. $\frac{1}{\lambda} = R \left(\frac{1}{n_j^2} - \frac{1}{n_i^2} \right) \quad n_j = 1, n_i = 10$

14. For hydrogen ionisation energy is

$$E_n = \frac{me^4}{8\varepsilon_0^2 h^2} = 13.6 \text{ eV}$$

For positronium, ionisation energy is

$$E'_n = \frac{m'e^4}{8\varepsilon_0^2 h^2} = \frac{m'me^4}{m8\varepsilon_0^2 h^2} = \frac{m'}{m} 13.6 \text{ eV}$$

A positronium atom is a system that consists of a positron and an electron that orbit each other. Therefore the reduced mass

$$m' = \frac{m M}{m + M} = \frac{m^2}{2m} = \frac{m}{2}$$

15. Using law of conservation of energy

$$\vec{p}_i = \vec{p}_i + \vec{p}_a$$

From law of conservation of energy

$$\frac{\vec{p}_i^2}{2m_a} = \frac{\vec{p}_i^2}{2m_a} + \frac{\vec{p}_a^2}{2m_e} \quad \text{replace } p_t = p_i - p_a$$

Solving we get $P_r = \frac{2p_i}{1 + \frac{m_e}{m_i}}$

$$K_i - K_f = K_f = \frac{p_i^2}{2m_e} - \frac{4m_e}{\left(1 + \frac{m_e}{m_i}\right)^2}$$

16. Potential energy at minimum separation

$$U = E - K = 9.6 - 4.8 = 4.8$$

Using $U = \frac{zZ^2 e^2}{4\pi\epsilon_0 r_{min}}$ $z = 2$, $Z = 47$

we get r_{min} where $\frac{e^2}{4\pi\epsilon_0} = 1.44 \text{ MeV fm}$

$$b = r_{min} \cdot \frac{Y_{min}}{\sqrt{v}} = r_{min} \sqrt{\frac{K_{min}}{K}}$$

Using $b = \frac{zZ^2}{4\pi\epsilon_0 2K} \cot \frac{\theta}{2}$

calculate θ

17. We have

$$\Delta E = E_2 - E_1 = (-13.6 \text{ eV}) 2^2 \left(\frac{1}{2^2} - \frac{1}{12} \right)$$

For Lyman $n_1 = 1$, to get longest wavelength $n_2 = 2$

$$\lambda = \frac{hV}{\Delta E} = \frac{1240 \text{ eV nm}}{40.8 \text{ eV}} = 30.4 \text{ nm}$$

For shortest wavelength $n_2 = \infty$

$$\Delta E = -(13.6 \text{ eV}) \cdot 2^2 \left(\frac{1}{\infty^2} - \frac{1}{n_1^2} \right)$$

$$\approx -55.4 \text{ eV}$$

$$\lambda = \frac{hc}{\Delta E} \approx 22.8 \text{ nm}$$

18. Energy of the incident photon

$$\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV nm}}{40.8 \text{ eV}} = 30.4 \text{ nm} = 21.0 \text{ eV}$$

out of this atom absorbs 13.6 eV

The remaining energy = $21 - 13.6 = 7.4 \text{ eV}$

19. a) $|E_1| = \frac{13.6 \text{ eV}}{9} = 1.51 \text{ eV}$

b) $|E_2| = \frac{13.6 Z^2}{2^2} = 13.6 \text{ eV}, Z = 2$

c) $|E_3| = \frac{13.6 Z^2}{4^2} = 7.65 \text{ eV}, Z = 3$

20. $R_H = R \left(1 + \frac{m}{M_H} \right) = R \left(1 + \frac{4.4858 \times 10^{-4}}{1.007824} \right) = 1.000544R$

$$R_D = R \left(1 + \frac{m}{M_D} \right) = R \left(1 + \frac{5.4858 \times 10^{-4}}{2.104102} \right) = 1.000272R$$

For the Balmer series

$$\lambda = \frac{1}{R} \frac{3^2 \cdot 2^2}{(3^2 - 2^2)}$$

Find $\lambda_D - \lambda_H = \left(\frac{1}{R_D} - \frac{1}{R_H} \right) \frac{3^2 \cdot 2^2}{(3^2 - 2^2)} = 0.178 \text{ nm}$