UNIT - II

2 INTERFERENCE BY DIVISION OF WAVE FRONT

Introduction

This section deals with the phenomenon of interference, one of the very important characteristics exhibited by waves. When two or more waves are allowed to pass through a continuous medium, they coincide in space and time, then we say that they interfere with each other. When two waves interfere the resultant effect can easily be explained by a simple law called the principle of superposition. This was first clearly stated and explained by an English physician and physicist Thomas Young (1773-1829) in 1802.

Principle of superposition

When two or more waves are allowed to pass simultaneously through a continuous medium, they superimpose one another then each wave produces its own displacement at any point and the resultant displacement will be the vector sum of their individual displacements. This is called the principle of superposition.

Let $\vec{y}_1, \vec{y}_2, \vec{y}_3 .....\vec{y}_n$ be the displacements of waves at any instant and the resultant displacement $\vec{y}$ can be written as

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + ...\vec{y}_n.$$

Superposition of two sinusoidal waves

Let us consider the superposition of two sinusoidal waves having the same frequency.

Let $\vec{y}_1 = a_1 \sin(kx - \omega t + \phi_1)$ ..... (1)

and $\vec{y}_2 = a_2 \sin(kx - \omega t + \phi_2)$ ..... (2)

represent the displacements produced by each wave. We assume that the displacements are in the same direction, however they have different amplitudes and different phases. According to superposition principle, the resultant displacement $\vec{y}$ would be given by

$$\vec{y} = \vec{y}_1 + \vec{y}_2$$

$$\vec{y} = a_1 \sin(kx - \omega t + \phi_1) + a_2 \sin(kx - \omega t + \phi_2)$$

$$\vec{y} = a_1 \sin(kx - \omega t)\cos\phi_1 + a_1 \cos(kx - \omega t)\sin\phi_1$$
$$+ a_2 \sin(kx - \omega t)\cos\phi_2 + a_2 \cos(kx - \omega t)\sin\phi_2$$

$$\vec{y} = (a_1 \cos\phi_1 + a_2 \cos\phi_2)\sin(kx - \omega t)$$
$$+ (a_1 \sin\phi_1 + a_2 \sin\phi_2)\cos(kx - \omega t) \qquad \text{..... (3)}$$

put

$$a_1 \cos\phi_1 + a_2 \cos\phi_2 = A \cos\phi \qquad \text{..... (4)}$$

and $$a_1 \sin\phi_1 + a_2 \sin\phi_2 = A \sin\phi \qquad \text{..... (5)}$$

Putting equations (4) and (5) in equation (3), we get

$$\vec{y} = A \cos\phi \sin(kx - \omega t) + A \sin\phi \cos(kx - \omega t)$$

$$\vec{y} = A \sin(kx - \omega t + \phi) \qquad \text{..... (6)}$$

where A is the amplitude and ϕ is the phase of the resultant wave.

Squaring and adding equations 4 and 5, we get

$$A^2 = a_1^2 + a_2^2 + 2\, a_1 a_2 \cos(\phi_1 - \phi_2) \qquad \text{..... (7)}$$

Dividing equation (5) by equation (4), we get

$$\tan\phi = \frac{a_1 \sin\phi_1 + a_2 \sin\phi_2}{a_1 \cos\phi_1 + a_2 \cos\phi_2} \qquad \text{..... (8)}$$

Special cases

Case I

When the phase difference between the two waves is $2n\pi$, where $n = 0, 1, 2, 3....$

i.e. $\phi_1 - \phi_2 = 2n\pi$, From equation (7), we get

$$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 = (a_1 + a_2)^2$$

$$\therefore \quad A_{max} = a_1 + a_2, \text{ if } a_1 = a_2 \equiv a,\ A_{max} = 2a.$$

This shows that the resultant amplitude is maximum when $\phi_1 - \phi_2 = 2n\pi$. This is called the constructive superposition. Thus the condition for constructive superposition is that the phase difference between two waves must be an integral multiple of 2π.

Case II

When $\phi_1 - \phi_2 = (2n+1)\pi$ where $n = 0, 1, 2, 3$ .....

From equation (7), we get

$$A_{min} = a_1 - a_2$$

If $a_1 = a_2 = a, \quad A_{min} = 0$

This shows that the resultant amplitude is minimum when the phase difference is $(2n+1)\pi$. This is called destructive superposition. Thus the condition for destructive superposition is that the phase difference between the two waves must be an odd integral multiple of π.

Expression for resultant intensity

Since intensity of light is directly proportional to square of the amplitude, equation (7) can be written as

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\phi_1 - \phi_2)$$

where I_1, I_2 and I are the intensities of first wave, second wave and the resultant wave respectively.

If $\phi_1 - \phi_2 = 2n\pi$, I will be maximum.

$$\therefore \quad I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

If $a_1 = a_2$ then $I_1 = I_2 \equiv I$

$$\therefore \quad I_{max} = 4I$$

If $\phi_1 - \phi_2 = (2n+1)\pi$, I will be minimum

$$\therefore \quad I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

If $a_1 = a_2$ then $I_1 = I_2 \equiv I$

$$\therefore \quad I_{min} = 0.$$

Example 1

Two waves having intensities in the ratio 1 : 9. Find the ratio of the intensity at minima to that at maxima.

Solution

We have $I \propto a^2$

$$\therefore \quad \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$$

$$\frac{I_1}{I_2} = \frac{1}{9} \text{ given}$$

$$\therefore \quad \frac{1}{9} = \frac{a_1^2}{a_2^2}$$

or $$\frac{a_1}{a_2} = \frac{1}{3}$$

$$a_2 = 3\,a_1$$

$$A_{max} = a_1 + a_2 = a_1 + 3a_1 = 4a_1$$

$$A_{min} = a_1 \sim a_2 = a_1 \sim 3a_1 = 2a_1$$

$$\therefore \quad \frac{I_{min}}{I_{max}} = \frac{A_{min}^2}{A_{max}^2} = \frac{(2a_1)^2}{(4a_1)^2} = \frac{1}{4}.$$

Example 2

Two sources of intensities I and 4I are superimposed. Obtain the intensities where the phase difference is a) $\frac{\pi}{2}$ b) π.

Solution

$$I_1 = I, \quad I_2 = 4I$$

Using $I' = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\phi_1 - \phi_2)$

a) When $\phi_1 - \phi_2 = \frac{\pi}{2}$

$$\therefore \quad I' = I + 4I + 2\sqrt{I \times 4I}\cos\frac{\pi}{2}$$

$$I' = 5I$$

b) When $\phi_1 - \phi_2 = \pi$

$$I' = I + 4I + 2\sqrt{I \times 4I}\cos\pi$$

$$I' = I + 4I - 4I$$

$$I' = I$$

Example 3

Two waves travelling along the same line are given by $\vec{y}_1 = 5\sin\left(kx - \omega t + \frac{\pi}{2}\right)$ and $\vec{y}_2 = 7\sin\left(kx - \omega t + \frac{\pi}{3}\right)$ are superimposed. Find the (a) resultant amplitude (b) the initial phase angle of the resultant and (c) the resultant equation of motion.

Solution

According to the superposition principle, the displacement of the resultant wave

$$\vec{y} = \vec{y}_1 + \vec{y}_2$$

$$\vec{y} = 5\sin\left(kx - \omega t + \frac{\pi}{2}\right) + 7\sin\left(kx - \omega t + \frac{\pi}{3}\right)$$

$$\vec{y} = 5\sin(kx - \omega t)\cos\frac{\pi}{2} + 5\cos(kx - \omega t)\sin\frac{\pi}{2}$$

$$+7\sin(kx - \omega t)\cos\frac{\pi}{3} + 7\cos(kx - \omega t)\sin\frac{\pi}{3}$$

$$\vec{y} = 5\cos(kx - \omega t) + \frac{7}{2}\sin(kx - \omega t) + 7\frac{\sqrt{3}}{2}\cos(kx - \omega t)$$

$$\vec{y} = \left(5 + \frac{7\sqrt{3}}{2}\right)\cos(kx - \omega t) + \frac{7}{2}\sin(kx - \omega t) \qquad \text{..... (1)}$$

Put $$5 + \frac{7\sqrt{3}}{2} = A\sin\theta \qquad \text{..... (2)}$$

and $$\frac{7}{2} = A\cos\theta \qquad \text{..... (3)}$$

Putting equations 2 and 3 in equation (1), we get

$$y = A\sin(kx - \omega t)\cos\theta + A\cos(kx - \omega t)\sin\theta$$

or $$y = A\sin(kx - \omega t + \theta) \qquad \text{..... (4)}$$

squaring and adding eqns 2 and 3, we get

$$\left(5 + \frac{7\sqrt{3}}{2}\right)^2 + \left(\frac{7}{2}\right)^2 = A^2$$

$\therefore$ $$A = 11.6$$

Dividing eqn 2 by 3, we get

$$\tan\theta = \frac{5+\frac{7\sqrt{3}}{2}}{\frac{7}{2}} = 3.16$$

$$\theta = 72^\circ \cdot 44^\circ = 72^\circ \cdot 26'$$

Substituting A and θ in eqn 4, we get the resultant equation

$$\vec{y} = 11.6(\sin kx - \omega t + 72^\circ \cdot 26')$$

Example 4

Two simple harmonic waves acting at right angles to each other have time periods in the ratio 1:2 show that the resultant curve is a parabola when the phase angle between the two waves is $\frac{\pi}{2}$.

Solution

Let the two simple harmonic waves be represented by

$$x = a_1 \sin\left[2(kx - \omega t) + \frac{\pi}{2}\right] \qquad \text{..... (1)}$$

and $$y = a_2 \sin(kx - \omega t) \qquad \text{..... (2)}$$

From eqn 1 $$x = a_1 \cos 2(kx - \omega t)$$

or $$\frac{x}{a_1} = \cos 2(kx - \omega t)$$

$$\frac{x}{a_1} = 1 - 2\sin^2(kx - \omega t) \qquad \text{..... (3)}$$

From eqn 2 $$\frac{y}{a_2} = \sin(kx - \omega t)$$

Using this eqn 3 becomes

$$\frac{x}{a_1} = 1 - 2\frac{y^2}{a_2^2}$$

$$\frac{y^2}{2a_2^2} = 1 - \frac{x}{a_1}$$

$$y^2 = \frac{a_2^2}{2}\left(1 - \frac{x}{a_1}\right)$$

$$y^2 = -\frac{a_2^2}{2}\left(\frac{x}{a_1} - 1\right)$$

$$y^2 = -\frac{a_2^2}{2a_1}(x - a_1)$$

This represents a parabola $y^2 \propto x$

Example 5

The sources of intensities I_1 and I_2 are superimposed so that the ratio of maximum to minimum intensity is found to be 25. Find I_1 / I_2.

Solution

$$\frac{I_{max}}{I_{min}} = 25 \text{ given}$$

we have $$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = 25$$

or $$\frac{a_1 + a_2}{a_1 - a_2} = 5 \qquad \text{or} \qquad \frac{a_1}{a_2} = \frac{3}{2}$$

$\therefore$ $$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{9}{4}$$

Interference

We have found that when two or more waves are superimposed the redistribution of light energy takes place. This modification in the distribution of light energy due to superposition of two or more waves is called interference. As a result of redistribution, at some points intensity of light will be maximum called constructive interference and at some other points intensity of light will be minimum called destructive interference. Though the resultant intensity at any point depends upon the amplitudes and phase relationship of the component waves it is not possible to produce interference patterns by taking two waves of different amplitudes and different phases. In order to produce sustained interference coherent sources of light must be used.

Two waves are said to be coherent if their phase difference is independent of *maintains a constant phase relation between themselves are coherent.* *time. Two sources emitting light of same frequency (wavelength), amplitude and* Two independent sources can never be coherent. Two coherent sources are to be derived from the same parent source. Two slits illuminated by a monochromatic source of light, a source of light and its reflected image, or two refracted images of the same source will act as two coherent sources. Two sources are said to be incoherent if their phase difference varies with time. Two sources derived from a light source emitting waves of various frequencies are incoherent. Thus necessary conditions for producing sustained interference are (i) two sources are coherent (ii) two sources should emit waves of the same wave length, and (iii) the amplitudes of the waves from the two sources should be equal.

If the amplitudes are not same the intensity of minimum will not be much different from that of maxima. Hence interference cannot be observed because the maxima and minima cannot be distinguished.

In addition to the following necessary conditions, following conditions must also be fulfilled.

(i) The separation between two sources must be small and the seperation between the sources and the screen must be suitable.

(ii) If the interfering waves are polarized then their state of polarization must be same.

(iii) The interfering waves must propagate along the same direction.

(iv) The two sources must be narrow, because a broad source is equivalent to a large number of point sources producing interferences between any two point sources lying side by side resulting in general illumination.

Theory of interference

We found that the superposition of two light waves results in the phenomenon of interference. A light is an electromagnetic wave which consists of time varying electric and magnetic fields vibrating at right angles to each other and also to the direction of the propagation of the wave. The electric field is represented by electric field vector $\vec{E}$ and the magnetic field vector is represented by the magnetic field vector $\vec{B}$.

Since the effect of electric field vector $\vec{E}$ dominates much over $\vec{B}\left(\dfrac{E}{c} = B\right)$ usually a light wave is represented by electric field vector $\vec{E}$.

For simplicity we assume a monochromatic plane wave moving along the z direction. The plane of polarisation of the wave is in the y-direction. i.e., Electric field vectors vibrate in the yz plane. Therefore our wave is plane polarised. Such a wave is mathematically represented as

$$E_y = E_o \sin(kz - \omega t + \delta) \qquad \text{..... (9)}$$

where E_o is the amplitude of the wave and δ is the initial phase.

Note : The real light waves emitted by common light sources are neither monochromatic nor plane. The common light sources emit continuous distribution of light waves. The sodium vapour lamp is nearly monochromatic since it emits two wavelengths 5893Å and 5896Å. Another one is red light emitted by helium-neon laser (6328Å).

Theory of interference

Consider two light waves

$$\vec{E}_A = E_1 \sin(kz - \omega t) \qquad \text{..... (10)}$$

$$\vec{E}_B = E_2 \sin(kz - \omega t + \delta) \qquad \text{..... (11)}$$

where δ is the phase difference between the two waves.

When these two waves are allowed to superimpose, the resultant wave is given by

$$\vec{E}_R = \vec{E}_A + \vec{E}_B$$

$$\vec{E}_R = E_1 \sin(kz - \omega t) + E_2 \sin(kz - \omega t + \delta)$$

$$\vec{E}_R = E_1 \sin(kz - \omega t) + E_2 \sin(kz - \omega t)\cos\delta + E_2 \cos(kz - \omega t)\sin\delta$$

$$\vec{E}_R = (E_1 + E_2 \cos\delta)\sin(kz - \omega t) + E_2 \cos(kz - \omega t)\sin\delta \qquad \text{..... (12)}$$

Put $$E_1 + E_2 \cos\delta = E\cos\phi \qquad \text{..... (13)}$$

$$E_2 \sin\delta = E\sin\phi \qquad \text{..... (14)}$$

Using this eqn 12 becomes

$$\vec{E}_R = E\cos\phi\sin(kz - \omega t) + E\sin\phi\cos(kz - \omega t)$$

$$\vec{E}_R = E\sin(kz - \omega t + \phi) \qquad \text{..... (15)}$$

It shows that E is the amplitude of the resultant wave.

To find E

Squaring and adding eqns 13 and 14, we get

$$E^2 = (E_1 + E_2\cos\delta)^2 + E_2^2 \sin^2\delta$$

$$E^2 = E_1^2 + 2E_1E_2\cos\delta + E_2^2\cos^2\delta + E_2^2\sin^2\delta$$

or $$E^2 = E_1^2 + E_2^2 + 2E_1E_2\cos\delta \qquad \text{.....(16)}$$

It shows that the square of the amplitude of the wave is not equal to the sum of the squares of amplitudes of superposing waves, there is an additional term $2E_1E_2\cos\delta$ called interference term. This term is responsible for the interference phenomenon.

Intensity distribution

The intensity of light wave is given by

$$I = \frac{1}{2}\varepsilon_o cE^2 \qquad \text{.....(17)}$$

so $$I \,\alpha\, E^2 \quad \text{i.e.} \quad I = kE^2$$

Using this in eqn 16, we get

$$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\delta \qquad \text{.....(18)}$$

when $\delta = 0$, I will be maximum

i.e. $$I_{max} = I_1 + I_2 + 2\sqrt{I_1I_2} \qquad \text{.....(19)}$$

when $\delta = \pi$, I will be minimum

i.e. $$I_{min} = I_1 + I_2 - 2\sqrt{I_1I_2} \qquad \text{.....(20)}$$

when $I_1 = I_2 = I_o$, from eqn 18 we have

$$I = 2I_o + 2I_o\cos\delta$$

$$I = 2I_o(1+\cos\delta)$$

$$I = 2I_o 2\cos^2\delta/2$$

$$I = 4I_o\cos^2\delta/2 \qquad \text{.....(21)}$$

$\therefore$ $I_{max} = 4I_o$ and $I_{min} = 0$.

Conditions of complete interference

We learned that interference is the result of superposition of two waves having same wavelength, amplitude and maintaining constant phase difference between two waves. When two waves having each of amplitude E_o are superimposed, the amplitude of the resultant wave is obtained from equation (16).

i.e., $$E = [E_o^2 + E_o^2 + 2E_oE_o\cos\delta]^{\frac{1}{2}}$$

where δ, the phase difference between the two waves

$$E = [2E_o^2 + 2E_o^2 \cos\delta]^{\frac{1}{2}} = [2E_o^2(1+\cos\delta)]^{\frac{1}{2}}$$

$$E = (2E_o^2 \cdot 2\cos^2\delta/2)^{\frac{1}{2}} = 2E_o \cos\delta/2$$

i.e., $E = 2E_o \cos\delta/2$

Since intensity of light I is proportional to square of the amplitude, we have

$$I \propto E^2 = 4E_o^2 \cos^2\delta/2 \quad \text{or} \quad I = 4I_o \cos^2\delta/2$$

Since δ varies with the position, in the region of interference the intensity varies from point to point according to $\cos^2\delta/2$

Condition for maximum intensity or constructive interference

When the phase difference between the waves reaching at any point is

$\delta = 2n\pi$, where $n = 0, 1, 2, 3....$, then the intensity at that point is maximum and is given by

$$I_{max} \propto 4E_o^2$$

or

$$I_{max} = k\,4E_o^2 \quad \text{or} \quad I_{max} = 4I_o$$

i.e., The condition for maximum intensity is that the phase difference between the two waves must be an integral multiple of 2π.

Condition in terms of path difference

We have

$$\text{Phase difference} = \frac{2\pi}{\lambda}\ \text{path difference}$$

$$\therefore\ \text{Path difference} = \frac{\lambda}{2\pi} \times \text{phase difference}$$

when phase difference $= 2n\pi$

$$\text{Path difference} = \frac{\lambda}{2\pi} \times 2n\pi = n\lambda$$

Thus the condition for maximum intensity is that the path difference between the two waves must be an integral multiple of wave length.

Condition for minimum intensity or destructive interference

When the phase difference between the two waves reaching at a point is

$\delta = (2n+1)\pi$, then the intensity at that point is minimum and is given by $I_{min} = 0$

i.e. The condition for minimum intensity is that the phase difference between the two waves must be an odd integral multiple of π.

In terms of path difference the condition for minimum intensity is that the path difference between the two waves must be an odd multiple of half wavelength.

Energy distribution

We have found that intensity at different points due to interference is given by

$$I = k4a^2 \cos^2 \delta/2 = 4I_o \cos^2 \delta/2$$

When a graph is drawn between δ on the horizontal axis and I along the vertical axis we get a graph as shown in figure.

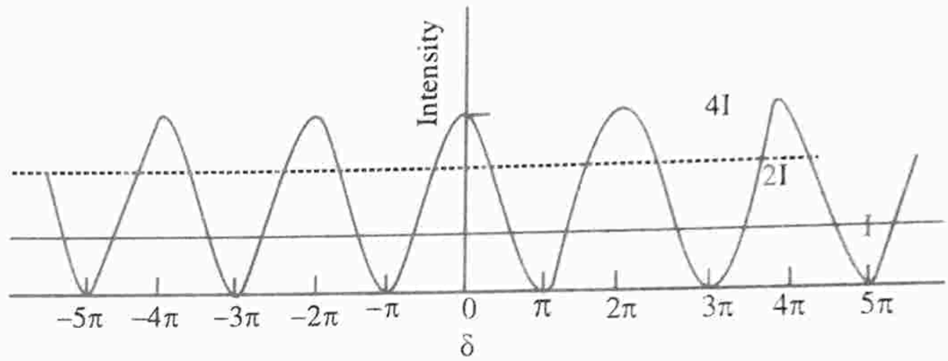
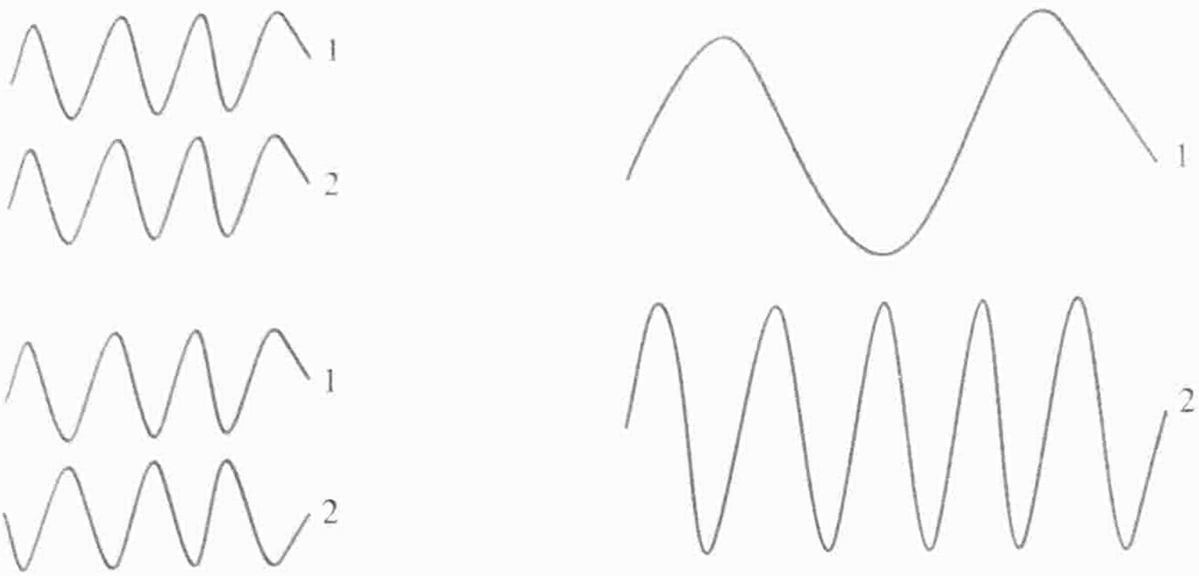


Figure 2.1

The curve is called intensity or energy distribution curve. From the intensity phase relationship we found that the intensity varies from a maximum value $4a^2$ to minimum value zero. Therefore the average intensity is proportional to $\frac{4a^2+0}{2} = 2a^2$. Before the superposition of the wave the total intensity of waves is proportional to $a^2 + a^2 = 2a^2$. This shows that the phenomenon of interference is in accordance with law of conservation of energy. It means that during interference energy is only transferred from the points of minimum intensity to the points of maximum intensity.

Something more about coherence

Light waves are said to be coherent if they are in phase with each other. For example if they maintain crest to crest and trough to trough correspondence as shown in figure. Two things are necessary for light waves to be coherent. Firstly, they must start with the same phase at the same position. Secondly their wave lengths must be the same or they will drift out of phase because the crests of the higher frequency will arrive ahead of the crests of the lower frequency wave.



(a) **Coherent waves** (b) **Incoherent waves**

Figure 2.2

The light that emerges from a conventional light source is a confused mixture of short waves which combine with each other in a random manner. The resultant light is incoherent and the wave front varies from point to point and changes from instant to instant. Coherence requires that there is a connection between the amplitude and phase of the light at one point and time, and the amplitude and phase of the light at another point and time. Accordingly we distinguish two classes of coherence namely temporal coherence and spatial coherence.

Temporal coherence refers to the constancy and predictability of phase as a function of time when the waves travel along the same path at slightly different times.

Spatial coherence refers to the phase relationship between waves travelling side by side at the same time but at some distance from one another.

Temporal coherence

The concept of temporal coherence can be easily understood with the help of the following example. Let us consider a single wave propagating along x-direction. Let us note the electric field at one point in space at two different times t_1 and t_2 as in figure above. Let the phase difference

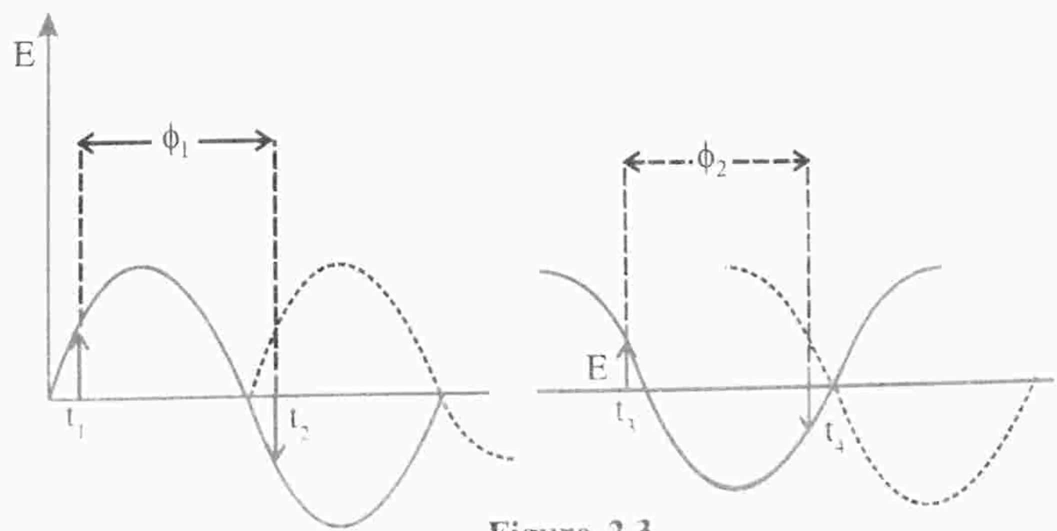


Figure 2.3

between the field E_1 at t_1 and the field E_2 at t_2 be ϕ_1. Let us again note electric fields at later times t_3 and t_4, where $(t_4 - t_3) = (t_2 - t_1)$. Let the phase difference be now ϕ_2.

If $\phi_1 = \phi_2$ and it is true for any time interval of same duration, then the wave is said to be temporally coherent. If the phase difference $\phi_2 \neq \phi_1$ and changes from interval to interval and in an irregular fashion, then the wave is said to be incoherent.

Temporal coherence is a characteristic of a single beam of light. We know that light is emitted by excited atoms of the source. An excited atom in the process of passing to a lower energy state gives up the excess energy. The transition lasts for a short time of 10^{-8}s. A light wave is emitted during this duration, which is not a continuous sine wave of infinite extension but is a wave packet. (See figure below)

The light from a source consists of such wave packets emanating from different atoms. Each wave packet has a sustained phase for only about 10^{-8}s after which there is a random phase. On an average, a light beam undergoes random phase changes about 10^8 times per second. Temporal coherence may be expected only for a short distance in space and for a brief time. In view of this temporal coherence is characterised by two parameters namely coherence length l_{coh} coherence time t_{coh}. Both the coherence length and coherence time measure how long light waves remain in phase as they travel in space. The coherence length depends on the central wave length λ and band width $\Delta\lambda$ of the wave packet.

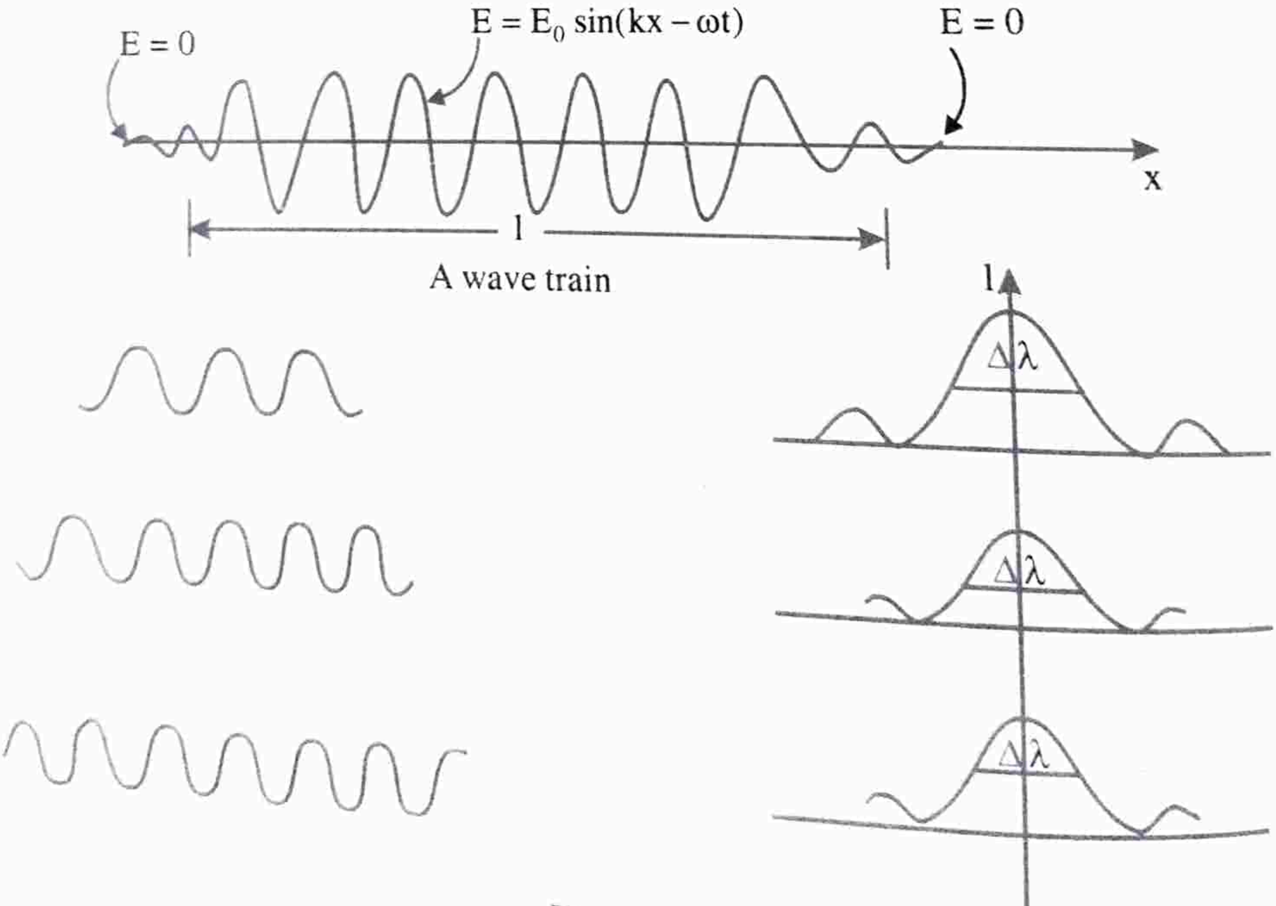


Figure 2.4

The coherence length is given by

$$l_{coh} = \frac{\lambda^2}{2\Delta\lambda} = \frac{c}{\Delta\upsilon}$$

For example light from a sodium vapour lamp has coherence length

$$l_{coh} = \frac{(5893\times10^{-10})^2}{2\times6\times10^{-10}} = 0.29\,\text{mm} \qquad \because \qquad \Delta\lambda = 6\text{Å}$$

The above discussion shows that temporal coherence is an effect due to the finite line width $\Delta\upsilon$ of the source. A strictly monochromatic wave $(\Delta\upsilon = 0)$ is an ideal harmonic wave of infinite extension and of infinite coherence length. A real light wave consists of a train of wave packets because of which its temporal coherence decreases. The broader the line width $\Delta\upsilon$, the shorter is the wave packet and its coherence length. In other words monochromaticity of a light beam is a measure of its temporal coherence.

Superposition of incoherent waves

Incoherent waves are waves which do not maintain a constant phase difference. Then the phase of the waves fluctuate irregularly with time and independently of each other. In case of light waves the phase fluctuates randomly at a rate about 10^8 per second. Out eye cannot detect this rapid changes. The detected intensity is always the average intensity, averaged over a time interval which is very much larger than the time of fluctuation. Thus

$$I_{av} = I_1 + I_2 + 2\sqrt{I_1 I_2} < \cos\delta >$$

or $$I_{av} = I_1 + I_2 \qquad \because \qquad < \cos\delta >= 0 .$$

Here the interference term is absent hence no interference.

Superposition of many coherent waves

When two coherent waves superimpose, we have

$$I_{max} = 4I_o = 2^2 I_o$$

If there are N coherent waves superimpose

$$I_{max} = N^2 I_o$$

and $$I_{min} = 0$$

Classification of interference phenomenon

In general there are two methods of obtaining coherent sources giving rise to two different classes of interference phenomenon, coherent sources can be produced from a single source either by division of wavefront or by division of amplitude.

(i) Division of wavefront

In this class the wavefront from a source is divided into two wavefronts either by reflections by mirrors or refraction through prisms. The two wavefronts thus formed travel unequal distances and finally brought together to produce interference. The devices used to obtain interference by division of wavefront are Youngs double slit, Fresnels two mirror, Fresnels biprism and Lloyd's mirror.

(ii) Division of amplitude

In this class the amplitude of the wave is divided into two either by partial reflection or refraction. The divided parts travel different paths and finally brought together to produce interference. This class of interference requires broad light sources. Newtons rings, Air wedge, Michelsons interferometer are the examples of division of amplitude devices.

Fresnels two mirror arrangement and biprism

Though Thomas Youngs double slit experiment proved the phenomenon of interference and the wave nature of light, this was questioned. The objection raised was that the bright fringes observed in Young's experiments were probably due to some complicated modification of light by the edges of the slits and not true interference. Few years later a French engineer Augustin Fresnel (1788-1827) devised several new experiments to support the validity of interference phenomenon exhibited by Young. Two of these experiments are Fresnels two mirror arrangement and Fresnels biprism.

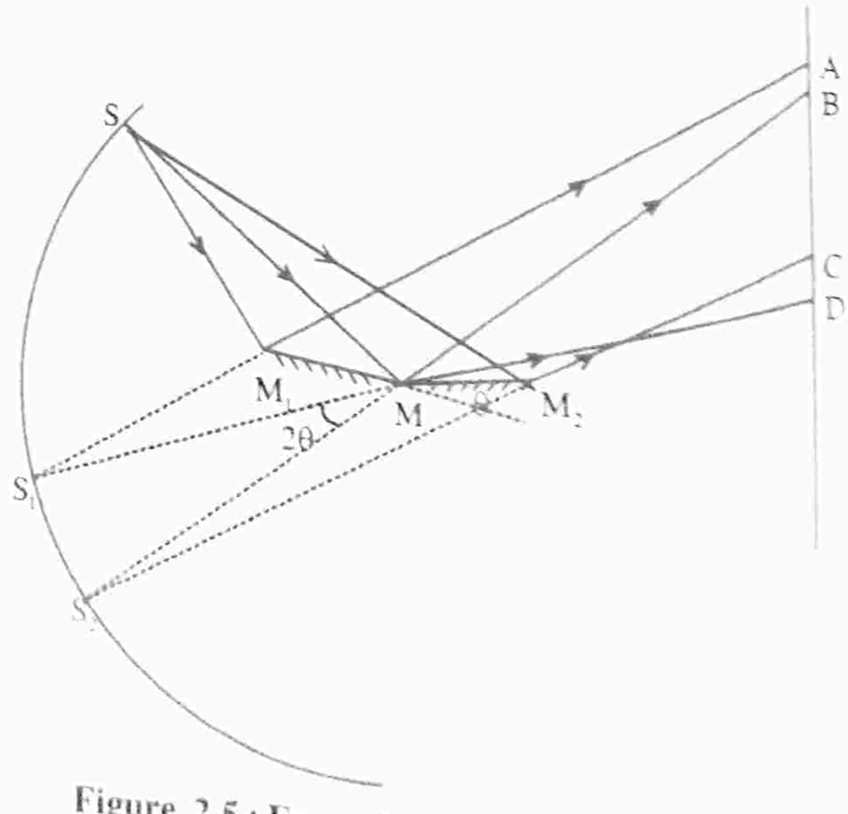


Figure 2.5 : Fresnels two mirror arrangement

Fresnels two mirror arrangement

It consists of two plane mirrors M_1M and MM_2 which are inclined to each other at a small angle θ and touching at the point M. S represents a narrow slit illuminated with monochromatic source placed perpendicular to the plane of the paper. A portion of the wave front from S gets reflected from M_1M and illuminates the region AD of the screen. Another portion of the wavefront gets reflected from the mirror MM_2 and illuminates the region BC of the screen. Since these two waves are derived from the same source they are coherent. Thus in the region BC, one observes interference fringes. The formation of images can also be understood as being due to the interference of the wavefronts from the virtual sources S_1 and S_2 of S formed by the mirrors MM_1 and MM_2 respectively. It can also be shown that S, S_1 and S_2 lie on a circle whose centre is at the point M. Since the angle between the two mirrors is θ, the angle S_1SS_2 is also θ and the angle S_1MS_2 is 2θ. Thus S_1S_2 is $2R\theta$, where R is the radius of the circle. (Arc = Radius × Angle).

Example 6

What must be the angle in degrees between the two Fresnel mirrors in order to produce fringes 1mm apart. If the slit is 40 cm from the mirror intersection and the screen is 1.5 m from the slit. Assume $\lambda = 5893\times10^{-10}$ m.

Solution

$$\beta = 1\text{mm} = 10^{-3}\text{m},\ R = 40\text{cm} = 0.40\text{m}$$

$$D = 1.5\text{m}$$

$$\therefore \quad d = 2R\theta$$

Using $$\beta = \frac{\lambda D}{d}$$

or $$d = \frac{\lambda D}{\beta}$$

$$\therefore \quad 2R\theta = \frac{\lambda D}{\beta}$$

$$\theta = \frac{\lambda D}{2R\beta} = \frac{5893\times10^{-10}\times1.5}{2\times0.4\times10^{-3}}$$

$$\theta = 11.05\times10^{-4} \text{ radian}$$

$$\theta = 11.05\times10^{-4}\times\frac{180}{\pi} = 0.063^\circ.$$

Fresnels biprism

This is another simple arrangement devised by Fresnel for the production of interference pattern by the principle of division of wavefront.

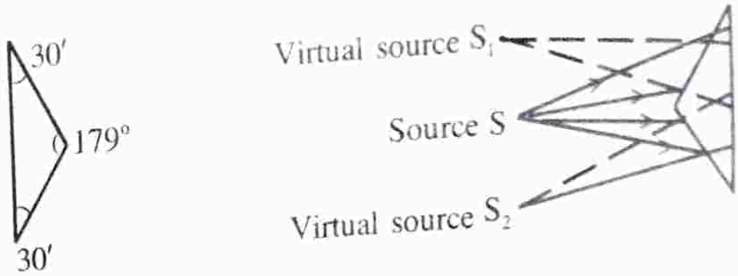


Figure 2.6 : Fresnels biprism

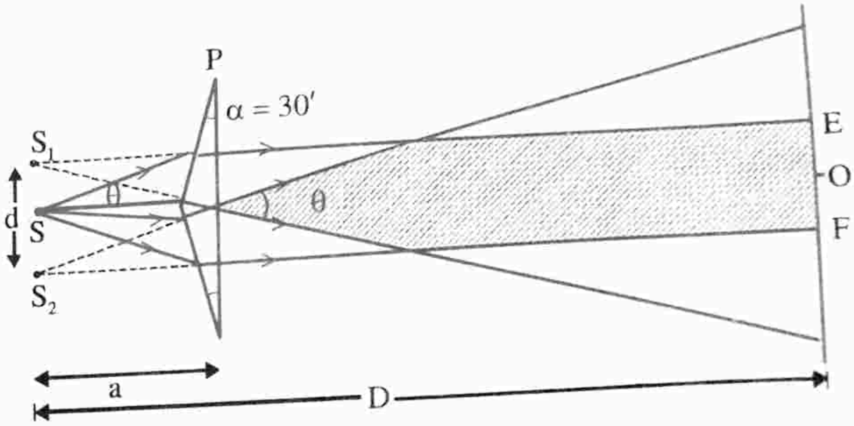


Figure 2.7

Fresnels biprism consists of two prisms of very small refracting angles (30′) placed base to base such that it acts as a single prism of obtuse angle of 179°. The experimental arrangement consists of a narrow vertical slit S illuminated by a monochromatic source of light. The light from S is made to fall symmetrically on the biprism P placed with its refracting edge parellel to the slit. When light falls on the biprism each half of it produces a virtual image of S by refraction. The distance between S and P is adjusted so that the two virtual images S_1 and S_2 can be made close together. The light from the two virtual sources S_1 and S_2 derived from the same source S act as coherent sources. The two cones of light diverging from S_1 and S_2 superimpose and the bright and dark interference fringes of equal width are obtained in the overlapping region (EF). The fringes can be observed on a screen kept at the overlapping region. In actual experiment the fringes are observed through an eyepiece.

If the distance between the sources S_1 and S_2 is d, D is the distance between the source and the screen and λ the wavelength of light then the fringe width (β) can be measured using $\beta = \frac{\lambda D}{d}$.

Determination of wavelength (λ)

Fresnels biprism can be used to determine the wavelength of a given source of monochromatic light using the equation $\lambda = \frac{\beta d}{D}$. To determine λ, the values of β, d and D are to be measured. The measurements are done as follows.

Experimental setup

The experimental arrangement consists of a long metal base fitted with four adjustable uprights called optic bench. The upright I carries a slit and the biprism is placed on the upright II. These uprights can be rotated in their own plane by means of rack and pinion arrangement so that the slit and the edge of the prism can be made exactly parallel. The upright II can also be moved horizontally perpendicular to the length of the bench. The upright III is used to carry a lens while measuring the distance between the two slits. The upright IV carries a micrometer eyepiece. The optical bench is provided with levelling screws.

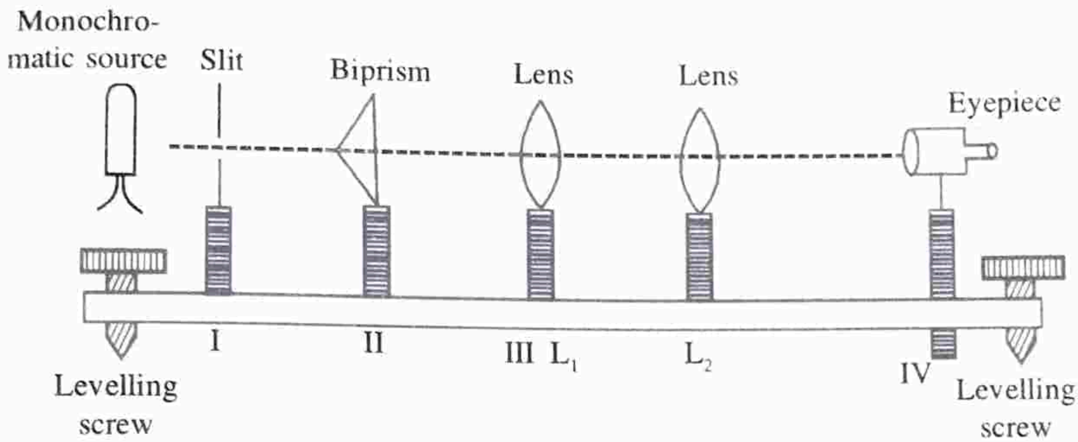


Figure 2.8

To start with the experiments the following adjustments are to be made.

(i) The optical bench is levelled horizontally with the levelling screws fitted in the bench.

(ii) The eyepiece is focussed on the cross wires and one of the wires is kept vertical.

(iii) The slit, eyepiece and the prism are adjusted to the same height.

(iv) The slit is illuminated by the light whose wavelength λ to be determined, then the slit is made vertical by viewing it through the eyepiece bringing close to the slit.

(v) When the slit is illuminated the interference fringes will be visible through the eyepiece. If the fringes are not seen, the biprism is rotated with help of the tangent screw so that the fringes are seen in the eyepiece. This will happen when the refracting edge of the biprism is parallel to the slit.

(vi) Adjust the slit and the edge of the biprism along the length of the bench. This adjustment is complete when on moving the eyepiece towards or away from the biprism the fringes donot shift laterally. This lateral shift can be removed by moving the biprism perpendicular to the bench with the help of the horizontal screw fitted on the upright II. Now the experimental set up is ready to take observations.

Measurement of β

To determine β, the vertical cross wire is made to coincide with the centre of one of the bright fringes. The reading of the micrometer is noted (x_0). Then the eyepiece is moved laterally slowly by counting a definite number (N) of bright fringes. Noted the reading of the micrometer again say (x_N). Then $\frac{x_N - x_0}{N}$ gives β.

Note : N could be 20, 25 or 30.

Measurement of D

The positions of the upright I carrying slit and the upright IV carrying eyepiece is noted from the scale of the bench. The difference between them gives the required value D.

Measurement of d

To measure the separation between the virtual sources d a convex lens of short focal length is placed on the up right III between the biprism and the eyepiece. Now the lens is brought close to the biprism at the position L_1 such that real images of S_1 and S_2 are formed in the field view of the eyepiece. The distance between these images is measured with the help of micrometer eyepiece. Let this be d_1. Now the lens is moved towards the eyepiece to the position L_2 such that the images of S_1 and S_2 are again obtained in the field view of the eyepiece. Again the distance between these images is measured. Let this distance be d_2. The two positions of the lens L_1 and L_2 are called the conjugate foci.

$\therefore$ We have $\frac{v}{u} = \frac{d_1}{d}$ for the position L_1 and $\frac{u}{v} = \frac{d_2}{d}$ for the position L_2. Multiplying the two equations, we get.

$$1 = \frac{d_1\, d_2}{d^2} \qquad \text{or} \qquad d = \sqrt{d_1\, d_2}$$

Using β, D and d, λ can be calculated

Measurement of d if angle of prism is known

Since the refracting angle α of the prism is very small the deviation produced is very small. The deviation produced is given by $\delta = (\mu - 1)\alpha$

where μ is the refractive index of the material of the prism. From the figure (2.7)

$$\frac{d}{a} = \theta \qquad \left(\frac{\text{Arc}}{\text{Radius}} = \text{Angle}\right)$$

But $\theta = 2\delta$

$\therefore$ $d = a2\delta$

or $d = 2a(\mu - 1)\alpha$,

where a is the distance between the slit and the prism.

Interference with white light

If white light is used to illuminate the slit in place of monochromatic light, then the central bright fringe is white because for that point, path difference is zero for all wavelengths from the two sources.

For different colours, the wavelengths are different and since μ depends on λ, the separation between the coherent sources $d = 2a(\mu - 1)\alpha$ is also different for different colours. As a result, in the field of view a central white fringe surrounded by a few coloured fringes on both sides are seen. The fringes of mixed colours are obtained due to overlapping of the maxima and minima of different colours at one place.

The outer edges of the bands are violet where as the inner edges are red. This is due to the fact that the fringe width for violet is minimum and maximum for red $\left(\beta \alpha \frac{\lambda}{\mu - 1}\right)$. Therefore, the first dark band of violet is obtained first and that of the red the last on either side of the central white. The inner edges of the first minimum of violet receives sufficient light intensity from red because maximum of red falls in its vicinity and the edge is reddish. Similary the first maximum of violet falls close to the inner edge of the minimum of red and hence the edge appears violet. For points at large distances from the centre, (after 8 or 10 frings) the maxima and minima due to large number of wavelengths overlap and a uniform illumination is obtained.

Example 7

Fresnels biprism of refractive index 1.5 has an angle of 1°. If the biprism is kept at a distance of 0.3 m from the slit illuminated by a monochromatic light of $\lambda = 6000\text{Å}$. Find the fringe width. D = 8m.

Solution

$$\mu = 1.5,\ \alpha = 1^\circ = \frac{\pi}{180}\ \text{rad},\ a = 0.3\text{m},$$

$$\lambda = 6000\text{Å} = 6 \times 10^{-7}\,\text{m}, \quad D = 8\text{m}$$

using $$d = 2a\,(\mu - 1)\alpha = 2 \times 0.3(1.5 - 1)\frac{\pi}{180}$$

$$d = \frac{\pi}{600}$$

$$\therefore \text{ Fringe width } \beta = \frac{\lambda D}{d} = \frac{6 \times 10^{-7} \times 8}{\pi} \times 600$$

$$= 9.17 \times 10^{-4}\,\text{m} = 0.917\,\text{mm}.$$

Example 8

A biprism of refractive index 1.5 is kept at a distance 0. 5 m from a slit illuminated by monochromatic light of wavelength $5890 Å$. If the distance between the successive fringes formed on the screen at a distance of 1 m from the biprism is $0.12 mm$. Find the vertex angle of the biprism.

Solution

$$\mu = 1.5, a = 0.5m, \lambda = 5890 Å = 5890 \times 10^{-10} m$$

$$D = 1 + 0.5 = 1.5m, \beta = 0.12 \text{ mm} = 0.12 \times 10^{-3} m$$

using $\beta = \frac{\lambda D}{d} = \frac{\lambda D}{2a(\mu - 1)\alpha}$

$$\alpha = \frac{\lambda D}{2a(\mu - 1)\beta}$$

$$\alpha = \frac{5890 \times 10^{-10} \times 1.5}{2 \times 0.5 \times (1.5 - 1) \times 0.12 \times 10^{-3}}$$

$$\alpha = 1.473 \times 10^{-2} \text{ rad}$$

$$\alpha = 1.473 \times 10^{-2} \times \frac{180}{\pi} = 0.8443^\circ$$

$\therefore$ The vertex angle of the biprism

$$= 180 - 2 \times 0.8441$$

$$= 178.312^\circ.$$

Example 9

A Fresnels biprism arrangement is set up with sodium light $(\lambda = 5893 Å)$ and in the field view of eyepiece we get 62 fringes. How many fringes shall we get if we replace the source by green light of wavelength $5461 Å$.

Solution

In biprism arrangement we have

$$\beta \propto \lambda$$

$$\therefore \quad \frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2}.$$

Let n_1 be the number of fringes contained in the field view of the eyepiece when light of wavelength λ_1 is used.

$\therefore$ Width of the field view $= n_1 \beta_1$ ..... (1)

n_2 be the number of fringes contained in the field view of the eyepiece when light of wave length λ_2 is used.

$\therefore$ Width of the field view $= n_2 \beta_2$ ...... (2)

Combining equations (1) and (2) we get

$$n_1 \beta_1 = n_2 \beta_2$$

$$\therefore \quad \frac{\beta_1}{\beta_2} = \frac{n_2}{n_1}$$

or
$$\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

$$\therefore \quad n_2 = \frac{\lambda_1}{\lambda_2} n_1 = \frac{5893 \times 10^{-10} \times 62}{5461 \times 10^{-10}}$$

$$n_2 = 67$$

Example 10

Interference fringes are produced by Fresnels biprism in the focal plane of a reading microscope which is 100 cm from the slit. A lens interposed between the biprism and the microscope given two images of the slit in two positions. If the images of the slit are 4.05 mm in one position 2.90 mm in the other position and the wavelength of sodium light is 5893Å. Find the fringe width.

Solution

$$D = 100\,\text{cm} = 1\text{m},\ d_1 = 4.05\text{ mm} = 4.05 \times 10^{-3}\text{ m}$$

$$d_2 = 2.90\,\text{mm} = 2.90 \times 10^{-3}\text{m},\ \lambda = 5893\text{Å} = 5893 \times 10^{-10}\text{m}$$

using $d = \sqrt{d_1 d_2} = \sqrt{2.90 \times 10^{-3} \times 4.05 \times 10^{-3}} = 3.427 \times 10^{-3}\text{m}$

$$\therefore \quad \text{Fringe width } \beta = \frac{\lambda D}{d} = \frac{5893 \times 10^{-10} \times 1}{3.427 \times 10^{-3}} = 1.72 \times 10^{-4}\text{ m}$$

$$\beta = 0.172\,\text{mm}.$$

Lateral displacement of fringes

The biprism experiment can be used to determine the thickness of a given thin sheet of transparent material such as glass or mica as follows.

Consider two virtual coherent monochromatic sources S_1 and S_2 separated by a distance d. The point O is equidistant from S_1 and S_2, where we obtain the central bright fringe.

$\therefore$ The optical path $S_1O = S_2O$.

Let a transparent plate G of thickness t and refractive index μ be introduced in the path of one of the beams as shown in figure below. the optical path lengths S_1O and S_2O are now not equal and the central bright fringe shifts to P from O.

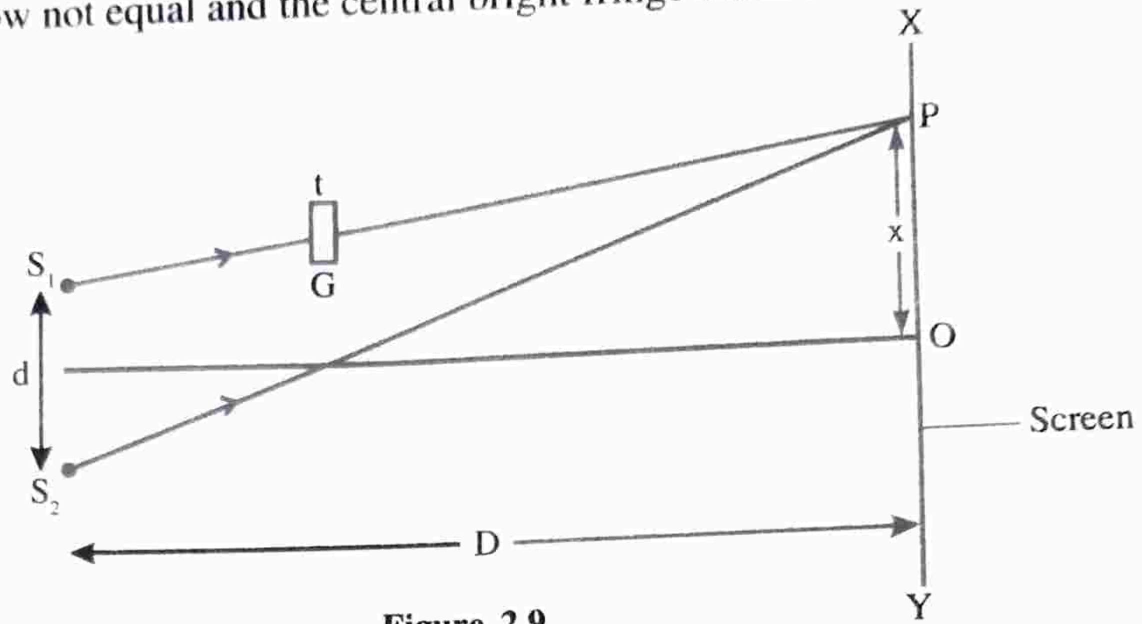


Figure 2.9

The light waves from S_1 to P, travel partly in air and partly through the sheet G. The distance travelled in air is $(S_1P - t)$ and that in the sheet is t.

$\therefore$ The optical path, $\Delta_{S_1P} = (S_1P - t) + \mu t$

$$\Delta_{S_1P} = S_1P + (\mu - 1)t$$

The optical path, $\Delta_{S_2P} = S_2P.$

$\therefore$ The optical path difference $= \Delta_{S_1P} - \Delta_{S_2P}$

since the central bright fringe is not at P, the optical path difference is zero.

i.e. $$\Delta_{S_1P} = \Delta_{S_2P}$$

or $$S_1P + (\mu - 1)t = S_2P$$

or $$S_2P - S_1P = (\mu - 1)t$$

we know that the path difference $= \dfrac{xd}{D}$ where x is the lateral shift of the fringe

Thus $$\frac{xd}{D} = (\mu - 1)t$$

$$\therefore \quad t = \frac{x\,d}{D(\mu - 1)}$$

Measuring x, d, D and knowing μ, the thickness of the sheet t can be calculated. (See Problem No. 24)

UNIVERSITY MODEL QUESTIONS

Section A

(*Answer questions in* **two** *or* **three** *sentences*)

Short answer type questions

1. State superposition principle.
2. What is meant by constructive superposition?
3. What is meant by destructive superposition?
4. Write down the expression for the resultant intensity of two waves and explain the symbols.
5. What are the conditions of obtaining constructive and destructive interference?
6. Define interference.
7. What are coherent sources? Give two examples.
8. What are incoherent sources? Give an example.
9. What are the necessary conditions for producing sustained interference?
10. Draw the intensity distribution curve of interference pattern.
11. Give one method of obtaining coherent sources.
12. Draw the diagram of Fresnel two mirror arrangement.
13. What is Fresnel's biprism? What is its use?
14. Define temporal and spatial coherence.
15. Write an expression for coherence length and explain the symbols.
16. Can two electric bulbs with point line filament of the same material each 15 watts and lying close to each other produce interference?
17. Why is the angle of Fresnel's biprism kept so small?
18. What becomes of the energy of light waves in destructive interference?

Section B

(*Answer questions in a* **paragraph** *of about half a page to one page*)

Paragraph / Problem type questions

1. Two sinusoidal waves of different amplitude, same frequency and same phase angle are superimposed, find the resultant amplitude.

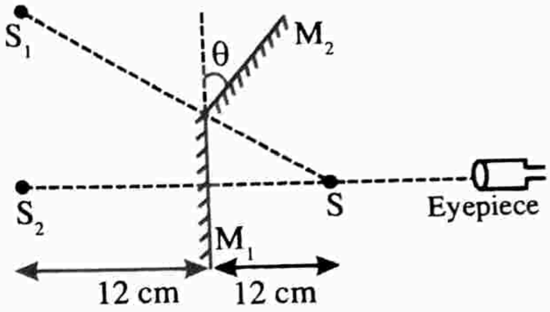
2. Two light waves of same intensities are superimposed, find the maximum and minimum intensities.
3. Show that the phenomenon of interference is in accordance with law of conservation of energy.
4. State and explain the phenomenon of interference.
5. Explain the temporal coherence.
6. Show that the coherence length of a light from a sodium vapour lamp is 0.029mm.
7. Explain Fresnel's two mirror arrangement.
8. Explain the interference with white light.
9. How will you determine the thickness of a given thin sheet of mica using biprism experiment?
10. How will you measure the separation between the two slits of Fresnel's biprism?
11. In Young's experiment, the interference pattern is found to have an intensity ratio between the bright and dark fringes as 9. What is the ratio of (a) intensities and (b) amplitudes of the two interfering waves. [a) 4 b) 2]
12. Find the maximum intensity in case of inference of n identical wave each of intensity I_0 if the interference is (a) coherent (b) incoherent. [a) n^2I_0 b) nI_0]
13. A biprism is kept at a distance of 5cm from slit illuminated by a monochromatic light of wavelength 5890Å. The width of the fringes formed on a screen at 75cm from the biprism is 9.424×10^{-2} cm. Find the separation between the coherent sources [$d = 0.5$ mm]
14. Interference fringes are observed with a biprism of refracting angle 1° and refractive index 1.5 on a screen 80 cm away from it. If the distance between the source and the biprism is 20 cm calculate the fringe width. $\lambda = 6900\text{Å}$. [2×10^{-4} m]
15. The distance between the slit and biprism and between the biprism and the screen are 50 cm each. The angle of biprism is 179° and its refractive index is 1.5. If the distance between successive fringes is 0.0135cm, calculate the wavelength of light used [5893×10^{-10} m]
16. In a biprism experiment at a certain position of the eyepiece, the fringe width obtained is 0.2 mm. When the eyepiece is moved away by 50cm, the fringe width becomes 3mm. If the distance between the two sources is 0.3cm, Find the wavelength of light used. [6×10^{-7} m]
17. Two waves travelling in the same line are given by $\vec{y}_1 = \sin\omega t$ and $y_2 = \cos\omega t$ are superimposed. Find the resultant wave [$\sqrt{2}\sin(\omega t + \frac{\pi}{4})$]
18. Two waves are represented by

$$\vec{y}_1 = 10\sin(3\pi t + \tfrac{\pi}{4}) \text{ and } \vec{y}_2 = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$$

Show that amplitudes are in the ratio 1:1

19. In a biprism experiment with sodium light $(\lambda = 5893Å)$ the micrometer reading is 2.32 mm when the eyepiece is placed at a distance of 100 cm from the source. If the distance between two virtual sources is 2 cm, find the new reading of micrometer when the eyepiece is moved such that 20 fringes cross the field of view
[2.91 or 1.73mm]

20. Two plane mirrors M_1 and M_2 are inclined to each other at angle θ and an illuminated slit S is placed infront of them at a distance 12 cm away from and parallel to the line of intersection of the mirror. An eyepiece is mounted at a distance 60 cm from the line of intersection of the mirrors. If the band width obtained is 0.16 mm and wavelength of light used is 5460Å. Find the value of θ.



$[\theta = 10.24 \times 10^{-3}\,\text{rad}]$

21. In Fresnel's biprism experiment, the fringe width of 0.185 mm is observed at a distance of 1m from the slit. The image of the coherent sources then produced at the same distance from the slit by placing a convex lens at 30 cm from the slit. The two images are separated by 0.7 cm. Calculate the wavelength of light used. [5550Å]

22. Interference fringed are produced in the focal plane of an eyepiece at a distance of 50 cm from the slit. The separation between the two images obtained for the two positions of the convex lens are 10.33 mm and 0.43 mm respectively. If the wavelengths of the light is 6300Å. Find the band width [0.0149 cm]

23. In a biprism experiment the cross wire of the eye piece is set at a position of the 8th dark band when the source used has a wavelength of 6360Å. When the source is changed the cross wire is found to be in the position of 8th bright band. Calculate the wavelength of the source [5962.5Å]

24. A thin plate of mica $(\mu = 1.58)$ is used to cover one slit of a double slit arrangement. The central point on the screen is occupied by the seventh fringe. If $\lambda = 5500Å$, calculate the thickness of the mica sheet. $[6.64 \times 10^{-6}\,\text{m}]$

25. In a biprism experiment the micrometer readings for the central bright and tenth order fringe are 2.37 mm and 3.55mm respectively. Wavelength of light used is 5890Å. Determine the position of the central and tenth order fringe when λ changes to 7500Å.
[no change, 0.387 cm]

Section C

(Answer questions in about **one** *or* **two** *pages)*

Long answer type questions (Essays)

1. Explain the superposition principle and obtain an expression for the resultant intensity of two waves. What are the conditions of maximum and minimum intensity.

2. Describe an experiment to determine the wave length of sodium light using Fresnel's biprism.

Hint to problems

11. a) $\frac{I_{max}}{I_{min}} = 9 = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$ gives $\frac{a_1}{a_2} = 2 \quad \therefore \quad \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = 4$

b) $\frac{a_1}{a_2} = 2$

12. a) In case of coherence interference at a given point, δ does not very with time

b) In case of incoherent interference at a given point, δ varies randomly with time, so $< \cos\delta >= 0$.

13. $d = \frac{\lambda D}{\beta}$, $D = 0.75 + 0.05 = 0.8m$

14. $\beta = \frac{\lambda D}{d}$ with $d = 2a(\mu - 1)\alpha$ where $D = 1m$.

15. $\lambda = \frac{\beta d}{D}$ with $d = 2a(\mu - 1)\alpha$, $\alpha = 30'$, $a = 0.5m$ and $D = 1m$.

16. $\beta_1 = \frac{\lambda D_1}{d}, \quad \beta_2 = \frac{\lambda D_2}{d}$

$\therefore \quad \beta_2 - \beta_1 = \frac{\lambda}{d}(D_2 - D_1) \qquad$ or $\quad \lambda = \frac{(\beta_2 - \beta_1)d}{(D_2 - D_1)}$, $D_2 - D_1 = 0.5m$

17. $y_1 + y_2 = \sin\omega t + \cos\omega t$

$$y_1 + y_2 = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\omega t + \frac{1}{\sqrt{2}}\cos\omega t\right)$$

$$= \sqrt{2}\left(\cos\frac{\pi}{4}\sin\omega t + \sin\frac{\pi}{4}\cos\omega t\right)$$

$$= \sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right)$$

18. $y_1 = 10\sin\left(3\pi t + \frac{\pi}{4}\right)$. The amplitude is 10

$y_2 = 5\sin 3\pi t + 5\sqrt{3}\cos 3\pi t$

$\therefore$ Amplitude $= \sqrt{5^2 + (5\sqrt{3})^2 + 2\times5\times5\sqrt{3}\cos 90} = 10$

19. $\beta = \dfrac{\lambda D}{d} = \dfrac{5893\times10^{-10}\times1}{2\times10^{-2}} = 2946.5\times10^{-8}$

$= 0.02946\text{mm}$

movement of eyepiece $= 0.02946 \times 20$

$\therefore$ Final reading $=$ Initial reading $\pm\ 0.5892$

$= 2.32 \pm 0.5892$

20. See example 6.

21. Let u and v be the distances of the virtual sources and their images from the convex lens. Let d be the separation of sources and d_1 be the separation of images using $\dfrac{u}{v} = \dfrac{d}{d_1}$

or $d = \dfrac{u}{v}d_1 = \dfrac{0.3}{0.7}\times0.007$

$\therefore \quad \lambda = \dfrac{\beta d}{D} = \dfrac{0.185\times10^{-3}\times0.003}{1}$

$= 5550\text{Å}$

22. $d = \sqrt{d_1 d_2} \quad d_1 = 10.33\times10^{-3}\text{m}, \quad d_2 = 0.043\times10^{-3}\text{m}.$

$\beta = \dfrac{\lambda D}{d} \quad \lambda = 6300\times10^{-10}\text{m}, \quad D = 0.5\text{m}$

23. The distance between the centre and the cross wire.

$7.5\beta_1 = 8\beta_2 \qquad 7.5\dfrac{\lambda_1 D}{d} = 8\dfrac{\lambda_2 D}{d} \qquad \lambda_2 = \dfrac{7.5}{8}\lambda_1$

24. The shift of the central fringe.

$x = \dfrac{\beta}{\lambda}(\mu - 1)t, \ x = 7\beta$

25. No change will occur in the position of the central fringe since it corresponds to zero path difference

$\beta = \dfrac{3.55 - 2.37}{10} = \dfrac{1.18}{10}\text{mm} = 0.118\text{mm}.$

$$\beta = \frac{\lambda D}{d} \qquad ..... (1)$$

$$\beta' = \frac{\lambda' D}{d} \qquad ..... (2)$$

$\therefore$ $\beta' = .15\text{mm}, \; 10\beta' = 1.5\,\text{mm}$

$\therefore$ position of tenth order fringe = 2.37 + 1.5 = 3.87mm

IMPORTANT FORMULAE

1. Two sinusoidal waves having same frequency but amplitudes a_1 and a_2 with phase angles ϕ_1 and ϕ_2 are superimposed.

The resultant amplitude, $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos(\phi_1 - \phi_2)}$

The phase angle of the resultant wave, $\tan\phi = \dfrac{a_1 \sin\phi_1 + a_2 \sin\phi_2}{a_1 \cos\phi_1 + a_2 \cos\phi_2}$

when $\phi_1 - \phi_2 = 2n\pi$, n = 0, 1, 2, ...

$$A_{max} = a_1 + a_2$$

when $\phi_1 - \phi_2 = (2n+1)\pi$, n = 0, 1, 2, ...

$$A_{min} = a_1 - a_2$$

Intensity, $I \propto a^2$,

$$\therefore \quad I = I_1 + I_2 + 2\sqrt{I_1 - I_2}\cos(\phi_1 - \phi_2)$$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

$$\frac{I_{max}}{I_{1min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

2. Intensity of light wave, $I = \frac{1}{2}\varepsilon_0 c E^2$

3. Relation between phase difference and path difference:

Phase difference = $\dfrac{2\pi}{\lambda}$ path difference.

4. Coherence length: $l_{coh} = \frac{\lambda^2}{2\Delta\lambda} = \frac{c}{\Delta\nu}$

5. Angle between two Fresnel mirrors: $\theta = \frac{\lambda D}{2R\beta}$

6. Fringe width: $\beta = \frac{\lambda D}{d}$ in air

   $\beta = \frac{\lambda D}{\mu d}$ in a medium.

   The distance of the nth fringe from the central fringe, $x = \frac{n\lambda D}{d}$

7. Separation between the virtual sources in Fresnel's biprism: $d = 2a(\mu - 1)\alpha$

8. When a transparent a sheet of thickness 't' is kept in the path of one of the two interfering beams, the lateral shift produced:

   $$x = \frac{(\mu - 1)tD}{d}$$

   Thickness of the sheet, $t = \frac{xd}{D(\mu - 1)}$

   path difference produced = $(\mu - 1)t$