4 PARTICLE ACCELERATORS

Introduction

Particle accelerators are devices by which the charged particles can be energised. The accelerators played a vital role in the development of nuclear and particle physics. Before the advent of this machines, the only sources of high energy charged particles required for the study of nuclear transmutation were the naturally radio active substances emitting α and β particles. It is due to the limitations both of energy and intensity of the beam of particles the usefulness of these sources are limited. It was J.D. Cockcroft and E.T. S Walton, students of Rutherford, constructed a particle accelerator for the first time which would accelerate protons to high enough energy to produce nuclear transmutation.

Cockcroft - Walton proton accelerator

In 1932 Cockcroft and Walton designed and operated a proton accelerator at Cambridge. The basic principle used in this was that of a voltage doubler. The experimental arrangement is shown in figure below. It consists of a voltage doubler circuit, an accelerator to be provided with proton source.

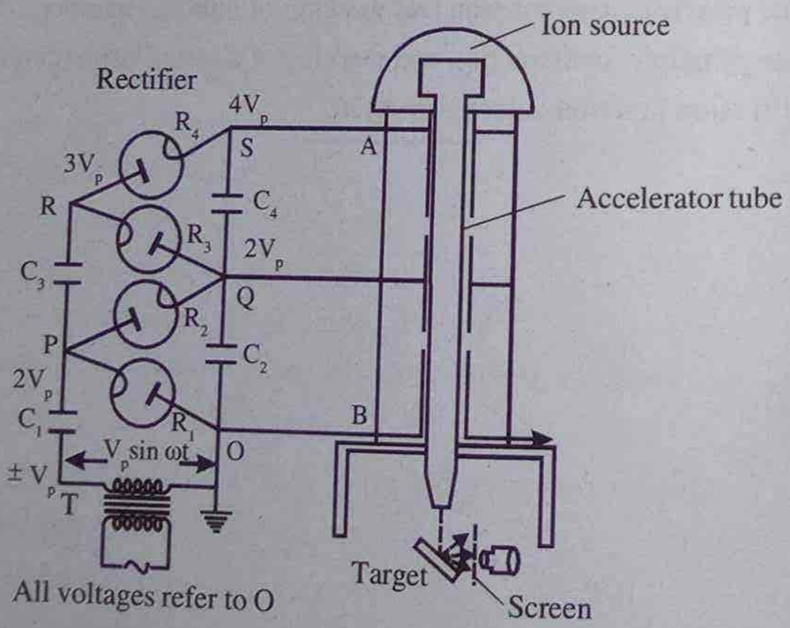


Figure 4.1: Schematic diagram of Cockcroft-Walton accelerator and discharge tube

Voltage doubler circuit

It is an electric circuit that generates a high voltage from low voltage. This voltage is used to power the accelerator. It is made up of a voltage multiplier ladder network of capacitors and rectifiers to generate voltage.

To understand the circuit operation, the circuit is powered by an alternating voltage V_i with a peak value of V_p and initially the capacitors are uncharged.

Switch on the input voltage V_i. When the input voltage V_i reaches its negative peak ($-V_p$) current flows through the rectifier R_1 to charge the capacitor to a voltage V_p. When V_i reverses polarity and reaches its peak value (V_p), it adds to the capacitor voltage to produce a voltage of $2V_p$, the upper plate of C_1. Since R_1 is reverse biased current flows from C_1 through the rectifier R_2, charging the capacitor C_2 to a voltage of $2V_p$. When V_i reverses polarity again current from C_2 flows through the rectifier R_3, charging the capacitor also to voltage of $2V_p$. When V_i reverses polarity again current flows from C_3 through the rectifier R_4, charging capacitor to a voltage of $2V_p$.

All the capacitors are charged to a voltage of $2V_p$ except C_1 which is charged to V_p. The key to the voltage doubler is that while the capacitors are charged in parallel, they are connected to the load in series. Since C_2 and C_4 are in series, between the output and ground, total output voltage is $4V_p$.

This circuit can be extended to any number of stages. The no load output voltage is twice the peak input voltage multiplied by the number of stages.

i.e., $$V_0 = 2NV_p = NV_{pp}$$

where V_{pp} is the peak to peak input voltage.

Cockcroft and Walton accelerator accelerated protons upto a voltage of 700 KeV. This accelerator performed the first nuclear disintegration by artificial means. They bombarded accelerated protons with lithium producing the reaction.

$$^1_1p + ^7_3Li \rightarrow ^4_2He + ^4_2He$$

This was the first experiment to show that one element (Lithium) could be artificially transformed into another element. The Cockcroft-Walton accelerators are still widely used today. sometimes as injectors to much larger accelerators.

Note: Since the C.W accelerator produces only a low voltage proton reactions were limited to light elements like lithium, boron, beryllium etc.

Van de Graaff generator

A Van de Graaff generator is a device used for building up high potential differences of the order of several million volts. Such high potential differences are used to accelerate charged particles needed for various experiments of nuclear physics. It was designed by Robert J Van de Graaff in 1929.

Principle

This generator is based on

i) The action of sharp points

ii) The property that charge given to a hollow conductor is transferred to outer surface and is distributed uniformly over it.

Construction

It consists of a large spherical conducting shell of radius of few metres. It is mounted on an insulating pillar of several metres high. An insulating belt is wound around the pullies P_1 and P_2. The belt is kept moving continuously over the pulley's with the help of a motor. B_1 and B_2 are two sharply pointed combs fixed as shown. B_1 is called spray comb and B_2 is called the collecting comb.

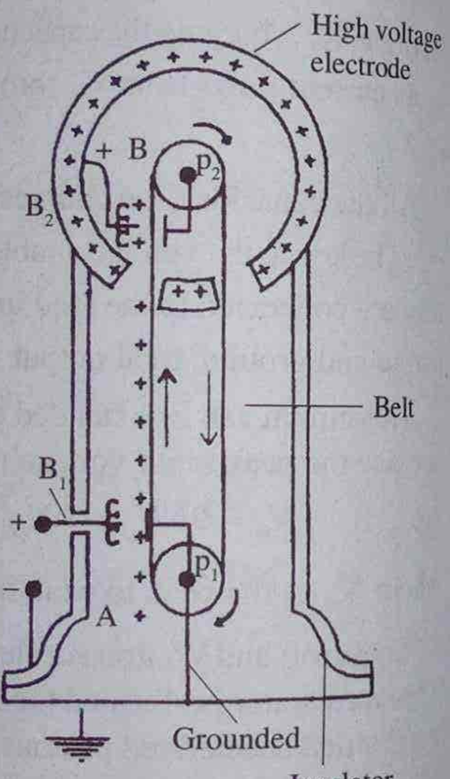


Figure 4.2

Working

The spray comb is given a positive potential (-10^4 volts) with respect to earth by high tension source (H.T.). It is due to discharging action of sharp points a positively charged wind is set up, which sprays positive charge on the belt. As the belt moves and reaches the sphere a negative charge is induced on the sharp end of collecting comb B_1 and an equal positive charge is induced on the farther end of B_2. The positive charge shift immediately to the outer surface of sphere. Due to discharging action of sharp points of B_2 a negatively charged wind is set up. This neutralises the positive charge on the belt. The uncharged belt returns down and collects the positive charge from B_1 which in turn is collected by B_2. This is repeated. Thus the positive charge on the sphere goes on increasing. As $V = \frac{q}{C}$ and capacity of the spherical shell is constant, potential V of the shell goes on increasing till it reaches the desired value. With increase in V beyond a certain value leakage of charge to the surrounding occurs. This sets a limit on the maximum potential that can be achieved. The leakage is minimised by keeping the generator inside a steel chamber filled with nitrogen or methane at high pressure.

Linear accelerator (LINAC)

Linear accelerators (also called linac) accelerate charged particles along a straight line in multiple steps by an oscillating electric field. The first working linear accelerator was built by R. Wideroe in 1928 used only three coaxial cylindrical electrodes and could accelerate K^+ and Na^+ ions to twice the energy available for a single traversal of the field. In 1931, E.O. Lawrence and D.H. Sloan at the university of California Berkeley built an accelerator using 10 accelerating electrodes to produce 1.25 MeV mercury ions. Since then a large number of linear accelerators both for atomic ions and electrons have been built in different parts of the world. Here we discuss a simple form of linear accelerator.

Construction

It consists of a series of coaxial cylindrical drift tubes along the axis of which the charged particles travel. One set of alternate electrodes is connected to one terminal of the rf supply system while the other set of alternate electrodes is connected to the other terminal of the rf supply. In a linear accelerator for ions, the successive electrode tubes are of gradually increasing lengths. The cylindrical drift tubes are arranged inside a glass vacuum chamber. An ion source which is to be a accelerated is kept at A.

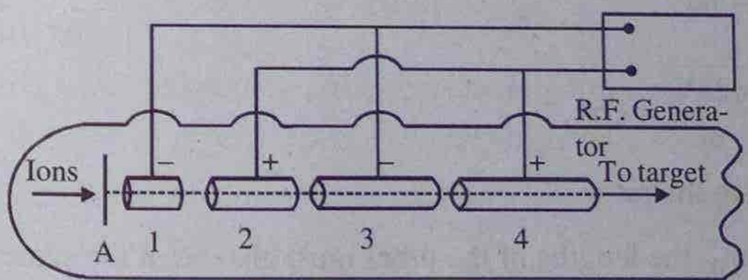


Figure 4.3

Principle of working

Suppose that the first drift tube is negative when the positive ion just leaves from A. On entering the tube the positive ion gets accelerated. The length of the first tube is adjusted in such way that the time taken by the positive ion to travel the entire length of the tube is exactly equal to the half the time period of the rf accelerating voltage . When this condition is met the second drift tube becomes negative when the positive ion just leaves the first drift tube. As a result the positive ion again gets accelerated on entering the second tube. As the ions travel down the tube they do not gain any energy because of the screening action of the tube. i.e., ions get accelerated while crossing the gap.

If V be the potential of the first drift tube with respect to A, then the velocity v_1 of the ion on reacting the first drift tube is given by.

$$\frac{1}{2}mv_1^2 = qV$$

or
$$v_1 = \sqrt{\frac{2qV}{m}}$$

where q is the charge of the ion and m is its mass.

While crossing the second gap between first drift tube and the second drift tube ions gain an additional amount of energy qV. If v_2 be the velocity of the ion on entering the second drift tube, then

$$\frac{1}{2}mv_2^2 = 2qV$$

or
$$v_2 = \sqrt{\frac{4qV}{m}} = \sqrt{2}v_1$$

The process continues and when the ion crosses the n^{th} gap, we have

$$\frac{1}{2}mv_n^2 = nqV$$

or
$$v_n = \sqrt{\frac{n2qV}{m}} = \sqrt{n}v_1$$

since velocities are in the ratio

$1:\sqrt{2}:\sqrt{3}:.......:\sqrt{n}$ the lengths of the tubes must also be in the same ratio.

It may also be noted that the final energy of the ions depends on the total number of gaps and the energy gained in each step.

Advantages of linear accelerators

(i) It produces a relatively high beam current

(ii) Extraction and injection of the high energetic beam is simple.

(iii) Since they travel in straight line, it has practically no radiation loss.

The limitation of linear accelerator is that the length of the accelerator is inconveniently large and it is very difficult to maintain vacuum in such a large chamber.

Linear accelerators are used for electron acceleration, proton acceleration, ion acceleration, etc. They are used as injectors for high energy machines such as proton synchrotrons.

Cyclotron

A cyclotron is a device developed by Lawrence and Living Stone in 1931 by which the positively charged particles like proton, deuteron, α particle etc. can be accelerated.

Principle

A charged particle moving perpendicular in a magnetic field, it describes a circular path. It can be accelerated by allowing it to pass through a suitably adjusted electric field.

Construction

It consists of two D-shaped hollow evacuated metal chambers D_1 and D_2 called dees. These dees are placed horizontally with their diametric edges parallel and slightly separated from each other. The dees are connected to high frequency oscillator which can produce a potential difference of the order 10^4 V at $\approx 10^7$ Hz frequency. The two dees are enclosed in a steel box and well insulated from it. The box is placed in a strong magnetic field produced by two pole pieces of strong electromagnet NS. The magnetic field is perpendicular to the plane of the dees. At P the positively charged particles which is to be accelerated is kept.

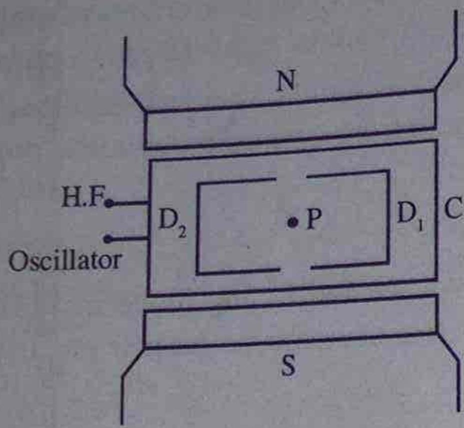
Working and theory

The positively charged particle to be accelerated is produced at P. Suppose at that instant D_1 is at negative potential and D_2 is at positive potential. Therefore the particle will be accelerated towards D_1. On reaching inside D_1 it moves with a constant speed v in a circular path of radius r.

$$\therefore \quad Bqv = \frac{mv^2}{r} \qquad \text{or} \quad r = \frac{mv}{qB}$$

Time taken by the particle to describe a semi circular path is given by

$$t = \frac{\pi r}{v} = \frac{m\pi}{qB}$$



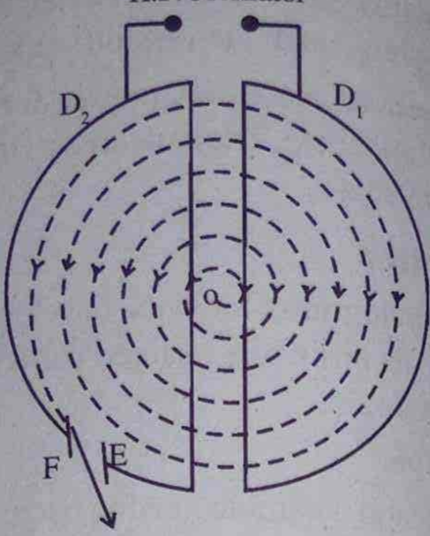


Figure 4.4

This time is independent of both the speed of the ion and radius of the circular path. In case the time during which the particle describes a semicircular path is equal to the time during which half cycle of oscillator is completed, then as the particle arrives in the gap between the dees, the polarity of the dees is reversed i.e., D_1 becomes positive and D_2 negative. Therefore particle is accelerated towards D_2 and it enters D_2 with greater speed which remains constant in D_2. The particle describe a semicircular path of greater radius. The process repeats and the particle goes on accelerating every time and acquires high energy. The accelerated particle can be removed out of the dees by applying the electric field across the deflecting plates E and F.

Maximum energy of the particle

Let v_0 be the maximum velocity and r_0 the radius of the circular path.

Then $$\frac{mv_0^2}{r_0} = qBv_0$$

or $$v_0 = \frac{qBr_0}{m}$$

$\therefore$ Maximum $$K.E = \frac{1}{2}mv_0^2 = B^2q^2\frac{r_0^2}{2m}$$

Cyclotron Frequency

If T is the time period of oscillating electric field then

$$T = 2t = \frac{2\pi m}{qB}$$

the cyclotron frequency is given by

$$\upsilon = \frac{1}{T} = \frac{qB}{2\pi m}$$

One limitation of cyclotron is that, it cannot accelerate a charged particle beyond a limit. This is because when the velocity of a charged particle increases its mass varies according to the equation $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$. As a result the resonance frequency also varies. This happens when v is comparable with c. The particles take longer time to describe semicircle. And it may become out of step with the accelerating voltage. That is instead of accelerating it may be retarded.

Note : Lawrence and living stone won the Nobel prize in 1934 for the construction of cyclotron.

Example 1

A cycloron is connected to the oscillator of frequency 15 MHz. What should be the operating magnetic field for accelerating protons. The radius of the dees is 60cm. Calculate the maximum kinetic energy of the proton in eV, the mass of the proton $= 1.67 \times 10^{-27}$ kg. $1 eV = 1.6 \times 10^{-19}$ J

Solution

$$\upsilon = 15 MHz = 15 \times 10^6 Hz$$

$$m = 1.67 \times 10^{-27} kg . q = 1.6 \times 10^{-19} C$$

we have $\upsilon = \frac{qB}{2\pi m}$

or $B = \frac{2\pi m \upsilon}{q}$

$$B = \frac{2 \times 3.14 \times 1.67 \times 10^{-27} \times 15 \times 10^6}{1.6 \times 10^{-19}} = 0.99 T$$

Maximum $K.E = \frac{B^2 q^2 r_0^2}{2m}$

$$= \frac{(0.99)^2 \times (1.6\times10^{-19})^2 \times (0.6)^2}{2\times1.67\times10^{-27}\times1.6\times10^{-19}} eV$$

$$= 16.9\times10^6 eV = 16.9 MeV$$

Example 2

An electron after being accelerated through a p d of 100V enters a uniform magnetic field of 0.004 T perpendicular to its direction. Calculate the radius of the path described by the electron.

Solution

V = 100 volt, B = 0.004 T

Let v be the velocity acquired by electron when accelerated under a p d, we have

$$\frac{1}{2}mv^2 = eV$$

or $$v = \sqrt{\frac{2eV}{m}}$$

we have $$r = \frac{mv}{qB} = \frac{\sqrt{2meV}}{qB}$$

$$= \frac{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times100}}{1.6\times10^{-19}\times0.004}$$

$$= 8.4\times10^{-3} m$$

Example 3

The electric field in a cyclotron is reversed every 9.372×10^{-8} s. It is used to accelerate deuterons, each of mass 3.34×10^{-27} kg and charge 1.6×10^{-19} C. Calculate the flux density of the magnetic field.

Solution

Time period of the electric field, $T = 2\times9.372\times10^{-8}$

$$= 18.744\times10^{-8} s$$

Using $$T = \frac{2\pi m}{qB}$$

$$\therefore \quad B = \frac{2\pi m}{qT} = \frac{2\times3.14\times3.34\times10^{-27}}{1.6\times10^{-19}\times18.744\times10^{-8}}$$

$$= 0.6993\,Wb/m^2$$

Example 4

Deuterons are accelerated in a fixed frequency cyclotron to a maximum dee orbit radius of 0.88m. The magnetic flux density across the dees has a mean value of 1.4T. Calculate energy of the emerging beam and the frequency of the dee voltage. $m = 3.34\times10^{-27}\,kg$

Solution

$q = 1.6\times10^{-19}\,C$, $r_o = 0.88m$, $B = 1.4T$

$$m = 3.34\times10^{-27}\,kg$$

Energy of the emergent beam

$$= \frac{B^2q^2r_0^2}{2m} = \frac{1.4^2\times(1.6\times10^{-19})^2\times0.88^2}{2\times3.34\times10^{-27}} = 5.816\times10^{-12}\,J$$

Frequency of the voltage, $\upsilon = \frac{qB}{2\pi m}$

$$\upsilon = \frac{1.6\times10^{-19}\times1.4}{2\times3.14\times3.34\times10^{-27}} = 1.067\times10^7\,Hz$$

$$= 10.67MHz.$$

Example 5

In a linear accelerator, proton accelerated thrice by a potential of 40kV leaves a tube and enters an accelerating space of length 30cm before entering the next tube. Calculate the frequency of the r.f. voltage

Solution

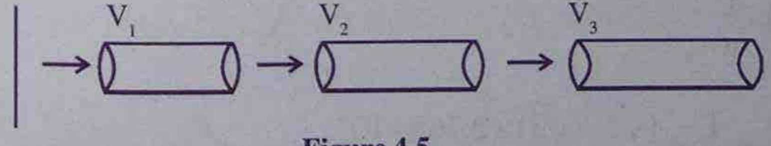


Figure 4.5

Let v_1 be the velocity of the proton on reaching the first drift tube, given by

$$\frac{1}{2}mv_1^2 = eV$$

or $$v_1 = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times40\times10^{3}}{1.67\times10^{-27}}} = 2.768\times10^{6}\,ms^{-1}$$

while crossing the second gap between first drift tube and the second drift tube protons gain an additional amount of energy eV. If v_2 be the velocity of the protons entering the second drift tube, then

$$\frac{1}{2}mv_2^2 = 2eV$$

$$v_2 = \sqrt{2}\,v_1$$

Similarly on entering the third tube v_3 be the velocity which is given by

$$v_3 = \sqrt{3}\,v_1$$

Time taken to cover the third gap (0.30m)

$$t = \frac{\text{dis tan ce}}{\text{average velocity}}$$

$$= \frac{0.3}{\left(\frac{v_2+v_3}{2}\right)} = \frac{0.6}{v_2+v_3} = \frac{0.6}{\sqrt{2}\,v_1+\sqrt{3}\,v_1}$$

$$t = \frac{0.6}{(\sqrt{2}+\sqrt{3})v_1} = \frac{0.6}{(\sqrt{2}+\sqrt{3})\times2.768\times10^{6}}$$

This time is equal to the half the period $\left(\frac{T}{2}\right)$ of the r.f voltage

i.e., $$t = \frac{T}{2}$$

$$\frac{1}{T} = \frac{1}{2t} = \frac{(\sqrt{2}+\sqrt{3})\times2.768\times10^{6}}{2\times0.6}$$

$$= 7.257\times10^{6}\,Hz$$

Synchrocyclotron

In the case of cyclotron the expression we used for the kinetic energy was the non-relativistic value $\frac{1}{2}mv^2$. For the early cyclotrons, working at low velocities, the non-relativistic value was enough. But for higher speeds relativistic effects will come into play.

For example if $v = 0.8c$, $\frac{v^2}{c^2} = 0.64$. So the mass changes according to the relation

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}} = \frac{m_0}{\sqrt{1-0.64}} = \frac{m_0}{0.6} = 1.66m_0$$

In this situation the frequency of oscillation $\upsilon = \frac{qB}{2\pi m}$ will change. As m increases, υ decreases. This will upset the resonance condition. Therefore gradually the particle gets out of phase with the frequency potential on the dees. The frequency on the dees must therefore be compensated for the gain in mass, this is carried out by a rotating variable condenser giving the imposed frequency modulation required. Ions can then be accelerated to very high velocities, and the cyclotron becomes a synchrocyclotron. The synchrocyclotron in the Lawrence radiation laboratory produced 380 MeV alpha particles and was later redesigned to give 720 MeV protons.

The difference between the cyclotron and the synchrotron is that in the former the output is continuous but in the case of latter it is in bursts lasting about 100 microseconds.

Electron accelerating machines

The betatron

The device cyclotron can accelerate protons, deutrons, α – particles to very high energies of the order of several million electron volts. It doesn't require excessive high voltage for producing high energy particles. But as stated earlier it has a very serious limitation arising from relativistic variation of mass with energy for very high energy particles. When the relativistic variation of mass comes into play for high velocities, the frequency of revolution of the particle (cyclotron frequency) becomes

$$\upsilon = \frac{qB}{2\pi m} = \frac{qBc^2}{2\pi mc^2} = \frac{qBc^2}{2\pi(m_0c^2 + E_k)}$$

$$\therefore \quad mc^2 = m_0c^2 + E_k$$

where m_0 is the rest mass of the particle and E_k its kinetic energy when the velocity is v.

When the speed of the particle increases, υ and the spiralling particle lags in phase behind the accelerating voltage. Hence the acceleration will stop. The relativistic mass increase with thus limits the maximum energy attainable for a particle. In the case of electron the relativistic mass increase over the rest mass is considerable, since being light, even for moderate energies of electron (>10 keV). Hence resonance condition fails. This shows that electron is impracticable to be accelerated by cyclotron. To overcome this difficulty D.W. Kerst at the university of Illinois in the U.S.A devised a new instrument known as betatron in 1940 for the acceleration of electrons which based on an altogether new principle.

To overcome the relativistic effect we have to synchronies the revolution of the particles with the alternations of the accelerating p.d applied. This can be done in two ways.

1. By reducing the frequency of the alternating p.d applied which is made equal to the revolution frequency of the particles. Remember that the decrease in frequency of the particles is brought by the relativistic effect.
2. By increasing the magnetic field as the particle accelerate so as to keep the particles in phase with constant frequency of the alternating p.d applied.

The second method is adopted in the case of betatron and a combination of both for synchrotron.

Principle of betatron

The principle on which the action of a betatron depends is that the electrons moving in a stable orbit of fixed radius are accelerated by increasing the magnetic flux through the orbit.

Let the radius of the orbit be r and ϕ the total magnetic flux through the orbit, while the flux density is B in a direction perpendicular to the plane of the orbit. The stable orbit must be such that the average magnetic field over the space enclosed by the orbit is twice the magnetic field at the orbit.

Proof

If the total flux ϕ is increased at the rate $\frac{d\phi}{dt}$, the induced emf is given by

$$V = -\frac{d\phi}{dt}$$

Let $\vec{B}$ be the average value of the magnetic flux density within the area enclosed by the stable orbit of radius r

Thus $\quad \phi = B \cdot \pi r^2 \qquad \left(\because \quad \frac{\phi}{A} = B\right)$

The work done on the electron in one revolution is therefore

$$-eV = 2\pi r\ F$$

where F is tangential force on the electron

or $$F = \frac{-eV}{2\pi r} = \frac{e}{2\pi r}\frac{d\phi}{dt} \qquad \text{..... (1)}$$

For the stability of the orbit, we must have

$$Bev = \frac{mv^2}{r}$$

or $$mv = Ber$$

Using $$F = \frac{dp}{dt} = \frac{d}{dt}(Ber) = er\ \frac{dB}{dt} \qquad \text{..... (2)}$$

It is due to increase in energy, the electron tends to move into an orbit of larger radius. If stability is to be maintained, this tendency is to be resisted. For this to happen equations 1 and 2 must be equal.

i.e., $$\frac{e}{2\pi r}\ \frac{d\phi}{dt} = er\ \frac{dB}{dt}$$

or $$\frac{d\phi}{dt} = 2\pi r^2\ \frac{dB}{dt} \qquad \text{..... (3)}$$

This is known as the betatron condition. It shows that the electrons will revolve in an orbit of constant radius r, provided the magnetic flux ϕ changes at twice the rate at which it would change if the magnetic field had been uniform throughout the area enclosed by the electron orbit.

Integrating equation (3) gives

$$\phi_2 - \phi_1 = 2\pi r^2 B$$

assuming that the magnetic induction increases from 0 to B at the electron trajectory.

For a uniform magnetic induction, on the other hand, the flux would be $\pi r^2 B$. So for the betatron condition to hold, the magnetic flux be twice that for a uniform magnetic induction.

Construction

Betatron consists of a highly evacuated annular tube either made of glass or ceramics called doughnut. The doughnut tube is placed between specially shaped pole pieces of an electromagnet energised by passing alternating current through exciting coils. This a.c is supplied from the normal so cycles/s in which the increasing magnetic flux is obtained for quarter cycles i.e., for $\frac{1}{200}$ s. As a consequence electrons are accelerated for periods of $\frac{1}{200}$ s and out put beam is obtained in pulses. The pole pieces of the magnet are made of laminated iron to reduce eddy current losses and they have been shaped such that the total flux through the orbit is twice the value that would have been if the field were uniform.

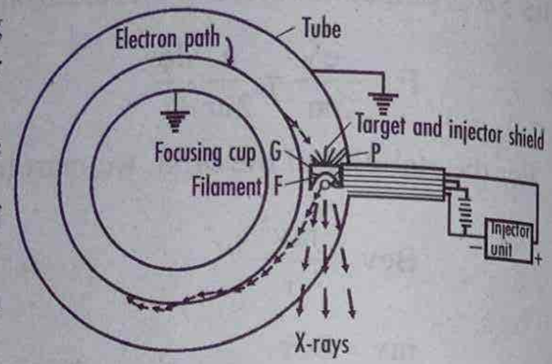


Figure 4.6

The electrons are emitted from a heated filament are first accelerated to about 50,000 volts and then injected into the doughnut at the instant when B = 0. The magnetic field then rises sinusoidally. The acceleration takes place during the first quarter of the time period T of the oscillating magnetic field. Exactly at the end of the acceleration period $\frac{T}{4}$ when B attains the peak value, an instantaneous current is passed through an auxillary coil which changes the magnetic field from the stable orbit value, as a result of which the electrons are deflected from stable orbit to hit a suitably located target, which makes the target emit x-rays within spread of energy from 0 upto the maximum electron energy by bremsstrahlung process. The process repeats itself at the interval of the time period T of the magnetic field so that the x-ray beam is obtained in pulses of very short duration during each interval T.

Energy of the electrons

Let the variation of the magnetic flux be $\phi = \phi_0 \sin \omega t$

Energy per turn $= eV = e\dfrac{d\phi}{dt}$

$$= e\frac{d}{dt}(\phi_0 \sin \omega t)$$

$$= e\phi_0\, \omega \cos \omega t$$

$\therefore$ Average energy per turn

$$\langle E\rangle = \int_0^{T/4} \frac{e\omega\phi_0 \cos \omega t\, dt}{\int_0^{T/4} dt}$$

Remember that the acceleration of the electrons takes places for quarter cycle.

$$\langle E\rangle = e\omega\phi_0 \int_0^{T/4} \frac{\cos \omega t\, dt}{\int_0^{T/4} dt}$$

$$\langle E\rangle = e\omega\phi_0 \frac{4}{T}\int_0^{T/4} \cos \omega t\, dt$$

$$\langle E\rangle = \frac{e\omega\phi_0}{\omega}\frac{4}{T}[\sin \omega t]_0^{T/4}$$

$$\langle E\rangle = e\,\phi_0 \frac{4}{T}\left[\sin \frac{2\pi}{T}\cdot\frac{T}{4} - 0\right]$$

$$= e\,\phi_0 \cdot \frac{4}{T} = \frac{e\,\phi_0 4}{2\pi/\omega}$$

$$= \frac{2e\,\phi_0\omega}{\pi}.$$

Since the speed of the electron in the orbit is almost constant $(\approx c)$ the total path travelled by the electron during acceleration is

$$L = c\cdot\frac{T}{4} \qquad (x = vt)$$

$$L = \frac{c\cdot 2\pi}{4\omega} = \frac{c\pi}{2\omega}$$

$\therefore$ The number of turns described by the electron

$$N = \frac{L}{2\pi r} = \frac{c\pi}{2\omega \cdot 2\pi r} = \frac{c}{4\omega r}$$

Hence final energy of the electron

$$= \frac{N \cdot 2e\omega}{\pi}\phi_0 = \frac{ec\ \phi_0}{2\pi r}$$

i.e., the final energy is determined by the peak value ϕ_0 of the magnetic flux and the radius of the orbit r.

For example $r = 0.5\text{m}$ and $B = 1\text{T}$

$$\therefore \qquad \phi_0 = 2\pi r^2 B = 2 \times \pi \times 0.5^2 \times 1$$

$$= 1.57 \text{ weber.}$$

$\therefore$ Total energy $E = \dfrac{ec\phi_0}{2\pi r}$

$$E = \frac{1.6 \times 10^{-19} \times 3 \times 10^8 \times 1.57}{2 \times 3.14 \times 0.5} = 2.4 \times 10^{-11}\text{J}$$

or

$$E = \frac{2.4 \times 10^{-11}}{1.6 \times 10^{-19}} = 150 \times 10^6 \text{eV} = 150\,\text{MeV}.$$

Note : In the first machine built by Kerst an electron beam of 2.3 MeV energy was obtained. One of the largest machine built at the Illinois university used a magnet weighing 356 tonne and the radius of the accelerator tube was 2.46m and obtained an energy of 340 MeV.

Example 6

The maximum magnetic induction in a betatron is 0.5T. If the radius of the dough-nut is 0.8m and the frequency of variation of the magnetic field is 50Hz. What is the energy gained per turn. What is the maximum energy of the electron

Solution

$$B = 0.5\text{J}, \quad r = 0.8\text{m} \quad \upsilon = 50\text{Hz}$$

Energy gained per t urn $= \dfrac{2e\,\phi_0\omega}{\pi}$

$$= \frac{2e\,2\pi r^2 B\,\omega}{\pi} = \frac{4\pi r^2 e\,B\,\omega}{\pi}$$

$$= 4er^2 B \cdot 2\pi \upsilon$$

$$= 4 \times 1.6 \times 10^{-19} \times 0.8^2 \times 0.5 \times 2 \times 3.14 \times 50 \mathrm{J}$$

$$= \frac{4 \times 1.6 \times 10^{-19} \times 0.8^2 \times 0.5 \times 2 \times 3.14 \times 50\,\mathrm{eV}}{1.6 \times 10^{-19}}$$

$$= 4 \times 0.8^2 \times 0.5 \times 2 \times 3.14 \times 50\,\mathrm{eV}$$

$$= 401.92 \mathrm{eV}.$$

Total number of revolutions

$$= \frac{c \cdot \frac{T}{4}}{2\pi r} = \frac{c \cdot 2\pi}{4\omega \cdot 2\pi r}$$

$$N = \frac{c}{4\omega r} = \frac{3 \times 10^8}{4 \times 2\pi \times 50 \times 0.8} = 2.985 \times 10^5$$

$\therefore$ Maximum energy $= N \times 401.92\,\mathrm{eV}$

$$= 2.985 \times 10^5 \times 401.92\,\mathrm{eV}$$

$$= 119.97 \times 10^6 \mathrm{eV}$$

$$= 119.97\,\mathrm{MeV}.$$

Use of betatron

1. The X-rays produced by the electrons accelerated in the betatron are used to produce nuclear reactions, such as the (γ, n), (γ, p), (γ, 2n), and (γ, n, p) reactions, or as highly penetrating radiations for the study of the properties of solids.
2. The high energy electrons are used for research in biological sciences.
3. The x-rays are used for the treatment of cancer and other diseases. Many hospitals in different parts of the world use this facility now a days. In India one such machine has been installed in christian missionary hospital at Vellore in Tamilnadu.

Electron synchrotron

Synchrotron is an accelerator which can accelerate charged particles giving energy of the order of several hundred million electron volts. Synchrotrons are of two types, electron synchrotron and proton synchrotron. Here we shall discuss only about electron synchrotron. By adapting the betatron to use the synchrotron principle, Goward and Barnes in 1946 in England developed a machine which is known as electron synchrotron. The name synchrotron was suggested by Mac Millan because the behaviour of the machine is similar in some respects to that of a synchronous motor.

Principle of electron synchrotron

In the synchrotron the frequency of the rf oscillator is matched with the revolution frequency of the electron. The frequency of the rf field is kept unchanged and the matching of the frequencies being achieved by changing the magnetic field with time.

The initial acceleration in the synchrotron is produced by betatron principle. The changing magnetic field through the orbit of the electrons induces an electric field along the orbit which accelerates the electrons upto about 2 MeV. At this point the betatron action ceases and electrons revolve in the orbit with almost constant velocity $v \approx c$ so the angular speed $\omega = \frac{c}{r}$ is also constant and hence the orbit radius r remains practically constant.

The electrons are injected at fairly at high speed from the electron gun so that the orbit radius increases only marginally afterwards. After the initial betatron acceleration, the electrons revolve in stable orbits with angular speed $\omega = \frac{Be}{m}$, where m is the relativistic mass and gain energy due to acceleration by the rf field within the accelerator tube. Since m increases with the gain of energy, B must be increased to maintain synchronism.

Construction

Synchrotron consists of a doughnut (a ring shaped accelerator tube of toroidal section made of glass or porcelain) in an a.c magnetic field. The magnet can be a ring of C-shaped units not filling the hole in the doughnut as the case with the betatron. In the central gap of the doughnut some flux bars serve as the central core of the magnet to start up the machine as betatron. These bars are made of high permeability metal and donot have to be large, they short the magnetic field at low injections but become saturated at high inductions and the transition from betatron action to synchrotron action can be made smoothly. Part of the interior of the doughnut is coated with copper or silver to given resonance cavity. A small break in the coating separates it into two parts. A high frequency electric field from a radio frequency oscillator is applied across this gap at the proper time in the magnetic cycle. When the accelerator is on, the electron is accelerated each time it crosses through the resonator.

The electron synchrotron accelerates electron in an orbit of constant radius by means of a radio frequency electric field applied across the gap. A ring shaped magnet provides the magnetic field over the doughnut shaped vacuum chamber. The pole faces are accurately shaped to provide a field which decreases with increasing radius.

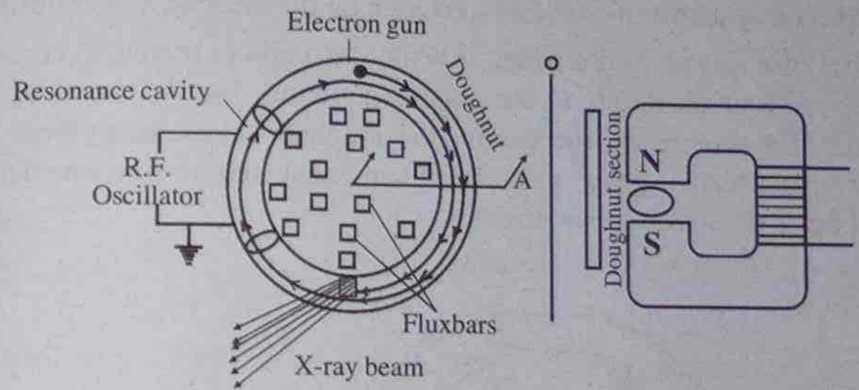


Figure 4.7

Working

Electrons are injected into the doughnut after a preliminary acceleration in an electrostatic field to 50 to 100 keV. When these are injected at the beginning of the magnetic cycle, the device operates as a betatron. The electrons travel in circular paths and increase their energy as the field increases until the electrons reach some 2MeV at which point the bars are magnetically saturated and are no longer able to induce an emf. This saturation causes the betatron mechanism to stop. At this point the synchrotron mechanism begins to operate. If the potential applied to the resonator operates at the proper frequency, the electrons are all kept in phase and receive increments of energy at each revolution, as they pass through the cavity. The high frequency field remains on while the magnetic field is increasing and is automatically cut off when the electrons have acquired their maximum energy. As the magnetic field still has a little increase to go before reaching its maximum,, the radius of the beam of constant velocity ($\approx c$) diminishes a little so that the beam spirals in slightly to strike a target projecting from the inner edge of the doughnut which gives off short wavelength x-rays. The rays emerge in pulses as in the betatron. The maximum energy of the electrons depends on the radius and on the maximum magnetic field. The electron energies obtained is of the order of 1 GeV, although a 6 GeV electron synchrotron is now operating in the Soviet Union.

Proton synchrotron

Proton synchrotrons are used to produce energies of the order of several giga electron volts ($\sim 10\,\text{GeV}$). This much energy is necessary to probe into the nucleus. If this is done with synchrocyclotron the size and cost would be enormous. To overcome this proton synchrotron was derived based on the electron synchrotron.

It consists of a ring shaped magnet in which the particle travels in constant radius. There are four quadrants to the magnet covering the annular doughnut as shown figure. The protons are injected into the doughnut at low energy from a linear accelerator or by a Van de graaff machine and are recovered by magnetic deflection as a pulsed beam after many revolutions.

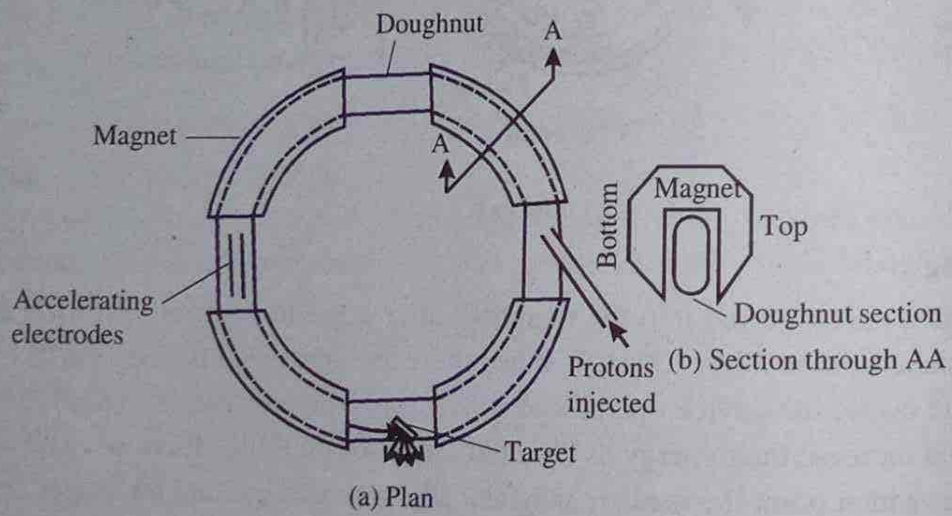


Figure 4.8: Proton Synchrotron

A high frequency resonator cavity accelerator is used in one of the straight parts with an increasing frequency corresponding to the increased speed of the protons. The field strength of the magnets is also increased to maintain the accelerated protons in a circular path of constant radius. The synchrotron action is applied at the beginning of each cycle. Energies attained are at the order of 10 GeV. The biggest present day machines are the Bevatron in the U.S.A which gives 6.4 GeV protons and the synchrophasotron of the U.S.S.R which operates at 10 GeV.

The alternating gradient synchrotron

It is a super proton synchrotron. We found that proton synchrotron is a constant gradient synchrotron, which yields an energy of about 10 GeV. If we want to have a proton energy of 50 GeV, such a machine would require a mass of 10^5 tonnes. It is found that the proton beam deviates appreciably from the circular path when radial magnetic field is constant. Since the whole doughnut is enclosed by the magnetic field in order to confine the beam as much as possible to a circular orbit, the size of the magnet becomes an important factor in the design of the big proton synchrotrons. For instance the equivalent mass of 50 GeV proton is 50 GeV + rest mass which is equal to 50 GeV + 0.938 GeV = 50.938 GeV.

Converting into atomic mass unit

$$m = 54.68\,u$$

or

$$m = 54.68 \times 1.66 \times 10^{-27}\,kg$$

$$m = 54.68 \times 1.66 \times 10^{-27}\,kg$$

Using $$R = \frac{mv}{qB} = \frac{90.78 \times 1.66 \times 10^{-27} \times 3\times10^{8}}{1.6\times10^{-19} \times 2}$$

Take B = 2 tesla

$$R = 85m$$

The orbital radius 85m and a magnetic field 2T is required.

To overcome this difficulty the alternating gradient synchrotron is devised. A method of overcoming this difficulty of beam wandering is to use magnetic fields with alternating gradients to focus the beam, i.e. in successive sections of the radial field, the gradient is first towards the centre and then outwards from the centre and soon. As the beam travels around its orbit it then passes through sections of the pole pieces of the magnet which cause the beam to be focused vertically and horizontally in rapid succession. At the same time the particles are defocused horizontally and vertically in rapid succession. All this is done by suitably arranging the magnetic field gradients to be inward and outward in the successive sections as described. Thus the field can be regarded as having an alternating gradient and this makes for the strong focusing and defocusing action. The A.G proton synchrotron in the Brook haven producing 33 GeV energy, which has more than 200 sections, each section is separated by a field free region and the total mass of the magnet is about 4000 tonnes the same as that of the C.G. proton synchrotron at Brookhaven. The A.G. Proton synchrotron at CERN (European Council for Nuclear research) at Geneva gives 25 GeV protons. A 70 GeV A.G. proton synchrotron has been operating at Serpukhov in the U.S.S.R.

After the ISR shut down in 1984 CERN's focus shifted to the large electron-positron collider (LEP) but ISR tunnel is still used for storage and magnetic purposes. The proton -antiproton collider was commissioned in the year 1981. After 10 years of functioning it was shutdown in the year 1991. After that CERN constructed still higher energy particle accelerator called relativistic heavy ion collider. it started operation in the year 2000 and still continuing. In the year 2009, large hadron collider (LHC) started its operation. We can see the details of LHC in the next section.

Intersecting beam accelerators

Physicist at CERN had planned to construct an intersecting beam accelerator in 1960s, based on the idea that smashing two particles beams head on would give higher energies than a single beam of particles with a fixed target. For this they constructed a specially designed device called intersecting storage ring (ISR). ISR composed of two interlaced rings each with a diameter 300 metres. Each ring contained a beam pipe surrounded by magnets to direct the circulating particles inside. Two protons can collide in the intersecting storage rings. The ISR begun its operation on 27th January 1971. It was the world's first hadron collider. ISR continued its operation until 1984.

Working of intersecting beam accelerator

Protons from a small proton linear accelerator (LINAC) A enter a booster accelerator. After boosting, protons enter into a proton synchrotron accelerator, where it is energised to about 30 GeV.

The energised protons, emerging from the proton synchrotron, are split magnetically at C. One beam goes through the ISR pipe in clockwise direction the other beam goes through the second ISR pipe in anticlockwise direction. The two beams collide at the eight intersections of two pipes of ISR. The schematic diagram of beam accelerator is given below.

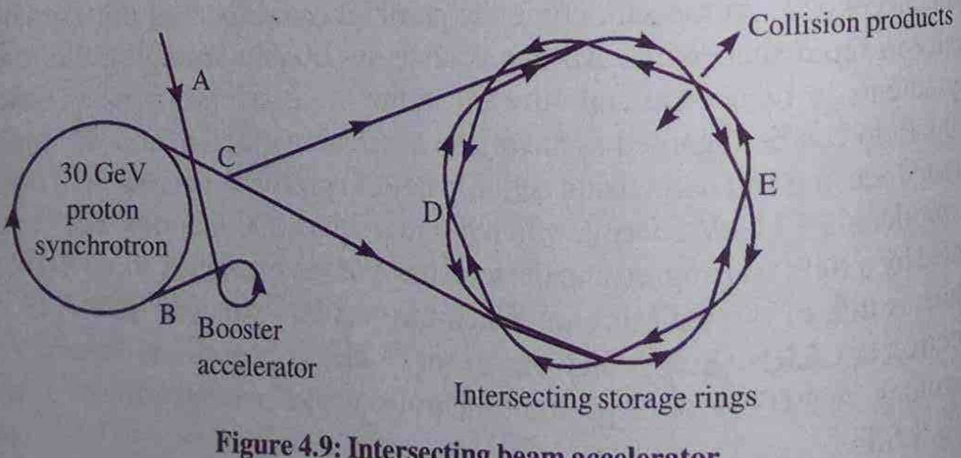


Figure 4.9: Intersecting beam accelerator

The maximum energy attained in ISR was 67GeV. This is equivalent to 2000 GeV beam hitting a stationary target. ISR paved the way for later accelerators as it hinted at the smaller elements of protons are now known to be quarks.

The growth and future of large accelerating machines

It has been realised that the mysteries of the ultimate structure of protons, neutrons and pions can only be solved by using intense beams of nuclear particles of very high energies as probes. The Cockcroft-Walton proton accelerator operated in 1932 the energy obtained was about 1MeV. The limiting value of this accelerator was 1 MeV. This is because at 1MeV the resistance of the accelerating tube materials break down. This led to the design of Van de graaff generator produced energy about 10 MeV.

In order to go beyond 10 MeV an entirely new method was required. The idea of resonance acceleration was conceived by Lawrence and living stone and mud the first cyclotron accelerator in 1932 which produced a proton beam about 1.2 MeV. In 1940 the betatron was designed and produced a beam of 2.3 MeV electrons. The energy limit of this was 25 MeV for protons and 300 MeV for electrons but linear accelerator was capable of producing 50 MeV for protons and about 1 GeV for electrons. The next break through was the construction of synchrotrons for both protons and electrons. The electron synchrotrons produced energies of the order of 350 MeV for electrons while the proton synchrotron produced energies upto 10 GeV. Finally the A.G. proton synchrotron was constructed and given output of 33 GeV, at the Brook Haven national laboratory. At present we arrived at the construction of 100 GeV energy output in the CERN (European Council for Nuclear research) at Geneva, Switzerland. So many European countries are contributing to this project. India had committed to contribute in kind 25 million U.S. dollars to L.H.C (Large Hadron Collider Project). CAT (Centre for Advanced Technology, Indore) is co-ordinating this activity.

In India major accelerator programmes are being persuaded at BARC (Baba Atomic Research Centre) TIFR (Tata Institute of Fundamental Research) at Mumbai, VECC (Variable Energy Cyclotron at Calcutta), NSC (Nuclear science centre) at Delhi.

CERN and its future project

The biggest accelerator of the world is at Geneva, Switzerland named CERN (European Council for Nuclear Research). The accelerator complex at CERN is a succession of machines that accelerates particles into increasingly high energies. Each machine booster the energy beam of particles before injecting the beam into the next machine in the sequence. The last element in the chain of machines is the large hadron collider (L.H.C). In the L.H.C beams are accelerated upto 6.5 TeV per beam. Most of other accelerators (totally eight) in the chain have their own experimental halls, where beams are used for experiments at lower energies.

Working

Basically it is a proton accelerator. The proton is a source sample bottle of hydrogen. An electric field is used to strip of electrons from the hydrogen atoms to yield protons. Linac-2 the first accelerator in the chain accelerates the protons to the energy 50 MeV. This beams is injected into the next proton synchrotron (PS) accelerator booster which accelerates the protons to 1.4 GeV. This will be followed by the another proton synchrotron which pushes the beam to 25 MeV. Protons are then sent to super proton synchrotron (SPS) where they are accelerated to 450 GeV. These protons are finally transferred to the two beam pipes of the LHC. The beam in one pipe circulate clockwise while the other one anticlockwise over a length of 27 kilometres. The LHC ring is installed underground at a depth of 100 metres crossing the borders between Switzerland and France. Proton beam takes 4 minutes and 20 seconds to fill each LHC ring, and in 20 minutes protons to reach their maximum kinetic energy 6.5 TeV. The beams circulated for many hours inside the LHC beam pipes under normal operating conditions. Two beams are brought into collision inside four detectors ALICE(a large ion collider experiment) ATLAS (atmospheric laboratory for applications and science) CMS (compact muon solenoid) and LHCb (large hadron collider b) where total energy at collision point is equal to 13 TeV.

The CERN is fully operational now. There are totally 17,500 scientists, engineers and technicians are working over there.

The future project of CERN is to construct an accelerator 4 times larger and 6 times powerful than the current LHC. The circle collider will be a total length of 100 kilometres and cross over Geneva and other two countries. The installation cost would be about 21 billion Euros and expected to be operational in the year 2040.

IMPORTANT FORMULAE

1. In a Cockcroft-Walton proton accelerator the output voltage (V_0) of the voltage multiplier

$$V_0 = 2NV_p$$

Where N is the number of stages and V_p is the peak input voltage.

2. In a linear accelerator, velocities are in the ratio

$$V_1 : V_2 : V_3 \,.............. = 1 : \sqrt{2} : \sqrt{3} : ....$$

The lengths of the tubes must also be in the same ratio.

3. Cyclotron:

a) Principle, $\dfrac{mv^2}{r} = Bqv$

b) Maximum kinetic energy of the particle, $K.E_{max} = \frac{B^2 q^2 r_0^2}{2m}$

c) Cyclotron frequency, $\upsilon = \frac{qB}{2m}$

d) Radius of the circular orbit $r = \frac{mv}{qB} = \frac{\sqrt{2meV}}{qB}$

4. Betatron:

a) Betatron condition, $\frac{d\phi}{dt} = 2\pi r^2 \frac{dB}{dt}$

b) Average energy per turn, $\langle E \rangle = \frac{2e\phi_0 \omega}{\pi}$

c) Number of revolutions described by the electron, $N = \frac{c}{4\omega r}$

d) Final energy of the electron $= \frac{ec\phi_0}{2\pi r} = ec\ Br$

UNIVERSITY MODEL QUESTIONS

Section A

(*Answer questions in* **two** *or* **three** *sentences*)

Short answer type questions

1. What are accelerators? What is their uses?
2. What is the principle of Cockcroft-Walton accelerator?
3. What is a voltage doubler?
4. Write down the reaction of the first artificially transformed element using Cockcroft-Walton accelerator?
5. What is a linear accelerator?
6. What are the limitations of linear accelerators?
7. What is the principle of linear accelerator?
8. What are the uses of linear accelerators?
9. What is the difference between a linear accelerator and a cyclotron?
10. What is the principle of cyclotron?
11. What is cyclotron frequency?
12. Write down an expression for cyclotron frequency and explain the symbols used?
13. Can electrons be accelerated by cyclotron. Why?
14. What is the difference between electrostatic accelerators and cyclic accelerators?
15. What is a synchrotron?
16. What is a betatron?
17. What is the basic difference between a cyclotron and a betatron?
18. Write down the betatron condition and explain the symbols used?

19. What is an electron synchrotron?
20. What is a proton synchrotron?
21. What is the use of proton synchrotron?
22. What is the difference between electron and proton synchrotron?
23. What is the disadvantage of proton synchrotron?
24. What is an intersecting beam accelerator?
25. What is the basic idea of intersecting beam accelerator?
26. Draw a diagram of intersecting beam accelerator and label it?
27. What is an ISR?
28. Give the names of three accelerators still in use:

Section B

(*Answer questions in a* **paragraph** *of about half a page to one page*)

Paragraph / Problem type questions

1. Explain the working of a voltage doubler?
2. Explain the principle of working of linear accelerator?
3. What are the advantages of linear accelerators?
4. What are the limitations of a cyclotron ?
5. A charged particle moving in a uniform magnetic field penetrates a layer of lead and thereby loses half of its kinetic energy. How does the radius of curvature of its path change?
6. A charged particle of mass m and charge q revolves with a velocity v in a magnetic field B. Show that the radius r is proportional to its momentum?
7. How do the relativistic effects limit the acceleration of electrons to high energies in cyclotron?
8. What is the principle of synchrotron?
9. Explain two ways in which relativistic effect in accelerators can be overcome.
10. Explain the principle of betatron?
11. What are the uses of betatron?
12. What is the principle of electron synchrotron?
13. Explain the principle of alternating gradient synchrotron?
14. What is the principle of intersecting beam accelerator?
15. In a cyclotron the strength of the magnetic field is 0.954 Wb m^{-2} and the dees are dimensioned so that the accelerated protons emerge tangentially at a distance of 0.15m from the centre. What should be the frequency of the oscillator and the energy in MeV of the emerging protons. $m_p = 1.67 \times 10^{-27}$ kg [14.55MHz, 0.98 MeV]
16. A cyclotron is to be designed to accelerate α-particles to a maximum energy of 32MeV. If the peak value of the accelerating voltage is 40 KeV, how many revolutions will be needed for a particle initially at rest before it acquires this amount of energy. If the induction associated with the magnetic field is 60 Wb m^{-2}. Compute the value of the diameter of the dees and frequency of the oscillator.

[n = 200 d = 1.18m υ = 12.2MHz]

17. A cyclotron used to produce a beam of protons has a magnetic field of flux density $1.80\ \text{Wb m}^{-2}$ and the extreme useful radius is 0.2m. Assume the specific charge of the proton $9.58 \times 10^7\ \text{C kgm}^{-1}$, obtain the values of a) energy of the protons b) the wave-length of the electron magnetic waves produced by the oscillator. $m_p = 1.67 \times 10^{-27}\ \text{kg}$.

$$\begin{bmatrix} \text{a) } E = 6.2\ \text{MeV} \\ \text{b) } \lambda = 10.92\text{m} \end{bmatrix}$$

18. An α-particle is accelerated by a potential difference of 10^4 V. Find the change in its direction of motion if it enters normally in a region of thickness 0.1m having trans-verse magnetic induction of 0.1 J. Mass of α-particle $= 6.4 \times 10^{-27}$. [30°]
19. Prove that the radius of curvature r of the path of a positive ion in a cyclotron is proportional to $\sqrt{n}$, where n is the number of times the ion, initially at rest has been accelerated across the space separating the two dees, provided the mass of the ion remains constant.
20. In a betatron the maximum magnetic field at orbit was $0.4\ \text{Wbm}^{-2}$ operating at 50 Hz with a stable orbit diameter of 1.524 m. Calculate the average energy gained per revolution and the final energy of the electrons. [291.9eV, 91.45MeV]
21. A cyclotron with dees of diameter 1.8 m has a magnetic field of 0.8T. Calculate the energy of the accelerated protons [25 MeV]
22. An electron and a proton have 1MeV energies each. Which one should be treated relativistically. [only electron]
23. In a betatron accelerator the maximum magnetic field at orbit was 0.4T, operating at 50 H_z with a stable orbit diameter of 1.5m. Calculate the average energy gained per revolution and the final energy of the electrons. [239.5eV, 91 MeV]
24. A cyclotron with dees of radius 0.9m has a magnetic field of 0.8T. Calculate the generator frequency required to accelerate the proton to 25 MeV. [6.17×10^7 Hz]
25. What radius is needed in proton synchrotron to attain particle energies of 10 GeV. Assume that magnetic field available is 1.5T [18m]

Section C

(*Answer questions in about* **one** *or* **two** *pages*)

Long answer type questions (Essays)

1. Explain the principle, construction and working of a Cockcroft-Walton accelerator.
2. Describe the working of Van de graaff generator with a neat diagram.
3. Explain the principle, construction and working of a linear accelerator?
4. Explain with a neat figure the working of a cyclotron?
5. Describe the principle, construction and working of a betatron?
6. Describe the principle, construction and working of an electron synchrotron?
7. Explain a proton synchrotron in detail?
8. Explain an alternating proton synchrotron in detail?
9. Write down an essay on the growth and future of accelerating machines?

10. Explain the principle, construction and working of an intersecting beam accelerator with a diagram?

Hints to problems

15. $\upsilon = \frac{qB}{2\pi m}$, $E = \frac{B^2 q^2 r_0^2}{2m}$, $r_0 = 0.15\text{m}$

16. a) The particle will be accelerated twice during each revolution. For one revolution energy acquired $= q.2V$. Let n be the required number of revolution to get 32 MeV

$$\therefore \quad n\, q.2V = 32\,\text{MeV}$$

$$q = 3.2\times10^{-19}\text{C},\ V = 40\times10^{3}\text{V}$$

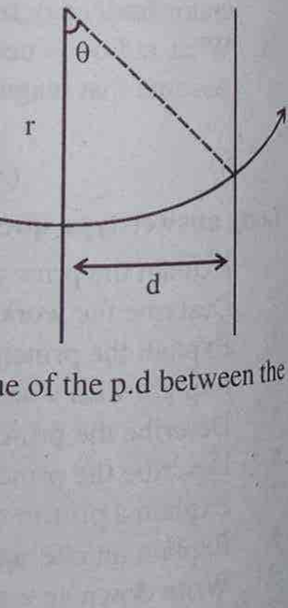
b) $\frac{B^2 q^2 r_0^2}{2m} = 32\times10^{6}\times1.6\times10^{-19}$ find r_0, $\because d = 2r_0$

c) $\upsilon = \frac{qB}{2\pi m}$

17. a) $E = \frac{B^2 q^2 r_0^2}{2m} = \frac{B^2 r_0^2}{2}\left(\frac{q}{m}\right)^2 m$

b) $\lambda = \frac{c}{\upsilon}$, $\upsilon = \frac{qB}{2\pi m}$

18. $r = \frac{mv}{qB} = \frac{\sqrt{2\,m\,k\cdot E}}{qB} = \frac{\sqrt{2m\,qV}}{qB}$ find r $\quad r = 0.2\text{m}$



$$\sin\theta = \frac{d}{r}$$

$$\theta = \sin^{-1}\frac{d}{r} = \sin^{-1}\left(\frac{0.1}{0.2}\right)$$

19. We have $\frac{mv^2}{r} = \text{Bev}$ or $mv = \text{Ber}$. If V be the maximum value of the p.d between the dees $\frac{1}{2}mv^2 = eVn$

or $\quad m^2v^2 = \frac{2eVn}{m}$ or $mv = \sqrt{\frac{2eVn}{m}}$

using $\quad r = \dfrac{mv}{Be} = \sqrt{\dfrac{2eVn}{m}} \cdot \dfrac{1}{Be}$

$$r = \frac{1}{B}\sqrt{\frac{2V}{me}} \cdot \sqrt{n}$$

or $\quad r \,\alpha\, \sqrt{n}$

20. See example 6. $N = 3.132 \times 10^5$

21. $KE_{max} = \dfrac{B^2 q^2 r_0^2}{2m} = \dfrac{0.8^2 \times (1.6 \times 10^{-19})^2 \times (0.9)^2}{2 \times 1.67 \times 10^{-27}} J$

$$= \frac{0.8^2 \times (1.6 \times 10^{-19})^2 \times 0.9^2}{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-13}} \text{ MeV}$$

$$= 24.83 \text{ MeV} \simeq 25 \text{ MeV}$$

22. $K.E = (m - m_0)c^2 = m_0 c^2 \left(\dfrac{m}{m_0} - 1\right)$

$$K.E = E_0 \left(\frac{m}{m_0} - 1\right), \quad E_0 = m_0 c^2 = \text{Rest energy}$$

$$\therefore \quad \frac{m}{m_0} = 1 + \frac{K.E}{E_0}$$

For a proton

$$\left(\frac{m}{m_0}\right)_{proton} = 1 + \frac{1MeV}{938MeV} = 1.00106$$

So proton with energy need not be considered relativistically.

For an electron

$$\left(\frac{m}{m_0}\right)_{electron} = 1 + \frac{1MeV}{0.51MeV} = 2.96$$

So electron with energy 1 MeV should be considered relativistically.

23. Here the electron must be treated relativistically

$$mv = \frac{E}{c}, \quad \text{E is the final energy.}$$

Using $\dfrac{mv^2}{R} = Bqr$

$\therefore \quad mv = BqR$

$$\frac{E}{c} = BqR$$

$\therefore \quad E = BqRc$

Substituting the values, we get E = 91 MeV.

Total number of revolutions

$$N = \frac{c}{4\omega R} = \frac{3\times10^8}{4\times2\pi\times50\times0.75} = 3.1\times10^5$$

$\therefore$ Average energy per revolution

$$= \frac{91\times10^6\,eV}{3.1\times10^5} = 293.5\ eV$$

24. $\upsilon = \dfrac{qB}{2m}$ ..... (1)

$$K.E = \frac{B^2q^2r_0^2}{2m} \qquad \text{..... (2)}$$

$\dfrac{Eq(1)}{Eq(2)}$ gives $\dfrac{\upsilon}{K.E} = \dfrac{1}{Bqr_0^2}$, $\upsilon = \dfrac{K.E}{Bqr_0^2}$

$$\upsilon = \frac{25\times10^6\times1.6\times10^{-19}}{0.8\times1.6\times10^{-19}\times(0.9)^2} = 6.17\times10^7\,Hz$$

25. $R = \dfrac{mv}{Bq}$, Equivalent mass of 10 GeV proton is equal to 10 GeV + rest mass.

m = 10.938 GeV

m = 11.75 u

$m = 11.75\times1.66\times10^{-27}\,kg$

$m = 1.95\times10^{-26}\,kg$

Take $v = 3\times10^8\,ms^{-1}$