3 INTERFERENCE BY DIVISION OF AMPLITUDE

Introduction

In the last section we dealt with interference produced by division of a wavefront. Here we are going to deal with the formation of interference by division of amplitude. If a plane wave falls on thin films of transparent material like soap bubbles or of a drop of oil spread on the surface of water, the wave reflected from the upper surface and that reflected from the lower surface get superposed to get interference pattern. These studies have many practical applications and also explain phenomenon of brilliant colours produced by a soap film exposed to white light (sunlight).

Interference by a plane film

When a plane wave is incident on a thin film of uniform thickness (like a film of oil spread on water surface, or soap film), then the waves reflected from the upper surface interfere with the waves reflected from the lower surface. Depending upon the path difference between the two interfering waves, brightness or darkness can be seen.

Note : A film of thickness in the range $0.5\mu m$ to $10\mu m$ is considered as a thin film.

Normal incidence of light on a plane film

Consider a thin transparent film of uniform thickness t, having refractive index μ is kept in air. A plane wave from a monochromatic light of wavelength λ is falling normally on the film after reflection from a mirror G placed at an angle 45° with respect to the film. Then the waves reflected from the upper surface interfere with the waves reflected from the lower surface.

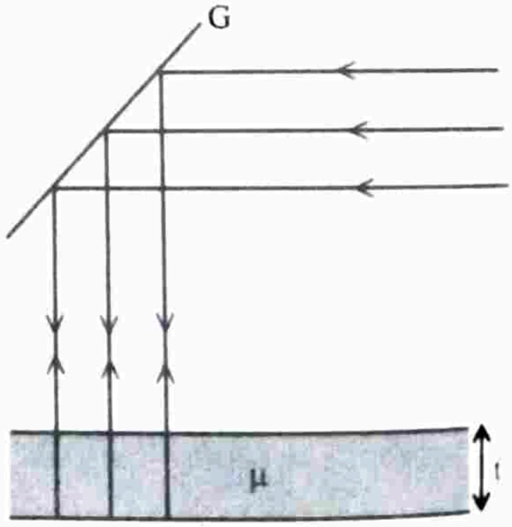


Figure 3.1

Clearly the wave reflected from the lower surface travels an additional optical path of $2\mu t$ to join the wave reflected from the upper surface. Since the film is placed in air, then the wave reflected from the upper surface of the film will undergo a sudden change in phase of π, which is equivalent to a path difference of $\frac{\lambda}{2}$.

So the effective path difference between the two rays reflected from upper and lower surfaces of the film is

$$\Delta = 2\mu t - \frac{\lambda}{2}$$

If this path difference is an integral multiple of λ, the film appears to be bright. So the condition for brightness becomes

$$2\mu t - \frac{\lambda}{2} = n\lambda, \text{ where } n = 0, 1, 2.....$$

or
$$2\mu t = \left(n + \frac{1}{2}\right)\lambda$$

If the path difference is an odd integral multiple of $\frac{\lambda}{2}$, the film appears to be dark so the condition for darkness becomes

$$2\mu t - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

or
$$2\mu t = n\lambda \quad \text{where } n = 1, 2, 3.....$$

Oblique incidence of light on a plane film

Consider a thin transparent film of uniform thickness t, having refractive index μ placed in air. A plane wave from a monochromatic light of wavelength λ is falling at an angle of incidence i. Part of a ray AB is reflected along BC, and part of it is transmitted into the film along BF. The transmitted ray BF makes an angle r with the normal to the surface at point G. The ray BF is in turn partially reflected back into the film along FD while a major part refracts into air along FK. Part of the reflected ray FD is transmitted at the upper surface and travels along DE. Since the surfaces of the film are parallel, the reflected rays BC and DE will be parellel to each other. The waves travelling along the paths BC and BFDE are derived from a single incident wave AB. Therefore they are coherent and can produce interference when they are made to superimpose.

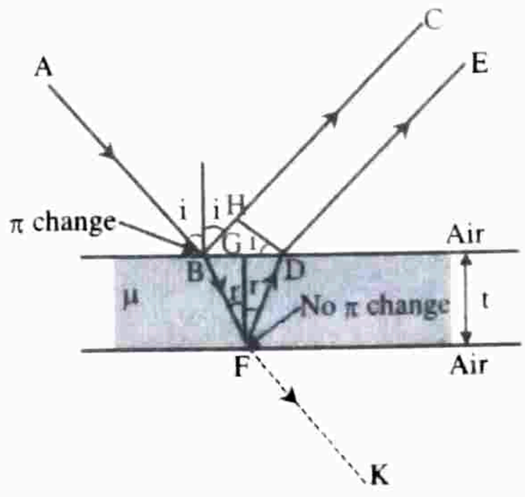


Figure 3.2

Let DH be the normal to BC. From H and D onwards, the rays HC and DE travel equal path.

$\therefore$ The optical path difference between the two reflected rays, $\Delta = \mu(BF + FD) - BH$

From the ΔBFG, $\frac{FG}{BF} = \cos r$

or $$BF = \frac{FG}{\cos r} = \frac{t}{\cos r}$$

$\therefore$ $$BF + FD = 2BF \quad (\because BF = FD)$$

or $$BF + FD = \frac{2t}{\cos r} \qquad .....(2)$$

From the ΔBHD, $\frac{BH}{BD} = \sin i$

or $$BH = BD \sin i = 2BG \sin i \quad (\because BD = 2BG)$$

From the Δ BFG, $\frac{BG}{BF} = \sin r$

or $$BG = BF \sin r = \frac{t}{\cos r} \sin r = t \tan r$$

$\therefore$ $$BH = 2\, t \tan r \sin i$$

Using $\frac{\sin i}{\sin r} = \mu$, $BH = 2t \tan r\, \mu \sin r$

$$BH = 2\mu t \frac{\sin^2 r}{\cos r} \qquad .....(3)$$

Using equations 2 and 3 in equation (1), we get

$$\Delta = \mu \frac{2t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r}$$

$$\Delta = \frac{2\mu t}{\cos r}(1 - \sin^2 r) = 2\mu t \cos r$$

Since the film is placed in air, then the wave reflected from the upper surface of the film will undergo a sudden change in phase of π, which is equivalent to a path difference of $\frac{\lambda}{2}$. There is no path difference due to transmission at D. So the effective path difference between the two rays from upper and lower surfaces of the film is

$$\Delta = 2\mu t \cos r - \frac{\lambda}{2}.$$

If this path difference is an integral multiple of λ, the film appears to be bright, so the condition for brightness becomes

$$2\mu t\cos r-\frac{\lambda}{2}=n\lambda$$

$$2\mu t\cos r=\left(n+\frac{1}{2}\right)\lambda \text{ where } n=0,1,2,3....$$

If the path difference is an odd integral multiple of $\frac{\lambda}{2}$, the film appears to be dark. So the condition for darkness becomes

$$2\mu t\cos r-\frac{\lambda}{2}=(2n-1)\frac{\lambda}{2}$$

$$2\mu t\cos r=n\lambda \text{ where } n=1,2,3....$$

The Cosine law

The effective path difference between the wave reflected from the lower surface and the upper surface of the film is given by

$$\Delta=2\mu t\cos r \qquad .....(4)$$

This is known as the cosine law.

Example 1

A soap film is illuminated by white light incident at angle of 30°. The reflected light is examined by a spectroscope in which dark band corresponding to the wave-length $6\times10^{-7}\text{m}$ is found. Calculate the smallest thickness of the film. $\mu=1.33$

Solution

$$i=30^\circ,\ \lambda=6\times10^{-7}\text{m} \text{ and } \mu=1.33$$

Dark band for reflected light is seen when

$$2\mu t\cos r=n\lambda$$

For minimum thickness $n=1$

$$\therefore \quad 2\mu t\cos r=\lambda$$

$$t=\frac{\lambda}{2\mu\cos r}=\frac{6\times10^{-7}}{2\times1.33\times\cos r}$$

But $\cos r=\sqrt{1-\sin^2 r}=\sqrt{1-\frac{\sin i^2}{\mu^2}} \qquad \left(\because \frac{\sin i}{\sin r}=\mu\right)$

$$=\sqrt{1-\frac{\sin^2 30}{1.33^2}}=0.927$$

$$\therefore \qquad t = \frac{6\times10^{-7}}{2\times1.33\times0.927} = 2.433\times10^{-7}\,\text{m}.$$

Example 2

A soap film 5×10^{-5}cm thick is viewed at an angle of 35° to the normal. Find the wavelength of light in the visible spectrum which will be absent from the reflected light $\mu = 1.33$.

Solution

$$t = 5\times10^{-5}\text{cm} = 5\times10^{-7}\text{m}, \quad i = 35^\circ, \text{ and } \mu = 1.33$$

using $\dfrac{\sin i}{\sin r} = \mu$ we have $\sin r = \dfrac{\sin i}{\mu} = \dfrac{\sin 35}{1.33}$

$\sin r = 0.431$

$$\therefore \qquad \cos r = \sqrt{1-\sin^2 r} = \sqrt{1-(0.431)^2} = 0.90$$

The condition for destructive interference is

$$2\mu t\cos r = n\lambda$$

$$\therefore \qquad \lambda = \frac{2\mu t\cos r}{n}$$

for $n = 1$ $\lambda_1 = 2\mu t\cos r = 2\times1.33\times5\times10^{-7}\times0.90$

$$\lambda_1 = 11.97\times10^{-7}\text{m}$$

Similarly

$$\text{for } n = 2, \ \lambda_2 = 5.985\times10^{-7}\text{m}$$

$$\text{for } n = 3, \ \lambda_3 = 3.99\times10^{-7}\text{m}$$

$$\text{for } n = 4, \ \lambda_4 = 2.99\times10^{-7}\text{m}.$$

λ_2 and λ_3 lie in the visible region, therefore the two wave lengths are absent in the reflected light.

Example 3

A parallel beam of sodium light $(\lambda = 5890\text{Å})$ strikes a film of oil floating on water. When viewed at angle of 30° from the normal 8^{th} dark band is seen. Determine the thickness of the film. (μ of the oil = 1.5).

Solution

$\lambda = 5890\,\text{Å} = 5890 \times 10^{-10}\,\text{m}$

$i = 30^\circ, \quad \mu = 1.5, \quad n = 8$

using
$$\mu = \frac{\sin i}{\sin r}$$

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 30}{1.5} = \frac{1}{2 \times \frac{3}{2}} = \frac{1}{3}$$

$$\therefore \quad \cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}}$$

$$= \frac{2\sqrt{2}}{3}$$

For the dark band, we have

$$2\mu t \cos r = n\lambda$$

$$\therefore \quad t = \frac{n\lambda}{2\mu \cos r} = \frac{8 \times 5890 \times 10^{-10}}{2 \times \frac{3}{2} \cdot \frac{2\sqrt{2}}{3}}$$

$$= 1.67 \times 10^{-4}\,\text{mm}$$

Example 4

When white light is incident on a soap film at an angle of $\sin^{-1}(4/5)$ and the reflected light on examination by a spectroscope shows dark bands. Two consecutive dark bands correspond to wavelengths $6.1 \times 10^{-7}\,\text{m}$ and $6 \times 10^{-7}\,\text{m}$. If μ of the film be $\frac{4}{3}$, calculate its thickness.

Solution

The path difference for dark band in reflected light is given by

$$2\mu t \cos r = n\lambda$$

For λ_1 we have $\quad 2\mu t \cos r = n\lambda_1 \qquad .....(1)$

For λ_2 we have $\quad 2\mu t \cos r = (n+1)\lambda_2 \qquad .....(2)$

comparing eqns (1) and (2), we have

$$n\lambda_1 = (n+1)\lambda_2$$

or $$n = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{6\times10^{-7}}{6.1\times10^{-7} - 6\times10^{-7}} = 60$$

Putting the value of n in eqn (1), we get

$$2\mu t\cos r = 60\times6.1\times10^{-7}$$

$\therefore$ $$t = \frac{60\times6.1\times10^{-7}}{2\mu\cos r} = \frac{60\times6.1\times10^{-7}}{2\times\frac{4}{3}\times\cos r}$$

But $i = \sin^{-1}\frac{4}{5}$ given

or $$\sin i = \frac{4}{5}$$

using $$\mu = \frac{\sin i}{\sin r}$$

or $$\sin r = \frac{\sin i}{\mu} = \frac{4}{5\times\frac{4}{3}} = \frac{3}{5}$$

$\therefore$ $$\cos r = \sqrt{1-\sin^2 r} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$$

Then $$t = \frac{60\times6.1\times10^{-7}}{2\times\frac{4}{3}\times\frac{4}{5}} = 1.716\times10^{-5}\,\text{m}$$

Reflection and transmission coefficient

For simplicity we assume that light (electromagnetic wave) is a plane wave travelling in the z-direction. The field vectors $\vec{E}_I$ and $\vec{B}_I$ characterise the incident wave travelling in the positive direction, $\vec{E}_R$ and $\vec{B}_R$ the reflected wave travelling in the $-z$ direction, $\vec{E}_T$ and $\vec{B}_T$ the wave transmitted into the second medium of refractive index μ_2. The interface between the dielectric media is taken at $z = 0$

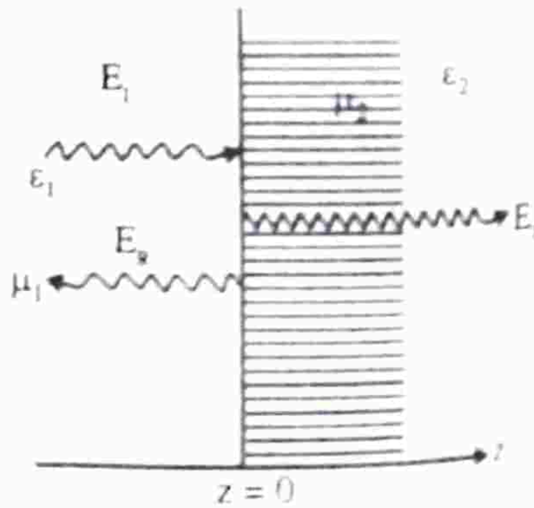


Figure 3.3

The incident, reflected and transmitted waves are given by

Incident $$\vec{E}_I = \hat{i}\ E_{Io}e^{i(k_1z-\omega t)} \qquad .....(5)$$

where $\hat{i}$ is the direction of the electric field vector, E_{Io} is the amplitude of the field and k_1 is wave vector in medium 1.

$$\vec{B}_I = \hat{j}\ B_{Io}e^{i(k_1z-\omega t)} \qquad .....(6)$$

$\hat{j}$ is the direction of the magnetic field vector and B_{Io} is the amplitude of the wave.

Reflected $$\vec{E}_R = \hat{i}E_{Ro}e^{-i(k_1z+\omega t)} \qquad .....(7)$$

and $$\vec{B}_R = -\hat{j}B_{Ro}e^{-i(k_1z+\omega t)} \qquad .....(8)$$

Transmitted $$\vec{E}_T = \hat{i}E_{To}e^{i(k_2z-\omega t)} \qquad .....(9)$$

and $$\vec{B}_T = \hat{j}B_{To}e^{i(k_2z-\omega t)} \qquad .....(10)$$

where E_{To} and B_{To} are the amplitudes of the field vectors and k_2 is the wave vector in the second medium.

The boundary conditions on $\vec{E}$ and $\vec{B}$ at $z = 0$ are the continuity of their tangent components

$$E_{It} + E_{Rt} = E_{Tt} \text{ (From eqns 5, 6, 7, 8, 9, 10)}$$

or $$E_{Io} + E_{Ro} = E_{To} \qquad .....(11)$$

and $$\frac{B_{Io}}{\mu_1} - \frac{B_{Ro}}{\mu_1} = \frac{B_{To}}{\mu_2} \qquad .....(12)$$

Using $B_{Io} = \dfrac{E_{Io}}{v_1}$, $B_{Ro} = \dfrac{E_{Ro}}{v_1}$ and $B_{To} = \dfrac{E_{To}}{v_2}$

∴ Eqn 12 becomes

$$\sqrt{\mu_1\varepsilon_1}\,\frac{E_{Io}}{\mu_1} - \sqrt{\mu_1\varepsilon_1}\,\frac{E_{Ro}}{\mu_1} = \frac{\sqrt{\mu_2\varepsilon_2}}{\mu_2}E_{To}$$

we have used $v = \frac{1}{\sqrt{\mu\varepsilon}}$ where μ is the permeability and ε is the permittivitty

or
$$\sqrt{\frac{\varepsilon_1}{\mu_1}}(E_{Io} - E_{Ro}) = \sqrt{\frac{\varepsilon_2}{\mu_2}}E_{To} \qquad .....(13)$$

solving eqns 11 and 13 we get

$$E_{Ro} = \frac{\sqrt{\frac{\varepsilon_1}{\mu_1}} - \sqrt{\frac{\varepsilon_2}{\mu_2}}}{\sqrt{\frac{\varepsilon_1}{\mu_1}} + \sqrt{\frac{\varepsilon_2}{\mu_2}}} E_{Io} \qquad .....(14)$$

and
$$E_{To} = \frac{2\sqrt{\frac{\varepsilon_1}{\mu_1}}}{\sqrt{\frac{\varepsilon_1}{\mu_1}} + \sqrt{\frac{\varepsilon_2}{\mu_2}}} E_{Io} \qquad .....(15)$$

The refractive index $n = \frac{c}{v} = \sqrt{\frac{\mu\varepsilon}{\mu_o\varepsilon_o}}$

Here $\mu_1 = \mu_2 = \mu_0$ for both the medium.

$\therefore \quad n = \frac{\varepsilon}{\varepsilon_o} \quad \therefore \quad n_1 \; \alpha \; \sqrt{\varepsilon_1}$ and $n_2 \; \alpha \; \sqrt{\varepsilon_2}$

Re-writing eqns 14 and 15 in terms of n_1 and n_2 we get

$$E_{Ro} = \frac{n_1 - n_2}{n_1 + n_2} E_{Io} \qquad .....(16)$$

when $n_2 > n_1$, E_{Ro} is negative showing that when a reflection occurs at a denser medium a phase change of π occurs.

$$E_{To} = \frac{2n_1}{n_1 + n_2} E_{Io} \qquad .....(17)$$

$\frac{E_{Ro}}{E_{Io}}$ is called the amplitude reflection coefficient r and $\frac{E_{To}}{E_{Io}}$ is called the amplitude transmission coefficient.

i.e.
$$r = \frac{n_1 - n_2}{n_1 + n_2} \quad .....(18)$$

$$t' = \frac{2n_1}{n_1 + n_2} \quad .....(19)$$

Note : We have been using the symbol μ for refractive index. Here only n is used not to confuse with permeability μ.

Non-reflecting films

Here we will see how to reduce the reflectivity of thin film surfaces. The technique of reducing reflectivity is known as blooming. Consider a light beam propagating in a medium of reflective index μ_1 is incident normally on a dielectric of refractive index μ_2. Let r and t be the amplitude reflection and transmission coefficients.

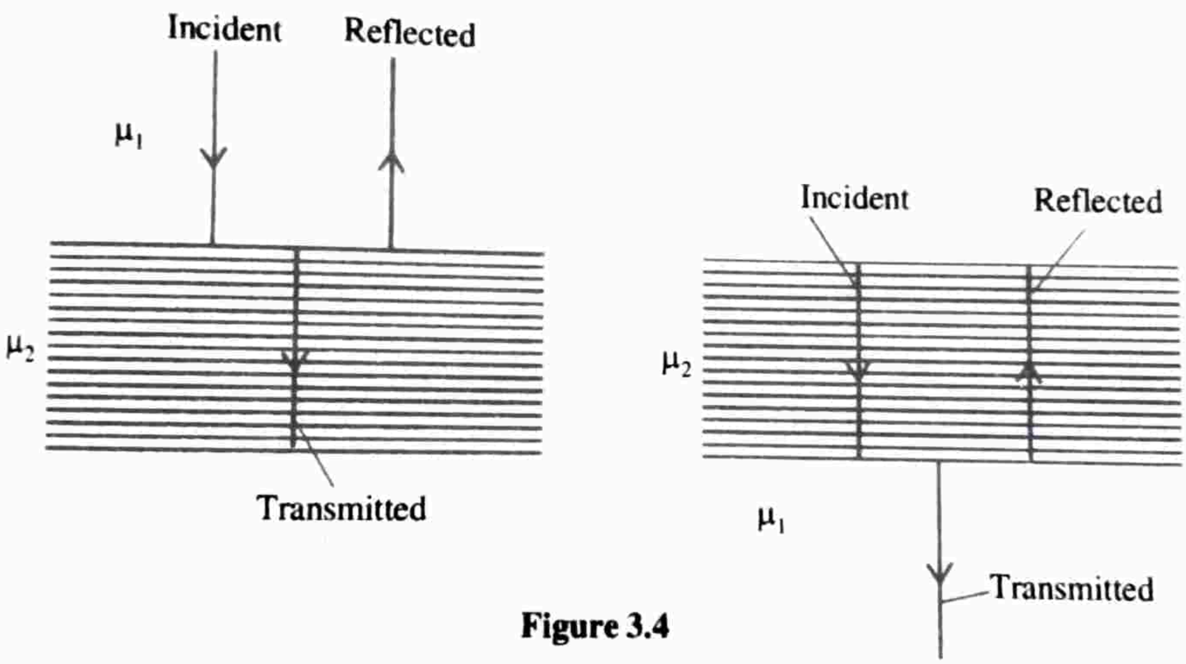


Figure 3.4

Then
$$r = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \quad .....(20)$$

and
$$t = \frac{2\mu_1}{\mu_1 + \mu_2} \quad .....(21)$$

If the light beam propagating in a medium of refractive index μ_2 is incident on a medium of refractive index μ_1, then the amplitude reflection coefficient r' and amplitude transmission coefficient t' are given by

$$r' = \frac{\mu_2 - \mu_1}{\mu_2 + \mu_1} \quad .....(22)$$

and $$t' = \frac{2\mu_2}{\mu_2 + \mu_1} \quad .....(23)$$

From eqn 22, we get

$$r' = -\frac{(\mu_1 - \mu_2)}{(\mu_1 + \mu_2)} = -r \quad .....(24)$$

We know that the intensity of light is directly proportional to the square of the amplitude. Hence the square of the amplitude reflection coefficient gives the fraction of intensity of light get reflected. It is called reflection coefficient or reflectivity and is denoted by R.

i.e. $$r^2 = R = \left(\frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}\right)^2 \quad .....(25)$$

Reduction of reflectivity of lens surfaces

As an application of the thin film interference phenomenon, here we will see how to reduce the reflectivity of lens surfaces. We know that in optional instruments there are many lens surfaces. Each surface reflect light and the loss of intensity of light will be large. For example the reflectivity for normal incidence is given by

$$R = \left(\frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}\right)^2$$

Suppose light falls on a crown glass surface from air then $\mu_1 = 1$ and $\mu_2 = 1.5$

$$\therefore \quad R = \left(\frac{1 - 1.5}{1 + 1.5}\right)^2 = 0.04$$

It shows 4% light is reflected from a crown glass from each surface.

If the glass is flint $\mu_2 = 1.67$

Then $$R = \left(\frac{1 - 1.67}{1 + 1.67}\right)^2 = 0.06$$

i.e. 6% light is reflected from a flint glass from each surface. If there are large num ber of surfaces the loss will be severe. In order to reduce these loses lens surfaces ar coated with a thick non-reflecting film whose refractive index is less than that o glass. Suppose a thin film of a transparent film is coated on the surface of a glas The material of the thin film is so selected that its refractive index is equal to th square root of the product of refractive indices of air and of glass. If μ_f is the refractive index of film and μ_g and μ_a are respectively the refractive indices of glass and air then (see example 5)

$$\mu_f \approx \sqrt{\mu_f \mu_a}$$

since $\mu_a = 1$

$$\mu_f = \sqrt{\mu_g} \qquad .....(26)$$

For crown glass $\mu_g = 1.5$ $\quad \therefore \quad \mu_f = \sqrt{1.5} = 1.22$

For flint glass $\mu_g = 1.67$ $\quad \therefore \quad \mu_g = \sqrt{1.67} = 1.29$

Such a value of μ_f results into reflection of equal quantities of light from the outer surface of the film and the boundary surface between the film and the glass. Since in both reflections the light travels from a rarer medium to a denser medium, then same phase change occurs in each reflections. If the thickness of the coating is $\frac{\lambda}{4\mu_f}$ (or $\frac{\lambda'}{4}$ where λ' is the wave length of light in the film) then the light reflected from these two surfaces (upper and lower) will have a path difference of $\frac{\lambda'}{2}$ or phase difference π. Then the conditions for complete destructive interference takes place.

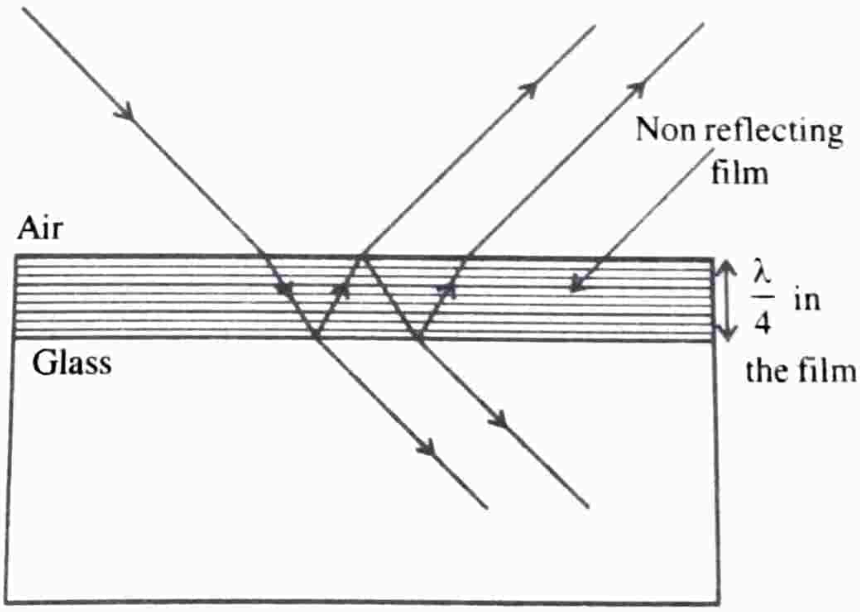


Figure 3.5

It should be remembered that the thickness of the film can be one quarter wavelength for one particular wavelength only. This is usually chosen in the yellow-green portion (5500Å) of the spectrum, because our eye is most sensitive for this region of spectrum. Thus when incident light is white some reflections take place for longer (red) and shorter (violet) wave lengths and the reflected light has a purple blue.

This method has been found useful in reducing reflection from a lens or a prism. Such a coating is therefore termed as non-reflecting film and the technique employed is called blooming.

Note : The best materials suitable for using as non-reflecting films are magnesium fluoride (MgF_2) with refractive index 1.38 and cryolite with refractive index 1.36.

Interference by a film with two non-parallel reflecting surfaces – wedge shaped film

Consider a wedge shaped thin air film enclosed between two plane surfaces OA and OB inclined at angle θ. As shown in figure the thickness of the film goes on increasing from O onwards.

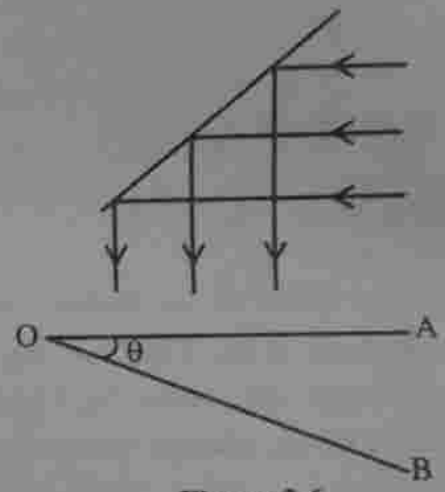


Figure 3.6

When a monochromatic parallel beam of light falls on such a film the rays reflected from upper and lower surface of the film interfere and produce fringes. These fringes are straight and are formed parallel to the edge of the film. (See footnote). The interfering rays do not enter the eye parallel to the edge but appear to diverge from a point near to the film. The effect is best observed when the angle of incidence is very small.

We know that the path difference between the reflected rays from a thin film is given by $2\mu t\cos r-\frac{\lambda}{2}$. If we are observing the reflected ray only in vertical direction then i will be small and consequently r will also be small and we can take $\cos r=1$.

$\therefore$ The path difference between the reflected rays $= 2\mu t-\frac{\lambda}{2}$.

For constructive interference

$$2\mu t-\frac{\lambda}{2}=n\lambda \quad (n=0, 1, 2,\ldots)$$

or
$$2\mu t=\left(n+\frac{1}{2}\right)\lambda \qquad \text{.....(27)}$$

We observe bright fringes at all places which satisfy the above condition.

For destructive interference

$$2\mu t-\frac{\lambda}{2}=(2n-1)\frac{\lambda}{2}$$

$$2\mu t=n\lambda \qquad \text{.....(28)}$$

We observe dark fringes at all places which satisfy the above condition.

Expression for fringe width in wedge shaped film

Suppose n^{th} dark fringe is formed at a distance x_n from O.

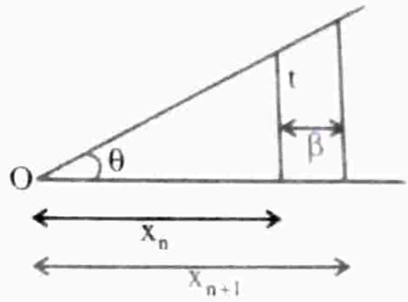


Figure 3.7

From the figure $\frac{t}{x_n} = \tan\theta$

or $t = x_n \tan\theta$

Using the condition for destructive interference (eqn 28), we get

$$2\mu x_n \tan\theta = n\lambda \quad .....(29)$$

Similarly for $(n+1)^{th}$ dark fringe, we get

$$2\mu x_{n+1} \tan\theta = (n+1)\lambda \quad .....(30)$$

eqn 30 – eqn 29 gives

$$2\mu \tan\theta (x_{n+1} - x_n) = \lambda$$

But $x_{n+1} - x_n = \beta$, the fringe width

Then $2\mu \tan\theta\beta = \lambda$

or $$\beta = \frac{\lambda}{2\mu \tan\theta}$$

If θ is small $\tan\theta \approx \theta$

$$\therefore \quad \beta = \frac{\lambda}{2\mu\theta} \quad .....(31)$$

Since the fringe width does not depend on n, all the fringes are of equal thickness. Fringe width increases in direct proportion to λ and in inverse proportion to wedge angle θ. If we determine β for any given wavelength λ, we can determine the angle of the wedge.

Note : This can be made use of for the determination of the wavelength of a given source of light and also can be used to determine the thickness of fine wires. (see example 7 and 9)

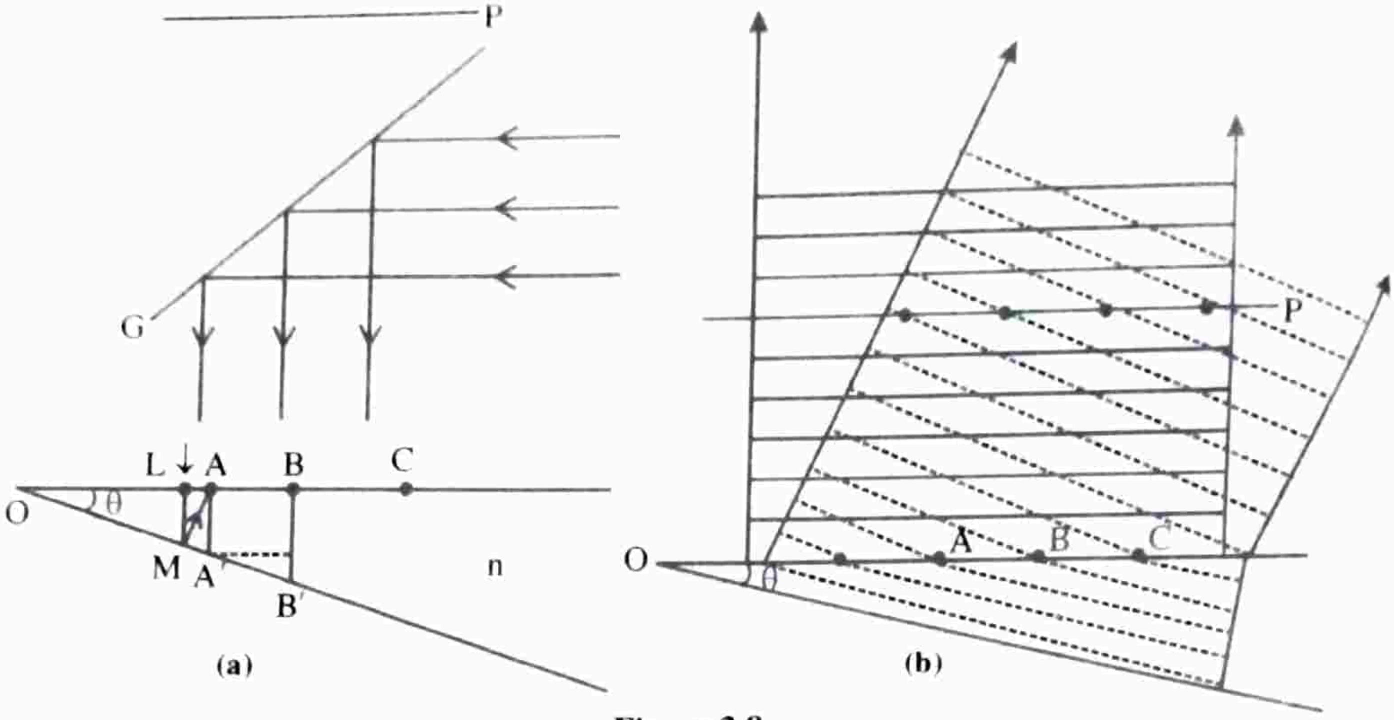


Figure 3.8

Suppose a parallel beam of light is incident normally on the upper surface of the film. In the successive positions of crests (at a particular instant of time) reflected from the upper surface and lower surface of the film are shown by solid and dashed lines respectively. Obviously a photographic plate P will record straight line interference fringes which will be parallel to the edge of the wedge. (The edge is the line passing through the point O and perpendicular to the plane of the paper).

Fringes produced by extended source of light

If the wedge is illuminated by extended source of light (white light) then at the edge of the film appears dark, and near to the edge a few coloured fringes are seen in the order of violet to red colour. Then there is overlapping of colours. And further for large thickness, the film becomes uniformly white. i.e. no fringe pattern will be observed.

Colours of thin films

It is a commonly observed phenomenon that a thin film of oil on water surface. oil spreads on roads, soap bubbles etc. when viewed under an extended source of white light (sun) appears to be beautifully coloured. It is due to the phenomenon of interference. When an observer views different regions of the film at different angles. depending on the angle and thickness of the film a particular wavelength in white light may satisfy the condition for constructive or destruction interference. The colours which satisfies the condition for constructive interference is present in the region. The colour which satisfies the condition for destructive interference is absent in the region and the corresponding complementary colour is present in the region. So different points on the film appear in different colours depending on the angle of reflected light coming to the observers eye from that point. As the observer change his position the thickness and angle of reflected light vary from point to point, different colours are seen.

Note : No colours are visible with thick films. Increasing the thickness of the film, causes the reflected rays to get far apart from each other. As the pupil of the eye is small only one of the two rays that produce interference can enter the eye at a time and interference cannot occur. To see the interference, the film should not be more than few wavelength thick.

Example 5

A non-reflecting film of refractive index μ_f is coated on a glass having refractive index μ_g. Show that $\mu_f = \sqrt{\mu_g \times \mu_a}$ where μ_a is the refractive index of air.

Solution

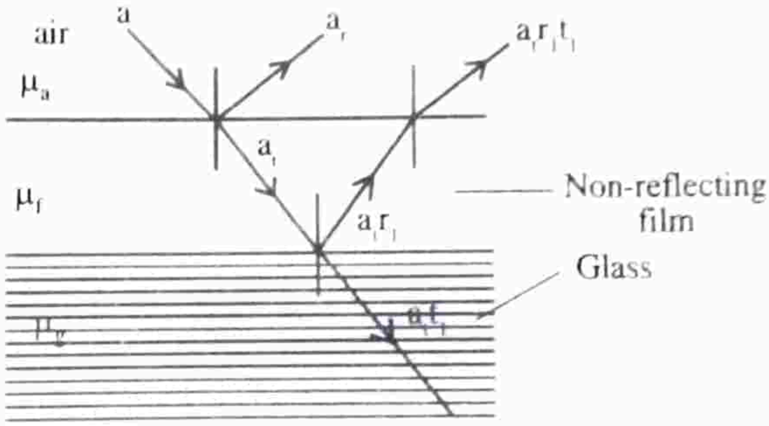


Figure 3.9

Let μ_a, μ_f and μ_g be the refractive indices of air, non-reflecting film and glass respectively. The amplitude reflection and transmission coefficients r and t are given by

$$r = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \quad \text{and}$$

$$t = \frac{2\mu_1}{\mu_1 + \mu_2}$$

Let a be the amplitude of the incident wave

∴ The amplitude of the reflected wave = a_r

$$= \frac{(\mu_a - \mu_f)}{(\mu_a + \mu_f)} a \qquad \text{.....(1)}$$

The amplitude of the transmitted wave in the film = a_t

$$= \frac{2\mu_a}{\mu_a + \mu_f} a \qquad \text{.....(2)}$$

The amplitude of the reflected wave from the glass surface = $a_t r_1$

$$= \frac{2\mu_a}{\mu_a + \mu_f} \cdot \frac{(\mu_f - \mu_g)}{(\mu_f + \mu_g)} \cdot a \qquad .....(3)$$

The amplitude of the transmitted wave from film to air = $a_t r_1 t_1$

$$= \frac{2\mu_a}{\mu_a + \mu_f} \frac{(\mu_f - \mu_g)}{(\mu_f + \mu_g)} \frac{2\mu_f}{\mu_f + \mu_a} a \qquad .....(4)$$

For complete destructive interference the amplitudes of the two waves in air (one reflected and the other transmitted) must be the same.

i.e. eqn (1) = eqn (4)

We get

$$\frac{\mu_a - \mu_f}{\mu_a + \mu_f} = \frac{4\mu_a\mu_f(\mu_f - \mu_g)}{(\mu_a + \mu_f)^2(\mu_f + \mu_g)}$$

Since $\frac{4\mu_a\mu_f}{(\mu_a + \mu_f)^2} \simeq 1$, we have

$$\frac{\mu_a - \mu_f}{\mu_a + \mu_f} = \frac{\mu_f - \mu_g}{\mu_f + \mu_g}$$

Cross multiplying we get

$$\mu_a\mu_f - \mu_f^2 + \mu_a\mu_g - \mu_f\mu_g = \mu_a\mu_f - \mu_a\mu_g + \mu_f^2 - \mu_f\mu_g$$

or $$2\mu_f^2 = 2\mu_a\mu_g$$

or $$\mu_f^2 = \mu_a\mu_g$$

$\therefore$ $$\mu_f = \sqrt{\mu_a \times \mu_g}$$

Note : It may be noted that the calculation is done for normal incidence but figure drawn is for oblique incidence. This is done for clarity.

Example 6

A light of wavelength 6000Å is falling on a glass plate $(\mu = 1.5)$. What is the refractive index and minimum coating thickness for minimum reflectivity.

Solution

$$\lambda = 6000 \times 10^{-10}\,\text{m}$$

$$\mu_g = 1.5$$

$$\therefore \qquad \mu_f = \sqrt{\mu_g} = \sqrt{1.5} = 1.225$$

minimum thickness of coating.

$$t = \frac{\lambda}{4\mu_f} = \frac{6000 \times 10^{-10}}{4 \times 1.225}$$

$$= 1.2245 \times 10^{-7}\,\text{m}$$

Example 7

In a wedge shaped film the distance between the successive fringes is measured to be 1.25 mm. The angle of the wedge is 40 seconds. Calculate the wavelength of light used. $\mu = 1.4$.

Solution

$$\beta = 1.25 \times 10^{-3}\,\text{m}, \ \theta = 40\text{s} = \frac{40}{3600} \text{ degree}$$

$$\theta = \frac{40}{3600} \times \frac{\pi}{180} \text{ radian } = 0.194 \times 10^{-3} \text{ radian}$$

$$\mu = 1.4$$

We have $\beta = \dfrac{\lambda}{2\mu\theta}$ or $\lambda = 2\beta\mu\theta$

$$\lambda = 2 \times 1.25 \times 10^{-3} \times 1.4 \times 0.194 \times 10^{-3}$$

$$\lambda = 6790 \times 10^{-10}\,\text{m}$$

Example 8

A wedge shaped air film is illuminated with a light of wavelength 4000Å. There are 15 fringes per cm. What is the angle of the wedge. Assume normal incidence.

Solution

$$\lambda = 4000 \times 10^{-10}\,\text{m}$$

Number of fringes per cm = 15

Number of fringes per m $= 15 \times 10^2$

$\therefore$ Fringe width, $\beta = \dfrac{1}{15 \times 10^2} = \dfrac{10^{-2}}{15}\,\text{m}$

Using $\beta = \dfrac{\lambda}{2\mu\theta}$

$$\therefore \quad \theta = \frac{\lambda}{2\mu\beta} = \frac{4000\times10^{-10}\times15}{2\times1\times10^{-2}}$$

$$= 3\times10^{-4} \text{ radian}$$

Example 9

Two plane glass surfaces in contact along one edge are separated at the opposite edge by a thin wire. If 20 fringes are observed between these edges in sodium light of normal incidence. What is the thickness of the wire. ($\lambda = 5890$Å)

Solution

If the thickness of the wire is t, then

$$2t = n\lambda$$

or $$t = \frac{n\lambda}{2} = \frac{20\times5890\times10^{-10}}{2} = 5.89\times10^{-6}\,\text{m}$$

Example 10

The thickness of a wedge shaped square cellophane film ($\mu = 1.5$) at two ends is t_1 and t_2 respectively. If the number of fringes seen on the film by the light of wavelength 6000Å is 10. Find the value of $t_2 - t_1$.

Solution

$\mu = 1.5$, $\lambda = 6000\times10^{-10}$m $n = 10$

If the side of the square film is x. then from the figure

$$\theta = \frac{t_2 - t_1}{x}$$

or $$t_2 - t_1 = x\theta$$

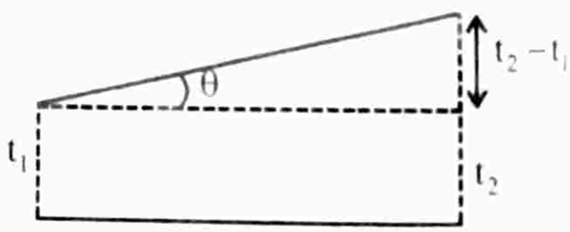


Figure 3.10

But $$\beta = \frac{x}{10} = \frac{\lambda}{2\mu\theta}$$

or $$\theta = \frac{10\lambda}{2\mu x}$$

$$\therefore \quad t_2 - t_1 = \frac{10\lambda}{2\mu} = \frac{5\lambda}{\mu} = \frac{5\times6000\times10^{-10}}{1.5}$$

$$= 2\times10^{-6}\,\text{m}.$$

Newtons rings

When we place a planoconvex on a plane glass plate, a thin film of air is formed between the lens and the glass plate. The thickness of the film at the point of contact is zero and increases as one moves away from the centre. When monochromatic light is allowed to fall on this arrangement normally (as shown in figure) interference takes place between the light reflected from the upper and lower surfaces of the air film and interference pattern can be observed using microscope. The fringes are in the form of concentric rings since the thickness of air film is constant over a circle having the point of lens-glass plate contact as the centre. These rings are called Newtons rings.

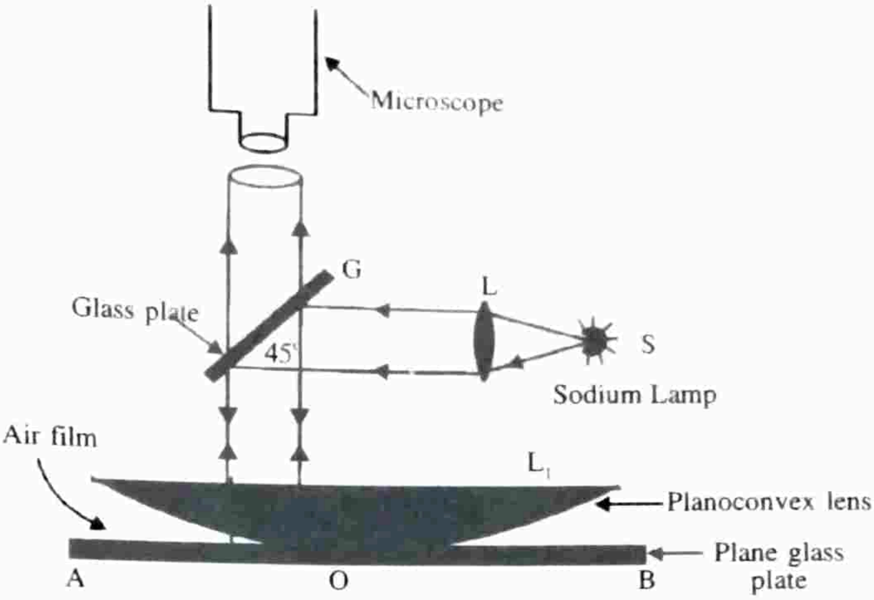


Figure 3.11

Theory

Let R be the radius of curvature of the lens. Let a dark fringe be located at Q. Let the thickness of the air film at Q be $PQ = t$, and radius of the circular fringe at Q be $OQ = r_n$. Using Pythagorus theorem we have

$$PM^2 = PN^2 + MN^2$$

i.e. $$R^2 = r_n^2 + (R-t)^2$$

$$R^2 = r_n^2 + R^2 - 2Rt + t^2$$

or $$r_n^2 = 2Rt - t^2$$

since $R \gg t,\ 2Rt \gg t^2$

hence t^2 can be neglected

$\therefore$ $$r_n^2 = 2Rt \qquad .....(1)$$

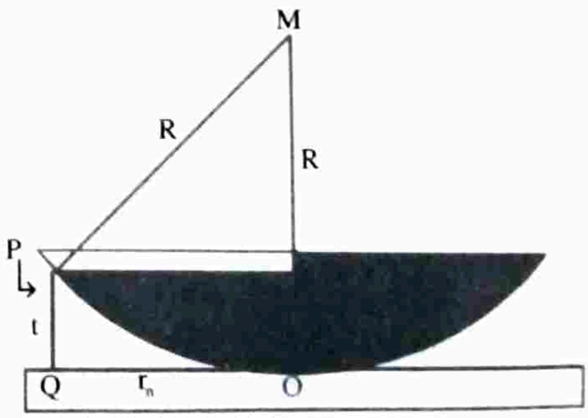


Figure 3.12

using the condition for darkness

$$2\mu t \cos r = n\lambda$$

$$\mu = 1 \text{ and } r = 0 \text{ (since incidence is normal)}$$

$\therefore$ $$2t = n\lambda$$

$\therefore$ equation 1, becomes

$$r_n^2 = n\lambda R \qquad .....(2)$$

or $r_n = \sqrt{n\lambda R}$ where $n = 1, 2, 3....$ it shows that the radii of dark rings are proportional to the square root of natural numbers.

$\therefore$ The diameter of the ring $D_n = 2r_n$

or $D_n = 2\sqrt{2Rt} = 2\sqrt{n\lambda R}, \qquad n = 1, 2, 3, ...$

Similarly the diameter of the $(n+1)^{th}$ bright ring will be

$$D_{n+1} = 2\sqrt{(n+\tfrac{1}{2})\lambda R}$$

or $$D_n = 2\sqrt{(n-\tfrac{1}{2})\lambda R} \text{ for } n^{th} \text{ ring.}$$

Since thickness of the air film at the centre is zero, the path difference at the centre is zero and a dark spot is surrounded by the first bright ring.

Let us consider the $(n+k)^{th}$ dark ring. The radius of the $(n+k)^{th}$ ring be r_{n+k}. Then

$$r_n^2 = n\lambda R \qquad .....(3)$$

$$D_n^2 = 4n\lambda R$$

$$D_{n+k}^2 = 4(n+k)\lambda R \qquad .....(4)$$

$\therefore$ $$D_{n+k}^2 - D_n^2 = 4k\lambda R$$

So the difference of the squares of diameters of $(n+k)^{th}$ ring and n^{th} ring is independent of n and only depends on k.

$\therefore$ $$\lambda = \frac{D_{n+k}^2 - D_n^2}{4kR}.$$

In this way the wave length of monochromatic light can be found using Newton's rings.

Note : If the medium other than air, say liquid, forms the film λ can be calculated by

$$\lambda = \mu \frac{D_{n+k}^2 - D_n^2}{4kR}$$ where μ is the refractive index of the liquid.

Determination of wavelength of monochromatic light

The experimental arrangement (see figure 1.11) for observing Newtons rings consists of a monochromatic light S whose wavelength λ to be determined. placed at the focus of a lens L. The rays emerging out through the lens are parallel. These horizontal parallel rays fall on a glass plate G kept at 45° and are partially reflected from it. These reflected rays fall normally on the planoconvex L_1 placed on the glass plate AB. It is due to the superposition of two reflected rays one from the top and the other from the bottom surfaces of the glass plate interference rings are formed. These rings are observed through a microscope arranged vertically above the glass plate G. The microscope is focussed well so that the rings are clearly seen. Then by working the tangential screw, the point of intersection of the cross wire is kept at the central dark. Then the microscope is moved to the left and to the right in order to ensure that about 30 dark rings are clearly seen.

Starting from the central spot the microscope is moved to the left by working the tangential screw. The tangential screw is slowly adjusted so that the cross wire is tangential to the 28th dark ring. The microscope reading on the horizontal scale is taken. Then by working the tangential screw the cross wire is kept tangential to the 26th, 24th, 22nd etc. dark rings up to the second ring on the left and the reading to each ring is taken. Then by working the tangential screw, the microscope is moved in the same direction until the cross wire is tangential to the second dark ring on the right. The corresponding reading is taken. Similarly the cross wire is kept tangential to the 4th, 6th, 8th etc. dark rings up to the 28th dark ring on the right. Then reading corresponding to each ring is taken. (The tangential screw should be worked only in one direction from the position of the 28th ring on the left to the position of the 28th ring on the right. This is to avoid back lash error).

The difference in readings on the left and right at each ring gives its diameters. The value of D^2 is calculated. The values of $D_{n+k}^2 - D_n^2$ are calculated for a value of $k = 10$. Then the mean value of $D_{n+k}^2 - D_n^2$ is found.

The focal length of the lens is determined by the plane mirror method as it is a long focus lens. For this the convex lens is placed in front of an illuminated wire gauze with a plane mirror held behind the lens. The distance of the lens is adjusted to get a clear image of the wire gauze side by side with it. The distance between the lens and wire gauze is measured. This gives the focal length of the lens. Repeat this three or four times take the average value. The radius of curvature of the lower surface of the lens is found by Boy's method. For this convex lens is placed in front of an illuminated wire gauze with a piece of black paper held behind the lens. The distance of the lens is adjusted to get a clear image at the wire gauze side by side with it. The distance d between the lens and wire gauze is measured. The experiment is repeated and the average value of d is found. Then the radius of curvature of the convex lens is calculated using the formula (Boy's method)

$$R = \frac{fd}{f-d}$$

The wavelength of the given light is hence calculated using the formula

$$\lambda = \frac{D^2_{n+k} - D^2_n}{4kR}, \text{ k may be taken to be 10.}$$

Determination of refractive index of liquid by Newtons rings

In the experimental arrangement shown above one or two drops of liquid whose refractive index is to be determined is put between lens and glass plate.

When the film is of refractive index μ, the diameter of the n^{th} dark ring is

$$D'^2_n = \frac{4n\lambda R}{\mu} \qquad .....(1)$$

For the $(n+k)^{th}$ dark ring

$$D'^2_{n+k} = \frac{4(n+k)}{\mu}\lambda R \qquad .....(2)$$

$$\therefore \qquad D'^2_{n+k} - D'^2_n = \frac{4k\lambda R}{\mu} \qquad .....(3)$$

For air

$$D^2_{n+k} - D^2_n = 4k\lambda R \qquad .....(4)$$

$\frac{\text{eqn 4}}{\text{eqn 3}}$ gives

$$\mu = \frac{D^2_{n+k} - D^2_n}{D'^2_{n+k} - D'^2_n} \qquad .....(5)$$

First the diameters of n^{th} and $(n+k)^{th}$ dark rings are measured when the film is air. And the diameters of same rings are measured when the film is of liquid. Then by using eqn (5), μ can be determined.

Testing of optical flatness

An important application of the principle involved in the Newton's rings experi ment lies in the determination of the optical flatness of a glass plate. When a beam of parallel monochromatic light is allowed to fall on a film with flat surfaces equal thickness fringes will be observed. In other words if the fringes obtained are not of equal thickness it means that the surfaces are not optically flat.

To test whether a given surface is optically flat or not, take an optically plane surface OA and surface OB to be tested. A wedge shaped air film of varying thickness is formed in between two surfaces. The fringes so obtained are observed through a microscope and if the fringes are not of equal thickness, the surface OB is not flat. Polishing the surface and the process is repeated till fringes of equal thickness are observed. This is how manufactures of optical flats detect even minute deviations from true optical flatness.

Example 11

Newtons rings are observed in reflected light of $\lambda = 5.9\times10^{-7}$ m. The diameter of the 10^{th} dark ring is 0.5 cm. Find the radius of curvature of the lens and the thickness of air film.

Solution

$$\lambda = 5.9\times10^{-7}\,\text{m},\ D_{10} = 0.5\,\text{cm} = 0.5\times10^{-2}\,\text{m},\ n = 10$$

We have

$$D_n^2 = 4\,n\,R\,\lambda$$

$$\therefore \qquad R = \frac{D_n^2}{4n\lambda} = \frac{0.5\times10^{-2}}{4\times10\times5.9\times10^{-7}} = 1.059\,\text{m}$$

Using $\quad 2t = n\lambda$

$$t = \frac{n\lambda}{2} = \frac{10\times5.9\times10^{-7}}{2}$$

$$= 29.5\times10^{-7}\,\text{m}.$$

Example 12

Newtons rings are formed with a monochromatic light between a flat glass plate and a convex lens are viewed normally. What will be the wavelength of light used if the diameter of the 20^{th} bright ring is 0.56cm, R = 1.2m.

Solution

$$D_{20} = 0.56\,\text{cm} = 0.56\times10^{-2}\,\text{m} \ \text{and}\ R = 1.2\,\text{m}.$$

The diameter of the n^{th} bright ring is given by

$$D_n = 2\sqrt{(n-\tfrac{1}{2})\lambda R}$$

Squaring and rearranging, we get

$$\lambda = \frac{D_n^2}{4(n-\frac{1}{2})R}$$

$$\lambda = \frac{(0.56\times10^{-2})^2}{4(20-\frac{1}{2})\times1.2} = \frac{0.56^2\times10^{-4}}{4\times19.5\times1.2}$$

$$\lambda = 3.35\times10^{-7}\,\text{m}.$$

Example 13

A Newton's ring arrangement is used with a source emitting two wave lengths $\lambda_1 = 6\times10^{-7}$m and $\lambda_2 = 4.5\times10^{-7}$m is found that the n^{th} dark ring due to λ_1 coincides with $(n+1)^{th}$ dark ring due to λ_2. Find the diameter of the n^{th} dark ring for λ_1. $R = 90$cm.

Solution

$\lambda_1 = 6\times10^{-7}$m, $\lambda_2 = 45\times10^{-7}$m and $R = 90\text{cm} = .90\text{m}$.

The diameter of n^{th} dark ring $D_n = 2\sqrt{nR\lambda}$

As the n^{th} dark ring due to λ_1 coincides with $(n+1)^{th}$ dark ring due to λ_2, the diameters are the same, then we have

$$2\sqrt{nR\lambda_1} = 2\sqrt{(n+1)R\lambda_2}$$

$\therefore \quad n\lambda_1 = (n+1)\lambda_2$

$$n\times6\times10^{-7} = (n+1)\times4.5\times10^{-7}$$

$$6n = 4.5n + 4.5$$

or $\quad 1.5n = 4.5$

$\therefore \quad n = 3.$

Then $\quad D_3 = 2\sqrt{3\times.90\times6\times10^{-7}} = 2.545\times10^{-3}\,\text{m}.$

Example 14

In the Newtons rings arrangement, the radius of curvature of the curved surface is 50 cm. The radii of the 9^{th} and 16^{th} dark rings are 0.18cm and 0.2235cm respectively. Calculate the wavelength.

Solution

$$R = 50\text{cm} = 0.5\text{m}$$

$$r_9 = 0.18\text{cm} = 0.18\times10^{-2}\text{m}$$

$\therefore \quad D_9 = 2 \times r_9 = 0.36 \times 10^{-2}\,m$

$r_{16} = 0.2235\,cm = 0.2235 \times 10^{-2}\,m$

$\therefore \quad D_{16} = 2 \times r_{16} = 0.4470 \times 10^{-2}\,m$

Using
$$\lambda = \frac{D_{n+k}^2 - D_n^2}{4kR}$$

$$\lambda = \frac{D_{16}^2 - D_9^2}{4 \times 7 \times R} = \frac{(0.4470 \times 10^{-2})^2 - (0.36 \times 10^{-2})^2}{4 \times 7 \times 0.5}$$

$$\lambda = \frac{(19.98 - 12.96) \times 10^{-6}}{14}$$

$$\lambda = 5.014 \times 10^{-7}\,m.$$

Example 15

In Newton's ring experiment the radius of curvature of a lens is 5m and its diameter is 2cm. Calculate the total number of rings formed. If the wavelength of incident monochromatic light is 5500Å. Also calculate the number of rings if the system is kept in water $(\mu = 1.33)$

Solution

$D_n = 2 \times 10^{-2}\,m,\; R = 5m,\; \lambda = 5500 \times 10^{-10}$

$\mu = 1.33,\; r_n = 1 \times 10^{-2}\,m$

$$r_n = \sqrt{n\lambda R}$$

or
$$n = \frac{r_n^2}{\lambda R} = \frac{(10^{-2})^2}{5500 \times 10^{-10} \times 5} = 36.36$$

Therefore 36 dark and 36 bright rings are formed

When the system is kept in water

$$r_n = \sqrt{\frac{n\lambda R}{\mu}}$$

or
$$n = \frac{r_n^2 \mu}{\lambda R} = 36.36 \times 1.33$$
$$= 48.35$$

Therefore in water 48 dark and 48 bright fringes are formed.

Interferometers

Instruments based on the principle of interference of light are called as interferometers. These instruments use large path differences between the two interfering beams to obtain interference. They are used to study the fine structure of spectral lines and variation of refractive indices of glass and also of liquids at different conditions such as pressure, temperature and density. The interferometers used for the study of refractive indices are called as refractometers.

Michelson's interferometer

Prof. A.A. Michelson (1852-1931) devised an interferometer based on the principle of interference called Michelson's interferometer. It consists of two highly polished front silvered plane mirrors M_1 and M_2 mounted mutually perpendicular to each other. The mirror M_1 is mounted on a fine pitch screw (capable of reading up to 10^{-5} cm accurately) by which M_1 can be moved to and fro exactly parallel to itself. The mirror M_2 is fixed and provided with levelling screws at its back for adjusting the plane of the mirror. G_1 and G_2 are optically plane and equally thick parallel plates of glass held parallel to each other and at 45° to mirrors M_1 and M_2. G_1 is semi silvered at its back surface and G_2 is plane. The plane of G_2 can be slightly adjusted by means of fine screw. The plate G_2 acts as a compensating plate and required essentially to work with white light source.

Working

Light from a monochromatic source S after being rendered parallel by a convex lens falls on the glass plate G_1. At G_1 the beam is divided into two parts, one part goes to M_1 after reflection and the other part is transmitted and goes to M_2. Both the rays falls on the mirrors normally and retrace their paths. The ray coming from M_1 is partially transmitted at G_1 and proceeds along AT. The ray coming from M_2 gets reflected at G_1 and proceeds along AT. Thus the telescope T receives two sources. Since these two are derived from the same source by the process of division of amplitude, they are coherent. The two beams entering the telescope differ in their path lengths. This path difference between the two interfering rays arises due to the difference in distances of M_1 and M_2 from G_1 and also due to the fact that the rays travelling from M_2 to G_1 suffer a phase change of π when it is reflected at G_1. The two beams incident on the telescope interfere with each other and fringes are observed in the field of view of telescope T.

Function of compensating plate G_2

The two rays travel through glass unequal distances before reaching the telescope. The ray going towards M_1 and reaching the telescope passes through the glass plate thrice while the other ray passes the glass plate only once. To equalise this a compensating plate G_2 is introduced between the plates.

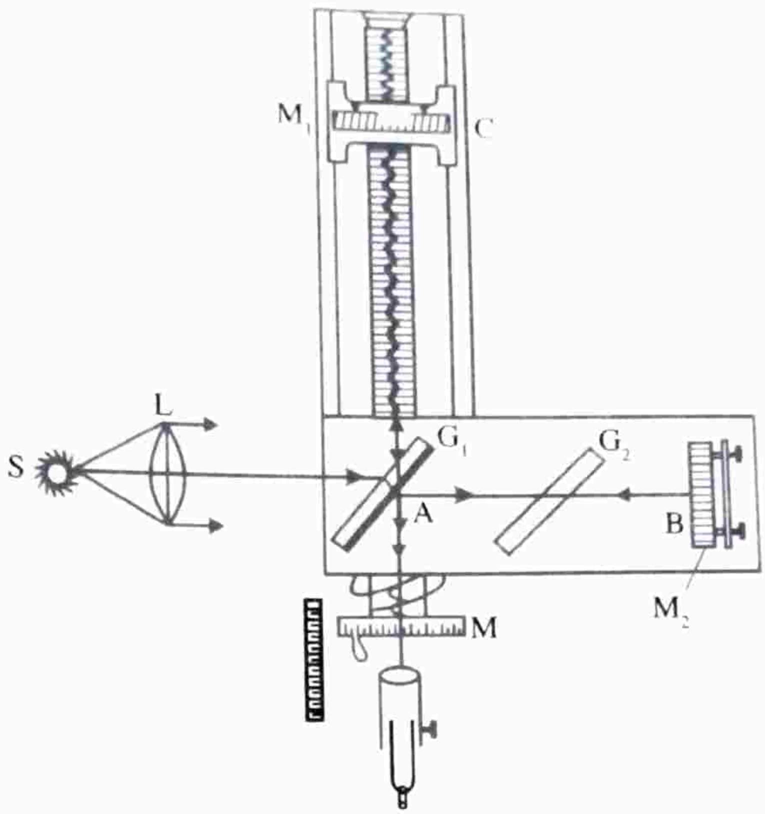


Figure 3.13

Adjustments of the setup

Michelson interferometer is to be adjusted before using it for doing experiments. The following adjustments are to be done to obtain fringes.

(i) The interferometer is illuminated with a broad source of monochromatic light. A sodium vapour lamp or mercury vapour lamp provide this. If the source is small a convex lens can be made use of to broaden the source.

(ii) The distances of the mirrors M_1 and M_2 from the back surface of the plate G_1 must be almost equal to within a millimetre.

(iii) The mirror M_2 is made perpendicular to the mirror M_1. To do this a pin is placed between the source S and the plate G_1. On looking through the telescope four images (two pairs) are seen. One pair is due to reflections from the silvered part of G_1 and the other pair is due to the reflection from the unsilvered part of G_1. By adjusting the screws at the back of one of the mirrors (M_1 or M_2), the weaker images are made to coincide with the stronger ones. Now only two images are seen through the telescope. After this being done the pin is removed.

When the adjustments are over circular fringes can be seen through the telescope. Further the centre of the fringe system can be brought in the field view by slightly adjusting the screws provided at the back of the mirror.

Types of fringes

The shape of the fringes formed in Michelson's interferometer depends on the inclination of M_1 and M_2. This can be easily understood from the figure below. Let

M_2' be the virtual image of mirror M_2 formed by the reflection in G_1 and S_0 be the image of the source formed in G_1. The interference pattern may be regarded due to the rays of light reflected from the surface of M_1 (real) and M_2' (virtual). Therefore the interference will be due to the air film enclosed between the two mirrors M_1 and M_2'. If S_1 and S_2 are the images of the source in the mirror M_1 and M_2'. Then we can consider that the whole arrangement produces two coherent sources by the process of division of amplitude.

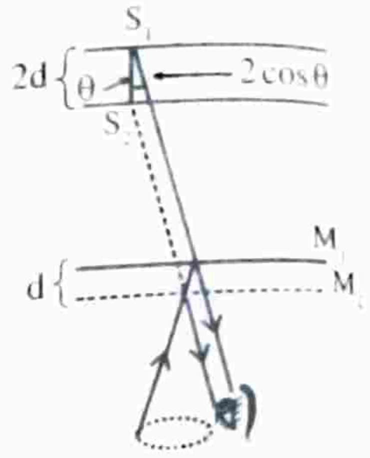


Figure 3.14

Circular fringes

When the mirrors M_1 and M_2 are exactly at right angles to each other, then the image M_2' is parallel to M_1. If the difference in the distances of mirror M_1 and M_2 from G_1 is d. Then there will be a air film of constant thickness d between M_1 and M_2 and the virtual coherent sources S_1 and S_2 will also be parallel but separated by a distance 2d. And the path difference between the rays coming from S_1 and S_2 and inclined at an angle θ is $2d\cos\theta$.

If the two rays entering the telescope satisfy the condition

$$2d\cos\theta - \frac{\lambda}{2} = n\lambda \text{ (see foot note)}$$

or

$$2d\cos\theta = \left(n + \frac{1}{2}\right)\lambda$$

Then the two rays are in a condition of constructive interference and therefore a bright circular fringe is seen at all those points which are at inclination θ.

If the path difference $\quad 2d\cos\theta - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$

$$2d\cos\theta = n\lambda$$

Then the two rays are destructively interfere and there a dark circular fringe is seen at the inclination θ.

When $\theta = 0$, then the path difference between the two rays is 2d.

If $2d = \left(n + \frac{1}{2}\right)\lambda$ the centre is bright and when $2d = n\lambda$ the centre is dark.

Note : Since the beam splitter (G_1) is just a simple glass plate, the beam reflected from the mirror M_2 will undergo a phase change π, this will introduce an additional path difference $\frac{\lambda}{2}$ in $2d\cos\theta$.

Localised fringes

When the mirrors M_1 and M_2 are kept not exactly at right angles to each other, then M_1 and M'_2 will not be parallel, hence space between them is a wedge shaped air film. Thus we get equally spaced (fringes of equal thickness) localised fringes. The fringes are nearly straight but slightly curved.

White light fringes

When white light is used instead of monochromatic light no fringes will be seen except for a path difference so small that it does not exceed a few wavelengths. When the path difference is very small the different colours overlap hence only first few coloured fringes are seen. Here the central fringe corresponding to zero path difference is dark.

Uses of Michelson's interferometer

Michelson's interferometer is used for variety of purposes. Here we discuss only three of them.

(i) Determination of the wavelength of monochromatic light

Make the necessary adjustments of the interferometer with monochromatic light whose wavelength is to be determined. By adjusting the mirror M_1 bright spot is obtained at the centre. Telescope is focused on the bright centre spot. The mirror M_1 is moved slowly backward or forward, the number of bright fringes crossing at the centre is counted. The distance through which mirror moved is read from the micrometer screw.

If the difference between the distances of M_1 and M_2 from G_1 is d, then for the dark spot at the centre, we have

$$2d = n\lambda$$

where n is the order of the fringe.

If now M_1 is further moved away by a distance $\frac{\lambda}{2}$, 2d changes by λ and again we get dark spot at the centre. Now the dark spot at the centre will be $(n+1)$ instead of n. Thus each time when M_1 is moved by a distance $\frac{\lambda}{2}$ one fringe will cross the field of view.

Now moving the mirror M_1 by a distance x, if the number of fringes which cross the field of view is N. Then

$$x = N \cdot \frac{\lambda}{2}$$

or $$\lambda = \frac{2x}{N}$$

Knowing N and x the wavelength can be calculated.

(ii) Determination of the difference between wavelengths of two close spectral lines

When the source of light emits composite light of wavelengths λ_1 and λ_2 are used to illuminate interferometer then each source produces its own fringes. As a result of superposition of these fringes the resultant fringes may or may not be clearly visible. When the bright fringe produced by one source falls on the bright fringe produced by the other source then the fringes observed are brightest and clear. This can be done by adjusting the mirror M_1. The displacement in the mirror M_1 for two successive clear positions is obtained on the micrometer screw. Let this distance be x. Then the path difference be 2x. If the number of fringes of wavelength λ_1 in this path be n and the number of fringes corresponding to fringes of wavelength λ_2 will be $n+1$.

$\therefore$ $$2x = n\lambda_1 = (n+1)\lambda_2$$

or $$n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

or $$2x = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2} \approx \frac{\lambda^2}{\lambda_1 - \lambda_2} \qquad (\because \quad \lambda_1\lambda_2 \simeq \lambda^2)$$

or $$\lambda_1 - \lambda_2 = \frac{\lambda^2}{2x}$$

Knowing λ (geometric mean of two wavelengths) and x, the difference in the wavelengths $\lambda_1 - \lambda_2$ can be calculated.

Note : In the experiment the reading of ten successive positions of maxima are taken and mean value of x is found.

(iii) Standardisation of the metre

One of the most outstanding uses of Michelson's interferometer is to measure length in terms of wavelengths very accurately. When the mirror M_1 is moved slowly and one position to another counting the number of fringes in monochromatic light which cross the centre of the field of view will give a measure of the distance the mirror has moved in terms of λ. Let x_1 be the position corresponding to the dark fringe of order n_1 and x_2 be the position corresponding to the dark fringe of order n_2.

we have

$$2x_1 = n_1\lambda$$

and
$$2x_2 = n_2\lambda$$

or
$$x_1 - x_2 = (n_1 - n_2)\frac{\lambda}{2}$$

Hence the distance moved equals the number of fringes counted multiplied by a half wavelength. The distance measured need not correspond to an integral number of half wavelengths. Fractional parts of a whole fringe displacement can easily be estimated to one fiftieth of a fringe. This gives the distance to an accuracy of one hundredth wavelength.

The most important measurement made with the interferometer by using the above principle was the comparison of the standard metre, kept at the International Bureau of weights and Measures at serves near Paris, with the wavelengths of intense red, green and blue lines of Cadmium. This was first performed by Michelson and Benoit in 1895. But it would be impossible to count directly the number of fringes for a displacement of the movable mirror from one end of the standard metre to the other. This is because for a path difference of more than 20 cm it is not possible to obtain fringes. To circumvent this difficulty Michelson and Benoit divided the metre into a number of substandards called etalons. Each etalon is twice the length of the preceding one. The length of the largest etalon was 10 cm and that of the shortest one was 0.039 cm. The smallest etalon was calibrated in terms of wavelengths by actually counting the number of fringes which cross the cross wire of the telescope. This etalon was then compared with the next larger etalon and so on until the longest etalon was reached. Finally the length of the longest etalon i.e. 10 cm was then compared with standard metre. With this Michelson and Benoit found that one standard metre consists of 1,553,164.13 wavelengths of red light of cadmium, 1,966,249.7 wavelengths of green light of cadmium and 2,083,372.7 wavelengths of blue light of cadmium at 15°C and 76 cms of mercury. The values are correct up to 1 part in 10^9. After this the international standard metre is redefined to be consists of 1,650,763.73 wavelengths of orange-red light of Krypton atom.

Example 16

In a Michelson interferometre it is found that 200 fringes cross the field of view when the movable mirror is displaced through 0.049 mm. Calculate the wavelength of light used.

Solution

$$N = 200, \ x = 0.049 \times 10^{-3}\,\text{m}$$

Using $$\lambda = \frac{2x}{N} = \frac{2 \times 0.049 \times 10^{-3}}{200}$$

$$= 4900 \times 10^{-10}\,\text{m}$$

Example 17

In a Michelson's interferometer the distance travelled by the mirror for two successive positions of maximum distances was 0.289mm. Calculate the difference between the wavelengths. $\lambda = 5890\,\text{Å}$.

Solution

We have $$\lambda_1 - \lambda_2 = \frac{\lambda^2}{2x}$$

$$= \frac{(5890 \times 10^{-10})^2}{2 \times 0.289 \times 10^{-3}}$$

$$= 6 \times 10^{-10}\,\text{m}\cdot$$

Example 18

In Michelson's interferometer experiment, calculate the various values of θ corresponding to bright rings for $d = 5 \times 10^{-3}\,\text{cm}$. Show that if d is decreased to $4.997 \times 10^{-3}\,\text{cm}$ the fringe corresponding to $n = 200$ disappears. What will be the corresponding value of θ. ($\lambda = 5 \times 10^{-5}\,\text{cm}$ given)

Solution

The condition for bright ring is

$$2d\cos\theta = \left(n + \frac{1}{2}\right)\lambda$$

or $$\cos\theta = \frac{\left(n + \frac{1}{2}\right)\lambda}{2d} = \frac{(2n+1)\lambda}{4d}$$

$$\cos\theta = \frac{(2n+1)5 \times 10^{-7}}{4 \times 5 \times 10^{-5}}$$

$$\cos\theta = \frac{2n+1}{400}$$

For the central bright, $\cos\theta = 1 \quad \because \quad \theta = 0$

i.e. $$\frac{2n+1}{400} = 1 \quad \text{or} \quad 2n + 1 = 400$$

or $$n = \frac{399}{2}$$

$$\theta = \cos^{-1}\frac{(2n+1)}{400}$$

Put 2n + 1 = 400, 399, 398 ..... etc. we get bright fringes at these angles.

$\theta = 0^\circ$, 4.052°, 5.731°,.... corresponding to 2n + 1 = 400, 399, 398 .....

(ii) Now $d = 4.997 \times 10^{-5}\,m$. The condition for dark is $2d\cos\theta = n\lambda$

Then $$\cos\theta = \frac{n\lambda}{2d} = \frac{n \times 5 \times 10^{-7}}{2 \times 4.997 \times 10^{-5}}$$

$$\cos\theta = \frac{n}{199.88} = \frac{n}{200}$$

For the central fringe $\theta = 0$ and $\cos\theta = 1$

$\therefore$ $$1 = \frac{n}{200}$$

or $$n = 200$$

It shows that for n = 200, we obtain the central fringe and the condition for darkness is satisfied. Hence the 200th fringe disppears.

To find the angles $$\theta = \cos^{-1}\left(\frac{n}{200}\right)$$

The angles correspond to n = 200, 199, 198, 197, ... are 0°, 5.73°, 8.11°, 9.936°....

UNIVERSITY MODEL QUESTIONS

Section A

(*Answer questions in* **two** *or* **three** *sentences*)

Short answer type questions

1. What is the interference of light by a plane film?
2. Why a thick film cannot produce interference when illuminated with white light?
3. What are the conditions for brightness and darkness of normal incidence of light on a plane film producing interference?
4. Write down the conditions of brightness and darkness of oblique incidence of light on plane film producing interference.

5. What are Newton's rings? Give two of its uses.
6. Explain why the centre of Newton's rings is dark for reflected light.
7. Why an extended source of light is essential to observe colours in thin films?
8. Write down the cosine law and explain the symbols used.
9. What is reflectivity? What is its relation to amplitude reflection coefficient?
10. What is a non-reflecting film? How it can be achieved?
11. What are the conditions to be satisfied for a non-reflecting material?
12. What is an air wedge?
13. What are the conditions for constructive and destructive interferences formed in a film with non-parallel reflecting surfaces?
14. What are the uses of interference on wedge shaped film?
15. An extended source of light falls on a wedge shaped film. Discuss about the fringes so formed.
16. What is an interferometer?
17. What are the uses of Michelson's interferometer?
18. What is the function of a compensating plate in Michelson's interferometer?
19. What are the conditions to be satisfied to obtain constructive and destructive interferences in Michelson's interferometer?
20. What is the principle behind the standardisation of metre using Michelson's interferometer?
21. Why an extended source of light is essential to observe colours in thin films?
22. What do you understand by
    (i) fringes of equal thickness (ii) fringes of equal inclination
23. State conditions for obtaining
    (i) circular fringes (ii) straight lines fringes
    (iii) white light fringes in Michelson's interferometer
24. What are the adjustments of the Michelson's interferometer for doing experiments with it?
25. Define standard metre in terms of wavelength of cadmium.
26. What are white light fringes?

Section B

(Answer questions in a **paragraph** *of about half a page to one page)*

Paragraph / Problem type questions

1. Derive the condition for brightness for normal incidence of light on a plane film.
2. Derive the cosine law.

3. Derive an expression for fringe width in wedge shaped film.
4. Explain the phenomenon of colours of thin films.
5. How will you determine the refractive index of a liquid by Newton's rings?
6. How will you test the optical flatness of a glass plate?
7. Prove that an excessive thin film appears black in the reflected light.
8. White light falls perpendicularly upon a film of soapy water whose thickness is 5×10^{-7} m and index of refraction 1.33. Which wavelength in the visible region will be reflected most strongly. [5320Å will be reflected]
9. Light of wavelength 5880Å is incident on a thin film of glass of $\mu = 1.5$ such that the angle of refraction in the plate is 60°. Calculate the smallest thickness of the plate which will make it dark by reflection. $[3.72\times10^{-7}\,m]$
10. When light is incident normally on a glass plate of thickness 0.5×10^{-6} m and index of refraction 1.5. Which wavelengths in the visible region (400nm to 700nm) are strongly reflected by the plate [429nm, 600nm]
11. Using light of $\lambda = 5.9\times10^{-7}$ m, it is found that in a thin film of air 7.4 fringes occur between two points. Deduce the difference of the film thickness between these points. $[2.183\times10^{-6}\,m]$
12. There is a wedge shaped air film between two glass plates. The plates are in contact along one edge and are separated by a wire of diameter 0.05 mm placed at a distance of 15 cm from the edge. Find the fringe width when light of wavelength 6000Å is incident normally. $[9\times10^{-4}\,m]$
13. A parallel beam of white light is allowed to fall on the slit of the collimator of a telescope and a thin film of air of thickness enclosed between two glass plates is placed in front of the slit with the surface of the air film perpendicular to the path of light. On seeing through the telescope 250 fringes are observed at the spectral region between wavelengths 4000Å and 6500Å. Calculate the thickness of the air film $[0.013\times10^{-2}\,m]$
14. The angle of an air wedge is 24′. Light having wavelengths 4732Å and 5460Å is incident on the wedge normally. Find the distance from the edges in contact at which the maxima due to each wavelength coincide for the first time in the reflected light. $[2.54\times10^{-2}\,m]$
15. A vertical soap film of length 10cm is viewed by reflected monochromatic light of wavelength 5893Å. Just before the film breaks there are 12 black and 11 bright fringes. Find the angle of wedge formed and thickness of film at the base just before it breaks. $[2.44\times10^{-4}\,cm]$
16. Newton's rings are formed by reflected light of wavelength 6250Å with a liquid between the plane and curved surfaces. If the diameter of the 10^{th} bright ring is 5mm and

radius of curvature of the curved surface is 1.2m. Calculate the refractive index of the liquid. [$\mu = 1.4$]

17. In the Newton's rings arrangement, the radius of the curvature of the curved side of the planoconvex lens is 1m. For $\lambda = 6\times10^{-7}$ m, what will be the radii of the 10^{th} and 20^{th} bright rings.
$\begin{bmatrix} r_{10} = 2.387\times10^{-3}\text{m} \\ r_{20} = 3.421\times10^{-3}\text{m} \end{bmatrix}$

18. The diameter of the n^{th} ring changes from 1.2cm to 1.04cm when the air space between the lens and the plate is replaced by a liquid. Find the refractive index of liquid. [$\mu = 1.13$]

19. Newton's rings are formed with red light of $\lambda = 670$nm. The radius of the 20^{th} ring is found to be 1.1×10^{-2}m. Find the radius of curvature of the 30^{th} ring. [$r_{30} = 1.347\times10^{-2}$m]

20. In the Michelson's interferometer arrangement, if one of the mirrors is moved by a distance of 0.08mm, 250 fringes cross the field of view. Calculate the wavelength. [6000 Å]

21. The Michelson's interferometer experiment is performed with a source which consists of two wavelengths 4882Å and 4886Å. Through what distance does the mirror have to be moved between two positions of the disappearance of the fringes. [0.298mm]

22. In Michelson's interferometer calculate the various values of θ corresponding to dark rings. For d = 0.3mm and $\lambda = 6\times10^{-5}$cm. [0°, 2.56°, 3.62°, 4.44°, 5.13° ...]

Section C

(Answer questions in about **one** *or* **two** *pages)*

Long answer type questions (Essays)

1. Derive an expression for the conditions of brightness and darkness produced under oblique incidence of light on a plane film producing interference due to reflected light.
2. Explain the formation of Newton's rings. How can these be used to determine the wavelength of monochromatic light.
3. What are Newton's rings? Derive an expression for the radii of rings.
4. Describe Michelson's interferometer. How will you determine the wavelength of monochromatic light with the help of Michelson's interferometer?
5. Describe the construction with diagram and outline the theory of Michelson's interferometer. Discuss the nature of interference pattern produced.
6. How can Michelson's interferometer is used for
   (i) Determining the difference of wavelengths of two nearby spectral lines.
   (ii) Calibrating standard metre.
7. Explain the theory of interference formed in a wedge shaped film. Derive an expression for fringe width. How will you determine the wavelength of given light using air wedge?

Hint to problems

1 to 7 see book work

8. The condition for brightness is

$$2\mu t\cos r = (2n+1)\frac{\lambda}{2}$$

$$\therefore \quad \lambda = \frac{4\mu t\cos r}{(2n+1)},\ r = 0 \quad \therefore \quad \cos r = 1$$

Put n = 0 n = 0, 1, 2, 3 in the above we get

$$\lambda_o = 26600\text{Å}$$
$$\lambda_1 = 8866\text{Å}$$
$$\lambda_2 = 5320\text{Å}$$
$$\lambda_3 = 3800\text{Å}$$

out of these only λ_2 lies in the visible spectrum. (See also example 2)

9. For the film to be appear dark by reflected light, we have

$$2\mu t\cos r = n\lambda$$

For smallest thickness n = 1

$$\therefore \quad t = \frac{\lambda}{2\mu\cos r} = \frac{5880\times10^{-10}}{2\times1.5\times\cos 60} = 3.72\times10^{-7}\text{m}$$

10. The condition for brightness is

$$2\mu t\cos r = (2n+1)\frac{\lambda}{2} \quad n = 0, 1, 2, \ldots$$

$$\cos r = 1 \quad \because \quad r = 0$$

$$\therefore \quad 2\mu t = (2n+1)\frac{\lambda}{2}$$

When $\lambda = 400$nm $n = 3.25$

When $\lambda = 700$nm $n = 1.66$

Hence n can take values 2 and 3.

When n = 2 $\lambda_1 = 429$nm, when n = 3, $\lambda_2 = 600$nm

11. We have $2\mu t = (2n+1)\frac{\lambda}{2}$ ....(1)

assumed normal incidence

If δt is the thickness for the shift of 7.4 fringes, then

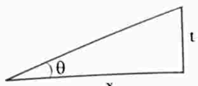
$$2\mu(t+\delta t) = [2(n+7.4)+1]\frac{\lambda}{2} \quad ....(2)$$

eqn 2 - eqn 1 $\Rightarrow 2\mu\delta t = 7.4\lambda$

$\therefore$ $$\delta t = \frac{7.4\lambda}{2\mu} = 2.183\times10^{-6}\,m$$

12. t be the diameter of the wire, then

$$\theta = \frac{t}{x} \qquad \left(\text{Angle} = \frac{\text{Arc}}{\text{Radius}}\right)$$



$$\beta = \frac{\lambda}{2\theta} = \frac{\lambda}{2t}x$$

$$\beta = \frac{6000\times10^{-10}\times15\times10^{-2}}{2\times0.05\times10^{-3}} = 9\times10^{-4}\,m$$

13. The condition for constructive interference in a thin film where the interference is due to transmitted rays is

$$2\mu t\cos r = n\lambda$$

or $$2t = n\lambda \quad (\because\ \mu=1,\ \cos r = 1)$$

$$2t = n_1\lambda_1$$

$$2t = n_2\lambda_2$$

or $n_1 = \frac{2t}{\lambda_1}$ and $n_2 = \frac{2t}{\lambda_2}$

$\therefore$ $$n_1 - n_2 = 2t\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$n_1 - n_2 = 250$, λ_1 and λ_2 are given find t.

14. For maximum intensity we have

$$2\mu t\cos r = \left(n+\frac{1}{2}\right)\lambda \qquad (\because\ \mu=1,\ r=0)$$

or $$2t = \left(n+\frac{1}{2}\right)\lambda_1 \quad ....(1)$$

$$2t = \left(n+\frac{3}{2}\right)\lambda_2 \quad ....(2)$$

$\therefore$ $n = 7$

using $t = x\theta$

Putting this in eqn (1) or (2), solve it for x.

$$x = 2.54\times10^{-2}\,m$$

15. $\beta = \frac{\text{length}}{\text{No. of fringes}}$ $\beta = \frac{10}{11}$cm

Using $\beta = \frac{\lambda}{2\mu\theta}$ find θ, $\theta = 2.44\times10^{-5}$ radian

When the film breaks the thickness at the top is zero

$\therefore$ **Thickness at the bottom** $t = x_o\theta$

$$= 10\times2.44\times10^{-5} = 2.44\times10^{-4}\,cm$$

16. **For the nth bright ring in air**

$$D_n^2 = 4R\left(n-\frac{1}{2}\right)\lambda_{air}$$

In liquid $$D_n'^2 = 4R\left(n-\frac{1}{2}\right)\lambda_{liquid}$$

$$\frac{\lambda_{air}}{\lambda_{liquid}} = \mu \qquad \therefore\ D_n'^2 = 4R\left(n-\frac{1}{2}\right)\frac{\lambda_{air}}{\mu}$$

or $$\mu = \frac{4R\left(n-\frac{1}{2}\right)\lambda_{air}}{D_n'^2}$$

17. $$r_n = \sqrt{\left(n-\frac{1}{2}\right)\lambda R}$$

18. $D_n^2 = 4nR\lambda$ **or** $D^2\ \alpha\ \lambda$

$$\frac{D_{air}^2}{D_{liquid}^2} = \frac{\lambda_{air}}{\lambda_{liquid}} = \mu$$

19. $r_n\ \alpha\ \sqrt{n}$

$\therefore$ $$\frac{r_{30}}{r_{20}} = \sqrt{\frac{30}{20}}$$

20. $N = 250,\ x = 0.08\times10^{-3}\,m$

$$\lambda = \frac{2x}{N} = \frac{2\times0.08\times10^{-3}}{250} = 6400\times10^{-10}\,m$$

21 We have $\lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{2x}$

$$2x = \frac{4886\times10^{-10}\times4882\times10^{-10}}{\lambda_1-\lambda_2}$$

$$2x = \frac{4886\times4882\times10^{-20}}{4\times10^{-10}}$$

$$= .5963\times10^{-3}\,m$$

Distance moved = x = 0.298mm

22. The condition for dark ring is given by

$$2d\cos\theta = n\lambda$$

or $$\cos\theta = \frac{n\lambda}{2d} = \frac{n\times6\times10^{-7}}{2\times0.3\times10^{-3}} = \frac{n}{1000}$$

i.e. $$\cos\theta = \left(\frac{n}{1000}\right)$$

For the central dark $\cos\theta = 1$ ($\because\ \theta = 0$)

$$1 = \frac{n}{1000} \Rightarrow n = 1000$$

Then n = 999, 998, 997, 996 ..... etc. give the other angles.

$$\theta = \cos^{-1}\left(\frac{n}{1000}\right)$$

For $n = 1000,\ \theta = 0^\circ$

$n = 999,\ \theta = 2.56^\circ$

$n = 998,\ \theta = 3.62^\circ$

$n = 997,\ \theta = 4.44^\circ$

$n = 996,\ \theta = 5.13^\circ$ etc.

IMPORTANT FORMULAE

1. Normal incidence of light on a plane film:

   (i) Condition for brightness: $2\mu t = \left(n+\frac{1}{2}\right)\lambda,\quad n = 0, 1, 2, \ldots\ldots$

   (ii) Condition for darkness: $2\mu t = n\lambda,\quad n = 1, 2, 3\ldots\ldots$

2. Oblique incidence of light on a plane film:

   (i) The optical path difference between the two reflected rays. $\Delta = 2\mu t\cos r$

   (ii) Condition for brightness: $2\mu t\cos r = \left(n+\frac{1}{2}\right)\lambda,\quad n = 0, 1, 2, 3\ldots\ldots$

   (iii) Condition for darkness: $2\mu t\cos r = n\lambda,\quad n = 1, 2, 3\ldots\ldots$

3. The cosine law:

   The effective path difference between the wave reflected from the lower surface and the upper surface of a film: $\Delta = 2\mu t\cos r$ known as cosine law.

4. Reflection coefficient: $R = \left(\frac{\mu_1-\mu_2}{\mu_1+\mu_2}\right)^2$

5. Transmission coefficient: $T = \frac{4\mu_1^2}{(\mu_1+\mu_2)^2}$

   Remember that $R + T = 1$

6. Interference by a film with two non-parallel reflecting surfaces - wedge shaped.

   (i) Condition for constructive interference: $2\mu t = \left(n+\frac{1}{2}\right)\lambda,\quad n = 0, 1, 2, 3\ldots\ldots$

   (ii) Condition for destructive intereference: $2\mu t = n\lambda,\quad n = 1, 2, 3\ldots\ldots$

   (iii) Expression for fringe width: $\beta = \frac{\lambda}{2\beta\theta}$

7. Newton's rings:

   (i) The diameter of the nth dark ring: $D_n = 2\sqrt{n\lambda R},\quad n = 1, 2, 3\ldots\ldots$

(ii) Wavelength of the light used: $\lambda = \dfrac{D_{n+k}^2 - D_n^2}{4kR}$ in air

$\lambda = \mu \dfrac{D_{n+k}^2 + D_n^2}{4kR}$ in medium

(iii) Radius of curvature of a convex lens: $R = \dfrac{fd}{f-d}$ Boy's method

8. **Michelson's interferometer:**

(i) Condition for constructive interference: $2d\sin\theta = \left(n + \dfrac{1}{2}\right)\lambda,\ n = 0, 1, 2 \ldots\ldots$

(ii) Condition for destructive interference: $2d\sin\theta = n\lambda,\ n = 1, 1, 3 \ldots\ldots$

(iii) Difference between the wavelength: $\lambda_1 - \lambda_2 = \dfrac{\lambda^2}{2x},\ \lambda = \sqrt{\lambda_1 \lambda_2}$