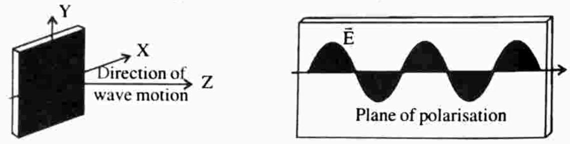
UNIT - IV

6 POLARISATION

Introduction

In the last few chapters we dealt with the phenomenon of interference and diffraction in detail which proved that light is a wave. But these phenomena donot tell us the nature of the wave whether it is longitudinal or transverse. The development of electromagnetic theory by Maxwell proved that light is an electromagnetic wave and its nature is transverse. The concept of transverse nature of light leads to the concept of polarisation. An understanding of polarisation is essential for understanding the propagation of electromagnetic waves guided through wave guides and optical fibres. Polarised light has many practical applications in industry and engineering. One of the most important applications is in liquid crystal displays (LCDs) which are widely used in wrist watches, calculators, TV screens etc.

We know that electromagnetic wave which consists of electric and magnetic field vectors oscillating in mutually perpendicular directions. These vectors are perpendicular to the direction of propagation of light. Even though light is made up of two field vectors, we prefer to describe the light wave in terms of the electric vector because of two reasons. The magnitude of the electric vector is much larger as compared to that of the magnetic vector ($E = cB$) and the effect of electric vector on the eye is much larger as compared to that of the magnetic vector.



Unpolarised light — Plane polarised light

Figure 6.1

An ordinary beam of light consists of a large number of waves emitted by the atoms or molecules of the light source. Each atom produces a wave with its own orientation of electric field vector $\vec{E}$. However, because all directions of vibrations of $\vec{E}$ are equally probable the resultant electromagnetic wave is a superposition of

waves produced by the individual atomic sources. This resultant wave is called unpolarised light and is symmetrical about the direction of wave propagation ad shown in figure i.e. *The light having oscillations in all directions is called unpolarised.*

Suppose the light is travelling in the Z-direction. Then the electric field vector oscillates in the X–Y plane. We may resolve the electric vectors along X and Y axes. In the figure given above XY plane is perpendicular to the plane of the paper.

Polarised light, polarisation and plane of polarisation

An unpolarised light having oscillations in all directions. However we can confine the vibrations of electric vector in one direction perpendicular to the direction of wave motion by using polaroids, nicol prism or tourmalline crystal. This light is said to be plane polarised or linearly polarised.

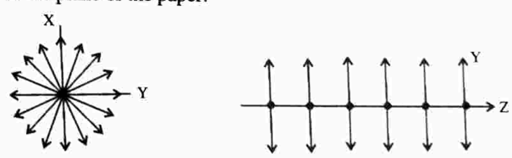
i.e. *The light in which the electric vector of all waves oscillates only in one direction is called plane polarised or linearly polarised.*

The phenomenon of confining the vibrations of wave in a specific direction perpendicular to the direction of wave motion is called polarisation . The plane containing the direction of vibration and wave motion is called plane of polarisation

Note : Some authors call this plane as 'plane of vibration which is not very proper.

Regarding polarisation it is worth noting that

(i) All the vibrations of an unpolarised light at a given instant (say X and Y axes) and hence an unpolarised light is equivalent to the superposition of two mutually perpendicular identical plane polarised light as shown in figure below. In the figure XY plane is perpendicular to the plane of the paper The double arrows represent the oscillations in the plane of the paper. The double arrows represent the oscillations in the plane of the paper. The dots represent the oscillations perpendicular to the plane of the paper.



Unpolarised light propagating along Z-axis — **Symbolic representation of unpolarised light**

Figure 6. 2

(ii) If, in case of unpolarised light, electric vector in some plane is either more or less than its perpendicular plane, the light is said to be partially polarised (see figure below)

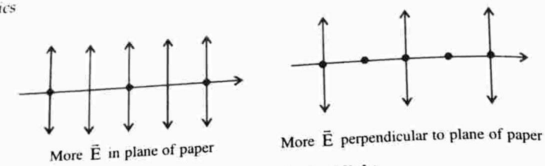


Figure 6.3 : Partially polarised light

(iii) If an unpolarised light is converted into plane polarised light, its intensity reduces to half.

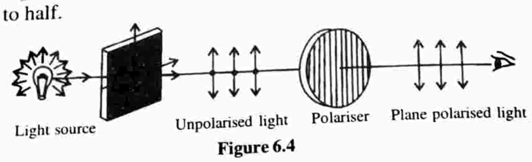


Figure 6.4

(iv) Polarisation is a convincing proof of transverse nature of wave as in transverse wave vibrations are in a plane perpendicular to the direction of wave motion and so the wave can be polarised . However, if the wave is longitudinal i.e. vibrations in a wave are along the direction of wave motion it cannot be polarised. This is why light can be polarised while sound cannot be.

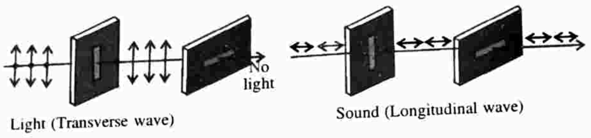


Figure 6.5

(v) In case of interference of polarised lights the interfering waves have same plane of polarisation, otherwise unpolarised or partially polarised light will result.

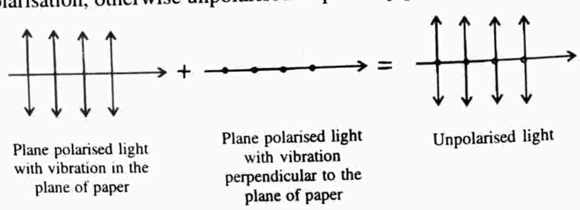


Figure 6.6

(vi) Our eye cannot distinguish between polarised and unpolarised light. Crystals such as calcite, quartz, tourmalline, nicol prism and polaroids are called polarisers when used to produce plane polarised light and analysers when used to analyse (i.e.identify) the given light.

When unpolarised light is seen through a single crystal intensity of emergent light reduces to half of the original intensity due to polarisation. On rotating the crystal, intensity of polarised light does not change. However when light transmitted from the first crystal is seen through another crystal, the intensity of the emergent light decreases as we rotate the second crystal. Here the first crystal is called as the polariser and the second crystal is called as the analyser.

Methods of obtaining plane polarised light

(a) polarisation by reflection

In 1811, Brewster discovered that when unpolarised light is reflected from a surface, the reflected light may be completely polarised, partially polarised or unpolarised. This depends on the angle of incidence. If the angle incidence is 0° or 90° the reflected light is unpolarised. For angle of incidence beween 0° and 90° the reflected beam is polarised to varying degree. *The angle of incidence at which the reflected light is completely polarised is called polarising angle* (i_p). It depends upon the wavelength of light used.

Brewsters law

It states that when unpolarised light is incident at polarising angle (i_p) *on an interface seperating air from a medium of refractive index* μ, *then* $\mu = \tan i_p$. *This is called Brewsters law.*

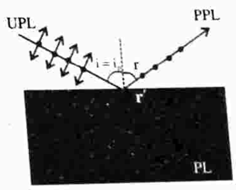


Figure 6.7

It has been found that when light is incident at polarising angle, the reflected ray and the refracted light are mutually perpendicular

i.e. $\quad r + r' = 90^\circ$ (see figure)

or $\quad r' = 90 - r = 90 - i_p$

According to snell's law

$$\mu = \frac{\sin i_p}{\sin r'} = \frac{\sin i_p}{\sin(90 - i_p)} = \frac{\sin i_p}{\cos i_p} = \tan i_p$$

For glass $\mu = 1.5$, then $i_p = \tan^{-1}\mu = \tan^{-1}1.5 = 56.3^\circ$

For water $\mu = 1.33$, then $i_p = \tan^{-1}1.33 = 53.1^\circ$

Example 1

When sunlight is incident on water surface at glancing angle of 37°, the reflected light is found to be completely plane polarised. Determine the refractive index of water and angle of refraction.

Solution

As the glancing angle is 37°, the angle of incidence $i = 90 - 37 = 53^\circ$

Since the reflected light is completely plane polarised, the light is incident at polarising angle

i.e. $i_p = i = 53^\circ$

Using Brewsters law $\mu = \tan i_p$

$$\mu = \tan 53^\circ = 1.327$$

In the case of Brewsters law reflected and refracted rays are perpendicular to each other

i.e. $r + r' = 90^\circ$

$$r = i = i_p = 53$$

$\therefore$ $r' = 90 - r = 90 - 53 = 37^\circ$

Example 2

If the critical angle for glass air boundary is 38°, calculate the polarising angle for glass.

Solution

Critical angle C = 38°

Using $\mu = \dfrac{1}{\sin C} = \dfrac{1}{\sin 38^\circ} = 1.62$

from Brewsters law $\mu = \tan i_p$

$\therefore$ $i_p = \tan^{-1}\mu = \tan^{-1}1.62 = 58.3^\circ$

(b) Polarisation by refraction

In this method a pile of glass plates is formed by taking 20 to 30 microscope slides and light is made to be incident at polarising angle (56.3°). In accordance with Brewsters law, the reflected light will be plane polarised with vibrations perpendicular to the plane of incidence (which is here plane of paper) and the transmitted light will be partially polarised. Since in one reflection about 15% of the light with vibration perpendicular to plane of paper is reflected. This process is repeated. When light passes through 15 to 20 glass plates all normal vibrations to the plane of incidence (plane of paper) will be removed by reflection. The refracted ray contains only vibrations parallel to the plane of incidence. In other words the refracted ray is completely plane polarised

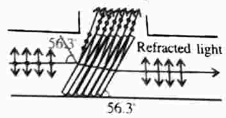


Figure 6.8

Intensity of light emerging from a polaroid

If a plane polarised light of intensity $I_o (= Ka^2)$ is incident on a polaroid and its vibrations of amplitude a make an angle θ with the transmission axis, then the component of vibrations parallel to transmission axis will be $a\cos\theta$ while perpendicular to it is $a\sin\theta$. Now, as polaroid will pass only these vibrations which are parallel to its transmission axis i.e. $a\cos\theta$, so the intensity of emergent light will be

$$I = K(a\cos\theta)^2 = Ka^2\cos^2\theta$$

$$I = I_o\cos^2\theta \qquad \text{...... (1)}$$

This law is called Malus law

If the incident light is unpolarised then as vibrations are equally probable in all directions, θ can have any value from 0 to 2π and hence

$$I = \frac{I_o}{2}\left[\therefore (\cos^2\theta)_{av} = \frac{1}{2}\right]$$

i.e. If an unpolarised light is allowed to pass through a polaroid or nicol prism, it becomes a plane polarised light whose intensity be comes half. If light of intensity I_1 emerging from one polaroid called polariser is incident on a second polaroid (usually called analyser) the intensity of the light emerging from the second polaroid in accordance with Malus law will be given by

$I_2 = I_1 \cos^2\theta'$, where θ' is the angle between the transmission axis of the two polaroids.

Example 3

The polariser and the analyser are crossed. Through what angle should the analyser be rotate so that 25% of light passes through the analyser.

Solution

When the polariser and the analyser are crossed $\theta = 90^\circ$

Suppose we rotate the analyser further by α,

then $\theta = 90 - \alpha$

we have $I = I_o \cos^2\theta$

$$I = \frac{25}{100} I_o \text{(given)}$$

$$\therefore \quad \frac{25}{100} I_o = I_o \cos^2(90-\alpha)$$

$$\frac{1}{4} = \cos^2(90-\alpha)$$

$$\frac{1}{2} = \cos(90-\alpha)$$

i.e. $90 - \alpha = 60^\circ$

$\alpha = 30^\circ$

Example 4

Two polaroids are oriented with their planes normal to incident light and transmission axis making an angle of 30° with each other. What fraction of incident unpolarised light is transmitted

Solution

If unpolarised light of intensity (I_o) is passed through a polaroid P_1 its intensity (I_1) will become half, so $I_1 = \frac{I_o}{2}$

Now this light will pass through the second polaroid P_2 whose axis is inclined at angle of 30° to the axis of P_1. So the intensity of light (I_2) emerging from P_2 will be

$$I_2 = I_1 \cos^2 30 = \frac{I_o}{2}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{8} I_o$$

so the fractional transmitted light, $\frac{I_2}{I_o} = \frac{3}{8} = 37.5\%$

Optically isotropic and anisotropic materials

A material which exhibits the index of refraction the same in all directions is called an optically isotropic material.

Glass, water, air, table salt (NaCl) etc. are examples of optically isotropic materials. When a light beam is incident on an isotropic material, it refracts as a single ray due to its single refractive index.

A material which exhibits the index of refraction different in different crystallographic axes is callled an optically anistropic material.

Calcite ($CaCO_3$), calomel (Hg_2Cl_2), ice (H_2O), quartz (SiO_2), sodium nitrate ($NaNO_3$), tourmalline (complex silicate), ruby (Al_2O_3), borax, epsom salt ($MgSO_4 \cdot 7H_2O$), mica, topaz, boron nitride etc. are examples of anisotropic materials.

When a light beam is incident on an anisotropic material, it undergoes double refraction. It is due to its different polarisations i.e. different refractive indices.

Depending upon the behaviour of light in different polarisations (different refractive indices) in anisotropic crystals,they are divided into two categories. They are (i) uniaxial crystals and (ii) biaxial crystals.

Uniaxial crystals

Anisotropic crystals which exhibit double refraction having two perpendicular polarisations, two refractive indices and one symmetry axis are called uniaxial crystals.

When a light beam is incident on an uniaxial anisotropic crystal, it splits into two. i.e. the beam under goes double refraction. One beam passes through the crystal without deviation is known as the ordinary ray (usually abbreviated as o-ray) and obeys snells laws of refraction. The corresponding refractive index is called ordinary refractive index and is denoted by μ_o. The other beam which passes deviated is known as the extraordinary (usually abbreviated as the e-ray)and does not obey snells laws. The corresponding refractive index is called as extra ordinary refractive index and is denoted by μ_e. The two beams are obviously linearly polarised in perpendicular directions.

However, in the crystal there is a direction or axis along which a beam of light may pass without undergoing double refraction. This direction or axis is called the optic axis. In uniaxial crystals there is only one axis of symmetry hence its name.

Several examples with corresponding refractive indices of uniaxial crystals are given in the table below.

Table 1 : Examples of uniaxial materials with refractive indices

No	Material	μ_o	μ_e
1	Ice (H_2O)	1.309	1.313
2	Magnesium Fluoride (MgF_2)	1.380	1.385
3	Quartz (SiO_2)	1.544	1.553
4	Sodium nitrate ($NaNO_3$)	1.587	1.336
5	Beryl $Be_3Al_2(SiO_3)_6$	1.602	1.557
6	Calcite ($CaCO_3$)	1.658	1.486
7	Tourmalline (Complex silicate)	1.669	1.638
8	Sapphire (Al_2O_3)	1.768	1.760
9	Ruby (Al_2O_3)	1.770	1.762
10	Calomel (Hg_2Cl_2)	1.973	2.656

Biaxial Crystals

Anisotropic crystals which exhibit double refraction having two perpendicular polarisations, three refractive indices and more than one symmetry axis are called biaxial crystals.

In a biaxial material though there are three refractive indices μ_α, μ_β, and μ_γ but only two rays which are called the fast and the slow ray. The slow ray is the ray that has the highest effective refractive index. Both rays do not obey snells laws of refraction. Examples of biaxial crystals are given below.

Table 2

No	Material	μ_α	μ_β	μ_γ
1	Epsom salt $MgSO_4 \cdot 7(H_2O)$	1.433	1.455	1.461
2	Borax	1.447	1.469	1.472
3	Ulexite	1.490	1.510	1.520
4	Mica, biotite	1.595	1.640	1.640
5	Topaz	1.618	1.620	1.627

The phenomenon of double refraction

The phenomenon of double refraction was first described by the Danish scientist Rasmus Bartholin in 1669.

When an unpolarised light beam is incident on anisotropic crystals like calcite, tourmalline quartz etc. it usually splits up into two linearly polarised beams. This phenomenon is called double refraction or birefringence.

To demonstrate double refraction, a pencil is viewed through a calcite crystal. Two images are seen (see figure below). On rotating the crystal one image remains stationary while the other image rotates round the stationary image. The stationary image is due to ordinary ray (o-ray) and the other image is due to extra ordinary ray (e-ray).

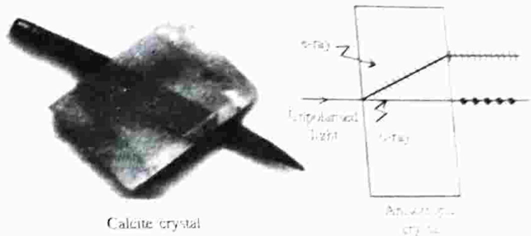


Figure 6.9

To test the polarisation of the refracted rays these images are viewed through a tourmalline plate. When the plate is rotated slowly, for a particular position the ordinary image becomes maximum bright and the extraordinary image becomes invisible. On further rotation of tourmalline plate through 90° the ordinary image becomes invisible and the extra ordinary image becomes maximum bright. This shows that the two refracted rays are plane polarised with their plane of polarisation perpendicular to each other.

Huygens explanation of double refraction

According to Huygens wave theory of light, the propagation of light is explained in terms of wave surfaces. A point source of light embedded in an isotropic substance such as glass emits spherical wave front. This wave front stimulates atoms which then acts as sources of secondary wavelets all of which are in phase. They expand in all directions with the same velocity. But a point source is embeded in a bire fringent crystal (double refracting) two wave surfaces will be formed simultaneously. As a result the beam will split into two rays. The wave surface corresponding to o-ray propagates with the same velocity in at directions and is therefore spherical. The wave surface corresponding to e-ray is an ellipsoid of revolution about the optic axis since e-ray travels with different velocities in different directions within the crystal. The two waves surfaces touch each other at two points and the line joining these points defines the direction of the optic axis. As light propagates through the crystal, the two surfaces travel in different directions in the crystal. Ultimately two refracted rays emerge from the crystal.

Because of two wavefronts two different types of situations arise. In one case spherical wave front is enclosed by the ellipsoidal wave front. These types of crystals are called negative crystals. In the other case ellipsoidal wavefront is enclosed by the spherical wavefront. These types of crystals are called positive crystals. (see figure below).

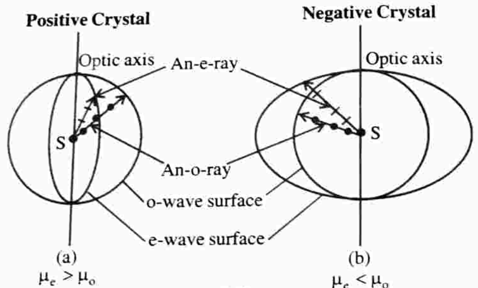


Figure 6.10

Polarisation by double refraction

By using the phenomenon of double refraction and isolating one ray from the other we can obtain plane polarised light which actually happens in Nicol prism. Nicol prism is made up of calcite crystal and in it e-ray is isolated from o-ray through total internal reflection of o-ray at canada balsam layer and then absorbing it at the blackened surface as shown in figure below.

Figure 6.11

Distinction between o-rays and e-rays

(i) o-rays obey snells laws of refraction, whereas e-rays do not.

(ii) Both o and e-rays are linearly polarised. They are polarised in mutually perpendicular directions

(iii) o-rays travel with same speed in all directions within the crystal. The e-rays travel with different speeds along different directions in the crystal

(iv) Refractive index of o-ray is a constant since it travels with same speed in all directions. On the other hand the refractive index of e-ray varies from direction to direction.

Negative and positive crystals

The phenomenon of double refraction or birefringence can occur only if the structure of the material is anisotropic. if the material has a single axis of anisotropy (uniaxial material) double refraction can be formalised by assigning two different refractive indices to the material for different polarisations. The birefrigence magnitude is then defined by

$$\Delta\mu = \mu_e - \mu_o$$

where μ_e and μ_0 are the refractive indices of e-ray and o-ray respectively.

Depending upon the value of $\Delta\mu$ crystals are divided into two.

When $\Delta\mu$ is positive i.e. $\mu_e > \mu_o$ then the crystal is called positive crystal. Calomel, ice, magnesium fluoride, quartz are examples of positive crystals (see table 1)

When $\Delta\mu$ is negative i.e. $\mu_e < \mu_o$, the crystal is called a negative crystal.

Beryl, calcite, sodium nitrate, tourmalline, ruby, sapphire are negative crystals. (see table 1)

For positive crystals we have $\mu_e > \mu_o$ i.e. $v_e < v_o$ (remember that $\mu = \frac{c}{v}$). It means that in positive crystals e-ray travels slower than o-ray.

For negative crystals we have $\mu_e < \mu_o$ i.e. $v_e > v_o$. It means that in negati crystals o-ray travels slower than e-ray.

It may be noted that the refractive indices μ_o and μ_e are defined in differe ways because o-ray travels with the same velocity in all directions it has const value. On the other hand the refractive index of e-ray varies from direction to dir tion.

The refractive index of o-ray is defined as

$$\mu_o = \frac{c}{v_o} = \frac{\text{velocity of light in vacuum}}{\text{velocity of o-ray in the crystal}}$$

The refractive index for e-ray is defined in different ways for positive and negative crystals. For a positive crystal.

$$\mu_e = \frac{c}{(v_e)_{min}} = \frac{\text{velocity of light in vacuum}}{\text{minimum velocity of e-ray in the crystal}}$$

For a negative crystal

$$\mu_e = \frac{c}{(v_e)_{max}} = \frac{\text{velocity of light in vacuum}}{\text{maximum velocity of e-ray in the crystal}}.$$

Wave plates or retarders

A wave plate is an optical device that can change the state of polarisation of an incident wave. It consists of a thin plate of birefringent crystal.

When plane polarisd light is incident on a wave plate it splits the light into two plane polarised light waves and one of the waves lags behind the other by a known amount. After emerging from the wave plate the two waves superimpose to produce a wave of different state of polarisation.

There are two types of wave plates

(i) quarter wave plate (ii) half wave plate.

Quarter wave plate and half wave plate

We have seen that when unpolarised light incident on an anisotropic crystal undergoes double refraction and produces two plane polarised waves. This is true even if the incident light is plane polarised provided incident electric vector makes an angle with the optic axis i.e. when a plane polarised light wave is incident on a birefringent crystal at an angle with the optic axis, then the polarised wave splits into two polarised waves namely e-ray and o-ray. These two rays travel along the crystal with different velocities. As a result when the waves emerge from the crystal, an optical path difference is developed between the two rays.

Let t be the thickness of the crystal, then the optical path for o-ray within the crystal $= \mu_o t$

Then optical path for e-ray within the crystal $= \mu_e t$

$\therefore$ The optical path difference between e-ray and o-ray $= (\mu_e \sim \mu_o)t$

$\therefore$ The phase difference between the two rays $= \frac{2\pi}{\lambda}(\mu_e \sim \mu_o)t$

The thickness of the wave plate is adjusted such that the optical path difference is equal to $\frac{\lambda}{4}$, such a wave plate is called quarter wave plate.

i.e. $$(\mu_e \sim \mu_o)t = \frac{\lambda}{4}$$

or $$t = \frac{\lambda}{4(\mu_e \sim \mu_o)}$$

In terms of phase difference, $t = \frac{\pi}{2(\mu_e \sim \mu_o)}$

Thus, *a quarter wave plate is a plate of birefringent crystal whose thickness is adjusted such that the optical path difference between the e-ray and o-ray emerging out of the crystal is* $\frac{\lambda}{4}$

A quarter wave plate is used in producing elliptically or circularly polarised light.

Note : If the ordinary and extra ordinary ray have a path difference of $\left(n+\frac{1}{4}\right)\lambda$ even then the quarter wave plate would introduced a path difference of $\frac{\lambda}{4}$ between ordinary and extra ordinary rays. Thus in general we have

$$(\mu_o \sim \mu_e)t = \left(n+\frac{1}{4}\right)\lambda$$

where n = 0, 1, 2, 3 ......

When n = 0, the thickness of the plate is minimum.

If the thickness of the wave plate is adjusted such that the optical path difference between the e-ray and the o-ray is equal to $\frac{\lambda}{2}$, then the wave plate is called half wave plate.

i.e. $$(\mu_e \sim \mu_o)t = \frac{\lambda}{2}$$

or $$t = \frac{\lambda}{2(\mu_e \sim \mu_o)}$$

In terms of phase difference $t = \frac{\pi}{(\mu_e \sim \mu_o)}$

The half wave plate is used to invert the handedness of elliptical or circular polarised light i.e. changing right to left and vice versa.

Note : If the ordinary and extra ordinary ray have a path difference of $\left(n+\frac{1}{2}\right)\lambda$ even then the half wave plate would introduce a path difference of $\frac{\lambda}{2}$ between ordinary and extra ordinary rays.

Thus in general we have

$$(\mu_o \sim \mu_e)t = \left(n+\frac{1}{2}\right)\lambda$$

where n = 0, 1, 2, ...

When n = 0, the thickness of the plate is minimum.

Phase retardation plate

Whenever plates placed in the path of light introduce certain phase difference (path difference) between ordinary and extra ordinary rays, such plates are called as phase retarding plates. Half wave and quarter wave plates are phase retarding plates. These plates introduce a phase angle between ordinary and extra ordinary rays through decreasing the speed of one of the rays and that is why they are called as phase retarding plates. Generally they are made of calcite and quartz. They are also made from mica sheets. Mica is a biaxial crystal but the angle between its two axes is very small. Whenever light enters such plates it is divided into ordinary and extra ordinary rays. The speed or the refractive indices of these two rays being different a path difference or a phase difference between the two is introduced by the plates.

Example 5

A quarter half wave plate is constructed from quartz crystal whose refractive indices are $\mu_e = 1.553$ and $\mu_o = 1.544$. Calculate the thickness of the plate for wavelength of 6500Å.

Solution

$$\mu_e = 1.553, \mu_o = 1.544, \lambda = 6500\times10^{-10}\text{m}$$

For a quarter wave plate $t = \frac{\lambda}{4(\mu_e - \mu_o)}$

$$t = \frac{6500\times10^{-10}}{4(1.553-1.544)}$$

$$= \frac{6.5\times10^{-7}}{4\times9\times10^{-3}} = \frac{6.5}{36}\times10^{-4}$$

$$= 1.8\times10^{-5}\text{m} = 1.8\times10^{-2}\text{mm}$$

Example 6

Calculate the thickness of ice required to act like a half wave plate for a wavelength of 590nm. $\mu_e = 1.313$, and $\mu_o = 1.309$

Solution

$$\lambda = 590\text{nm} = 590\times10^{-9}\text{m}$$
$$\mu_e = 1.313, \mu_o = 1.309$$

For a half wave plate $t = \frac{\lambda}{2(\mu_e - \mu_o)} = \frac{590\times10^{-9}}{2(1.313-1.309)}$

$$= \frac{590\times10^{-9}}{2\times4\times10^{-3}} = 7.375\times10^{-5}\text{m}$$

$$= 7.375\times10^{-2}\text{mm}\cdot$$

Example 7

Calculate the thickness of the doubly refracting crystal required to introduce a path difference of $\frac{\lambda}{2}$ between the ordinary and extra ordinary ray when $\lambda = 6000$Å, $\mu_o = 1.55$ and $\mu_e = 1.54$.

Solution

To introduce a path difference of $\frac{\lambda}{2}$, we use a half wave plate, so

$$t = \frac{\lambda}{2(\mu_o - \mu_e)} = \frac{6000\times10^{-10}}{2(1.55-1.54)}$$

$$= 3\times10^{-5}\text{m}\cdot$$

Example 8

The faces of a quartz plate are parallel to the optic axis of the crystal. (i) what is the thinnest possible plate that would serve to put the ordinary and extra ordinary rays of $\lambda = 5890$Å a half wave apart on their exit. (ii) What amplitudes of this thickness would give the same result. $\mu_o = 1.553$ and $\mu_e = 1.544$.

Solution

(i) Since the quartz plate behaves as a half wave plate, we have

$$t = \frac{\lambda}{2(\mu_o - \mu_e)} = \frac{5890 \times 10^{-10}}{2(1.553 - 1.544)}$$

$$= 3.27 \times 10^{-5}\,\text{m}.$$

(ii) The plates which would introduce a path difference of $\frac{\lambda}{2}$ or its odd multiples of $\frac{\lambda}{2}$ would produce the same result i.e., t, 3t, 5t, ... etc. Hence t are 3.27×10^{-5} m, 9.81×10^{-5} m ......

Example 9

Quartz plate of thickness 0.03mm works as a phase retardation plate for light of wave length 6000Å. Calculate the value of retardation in phase. $\mu_o = 1.55$ and $\mu_e = 1.54$.

Solution

We have

Path difference $= (\mu_o - \mu_e)t$

Using

$$\text{phase difference} = \frac{2\pi}{\lambda} \times \text{path difference}$$

$$= \frac{2\pi}{\lambda}(\mu_o - \mu_e)t$$

$$= \frac{2\pi(1.55 - 1.54)}{6000 \times 10^{-10}} \times 0.3 \times 10^{-3}$$

Phase difference $= \pi$ radians.

Production of circularly and elliptically polarised light wave

When a plane polarised light is incident on a wave plate at an angle with the optic axis, it splits into two rays (e-ray and o-ray). When e-ray and o-ray emerge out of wave plate there is a path difference between them. When these two waves superimpose we get circularly, elliptically or plane polaraised light wave depending upon the wave plate and the angle of incidence of plane polarised light. Let E_o be the

amplitude of the electric field vector of the plane polarised light falling on the wave plate at angle ϕ with the Z axis (we assume the Z axis to be along the optic axis). Such a beam can be assumed to be a superposition of two linearly polarised beams (vibrating in phase) polarised along Y and Z directions with amplitude $E_o \sin\phi$ and $E_o \cos\phi$ respectively. The Z component whose amplitude $E_o \cos\phi$ passes through as an extra ordinary beam propagating with velocity $\frac{c}{\mu_e}$. The Y component whose amplitude $E_o \sin\phi$ passes through as an ordinary beam propagating with velocity $\frac{c}{\mu_o}$. Since $\mu_e \neq \mu_o$ the two beams will propagate with different velocities, as a result when they come out of the crystal they will have a phase difference.

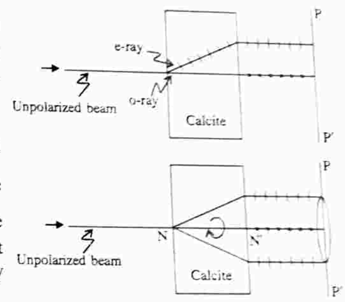


Figure 6.12

Let the plane x = 0 represent the surface of the crystal on which the plane polarised beam is incident. The Y and Z component of the incident beam can be written in the form

$$E_y = E_o \sin\phi \cos(kx - \omega t) \qquad \text{......(1)}$$

$$E_z = E_o \cos\phi \cos(kx - \omega t) \qquad \text{......(2)}$$

where $k = \frac{\omega}{c}$, the wave vector

Thus at x = 0 we have

$$E_y(x=0) = E_0 \sin\phi \cos\omega t$$

$$E_z(x=0) = E_0 \cos\phi \cos\omega t$$

Inside the crystal the two components will be given by

$$E_y = E_o \sin\phi \cos(k_1 x - \omega t) - \text{o-ray}$$

$$E_z = E_o \cos\phi \cos(k_2 x - \omega t) - \text{e-ray}$$

where k_1 and k_2 are the wave vector of o-ray and e-ray respectively

$k = \frac{2\pi}{\lambda}$, λ is the wavelength in air

$$k_1 = \frac{2\pi}{\lambda_1},\ \lambda_1 \text{ is the wavelength of o-ray in the crystal}$$

but $\lambda_1 = \dfrac{\lambda}{\mu_o}$

$$\therefore \quad k_1 = \frac{2\pi}{\frac{\lambda}{\mu_o}} = \mu_o \frac{2\pi}{\lambda} = \mu_o k$$

Similarly $k_2 = \mu_e k$

$$\therefore \quad E_y = E_o \sin\phi \cos(\mu_o kx - \omega t) \quad \text{...... (3)}$$

$$E_z = E_o \cos\phi \cos(\mu_e kx - \omega t) \quad \text{...... (4)}$$

If the thickness of the crystal is d, then at the emerging surface, we have

$$E_y = E_o \sin\phi \cos(\mu_o kd - \omega t)$$

$$E_z = E_o \cos\phi \cos(\mu_e kd - \omega t)$$

put $\mu_o kd = \theta_o$, $\mu_e kd = \theta_e$ and $\theta_o - \theta_e = \theta$

Then
$$E_y = E_o \sin\phi \cos(\theta_0 - \omega t) \quad \text{...... (5)}$$

$$E_z = E_o \cos\phi \cos(\theta_e - \omega t) \quad \text{...... (6)}$$

The orgin of time is shifted from t to $t + \dfrac{\theta_e}{\omega}$. The above equations can be written in a simple form as

$$E_y = E_o \sin\phi \cos(\theta - \omega t) \quad \text{...... (7)}$$

$$E_z = E_o \cos\phi \cos\omega t \quad \text{...... (8)}$$

These two are the expression for the rays (o-ray and e-ray) coming out of the plate. When they superimpose depending upon the conditions on ϕ and θ we get different polarisations.

Case I

When $\phi = \dfrac{\pi}{4}$ and $\theta = \dfrac{\pi}{2}$ (quarter wave plate)

Equations 7 and 8 become

$$E_y = E_o \sin\frac{\pi}{4} \cos\left(\frac{\pi}{2} - \omega t\right)$$

i.e.
$$E_y = \frac{E_o}{\sqrt{2}} \sin\omega t \quad \text{...... (9)}$$

and
$$E_z = E_o \cos\frac{\pi}{4} \cos\omega t$$

i.e.
$$E_z = \frac{E_o}{\sqrt{2}} \cos\omega t \quad \text{...... (10)}$$

Squaring and adding equations 9 and 10, we get

$$E_y^2 + E_z^2 = \frac{E_o^2}{2}$$

This is the equation of a circle. This means that we get a circularly polarised light wave.

Case II

When $\phi \neq 45^\circ$ and $\theta = \dfrac{\pi}{2}$ (quarter wave plate)

Equations 7 and 8 become

$$E_y = E_o \sin\phi \sin\omega t$$

and
$$E_z = E_o \cos\phi \cos\omega t$$

put $E_o \sin\phi = A$ and $E_o \cos\phi = B$, then squaring and adding the above equations we get

$$\frac{E_y^2}{A^2} + \frac{E_z^2}{B^2} = 1$$

This is an equation of an ellipse with semi axes as A and B.

This means that we get an elliptically polarised light wave in the Y – Z plane.

Case III

When $\phi = \dfrac{\pi}{4}$, $\theta = \pi$ (half wave plate)

Equations 7 and 8 become

$$E_y = \frac{E_0}{\sqrt{2}} \cos(\pi - \omega t) = -\frac{E_0}{\sqrt{2}} \cos\omega t \quad \text{...... (11)}$$

and $$E_Z = \frac{E_0}{\sqrt{2}}\cos\omega t \qquad \text{...... (12)}$$

which represents a linearly polarised wave with the direction of polarisation making an angle 135° with the z-axis

It means that if the linearly polarised beam making an angle of 45° with the z-axis is incident on a half wave plate, the plane of polarisation gets rotated by 90°.

Production of circularly polarised light

A monochromatic unpolarised light is allowed to pass through a polariser (such as polaroids, nicol prism, tourmalline etc.) the emergent light will be plane polarised. This plane polarised light is made to incident on a quarter wave plate at an angle 45° with the optic axis. The plane polarised light incident on the quarter wave plate splits into two rays o-ray and e-ray. When these rays come out of the crystal, they will have a path difference of $\frac{\lambda}{4}$ or phase difference of $\frac{\pi}{2}$. The two rays are linearly polarised in mutually perpendicular directions. When they combine they produce circularly polarised light.

Production of elliptically polarised light

A monochromatic unpolarised light is allowed to pass through a polariser (such as polaroids, nicolprism, tourmalline crystal etc), the emergent light will be plane polarised. This plane polarised light is made to incident on a quarter wave plate at an angle not equal to 45° with the optic axis. The plane polarised light incident on the quarter wave plate splits into two rays o-ray and e-ray. When the rays come out of the crystal they will have different amplitudes and a path difference of $\frac{\lambda}{2}$ or phase difference of $\frac{\pi}{2}$. When they combine, the resulting light will be elliptically polarised.

Detection of plane, circularly and elliptically polarised light

We have seen that there are three types of polarisations. They are (i) plane polarisation (ii) circular polarisation and (iii) elliptical polarisation. But our eye cannot distinguish the different polarisations. However, using a polariser and a quarter wave plate the type of polarisation can be determined.

The light of unknown polarisation is allowed to fall normally on a polariser. Then the polariser is slowly rotated through a full circle and the intensity of the transmitted light is observed. If the intensity of the transmitted light is extinguished twice in one full rotation of the polariser, then the incident light is plane polarised.

If the intensity of the transmitted light varies between a maximum and minimum value but does not become extinguished in any position of the polariser, then the incident light is either elliptically polarised or partially polarised.

If the intensity of the transmitted light remains constant on rotation of the polariser, then the incident light is circularly polarised or unpolarised.

In the above two cases, to confirm whether it is partially ,circularly or elliptically polarised we can make use of a quarter wave plate.

To distinguish between elliptically polarised and partially polarised, the light is first made to be incident on the quarter wave plate and then passes through the polariser. If the incident light is elliptically polarised, the quarter wave plate converts it into a plane polarised beam. When this plane polarised light passes through the polariser it would be extinguished twice in one full rotation of the polarizer. On the other hand, if the transmitted light intensity varies between a maximum and a minimum without becoming zero, then the incident light is partially polarised.

To distinguish between circularly polarised and partially polarised, the light is first made to be incident on the quarter wave plate and then it is allowed to pass through the polariser. If the incident light is circularly polarised the quarter wave plate converts it into plane polarised light. When this plane polarised light passes through the polariser, it would be completely extinguished twice in one full rotation of the polariser. On the other hand if the intensity of the transmitted light stays constant, then the incident light is unpolarised.

Example 10

Plane polarised light falls normally on a quarter wave plate. What will be the nature of the emergent light if the plane of polarisation of the incident light makes 45° with the principle plane of quarter wave plates.

Solution

The incident beam can be represented by

$$E_x = E_o \sin\phi\cos(kx - \omega t)$$

i.e., $$E_x = E_o \sin 45\cos(kx - \omega t)$$

$$E_x = \frac{E_o}{\sqrt{2}}\cos(kx - \omega t)$$

similarly $$E_y = \frac{E_o}{\sqrt{2}}\sin(kx - \omega t)$$

Squaring and adding eqns (1) and (2) we get

$$E_x^2 + E_y^2 = \frac{E_o^2}{2}$$

This is the equation of a circle. This means that we get a circularly polarised light wave.

Example 11

Calculate the thickness of calcite sheet required to change plane polarised light into circularly polarised light. $\mu_o = 1.658$, $\mu_e = 1.486$ and $\lambda = 6000\text{Å}$.

Solution

A plane polarised light on traversing through the plate will become circularly polarised light when

$$t(\mu_o - \mu_e) = \frac{n\lambda}{4} \text{ for odd n.}$$

$$\therefore \quad t = \frac{n\lambda}{(\mu_o - \mu_e)4}$$

$$t = \frac{n \times 6000 \times 10^{-10}}{4(1.658 - 1.486)} = 8.72 \times 10^{-7}\text{m}$$

For $n = 1 \quad t_1 = 8.72 \times 10^{-7}\text{m}$

$n = 3 \quad t_2 = 26.16 \times 10^{-7}\text{m}$

$n = 5 \quad t_3 = 43.6 \times 10^{-7}\text{m}$ and so on.

A sum up of different polarised lights

1. Unpolarised light

It consists of sequence of wave trains all oriented at random. It is considered as the resultant of two optical vector components which are incoherent.

2. Linearly polarised light

It is the resultant of two linearly polarised waves.

3. Partially polarised light

It is a mixture of linearly polarised light and unpolarised light.

4. Elliptically polarised light

It is the resultant of two coherent waves having different amplitudes and a constant phase difference of 90°. In elliptically polarised light, the magnitude of electric vector $\vec{E}$ changes with time and $\vec{E}$ rotates about the direction of propagation.

5. Circularly polarised light

It is the resultant of two coherent waves having same amplitude and a constant phase of 90°. In circularly polarised light the magnitude of electric vector $\vec{E}$ remains constant but rotates about the direction of propagation such that it sweeps a circular helix in space.

Optical activity

When a beam of plane polarised light is allowed to pass through certain materials like quartz, the plane of polarisation gets rotated about the direction of the beam. This is called optical rotation.

Thus, *optical rotation is the turning of plane of linearly polarised light about the direction of motion as the plane polarised light travels through certain material.*

The ability to rotate the plane of polarisation of plane polarised light by certain substances is called optical activity.

Substances showing this ability are called optically active substances. Quartz, cinnabar are examples of optically active crystals. While a aqueous solutions of sugar, tartaric acid, turpentine, quinine in alcohol are optically active solutions. Depending upon the sense of rotation optical rotation is divided into two.

(i) Dextro rotation

(ii) Laevo rotation

Dextro rotation

If the plane of polarisation gets turned towards right (clock wise) when viewed against the direction of light, the optical rotation is called dextro rotation (right handed).

Laevo rotation

If the plane of polarisation gets turned towards left (counter clockwise) when viewed against the direction of light, the optical rotation is called laevo rotation (left handed).

A substance with dextro rotation is called dextro rotory and that with laevo rotation is called levo rotary. Cane sugar, sodium chloride, glucose (dextrose) are dextro rotary and fruit sugar, turpentine, fructose (levulose) are levo rotary.

Note

The word dextro comes from Latin word for right and laevo comes from Latin for left.The history of optical rotation actually dates back to 1811, when French physicist Fracois Jean Dominique Arago observed optical rotation in quartz. Around this same time Jean Bapiste Biot also observed the effect in liquids like turpentine. In 1822 the English astronmer Sir Jonn F.W. Hershel discovered that different crystal forms of quartz rotated the linear polarisation in different directions. In 1849 Louis pasteur resolved a problem concerning the nature of tartaric acid. A solution of this compound derived from living things rotated the plane of polarisation of light passing through it. But tartaric acid derived by chemical synthesis has no such effect, even though its reactions were identical and its elements of composition the same. Pasteur noticed that the crystals came in two assymetric forms that were mirror images of one another. One crystal showing dextro effect and the other laevo effect. An equal mix of the two had no polarising effect. Later it was proved that this optical rotation effect could be explained in terms of chemical bonds between the atoms which lead to a better understanding of the three dimensional nature of molecules.

Specific rotation

The intensity of optical activity is expressed in terms of a quantity called specific rotation.

Specific rotation is a standard measure of the degree to which a substance is dextro rotary or laevo rotary and is defined by an equation that relates the angle through which the plane is rotated, the length of the light path through the sample and the density of the sample (or its concentration if it is present in a solution)

It is observed that the angle (θ) through which the plane of polarisation rotated is directly proportional to

(i) length of the light path through the sample l, and

(ii) the concentration of the solution c.

Then we have

$$\theta \propto lc$$

or

$$\theta = Slc \qquad \text{...... (1)}$$

S is the constant of proportionality called specific rotation. It is a constant only for a given wavelength λ and at a given temperature T. Thus specific rotation is usually denoted by S_λ^T

From equation (1) we can write

$$S_\lambda^T = \frac{\theta}{lc},$$

where θ is the observed angle of rotation in degrees, l is the length measured in decimeter and c is the concentration in gcm^{-3}.

Thus, the specific rotation for a given wavelength of light at a given temperature is defined conventionally as the rotation produced by one decimeter long column solution containing one gram of optically active material per cubic centimeter of solution.

When the length of the solution is measured in centimeter, we have

$$S_\lambda^T = \frac{10\theta}{lc}$$

Polarimeter

A polarimeter is an instrument used for determining the optical rotation of solutions. When used for determining the quantity of sugar in a solution it is called a saccharimeter.

Construction

A polarimeter consists of a convex lens L, a nicol prism(polariser) (N_1), a half shade plate (HSP), a glass tube (G), a nicol prism (analyser) (N_2) and a eyepiece (T) arranged as shown in figure. All these are arranged inside a long tube. The half shade plate is used for accurately adjusting the two nicol prisms for crossed positions. The analyser nicol prism can be rotated and the angle of rotation can be measured from the circular scale attached to it.

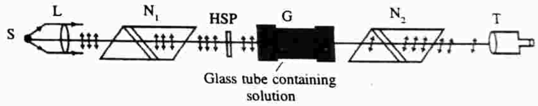


Figure 6.13

Experimental arrangements

A monochromatic source of light (S) is kept in front of the convex lens L. The light from the monochromatic source S is rendered parallel by the lens L. When it is passed through the nicol prism N_1 (polariser) a plane polarised light is produced. This polarised light is passed through the half shade plate (HSP), glass tube G and the nicol prism (analyser) N_2. The final image is viewed through the eyepiece. Looking through the eyepiece T, the analyser is rotated until both halves of the half shade are equally dark. The reading on the circular scale on the analyser is noted as θ_1. At this position the two prisms are crossed. Now the glass tube is filled with the given solution whose specific rotation is to be determined. When viewed through the eyepiece, two halves of the half shade plate is found to be partially illuminated. The analyser is again rotated until both the halves of the half shade plate become equally dark. The reading on the circular scale is noted as θ_2. The difference $\theta_1 - \theta_2$ will give the angle of rotation θ of the plane of polarisation. To repeat the experiment, different concentrations of solution are taken and the corresponding angle of rotation are determined. A graph is plotted between concentration c and the angle of rotation θ. The graph is found to be a straight line. From the graph, slope $\left(\frac{\theta}{c}\right)$ is determined. Using the relation $S = \frac{10\theta}{lc}$ specific rotation can be calculated by measuring the length of the tube in cm.

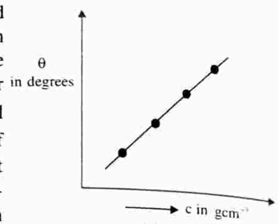


Figure 6.14

Laurents half shade device

It consists of a semi circular quartz plate ACB joined to a semicircular plate ADB of glass. The optic axis of the quartz plate is parallel to the line of separation AB. The thickness of quartz plate is such that, it introduces a phase difference of 180° between the e-ray and o-ray passing through it hence called half wave plate. The thickness of the glass plate is such that it transmits or absorbs the same amount of light as done by the quartz half wave plate. Plane polarised light is allowed to fall on the half shade device normally. One half of the incident light passes through the quartz plate ACB and the other half through the glass plate ADB. The plane polarised light incident normally on the half shade plate has vibrations along OP. On passing through the glass, half the vibrations will remain along OP but on passing through the quartz half, the vibrations will split into e and o rays. The o-vibrations are along OD and e vibration are along OA. The half wave plate introduces a phase difference of 180° between the two vibrations. The vibrations of o-ray will occur along OC instead of OD on emerging from the plate. Therefore the resultant vibrations will be along OQ. Where as the vibrations of the beam emerging from glass plate will be along OP. In effect, the half wave

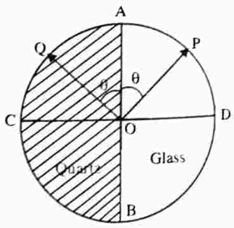


Figure 6.15

plate turns the plane of polarisation of the incident light through an angle 2θ. If the principle plane of the nicol N_2 is alligned parallel to OP, the plane polarised light emerging from the glass tube will pass through the glass plate of the half shade plate and that appears brighter. On the other hand light coming out of the quartz plate is partially obstructed and the corresponding field view appears less bright. If the principal plane of N_2 is parallel to OQ the quartz half will appear brighter than the glass half. Thus the two halves of the plate are unequally illuminated. When the principle plane of N_2 is parallel to AB, the two halves appear equally bright and when it is parallel to CD, the two halves are equally dark. This mechanism will ensure us to make the two nicol prisms at crossed positions.

Biquartz

Biquartz is an optical device used in polarimeter instead of half shade device. The action of biquartz is the same as that of half shade plate but with different manner. It consists of two semicircular plates of quartz each of same thickness (3.75mm) joined along the diameter AB. One half consists of dextro rotatory quartz while the other is laevo rotatory. If white light is used, yellow light is repressed by the biquartz plate and both the halves will have the shade of yellow colour. This can be adjusted by rotating the analyser N_2 to a particular position. When the analyser is rotated to one side from this position. One half of the field of view appears blue, while the other half appears red. If the analyser is rotated in the opposite direction, the first half which was blue earlier now appears as red and the second half which was red earlier now appears as blue. Therefore by adjusting the position of the analyser, the field of view appears equally bright with the shadow of yellow colour.

Figure 6.16

Applications of study of optical rotation

(i) It is used in the sugar industry to measure syrup concentration
(ii) In optics to manipulate polarisation
(iii) In chemistry to characterise substances in solution
(iv) In optical mineralogy to help identify certain minerals in thin sections
(v) In medicine to measure blood sugar concentration in diabetic people and
(vi) to determine the impurifies present in solutions (see example 9)

Example 12

Sugar solution of concentration $0.2 gcm^{-3}$ contained in a tube of length 18 cm rotates the plane of polarisation by 23.4°. Calculate the specific rotation of sugar solution.

Solution

$c = 0.2 gcm^{-3}$, $l = 18$ cm, $\theta = 23.4°$

Using $S = \frac{10\theta}{lc} = \frac{10 \times 23.4}{18 \times 0.2} = 65°$

Example 13

A 20 cm long tube containing 50 cm^3 of sugar solution produces an optical rotation of 10°. Calculate the quantity of sugar contained in the solution. Specific rotation of sugar is 65°.

Solution

$l = 20$ cm, $V = 50 cm^3$, $\theta = 10°$, $S = 65°$

Using $S = \frac{10\theta}{lc}$

We have $c = \frac{10\theta}{lS} = \frac{10 \times 10}{20 \times 65} = 0.0769 gcm^{-3}$

$\therefore$ Mass of sugar in Vcm^3 solution = $0.0769 \times V$

$0.0769 \times 50 = 3.845g$

Example 14

A sample of turpentine give 52° optical rotation for 18 cm length of the solution. Calculate the percentage of impurity of the turpentine. For the pure turpentine $S = 37°$ and density $0.87 gcm^{-3}$.

Solution

$\theta = 52°$, $l=18cm$, $S = 37°$

Using $S = \frac{10\theta}{lc}$, we have

$$c = \frac{10\theta}{lS} = \frac{10 \times 52°}{18 \times 37} = 0.781 gcm^{-3}$$

since the pure solution contains $0.87 gcm^{-3}$, the impurity content

$= 0.87 - 0.781 = 0.089$

$\therefore$ Percentage of impurity $= \frac{0.089}{0.87} \times 100$

$= 10.23\%$

Example 15

Specific rotation of sugar solution is 65°. If the glass tube of the saccharimeter having length 20 cm contains sugar solution of concentration $0.1 gcm^{-3}$, through what angle the plane of polarisation turned.

Solution

$S = 65°$, $l = 20$ cm, $c = 0.1 gcm^{-3}$

Using $S = \frac{10\theta}{lc}$, we have $\theta = \frac{Slc}{10}$

$\theta = \frac{65 \times 20 \times 0.1}{10} = 13°$

UNIVERSITY MODEL QUESTIONS

Section A

(*Answer questions in* **two** *or* **three** *sentences*)

Short answer type questions

1. What is meant by polarisation of light?
2. What is meant by plane polarised light ?
3. Distinguish between ordinary light and plane polarised light.
4. What is polarising angle? How it is related to the refractive index of the medium?
5. State and explain Brewsters law.
6. Distinguish between e-rays and o-rays.
7. What are negative and positive crystals and give two examples for each?
8. What is a wave plate?
9. What is a quarter wave plate?
10. How will you distinguish plane polarised, circularly polarised and elliptically polarised lights?
11. Define (i) optical rotation and (ii) optical activity.
12. Define specific rotation. What is its unit?

13. Define dextrorotary and laevorotary substances. Give two example for each.
14. What are the factors on which the optical rotation of an optically active solution depend?
15. What is a polarimeter?
16. Mention any three applications of optical rotation.
17. What are uniaxial and biaxial crystals? Give two example for each.
18. Draw the diagram of a polarimeter.
19. Define Malus law. At what conditions maximum and minimum intensity of light will be obtained.
33. Define refractive index of (i) o-ray (ii) e-ray.
34. Draw the diagram of Huygens wave surfaces produced by a point source embeded in a birefrigent crystal.
35. Define the optic axis in the case of a double refraction crystal.
36. Name the five types of polarised light.
37. What is a phase retardation plate why it is called so?
38. What is an unpolarised light?

Section B

(Answer questions in a **paragraph** *of about half a page to one page)*

Paragraph / Problem type questions

1. Derive Brewster's law.
2. Show that the reflected and refracted rays are at right angle to each other when rays are incident at polarising angle.
3. What is meant by double refraction? How will you demonstrate it experimentally?
4. Explain the phenomenon of polarisation by double refraction.
5. What is a wave plate and explain its function ?
6. How a quarter wave plate is constructed ?
7. How a halfwave plate is constructed?
8. Explain how circularly polarised light can be produced.
9. What is an elliptically polarised light? How it can be produced ?
10. Explain the working of quarter wave plate.
11. Explain the working of half wave plate.
12. Explain the working of Laurent's half shade.
13. Explain the working of biquartz.
14. Explain the phenomenon of double refraction according to Huygen.
15. How a plane polarised light is converted into a circularly polarised light?

16. A ray of light in air is incident on a glass plate at the polarising angle. It suffers a deviation of 26° on entering glass. Calculate the refractive index of glass. [1.6]
17. Two nicols are crossed each other. Now one of them is rotated through 60°. What percentage of incident unpolarised light will pass through the system. [37.5]
18. A plane polarised light is incident perpendicularly on a quartz plate cut with faces parallel to optic axis. Find the thickness of quartz plate which introduces a phase difference of 45° between e and o-rays $\mu_e = 1.553$ $\mu_o = 1.544$ and $\lambda = 5400Å$ [7.5μm]
19. Calculate the least thickness of a sapphire plate which would convert plane polarised light into circularly polarised light. Given $\mu_o = 1.768$, $\mu_e = 1.760$ and wavelength of light is 590 mm [0.184μm]
20. Calculate the thickness of ice capable of inverting a circularly polarised light $\mu_o = 1.309$, $\mu_e = 1.313$, $\lambda = 590\,nm$. [0.7375μm]
21. Calculate the thickness of (i) a quarter wave plate and (ii) a half waveplate. Given that $\mu_o = 1.973$, $\mu_e = 2.656$ and $\lambda = 590\,nm$. [(i) 21.6μm (ii) 43.2μm]
22. Specific rotation of sugar solution is 65° and its density is $0.2 gcm^{-3}$. A sample of sugar is adultered, and gives a rotation of 20° for 20 cm length of the solution. Calculate percentage purity of sample. [75%]
23. Calculate the specific rotation of turpentine. If the plane of polarisation is turned through 64°. The length of the tube is 20 cm and concentration is $0.87 gcm^{-3}$. [36.8°]
24. Calculate the thickness of half wave plate for sodium light ($\lambda = 5893Å$), given $\mu_o = 1.54$ and ratio of velocity of ordinary and extra ordinary components is 1.007. [26.79μm]
25. Quartz plate of thickness 0.03 mm works as a phase retardation plate for light of wavelength 6000Å. Calculate the value of retardation in phase. [π rad]

Section C

(Answer questions in about **one** *or* **two** *pages)*

Long answer type questions (Essays)

1. Explain with theory the production of circularly polarised and elliptically polarised light waves.
2. Explain the construction and working of Laurents half shade polarimeter to measure specific rotation.

Hints to problems

1 to 15 see book work

1. Deviation, $d = i_p - r' = i_p - (90 - i_p)$ $\because$ $i_p + r' = 90$

$$d = 2i_p - 90$$

or $$i_p = \frac{d + 90}{2} = 58 \quad \text{use } \mu = \tan i_p$$

2. See example 4

3. Phase difference = $\frac{2\pi}{\lambda} \times$ path difference

$\therefore$ Path difference = $\frac{\lambda}{8}$ use $(\mu_e - \mu_o)t = \frac{\lambda}{8}$

4. To convert into circularly polarised light, the path difference must be $\frac{\lambda}{4}$. Thus

$(\mu_o - \mu_e)t = \frac{\lambda}{4}$

5. To invert the handedness, a half wave plate is used. Thus we have $(\mu_e - \mu_o)t = \frac{\lambda}{2}$

6. (i) $t = \frac{\lambda}{4(\mu_e - \mu_o)}$ (ii) $t = \frac{\lambda}{2(\mu_e - \mu_o)}$

7. See example 14. $\frac{0.15}{0.20} \times 100$

8. See example 12.

9. $t = \frac{\lambda}{2(\mu_e - \mu_o)}$, $\frac{\mu_e}{\mu_o} = 1.007$, $\therefore \mu_e = 1.551$

10. Phase difference $= \frac{2\pi}{\lambda} \times$ path difference $= \frac{2\pi}{\lambda}(\mu_e - \mu_o)t$

IMPORTANT FORMULAE

1. Brewster's law: $\mu = \tan i_p$

2. Malu's law: $I = I_0 \cos^2 \theta$

3. Birefrigence magnitude, $\Delta\mu = \mu_e - \mu_0$

If $\mu_e > \mu_0$, positive crystal and $v_e < v_0$

If $\mu_e < \mu_0$, negative crystal $v_e > v_0$.

4. For quarter wave plate: $t = \frac{\lambda}{4(\mu_e \sim \mu_0)}$

or $t = \frac{\pi}{2(\mu_e \sim \mu_0)}$

5. For half wave plate: $t = \frac{\lambda}{2(\mu_e \sim \mu_0)}$

or $t = \frac{\pi}{(\mu_e \sim \mu_0)}$

6. Specific rotation: $S_\lambda^T = \frac{10\theta}{lc}$

θ is the angle in degrees, l is the length measured in centimeter and c is the concentration in gcm^{-3}.