UNIT THREE

SECOND LAW OF THERMODYNAMICS

Conversion of work into heat and vice versa

First law of thermodynamics enables us to convert work into heat and vice versa in principle. How to achieve it experimentally is the discussion of this section.

Work into heat

According to first law of thermodynamics we have

$$Q = U_f - U_i + W$$

If a system is subjected to a process such that it is brought back to the initial state, then $U_f - U_i = 0$

$$\therefore \quad W = Q$$

This shows that work can be completely converted into heat and the efficiency is 100%. The process can be continued indefinitely since in each process system comes back to the initial state. This indicates that work can be transformed into heat indefinitely.

How to achieve this in practice is our problem. For example when two stones are rubbed under water, it is due friction, heat is produced.

As a result temperature of water rises. If the mass of water is very large there will be no appreciable change of temperature of water. Here water acts like a reservoir. Since the state of system (stones) is same at the beginning and end. The net result of the process is the conversion of work into heat.

That is to convert work into heat we require three things (i) a system (ii) cyclic process and (iii) a reservoir.

Heat into work

Consider a gas enclosed in a cylinder provided with a movable piston. When it is heated the cylinder at the bottom, the gas expands and pushes the piston upward doing work. Our problem how to get work indefinitely without any changes in the system. One way is supply the heat at constant temperature. According to first law

$$Q = U_f - U_i + W$$

or $$Q = W \qquad (\because \quad U_f - U_i = C_V dT = 0)$$

i.e. heat is converted into work completely. But in this process, volume increases and pressure decreases. This process can continue only up to pressure reaches atmospheric pressure. i.e., the extraction of work stops. Therefore the process of isothermal expansion cannot be used to extract work indefinitely.

So to get work from heat we need a series of processes in which a system is brought back to its initial state (cyclic process). In each cycle work is done by absorbing heat. For this to takes place we require two reservoirs one at high temperature called source and the other at low temperature called sink.

Let Q_H be the heat exchanged between the source and the system and Q_L be the heat exchanged between the sink and the system. W be the work done during each cycle. To realise this in process heat must be extracted from the source by the system. The system performs work in each cycle then the remaining heat will be rejected by into sink. A device does this process is called heat engine.

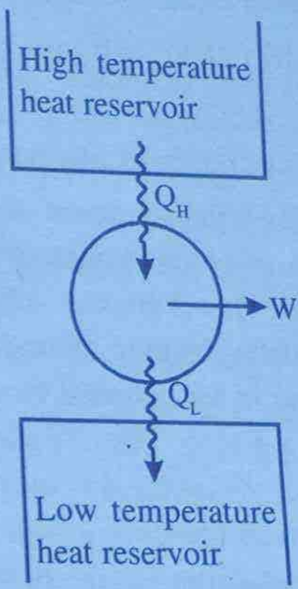


Figure 3.1: Schematic representation of heat engine

Heat engine

Any device which converts heat energy into mechanical energy is called heat engine.

Thermal efficiency of heat engine

The heat extracted (Q_H) from the source during each cycle is called the input whereas work obtained (W) during each cycle is called the output. The thermal efficiency (η) of the engine is defined as the ratio between the workout put (W) to the heat input Q_H.

i.e. $$\eta = \frac{\text{work output}}{\text{heat input}}$$

$$\eta = \frac{W}{Q_H} \qquad \text{..... (1)}$$

According to first law of thermodynamics

$$Q_H - Q_L = U_f - U_i + W$$

For a cyclic process

$$Q_H - Q_L = W$$

$$\therefore \quad \eta = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H} \qquad \text{..... (2)}$$

This equation shows that the efficiency of the engine is always less than one (100%). If $Q_L = 0,\ \eta = 1 = 100\%$

It shows that the efficiency of the heat engine is 100% only if engine works without rejecting any heat to the sink. This cannot be realised in actual practice.

The transformation of heat into work is usually accomplished in practice by two types of engines namely internal combustion engine, such as the gasoline engine and the diesel engine, and external combustion engine such as the steam engine and the stirling engine. In both the engines working is almost the same. In general a heat engine in its simplest form consists of a cylinder closed at one end and provided with a piston. The cylinder contains gas or mixture of gas the as the working substance. When the gas in the cylinder is ignited the temperature and pressure of the system increases to a high value. The high pressure provides force to perform work. In the gasoline and the diesel engines combustian takes inside the cylinder are called internal combustian engines, where as the combustian takes place outside the cylinder are called external combustian engines.

Heat engine : Kelvin-Planck statement of the second law

We found that there are different kinds of heat engines. Here we discuss the fundamental theory of operation of heat engines. The second law of thermodynamics is based upon the operations of heat engines. Though second law is an independent law stands by itself it emerges from the draw back of first law.

Second law of thermodynamics

The first law of thermodynamics gives the relation between heat and mechanical energy and that one of them can be converted into another. But it has two major limitations.

(i) It does not give information regarding the direction in which change will take place, and

(ii) it does not specify the limit to which heat can be converted to work.

To circumvent the difficulties not explained by first law, second law was formulated. It was not derived on any theoretical basis but on the basis of experiments.

There are number of ways in which the second law of thermodynamics can be stated. Here we explain four of them, one due to Lord Kelvin, other due to Clausius and third one due to Planck.

Kelvin's statement

"It states that it is impossible to get continuous supply of work by cooling a body to a temperature lower than that of the coldest of its surroundings".

Clausius's statement

"It states that it is impossible to transfer heat from a cold body to a hot body without doing external work".

This statement is based upon the performance of refrigerator (A heat engine working in reverse direction). This means that the natural flow of heat is always from a hot body to a cold body. If heat is to be transferred from a cold body to a hot body work will have to be done by an external agency.

Planck's statement

"It is impossible to construct an engine which working in a complete cycle will produce no effect other than the raising of a weight and cooling of a heat reservoir."

Thus, it is impossible to construct an engine which working in a complete cycle will produce no effect other than the absorption of heat from a reservoir and its conversion into an equivalent amount of work. i.e. perpetual motion of second kind is impossible. The engine must reject a part of the heat absorbed to a sink at lower temperature.

The statement of Kelvin and that of Planck can be combined into one equivalent statement known as Kelvin-Planck's statement of the second law of thermodynamics.

Kelvin-Planck statement

"It is impossible to construct an engine which, operating in a cycle, has the sole effect of extracting heat from a reservoir and performing an equivalent amount of work".

It may be noted that second law is not a deduction from first law, it is a separate law of nature. The first law denies the possibility of creating or destroying energy. The second law denies the possibility of utilising energy in a particular way. The continuous operation of a machine that creates its own energy thus violates the first law is called perpetual motion machine of the first kind. The operation of a machine that utilises the internal energy of only one reservoir thus violating the second law is called perpetual machine of the second kind.

Refrigerator-Clausius' statement of second law

A heat engine is a device that takes a working substance through a cycle of operation during which some heat is absorbed by the system from a higher temperature heat reservoir and doing work by the system on the surroundings and the remaining heat is rejected to the lower temperature heat reservoir.

Refrigerator

A refrigerator is a heat engine working in the reverse order and works on the principle of second law. A device in which some heat is absorbed by the system from a heat reservoir at low temperature, a larger amount of heat is rejected to a reservoir at a high temperature by doing work on the system by the surroundings. A device that performs a cycle in this way is called a refrigerator. The working instance (system) is called a refrigerant. Refrigerators are used for climate control. Air conditioner and heat pump are the two examples.

Let Q_L be the amount of heat absorbed by the refrigerant from the lower temperature reservoir, Q_H be the amount of heat rejected by the refrigerant to the high temperature reservoir and W be the network done on the refrigerant by the surroundings. A schematic diagram of refrigerator is shown in figure 3.2.

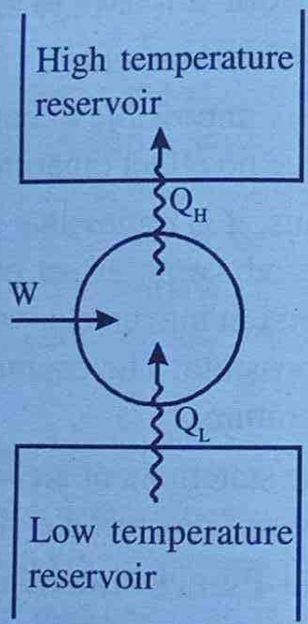


Figure 3.2: Schematic representation of the refrigerator

According to first law, we have

$$Q_H - Q_L = U_f - U_i + W$$

As the refrigerant undergoes a cycle, change in internal energy is zero.

Thus $$Q_H - Q_L = W$$

or $$Q_H = Q_L + W$$

Here $Q_H > Q_L$, absorbing a small quantity of heat from the low temperature reservoir and rejecting large amount of heat to the reservoir at high temperature the above equation shows that work is to be done on the refrigerant. In other words work is necessary to transfer heat from cold body to a hot body. The negative statement of this leads us to the Clausius statement of second law. The statement is already given.

Equivalence of Kelvin-Planck and Clausius statement

Kelvin-Planck statement and Clausius statement are two different statements of second law. Two statements are said to be equivalent when the truth of one implies the truth of the second, and the truth of the second implies the truth of the first.

Let K represent the truth of Kelvin-Planck statement and C represent the truth of Clausius statement. We use two symbols $\equiv$ and $\supset$. The symbol $\equiv$ denotes equivalent. The symbol $\supset$ means to imply.

In symbolic language the equivalence of two statements can be put in the following way

$$K \equiv C$$

If $K \supset C$ and $C \supset K$.

The equivalence of two statements can be put in another way. Two statements are said to be equivalent when the falsity one implies the falsity of the second and the falsity of second implies falsity of the first.

Let $-K$ represent the falsity of Kelvin-Planck statement and $-C$ represent the falsity of Clausius statement

In symbolic language it can be put as

$$K \equiv C$$

If $-K \supset -C$ and $-C \supset -K$.

To demonstrate the equivalence of Kelvin-Planck statement and Clausius statement we will use the second definition of equivalence.

1. To prove that $-C \supset -K$

Consider a refrigerator. Let Q_L be amount of heat absorbed from the low temperature reservoir and Q_H be the amount of heat rejected to the higher temperature reservoir without doing work W on the refrigerant. (see schematic representation in Fig. 3.3). This violates the Clausius statement. Suppose a heat engine operates between the same two reservoirs such that the same Q_L is rejected to the low temperature reservoir.

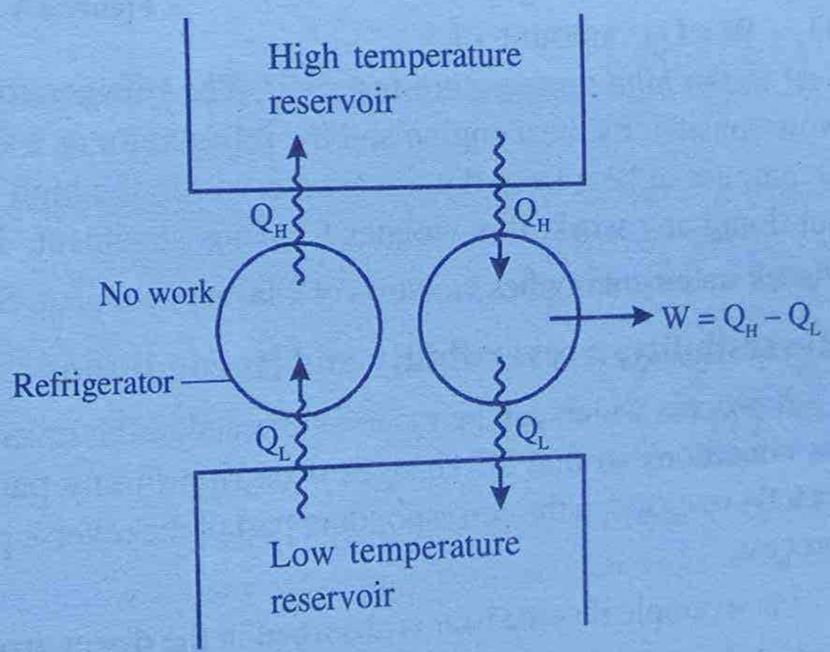


Figure 3.3

The heat engine does not violate any law. Now consider the heat engine and the refrigerator as a single machine that takes $Q_H - Q_L$ amount of heat from the high temperature reservoir and converts all this heat into work without producing any change in the low temperature reservoir. This violates Kelvin-Planck statement. So we proved that $-C \supset -K$.

2. To prove that $-K \supset -C$

Consider an engine working between high temperature reservoir and low temperature reservoir. Suppose the engine absorbs Q_H amount of heat from the high temperature reservoir and does W amount of work and no heat is transferred to low temperature reservoir. It violates Kelvin-Planck statement (see schematic representation in figure 3.4). Suppose a refrigerator operates between the same two reservoirs such that Q_L amount of heat is absorbed from the low temperature reservoir by using up the work done by the engine and rejects $Q_L + W = Q_H$ amount of heat to the high temperature reservoir. The refrigerator does not violate any law. Now consider the heat engine and the refrigerator as a single machine that transfers Q_L amount of heat from the lower reservoir to the high temperature reservoir without doing any work. this violates Clausius statement. That is violation of Kelvin-Planck statement implies violation of Clasius statement. So we proved that $-K \supset -C$.

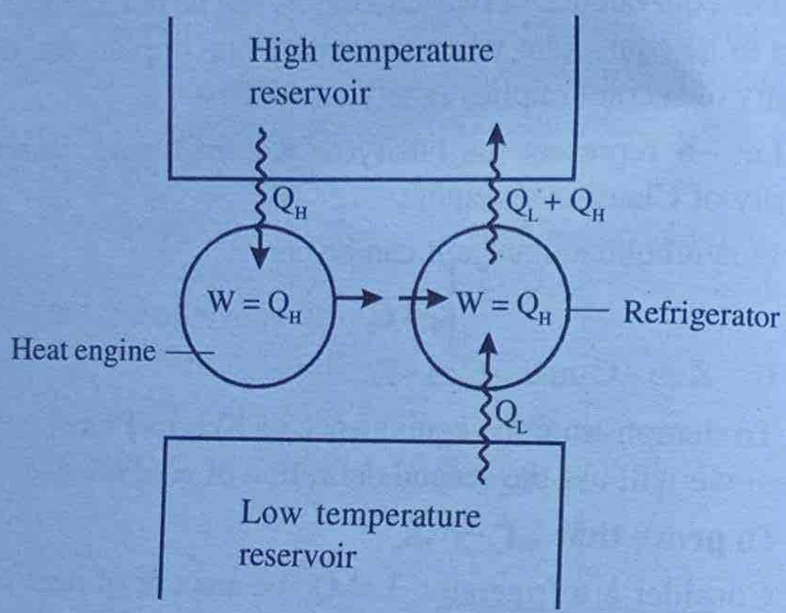


Figure 3.4

Reversibility, irreversibility and its conditions

A process which can be made to proceed in the reverse direction by variation in its conditions so that all changes occurring in any part of the direct process are exactly reversed in the corresponding part of the reverse process is called a reversible process.

For example if some heat is absorbed in the direct process the same amount must exactly be given out in the reverse process. If work is done by some substance during the direct process an equal amount of work must be done on it during the reverse process.

For a reversible process to take place,

(i) dissipative forces like friction, inelasticity, viscosity, electrical resistance etc., should be absent

(ii) the system should always be in mechanical and thermal equilibrium with the surroundings.

(iii) must be quasi-static.

1. Take some gas enclosed in an insulated cylinder fitted with well lubricated piston. Compress the gas very slowly by applying on the piston a pressure slightly exceeding the pressure exerted by the gas. Some work is done on the gas by compressing it. Now reduce the pressure on the piston so that it is slightly smaller than the pressure exerted by the gas. The gas will expand and do almost the same work during expansion as was done on it during compression.
2. Take an elastic spring. Compress it gradually by applying some force on it. Some work is done on the spring. Now reduce the force of compression. The spring will expand and so almost the same work as before.
3. The infinitesimally slow isothermal expansion and compression of a gas is a reversible process.

In fact all isothermal and adiabatic operations are reversible when carried out very slowly. It must be remembered that every reversible process must be a quasi-static process. The reverse of this is not true that is every quasi-static process need not be reversible.

Irreversible process

In nature all changes are irreversible because the conditions for the reversible process cannot be satisfied.

Any process which cannot be made to proceed in the reverse direction is called an irreversible process.

Examples

1. Transfer of heat between two bodies at different temperatures.
2. Two gases when left to themselves tend to mix together. But the reverse process, i.e. their mutual separation is not possible.
3. Rusting of iron is an irreversible process. Rusting is a chemical change during which iron gets converted into iron oxide. It cannot by itself come to its original state.
4. Spontaneous expansion of a gas into an evacuated space.

5. Transfer of electricity through a resistor.

Note: It may be noted that in a reversible process both the system and the local surroundings must be restored to their initial states without producing any changes in the rest of the universe. Here universe does not imply cosmic or celestial, simply means surroundings and auxillary surroundings of the system.

Carnot engine and Carnot cycle

Nicolas Leonard Sadi Carnot, a brilliant young French engineer (who died young at 26 in 1832) in the year 1824 proposed an ideal heat engine free from all imperfections of actual heat engine. It consists of a cycle of operations, hence the name Carnot's cycle. Though this can never be realised in practice it provides the best guide for the construction of actual heat engines and efficiency improvement. This ideal heat engine (Carnot engine) consists of four parts.

(i) Insulated cylinder

It is a cylinder with non-conducting walls and conducting bottom. A perfect gas is used as a working substance. The cylinder is fitted with a perfectly non-conducting and frictionless piston.

(ii) Source

It is a hot body of infinite thermal capacity at temperature T_1. Any amount of heat can be drawn from the source without changing its temperature T_1.

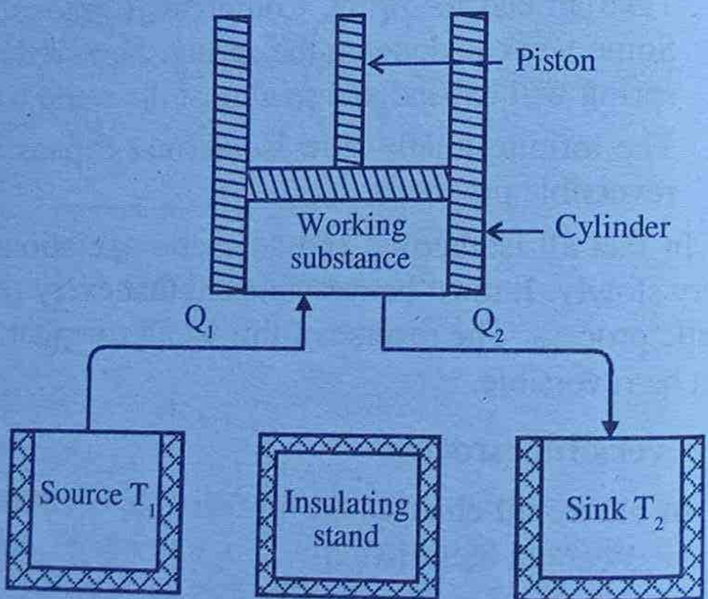


Figure 3.5

(iii) Sink

It is a cold body of infinite thermal capacity at temperature T_2. Any amount of heat can be added to it without changing its temperature T_2.

(iv) Insulating stand

This is a perfectly insulating stand so that the gas can undergo adiabatic changes when the cylinder is placed on it.

Working of Carnot engine

In order to obtain continuous supply of work, the working substance is subjected to the following cycle of operations known as Carnot's cycle.

Carnot's cycle of operations

Consider that the cylinder contains one mole of a perfect gas as the working substance.

1. Isothermal expansion

To start with the cylinder containing the gas at T_1K is placed on the source. The initial pressure and volume of the gas are P_1 and V_1 respectively. It is represented by the point A in the indicator diagram. The gas is allowed to expand isothermally at T_1K until its pressure becomes P_2 and volume V_2. It is represented by the point B in the indicator diagram. Let Q_1 be the amount of heat absorbed from the source during isothermal expansion. The work done by the gas during this expansion is given by

$$Q_1 = W_1 = \int_{V_1}^{V_2} PdV = RT_1 \ln\frac{V_2}{V_1} = \text{Area ABGEA} \qquad \text{..... (3)}$$

2. Adiabatic expansion

The cylinder is now removed from the source and is placed on the non-conducting platform until its temperature falls to T_2K. Let P_3 be the pressure and V_3 be the volume of the gas now. This is represented by the curve BC on the indicator diagram. The work done by the gas during this process (adiabatic) is given by

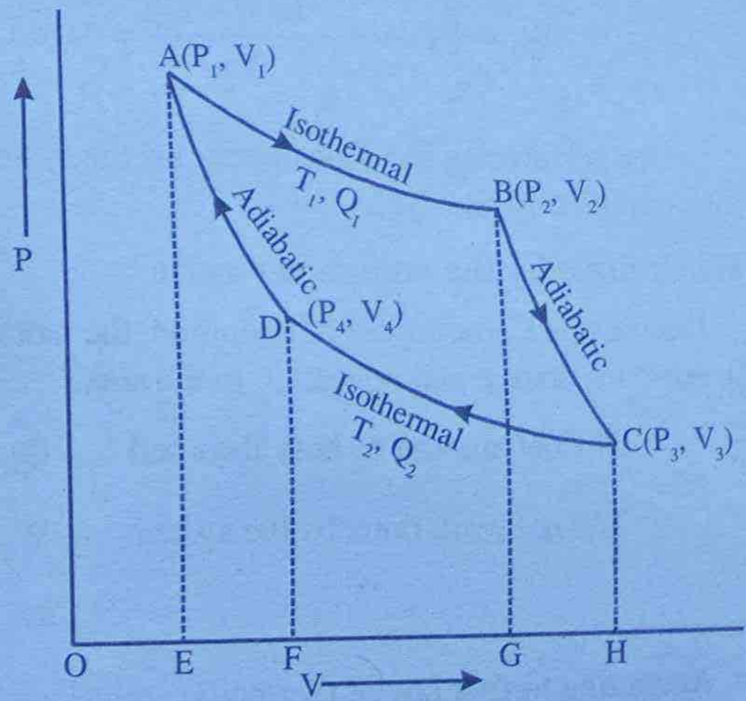


Figure 3.6

$$W_2 = \int_{V_2}^{V_3} PdV = \frac{R(T_1 - T_2)}{\gamma - 1} = \text{Area BCHGB} \qquad \text{..... (4)}$$

3. Isothermal compression

The cylinder is now removed from the insulating stand and is placed over the sink at T_2K and the gas is compressed isothermally. Let P_4 be the pressure and V_4 be the volume of the gas after compression. The process is represented by the curve

CD. Let Q_2 be the quantity of heat rejected to the sink by the gas during isothermal compression. The work done on the gas is given by

$$Q_2 = W_3 = \int_{V_3}^{V_4} PdV = -RT_2 \ln\frac{V_3}{V_4} = \text{Area CHFDC} \quad \text{..... (5)}$$

–ve sign indicates that work is done on the gas.

4. Adiabatic compression

Finally the cylinder is removed from sink and is placed on the non-conducting platform and the gas is compressed adiabatically till the initial condition of the gas i.e. the state $A(P_1, V_1, T_1)$, is regained. This operation is represented by the curve (adiabatic) DA. In this process work done is represented by

$$W_4 = \int_{V_4}^{V_1} PdV = -\frac{R(T_1 - T_2)}{\gamma - 1} = \text{Area DFEAD} \quad \text{..... (6)}$$

After performing the four operations the system comes back to the initial state thus completing one cycle.

Work done by the engine per cycle

During one Carnot cycle of operation, the working substance absorbs an amount Q_1 from the source and rejects Q_2 to the sink.

$\therefore$ The net amount of heat absorbed $= Q_1 - Q_2$

The net work done by the system $= W_1 + W_2 + W_3 + W_4$

$$= W_1 + W_3 \quad (\because\ W_2 = -W_4)$$

According to first law of thermodynamics we have $đQ = dU + đW$

In a cyclic process $dU = 0$

$\therefore$

$$đQ = đW$$

i.e. Heat absorbed = work done

$$Q_1 - Q_2 = W_1 + W_3$$

or

$$Q_1 - Q_2 = RT_1 \ln\frac{V_2}{V_1} - RT_2 \ln\frac{V_3}{V_4} \quad \text{..... (7)}$$

Since the points A and D are at the same adiabatic DA

$$T_1V_1^{\gamma-1} = T_2V_4^{\gamma-1}$$

or
$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_4}\right)^{\gamma-1} \qquad \text{..... (8)}$$

Similarly, points B and C are at the same adiabatic BC

$$T_1V_2^{\gamma-1} = T_2V_3^{\gamma-1}$$

or
$$\frac{T_2}{T_1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1} \qquad \text{..... (9)}$$

Comparing eqs 8 and 9, we get

$$\left(\frac{V_1}{V_4}\right)^{\gamma-1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1}$$

or
$$\frac{V_1}{V_4} = \frac{V_2}{V_3}$$

or
$$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

Substituting this in eq 7, we get

$$Q_1 - Q_2 = RT_1 \ln\frac{V_2}{V_1} - RT_2 \ln\frac{V_2}{V_1}$$

$$Q_1 - Q_2 = R(T_1 - T_2)\ln\frac{V_2}{V_1}$$

$\therefore$ Work done $= Q_1 - Q_2 = R(T_1 - T_2)\ln\dfrac{V_2}{V_1}$ ..... (10)

Efficiency

By definition

$$\text{efficiency,} \quad \eta = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = \frac{R(T_1 - T_2)\ln\frac{V_2}{V_1}}{RT_1 \ln\frac{V_2}{V_1}}$$

$$\eta = \frac{T_1 - T_2}{T_1}$$

or $$\eta = 1 - \frac{T_2}{T_1} \qquad \text{.....(11)}$$

This shows that η depends only upon T_1 and T_2 i.e. independent of working substance. For 100% efficiency T_2 must be OK. i.e. the temperature of the sink must be at absolute zero. Since this is impossible, attaining 100% efficiency is impossible. When $T_1 = T_2$, we have $\eta = 0$ i.e. the engine does not work.

Example 1

Calculate the efficiency of an engine that absorbs heat at 600K and exhausts it at 400K.

Solution

$T_1 = 600\text{K},\ T_2 = 400\text{K}$

$\therefore$ Efficiency, $$\eta = \frac{T_1 - T_2}{T_1} = \frac{600 - 400}{600}$$

$$\eta = \frac{2}{6} = \frac{1}{3}$$

$\therefore$ $$\eta = 33.33\%$$

Example 2

A carnot engine takes 200 calories of heat from a source at temperature 400K and rejects 150 calories of heat to the sink. What is the temperature of the sink. Also calculate the efficiency of the engine.

Solution

$Q_1 = 200$ calories, $Q_2 = 150$ calories, $T_1 = 400\text{K}$

Efficiency, $$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{200 - 150}{200}$$

$$\eta = \frac{1}{4}$$

i.e. $$\eta = 25\%$$

Using $$\eta = 1 - \frac{T_2}{T_1}$$

$$\frac{1}{4} = 1 - \frac{T_2}{400}$$

or $$\frac{T_2}{400} = 1 - \frac{1}{4} = \frac{3}{4}$$

$\therefore$ $$T_2 = 300K$$

Example 3

Three designs are proposed for an engine which is to operate between 500K and 300K. Design A is claimed to produce 3000J of work per kcal of heat input, B is claimed to produce 2000J and C, 1000J. Which design would you choose.

Solution

$T_1 = 500K, \; T_2 = 300K$

$\therefore$ $$\eta = \frac{T_1 - T_2}{T_1} = \frac{500 - 300}{500} = 0.4$$

claimed efficiencies of the proposed engines are

$$\eta[A] = \frac{\text{Work output}}{\text{Heat input}} = \frac{3000}{4185} = 0.72$$

$$\eta[B] = \frac{2000}{4185} = 0.48$$

$$\eta[C] = \frac{1000}{4185} = 0.24$$

Since A and B claim efficiencies greater than that of the ideal engine which is not possible, we choose design C.

Example 4

A reversible engine converts one fifth of heat which it absorbs at source into work. When the temperature of the sink is reduced by 77°C, its efficiency is doubled.

Compute the temperature of the source and sink. $\eta = \frac{1}{5}$

Solution

We have $$\eta = 1 - \frac{T_2}{T_1}$$

$$\frac{1}{5} = 1 - \frac{T_2}{T_1} \qquad \text{..... (12)}$$

When T_2 is lowered by 77°C (77K), the efficiency is doubled.

$$\therefore \qquad \frac{2}{5} = 1 - \frac{(T_2 - 77)}{T_1}$$

$$\frac{2}{5} = 1 - \frac{T_2}{T_1} + \frac{77}{T_1}$$

$$\frac{2}{5} = \frac{1}{5} + \frac{77}{T_1}$$

or $$\frac{1}{5} = \frac{77}{T_1}$$

$$T_1 = 77 \times 5 = 385\text{K}.$$

Putting this in eq 12, we get

$$\frac{1}{5} = 1 - \frac{T_2}{385}$$

$$\frac{T_2}{385} = 1 - \frac{1}{5} = \frac{4}{5}$$

$$\therefore \qquad T_2 = \frac{385 \times 4}{5} = 308\text{K}$$

Carnots refrigerator

Carnots cycle is perfectly reversible. It can work as a heat engine and also as a refrigerator.

When it works as a heat engine it absorbs Q_1 amount of heat from the source at temperature T_1, does an amount of work W and rejects Q_2 amount of heat to the sink at a lower temperature $T_2 (T_1 < T_2)$. See figure (a)

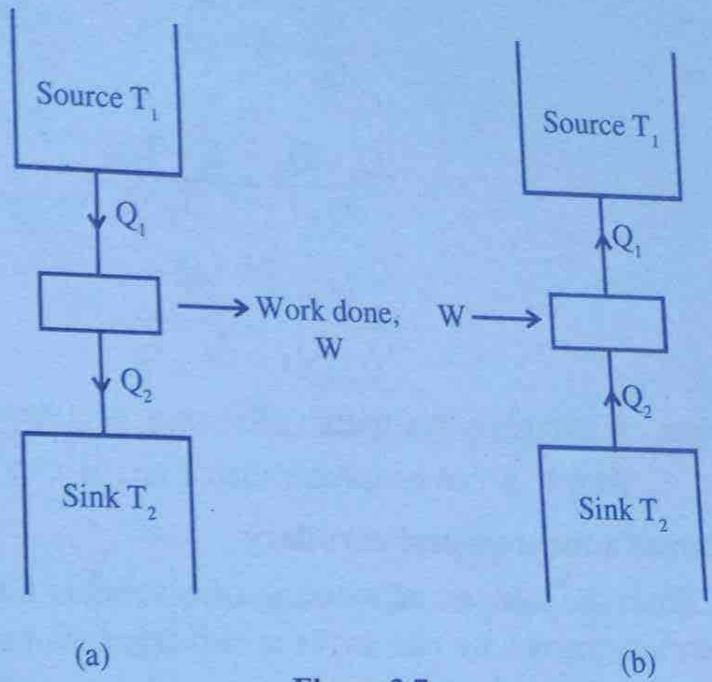


Figure 3.7

Refrigerator

A refrigerator is a heat engine working in the reverse direction. It works on the principle of second law of thermodynamics. When it works as a refrigerator, it absorbs Q_2 amount of heat from the sink at temperature T_2 by doing W amount of work on the system by means of external agency then reject Q_1 amount of heat $(Q_1 = Q_2 + W)$ to the source at temperature $T_1 (T_1 > T_2)$. See figure above (b). In this case heat flows from lower temperature to higher temperature with the help of external work done. In each cycle Q_2 amount of heat is removed from the sink. This is the principle of refrigerator.

Coefficient of performance of a refrigerator (β)

The coefficient of performance (β) of a refrigerator is defined as the ratio of the amount of heat Q_2 removed from the sink in each cycle to the work W done in each cycle.

i.e.
$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

If the working substance is a perfect gas, we have

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

or
$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

or
$$\frac{Q_1}{Q_2}-1=\frac{T_1}{T_2}-1$$

$$\frac{Q_1-Q_2}{Q_2}=\frac{T_1-T_2}{T_2}$$

$\therefore$
$$\frac{Q_2}{Q_1-Q_2}=\frac{T_2}{T_1-T_2}$$

Note : Remember that unlike efficiency of a heat engine which cannot be greater than 1, β can be greater than 1 i.e., β can be more than 100%.

Carnot's theorem and corollary

From the analysis of working of reversible Carnot's engine and second law of thermodynamics we can arrive at two conclusions. These two are taken together to constitute Carnot's theorem.

According to Carnot's theorem "No engine can be more efficient than a reversible carnot engine working between the same limits of temperature (source and sink) and all reversible engines operating between the same limits of temperature have the same efficiency".

Proof

Consider two engines one reversible (R) and the other one irreversible (I). Both the engines working between the temperatures T_1 (source) and T_2 (sink). Let the amounts of their working substances be so adjusted that the work performed per cycle by engine is the same say W.

If the engine R absorbs Q_1 amount of heat and rejects Q_2 amount of heat, then its efficiency

$$\eta=\frac{W}{Q_1}$$

If the engine I absorbs Q_1' amount of heat and rejects Q_2' amount of heat then its efficiency, $\eta'=\frac{W}{Q_1'}$

Suppose that engine I is more efficient that engine R

i.e.
$$\eta'>\eta$$

or
$$\frac{W}{Q_1'} > \frac{W}{Q_1}$$

i.e.
$$Q_1 > Q_1'$$

$\therefore$ $Q_1 - Q_1'$ is a positive quantity.

Let the two engines coupled together (see figure 3.8) such that engine I drives R backward. Then R performs as a refrigerator driven by I. Therefore engine R extracts $Q_1 - W$ amount of heat from the sink and work W being done on it by the engine I, transfers Q_1 amount of heat to the source.

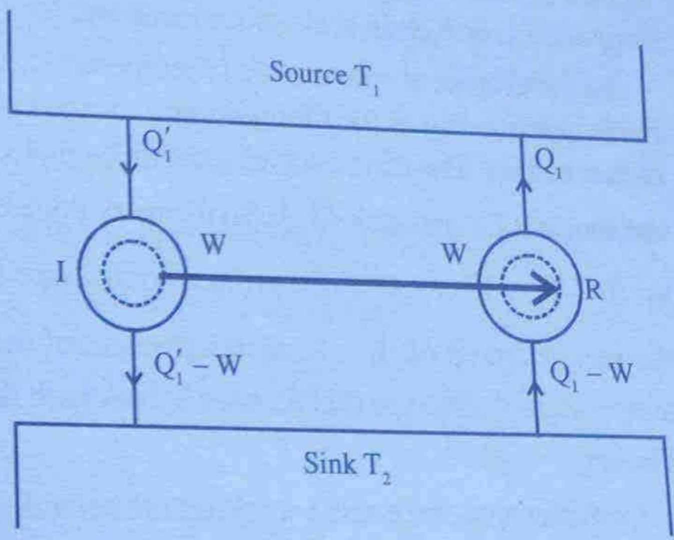


Figure 3.8

The source thus loses Q_1' amount of heat and gains Q_1 amount of heat, therefore source gains $Q_1 - Q_1'$ amount of heat ($\because Q_1 > Q_1'$). The sink gains $Q_1' - W$ amount of heat and loses $Q_1 - W$ amount of heat so the sink loses $(Q_1 - W) - (Q_1' - W) = Q_1 - Q_1'$ amount of heat.

The above discussion shows that the heat gained by the source is equal to heat lost by the sink. Thus we can say that the coupling of engines R and I behaves like a self acting machine which requires no external agency to transfer heat from the sink to the source. This is against second law of thermodynamics. This implies that our basic assumption that the irreversible engine (I) is more efficient than the reversible engine (R) is wrong. In other words no engine operating between a given source and sink can be more efficient than a reversible engine operating between the same source and sink. This is the same thing as saying that the efficiency of a reversible engine operating between a given source and sink is maximum.

To prove the second part of the theorem we consider two reversible engines R_1 and R_2 and assume that R_2 is more efficient than R_1. Proceeding as before we can show that R_2 cannot be more efficient than R_1. Therefore all engines working between the same two temperatures have the same efficiency.

Thermodynamic scale of temperature

To measure temperature we make use of thermometers. In different thermometers different thermometric substances are used. Depending upon the nature of the substances used we get different scales of temperatures. Hence temperature measured with different thermometers do not agree with one another i.e., there is no ideal thermometric scale to measure temperature. To overcome this difficulty Lord Kelvin in 1848 suggested a new scale of temperature known as absolute scale of temperature or Kelvin scale of temperature.

To develop an absolute scale of temperature, the measuring temperature must be made independent of the thermometric substance. We have one such system. That is carnot engine. The efficiency of a carnot engine depends only on the temperature of the source (T_1) and sink (T_2). So the work done by the carnot's engine depends only on $T_1 - T_2$. This idea can be utilised to measure $T_1 - T_2$. Here it may be noted that the measurement of $T_1 - T_2$ is independent of the working substance. The scale of temperature defined in this manner agrees with the ideal gas scale.

Theory

Consider a carnot engine working between T_1 and T_2 temperature measured on any arbitrary scale. Let Q_1 be the amount of heat absorbed at T_1 and Q_2 be the amount of heat rejected at T_2.

We have $$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} = f(T_1, T_2)$$

$$\therefore \quad \frac{Q_2}{Q_1} = 1 - f(T_1, T_2)$$

or $$\frac{Q_1}{Q_2} = \frac{1}{1 - f(T_1, T_2)} = F(T_1, T_2) \qquad \text{..... (i)}$$

Where F is some other function of T_1 and T_2 similary, if the carnot engine working between T_2 and $T_3 (T_2 > T_3)$ absorbing a heat Q_2 and rejecting Q_3. We can write

$$\frac{Q_2}{Q_3} = F(T_2, T_3) \qquad \text{..... (ii)}$$

If it works between T_1 and $T_3 (T_1 > T_3)$, then

$$\frac{Q_1}{Q_3} = F(T_1, T_2) \qquad \text{..... (iii)}$$

eq (i) × eq (ii) gives

$$\frac{Q_1}{Q_2} \times \frac{Q_2}{Q_3} = F(T_1, T_2) \times F(T_2, T_3)$$

i.e.,

$$\frac{Q_1}{Q_3} = F(T_1, T_2) \times F(T_2, T_3)$$

$$F(T_1, T_3) = F(T_1, T_2) \times F(T_2, T_3)$$

This is called function equation. In the above equation L.H.S contains no T_2. So R.H.S. should be independent of T_2. This is possible if we choose

$$F(T_1, T_2) = \frac{\phi(T_1)}{\phi(T_2)} \quad \text{and} \quad F(T_2, T_3) = \frac{\phi(T_2)}{\phi(T_3)}$$

where ϕ is another function of temperature.

$\therefore$

$$F(T_1, T_3) = \frac{\phi(T_1)}{\phi(T_2)} \times \frac{\phi(T_2)}{\phi(T_3)} = \frac{\phi(T_1)}{\phi(T_2)}$$

since $T_1 > T_2$ and $T_1 > T_2$, the function $\phi(T_1) > \phi(T_2)$. This $\phi(T)$ is a linear function of T and can be used to measure temperature. Thus Lord Kelvin suggested that $\phi(T)$ should be taken proportional to T.

i.e.,

$$\phi(T_1) \times T_1 \text{ and } \phi(T_2) \propto T_2 .$$

Now we have

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

or

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

This equation shows that the ratio of the two temperatures on this scale is equal to the ratio of the heat absorbed to the heat rejected. This temperature scale is called Kelvin's thermodynamic scale of temperature.

Example 5

A Carnot's refrigerator takes heat from water at 0°C and rejects it to a room at temperature 27°C. 1 kg of water at 0°C is to be changed into ice at 0°C. How many calories of heat are rejected to the room. What is the work done by the refrigerator in this process. What is the coefficient of performance of the machine (1 calorie = 4.2J).

Solution

$$T_2 = 0^\circ C = 273K, \; T_1 = 27^\circ C = 300K$$

The latent heat of ice $= 80$ cal/g

$$= 80 \times 10^3 \text{cal/kg}$$

It means that 80×10^3 calories of heat is to be removed from 1 kg of water to make it ice at 0°C.

i.e.

$$Q_2 = 80 \times 10^3 \text{cal}$$

Heat rejected to the room $= Q_1$

Using

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

or

$$Q_1 = \frac{T_1}{T_2} Q_2 = \frac{300}{273} \times 80 \times 10^3$$

$$= 87900 \text{ cal}$$

Work done by the refrigerator

$$W = Q_1 - Q_2 = 87,900 - 80,000$$

$$= 7900 \text{ cal}$$

or

$$W = 7900 \times 4.2J$$

$$= 3.183 \times 10^4 J$$

Coefficient of performance $\beta = \dfrac{T_2}{T_1 - T_2}$

$$\beta = \frac{273}{300 - 273} = \frac{273}{27} = 10.11$$

Example 6

When a refrigerator is switched off, the ice stored in a cold storage melts at the rate of 36 kg/hour when the external temperature is 30°C. Find the minimum output power of the motor of the refrigerator required to prevent the ice from melting. L = 80 cal/g, 1 calorie = 4.2J

Solution

ice melts in one hour = 36 kg

$\therefore$ Heat released $= 36\times80\times10^3$ cal

i.e. To prevent melting $36\times80\times10^3$ calories of heat must be removed.

i.e. $$Q_2 = 36\times80\times10^3\,\text{cal}$$

$$T_1 = 30^\circ C = 303K,\ T_2 = 273K$$

Using $$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

or $$Q_1 = \frac{T_1}{T_2}Q_2 = \frac{303}{273}\times36\times80\times10^3$$

$$Q_1 = 1.11\times36\times80\times10^3$$

$\therefore$ Work done, $$W = Q_1 - Q_2 = 0.11\times36\times80\times10^3\,\text{cal}$$

$$W = 31.68\times10^4\,\text{cal}$$

$$W = 31.68\times10^4\times4.2J$$

$$W = 133.056\times10^4\,J$$

This is the work done in one hour

$\therefore$ Work done in one second

$$P = \frac{133.056\times10^4}{60\times60}$$

$$P = 369.6\ \text{watt}$$

Example 7

An ideal refrigerator takes heat from a cold body and rejects to a hot reservoir at

300K. Calculate the amount of work which must be done in order to remove one calorie of heat when the cold body is at (i) 290K (ii) 100K (iii) 1K (iv) 10^{-4} K. What does this problem reveal?

Solution

$T_1 = 300K,\ Q_2 = 1 \text{ calorie} = 4.2J$

Work done $\qquad W = Q_1 - Q_2$

Using $\qquad \dfrac{Q_1}{T_1} = \dfrac{Q_2}{T_2}$

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

or $\qquad \dfrac{Q_1 - Q_2}{Q_2} = \dfrac{T_1 - T_2}{T_2}$

$\therefore \qquad W = Q_1 - Q_2 = \dfrac{T_1 - T_2}{T_2} Q_2$

(i) $\qquad W_1 = \dfrac{T_1 - T_2}{T_2} Q_2 = \dfrac{(300 - 290)}{290} \times 4.2$

$W_1 = 0.1448J$

(ii) $\qquad W_2 = \dfrac{T_1 - T_2}{T_2} Q_2 = \dfrac{(300 - 100)}{100} \times 4.2$

$W_2 = 8.4J$

(iii) $\qquad W_3 = \dfrac{T_1 - T_2}{T_2} Q_2 = \dfrac{(300 - 1)}{1} \times 4.2$

$W_3 = 1260J$

(iv) $\qquad W_4 = \dfrac{T_1 - T_2}{T_2} Q_2 = \dfrac{(300 - 10^{-4})}{10^{-4}} \times 4.2$

$W_4 = 125999995.8J$

The problem reveals that when the temperature of the body becomes lower and lower, more and more work has to be done.

To complete the definition of the thermodynamical scale, we assign the arbitrary value of 273.16 K to the triple point of water T_{TP}.

Thus $$T_{TP} = 273.16\,K$$

For a Carnot engine working between reservoirs at the temperatures T and T_{TP} we have

$$\frac{Q}{Q_{TP}} = \frac{T}{T_{TP}}$$

$$\frac{Q}{Q_{TP}} = \frac{T}{273.16}$$

or $$T = 273.16\frac{Q}{Q_{TP}} \qquad \text{..... (A)}$$

From unit one recall that the equation for the ideal gas temperature, we have

$$T = 273.16\ \mathrm{Lt}_{P_{TP}\to 0}\frac{P}{P_{TP}} \qquad \text{..... (B)}$$

Here pressure is the thermodynamics property used to measure ideal gas temperature. In the limiting case $P_{TP} \to 0$ temperature measurement is independent of the nature of the gas. Comparing equations A and B we can infer that Q takes the role of P. i.e. Q plays the role of thermometric property for a Carnot cycle and at the same time it is independent of the nature of the working substance. Thus thermodynamic scale of temperature is independent of the nature of the working substance. It is essential for a standard reference chosen. It will be soon proved that the thermodynamic scale of temperature and ideal gas temperature scale are numerically equal.

Absolute zero and Carnot efficiency

Recall the relation

$$T = 273.16\frac{Q}{Q_{TP}}$$

when Q is small, T is also small. The smallest possible value of Q is zero and the corresponding value of T is absolute zero.

When a system undergoes a reversible isothermal process without transfer of heat, the temperature at which this process takes place is called absolute zero.

The definition of absolute zero holds for all substances and is independent of specific properties of a substance arbitrarily chosen.

The efficiency of a reversible Carnot engine is given by

$$\eta_R = 1 - \frac{Q_L}{Q_H}$$

where Q_H is the heat absorbed from a hot reservoir and Q_L is the heat rejected to cold reservoir.

But we know that $\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$

$\therefore$ The efficiency in terms of absolute temperature is

$$\eta_R = 1 - \frac{T_L}{T_H}$$

For η_R to be 100%, T_L must be zero. That is the low temperature reservoir must be at absolute zero in order to convert all heat into work. Since nature does not provide us with a reservoir at absolute zero, a heat engine with 100% efficiency is not possible

Equality of ideal gas and thermodynamic scale

Let θ represent the ideal gas temperature and T represent thermodynamic scale of temperature. Consider a P-V diagram of a Carnot cycle of an ideal gas.

During the isothermal expansion process $(1 \rightarrow 2)$ the heat absorbed Q_1 can be calculated from first law

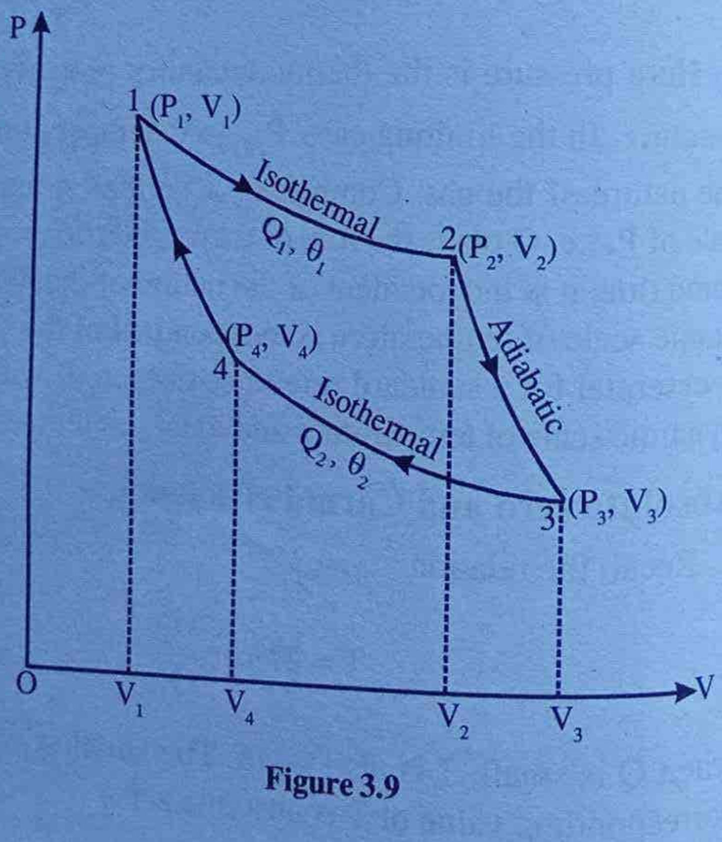


Figure 3.9

$$đQ = C_V d\theta + PdV \qquad (d\theta = 0 \text{ isothermal})$$

$$đQ = PdV$$

$$Q_1 = \int_{V_1}^{V_2} PdV$$

Using $PV = nR\theta_1$, gives $P = \frac{nR\theta_1}{V}$

$$\therefore \quad Q_1 = nR\theta_1 \ln\frac{V_2}{V_1} \qquad \text{..... (1)}$$

Similarly for the isothermal process $(3 \rightarrow 4)$, heat rejected is

$$Q_2 = nR\theta_2 \ln\frac{V_3}{V_4} \qquad \text{..... (2)}$$

$$\therefore \quad \frac{Q_1}{Q_2} = \frac{\theta_1 \ln\frac{V_2}{V_1}}{\theta_2 \ln\frac{V_3}{V_4}} \qquad \text{..... (3)}$$

For the adiabatic expansion process $(2 \rightarrow 3)$

$$đQ = C_V d\theta + PdV \qquad (đQ = 0 \text{ adiabatic})$$

$$C_V d\theta = -PdV$$

Using $PV = nR\theta$ gives $P = \frac{nR\theta}{V}$

$$\therefore \quad C_V d\theta = -\frac{nR\theta}{V} dV$$

or

$$C_V \frac{d\theta}{\theta} = -nR\frac{dV}{V} \text{ integrating}$$

$$\int_{\theta_1}^{\theta_2} \frac{C_V}{\theta} d\theta = -nR\int_{V_2}^{V_3} \frac{dV}{V}$$

or $$\int_{\theta_1}^{\theta_2} C_V \frac{d\theta}{\theta} = -nR \ln \frac{V_3}{V_2} = nR \ln \frac{V_2}{V_3} \qquad \text{..... (4)}$$

Similarly for the adiabatic process $(4 \rightarrow 1)$

$$\int_{\theta_2}^{\theta_1} C_V \frac{d\theta}{\theta} = -nR \int_{V_4}^{V_1} \frac{dV}{V}$$

or $$\int_{\theta_1}^{\theta_2} C_V \frac{d\theta}{\theta} = nR \ln \frac{V_1}{V_4} \qquad \text{..... (5)}$$

comparing equations 4 and 5, we get

$$\frac{V_2}{V_3} = \frac{V_1}{V_4}$$

or $$\frac{V_2}{V_1} = \frac{V_3}{V_4} \qquad \text{..... (6)}$$

Substituting equation 6 in equation 3 yields

$$\frac{Q_1}{Q_2} = \frac{\theta_1}{\theta_2}$$

From thermodynamic scale of temperature we already have

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$\therefore$ $$\frac{T_1}{T_2} = \frac{\theta_1}{\theta_2}$$

If θ_1 and T_1 refer to any temperature and θ_2 and T_2 refer to triple point of water

$$\frac{T}{T_{TP}} = \frac{\theta}{\theta_{TP}}$$

since $T_{TP} = \theta_{TP} = 273.16$ K, we get

$$\theta = T$$

This shows that the ideal gas temperature (θ) and the thermodynamic temperature scale (T) are numerically equal.

Example 8

In the PV diagram shown below, find the efficiency of the engine.

Solution

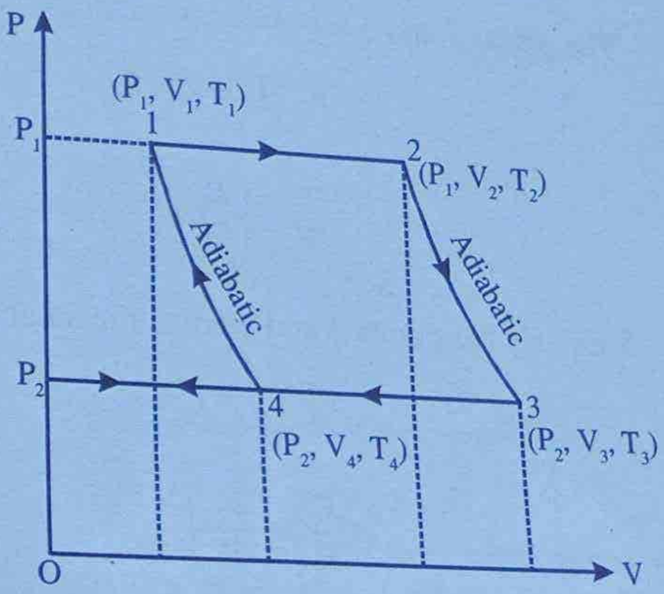


Figure 3.10

Heat is supplied only by isobaric processes.

During the isobaric process $(1 \to 2)$, $\frac{V}{T} =$ constant

when V increases T also increases

$$\therefore \quad dQ_1 = C_P(T_2 - T_1)$$

During the isobaric process $(3 \to 4)$ heat

$$dQ_2 = C_P(T_4 - T_3)$$

Work done by isobaric process $(1 \to 2)$

$$W_1 = P_1(V_2 - V_1) = R(T_2 - T_1)$$

Work done by isobaric process $(3 \to 4)$

$$W_2 = P_2(V_4 - V_3) = R(T_4 - T_3)$$

Work done by adiabatic process $(2 \to 3)$

$$W_3 = \frac{R}{\gamma - 1}(T_3 - T_2)$$

Work done in adiabatic process $(4 \to 1)$

$$W_4 = \frac{R}{\gamma - 1}(T_1 - T_4)$$

$\therefore$ Total work done

$$= W_1 + W_2 + W_3 + W_4$$

$$= R(T_2 - T_1) + R(T_4 - T_3) + \frac{R}{\gamma - 1}(T_3 - T_2) + \frac{R}{\gamma - 1}(T_1 - T_4) \qquad \text{..... (1)}$$

The points 2 and 3 are at the same adiabatic points.

$$P_1^{1-\gamma} T_2^{\gamma} = P_2^{1-\gamma} T_3^{\gamma}$$

$$\therefore \qquad \frac{T_2}{T_3} = \left(\frac{P_2}{P_1}\right)^{\frac{1-\gamma}{\gamma}} \qquad \text{..... (2)}$$

Similarly the points 4 and 1 are at the adiabatic points, we have

$$P_2^{1-\gamma} T_4^{\gamma} = P_1^{1-\gamma} T_1^{\gamma}$$

or

$$\frac{T_1}{T_4} = \left(\frac{P_2}{P_1}\right)^{\frac{1-\gamma}{\gamma}} \qquad \text{..... (3)}$$

Comparing eqs 2 and 3, we get

$$\frac{T_2}{T_3} = \frac{T_1}{T_4} \quad \text{or} \quad \frac{T_1}{T_2} = \frac{T_4}{T_3}$$

Simplifying eq (1), we get

$$W = R\gamma(T_2 - T_1)\frac{\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{1-\gamma}{\gamma}}\right]}{\gamma - 1}$$

$$\eta = \frac{\text{work done}}{\text{heat input}} = \frac{R\gamma(T_2 - T_1)\frac{\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{1-\gamma}{\gamma}}\right]}{\gamma - 1}}{C_P(T_2 - T_1)}$$

put

$$C_P = \frac{R\gamma}{\gamma - 1}$$

$$\eta = 1 - \left(\frac{P_2}{P_1}\right)^{\frac{1-\gamma}{\gamma}}$$

Note: Do the entire problem in one step. Example 11.b.

Example 9

Assuming constant heat capacities, from the given PV diagram show that

$$\eta = 1 - (\gamma - 1)\frac{\left(\frac{V_1}{V_2} - 1\right)}{\left(\frac{P_3}{P_2} - 1\right)}$$

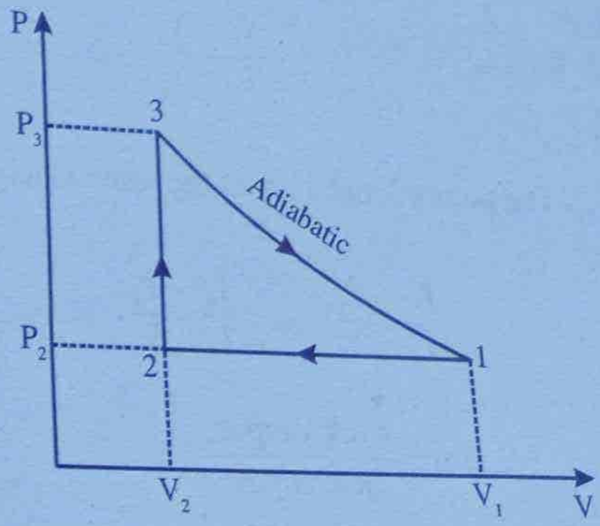


Figure 3.11

Solution

In the isobaric process $(1 \rightarrow 2)\,\frac{V}{T}$ = constant, as V decreases, T also decreases, so heat is rejected. Work done during this process

$$W_1 = C_P \cdot (T_2 - T_1)$$

During isochoric process $(2 \rightarrow 3)\,\frac{P}{T}$ = constant, as P increases, T also increases. So heat is absorbed by the system.

$$Q_1 = C_V(T_3 - T_2)$$

In the adiabatic process, $(3 \rightarrow 1)$ there is no change in heat. Work done by the system is

$$W_2 = \frac{R}{\gamma - 1}(T_3 - T_1)$$

$\therefore$ The network done $= W_1 + W_2$

$$= C_P(T_2 - T_1) + \frac{R}{\gamma - 1}(T_3 - T_1)$$

The points 1 and 2 are at the same isobaric points. Using $\frac{V}{T} =$ constant, we get

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad \text{or} \quad \frac{T_1}{T_2} = \frac{V_1}{V_2}$$

The points 2 and 3 are at the same isochoric points. Using $\frac{P}{T} =$ constant

$$\frac{P_2}{T_2} = \frac{P_3}{T_3} \quad \text{or} \quad \frac{T_3}{T_2} = \frac{P_3}{P_2}$$

$$\eta = \frac{\text{work output}}{\text{heat input}}$$

$$\eta = \frac{C_P(T_2 - T_1) + \frac{R}{\gamma - 1}(T_3 - T_1)}{C_V(T_3 - T_2)}$$

$$\eta = \frac{\gamma(T_2 - T_1) + (T_3 - T_1)}{T_3 - T_2} \qquad \left(\because\ C_P = \frac{R\gamma}{\gamma - 1} \text{ and } C_V = \frac{R}{\gamma - 1}\right)$$

$$\eta = \frac{\gamma(T_2 - T_1) + (T_3 - T_2) + (T_2 - T_1)}{T_3 - T_2}$$

$$\eta = \frac{(T_2 - T_1)(\gamma - 1)}{T_3 - T_2} + 1$$

$$\eta = \frac{T_2\left(1 - \frac{T_1}{T_2}\right)(\gamma - 1)}{T_2\left(\frac{T_3}{T_2} - 1\right)} + 1$$

$$\eta = \frac{\left(1 - \frac{T_1}{T_2}\right)(\gamma - 1)}{\frac{T_3}{T_2} - 1} + 1$$

Substituting for $\frac{T_1}{T_2} = \frac{V_1}{V_2}$ and $\frac{T_3}{T_2} = \frac{P_3}{P_2}$

$$\therefore \quad \eta = \frac{\left(1 - \frac{V_1}{V_2}\right)(\gamma - 1)}{\frac{P_3}{P_2} - 1} + 1$$

or

$$\eta = 1 - \frac{\left(\frac{V_1}{V_2} - 1\right)(\gamma - 1)}{\left(\frac{P_3}{P_2} - 1\right)}$$

Example 10

Take an ideal monatomic gas $\left(\gamma = \frac{5}{3}\right)$ around the Carnot cycle, where $T_1 = 600\text{K}$ and $T_2 = 300\text{K}$.

Point 1 at the beginning of the adiabatic compression $P_1 = P_0$ (atmospheric pressure) and volume 50 litres. Point 3 has a volume $V_3 = 75$ litres. Carnot cycle is shown in figure below. Calculate the values of volume and pressure at all four points.

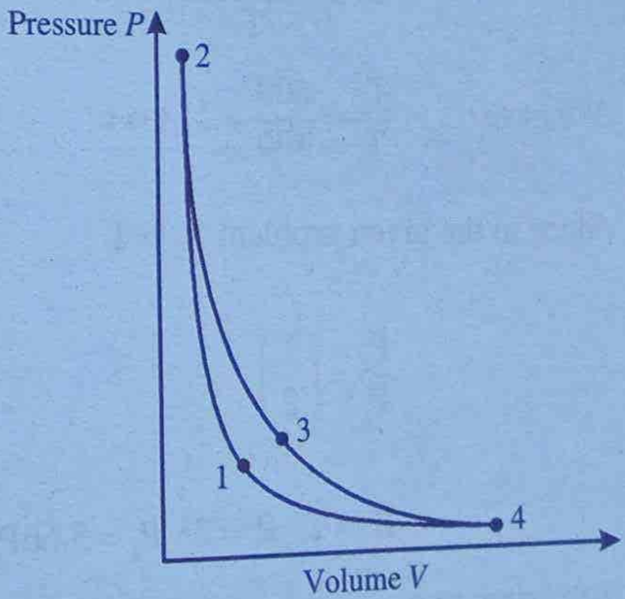


Figure 3.12

Solution

$P_1 = P_0 = 1\,\text{atm}$, $\quad V_1 = 50$ litres, $V_3 = 75$ litres $\gamma = \frac{5}{3}$, $T_1 = 600\,\text{K}$ and $T_2 = 300\,\text{K}$

Points 1, 2, 3 and 4 are designated as follows

Point 1 as P_1, V_1, T_1

Point 2 as P_2, V_2, T_2

Point 3 as $P_3, V_3, T_3 = T_2$

(Since points 2 and 3 are isothermal points)

Point 4 as $P_4, V_4, T_4 = T_1$ (since points 4 and 1 are isothermal points)

The process $1 \rightarrow 2$ is adiabatic compression $T_2 > T_1$

Applying $P^{1-\gamma}T^{\gamma} =$ constant to the adiabatic points 1 and 2, we have

$$P_1^{1-\gamma}T_1^{\gamma} = P_2^{1-\gamma}T_2^{\gamma}$$

$$\left(\frac{P_2}{P_1}\right)^{1-\gamma} = \left(\frac{T_1}{T_2}\right)^{\gamma}$$

so $$\frac{P_2}{P_1} = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{1-\gamma}} \qquad \frac{\gamma}{1-\gamma} = \frac{5}{3\left(1-\frac{5}{3}\right)} = -\frac{5}{2}$$

$$\frac{P_2}{P_1} = \left(\frac{T_1}{T_2}\right)^{-\frac{5}{2}}$$

We have $\frac{T_2}{T_1} = \frac{600}{300} = 2$ given

Since in the given problem $T_2 > T_1$

$$\therefore \quad \frac{P_2}{P_1} = \left(\frac{1}{2}\right)^{-\frac{5}{2}} = 2^{\frac{5}{2}}$$

$$P_2 = 2^{\frac{5}{2}}\, P_1 = 2^{\frac{5}{2}}\, P_0 = 5.66P_0.$$

Applying $PV^{\gamma} =$ constant to the adiabatic points 1 and 2, we get

$$P_1V_1^\gamma = P_2V_2^\gamma$$

$$P_0 50^\gamma = 5.66P_0V_2^\gamma$$

$$V_2^\gamma = \frac{50^\gamma}{2^{\frac{5}{2}}} \text{ gives } V_2 = \frac{50}{\left(2^{\frac{5}{2}}\right)^{\frac{1}{\gamma}}}$$

$$V_2 = \frac{50}{2^{\frac{5}{2}}} = 17.677 \text{ litres.}$$

The process $2 \rightarrow 3$ is isothermal expansion. So points 2 and 3 are isothermal points.

Applying PV = constant to the points we get

$$P_2V_2 = P_3V_3$$

$$P_3 = \frac{P_2V_2}{V_3} = 2^{\frac{5}{2}} P_0 \cdot \frac{50}{2^{\frac{3}{2}}}\frac{1}{75}$$

$$P_3 = 2P_0 \cdot \frac{50}{75} = \frac{4}{3}P_0 = 1.33P_0.$$

The process $3 \rightarrow 4$ is adiabatic expansion, so points 3 and 4 are adiabatic points.

Applying $P^{1-\gamma}T^\gamma =$ constant to the points 3 and 4, we get

$$P_3^{1-\gamma}T_2^\gamma = P_4^{1-\gamma}T_1^\gamma$$

$$\therefore \quad \left(\frac{P_4}{P_3}\right)^{1-\gamma} = \left(\frac{T_2}{T_1}\right)^\gamma$$

$$\frac{P_4}{P_3} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{1-\gamma}} = (2)^{\frac{\gamma}{1-\gamma}} = 2^{-\frac{5}{2}}$$

Thus $$P_4 = 2^{-\frac{5}{2}} \cdot P_3 = \frac{\frac{4}{3}P_0}{2^{\frac{5}{2}}} = 0.235\ P_0.$$

Finally applying PV^{γ} = constant to the adiabatic points 3 and 4, we get

$$P_3V_3^{\gamma} = P_4V_4^{\gamma}$$

$$V_4^{\gamma} = \frac{P_3V_3^{\gamma}}{P_4}$$

$$V_4 = \left(\frac{P_3}{P_4}\right)^{\frac{1}{\gamma}} \cdot V_3 = \left[\frac{4\ P_0}{3(0.235)P_0}\right]^{\frac{3}{5}} \cdot 75$$

$$V_4 = \left(\frac{1.33}{0.235}\right)^{\frac{3}{5}} 75 = 212 \text{ litres.}$$

Example 11

A Carnot engine absorbs 100J of heat from a reservoir at the temperature of the boiling point of water and rejects heat to a reservoir at the temperature of the triple point of water. Find the heat rejected, the work done by the engine and the thermal efficiency.

Solution

$$Q_H = 100\text{J}$$

$$T_H = 100 + 273 = 373\text{K}$$

$$T_L = 273.16\text{K}$$

$\therefore$ Efficiency, $\eta = 1 - \frac{T_L}{T_H}$

$$\eta = 1 - \frac{273.16}{373} = 0.268$$

Using $$\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$$

$\therefore$ $$Q_L = \frac{T_L}{T_H} Q_H = \frac{273.16}{373} \times 100$$

$$Q_L = 73.2\text{J}$$

Work done by the engine, $W = Q_H - Q_L$

$$W = 100 - 73.2 = 26.8\,\text{J}$$

Example 12

Hydrogen is used in a Carnot cycle as a working substance. Find the efficiency of the cycle if as a result of an adiabatic expansion.

(a) the gas volume increase by 2 times.

(b) the pressure decreases by two times.

Solution

(a) $$\eta = 1 - \frac{T_2}{T_1}$$

At the two adiabatic points

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

or $$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{1}{2}\right)^{\gamma-1} = \left(\frac{1}{2}\right)^{1.4-1}$$

$$\frac{T_2}{T_1} = (0.5)^{0.4} = 0.7578$$

$\therefore$ $$\eta = 1 - 0.7578 = 0.24 = 24\%$$

(b) At the two adiabatic points

$$P_1^{1-\gamma} T_1^{\gamma} = P_2^{1-\gamma} T_2^{\gamma}$$

$\therefore$ $$\frac{T_2}{T_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1-\gamma}{\gamma}} = (2)^{\frac{1-1.4}{1.4}} = (2)^{-\frac{0.4}{1.4}}$$

$$\frac{T_2}{T_1} = 0.82$$

$\therefore$ $$\eta = 1 - 0.82 = 0.18 = 18\%.$$

IMPORTANT FORMULAE

1. Thermal efficiency of heat engine:

$$\eta = \frac{\text{work output}}{\text{heat input}} = \frac{W}{Q}$$

$$\eta = \frac{Q_H - Q_L}{Q_H}$$

$$\eta = \frac{T_H - T_L}{T_H}$$

2. Coefficient of performance of refrigerator:

$$\beta = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L}$$

3. Thermodynamic scale of temperature:

$$T = 273.16 \frac{Q}{Q_{TP}}$$

4. Ideal gas temperature:

$$T = 273.16 \, Lt_{P_{TP} \to 0} \left(\frac{P}{P_{TP}} \right)$$

UNIVERSITY MODEL QUESTIONS

Section A

(*Answer questions in about* **two** *or* **three** *sentences*)

Short answer type questions

1. What are the three basic essential things required to convert work into heat?
2. What is heat engine?
3. What is the principle of heat engine?
4. Define the efficiency of a heat engine.
5. A heat engine cannot attain 100% efficiency. Explain why?
6. Distinguish between internal and external comustian engines.
7. Give two examples each of internal and external combusitan engines.
8. Give Kelvin's statement of second law of thermodynamics.
9. Give Planck's statement of second law of thermodynamics.
10. Give Kelvin-Planck statement of second law of thermodynamics.

11. Give Clausius' statement of second law of thermodynamics.
12. What are the limitations of first law of thermodynamics?
13. Distinguish between first and second law of thermodynamics.
14. What is perpetual motion machine of the first kind?
15. What is the perpetual motion machine of the second kind?
16. What is the principle of refrigerator?
17. When two statements are said to be equivalent?
18. What is a reversible process? Give two examples.
19. What are the conditions to be satisfied for a reversible process?
20. What is an irreversible process? Give two examples.
21. Define the coefficient of performance of refrigerator.
22. What is Carnot's theorem?
23. Define absolute zero of thermodynamic scale.

Section B

(*Answer questions in a* **paragraph** *of about half a page to one page*)

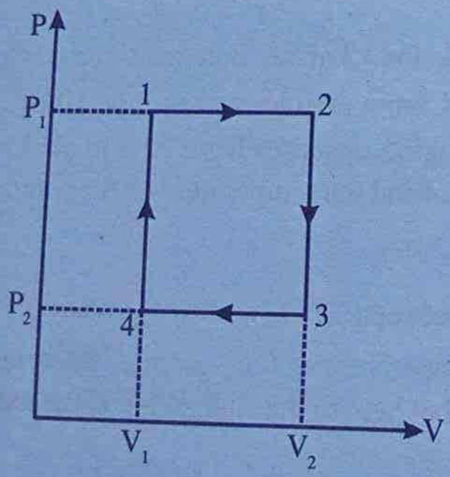
Paragraph / Problem type questions

1. Briefly explain how to convert work into heat.
2. Briefly explain how to convert heat into work.
3. Under isothermal process it is not possible to convert heat into work indefinitely. Explain.
4. Briefly explain the mechanism of refrigerator that leads to Clausius statement of second law.
5. Prove that the Kelvin-Planck and Clausius statements are equivalent.
6. Show that the efficiency of Carnot engine can never be 100%.
7. The efficiency of an ideal engine increases from 20% to 30% when the temperature of the sink is lowered by 40°C. Find the temperature of the source and sink.

   $[T_1 = 400\,K,\ T_2 = 320\,K]$
8. A Carnot engine working between 127°C and 27°C. What is the efficiency [25%]
9. A Carnot engine whose temperature of the source is 400K takes 800 J of heat at this temperature and reject 600J of heat to the sink. What is the temperature of the sink and efficiency of the engine. $[T_2 = 300\,K,\ \eta = 25\%]$
10. A Carnot engine whose low temperature reservoir is at 7°C has an efficiency of 50%. It is desired to increase the efficiency to 70%. By how many degrees should the temperature of high temperature reservoir be increased. [373.3]
11. The efficiency of a ideal engine is 0.2. If the temperature of the sink is lowered by 20°C, the efficiency becomes 0.25. Find the temperature of the source and sink.

   [400K, 320K]

12. A Carnot engine working between a source at 400K and a sink at T_2K has an efficiency of 50%. If the temperature of both source and sink are increased by 100K. What is the efficiency of the engine. [0.4]
13. In a refrigerator heat from inside at 277K is transferred to a room at 300K. How many joules heat will be delivered to the room for each joule of electrical energy consumed ideally. Also compute the coefficient of performance of this refrigerator. [13.04J, 12.04]
14. A Carnot engine working as a refrigerator between 260K and 300K receives 500 calories of heat from the reservoir at the lower temperature. Calculate the amount of heat rejected to the reservoir at the higher temperature. Calculate also the amount of work done in each cycle to operate the refrigerator. [576.9 cal, 323.1J]
15. An inventor claims to have developed an engine that takes 100000 J at temperature of 400 K rejects 40,000 J at a temperature of 200 K and delivers 15 Kwh of work. Would you advise investing money to put his on the market? [No]
16. Which is the more effective way to increase the thermal efficiency of a Carnot engine to increase T_H keeping constant or to decrease T_L, keeping T_H constant? [Decrease T_L]
17. A Carnot engine whose efficiency is 10% is used as a refrigerator. Find the coefficient of performance.
18. What amount of heat is transferred to N_2 in the isobaric process to perform work 2J [7J]
19. In the PV diagram shown calculate the thermal efficiency. $T_2 \rightarrow nT_1$



$$\left[\eta = 1 - \frac{n+\gamma}{1+\gamma n}\right]$$

Section C

(Answer questions in about **two** *pages)*

Long answer type questions (Essays)

1. Describe Carnot's cycle and obtain an expression for the efficiency of an ideal heat engine in terms of temperatures.

2. State and prove Carnot's theorem.
3. Prove that ideal gas temperature and thermodynamic temperature scale are numerically equal.

Hints to problems

7. See example 4

8. $\eta = 1 - \frac{T_2}{T_1}$

9. $\eta = 1 - \frac{Q_2}{Q_1}, \quad \eta = 1 - \frac{T_2}{T_1}$

10. $T_2 = 280K, \ \eta_1 = \frac{1}{2} \quad$ find $T_1 \cdot T_1 = 560\,K$

$$\eta_2 = 1 - \frac{T_2'}{T_1'} \quad 0.7 = 1 - \frac{280}{T_1'} \quad T_1' = 933.3\,K$$

$$\therefore \quad T_1' - T_1 = 933.3 - 560 = 373.3K$$

11. $\eta = \frac{T_1 - T_2}{T_1}$

$$0.2 = \frac{T_1 - T_2}{T_1}$$

$$0.25 = \frac{T_1 - (T_2 - 20)}{T_1}$$

$$0.25 = \frac{T_1 - T_2}{T_1} + \frac{20}{T_1}$$

$$0.25 = 0.2 + \frac{20}{T_1}$$

or $T_1 = 400K \qquad \therefore \quad T_1 = 320K$

12. $\eta = 1 - \frac{T_2}{T_1}$

$$\frac{1}{2} = 1 - \frac{T_2}{400} \qquad \text{......(1)}$$

$$\eta = 1 - \frac{T_2 + 100}{500} \qquad \text{......(2)}$$

From eq (1), we get $T_2 = 200K$. Put this in eq (2), we get $\eta = \frac{2}{5} = 0.4$

13. $T_2 = 277K,\ T_1 = 300K$

$W = 1J$

Using $\frac{Q_1}{Q_2} = \frac{T_1}{T_2} = \frac{300}{277}$

or $\frac{Q_1 - Q_2}{Q_2} = \frac{T_1 - T_2}{T_2}$

$$\frac{W}{Q_2} = \frac{T_1 - T_2}{T_2}$$

or $Q_2 = \frac{T_2 W}{T_1 - T_2} = \frac{277 \times 1}{300 - 277} = \frac{277}{23} = 12.04$

$\therefore$ Heat transferred $Q_1 = Q_2 + W = 13.04$

$$\beta = \frac{T_2}{T_1 - T_2}$$

14. $T_1 = 300K,\ T_2 = 260K$

$$Q_2 = 500 \text{ cal}$$

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \quad \text{or} \quad Q_1 = \frac{T_1}{T_2} Q_2$$

$$Q_1 = \frac{300}{260} \times 500 = 576.9$$

Work done $= Q_1 - Q_2 = 576.9 - 500$

$= 76.9 \text{ cal} = 76.9 \times 4.2$

$= 323.1 \text{ J}$

15. $T_H = 400K,\ \ T_L = 200K,\ \ W = 15 \times 10^4 \text{ Wh}$

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{200}{400} = 1 - \frac{1}{2} = \frac{1}{2} = 5\%$$

Work done $= Q_H - Q_L = 100000 - 40,000 = 60,000$

$$\therefore \quad \eta' = \frac{Q_H - Q_L}{Q_H} = \frac{60,000}{100000} = 0.6 = 60\%.$$

since $\eta' = \eta$, so impossible.

16. $\eta = \frac{T_H - T_L}{T_H}$, Let ΔT be the increase in T_H, keeping T_L constant

$$\therefore \quad \eta_1 = \frac{(T_H + \Delta T) - T_L}{(T_H + \Delta T)} = \frac{T_H - T_L}{T_H + \Delta T} + \frac{\Delta T}{T_H + \Delta T} \quad \text{..... (1)}$$

Let ΔT be the decrease in T_L, keeping T_H constant

$$\eta_2 = \frac{T_H - (T_L - \Delta T)}{T_H} = \frac{T_H - T_L}{T_H} + \frac{\Delta T}{T_H} \quad \text{..... (2)}$$

Comparing eqs 1 and 2

$$\eta_2 > \eta_1.$$

So it is go for the second.

17. $\beta = \frac{T_2}{T_1 - T_2} = \frac{1}{\frac{T_1}{T_2} - 1}$

Using $\eta = 1 - \frac{T_2}{T_1}$ gives $\frac{T_2}{T_1} = 1 - \eta$

$$\therefore \quad \beta = \frac{1}{\frac{1}{1-\eta} - 1} = \frac{1-\eta}{\eta} = \frac{1-0.1}{0.1} = 9$$

18. $dW = PdV = nRdT$ ..... (1)

$dQ = nC_P dT$ ..... (2)

$$\therefore \quad \frac{dW}{dQ} = \frac{R}{C_P} = \frac{R(\gamma - 1)}{R\gamma} = \frac{\gamma - 1}{\gamma}$$

$$\therefore \qquad dQ = \frac{\gamma}{\gamma - 1} dW = \frac{\frac{7}{5}}{\frac{7}{5} - 1} \times 2 = 7\ \text{J}$$

19. $1 \to 2$ heat supplied $Q_1 = C_p(T_2 - T_1)$

$4 \to 1$ heat supplied $Q_2 = C_V(T_1 - T_4)$

$\therefore$ Total heat supplied $C_P(T_2 - T_1) + C_V(T_1 - T_4)$

In the isobaric process $(1 \to 2)$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{V_1}{T_1} = \frac{V_2}{nT_1} \qquad \therefore \qquad \frac{V_2}{V_1} = n \qquad \text{..... (0)}$$

In the isochoric process $(2 \to 3)$

$$\frac{P_1}{T_2} = \frac{P_2}{T_3}$$

$$\frac{P_1}{nT_1} = \frac{P_2}{T_3} \quad \text{gives} \quad \frac{P_1}{P_2} = \frac{nT_1}{T_3} \qquad \text{..... (1)}$$

In the isochoric process $(4 \to 1)$

$$\frac{P_1}{T_1} = \frac{P_2}{T_4} \quad \text{gives} \quad \frac{P_1}{P_2} = \frac{T_1}{T_4} \qquad \text{..... (2)}$$

Comparing eqs 1 and 2, $T_4 = \frac{T_1}{n}$

$$\eta = \frac{\text{work out}}{\text{heat input}} = \frac{\text{area}}{\text{heat input}} = \frac{(V_2 - V_1)(P_1 - P_2)}{C_P(T_2 - T_1) + C_V(T_1 - T_4)}$$

$$\text{Heat input} \quad = \frac{R\gamma}{\gamma - 1}(nT_1 - T_1) + \frac{R}{\gamma - 1}\left(T_1 - \frac{T_1}{n}\right)$$

$$= \frac{RT_1}{\gamma - 1}\left[\gamma(n-1) + \left(1 + \frac{1}{n}\right)\right]$$

$$= \frac{RT_1}{\gamma - 1}\left[\gamma(n-1) + \left(\frac{(n-1)}{n}\right)\right]$$

$$\text{Work output} = P_1V_2 - P_1V_1 + P_2V_1 - P_2V_2$$

$$= RnT_1 - RT_1 + \frac{RT_1}{n} - RT_1$$

$$= RT_1(n-1) - \frac{RT_1}{n}(n-1)$$

$$= \frac{RT_1(n-1)^2}{n}$$

$$\therefore \qquad \eta = \frac{RT_1(n-1)^2}{n\frac{RT_1}{\gamma - 1}\left[\gamma(n-1) + \frac{(n-1)}{n}\right]}$$

$$\eta = \frac{(n-1)(\gamma-1)}{1+n\gamma} = 1 - \frac{(n+\gamma)}{(1+\gamma n)}$$