1

PARTICLE LIKE PROPERTIES OF ELECTROMAGNETIC RADIATION

Intrduction

We are very much familiar about the concept of particle and wave in mechanics. A particle is a point object endowed with mass. i.e., it has position and mass. Moreover several attributes such as charge, momentum, energy can be assigned to it, where as a wave is the propagation of energy from one place to another without the actual movement of the medium through which energy propagates. Several attributes such as wave length, frequency, momentum etc. can be assigned to it. The definition of particle and wave are clear cut and there is no ambiguity about the concept of these in the realm of macroscopic (everyday) world. However when we come to the microscopic world there are neither particles nor waves in accordance with the definitions stated above. For example electrons are considered as particles because they possess charge and mass and behave according to the laws of particle mechanics. However we shall see later that electrons behave like waves.

In other words electrons behave like particles in certain situations and behave like waves in some other situations. This is called the wave-particle duality. This is why now days electron is called as a wavicle. Now we take another example. We consider electromagnetic wave as a wave because it has all the properties of a wave and they exhibit the phenomenon of diffraction, interference, polarization etc. Though it is a wave, electromagnetic wave behaves like stream of particles (photons). i.e., a wave exhibits particle properties. This is the subject matter of this chapter.

This chapter begins with the basic ideas of wave mechanics. The system of mechanics associated with quantum systems is called wave mechanics because it deals with the wave like properties of particles. One consequence of wave mechanics is the breakdown of the classical distinction between particles and waves. In particular this chapter deals with three early experiments (photoelectric effect, thermal radiation and Compton scattering) that provided evidence that light has properties of particles. In wave nature of light the energy associated with it is spreading over a wavefront however if light is considered as a particle, the energy is delivered in concentrated bundles like particles; a discrete bundle (quantum) of electromagnetic energy is known as a photon. Before delve into the experimental evidence that supports the existence of photon and the particle like behaviour of light we review some of the properties of electromagnetic waves.

Review of electromagnetic waves

Systematic study of electricity and magnetism began around 1785 with Columb's observation on the forces between charged bodies and the Gauss's law in electrostatics. In 1819 Oersted discovered that a time varying current results in a magnetic field. After that Biot, Savart and Ampere in 1820 established the relation connecting between the magnetic field and current. In 1831 Faraday discovered that a time varying magnetic field results in an electric field. These discoveries paved the way for Maxwell to discover electromagnetic waves. Maxwell from his theoretical study pointed out that a change in either field produces the other field. This idea led Maxwell to conclude that the variation in electric and magnetic field vectors perpendicular to each other leads to the production of electromagnetic disturbances in space. These disturbances have the properties of wave and can travel in space without any material medium. These waves are called electromagnetic waves.

Thus, **the electromagnetic waves are those waves in which there are sinusoidal variation of electric and magnetic field vectors at right angles to each other as well as at right angles to the direction of wave propagation and can travel in space without any material medium.**

Though Maxwell theoretically predicted the existence of electromagnetic waves in 1865, we had to wait 23 years for the experimental confirmation of the existence of electromagnetic waves. In 1888 German physicist Heinrich Rudolf Hertz produced electromagnetic waves of wave length 6m using an oscillary circuit. The principle used is that an oscillating electric charge radiates out electromagnetic radiation. The radiation carries energy and this energy is being supplied at the cost of kinetic energy of the oscillating charge. An oscillating charge gives out radiation which is appreiable only when the extend to which the charge oscillates is comparable to the wave length of the radiation. For example when charge oscillating with a frequency of $1500 s^{-1}$ would radiate electromagnetic waves of wave length $\lambda = \frac{c}{\nu} = \frac{3\times10^8}{1500} = 200\,\text{km}$. This shows that to radiate sufficient amount of energy by a charge vibrating with a frequency of $1500 s^{-1}$, it must oscillate over a distance of 200 km. With this in mind Hertz designed an experimental set up.

Hertz experiment

It consists of two large square metal plates A and B connected to source of high voltage. The plates are connected through thick copper wires to two polished brass knobs k_1 and k_2. The voltage is high enough such that the air in the gap between the knobs gets ionized and provides a path for discharge of the plates. These two plates form a condenser of low capacity. According to the relation $\nu = \frac{1}{2\pi\sqrt{LC}}$, C being small, will result in high frequency oscillations.

When the potential difference between the plates is very high the air gap between the plates becomes conducting, consequently condenser discharges producing electro magnetic waves. Owing to the discharge of the condenser the p.d between the plates falls and the air ceases to be conducting. Again condenser is charged by the induction coil which provides the p.d to the plates and it gets discharged again through the air gap producing electromagnetic waves. This process of charging and discharging of the condenser is continuous in the out put. Hertz was able to produce e.m. waves of wavelength around 6m.

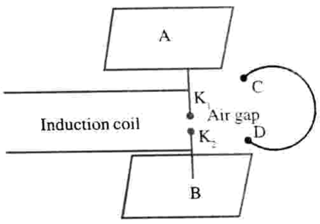


Figure 1.1

The em waves are detected by using an unclosed metallic ring having small metallic spheres C and D with some gap. It is held in a position such that the magnetic field produced by the oscillating current is perpendicular to the plane of the ring. The oscillating magnetic field linked with the ring produces large induced e.m.f. which causes a spark to appear across the spheres C and D.

After the discovery of Hertz, in 1895 an Indian physicist Jagadish Chandra Bose was able to produce electromagnetic waves of wavelength 5mm to 25mm. But his experiment was confined to laboratory only.

In 1899 Guylielimo Marconi was the first to send electromagnetic waves upto a few kilometers and established wireless communication across English channel, a distance of about 50 km.

Mathematical aspects of electromagnetic waves

A charge at rest produces electric field. For example the electric field $(\bar{E})$ at a distance r from a point charge q is

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} \quad ..... (1)$$

A charge in uniform motion produces both electric and magnetic fields. The expression for electric field is same as eq (1). The expression for magnetic field $(\vec{B})$ due to a long straight wire carrying current I at a distance r is

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{r} \quad ..... (2)$$

If the charges are accelerated or if the current varies with time, an electromagnetic wave is produced, in which $\vec{E}$ and $\vec{B}$ vary not only with $\vec{r}$ but also with t. Depending on the properties of the source of the wave and of the medium through which wave travels, waves can be expressed in different mathematical forms. For simplicity we take a plane wave. **A plane wave is a wave whose wave fronts are plane and having constant momentum and energy.** A plane electromagnetic wave travelling in z-direction is mathematically represented by

$$\vec{E} = \vec{E}_0 \sin(kz - \omega t) \text{ and } \vec{B} = \vec{B}_0 \sin(kz - \omega t) \quad ..... (3)$$

where $k\left(\frac{2\pi}{\lambda}\right)$ is the wave vector and $\omega(2\pi\nu)$ is the angular frequency.

The relation between ω and k is $\frac{\omega}{k} = c$, c is the speed of electromagnetic wave.

The polarization of the wave is represented by the vector $\vec{E}_0$. The polarization vector defines the plane of vibration and is perpendicular to the direction of propagation. Once the direction of propagation (in our case it is along z direction) of wave and polarisation of $\vec{E}_0$ (say along x direction) is fixed, then the direction $\vec{B}_0$ will be automatically fixed in such a way that $\vec{E}_0 \times \vec{B}_0$ point in the direction of travel.

Let $\quad \vec{E} = E_0\hat{i}$

then $\quad \hat{E}_0 \times \hat{B}_0 = \hat{k}$

$$\hat{i} \times \hat{B}_0 = \hat{k}$$

This implies that $\hat{B}_0$ is along $\hat{j}$ direction

i.e. $\quad \vec{B}_0 = B_0\hat{j}$

Moreover the magnitude of $\vec{B}_0$ is given by

$$\frac{E_0}{B_0} = c \quad ..... (4)$$

An electromagnetic wave carries energy and propagates from one place to another. The energy transmitted per second per unit area is specified by the poynting vector $\vec{N}$. It is given by

$$\vec{N} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) \quad ..... (5)$$

For the plane wave, this becomes

$$\vec{N} = \frac{1}{\mu_0} E_0\hat{i}\sin(kz - \omega t) \times B_0\hat{j}\sin(kz - \omega t)$$

$$\vec{N} = \frac{1}{\mu_0} E_0 B_0 \sin^2(kz - \omega t)(\hat{i} \times \hat{j})$$

$$\vec{N} = \frac{E_0 B_0}{\mu_0}\sin^2(kz - \omega t)\hat{k} \quad ..... (6)$$

The unit of $\vec{N}$ is Wm^{-2}.

The power associated with this electromagnetic wave is

$$P = NA = \frac{E_0 B_0 A}{\mu_0}\sin^2(kz - \omega t)$$

Using $\quad \frac{E_0}{c} = B_0$

$$P = \frac{E_0^2 A}{\mu_0 c}\sin^2(kz - \omega t)$$

Associated with power we can introduce one more property of waves called intensity. Intensity is the average power per unit area.

$$P_{av} = \frac{E_0^2 A}{\mu_0 c} \cdot \frac{1}{2} \qquad \left(\because \overline{\sin^2(kz - \omega t)} = \frac{1}{2}\right)$$

$$\therefore \quad I = \frac{P_{av}}{A} = \frac{E_0^2}{2\mu_0 c} \qquad \text{..... (8)}$$

Interference and diffraction

One important property of waves which makes waves a unique physical phenomenon is the principle of superposition. **According to this principle when two or more waves are allowed to pass simultaneously through a continuous media, they superimpose one another then each wave produces its own displacement at any point and the resultant displacement will be the vector sum of their individual displacements.**

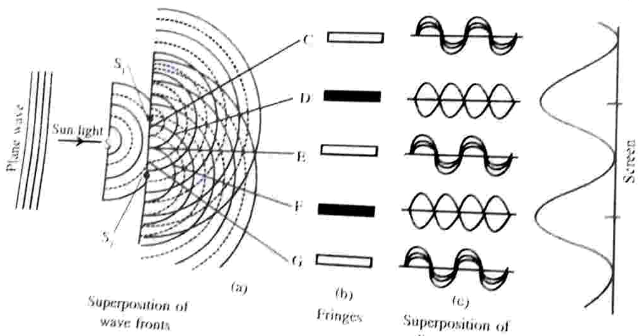
Let $\vec{y}_1, \vec{y}_2, \ldots\ldots \vec{y}_n$ be the displacements of waves at any instant and the resultant displacement $\vec{y}$ can be written as

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \cdots\cdots \vec{y}_n \qquad \text{..... (9)}$$

This unique property of waves leads to the phenomenon of interference and diffraction.

Interference of light

When there is a single source of light the distribution of light energy in the surrounding medium is uniform in all directions. When two light waves of same ampli-



(a) Superposition of wave fronts (b) Fringes (c) Superposition of displacement

Figure 1.2

tude, same wavelength and in the same phase (or with constant phase difference) the distribution of energy does not remain uniform in all directions. At some points where the crest of one wave falls on the crest of the other, resultant amplitude is maximum. Hence intensity of light is maximum i.e. such points appear to be bright. It is called constructive interference. At certain other points crest of one wave falls on the trough of the other. Therefore resultant amplitude becomes minimum. Such points appear to be dark. It is called destructive interference. This kind of modification in the distribution of light energy due to superposition of waves is called interference.

English physicist Thomas Young was the first to study and demonstrated experimentally the interference of light.

Figure 1.2 illustrate the experimental arrangement. The light waves coming out of two slits S_1 and S_2 undergo diffraction by each of the slits. It is due to diffractions the light waves coming out of the two slits cover much large area on the screen than the geometrical shadow of the slit. This causes the light from the two slits to overlap on the screen, producing the interference. Now consider a point P on the screen at a distance x from the centre of the screen (OP = x). Suppose a wave crest passing through one slit (S_1) and another wave crest passed through slit (S_2) at an earlier time reach the point simultaneously. Then this point appears to be bright. This is known as constructive interference. This occurs when the path difference between the two waves must be an integral multiple of λ.

From the figure given below, we have

$$S_2P - S_1P = n\lambda, \; n = 0, 1, 2, \ldots\ldots \qquad \text{..... (10)}$$

It is also possible for the crest of the wave from one slit arrive at P on the screen simultaneously with the trough of the wave from the other slit. When this happens, the two waves cancel, giving dark regions on the screen. This is known as destructive interference. This occurs when the path difference between the two waves must be an half integral multiple of λ.

i.e.,
$$S_2P - S_1P = \left(n + \frac{1}{2}\right)\lambda, \; n = 0, 1, 2, \ldots\ldots \qquad \text{..... (11)}$$

Now we can find the locations on the screen where constructive and destructive interference take place.

Let d be the separation of slits and let D be the distance from the slits to the screen. Let $OP = x_n$ be the distance from the centre to the nth interference

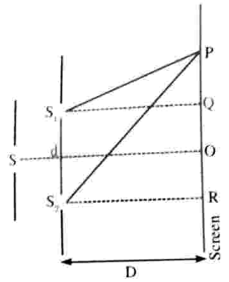


Figure 1.3

$$OP = x_n$$
$$PQ = OP - OQ$$
$$= x_n - \frac{d}{2}$$
$$PR = PO + OR$$
$$= x_n + \frac{d}{2}$$

Then $$PQ = x_n - \frac{d}{2}$$

$$PR = x_n + \frac{d}{2}$$

$\therefore$ $$S_2P^2 - S_1P^2 = (S_2R^2 + PR^2) - (S_1Q^2 + PQ^2)$$
$$= (D^2 + PR^2) - (D^2 + PQ^2)$$
$$= PR^2 - PQ^2$$

$$S_2P^2 - S_1P^2 = \left(x_n + \frac{d}{2}\right)^2 - \left(x_n - \frac{d}{2}\right)^2 = 2x_n d$$

$$(S_2P + S_1P)(S_2P - S_1P) = 2x_n d$$

$$S_2P - S_1P = \frac{2x_n d}{S_2P + S_1P}$$

But $$S_2P \approx S_1P = D$$

$\therefore$ $$S_2P - S_1P = \frac{2x_n d}{2D} = \frac{x_n d}{D}$$

or path difference $= \dfrac{x_n d}{D}$

For constructive interference

$$\frac{x_n d}{D} = n\lambda \quad \text{or} \quad x_n = \frac{n\lambda D}{d} \qquad .....(12)$$

For destructive inference

$$\frac{x_n d}{D} = \left(n + \frac{1}{2}\right)\lambda \quad \text{or} \quad x_n = \left(n + \frac{1}{2}\right)\frac{\lambda D}{d} \qquad .....(13)$$

The separation between consecutive bright bands or dark bands is known as fringe width β.

Thus $$\beta = x_{n+1} - x_n$$

$$\beta = (n+1)\frac{\lambda D}{d} - \frac{n\lambda D}{d} = \frac{\lambda D}{d} \text{ for bright band.} \qquad .....(14a)$$

$$\beta = \left(n + 1 + \frac{1}{2}\right)\frac{\lambda D}{d} - \left(n + \frac{1}{2}\right)\frac{\lambda D}{d}$$

$$\beta = \frac{\lambda D}{d} \text{ for dark band.} \qquad .....(14b)$$

Note: The phenomenon of interference is exhibited by all electromagnetic waves but also mechanical waves such as sound waves or water waves.

Example 1

A double slit experiment is performed with sodium light ($\lambda = 589\text{nm}$). The slits are separated by 1.05 mm and the screen is 2.357 m from the slits. Find the separation between adjacent maxima on the screen.

Solution

$$\lambda = 589 \times 10^{-9}\text{m}, \quad d = 1.05 \times 10^{-3}\text{m}$$
$$D = 2.357\text{m}$$

Using $$\beta = \frac{\lambda D}{d} = \frac{589 \times 10^{-9} \times 2.357}{1.05 \times 10^{-3}}$$

$$\beta = 1.322\,\text{mm}.$$

Diffraction of light

Apart from Young's double slit experiment there is one more device which we are familiar with is diffraction grating element for observing interference of light. Wavefronts which pass through grating element containing many slits recombine to produce interference. The experimental set up is given in figure 1.4. Interference maxima corresponding to different wavelengths appear at different angles θ is given by

$$d \sin\theta = n\lambda \qquad \ldots\ldots (15)$$

where d is the slit spacing and n is the order of the spectrum (n = 1, 2, 3, ...).

The advantage of diffraction grating is its superior resolution. This enables us to get very good separation of wavelengths that are close to one another. Thus grating element is a very useful device for measuring wavelengths.

For visible light the separation between the slits $d = \frac{n\lambda}{\sin\theta}$ is of the order of few times the wavelength. This can be achieved easily.

Suppose we are using X-rays instead of visible light, then

$$d = \frac{n\lambda}{\sin\theta} \quad (\lambda \text{ for X-rays} \simeq 0.1\text{nm})$$

$$d = \frac{n}{\sin\theta} \times 0.1 \times 10^{-9}$$

$$d = 1\text{nm}. \left(\frac{n}{\sin\theta} = 10 \text{ maximum}\right)$$

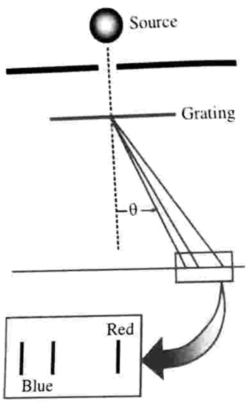


Figure 1.4
The use of a diffraction grating to analyze light into its constituent wavelengths

This shows that we need to construct a grating element with slit separation less than 1nm. This is not usually possible. But the spacing between atoms of most of the crystals is of the order of 1 nm. Here comes crystal diffraction of X-rays.

> Max van lane (1879-1960 Germany) developed the method of X-ray diffraction for the study of crystal structures for which he received the 1914 Nobel prize. But the law for X-ray diffraction was developed by Lawrence Bragg (1890-1977, England) while he was a student. Bragg shared the 1915 Nobel prize with his father, William Bragg for their research on the use of X-rays to determine crystal structures.

X-ray diffraction

X-rays are electromagnetic waves and they should exhibit the phenomenon of diffraction. However unlike visible, X-rays cannot be diffracted by devices such as ruled diffraction gratings because of their shorter wavelengths (0.1 nm order). In 1921 German physicist Max Von Laue suggested that a crystal which consisted of a 3D array of regularly spaced atoms could serve the purpose of grating. This is possible because all the atoms in a single crystal are regularly arranged with interatomic spacing of the order of a few angstroms and this is compatible with the conditions required to be satisfied for diffraction to take place.

On the suggestion of Laue, his associates, Friedrich and Knipping later successfully demonstrated the diffraction of X-rays from a thin crystal of zinc blende (ZnS). The diffraction pattern obtained on a photographic film consisted of a central spot and a series of dark spots arranged in a definite pattern around the central spot. Such a pattern is called the Laue's pattern and reflects the symmetry of the crystal. After that the phenomenon of X-ray diffraction has become an invaluable tool to determine the structure of crystals. It is also used to determine the wavelengths of X-rays.

Braggs' law

In 1912 W.H. Bragg and W.L. Bragg put forward a model which generates the conditions for diffraction in a simple way. According to their model a crystal is aggregate of large number of parallel atomic planes. If X-rays are considered to be reflected by such a set of parallel planes followed by the constructive interference of the resulting reflected ways, the diffraction pattern is obtained. Thus the problem of diffraction of X-rays by the atoms converted into the problem of reflection of X-rays by the parallel atomic planes. Based on these considerations, Braggs derived a simple mathematical relationship which is the condition for the reflection to occur. This condition is known as the Braggs' law.

Derivation of Braggs' law

Consider a set of parallel atomic planes with interplanar spacing d. Let a parallel beam of X-rays of wavelength λ be incident on these parallel planes at a glancing angle θ such that the rays lie in the plane of the paper. Consider two such rays 1 and 2 which strike the first two planes and get partially reflected at the same angle θ. The diffraction is the consequence of construc-

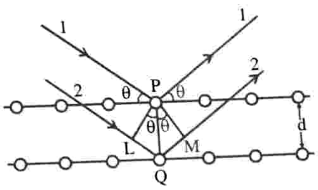


Figure 1.5
Bragg's reflection of X-rays from the atomic planes

tive interference of these reflected rays. Let PL and PM be the perpendiculars drawn from the point P on the incident and reflected portions of ray 2.

The path difference between the rays 1 and 2

$$= LQ + QM$$

From the figure we have

$$\frac{LQ}{PQ} = \sin\theta$$

or $$LQ = PQ\sin\theta = d\sin\theta$$

and also $$QM = d\sin\theta.$$

$\therefore$ The path difference $= 2d\sin\theta$

For constructive interference of rays, the path difference must be an integral multiple of wavelength λ.

i.e. $$2d\sin\theta = n\lambda, \text{ where n is an integer} \qquad \text{..... (16)}$$

This equation is called Bragg's law. For $n = 0$, we get the zeroth order reflection which occurs for $\theta = 0$ i.e., in the direction of incident ray and hence cannot be observed experimentally. The diffractions corresponding to $n = 1, 2, 3, \ldots$ etc. are called first, second, third, ... etc. order diffractions.

The highest possible order is determined by the condition that $\sin\theta \le 1$ and $\lambda \le d$ for Bragg reflection to occur. Taking $d \approx 10^{-10}$m, we get $\lambda \le 10^{-10}$m or 1Å. X-rays having wavelengths in this range are, therefore, preferred for analysis of crystal structures.

All experiments we discussed so far depend upon the wave properties of electromagnetic radiation. However there are other experiments such as photoelectric effect, compton effect, thermal radiation, thermionic emission etc. cannot be explained if we regard electromagnetic radiation as waves.

Example 2

A single crystal of table salt (NaCl) is irradiated with a beam of X-rays of wavelength 0.250 nm and the first Bragg reflection is observed at angle of 26.3°. (a) What is the angle of atomic spacing of NaCl (b) What angle of incidence will produce the second order Bragg peak.

Solution

Using $2d\sin\theta = n\lambda$

a) $$d = \frac{n\lambda}{2\sin\theta} = \frac{1\times 0.25\times 10^{-9}}{2\times \sin 26.3} = 0.282\,\text{nm}$$

b) $$\sin\theta = \frac{n\lambda}{2d} = \frac{2\times 0.25\times 10^{-9}}{2\times 0.282\times 10^{-9}} = 0.886$$

$$\therefore \quad \theta = \sin^{-1}(0.886) = 62.37^\circ$$

Photoelectric effect

The photoelectric effect was first observed by Hertz in 1887 while during his experiments on electromagnetic waves. Hertz noticed that sparks occurred more steadly in the air gap of his transmitter when ultraviolet light was directed at one of the metal balls. Hertz could not pursue his observation. But his student Philip Lenard studied the observation in detail and discovered that the cause of the spark was electrons emitted when the frequency of the electromagnetic wave was sufficiently high. This phenomenon is known as photoelectric effect.

i.e., **photoelectric effect is the phenomenon of emission of electrons from the surface of metals when light of radiation of suitable frequency falls on them. The emitted electrons are called photo electrons and the current so produced is photoelectric current.**

Different metals emit photoelectrons when they are exposed to radiations of suitable frequencies or wavelengths. For example alkali metals like lithium, sodium, potassium, cesium etc. show photoelectric effect with visible light, where as the metals like zinc, cadmium, magnesium etc. are sensitive only to ultraviolet light.

Philip Lenard failed to explain the phenomenon of photoelectric effect. A few year later in 1905 Einstein realized that the photoelectric effect could be understood if the energy of light is concentrated in small packets called photons (light quantum is called as photon). The term photon was coined by the chemist Gilbert Lewis in 1926. Each photon of light of frequency ν has the energy $h\upsilon$, the same as Planck's quantum energy. With this idea Einstein explained photoelectric effect i.e., on the basis of quantum theory he could do this. For this Einstein was awarded Nobel prize.

Cause of photoelectric effect

When light is incident on a metal surface, the photons penetrate the surface and are absorbed by the orbital electrons. One electron absorbs only one photon. The energy of the photon is converted into kinetic energy of the electron. As a result the electron is able to come out of the metallic surface producing photoelectric effect.

Experimental study of the photoelectric effect

The experimental study of photoelectric effect was carried out by philip Lenard. His experimental set up consists of an evacuated quartz tube TT, filled with electrodes E (emitter) and C (collector). The electrode E is made of some photo sensitive metal. The light is incident on the electrode E through the window W.A potential difference is applied across the electrodes with the help of a potential dividing arrangement and the reversing key as shown in figure. This enables us to apply variable potential difference between E and C. The photo electrons emitted from the emitter E are collected by the electrode C. The photoelectric current in the circuit is measured with the ammeter shown in the figure. With this arrangement we can study the effect of different factors on the photoelectric effect.

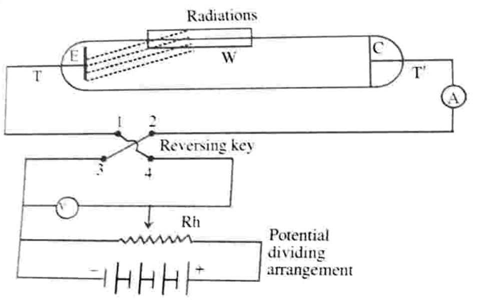


Figure 1.6

Factors on which the photoelectric current depends

The photoelectric current depends on the following factors.

1) Intensity of radiations. 2) Potential difference between the emitter and collector. 3) Frequency of incident radiations and of the nature of the photo sensitive metal.

1. Effect of intensity of incident radiations

Maintain the collector C at a definite positive potential with respect to emitter E by adjusting the potential divider. Using the incident radiations at a fixed frequency

it is found that the photoelectric current increases linearly with the intensity of incident light. It means that the number of photo electrons emitted from sensitive plate is directly proportional to the intensity of incident radiation.

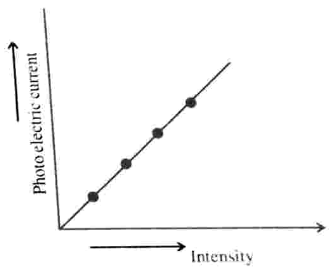


Figure 1.7

2. Effect of potential

If the frequency and the intensity of the incident radiation are kept fixed, it is found that the photoelectric current increases gradually with the increase in positive potential on plate C until a stage comes at which the photoelectric current becomes maximum for a certain positive potential of C. At this stage all the photo electrons emitted from the plate E are reaching the plate C. Now the photoelectric current attains saturation value and it does not increase further for any increase in the positive potential of plate C.

If we apply a negative potential to plate C with respect to plate E and increase it gradually we note that photoelectric current decreases rapidly until it becomes zero. The minimum negative potential given to plate C at which the photoelectric current becomes zero is called stopping potential or cut off potential.

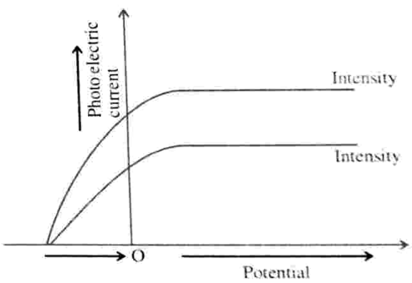


Figure 1.8

If the experiment is repeated with the radiation of same frequency but of higher intensity we note that the saturation current is more but stopping potential is same. This means that for a given frequency of incident radiation, the stopping potential is independent of its intensity.

For a given frequency of incident radiation, the stopping potential V_0 is the measure of the maximum kinetic energy of the photo electron that is just stopped from reaching the plate C.

If m is the mass and v_{max} is the maximum velocity of photo electrons emitted then

Maximum K.E. of electron $= \frac{1}{2} mv^2_{max}$

If e is the charge of the electron and V_0 is the stopping potential, then work done by the retarding potential in stopping the electron $= eV_0$

We have $\frac{1}{2} mv^2_{max} = eV_0$

This relation shows that the maximum velocity of the emitted photo electron is independent of the intensity of the incident light.

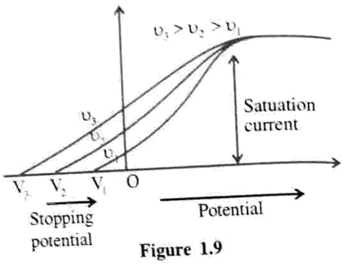


Figure 1.9

Effect of frequency of incident radiation

To make this study, we take radiations of different frequencies but of the same intensity. For each radiation we study the variation between photoelectric current and potential of plate C. We note that the saturation current is the same for the radiation of different frequencies but the value of stopping potential is different for the incident radiation of different frequencies. Greater is the frequency of the incident radiation more is the value of stopping potential i.e., higher is the energy of the photo electrons emitted.

There is a certain minimum frequency v_0 called (Threshold frequency) of the incident radiation for which the stopping potential is zero. It means that for the threshold frequency of incident radiation the emission of photo electrons is just possible, but for the incident radiation of frequency less than threshold frequency no emission of photo electron is possible.

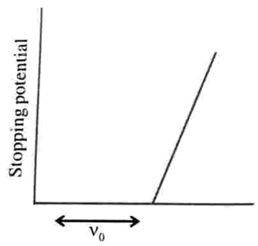


Figure 1.10

Laws of photoelectric emission

1. For a given metal and frequency of radiation the number of photo electrons ejected is directly proportional to the intensity of the incident light.
2. For a given metal there exists a certain minimum frequency of the incident radiation below which no emission of photo electrons take place. This frequency is called threshold frequency.

3. Above the threshold frequency the maximum kinetic energy of the emitted photo electron is independent of the intensity of the incident light but depends only upon the frequency of the incident light.
4. The photoelectric emission is an instantaneous process. The time lag between the incidence of radiation and emission of photo electrons is very small, less than even 10^{-9} seconds.

Failure of classical electromagnetic theory

1. According to electromagnetic theory, photoelectric effect should occur for any frequency of light, provided the intensity can cause vibrations of electrons to the amplitude required so as to free them from the metal. But experimentally, for each metal, there exists a threshold frequency (v_0) below which no emission should occur. Thus, electromagnetic theory fails to explain observation (2).
2. The electric and magnetic fields of the incident light wave exert forces to liberate electrons from the metal surface. So the light of higher intensity consisting of stronger fields should give higher velocity to the photoelectrons. In other words, the energy of the photoelectrons should depend on the intensity of incident radiation. But this is just contrary to observation (3).
3. According to the wave theory the energy of the incident light is distributed over the entire wavefront. So the energy incident on any electron is very small and some time will be required by an electron to absorb enough energy to escape from the metal surface. But experiment shows that there is no time lag between the incidence of radiation and emission of electrons. So classical electromagnetic theory fails to account observation (4).

Thus, wave theory fails to account the characteristics of photoelectric effect.

Einsteins photoelectric equation

Albert Einstein used the law of conservation of energy and Plancks quantum hypothesis to explain the photoelectric emission. Einstein assumed that one photo electron is ejected from a meatal surface if one photon of suitable light radiation falls on it.

Consider a photon of light of frequency v incident on a photo sensitive meta surface. The energy of the photon $(h\upsilon)$ is used for two purposes.

(i) A part of the incident energy of the photon is used in liberating the electro from the metal surface which is equal to the work function w_0 of the meta

(ii) The rest of the energy of the photon is used in imparting kinetic energy to the emitted photo electron.

If v is the velocity of the emitted photo electron and m is its mass, then

K.E of the photo electron $= \frac{1}{2}mv^2$

According to law conservation of energy, we have

$$h\nu = w_0 + \frac{1}{2}mv^2$$

or

$$\frac{1}{2}mv^2 = h\nu - w_0$$

This is called Einsteins photoelectric equation

If the incident photon of threshold frequency ν_0, then the incident photon of energy $h\nu_0$ is just sufficient to eject the electron from the metal surface without imparting it any kinetic energy. Hence $h\nu_0$ must be equal to work function w_0 of the metal i.e., $h\nu_0 = w_0$

$\therefore$ we have

$$\frac{1}{2}mv^2 = h\nu - h\nu_0 \qquad \text{..... (17)}$$

If λ is the wave length of the incident photon and λ_0 is the threshold wave length for the metal surface

We get

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

$$\frac{1}{2}mv^2 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

Explanation of laws of photoelectric emission from Einsteins photoelectric equation

1. Since each photon ejects one photo electron from a metal surface, the number of photo electrons emitted depends upon the number of photons falling on the metal surface which in turn depends upon the intensity of light. If the intensity of the light is increased the number of incident photons increases, which results in an increase in the number of photo electrons ejected. This is the first law of photoelectric emission.
2. The velocity and hence kinetic energy of photoelectrons emitted depends on the frequency of the incident light but is independent of the light intensity. This is the second law of photoelectric emission.
3. If $\nu < \nu_0$, kinetic energy is negative, which is impossible. This means that kinetic energy depends only on the frequency (or wave length) of incident light. If the intensity of incident light radiation is increased the number of incident photons falling on the metal surface increases but the energy of each photon remains the same. This is the third law of photoelectric emission.
4. The phenonon of photoelectric emission is due to elastic collision between a photon and an electron inside the metal. As a result of it the absorption of energy by the electron of metal from the incident photon is a single event which involves transfer of energy at once without any time lag. Due to this there is no time lag between the incident photon and ejected photo electron. This is the fourth law of photoelectric emission.

Therefore, the photoelectric emission is possible only if the incident light is in the form of packets of energy, each having a definite value, more than the work function of the metal. This shows that light is of particle nature. It is due to this reason that photoelectric emission was accounted by quantum theory of light.

Example 3

Find the wavelength and frequency of 1 MeV photon. $h = 6.6 \times 10^{-34}$ Js

Solution

$$\varepsilon = 1\,\text{MeV} = 10^6\,\text{eV} = 10^6 \times 1.6 \times 10^{-19}\,\text{J} = 1.6 \times 10^{-13}\,\text{J}$$

Using $\varepsilon = h\nu$

$$\nu = \frac{\varepsilon}{h} = \frac{1.6 \times 10^{-13}}{6.6 \times 10^{-34}} = 2.424 \times 10^{20}\,\text{Hz}$$

wavelength

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{2.424 \times 10^{20}} = 1.238 \times 10^{-12}\,\text{m}$$

Example 4

A 1kW radio transmitter operates at a frequency of 880kHz. How many photons per second does it emit.

Solution

Power, $P = 1kW = 10^3 W$

Frequency $\nu = 880kHz = 830 \times 10^3 Hz$

Using $\varepsilon_n = nh\nu$

or $\frac{\varepsilon_n}{t} = \frac{n}{t} h\nu$

$$P = \frac{n}{t} h\nu$$

or $\frac{n}{t} = \frac{P}{h\nu} = \frac{10^3}{6.62 \times 10^{-34} \times 880 \times 10^3}$

$$= \frac{10^3 \times 10^{31}}{6.62 \times 880} = \frac{10^{33}}{6.62 \times 88}$$

$$= 1.72 \times 10^{30} \text{ photons/s.}$$

Example 5

Work function of sodium is 2.3eV. Does sodium show photo electric emission for orange light of wavelength, $\lambda = 6800Å$. Given $h = 6.62 \times 10^{-34}$ Js.

Solution

$w_0 = 2.3eV$, $\lambda = 6800Å = 6800 \times 10^{-10} m$

Energy of the photon of orange light

$$\varepsilon = h\nu = \frac{hc}{\lambda}$$

$$\varepsilon = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{6800 \times 10^{-10}} = 2.921 \times 10^{-19}$$

or $\varepsilon = \frac{2.921 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.83eV.$

Since the energy is less than the work function w_0, photoelectric emission doesnot take place.

Example 6

Light of wavelength 5000Å falls on a sensitive plate with photoelectric work function 1.9eV. Find (i) energy of the photon (ii) kinetic energy of photoelectrons emitted and (iii) stopping potential. $h = 6.62 \times 10^{34}$ J.

Solution

$$w_0 = 1.9eV, \lambda = 5000 \times 10^{-10} m$$

(i) Energy of the photon, $\varepsilon = \frac{hc}{\lambda}$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10} \times 1.6 \times 10^{-19}}$$

$$= 2.482eV$$

(ii) K.E. of photoelectron $= h\upsilon - w_0$

$$= \varepsilon - w_0 = 2.482 - 1.19$$

$$= 0.582eV$$

(iii) If V_0 is the stopping potential, then

K.E. of the photoelectron, $K.E = eV_0$

$$\therefore \quad V_0 = \frac{K.E}{e} = \frac{0.582eV}{e} = 0.582V$$

Example 7

In a photoelectric experiment, it was found that the stopping potential decreases from 1.85V to 0.82V as the wave length of incident light is varied from 300nm to 400nm. Calculate the value of the plancks constant.

Solution

The maximum K.E. of a photoelectron is

$$KE_{max} = \frac{hc}{\lambda} - \frac{w_0}{e}$$

and the stopping potential is

$$V = \frac{KE_{max}}{e} = \frac{hc}{e\lambda} - \frac{w_0}{e}$$

If V_1 and V_2 are the stopping potentials at wavelengths λ_1 and λ_2 respectively,

$$V_1 = \frac{hc}{e\lambda_1} - \frac{w_0}{e} \text{ and}$$

$$V_2 = \frac{hc}{e\lambda_2} - \frac{w_0}{e}$$

$$V_1 - V_2 = \frac{hc}{e}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$h = \frac{e(V_1 - V_2)}{c\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right]} = \frac{1.85 - 0.82}{3\times10^8\left[\frac{1}{300\times10^{-9}} - \frac{1}{400\times10^{-9}}\right]}$$

$$= 4.12\times10^{-15}\,\text{eVs}$$

or $$h = 4.12\times10^{-15}\times1.6\times10^{-19}\,\text{Js}$$

$$h = 6.592\times10^{-34}\,\text{Js}$$

Example 8

Calculate the velocity of photoelectron when a light of wavelength 2000Å is incident on a metal surface. Work function of the metal is 2.5eV, $m_e = 9.1\times10^{-31}$ kg.

Solution

$$\lambda = 2000\text{Å} = 2000\times10^{-10}\,\text{m} = 2\times10^{-7}\,\text{m}$$

$$w_0 = 2.5\text{eV} = 2.5\times1.6\times10^{-19}\,\text{J}$$

$$= 4\times10^{-19}\,\text{J}$$

Using $$\frac{1}{2}mv^2_{max} = h\upsilon - w_0 = \frac{hc}{\lambda} - w_0$$

$$\frac{1}{2}mv^2_{max} = \frac{6.62\times10^{-34}\times3\times10^8}{2\times10^{-9}} - 4\times10^{-19}$$

$$\frac{1}{2}mv^2_{max} = 5.9\times10^{-19}$$

$$\therefore \quad v_{max} = \sqrt{\frac{2\times5.9\times10^{-19}}{9.1\times10^{-31}}} = 1.134\times10^6\,\text{ms}^{-1}$$

Thermal radiation

The electromagnetic radiation emitted by hot bodies is called thermal radiation. This also cannot be explained by the classical wave theory. At room temperature a hot body emits thermal radiation in the infra red region which is not sensitive to our eye. As we heat bodies to higher temperatures they emit radiation in the visible region.

During the late 19th century several experiments were conducted to explain the spectrum given out by thermal radiation. At that time the theories available were classical theories of thermodynamics and electromagnetism. These theories failed or only partially successful in explaining the spectrum of thermal radiation. However on the basis of newly advented quantum theory, spectra of thermal radiations were successfully explained.

Experimental setup to study thermal radiation

A hot body is maintained at a constant temperature. It emits radiations of different wavelengths. These radiations are allowed to pass through a prism. It is due to dispersion different wavelengths fall at different angles. By using a detector such as infra red spectrometer or bolometer the intensities of radiations of different wavelengths are measured. A graph is plotted between intensity of radiation and wavelength we get a graph as shown in figure.

We can repeat the experiment for many different temperatures and each time we plot intensity versus wavelength.

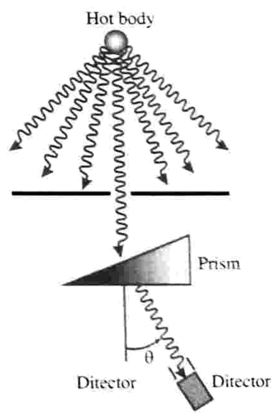


Figure 1.11: Experimental set up to study thermal radiation

From the graph we can draw to important characteristics of radiation intensity.

1. As temperature increases, the area under each curve increases. The area under the curve gives the total intensity of radiated over all wavelengths. From careful measurement, we can find that total intensity (I) is directly proportional to the fourth power of the temperature in kelvin.

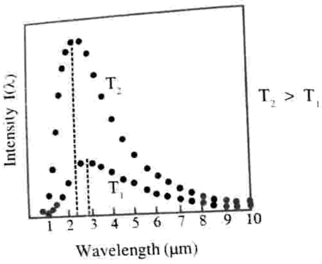


Figure 1.12
Intensity distribution versus wavelength at two different temperatures

i.e., $I \propto T^4$

or $I = T^4$ ..... (18)

This is known as Stefan's law and constant σ is called Stefan-Boltzmann constant.

$$\sigma = 5.67037 \times 10^{-8}\,\mathrm{Wm^{-2}K^{-4}}$$

2. As temperature increases, the wavelength corresponding to maximum intensity (λ_m) also increases. It is found that

$$\lambda_{max} \propto \frac{1}{T}$$

or $\lambda_{max} T = \text{constant}$

$$\lambda_{max} T = 2.898 \times 10^{-3}\,\mathrm{mK} \quad ..... (19)$$

This is known as Wein's displacement law. The term displacement comes λ_{max} is displaced as temperature varies.

The above relation is consistent with every day experience. A hot body begins to glow with a red colour. When temperature increases, wavelength λ_{max} decreases. As a result as temperature goes on increasing we can see yellow, green, blue, violet respectively. Violet colour corresponds to maximum temperature. Look at bluish-violet colour of your gas stove. The term white hot refers to a body that is hot enough to produce the mixture of all visible colours to make it white.

Example 9

A cavity is maintained at a temperature of 1650K. At what rate does energy escape from the interior of the cavity through a hole in its wall of diameter 1.00 mm.

Solution

Energy emitted per second per area,

$$I = \sigma T^4$$

$\therefore$ Energy emitted per second $= \sigma T^4 \times \pi r^2$

$$= 5.67 \times 10^{-8} \times (1650)^4 \times 3.14 \times (0.5 \times 10^{-3})^2$$

$$= 0.33\,\mathrm{W}$$

Black body radiation

In general the study of thermal radiation emitted by a hot body is extremely complicated. This because the emission of radiation depends on the surface properties of the body, the reflecting nature of the body to its surroundings. To simplify our study we consider a special type of body called Black body.

A body that absorbs the radiation of all wavelengths falling on it is called a black body. Such body will emit radiation at the fastest rate. The radiation emitted by a black body is called black body radiation.

Black body

A black body consists of a hollow metallic spherical cavity or a hollow metal box suitably insulated and heated to a temperature. The cavity is filled with electromagnetic radiation in thermal equilibrium with its walls. The radiation is emitted and reflected by the walls. A small hole is made on the metallic sphere which allows some of the radiation to escape. The hole that leaks out radiation is the black body.

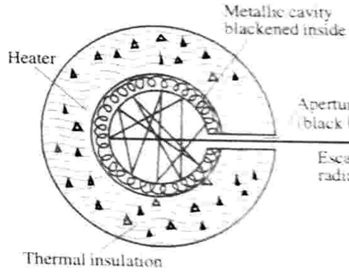


Figure 1.13

Note : Lamp black and platinum black are almost perfect black bodies. Lamp black absorbs 77% and platinum black about 98%. The term black body has no relation to its blackness. The lamp black even when white hot is, technically speaking, a black body. The white hoar-frost considered as a radiator is also a black body. Different scientists devised artificial black bodies. Some of them are Lummer and pringsheim black body, Wien's black body, Fery's black body etc.

Let u_λ be the energy density per unit wavelength within the metallic cavity.

$\therefore$ The energy density of the electromagnetic radiation with wavelengths between λ and $\lambda + d\lambda$ is $u_\lambda d\lambda$. the corresponding intensity (power per unit area) emerging from the hole is given by

$$I(\lambda) = \frac{c}{4} u(\lambda) \qquad \text{..... (20)}$$

The total intensity emitted in the region between wavelength λ_1 and λ_2 is

$$I(\lambda_1, \lambda_2) = \int_{\lambda_1}^{\lambda_2} I(\lambda) d\lambda \qquad \text{..... (21)}$$

The total radiant intensity emitted is

$$I = \int_0^\infty I(\lambda) d\lambda \qquad \text{..... (22)}$$

This is found to be

$$I = \sigma T^4$$

Note

Intensity, $\quad I = \frac{P_{av}}{A}$

Energy density $\quad u = \frac{\text{Energy}}{\text{Volume}}$

or $\quad uc = \frac{\text{Energy}}{\text{Area} \times \text{Length}} \times \frac{\text{length}}{\text{time}}$

$$uc = \frac{\text{Energy}}{A \times \text{time}}$$

$$\therefore \quad I = \frac{(E_{av})}{\text{time } A} = \frac{1}{2} uc$$

$\frac{1}{2}$ comes from the average of energy. This is the total intensity of radiation inside the metallic cavity. Out of this only half of the intensity coming out of the hole, the other half moving away from the wall.

$\therefore$ The radiant intensity coming out of the hole (Black body) is $I = \frac{1}{4} uc$

Classical theory of thermal radiation

On the basis of classical theories of electromagnetism and thermodynamics the intensity of radiation emitted by a black body was calculated by Rayleigh and Jean. For this they assumed that the radiation, inside a cavity at absolute temperature whose walls are perfect reflectors, to be a series of standing waves. This is a three dimensional generalisation of standing waves in a stretched string. The number of standing waves per unit volume within $d\upsilon$ was calculated to be

$$g(\upsilon) d\upsilon = \frac{8\pi\upsilon^2}{c^3} d\upsilon$$

(For the derivation see appendix A)

or $$g(\lambda) d\lambda = \frac{8\pi}{\lambda^4} d\lambda \qquad \text{..... (23)}$$

$$\left(\upsilon = \frac{c}{\lambda}, \ d\upsilon = -\frac{c d\lambda}{\lambda^2} \right)$$

From eq 23, we can calculate the energy density $u(\lambda)$.

Energy density within $d\lambda$ = number of standing waves per unit volume within $d\lambda$ × average energy per standing wave.

i.e. $\quad u(\lambda) d\lambda = \frac{8\pi}{\lambda^4} d\lambda \times$ average energy per standing wave.

According to equipartition theorem the average energy associated with one degree of freedom is $\frac{1}{2} kT$. Our standing wave considered as an oscillator possesses two degrees of freedom. Thus average energy per standing wave is $2 \times \frac{1}{2} kT = kT$, where k is the Boltzmann's constant and T is the temperature in kelvin. Therefore

$$u(\lambda)d\lambda = \frac{8\pi}{\lambda^4}kTd\lambda \qquad \text{..... (24)}$$

The corresponding intensity

$$I(\lambda) = \frac{c}{4}u(\lambda) = \frac{c}{4}\cdot\frac{8\pi}{\lambda^4}kT$$

$$I(\lambda) = \frac{2\pi c}{\lambda^4}kT \qquad \text{..... (25)}$$

This result is known as Rayleigh-Jeans formula.

Now we can compare eq. 25 with the experimental results. At long wavelength region our equation 25 tallies with experimental results. i.e. as $\lambda \to \infty$, $I \to 0$ as seen from the graph (Fig 1.12). But at shorter wavelength ($\lambda \to 0$) Rayleigh-Jeans formula fails to account experimental curve. As $\lambda \to 0$, eq. 25 predicts $I \to \infty$. The failure of Rayleigh-Jeans formula at short wavelengths is known as ultraviolet catastrophe. Actually this says that (as $\lambda \to 0$, $I \to \infty$) the whole intensity of the black body is concentrated in the low wavelength region (ultraviolet region). Above discussion shows that classical theory does not hold good to explain the spectrum of black body radiation.

Quantum theory of Black body radiation

Rayleigh-Jeans formula failed to explain the spectral distribution of black body radiation. They explained everything within the frame work of classical physics. According to classical physics the oscillators in the cavity walls should have continuous energy distribution. This assumption made them went wrong. To circumvent the failure of Rayleigh-Jeans, in 1900 German physicist Max planck made a daring assumption that the oscillators in the cavity walls could not have a continuous distribution of possible energies, but it must have specific energies.

According to Planck an oscillator emits radiation of energy $h\nu$ when it jumps from one energy state to the next lower state. When it jumps from lower state to the next higher state it absorbs radiation of energy $h\upsilon$. In other words emission and absorption take place not in continuous manner as predicted by classical physics but in discrete manner, each discrete bundle carries an energy $h\nu$, where h is called Plancks constant whose value is

$$h = 6.626\times10^{-34}\,Js$$

Each discrete bundle of energy is called as a quantum of energy. (quantum is a Latin word for how much).

If there are n oscillators in a cavity, the energy of the oscillator is given by

$$\varepsilon_n = nh\nu \quad n = 0, 1, 2, 3 \text{ ....} \qquad \text{..... (26)}$$

with the oscillator energies is limited to be $nh\upsilon$, the average energy per oscillator is calculated to be

$$\bar{\varepsilon} = \frac{h\nu}{e^{h\nu/kT}-1} \qquad \text{..... (27)}$$

(For the derivation see appendix B).

Using this the energy density $u(\upsilon)d\upsilon$ in the cavity in the frequency interval υ and $\upsilon + d\upsilon$ becomes

$u(\upsilon)d\upsilon$ = Number of oscillators per unit volume × The average energy per oscillator

i.e.,
$$u(\upsilon)d\upsilon = \frac{8\pi\upsilon^2 d\upsilon}{c^3}\times\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$$

or
$$u(\upsilon)d\upsilon = \frac{8\pi h}{c^3}\frac{\nu^3 d\upsilon}{e^{\frac{h\nu}{kT}}-1} \qquad \text{..... (28a)}$$

This is the Planck's radiation law in terms of frequency.

This law could be able to explain the spectral distribution of radiation with amazing success. Using $\nu = \frac{c}{\lambda}$, we can write

$$u(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1} \qquad \text{..... (28b)}$$

This is the Planck's radiation law in terms of wavelength.

From eqn. 28(b), we can very well calculate the intensity of radiation emitted by Black body.

We have

i.e.,
$$I(\lambda) = \frac{c}{4}u(\lambda)$$

$$I(\lambda) = \frac{c}{4}\frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}$$

$\therefore$
$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1} \quad .....(29)$$

When a graph is drawn between I versus λ, it is found that it is in perfect agreement with our experimental curve (Fig. 1.12).

The very interesting thing is that Planck's formula attributed particle properties to waves.

It may also be noted that at high frequencies $h\nu \gg kT$ then $e^{\frac{h\nu}{kT}} \to \infty$. From eqn 28(a) it is seen that $u(\upsilon)d\upsilon \to 0$ as experimentally observed. In other words Planck's formula shows no ultraviolet catastrophe. Moreover in the low frequency region as an approximation Planck's formula becomes Rayleigh-Jeans formula.

Deduction of Rayleigh-Jeans law from Planck's law

In the short frequency region

$$h\nu \ll kT \text{ or } \frac{h\nu}{kT} \ll 1$$

$$\frac{h\nu}{e^{\frac{h\nu}{kT}}-1} \approx \left(1+\frac{h\nu}{kT}-1\right) = \frac{h\nu}{kT} \quad .....(30)$$

since $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots$

when $x \ll 1$

$$e^x \approx 1 + x$$

Substituting eqn 7 in eqn 6(a), we get

$$u(\upsilon)d\upsilon = \frac{8\pi h}{c^3}\frac{\upsilon^2 d\upsilon}{(h\nu/kT)}$$

$$u(\upsilon)d\upsilon = \frac{8\pi kT}{c^3}\upsilon^2 d\upsilon$$

This is nothing but our Rayleigh-Jeans formula.

In the long wavelength region $\frac{hc}{\lambda} \ll kT$ or $\frac{hc}{\lambda kT} \ll 1$

$$\therefore \quad e^{\frac{hc}{\lambda kT}} \simeq 1 + \frac{hc}{\lambda kT}$$

$$e^{\frac{hc}{\lambda kT}} - 1 \simeq \frac{hc}{\lambda kT}$$

Put this in eq (6b), we get

$$u(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}d\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{\left(\frac{hc}{\lambda kT}\right)} = \frac{8\pi kT}{\lambda^4}d\lambda$$

$$\therefore \quad I(\lambda)d\lambda = \frac{c}{\lambda}u(\lambda)d\lambda = \frac{2\pi}{\lambda^4}ckT$$

This is Rayleigh-Jeans formula in terms of wavelength.

Deduction of Wien's displacement law from Planck's law

From Planck's law we have

$$u_\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}$$

Taking log on both sides, we get

$$\ln u_\lambda = \ln(8\pi hc) - 5\ln\lambda - \ln\left(e^{\frac{hc}{\lambda kT}}-1\right)$$

Differentiating on both sides with respect to λ, we get

$$\frac{1}{u_\lambda}\frac{du_\lambda}{d\lambda} = 0 - \frac{5}{\lambda} - \frac{1}{e^{\frac{hc}{\lambda kT}}-1}e^{\frac{hc}{\lambda kT}} \times -\frac{hc}{\lambda^2 kT}$$

$$\frac{1}{u_\lambda}\frac{du_\lambda}{d\lambda} = \frac{1}{\lambda}\left[-5 + \frac{hc}{\lambda kT}\frac{e^{\frac{hc}{\lambda kT}}}{e^{\frac{hc}{\lambda kT}}-1}\right]$$

For $\lambda = \lambda_m$, $\frac{du_\lambda}{d\lambda} = 0$

$$-5 + \frac{hc}{\lambda_m kT} \frac{e^{\frac{hc}{\lambda_m kT}}}{e^{\frac{hc}{\lambda_m kT}} - 1} = 0$$

Put $\frac{hc}{\lambda_m kT} = x$, we have

$$-5 + \frac{x e^x}{e^x - 1} = 0$$

$$\frac{x e^x}{e^x - 1} = 5$$

Solving this equation, we get

$$x = 4.9651$$

or $$\frac{hc}{\lambda_m kT} = 4.9651$$

or $$\lambda_m T = \frac{hc}{4.9651 \times k}$$

$$\lambda_m T = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{4.9651 \times 1.38 \times 10^{-23}} = 2.898 \times 10^{-3}\,\text{mK}$$

This is Wien's displacement law.

Example 10

An analyser for thermal radiation is set to accept wavelengths in an interval of 1.55nm. What is the intensity of the radiation in that interval at a wavelength of 875nm emitted from a glowing object whose temperature is 1675K.

Solution

$d\lambda = 1.55 \times 10^{-9}\,\text{m}$, $hc = 1240\,\text{eVnm}$

$\lambda = 875 \times 10^{-9}\,\text{m}$ $kT = 8.6174 \times 1675 = 0.1443\,\text{eV}$

Using $$I(\lambda)d\lambda = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} d\lambda$$

$$I(\lambda)d\lambda = \frac{2 \times 3.14 \times 6.626 \times 10^{-34} \times (3 \times 10^8)^2}{(875 \times 10^{-9})^5 \left(e^{\frac{1240}{875 \times 0.1443}} - 1\right)}$$

$$= 61.4\,\text{W/m}^2$$

Example 11

Assuming that the sun to radiate like a black body at temperature of 6000K, what is the intensity of radiation emitted in the range 550.0nm to 552.0nm. What fraction of the total solar radiation does this represent.

Solution

$$d\lambda = 552 - 550 = 2\text{nm}$$

$$\lambda = \frac{552 + 550}{2} = 551\,\text{nm}$$

$$kT = 8.6174 \times 6000 = 0.517\text{eV}$$

$$hc = 1240\,\text{eVnm}$$

Using $$I(\lambda)d\lambda = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\left(e^{\frac{hc}{\lambda kT}} - 1\right)}$$

$$I(\lambda)d\lambda = \frac{2 \times 3.14 \times 6.626 \times 10^{-34} \times (3 \times 10^8)^2}{(551 \times 10^{-9})^5 \left(e^{\frac{1240}{551 \times 0.517}} - 1\right)}$$

$$I(\lambda)d\lambda = 1.9 \times 10^5\,\text{Wm}^{-2}$$

The total radiant intensity emitted by the sun

$$I = \sigma T^4 = 5.67 \times 10^{-8} \times (6000)^4$$

$$I = 7.35 \times 10^7\,\text{Wm}^{-2}$$

$$\therefore \quad \text{Fraction} = \frac{1.9\times10^5}{7.35\times10^7} = 0.00259$$

Compton effect

Introduction

According to Einstein light consists of quanta (photons). If so when two lights are allowed to collide they must scatter like billiard balls. When this experiment was carried out, unfortunately, it is seen that light cannot be scattered by light so this experiment was ruled out. But Aurthur Compton of Chicago advanced one more step and thought, why not then bounce a photon off an electron. Compton worked out the theory then he performed an experiment to verify this. The result was miraculous. The theoretical prediction and experimental result were one and the same. The experimental support of the theory indicates very convincingly that light quantum is a reality once again proving that light has a dual character (wave and particle).

Compton effect

Compton effect is the scattering of a photon by an electron. As a result of scattering the scatterd photon has less energy (longer wavelength) than the incident photon. In other words there is a change in wavelength of the photon due to scattering. For the derivation of the change in wavelength he assumed that the collision between photon and electron is elastic. Elastic collision means

(i) the sum of the momenta of the colliding particles before the collision must be the same as that after the collision and

(ii) the sum of their energies before collision is equal to sum of their energies after collision.

Compton supposed that a photon of momentum $\frac{h}{\lambda}$ corresponding to a frequency ν and wavelength λ collided with an electron and that after the collision the photon acquired a new momentum $\frac{h}{\lambda'}$ corresponding to a frequency ν' and wavelength λ'. It is due to collision, electron received a powerful kick from the photon and absorbed some energy from it. The electron too would change its energy and momentum. Since the incident photon has given some of its energy to the electron, the scattered photon would naturally have a lesser energy than the incident photon. In turn this would mean that the scattered photon would have a lower frequency (i.e., a longer wavelength). Compton showed that the wavelength change $\Delta\lambda$ is given by

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1-\cos\phi) \qquad \text{..... (31)}$$

Proof

Consider a photon having energy $h\nu$ and moment $\frac{h}{\lambda}$ collides with an electron, at rest having energy mc^2, where m is the mass of the electron, as shown in figure. After collision the photon moves with an energy $h\nu'$ and momentum $\frac{h}{\lambda'}$ at an angle ϕ with the direction of incident photon. The electron scattered wtih an energy E_e and momentum p at an angle θ with the initial direction.

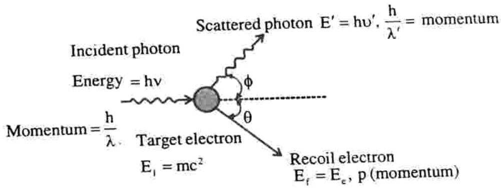


Figure 1.14

According to the law of conservation of energy

Total energy before collision = Total energy after collision

i.e., Energy of the incident photon + Energy of the electron = Energy of the scattered photon + energy of the scattered electron

i.e., $$h\nu + mc^2 = h\nu' + E_e \qquad \text{..... (32)}$$

$\therefore$ $$E_e = h\nu - h\nu' + mc^2$$

or $$E_e = h(\nu - \nu') + mc^2 \qquad \text{..... (33)}$$

According to the law of conservation of momentum, (we have two equations one along the horizontal direction and the other along the vertical direction since momentum is a vector quantity)

Total momentum before collision along the horizontal direction = Total momentum after collision along the horizontal direction.

i.e., (Momentum of the photon + Momentum of the electron) before collision along the horizontal direction = (Momentum of the photon + momentum of the electron) after collision along the horizontal direction.

i.e., $$\frac{h}{\lambda}+0=\frac{h}{\lambda'}\cos\phi+p\cos\theta$$

where p is the momentum of the electron

or $$\frac{h}{\lambda}=\frac{h}{\lambda'}\cos\phi+p\cos\theta$$

or $$\frac{h}{\lambda}-\frac{h}{\lambda'}\cos\phi=p\cos\theta \qquad \text{..... (34)}$$

Similary along the vertical direction, we get

$$0+0=\frac{h}{\lambda'}\sin\phi-p\sin\theta$$

or $$\frac{h}{\lambda'}\sin\phi=p\sin\theta \qquad \text{..... (35)}$$

Squaring and adding eqns 34 and 35, we get

$$\left(\frac{h}{\lambda}-\frac{h}{\lambda'}\cos\phi\right)^2+\left(\frac{h}{\lambda'}\sin\phi\right)^2=p^2\cos^2\theta+p^2\sin^2\theta$$

$$\frac{h^2}{\lambda^2}-\frac{2h^2}{\lambda\lambda'}\cos\phi+\frac{h^2}{\lambda'^2}=p^2 \qquad \text{..... (36)}$$

According to relativity, the relation between energy and momentum of an electron is given by

$$E_e=\sqrt{p^2c^2+m^2c^4}$$

squaring we get

$$E_e^2=p^2c^2+m^2c^4 \qquad \text{..... (37)}$$

Substituting eqns 33 and 36 in eqn 37, we get

$$[h(\upsilon-\upsilon')+mc^2]^2=\left(\frac{h^2}{\lambda^2}-\frac{2h^2}{\lambda\lambda'}\cos\phi+\frac{h^2}{\lambda'^2}\right)c^2+m^2c^4$$

or $$h^2(\upsilon-\upsilon')^2+2h(\upsilon-\upsilon')mc^2+m^2c^4=\left(\frac{h^2}{\lambda^2}-\frac{2h^2}{\lambda\lambda'}\cos\phi+\frac{h^2}{\lambda'^2}\right)c^2+m^2c^4$$

or $$h^2(\upsilon-\upsilon')^2+2h(\upsilon-\upsilon')mc^2=\left(\frac{h^2}{\lambda^2}-\frac{2h^2}{\lambda\lambda'}\cos\phi+\frac{h^2}{\lambda'^2}\right)c^2$$

Dividing throughout by c^2, we get

$$h^2\left(\frac{\upsilon}{c}-\frac{\upsilon'}{c}\right)^2+2h(\upsilon-\upsilon')m=\frac{h^2}{\lambda^2}-\frac{2h^2}{\lambda\lambda'}\cos\phi+\frac{h^2}{\lambda'^2}$$

$$h^2\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)^2+2hc\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)m=\frac{h^2}{\lambda^2}-\frac{2h^2}{\lambda\lambda'}\cos\phi+\frac{h^2}{\lambda'^2}$$

$$h^2\left(\frac{1}{\lambda^2}-\frac{2}{\lambda\lambda'}+\frac{1}{\lambda'^2}\right)+2hmc\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)=\frac{h^2}{\lambda^2}-\frac{2h^2}{\lambda\lambda'}\cos\phi+\frac{h^2}{\lambda'^2}$$

$$\frac{h^2}{\lambda^2}-\frac{2h^2}{\lambda\lambda'}+\frac{h^2}{\lambda'^2}+2hcm\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)=\frac{h^2}{\lambda^2}-\frac{2h^2}{\lambda\lambda'}\cos\phi+\frac{h^2}{\lambda'^2}$$

or $$-\frac{2h^2}{\lambda\lambda'}+2hcm\frac{(\lambda'-\lambda)}{\lambda\lambda'}=-\frac{2h^2\cos\phi}{\lambda\lambda'}$$

Multiplying throughout by $\frac{\lambda\lambda'}{2}$, we get

$$-h^2+hcm\ (\lambda'-\lambda)=-h^2\cos\phi$$

or $$hcm\ (\lambda'-\lambda)=h^2-h^2\cos\phi$$

$$hcm\ (\lambda'-\lambda)=h^2(1-\cos\phi)$$

or $$(\lambda'-\lambda)=\frac{h}{mc}(1-\cos\phi)$$

$$\Delta\lambda=\frac{h}{mc}(1-\cos\phi) \qquad \text{..... (38)}$$

The change in wavelength is called compton shift. The term $\frac{h}{mc}$ is called the compton wavelength of the scattered particle and is denoted by λ_c. This constant which is characteristic of the X-ray scattering.

i.e., $\lambda_c = \frac{h}{mc}$

$\therefore$ Equation 38 becomes

$$\Delta\lambda = \lambda_c(1-\cos\phi) \qquad \text{..... (39)}$$

The compton wavelength of electron is

$$\lambda_c = \frac{h}{mc} = \frac{6.62\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}} = 2.425\times10^{-12}\,\text{m}$$

$\lambda_c = 2.425\text{pm}$. It is a universal constant.

It may also be noted from eqn 39 that the change in wavelength $(\lambda'-\lambda)$ is when $\phi = 0^\circ$, $\Delta\lambda = 0$, when $\phi = \frac{\pi}{2}$, $\Delta\lambda = \lambda_c$ and when $\phi = 180^\circ$, $\Delta\lambda$ is maximum. i.e.,

$$(\Delta\lambda)_{max} = \lambda_c(1--1) = 2\lambda_c$$

or $\qquad (\Delta\lambda)_{max} = 2\times2425 = 4.85\,\text{pm}$

It shows that the maximum wavelength change possible is 4.85pm.

Experimentally we observe the fractional shift $\frac{\Delta\lambda}{\lambda}$. Therefore the maximum fractional shift observed is $\frac{(\Delta\lambda)_{max}}{\lambda} = \frac{4.85}{\lambda}$. For visible light $(\lambda = 500\text{nm})$, The fractional shift observed is

$$= \frac{4.85}{\lambda}\text{pm} = \frac{4.85\times10^{-12}}{500\times10^{-9}} = 10^{-5}$$

i.e., the fractional shift observed is only 10^{-5} or 0.001%. This shift is very difficult to detect.

For X-ray $(\lambda = 0.1\text{nm})$, the fractional shift observed is

$$\frac{\Delta\lambda_{max}}{\lambda} = \frac{4.85\times10^{-12}}{0.1\times10^{-9}} = 4.85\times10^{-2}$$

i.e., fractional shift observed is 4.85%. This could be easily observed. Thus we conclude that compton effect is importance only for X-ray photons and is very small for visible light.

Compton performed an experiment in which he scattered X-rays of wavelength

0.071nm from carbon at several angles and verified his formula. Finally we can say that compton effect demonstrated the quantum concept and the particle nature of the photon.

Note:

(i) Relation between υ and υ': $\frac{\upsilon}{\upsilon'} = 1 + \frac{h\upsilon}{mc^2}(1-\cos\phi)$

$$\frac{\upsilon}{\upsilon'} = 1 + \alpha(1-\cos\phi)$$

(ii) Relation between θ and ϕ: $\cot\theta = (1+\alpha)\tan\frac{\phi}{2}$

(iii) Kinetic energy of the recoil electron

$$KE = h\upsilon - h\upsilon' = h\upsilon\frac{\alpha(1-\cos\phi)}{1+\alpha(1-\cos\phi)} = \frac{h\upsilon 2\alpha\cos^2\theta}{(1+\alpha)^2 - \alpha^2\cos^2\theta}$$

Experimental demonstration of the Compton effect

The experimental arrangement for investigating Compton effect designed by Compton consists of an intense X-ray from the molybdenum target of a coolidge tube. The X-ray is allowed to pass through a suitable filtter such that it allows only K_α line to pass through it. This monochromatic X-ray is allowed to fall on a carbon block C and scattered in all directions. The radiation scattered at a scattering angle ϕ is analyzed by a crystal spectrometer. From the crystal spectrometer the X-rays

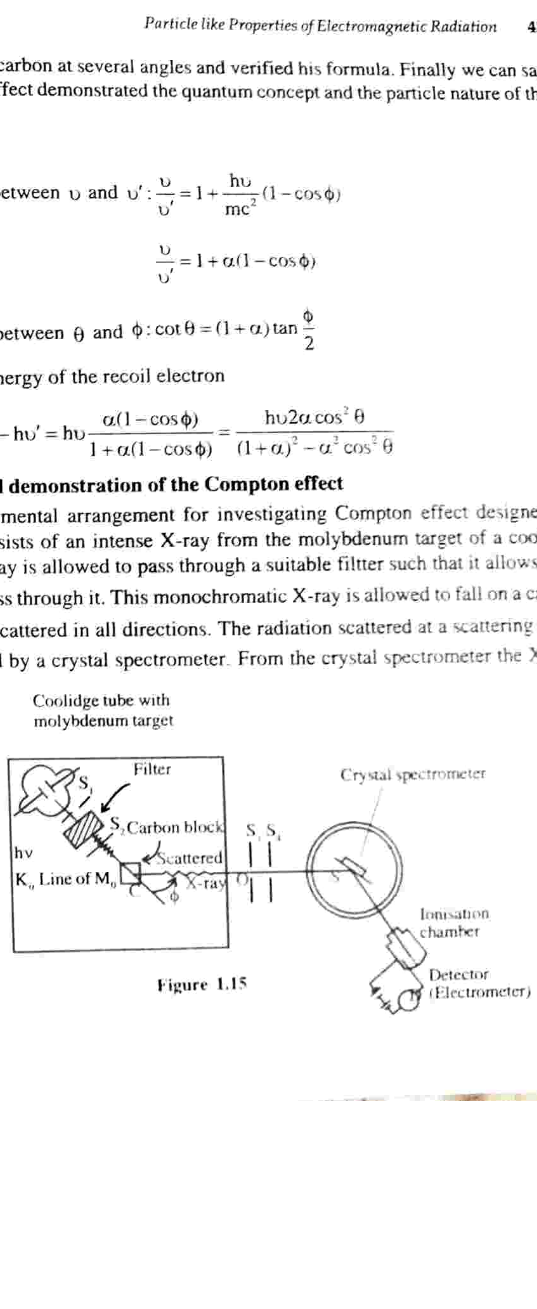


Figure 1.15

get diffracted and allowed to enter an ionisation chamber as shown in figure. X-rays produce ionisation in the chamber and the circuit external to the chamber measures the ionisation current with the help of an electrometer. This current will be proportional to the intensity of X-rays.

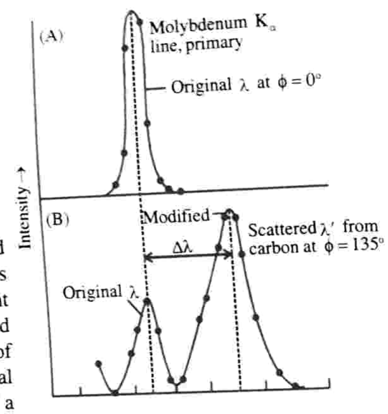


Figure 1.16

Compton found that the scattered radiation was less penetrating (of less frequency) than the primary incident radiation and that this contained radiation of two wavelengths. One of these was identical with the original one (see fig.) and the other was of a longer wavelength depending on the scattering angle. Results are plotted on a graph with intensity of ionisation (proportional to X-ray intensity) along the vertical axis and the wavelength along the horizontal axis. The difference between the two peaks (one corresponds to original λ and the other corresponds to the new scattered longer wavelength λ') gives the change in wavelength $\lambda' - \lambda = \Delta\lambda$. This was in perfect agreement with what was experimentally observed.

Example 12

X-rays of wavelength 0.140nm are scattered from a block of carbon. What will be the wavelength of X-rays scattered at

a) $0°$ b) $90°$ and c) $180°$

Solution

We have $\lambda' - \lambda = \dfrac{h}{mc}(1 - \cos\phi)$

or $$\lambda' = \lambda + \frac{h}{mc}(1 - \cos\phi)$$

a) For $\phi = 0°$

$$\lambda' = \lambda + \frac{h}{mc}(1 - 1) = \lambda$$

i.e., $\lambda' = 0.140\text{nm}$

b) For $\phi = 90°$

$$\lambda' = \lambda + \frac{h}{mc}$$

$$\lambda' = 0.140 \times 10^{-9} + \frac{6.62 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^{8}}$$

$$\lambda' = 0.140 \times 10^{-9} + 2.425 \times 10^{-12}$$

$$\lambda' = 0.140 \times 10^{-9} + 0.002425 \times 10^{-9}$$

$$\lambda' = 0.1421\text{nm}$$

c) For $\phi = 180°$, $\cos\phi = -1$

$$\lambda' = \lambda + \frac{h}{mc}(1 - -1)$$

$$\lambda' = \lambda + \frac{2h}{mc}$$

$$\lambda' = 0.140\text{nm} + 2 \times 0.002425\text{nm}$$

$$\lambda' = 0.145\text{nm}.$$

Example 13

The wavelength of a photon is equal to the compton wavelength of electron. What is its energy in MeV.

Solution

$$\lambda = \frac{h}{mc} \text{ given}$$

$\therefore$ Energy of the photon, $\varepsilon = h\nu = \dfrac{hc}{\lambda} = \dfrac{hc}{h/mc} = mc^2$

$$\varepsilon = 9.1 \times 10^{-31} \times (3 \times 10^{8})^2$$

$$\varepsilon = 81.9\times10^{-15}\,\text{J}$$

or $$\varepsilon = \frac{81.9\times10^{-15}}{1.6\times10^{-19}}\,\text{eV} = 51.18\times10^{4}\,\text{eV}$$

$$\varepsilon = \frac{51.18\times10^{4}}{10^{6}}\,\text{MeV} = 0.511\,\text{MeV}.$$

Example 14

An X-ray beam has an energy of 40keV. Find the maximum possible kinetic energy of compton scattered electrons

Solution

We have K.E $= h\nu - h\nu'$

For maximum possible kinetic energy (K.E) for a given $h\nu$, $h\nu'$ must be minimum i.e., λ' must be maximum.

We have $$\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi)$$

or $$\lambda' = \lambda + \frac{h}{mc}(1-\cos\phi)$$

For λ' to be maximum $\cos\phi$ must be -1

$$\therefore \quad \lambda' = \lambda + \frac{2h}{mc}$$

Using $$\varepsilon = h\nu = \frac{hc}{\lambda}$$

or $$\lambda = \frac{hc}{\varepsilon} = \frac{6.62\times10^{-34}\times3\times10^{8}}{40\times10^{3}\times1.6\times10^{-19}}$$

$$\lambda = 31\times10^{-12}\,\text{m}.$$

$$\therefore \quad \lambda' = 31\times10^{-12} + 4.85\times10^{-12}$$

$$\lambda' = 35.85\times10^{-12}$$

$$\therefore \quad \nu' = \frac{c}{\lambda'} = \frac{3\times10^{8}}{35.85\times10^{-12}}$$

or $$h\nu' = \frac{6.62\times10^{-34}\times3\times10^{8}}{35.85\times10^{-12}} = 0.554\times10^{-14}\,\text{J}$$

or $$h\nu' = \frac{0.554\times10^{-14}}{1.6\times10^{-16}}\,\text{eV}$$

$$h\nu' = 34.625\,\text{keV}$$

$\therefore$ Change in K.E $= h\nu - h\nu'$

$$= 40 - 34.625$$

$$= 5.375\ \text{keV}.$$

Note : See also example 11

Example 15

X-rays with an energy of 50 keV are scattered by 45°. Find the frequency of the scattered photon.

Solution

The energy of the photon, $\varepsilon = \dfrac{hc}{\lambda}$

$$\lambda = \frac{hc}{\varepsilon} = \frac{6.62\times10^{-34}\times3\times10^{8}}{50\times10^{3}\times1.6\times10^{-19}}$$

$$\lambda = 24.82\times10^{-12}\,\text{m}$$

Using $$\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi)$$

$$\frac{h}{mc} = 2.425\times10^{-12} \text{ and } \cos\phi = \cos 45 = \frac{1}{\sqrt{2}}$$

$$\therefore \quad \lambda' - \lambda = 2.425\times10^{-12}\left(1-\frac{1}{\sqrt{2}}\right)$$

$$\lambda' - \lambda = 2.425\times10^{-12}\,\frac{(\sqrt{2}-1)}{\sqrt{2}}$$

$$\lambda' - \lambda = 0.71 \times 10^{-12}$$

or

$$\lambda' = \lambda + 0.71 \times 10^{-12}$$

$$\lambda' = 24.82 \times 10^{-12} + 0.71 \times 10^{-12}$$

$$\lambda' = 25.53 \times 10^{-12}$$

$$\nu' = \frac{c}{\lambda'} = \frac{3 \times 10^8}{25.53 \times 10^{-12}}$$

$$\nu' = 1.18 \times 10^{19}\,\text{Hz}$$

Example 16

A photon of frequency ν is scatted by an electron initially at rest. Verify that the maximum kinetic energy of the recoil electron is $K.E_{max} = \frac{2h^2\nu^2}{mc^2}\left(1+\frac{2h\nu}{mc^2}\right)^{-1}$.

Solution

We have

$$K.E = h\nu - h\nu' = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = hc\frac{(\lambda'-\lambda)}{\lambda\lambda'}$$

or

$$K.E = \frac{hc(\lambda'-\lambda)}{\frac{c}{\nu}\,\frac{c}{\nu'}} \qquad \left(\because \lambda = \frac{c}{\nu},\ \lambda' = \frac{c}{\nu'}\right)$$

$$K.E = \frac{h\nu\nu'}{c}(\lambda'-\lambda)$$

Substituting for $(\lambda'-\lambda) = \frac{h}{mc}(1-\cos\phi)$

$$\therefore \quad K.E = \frac{h^2}{mc^2}\nu\nu'(1-\cos\phi)$$

K.E is maximum when $\phi = \pi$

$$\therefore \quad (K.E)_{max} = \frac{2h^2\nu\nu'}{m\,c^2} \qquad \text{..... (1)}$$

or

$$h\nu - h\nu' = \frac{2h^2\nu\nu'}{m\,c^2}$$

or

$$h\nu = h\nu' + \frac{2h^2\nu\nu'}{mc^2}$$

$$\nu = \nu'\left(1+\frac{2h\nu}{mc^2}\right)$$

$$\therefore \quad \nu' = \frac{\nu}{\left(1+\frac{2h\nu}{mc^2}\right)}$$

Put this in eqn (1), we get

$$(K.E)_{max} = \frac{2h^2\nu^2}{mc^2}\cdot\frac{1}{\left(1+\frac{2h\nu}{mc^2}\right)}$$

$$(K.E)_{max} = \frac{2h^2\nu^2}{mc^2}\left(1+\frac{2h\nu}{mc^2}\right)^{-1}$$

Example 17

At what scattering angle will incident 100keV X-rays leave a target with an energy of 90keV.

Solution

Energy of incident photon $\varepsilon = 100\text{keV}$

i.e.,

$$\frac{hc}{\lambda} = 100\text{keV} = 100 \times 10^3 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-14}$$

or

$$\lambda = \frac{hc}{1.6 \times 10^{-14}} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-14}}$$

$$\lambda = 12.4\text{pm}$$

Energy of the scattered photon,

$$\varepsilon' = 90\text{keV}$$

or $$\frac{hc}{\lambda'} = 90 \times 10^3 \times 1.6 \times 10^{-19}$$

$$\frac{hc}{\lambda'} = 1.44 \times 10^{-14}$$

or $$\lambda' = \frac{hc}{1.44 \times 10^{-14}} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{1.44 \times 10^{-14}}$$

$$\lambda' = 13.79\text{pm}$$

Using $$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$$

$$13.79 - 12.4 = 2.425(1 - \cos\phi)$$

$$1.39 = 2.425(1 - \cos\phi)$$

$$\frac{1.39}{2.425} = 1 - \cos\phi$$

or $$\cos\phi = 1 - \frac{1.39}{2.425} = 0.426$$

$\therefore$ $$\phi = \cos^{-1}(0.426)$$

$$\phi = 64.7^\circ$$

Other photon processes

The phenomenon of photoelectric effect, thermal radiation and Compton effect conclusively proved that electromagnetic radiations exhibit particle like behaviour. In other words electromagnetic radiations have energy in discrete quanta known as particles. There are other numerous processes which could be explained only on the basis of particle nature of electromagnetic radiation rather than wave nature. Here we discuss some of them.

Interactions of photons with atoms

Atoms emit electromagnetic radiations in the form of discrete energy called photons. When an atom emits a photon of energy $E(h\nu)$, the atom loses an equivalent amount of energy. Consider an atom at rest having energy E_i. The atom emits a photon of energy E. After the emission the atom is left with a final energy E_f. It is

due to conservation of momentum, when the atom emits a photon, to conserve momentum atom must recoil. Let K be the recoil energy. Then conservation of energy gives

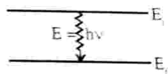
$$E_i = E_f + K + E$$

or $$E = E_i - E_f + K$$

Usually K is negligibly small.

Thus $$E = E_i - E_f$$

This shows that when an atom jumps from initial state i (higher state) to final state f lower state, it emits a photon of energy equal to difference in energy of the two states



In the reverse process an atom initial at rest has energy E_i. Suppose it absorbs a photon of energy $E(h\nu)$. The atom gains energy. The atom must also acquire a small recoil energy in order to conserve momentum.

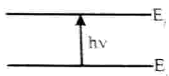
Then conservation of energy gives

$$E_i + E = E_f + K$$

or $$E = E_f - E_i + K$$

If K is negligible

$$E = E_f - E_i$$



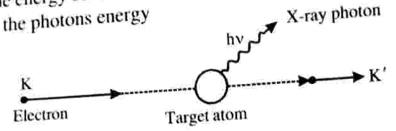
This shows that when atom absorbs an energy E, the atom will go from the initial state (lower state) to final state (higher state).

This has been verified experimentally. The photon emission and photon absorption experiments enabled us to gather information regarding internal structure of atoms.

Bremsstrahlung and X-ray production

According to electromagnetic theory an accelerated or decelerated charge radiates electromagnetic waves. Quantum theory says these electromagnetic radiations are photons. i.e. an accelerated or decelerated charge emits photons.

Consider a beam of electrons accelerated by a potential difference V. Thus electrons acquire a kinetic energy of K. When these electrons are allowed to strike a target, they are slowed down and eventually comes to rest, because they make collisions with the atoms of the target material. In such a collision momentum is transferred to the atom and electrons get decelerated there by emitting photons. It is due to momentum transfer atom gets recoil kinetic energy. Since the atom is so massive this energy can be neglected. If the electron has a kinetic energy K before the collision and K′ be the energy of the electron after collision. The energy lost by electron is converted into the photons energy



i.e. $$K - K' = h\nu = \frac{hc}{\lambda}$$

If the electron comes to rest after collision $K' = 0$, thereby entire kinetic energy is given to photons. In this situation λ is minimum

i.e. $$K = \frac{hc}{\lambda_{min}}$$

or $$\lambda_{min} = \frac{hc}{K}$$

If the accelerating voltage is 10,000 V

$$\therefore \quad \lambda_{min} = \frac{1240\,\text{eVnm}}{10000\,\text{eV}} \approx 0.1\text{nm}$$

This wavelength corresponds to X-rays. Experiments show that this X-ray spectrum is continuous in its nature. This is called bremsstrahlung.

Fast moving electrons when suddenly stopped results in the production of continuous X-rays is called bremsstrahlung. It is a German word for braking radiation.

Note: Whenever we say X-rays, usually they are the radiations emitted in atomic transition. Unlike bremsstrahlung ordinary X-rays are discrete.

Example 18

What is the minimum X-ray wavelength produced in bremsstrahlung by electrons that have been accelerated through 2.50×10^4 V

Solution

We have, $$\lambda_{min} = \frac{hc}{K}$$

$$\lambda_{min} = \frac{1240\,\text{eVnm}}{2.50 \times 10^4\,\text{eV}}$$

$$\lambda_{min} = 0.0496\text{ nm}$$

Example 19

An atom absorbs a photon of wavelength 375nm and immediately emits another photon of wavelength 580nm. What is the net energy absorbed by atom in this process.

Solution

Energy absorbed $$E_1 = \frac{hc}{\lambda} = \frac{1240\,\text{eVnm}}{375\text{ nm}}$$

$$E_1 = 3.3066\,\text{eV}$$

Energy released $$E_2 = \frac{hc}{\lambda_2} = \frac{1240\,\text{eVnm}}{580\text{ nm}}$$

$$E_2 = 2.1379\,\text{eV}$$

$\therefore$ Net energy absorbed $= 3.3066 - 2.1379$

$= 1.1687\text{eV}$

Pair production and pair annihilation

We have seen that during collision between photon and electron all of the photons energy can give it to an electron in the case of photoelectric effect and part of the photons energy to the electron in the case of compton effect. Naturally a question arises. Can the kinetic energy of a photon be converted into particle mass and vice versa. The answer is yes.

Conversion of photon energy into mass

A photon of sufficient energy materialises into electron and a positron is called pair production.

$$2\gamma \rightarrow e^- + e^+$$

Some important points regarding pair production worth noting

1. During pair production charge is conserved. Since an electron has negative charg the partner must be a positively charged particle to balance charge conservatio

2. Positively charged electron e^+ is called a positron. It has the same mass as the electron but an opposite charge.
3. Pair production can occur only when photon passes through matter because energy and momentum are not conserved when the reaction takes place in empty space. The missing momentum in this process must be supplied by interaction with a massive object such as nucleus.
4. When pair production take place inside an atom, the electric field of nucleus is large. The nucleus recoils and takes away a negligible amount of energy but a considerable amount of momentum. According to law of conservation of energy

Energy of the photon = Energy of the electron + Energy of the positron + K.E of the nucleus.

i.e., $h\nu = \varepsilon_- + \varepsilon_+ + KE$ of nucleus

To create the rest masses

$h\nu = mc^2 + mc^2 + K.E$ of nucleus

i.e., $h\nu > 2mc^2 = 1.02\,MeV$.

It shows that pair production requires a minimum photon energy of $1.02\,MeV$. When calculate the wavelength of this photon, we get

$$\lambda = \frac{c}{\nu} = \frac{hc}{h\nu} = \frac{hc}{2mc^2}$$

$$\lambda = \frac{hc}{1.02\times10^6\times1.6\times10^{-19}}$$

$$\lambda = \frac{6.62\times10^{-34}\times3\times10^8}{1.02\times10^6\times1.6\times10^{-19}} = 1.217\times10^{-12}\,m$$

Electromagnetic waves with this wavelength are called gamma rays. It is denoted by γ.

Conversion of mass into energy

When a positron passes through matter it loses its kinetic energy due to atomic collisions. A positron and an electron, due to their mutual electric attraction, may form an atom-like configuration called positronium, where they rotate around each other about the centre of mass. Ultimately the electron and positron come close together and annihilate each other in a time of the order of 10^{-10} s, producing electromagnetic radiation (photons).

Positronium, before decay (schematic only)
(A)

After annihilation
(B)

Figure 1.17

The process $e^+ + e^- \rightarrow \gamma + \gamma$ is called pair annihilation.

A positronium "atom" in free space must emit at least two photons in order to conserve energy and momentum. The photons emerge in opposite directions with equal energies. Because initially positronium is at rest, the initial momentum is assumed to be zero as shown in the figure.

For the process $(e^+e^-)\text{atom} \rightarrow \gamma + \gamma$

From conservation of energy: $2m_ec^2 = h\nu_1 + h\nu_2$ ..... (1)

From conservation of momentum : $0 = \dfrac{h\nu_1}{c} - \dfrac{h\nu_2}{c}$ ..... (2)

Hence,

Thus Equation (1) becomes $2m_ec^2 = 2h\nu$

or $h\nu = m_ec^2 = 0.511\,MeV$ ..... (3)

Hence the two photons originating from positronium annihilation move in opposite directions, each with energy 0.511 MeV. This fact has been observed experimentally.

Pair annihilation has useful applications. Positron Emission Tomography (PET) scanning is a standard diagnostic technique in medicine. A positron-emitting radioactive chemical (containing a nucleus such as ^{15}O, ^{11}C, ^{13}N or ^{18}F) is injected into the body. From the point of concentration of chemicals two characteristic annihilation photons are emitted. By measuring the directions of two gamma-ray photons the point from which the photons originate is identified by measuring the directions of two gamma-ray photons of the correct energy. Measurement of blood flow in the brain is an example of a diagnostic tool used in the evaluation of strokes, brain tumours, and other brain lesions.

Note : It may be noted that no nucleus or other particle is needed for pair annihilation to take place.

Example 20

A positron collides head on with an electron and both are annihilated. Each particle had a kinetic energy of 1MeV. Find the wavelength of the resulting photons.

Solution

We have $e^+ + e^- \rightarrow 2\gamma$

Total energy of $e^+ = mc^2 + K.E$

$= 0.51 MeV + 1 MeV = 1.51 MeV$

Total energy of $e^- = mc^2 + K.E$

$= 0.51 MeV + 1 MeV = 1.51 MeV$

$\therefore$ Total energy of $2\gamma = 1.51 + 1.51$

$= 3.02\,MeV$

or Energy of the photon $= \frac{3.02}{2} MeV$

$= 1.51 MeV$

$\therefore$ Wavelength, $\lambda = \frac{c}{v} = \frac{hc}{hv} = \frac{hc}{1.51 MeV}$

$$\lambda = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{1.51 \times 10^6 \times 1.6 \times 10^{-19}}$$

$$\lambda = 0.822 \times 10^{-12} m$$

Example 21

Show that the minimum energy of a photon to create an electron-positron pair in the presence of a stationary nucleus of mass M is $2mc^2\left(1 + \frac{m}{M}\right)$, where m is the electron mass.

Solution

We have $hv = \varepsilon_+ + \varepsilon_- + KE$ of the nucleus

$$hv = mc^2 + mc^2 + \frac{1}{2}Mv^2$$

$$hv = 2mc^2 + \frac{1}{2}Mv^2 \qquad \text{..... (1)}$$

From the law of conservation of momentum we have

$$mc + mc = Mv$$

i.e., $2mc + Mv$

or $v = \frac{-2mc}{M}$

Putting this in equation (1), we get

$$hv = 2mc^2 + \frac{1}{2}M\left(\frac{-2mc}{M}\right)^2$$

$$hv = 2mc^2 + \frac{2m^2c^2}{M}$$

$$hv = 2mc^2\left(1 + \frac{m}{M}\right)$$

Photon absorption

So far we have been dealing with photoelectric effect, Compton effect and pair production. In all the three cases photons of light interact with matter there by loosing energy. Depending upon the photon energy and the atomic number of the absorber the cause of energy loss will be different. At low photon energies the energy lose is due to photo electric effect. When the photon energy increases the lose will be due to Compton effect. For still more photon energy it is due to pair production the energy is lost.

What is a photon

Photon is a quantum particle associated with electromagnetic field we can describe photon by giving their basic properties. They are

1. Photons move with speed of light
2. Photons have zero rest mass and zero rest energy. Its mass is $m = \frac{hv}{c^2}$
3. Photons carry energy and momentum. $E = hv$ and $p = \frac{h}{\lambda}$
4. Photons can be created or destroyed when radiation is emitted or absorbed. Photon number is not conserved in nature.
5. Photons behave like particles during collision with electrons.

Numerous experiments were conducted to test whether this dual nature is an intrinsic property of light or of our apparatus. All experiments conclusively proved that wave-particle duality is the intrinsic property of light. The wave nature and particle nature are both present simultaneously in the light. Light is not either particles or waves. It is somehow both particles and waves and shows only one aspect depending on the kind of experiments we conduct. A particle type experiment shows the particle nature, while a wave type experiment shows the wave nature.

Nature of light-wave particle duality

What is the nature of light ? Is it a wave or a particle or both. Now we are in a position to answer this question. Hertz experiment and Young's experiment confirmed that light is a wave. But Planck's radiation law and the phenomenon of photoelectric effect confirmed that light consists of particles. From this we can think of light as having a dual character i.e., both particle and wave. This is called the wave-particle duality of light. Now a days light is called as a wavicle. The term is a beautiful combination of wave and particle. The wave theory of light explains light as a wave where as quantum theory explains light as a quanta (stream of particles). The wave theory and quantum theory complement each other. Both theories are required to explain all the phenomena exhibited by light. Wave theory explains certain aspects of light and quantum theory explains some other aspects. Thus we can say wave theory and quantum theory are partial theories of light. Both the theories together give a complete theory of light. Finally we can say the true nature light is both wave character and particle character.

IMPORTANT FORMULAE

1. The electfic field due to a point charge at a distance r

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

2. Magnetic field due to a straight wire carrying steady current at distance r is

$$\vec{B} = \frac{\mu_0 I}{4\pi r}\hat{r}$$

3. A plane wave moving in z-direction is represented by

$$\vec{E} = E_0\hat{i}\sin(kz - \omega t)$$

$$\vec{B} = B_0\hat{j}\sin(kz - \omega t)$$

where $k = \frac{2\pi}{\lambda}$, $\omega = 2\pi\nu$ and $\frac{\omega}{k} = c$

4. Poynting vector $\vec{N} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})Wm^{-2}$

5. Intensity of light, $I = \frac{P_{av}}{A}$

For a plane wave $I = \frac{E_0^2}{2\mu_0 c} = uc$

u is the energy density and $c = \frac{E_0}{B_0}$.

6. Young's experiment: The distance of maxima from the centre of the screen

$$x_n = \frac{n\lambda D}{d} \quad n = 1, 2, 3, \ldots\ldots$$

For minima $x_n = \left(n + \frac{1}{2}\right)\frac{\lambda D}{d}$ $n = 1, 2, 3, \ldots\ldots$

Band width $\beta = \frac{\lambda D}{d}$

7. Diffraction by grating

$$d\sin\theta = n\lambda, \; n = 1, 2, 3, \ldots\ldots$$

where d is the split spacing and n is the order of the spectrum.

8. Bragg's law for X-ray diffraction

$$2d\sin\theta = n\lambda, \; n = 1, 2, 3, \ldots\ldots$$

where d is the interplanar distance, θ is the glancing angle and n is the order of the spectrum.

9. Energy of a photon $E = h\nu = \frac{hc}{\lambda}$

$$hc = 1240\,eVnm$$

10. Maximum kinetic energy of photo electrons $K_{max} = eV_0$ where V_0 is the stopping potential.

11. Photoelectric equation

$$h\nu - h\nu_0 = \frac{1}{2}mv^2$$

or
$$\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = \frac{1}{2}mv^2$$

$h\nu_0 = w_0$ work function

12. Cut off wavelength $\lambda_c = \frac{hc}{w_0}$

13. Stefan's law: $I = \sigma T^4$
I is the intensity, σ Boltzmann's constant and T is the temperature in kelvin.
$$\sigma = 5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$$

14. Wein's displacement law $\lambda_m T = 2.8978 \times 10^{-3}$ mK

15. Rayleigh Jeans formula
$$I(\lambda) = \frac{2\pi c}{\lambda^4}kT$$
$$u(\lambda) = \frac{8\pi kT}{c^3}\nu^2$$
Intensity $\quad I = \frac{c}{4}u$

16. Planck's radiation law
$$u(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^2 d\nu}{e^{\frac{h\nu}{kT}} - 1}$$
$$u(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$
$$I(\lambda)d\lambda = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

17. Compton shift, $\lambda - \lambda' = \Delta\lambda = \lambda_c(1 - \cos\phi)$

$$\lambda_c = \frac{h}{mc}$$

18. Relation between ν and ν'
$$\frac{\nu}{\nu'} = 1 + \frac{h\nu}{mc^2}(1 - \cos\phi)$$

19. Relation between θ and ϕ
$$\cot\theta = (1+\alpha)\tan\frac{\phi}{2},\ \alpha = \frac{h\nu}{mc^2}$$
where θ is the scattering angle of electron and ϕ that of photon.

20. Kinetic energy of recoil $K = h\nu - h\nu' = h\nu\frac{\alpha(1-\cos\phi)}{1+\alpha(1-\cos\phi)}$

21. Bremsstrahlung: $\lambda_{min} = \frac{hc}{K} = \frac{hc}{eV}$

22. Pair production: $h\nu = \varepsilon_- + \varepsilon_+ +$ kinetic energy of nucleus.

23. Pair annihilation: $\varepsilon_+ + \varepsilon_- \rightarrow 2\gamma$

UNIVERSITY MODEL QUESTIONS

Section A

(*Answer questions in* **two** *or* **three** *sentences*)

Short answer type questions

1. Define (a) particle (b) wave in classical mechanics.
2. Distinguish between particle and wave in the realm of microscopic world.
3. What is wave mechanics?
4. Write down one consequence of wave mechanics.
5. Write down the name of three early experiments that provided evidence that light has properties of particles.
6. What is a photon?
7. What is an electromagnetic wave?
8. Give some of the properties of electromagnetic wave
9. Define poynting vector.
10. What is the cause of electromagnetic wave?

11. What is a plane wave?
12. Define the intensity of a wave.
13. What is meant by superposition principle of waves?
14. What is interference?
15. Distinguish between constructive and destructive interference.
16. What are the conditions to be satisfied for constructive and destructive interferences?
17. Write down Bragg's law of X-ray diffraction and explain the symbols.
18. Write down three experiments which favour particle nature of electromagnetic radiation.
19. What is thermal radiation?
20. What is black body radiation?
21. What is a black body? Give two examples.
22. Show graphically the variation of intensity of spectral line distribution with wavelength
23. Write down Rayleigh-Jeans formula and explain the symbols.
24. What is meant by ultraviolet catastrophe?
25. What is equipartition of energy theorem?
26. What are the two assumptions made by Rayleigh-Jean formula to derive their formula?
27. Where did Rayleigh-Jean go wrong while deriving their formula?
28. What are the two assumptions made by Planck for deriving Planck's radiation law?
29. Write down Planck's radiation formula and explain their symbols.
30. Write down Wien's displacement law and explain their symbols.
31. What is Wien's displacement law?
32. What is photoelectric work function? Give its expression and explain the symbols.
33. What is photoelectric effect?
34. State two laws of photoelectric effect.
35. Write down Einstein's photoelectric equation and explain the symbols.
36. What is cut-off or stopping potential?
37. What is Compton effect?
38. What is Compton shift? Write down an expression for it.
39. What is the difference between photoelectric effect and Compton effect
40. What is Compton wavelength? What is its physical significance?
41. Why does Compton effect not occur with visible light?
42. Why it is preferable to use short wavelength for incidents radiation in Compton effect?
43. What is Bremsstrahlung?
44. What is the difference between ordinary X-rays and Bremsstrahlung X-rays?
45. What is a photon? Give two of its properties.

Section B

(Answer questions in a **paragraph** *of about half a page to one page)*

Paragraph / Problem type questions

1. Show that Planck's radiation formula has no ultraviolet catastrophe.
2. Show that Rayleigh-Jean formula is an approximation of Planck's radiation law.
3. Deduce Wien's distribution law from Planck's law.
4. Deduce Wien's displacement law from Planck's law.
5. Show that for a continuous distribution of energies the average energy density becomes kT.
6. Draw an experimental set up for the demonstration of Compton effect.
7. Find the energy of 400nm (violet) and 700nm (red) photons (3.013eV, 1.773eV)
8. The human eye can detect 1.986×10^{-17} J of e.m energy. How many 550nm photons this represent. (55 photons)
9. Find the number of photons emitted per second by a 25W source of monochromatic light of wavelength 6000Å. $h = 6.62 \times 10^{-34}$ Js. $(7.55 \times 10^{19}$ photons/s)
10. Calculate the longest wave length of the incident radiation which will eject photoelectrons from a metal whose work function is 6.3eV. $(1.972 \times 10^{-7}$ m)
11. The photon electrons emitted by a radiation of frequency 3.65×10^{15} Hz are brought to rest by applying a retarding potential of 10 volts. Find the threshold frequency of that surface. $(1.225 \times 10^{15}$ Hz)
12. Calculate the maximum velocity of the photoelectrons from an emitter if the incident frequency is 5×10^{14} Hz and threshold frequency 3×10^{14} Hz $\cdot$ $m_e = 9.1 \times 10^{-31}$ kg. $(5.386 \times 10^{5}\ ms^{-1})$
13. What is the frequency of an X-ray photon whose momentum is 1.1×10^{-23} kgms^{-1} $(498 \times 10^{18}$ Hz)
14. A monochromatic X-ray beam whose wavelength is 55. 8pm is scattered through 46°. Find the wavelength of scattered beam. (56.54 pm)
15. A beam of X-rays is scattered by a target. At 45° from the beam direction the scattered X-rays have a wavelength of 2.2 pm. What is the wavelength of X-rays in the direct beam. (1.5pm)
16. A photon of energy E is scattered by a particle of rest energy E_o. Find the maximum kinetic energy of the recoiling particle in terms of E and E_o. $\left(\frac{2E^2}{2E + E_o}\right)$

17. Find the change in wavelength of 80 pm X-rays that are scattered 120° by a target. Find the angle between directions of the recoil electron and incident photon.
($3.6375\,\text{pm}$, 26.2645°)

18. Consider a hypothetical annihilation of a stationary electron with a stationary positron. What is the wavelength of resulting radiation. $\left(\lambda = \dfrac{h}{mc}\right)$

*19. A beam of light of wavelength 400nm and power 1.85mW is directed at the cathode of a photoelectric cell (given hc = 1240 eV-nm, e = 1.6×10^{-19} C). If only 10% of the incident photons effectively produce photoelectrons, find the current due to these electrons. If the wavelength of light is now reduced to 200 nm keeping its power the same, the kinetic energy of the electrons is found to increase by a factor of 5. What are the stopping potentials for the wavelengths. (IIT-JAM 2007)

*20. Light described by the equation

$$E = 90(V/m)[\sin(6.28\times10^{15}s^{-1})t + \sin(12.56\times10^{15}s^{-1})t]$$

is incident on a metal surface. The work function of the metal is 2.0 eV. Calculate the maximum kinetic energy of the photo electrons. (IIT-JAM 2011)

*21. Electric field component of an electromagnetic radiation varies with time as $E = a(\cos\omega_0 t + \sin\omega t\cos\omega_0 t)$, where a is a constant and values of ω and ω_0 are $1\times10^{15}s^{-1}$ and $5\times10^{15}s^{-1}$ respectively. This radiation falls on a metal of work function 2 eV. Calculate the maximum kinetic energy of photons in eV. (IIT-JAM 2013)

*22. In a photoelectric effect experiment, ultraviolet light of wavelength 320 nm falls on the photo cathode with work function 2.1eV. Calculate the stopping potential
(IIT-JAM 2014)

*23. In a photoelectric experiment both sodium (work function = 2.3 eV) and tungsten (work function = 4.5 eV) metals were illuminated by an ultraviolet light of same wavelength. If the stopping potential for tungsten is measured to be 1.8V, what is the stopping potential for sodium. (IIT-JAM 2016)

*24. A photon of energy E_{ph} collides with an electron at rest and gets scattered at an angle 60° with respect to the direction of the incident photon. The ratio of the relativistic kinetic energy T of the recoiled electron and the incident photon energy E_{ph} is 0.05.

a) Determine the wavelength of the incident photon in terms of Compton wavelength $\left(\lambda_c = \dfrac{h}{m_e c}\right)$

b) What is the total energy E_e of the recoiled electron in units of its rest mass.
(IIT-JAM -2006)

* Harder problems. For IIT-JAM aspirants.

*25 a) A photon of initial momentum p_0 collides with an electron of rest mass 'm_0' moving with relativistic momentum P and energy E. The change in wavelength of the photon after scattering by an angle θ is given by, $\Delta\lambda = 2c\lambda_0 \dfrac{p_0 + P}{E - cP}\sin^2\dfrac{\theta}{2}$, where c is the speed of the light and λ_0 is the wavelength of the incident photon before scattering. What will be the value of $\Delta\lambda$ when the electron is moving in a direction opposite to that of the incident photon with momentum P and energy E? Show that the value of $\Delta\lambda$ becomes independent of the wavelength of the incident photon when the electron is at rest before collision.

b) In a Compton experiment, the ultraviolet light of the wavelength 2000Å is scattered from an electron at rest. What should be the minimum resolving power of an optical instrument to measure the Compton shift, if the observation is made at 90° with respect to the direction of the incident light? (IIT-JAM -2010)

*26. X-ray of wavelength 0.24 nm are Compton scattered and the scattered beam is observed at an angle of 60° relative to the incident beam. The Compton wavelength of the electron is 0.00243 nm. The kinetic energy of scattered electrons in eV is ———
(IIT-JAM-2015)

*27. X-rays of 20 keV energy is scattered inelastically from a carbon target. The kinetic energy transferred to the recoiling electron by photons scattered at 90° with respect to the incident beam is ——— keV.
(Planck constant = 6.6×10^{-34} Js, speed of light = 3×108 m/s, electron mass = 9.1×10^{-31}, electronic charge = 1.6×10^{-19} C). (IIT-JAM-2016)

Section C

(Answer questions in about **two** *pages)*

Long answer type questions (Essays)

1. Describe the phenomenon of photoelectric effect. Give the quantum interpretation of the effect. Write down photoelectric equation and explain the terms used.
2. Explain what is meant by compton effect. Derive an expression for compton shift.
3. Derive Planck's radiation law.

Hint to problems

7. $\varepsilon = h\nu = \dfrac{hc}{\lambda} = \dfrac{6.62\times10^{-34}\times3\times10^{8}}{400\times10^{-98}}$

$\varepsilon = 4.965\times10^{-19}\,\text{J}$

* Harder problems. For IIT-JAM aspirants.

$$\varepsilon = \frac{4.965 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.103\text{eV}$$

Similary the second one

8. $\varepsilon_\lambda = nh\nu = nh\frac{c}{\lambda}$

$$1.986 \times 10^{-17} = \frac{n \times 6.62 \times 10^{-34} \times 3 \times 10^8}{550 \times 10^{-9}}$$

9. $\varepsilon_\lambda = nh\nu = n\frac{hc}{\lambda}$

or $\qquad \frac{\varepsilon_n}{t} = \frac{n}{t}\frac{hc}{\lambda}$

$$25 = \frac{n}{t}\frac{6.62 \times 10^{-34} \times 3 \times 10^8}{6000 \times 10^{-10}}$$

Find $\frac{n}{t}$.

10. $w_0 = h\nu_0 = \frac{hc}{\lambda_0}$

Find λ_0 which gives the longest wavelength corresponding to the threshold frequency.

11. We have $\frac{1}{2}mv^2 = h\nu - h\nu_0$

$\frac{1}{2}mv^2 = 10\text{eV} = 10 \times 1.6 \times 10^{-19}\text{J}$

$\nu = 3.65 \times 10^{15}\text{Hz}$

Find ν_0

12. $\frac{1}{2}mv^2 = h\nu - h\nu_0$. Find ν

13. $p = \frac{h}{\lambda}$ or $\frac{1}{\lambda} = \frac{p}{h}$

or $\qquad \frac{c}{\lambda} = \frac{pc}{h}$

or $\qquad \nu = \frac{pc}{h} = \frac{1.1 \times 10^{-23} \times 3 \times 10^8}{6.62 \times 10^{-34}}$

14. $\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$, $\frac{h}{mc} = 2.425\,\text{pm}$

$\phi = 46°$ and $\lambda = 55.8\text{pm}$, then find λ'

15. $\lambda' = 2.2\text{pm}$ $\phi = 45°$ $\frac{h}{mc} = 2.425\text{pm}$

Using $\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$, find λ.

16. Same as example 16

From example 16, we have

$$K.E_{max} = \frac{2h^2\nu^2}{mc^2\left(1 + \frac{2h\nu}{mc^2}\right)}$$

Put $h\nu = E$ and $mc^2 = E_0$

17. $\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$ $\phi = 120°$, $\cos 120 = -\frac{1}{2}$

$\lambda' - \lambda = \frac{3}{2}\cdot\frac{h}{mc} = \frac{3}{2} \times 2.425\,\text{pm} = 3.6375\,\text{pm}$

$\therefore \qquad \lambda' = \lambda + 3.6375$

$\lambda' = 80 + 3.6375 = 83.6375\,\text{pm}$

From eqn 34 and 35 we get

$$\tan\theta = \frac{\sin\phi}{\lambda'\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\cos\phi\right)} = \frac{\sin\phi}{\frac{\lambda'}{\lambda} - \cos\phi}$$

knowing ϕ, λ and λ', θ can be calculated

18. $\frac{hc}{\lambda} + \frac{hc}{\lambda} = mc^2 + mc^2$

*19. Energy of one photon, $E = \frac{hc}{\lambda} = \frac{1240}{400} = 3.1\text{eV}$

Number of photons per second

$P = nh\nu$

$$n = \frac{p}{h\nu} = \frac{p}{(hc/\lambda)} = \frac{1.55\times10^{-3}}{3.1\times1.6\times10^{-19}}$$

Number of photons ejected is 10% of number of photons falling per second

$$N = \frac{10}{100}\times n$$

So current constituted, $I = Ne = 50\mu A$

K.E of ejected electron = Energy of incident photon-work function

$$K.E = \frac{hc}{\lambda} - w_0 \qquad \text{...... (1)}$$

$\lambda = 400nm$ is reduced to 200 nm. K.E increased by 5

$$5K.E = \frac{hc}{\lambda/2} - w_0 \qquad \text{...... (2)}$$

Eq (1) - Eq (2) gives $K.E = 0.775eV$

Stopping potential for 400 nm, $V_0 = \frac{K.E}{e} = \frac{0.775}{1.6\times10^{-19}}eV = 0.775eV$

Stopping potential for 200 nm = $\frac{5K.E}{e}$

$= 3.75eV.$

*20. $\omega_1 = 6.28\times10^{15}$ $\quad \omega_2 = 12.56\times10^{15}$

Photon of higher frequency will give higher K.E

$$(K.E)_{max} = h\nu_2 - w_0 = \frac{h\omega_2}{2\pi} - w_0$$

$$= \frac{4.14\times10^{-15}\times12.56\times10^{15}}{2\times3.14} - 2 = 6.28eV$$

*21. $E = a(\cos\omega_0 t + \sin\omega t\cos\omega_0 t)$

$$E = a\left[\cos\omega_0 t + \frac{1}{2}\sin(\omega+\omega_0)t + \frac{1}{2}\sin(\omega-\omega_0)t\right]$$

For maximum kinetic energy

$$h\nu = \frac{h}{2\pi}(\omega+\omega_0)$$

$$(K.E)_{max} = h\nu - w_0 = 3.9375 - 2 = 1.94eV.$$

*22. $(K.E)_{max} = h\nu - w_0$

or $eV_0 = \frac{hc}{\lambda} - w_0$

$$eV_0 = \frac{hc}{\lambda} - w_0 = \frac{1240eV.nm}{320eV} - 2.1eV$$

$eV_0 = 1.8eV \Rightarrow V_0 = 1.8$ volts.

*23. $eV_0 = h\nu - w_0$

For tungsten $eX1.8 = h\nu - 4.5$

$1.8eV = h\nu - 4.5$

$h\nu = 6.3eV$

For sodium $eV_s = 6.3 - 2.3 = 4eV$

$\therefore$ $V_s = 4$ volt.

*24. (a) $\frac{T}{E_{ph}} = 0.05;\ T = 0.05E_{ph} = 0.25\frac{hc}{\lambda}$

But $T = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$

$$0.05\frac{hc}{\lambda} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

$$\frac{0.05}{\lambda} = \left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

Using $\lambda' - \lambda = \frac{h}{m_e c}(1-\cos60) = \frac{h}{2m_e c}$

or $\lambda' - \lambda + \frac{\lambda_c}{2}$

Thus $\lambda = 9.5\lambda_c$

(b) Total energy of recoiled electron, E_e

= Rest mass energy + K.E of electron.

$$E_e = m_e c^2 + 0.05 E_{ph}$$

$$= m_e c^2 + 0.05\frac{hc}{\lambda} \qquad \left(\because T_c = \frac{h}{m_e c}\right)$$

$$= m_e c^2 + \frac{1}{190} m_e c^2 = \frac{191}{190} m_e c^2$$

*25. (a) The change in wavelength of photon after scattering

$$\Delta\lambda = 2c\lambda_0 \frac{(p_0 + P)}{(E - cP)} \sin^2\left(\frac{\theta}{2}\right)$$

If electron is at rest, then $P = 0$

$$\Delta\lambda = 2c\lambda_0 \frac{p_0}{E} \sin^2\left(\frac{\theta}{2}\right)$$

Now, $p_0 = \frac{h}{\lambda}, E = m_0 c^2 =$ Rest mass energy

So, $\frac{p_0}{E} = \frac{h}{m_0 c^2 \lambda_0}, \Delta\lambda = 2c\lambda_0 \frac{h}{m_0 c^2 \lambda_0} \sin^2\left(\frac{\theta}{2}\right) = \frac{h}{m_0 c}(1 - \cos\theta)$

(b) For UV light of wavelength 2000Å scattered at 90°

Compton shift, $\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\theta) \Rightarrow \Delta\lambda = \frac{h}{m_0 c}(1 - 0) = \frac{h}{m_0 c} = 0.02426\text{Å}$

*26. Incident wavelength $\lambda = 0.24\text{nm}$

Angle of scattering, $\phi = 60°$

Compton wavelength $\lambda_c = \frac{h}{m_0 c} = 0.00243\text{nm}$

Kinetic energy of the scattered electron $E_k =$ loss in energy of X-ray photon

$= h\nu - h\nu'$

$$\Rightarrow E_k = \frac{hc}{\lambda} - \frac{hc}{\lambda'}$$

Scattered wavelength, $\lambda' = \lambda + \Delta\lambda = \lambda + \lambda_c(1 - \cos\phi)$

$$= (0.24 + 0.001215)\text{nm} = 0.241215\text{nm}$$

Therefore, $E_k = \frac{(1240\text{eV} - \text{nm})}{0.24\text{nm}} - \frac{(1240\text{eV} - \text{nm})}{0.241215\text{nm}} = 26\text{eV}$

*27. In Compton effect experiment, the kinetic energy transfered to the recoil electron by photons is

$$E_k = \frac{\frac{h^2\nu^2}{m_0 c^2}(1 - \cos\varphi)}{1 + \frac{h\nu}{m_0 c^2}(1 - \cos\varphi)}$$

where $h\nu$ = energy of X-ray photon $= 20\text{keV}$, $m_0 c^2$ = rest mass energy of the electron $= 511\text{keV}$

$\varphi =$ angle of scattering $= 90°$.

Therefore, $E_k = 0.75\text{keV}$.