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Invited Review

The single-machine total tardiness scheduling problem: Review and extensions

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ABSTRACT

We review the latest theoretical developments for the single-machine total tardiness $1//\bar{T}$ scheduling problem and propose extensions to some of them. We also review (and in some cases extend) exact algorithms, fully polynomial time approximation schemes, heuristic algorithms, special cases and generalizations of the $1//\bar{T}$ problem. Our findings indicate that the $1//\bar{T}$ problem continues to attract significant research interest from both a theoretical and a practical perspective. Even though the problem is ordinary NP-hard, the current state-of-the-art algorithms are capable of solving problems with up to 500 jobs.

1. Introduction

The single-machine total (average) tardiness scheduling problem is defined as follows: There are n jobs available at time zero to be processed on a continuously available single machine which can process at most one job at a time; job j has a processing time p_j and a due date d_j. The tardiness of job j is defined as $T_j = \max(0, C_j - d_j)$ where C_j is the completion time of job j in a given sequence. The objective is to determine a job sequence σ such that the total tardiness $T_\sigma = \sum_{j=1}^{n} T_j$ is minimized. Total tardiness is a regular (non-decreasing) criterion of the job completion times, therefore, there is no inserted idle time in an optimal sequence. Also, any preemptive sequence can be converted to a non-preemptive one with no greater total tardiness (McNaughton, 1959).

Using the well known three-field classification of scheduling problems (Graham et al., 1979 in which the first field specifies the machine environment, the middle field states any special job characteristics and the third field denotes the objective function), the single machine total (average) tardiness scheduling problem can be stated as the $1//\bar{T}$ problem.

The $1//\bar{T}$ problem is ordinary NP-hard (Du and Leung, 1990). Lawler (1977) developed a decomposition-based pseudo-polynomial algorithm for the $1//\bar{T}$ problem which runs in $O(n^4P)$ time (where $P = \sum_{j=1}^{n} p_j$). Lawler's pseudo-polynomial algorithm in conjunction with Du and Leung's (1990) ordinary NP-hardness proof provide a sharp boundary on the complexity of the $1//\bar{T}$ problem.

The $1//\bar{T}$ problem is one of the most widely researched problems in the scheduling literature. Koulamas (1994) surveyed both exact and approximation algorithms for the $1//\bar{T}$ problem. There are significant theoretical developments during the last fifteen years or so, the impact of which on the design of algorithms for the $1//\bar{T}$ problem was not assessed in the more recent survey of Sen et al. (2003). More important, some of these developments can be extended as shown in the rest of the paper. For these reasons, a survey of the recent research on the $1//\bar{T}$ problem focused on theoretical developments and their extensions is quite timely.

The rest of the paper is organized as follows. Section 2 presents the latest theoretical developments for the $1//\bar{T}$ problem along with extensions for some of them. The latest exact algorithms and fully polynomial time approximation schemes for the $1//\bar{T}$ problem are reviewed in Sections 3 and 4 respectively. Section 5 surveys heuristic algorithms for the $1//\bar{T}$ problem with an emphasis on the discussion of their worst-case ratio bounds. Special cases and generalizations of the $1//\bar{T}$ problem are reviewed in Sections 6 and 7 respectively. Section 8 focuses on the $1//\bar{T}$ problem with assignable due dates and the conclusions of this research are summarized in Section 9.

2. Theoretical developments

The most significant theoretical developments for the $1//\bar{T}$ problem are Emmons's (1969) dominance conditions and Lawler's (1977) decomposition principle. Emmons' dominance conditions establish precedence relations among the jobs in an optimal sequence. These relationships have been used extensively to curtail the solution space in enumeration methods.

Let B_i and A_i be the set of jobs sequenced before and after job i respectively. Let $A'_i = \{i : i \notin A_i\}$. Emmons' first theorem states that if $p_i \leqslant p_j$ and $d_i \leqslant \max\left(\sum_{k \in B_j} p_k + p_j, d_j\right)$, then i precedes j in at least one optimal sequence. Emmons' second theorem states that if $p_i \leqslant p_j, d_i > \max\left(\sum_{k \in B_j} p_k + p_j, d_j\right)$ and $d_i + p_i \geqslant \sum_{k \in A_j} p_k$, then j precedes i in at least one optimal sequence. Emmons' first theorem gives the necessary conditions for a shorter job to precede a longer one in an optimal sequence while the second theorem gives the necessary conditions for a longer job to precede a shorter one in

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an optimal sequence. Emmons also derived a third theorem for a shorter job to precede a longer one; that theorem found limited use in applications.

The major issue regarding the use of Emmons' theorem is the prevention of precedence cycles. Emmons addressed this issue by stating that the precedence relations obtained by implementing his theorems can be accumulated. Szwarc (1998) showed that Emmons' conditions are transitive. Yu (1996a) showed that the partial orders accumulated by implementing Emmons' theorems are consistent provided that the accumulation starts from "null". Tansel and Sabuncuoglu (1997) presented a geometric interpretation of Emmons' theorems which can be used to provide some insights into problem "hardness". Emmons' theorems were recently generalized by Kanet (2007), so they can be applied to the single-machine weighted total tardiness $1//\bar{T}_w$ problem with arbitrary weights.

We now turn our attention to surveying the recent theoretical developments related to Lawler's (1977) decomposition theorem. To explain the decomposition theorem, let us assume that the jobs are numbered in the Earliest Due Date (EDD) order so that $d_1 \leqslant d_2 \leqslant \ldots \leqslant d_n$ and $p_i \leqslant p_{i+1}$ when $d_i = d_{i+1}, i = 1, \ldots, n-1$. Let $p_j = \max_{i=1,\ldots,n}(p_i)$. According to Lawler (1977), the $1//\bar{T}$ problem decomposes with the longest job j in some position $k, j \leqslant k \leqslant n$, that is, the search for an optimal solution can be restricted to schedules in which jobs $\langle 1, \ldots, j-1, j+1, \ldots, k\rangle$ are scheduled in the first $k-1$ positions, job j is scheduled in position k and jobs $\langle k+1, \ldots, n\rangle$ are scheduled in the last $n-k$ positions. Lawler's (1977) decomposition theorem utilizes the following rightmost assumption:

Rightmost assumption: The longest job j is completed as late as possible in an optimal sequence.

Early research focused on trimming the list of candidate positions k for the longest job j so that the search for an optimal solution is curtailed. It was shown that the problem does not decompose with job j in position k (under the rightmost assumption) if any of the following conditions hold:

$$\sum_{i=1}^{k} p_k \geqslant d_{k+1}, \quad j \leqslant k \leqslant n-1; \quad (\text{Lawler, 1977}), \tag{1}$$

$$\sum_{i=1}^{k-1} p_k < d_k, \quad j+1 \leqslant k \leqslant n; \quad (\text{Potts and Van Wassenhove, 1982}), \tag{2}$$

$$\sum_{i=1}^{k} p_k < d_i + p_i \quad \text{for some } i, j+1 \leqslant i \leqslant k-1, \quad j+1 \leqslant k \leqslant n; \quad (\text{Szwarc, 1993}). \tag{3}$$

Della Croce et al. (1998) proposed a variant of Lawler's (1977) decomposition theorem for the job with the minimum due date resulting in a double decomposition theorem in which candidate positions are specified for both the longest job and the minimum due date job.

The most significant theoretical development since Lawler's (1977) decomposition theorem is the derivation of alternative decomposition conditions by Chang et al. (1995) who replaced Lawler's (1977) rightmost assumption with the following leftmost assumption:

Leftmost assumption: The longest job j is completed as early as possible in an optimal sequence.

Let EDD(j,k)be the modified EDD sequence in which the longest job j is moved to position k $(j < k \leqslant n)$, jobs $\langle j+1, \ldots, k\rangle$ are moved one position earlier to positions $\langle j, \ldots, k-1\rangle$ respectively, and the remaining jobs retain their position in the EDD sequence. Let $TT(j,k)$ be the total tardiness of the EDD(j,k) sequence. Then, according to Chang et al. (1995), the problem does not decompose with job j in position k (under the leftmost assumption) if any of the following conditions hold:

$$TT(j,i) \leqslant TT(j,k) \quad \text{for some } j+1 \leqslant k \leqslant n; \tag{4}$$

$$TT(j,k+1) < TT(j,k), \quad j \leqslant k \leqslant n-1; \tag{5}$$

Chang et al. (1995) stated that condition (5) is equivalent to $\sum_{i=1}^{k} p_k > d_{k+1}$, which is condition (1) expressed as a strong inequality; Chang et al. (1995) also stated that condition (4) for $i = k-1$, that is condition $TT(j,k-1) \leqslant TT(j,k)$ is equivalent to $\sum_{i=1}^{k-1} p_k \leqslant d_k$, which is condition (2) expressed as a weak inequality. Conditions (4) and (5) are more powerful than conditions (1) and (2) because they contain more inequalities. Szwarc et al. (1999) showed by counterexample that condition (3) is not subsumed by conditions (4) and (5) and subsequently Szwarc (2007) restated condition (3) as a weak inequality under the leftmost assumption. Szwarc (2007) also restated and proved Lawler's (1977) decomposition theorem under the leftmost assumption. Therefore, it is possible to obtain conditions similar to inequalities (4) and (5) under the rightmost assumption. These conditions will state that the $1//\bar{T}$ problem does not decompose with job j in position k (under the rightmost assumption) if any of the following inequalities hold:

$$TT(j,i) < TT(j,k) \quad \text{for some } j+1 \leqslant k \leqslant n; \tag{6}$$

$$TT(j,k+1) \leqslant TT(j,k), \quad j \leqslant k \leqslant n-1; \tag{7}$$

If some of the conditions (4)–(7) are satisfied as equalities, then the leftmost and rightmost assumptions yield different optimal solutions. As an example, consider the problem instance in Szwarc (2007) with $n = 3$, $p_{\langle 1,2,3\rangle} = \langle 10, 9, 8\rangle$ and $d_{\langle 1,2,3\rangle} = \langle 4, 10, 20\rangle$ in which $j = 1$ and $TT(1,1) = TT(1,2) = 22$. Under the leftmost assumption, conditions (4) and (5) yields as the optimal sequence with $T_{\langle 1,2,3\rangle} = 22$. Under the rightmost assumption, conditions (6) and (7) yields $\langle 2, 1, 3\rangle$ as the optimal sequence with $T_{\langle 2,1,3\rangle} = 22$. On the other hand, if conditions (4)–(7) are all satisfied as inequalities, then the leftmost and rightmost assumptions yield the same optimal solution. This observation can be used to obtain an informal proof of the validity of conditions (6) and (7) (under the rightmost assumption) by contradiction. Assume that conditions (6) and (7) are not valid. Then, consider a problem instance with input data such that conditions (4) and (5) are all satisfied as inequalities under the leftmost assumption. In that case, conditions (6) and (7) are also satisfied as inequalities under the rightmost assumption and coincide with the corresponding (4) and (5) inequalities. Consequently, the validity of conditions (4) and (5) under the leftmost assumption (proved by Chang et al., 1995) leads to a contradiction if one assumes the non-validity of conditions (6) and (7) under the rightmost assumption.

The findings of this section are summarized in Table 1. In summary, it can be stated that the above theoretical developments provide conditions for the position of the longest job j in an optimal solution (under either the leftmost or the rightmost assumption) effectively decomposing the problem into two mutually exclusive sub-problems for each eligible position for the longest job j. This in turn leads to the development of optimal algorithms if all eligible positions for the longest job j are considered and to the development of heuristic algorithms if some (but not all) positions for the longest job j are considered. The detailed design of such algorithms is presented in Sections 3 and 5 respectively.

We close this section by clarifying a statement in Du and Leung (1990) regarding the symmetry between the $1//\bar{T}$ and the total earliness $1//\bar{E}$ problem in which the earliness of a job j is defined as $E_j = \max(0, d_j - C_j)$ and the objective is to minimize the total earliness $E = \sum_{j=1}^{n} E_j$. Du and Leung's statement is valid for the $1/nmit/\bar{E}$ problem (with no inserted idle time) but not in general for the $1//\bar{E}$ problem.

Table 1
Decomposition conditions for the $1//\bar{T}$ problem.

Decomposition assumption	Reference	Inequality number[a]
Rightmost	Lawler (1977)	(1)
Rightmost	Potts and Van Wassenhove (1982)	(2)
Rightmost	Szwarc (1993)	(3)
Leftmost	Chang et al. (1995)	(4) and (5)
Leftmost	Szwarc (2007)	(3) expressed as a weak inequality
Rightmost	This paper	(6) and (7)

[a] Indicates the inequality number in the paper corresponding to the derived condition.

Table 2
A summary of the exact BB algorithms.

Algorithm	Use of lower bound	Use of induced due dates	Max problem size solved (n value)
Tansel et al. (2001)	Yes	Yes	300[a]
Szwarc et al. (2001)	No	Yes	500
Szwarc et al. (1999)	Yes	Yes	150
Hirakawa (1999)	No	No	100
Della Croce et al. (1998)	Yes	No	150
Szwarc and Mukhopadhyay (1996)	Yes	No	150
Kondakci et al. (1994)	Yes	No	35

[a] 400-job and 500-job problems were also solved with 95% and 87.5% success rates respectively.

3. Exact algorithms

The majority of the exact algorithms for the $1//\bar{T}$ problem included in the survey of Koulamas (1994) utilize either dynamic programming (DP), branch and bound (BB) or a hybrid DP/BB approach. The most effective of these algorithms is the BB algorithm of Potts and Van Wassenhove (P–W, 1982) capable of solving problems with up to 100 jobs. The P–W algorithm decomposes the problem until the generated sub-problems are small enough so they can be solved by dynamic programming and it does not utilize any lower bounds.

In contrast, almost all of the next generation BB algorithms (developed after 1994) continue the decomposition until trivial sub-problems (with one or two jobs) are obtained. The majority of these BB algorithms utilize lower bounds to prune partial solutions. The lower bounds used are based on Chu's (1992) lower bound computed as $LB = \sum_{i=1}^{n} \max(C_{[i]} - d_i)$ where $C_{[i]}$ are the job completion times for the shortest processing time (SPT) sequence and d_i are the job due dates arranged in the non-decreasing EDD order. One should exercise care when implementing this lower bound to a partial solution as Biskup and Piewitt (2000) point out in their comments about the BB algorithm of Kondakci et al. (1994). Szwarc et al. (2001) observed that the deletion of this lower bound improves the performance of their BB algorithm. This is because most of the sub-problems pruned by the lower bound appear later in other branches of the tree and must be solved anyway. Hirakawa (1999) does not use a lower bound in his BB algorithm.

Another popular feature among many of these BB algorithms (Szwarc et al., 1999; Szwarc et al., 2001; Tansel et al., 2001) is the utilization of induced job due dates. The induced due dates are computed after certain jobs have been shown to precede/follow other jobs as a result of Emmons' dominance conditions and/or the decomposition theorem.

Almost all of these BB algorithms use the latest theoretical developments available at the time of their development. Tansel et al. (2001) incorporate one of Chang et al.'s (1995) findings that the $1//\bar{T}$ problem decomposes unconditionally when the minimum $TT(j,k)$ value for all $j \leqslant k \leqslant n$ occurs for some $k \leqslant n-1$ (the key position value).

Della Croce et al. (1998) proposed a BB algorithm utilizing their double decomposition theorem mentioned in the previous section. Szwarc et al. (2001) observed that the addition of this double decomposition rule may actually worsen the performance of a BB algorithm.

Table 2 presents a summary of the BB algorithms reviewed in this section. It should be noted that the most effective algorithms listed in Table 2 implement the decomposition principle and/or Emmons' dominance conditions intelligently so that just one branch emanates from the active node whenever possible. The "split" rule of Szwarc and Mukhopadhyay (1996) is an example of such an implementation.

In summary, the best exact algorithms for solving the $1//\bar{T}$ problem are BB algorithms utilizing the latest developments of the decomposition theorem outlined in the previous section. The performance of these algorithms can be enhanced by utilizing induced due dates computed after certain jobs have been shown to precede/follow other jobs. The branching algorithm of Szwarc et al. (2001) is an example of such an algorithm and can handle problems with up to 500 jobs.

4. Fully polynomial time approximation schemes

It is well known that dynamic programming (DP) formulations normally extend to fully polynomial time approximation schemes (FPTAS). Woeginger (2000) derives the necessary conditions to guarantee the existence of a FPTAS. According to Woeginger, there are two standard approaches to extend a DP formulation to a FPTAS. The first approach rounds the input data of the problem instance; the second approach iteratively "thins" out the state space of the DP in order to collapse states that are close to each other and therefore reduce the size of the state space to polynomial.

In the case of the $1//\bar{T}$ problem, Lawler (1982) presented a FPTAS by utilizing the rounding approach. Lawler (1982) implemented his DP algorithm with rescaled (rounded) job processing and rescaled (non-rounded) due dates using a scale factor proportional to the desired level of approximation ε. The resulting algorithm has an $O(\frac{n^6 UB}{\varepsilon LB})$ running time and supplies an $(1+\varepsilon)$ approximate solution where LB, UB are a lower bound and an upper bound respectively for the optimal T^*value. Using $LB = T_{\max}$ and $UB = nT_{\max}$, (where $T_{\max}$ is the maximum tardiness value of the EDD sequence), Lawler (1982) obtained an $(1+\varepsilon)$ approximate solution in $O(\frac{n^7}{\varepsilon})$ time.

Koulamas (2009) suggests an alternative rounding procedure with rescaled (non-rounded) job processing and rescaled (rounded) due dates which leads to a faster FPTAS running in $O(\frac{n^5 UB}{\varepsilon LB})$ time or in $O(\frac{n^6}{\varepsilon})$ time when $LB = T_{\max}$ and $UB = nT_{\max}$ are used. The computational savings stem from the observation that a rounded processing time influences the job's completion time and therefore potentially the tardiness of all jobs while a rounded due date influences only the tardiness of the job in question; the former approach impacts the running time of the FPTAS by an $O(n^2)$ factor while the latter approach impacts the running time of the FPTAS only by an $O(n)$ factor.

Kovalyov (1995) proposed a bound improvement procedure which can by applied to a set of LB, UB values in conjunction with an appropriate algorithm yielding a new set of bounds, LB^* and UB^* respectively, such that $UB^* = 3LB^*$. Kovalyov (1995) implemented his bound improvement procedure with the $LB = T_{\max}$ and $UB = nT_{\max}$ bounds (assuming that $n > 3$) in conjunction with Lawler's (1982) FPTAS and improved the running time of the FPTAS to $O(n^6 \log n + \frac{n^6}{\varepsilon})$. Koulamas (2009) implemented Kovalyov's (1995) bound improvement procedure in conjunction with his alternative

Table 3
A summary of the FPTAS for the $1//\bar{T}$ problem.

FPTAS	Running time
Lawler (1982)	$O(\frac{n^7}{\varepsilon})$
Lawler (1982) + Kovalyov (1995)	$O(n^6 \log n + \frac{n^6}{\varepsilon})$
Koulamas (2009)	$O(\frac{n^6}{\varepsilon})$
Koulamas (2009) + Kovalyov (1995)	$O(n^5 \log n + \frac{n^5}{\varepsilon})$

rounding scheme resulting in an even faster FPTAS running in $O(n^5 \log n + \frac{n^5}{\varepsilon})$ time. The above results are summarized in Table 3.

It is clear from Table 3 that the fastest FPTAS for the $1//\bar{T}$ problem can be obtained by implementing Lawler's (1977) pseudo-polynomial algorithm utilizing Koulamas's (2009) rounding procedure with rescaled (non-rounded) job processing and rescaled (rounded) due dates in conjunction with Kovalyov's (1995) bound improvement procedure.

5. Approximation algorithms

The complexity status of the $1//\bar{T}$ problem justified the development of fast approximation algorithms to solve it. These approximation algorithms can be broadly classified into construction, local search, decomposition-based and meta-heuristic algorithms. Construction heuristics build up the schedule one job at a time using some type of dispatching rule while local search heuristics attempt to improve an existing schedule by pair-wise job interchanges. A decomposition-based heuristic (DEC/H) utilizes the decomposition theorem in a heuristic fashion. It employs a simple construction heuristic H to compute the total tardiness of the two sub-problems generated for each eligible position k for the longest job j. The minimum total tardiness position k is selected and decomposition continues until trivially solvable one-job/two-job sub-problems are generated. The meta-heuristics employ generalized search techniques such as simulated annealing, ant colony optimization (ACO), neural networks and genetic algorithms.

The evaluation of these heuristic algorithms has been mainly experimental with very few analytical results on their worst-case performance bounds. Comprehensive experimental results were presented in Potts and Van Wassenhove (1991) and in Koulamas (1994). The only analytical results until 1994 are the ones by Chang et al. (1990) on the performance of local search heuristics and their extension in Koulamas (1994). The more recent survey of Sen et al. (2003) contains neither experimental nor analytical results for the more recent heuristic algorithms surveyed in their paper. We present such results in the remainder of this section.

5.1. Analytical results

Very few worst-case ratio bounds (w.c.r.b) have been derived for heuristic algorithms for the $1//\bar{T}$ problem. For ease of exposition, let $r_H = \frac{T_H}{T^*}$ denote the w.c.r.b. of a heuristic algorithm H, where T_H, T^* denote the total tardiness of the heuristic and the optimal solution respectively. It is possible that r_H may not be defined when $T^* = 0$, therefore it is assumed that the EDD order yields a sequence with $T_{\max} > 0$ which is a lower bound to the optimal T^* value (Lawler, 1982).

Lawler (1982) showed that $r_{\mathrm{EDD}} \leqslant n$ and Della Croce et al. (2004) proved that this bound is tight. One can follow the approach of Cheng et al. (2005) for the $1//\bar{T}_w$ problem and introduce the induced due dates $d'_j = \max\{p_j, d_j\}$ for all $j = 1, \ldots, n$; let EDD′ denote the earliest induced-due-date sequence. Cheng et al. (2005) proved the tight w.c.r.b. of $r_{\mathrm{EDD'}} \leqslant n - 1$.

Yu's (1996b, pp. 109–111) showed that $r_{\mathrm{DEC/EDD}} \leqslant n - 1$ where DEC/EDD is the decomposition heuristic utilizing the EDD heuristic to sequence the jobs in the generated sub-problems. If DEC/EDD is implemented without the refinements to the decomposition theorem, then the above bound is shown to be tight (Yu, 1996b). The w.c.r.b for the DEC/EDD heuristic can be improved by utilizing the induced due dates $d'_j = \max\{p_j, d_j\}$ in place of the original ones; then, one can prove the w.c.r.b. of $r_{\mathrm{DEC/EDD'}} \leqslant n - 2$ by substituting the inequality $r_{\mathrm{EDD'}} \leqslant n - 1$ in the place of the inequality $r_{\mathrm{EDD}} \leqslant n$ in the proof of the w.c.r.b. of the DEC/EDD heuristic.

The only other available w.c.r.b. reported in the literature is $r_{\mathrm{WI}} \leqslant \frac{n}{2}$ for the WI heuristic of Wilkerson and Irwin (1971). This w.c.r.b. is reported on p. 114 in Yu (1996b) citing a report in Chinese which is not available in the open literature. If this bound is assumed valid, then it is also tight (Della Croce et al., 2004). Cheng (1992), Alidaee and Gopalan (1997) showed that the WI heuristic, the PSK heuristic of Panwalkar et al. (1993) and the modified due date (MDD) heuristic of Baker and Bertrand (1982) are all equivalent because they differ only in their tie-breaking rules. Using these findings, Della Croce et al. (2004) extend their $r_{\mathrm{MDD}} \geqslant \frac{n}{2}$ lower bound to the $r_{\mathrm{PSK}} = r_{\mathrm{WI}} \geqslant \frac{n}{2}$ lower bounds; Della Croce et al. (2004) also showed that $r_{\mathrm{COVERT}} \geqslant \frac{n}{2}$ and that $r_{\mathrm{NBR}} \geqslant \frac{n}{6}$ where COVERT is Carroll's (1965) construction heuristic and NBR is the net benefit of relocation heuristic of Holsenback and Russell (1992).

The MDD, PSK and WI heuristics can be embedded in the decomposition heuristic yielding the DEC/MDD, DEC/PSK and DEC/WI decomposition heuristics respectively. If the $r_{\mathrm{WI}} \leqslant \frac{n}{2}$ w.c.r.b. is assumed valid, then it can be easily shown that $r_{\mathrm{DEC/MDD}} = r_{\mathrm{DEC/PSK}} = r_{\mathrm{DEC/WI}} \leqslant \frac{n-1}{2}$ (Della Croce et al., 2004). Della Croce et al. (2004) also proved that $r_{\mathrm{DEC/MDD}} = r_{\mathrm{DEC/PSK}} = r_{\mathrm{DEC/WI}} \geqslant \frac{n}{3}$. One may conjecture that these lower bounds are a consequence of the $r_{\mathrm{MDD}} = r_{\mathrm{PSK}} = r_{\mathrm{WI}} \geqslant \frac{n}{2}$ lower bounds. This conjecture motivates the consideration of an alternative DEC/LB decomposition heuristic in which the position k for the longest job j is selected by computing a lower bound on the total tardiness for each of the generated sub-problems. We analyzed the DEC/LB decomposition heuristic using Chu's (1992) lower bound and observed that the $r_{\mathrm{DEC/LB}} \geqslant \frac{n}{3}$ lower bound holds. The above findings are summarized in Table 4 (except for the ones depending on the $r_{\mathrm{WI}} \leqslant \frac{n}{2}$ w.c.r.b. which cannot be independently validated).

5.2. Experimental results

Despite the somehow disappointing lower bounds on the w.c.r.b. of the decomposition heuristics, they perform relatively well experimentally as demonstrated by Potts and Van Wassenhove (1991), Koulamas (1994). This observation motivated the experimentation with additional decomposition-based heuristics. Tansel et al. (2001) proposed a decomposition-based heuristic in which induced due dates are utilized and the leftmost position k for the longest job j is always selected. Naidu et al. (2002) observed that the MDD heuristic has a decomposition structure and proposed a decomposition heuristic in which the position k for the longest job j is selected based on reducible tardiness; in the case of multiple eligible k values, they select the leftmost one. Tansel et al.

Table 4
Worst-case ratio bounds of heuristic algorithms for the $1//\bar{T}$ problem.

Algorithm	Reference[a]	Lower bound	Upper bound
EDD	Lawler (1982), Della Croce et al. (2004)	n	n
EDD′	Cheng et al. (2005)	$n-1$	$n-1$
DEC/EDD	Yu (1996b)	$n-1$	$n-1$
DEC/EDD′	This paper		$n-2$
MDD, PSK, WI, COVERT	Della Croce et al. (2004)	$\frac{n}{2}$	
NBR	Della Croce et al. (2004)	$\frac{n}{6}$	
DEC/MDD, DEC/PSK, DEC/WI	Della Croce et al. (2004)	$\frac{n}{3}$	
DEC/LB	This paper	$\frac{n}{3}$	

[a] Indicates the reference in which the bound was obtained.

(2001), Naidu et al. (2002) do not compare their decomposition heuristics directly against other decomposition heuristics and, based on their reported results, there is no significant deviation from earlier assessments on the experimental performance of decomposition heuristics.

Russell and Holsenback (1997a) compared the PSK and NBR heuristics and concluded that, in general, NBR performs better than PSK even though in some problem instances (depending on the range of the due date values) PSK performs better than NBR. A modified version of the NBR heuristic (the M-NBR heuristic) developed by Russell and Holsenback (1997b) improves on the performance of the NBR heuristic at no additional computational effort. Since PSK and NBR have modest running times, it is reasonable to combine them into a composite heuristic to obtain the better of the two solutions. More recently, Fadlalla et al. (2004) developed a construction heuristic in which the sequence is built-up recursively starting with the last position. Panneerselvam (2006) developed a construction heuristic in which the jobs are added to the sequence according to the minimum slack per processing time rule.

We now turn our attention to the implementation of meta-heuristics for the $1//\bar{T}$ problem. In general, meta-heuristics are more suitable to scheduling problems with less structure than the $1//\bar{T}$ problem such as the $1//\bar{T}_w$ problem. This is because the existence of the decomposition theorem enables the development of the decomposition heuristics for the $1//\bar{T}$ problem which perform as well as the meta-heuristics with smaller computational requirements. Among the meta-heuristics, simulated annealing seems to be the most popular one for the $1//\bar{T}$ problem; it was implemented by Potts and Van Wassenhove (1991), Antony and Koulamas (1996), Ben-Daya and Al-Fawzan (1996); all of these researchers observed experimentally that, if simulated annealing is allowed to run long enough with proper fine-tuning of its parameters, then it produces solutions very close to the optimal one. This is because of the existence of theoretical results specifying conditions under which simulated annealing solutions converge to optimal ones with probability one. However, these conditions usually imply very long running times.

The ant colony optimization (ACO) meta-heuristic was applied to the $1//\bar{T}$ problem by Bauer et al. (1999), Cheng et al. (2009) proposed a hybrid extension of the ACO heuristic in which the decomposition theorem is incorporated into the ACO meta-heuristic. A neural network model was applied to the $1//\bar{T}$ problem by Sabuncuoglu and Gurgun (1996). Bean (1994) implemented a genetic algorithm to the $P//\bar{T}$ (the identical parallel machine generalization of the $1//\bar{T}$ problem). All these meta-heuristics can supply near optimal solutions if they are allowed to run long enough.

It is well known that meta-heuristics outperform local search heuristics due to the inability of local search algorithms to "shoot out" of local optima. One way to alleviate this problem is to explore a neighborhood of exponential size in polynomial time using dynamic programming. This approach was used by Congram et al. (2002) in their iterated dynasearch algorithm and was shown to outperform a tabu search algorithm for the weighted total tardiness $1//\bar{T}_w$ problem. Based on these findings, it is expected that Congram's et al. algorithm will outperform certain meta-heuristic algorithms if applied to the $1//\bar{T}$ problem.

We close this section by concluding that the structure of the $1//\bar{T}$ problem enables the development of problem-specific heuristic algorithms such as decomposition-based heuristics. Even though these heuristics may have disappointing w.c.r.b., they perform well in practice (and comparable to meta-heuristics) with small computational requirements. Therefore, from a solution quality/ computational time tradeoff standpoint, a decomposition heuristic utilizing the composite PSK/NBR heuristic to solve the generated sub-problems can be considered a very good heuristic algorithm for the $1//\bar{T}$ problem.

6. Special cases of the $1//\bar{T}$ problem

The complexity status of the $1//\bar{T}$ problem motivated researchers to identify special cases which can be solved in polynomial time by restricting either the values or the range of the job parameters. Emmons (1969) derived the necessary conditions on the p_j, d_j values so that either the SPT or the EDD sequence is optimal. Emmons (1969) also derived the necessary conditions for a certain job to be either first or last in an optimal sequence. Koulamas (1997) proposed another polynomially solvable $1//\bar{T}$ case by utilizing the observation that, according to the decomposition theorem, two adjacent positions k are never considered for scheduling the longest job j.

The alternative approach of restricting the number of distinct d_j values was followed by Tian et al. (2005); they considered the case of m distinct d_j values and proposed an $O(m^{4-m}n^m)$ algorithm by modifying Lawler's (1977) algorithm. In the case of just two distinct d_j values, the running time of their algorithm is further reduced to $O(n \log n)$.

It should be pointed out that certain restrictions on the p_j, d_j values do not make the problem much easier. For example, the $1//\bar{T}$ problem with opposing p_j, d_j values (in which $p_1 \geqslant \ldots \geqslant p_n$ and $d_1 \leqslant \ldots \leqslant d_n$) is ordinary NP-hard (Gafarov and Lazarev, 2006). In that case, Lazarev (2007) proposes an $O(n^2P)$ time algorithm; he also identifies additional conditions on the range of due dates which make the problem polynomially solvable. Using the approach of Koulamas (2009) (in conjunction with Kovalyov's bound improvement procedure) a FPTAS can be developed for Lazarev's (2007) algorithm running in $O(n^3 \log n + \frac{n^3}{\varepsilon})$ time.

We close this section be pointing out that a natural way to induce the solvability of the $1//\bar{T}$ problem in polynomial time is to impose conditions on the problem parameters so that the SPT and EDD sequences coincide.

7. Generalizations of the $1//\bar{T}$ problem

The complexity status of the $1//\bar{T}$ problem implies that any generalization will lead to a problem which is at least as hard or even harder. For example, the $1//\bar{T}_w$ problem with arbitrary general weights is strongly NP-hard (Lawler, 1977). There is a stream of research focused on identifying generalizations of the $1//\bar{T}$ problem which can be solved by algorithms having the same computational requirements as the one for the $1//\bar{T}$ problem. The objective of this section is to survey such papers published after the survey of Koulamas (1994). We do not consider the $1//\bar{T}_w$ problem because it is not the focus of this survey. The reader interested in the $1//\bar{T}_w$ problem is directed to the surveys of Abdul-Razaq et al. (1990), Sen et al. (2003).

The $1/r/\bar{T}$ problem (with arbitrary job release times r_j) is known to be strongly NP-hard (Rinnooy Kan, 1976); actually, Rinnooy Kan's (1976, p. 83), Lenstra et al. (1977) proved that the $1/r/\bar{C}$ problem is strongly NP-hard which implies that the $1/r/\bar{T}$ problem is also strongly NP-hard. Koulamas and Kyparisis (2001) showed that even the $1/(r,d)/\bar{T}$ problem (with agreeable release times and due dates in which if $r_i \leqslant r_j$, then $d_i \leqslant d_j$ for any two jobs i,j) is strongly NP-hard. Koulamas and Kyparisis (2001) also showed that an optimal $1/r/\bar{T}$ schedule need not be a non-preemptive one.

The $1/\bar{d}/\bar{T}$ problem (with arbitrary deadlines $\bar{d}_j$) was thoroughly studied by Tadei et al. (2002). It is assumed that $\bar{d}_j \geqslant d_j$ for all jobs $j = 1, \ldots, n$ and that the earliest deadline (EDL) sequence supplies a feasible schedule. Tadei et al. (2002) also considered the $1/(d,\bar{d})/\bar{T}$ problem (with agreeable due dates d_j and deadlines $\bar{d}_j$ in which if $d_i \leqslant d_j$, then $\bar{d}_i \leqslant \bar{d}_j$ for any two jobs i,j). They concluded that the $1/(d,\bar{d})/\bar{T}$ problem cannot be solved by a pseudo-polynomial algorithm similar to Lawler's (1977) algorithm. Tadei et al. (2002)

showed that the $1/(p, \bar{d})/\bar{T}$ problem (with agreeable processing times and deadlines in which $\bar{d}_i \leqslant \bar{d}_j$ if $p_i \leqslant p_j$ for any two jobs i, j) can be solved in $O(n^4P)$ time by Lawler's (1977) algorithm and stated that a FPTAS can be developed in $O(\frac{n^7}{\varepsilon})$ time.

Tadei et al. (2002) also showed that the $1/(\bar{d}, d)/\bar{T}$ problem (with opposing $d_j, \bar{d}_j$ values in which if $d_i > d_j$, then $\bar{d}_i < \bar{d}_j$ for any two jobs i, j) can be solved in $O(n^6P)$ time by a variant of Lawler's (1977) algorithm and stated that a FPTAS can be developed in $O(\frac{n^9}{\varepsilon})$ time.

The FPTAS for the $1//\bar{T}$ problem utilizes the EDD sequence to obtain a lower bound on the optimal T^* value. It is easy to show by counterexample that the EDD sequence may be infeasible for the $1/(p, \bar{d})/\bar{T}$ and $1/(\bar{d}, d)/\bar{T}$ problems; e.g., consider a two-job problem instance with $p_{\langle 1,2\rangle} = \langle 4, 6\rangle, d_{\langle 1,2,3\rangle} = \langle 4, 10, 20\rangle$ and $\bar{d}_{\langle 1,2\rangle} = \langle 9, 11\rangle$. Consequently, the $O(n \log n)$ algorithm of Koulamas and Kyparisis (2001) for solving the single-machine maximum tardiness $1/\bar{d}/T_{\max}$ problem with deadlines should be utilized to supply both a lower bound (T^*_{LB}) and an upper bound (nT^*_{LB}) on the optimal T^* value. These bounds should be used in conjunction with Koulamas' (2009) alternative rounding scheme and Kovalyov's (1995) bound improvement procedure to develop FPTAS for the $1/(p, \bar{d})/\bar{T}$ and the $1/(\bar{d}, d)/\bar{T}$ problems running in $O(n^5 \log n + \frac{n^5}{\varepsilon})$ and $O(n^7 \log n + \frac{n^7}{\varepsilon})$ time, respectively.

We close this section by mentioning that Tadei et al. (2002) showed that the $1/\bar{d}/\bar{T}$ problem can be solved in $O(n \log n)$ time when the SPT sequence coincides with the EDD sequence. This finding also implies the solvability of the problem when $p_j = p$ ($j = 1, \ldots, n$). In that case, any sequence is an SPT sequence and therefore the EDD sequence coincides with the SPT sequence yielding the $1/\bar{d}/\bar{T}$ problem solvable in $O(n \log n)$ time when $p_j = p$ ($j = 1, \ldots, n$). The above result indicates that in the presence of additional parameters (such as deadlines) the $1//\bar{T}$ problem becomes harder. One way to simplify it is to require either equal job processing times or equal job due dates.

8. The $1//\bar{T}$ problem with assignable due dates

One way to simplify machine scheduling problems with due date-related objectives is to treat the due dates as decision variables to be determined in conjunction with an optimal sequence. This approach was proposed by Panwalkar et al., 1982 (in the context of a common due date) and Hall, 1986 (in the context of generalized due dates). Shabtay and Steiner (2006) considered assignable due dates with the weighted number of tardy jobs objective; their approach can be extended to the $1//\bar{T}$ problem as shown next.

Let c be the unit due date assignment cost; then, the total cost objective function can be written as $TC = c\sum_{i=1}^{n} d_{[i]} + \sum_{j=1}^{n} \max(0, C_{[i]} - d_{[i]})$ where $[i]$ denotes the i job in the sequence. If job i is non-tardy, then $d_{[i]} \geqslant C_{[i]}$. It is easy to observe that if $c \leqslant 1$, then all jobs should be non-tardy with minimal total due date assignment costs. This can be accomplished by sequencing the jobs in the SPT order with $d_{[i]} = C_{[i]}$ for all $i = 1, \ldots, n$. On the other hand, if $c > 1$, then all jobs should be tardy with minimal total tardiness and with the shortest possible due date values. This can be accomplished by sequencing the jobs in the SPT order with $d_{[i]} = 0$ for all $i = 1, \ldots, n$.

Shabtay and Steiner (2007) also considered due date assignment/sequencing problems in conjunction with resource allocation decisions. They concluded that the resource allocation problem and the due date assignment/sequencing problem can be solved sequentially. It is easy to observe that this sequential approach applies to the $1//\bar{T}$ problem as well.

We close this section by mentioning that Qi et al. (2002) considered a different version of the $1//\bar{T}$ problem with assignable due dates in which n distinct due date values are given to be assigned to the n jobs by the scheduler. Qi et al. (2002) concluded that the earliest due date should be assigned to the shortest job and so on; the resulting SPT/EDD sequence is known to be optimal for the $1//\bar{T}$ problem (Emmons, 1969).

The above findings demonstrate that one way to simplify a hard problem is to treat some of its parameters as decision variables to be determined. As an example, the ordinary NP-hard $1//\bar{T}$ problem becomes polynomially solvable when the due dates are treated as decision variables.

9. Conclusions

We surveyed the recent developments for the $1//\bar{T}$ problem. Our findings show that the $1//\bar{T}$ problem continues to attract significant research interest. Even though the problem is ordinary NP-hard, the current state-of-the-art algorithms are capable of solving problems with up to 500 jobs. In addition to reviewing the recent developments for the $1//\bar{T}$ problem, we also presented the following extensions:

1. Additional conditions under the rightmost assumption for the decomposition theorem which are comparable to the corresponding ones under the leftmost assumption.
2. An improved worst-case ratio bound for the DEC/EDD heuristic by replacing the EDD sequence with the earliest induced due date EDD′ sequence.
3. An appropriate lower bound and improved running times for the FPTAS of the $1/(p, \bar{d})/\bar{T}$ and the $1/(\bar{d}, d)/\bar{T}$ problems.
4. Solutions for the $1//\bar{T}$ problem with assignable due dates.

Future research should focus on incorporating the recent theoretical findings on the design of both exact and approximation algorithms for the $1//\bar{T}$ problem. Exact algorithms should focus on utilizing these findings to compute stronger induced due dates. Approximation algorithms should focus on incorporating the $1//\bar{T}$ problem structure (especially its decomposition theorem) into general purpose meta-heuristics. Another avenue of future research is to embed the structure of the $1//\bar{T}$ problem into algorithms designed to solve more complex total tardiness problems such as the identical parallel machine total tardiness ($P//\bar{T}$) problem and/or the job shop total tardiness ($J//\bar{T}$) problem. In the case of the $P//\bar{T}$ problem, the additional theoretical findings for the $1//\bar{T}$ problem can be used to optimize more efficiently the sequence on each individual machine after the jobs have been allocated to the machines. In the case of the $J//\bar{T}$ problem, the theoretical developments for the $1//\bar{T}$ problem can be used to identify more efficiently which individual machine is a bottleneck for the overall problem; the sequence on the identified bottleneck machine can then be further improved using an effective solution algorithm for the $1//\bar{T}$ problem leading to an improved sequence for the overall $J//\bar{T}$ problem.

Finally, another line of future research should focus on imposing additional conditions on the problem parameters so that the $1//\bar{T}$ problem becomes polynomially solvable.

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