Latihan 2

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1. $T(n) = \dots ?$

   $T(0) = 0$

   $T(1) = 0$

   $T(2) = 1$

   $T(3) = 3$

Deret aritmatik, maka

$$\boxed{T(n) = \frac{n^2 - n}{2}}$$

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1. $T(n) = 5 = O(1)$.

   ↳ Ambil $\boxed{c = 5}$ & $\boxed{n_0 = 0}$ → maka

   $5 = 5 \cdot 1$

   $T(n) = 5 = O(1)$

   $= O(n^2)$

2. $T(n) = \frac{n(n-1)}{2} + n - 1 = O(n^2)$, untuk $n \geq 1$

$\hookrightarrow \frac{n(n-1)+n-1}{2} \rightarrow \frac{n}{2} \cdot \frac{n-1}{2} + (n-1) \leq \frac{n^2}{2} + \frac{n^2}{2} + \frac{n^2}{2}$

Maka $\frac{3n^2}{2} \rightarrow O(n^2)$ $\boxed{\begin{aligned} c &= 3 \\ n_0 &= 1 \end{aligned}}$

3. $T(n) = 6 \times 2^n + 2n^2 = O(2^n)$

$\hookrightarrow$ untuk $n \geq 1$, maka $2^n \geq n^2$ (kecuali $n = 3$)

$6 \times 2^n + 2n^2 \leq 6 \times 2^n + 2 \cdot 2^n = 8 \cdot 2^n$

$\boxed{C = 8 \ \& \ n_0 = 1}$

4. $T(n) = 1 + 2 + \ldots + n = O(n^2)$,

$\hookrightarrow 1 + 2 + \ldots + n \rightarrow \frac{n}{2}(1+n) \rightarrow \frac{n}{2} + \frac{n^2}{2}$

$\hookrightarrow$ untuk $n \geq 0$, $n^2 \geq n$, maka:

$\frac{n}{2} + \frac{n^2}{2} \leq \frac{n^2}{2} + \frac{n^2}{2} : O(n^2)$ dengan $\boxed{C = 1 \ \& \ n_0 = 0}$

5. $T(n) = n! = O(n^n)$

$\hookrightarrow$ untuk $n \geq 1$, $n \geq 1$, maka:

$1 \cdot 2 \cdot 3 \cdot \ldots \cdot n \leq n \cdot n \cdot n \cdot \ldots \cdot n$ ~~dengan~~ $= O(n^n)$

dengan $\boxed{C = 1 \ \& \ n_0 = 1}$

6. $T(n) = 1^k + 2^k + \ldots n^k = O(n^{k+1})$

$\hookrightarrow$ untuk $n \geq 1$, maka

$1^k + 2^k + \ldots n^k \leq n^k + n^k + \ldots n^k = n^k \cdot n = O(n^{k+1})$

dengan $\boxed{C = 1 \ \& \ n_0 = 1}$

7. $T(n) = 5 \log(3^n) = O(n)$

↳ untuk $n \geq 0$, $\log(3^n) \leq n$, maka

$5 \log(3^n) \leq 5n = O(n)$, dengan $\boxed{c = 5 \text{ dan } n_0 = 0}$

8. $T(n) = \log(n!) = O(n \log(n))$.

↳ untuk $n \geq 1$, $\log(n!) \leq n \log(n)$, maka

$\log(n!) \leq n \log n = O(n \log(n))$, dengan $\boxed{c = 1 \text{ \& } n_0 = 1}$