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Engineering 447 Control Systems — Homework #2

1. Find the system transfer function for the following system differential equation where $x(t)$ is the input to the system and $y(t)$ is the system output:

$$\frac{d^3y}{dt^3} + 5\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 6y = 2\frac{dx}{dt} + x$$

- Apply laplace transform

$$s^3 y(s) + 5s^2 y(s) + 10 s \cdot y(s) + 6y(s) = 2s\,x(s) + x(s)$$

$$y(s)(s^3 + 5s^2 + 10s + 6) = (2s+1)\,X(s)$$

$$\frac{y(s)}{X(s)} = \frac{2s+1}{s^3 + 5s^2 + 10s + 6} \longleftarrow \text{Transfer Function}$$

2. Given that the input $x(t)$ is a step function of magnitude 2 [$x = 2u(t)$], find the output $y(t)$ by finding the inverse Laplace transform of $Y(s)$ by the method of partial fraction expansion (by hand).

$$X(t) = 2u(t) \qquad \text{Let System} = \frac{1}{s+1} \quad \text{Assumption}$$

$$X(s) = \frac{2}{s}$$

$$2/s \longrightarrow \boxed{1/s+1} \longrightarrow y(s)$$

$$y(s) = \frac{2}{s(s+1)} \Longrightarrow \frac{2}{s(s+1)} = \frac{2}{s} - \frac{2}{s+1}$$

$$y(s) = \frac{2}{s} - \frac{2}{s+1} \qquad y(t) = [2 - 2e^{-t}]\,u(t)$$

3. Use symbolic MATLAB, as explained in Handout 3A, to:
   a. Check the partial fraction expansion terms from your hand calculations in problems 2 above.
   b. Find $y(t)$ by the `ilaplace` command and compare to your answers from problems 2.

(a)

```
b=[4 2];
a=[1 5 10 6 0];
[r,p,k]=residue(b,a)
r=
  -0.500+0.4714i
  -0.500-0.4714i
   0.6667
   0.3333
p= -2.0+1.4142i
   -2.0-1.4142i
   -1.0000
    0
k=[]
```

(b)

```
syms a s b
a=0;
b=2;
F=2/s;
ilaplace(F,b)

ans=2
```

```
syms a s b
a=-1;
b=2;
F=2/(s+1);
ilaplace(F,b)

ans= 2*exp(-2)
```

4. Given the following Laplace transform of a function which has a delay time of 2 seconds:

$$G(s) = \frac{10e^{-2s}}{s(s+3)(s+2)^2}$$

a. Find the magnitude and angle (in degrees) of $G(j\omega) = |G(j\omega)|\angle G(j\omega)$ by MATLAB:

i. $s = 0.1j$

ii. $s = 1j$

iii. $s = 10j$

b. Do case (ii) using your calculator ($s = j$, only).

ⓐ

i.
```
G = (10*exp(-2*s))/(s*(s+3)*(s+2)^2)
s = 0.1*i;
abs(G)
angle(G)
```
⟹
```
G = -2.7176 - 7.8509i
ans = 8.3079
ans = -1.9040
```

ii.
```
G = (10*exp(-2*s))/(s*(s+3)*(s+2)^2)
s = 1i;
abs(G)
angle(G)
```
⟹
```
G = 0.0678 + 0.6288i
ans = 0.6325
ans = 1.4633
```

iii.
```
G = (10*exp(-2*s))/(s*(s+3)*(s+2)^2)
s = 10ji
abs(G)
angle(G)
```
⟹
```
G = 8.23x10^-4 - 4.12x10^-4 i
ans = 9.2099x10^-4
ans = -0.4642
```

ⓑ

$$G(s) = \frac{10e^{-2s}}{s(s+3)(s+2)^2}$$

$s = j$

$$\Longrightarrow \quad G(s) = \frac{10e^{-2j}}{j(j+3)(j+2)^2}$$

$$G(s) = \frac{10(-0.416 - j0.91)}{j(j+3)(3+j4)}$$

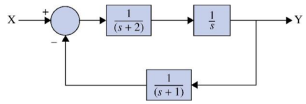
$$G(s) = \frac{-4.16 - j\,9.1}{-15 + j5}$$

$$G(s) = 0.0676 + j\,0.6292$$

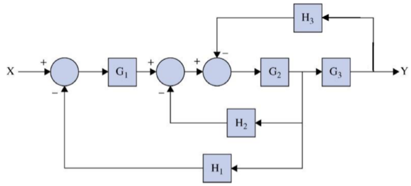
$$G(s) = 0.6328 \angle 1.4637 \text{ rad}$$

5. Reduce the block diagrams below to find the equivalent transfer function $G(s) = \frac{Y(s)}{X(s)}$.

a.



b.



(Part b Hint: The two adjacent summing junctions may be combined into one. Hint: Are all loops and the forward path touching?)

(a)

$$\frac{\frac{1}{s+2}\cdot\frac{1}{s}}{1+\left(\frac{1}{s+2}\right)\left(\frac{1}{s+1}\right)\left(\frac{1}{s}\right)} = \frac{1}{\cancel{s(s+2)}} \times \frac{\cancel{s}(s+1)\cancel{(s+2)}}{(s+2)(s+1)s+1}$$

$$= \frac{s}{(s^2+2s)(s+1)+1}$$

$$= s / s^3 + 3s^2 + 2s + 1$$

(b)

$$= \frac{G_1 G_2 G_3}{1-[(-G_2H_2) + (-G_1G_2H_1) - (-G_2G_3H_3)]} = \frac{G_1G_2G_3}{1 + G_2H_2 + G_1G_2H_1 + G_2G_3H_3}$$