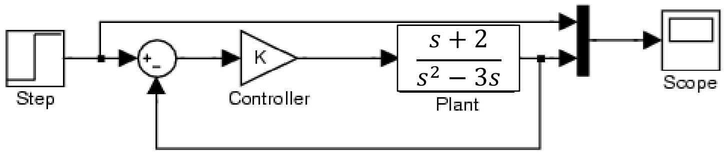
Two Simulink Experiments: Stabilizing an Unstable System and a Nonlinear System vs. Its Linearized Version

1. Stabilizing an Unstable System by Closed Loop Proportional Control:

Load Simulink by typing "simulink" at the MATLAB prompt. Once Simulink has loaded, create a new blank model by going to: File → New → Model (or alternatively press CTRL+N). Next, begin placing components in the empty window and construct system below. Note that "Transfer Fcn" is under "Continuous" and the multiplexer at the scope is under "Signal Routing" in the Simulink Library Browser window:



Task 1:
- Set $K = 1$ in the proportional controller (the gain block). Run the simulation. Double-click on the scope to see the output of the simulation. Is the output stable? Increase the gain to 2 and re-run the simulation. Continue increasing the gain to 10, and observe the results. Discuss your observations in the report.
- For what range of values of K is the system stable? For what value of K is the system limitedly stable? (A linear system with continuously growing oscillatory output is unstable; one with oscillatory output that neither grows nor dies out (like a pure sine wave) is **limitedly stable** (also called marginally stable).
- Present block diagram and your discussions in the report as how and why changing K affects system stability.

Task 2:
- Set the gain $K = 8$, and replace the step input with a ramp input. Re-run the simulation.
- The steady-state error is defined as the difference between the input and output signals when $t \rightarrow \infty$. Using the mouse, zoom in on the scope output at $t = 10$ seconds. Make an estimate of the steady state error of this system due to a ramp input. Present a plot in report.
- Research "System Type" and explain in your report as what it is. The above plant transfer function is of type 1 (why? Explain). What would you expect the steady state error of this system be to a step input?

Task 3:
- What is the closed-loop transfer function for this system (leave K as a variable)?
- What is the closed loop characteristic equation as a function o K?
- What are the locations of the closed loop poles as K varies?
- Also, evaluate the expression for the poles locations for any value of $K =$ "unstable", $K =$ "limitedly stable" and $K =$ "stable". Leave K as a variable in all of the above.

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2. Behavior of a Nonlinear System vs. Behavior of Its Linearized Version:

Nonlinear equation of motion for a damped pendulum is as follows:

$$\ddot{\theta} + \frac{c}{ml}\dot{\theta} + \frac{g}{l}\sin\theta = \frac{T_c}{ml^2}$$

$\theta =$ angular position of the pendulum with respect to the vertical.
$l =$ length of massless pendulum arm.
$m =$ mass of weight attached to end of arm.
$c =$ linear damping coefficient at pendulum hinge.
$T_c =$ input torque acting on pendulum at hinge.
$g = 9.81$ (m/s^2) acceleration of gravity.

Task 4:

- Construct a Simulink model for the above differential equation with input T_C and output θ in a new file. Use only integrators, a summing junction and gain blocks plus a "sin" block which you may find in the Simulink Library Browser under "Math Operations" → "trigonometric function". In the gain blocks define the gain values as functions of m, l, c, g; then, assign the following values in an m-file: $l = 2.5, m = 0.75, c = 0.15, g = 9.81$. Once the m-file is run, MATLAB would substitute for these values in the gain blocks. This helps avoid computational mistakes and provides an easy way to change system properties.
- Present your Simulink diagram in the report.

Task 5:

- Simulate for 50 seconds and provide a plot of the response of this system to an input T_C pulse having amplitude 20 (Nm) and width 0.1 seconds. Simulation time may be set by typing in the "Simulation Stop Time" box below the main menu (top right). There are several ways to generate the above pulse; one easy way is to use a "pulse generator" (under "Sources") with period = 100 and pulse width = 0.1 (the second pulse would never occur in the 50 second simulation). Double click on the pulse generator block; note that pulse width is set as a % of period; set the percentage correctly to have 0.1 seconds of pulse width.

Task 6:

- Replace the non-linear sine term with its linearized value which is the first order approximation of $\sin\theta$. Consider the Taylor expansion:

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots$$

Small-angle linear approximation: $\sin\theta \approx \theta$

- Simulate and provide a plot of the response of the linearized system to same pulse as in Task 5.
- Compare the plots in Tasks 5 and 6, is the linear approximation a good approximation? Discuss.

Task 7:

- Put the nonlinear sine block back into the simulation; increase pulse amplitude to 200 with same pulse width of 0.1 seconds. Simulate for 50 seconds and provide a plot of the system response. To what value is the output converging? Discuss what is happening.
- In this system, multiples of 2π encode to the same location due to the $\sin\theta$. Is there any difference between $\theta = 0$ and $\theta = 2\pi$? Is there any difference between a system trajectory that starts at $\theta = 0$ and goes to $\theta = 0$ and one that starts at $\theta = 0$ and goes to $\theta = 2\pi$? Explain.

Task 8:

- Replace sine block with the linear approximation and provide a plot of the linearized system response to the pulse in Task 7 (amplitude=200, width=0.1; simulate for 50 s). Compare Tasks 7 and 8 response plots.
- To what value is θ converging now? Why? Is the linear approximation a good approximation here?

Report Due Monday October 24, 11:59pm, Upload at iLearn in pdf

Concentrate on the "Tasks", provide answers to the questions and supply graphs and Simulink block diagrams. **Please make observations and try to explain them**. All figures and plots should be properly labeled and referenced.

Your report should include the following sections:

(1) Introduction.
(2) Problem definition.
(3) A brief explanation of the experiments (Tasks) followed by the results and discussions for each Task.
(4) Conclusions (state important observations here).
(5) List of references if any.
(6) Appendix (if you feel necessary).