Engineering 446

Control Systems Laboratory

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Lab Assignment #7
Inverted Pendulum

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Introduction

The goal of this laboratory assignment is to achieve simultaneous control of both the angular position of a pendulum and horizontal position of a car on the track using full state variable feedback. In this model this system will be linearized by assuming small angle perturbations. The model for the system will have a pendulum which is attached to a movable cart. In the system N and P will be the vertical and horizontal components. The values of various system parameters are listed in a table that is provided in the laboratory assignment. These values will be used to set up the system and model within Matlab and Simulink.

Problem Definition

We will be required to derive the equation of motion of the inverted pendulum cart system while showing the deviations. There is one way that we can consider to do this which is by using a free body diagram of the car and the pendulum separately and then writing their equations of motion. The user will be required to draw the separate free body diagram and then write the equations of motion for X and Y directions of the pendulum. The user will take note that the X component of the pendulum mass position consists of two parts. User user will differentiate twice the equations that are given in the laboratory excitement to find the X and Y components of the acceleration of the pendulum mass.

Explanation of the experiments

The goal of this lab is to ultimately present the work and the results as well as the plot of the six part procedure indicated laboratory assignment. It is always convenient to use matlab and Simulink to create a simple System using some injunctions Game an integrator blocks whose parameters are conveniently provided in this particular laboratory assignment.

In this laboratory assignment to user will achieve simultaneous control of both the angular position of the pendulum as well as the horizontal position of the car on the track using full state variable feedback. I will first derive the equations of the inverted pendulum-cart system. Next I will substitute the model parameters and obtain the state-space model for the complete system., Then I will determine the Eigen values of the state Matrix A, I will then find the system Reponses to a unit step input in the applied motor voltage, then I will use a full state variable feedback controller to achieve the desired performance specifications mentioned in the lab, lastly I will calculate N using the K matrix and find new A and B matrices for after the state feedback and use MATLAB to calculate the closed loop transfer function of cart position with respect to reference input r.

Models/Calculations/ Simulation Results

Part 1:

In the first section of this laboratory assignment we derive the equation of motion of the inverted pendulum cart system. The appendix provides a full derivation. After adding the equations of motion of the inverted pendulum car system to Matlab we get the following results:

$$\ddot{x} + 4/3\ Lp\theta - g\theta = 0$$

Part 2:

The numerical values of A,B,C,D matrices are found in this section...

A= 0 1.0000 0 0

0 -6.8312 -1.4967 0

0 0 0 1.0000

0 15.5161 25.6815 0

B= 0 1.5244 0 -3.4625

C= 1 0 0 0 0 0 1 0

D= 0 0

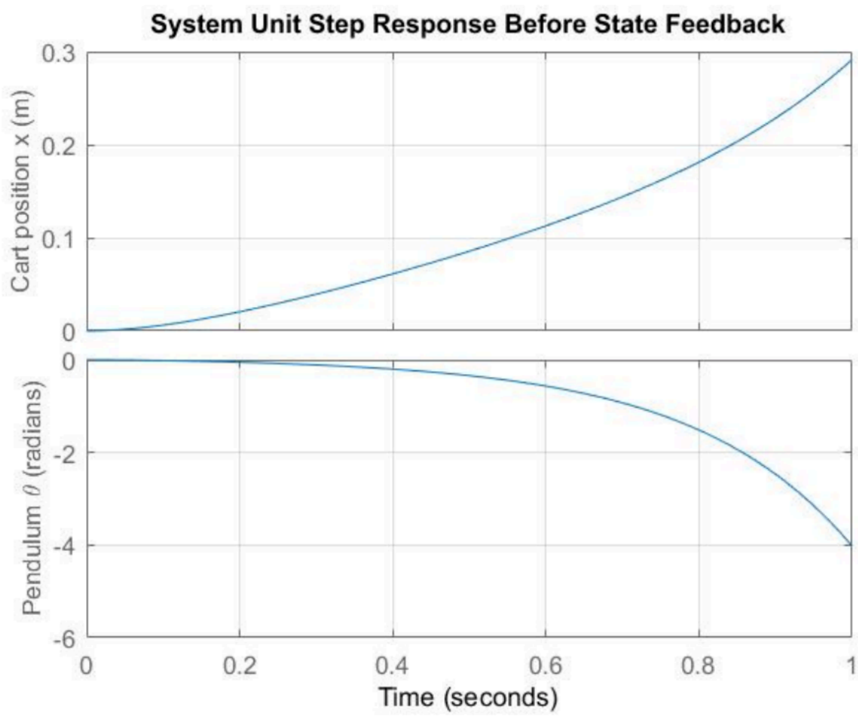
Part 3:

The eigenvalues of the State matrix A are found. The Eigens are also the poles of the system.

The poles are found to be 0, -7.6,-4.1,4.86. These values let us know that the system is unstable since one of the values is in the right hand portion, ie positive.

Part 4:

This portion of the laboratory assignment requires the user to find the system response to a unit step input. Using Matlab to generate the plots we get….



We can see that as the cart moves the position of the pendulum also moves to balance out the cart. If you go on the physical system for infinite time you will eventually pass and just simulate going around in the full circle forever.

Part 5:

In this section a full state variable feedback controller is implemented to achieve the desired gain performance specifications.

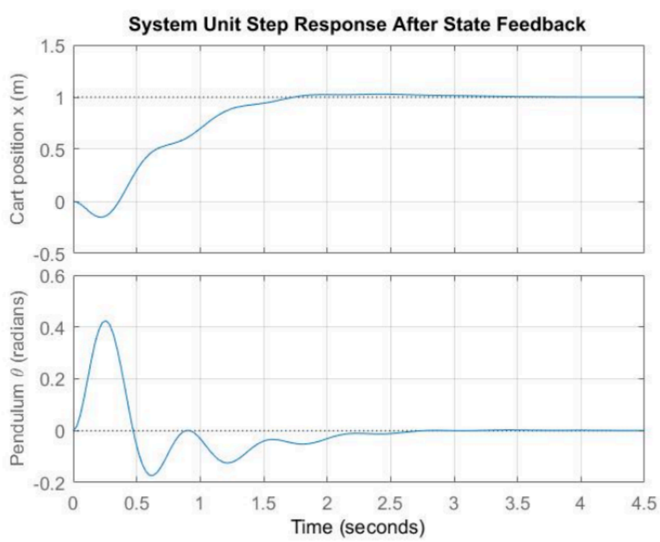
We get the following answer: K = -12.9639, --14.7177, -47.7877, -6.5285

This is the feedback gain matrix that will achieve u=Nr-Kx.

… These K values give us the best performance for specifications in the lab.

Part 6:

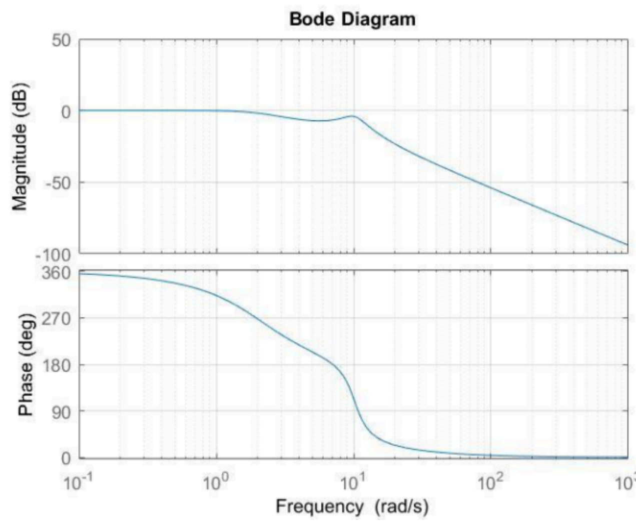
A. And is calculated from the equation given in the laboratory assignment to find new matrices after the state feedback. The stabilize system is plotted using unit step response to generate the following…

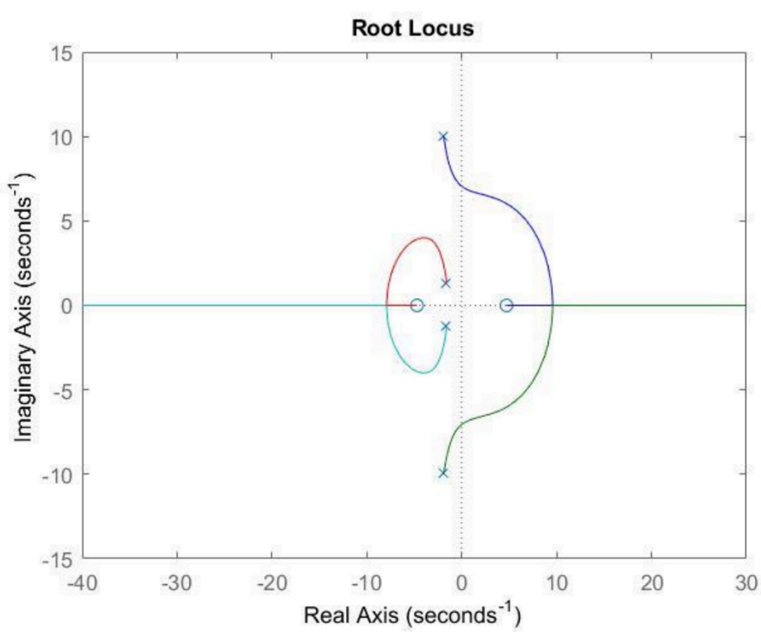


this gives the indication that if the cart was to move the pendulum with balance it out and once the cart stays at the same position the pendulum would stay balanced.

B. The transfer function of the cart position is calculated with respect to the referenced input....

$$T(s) = \frac{-19.76\,s^2 + 3.51e\text{-}14\,s + 440.3}{s^4 + 7\,s^3 + 120\,s^2 + 347.7\,s + 440.3}$$





Conclusions

This was a very cool laboratory assignment where we learned how to model an inverted pendulum and also to see how a moving car can change the pendulum and then have the system react.   The first step was deriving the equation of motion for the inverted pendulum card system. This was done in Matlab using the equations found by the matrix as well as the values of the system poles. The system response to a unit staff input was found and then when the car moves the pendulum moves in the system controls the system in order to create Control. Finally, in the very last part of this laboratory assignment we found a transfer function and plotted the Bodhi and root locus to see what value of K for the system is required in order to create stability.

List of References

"Simulink Basics Tutorial", SFSU 446 Laboratory manual book. Provided by University.

Appendix

```
syms M m Lp Kt Km Kg Rm r Jm g;
syms w wd;
% variables: xd = dx/dt; xdd = d2x/dt2; Vin = motor input
voltage
syms x xd xdd th thd thdd Vin
```

```
w = xd * Kg / r
wd = xdd * Kg / r
I = (Vin - Km*w)/Rm
T = Kt * I
% Force on cart:
F = (T - wd * Jm)*Kg/r
% Equations of motion:
Eq1 = (M + m) * xdd + m * Lp * thdd - F
Eq2 = xdd + (4/3) * Lp * thdd - g * th
xvec = [x; xd; th; thd] % d = derivative with respect to time.
xvecd = [xd; xdd; thd; thdd] % dd = 2nd derivative with respect
to time.
[xdd_, thdd_] = solve(Eq1 == 0, Eq2 == 0, xdd, thdd)
xvecd = [xd; xdd_; thd; thdd_]
%part 2
AA = jacobian(xvecd,xvec)
A = subs(AA,{M, m, Lp, Kt, Km, Kg, Rm, r, Jm, g},{.94, .23,...
 .3302, .00767, .00767, 3.71, 2.6, .00635, 3.9e-7, 9.81})
A = double(A)
BB = jacobian(xvecd,Vin)
B = subs(BB,{M, m, Lp, Kt, Km, Kg, Rm, r, Jm, g},{.94, .23,...
 .3302, .00767, .00767, 3.71, 2.6, .00635, 3.9e-7, 9.81})
B = double(B)
y = [x; th] % Define output vector y.
% No need to do substitution for C and D, they contain only 1's
and 0's.
C = jacobian(y,xvec);
C = double(C)
D = jacobian(y,Vin);
D = double(D)
%part 3
eig(A) % All must be in left half plane to be stable. Is it?
% Controlability:
Co = ctrb(A,B)
% Must have rank 4 to be controlable.
rank(Co)
% Observability:
Ob = obsv(A,C)
rank(Ob)
% Part 4: System unit step response before stabilization.
SysBefore = ss(A,B,C,D)      % Create state space system in
MATLAB.

figure
step(SysBefore, 1)           % Step response for 1 second.
grid ON
title ('System Unit Step Response Before State Feedback')
```

```
ylabel(' ')
h = get(gcf,'children') % Get handle to the subplots in figure
to edit y labels.
h(2).YLabel.String = 'Pendulum \theta (radians)'
h(3).YLabel.String = 'Cart position x (m)'
% Part 5: Pole Placement by State Variable Feedback (Non-
symbolic Method).
P = [-1.9+10j, -1.9-10j, -1.6-1.3j, -1.6+1.3j]
K = acker(A, B, P)
% Part 6(A): Calculation of N and Step Response after
stabilization.
Anew = A - B * K

N = -1 / ([1  0  0  0] * Anew^-1 * B)

Bnew = N * B

% Unit Step Response after State Variable Feedback:
SysAfter = ss(Anew,Bnew,C,D)

figure
step(SysAfter)  % Make sure you edit the ylabels of subplots.

grid on
title ('System Unit Step Response After State Feedback')
ylabel(' ')
h = get(gcf,'children') % Get handle to the subplots in figure.
h(2).YLabel.String = 'Pendulum \theta (radians)'
h(3).YLabel.String = 'Cart position x (m)'

%%
% Part 6(B): Bode and Root Locus of X(s)/R(s).

% Find transfer function matrix of system after state feedback:
TransF =  tf(SysAfter)

% Two transfer functions are returned in T, the first one is for
cart position:
XsRs = TransF(1)

figure
bode(XsRs)       % Bode plots.
grid on

figure
rlocus(XsRs)     % Root Locus.
```