San Francisco State University

Engineering 315

Laboratory #2.5

Modeling Electrical and Mechanical Systems

Purpose

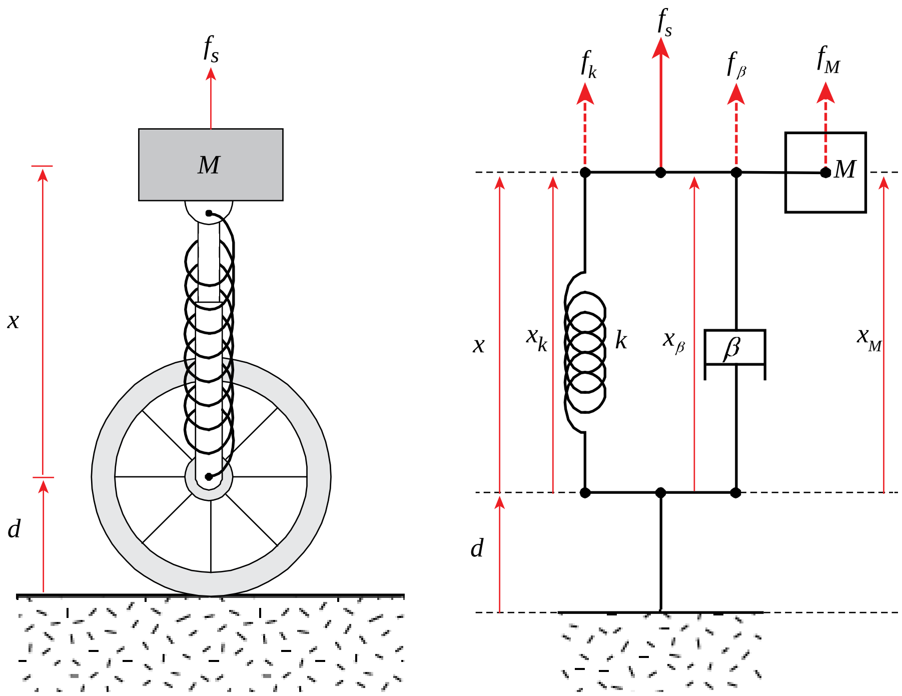
In this lab, you simulate the response of some of the mechanical systems that we introduced in the notes.

Background reading is [Unit #2.5] of the notes. In particular read and understand the sections on the suspension and the skier.

Background

Suspension

From the notes, wheel can be modeled by its mechanical schematic, as shown below, where *M* is the mass in kg, β is the damping in N-sec/m, and *k* is the spring constant in N/m.

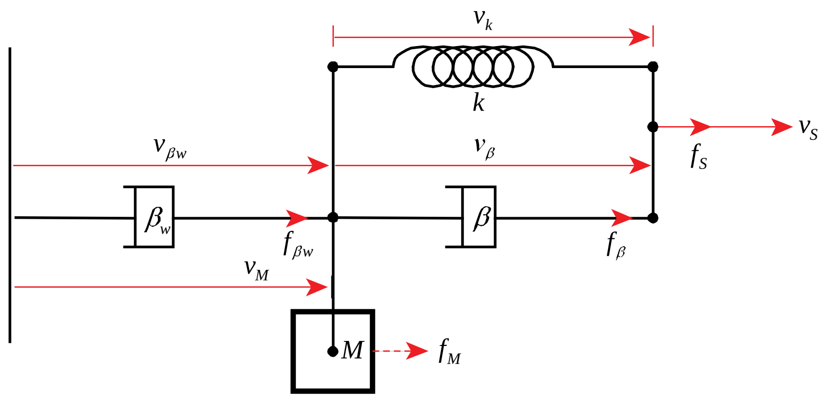


Solving the schematics for the displacement, *x*, results in the differential equation

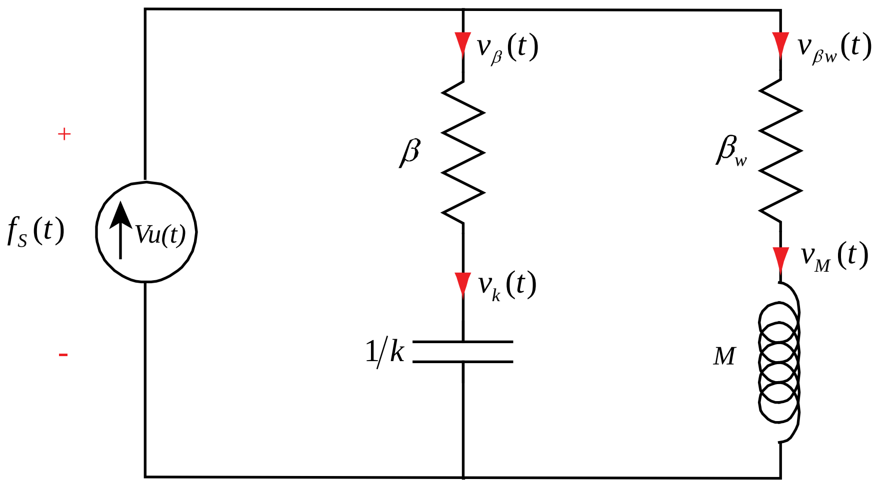
$$x''(t)+\frac{\beta}{M}x'(t)+\frac{k}{M}x(t)=\frac{f_s}{M}u(t).$$

Skiier

From the notes, the mechanical schematic of the skier is shown below.



M is the mass of the skier; *k* and β are the spring constant and damping of the rope connecting her to the boat, which moves with constant velocity, v_S . β_w is the drag of the water. In the notes we showed that this mechanical schematic is equivalent to the following electrical schematic:



Assignment

<u>Suspension</u>

You will write a function to plot the displacement of the suspension, *x*, given values of M, *k* and β in response to a step change in force, f_S .

```
function susp(M, k, beta)
% SUSP Simulate suspension
%       susp(M, k, beta)
%
%       Produces a plot of the displacement of the suspension, x,
%       as a function of time, given parameters
%       M (mass), k (spring constant) and beta (damping)
%
%       Your Name (date)
```

To do so, you will do the following:

1) Create a time vector, `t,` from -1 to 2 seconds in 1 msec increments.
2) Create a unit force vector, `f`, where $f(t) = u(t)$.
3) Use the Matlab command `tf` to create system function that represents the differential equation. For the suspension, the syntax is

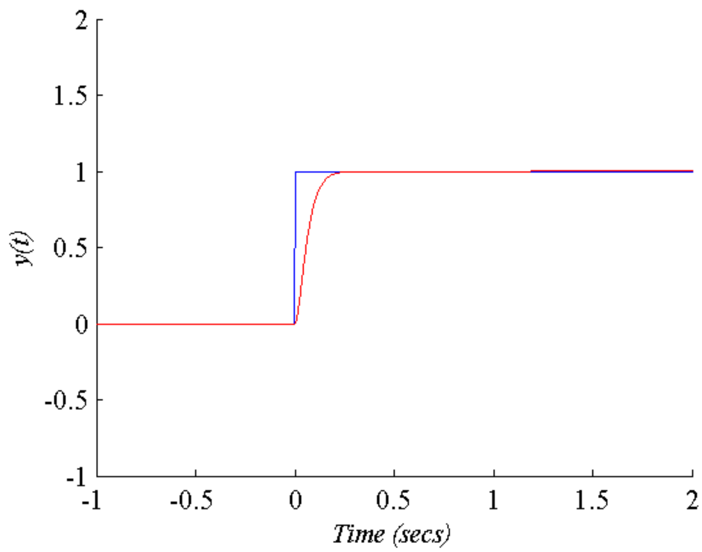
```
sys = tf(F/M, [1 beta/M k/M]);
```

where we will set $F = k$ in order to normalize the force, so we can plot both the foce and the displacement on the same axis. Note that the `tf` command takes two arguments. The first argument (`F/M` in this case) represents the right side of the differential equation. The second argument (the vector `[1 beta/M k/M]`) represents the coefficients of the left side of the equation

4) Simulate the output of the differential equation – the normalized displacement, $y(t)$ – using Matlab's lsim command:

```
y = lsim(sys, f, t);
```

5) Plot both the normalized force vector, `f`, in blue and the normalized displacement vector, `y`, in red on the same axis. The results of `susp(1, 900, 60)` should be as follows:



6) Note that you will get a warning, "`Warning: Simulation will start at a nonzero initial time`." You can either ignore it, or you can put the following line in your code before you call `lsim`:

```
warning off
```

7) Create plots for $M = 1$, $k = 900$ and values of $\beta = 6$ (very underdampled), 30 (somewhat underdamped), 60 (critically damped) and 400 (overdamped), preferable on the same axis using different colors. Explain what is going on.
8) Create plots for $M = 1$, $\beta = 60$ and $k = 90$, 900 and 9000, preferable on the same axis using different colors. Explain what is going on.
9) Create plots for $\beta = 60$, $k = 900$ and $M = 0.1$, 1 and 10, preferable on the same axis using different colors. Explain what is going on.
10) Compare plots of `susp(1, 900, 60)` and `susp(10, 9000, 600)`. Explain why they are similar or different.

Skier

Write a function to plot the velocity of the skier, v, (equivalent to $v_M(t)$ in the schematic, above), given values of M, k and β and β_w in response to a step change in velocity, $v_S(t) = Vu(t)$ of the boat.

```
function skier(M, k, beta, betaw)
% SKIER Simulate velocity of skier
%       skier(M, k, beta, betaw)
%
%       Produces a plot of the velocity of the skier, v,
```

```
%       as a function of time, given parameters
%       M (mass), k (spring constant) and beta (damping of rope)
%       and betaw (drag of water)
%
%       Your Name (date)
```

To do so, you will do the following:

1) Figure out the differential equation for $v_M(t)$ in the mechanical schematic.
2) You should just be able to clone and modify your `susp` program to simulate the new differential equation.
3) Create a time vector, `t,` from -1 to 5 seconds in 1 msec increments.
4) Create a velocity vector, `v`, where $v(t)=Vu(t)$, and $V=20$ m/sec.
5) Start with the values of parameters shown in the notes for a system with a low value of spring constant and no water drag, namely: $M=50$, $\beta=200$, $\beta_w=0$ and $k=150$. Plot $v_S(t)$ and $v_M(t)$ on the same axis. Show that you get the same plot as the notes.
6) Repeat with the same system, but with a high spring constant, $k=1500$.
7) Now add the resistance of the water. Take $M=50$, $\beta=200$ and $k=150$ and plot $v_S(t)$ and $v_M(t)$ for values of $\beta_{w=}=50$ and 500. If you've done it right, you should notice that the velocity of the skier jumps up relatively quickly to about $V\beta/(\beta+\beta_w)\simeq 5.7$ and then rises more slowly in an almost linear fashion. Note that in this case, the mass of the skier is almost unimportant. Make M a lot smaller (e.g. $M=10$) and you'll see that just the slope of the initial jump increases.
8) Extra credit: Simulate the circuit when $M=0$. You can't do this simply by substituting the value $M=0$ into your `skier` function (why)? Instead, you have to reanalyze the electrical schematic with $M=0$ and write a new differential equation of the system in the absence of mass, M. When you simulate this, you should find that the initial jump in the skier's velocity is exactly equal to $V\beta/(\beta+\beta_w)$.
