Relating Gradient Descent to Potential Field

Arun Kumar Singh

University of Tartu *arun.singh@ut.ee, aks1812@gmail.com*

October 12, 2021

Optimization: Basic Definition

$$\min_{\boldsymbol{\xi}} f(\boldsymbol{\xi}) \tag{1}$$

$\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3, \dots \xi_n)$, $f(\boldsymbol{\xi})$ is a scalar valued function.
E.g

$$f(\boldsymbol{\xi}) = \xi_1^3 + sin(\xi_2) - \xi_3 \tag{2}$$

Solving optimization problem amounts to obtaining $x_1, x_2, x_3, \dots x_n$ for which the function $f(\boldsymbol{\xi})$ has the minimum value.

Solving Optimization Problem: The Magic Bullet, Gradient Descent

- Gradient Descent is the most generic method for solving the optimization problem
- Can be described by the following set of equations

$$(\boldsymbol{\xi})^{k+1} = \boldsymbol{\xi}^k - \eta \overbrace{\nabla f(\boldsymbol{\xi})}^{v_x, v_y, \dot{\boldsymbol{\theta}}}, \tag{3}$$

where,

- η is called the learning rate (similar to Δt)
- ∇ represents gradient operator.
- k is the iteration index.
- We repeat the loop till either (i): $f(\boldsymbol{\xi})$ has reached a certain threshold or (ii) it has stopped decreasing.

Potential Field As Online Gradient Descent

- In motion planning example $\boldsymbol{\xi} = (x, y)$ and $f(\boldsymbol{\xi}) = c_g(x, y) + c_o(x, y)$.
- For manipulator inverse kinematics $\boldsymbol{\xi} = (\theta_1, \theta_2, \ldots, \theta_n)$ and $f(\boldsymbol{\xi}) = c_g(\theta_1, \theta_2, \ldots, \theta_n)$
- In typical, gradient descent you would the run the complete loop and extract the final solution.
- In potential field method, the robot keeps moving with the solution obtained at each iteration.