Q-learning

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What is Q-learning

The most popular *Dynamic Programming* approach to solve an RL problem

- Is based on Bellman principle of optimality
- Relies on definition of Quality function (also called state-action value function)

Three main components of an RL agent

- ~~Policy: The agent's decision~~
- Value function: how good the agent does in a state
- ~~Model: The agent's interpretation of the environment~~

Use Bellman's principle of optimality and

- estimate/evaluate the Quality function for all actions in all states
- choose an action which has the best Quality in the given state

Q function or state-action value function: The expected reward of MDP starting from state s, taking an arbitrary action a and then following the policy π.

$$Q(s, a) = r(s, a) + \gamma \, \mathbf{E}[Q(s', \pi(s'))] \tag{1}$$

Policy: The action maximizes the expected reward starting in s

$$\pi = \arg\max_a Q(s, a). \tag{2}$$

Bellman principle of optimality

Already in Bellman form!

$$Q(s,a) = r(s,a) + \gamma\, \mathbf{E}[Q(s', \pi(s'))]$$

Be careful!

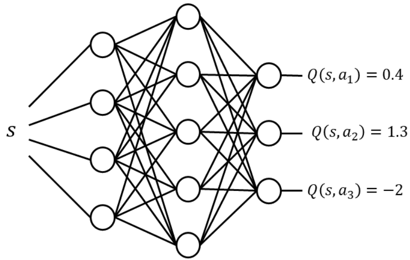
You need to solve an optimization problem!

$$\pi = \arg\max_a Q(s, a).$$

For discrete and continuous action space, the structure of $Q(s, a)$ should be selected carefully to avoid advanced optimization techniques.

Defining Q function in discrete case

- The function takes s as the input and generates $Q(s, a)$ for all possible actions.
- By feeding s the Q function is determined for all possible actions
- The actions are the indices for the vector.
- Policy is the index in which $Q(s, a)$ is maximized.



Defining Q function in continuous action space case

- The Q function takes state and action as inputs and generates a scalar output
- The policy is obtained by mathematical optimization
- Example: Quadratic Q

$$Q(s, a) = \begin{bmatrix} s^\dagger & a^\dagger \end{bmatrix} \begin{bmatrix} g_{ss} & g_{sa} \\ g_{sa}^\dagger & g_{aa} \end{bmatrix} \begin{bmatrix} s \\ a \end{bmatrix} \tag{3}$$

The policy is

$$\pi = -g_{aa}^{-1} g_{sa}^\dagger \, s. \tag{4}$$

Our guess of Q function does not satisfy Bellman and there is an error

$$e = r(s,a) + \gamma\, Q(s', \pi(s')) - Q(s,a). \tag{5}$$

How to build this error? For each sample point s_t, a_t, r_t, s_{t+1}, do the following

- Find $Q(s_t, a_t)$
- Find $Q_{target}(r_t, s_{t+1}) = r_t + \gamma\ \arg_a \max Q(s_{t+1}, a)$
- Define the error $e_t = Q_{target}(r_t, s_{t+1}) - Q(s_t, a_t)$.
- Minimize the mean square error $\frac{1}{2}\sum_{t=1}^{T} e_t^2$.

Temporal Difference (TD) learning!

How to select a in Q-learning?!??

Example: Eating in town

- **Exploitation:** Go to your favourite restaurant
- **Exploration:** Select a random restaurant

In RL

- **Exploitation only:** will get stuck in a local optimum forever
- **Exploration only:** will try only random things

It is important to balance Exploration *vs.* Exploitation

How to generate a in discrete action space case?

Set a level $0 < \epsilon < 1$ and generate a random number $r \sim [0,\ 1]$

$$a = \begin{cases} \text{random action} & \text{if } r \ < \ \epsilon, \\ \arg\max_a Q(s,a) & \text{Otherwise.} \end{cases}$$

How to generate a in discrete action space case?

Generate a random number $r \sim \mathcal{N}(0, \sigma^2)$

$$a = \arg\max_a Q(s, a) + r.$$

Putting all together

We build/select a network to represent $Q(s, a)$. Then, we iterate:

1. Collect data
   - Observe the state s and select the action a.
   - Apply a and observe r and the next state s'.
   - Add s, a, r, s' to the history.
2. Update the parameter θ
   - We minimize the mean squared error using the history of data.

Q-learning

- Model-free
- Based on Bellman's principle of optimality
- The first approach to try
- Usually good results
- Take a look at explanation and implementation on my Github,

  Crash_course_on_RL/q_notebook.ipynb

Email your questions to

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