Policy Iteration on Linear Quadratic Problem

LINKÖPINGS UNIVERSITET

Farnaz Adib Yaghmaie

Linkoping University, *Sweden*
farnaz.adib.yaghmaie@liu.se

April 6, 2021

- **Dynamics:**

$$s_{t+1} = As_t + Bu_t + w_t$$

- **State and action:**

$$s_t \in \mathbb{R}^n,$$
$$u_t \in \mathbb{R}^m$$

- **Cost function ($\equiv$ negative of reward):**

$$c_t = s_t^\dagger Q s_t + u_t^\dagger R u_t, \quad Q \geq 0, R > 0$$

Solvability Criterion: Minimize $V(s)$ with respect to the policy π

$$V(s_t) = \mathbf{E}[\sum_{k=t}^{+\infty}(c_k - \lambda)|s_t]$$

where λ is the average cost

$$\lambda = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} c_t$$

Note that minimizing $V(s_t)$ and λ are equivalent.

The agents learn a quadratic Q function

$$Q(s,a) = \begin{bmatrix} s^\dagger & a^\dagger \end{bmatrix} \begin{bmatrix} g_{ss} & g_{sa} \\ g_{sa}^\dagger & g_{aa} \end{bmatrix} \begin{bmatrix} s \\ a \end{bmatrix} = z^\dagger G z \tag{1}$$

The policy is given by optimizing the Q function

$$\pi = -g_{aa}^{-1} g_{sa}^\dagger \, s = Ks \tag{2}$$

1. Compute the empirical average cost $\lambda = \frac{1}{T}\sum_{t=1}^{T} c_t$
2. Collect data
   - Observe s and select a

$$a = Ks + r, \quad r \sim \mathcal{N}(0, \sigma^2).$$

   - Apply a and observe r.
   - Add s, a, r to the history.
3. Estimated the kernel of Q by Least Squares Temporal Difference (LSTD)

$$vecs(G)=(\tfrac{1}{T}\sum_{t=1}^{T}\Psi_t(\Psi_t-\Psi_{t+1})^{\dagger})^{-1}(\tfrac{1}{T}\sum_{t=1}^{T}\Psi_t(c_t-\lambda)) \tag{3}$$

where $z_t = [s_t^{\dagger},\ a_t^{\dagger}]^{\dagger}$, $\Psi_t = vecv(z_t)$.

Try the following:

- Run

  Crash_course_on_RL/q_on_lq_notebook.ipynb

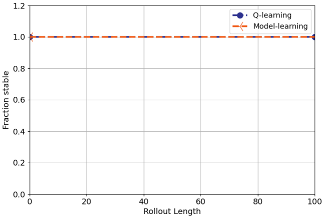
  and verify the median of the error in K is $\sim 0.01\%$.

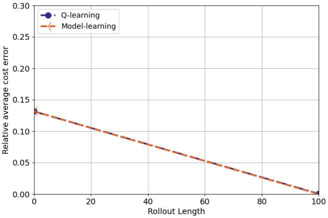
- Set

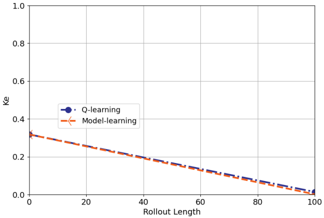
  'explore_mag=0.0' in 'My_q_learning.ql'

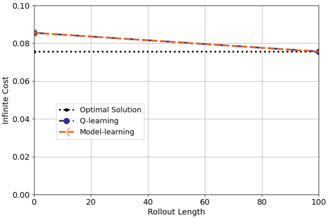
  and verify that the agent cannot solve the problem! you don't get any stable controller.

- Make sure you understand the code!


Q-learning
Model-learning
Fraction stable
Rollout Length
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0
20
40
60
80
100


Q-learning
Model-learning
Relative average cost error
Rollout Length
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0
20
40
60
80
100


Q-learning
Model-learning
Ke
Rollout Length
0.0
0.2
0.4
0.6
0.8
1.0
0
20
40
60
80
100


Optimal Solution
Q-learning
Model-learning
Infinite Cost
Rollout Length
0.00
0.02
0.04
0.06
0.08
0.10
0
20
40
60
80
100

Important observations

- Q-learning performs superb on the LQ problem
- No hyper-parameter to tune

Email your questions to

farnaz.adib.yaghmaie@liu.se