Unit 1 - Vectors and spaces

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1 Lesson 1: Vectors

1.1 Vector intro for linear algebra

A vector has magnitude and direction. If someone is going at "$5km/h$", that isn't a vector. However, if someone is going at "$5km/h$ east", since there is magnitude and direction, that is a vector.

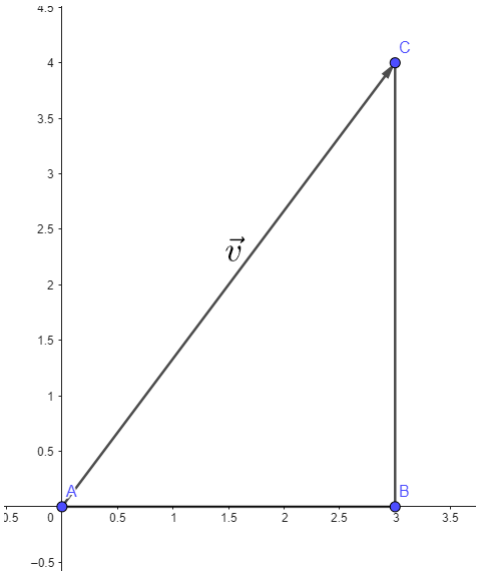
A vector will have a x and y component, and can be described like this:

$$\vec{v} = (x, y)$$

For the sake of this course, the matrix representation will be used more commonly. That is:

$$\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix}$$

A vector $\vec{v} = (3, 4)$ could be drawn the following way:



Notice that it's magnitude, per the Pythagorean Theorem, will be 5.

1.2 Real coordinate spaces

We can represent the 2-dimensional real coordinate space (a.k.a the cartesian plane) as $\mathbb{R}^2$. That will be all possible real-valued "2-tuples". In our case, these "2-tuples" are the vectors that have two values. All possible combinations of x and y, both real, make $\mathbb{R}^2$.

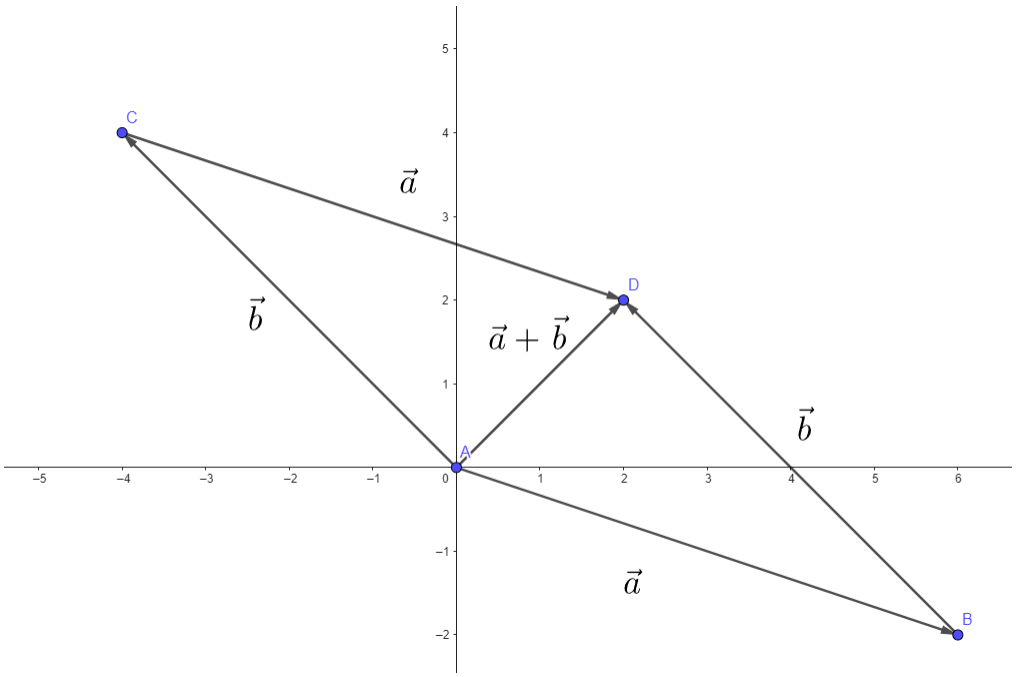
$\mathbb{R}^3$ would be the $3D$ real coordinate space, with all real-valued 3-tuples. Or, 3 dimensional vectors. In general, $\mathbb{R}^n$ represents an n-dimensional real coordinate space with n-tuples.

1.3 Adding vectors algebraically & graphically

Let $\vec{a} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} -4 \\ 4 \end{bmatrix}$

$$\vec{a} + \vec{b} = \begin{bmatrix} 6 + (-4) \\ -2 + 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Graphically, this is:



Theorem 1.1. The sum of two vectors $\vec{a}$ and $\vec{b}$ such that $\vec{a} = (x_1, y_1)$ and $\vec{b} = (x_2, y_2)$ is $\vec{a} + \vec{b} = (x_1 + x_2, y_1 + y_2)$