

# Unit 1 - Vectors and spaces

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## 1 Lesson 1: Vectors

### 1.1 Vector intro for linear algebra

A vector has magnitude and direction. If someone is going at " $5km/h$ ", that isn't a vector. However, if someone is going at " $5km/h$  east", since there is magnitude and direction, that is a vector.

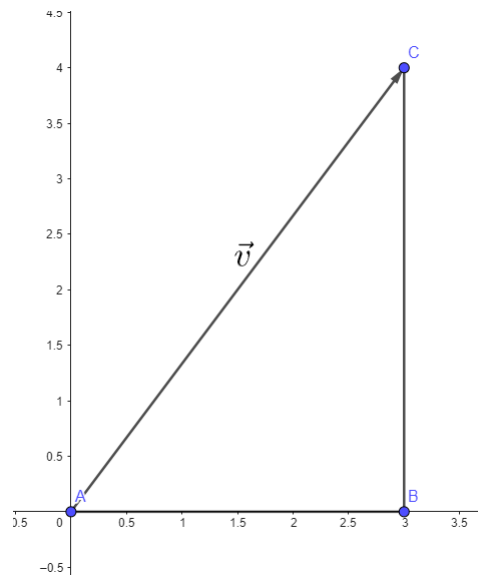
A vector will have a  $x$  and  $y$  component, and can be described like this:

$$\vec{v} = (x, y)$$

For the sake of this course, the matrix representation will be used more commonly. That is:

$$\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix}$$

A vector  $\vec{v} = (3, 4)$  could be drawn the following way:



Notice that its magnitude, per the Pythagorean Theorem, will be 5.

## 1.2 Real coordinate spaces

We can represent the 2-dimensional real coordinate space (a.k.a the cartesian plane) as  $\mathbb{R}^2$ . That will be all possible real-valued "2-tuples". In our case, these "2-tuples" are the vectors that have two values. All possible combinations of  $x$  and  $y$ , both real, make  $\mathbb{R}^2$ .

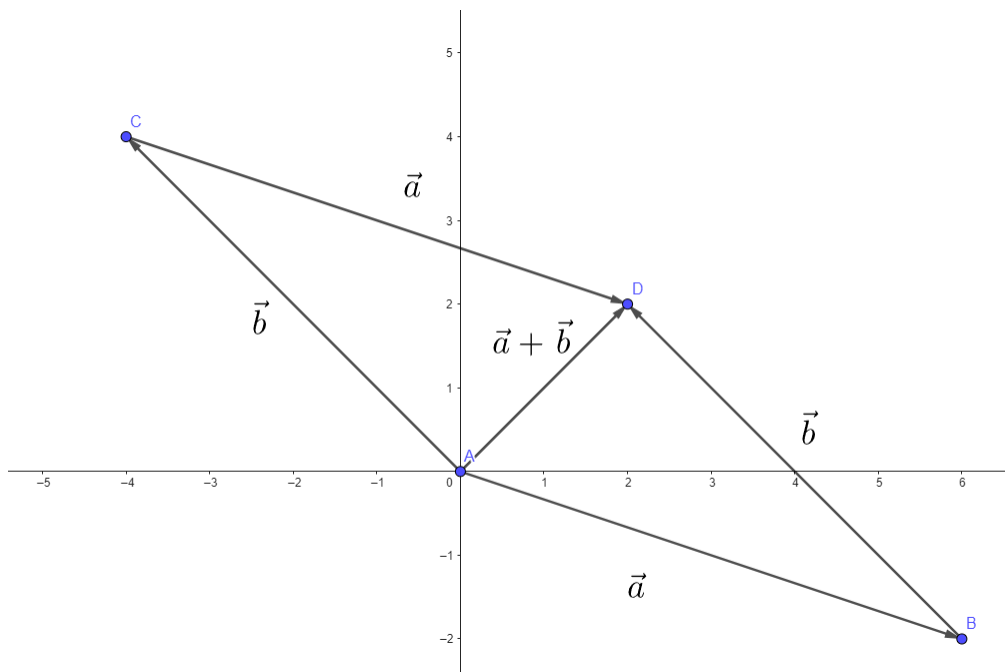
$\mathbb{R}^3$  would be the 3D real coordinate space, with all real-valued 3-tuples. Or, 3 dimensional vectors. In general,  $\mathbb{R}^n$  represents an n-dimensional real coordinate space with n-tuples.

## 1.3 Adding vectors algebraically & graphically

Let  $\vec{a} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$  and  $\vec{b} = \begin{bmatrix} -4 \\ 4 \end{bmatrix}$

$$\vec{a} + \vec{b} = \begin{bmatrix} 6 + (-4) \\ -2 + 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Graphically, this is:



**Theorem 1.1.** The sum of two vectors  $\vec{a}$  and  $\vec{b}$  such that  $\vec{a} = (x_1, y_1)$  and  $\vec{b} = (x_2, y_2)$  is  $\vec{a} + \vec{b} = (x_1 + x_2, y_1 + y_2)$ .