TP2 OAP Report

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0.1 First Practical Session

In this session, we tackled the problem starting with the understanding of the situation in which it takes place.

Context

The problem consists of a situation in which a client(a beer producer called Efes) desires to figure out how to minimize their costs of malt transport, malt import and beer distribution while guaranteeing the distribution of the forecasted demands of their beer. The following picture shows a map of the different malt production sites, malt import sites, beer production sites and distribution centers.

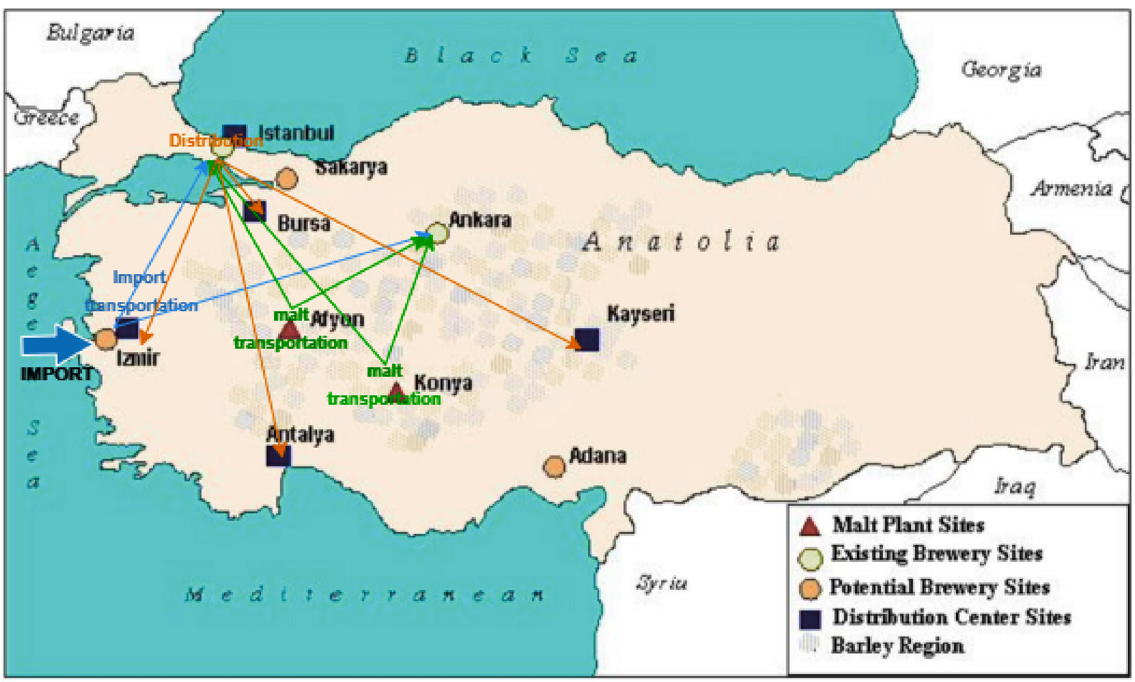

Figure 1: Map of different sites and their relations

The client provided their plan for next year, as shown in figure 2, and we need to test if their plan is optimal.In case it isn't, we need to provide them with an optimal one.

Amount of malt shipped from plant i to brewery j in year 1 (1000 ton)

	Istanbul	Ankara	Total	Capacity
Afyon	0	24	24,00	30,00
Konya	2,42	0	2,42	68,00
Import	20	0	20,00	20,00
Total malt	22,42	24		
Total beer	202	200	(Million lt)	

Amount of beer shipped from brewery j to distribution center k in year 1 (Million liters)

	Istanbul	Izmir	Antalya	Bursa	Kayseri	Export (Izmir)	Total	Capacity
Istanbul	103	49	50	0	0	0	202	220
Ankara	0	25	0	60	102	13	200,00	200
Total	103	74	50	60	102	13		
Demands	103	74	50	60	102	13		
						Total shipping cost		11,244
								(Million dollars)

Figure 2: Plan for Year 1 provided by Efes

Linear Model Year 1

In order for us to find the optimal solution, we need to describe the client's data with mathematical expressions and equations. Consequently, we need to define the different constants and variables that will be included in our model.
Let D_{ij} be the cost of distributing beer from i to j, i being a brewery $i \in Brew = \{Ist, Ank\}$ and j being a distribution site $j \in Dist = \{Ist, Izm, Ant, Bur, Kay, Exp\}$.
Let $D = (D_{ij})_{i \in Prod, j \in Dist}$ be the matrix representing the costs of distribution (Million \$/Million liters).

Let $T_{i \to j}$ be the cost of transporting malt from i to j, i being the malt production/import site $i \in Prod = \{Afy, Kon, Imp\}$ and j being a brewery $j \in Brew$.
Let $T = (T_{ij})_{i \in Prod, j \in Brew}$ be the matrix representing the costs of transporting/importing malt (Million \$/1000 tons).

Let $B_{i \to j}$ be the quantity of beer distributed from i to j, i being a brewery $i \in Brew = \{Ist, Ank\}$ and j being a distribution site $j \in Dist = \{Ist, Izm, Ant, Bur, Kay, Exp\}$.
Let $B = (B_{ij})_{i \in Prod, j \in Dist}$ be the matrix representing the quantities of beer to be stored in each distribution center (Million liters).

Let $M_{i \to j}$ be the quantity of malt to be transported from i to j, i being the malt production/import site $i \in Prod = \{Afy, Kon, Imp\}$ and j being a brewery $j \in Brew$.
Let $M = (M_{ij})_{i \in Prod, j \in Brew}$ be the matrix representing the quantities of malt to be transported to each brewery (1000 tons).

The purpose of our model being the minimization of the total cost of distribution and transport, our model needs to minimize the following function:

$$Z = \sum_{i \in Brew, j \in Dist} D_{ij} \cdot B_{ij} + \sum_{i \in Prob, j \in Brew} T_{ij} \cdot M_{ij}$$

In other terms, using the matrices D, B, T and M:

$$Z = \sum_{i \in Brew} D[i,:] \cdot B^T[:,i] + \sum_{i \in Prod} T[i,:] \cdot M^T[:i]$$

Dist. Centers	Year 1
Istanbul	103
Izmir	74
Antalya	50
Bursa	60
Kayseri	102
Export(Izmir)	13

Figure 3: Demand for Year 1

M lts of beer brewered from 1000 ton of malt	
Domestic	8,333
Import	9,091

Figure 4: Malt yield

Annual capacity of malt plants	(1000 tons/year)
Afyon	30
Konya	68
Import (Izmir)	20

Figure 5: Capacity of malt plants

Annual capacity of breweries			
		(Million litres/year)	
Existing Breweries		Current	
	Istanbul	220	
	Ankara	200	
Potential Breweries		New	Expansion
	Izmir	70	50
	Sakarya	70	50
	Adana	70	50

Figure 6: Capacity of breweries

Knowing that without constraints on our variables(B and M), the optimal solution would be to not produce nor malt nor beer, that way we would have $Z = 0$. Which is why the client needs to provide constraints. In this case, the constraints provided are:

From these constraints we conclude the following conditions:

For all $i \in Brew$: $\sum_{j \in Dist} B_{ij} = 8.333 \cdot (M_{i,Afy} + M_{i,Kon}) + 9.091 \cdot M_{i,Imp}$ And: For all $i \in Prod$ $\sum_{jinBrew} M_{ij} \leqslant MC[i]$ MC being the malt capacity matrix:

$$MC = \begin{bmatrix} 30 \\ 68 \\ 20 \end{bmatrix}$$

And: For all $i \in Brew$: $\sum_{j \in Dist} B_{ij} \leqslant BC[i]$;BC being the beer capacity matrix:

$$BC = \begin{bmatrix} 220 \\ 200 \end{bmatrix}$$

Finally, for all $j \in Dist$: $\sum_{i \in Brew} B_{ij} \geqslant Demand[j]$; Demand being the matrix representing the forcasted demand for year 1.
The final model representing the issue is:
Decision variables:$B = (B_{ij})_{i \in Prod, j \in Dist}$ and $M = (M_{ij})_{i \in Prod, j \in Brew}$.

Objective function:

$$Minimize : Z = \sum_{i \in Brew} D[i,:] \cdot B^T[:,i] + \sum_{i \in Prod} T[i,:] \cdot M^T[:i]$$

Subject To:
For all $i \in Brew$: $\sum_{j \in Dist} B_{ij} = 8.333 \cdot (M_{i,Afy} + M_{i,Kon}) + 9.091 \cdot M_{i,Imp}$
For all $i \in Prod$ $\sum_{jinBrew} M_{ij} \leqslant MC[i]$
For all $i \in Brew$: $\sum_{j \in Dist} B_{ij} \leqslant BC[i]$
For all $j \in Dist$: $\sum_{i \in Brew} B_{ij} \geqslant Demand[j]$
B$\geqslant$ 0,M $\geqslant 0$

Implementation in IBM OPL CPLEX

The corresponding code for our model in IBM OPL CPLEX is as follows:

```
/*********************************************
 * OPL 20.1.0.0 Model

 *********************************************/


dvar float+ B[i in 1..6,j in 1..2];
dvar float+ M[i in 1..2,j in 1..3];

float D[i in 1..2, j in 1..6]=...;
float T[i in 1..3, j in 1..2]=...;
float Demand[i in 1..6]=...;
minimize sum(i in 1..6)(D[1,i]*B[i,1]+D[2,i]*B[i,2])+sum(i in 1..2)
(T[1,i]*M[i,1]+T[2,i]*M[i,2]+T[3,i]*M[i,3]);

subject to{
  sum(i in 1..6)(B[i,1])==8.333*(M[1,1]+M[1,2])+9.091*M[1,3];
  sum(i in 1..6)(B[i,2])==8.333*(M[2,1]+M[2,2])+9.091*M[2,3];
  M[1,1]+M[2,1]<=30;
  M[1,2]+M[2,2]<=68;
  M[1,3]+M[2,3]<=20;
  sum(i in 1..6)(B[i,1])<=220;
  sum(i in 1..6)(B[i,2])<=200;
  forall (i in 1..6)(B[i,1]+B[i,2]>=Demand[i]);
}
```

Figure 7: Model code

```
/*********************************************
 * OPL 20.1.0.0 Data

 *********************************************/

D=[[0 0.04 0.052 0.017 0.055 0.042],[0.032 0.041 0.039 0.027 0.023 0.043]];
T=[[0.026 0.017],[0.037 0.017],[0.032 0.033]];
C=[103 74 50 60 102 13];
```

Figure 8: Data code

Running this model with CPLEX gives the following output:

```
// solution (optimal) with objective 9.90828153126125
// Quality There are no bound infeasibilities.
// Maximum reduced-cost infeasibility = 6,93889e-18
// Maximum Ax-b residual              = 0
// Maximum c-B'pi residual            = 3,46945e-18
// Maximum |x|                        = 103
// Maximum |slack|                    = 49,7581
// Maximum |pi|                       = 0,0451201
// Maximum |red-cost|                 = 0,033
// Condition number of unscaled basis = 1,0e+02
//

B = [[103
             0]
             [26 48]
             [0 50]
             [60 0]
             [0 102]
             [13 0]];
M = [[24.241 0 0]
             [5.759 18.242 0]];
```

Figure 9: Output

0.2 Second Practical Session

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