

CS498: Algorithmic Engineering

Lecture 1

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Outline

- 1 Course Logistics
 - Differences from CS374 and CS473
 - Content and Types of Projects in Class
 - Prerequisites
 - Grading
 - LLM Usage Policy
- 2 History of Linear Programming
- 3 Linear Programming: The Basics
- 4 The Engineer's Diet Dilemma
- 5 Interpreting and Debugging Gurobi Output

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From Proofs to Solvers

While standard algorithms courses focus on proving what is *computable*,
this course focuses on implementing what is *necessary*.

Relation to CS 374

CS 374: The Vocabulary

- Focuses on **Reductions**:
Transforming Problem A to B to prove either tractability (e.g. problem is P) or hardness.
- **Goal**: Determine theoretical tractability (P vs NP).
- **Output**: A formal proof.
- **Example Concept**:
3-SAT is NP-Complete, No Polynomial time algorithms, unless $P = NP$

CS 498: The Application

- Focuses on **Modelling**:
Transforming Problem A to B so a solver for B can handle it.
- **Goal**: Practical solutions for real-world instances.
- **Output**: A Python script or implementation of an Algorithm to solve the problem.
- **Example Concept**:
How to Solve This 3-SAT Instance (1M vars) in $< 5s$

Relation to CS 473

- **CS 473** analyzes the *internal mathematics of the engine*.
- Advanced algorithmic techniques (example: randomization, flow, advanced dynamic programming).
- Focus on proving efficiency (run-time) and approximation guarantees (bounds).
- **CS 498** teaches you how to *drive the car*.
- We treat powerful solvers, that researchers have spent decades working on, as black boxes to be mastered.
- Focus on modeling complex constraints rather than implementing the solver itself.
- We still explain the theory behind the solvers, but the focus is on basics of theory

The “NP-Hard” Perspective

CS 374: “Stop”

Proving a problem is NP-Hard is the end of the conversation. It means an **efficient worst-case algorithm does not exist**.

CS 473: “Detour”

Accept that exact **provable** solutions are impossible. Pivot to designing algorithms that provide **guaranteed approximations**.

CS 498: “Launch”

NP-Hardness is a worst-case warning, not a law of physics. Use SAT/SMT solvers to **crush real-world instances**. No more Grantees.

Modern Tooling Stack

We move beyond “pseudocode” to industrial-grade Python libraries used in Operations Research and Deep Learning.

- **Optimization:**

Gurobi, Pyomo (Linear & Integer Programming)

- **Logic & Verification:**

Z3, PySAT (SMT & SAT Solvers)

- **Differentiation:**

PyTorch (Autograd & Neural Networks)

Course Comparison Matrix

Feature	CS 374 / 473	CS 498
Primary Goal	Proofs & Analysis	Models & Implementations
Hardness	Prove it's impossible in worst case	Use solvers to solve your instance anyway
Key Tools	Pencil, Paper, LaTeX	Gurobi, Z3, PyTorch, ...
Style	Purely Theoretical	Hybrid (Basics of Theory + Implementation)

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Content and Types of Projects in Class

Part I: Discrete Optimization

- Linear & Integer Programming
- Modeling with Gurobi
- Supply chain & Network models

Part II: Differentiable Systems

- First order and Second Order Optimization Techniques
- Genetic Algorithms and Metaheuristics.
- PyTorch & Autograd internals
- Convex and Non-convex optimization

Part III: Formal Methods

- SAT & SMT Solvers (Z3, PySAT)
- Logic encodings (Sudoku, Scheduling)
- Automated verification and Solving Puzzles with SAT/SMT Solvers

Parts IV & V: Synthesis

- Data-Driven Optimization (aka Data Science)
- LLMs as Reasoning Engines, prompting, consistency, etc.
- Program Synthesis.

Projects I: Optimization & Logic

1. The “Fleetline” Challenge (Week 3)

- ▶ A real problem from a YC startup (Who are giving us a guest lecture).
- ▶ Problem: You must assign truck drivers to loads to maximize revenue while respecting complex human constraints (e.g., “*Driver A must be back in Chicago by Friday for their daughter’s rehearsal*”).

2. SMT for Scheduling (Week 9)

- ▶ Solve very complex Puzzles beyond most humans reach using SMT solvers.
- ▶ Learn to encode massive real-world scheduling conflicts into SMT Solvers (Z3).

Projects II: AI & Neurosymbolic Agents

3. Evolution & Gradients (Week 6-7)

- ▶ Solve NP-Hard problems (like TSP) using Genetic Algorithms and Metaheuristics.
- ▶ Look under the hood of Deep Learning by building your own Autodiff engine from scratch before training real neural-nets in PyTorch.

4. Data-Driven Pipelines (Week 11)

Build end-to-end ML pipelines (feature engineering, regression) to predict real-world parameters (e.g., housing prices) and integrate them directly into optimization objectives.

5. The AIME Agent (Week 14)

Build a neurosymbolic reasoning agent to solve **Math Olympiad (AIME)** problems. You will engineer prompts, implement self-consistency checks, and use open-source LLMs to tackle high-level reasoning tasks.

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Prerequisites

1. Theory

- **CS 374** is assumed.
- We won't reteach NP-Hardness; we assume you know what it implies.

2. Attitude

- **Coding Heavy:** This is an engineering class.
- **Resilience:** You must be willing to read documentation, debug strange library errors, and explore new tools.

3. Coding

Python Literacy Check

```
import numpy as np

A = np.array([[1, 2], [3, 4]])
b = np.array([5, 6])

# If you know what this does

x = np.linalg.solve(A, b)

# or can look it up quick
# ...you're Gucci.
```

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Grading Structure

60%

Weekly Homeworks

- **Groups of 2-4.**
- *The more, the merrier.*
- High volume of problems; working alone is a competitive disadvantage.

10%

3 "Pulse Checks"

- After Parts I, II, III.
- In-class, short, individual quizzes.
- **Goal:** Check if you are alive.
- If you understand the bare minimum, you get 100%.

30%

Individual Final Project

- **Algorithmic Engineering.**
- Build a system, implement a paper, or optimize a complex pipeline.
- Compare performance (speed/quality).

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LLM Usage Policy: “Productivity, not Replacement”

The Rule:

- You **CAN** use LLMs (ChatGPT, Gemini, Claude, Copilot).
- You **MUST** acknowledge usage and explain exactly what you asked the LLM to do.

The “Random Audit”:

- Each week, random students will be asked to **explain their code/solutions** in person.
- If you blind-copied without understanding → **Big Problems.**

LLM Usage Policy: “Productivity, not Replacement”

Kosher vs. Not Kosher

✓ Good:

- “Write a Python function to parse this DIMACS file format.”
- “Here is an Algorithm to solve this problem ... Encode the constraint this way ... Please implement my idea in Python.”

X Bad:

“Here is the PDF of the homework, solve Problem 3 for me.”

Questions?

Ready to build?

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The Toy Factory Example

Scenario: You build two products: **Widgets** (x_1) and **Gadgets** (x_2).

Profits:

- Widget: \$3 profit
- Gadget: \$4 profit

The LP Model:

$$\begin{array}{ll}\max & 3x_1 + 4x_2 \\ \text{s.t.} & 1x_1 + 2x_2 \leq 10 \\ & 2x_1 + 1x_2 \leq 15 \\ & x_1, x_2 \geq 0\end{array}$$

Constraints:

- **Metal:** Have 10kg. Widget uses 1, Gadget uses 2.
- **Wood:** Have 15kg. Widget uses 2, Gadget uses 1.

Act I: The Dark Ages (Pre-1947)

Before 1947, the idea of writing a massive planning problem as a single mathematical equation was unknown.

- **Fourier (1823):** Solved small systems of inequalities.
- **Leonid Kantorovich (1939):** Invented LP in the USSR to optimize plywood production.
- **The Tragedy:** The Soviet government ignored him. His work remained unknown to the West for decades.

Motzkin's Thesis (1936)

Listed only **42 papers** in all of history on linear inequalities. Today, there are tens of thousands per year.

Act II: WWII & George Dantzig

The Setup:

- George Dantzig spent WWII planning US Air Force logistics by hand.
- 1946: The Air Force asks: “**Can you mechanize the planning process?**”
- He built a dynamic model of resources and activities, but something was missing.
- Dantzig realized he needed an **Explicit Objective Function** to optimize on top of his linear constraints.
- But how to solve a system with thousands of linear constraints and linear objective? He needed help.

Act III: Meeting Von Neumann (Oct 1947)

Dantzig visits **John von Neumann** at Princeton.

- 1 Dantzig starts explaining his Air Force model in tedious detail.
- 2 Von Neumann cuts him off: “**Get to the point.**”
- 3 Dantzig writes the linear programming problem on the board.

The Revelation

Von Neumann stands up: “**Oh—that!**”

He proceeds to lecture Dantzig for 90 minutes on **Duality** and **Geometry**.

Von Neumann had already derived the theory of LP while inventing Game Theory.

Act IV: The Mic Drop

Conference, 1948: Dantzig presents LP to a room of heavyweights.

Harold Hotelling (Economics Giant) stands up:

“But we all know the world is non-linear.”

Dantzig freezes. The room goes silent. Then Von Neumann raises his hand:

***“If the axioms of linear programming fit your problem, use it.
If not, don’t.”***

He sat down. The field of Linear Programming was born.

For more historical readings, read “REMINISCENCES ABOUT THE ORIGINS OF LINEAR PROGRAMMING” by Dantzig himself!

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The Canonical Form

Every LP can be written in Matrix Notation: $\max \mathbf{c}^T \mathbf{x}$ s.t. $\mathbf{Ax} \leq \mathbf{b}$.

For our Factory:

$$\underbrace{\begin{bmatrix} 3 \\ 4 \end{bmatrix}}_{\mathbf{c}^T} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{x}} \quad \text{subject to} \quad \underbrace{\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{x}} \leq \underbrace{\begin{bmatrix} 10 \\ 15 \\ 0 \\ 0 \end{bmatrix}}_{\mathbf{b}}$$

- **x**: Decision Variables (The knobs we turn).
- **c**: Objective Coefficients (Profits/Costs).
- **A**: Constraint Matrix (Resource usage).
- **b**: Right-Hand Side (Capacities).

Pathologies: When things go wrong

Before we solve it, what if we *can't*?

1. Infeasibility

No solution satisfies all constraints.

$$x \leq 2 \quad \text{AND} \quad x \geq 3$$

The feasible region is **Empty**.

Gurobi: Model is infeasible.

2. Unboundedness

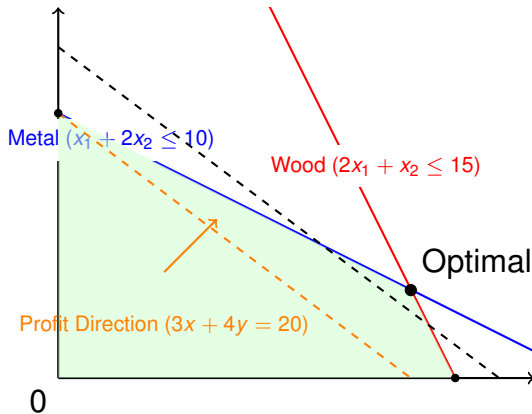
The region is open in the direction of improvement.

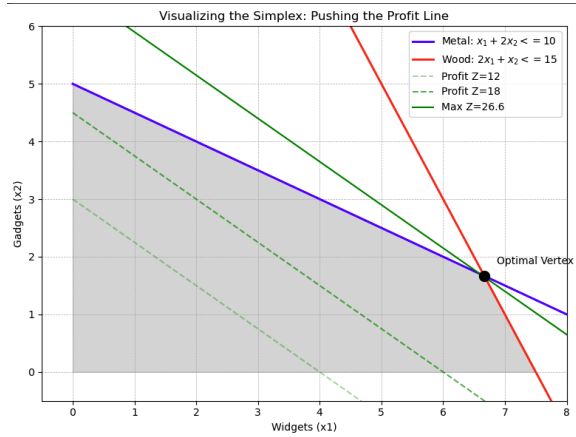
$$\max x \quad \text{s.t.} \quad x \geq 5$$

You can increase profit to ∞ .

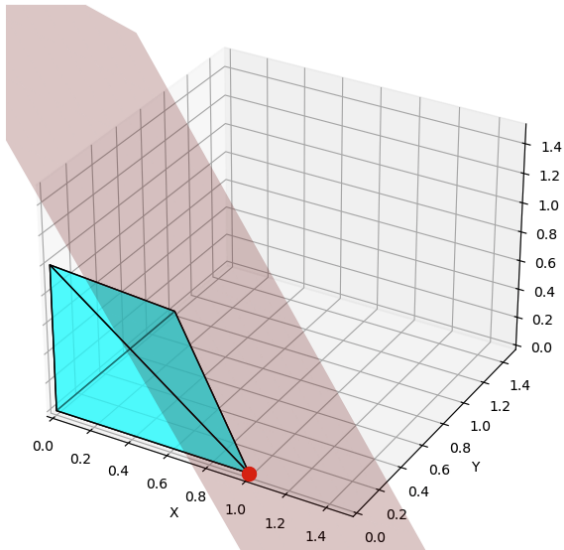
Gurobi: Model is unbounded.

Geometry: The Feasible Region





3D Optimization: Plane Tangent to Vertex



The Fundamental Theorem of LP

Theorem

If a Linear Program has an optimal solution, there exists a **Vertex** (corner point) of the feasible region that is optimal.

Proof Sketch (Convexity Argument):

- 1 Any point x in the polytope is a weighted average (convex combination) of the polytope's vertices v_1, \dots, v_k : $x = \sum \alpha_i v_i$ with $\sum_i \alpha_i = 1, \alpha_i \geq 0$.
- 2 The objective $f(x) = c^T x$ is linear.
- 3 Linearity means $f(x) = f(\sum_i \alpha_i v_i) = \sum_i \alpha_i f(v_i)$.
- 4 An average cannot be larger than the maximum of its components.
- 5 Therefore, $f(x) \leq \max_i f(v_i)$. The max is at a corner!

From Math to Code (Gurobi)

We don't solve these LPs by hand. We assume the **Solver** is a black box.

```
import gurobipy as gp
from gurobipy import GRB

m = gp.Model("factory")

# Variables
x1 = m.addVar(name="widgets")
x2 = m.addVar(name="gadgets")

# Objective
m.setObjective(3*x1 + 4*x2, GRB.MAXIMIZE)

# Constraints
m.addConstr(1*x1 + 2*x2 <= 10, "metal")
m.addConstr(2*x1 + 1*x2 <= 15, "wood")

m.optimize()

print(x1.X, x2.X)
```

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The Scenario

OptiMeal Inc. has a conflict:

- **Finance Team:** “Cut costs! Food is too expensive.”
- **Nutritionists:** “We need to meet daily health requirements.”

Your Mission:

- Use LP to design the cheapest daily meal plan.
- You can eat fractional servings (e.g., 0.5 bananas).
- **Objective:** Min Cost.
- **Constraints:** Calorie floor, Protein floor, Sugar ceiling, etc.

The Data (Nutrition & Costs)

Food	Cost (\$)	Cal	Prot (g)	Carb (g)	Sugar (g)	Fiber (g)	Fat (g)
Chicken	1.80	128	24.0	0.0	0.0	0.0	2.7
Banana	0.30	105	1.3	27.0	14.0	3.1	0.4
Yogurt	0.90	104	5.9	7.9	7.9	0.0	5.5
Beans	1.10	120	8.0	21.0	1.0	7.0	0.5
Spinach	0.40	7	0.9	1.1	0.1	0.7	0.1
Almonds	0.70	160	6.0	6.0	1.0	3.0	14.0

Requirements:

- Calories ≥ 2000
- Protein $\geq 100g$
- Fiber $\geq 50g$
- Sugar $\leq 50g$
- Fat $\leq 120g$
- Sodium $\leq 2300mg$

The Mathematical Model

Let x_j be the number of servings of food j . Let c_j be the cost of food j . Let a_{ij} be the amount of nutrient i in food j .

$$\min \sum_{j \in \text{Foods}} c_j x_j \quad (\text{Minimize Cost})$$

$$\text{s.t. } \sum_j \text{Cal}_j \cdot x_j \geq 2000$$

$$\sum_j \text{Prot}_j \cdot x_j \geq 100$$

$$\sum_j \text{Sugar}_j \cdot x_j \leq 50$$

...

$$x_j \geq 0$$

Implementation in Gurobi

```
params = {
    "Chicken" : { "price": 1.80, "protein": 24.0, "sugar": 0.0, "...": "..." },
    "Banana"   : { "price": 0.30, "protein": 1.3,  "sugar": 14.0, "...": "..." },
    "Yogurt"   : { "price": 0.90, "protein": 5.9,  "sugar": 7.9,  "...": "..." },
    "Beans"    : { "price": 1.10, "protein": 8.0,  "sugar": 1.0,  "...": "..." },
    ...
}
foods = list(params.keys())
# Variables: x[food]
x = m.addVars(foods, lb=0.0, name="servings")
# Objective: Minimize Cost
obj_expr = 0
for food in foods:
    obj_expr += params[food]["price"] * x[food]
m.setObjective(obj_expr, GRB.MINIMIZE)
# Constraints (Example: Protein & Sugar)
const_protein = m.addConstr(
    gp.quicksum( params[fd]["protein"] * x[fd] for fd in foods) >= 100, "min_protein"
)
const_sugar = m.addConstr(gp.quicksum(params[food]["sugar"] * x[food] for food in foods) <= 50, "max_sugar")
# ... rest of the requirements ...
m.optimize()
```

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Reading the Tea Leaves (Gurobi Output)

When you run `m.optimize()`, Gurobi populates attributes on the objects.

Model Attributes:

- `m.Status`: Did it work?
(2=Opt, 3=Infeas, 5=Unbdd)
- `m.ObjVal`: The total profit/cost (Z).

Variable Attributes:

- `var.X`: The optimal value
($x_1 = 6.66$).
- `var.RC`: **Reduced Cost**. How much the objective coefficient must improve before this variable becomes non-zero (More next week).

Constraint Attributes:

- `constr.Slack`: Difference between LHS and RHS.
- `constr.Pi` (π): **Shadow Price**.
"If I had 1 more unit of Metal, how much more profit would I make?". More on this next week!

Warning

Attributes like `.X` and `.Pi` are only available if `m.Status == 2` (Optimal). Always check status first!

Infeasibility Diagnosis

```
import gurobipy as gp
import gurobipy

m = gp.Model("Infeasible")
x = m.addVar(name="x")
m.setObjective(-1*x, gp.GRB.MAXIMIZE)
m.addConstr(x>=3)
m.addConstr(x<=2)
m.optimize()
print("Optimize status:", m.Status)
```

Gurobi Optimizer version 12.0.3 build v12.0.3rc0 (mac64[arm]
- Darwin 23.1.0 23B2073)

CPU model: Apple M3 Max

Thread count: 14 physical cores, 14 logical processors, using up to 14 threads

Optimize a model with 2 rows, 1 columns and 2 nonzeros

Model fingerprint: 0xf5b06d2b

Coefficient statistics:

Matrix range [1e+00, 1e+00]

Objective range [1e+00, 1e+00]

Bounds range [0e+00, 0e+00]

RHS range [2e+00, 3e+00]

Presolve time: 0.00s

Solved in 0 iterations and 0.00 seconds (0.00 work units)

Infeasible model

Optimize status: 3

What about Larger Models?

```
import gurobipy as gp
```

```
m = gp.Model("TrickyInfeasible")
```

```
# Variables
```

```
x = m.addVar(lb=-0, ub=8, name="x")
```

```
y = m.addVar(lb=-0, ub=8, name="y")
```

```
# Arbitrary bounded objective
```

```
m.setObjective(x + y, gp.GRB.MINIMIZE)
```

```
#Constraints
```

```
m.addConstr(2*x + y <= 4, name="c1_budget1")
```

```
m.addConstr(x + 2*y <= 4, name="c2_budget2")
```

```
m.addConstr(x + y >= 5, name="c3_demand")
```

```
m.addConstr(x <= 8, name="c4_x_cap")
```

```
m.addConstr(y <= 8, name="c5_y_cap")
```

```
m.optimize()
```

```
print("Optimize status:", m.Status)
```

Optimize a model with 5 rows, 2 columns and 8 nonzeros

Model fingerprint: 0x00fc1d77

Coefficient statistics:

Matrix range [1e+00, 2e+00]

Objective range [1e+00, 1e+00]

Bounds range [8e+00, 8e+00]

RHS range [4e+00, 8e+00]

Presolve removed 2 rows and 0 columns

Presolve time: 0.01s

Solved in 0 iterations and 0.01 seconds (0.00 work units)

Infeasible model

Optimize status: 3

Irreducible Infeasible Subsystem (IIS)

What is an IIS?

- When a model is **infeasible**, the full set of constraints cannot all be satisfied simultaneously.
- An **IIS** is a *minimal subset of constraints and bounds* that is still infeasible.
- “Minimal” = removing *any* constraint from that subset makes it feasible again.
- IISs help pinpoint the true source of infeasibility in large models.

Good News

Gurobi can compute an IIS for you automatically!

Computing an IIS in Gurobi

If the model is infeasible, we can ask Gurobi to identify the conflicting constraints.

```
if m.Status == GRB.INFEASIBLE:
    print("\nModel is infeasible; computing IIS...")
    m.computeIIS()

    print("Constraints in the IIS:")
    for c in m.getConstrs():
        if c.IISConstr:  # True if part of the IIS
            print(f" {c.ConstrName}")
```

Example IIS Output

Model **is** infeasible; computing IIS...

Iteration	Objective	Primal Inf.	Dual Inf.	Time
0	0.0000000e+00	2.500000e+00	0.000000e+00	0s

IIS computed: 3 constraints **and** 0 bounds

IIS runtime: 0.00 seconds (0.00 work units)

Constraints **in** the IIS:

c1_budget1
c2_budget2
c3_demand

- Remember, these constraints correspond to $2x + y \leq 4$, $x + 2y \leq 4$, and $x + y \geq 5$. Adding the first 2 inequalities contradicts the third.
- These are the **minimal conflicting constraints**.
- Removing any one of them would make the model feasible.
- Great for isolating modeling mistakes in large LPs/MIPs.

Unbounded LPs and Infinite Directions

Unbounded LP = The objective can grow without limit while staying feasible.

Gurobi not only detects unboundedness, it returns an *unbounded ray*.

- An **unbounded ray** is a vector d such that:

$$x + \lambda d \text{ is feasible for all } \lambda > 0$$

and the objective coefficient $c^T d > 0$ (for maximization).

- Gurobi provides this via the attribute:

`var.UnbdRay`

- Nonzero components of the ray indicate which variables “run off to infinity.”

Interpretation

The unbounded ray shows *how* the LP escapes to infinity.

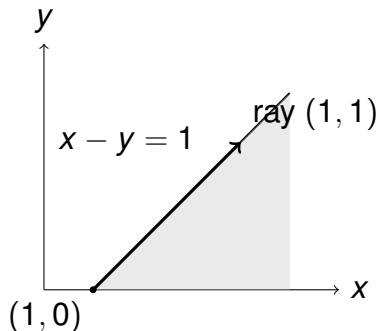
Example of an Unbounded LP?

Example (Maximization):

$$\begin{array}{ll}\max & x + y \\ \text{s.t.} & x - y \geq 1 \\ & x, y \geq 0\end{array}$$

- Feasible region goes to ∞ .
- Objective increases without bound.
- No vertex optimum exists.

Geometry of the Unbounded Ray



Feasible region:

$$x - y \geq 1, \quad x \geq 0, \quad y \geq 0.$$

- From the feasible point $(1, 0)$ we can move along $(x, y) = (1, 0) + \lambda(1, 1) = (1 + \lambda, \lambda)$, $\lambda \geq 0$.
- The objective $x + y$ grows without bound:

$$1 + 2\lambda \rightarrow \infty.$$

- Gurobi's `UnbdRay` returns this direction.

Gurobi Example: Unbounded Model + Ray

```
import gurobipy as gp
from gurobipy import GRB

m = gp.Model("Unbounded")
x = m.addVar(lb=0, name="x")
y = m.addVar(lb=0, name="y")
m.setObjective(x + y, GRB.MAXIMIZE)
m.addConstr(x - y >= 1, name="c1_skew")
# KEY: ask Gurobi to compute ray info
m.setParam(GRB.Param.InfUnbdInfo, 1)
m.optimize()

print("Status:", m.Status)
if m.Status == GRB.UNBOUNDED:
    print("\nUnbounded Ray:")
    for v in m.getVars():
        print(f"{v.VarName}: {v.UnbdRay}")
```

Status: 5

Unbounded Ray:

x: 1.0

y: 1.0

- The ray $(1, 1)$ means both x and y can increase indefinitely.
- The constraint $x - y \geq 1$ stays satisfied for all $(x, y) = (1, 0) + \lambda(1, 1)$.
- Objective grows as $x + y \rightarrow +\infty$.

TODOs after Lecture.

- **Install Gurobi:** Get your academic license working.
- Code and Solve **The Diet Problem** in HW1.
- Use **Tools** like `m.computeIIS()` and `var.UnbdRay` to find the conflict in toy infeasible models and unbounded models.