Assignment 1

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1.Abstract

This assignment centers on the modeling and analysis of wireless communication channels, with a particular focus on path loss characterization, large-scale fading, and small-scale fading. The study utilizes data collected from an outdoor measurement campaign conducted at LTH in Lund and primarily employs MATLAB for data processing and graphical simulations.

Key computations include channel gain, small-scale averaged gain, theoretical free-space channel gain, average received power, and the path loss exponent. In the large-scale fading analysis, the goodness of fit between the modeled CDF and the empirical CDF was evaluated. For small-scale fading, the modeled CDF was compared against the Rayleigh CDF and Rice CDF to assess their respective fitting accuracy.

2.Introduction

In wireless communication, signal propagation from the transmitter to the receiver is influenced by multiple factors. These influences can be classified into Path Loss, Large-Scale Fading, and Small-Scale Fading, each representing a distinct aspect of wireless channel behavior.

In this assignment, the data was collected from an outdoor measurement campaign at LTH in Lund using the RUSK Lund channel sounder at a carrier frequency of 2.6 GHz. The measurement took place in a semi-urban microcell environment at the Faculty of Engineering, with a base station (BS) equipped with a single vertically polarized antenna. The transmit antenna was positioned outside a second-floor window of the Study Center, approximately 6 meters above ground level. The receiver followed a predefined 490-meter route around the lake. For this assignment, only the base station located at the Study Center (BS-S) will be considered.

Path loss describes the gradual attenuation of received signal power as a function of increasing distance and environmental influences in wireless communication. In general, received power decays exponentially with distance. A higher path loss results in a reduced coverage area, which ultimately determines the communication range.

Large-scale fading refers to the slow variations in signal power caused by interactions with objects significantly larger than the wavelength, such as terrain, buildings, and other obstacles. This phenomenon manifests as random fluctuations in path loss and is commonly referred to as shadowing. Specifically, signal strength is higher in open areas, weaker behind buildings, and undergoes significant attenuation in tunnels or underground environments.

Small-scale fading, in contrast, represents rapid fluctuations in received signal power over short spatial distances or brief time intervals, primarily due to multipath propagation. As signals traverse multiple paths before reaching the receiver, they interfere constructively or destructively, leading to rapid variations in received power.

Each of these three propagation effects corresponds to distinct mathematical models, which will be discussed in detail in Part 3.

3. Methodology and Technical Results

3.1 T1 and T2

The code given used following formula to calculate the channel gain:

$$Channel\ Gain = 10\ log_{10}(P_{Rx}) \tag{1}$$

$$P_{RX} = \left|RxSignal_{amp}\right|^2 \tag{2}$$

Where P_{Rx} represents the received signal power. The received power is obtained by computing the squared magnitude of the complex amplitude of the received signal. Since the transmitted power has been removed, the resulting values directly represent the channel gain in dB.

To mitigate the effects of small-scale fading and emphasize large-scale fading trends, the received power and signal amplitude are processed through local averaging and normalization. The code given calculates the small-scale averaged received power over a window of 11 samples (5 preceding, 1 current, and 5 subsequent samples) as:

$$P_{SSA}(i) = \frac{1}{11}\sum_{x=i-5}^{i+5} P_{RX}(x) \tag{3}$$

Additionally, the small-scale fading amplitude is computed by normalizing the received signal amplitude with its local average over the same window. This normalization helps to isolate small-scale fading variations by removing the influence of large-scale fading:

$$SSF_{amp}(i) = \frac{|RxSignal_{amp}(i)|}{\frac{1}{11}\sum_{x=i-5}^{i+5} RxSignal_{amp}(x)} \tag{4}$$

For theoretical comparison, the code given computes the free-space channel gain using the free-space path loss model:

$$PL_{FS} = -20\log_{10}\left(\frac{4\pi d}{\lambda}\right) \tag{5}$$

where d is the transmitter-receiver separation distance, and λ is the wavelength of the transmitted signal, given by $\lambda = c/f$, with c as the speed of light and f as the carrier frequency.

The small-scale averaged channel gain is assumed to follow the log-distance path loss model, which is mathematically expressed as:

$$\bar{P}(d) = P(d_0) - 10nlog_{10}\left(\frac{d}{d_0}\right) \tag{6}$$

where $P(d_0)$ is the average received power (in dB) at a reference distance of $d_0 = 1m$, and n is the pathloss exponent. We use function polyfit function in Matlab to fit the data in $\bar{P}(d)$ in a least-squares sense [1].

From the regression analysis, **the pathloss exponent $\boldsymbol{n = 2.24}$, and pathloss**

at $d_0 = 1m$: $P(d_0) = -41.44\ dB$.

Based on the given data, the gains computed are shown in Figure 1.

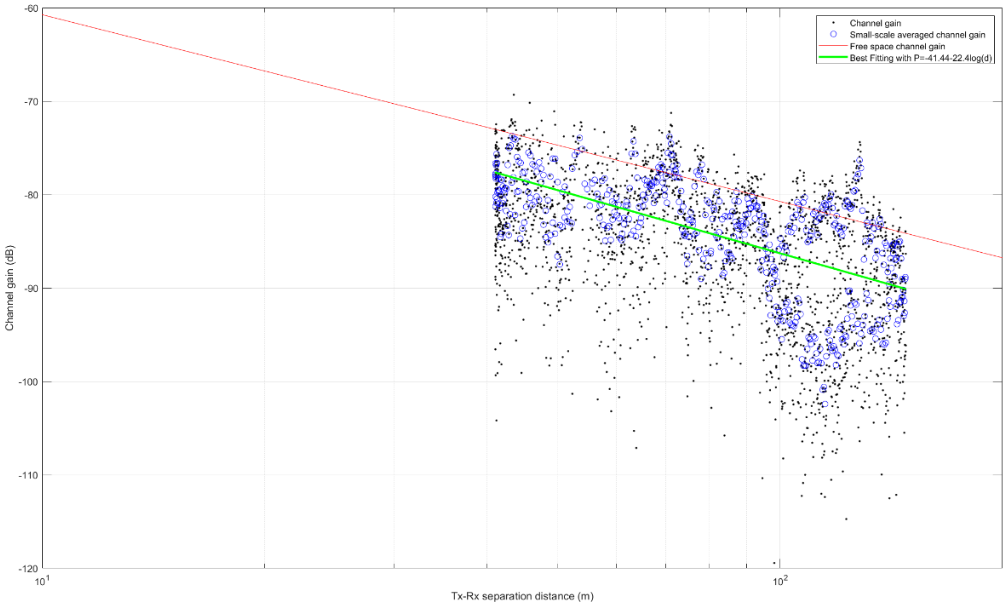

Fig 1. Comparison of Measured Channel Gain, Small-Scale Averaged Channel Gain, and Path Loss Models

According to the Power-Distance Law [2], the path loss exponent in free-space is typically $n = 2$. In LOS with multipath, it is slightly lower than 2, while in NLOS, it ranges from 2 to 5, depending on obstruction severity. A distinction exists between OLOS and severe NLOS. OLOS occurs when small objects partially obstruct LOS, with $n \approx 2 - 3$. Severe NLOS involves large obstructions or dense clutter, resulting in $n \approx 3 - 5$.

It can be characterized that it's **obstructed line-of-sight (OLOS) propagation** because of the propagation condition in the measurement based on the $\boldsymbol{n = 2.24}$ calculated above**.**

3.2 T3

The large-scale fading can be estimated with the following formula:

$$\widehat{LSF}|_{dB} = P_{SSA} - \bar{P}(d) \tag{7}$$

Where $\bar{P}(d)$ is the average channel gain we get from equation (1), P_{SSA} is the distance dependent average channel gain from the measured small scale averaged power.

The cdfplot function in MATLAB was utilized to visualize the empirical cumulative distribution function (CDF) of the $\widehat{LSF}|_{dB}$, derived from the measured received power.

In dB units, the large-scale fading can often be modeled by a normal distribution. Consequently, the theoretical CDF of the large-scale fading model follows a normal distribution, which is mathematically expressed as:

$$\frac{1}{2}[1+\mathrm{erf}\left(\frac{x-\mu_{LSF}}{\sigma_{LSF}\sqrt{2}}\right)] \tag{8}$$

Where μ_{LSF} is the mean of the large-scale fading and σ_{LSF}^2 is the variance of the large-scale fading, which can be calculated by following maximum-likelihood equations of a normal distribution:

$$\mu_{LSF}=\frac{1}{N}\sum_{i=1}^{N}LSF(i) \tag{9}$$

$$\sigma_{LSF}^2=\frac{1}{N}\sum_{i=1}^{N}(LSF(i)-\mu_{LSF})^2 \tag{10}$$

Figure 2 presents the empirical CDF (blue curve) alongside the modeled CDF (red curve). The close alignment between the two curves indicates that **the normal distribution provides a reasonable approximation of the large-scale fading characteristics**.

We used Kolmogorov-Smirnov (KS) test to test the goodness of fit between the empirical CDF and modeled CDF, i.e., it tests the hypothesis that the empirical samples come from the theoretical distribution (with the estimated optimum parameters) [2]. We used the kstest function in Matlab to implement it. The large-scale fading KS Test Results is $h=0,\ p=0.109327,\ ksstat=0.054502,\ cv=0.061440$, where h is hypothesis test result, p is the probability of observing a test statistic that is as extreme as, or more extreme than, the observed value under the null hypothesis, $ksstat$ is Test statistic of the hypothesis test, cv is critical value [3].

These results indicate that the normal distribution agrees well with the empirical large-scale fading data, which is align with figure 2.

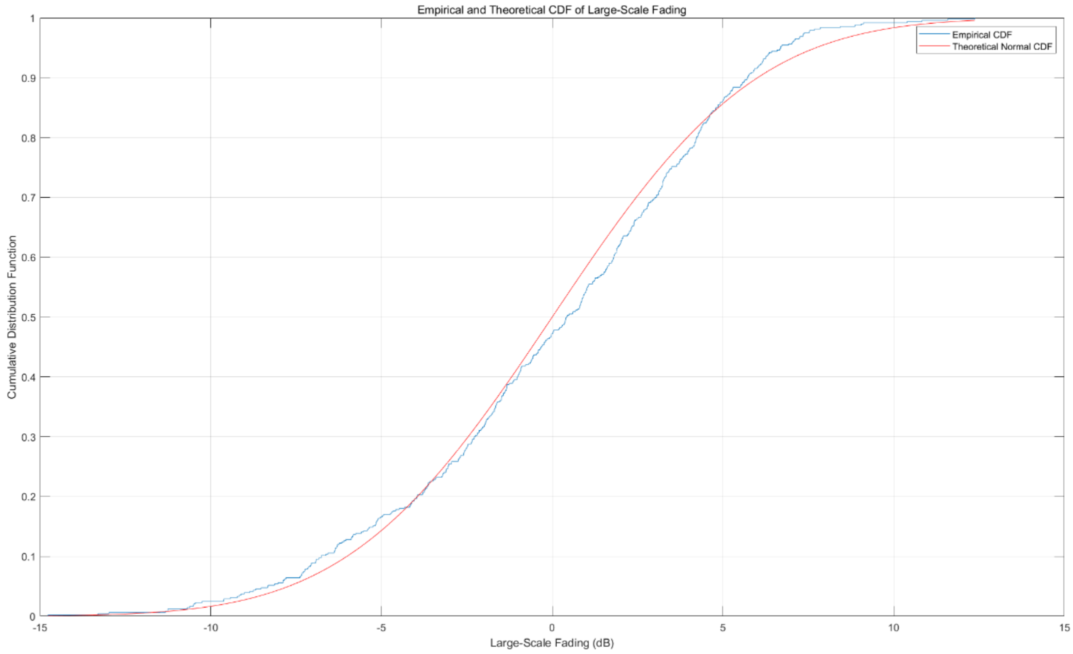

Fig 2. Empirical and Theoretical CDF of Large-Scale Fading

Based on the measured data, the probability of large-scale fading **exceeding 8 dB above the mean is 0.0166, whereas the probability derived from the theoretical model is 0.0441**. **These results are not that comparable**. We can see from the figure 2 that the empirical curve deviates from the theoretical normal CDF between 5 dB and 10 dB, particularly for large positive deviations. While the theoretical model captures the general trend of the large-scale fading characteristics, its limitations in modeling tail behavior lead to discrepancies in probability predictions for extreme values.

3.3 T4

The cdfplot function in Matlab was used to plot the empirical cumulative distribution function (CDF) of the SSF_{amp} we measured. Subsequently, the empirical CDF was compared with the theoretical CDF of the Rayleigh distribution:

$$cdf(r) = 1 - \exp\left(-\frac{x^2}{2\sigma_{SSF}^2}\right) \tag{11}$$

Where σ_{SSF}^2 is the estimate of the square of the scale parameter for the Rayleigh distribution, based on the measured small-scale fading:

$$\sigma_{SSF}^2 = \frac{1}{2N}\sum_{i=1}^{N} SSF_{amp}(i)^2 \tag{12}$$

The empirical CDF in blue curve and modeled CDF of the Rayleigh distribution in red curve are shown in figure 3. It can be concluded that the modeled CDF of the Rayleigh distribution **doesn't have a good fit** with the empirical CDF for the small-scale fading.

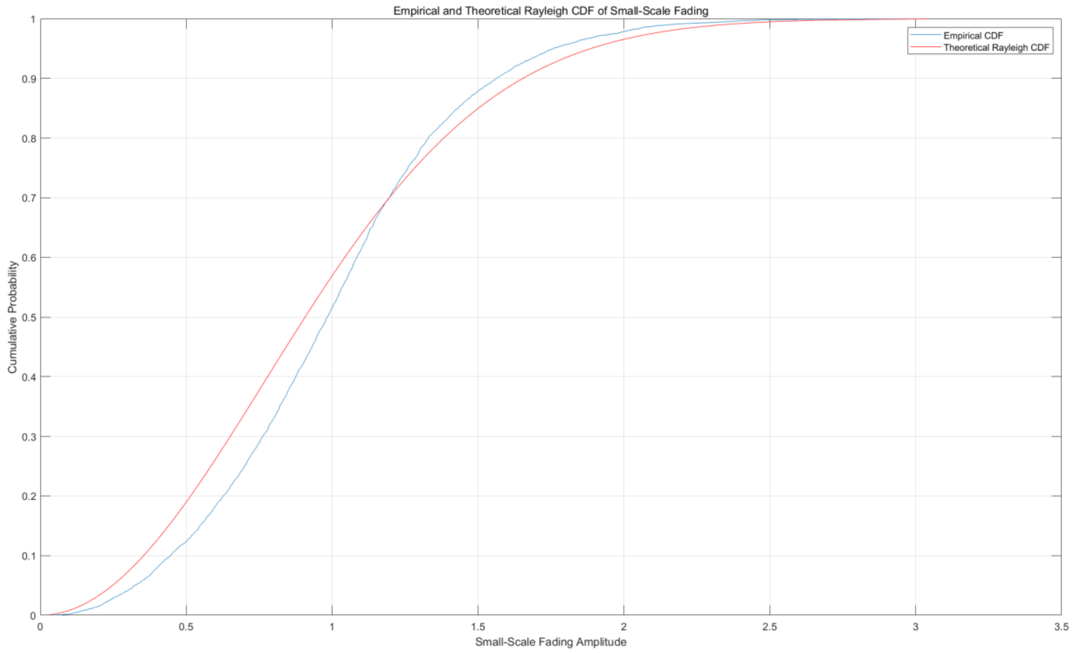

Fig 3. Empirical and Theoretical Rayleigh CDF of Small-Scale Fading

3.4 T5

If there is a dominant component present in the measurement, then a Ricean distribution could perhaps be a more valid model for the small-scale fading. The CDF of the Rice distribution is given by:

$$1 - Q\left(\frac{\upsilon}{\sigma_{Rice}}, \frac{x}{\sigma_{Rice}}\right) \tag{14}$$

Where υ is the amplitude of the dominant component, σ_{Rice} is standard deviation of the Rice distribution, and we are given estimates for the Rice distribution that $\upsilon = 0.84185$ and $\sigma_{Rice} = 0.489$. Q is the Marcum Q-function given by:

$$Q(a,b) = e^{-\frac{a^2+b^2}{2}} \sum_{n}^{\infty} \left(\frac{a}{b}\right)^n I_n(ab) \tag{15}$$

$I_n(\cdot)$ is the modified Bessel function of the first kind, order n. It is given by:

$$I_n(x) = \frac{1}{\pi} \int_0^{\pi} e^{x cos\theta} \cos(n\theta)\, d\theta \tag{16}$$

where n is integer.

Figure 4 presents a comparison between the empirical CDF and the modeled CDFs of the Rayleigh and Rice distributions. The results indicate that **the Rice distribution provides the best fit to the empirical CDF**. In contrast, a discrepancy is observed between the Rayleigh model and the measured data.

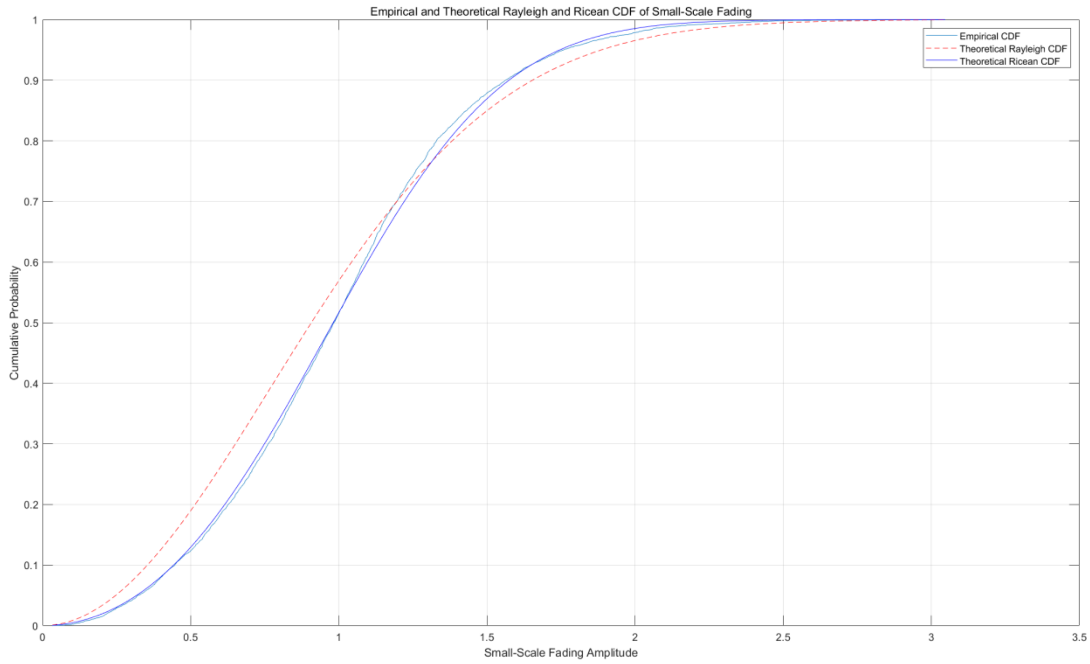

Fig.4 Empirical and Theoretical Rayleigh and Ricean CDF of Small-Scale Fading

However, The KS test results for the small-scale fading indicate that neither the Rayleigh nor the Rice distribution provides an adequate fit for the empirical data. Specifically, for the Rayleigh distribution $h = 1$, $p = 0.000000$, $ksstat = 0.089329$, $cv = 0.018600$, showing a significant deviation between the empirical CDF and the Rayleigh model. Similarly, for the Rice distribution, $h = 1$, $p = 0.002702$, $ksstat = 0.024903$, $cv = 0.018600$, also rejecting the null hypothesis, albeit with a higher p-value compared to the Rayleigh distribution. **This indicates a considerable difference between the two models**, with the Rayleigh distribution deviating more significantly from the empirical data than the Rice distribution.

Based on the figure 4, it is evident that the Rice distribution aligns more closely with the empirical CDF compared to the Rayleigh distribution. While the KS test results show that both distributions fail to provide an adequate fit to the data. This discrepancy between the statistical test results and the visual fit may be attributed to the large sample size of the small-scale fading data (5313 samples), which increases the sensitivity of the KS test to even minor deviations. Consequently, while the KS test formally rejects both models, the Rice distribution is a more reasonable approximation of the empirical CDF, as reflected in the visual comparison.

4. Conclusions

This study demonstrates that path loss and fading models can effectively characterize wireless signal propagation when properly estimated and validated against empirical data.

In task 1 and 2, The measured channel gain, small-scale averaged gain, and path loss models were analyzed. From the regression analysis, the pathloss exponent $n = 2.24$, and pathloss at $d_0 = 1m$: $P(d_0) = -41.44\ dB$, indicating obstructed line-of-sight (OLOS) propagation.

In task 3, the empirical CDF of large-scale fading closely follows a normal distribution, validating its statistical representation.

In task 4, comparison of small-scale fading CDF with the Rayleigh distribution showed a mismatch, suggesting the presence of a dominant component.

In task 5, the Ricean distribution provided a significantly better fit than Rayleigh, confirming a dominant component and making it the more suitable model for small-scale fading characterization.

This real-world scenario aligns with the characteristics of a generalized Rice distribution, often referred to as the two-path model with diffuse power. This distribution is particularly suitable when there are two strong signal components, such as a LOS path and a ground-reflected path, in addition to a number of small components [2]. At the measurement site, the primary signal paths likely include the LOS and reflections from the ground or lake surface. However, the LOS path could be obstructed by trees, buildings, or other objects, making the multipath environment more complex. As a result, the pathloss exponent $n = 2.24$ indicates that the propagation condition is obstructed line-of-sight (OLOS) propagation, which is align with the real-world scenario.

Such real-world complexities, including mixed multipath propagation, shadowing, and interference, introduce deviations from idealized theoretical models. As a result, while the KS test results suggest that neither the Rice nor Rayleigh distribution perfectly fits the empirical data. Nevertheless, the visual alignment of the empirical CDF with the Rice distribution from figure 4 suggests that it provides a reasonable approximation of the measured data, despite being statistically rejected.

5. References

[1] MathWorks, Polynomial Curve Fitting (polyfit), MATLAB Documentation, 2025. [Online]. Available: https://ww2.mathworks.cn/help/matlab/ref/polyfit.html.

[2] Molisch, Andreas F. Wireless communications: from fundamentals to beyond 5G. John Wiley & Sons, 2022.

[3] MathWorks, Kolmogorov-Smirnov Test (kstest), MATLAB Documentation, 2025. [Online]. Available: https://ww2.mathworks.cn/help/stats/kstest.html.

6. Appendix

```
clear; %Clear all variables in the workspace
close all; %Closes all figure windows
clc %Clear command window
```

```
%Load the data for Assignment 1, which
%contains the variables ComplexAmplitude and TxRxDist:
load('Assignment1');

%T1:
RxPow = (abs(ComplexAmplitude).^2); %Received power

%(note: the Tx power has been removed, so the data is in dB, not in dBm!)

Dist_dB = 10 * log10(TxRxDist); %Tx-Rx separation in dB.

D_win = [];
P_SSA = [];
ssfamplitude = []; %preallocation of variables in the for-loop
for i = 6:5:length(ComplexAmplitude) - 5
    D_win = [D_win; mean(TxRxDist(i-5:i+5))]; %Average TxRxDistance for the small-scale averaged power
    P_SSA = [P_SSA; mean(RxPow(i-5:i+5))]; %Small scale averaged power.

    ratio = (abs(ComplexAmplitude(i-5:i+5))) ./ (mean(abs(ComplexAmplitude(i-5:i+5))));
    ssfamplitude = [ssfamplitude, ratio]; %Small-scale fading amplitude based on the small-scale averaged power.
end
figure(1) %new figure window
semilogx((TxRxDist), 10*log10(RxPow), 'k.') %plot distance against received power, with logarithmic x-axis.
xlabel('Tx-Rx separation distance (m)') %label for the x-axis
ylabel('Channel gain (dB)') %label for the y-axis.
grid on %add grid
```

```
hold on %hold on, to be able to plot another figure, while keeping the old plot.
semilogx((D_win), 10*log10(P_SSA), 'bo') %Plot small-scale averaged channel gain

%A plot of the free-space pathloss at 2.4 GHz, using the same figure as above:
%I changed the frequency to 2.6e9 for its said that using a carrier frequency of 2.6 GHz
semilogx(([10, 200]), -20*log10(4*pi*[10, 200]/(3e8 / 2.6e9)), 'r-')
%A legend of for window 1:
legend('Channel gain', 'Small-scale averaged channel gain', 'Free space channel gain')
%-------------------------------------------------

%T2: Find the ordinary least-squares estimate of PL(d0)
%and pathloss exponent n:

%Plot the average received power using the pathloss estimates found above:

%\bar{P}(d)|_{dB}=(-10n)log_10(d)+P(d_0)=p_1*x+p_2
x = log10(D_win);
y = 10 * log10(P_SSA);

% Perform least-squares estimation
% p(x)=p_1*x+p_2
[p, S] = polyfit(x, y, 1); % Fit a first-degree polynomial (linear regression)

n = -p(1) / 10;
PL_d0 = p(2);

fprintf('pathloss exponent: n = %.2f\n', n);
```

```
fprintf('pathloss at d0: PL(d0) = %.2f dB\n', PL_d0);

semilogx(D_win, polyval(p, x), 'g-', 'LineWidth', 2);

legend('Channel gain', 'Small-scale averaged channel gain', 'Free space channel gain', 'Best Fitting with P=-41.44-22.4log(d)');

%--------------------------------------------------

%T3: In a new window, plot the empirical cdf of the large-scale fading from
%the measurement:

%Plot the CDF for the normal distribution based on the maximum-likelihood estimate
%for a normal distribution:
LSF = 10 * log10(P_SSA) - polyval(p, x);
figure(2)
cdfplot(LSF);
hold on;

% mu_LSF = mean(LSF);
mu_LSF = sum(LSF) / length(LSF);
% sigma_LSF = std(LSF,1);
sigma_LSF = sqrt(sum((LSF - mu_LSF).^2)/length(LSF));



x_LSF_cdf = linspace(min(LSF), max(LSF), length(LSF));

normal_CDF = 0.5 * (1 + erf((x_LSF_cdf - mu_LSF)/(sigma_LSF * sqrt(2))));
plot(x_LSF_cdf, normal_CDF, 'r-');
```

```
% test = cdf('Normal',LSF,mu_LSF,sigma_LSF);
% plot(sort(LSF), sort(test), 'g-');

xlabel('Large-Scale Fading (dB)');
ylabel('Cumulative Distribution Function');
legend('Empirical CDF', 'Theoretical Normal CDF');
grid on;
title('Empirical and Theoretical CDF of Large-Scale Fading');

empirical_prob_LSF = sum(LSF > mu_LSF+8) / length(LSF);
theoretical_prob_LSF = 1 - (0.5 * (1 + erf((mu_LSF + 8 - mu_LSF)/(sigma_LSF * sqrt(2)))));

fprintf('Empirical probability of LSF > %.2f dB: %.4f\n', mu_LSF+8, empirical_prob_LSF);
fprintf('Theoretical probability of LSF > %.2f dB: %.4f\n', mu_LSF+8, theoretical_prob_LSF);

% Kolmogorov-Smirnov(KS) test
[h_KS_LSF_Norm, p_KS_LSF_Norm, KSstat_LSF_Norm, cv_KS_LSF_Norm] = kstest(LSF, 'CDF', [x_LSF_cdf(:), normal_CDF(:)]);
% [h_KS_LSF_Norm, p_KS_LSF_Norm, KSstat_LSF_Norm, cv_KS_LSF_Norm] = kstest(LSF, 'CDF', test_cdf);
% test_cdf = [LSF(:), cdf('Normal', LSF, mu_LSF, sigma_LSF)];
fprintf('large-scale fading KS test results:\n');
fprintf('h=%d, p = %.6f, ksstat = %.6f, cv = %.6f\n', h_KS_LSF_Norm, p_KS_LSF_Norm, KSstat_LSF_Norm, cv_KS_LSF_Norm);

% Akaike Information Criterion(AIC)
```

```
%------------------------------------------------

%T4: In a new window, plot the empirical cdf of the small scale fading amplitude
figure(3)
SSFamp = ssfamplitude(:);
cdfplot(SSFamp);
hold on;

hat_sigma2_R = 1 / (2 * length(SSFamp)) * sum(SSFamp.^2);
x_SSF_cdf = linspace(min(SSFamp), max(SSFamp), length(SSFamp));
rayleigh_CDF = 1 - exp(-x_SSF_cdf.^2/(2 * hat_sigma2_R));
plot(x_SSF_cdf, rayleigh_CDF, 'r-');

xlabel('Small-Scale Fading Amplitude');
ylabel('Cumulative Probability');
legend('Empirical CDF', 'Theoretical Rayleigh CDF');
title('Empirical and Theoretical Rayleigh CDF of Small-Scale Fading');
grid on;



%------------------------------------------------

%Use the maximum-likelihood expression for the
%Rayleigh distribution to find an estimate of the scale
%parameter sigma:

%------------------------------------------------
%Plot the cdf of the Rayleigh distribution, using
% the estimate obtained in T5:
```

```
%------------------------------------------------

%------------------------------------------------
%T5: Plot the cdf for the Rice distribution with the
%parameter estimates given in the assignment. Hint:
%Type "help marcumq".

figure(4)
sigRice = 0.489;
upsilon = 0.84185;
ricean_CDF = 1 - marcumq(upsilon/sigRice, x_SSF_cdf/sigRice);

cdfplot(SSFamp);
hold on;

plot(x_SSF_cdf, rayleigh_CDF, 'r--');

plot(x_SSF_cdf, ricean_CDF, 'b-');

xlabel('Small-Scale Fading Amplitude');
ylabel('Cumulative Probability');
legend('Empirical CDF', 'Theoretical Rayleigh CDF', 'Theoretical Ricean CDF');
title('Empirical and Theoretical Rayleigh and Ricean CDF of Small-Scale Fading');
grid on;

% Kolmogorov-Smirnov(KS) test
[h_KS_SSF_Ray, p_KS_SSF_Ray, KSstat_SSF_Ray, cv_KS_SSF_Ray] = kstest(SSFamp, 'CDF', [x_SSF_cdf(:),rayleigh_CDF(:)]);
% test_cdf_SSF_Ray = [SSFamp, cdf('Rayleigh', SSFamp, sqrt(hat_sigma2_R))];
```

```
% [h_KS_SSF_Ray, p_KS_SSF_Ray, KSstat_SSF_Ray, cv_KS_SSF_Ray] = kstest(SSFamp, 'CDF', test_cdf_SSF_Ray);
% plot(sort(SSFamp), sort(cdf('Rayleigh', SSFamp, sqrt(hat_sigma2_R))), 'g--');

[h_KS_SSF_Rice, p_KS_SSF_Rice, KSstat_SSF_Rice, cv_KS_SSF_Rice] = kstest(SSFamp, 'CDF', [x_SSF_cdf(:),ricean_CDF(:)]);
% test_cdf_SSF_Rice = [SSFamp, cdf('Rician', SSFamp, upsilon, sigRice)];
% [h_KS_SSF_Rice, p_KS_SSF_Rice, KSstat_SSF_Rice, cv_KS_SSF_Rice] = kstest(SSFamp, 'CDF', test_cdf_SSF_Rice);
% plot(sort(SSFamp), sort(cdf('Rician', SSFamp, upsilon, sigRice)), 'g');

fprintf('\nsmall-scale fading (Rayleigh) KS test results:\n');
fprintf('h=%d, p = %.6f, ksstat = %.6f, cv = %.6f\n', h_KS_SSF_Ray, p_KS_SSF_Ray, KSstat_SSF_Ray, cv_KS_SSF_Ray);
fprintf('\nsmall-scale fading (Rice) KS test results:\n');
fprintf('h=%d, p = %.6f, ksstat = %.6f, cv = %.6f\n', h_KS_SSF_Rice, p_KS_SSF_Rice, KSstat_SSF_Rice, cv_KS_SSF_Rice);

%------------------------
```