Relation and Function

Relation and Function are two ways of establishing links between two sets in mathematics.
Relation and Function in maths are analogous to the relation that we see in our daily lives i.e., two persons are related by the relation of father-son, mother-daughter, brother-sister, and many more. On a similar pattern, two numbers can be related to each other as one number is the square of another number and many more.
A function is a special kind of relation that is defined as a unique relation between two mathematical entities.

What is Relation and Function?

Relation and Functions are the ways of mapping the establishing link between two entities in mathematics. They are used to establish mathematical relations between two terms. Relation and Function are studied under algebra and also used in calculus to find integration and differentiation.

Relation Definition

Relation is defined on a non-empty set A to no-empty set B such that Relation from A to B is a subset of Cartesian Product of A and B i.e. $R \subseteq A \times B$.

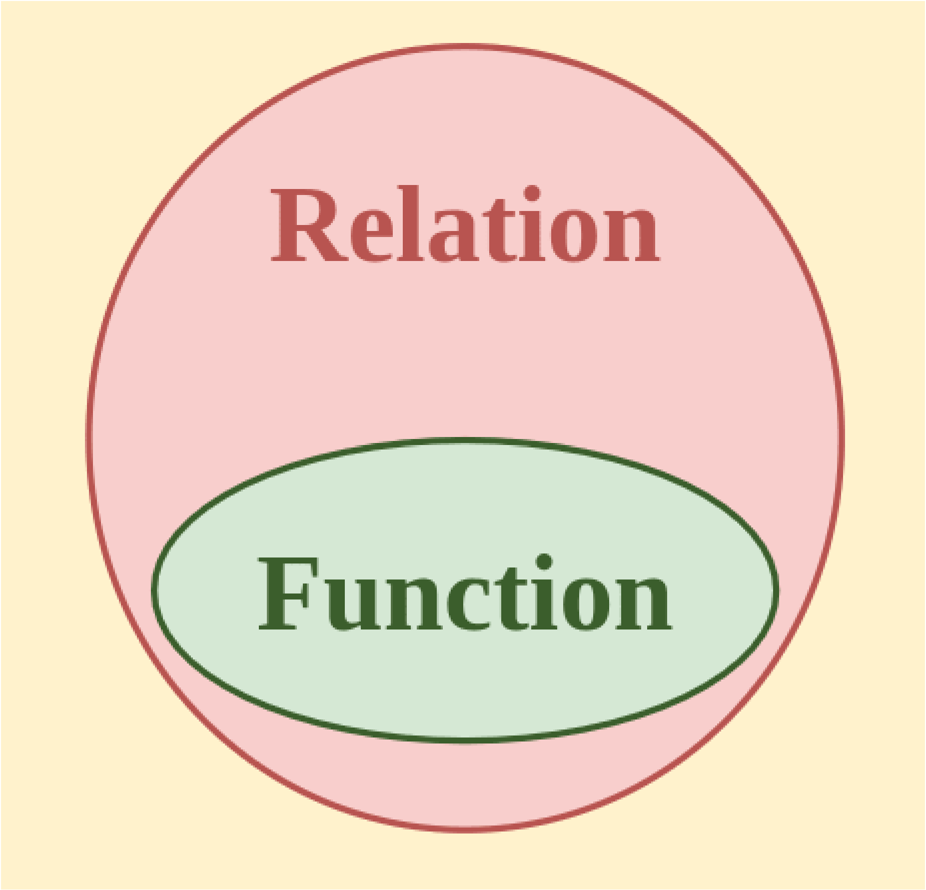
For Example if **A = {a, b, c}** and **B = {p, q, r}**,
Then **A × B = {(a, p), (a, q), (a, r), (b, p), (b, q), (b, r), (c, p), (c, q), (c, r)}**.
If there is another set are that is defined as **R = {(a, p), (b, q), (c, r)}** then we see that R is a subset of **A × B**.
Hence, R is a relation from set A to set B.

Function Definition

Function is a special type of relation in which the two entities are exclusively related to each other only. A relation from set A to set B is defined as function if all the elements of set A are related to at least one element of B and no two elements of B are related to a single element of A.

Here, the element of set A are called pre-image and the element of set B are called image. Hence a function from A to B is defined only when each pre-image in set A has an image in B and no two different images in B has a single pre-image in A.

It should be noted that every function is a relation but not every relation is a function.



Representation of Relation and Function

Relation and Function are in general the same with some basic difference. They both take input, process it and relates to output. They can be represented in the following forms:

Roster Form

In roster form the elements of two sets among which relation is defined are written in the form of ordered pair.

For Example, if A = {-1, 0, 1, 2} and B = {0, 1, 2, 3, 4} and set A is related to set B as $a^2 = b$ where a is an element of set A and b is an element of set B then R = {(-1, 1), (0, 0), (1, 1), (2, 4)}.

Set Builder Form

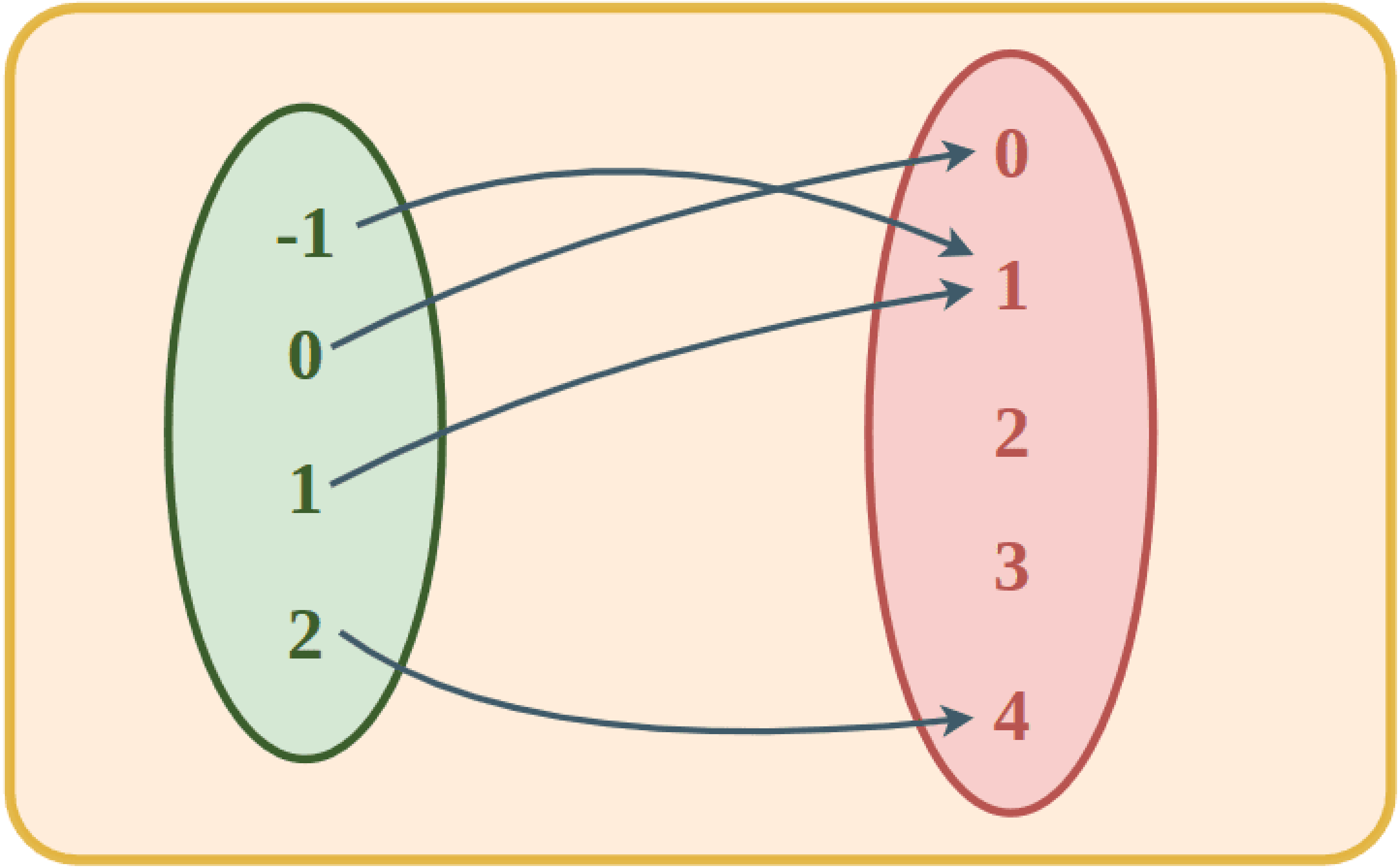
In set builder form the relation is not written in expanded pair form rather it is written in compressed form using an algebraic expression to define the relation between two sets.

For Example, if A = {-1, 0, 1, 2} and B = {0, 1, 2, 3, 4} and set A is related to set B as $a^2 = b$ where a is an element of set A and b is an element of set B then Relation in Set Builder form is given as $R = \{(a, b): a \in A, b \in B \text{ and } b = a^2\}$

Arrow Diagram

In arrow diagram the relation is shown using connecting the elements of the sets which are contained in the box using arrows.

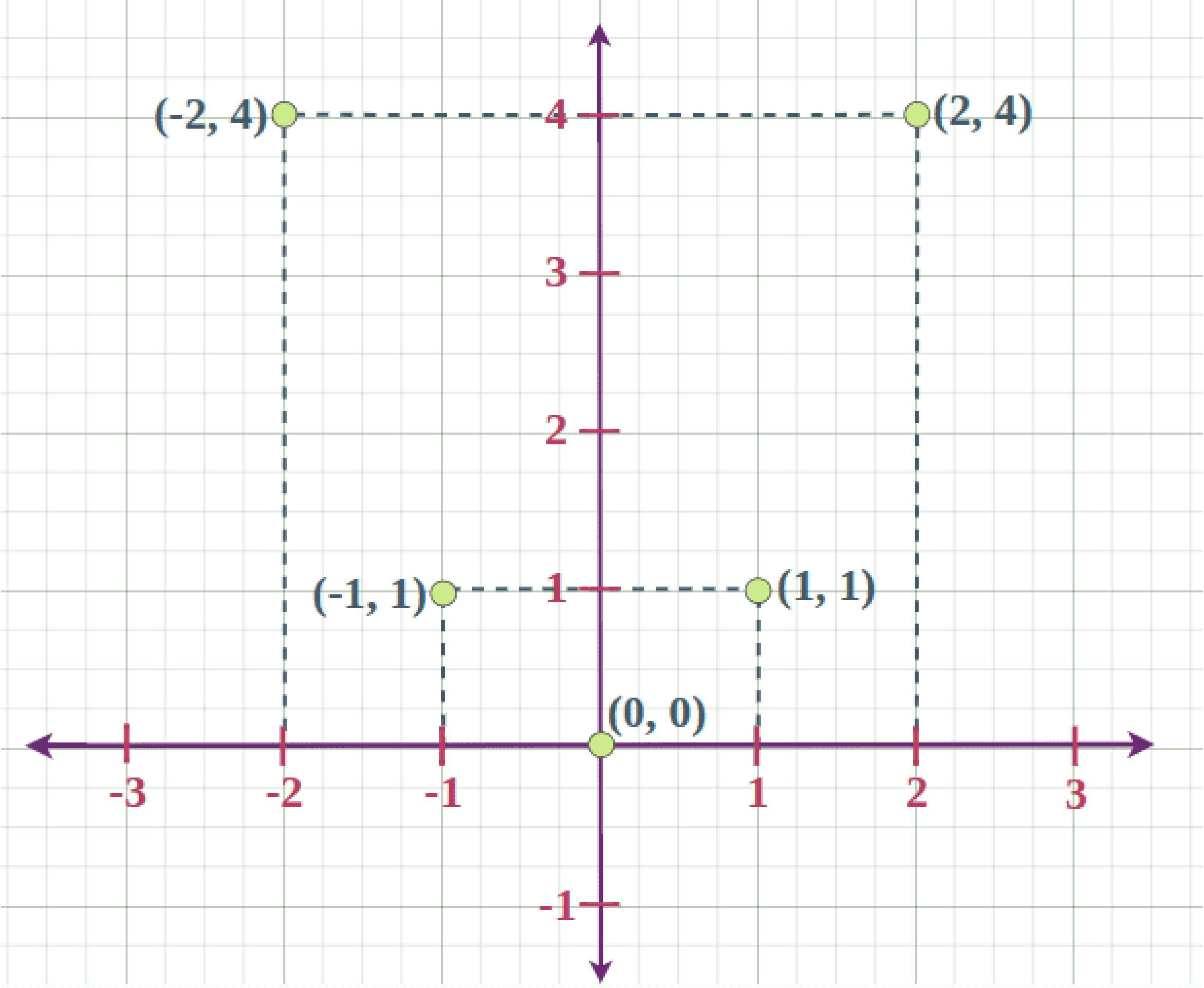
For Example, if A = {-1, 0, 1, 2} and B = {0, 1, 2, 3, 4} and set A is related to set B as $a^2 = b$ where a is an element of set A and b is an element of set B then Relation in Arrow Diagram is given as follows:



Lattice Diagram

In lattice diagrams the elements which are linked to each other by a relation are plotted on cartesian plane.

For Example, if A = {-1, 0, 1, -2, 3} and B = {0, 1, 2, 3, 4, 9} and set A is related to set B as $a^2 = b$ where a is an element of set A and b is an element of set B then Relation in Lattice Diagram is given as follows.



Terms Related to Relation and Function

Some of the commonly used terms associated with Relation and Function are discussed below:

Domain

Domain of Relation or a function is the set of inputs for which the outputs are obtained. For Example, in A = {-1, 0, 1, 2} and B = {0, 1, 2, 3, 4} and set A is related to set B as $a^2 = b$, the set A is the domain of the relation.

Codomain

Codomain is the set of outputs or the image of the relation and function. Codomain may contain exact or more number of elements than the output. For Example, in A = {-1, 0, 1, 2} and B = {0, 1, 2, 3, 4} and set A is related to set B as $a^2 = b$, set B is the codomain. In set B there is element 3 which is not a perfect square hence it will not have a pre-image.

Range

Range is the set of all outputs which has a pre image. In range all elements are related. Hence, it has has exact number of elements for which relation is defined. Thus, Range is subset of codomain. For Example, in A = {-1, 0, 1, 2} and B = {0, 1, 2, 3, 4} and set A is related to set B as $a^2 = b$, Range is {0, 1, 2, 4}.

Cartesian Product

Let's assume A and B to be two non-empty sets, the sets of all ordered pairs (x, y) where $x \in A$ and $y \in B$ is called a Cartesian product of the sets. $A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}$

Types of Relation and Function

Relation and Function are classified on the basis of the input it take and output it gives for a given relation. The different types of Relation and Function are discussed separately below:

T

ypes of Relation

There are eight different types of relations which are listed below:

- ***Empty Relation-*** *There is no relation between any elements of a set.*
- ***Universal Relation-*** *Every element of the set is related to each other.*
- ***Identity Relation-*** *In an identity relation, every element of a set is related to itself only.*
- ***Inverse Relation-*** *Inverse relation is seen when a set has elements that are inverse pairs of another set.*
- ***Reflexive Relation-*** *In a reflexive relation, every element maps to itself.*
- ***Symmetric Relation-*** *In asymmetric relation, if a=b is true then b=a is also true.*
- ***Transitive Relation-*** *For transitive relation, if $(x, y) \in R$, $(y, z) \in R$, then $(x, z) \in R$.*
- ***Equivalence Relation-*** *A relation that is symmetric, transitive, and reflexive at the same time.*

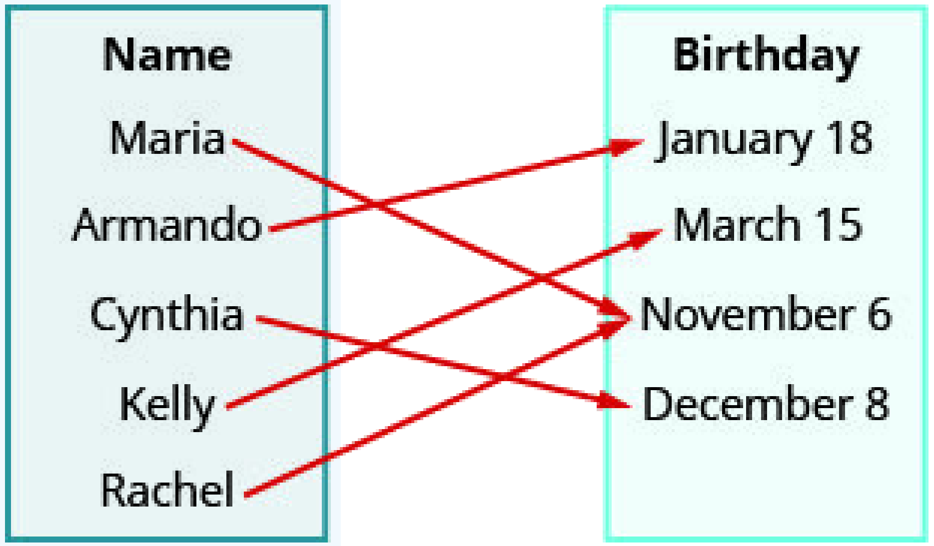
Types of Function

We know that in a function no two images can have one common pre-image and all the pre-images must have an image. A function ‘f’ defined from A × B is classified as follows:

- ***One-One Function(Injection):*** *A function ‘f’ from A to B is said to be One-One or Injection if each element of A is mapped with a different element in B. One-One Function is also called an Injective Function.*
- ***Many-One Function:*** *A function ‘f’ from A to B is said to be Many-One or Injection if two or more elements of A is mapped with a common element in B. It means two elemts in A can have common image in B.*
- ***Onto Function(Surjection):*** *A function ‘f’ from A to B is said to be Onto Function if all the elements of set B has a pre-image in set A i.e. no element in set B remains unmapped.*
- ***Into Function:*** *A function ‘f’ from A to B is said to be Into Function if at least one image in set B does not have a pre-image in set A i.e. one element of set B remains unmapped.*
- **One-One Onto Function:** A function ‘f’ from A to B is said to be One-One Onto function if all the elements of set A has a unique image in set B and all the elements of set B has a pre-image in set A. This type of function exhibits characteristics of One One Function and Onto Function. One-One Onto Function is also called Bijection Function.

Practice

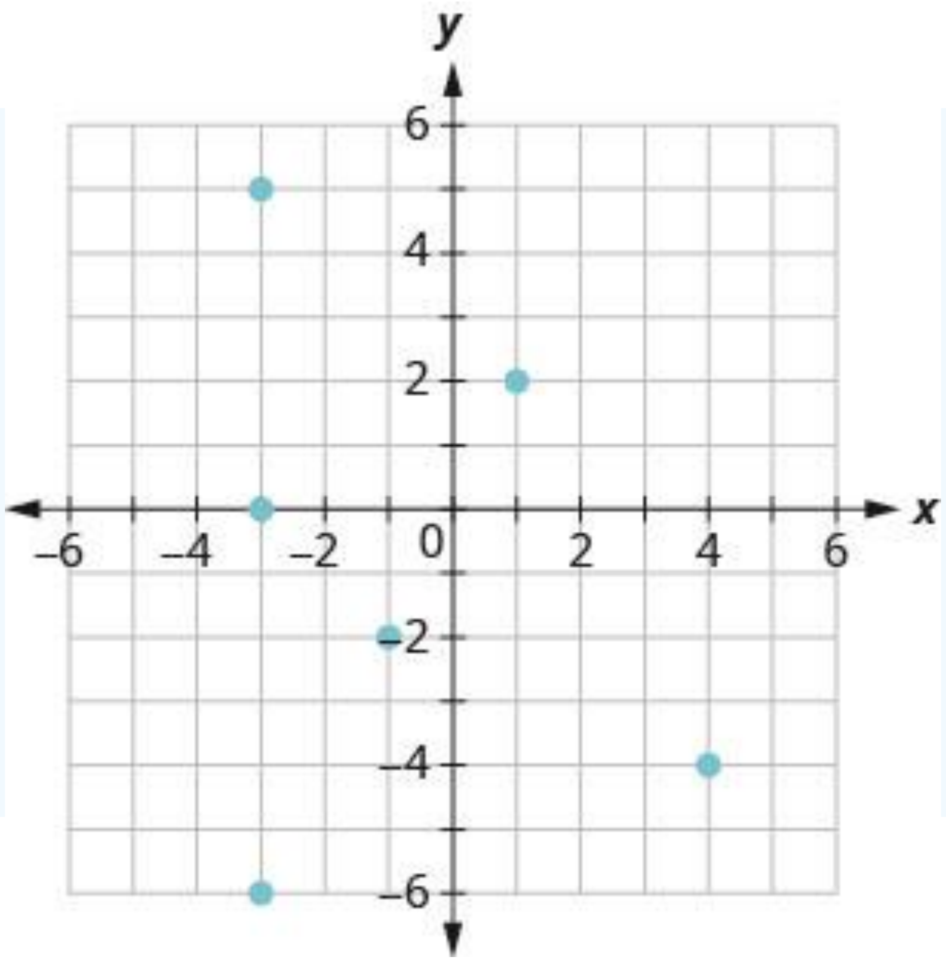
Use the mapping of the relation shown to
1.list the ordered pairs of the relation
2.find the domain of the relation
3.find the range of the relation.



Answer

ⓐ (Maria, November 6), (Arm and o, January 18), (Cynthia, December 8), (Kelly, March 15), (Rachel, November 6)
ⓑ {Maria, Arm and o, Cynthia, Kelly, Rachel}
ⓒ{November 6, January 18, December 8, March 15}

Use the graph of the relation to
1.list the ordered pairs of the relation
2.find the domain of the relation
3.find the range of the relation.



Answer

ⓐ (−3,0),(−3,5),(−3,−6),(−3,0),(−3,5),(−3,−6),
(−1,−2),(1,2),(4,−4)(−1,−2),(1,2),(4,−4)

ⓑ {−3,−1,1,4}{−3,−1,1,4}

ⓒ {−6,0,5,−2,2,−4}

Use the set of ordered pairs to (i) determine whether the relation is a function (ii) find the domain of the relation (iii) find the range of the relation.

1.{(−3,27),(−2,8),(−1,1),(0,0),(1,1),(2,8),(3,27)}$\{(-3,27),(-2,8),(-1,1),(0,0),(1,1),(2,8),(3,27)\}$

2.{(9,−3),(4,−2),(1,−1),(0,0),(1,1),(4,2),(9,3)}$\{(9,-3),(4,-2),(1,-1),(0,0),(1,1),(4,2),(9,3)\}$

Answer

ⓐ {(−3,27),(−2,8),(−1,1),(0,0),(1,1),(2,8),(3,27)}$\{(-3,27),(-2,8),(-1,1),(0,0),(1,1),(2,8),(3,27)\}$

(i) Each *x*-value is matched with only one *y*-value. So this relation is a function.

(ii) The domain is the set of all *x*-values in the relation.

The domain is: {−3,−2,−1,0,1,2,3}$\{-3,-2,-1,0,1,2,3\}$.

(iii) The range is the set of all *y*-values in the relation. Notice we do not list range values twice.

The range is: {27,8,1,0}$\{27,8,1,0\}$.

ⓑ {(9,−3),(4,−2),(1,−1),(0,0),(1,1),(4,2),(9,3)}$\{(9,-3),(4,-2),(1,-1),(0,0),(1,1),(4,2),(9,3)\}$

(i) The *x*-value 9 is matched with two *y*-values, both 3 and −3-3. So this relation is not a function.

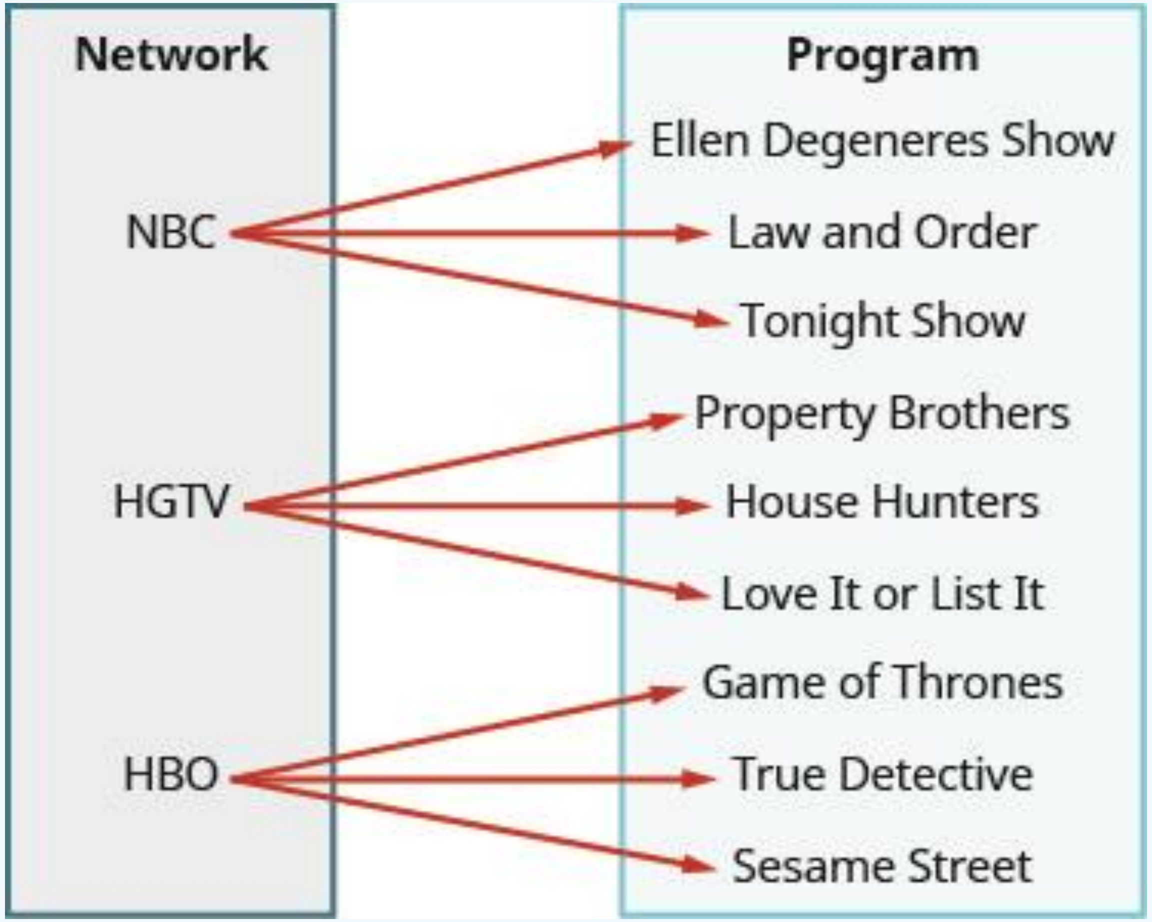
(ii) The domain is the set of all *x*-values in the relation. Notice we do not list domain values twice.

The domain is: {0,1,2,4,9}$\{0,1,2,4,9\}$.

(iii) The range is the set of all *y*-values in the relation.

The range is: {−3,−2,−1,0,1,2,3}$\{-3,-2,-1,0,1,2,3\}$.

use the mapping to ⓐ determine whether the relation is a function ⓑ find the domain of the relation ⓒ find the range of the relation.



Answer

ⓐ no

ⓑ {NBC, HGTV, HBO}

ⓒ {Ellen Degeneres Show, Law and Order, Tonight Show, Property Brothers, House Hunters, Love it or List it, Game of Thrones, True Detective, Sesame Street}

Exercises

1. Verify if $f(x) = x^2$ is a function or not.
2. Plot the graph of function $f(x) = |x|$.
3. Use a mapping diagram to determine if the relation $R=\{(0,1),(1,3),(3,4),(5,2)\}$ is a function
4. For the function $f(x)=|x-4|+1$, evaluate the following:

   $f(-5)$

   $f(-1)$

   $f(0)$