

A decorative graphic on the left side of the slide consists of a cluster of colorful 3D cubes and 2D squares in various colors (yellow, orange, red, blue, green, purple) and sizes, arranged in a way that suggests a dynamic, falling or exploding effect.

PERSAMAAN LANJAR (LINEAR)

Teknik Informatika
Politeknik Negeri Malang
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Persamaan Lanjar



- Persamaan lanjar atau linear adalah persamaan matematis yang menggambarkan hubungan antara variabel yang memiliki pangkat tertinggi satu. Dalam bentuk umum, persamaan linear dapat dituliskan sebagai:

$$ax + b = 0$$

atau dalam bentuk yang lebih umum untuk dua variabel:

$$ax + by + c = 0$$

di mana:

- a, b, dan c adalah konstanta (angka tetap),
- x dan y adalah variabel,
- a dan b tidak boleh keduanya sama dengan nol

Persamaan Lanjar



Persamaan lanjar merupakan suatu bentuk persamaan-persamaan yang menyajikan banyak variabel bebas

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \dots + a_{3n}x_n = b_3$$

.....

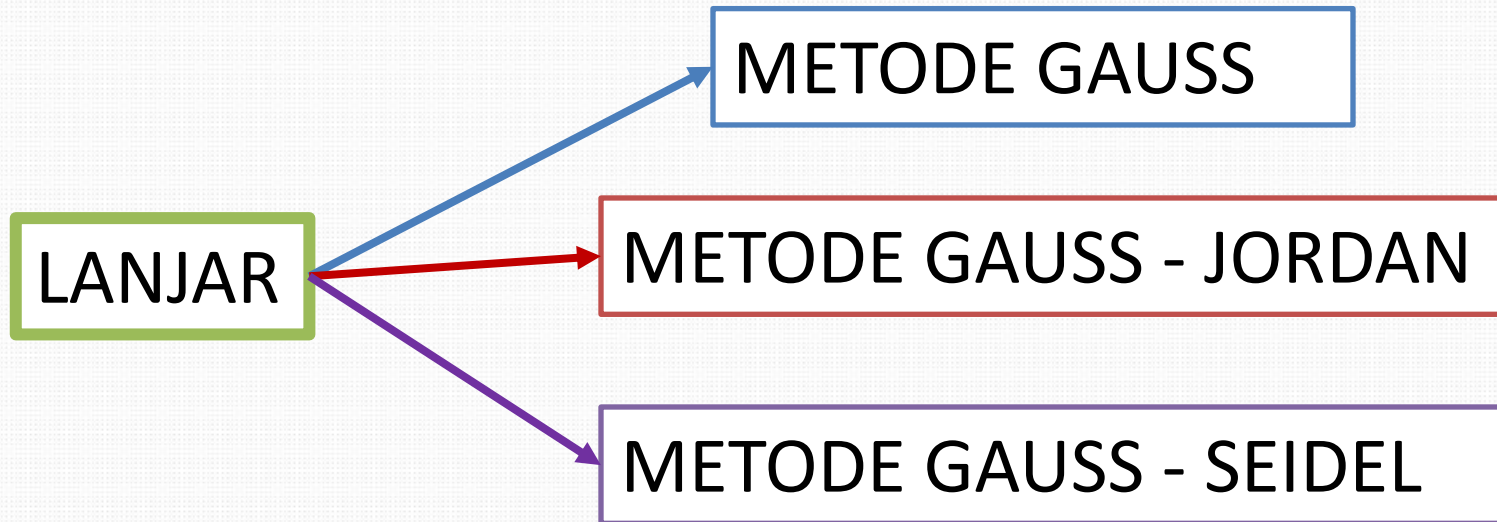
$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \dots + a_{mn}x_n = b_m$$



$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{bmatrix}$$



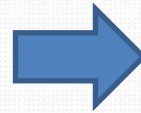
Penyelesaian





Metode Gauss

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} & b_2 \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} & b_3 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} & b_n \end{bmatrix}$$



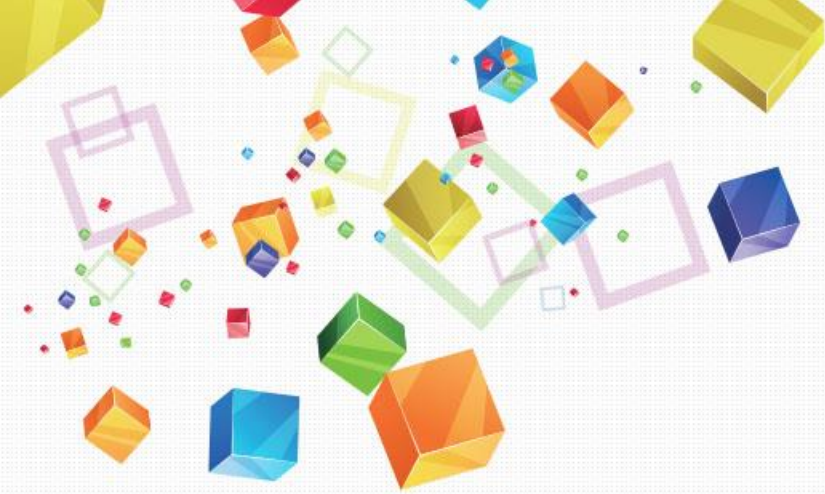
$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & \dots & c_{1n} & d_1 \\ 0 & c_{22} & c_{23} & \dots & c_{2n} & d_2 \\ 0 & 0 & c_{33} & \dots & c_{3n} & d_3 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & c_{nn} & d_n \end{bmatrix}$$

Proses eliminasi pada metode ini terdiri atas tiga Operasi Baris Elementer (OBE)



1. Pertukaran: urutan dua persamaan dapat ditukar karena pertukaran tersebut tidak berpengaruh pada solusi akhir.
2. Penskalaan: persamaan dapat dikali dengan konstanta bukan nol, sebab perkalian tersebut tidak berpengaruh pada solusi akhir.
3. Penggantian: persamaan bisa diganti dengan penjumlahan persamaan itu dengan penggandaan persamaan lain.





Contoh 1



$$x_1 + 2x_2 + 3x_3 = 6$$

$$2x_1 + 5x_2 + 10x_3 = 17$$

$$x_1 + 3x_2 + 10x_3 = 18$$

$$\begin{bmatrix} 1 & 2 & 3 & 6 \\ 2 & 5 & 10 & 17 \\ 1 & 3 & 10 & 18 \end{bmatrix}$$



$$\begin{aligned} B_2 &= B_2 - 2B_1 \\ B_3 &= B_3 - B_1 \end{aligned}$$

$$\begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & 1 & 4 & 5 \\ 0 & 1 & 7 & 12 \end{bmatrix}$$



$$B_3 = B_3 - B_2$$

$$\begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & 3 & 7 \end{bmatrix}$$

$$x_3 = \frac{7}{3} = 2\frac{1}{3}$$

$$x_2 + 4x_3 = 5$$

$$x_2 = 5 - 4x_3$$

$$= \left(5 - 4 \cdot 2\frac{1}{3} \right)$$

$$= 5 - 4 \cdot \frac{7}{3}$$

$$= 5 - \frac{28}{3} = -\frac{13}{3}$$

$$x_1 + 2x_2 + 3x_3 = 6$$

$$x_1 = 6 - 2x_2 - 3x_3$$

$$= 6 - 2\left(-\frac{13}{3}\right) - 3\left(\frac{7}{3}\right)$$

$$= 6 + \frac{26}{3} - \frac{21}{3}$$

$$= \frac{23}{3} = 7\frac{2}{3}$$

Contoh 2

$$x + 2y + z = 6$$

$$x + 3y + 2z = 9$$

$$2x + y + 2z = 12$$

Tentukan Nilai x, y dan z



$$\begin{bmatrix} 1 & 2 & 1 & 6 \\ 1 & 3 & 2 & 9 \\ 2 & 1 & 2 & 12 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & 6 \\ 0 & 1 & 1 & 3 \\ 0 & -3 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & 6 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 3 & 9 \end{bmatrix}$$

$$B_3 = B_3 - 2B_1$$

$$B_2 = B_2 - B_1$$

$$B_3 = B_3 + 3B_2$$

$$\begin{aligned} 3z &= 9 \\ z &= \frac{9}{3} = 3 \end{aligned}$$

$$\begin{aligned} y + z &= 3 \\ y &= 3 - z \\ &= 3 - 3 \\ &= 0 \end{aligned}$$

$$\begin{aligned} x + 2y + z &= 6 \\ x &= 6 - 2y - z \\ x &= 6 - 2(0) - 3 \\ &= 6 - 0 - 3 \\ &= 3 \end{aligned}$$

Jadi nilai dari $x=3$, $y=0$, dan $z=3$





Post Test

Tentukan himpunan penyelesaian dari sistem persamaan berikut dengan metode Gauss!

$$\begin{aligned}x_1 + 2x_2 &= 3 \\ 4x_1 + 10x_2 &= 20\end{aligned}$$

$$\begin{aligned}3x_1 + 2x_2 &= 10 \\ 9x_1 - 7x_2 &= 43\end{aligned}$$

$$\begin{aligned}2x - 4y + 3z &= -19 \\ 3x - y + 2z &= -11 \\ x + 2y - 5z &= 18\end{aligned}$$

$$\begin{aligned}2x - y + 2z &= 9 \\ x - 6y - 3z &= -28 \\ 3x + 2y + z &= 16\end{aligned}$$



Post Test



Tentukan himpunan penyelesaian dari sistem persamaan berikut dengan metode Gauss!

$$\begin{aligned} 4x_1 + 2x_2 &= 20 \\ 2x_1 + 8x_2 &= 24 \end{aligned}$$

$$\begin{aligned} 2x_1 + x_2 &= 10 \\ 2x_1 + 8x_2 &= 24 \end{aligned}$$

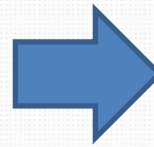
$$\begin{aligned} x - 4y - 4z &= 3 \\ 2x + 9y + z &= 13 \\ -x + 3y + 2z &= 1 \end{aligned}$$

$$\begin{aligned} 3x + 2y - 5z &= 2 \\ 2x - y + 4z &= 4 \\ -5x + 3y - z &= 0 \end{aligned}$$



Metode Gauss - Jordan

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} & b_2 \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} & b_3 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} & b_n \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 & 0 & \dots & 0 & d_1 \\ 0 & 1 & 0 & \dots & 0 & d_2 \\ 0 & 0 & 1 & \dots & 0 & d_3 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & d_n \end{bmatrix}$$





Contoh 1

$$\begin{aligned}x_1 + 2x_2 &= 3 \\ 2x_1 + 5x_2 &= 10\end{aligned}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 10 \end{bmatrix} \xrightarrow{B_2 = B_2 - 2B_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \end{bmatrix} \xrightarrow{B_1 = B_1 - 2B_2} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 4 \end{bmatrix}$$

$$B_2 = B_2 - 2B_1$$

$$B_1 = B_1 - 2B_2$$



$$x_1 = -5 \text{ dan } x_2 = 4$$



Contoh 2

$$\begin{aligned}x + y + z &= 6 \\x + 2y + 3z &= 14 \\x + 4y + 9z &= 36\end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 9 & 36 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 1 & 4 & 9 & 36 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 3 & 8 & 30 \end{bmatrix}$$

$$B_2 = B_2 - B_1$$

$$B_3 = B_3 - B_1$$



$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 2 & 6 \end{bmatrix}$$

$$B_3 = B_3 - 3B_2$$



$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$B_1 = B_1 - B_2$$

$$B_1 = B_1 + B_3$$



$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$B_2 = B_2 - 2B_3$$

$$x = 1$$

$$y = 2$$

$$z = 3$$



Post Test



Tentukan himpunan penyelesaian dari sistem Persamaan Berikut menggunakan metode Gauss Jordan!

$$\begin{aligned}x_1 + 2x_2 &= 3 \\ 4x_1 + 10x_2 &= 20\end{aligned}$$

$$\begin{aligned}4x_1 + 2x_2 &= 20 \\ x_1 + 4x_2 &= 12\end{aligned}$$

$$\begin{aligned}2x_1 + x_2 &= 10 \\ 2x_1 + 8x_2 &= 24\end{aligned}$$

$$\begin{aligned}2x - 4y + 3z &= -19 \\ 3x - y + 2z &= -11 \\ x + 2y - 5z &= 18\end{aligned}$$

$$\begin{aligned}2x - y + 2z &= 9 \\ x - 6y - 3z &= -28 \\ 3x + 2y + z &= 16\end{aligned}$$



Metode Gauss - Seidel



$$\begin{array}{ccccccccccccccccc} a_{11} & x_1 & + & a_{12} & x_2 & + & a_{13} & x_3 & + & \dots & + & a_{1n} & x_n & = & b_1 \\ a_{21} & x_1 & + & a_{22} & x_2 & + & a_{23} & x_3 & + & \dots & + & a_{2n} & x_n & = & b_2 \\ a_{31} & x_1 & + & a_{32} & x_2 & + & a_{33} & x_3 & + & \dots & + & a_{3n} & x_n & = & b_3 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & x_1 & + & a_{n2} & x_2 & + & a_{n3} & x_3 & + & \dots & + & a_{nn} & x_n & = & b_n \end{array}$$

$$x_1 = \frac{1}{a_{11}} (b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n)$$

$$x_2 = \frac{1}{a_{22}} (b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n)$$

.....

$$x_n = \frac{1}{a_{nn}} (b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{nn-1}x_{n-1})$$



Contoh 1



$$\begin{aligned} 2x_1 + x_2 &= 10 \\ x_1 + 4x_2 &= 12 \end{aligned} \quad \text{dengan nilai awal: } x_1 = 1 \text{ dan } x_2 = 1.$$

Buatlah persamaan di atas menjadi:

$$\begin{aligned} x_1 &= (10 - x_2)/2 \\ x_2 &= (12 - x_1)/4 \end{aligned}$$

$$\begin{aligned} x_1 &= (10 - 1)/2 = 4.5 \\ x_2 &= \frac{1}{4}(12 - 4.5) = 1.875 \end{aligned}$$

$$\begin{aligned} x_1 &= (10 - 1.875)/2 = 4.0625 \\ x_2 &= \frac{1}{4}(12 - 4.0625) = 1.984375 \end{aligned}$$

$$\begin{aligned} x_1 &= (10 - 1.984)/2 = 4.008 \\ x_2 &= \frac{1}{4}(12 - 4.008) = 1.998 \end{aligned}$$

$$\begin{aligned} x_1 &= (10 - 1.998)/2 = 4.001 \\ x_2 &= \frac{1}{4}(12 - 4.001) = 1.99975 \end{aligned}$$

$$\begin{aligned} x_1 &= (10 - 1.99975)/2 = 4.000125 \\ x_2 &= \frac{1}{4}(12 - 4.000125) = 1.9999687 \end{aligned}$$

$$\begin{aligned} x_1 &= (10 - 1.9999687)/2 = \mathbf{4.000016} \\ x_2 &= \frac{1}{4}(12 - 4.000016) = \mathbf{1.999996} \end{aligned}$$





Referensi

- Munir, Rinaldi. 2008. Metode Numerik Revisi Kedua. Informatika Bandung: Bandung
- Cahya Rahmad, ST, M.Kom. Dr. Eng, “Diktat Kuliah Matematika Numerik”, Program Studi Manajemen Informatika, Politeknik Negeri Malang





TERIMA KASIH

