EE498
Control System Design and Simulation

Part 1)

a) Determine a suitable system model for the system.

We will be using the Overreacting Lateral Dynamic Model as discussed in the journal paper, the relevant state equations along with the description of the parameters if given below:

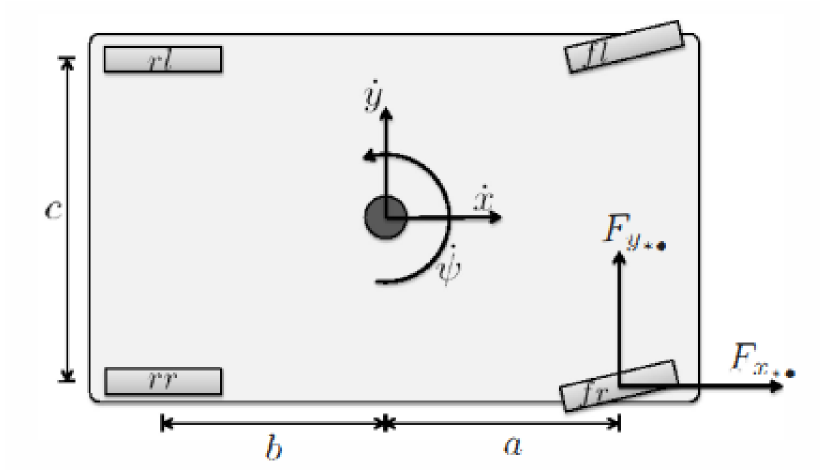


Fig 1.1 4-Wheel Front Axle Drive Model for the computation of the longitudinal and lateral load transfers.

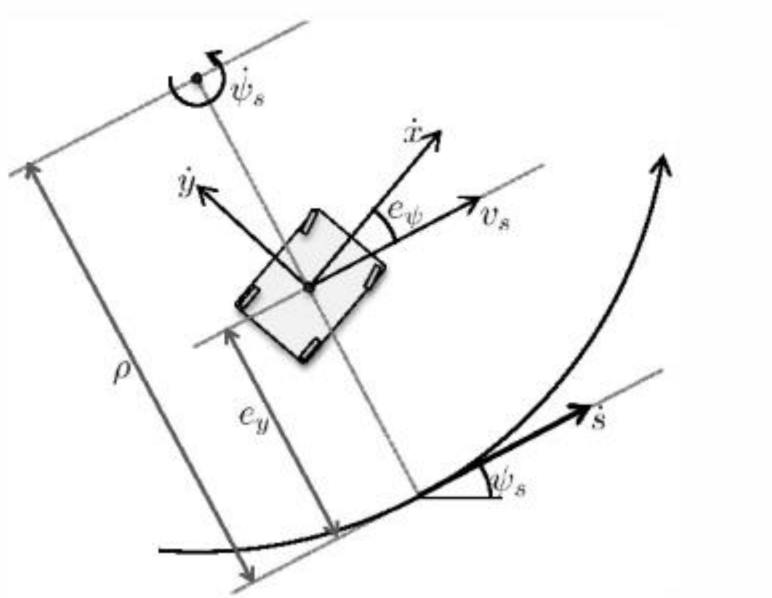


Fig 1.2 The curvilinear coordinate system. The dynamics are derived from a curve defining the centerline of a track. The coordinate s defines the arclength along the track.

The set of differential equations governing the model is given by these equations:

$$m\ddot{x} = F_{x_{fl}} + F_{x_{fr}} + F_{x_{rl}} + F_{x_{rr}} - k_d\dot{x}^2 \qquad (1a)$$

$$m\ddot{y} = -m\dot{x}\dot{\psi} + F_{y_{fl}} + F_{y_{fr}} + F_{y_{rl}} + F_{y_{rr}} \qquad (1b)$$

$$I\ddot{\psi} = a(F_{y_{fl}} + F_{y_{fr}}) - b(F_{y_{rl}} + F_{y_{rr}}), \qquad (1c)$$

For detailed analysis of the variables used above, please refer to the paper in the reference section.

To apply MPC we need to linearize the model which can be done as described in the paper, thus the new set of differential equations are given as follows:

$$m\ddot{y} = -m\dot{x}\dot{\phi} + C_{f_L}\alpha_f + C_{r_L}\alpha_r$$
$$I\phi = aC_{f_l}\ddot{\alpha}_f - bC_{r_U}\alpha_r$$
$$\dot{e}_\phi = \dot{\phi} - \dot{x}\phi_r$$
$$\dot{e}_y = \dot{y} + \dot{x}e_\phi$$
$$\dot{\delta} = u$$

Where $\alpha_f = \frac{\dot{y}+a\dot{\phi}}{\dot{x}} - \delta,\ \alpha_r = \frac{\dot{y}-b\dot{\phi}}{\dot{x}}$ are used for simplicity, the state vector equations then could be formulated as follows:

$$\begin{bmatrix} \ddot{y} \\ \ddot{\phi} \\ \dot{e}_\phi \\ \dot{e}_y \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} \frac{C_{f_U}+C_{r_U}}{\dot{x}m} & -\dot{x} + \frac{C_{f_U}a - C_{r_U}b}{\dot{x}m} & 0 & 0 & -\frac{C_{f_U}}{m} \\ \frac{C_{f_U}a - C_{r_L}b}{\dot{x}I} & \frac{C_{f_U}a^2 + C_{r_L}b^2}{\dot{x}I} & 0 & 0 & -\frac{aC_{f_U}}{I} \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & \dot{x} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{y} \\ \dot{\phi} \\ e_\phi \\ e_y \\ \delta \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -\dot{x}\phi_r \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ 1 \end{bmatrix}$$

For the output of the system, we want the lateral position of the car to be always at the center thus implying that the output should approach zero with respect to the reference at steady state thus $y = e_y$ is the output with the maximum weight in the cost matrix such that lane keeping is preferred always while considering the states. The relevant matrix equation can be formulated as follows:

$$y = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{y} \\ \dot{\phi} \\ e_\phi \\ e_y \\ \delta \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ 1 \end{bmatrix}$$

b) Explain which of the system features makes MPC suitable to solve the control problem.

Firstly, by linearizing the states of the previous model we are able to effectively solve the MPC problem, one key thing to note is the $-\dot{x}\phi_r$ value which is not a part of the state vector has to be considered thus it is modelled as an external input to the systems where $\phi_r = \frac{d\phi_s}{ds}$ can be linearized as $\phi_r = \frac{1}{\rho}, where\ \rho =$ radius of the curvature. Since we are tackling with low curvature roads, I have kept the radius of the curvature to be large enough such that we model the system accurately as possible, therefore $\rho \approx 300\ m$ is used in all the simulations. Also, since we want the model to be focusing on lane keeping, we want the state e_y to be minimized as much as possible preferably to be zero, thus the deviation from the reference can be considered as limits being imposed onto the system. These restrictions and assumptions are used as system features to solve the control problem.

c) Write down your controller design formulation:
- Discuss how you decide on the cost matrices.

The cost matrix is calculated by the standard equation of $Q = C^TC$, where the C matrix is associated with the output y/

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- Discuss how you decide on the constraints.

First we have the Car Mechanical and Model constraints:

1) Actuator limits which are the limits imposed on the steering angle itself and the rate of change of the steering angle. These bounds can be represented as linear constraints on the input and the state vector: $-\delta_{lim} \leq \delta_{k,j}^{*,\beta} \leq \delta_{lim}$ for our region of operation, we are using $\delta_{lim} = \frac{\pi}{6}$.
2) Slip angles bounds: The overreacting lateral dynamics model are valid for values of α_f and α_r . This requirement can be expressed as linear constraints on the state vector: $\alpha_{f,lim} \geq \left|\frac{\dot{y}_{k,j}^{*,\beta} + a\dot{\phi}_{k,j}^{*,\beta}}{\dot{x}_{k,j}^{*,\beta}} - \delta_{k,j}^{*,\beta}\right|$ and $\alpha_{r,lim} \geq \left|\frac{\dot{y}_{k,j}^{*,\beta} - b\dot{\phi}_{k,j}^{*,\beta}}{\dot{x}_{k,j}^{*,\beta}}\right|$ for our region of operation we have used $\alpha_f = \alpha_r = \frac{\pi}{6}$
3) Lane boundaries: Lane boundaries can be easily introduced in the model as constraints on the lateral position error ${e_y}_{k,j}^{*,\beta}$ as follows: $\left|{e_y}_{k,j}^{*,\beta}\right| \leq \frac{1}{2} l_w$ where l_w denotes the lane width. For our region of operation, we have used $l_w = 4$ therefore the value of $\left|{e_y}_{k,j}^{*,\beta}\right| \leq 1.5$
4) Obstacle Avoidance Constraints: Finally, we show how static and moving obstacles can be expressed as additional linear constraints on the lateral position error $\left|{e_y}_{k,j}^{*,\beta}\right|$. We introduce the following assumptions that the current and the future positions of all obstacles in the proximity of the vehicle are known as a function of time and the controller knows the side of the obstacle on which it is safe to pass.
Therefore, we have the following inequalities:

$$\left|{e_y}_{k,j}^{*,\beta}\right|_{obstacle} \leq \left|{e_y}_{k,j}^{*,\beta}\right|_{state} \leq 1.5 \quad or \quad -1.5 \leq \left|{e_y}_{k,j}^{*,\beta}\right|_{state} \leq \left|{e_y}_{k,j}^{*,\beta}\right|_{obstacle}$$

Next, we have the external input offset that we have in the model which can be modelled by the matrix equation as follows: $\vec{u} = \begin{bmatrix} u \\ 1 \end{bmatrix}$, this is the input vector which can be extracted by the equation $u_{ext} = [0 \quad 1] \begin{bmatrix} u \\ 1 \end{bmatrix} = 1$.

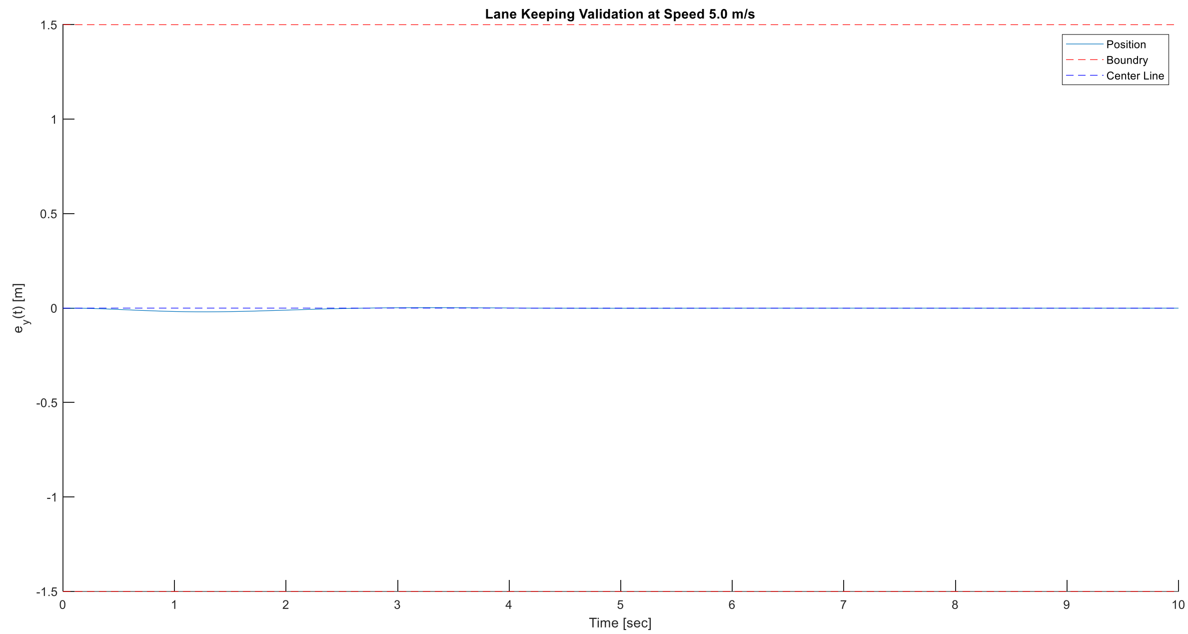
- Include possible disturbances in your model and study their effect on the feedback loop.

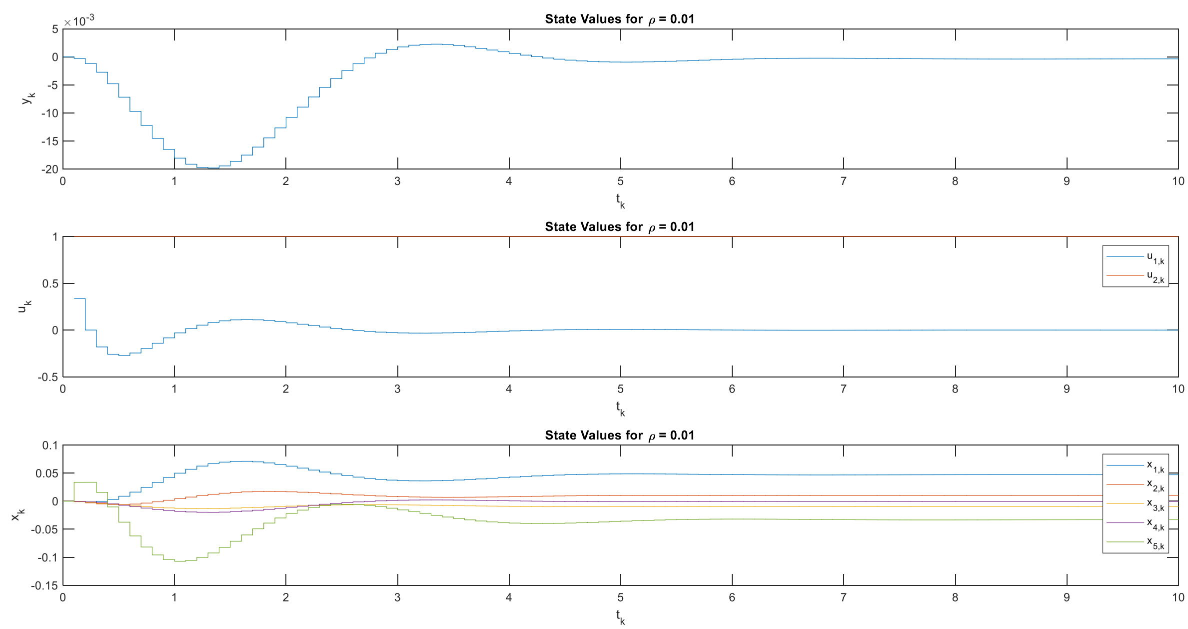
One of the possible disturbances is the road curvature which has been explained above, we are assuming that the curvature is static throughout the direction of travel which may not be true, also for heavy vehicle the rate of mass being consumed is also another factor to be discussed as it may affect the boundaries for the slip angles. The elevation of any path is also not considered therefore the reference speed $\dot{x}$ can have disturbances based on elevation and as well as air resistance.

d) Now we'll start validation of the MPC problem using Simulink

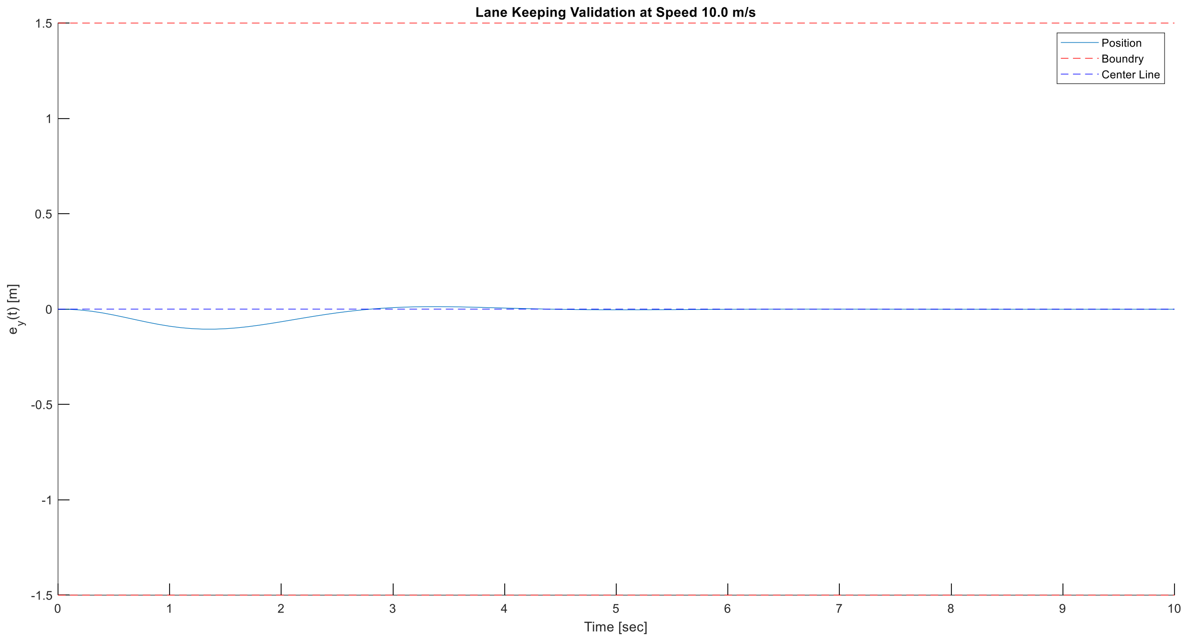
- Lane Keeping Validation

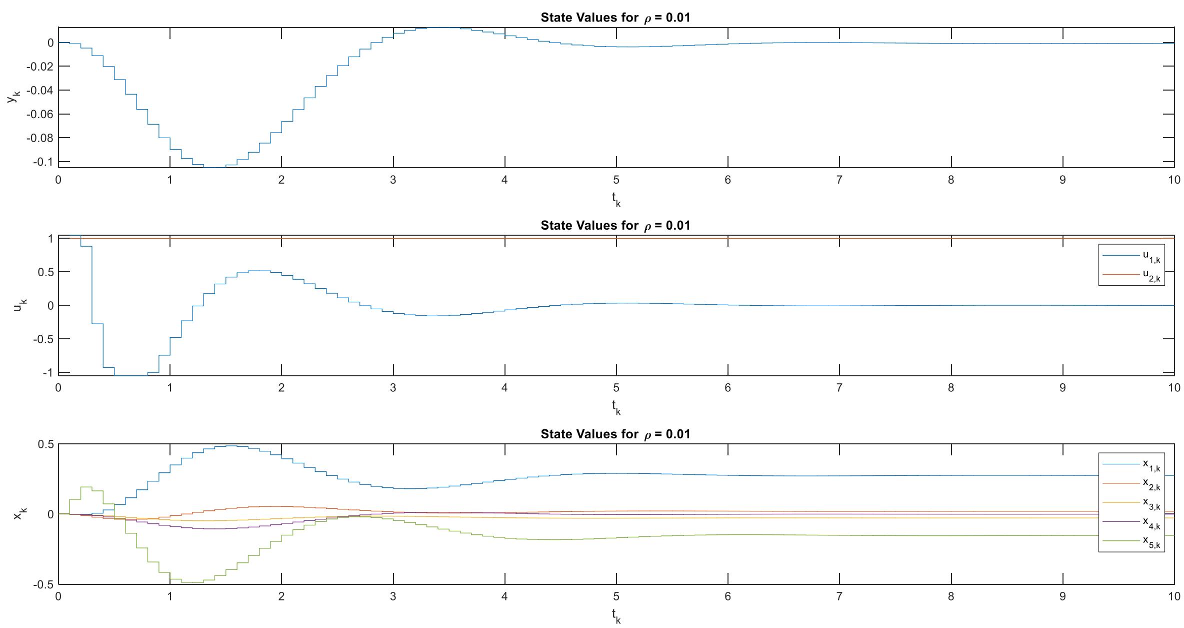
  1) At $x = 5\ m/s$





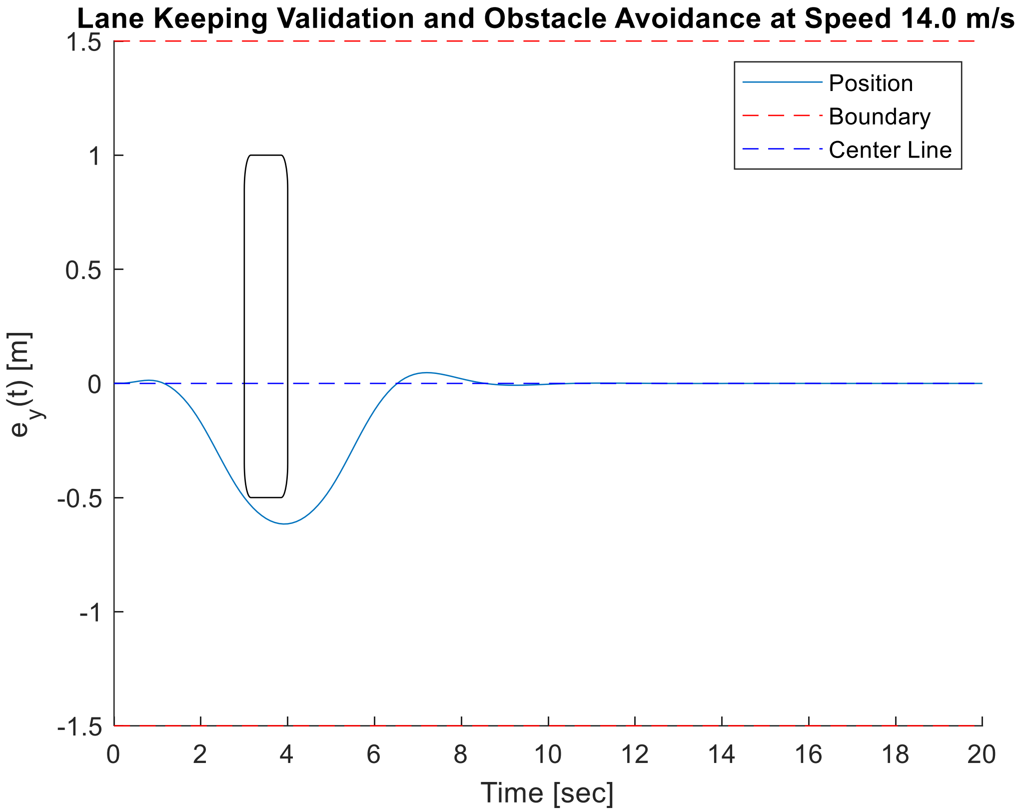
2) At $x = 10\ m/s$

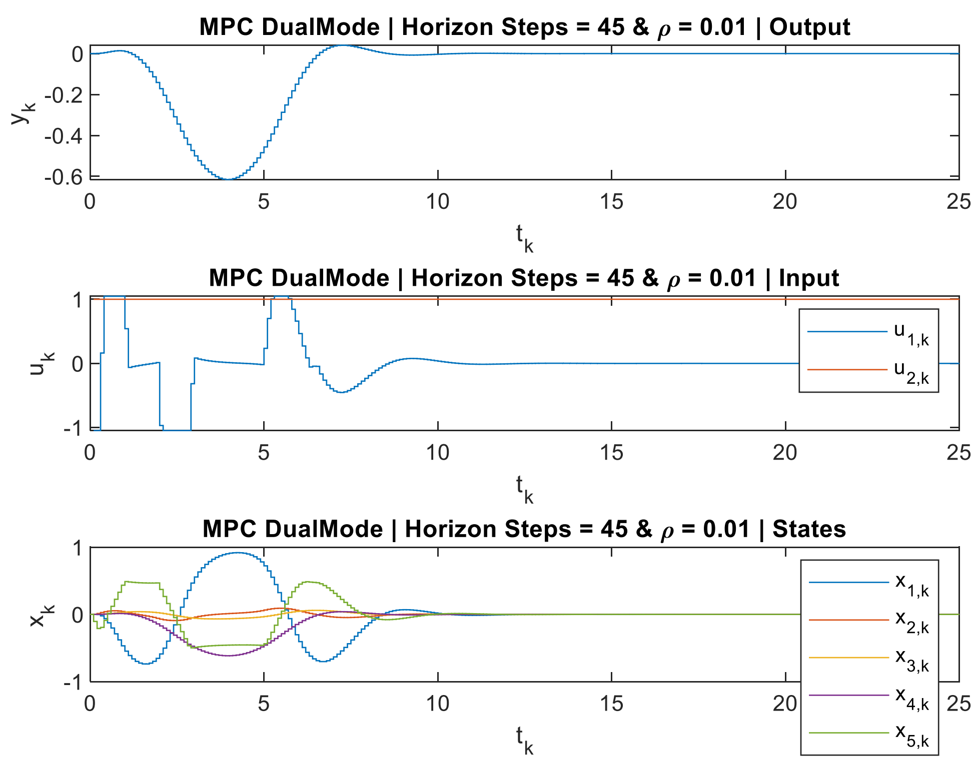




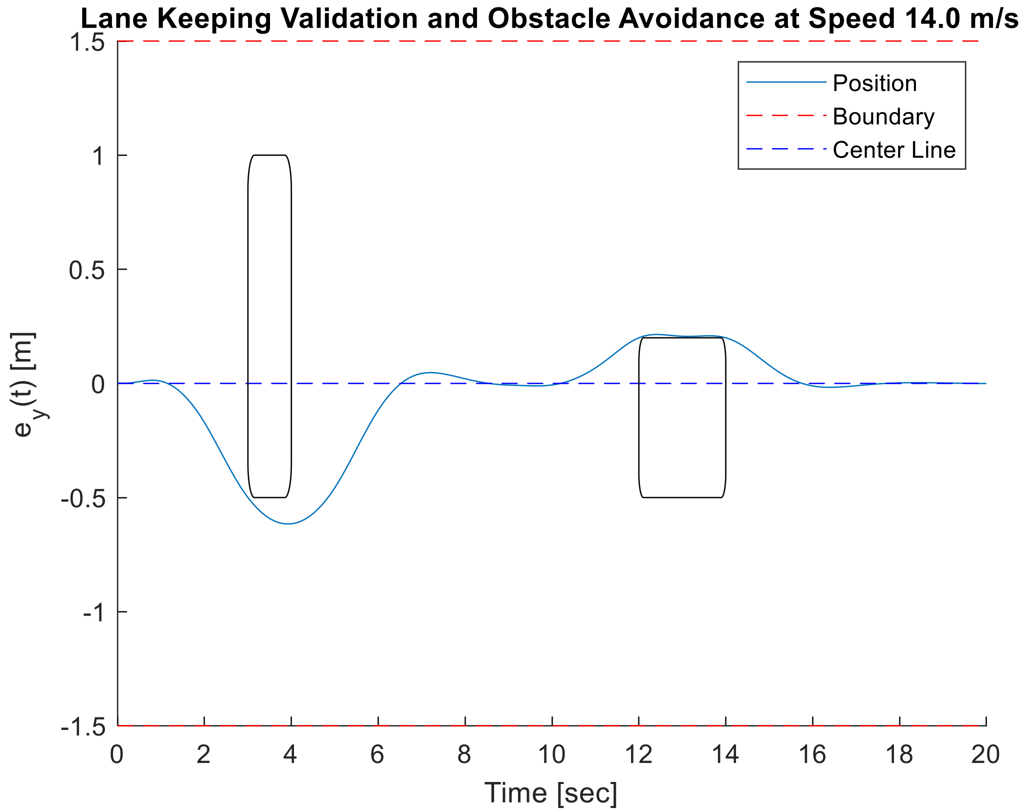
- Obstacle Avoidance Validation

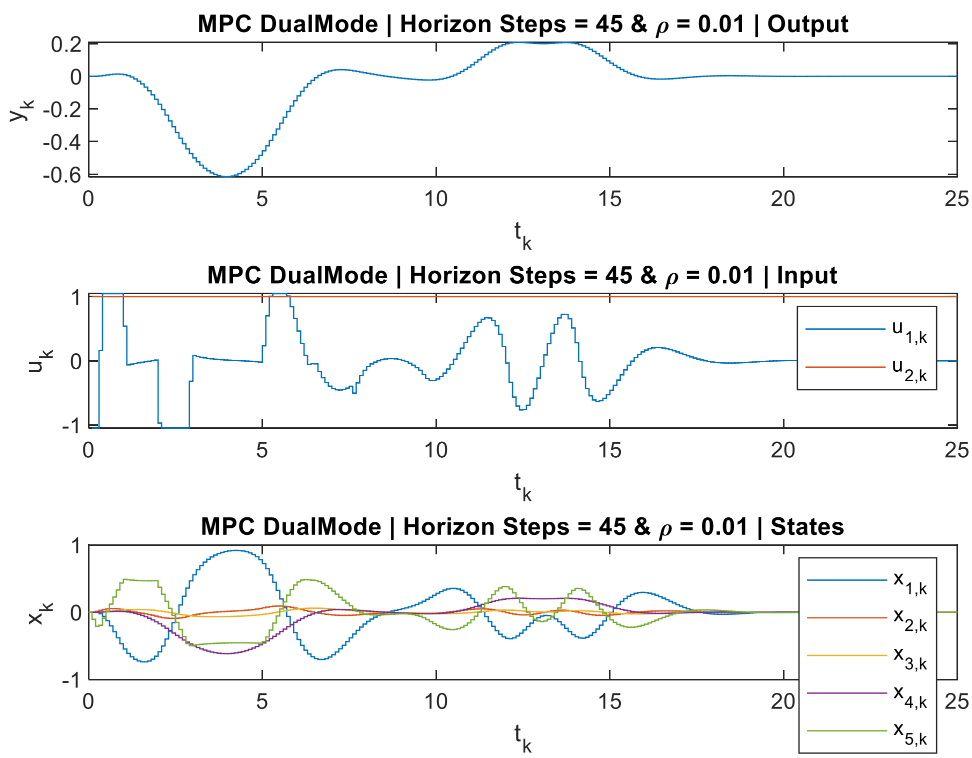
  1) Using a single Obstacle





2) Using 2 Obstacles


Lane Keeping Validation and Obstacle Avoidance at Speed 14.0 m/s
Position
Boundary
Center Line
$e_y(t)$ [m]
Time [sec]


MPC DualMode | Horizon Steps = 45 & ρ = 0.01 | Output
y_k
t_k
MPC DualMode | Horizon Steps = 45 & ρ = 0.01 | Input
u_k
$u_{1,k}$
$u_{2,k}$
t_k
MPC DualMode | Horizon Steps = 45 & ρ = 0.01 | States
x_k
$x_{1,k}$
$x_{2,k}$
$x_{3,k}$
$x_{4,k}$
$x_{5,k}$
t_k