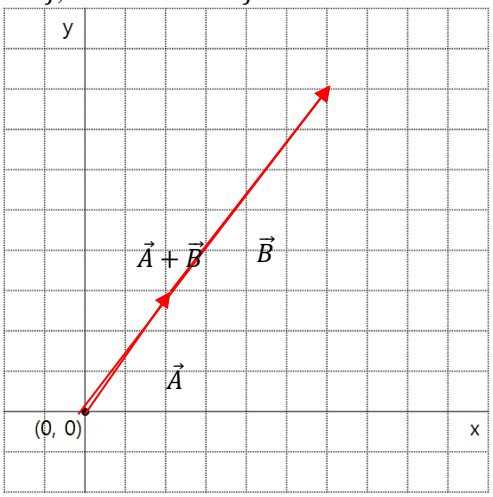
2023-1 Solid Mechanics Midterm Exam

(2023.04.25)

『Please write all the answers on this test sheets. You can also use the back side for the answers.』

1. There are two vectors: $\vec{A} = 2\vec{\imath} + 3\vec{\jmath}$, and $\vec{B} = 4\vec{\imath} + 5\vec{\jmath}$. Draw $\vec{A} + \vec{B}$ on the 1-unit graph paper. [5 Points]



2. Calculate the inner product of $\vec{A} \cdot \vec{B}$ (of Problem 1). [5 Points]

$\vec{A} \cdot \vec{B} = (2\vec{i} + 3\vec{j}) \cdot (4\vec{i} + 5\vec{j}) = (2)(4)\vec{i}\cdot\vec{i} + (2)(5)\vec{i}\cdot\vec{j} + (3)(4)\vec{j}\cdot\vec{i} + (3)(5)\vec{j}\cdot\vec{j}$

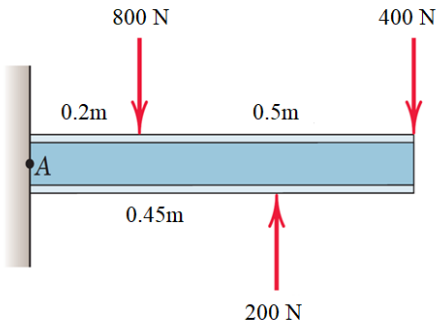
$= 8 + 15 = \underline{23}$

3. Calculate the cross product of $\vec{A} \times \vec{B}$ (of Problem 1). [5 Points]

$\vec{A} \times \vec{B} = (2\vec{i} + 3\vec{j}) \times (4\vec{i} + 5\vec{j}) = (2)(4)\vec{i}\times\vec{i} + (2)(5)\vec{i}\times\vec{j} + (3)(4)\vec{j}\times\vec{i} + (3)(5)\vec{j}\times\vec{j}$

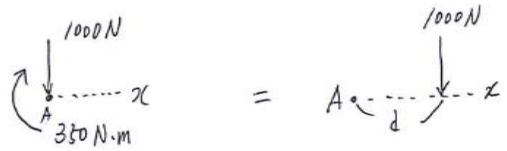
$= (10 - 12)\vec{k} = \underline{-2\vec{k}}$

4. Express the force-couple system at point A, and calculate the distance x to express the force-couple system as only one resultant force. [Total 12 Points (Answer: 3 Points, Equations: 9 Points)]



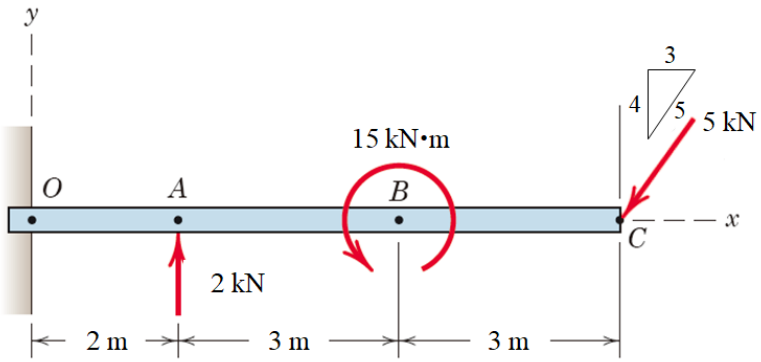
At A: $R = \Sigma F = 800 + 400 - 200 = 1000\ N\ (\downarrow)$

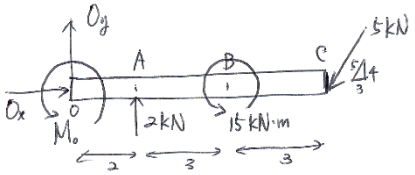
$\curvearrowleft M_A = 800(0.2) + 400(0.7) - 200(0.45) = 350\ N\cdot m$



$1000(d) = 350 \quad \therefore d = 0.35\ m$

5. Calculate the reaction forces at point O. (Neglect the mass of the beam.) [Total 12 Points (FBD: 4 Points, EoM: 6 Points, Answer: 2 Points)]





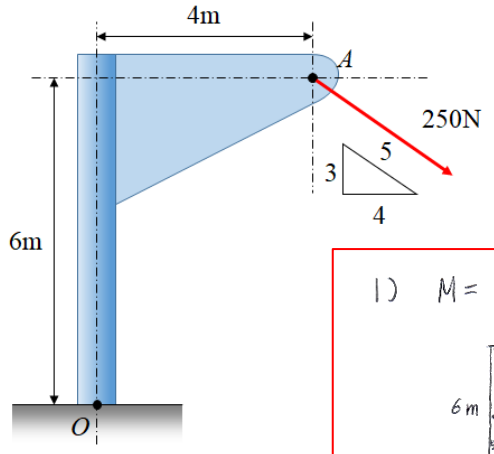
$\Sigma F_x = 0: \quad O_x - 3 = 0 \quad \therefore O_x = 3kN$

$\Sigma F_y = 0: \quad O_y + 2 - 4 = 0 \quad \therefore O_y = 2kN$

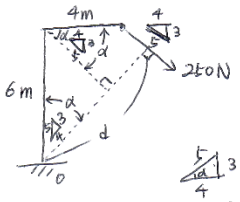
$\Sigma M_o = 0: \quad M_o + 2(2) + 15 - 8(4) = 0$

$\therefore M_o = 13\ kN\cdot m$

6. Calculate the magnitude of the moment about the base point O of the 600-N force in five different ways. [Total 15 Points (3 Points each)]



1) $M = Fd$ 이용.



$d = 4\sin\alpha + 6\cos\alpha$

$= 4\left(\frac{3}{5}\right) + 6\left(\frac{4}{5}\right)$

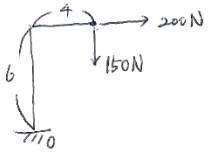
$= \frac{36}{5}\,(m)$

$\therefore M_o = Fd = 250\left(\frac{36}{5}\right)$

$= 1800\,(N\cdot m)$ CW

$\left(\sin\alpha = \frac{3}{5},\ \cos\alpha = \frac{4}{5}\right)$

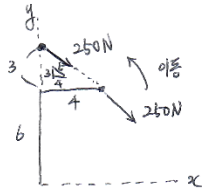
2) 250 N을 F_x와 F_y로나눔.

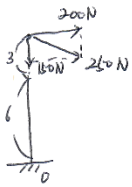


$\circlearrowleft M_o = 200(6) + 150(4)$

$= 1800\,(N\cdot m)$ CW

3) 250 N을 $x=0$ 위치로 이동.

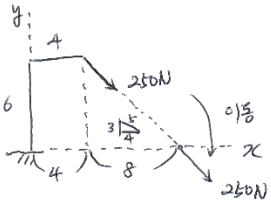


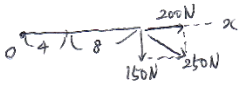


F_x 성분만 모멘트를 생성.

$M_o = 200(9) = 1800\,(N\cdot m)$CW

4) 250 N을 $y=0$ 위치로 이동.





F_y 성분만 모멘트를 생성.

$M_o = 150(12) = 1800\,(N\cdot m)$ CW

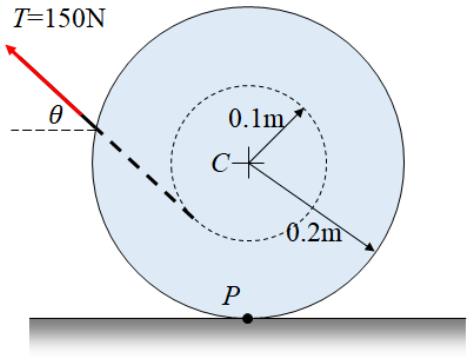
5) $\vec{M} = \vec{r} \times \vec{F}$ 이용.

$\vec{M}_o = (4\vec{i} + 6\vec{j}) \times (200\vec{i} - 150\vec{j})$

$= -600\vec{k} - 1200\vec{k} = -1800\vec{k}$

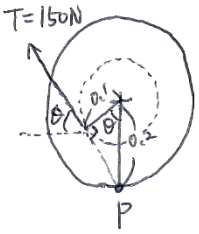
$\therefore M_o = 1800\,(N\cdot m)$ CW

7. Pull the string tightly wound around the inner hub of the drum with a force T of 150N. (1) Find the moment of T with respect to the center C of the drum. (2) Find the angle θ that makes the moment about the contact point P zero. [Total 10 Points (5 Points each)]



(1) $\oplus M_C = 150\,(0.1) = 15\ (N\cdot m)\ CW$

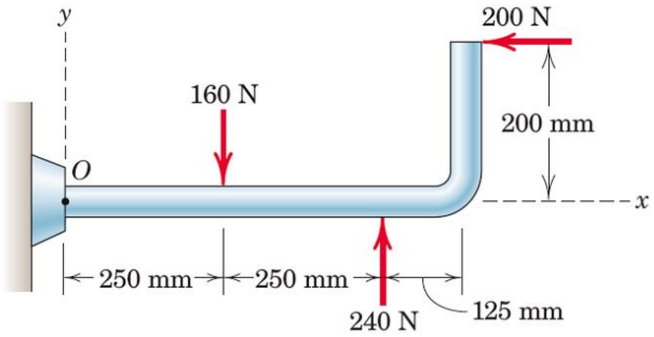
(2)



$\cos\theta = \frac{0.1}{0.2} = \frac{1}{2}$

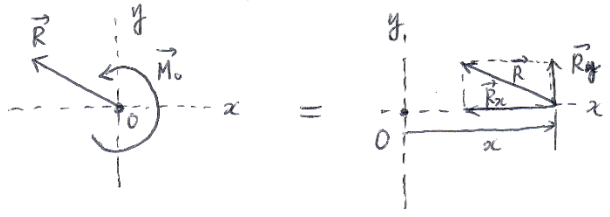
$\therefore \theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$

8. Replace the three forces acting on the bent pipe with one equivalent force R. Find the distance x between the point O and the point on the x-axis through which the line of action of the resultant force R passes. [Total 12 Points (Answer: 4 Points, Equations: 8 Points)]



$\vec{R} = -200\,\vec{i} + 80\,\vec{j}\ (N) \quad \cdots (*)$

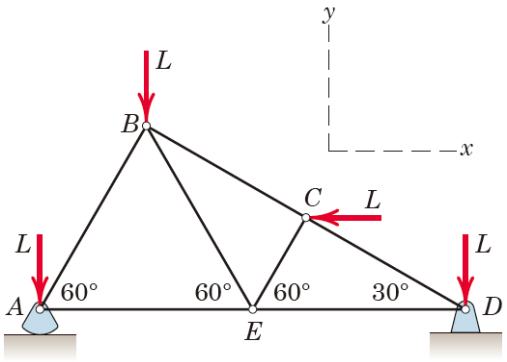
$\oplus \vec{M}_o = -160\,(0.25) + 240\,(0.5) + 200\,(0.2) = 120\ (N\cdot m)$



$R_y\, x = M_o$

$x = \frac{M_o}{R_y} = \frac{120}{80} = 1.5\ (m)$

9. A simple asymmetric simple truss is loaded as shown. Determine the reactions at A and D. Neglect the weight of the structure. [Total 12 Points (FBD: 4 Points, EoM: 6 Points, Answer: 2 Points)]



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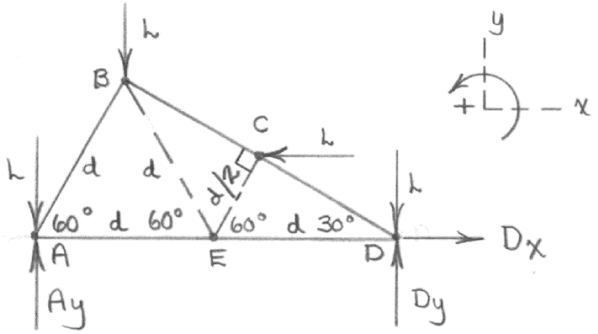


$\Sigma F_x = 0$: $D_x - L = 0$, $D_x = L$

$\Sigma F_y = 0$: $A_y + D_y - 3L = 0$

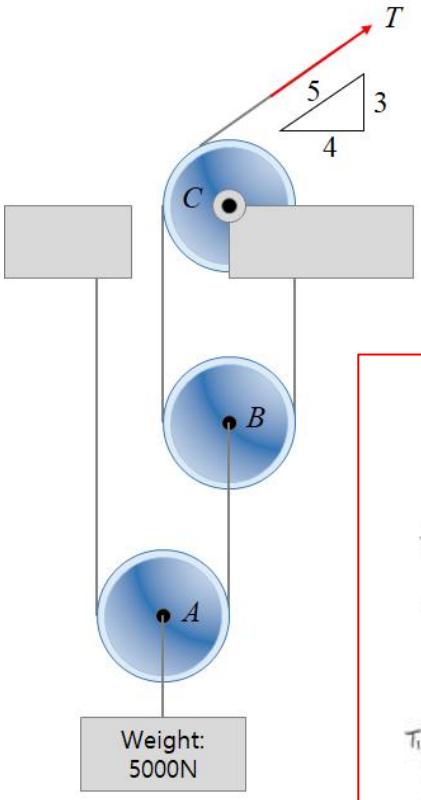
$\Sigma M_A = 0$: $D_y(2d) + L\left(\frac{d}{2}\frac{\sqrt{3}}{2}\right) - L\left(\frac{d}{2}\right) - L(2d) = 0$

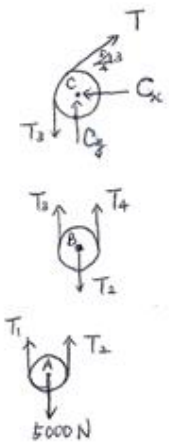
Solving the last 2 equations: $A_y = \frac{L}{4}\left(7 + \frac{\sqrt{3}}{2}\right)$

$D_y = \frac{L}{4}\left(5 - \frac{\sqrt{3}}{2}\right)$

(or $A_y = 1.967L$, $D_y = 1.033L$)

10. Calculate the tension T and the total force acting on the bearing of the pulley C in the cable supporting the pulley with a weight of 5000 N as shown in the figure. (Assume that the pulley can rotate freely for each bearing, and the weight of each part is smaller than the load of the load.) [Total 12 Points (Tension: 6 Points, Forces at C: 6 Points)]





A 도르래 도르래 반지름을 r이라 하자.

$$\begin{cases} \curvearrowleft \Sigma M_A = 0 : & T_2 r - T_1 r = 0 \\ & \therefore T_1 = T_2 \\ \Sigma F_y = 0 : & T_1 + T_2 - 5000 = 0 \\ & \therefore T_1 = T_2 = 2500\ (N) \end{cases}$$

B 도르래

$$\begin{cases} \curvearrowleft \Sigma M_B = 0 : & T_4 r - T_3 r = 0 \\ & \therefore T_3 = T_4 \\ \Sigma F_y = 0 : & T_3 + T_4 - T_2 = 0 \\ & \therefore T_3 + T_4 = 2500 \\ & \therefore T_3 = T_4 = 1250\ (N) \end{cases}$$

C 도르래

$$\begin{cases} \curvearrowleft \Sigma M_C = 0 : & T_3 r - T r = 0 \\ & \therefore T = T_3 = 1250\ (N) \quad \cdots ⑧ \\ \Sigma F_x = 0 : & 1250\left(\frac{4}{5}\right) - C_x = 0 \\ & \therefore C_x = 1000\ (N) \\ \Sigma F_y = 0 : & C_y + 1250\left(\frac{3}{5}\right) - 1250 = 0 \\ & \therefore C_y = 500\ (N) \end{cases}$$

$$\therefore C = \sqrt{C_x^2 + C_y^2} = \sqrt{1000^2 + 500^2}$$

$$= 500\sqrt{5}\ (N)$$