Introduction to Discrete Math

Felipe P. Vista IV

Course Outline

- Mathematical Thinking
  - Convincing Arguments, Find Example, Recursion, Logic, Invariants
- Probability & Combinatronics
  - Counting, Probability, Random Variables
- Graph Theory
  - Graphs (cycles, classes, parameters)
- Number Theory & Cryptography
  - Arithmetic in modular form
  - Intro to Cryptography

Mathematical Thinking – Combinatronics & Probability

Probability

WHAT IS PROBABILITY

- Paradox of Probability Theory
- Galton Board
- Natural Sciences and Mathematics
- Rolling Dice
- More Examples

Predicting Unpredictable

- tossing a coin

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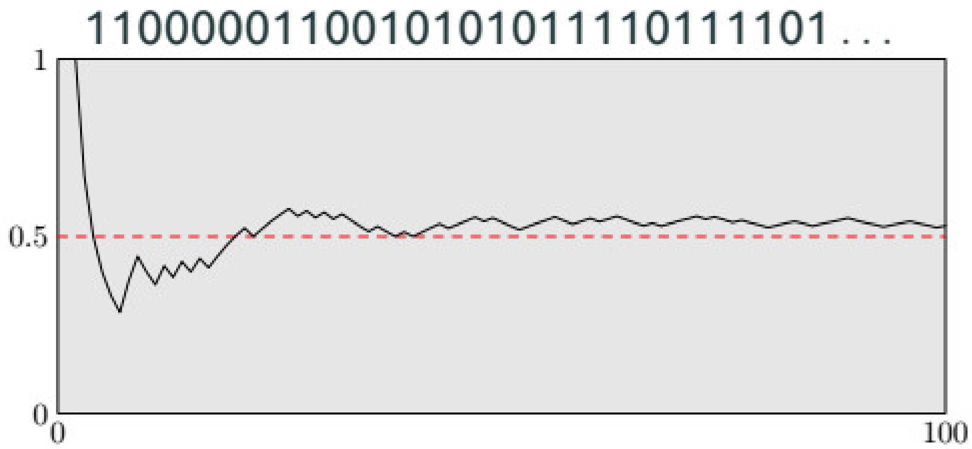
- zeroes and ones appear equally open
- frequency of *1's:* $(\#ones)/(length) \approx 1/2$

Random Bits

Hundred



By: George Marsaglia, 1995

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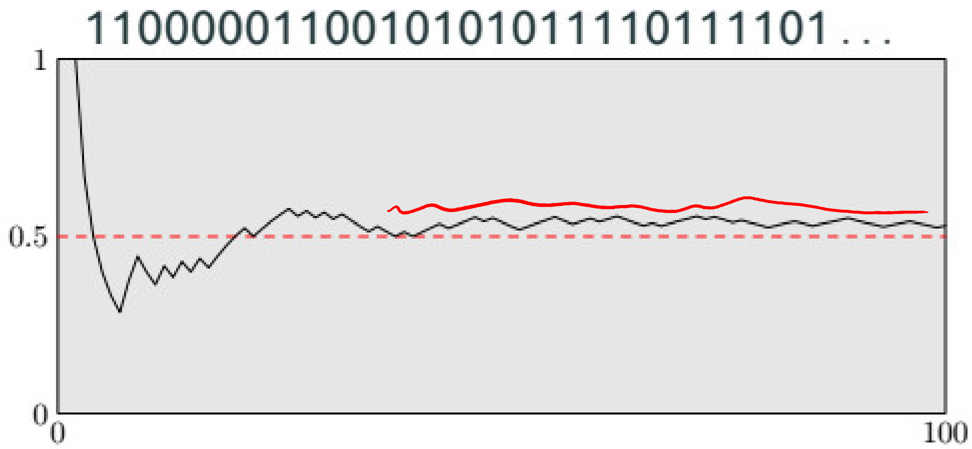

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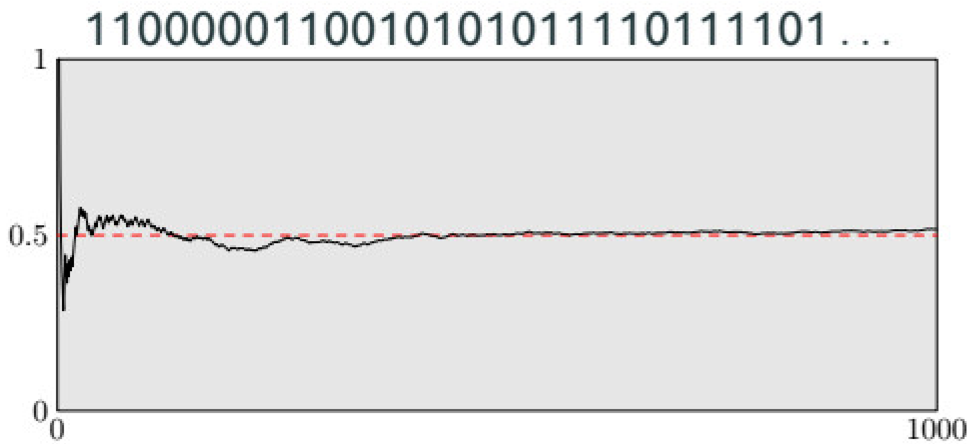

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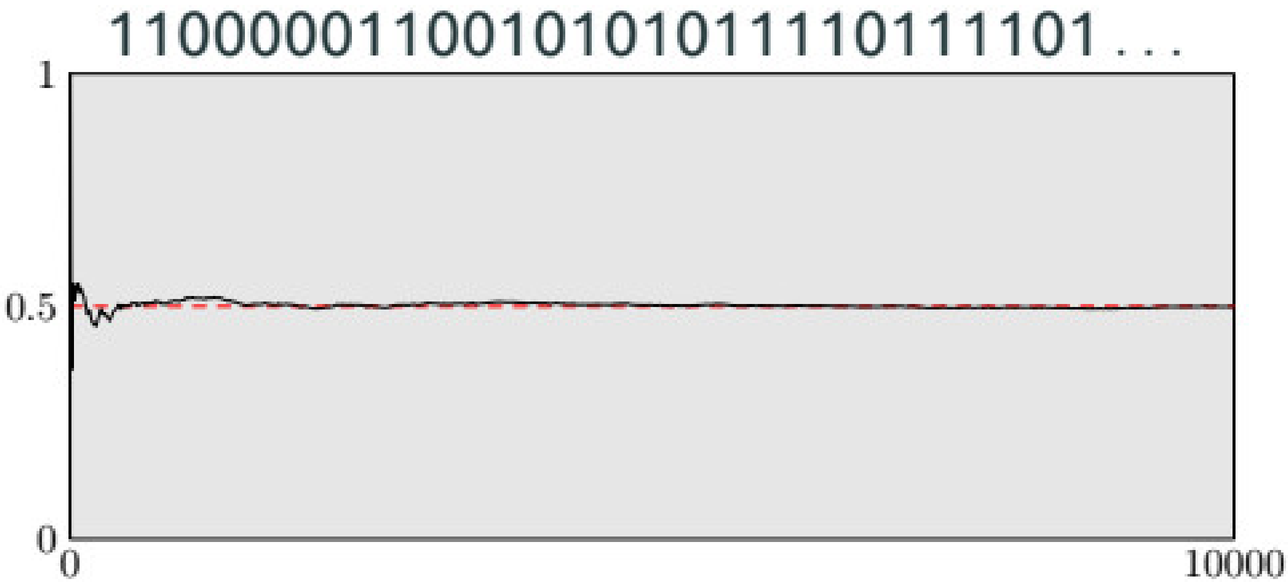

Ten Thousand

By: George Marsaglia, 1995

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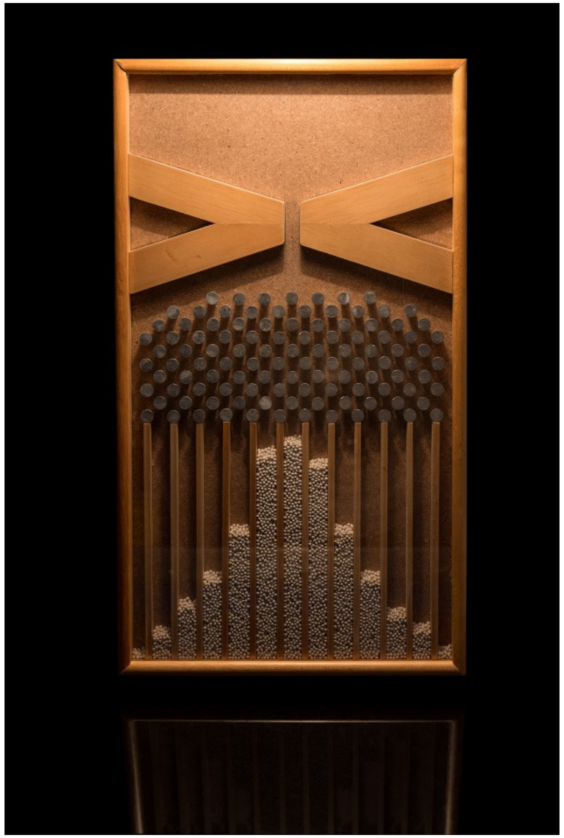
Bean Machine aka Galton Board



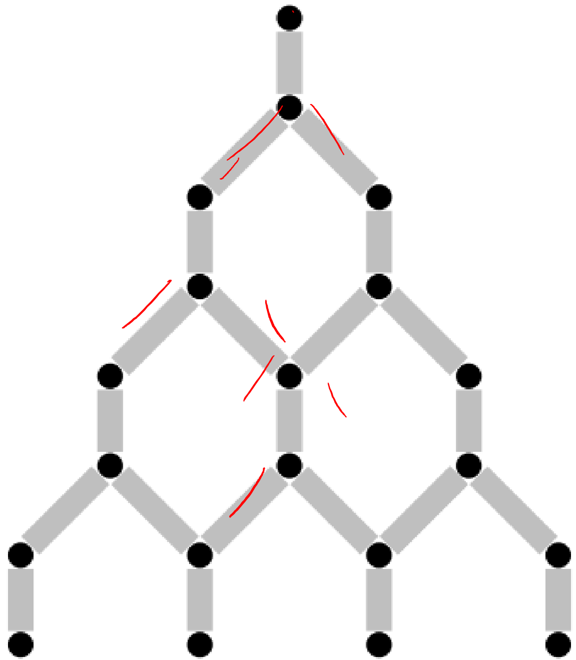
https://en.wikipedia.org/wiki/Bean_machine#/media/File:Galton_box.jpg

Bean Machine aka Galton Board

https://upload.wikimedia.org/wikipedia/commons/transcoded/d/dc/Galton_box.webm/Galton_box.webm.1080p.webm

Analysis

Assume that at each level, the beans are split evenly



Galcon and Pascal

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$$
\begin{array}{ccccccccl}
 & & & 1 & & & & & = z_0 \\
 & & 1/2 & & 1/2 & & & & = z_1 \\
 & 1/4 & & 2/4 & & 1/4 & & & = z_2 \\
1/8 & & 3/8 & & 3/8 & & 1/8 & & = z_3 \\
 & & & \cdots & & & & & = z_n
\end{array}
$$

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$$\frac{Pascal\ Triangle}{2^n} = \binom{n}{k} \Big/ 2^n$$

Concentration

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- *100* layers, bins *0* … *100*

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  - What fraction in bins $400 \ldots 600$?

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  - Natural Science: do real coins behave according to model
  - Mathematics: the implications of the model

* implication – conclusion that can be drawn

Tossing Two Coins

- Probability of 1 head or 1 tail?

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    - Janin is more right

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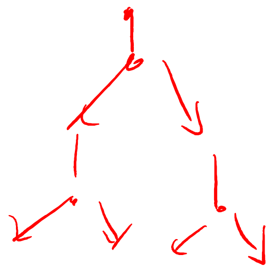

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  - Etc…

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Rolling a Dice

https://wherethewindsblow.com/wp-content/uploads/2019/04/DSC_2954-x600.jpg

- Natural Sciences:

* equiprobable – equally likely to occur

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  - A multiple of *3* appears in *1/3* of the cases $\{3, 6\} = \frac{2}{6} = \frac{1}{3}$ 33.3%

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Computing Probabilties

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fa fes o d 3 1 die $\{1, 2, \underline{3}, 4, 5, \underline{6}\}$

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$$p = \frac{15}{36}$$

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$$p = \frac{15}{36} = 0.4166 \approx 41.66\%$$

Independence

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- More than equiprobability for both dice

Independence

11

22

33

44

55

66

- More than equiprobability for both dice $\frac{6}{36} = \frac{1}{6} = 16.66\%$

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- More than equiprobability for both dice
- Simultaneous and sequential setting
- Equiprobable model is usually OK for both settings

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- Tossing a coin n times

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- Assumption : *equiprobable*

Sequence of Coin Tosses

- Tossing a coin n times
- Outcome: sequence of n bits
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- Probability of "*all heads*":

Sequence of Coin Tosses

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Sequence of Coin Tosses

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- Outcome: sequence of n bits
- 2^n outcomes
- Assumption : *equiprobable*
- Probability of "*all heads*": ½
- Event:

Sequence of Coin Tosses

- Tossing a coin n times
- Outcome: sequence of n bits
- 2^n outcomes
- Assumption : *equiprobable*
- Probability of "*all heads*": ½
- Event: "*first bit = last bit*"

Sequence of Coin Tosses

- Tossing a coin n times
- Outcome: sequence of n bits
- 2^n outcomes
- Assumption : *equiprobable*
- Probability of "*all heads*": $\frac{1}{2}$
- Event: "$first\ bit = last\ bit$"
  - Probability :

Sequence of Coin Tosses

- Tossing a coin n times
- Outcome: sequence of n bits
- 2^n outcomes
- Assumption : *equiprobable*
- Probability of "*all heads*": ½
- Event: "*first bit = last bit*"
  - Probability : ½

Sequence of Coin Tosses

- Tossing a coin n times
- Outcome: sequence of n bits
- 2^n outcomes
- Assumption : *equiprobable*
- Probability of "*all heads*": ½
- Event: "*first bit = last bit*"
  - Probability : ½
- "*number of heads is even*"

Sequence of Coin Tosses

- Tossing a coin n times
- Outcome: sequence of n bits
- 2^n outcomes
- Assumption : *equiprobable*
- Probability of "*all heads*": ½
- Event: "*first bit = last bit*"
  - Probability : ½
- "*number of heads is even*"
  - Probability :

Sequence of Coin Tosses

- Tossing a coin n times
- Outcome: sequence of n bits
- 2^n outcomes
- Assumption : *equiprobable*
- Probability of "*all heads*": ½
- Event: "*first bit = last bit*"
  - Probability : ½
- "*number of heads is even*"
  - Probability : ½

Galton Board Revisited

- Outcomes :

Galton Board Revisited

- Outcomes : Sequences of L/R with length n

Galton Board Revisited

- Outcomes : Sequences of L/R with length n
- 2^n outcomes

Galton Board Revisited

- Outcomes : Sequences of L/R with length n
- 2^n outcomes
- *Probability space*

Galton Board Revisited

- Outcomes : Sequences of L/R with length n
- 2^n outcomes
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Galton Board Revisited

- Outcomes : Sequences of L/R with length n
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Galton Board Revisited

- Outcomes : Sequences of L/R with length n
- 2^n outcomes
- *Probability space*
- Assumption : *equiprobable*
- Event :
-

Galton Board Revisited

- Outcomes : Sequences of L/R with length n
- 2^n outcomes
- *Probability space*
- Assumption : *equiprobable*
- Event : $\#R \in [0.4n, 0.6n]$

Galton Board Revisited

- Outcomes : Sequences of L/R with length n
- 2^n outcomes
- *Probability space*
- Assumption : *equiprobable*
- Event : $\# R \in [0.4n, 0.6n]$
- *probability* = *(#favourable)/(#total)*
-

Galton Board Revisited

- Outcomes : Sequences of L/R with length n
- 2^n outcomes
- *Probability space*
- Assumption : *equiprobable*
- Event : $\#R \in [0.4n, 0.6n]$
- *probability = (#favourable)/(#total)*
- $\left(\sum_{k\in[0.4n,0.6n]} \binom{n}{k} \Big/ 2^n\right)$

Galton Board Revisited

- Outcomes : Sequences of L/R with length n
- 2^n outcomes
- *Probability space*
- Assumption : *equiprobable*
- Event : $\#R \in [0.4n, 0.6n]$
- *probability = (#favourable)/(#total)*
- $\sum_{k \in [0.4n, 0.6n]} \binom{n}{k} \Big/ 2^n$

Probability Theory = Combinatronics?

- Not completely true

Probability Theory = Combinatronics?

- Not completely true
- Only the mathematical part

Probability Theory = Combinatronics?

- Not completely true
- Only the mathematical part
- Independence

Probability Theory = Combinatronics?

- Not completely true
- Only the mathematical part
- Independence
- Non-uniform distributions

Probability Theory = Combinatronics?

- Not completely true
- Only the mathematical part
- Independence
- Non-uniform distributions
- Unknown distributions

Probability Theory = Combinatronics?

- Not completely true
- Only the mathematical part
- Independence
- Non-uniform distributions
- Unknown distributions
- Continuous distributions

Thank you.