Introduction to Discrete Math

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Course Outline

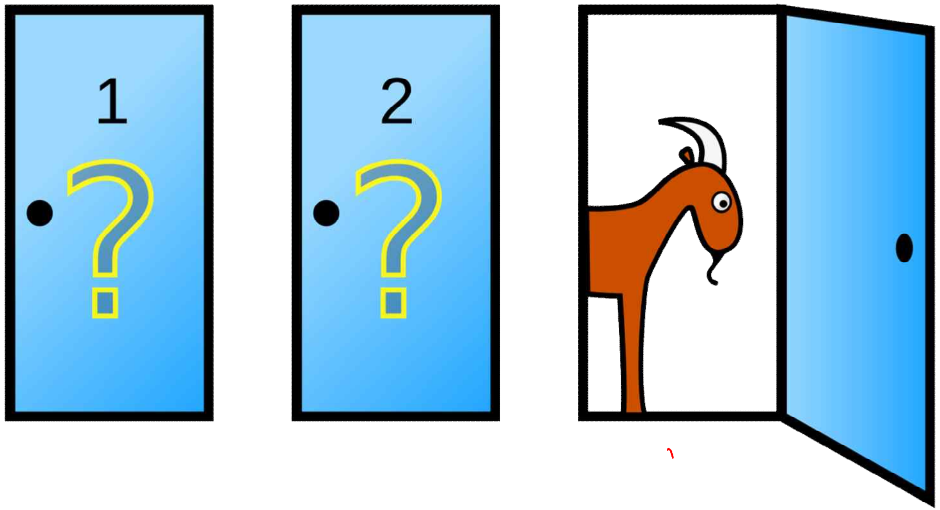
- Mathematical Thinking
  - Convincing Arguments, Find Example, Recursion & Induction, Logic, Invariants
- Probability & Combinatronics
  - Basic Counting, Binomial Coeff, Advanced Counting, Probability, Random Variables

Probability & Combinatronics - Probability

MONTY HALL PARADOX

- **Three "convincing" arguments**
- Our Position

Monty Hall Problem in Wikipedia

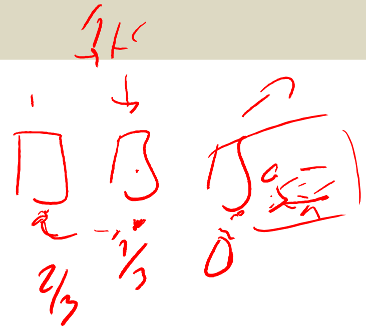


In search of a new car, the player picks a door, say 1. The game host then opens one of the other doors, say 3, to reveal a goat and offers to let the player switch from door 1 to door 2.

Game Setting

- TV show with a host and a guest
- there are three identical doors
  - *a prize behind one of them*
- randomly chosen
  - guest guesses the door with a prize then host opens the other door with no prize
  - (always exists; randomly chosen if two)
- guest can keep the guess or change it
  - door opens and prize is given (if the final guess is correct)

Conflicting Arguments



- why keep the guess: 1st choice ?

  the opening of the other door does not prove anything since an empty door always exists - so why change the guess?

- why make a new random guess among two doors:

  now there are two doors where the prize can be; we do not know where it is, so we can only make a random guess

- why change the door:

  the first door has the prize with probability 1 / 3. so the other one has it with better probability 2 / 3

Which argument convinces you more?

First Argument

* The opening of the other door does not prove anything since an empty door always exists - so why change the guess?

You do not need to prove something to influence the probabilities; the information may be indecisive, but still valuable.

Second Argument

* Now there are two doors where the prize can be; we do not know where it is, so we can only make a random guess

There are several wrong assumptions in this argument. In fact, if you want to guess the result of a random process with known probability, you should **not** imitate this process:

- *If you have a coin that gives 'head' in 70% of cases, you should always bet on 'head'.*

Third Argument

1st choice

* The first door has the prize with probability 1 / 3, so the other one has it with better probability 2 / 3.

When we speak about the probability, we have in mind some experiment. Opening the empty door changes the experiment - why does it not change the probabilities?

Spoiler Alert!

We strongly encourage you to stop here before we explain the correct answer (or 'our position on what is the correct answer').

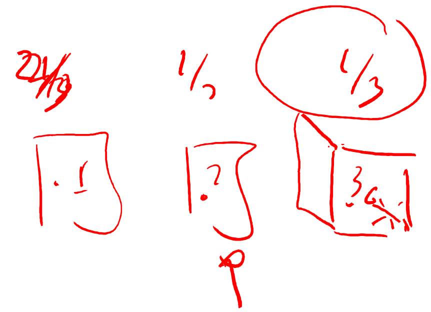
- Three “convincing” arguments

- **Our Position**

What Do We Think

- The short (wikipedia) setting is not a correct setting
- In our elaborated setting, the third solution is correct: changing the door increases the chances of winning by factor 2

How to Deal with Paradoxes



- "think about repetitive experiment"
  - *not possible → the question is bad*
  - *this case: TV show each day*
- prize is behind 1, 2, 3 in 1/3 of cases
- first guess correct in 1/3 of cases (independence)
  - *if so, the 'keep' strategy wins (in 1/3 of cases)*
  - *if not, the 'change' strategy is sure to win*
- hence, 'change' wins in 2/3 cases!
  - *'choose random' wins in 1/2 cases*

Shorter is Not Always Better

- "In search of a new car, the player picks a door, say 1. The game host then opens one of the other doors, say 3, to reveal a goat and offers to let the player pick door 2 instead of door 1."
- a sequence of events described, consistent with the following instruction for the host:
  - *"if player makes a false guess the first time, just open this door;*
  - *"if she makes a correct guess, open another empty door and suggest to change the guess"*
- obviously in this setting the suggestion to change the guess indicates that it was correct: keep it!

Thank you.