chapter 11.9 Exercise 2

$$f(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-iwx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} e^{2ix} e^{-iwx} dx \Rightarrow f(w) = \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} e^{(2-w)ix} dx$$

$$f(w) = \frac{1}{\sqrt{2\pi}(2-w)i} \left[e^{(2-w)ix} \right]_{-1}^{1} = \frac{1}{\sqrt{2\pi}(2-w)i} \left(e^{(2-w)i} - e^{-(2-w)i} \right)$$

$$= \frac{2i\sin(2-w)}{\sqrt{2\pi}(2-w)i} = \sqrt{\frac{2}{\pi}} \frac{\sin(2-w)}{2-w}$$

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chapter 11.9 Exercise 7

$$f(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-iwx} dx = \frac{1}{\sqrt{2\pi}} \left[\int_{0}^{a} x e^{-iwx} dx \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{iae^{-iwa}}{w} + \frac{e^{-iwa}}{w^2} - \frac{1}{w^2} \right]$$

$$= \frac{e^{-iwa}(1+iaw) - 1}{\sqrt{2\pi}\, w^2}$$

Chapter 11.8 Exercise 1

$$f_c(w) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos wx \, dx = \sqrt{\frac{2}{\pi}} \left[\int_0^1 (1) \cos wx \, dx + \right.$$

$$\left. + \int_1^2 (-1) \cos wx \, dx + \int_2^{\infty} 0 \, dx \right] =$$

$$= \sqrt{\frac{2}{\pi}} \left[\left\{ \frac{\sin wx}{w} \right\}_0^1 + \left\{ -\frac{\sin wx}{w} \right\}_1^2 \right] = \sqrt{\frac{2}{\pi}} \left[\frac{2\sin w - \sin 2w}{w} \right]$$

Chapter 11.8 Exercise 9

$$f_s(w) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin wx \, dx$$

$$f_s(e^{-ax}) = \sqrt{\frac{2}{\pi}} \left[\int_0^{\infty} e^{-ax} \sin wx \, dx \right] = \sqrt{\frac{2}{\pi}} \left[\frac{e^{-ax}}{a^2+w^2} \{ -a \sin wx \right.$$

$$\left. - w \cos wx \} \right]_0^{\infty} = \sqrt{\frac{2}{\pi}} \left[\frac{e^{-\infty}}{a^2+w^2} \{ -a \sin \infty - w \cos \infty \} - \frac{e^0}{a^2+w^2} \{ 0 - w \} \right.$$

$$= \sqrt{\frac{2}{\pi}} \frac{w}{a^2+w^2}$$

11.7 chapter Exercise 7

$$f(x) = \begin{cases} 1 & \text{if } 0<x<1 \\ 0 & \text{if } x>0 \end{cases}$$

$f(x) = \int_0^\infty A(\omega)\cos\omega x\,d\omega$ where $A(\omega) = \frac{2}{\pi}\int_0^\infty f(v)\cos wv\,dv$

$$A(\omega) = \frac{2}{\pi}\left[\int_0^1 1\cdot\cos wv\,dv + \int_1^\infty 0\cdot\cos wv\,dv\right]$$

$$= \frac{2}{\pi}\left[\frac{\sin wv}{w}\right]_0^1 = \frac{2}{\pi}\left[\frac{\sin w}{w}\right]$$

$$f(x) = \frac{2}{\pi}\int_0^\infty \frac{\sin w}{w}\cos xw\,dw$$

11.7 chapter Exercise 10

$$f(v) = \begin{cases} a^2 - v^2, & 0<v<a \\ 0, & v>a \end{cases}$$

$$A(w) = \frac{2}{\pi}\left[\int_0^a (a^2-v^2)\cos wv\,dv + \int_a^\infty 0\cdot\cos wv\,dv\right]$$

$$= \frac{2}{\pi}\left[\int_0^a a^2\cos wv\,dv - \int_0^a v^2\cos wv\,dv\right]$$

$$\int_0^a a^2\cos wv\,dv = a^2\int_0^a \cos wv\,dv = a^2\frac{\sin wv}{w}\Big]_0^a = a^2\frac{\sin wa}{w}$$

$$A(w) = \frac{2}{\pi}\left[\frac{2\sin aw}{w^3} - \frac{2a\cos aw}{w^2}\right]$$

$$f(x) = \frac{4}{\pi}\int_0^\infty \left[\frac{\sin aw}{w^3} - \frac{a\cos aw}{w^2}\right]\cdot\cos wx\,dw$$

11.2 chapter Exercise 26

$f(x) = a_0 + \sum a_n \cos(\pi n)$ $\qquad a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)\,dx$

$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$ $\qquad a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)\,dx = \frac{1}{\pi}\left\{\left[\frac{x^2}{2}\right]_0^{\pi/2} + \frac{\pi}{2}\left[x\right]_{\pi/2}^{\pi}\right\}$

$= \frac{1}{\pi}\left[\frac{\pi^2}{8} + \frac{\pi^2}{4}\right] = \frac{3\pi}{8}$

$f(x) = \sum_{n=1}^{\infty} b_n \sin n\pi x$ $\qquad b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$

$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx = \frac{2}{\pi}\left\{\left[x\frac{-\cos nx}{n} - \frac{-\sin nx}{n^2}\right]_0^{\pi/2} +\right.$

$\left. + \frac{\pi}{2}\left[\frac{-\cos nx}{n}\right]_{\pi/2}^{\pi}\right\} = \frac{2}{\pi}\left\{\frac{1}{n^2}\left(\sin\frac{n\pi}{2}\right) - \frac{\pi}{2n}(-1)^n\right\}$

$f(x) = \sum_{n=1}^{\infty} b_n \sin nx = \frac{2}{\pi}\left\{\left(\frac{\pi}{2}+1\right)\sin x - \frac{\pi}{4}\sin 2x + \left(\frac{\pi}{6} - \frac{1}{9}\right)\sin 3x - \right\}$

$= \left(1 + \frac{2}{\pi}\right)\sin x + \frac{1}{\theta}\sin 2x + \left(\frac{1}{3} - \frac{2}{9\pi}\right)\sin 3x + \frac{1}{4}\sin 4x + \dots$

11.2 chapter Exercise 24

$$f(x) = \begin{cases} 0, & 0<x<2 \\ 1, & 2<x<4 \end{cases} \quad f(x) = a_0 + \left[\sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{L}x\right]$$

$$a_0 = \frac{1}{L}\int_0^L f(t)\,dx = \frac{1}{4}\int_0^4 f(x)\,dx = \frac{1}{4}[x]_2^4 = \frac{1}{4}\times 2 = \frac{1}{2}$$

$$\frac{1}{2}\int_2^4 \cos\left(\frac{n\pi x}{2}\right)dx = \frac{4}{2n\pi}\left[\sin\left(\frac{n\pi 4}{4}\right) - \sin\left(\frac{n\pi 2}{4}\right)\right] =$$

$$= \frac{2}{n\pi}\left[\sin(n\pi) - \sin\left(\frac{n\pi}{2}\right)\right] = -\frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty}\left(-\frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)\right)\cos\left(\frac{n\pi x}{4}\right)$$

$$f(x) = \frac{1}{2} - \sum_{n=1}^{\infty}\left(-\frac{2}{n\pi}\sin\frac{n\pi}{2}\right)\cos\left(\frac{n\pi x}{\cancel{4}}\right)$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx = \frac{1}{2}\int_0^2 (0)\sin\left(\frac{n\pi x}{4}\right) + \frac{1}{2}\int_2^4 (1)\sin\frac{n\pi x}{4}\,dx$$

$$= \frac{1}{2}\int_2^4 \sin\left(\frac{n\pi x}{4}\right)dx$$

$$f(x) = \sum_{n=1}^{\infty}\left(\frac{2}{n\pi}\left[(-1)^{n+1} + \cos\left(\frac{n\pi}{2}\right)\right]\right)\sin\left(\frac{n\pi x}{4}\right)$$

11.1 chapter Exercise 18

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$a_0 = \frac{1}{2\pi} x \Big|_0^{\pi} = \frac{1}{2}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$a_n = \int_0^{\pi} \cos(nx)(\pi - x) dx = \frac{1}{\pi} \frac{\sin nx}{n} \Big|_0^{\pi} = 0$$

$$b_n = \frac{1}{\pi} \left(\int_{-\pi}^{0} 0 \cdot \sin(nx) dx + \int_0^{\pi} \sin(nx) dx \right) = \frac{1-(-1)^n}{\pi n}$$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1-(-1)^n}{\pi n} \sin(nx)$$

$$b_1 = \frac{2}{\pi} \quad b_2 = 0$$

$$b_3 = \frac{2}{3}\pi \quad b_4 = 0$$

$$f(x) = \frac{1}{2} + \frac{2}{\pi} \left(\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x \dots \right) \quad b_5 = \frac{2}{5}\pi$$

11.1 chapter Exercise 16

$$f(x) = \begin{cases} 0, & -\pi < x \leq \pi/2 \\ x, & -\pi/2 < x < \pi/2 \\ 0, & \pi/2 \leq x < \pi \end{cases} \qquad f(x) = a_0 + \sum_{n=1}^{+\infty} (a_n\cos(nx) + b_n\sin(nx)$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx \qquad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$$

$a_0 = 0$

$$\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx = 2\int_0^{\pi}\sin(nx)\,dx$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin(nx)\,dx = \frac{2}{\pi}\left(\int_0^{\pi/2} x\sin(nx)\,dx + \int_{\pi/2}^{\pi} 0\,dx\right)$$

$$= \frac{2}{\pi}\left(\frac{-1}{n}x\cos(nx)\Big|_0^{\pi/2} + \frac{1}{n}\int_0^{\pi/2}\cos(nx)dx\right)$$

$$= -\frac{1}{n}\cos\left(n\frac{\pi}{2}\right) + \frac{2}{n^2\pi}\sin\left(n\frac{\pi}{2}\right)$$

$$b_{2k} = \frac{(-1)^{k+1}}{2k} \qquad b_{2k-1} = \frac{2(-1)^{k+1}}{(2k-1)^2\pi}$$

$$f(x) = \sum_{n=1}^{+\infty}\left(-\frac{1}{n}\cos\left(n\frac{\pi}{2}\right) + \frac{2}{n^2\pi}\sin\left(n\frac{\pi}{2}\right)\right)\sin(nx)$$

$$= \sum_{k=1}^{+\infty}(-1)^{k+1}\left(\frac{1}{2k}\sin(2kx) + \frac{2}{(2k-1)^2\pi}\sin(2k-1)x\right)$$

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