Introduction to Discrete Math

Felipe P. Vista IV

Chonbuk National University

Global Frontier Colllege

Introduction

- Class Administrative Matters

- Introduction

Introduction to Data Structure

CLASS ADMIN MATTERS

Who me?

- Faculty member GFC – School of International Eng'g & Science
  - Network System Control Lab – Electronic Eng'g Dept., JBNU
- PhD in Electronic Engineering, JBNU
- Worked at Industry & Government of Philippines
  - Mostly systems using commercial/opensource DBs/DBMSs
- Research Interests:
  - Systems Design, Software Development, Fuzzy Logic, Sensor Fusion, Embedded Systems, Navigation systems, Marine Information System, Signal Processing, Augmented Reality, MRI Systems.

Class Information

- Class Schedule
  - Mod: 09:00(9am) – 11:00(11am); Wed: 09:00(9am) – 10:00(10am)
- Textbook
  - *"Discrete Mathematics, An Open Introduction, 3rd edition"*, Oscar Levin, ISBN-13: 978-1534970748 (CreateSpace Independent Publishing Platform)
  - *"Discrete Mathematics for Computer Science, 1st Edition"*, Stein, Drysdale, Bogart, ISBN-13: 9781534970748 (Addison-Wesley)

Class Information

- Mode of instruction
  - Video Upload/ Online lecture via ZOOM
- Assignments
  - Given during lecture or posted at IELMS
- MidTerms and Finals
  - Online via IELMS

Grading

- Midterms : 20%
- Finals : 30%
- Attendance/participation : 20%
- Assignment : 30%

Grading

- Mid Terms (20%) and Finals (30%)
  - Enough time will be given
  - It is ok to discuss with classmates but **submit your own solution!**
  - Discussing is ok, cheating is "no-no"→ candidate for automatic "F"

  - Late submission = less points, maximum 90%-95% per item/number
    - In case we have to do tests online
  - Wrong answer, is 10% = "F" per item/number
  - Submission but no answer is automatic 0% = "F" per item/number
  - Non submission is automatic 0% = "F" per item/number

Grading

- Attendance/participation : 20%
- Attendance (8)
  - more than 15 mins late = *absent*, and 3 late = 1 *absent*
  - more than 3 absences = *problem (very biiig)*
  - Everybody start with 8 points for attendance
    - Become less if too much absences, ex: 70% of 8 = 5.6 points
- Participation(12)
  - answer/raise questions during lecture to get points
  - everybody starts with 12 points for participation
    - Become less if you have less than 6 class participation, ex: 70% of 12 = 8.4

Grading

- Assignment : 30%
  - It will take some time
  - Mostly practical, to help learn
  - It is ok to discuss with classmates but **do it yourself!**
  - Assignments usually due one week after posting,

  - Late submission = less points, maximum 90%-95% per item/number
  - Wrong answer, is 10% = "F" per item/number
  - Submission but no answer is automatic 0% = "F" per item/number
  - Non submission assignment is automatic 0% = "F" per item/number

Grading

- Midterms : 20%
- Finals : 30%

MIDTERMS

1	2	3	4	5	6	7	8	9	10	Overall Score
10	5	10	10	10	10	9	8	5	5	82.00
7	9	10	10	10	10	7	9	10	10	92.00
5	5	10	10	10	8.5	5	8	9	8.5	79.00
										0.00
10	9	8.5	5	9	10	5	5	10	5	76.50
5	5	10	10	10	10	8.5	5	8.5	8.5	80.50
5	5	10	10	10	8.5	8.5	9.25	10	7	83.25
10	10	10	10	10	10	7	9	9	10	95.00
10	10	10	10	10	10	10	9.75	10	10	99.75
10	7	10	10	10	9	8	10	10	10	94.00

FINAL EXAMS

1	2	3	4	5	6	7	8	9	10	Overall Score
10	9	10	9	9	10	10	7	10	10	94.00
7	7	10	7	7	10	7	7	7	10	79.00
10	10	10	5	8	7	5	7	5	5	72.00
										0.00
10	7	10	8	5	5	5	5	5	5	65.00
5	5	5	5	5	5	5	5	5	5	50.00
10	10	10	10	9	9	10	9	10	5	92.00
10	8	10	9	5	5	5	5	10	9	76.00
10	10	10	7	10	10	10	10	10	10	97.00
10	10	10	10	10	10	10	9	10	10	99.00

Grading

- Attendance/participation : 20%

Week14			Week15 (Finals)			Raw Score	Grade Equivalent
16-Jun	16-Jun	18-Jun	23-Jun	23-Jun	25-Jun		
						0.00	100
						3.00	70
						1.00	100
1	1	1	1	1	1	20.00	0
						1.00	100
						3.00	70
		1				2.00	100
		1				3.00	70
						0.00	100
						2.00	100

					CLASS PARTICIPATION					
Week14		Week15 (Finals)			No of Times participated	Improtant ones missed	Grades I	Regular Conversation	Addl Points based on Regular Conversation	Grade I + Addl Points
16-Jun	18-Jun	23-Jun	23-Jun	25-Jun		(Total of 5)			(0.5 points per)	
	1				7.00	0	100	2	1	101
					4.00	3	97	2	1	98
					1.00	5	95	1	0.5	95.5
					0.00	5	95	0	0	0
					2.00	5	95	2	1	96
					2.00	4	96	1	0.5	96.5
					0.00	5	95	0	0	95
					2.00	5	95	2	1	96
	1				11.00	0	100	6	3	103
					4.00	5	95	4	2	97

Grading

- Assignment : 30%

ASSIGNMENTS										
1	2	3	4	5	6	7	8	9	10	Overall Score
	90	80	86	95	95	95			95	**90.86**
	100	95	97	100	100	70			100	**94.57**
	90	85	84	85	85	70			70	**81.29**
										0.00
	90	90	94	90	90	88			100	**91.71**
	70	70	70	70	70	70			70	**70.00**
	90	85	84	85	80	70			70	**80.57**
	89	90	96	90	90	98			100	**93.29**
	100	100	98	90	100	90			100	**96.86**
	100	100	97	100	100	100			100	**99.57**

Midterms (30%)	Finals (35%)	Attendance/Participation (20%)		Assignment (15%)	Overall Score	Equivalent Score
		Attendance (5%)	Participation (15%)			
82.00	94.00	100.00	101.00	90.86	91.28	Ao
92.00	79.00	70.00	98.00	94.57	87.64	B+
79.00	72.00	100.00	95.50	81.29	80.42	Bo
0.00	0.00	0.00	0.00	0.00	0.00	F
76.50	65.00	100.00	96.00	91.71	78.86	C+
80.50	50.00	70.00	96.50	70.00	70.13	Co
83.25	92.00	100.00	95.00	80.57	88.51	B+
95	76.00	70.00	96.00	93.29	86.99	B+
99.75	97.00	100.00	103.00	96.86	98.85	A+
94.00	99.00	100.00	97.00	99.57	97.34	A+

Student Responsibilities

- Download/Install ZOOM app for online lecture
  - Zoom profile must be your OASIS ID+name similar to OASIS
  - Ex.: 202061234(YourName)
- Regularly login and check on-line learning system for updates, notifications
  - https://ieilmsold.jbnu.ac.kr
  - Presentations & lecture videos will be uploaded after class
- Regularly check Kakao Group Chat
  - Everybody must have a Kakao talk account
  - Search & add account "**botjok**" then you will be added to the group chat

Course Outline

- Mathematical Thinking
  - Convincing Arguments, Find Example, Recursion, Logic, Invariants
- Probability & Combinatronics
  - Counting, Probability, Random Variables
- Graph Theory
  - Graphs (cycles, classes, parameters)
- Number Theory & Cryptography
  - Arithmetic in modular form
  - Intro to Cryptography

Reminder

- Everybody, make sure that your name in ZOOM is in the following format:
  - Ex: 202054321 Juan Dela Cruz

Not changing your name to this format

* you might be marked Absent * → absent?

Intro to Discrete Structure

CLASS PROPER

Course Outline

- Mathematical Thinking
  - Convincing Arguments, Find Example, Recursion, Logic, Invariants
- Probability & Combinatronics
  - Counting, Probability, Random Variables
- Graph Theory
  - Graphs (cycles, classes, parameters)
- Number Theory & Cryptography
  - Arithmetic in modular form
  - Intro to Cryptography

Mathematical Thinking – Arguments & Reasoning

PROOFS

Arguments & Reasoning

- Proofs?
- Proof by example
- Impossibility

Do we need Proofs?

- main tool for mathematicians!!!
- not really useful for a programmer?
  - proof that a video game is correct?
  - maybe also of search engine(s)?
- take note though
  - real-time operating system
  - protocol for encryption

What is Proof?

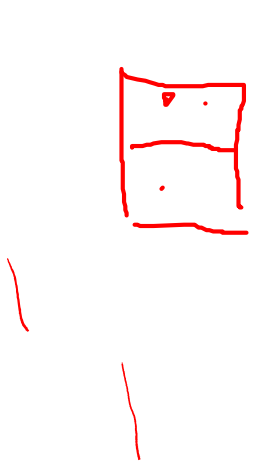
- a long formal meaningless manipulation?
- no, it is an argument that is so convincing you will use it to convince others
- it is a result of and a sign of understanding
- proofs can be fun :D

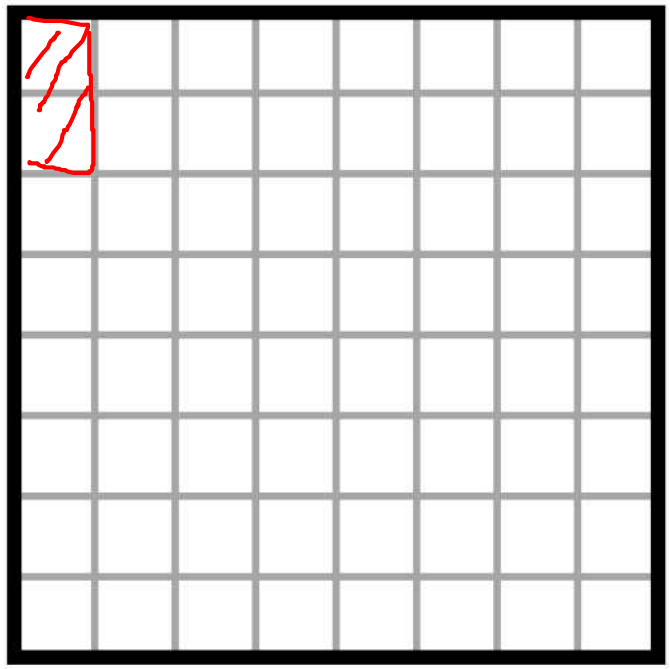
Topic Objectives or How To's

- how to understand proofs by learning through examples
- how to create proofs
- how to explain proofs
- how to appreciate and like proofs

Let's start

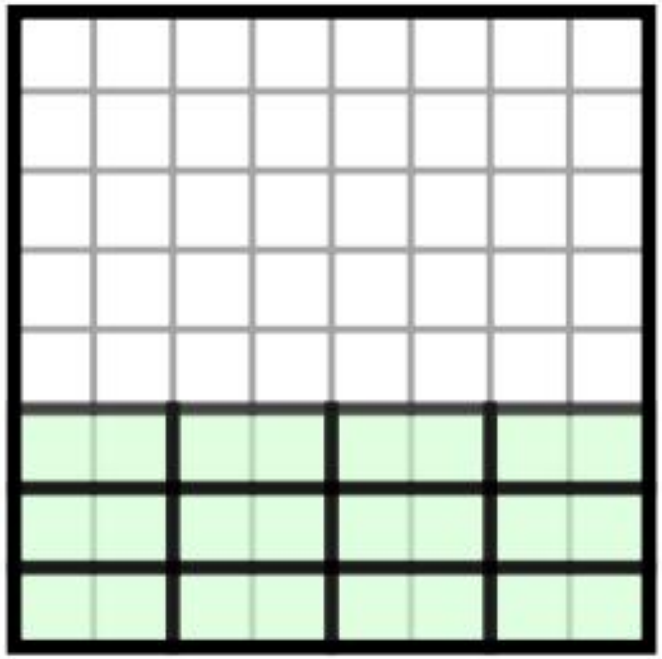
Can you completely tile an 8x8 chessboard with 1x2 domino tiles?





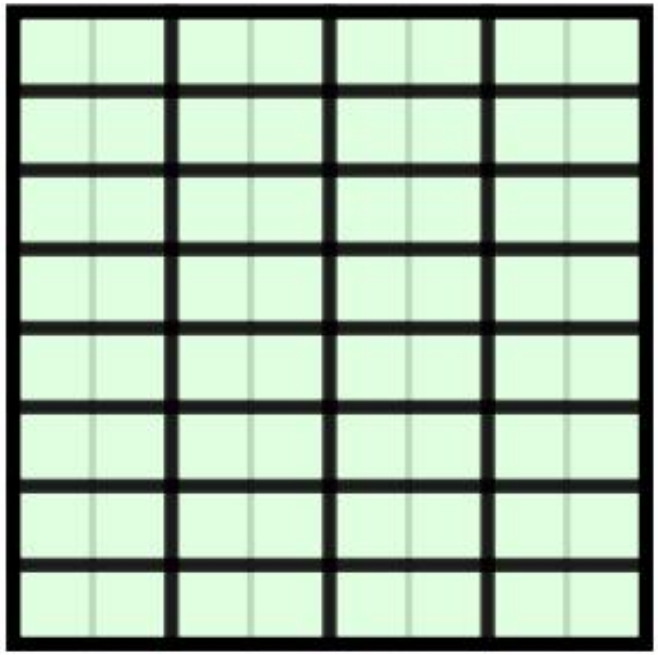
Let's start

Can you completely tile an 8x8 chessboard with 1x2 domino tiles?



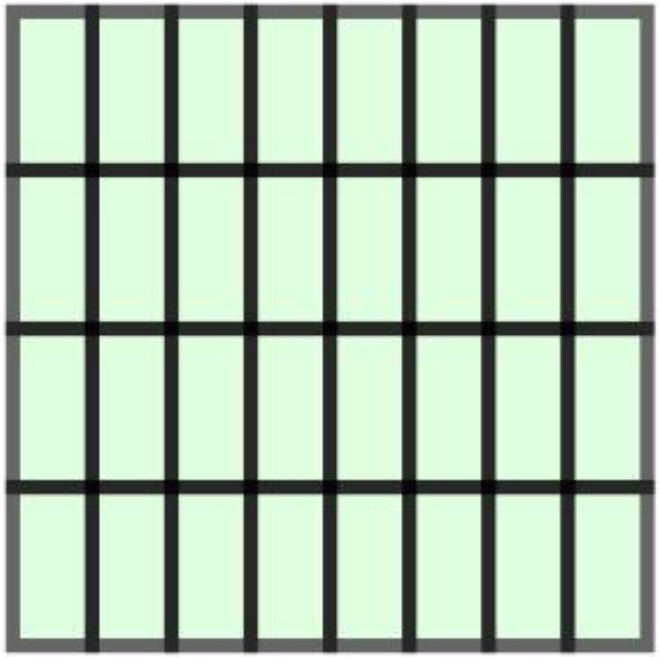
Let's start

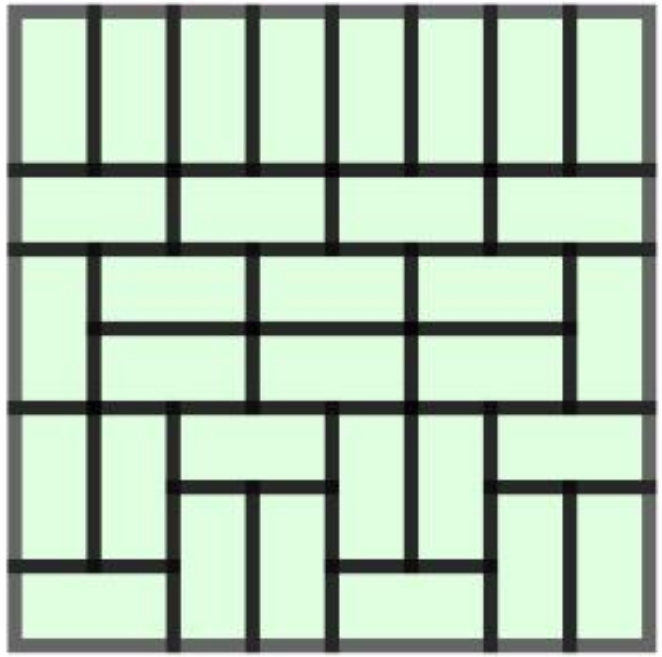
Can you completely tile an 8x8 chessboard with 1x2 domino tiles?



Let's start

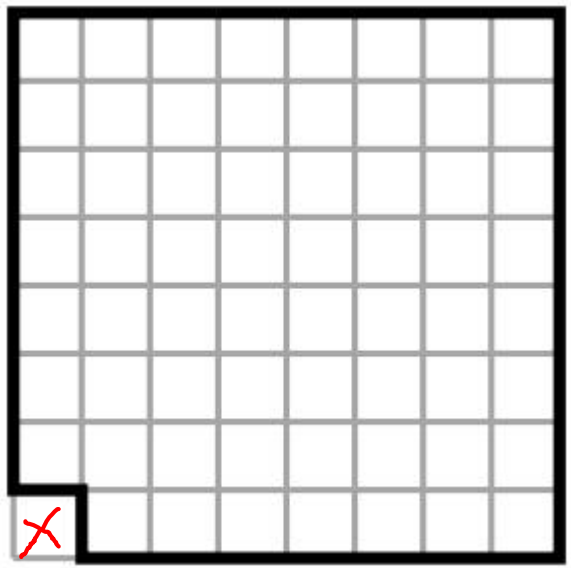
The tiling given is enough for the proof. But there are other possibilities too:





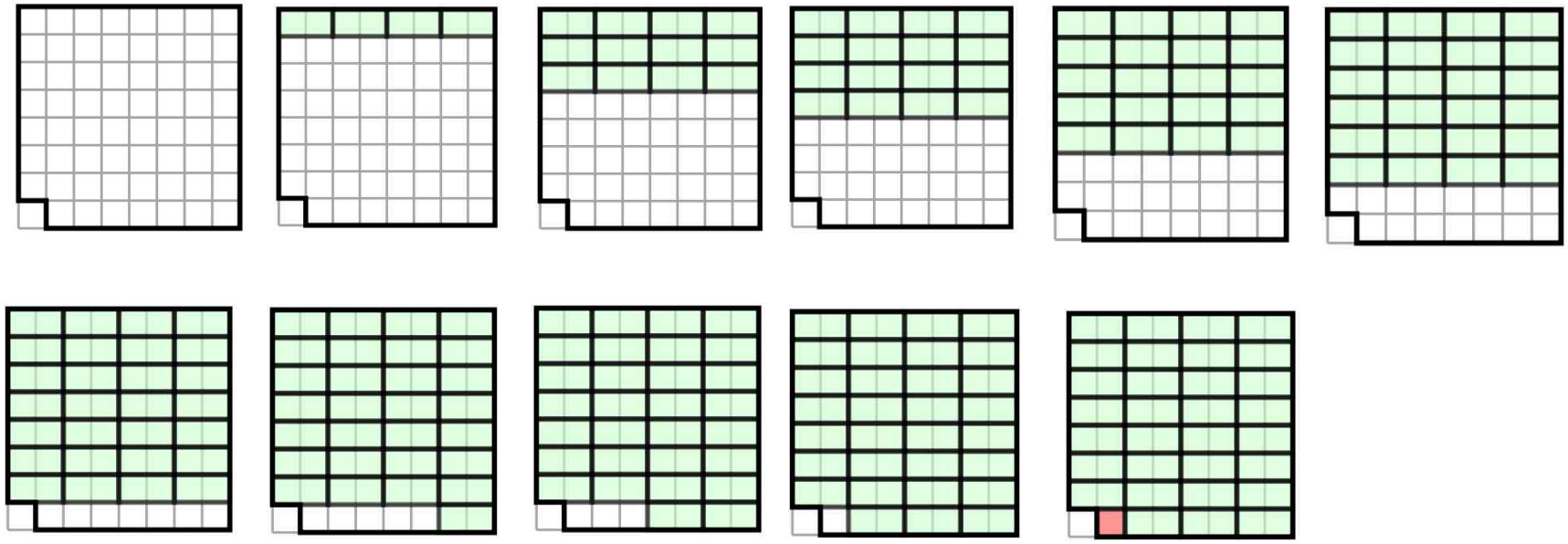
Can This Board Be Tiled?

Take note of missing box at lower-left



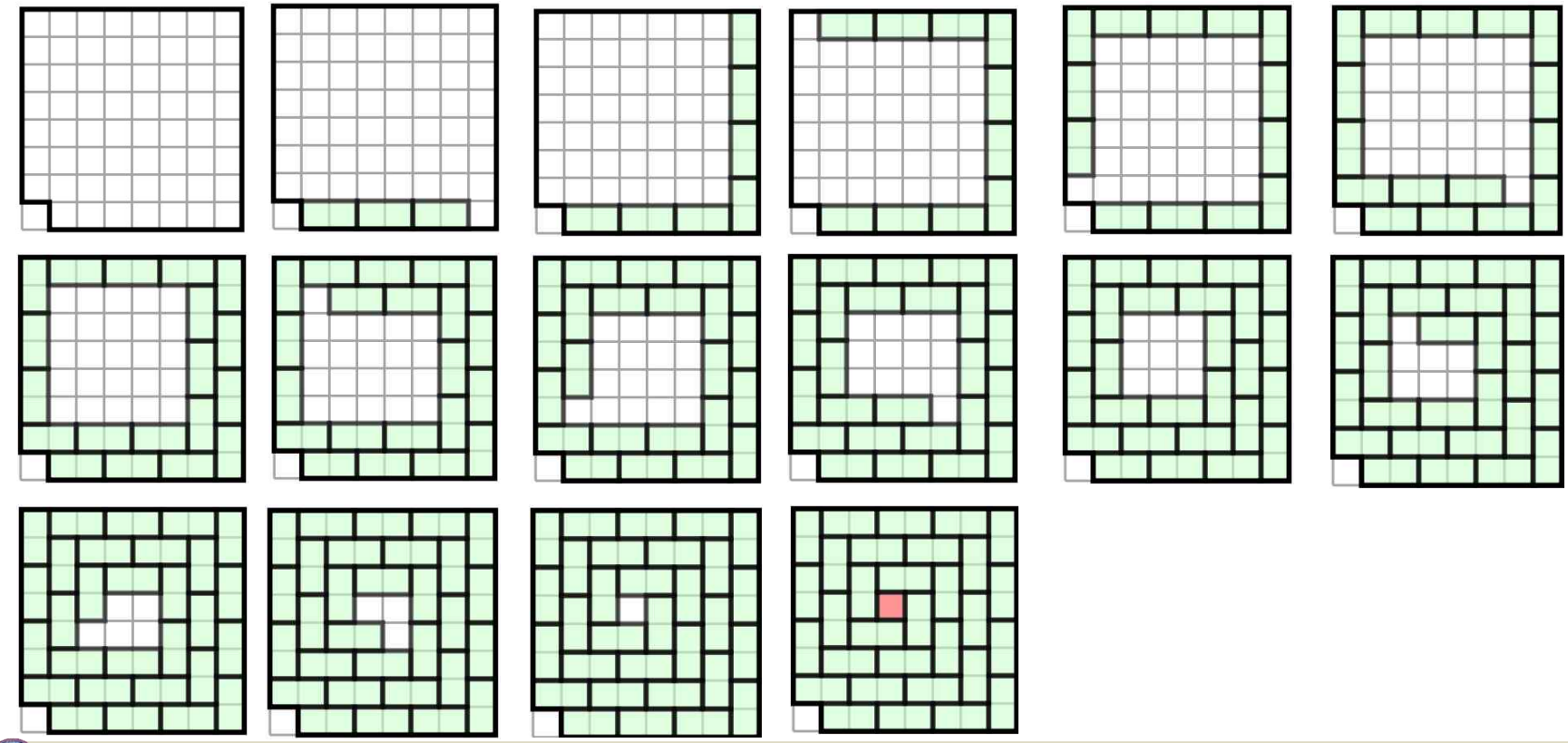
Can This Board Be Tiled?

Take note of missing box at lower-left



Can This Board Be Tiled?

How about trying another way?

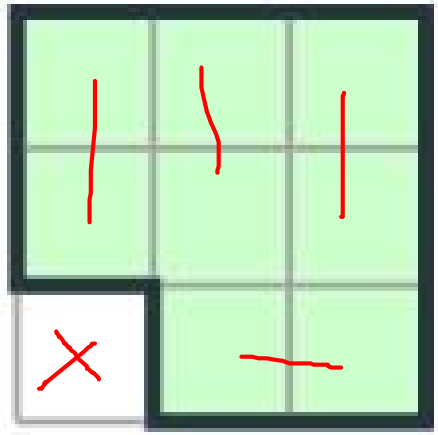


What Does This Mean?

- did we find a tiling that works and prove it exists?
  - NO
- did we prove that tiling does not exist?
  - NO
- challenge!!

Challenge!!

- Can you tile this 3x3 shape (missing corner square) using 1x2 tiles?

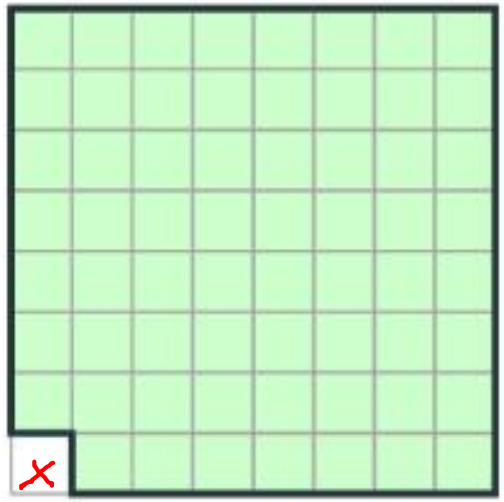


Yes, in many ways.
One example is using vertical tiles except for the lower two squares where we use a horizontal tile

Yes No

Challenge!!

- Can you tile this 8x8 shape with missing corner?

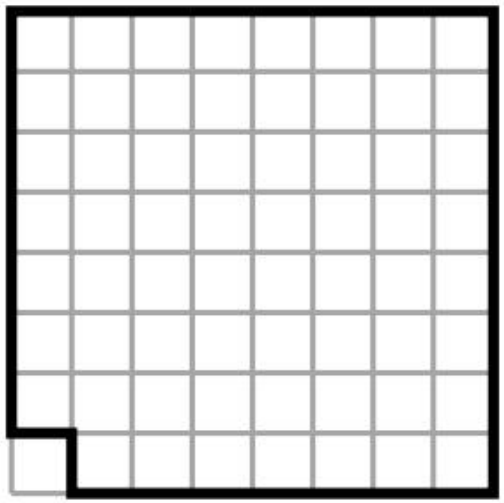


No, it is impossible!
Remember the activity earlier?
Let's check the proof from
the next slides.

Yes No

Proof of Impossibility

Can we tile this 8x8 board?

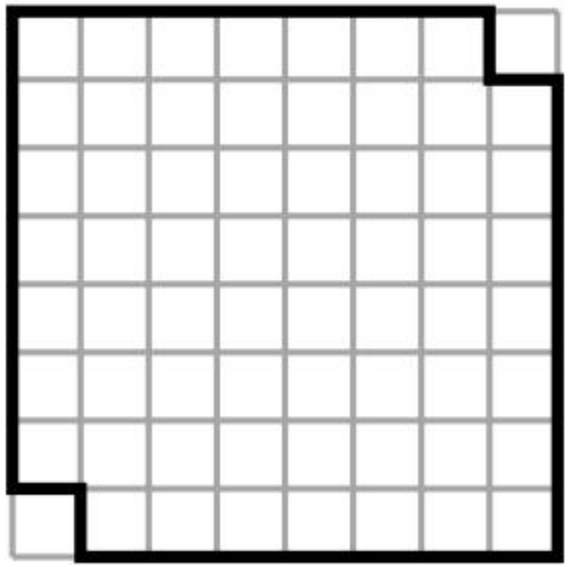


Proof of Impossibility

- one cell will always be untiled
- there are an odd number of cells to be tiled
  - 8 x 8 -1 = 63 cells
- 62 cells can be covered by total of 31 tiles
  - 31 x 2 = 62
- Hence, the objective is proven to be impossible

Proof of Impossibility

How about if two corners are cut from opposite sides?

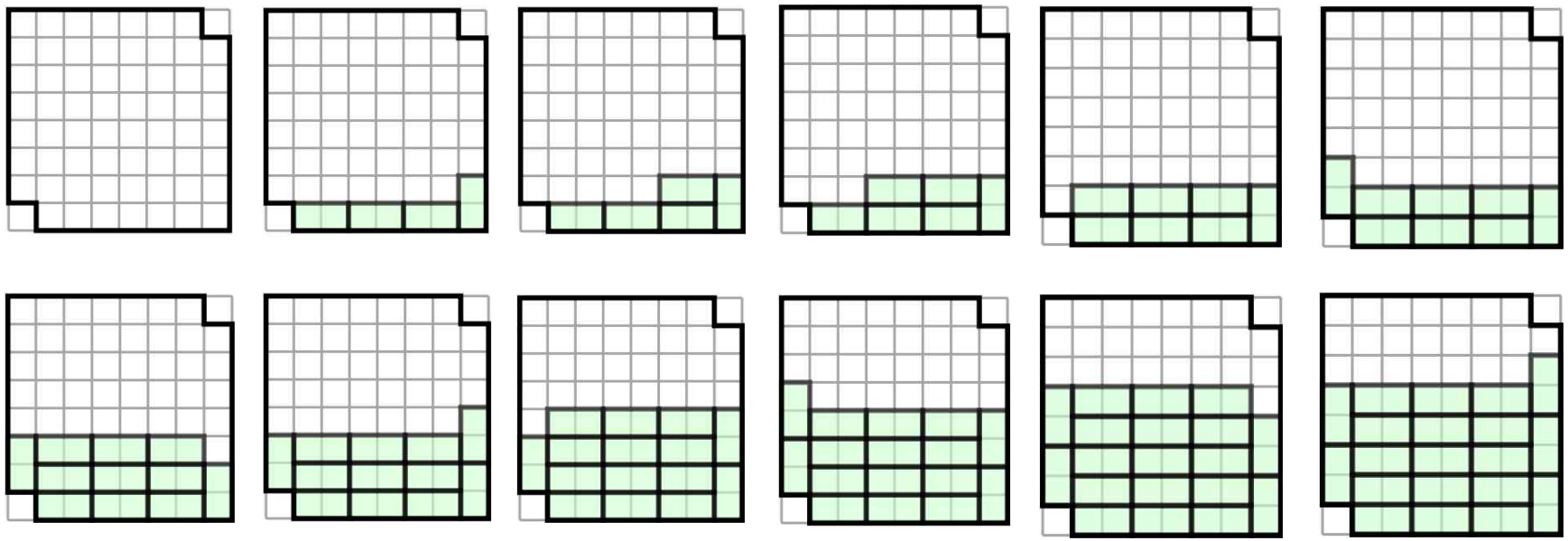


Proof of Impossibility

- there are even number of cells to be tiled
  - $8 \times 8 - 2 = 62$ cells
- 62 cells can be covered by total of 31 tiles
  - $31 \times 2 = 62$
- tiling is possible then? Let's check.

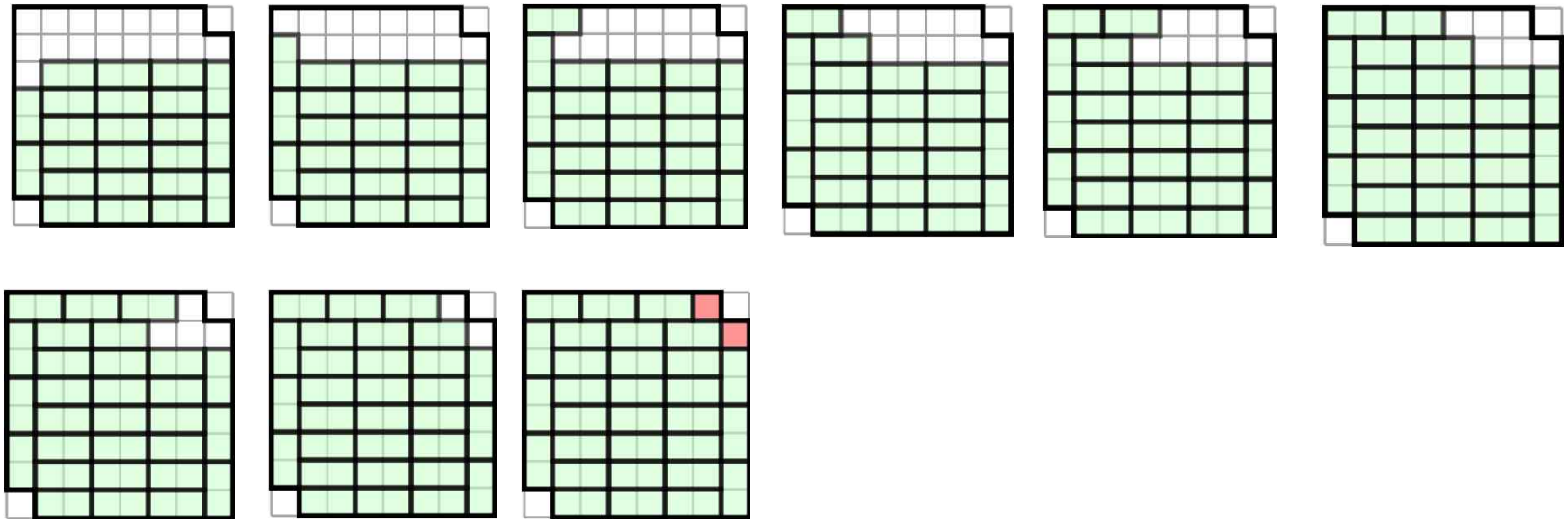
Can This Board Be Tiled?

Two corners are cut from opposite sides



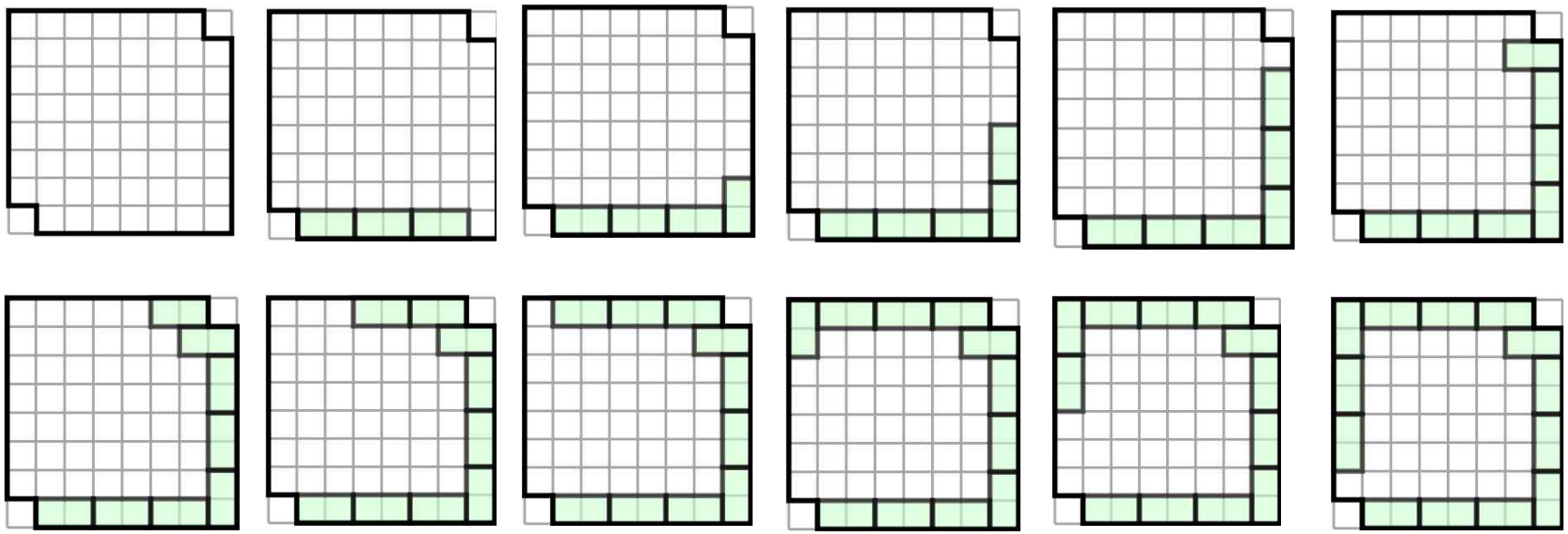
Can This Board Be Tiled?

Two corners are cut from opposite sides



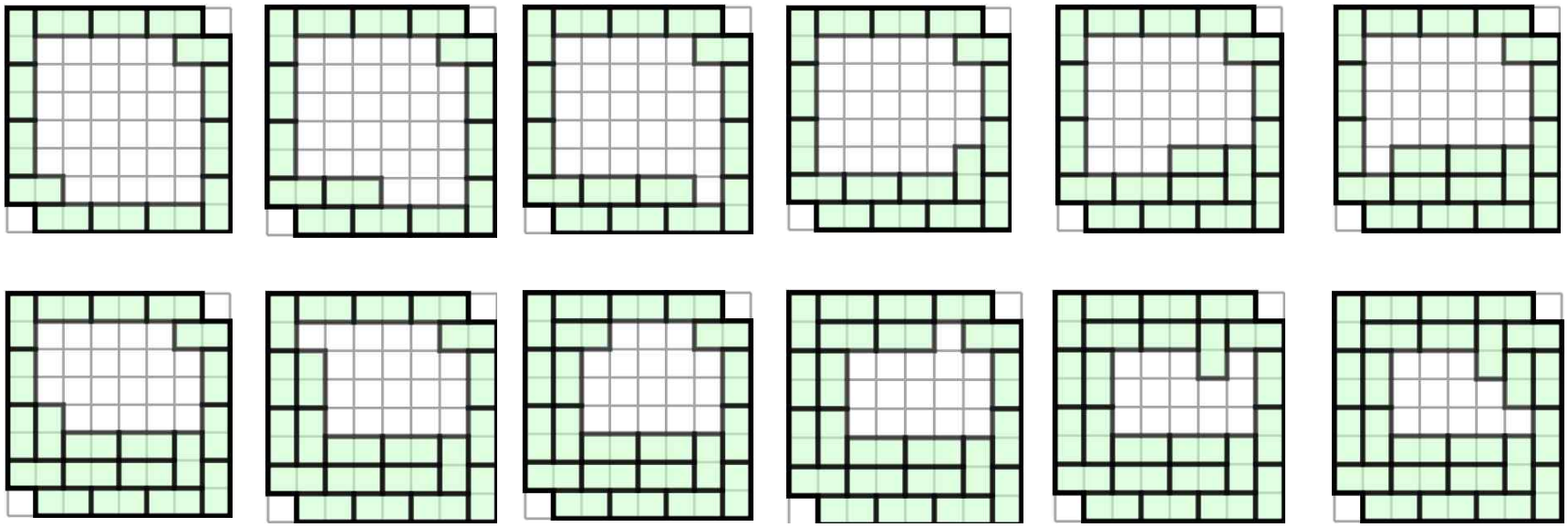
Can This Board Be Tiled?

Let us try another way



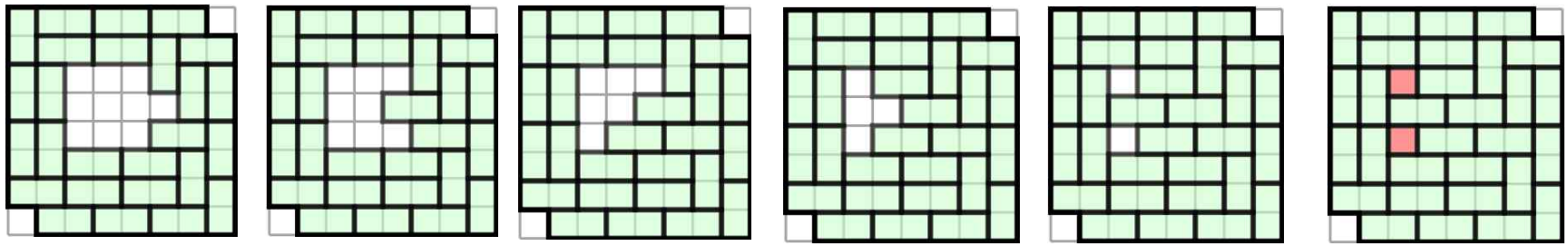
Can This Board Be Tiled?

Let us try another way



Can This Board Be Tiled?

Let us try another way



Still want to try and find another way???

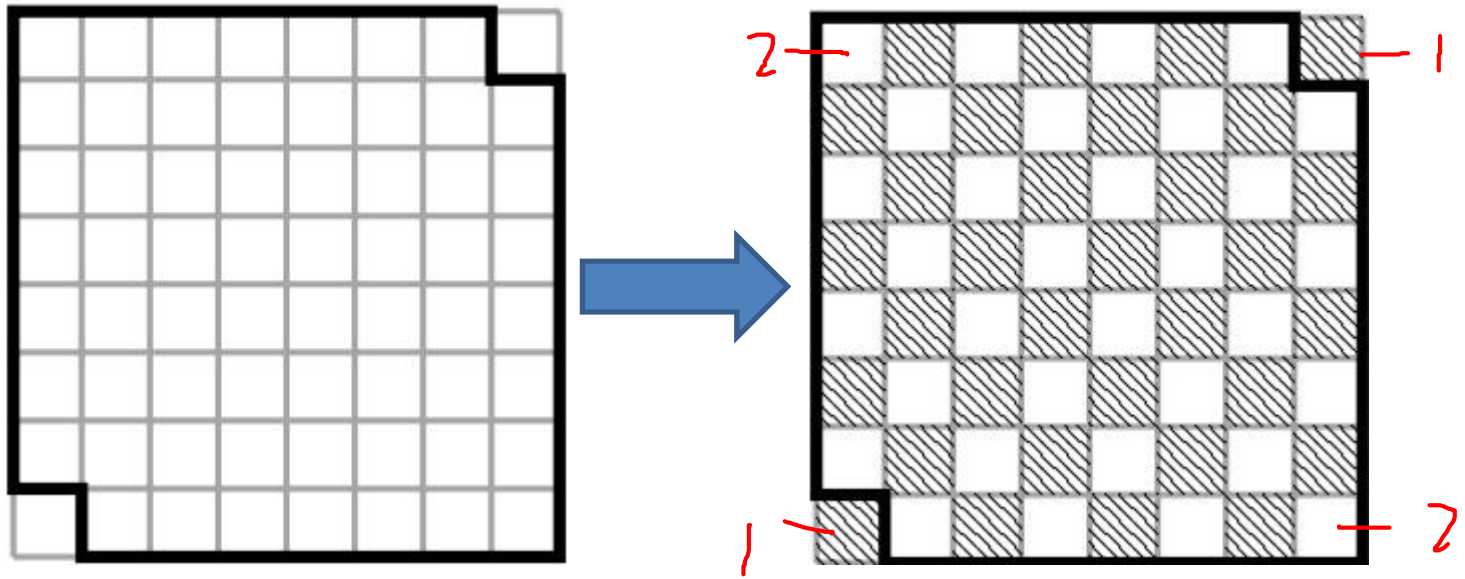
You think is it possible to tile this 8x8 shape with 2 cells cut at opposite ends?

It is impossible…

Proof of Impossibility – Part II

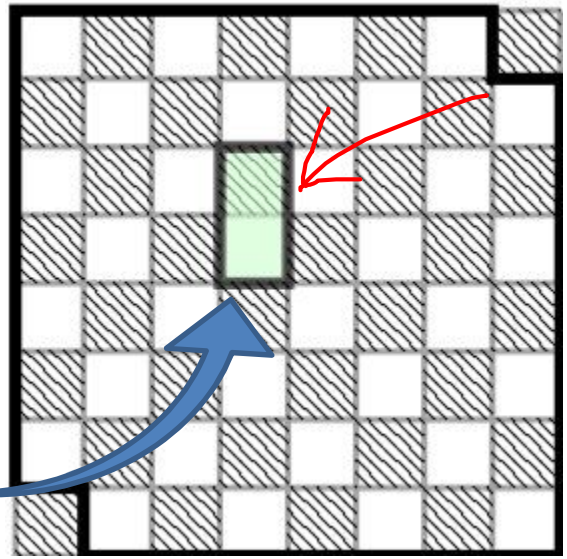
Can we tile this 8x8 board?

Let us treat the 8x8 shape as a
8x8 chess board w/ black & white colors



Proof of Impossibility – Part II

a 1x2 domino will always cover two colors (black & white)



- 8x8 = 64 total
- 32 whites & 32 black
- minus two blacks
- 32 whites & 30 blacks
- ⇒ two whites remain

Proof of Impossibility

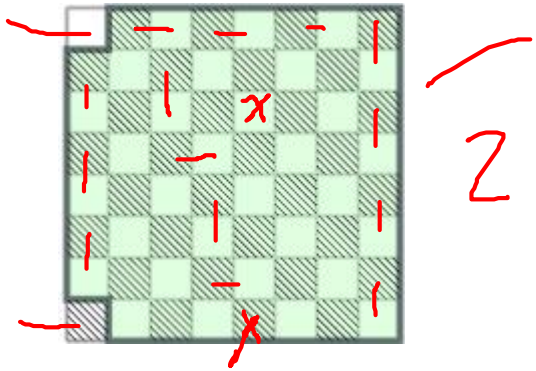
Theorem. A chess board (8x8) without two opposite corners cannot be tiled by 1x2 dominos

Proof.

- black and white cells
  - each row & column has 4 blacks and 4 whites
- opposite corners are black (or white, depending on choice)
- 1x2 domino has two different colors
- board has 30 black & 32 whites available
  - two whites at the least will remain
- q.e.d. (quod erat demonstradum)-> (that which is to be demonstrated)

Points to ponder

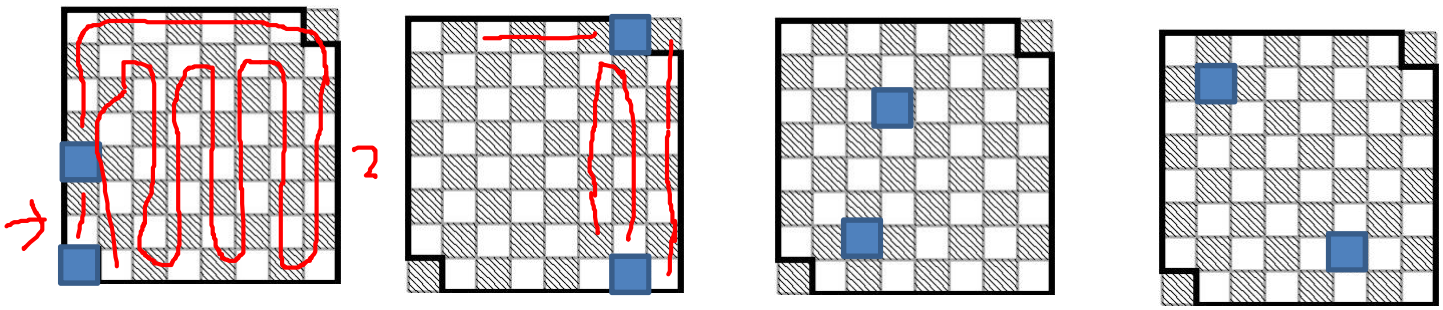
- Can we tile the board if we cut two non-opposite corners?



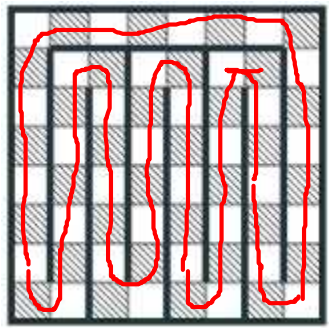
- yes, there are many possible layouts & using only vertical tiles is one of them
- it is very important to show an example, not just count black & white cells
- take note:
  - we know that **if** tiling exists, **then** num of black & white tiles is the same
  - but!!! it is not guaranteed that **if** num of black & white tiles are the same, **then** tiling exists

Points to ponder

- how about if we cut any two cells of different colors?



Hint: "snake" in the board



- if you delete 2 cells from the snake, it is split into 2 parts
- since colors are different, each part has even number of cells (one could also be empty)
- hence, the parts of the "snake" can be cut into blocks with length of 2 cells, i.e. tiles.

Mathematical Thinking – Arguments & Reasoning

EXISTENTIAL PROOFS

Arguments & Reasoning – Existence Proofs

- When One Example is Ok
- Splitting/Cutting Figures
- Making it Fun
- What Are Your Rights?
- You Can't Always Win

Know What You're Searching For

Ex. Find a "yeti" or "abominable snowman"!

Can be this:

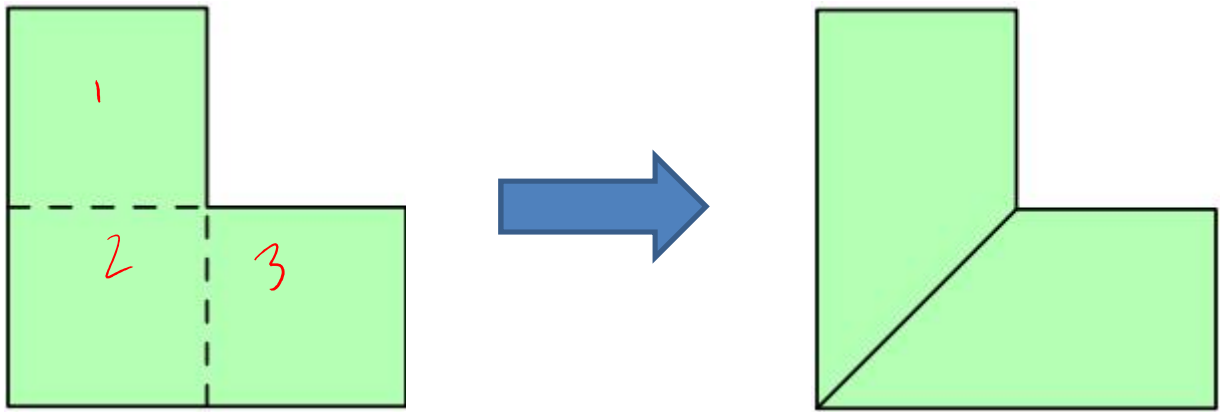
Or maybe this?

Proofs For Existential Statements

- understand what the proof looks like
- it depends on what you want to prove
- Existential Statement
  - **claim**: object with given properties exists
  - **proof**: show an example
    - one example is enough

Cutting Figures

- Prove that the given figure can be split into 2 congruent pieces
  - **congruent** pieces: having the same size and shape

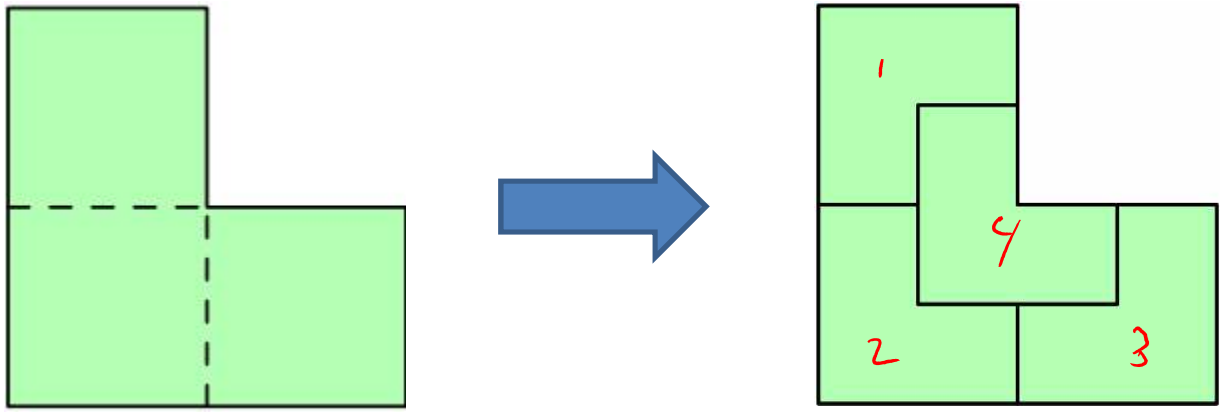


* Splitting it into 3 congruent pieces is relatively easy, but how about into 4?

Cutting Figures

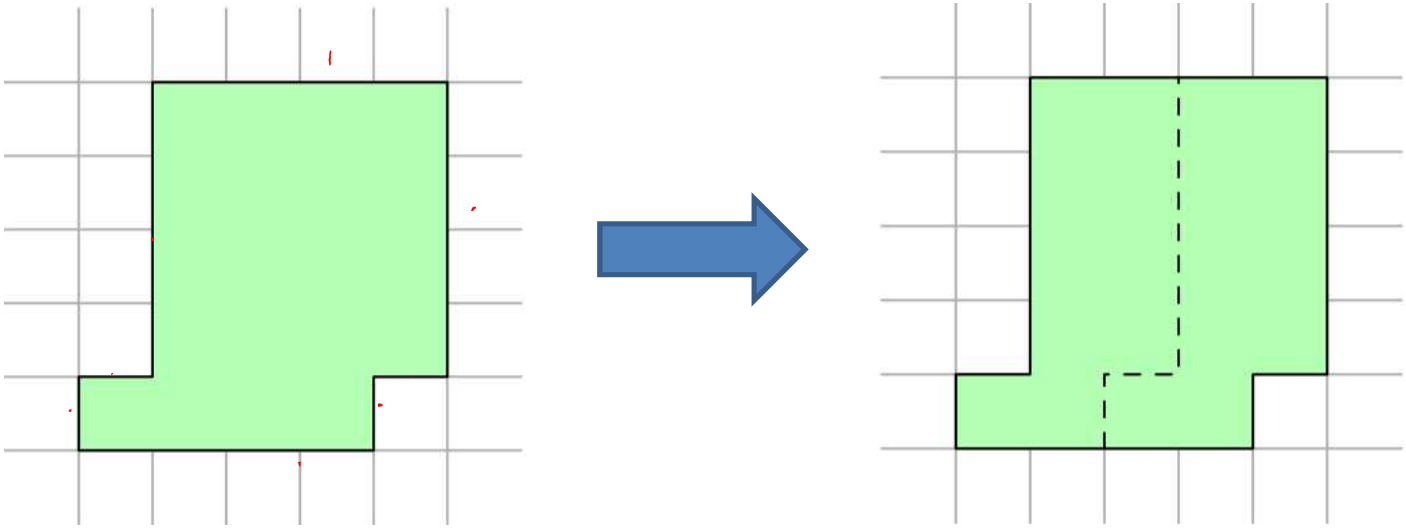
- Prove that the given figure can be split into 4 congruent pieces

Hint: These pieces should still be "L-shaped" but smaller 



Cutting Figures

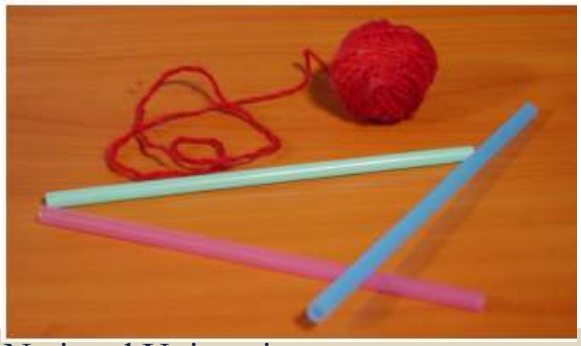
- Splitting the octagon into 2 congruent pieces
  - interesting but more difficult



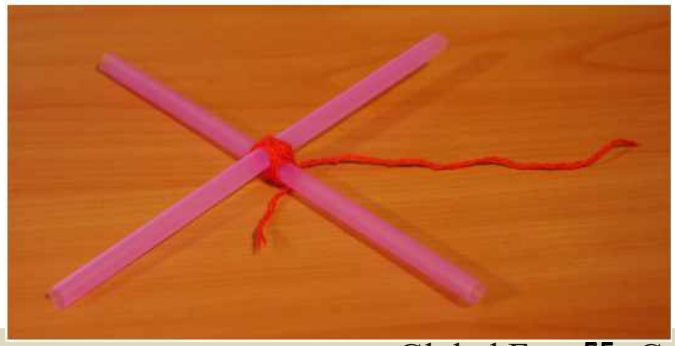
* How about into 3? Try to think about it.

Making It Fun

- **Tensegrity**
  - floating compression (tensional integrity)
  - from tension & integrity (Buckminster Fuller)
- create structure using thread and straws
  - connected by threads, straws don't touch each other
  - solid (rigid) that the pieces can't move, only the structure

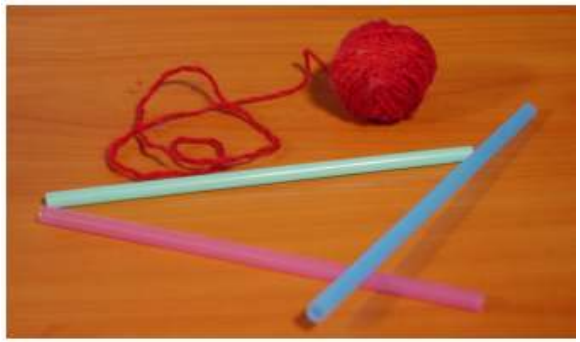


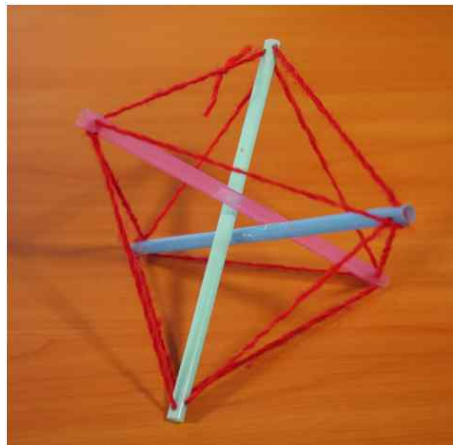
This is not allowed

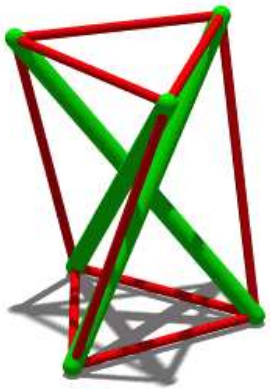


Making It Fun

- Tensegrities



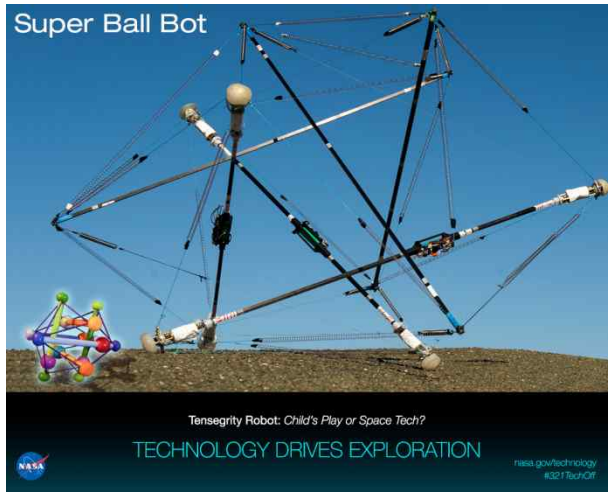




Tensegrity for Real

- Kurilpa Bridge
  - world's largest tensegrity bridge in Brisbane, Au

- NASA SUPERball Robot
  - all-in-one landing & mobility platform



Protect Your Sources

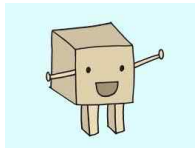
- Ex: Find a two digit number that becomes 7 times smaller when first digit is deleted
  - not that difficult since not that many options
  - check numbers less than 100 & is divisible by 7:
    - 14, 21, 28, 35, 42, 49, 56, 63
    - Found the answer?
      - it is 35 ⇒ (3)(5 x 7 = 35)
- What if find a number that becomes **57 times** smaller?
  - Next day you might say: 7125 ⇒ (7) (125 x 57 = 7125)
    - No need to explain how you found it (from students view)

Not So With Teachers

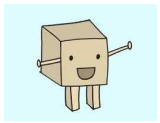
- 7125 ⇒ (7) (125 x 57 = 7125), need to explain
  - let ab…z = 57 * b…z
  - let X = b…z, with k digits
  - $a * 10^k + X = 57 * X$
  - $a * 10^k = 57X - X \Rightarrow 56X \Rightarrow 7 * 8 * X$
  - 10^k is $2^k * 5^k$ and 7can't be in it hence, since a should be divisible by 7, then a=7 since one digit only
  - $a * 10^k = 7 * 8 * X$, cancel out a & 7, since a=7
  - $10^k = 8 * X$, 10^k must be multiple of 8, hence 10^3 since k=3 digits
  - $10^k = 1000 = 8 * X$
  - $X = 1000/8 = 125$
  - also 71250 ⇒ (7)(1250 * 57 = 71250), etc…

You Can't Always Win – Splitting Weights

Two identical backpacks for you and a friend



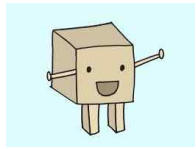
Three various things: 1, 2, 3

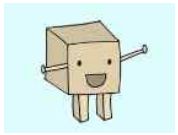


- split things into two equal weights to carry each
  - $1 + 2 = 3$, for convenience use $\pm$ sign so that total sum $= 0$
  - $\pm 1 \pm 2 \pm 3 = 0$
  - $+1 + 2 - 3 = 0$

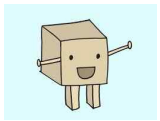
Splitting Weights (Obstacle)

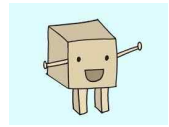
Two identical backpacks for you and a friend





Six various things:





- split things into two equal weights to carry each
  - total weight: $1+2+3+4+5+7 = 22$; $22/2 = 11$
  - find group of weights with 11 total
  - easy: $4+7 = 11$
    - also $1+2+3+5 = 11$

Splitting Weights (Obstacle)

- if weights are 1, 2, 3, 4, 5, 6
  - total weight: 1+2+3+4+5+6 = 21; not multiple of 2
  - impossible objective
- if weights are 2, 4, 6, 8, 10, 12
  - just changed the units by doubling
  - total weight is 42, multiple of 2 but 42/2=21, no odd valued individual weight
  - still impossible objective

Splitting Weights (Bad News)

- if weights are 1, 2, 3, 4, 5, 17
- total weight: $1+2+3+4+5+17 = 32$
- sum is multiple of 2 but $32/2 = 16$
- one weight has value of 17, too big for half of total
- this is another obstacle
  - obstacles are of different types
  - no complete list of obstacles
    - for this splitting weight problem, nobody knows the complete list of obstacles

Existence Proofs

- structure of the proof reflects structure of claim
  - depends on what you want to prove
- claim: an object with some property exists
- proof: an example
  - if problem is existential, only one example needed to prove
- no need to disclose sources
  - under no obligation to disclose how you found example
  - if you cheated or asked a friend, it is still good proof mathematically but problematic morally/ethically the way it was obtained
- beware: claim may be false!
  - prepare that it may not exist at all and not spend all time looking for it
  - some cases are very difficult

Thank you.