CHAPTER 6
FRICTION

Courtesy of Alyse Gagne

CHAPTER OUTLINE

Article 6/1 Introduction

- Introduction and Overview

Article 6/2 Types of Friction

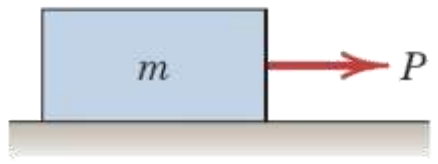
- Dry Friction

- Fluid Friction

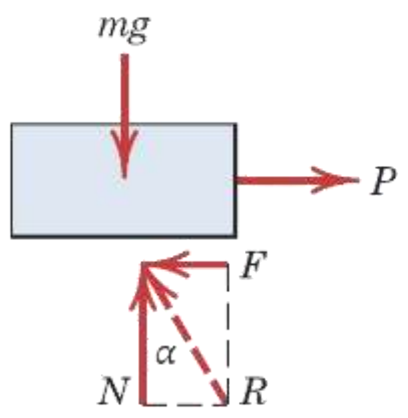
- Internal Friction

Article 6/3 Dry Friction

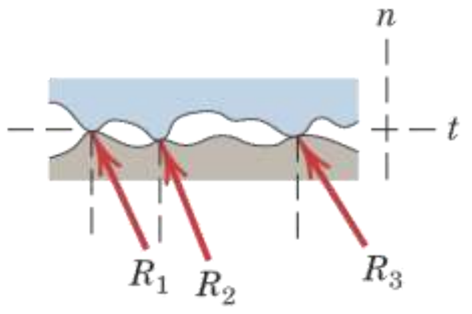
- Mechanism of Dry Friction
  - Block on a Rough Surface



  - Free-Body Diagram

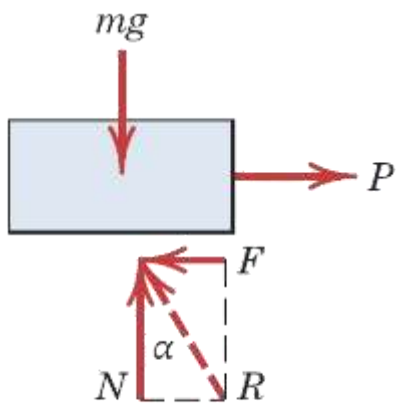


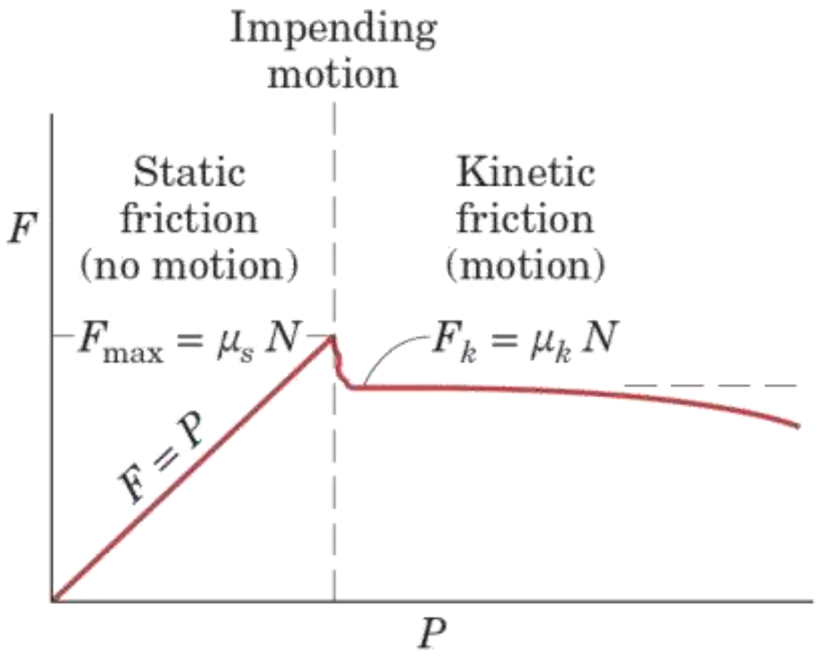
  - Close-Up of Mating Surfaces



Article 6/3 – Plot of Friction Force

- Regions of Significance
  - Static Friction Range, $F < F_{max}$
  - Impending Motion, $F = F_{max} = \mu_s N$
  - Kinetic Friction, $F = F_k = \mu_k N$





Article 6/3 – Friction Coefficients

- Coefficient of Static Friction, μ_s

- Coefficient of Kinetic Friction, μ_k

- Table of Values

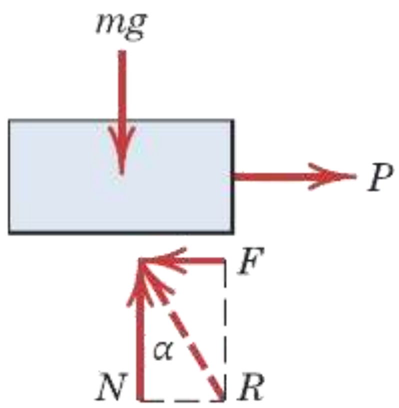
  Exact values for a specific problem can vary significantly from the values presented.

Contacting Surface	Typical Values of Coefficient of Friction	
	Static, μ_s	Kinetic, μ_k
Steel on steel (dry)	0.6	0.4
Steel on steel (greasy)	0.1	0.05
Teflon on steel	0.04	0.04
Steel on babbitt (dry)	0.4	0.3
Steel on babbitt (greasy)	0.1	0.07
Brass on steel (dry)	0.5	0.4
Brake lining on cast iron	0.4	0.3
Rubber tires on smooth pavement (dry)	0.9	0.8
Wire rope on iron pulley (dry)	0.2	0.15
Hemp rope on metal	0.3	0.2
Metal on ice		0.02

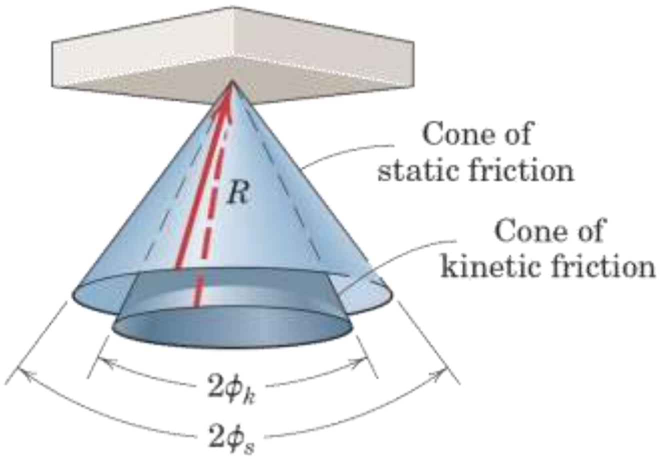
Article 6/3 – Friction Angles

- Static Friction Angle, ϕ_s
  - $\tan \phi_s = \mu_s$

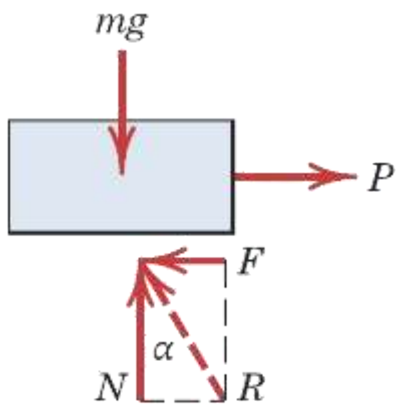
- Kinetic Friction Angle, ϕ_k
  - $\tan \phi_k = \mu_k$



- Friction Cones

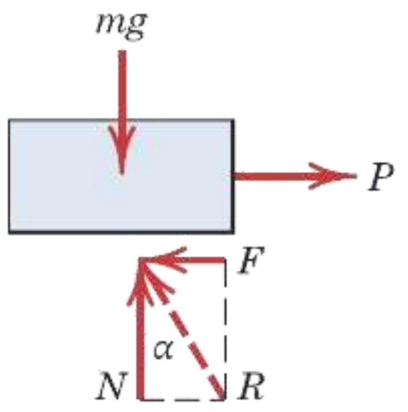


Article 6/3 – Factors Affecting Friction



- Contact Area
- Molecular Attraction
- Temperature
- Adhesion
- Relative Hardness of Mating Areas
- Surface Films

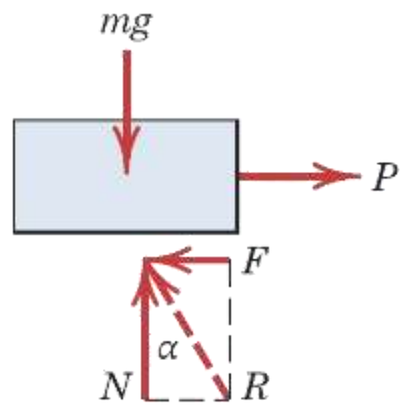
Article 6/3 – Types of Friction Problems (1 of 2)



- Type I: Impending Motion
  - Body is on the verge of slipping.
  - Equilibrium holds for the body.
  - $F = F_{\text{max}} = \mu_s N$

- Type II: Relative Motion Exists
  - Body is slipping.
  - Equilibrium does not hold in the direction of slip.
  - $F = F_k = \mu_k N$

Article 6/3 – Types of Friction Problems (2 of 2)



- Type III: Unknown – Body may or may not be slipping
  - Analysis Steps
    1. Assume equilibrium of the body.
    2. Solve for the necessary equilibrium friction force *F*.
    3. Check the assumption of equilibrium.
       a) If $F < (F_{max} = \mu_s N)$:

          Friction force necessary for equilibrium can be supported, and the body is in static equilibrium as assumed. The *actual* friction force *F* is *less than* the limiting value F_{max}.

       b) If $F = (F_{max} = \mu_s N)$:

          Motion is impending, but the assumption of equilibrium still holds.

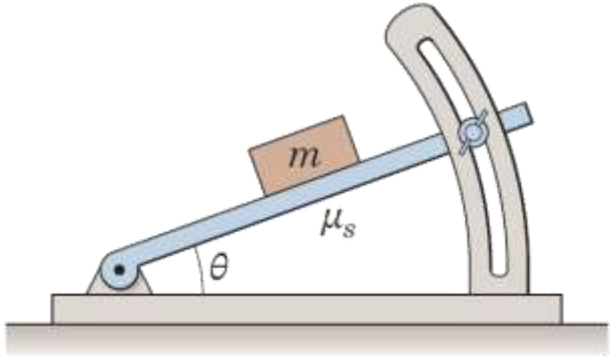
       c) If $F > (F_{max} = \mu_s N)$:

          The surfaces cannot support more force than the maximum $\mu_s N$, so the assumption of equilibrium is invalid. Motion occurs and $F = (F_k = \mu_k N)$.

Article 6/3 – Sample Problem 6/1 (1 of 2)

- Problem Statement

  Determine the maximum angle θ which the adjustable incline may have with the horizontal before the block of mass m begins to slip. The coefficient of static friction between the block and the inclined surface is μ_s.



Article 6/3 – Sample Problem 6/1 (2 of 2)

- Equilibrium Conditions

Equilibrium in the x- and y-directions requires ①

$[\Sigma F_x = 0]$ $\quad mg \sin\theta - F = 0 \qquad F = mg \sin\theta$

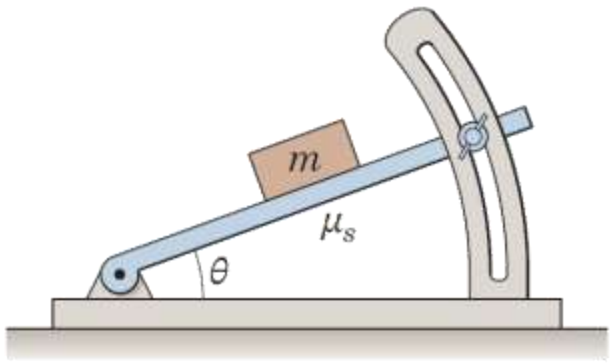
$[\Sigma F_y = 0]$ $\quad -mg \cos\theta + N = 0 \qquad N = mg\cos\theta$

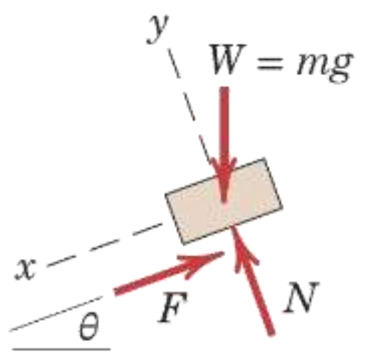
Dividing the first equation by the second gives $F/N = \tan\theta$. Since the maximum angle occurs when $F = F_{max} = \mu_s N$, for impending motion we have

$$\mu_s = \tan\theta_{max} \qquad \text{or} \qquad \theta_{max} = \tan^{-1}\mu_s \quad ② \qquad \textit{Ans.}$$

① We choose reference axes along and normal to the direction of F to avoid resolving both F and N into components.

② This problem describes a very simple way to determine a static coefficient of friction. The maximum value of θ is known as the *angle of repose*.

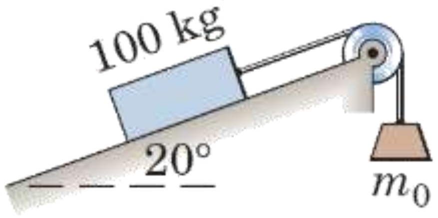




Article 6/3 – Sample Problem 6/2 (1 of 3)

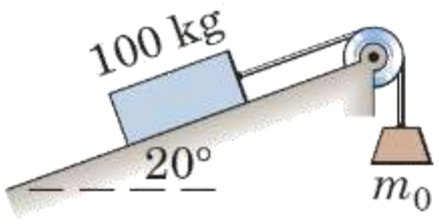
- Problem Statement

  Determine the range of values which the mass m_0 may have so that the 100-kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is 0.30.



Article 6/3 – Sample Problem 6/2 (2 of 3)

- Case I: Motion Impends Up the Plane

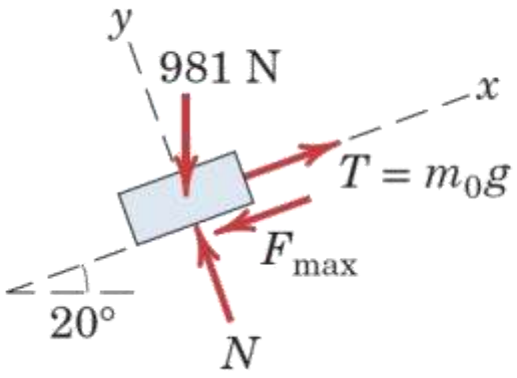


The maximum value of m_0 will be given by the requirement for motion impending up the plane. The friction force on the block therefore acts down the plane, as shown in the free-body diagram of the block for Case I in the figure. With the weight $mg = 100(9.81) = 981$ N, the equations of equilibrium give

$[\Sigma F_y = 0]$ $\quad N - 981 \cos 20° = 0 \qquad N = 922 \text{ N}$

$[F_{max} = \mu_s N]$ $\quad F_{max} = 0.30(922) = 277 \text{ N}$

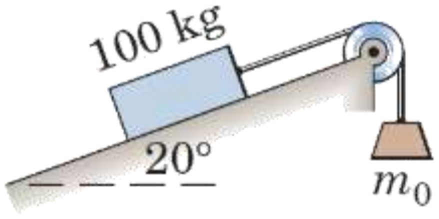
$[\Sigma F_x = 0]$ $\quad m_0(9.81) - 277 - 981 \sin 20° = 0 \qquad m_0 = 62.4 \text{ kg}$ *Ans.*



Case I

Article 6/3 – Sample Problem 6/2 (3 of 3)

- Case II: Motion Impends Down the Plane

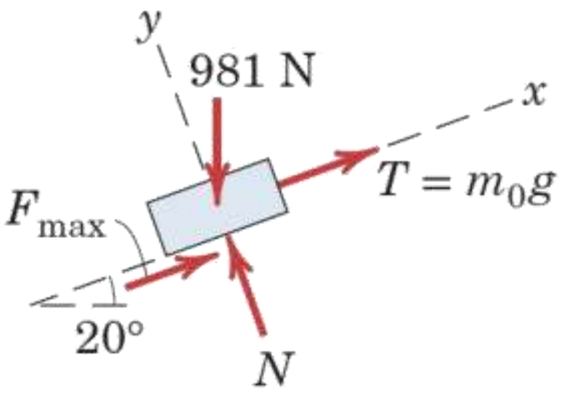


The minimum value of m_0 is determined when motion is impending down the plane. ① The friction force on the block will act up the plane to oppose the tendency to move, as shown in the free-body diagram for Case II. Equilibrium in the x-direction requires

$$[\Sigma F_x = 0] \quad m_0(9.81) + 277 - 981 \sin 20° = 0 \qquad m_0 = 6.01 \text{ kg} \qquad \textit{Ans.}$$

Thus, m_0 may have any value from 6.01 to 62.4 kg, and the block will remain at rest.

In both cases equilibrium requires that the resultant of F_{max} and N be concurrent with the 981-N weight and the tension T.



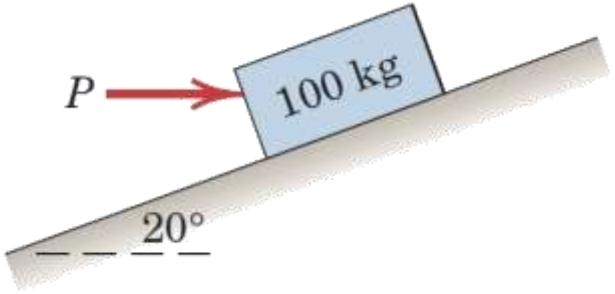
Case II

① We see from the results of Sample Problem 6/1 that the block would slide down the incline without the restraint of attachment to m_0 since $\tan 20° > 0.30$. Thus, a value of m_0 will be required to maintain equilibrium.

Article 6/3 – Sample Problem 6/3 (1 of 4)

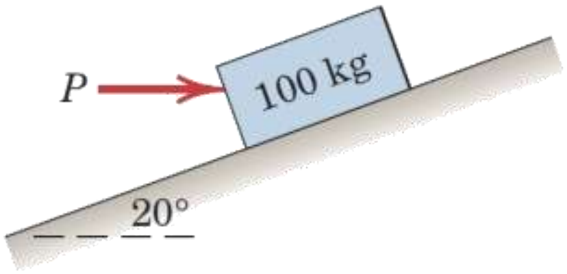
- Problem Statement

  Determine the magnitude and direction of the friction force acting on the 100-kg block shown if, first, P = 500 N and, second, P = 100 N. The coefficient of static friction is 0.20, and the coefficient of kinetic friction is 0.17. The forces are applied with the block initially at rest.



Article 6/3 – Sample Problem 6/3 (2 of 4)

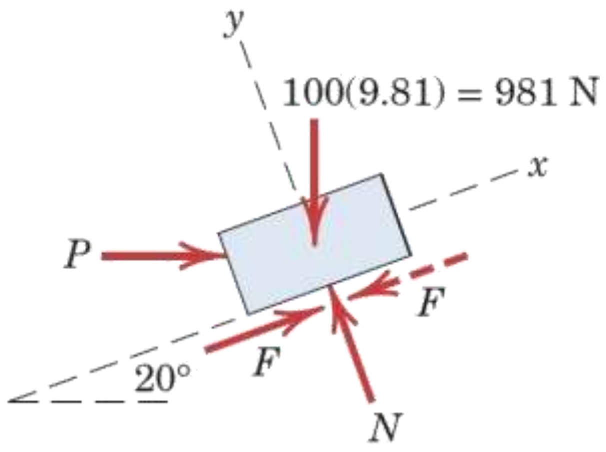
- Equilibrium Conditions



There is no way of telling from the statement of the problem whether the block will remain in equilibrium or whether it will begin to slip following the application of P. It is therefore necessary that we make an assumption, so we will take the friction force to be up the plane, as shown by the solid arrow. From the free-body diagram a balance of forces in both x- and y-directions gives

$[\Sigma F_x = 0]$ $\qquad P \cos 20^\circ + F - 981 \sin 20^\circ = 0$

$[\Sigma F_y = 0]$ $\qquad N - P \sin 20^\circ - 981 \cos 20^\circ = 0$



Article 6/3 – Sample Problem 6/3 (3 of 4)

- Case I: P = 500 N

Substitution into the first of the two equations gives

$$F = -134.3 \text{ N}$$

The negative sign tells us that *if* the block is in equilibrium, the friction force acting on it is in the direction opposite to that assumed and therefore is down the plane, as represented by the dashed arrow. We cannot reach a conclusion on the magnitude of F, however, until we verify that the surfaces are capable of supporting 134.3 N of friction force. This may be done by substituting $P = 500$ N into the second equation, which gives

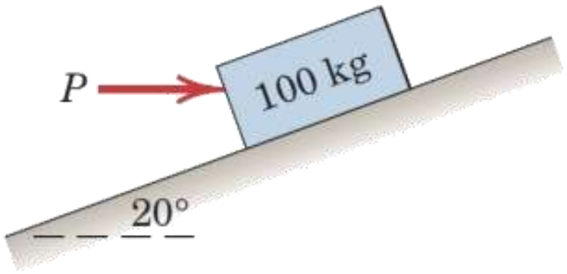
$$N = 1093 \text{ N}$$

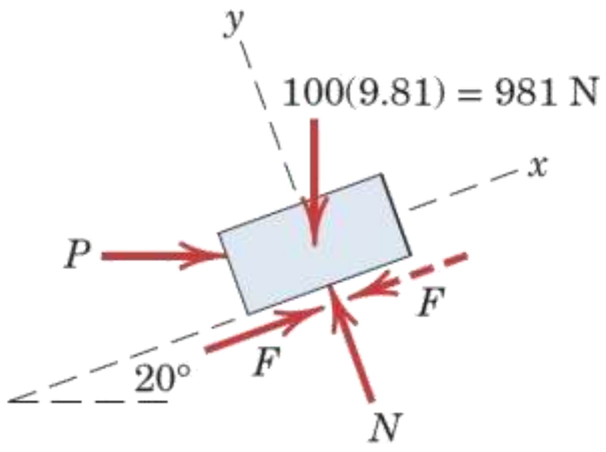
The maximum static friction force which the surfaces can support is then

$[F_{max} = \mu_s N]$ $\qquad F_{max} = 0.20(1093) = 219 \text{ N}$

Since this force is greater than that required for equilibrium, we conclude that the assumption of equilibrium was correct. The answer is, then,

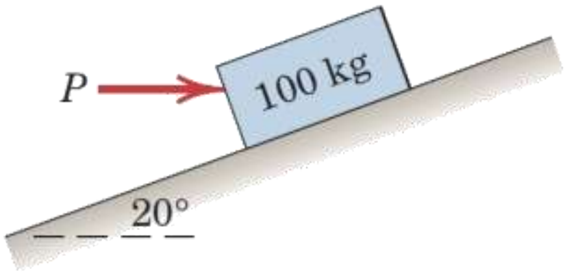
$$F = 134.3 \text{ N down the plane} \qquad \textit{Ans.}$$





Article 6/3 – Sample Problem 6/3 (4 of 4)

- Case II: P = 100 N



Substitution into the two equilibrium equations gives

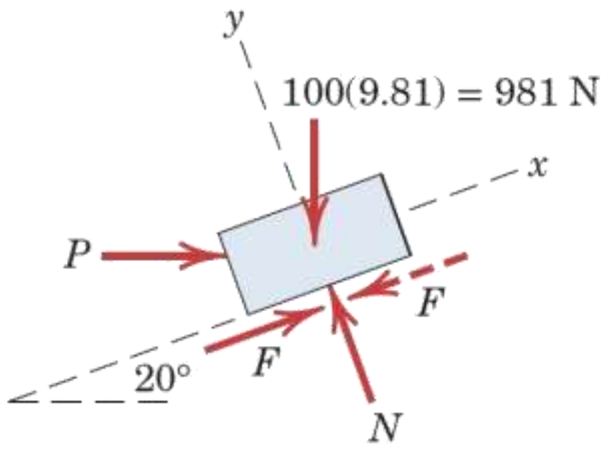
$$F = 242 \text{ N} \qquad N = 956 \text{ N}$$

But the maximum possible static friction force is

$[F_{max} = \mu_s N]$ $\qquad F_{max} = 0.20(956) = 191.2 \text{ N}$

It follows that 242 N of friction cannot be supported. Therefore, equilibrium cannot exist, and we obtain the correct value of the friction force by using the kinetic coefficient of friction accompanying the motion down the plane. Hence, the answer is

$[F_k = \mu_k N]$ $\qquad F = 0.17(956) = 162.5$ N up the plane ① *Ans.*

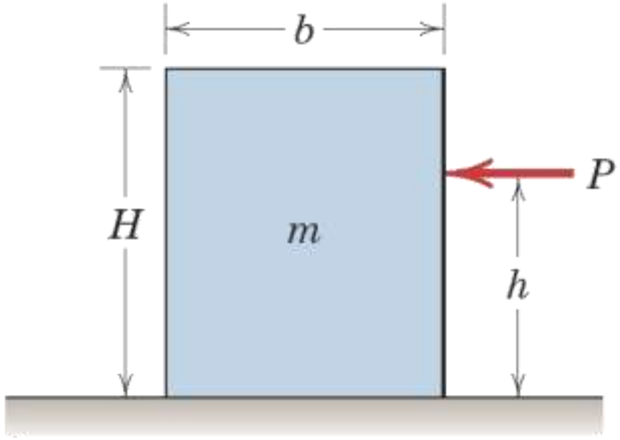


① We should note that even though ΣF_x is no longer equal to zero, equilibrium does exist in the y-direction, so that $\Sigma F_y = 0$. Therefore, the normal force N is 956 N whether or not the block is in equilibrium.

Article 6/3 – Sample Problem 6/4 (1 of 3)

- Problem Statement

  The homogeneous rectangular block of mass m, width b, and height H is placed on the horizontal surface and subjected to a horizontal force P which moves the block along the surface with a constant velocity. The coefficient of kinetic friction between the block and the surface is μ_k. Determine (a) the greatest value which h may have so that the block will slide without tipping over and (b) the location of a point C on the bottom face of the block through which the resultant of the friction and normal forces acts if $h = H/2$.



Article 6/3 – Sample Problem 6/4 (2 of 3)

- Part (a) Maximum Value of h

With the block on the verge of tipping, we see that the entire reaction between the plane and the block will necessarily be at A. The free-body diagram of the block shows this condition. Since slipping occurs, the friction force is the limiting value $\mu_k N$, and the angle θ becomes $\theta = \tan^{-1} \mu_k$. The resultant of F_k and N passes through a point B through which P must also pass, since three coplanar forces in equilibrium are concurrent. ① Hence, from the geometry of the block

$$\tan\theta = \mu_k = \frac{b/2}{h} \qquad h = \frac{b}{2\mu_k} \qquad \textit{Ans.}$$

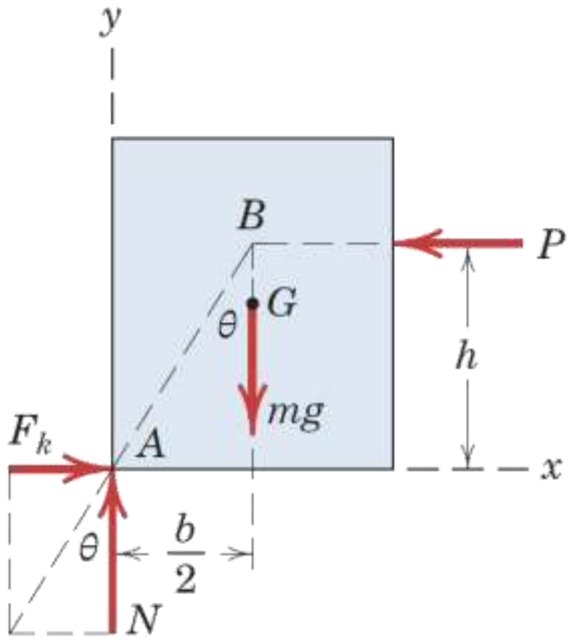
If h were greater than this value, moment equilibrium about A would not be satisfied, and the block would tip over.

Alternatively, we may find h by combining the equilibrium requirements for the x- and y-directions with the moment-equilibrium equation about A. Thus,

$$[\Sigma F_y = 0] \qquad N - mg = 0 \qquad N = mg$$

$$[\Sigma F_x = 0] \qquad F_k - P = 0 \qquad P = F_k = \mu_k N = \mu_k mg$$

$$[\Sigma M_A = 0] \qquad Ph - mg\frac{b}{2} = 0 \qquad h = \frac{mgb}{2P} = \frac{mgb}{2\mu_k mg} = \frac{b}{2\mu_k} \qquad \textit{Ans.}$$



① Recall that the equilibrium equations apply to a body moving with a constant velocity (zero acceleration) just as well as to a body at rest.

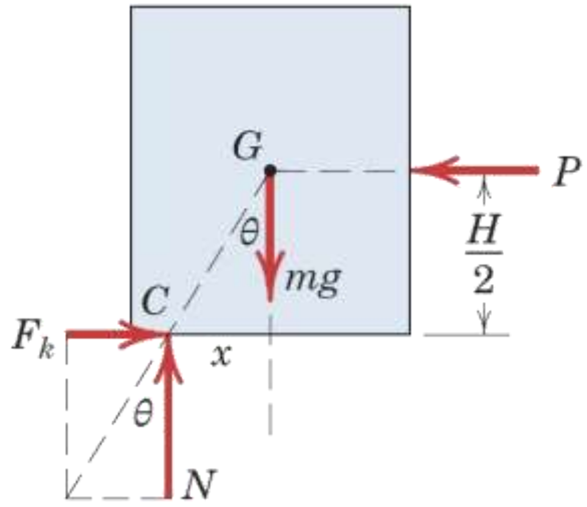
Article 6/3 – Sample Problem 6/4 (3 of 3)

- Part (b) $h = H/2$

With $h = H/2$ we see from the free-body diagram for case (b) that the resultant of F_k and N passes through a point C which is a distance x to the left of the vertical centerline through G. The angle θ is still $\theta = \phi = \tan^{-1} \mu_k$ as long as the block is slipping. Thus, from the geometry of the figure we have

$$\frac{x}{H/2} = \tan\theta = \mu_k \qquad \text{so} \qquad x = \mu_k H/2 \quad ② \qquad \textit{Ans.}$$

If we were to replace μ_k by the static coefficient μ_s, then our solutions would describe the conditions under which the block is (a) on the verge of tipping and (b) on the verge of slipping, both from a rest position.

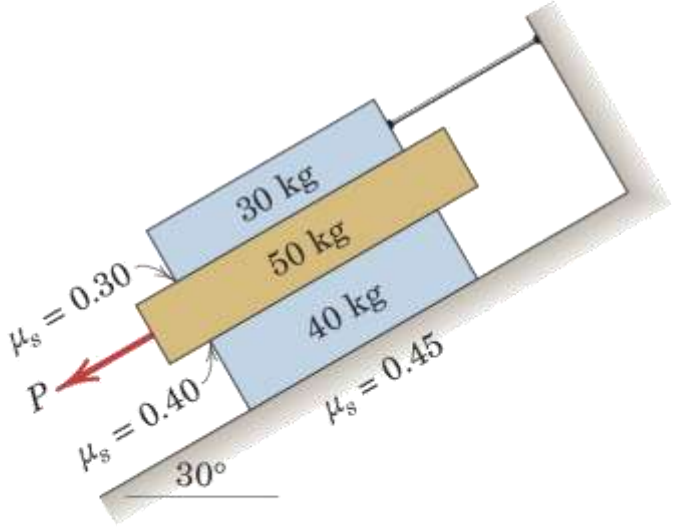


② Alternatively, we could equate the moments about G to zero, which would give us $F(H/2) - Nx = 0$. Thus, with $F_k = \mu_k N$ we get $x = \mu_k H/2$.

Article 6/3 – Sample Problem 6/5 (1 of 4)

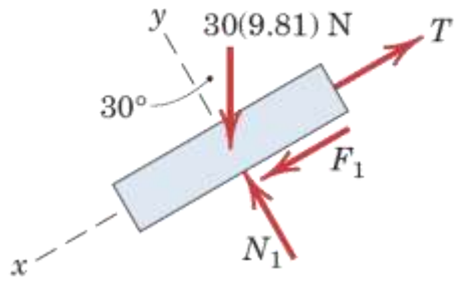
- Problem Statement

  The three flat blocks are positioned on the 30° incline as shown, and a force *P* parallel to the incline is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The coefficient of static friction for each of the three pairs of mating surfaces is shown. Determine the maximum value which *P* may have before any slipping takes place.



Article 6/3 – Sample Problem 6/5 (2 of 4)

- Free-Body Diagrams



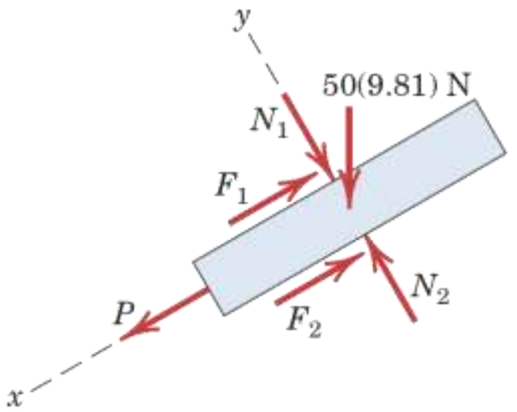
- Normal Force Calculations

The normal forces, which are in the y-direction, may be determined without reference to the friction forces, which are all in the x-direction. Thus,

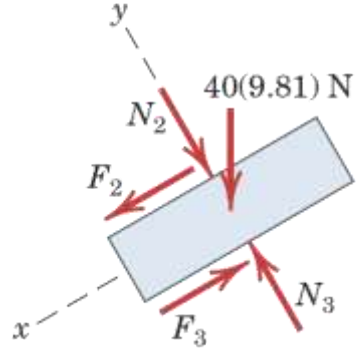
$[\Sigma F_y = 0]$ (30-kg) $N_1 - 30(9.81)\cos 30° = 0$ $\qquad N_1 = 255$ N

(50-kg) $N_2 - 50(9.81)\cos 30° - 255 = 0$ $\qquad N_2 = 680$ N

(40-kg) $N_3 - 40(9.81)\cos 30° - 680 = 0$ $\qquad N_3 = 1019$ N



① In the absence of friction the middle block, under the influence of P, would have a greater movement than the 40-kg block, and the friction force F_2 will be in the direction to oppose this motion as shown.



Article 6/3 – Sample Problem 6/5 (3 of 4)

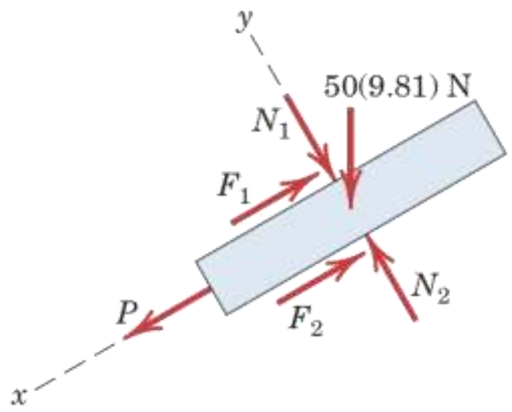
- Assume 50-kg Block Slips

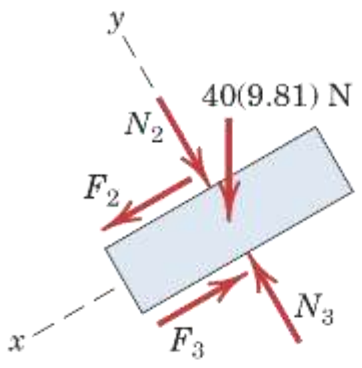
We will assume arbitrarily that only the 50-kg block slips, so that the 40-kg block remains in place. Thus, for impending slippage at both surfaces of the 50-kg block, we have

$[F_{max} = \mu_s N] \qquad F_1 = 0.30(255) = 76.5 \text{ N} \qquad F_2 = 0.40(680) = 272 \text{ N}$

The assumed equilibrium of forces at impending motion for the 50-kg block gives

$[\Sigma F_x = 0] \qquad P - 76.5 - 272 + 50(9.81) \sin 30° = 0 \qquad P = 103.1 \text{ N}$





Article 6/3 – Sample Problem 6/5 (4 of 4)

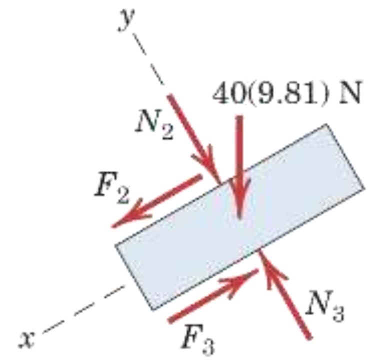
- Check Assumption

We now check on the validity of our initial assumption. For the 40-kg block with $F_2 = 272$ N the friction force F_3 would be given by

$$[\Sigma F_x = 0] \qquad 272 + 40(9.81) \sin 30° - F_3 = 0 \qquad F_3 = 468 \text{ N}$$

But the maximum possible value of F_3 is $F_3 = \mu_s N_3 = 0.45(1019) = 459$ N. Thus, 468 N cannot be supported and our initial assumption was wrong. We conclude, therefore, that slipping occurs first between the 40-kg block and the incline. With the corrected value $F_3 = 459$ N, equilibrium of the 40-kg block for its impending motion requires

$$[\Sigma F_x = 0] \qquad F_2 + 40(9.81) \sin 30° - 459 = 0 \qquad F_2 = 263 \text{ N} \quad ②$$



② We see now that F_2 is less than $\mu_s N_2 = 272$ N.

- Revise Calculation for *P*

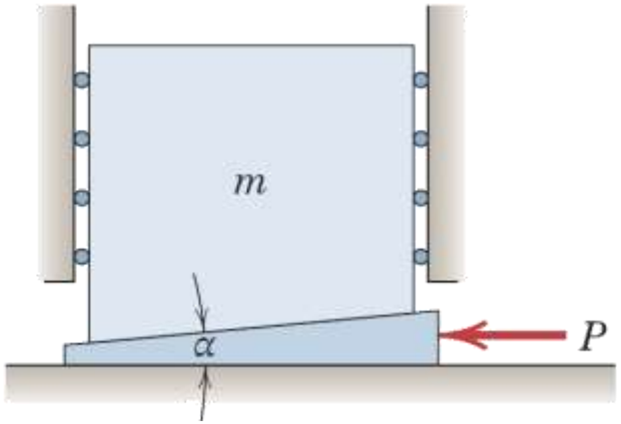
Equilibrium of the 50-kg block gives, finally,

$$[\Sigma F_x = 0] \qquad P + 50(9.81) \sin 30° - 263 - 76.5 = 0$$

$$P = 93.8 \text{ N} \qquad \textit{Ans.}$$

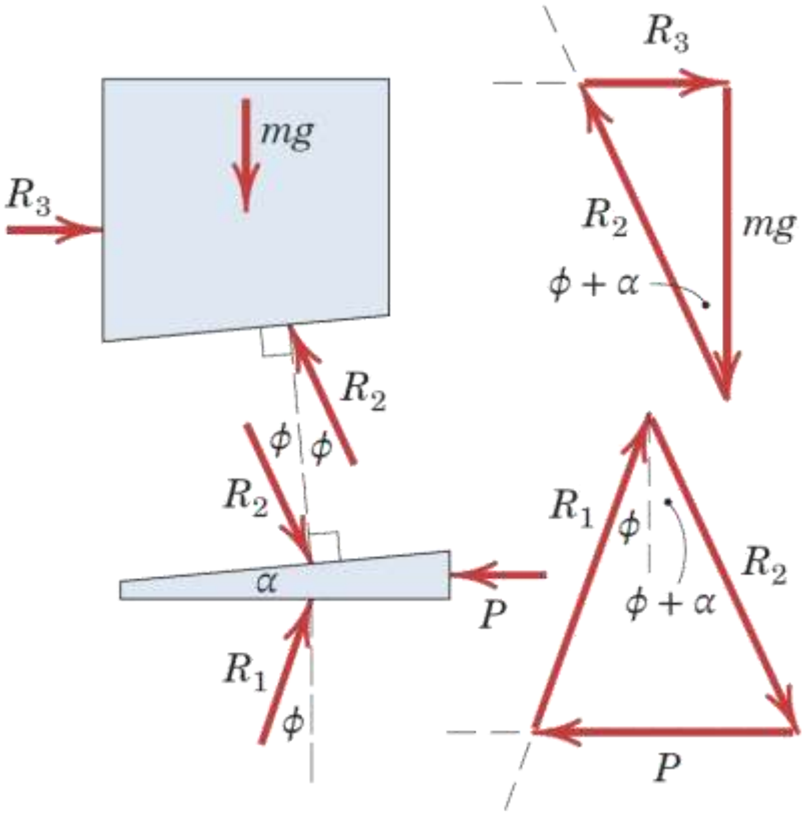
Article 6/4 Wedges

- Overview and Illustration



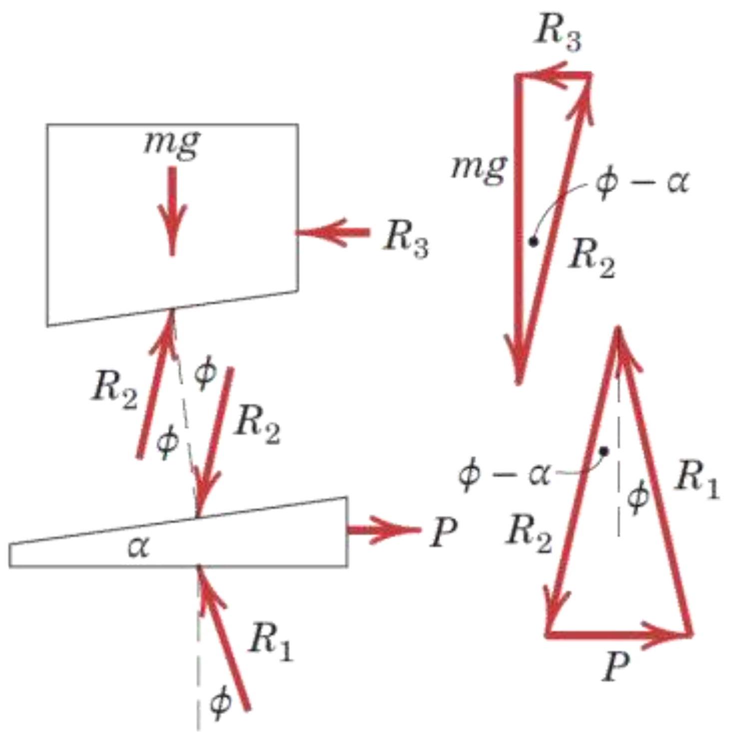
Article 6/4 – Raising a Load

- Free-Body Diagrams
- Wedge Angle, α
- Force Polygons



Article 6/4 – Lowering a Load

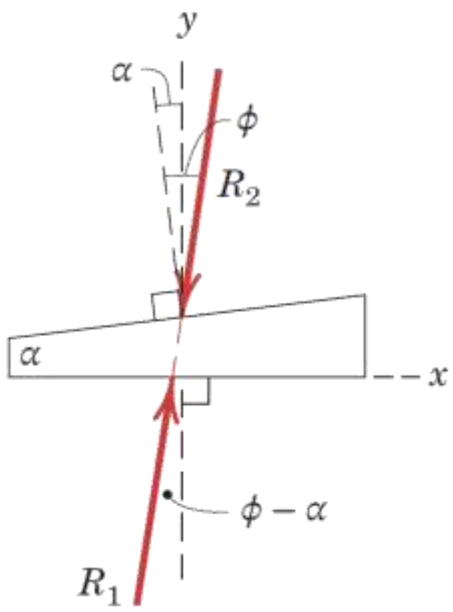
- Free-Body Diagrams
- Wedge Angle, α
- Force Polygons



Article 6/4 – Self-Locking of a Wedge (1 of 2)

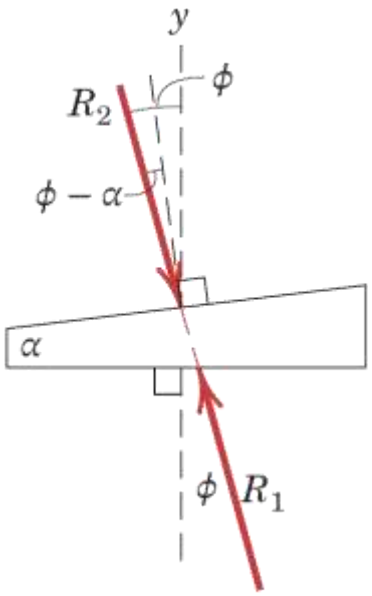
- Explanation of Self-Locking

- Impending Slip at the Upper Surface

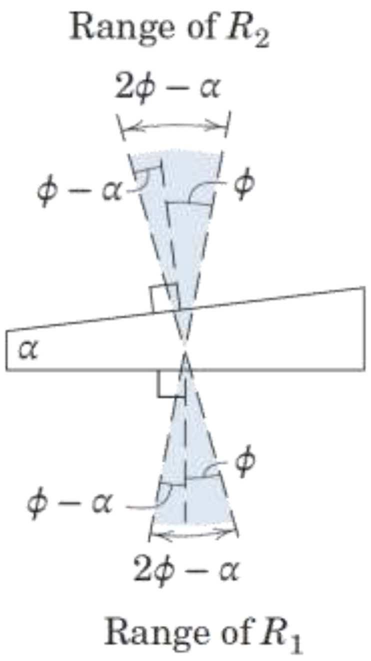


Article 6/4 – Self-Locking of a Wedge (2 of 2)

- Impending Slip at the Lower Surface

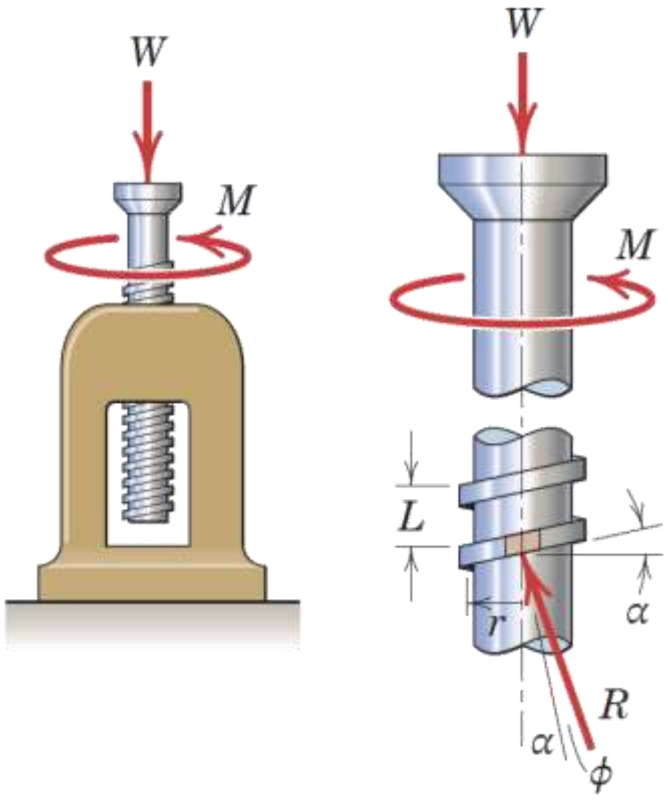


- Range for No Slip



Article 6/5 Screws

- Introduction
- Force Analysis
- Applied Moment, $M = Wr \tan(\alpha + \phi)$
- Lead, L
- Helix Angle, $\tan \alpha = L/2\pi r$

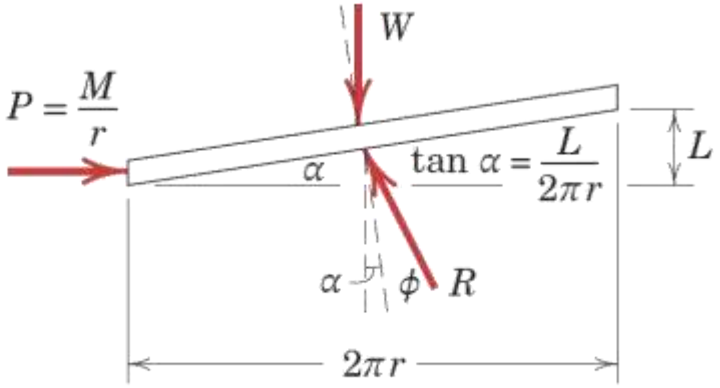


Article 6/5 – Unwrapped Thread View

- Applied Moment, $M = Wr \tan(\alpha + \phi)$

- Helix Angle, $\tan \alpha = L/2\pi r$

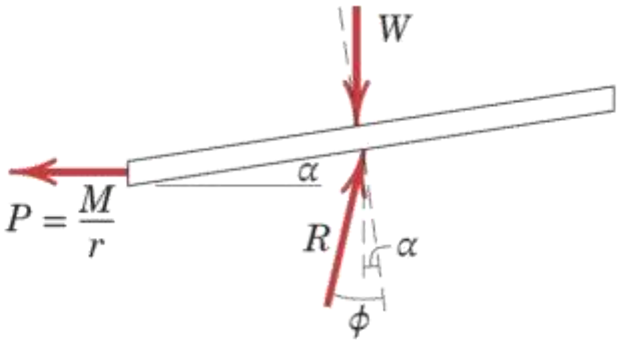
- Equivalent Force, $P = M/r$



(a) To raise load

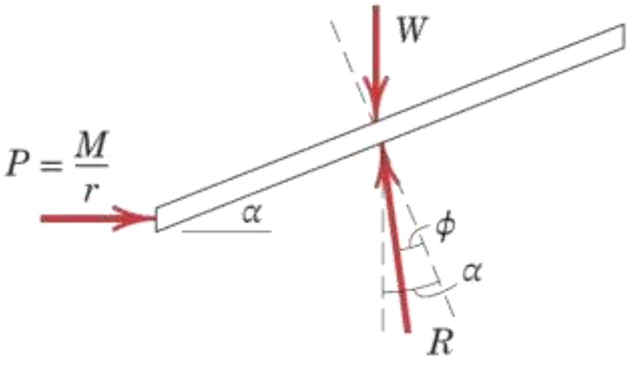
Article 6/5 – Conditions for Unwinding

- Case I: $\alpha < \phi$



(b) To lower load ($\alpha<\phi$)

- Case II: $\alpha > \phi$

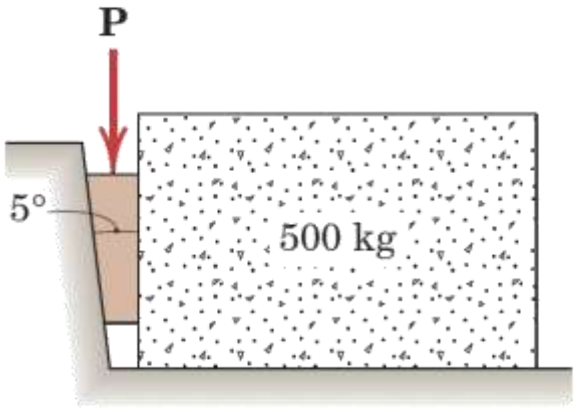


(c) To lower load ($\alpha>\phi$)

Article 6/5 – Sample Problem 6/6 (1 of 4)

- Problem Statement

  The horizontal position of the 500-kg rectangular block of concrete is adjusted by the 5° wedge under the action of the force **P**. If the coefficient of static friction for both wedge surfaces is 0.30 and if the coefficient of static friction between the block and the horizontal surface is 0.60, determine the least force *P* required to move the block.



Article 6/5 – Sample Problem 6/6 (2 of 4)

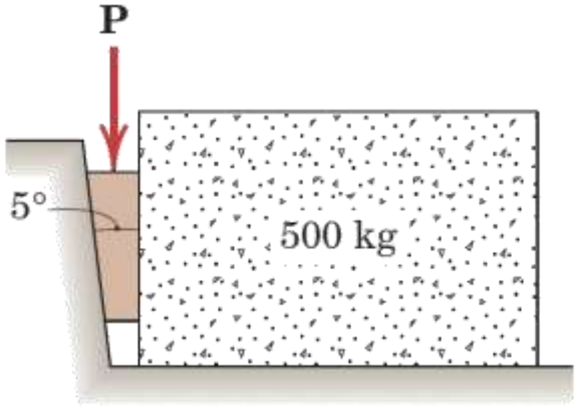
- Solution

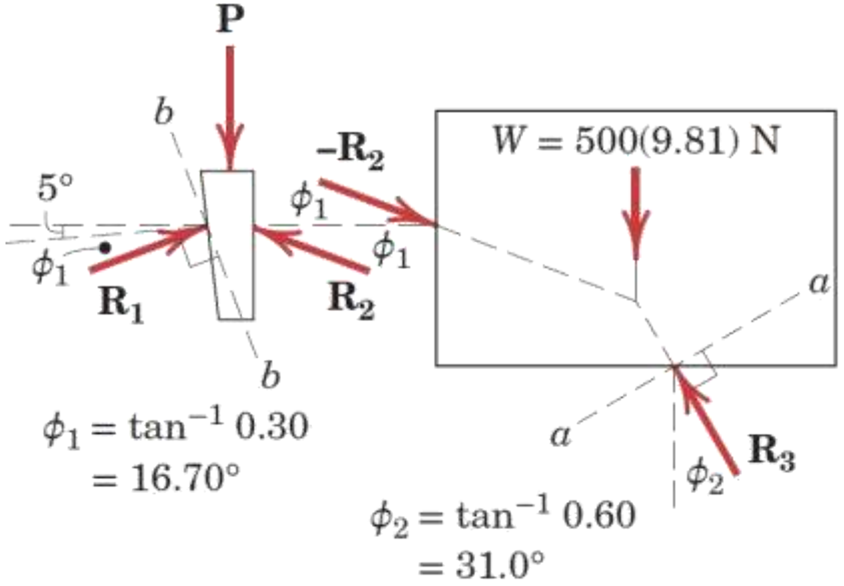
The free-body diagrams of the wedge and the block are drawn with the reactions $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ inclined with respect to their normals by the amounts of the friction angles for impending motion. ① The friction angle for limiting static friction is given by $\phi = \tan^{-1} \mu$. Each of the two friction angles is computed and shown on the diagram.

We start our vector diagram expressing the equilibrium of the block at a convenient point A and draw the only known vector, the weight $\mathbf{W}$ of the block. Next we add $\mathbf{R}_3$, whose 31.0° inclination from the vertical is now known. The vector $-\mathbf{R}_2$, whose 16.70° inclination from the horizontal is also known, must close the polygon for equilibrium. Thus, point B on the lower polygon is determined by the intersection of the known directions of $\mathbf{R}_3$ and $-\mathbf{R}_2$, and their magnitudes become known.

For the wedge we draw $\mathbf{R}_2$, which is now known, and add $\mathbf{R}_1$, whose direction is known. The directions of $\mathbf{R}_1$ and $\mathbf{P}$ intersect at C, thus giving us the solution for the magnitude of $\mathbf{P}$.

① Be certain to note that the reactions are inclined from their normals in the direction to oppose the motion. Also, we note the equal and opposite reactions $\mathbf{R}_2$ and $-\mathbf{R}_2$.





- Algebraic Solution

The simplest choice of reference axes for calculation purposes is, for the block, in the direction *a*-*a* normal to $\mathbf{R}_3$ and, for the wedge, in the direction *b*-*b* normal to $\mathbf{R}_1$. ② The angle between $\mathbf{R}_2$ and the *a*-direction is 16.70° + 31.0° = 47.7°. Thus, for the block

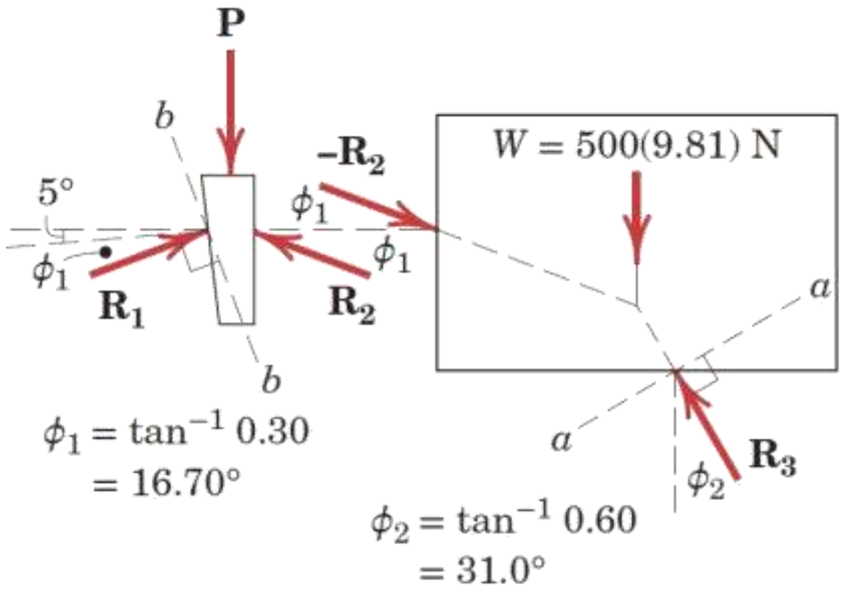
$$[\Sigma F_a = 0] \qquad 500(9.81)\sin 31.0° - R_2 \cos 47.7° = 0$$

$$R_2 = 3750 \text{ N}$$

For the wedge the angle between $\mathbf{R}_2$ and the *b*-direction is $90° - (2\phi_1 + 5°) = 51.6°$, and the angle between $\mathbf{P}$ and the *b*-direction is $\phi_1 + 5° = 21.7°$. Thus,

$$[\Sigma F_b = 0] \qquad 3750 \cos 51.6° - P \cos 21.7° = 0$$

$$P = 2500 \text{ N} \qquad \textit{Ans.}$$

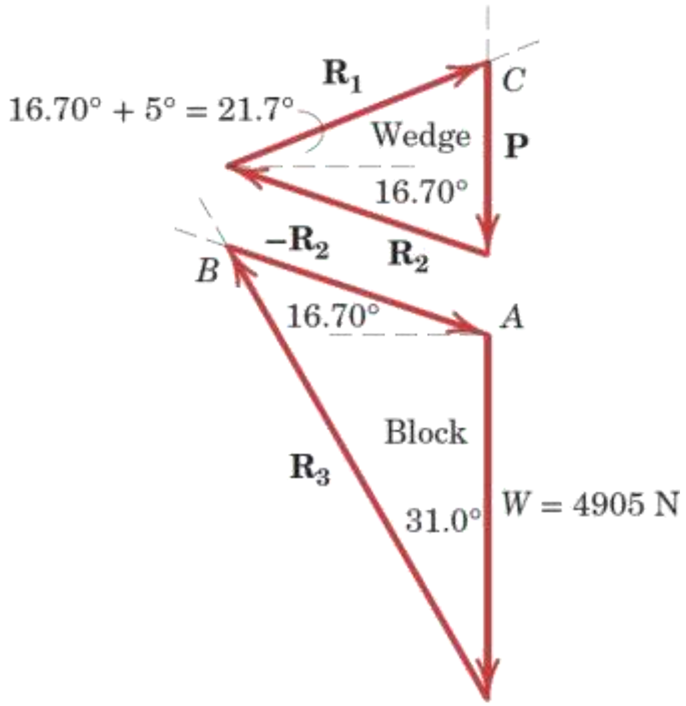


② It should be evident that we avoid simultaneous equations by eliminating reference to $\mathbf{R}_3$ for the block and $\mathbf{R}_1$ for the wedge.

Article 6/5 – Sample Problem 6/6 (4 of 4)

- Graphical Solution

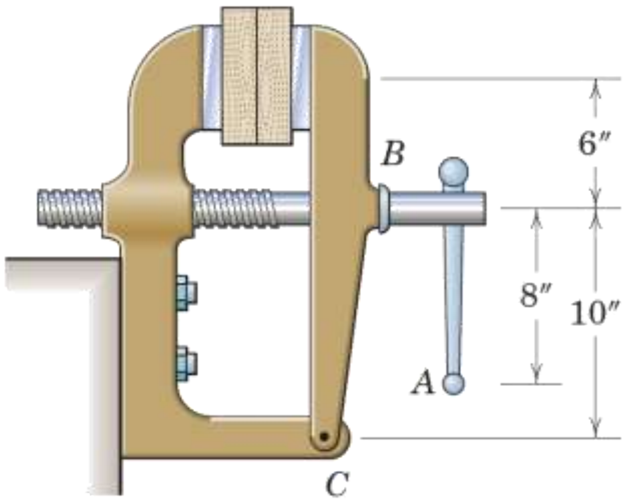
The accuracy of a graphical solution is well within the uncertainty of the friction coefficients and provides a simple and direct result. By laying off the vectors to a reasonable scale following the sequence described, we obtain the magnitudes of **P** and the **R**'s easily by scaling them directly from the diagrams.



Article 6/5 – Sample Problem 6/7 (1 of 4)

- Problem Statement

The single-threaded screw of the vise has a mean diameter of 1 in. and has 5 square threads per inch. The coefficient of static friction in the threads is 0.20. A 60-lb pull applied normal to the handle at *A* produces a clamping force of 1000 lb between the jaws of the vise. (*a*) Determine the frictional moment M_B, developed at *B*, due to the thrust of the screw against the body of the jaw. (*b*) Determine the force *Q* applied normal to the handle at *A* required to loosen the vise.



Article 6/5 – Sample Problem 6/7 (2 of 4)

- Equilibrium of the Jaw

$[\Sigma M_C = 0] \qquad 1000(16) - 10T = 0 \qquad T = 1600 \text{ lb}$

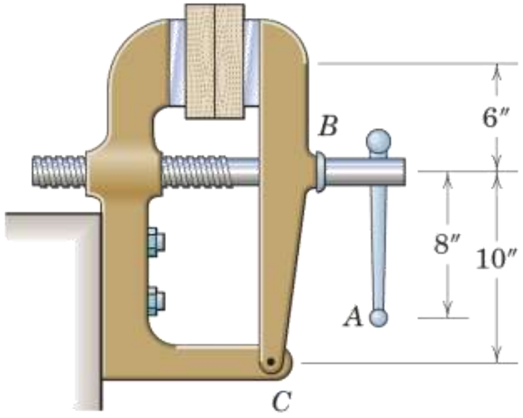
The helix angle α and the friction angle ϕ for the thread are given by

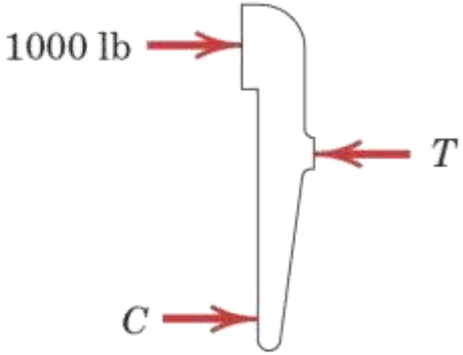
$$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{1/5}{2\pi(0.5)} = 3.64^\circ \quad ①$$

$$\phi = \tan^{-1}\mu = \tan^{-1}0.20 = 11.31^\circ$$

where the mean radius of the thread is $r = 0.5$ in.

① Be careful to calculate the helix angle correctly. Its tangent is the lead L (advancement per revolution) divided by the mean circumference $2\pi r$ and not by the diameter $2r$.





Article 6/5 – Sample Problem 6/7 (3 of 4)

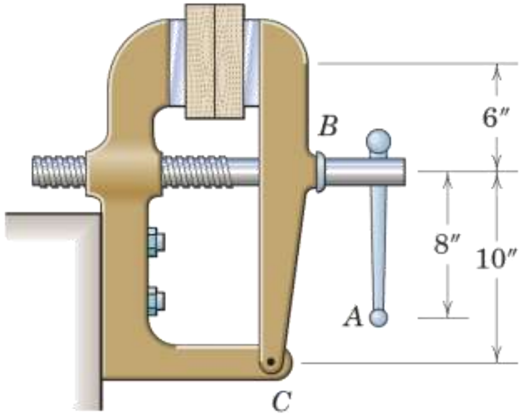
- (a) To Tighten

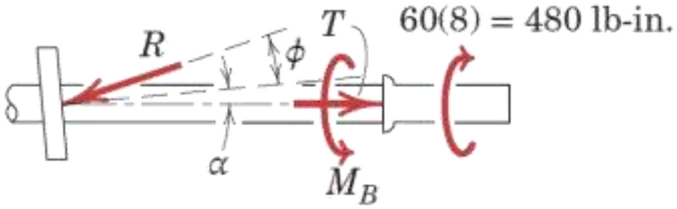
The isolated screw is simulated by the free-body diagram shown where all of the forces acting on the threads of the screw are represented by a single force R inclined at the friction angle ϕ from the normal to the thread. The moment applied about the screw axis is $60(8) = 480$ lb-in. in the clockwise direction as seen from the front of the vise. The frictional moment M_B due to the friction forces acting on the collar at B is in the counterclockwise direction to oppose the impending motion. From Eq. 6/3 with T substituted for W, the net moment acting on the screw is

$$M = Tr \tan(\alpha + \phi)$$

$$480 - M_B = 1600(0.5) \tan(3.64° + 11.31°)$$

$$M_B = 266 \text{ lb-in.} \qquad \textit{Ans.}$$





(a) To tighten

Article 6/5 – Sample Problem 6/7 (4 of 4)

- (*b*) To Loosen

The free-body diagram of the screw on the verge of being loosened is shown with R acting at the friction angle from the normal in the direction to counteract the impending motion. ② Also shown is the frictional moment $M_B = 266$ lb-in. acting in the clockwise direction to oppose the motion. The angle between R and the screw axis is now $\phi - \alpha$, and we use Eq. 6/3*a* with the net moment equal to the applied moment M' minus M_B. Thus

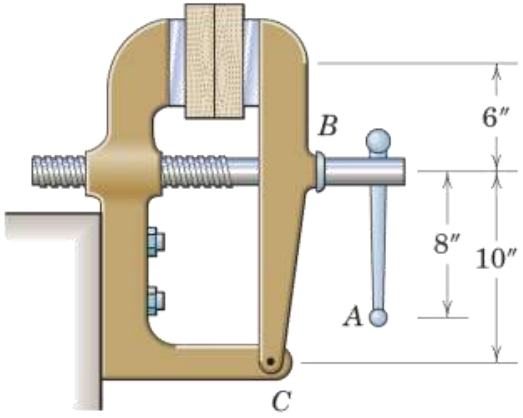
$$M = Tr \tan(\phi - \alpha)$$

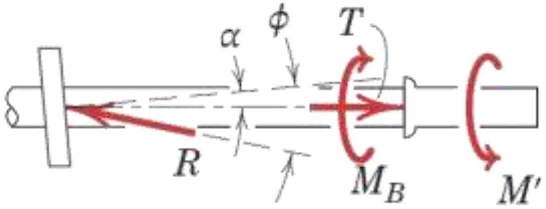
$$M' - 266 = 1600(0.5) \tan(11.31° - 3.64°)$$

$$M' = 374 \text{ lb-in.}$$

Thus, the force on the handle required to loosen the vise is

$$Q = M'/d = 374/8 = 46.8 \text{ lb} \qquad \textit{Ans.}$$



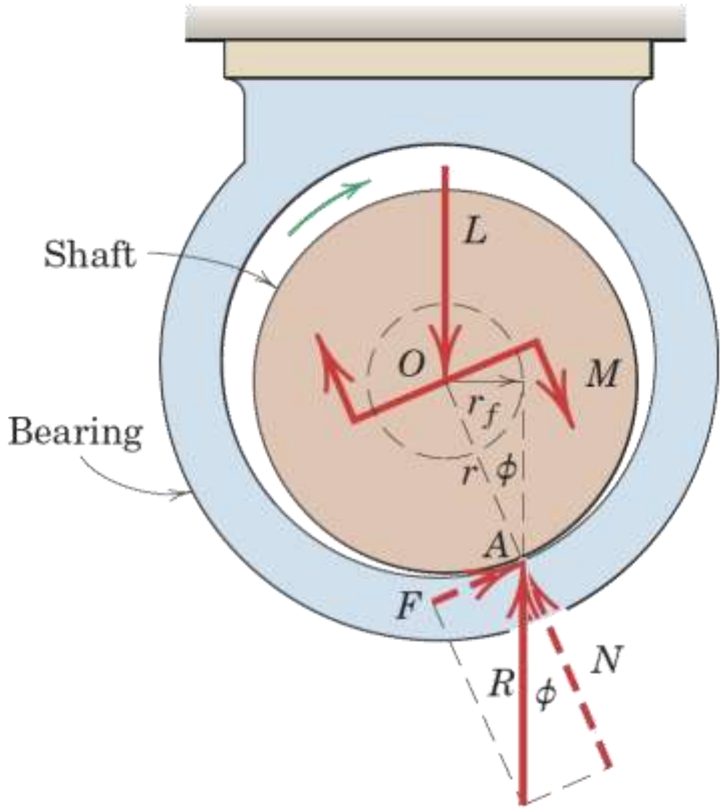


(*b*) To loosen

Article 6/6 Journal Bearings

- Purpose
- Illustration
- Load Analysis
- Torque to Maintain Rotation

  $M = Lr \sin \phi = \mu Lr$



Article 6/7 Thrust Bearings; Disk Friction

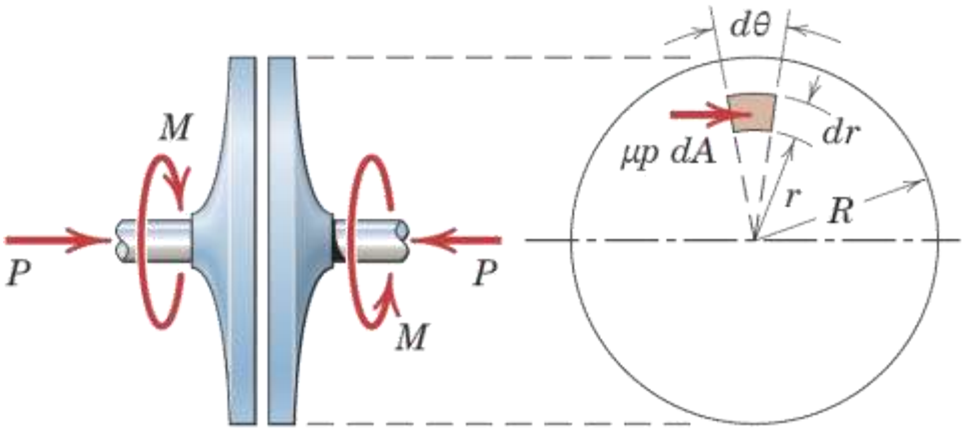
- Purpose

- Example

Westend61 GmbH/Alamy Stock Photo

Article 6/7 – Disk Friction Load Analysis

- Free-Body Diagram
  - Axial Force, P
  - Transmitted Torque, M
  - Pressure Between the Plates, $p = P/\pi R^2$



- Friction Force on a Differential Area

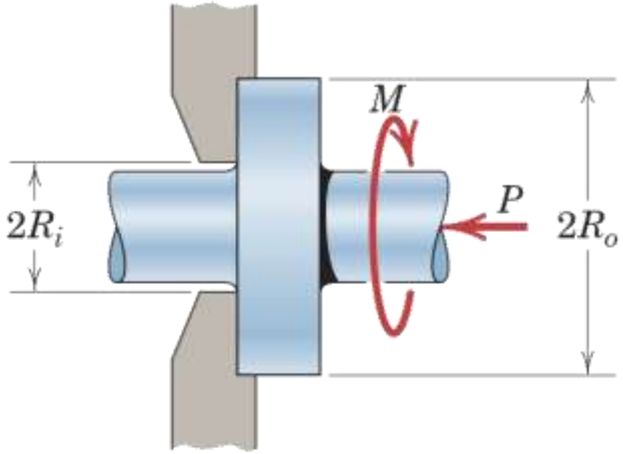
  $dF = \mu p\ dA$

- Integration over the Disk Area

  $M = 2/3\ \mu PR$

Article 6/7 – Collar Bearing Load Analysis

- Example with a Collar Bearing
  - Inside Contact Radius, R_i
  - Outside Contact Radius, R_o



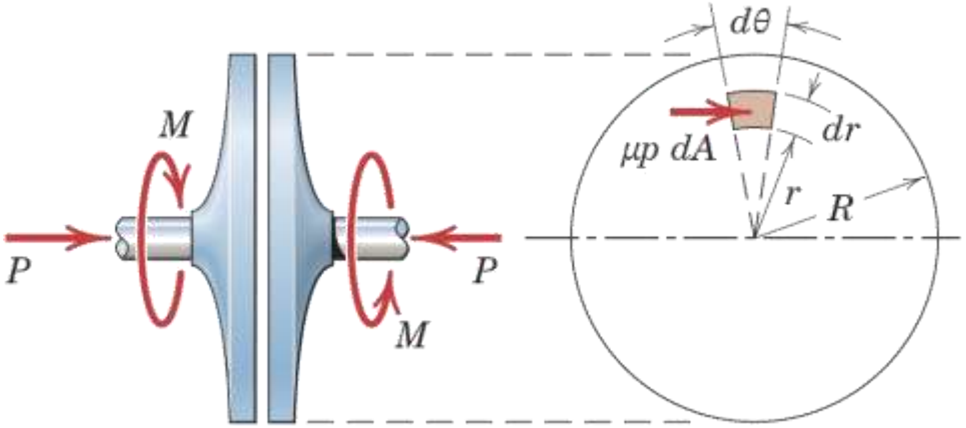
- Integration over the Disk Area

$$M = 2/3\, \mu P(R_o^3 - R_i^3)/(R_o^2 - R_i^2)$$

Article 6/7 – Effect of Wear-In

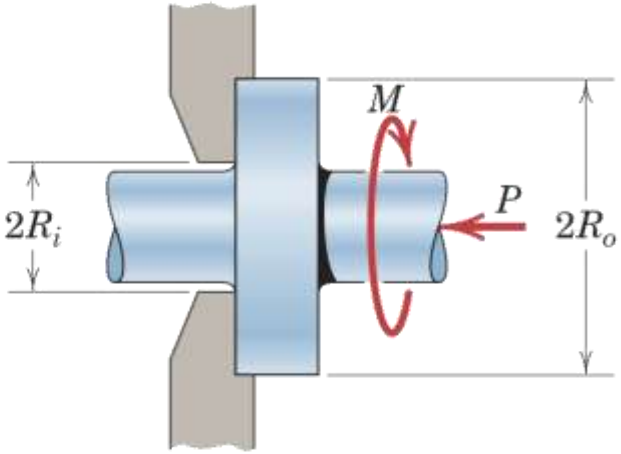
- Solid Disks

  $M = 1/2\ \mu PR$



- Ring Disks

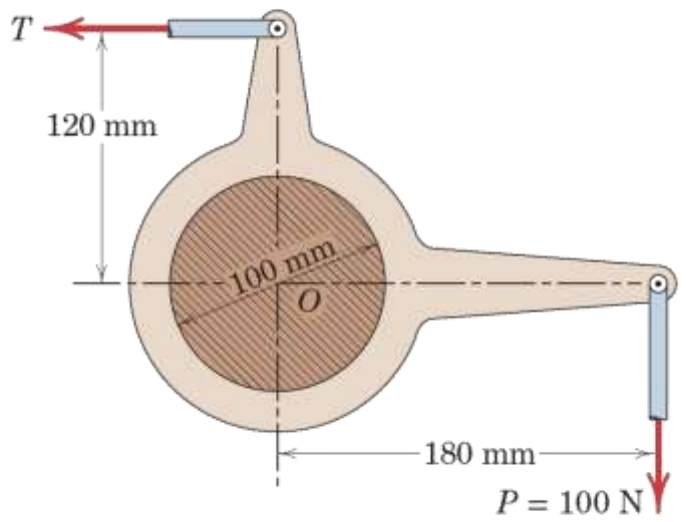
  $M = 1/2\ \mu P(R_o + R_i)$



Article 6/7 – Sample Problem 6/8 (1 of 4)

- Problem Statement

  The bell crank fits over a 100-mm-diameter shaft which is fixed and cannot rotate. The horizontal force T is applied to maintain equilibrium of the crank under the action of the vertical force P = 100 N. Determine the maximum and minimum values which T may have without causing the crank to rotate in either direction. The coefficient of static friction μ between the shaft and the bearing surface of the crank is 0.20.



Article 6/7 – Sample Problem 6/8 (2 of 4)

- Solution

Impending rotation occurs when the reaction R of the fixed shaft on the bell crank makes an angle $\phi = \tan^{-1} \mu$ with the normal to the bearing surface and is, therefore, tangent to the friction circle. Also, equilibrium requires that the three forces acting on the crank be concurrent at point C. These facts are shown in the free-body diagrams for the two cases of impending motion.

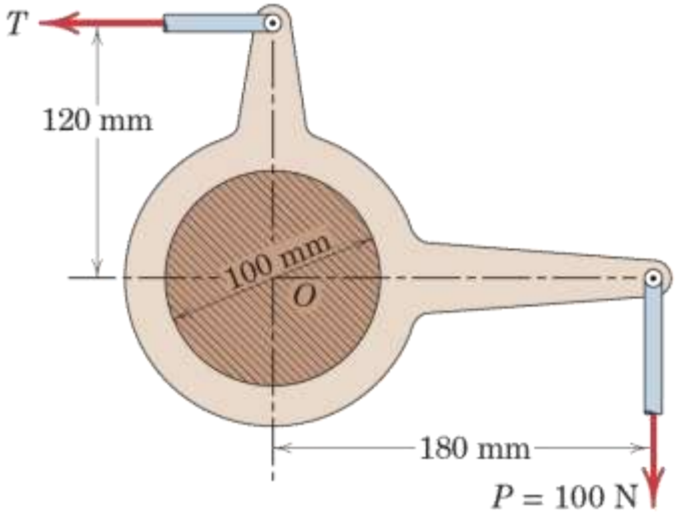
The following calculations are needed:

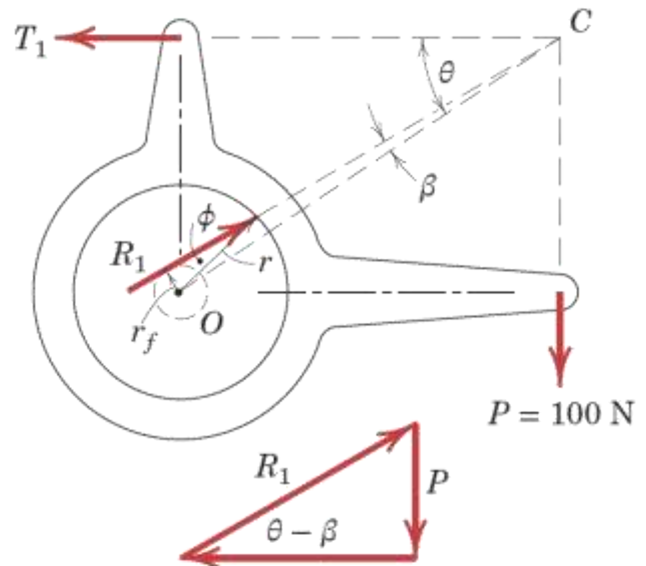
Friction angle $\phi = \tan^{-1} \mu = \tan^{-1} 0.20 = 11.31°$

Radius of friction circle $r_f = r \sin \phi = 50 \sin 11.31° = 9.81$ mm

$$\text{Angle } \theta = \tan^{-1} \frac{120}{180} = 33.7°$$

$$\text{Angle } \beta = \sin^{-1} \frac{r_f}{OC} = \sin^{-1} \frac{9.81}{\sqrt{(120)^2 + (180)^2}} = 2.60°$$



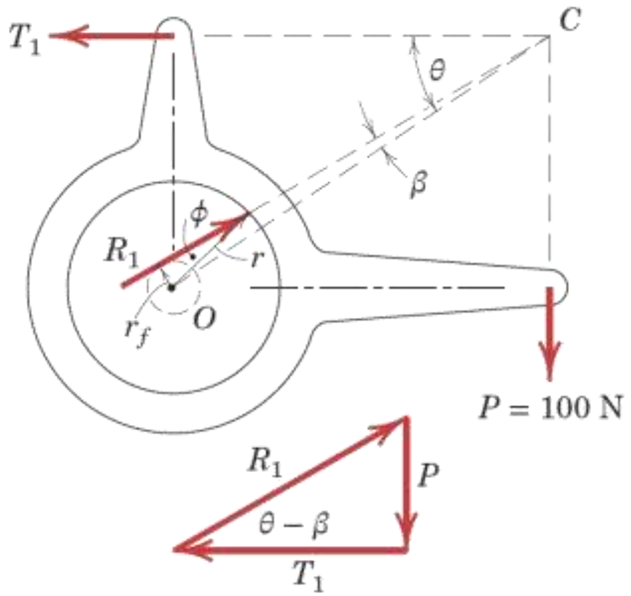


Article 6/7 – Sample Problem 6/8 (3 of 4)

- Impending Counterclockwise Motion

$$T_1 = P \cot(\theta - \beta) = 100 \cot(33.7° - 2.60°)$$

$$T_1 = T_{max} = 165.8 \text{ N} \qquad \textit{Ans.}$$



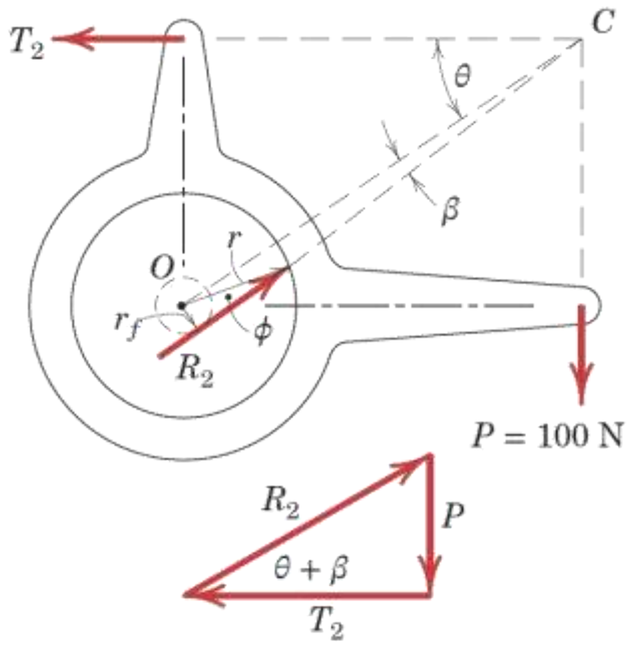
(a) Counterclockwise motion impends

Article 6/7 – Sample Problem 6/8 (4 of 4)

- Impending Clockwise Motion

$$T_2 = P \cot(\theta + \beta) = 100 \cot(33.7° + 2.60°)$$

$$T_2 = T_{\min} = 136.2 \text{ N} \qquad \textit{Ans.}$$



(b) Clockwise motion impends

Article 6/8 Flexible Belts

- Introduction

- Illustration

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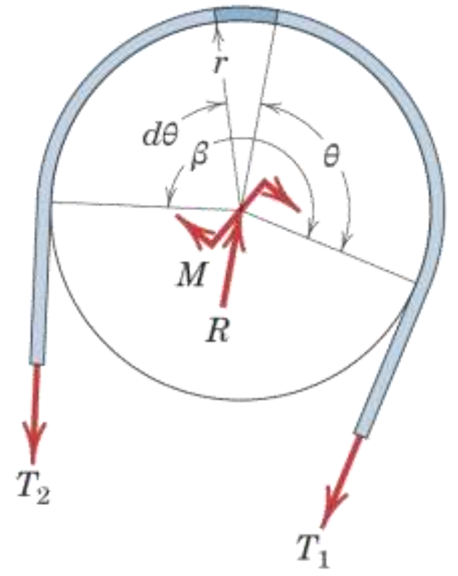
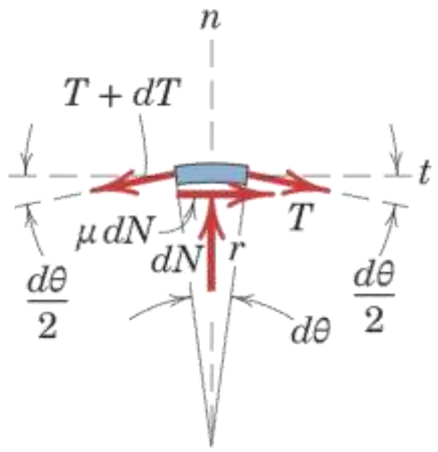
Article 6/8 – Derivation (1 of 2)

- Situation of Interest



- Free-Body Diagram of a Belt Segment

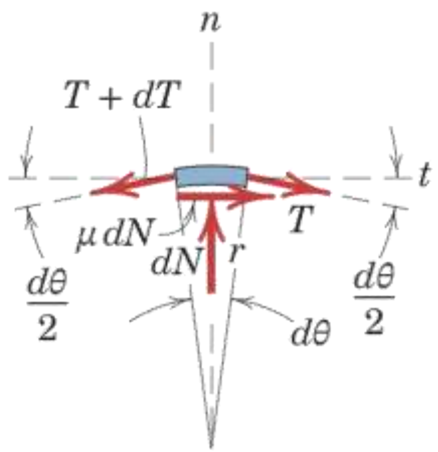


Article 6/8 – Derivation (2 of 2)

- Equilibrium Conditions

$$T \cos \frac{d\theta}{2} + \mu \, dN = (T + dT) \cos \frac{d\theta}{2} \qquad dN = (T + dT) \sin \frac{d\theta}{2} + T \sin \frac{d\theta}{2}$$

$$\mu \, dN = dT \qquad dN = T \, d\theta$$

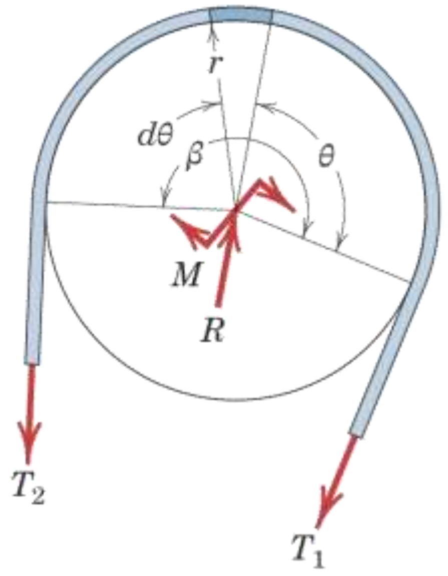


- Combine and Integrate

$$dT/T = \mu \, d\theta \quad \rightarrow \quad T_2 = T_1 e^{\mu\beta}$$

Article 6/8 – The Belt Friction Equation

- Comments about the Equation: $T_2 = T_1 e^{\mu\beta}$
  - T_2 = the larger of the two belt tensions (it points in the direction the belt will tend to move in along the surface).
  - T_1 = the smaller of the two belt tensions (it opposes the motion of the belt along the surface).
  - μ = the coefficient of friction between the belt and the surface.
  - β = the total angle of belt contact with the surface. It must be expressed in radians.

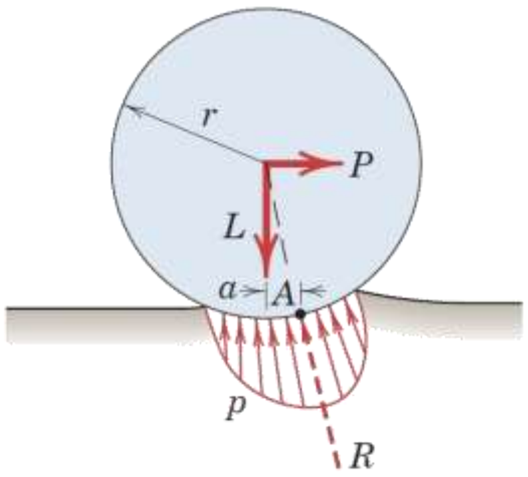


Article 6/9 Rolling Resistance

- Introduction

- Illustration

- Load Analysis
  - Load on the Wheel, L
  - Force to Maintain Speed, P
  - Distribution of Pressure over the Contact Area, p
  - Resultant Pressure-Force Distribution, R
  - Location of Resultant Pressure Force, A
  - Forward Shift of the Pressure Center from the Wheel Center, a

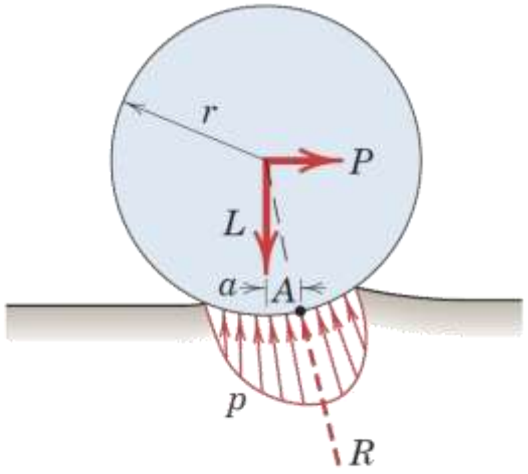


Article 6/9 – Load Analysis

- Moment Sum about the Pressure Center

  $P = a/rL = \mu_r L$

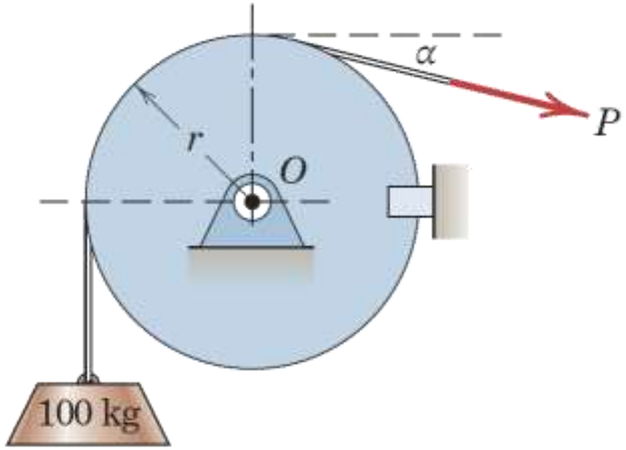
- Coefficient of Rolling Resistance, μ_r



Article 6/9 – Sample Problem 6/9 (1 of 3)

- Problem Statement

  A flexible cable which supports the 100-kg load is passed over a fixed circular drum and subjected to a force P to maintain equilibrium. The coefficient of static friction μ between the cable and the fixed drum is 0.30. (*a*) For $\alpha = 0$, determine the maximum and minimum values which P may have in order not to raise or lower the load. (*b*) For $P = 500$ N, determine the minimum value which the angle α may have before the load begins to slip.



Article 6/9 – Sample Problem 6/9 (2 of 3)

- Part (a) $\alpha = 0$

With $\alpha = 0$ the angle of contact is $\beta = \pi/2$ rad. ① For impending upward motion of the load, $T_2 = P_{max}$, $T_1 = 981$ N, and we have

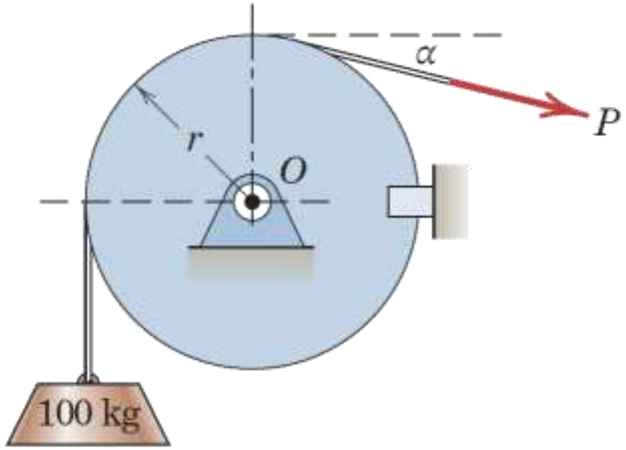
$$P_{max}/981 = e^{0.30(\pi/2)} \qquad P_{max} = 981(1.602) = 1572 \text{ N} \quad ② \quad \textit{Ans.}$$

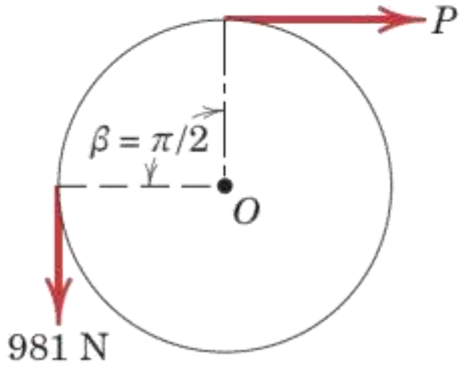
For impending downward motion of the load, $T_2 = 981$ N and $T_1 = P_{min}$. Thus,

$$981/P_{min} = e^{0.30(\pi/2)} \qquad P_{min} = 981/1.602 = 612 \text{ N} \qquad \textit{Ans.}$$

① We are careful to note that β must be expressed in radians.

② In our derivation of Eq. 6/7 be certain to note that $T_2 > T_1$.





(a) $\alpha = 0$

Article 6/9 – Sample Problem 6/9 (3 of 3)

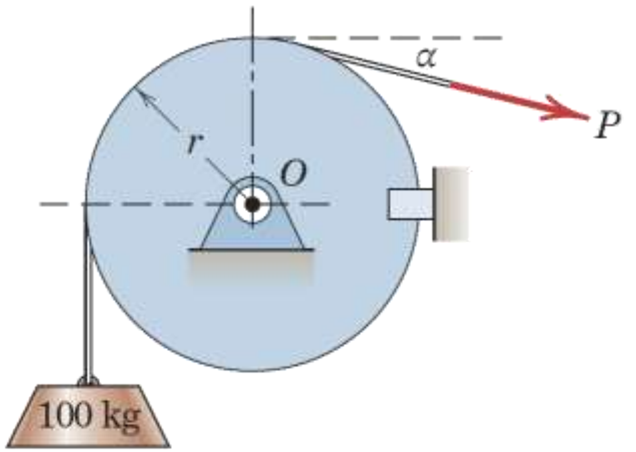
- Part (b) $P = 500$ N

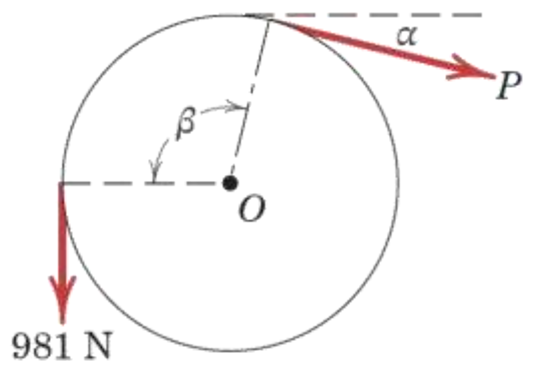
With $T_2 = 981$ N and $T_1 = P = 500$ N, Eq. 6/7 gives us

$$981/500 = e^{0.30\beta} \qquad 0.30\beta = \ln(981/500) = 0.674$$

$$\beta = 2.25 \text{ rad} \qquad \text{or} \qquad \beta = 2.25\left(\frac{360}{2\pi}\right) = 128.7°$$

$$\alpha = 128.7° - 90° = 38.7° \quad ③ \qquad \textit{Ans.}$$





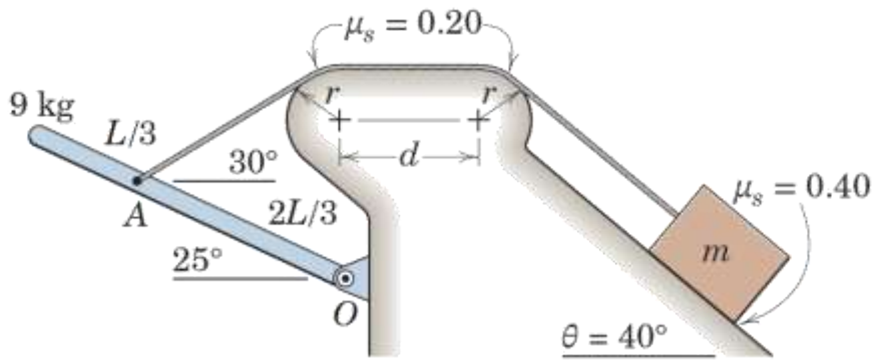
(b) $P = 500$ N

③ As was noted in the derivation of Eq. 6/7, the radius of the drum does not enter into the calculations. It is only the angle of contact and the coefficient of friction which determine the limiting conditions for impending motion of the flexible cable over the curved surface.

Article 6/9 – Sample Problem 6/10 (1 of 4)

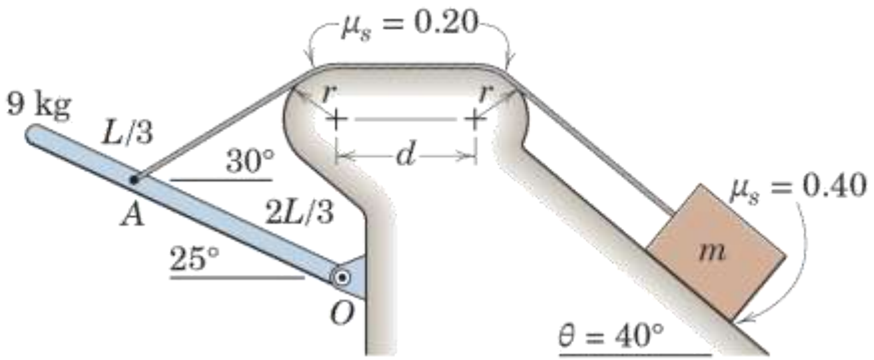
- Problem Statement

  Determine the range of mass *m* over which the system is in static equilibrium. The coefficient of static friction between the cord and the upper curved surface is 0.20, while that between the block and the incline is 0.40. Neglect friction at the pivot *O*.



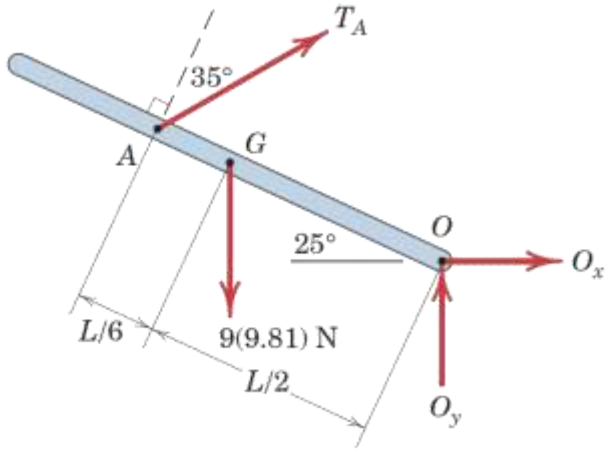
Article 6/9 – Sample Problem 6/10 (2 of 4)

- Equilibrium of Slender Bar



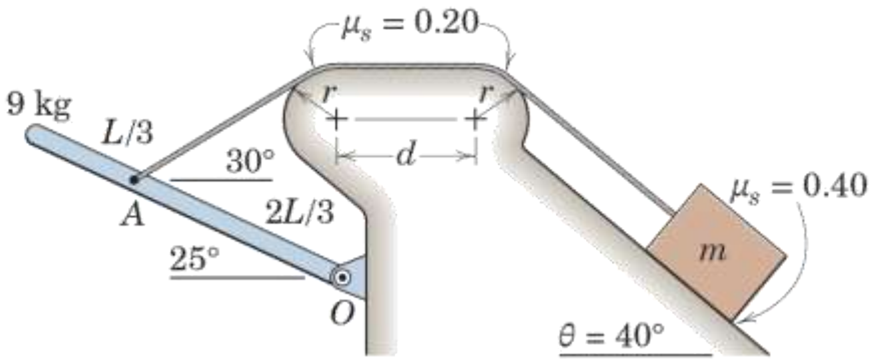
$$[\Sigma M_O = 0] \qquad -T_A\left(\frac{2L}{3}\cos 35°\right) + 9(9.81)\left(\frac{L}{2}\cos 25°\right) = 0$$

$$T_A = 73.3 \text{ N}$$



Article 6/9 – Sample Problem 6/10 (3 of 4)

- Case I: Motion Impends Up the Incline



The tension $T_A = 73.3$ N is the larger of the two tensions associated with the rough rounded surface. From Eq. 6/7 we have

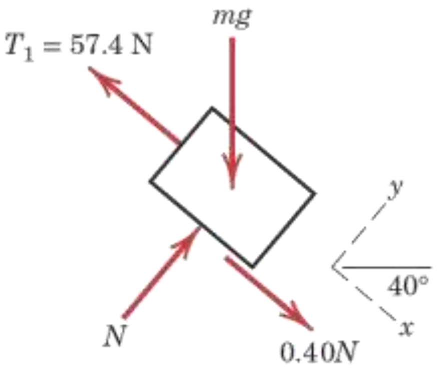
$[T_2 = T_1 e^{\mu_s \beta}]$ $\qquad 73.3 = T_1 e^{0.20[30° + 40°]\pi/180°}$ $\qquad T_1 = 57.4$ N ①

From the FBD of the block for Case I:

$[\Sigma F_y = 0]$ $\qquad N - mg\cos 40° = 0 \qquad N = 0.766mg$

$[\Sigma F_x = 0]$ $\qquad -57.4 + mg\sin 40° + 0.40(0.766mg) = 0$

$$mg = 60.5 \text{ N} \qquad m = 6.16 \text{ kg}$$

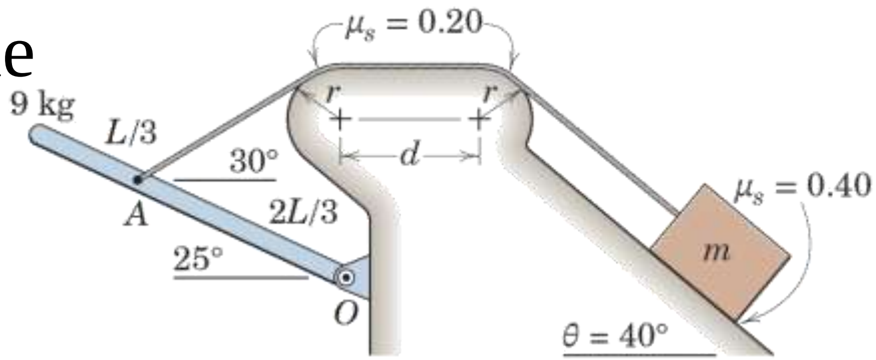


Case I

① Only the total angular contact enters Eq. 6/7 (as β). So our results are independent of the quantities r and d.

Article 6/9 – Sample Problem 6/10 (4 of 4)

- Case II: Motion Impends Down the Incline



The value $T_A = 73.3$ N is unchanged, but now this is the smaller of the two tensions in Eq. 6/7.

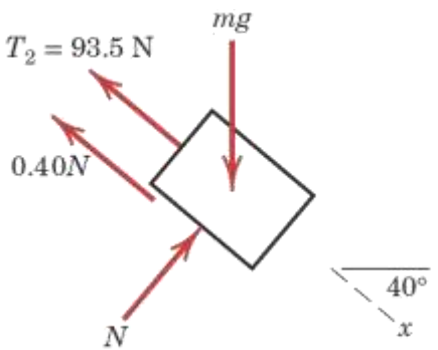
$[T_2 = T_1 e^{\mu_s \beta}]$ $\quad T_2 = 73.3e^{0.20[30°+40°]\pi/180°}$ $\quad T_2 = 93.5$ N

Considering the FBD of the block for Case II, we see that the normal force N is unchanged from Case I.

$[\Sigma F_x = 0]$ $\quad -93.5 - 0.4(0.766mg) + mg \sin 40° = 0$

$$mg = 278 \text{ N} \qquad m = 28.3 \text{ kg}$$

So the requested range is $6.16 \leq m \leq 28.3$ kg. ② *Ans.*



Case II

② Re-solve the entire problem if the ramp angle θ were changed to $20°$, with all other given information remaining constant. Be alert for a surprising result!