Problem #5 in section 1.3

$$yy' + 36x = 0$$

$$y' = \frac{dy}{dx} \qquad y\frac{dy}{dx} = -36x$$

$$y\,dy = -36x\,dx$$

$$\int y\,dy = \int -36x\,dx$$

$$\frac{y^2}{2} = -36\left(\frac{x^2}{2}\right) + C \qquad \boxed{y^2 = -36x^2 + C_1}$$

$$2y\,dy = -36(2x)\,dx$$

$$\frac{dy}{dx} = -\frac{72x}{2y} \qquad y' = -36\frac{x}{y}$$

$$y\left(-36\frac{x}{y}\right) + 36x = 0 \qquad -36x + 36x = 0$$

Problem #15 in section 1.3

$$\frac{dy}{dx} = -\frac{4x}{y} \qquad y(2) = 3$$

$$y\,dy = -4x\,dx$$

$$\int y\,dy = \int -4x\,dx$$

$$\frac{y^2}{2} = -2x^2 + C$$

$$\frac{y^2}{2} + 2x^2 = \frac{25}{2}$$

$$\boxed{y^2 + 4x^2 = 25}$$

Problem #5 in section 2.4

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} \qquad m = 5\,kg \qquad k_1 = 20\,nt/m$$

$$f = \frac{1}{2(3.14)}\sqrt{\frac{20}{5}} = \frac{2}{6.28} \approx 0.31\ Hz$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}} = \frac{1}{6.28}\sqrt{9} \approx 0.47\ Hz$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}} = \frac{1}{6.28}\sqrt{\frac{65}{5}} = \frac{3.60}{6.28} = 0.57\ Hz$$

$$k_2 = 45\,nt \qquad k_1 = 20\,nt/m$$

Problem #5 in section 2.5

$$4x^2y'' + 5y = 0 \qquad y = x^q \qquad y' = qx^{q-1}$$

$$y'' = q(q-1)x^{q-2}$$

$$4q(q-1)x^{q-2}x^2 + 5x^q = 0$$

$$4q(q-1)x^q + 5x^q = 0$$

$$q = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(5)}}{2\cdot 4} = \frac{4 \pm \sqrt{-64}}{8} = \frac{4 \pm i\sqrt{64}}{8}$$

$$= \frac{1}{2} \pm i \qquad \boxed{m_1 = \frac{1}{2} + i} \qquad \boxed{m_2 = \frac{1}{2} - i}$$

$$y = x^{\frac{1}{2}}(c_1\cos\ln x + c_2\sin(\ln x)) = \sqrt{x}(c_1\cos\ln x + c_2\sin\ln x)$$

$$y = \sqrt{x}(c_1\cos(\ln x) + c_2\sin(\ln x))$$

Problem #23 in section 2.2

$y'' + y' - 6y = 0 \qquad \lambda^2 + \lambda - 6 = 0$

$\lambda = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-6)}}{2 \times 1} = \frac{-1 \pm 5}{2} \qquad \lambda = -3, 2$

$y(x) = c_1 e^{-3x} + c_2 e^{2x}$

$10 = c_1 e^{0} + c_2 e^{0} \qquad 10 = c_1 + c_2$

$y'(x) = -3c_1 e^{-3x} + 2c_2 e^{2x} \qquad 0 = -3c_1 + 2c_2$

$-3c_1 + 2c_2 = 0 \rightarrow \quad -3c_1 + 2c_2 = 0 \qquad \boxed{c_2 = 6}$

$c_1 + c_2 = 10$

$\boxed{c_1 = 4}$

$y = -12e^{-3x} + 12e^{2x}$

$y'' = 36e^{-3x} + 24e^{2x}$

$y'' + y' - 6y = 36e^{-3x} + 24e^{2x} - 12e^{-3x} + 12e^{2x} - 6(4e^{-3x} + 6e^{2x})$

$= (36 - 36)e^{-3x} + (36 - 36)e^{-2x} = 0$

Problem #3 in Section 2.3

$(D-2I)^2 \qquad (D-2I)^2(e^{2x}) = (D^2 - 4D + 4)(e^{2x})$

$= \frac{d^2}{dx^2}(e^{2x}) - 4\frac{d}{dx}(e^{2x}) + 4e^{2x} = \frac{d}{dx}(2e^{2x}) - 4(2e^{2x}) + 4e^{2x}$

$8e^{2x} - 8e^{2x} = 0 \qquad (D-2I)^2(xe^{2x}) = (D^2 - 4D + 4)(xe^{2x})$

$= 4xe^{2x} - 4xe^{2x} + 4e^{2x} - 4e^{2x} = 0.$

$(D-2I)^2(e^{-2x}) = (D^2 - 4D + 4)(e^{-2x}) \rightarrow 16e^{-2x}$

$= \frac{d}{dx}(-2e^{-2x}) - 4(-2e^{-2x}) + 4e^{-2x} = 16e^{-2x}$

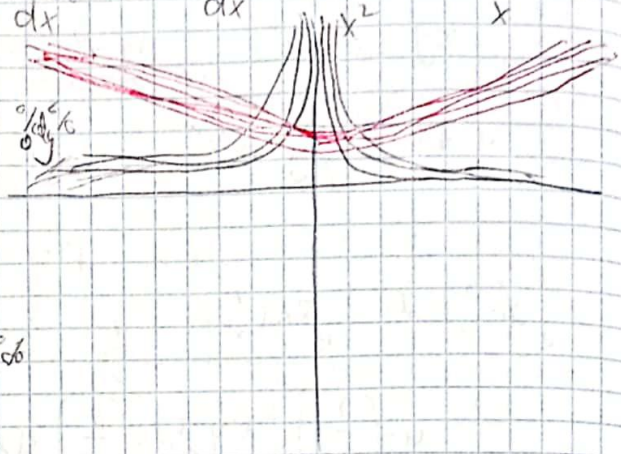
Problem #7 in section 1.6

$y = \frac{c}{x^3}$ $\quad 2x(y) + x^2 \frac{dy}{dx} = 0$ $\quad \frac{dy}{dx} = -\frac{2xy}{x^2} = -\frac{2y}{x}$

$-\frac{dx}{dy} = \frac{-2y}{x}$ $\quad x\,dx = 2y\,dy$

$\int x\,dx = \int 2y\,dy$

$\frac{x^2}{2} + \frac{c_0}{2} = y^2$ $\quad 2y^2 - x^2 = c_0$



Problem #15 in section 2.1

$4y'' + 25y = 0$ $\quad y(0) = 3$ $\quad y'(0) = -2.5$

$\cos 2.5x$, $\sin 2.5x$

$y = \cos 2.5x$

$y' = -2.5 \sin 2.5x$ $\quad y'' = -(2.5)(2.5)\cos 2.5x = -6.25 \cos 2.5x$

$4(-6.25\cos 2.5) + 25\cos 2.5x = -25\cos 2.5x + 25\cos 2.5x = 0$

$y = \sin 2.5x$ $\quad y' = 2.5 \sin 2.5x$ $\quad y'' = -(2.5)(2.5)\sin 2.5x$

$4(-6.25\sin 2.5x) + 25\sin 2.5x = -25\sin 2.5x + 25\sin 2.5x = 0$

$\frac{\cos 2.5x}{\sin 2.5x} = \cot 2.5x \neq \text{const}$ $\quad 3.0 = c_1 \cos(2.5(0)) + c_2 \sin 2.5(0)$

$= c_1(1) + c_2(0) = c_1$

$-2.5 = 2.5c_2$ $\quad c_1 = 3$

$c_2 = -1$ $\quad \boxed{y = 3\cos 2.5x - \sin 2.5x}$

Problem #7 in section 1.5

$$xy' - 2y = x^3 e^x$$

$$xy' - 2y = x^3 e^x \qquad y' - \frac{2}{x}y = x^2 e^x$$

$$y' + \left(-\frac{2}{x}\right)y = x^2 e^x \qquad y' + p(x)y = g(x)$$

$$p(x) = -\frac{2}{x} \qquad g(x) = x^2 e^x$$

$$e^{\int -\frac{2}{x}\,dx} = e^{\ln\left(\frac{1}{x^2}\right)} = \frac{1}{x^2}$$

$$y\left(\frac{1}{x^2}\right) = \int (x^2 e^x)\left(\frac{1}{x^2}\right) dx + C$$

$$y\left(\frac{1}{x^2}\right) = e^x + C \qquad \boxed{y = x^2(e^x + C)}$$

Problem #5 in section 1.4

$$(x^2 + y^2)\,dx - 2xy\,dy = 0$$

$$M = x^2 + y^2 \qquad N = -2xy$$

$$\frac{\partial M(x,y)}{\partial y} = \frac{\partial N(x,y)}{\partial x}$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(x^2 + y^2)$$

$$\frac{dM}{dy} \neq \frac{dN}{dx}$$

Not exact

$$(x^2 + y^2)\,dx - 2xy\,dy = 0$$

$$x^2 dx - (x\,d(y^2) - y^2 dx) = 0 \Big/ \frac{1}{x^2}$$

$$dx - \left(\frac{x\,d(y^2) - y^2 dx}{x^2}\right) = 0 \qquad dx - d\left(\frac{y^2}{x}\right) = 0$$

$$\int dx - \int d\left(\frac{y^2}{x}\right) = 0 \qquad x - \frac{y^2}{x} + C = 0 \qquad x^2 - y^2 + Cx = 0$$

$$y^2 = x^2 + Cx \qquad \boxed{y = \sqrt{x^2 + Cx}}$$

Thank you

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