Introduction to Discrete Math

Felipe P. Vista IV

Course Outline

- Mathematical Thinking
  - Convincing Arguments, Find Example, Recursion & Induction, Logic, Invariants
- Probability & Combinatronics
  - Basic Counting, Binomial Coeff, Advanced Counting, Probability, Random Variables

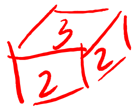
Probability & Combinatronics – Random Variables

EXPECTATION IS NOT ALL

- A Dice Game

A Dice Game

- Suppose Mikki and Matt are playing a game
- Each of them has an (unconventional) dice



- Numbers on Mikki's dice are 2, 2, 2, 2, 3, 3

- Numbers on Matt's dice are 1, 1, 1, 1, 6, 6
- Mikki and Matt throw their dices → the one with the larger number on the dice wins

Mikki				**Matt**
Numbers: 2, 2, 2, 2, 3, 3	→	**2**	**6** ←	Numbers: 1, 1, 1, 1, 6, 6

A Dice Game

- Suppose Mikki and Matt are playing a game
- Each of them has an (unconventional) dice
- Numbers on Mikki's dice are 2, 2, 2, 2, 3, 3
- Numbers on Matt's dice are 1, 1, 1, 1, 6, 6
- Mikki and Matt throw their dices → the one with the larger number on the dice wins

Mikki

Numbers:

2, 2, 2, 2, 3, 3 → **2**

Matt Wins!

Matt

Numbers:

1, 1, 1, 1, 6, 6 → **6**

- If they play the game many times, who will win more often?

Who Has Better Expected Value

Mikki	Matt
Numbers:	Numbers:
2, 2, 2, 2, 3, 3	1, 1, 1, 1, 6, 6

- Let's see who has better expected value of a dice throw
- Mikki has: $2\times\frac{2}{3}+3\times\frac{1}{3}=\frac{7}{3}$
- Matt has: $1\times\frac{2}{3}+6\times\frac{1}{3}=\frac{8}{3}$

Who Has Better Expected Value

Mikki	**Matt**
Numbers:	Numbers:
2, 2, 2, 2, 3, 3	1, 1, 1, 1, 6, 6

- Let's see who has better expected value of a dice throw
- Mikki has: $2\times\frac{2}{3}+3\times\frac{1}{3}=\frac{7}{3}$ ✗
- Matt has: $1\times\frac{2}{3}+6\times\frac{1}{3}=\frac{8}{3}$ ✓
- Matt has better expected value

But, who wins more often?

Mikki	**Matt**
Numbers:	Numbers:
2, 2, 2, 2, 3, 3	1, 1, 1, 1, 6, 6

- Note that the winner depends only on Matt's throw:
  If he throws "1" he definitely loses, else if a "6", he wins! = 1/3
- Matt throws a "1" with probability of 2/3 = lose

But, who wins more often?

Mikki	**Matt**
Numbers:	Numbers:
2, 2, 2, 2, 3, 3	1, 1, 1, 1, 6, 6
	2/3 1/3

- Note that the winner depends only on Matt's throw:
  If he throws "1" he definitely loses, else if a "6", he wins!
- Matt throws a "1" with probability of 2/3
- So Matt loses (substantially) more often, despite a greater expected value ☹

How about the Expected Value?

Mikki	**Matt**
Numbers:	Numbers:
2, 2, 2, 2, 3, 3	1, 1, 1, 1, 6, 6

- Where did the large expected value go?
  Why does it now help Matt to win? 6 >> 2 or 3
- When Matt wins, he wins by big margin: "6" against "2" or "3";
  If Matt loses, he loses slightly: "1" against "2" or "3" 1 < 2 or 3
- But he does not get credit for difference between the numbers

Conclusion

Mikki	**Matt**
Numbers:	Numbers:
2, 2, 2, 2, 3, 3 2	1, 1, 1, 1, 6, 6

$2 \times \frac{2}{3} + 3 \frac{1}{3} =$

- This example shows that the expected value does not tell us everything about random variable
- A random variable with "better" expected value can be "worse" because of some other properties

Thank you.