chapter 13.2 Problem 5

$$\frac{\sqrt{2}+i/3}{-\sqrt{8}-2i/3}\cdot\frac{3\sqrt{2}+1}{-3\sqrt{8}-2i}=\frac{-36+6\sqrt{2}i-3\sqrt{8}i+2i^2}{(3\sqrt{8})^2+2^2}=\boxed{-\frac{1}{2}}$$

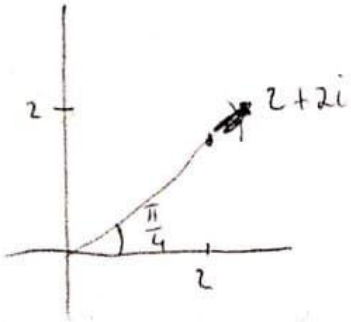
$$r^2=x^2+y^2=\left(\frac{1}{2}\right)^2=\boxed{\frac{1}{2}} \qquad z=\frac{1}{2}(\cos\pi+i\sin\pi)$$

$$\theta=\tan^{-1}\left(\frac{y}{x}\right)\cdot\tan^{-1}(0)=\pi$$

chapter 13.2 Problem 17

$$\sqrt{8}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right) \qquad z=\sqrt{8}\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right)$$

$$z=2+2i$$



chapter 13.3 Problem 10

$$f(4-3i)=5(4-3i)^2-10(4-3i)+3+2i=35-120i-48+36i+3+2i$$

$$=-10-82i$$

Chapter 13.1 Problem 8

$z_1 = -2+11i, \quad z_2 = 2-i$

$z_1 \cdot z_2 = (-2+11i)(2-i) = -4+2i+22i-11i^2 = 7+24i$

$\overline{z_1 z_2} = \overline{7+24i} = 7-24i$

$\overline{z_1}\,\overline{z_2} = 7-24i$

Chapter 13.1 Problem 15

$z_1 = -2+11i \quad z_2 = 2-i$

$z_1+z_2 = (-2+11i)+(2-i) = 10i$

$z_1 - z_2 = (-2+11i)-(2-i) = -4+12i$

$$\frac{4(z_1+z_2)}{(z_1-z_2)} = \frac{4(10i)}{-4+12i} = \frac{-160i-480i^2}{16-144i^2}$$

$$= \frac{160(3-i)}{160} = \underline{3-i}$$

Chapter 13.5 problem 4

$e^z = e^{0.6-1.8i} = e^{0.6}(\cos 1.8 - i\sin(1.8))$ u+iv

$|e^z| = |e^{0.6} \cdot e^{1.8i}| = e^{0.6}|e^{-1.8i}| \cdot e^{0.6}$

Chapter 13.5 Problem 13

$z = 1+i$

$r = \sqrt{x^2+y^2} = \sqrt{2}$ $\quad \theta = \tan^{-1}\left(\frac{y}{x}\right) = \frac{\pi}{4}$

$r(\cos\theta + i\sin\theta) = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ $\quad z = re^{i\theta} = \sqrt{2}e^{i\pi/4}$

Chapter 13.6 Problem 7

$\cos iz = \cosh z$

$\cos i = \cos ii = \cosh 1 \approx 1.543$

$\sin i = \sin 1i = i\sinh 1 \approx 1.175i$

chapter 13.6 Problem 18

$\cosh z = \frac{e^z + e^{-z}}{2}$ $\quad -1 = \frac{e^z + e^{-z}}{2}$ $\quad e^{2z} + 2e^z + 1 = 0$

$e^{x+iy} = -1$ $\quad e^x\cos y = -1$ $\quad e^x \sin y = 0$

$e^x = 1 \rightarrow x = 0$ $\quad \boxed{0 \pm i(2n+1) \quad n = 0, 1, 2}$

Chapter 13.3 Problem 19

$$S'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

$$f'(3+4i) = \lim_{z \to 3+4i} \frac{f(z) - f(3+4i)}{z - (3+4i)} = \lim_{z \to 3+4i} (z - 4i + 3)\left[(z-4i)^2 + 3^2\right]$$

$$\left[(z-4i)^4 + 3^4\right] = (3+4i-4i+3)\left[(3+4i-4i)^2 + 3^2\right]$$

$$\left[(3+4i-4i)^4 + 3^4\right] = 6(18)(162) = 17496$$

$$f'(3+4i) = 17496$$

Chapter 13.4 Problem 4

$$\frac{\partial}{\partial y}(u(x,y)) = \frac{\partial}{\partial x}(e^x \cos y) \qquad u_y = \cos y \frac{\partial}{\partial x}(e^x)$$

$$u_x = e^x \cos y$$

$$\frac{\partial}{\partial y}(v(x,y)) = \frac{\partial}{\partial y}(-e^x \sin y) \qquad v_y = -e^x \frac{\partial}{\partial y}(\sin y) \quad v_y = -e^x \cos y$$

$u_x \neq v_y$ not analytic

Chapter 13.4 Problem 12

$u_{xx} + u_{yy} = 0 \quad v_{xx} + v_{yy} = 0 \ldots$

$u_x = 2x \quad u_y = 2y$

$u_{xx} = 2 \quad u_{yy} = 2$

$u_{xx} + u_{yy} \neq 0$ not harmonic

Chapter 13.7 Problem 15

$|z| = |e^i| = 1$

$\operatorname{Arg} z = \operatorname{Arg} e^i = 1$ $\quad \operatorname{Ln} z = \ln|z| + i \operatorname{Arg} z = \ln 1 + 1i = i$

$\ln z = \operatorname{Ln} z \pm 2n\pi i$. $\quad \ln e^i = i \pm 2n\pi i = (1 \pm 2n\pi) i$

Chapter 13.7 Problem 24

$(1-i)^{1+i} = e^{\ln(1-i)^{1+i}} = e^{(1+i)\ln(1-i)}$

$|z| = |1-i| = \sqrt{2}$ $\quad \operatorname{Arg} z = \operatorname{Arg}(1-i) = \tan^{-1}\left(\frac{-1}{1}\right) = -\frac{\pi}{4}$

$\operatorname{Ln} z = \ln|z| + i \operatorname{Arg} z = \ln\sqrt{2} - \frac{\pi i}{4}$

$\ln z = \operatorname{Ln} z \pm 2n\pi i$ $\quad \ln(1-i) = \ln\sqrt{2} - \frac{\pi i}{4} \pm 2n\pi i$

$(1-i)^{1+i} = e^{(1+i)\ln(1-i)}$

$e^{(1+i)\left[\ln\sqrt{2} - \frac{\pi i}{4}\right]} = e^{\ln\sqrt{2} + \frac{\pi}{4} + i\left(\ln\sqrt{2} - \frac{\pi}{4}\right)}$

$= e^{\ln\sqrt{2} + \frac{\pi}{4}} \left[\cos\left(\ln\sqrt{2} - \frac{\pi}{4}\right) + i \sin\left(\ln\sqrt{2} - \frac{\pi}{4}\right)\right]$

$\approx 2.807 - 1.317\, i$

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