CHAPTER 1
INTRODUCTION TO STATICS

CHAPTER OUTLINE

1/1 Mechanics

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1/3 Scalars and Vectors

1/4 Newton’s Laws

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1/7 Accuracy, Limits, and Approximations

1/8 Problem Solving in Statics

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Article 1/1 Mechanics

- Mechanics deals with the effects of forces on objects.
- No other subject plays a greater role in engineering analysis.
- Mechanics is the oldest of the physical sciences.
- This course deals with the development and application of the principles of mechanics.
- Statics is concerned with the equilibrium of bodies under the action of forces.

Article 1/2 Basic Concepts

- Space
- Time
- Mass
- Force
- Particle
- Rigid Body

Article 1/3 Scalars and Vectors

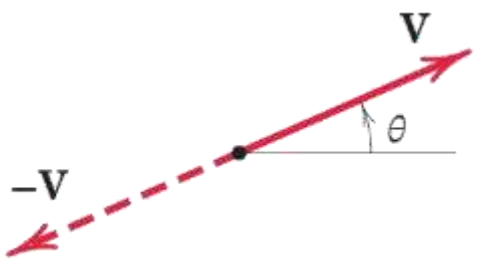
- Scalar Quantity
  - A quantity with only a magnitude.
  - Examples: time, volume, density, speed, energy, and mass

- Vector Quantity
  - Quantity with both a magnitude and a direction.
  - Obeys the parallelogram law of addition.
  - Examples: displacement, velocity, acceleration, force, moment, and momentum.

Article 1/3 – Types of Vectors in Mechanics

- Types of Vectors
  - Free Vector – one whose action is not confined to or associated with a unique line in space.
  - Sliding Vector – has a unique line of action in space but not a unique point of application.
  - Fixed Vector – has a unique line of action and point of application.

Article 1/3 – Conventions for Equations and Diagrams

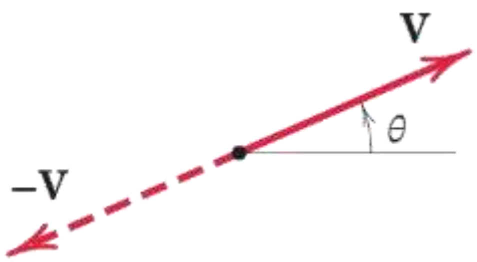


- Vector Representation
  - Line Segment with an Arrowhead to Indicate Direction
  - Written in Bold, Roman Type, e.g., **V**
  - Magnitude is Written in Lightface, Italic type, e.g., *V*

- Always Distinguish between Scalars and Vectors
  - Use an underline, over-arrow, under-squiggle, etc., to represent vectors.
  - Failure to do so causes many mistakes in mechanics.

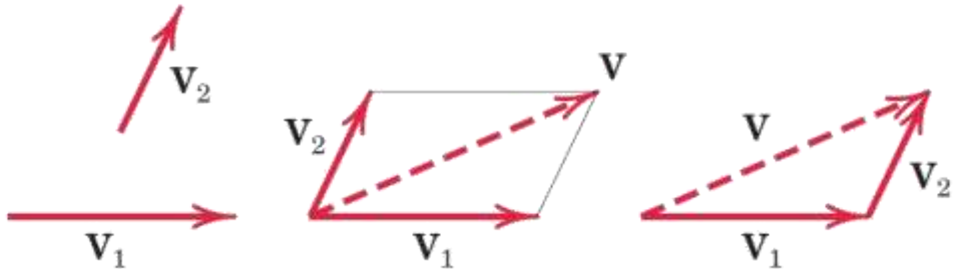
Article 1/3 – Working with Vectors (1 of 4)

- Drawing Vectors
  - Line Segment with an Arrowhead to Indicate Direction
  - Reference Angle θ

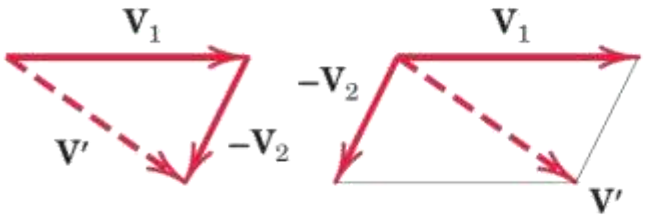


Article 1/3 – Working with Vectors (2 of 4)

- Parallelogram Law of Addition – Vector Sum $\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$
  - Two Vectors, $\mathbf{V}_1$ and $\mathbf{V}_2$, treated as free vectors, may be replaced by their equivalent vector $\mathbf{V}$, which is the diagonal of the parallelogram formed by $\mathbf{V}_1$ and $\mathbf{V}_2$. This is called a *vector sum*.

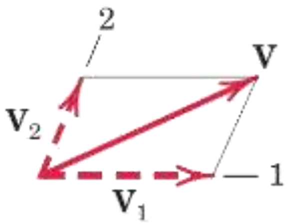


- Vector Difference $\mathbf{V}' = \mathbf{V}_1 – \mathbf{V}_2$ (Adding a Negative)

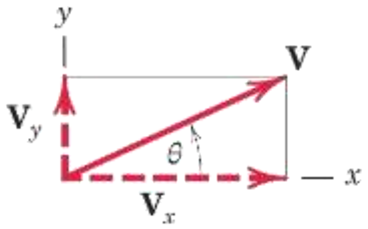


Article 1/3 – Working with Vectors (3 of 4)

- Vector Components

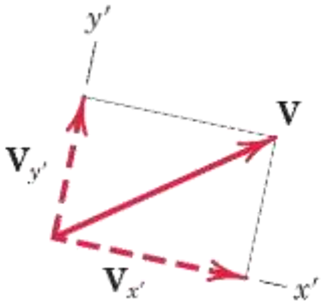


- Rectangular Components (*x*-*y*)



$$\theta = \tan^{-1}\frac{V_y}{V_x}$$

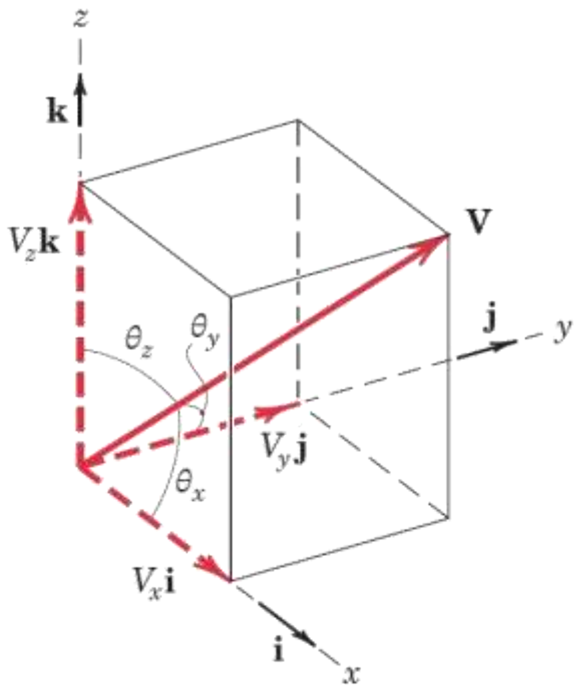
- Axis Orientation



Article 1/3 – Working with Vectors (4 of 4)

- Unit Vector Representation, $\mathbf{V} = V\mathbf{n}$
  - A unit vector $\mathbf{n}$ has a magnitude of one (unity) and points in the direction of a vector.

- Three-Dimensional Vectors and Direction Cosines
  - $\mathbf{V} = V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k}$
    - $V_x = V \cos \theta_x = Vl$ where $l = \cos \theta_x$
    - $V_y = V \cos \theta_y = Vm$ where $m = \cos \theta_y$
    - $V_z = V \cos \theta_z = Vn$ where $n = \cos \theta_z$
  - Pythagorean Theorem (Vector Magnitude)
    - $V^2 = V_x^2 + V_y^2 + V_z^2$
    - $l^2 + m^2 + n^2 = 1$



Article 1/4 Newton's Laws

- Law I

  A particle remains at rest or continues to move with *uniform velocity* (in a straight line with a constant speed) if there is no unbalanced force acting on it.

- Law II

  The acceleration of a particle is proportional to the vector sum of forces acting on it and is in the direction of this vector sum.

- Law III

  The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and *collinear* (they lie on the same line).

Article 1/5 Units

- Fundamental Quantities of Mechanics and their Units

Quantity	Dimensional Symbol	SI Units			U.S. Customary Units		
			Unit	Symbol		Unit	Symbol
Mass	M	Base units	kilogram	kg		slug	—
Length	L	Base units	meter	m	Base units	foot	ft
Time	T	Base units	second	s	Base units	second	sec
Force	F		newton	N	Base units	pound	lb

Article 1/5 – SI Units

- Base Units
  - kilogram (kg)
  - meter (m)
  - second (s)

- Derived Unit: Newton (N)
  - Force Unit
  - $N = kg \cdot m/s^2$

Article 1/5 – U.S. Customary Units

- Base Units
  - pound (lb)
  - foot (ft)
  - second (sec)

- Derived Unit: slug (slug)
  - Mass Unit
  - slug = lb-sec^2/ft
  - Gravitational System

Article 1/5 – Primary Standards (1 of 2)

- Mass
  - Mass of a specific platinum iridium cylinder kept at the International Bureau of Weights and Measures near Paris, France.

- Length
  - The distance traveled by light in a vacuum in (1/299 792 458) second.

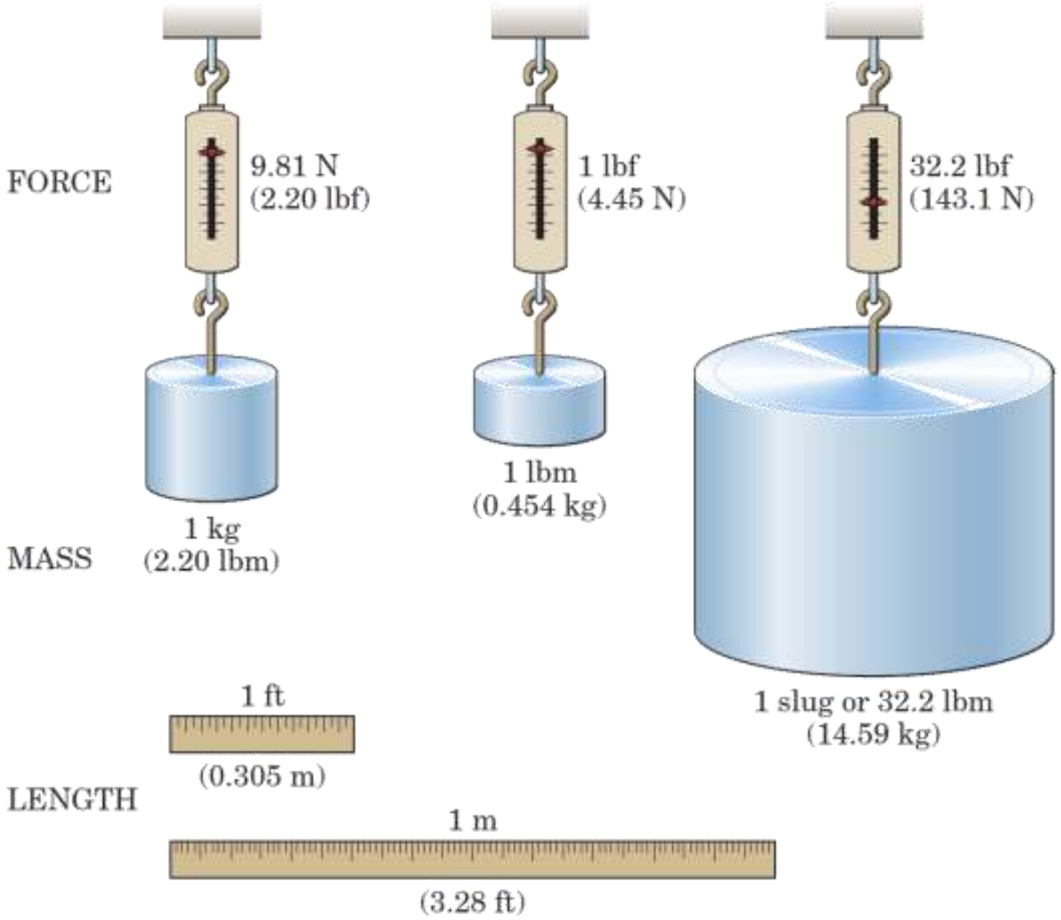
- Time
  - The duration of 9 192 631 770 periods of the radiation of a specific state of a cesium-133 atom.

Article 1/5 – Primary Standards (1 of 2)

- Acceleration of Gravity
  - SI Units: $g = 9.806\ 65\ \text{m/s}^2$
  - U.S. Units: $g = 32.1740\ \text{ft/sec}^2$
- Values for Most Problems in Mechanics
  - SI Units: $g = 9.81\ \text{m/s}^2$
  - U.S. Units: $g = 32.2\ \text{ft/sec}^2$

Article 1/5 – Unit Conversions

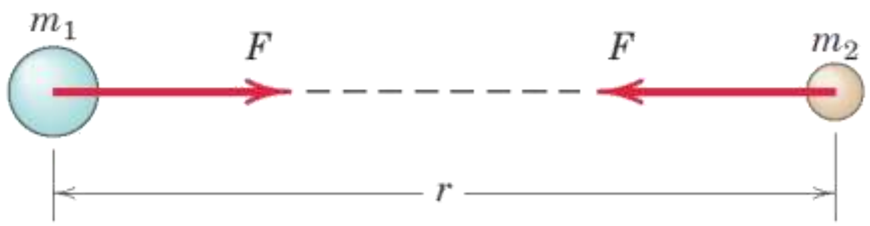
- Visual Comparison of Force, Mass, and Length



A comprehensive list of unit conversion is available in Table D/5 of Appendix D.

Article 1/6 Law of Gravitation

- Mathematical Expression



$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

F = the mutual force of attraction between two particles

G = a universal constant known as the constant of gravitation

m_1, m_2 = the masses of the two particles

r = the distance between the centers of the particles

- Constant of Gravitation, G
  - SI Units: $G = 6.673(10^{-11})\ \mathrm{m^3/(kg \cdot s^2)}$
  - U.S. Units: $G = 3.439(10^{-8})\ \mathrm{ft^4/(lb\text{-}sec^4)}$

Article 1/6 – Gravitational Attraction of the Earth

- Apparent Weight, $W = mg$

- SI Problems
  - Mass m is always in kilograms (kg) and is almost always provided in the book.
  - Acceleration of gravity $g = 9.81\ \text{m/s}^2$ (unless stated otherwise).
  - Weight W is in newtons (N).
  - Kilogram (kg) is not a force!

- U.S. Problems
  - Mass m is always in slugs (slugs) and is almost never provided in the book.
  - Acceleration of gravity $g = 32.2\ \text{ft/sec}^2$ (unless stated otherwise).
  - Weight W is in pounds (lb) and is usually what you are provided.
  - Pound (lb) is not a mass!

Article 1/7 Accuracy, Limits, and Approximations

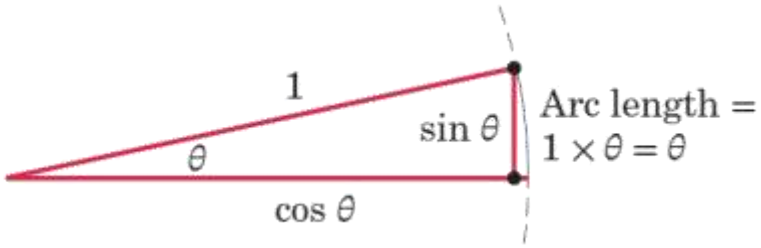
- Significant Figure Use in the Textbook
  - The textbook assumes that all given numbers are exact for simplicity.
  - If the first non-zero digit is a one (1), the textbook will record four (4) significant figures in the answer.
  - If the first non-zero digit is a two through nine (2-9), the textbook will record three (3) significant figures in the answer.
  - Retain all significant figures on intermediate calculations in a calculator, but only record answers or calculations according to the convention listed above.

Article 1/7 – Differentials

- Order of Differential Quantities

Article 1/7 – Small-Angle Approximations

- Small Angle Approximations (with radians)



  - $\sin\theta \cong \tan\theta \cong \theta$ and $\sin d\theta \cong \tan d\theta \cong d\theta$
  - $\cos\theta \cong 1$ and $\cos d\theta \cong 1$
  - Sample Calculation
    - 1° = 0.017 453 rad, sin 1° = 0.017452, tan 1° = 0.017455, cos 1° = 0.999848
    - The percent error for the sine function is only 0.51% at 10°.
  - If more accuracy is required, retain the first two terms in the series expansion of the function.
  - To convert from degrees to radians, multiply the angle in degrees by $\pi/180°$.

Article 1/8 Problem Solving in Statics (1 of 4)

- Dual Thought Process in Statics
  - Think about the **physical situation** and the corresponding **mathematical description**.

- Make Appropriate Assumptions

- Use Graphics
  1. Representing a problem geometrically helps us with its physical interpretation. This is especially true for three-dimensional problems.
  2. Graphical solutions can often be obtained more readily than with a direct mathematical solution.
  3. Charts and graphs are valuable aids for representing results.

Article 1/8 Problem Solving in Statics (2 of 4)

- Formulating Problems and Obtaining Solutions

1. Formulate the problem
   a) State the given data.
   b) State the desired result.
   c) State your assumptions and approximations.
2. Develop the solution
   a) Draw any diagrams you need to understand the relationships.
   b) State the governing principles to be applied to your solution.
   c) Make your calculations.
   d) Ensure that your calculations are consistent with the accuracy justified by the data.
   e) Be sure that you have used consistent units throughout your calculations.
   f) Ensure that your answers are reasonable in terms of magnitudes, directions, common sense, etc.
   g) Draw conclusions.

Article 1/8 Problem Solving in Statics (3 of 4)

- The Free-Body Diagram
  - Isolation of a Body from all other Interacting Bodies
  - Developed Fully in Chapter 3
  - Single Most Important Step in Equilibrium Problems

- Numerical Values versus Symbols
  - Symbolic Solutions Advantages
    - Helps to focus attention on the connection between the physical situation and its related mathematical description.
    - Can be used repeatedly for obtaining answers to the same type or problem but having different units or numerical values.
    - Enables dimensional checks at every step to ensure dimensional homogeneity.

Article 1/8 Problem Solving in Statics (4 of 4)

- Solution Methods

1. Obtain mathematical solutions by hand, using either algebraic symbols or numerical values. We can solve most problems this way.

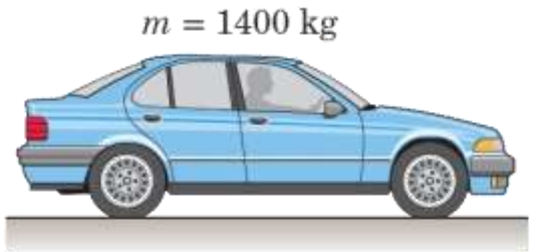
2. Obtain graphical solutions for certain problems.

3. Solve problems by computer. This is useful when a large number of equations must be solved, when a parameter variation must be studied, or when an intractable equation must be solved.

Article 1/9 – Sample Problem 1/1 (1 of 2)

- Problem Statement

  Determine the weight in newtons of a car whose mass is 1400 kg. Convert the mass of the car to slugs and then determine its weight in pounds.



Article 1/9 – Sample Problem 1/1 (2 of 2)

- Solution

$$W = mg = 1400(9.81) = 13\ 730 \text{ N} \quad ① \qquad Ans.$$

From the table of conversion factors in Table D/5 of Appendix D, we see that 1 slug is equal to 14.594 kg. Thus, the mass of the car in slugs is

$$m = 1400 \text{ kg}\left[\frac{1 \text{ slug}}{14.594 \text{ kg}}\right] = 95.9 \text{ slugs} \quad ② \qquad Ans.$$

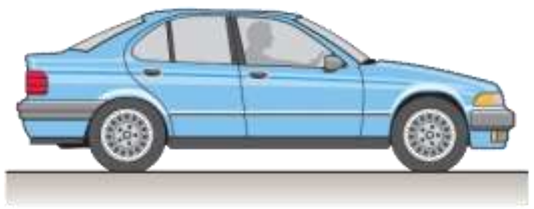
Finally, its weight in pounds is

$$W = mg = (95.9)(32.2) = 3090 \text{ lb} \quad ③ \qquad Ans.$$

As another route to the last result, we can convert from kg to lbm. Again using Table D/5, we have

$$m = 1400 \text{ kg}\left[\frac{1 \text{ lbm}}{0.45359 \text{ kg}}\right] = 3090 \text{ lbm}$$

③ Note that we are using a previously calculated result (95.9 slugs). We must be sure that when a calculated number is needed in subsequent calculations, it is retained in the calculator to its full accuracy, (95.929834 . . .), until it is needed. This may require storing it in a register upon its initial calculation and recalling it later. We must not merely punch 95.9 into our calculator and proceed to multiply by 32.2—this practice will result in loss of numerical accuracy. Some individuals like to place a small indication of the storage register used in the right margin of the work paper, directly beside the number stored.



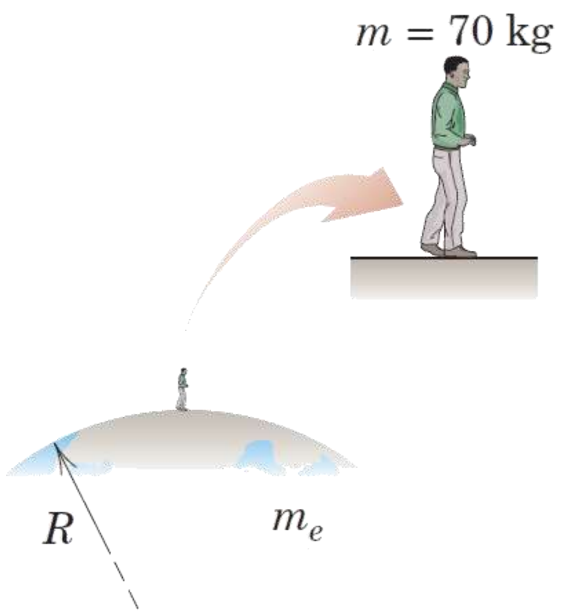
① Our calculator indicates a result of 13 734 N. Using the rules of significant-figure display used in this textbook, we round the written result to four significant figures, or 13 730 N. Had the number begun with any digit other than 1, we would have rounded to three significant figures.

② A good practice with unit conversion is to multiply by a factor such as $\left[\frac{1 \text{ slug}}{14.594 \text{ kg}}\right]$, which has a value of 1, because the numerator and the denominator are equivalent. Make sure that cancellation of the units leaves the units desired; here the units of kg cancel, leaving the desired units of slug.

Article 1/9 – Sample Problem 1/2 (1 of 2)

- Problem Statement

  Use Newton's law of universal gravitation to calculate the weight of a 70-kg person standing on the surface of the earth. Then repeat the calculation by using $W = mg$ and compare your two results. Use Table D/2 as needed.



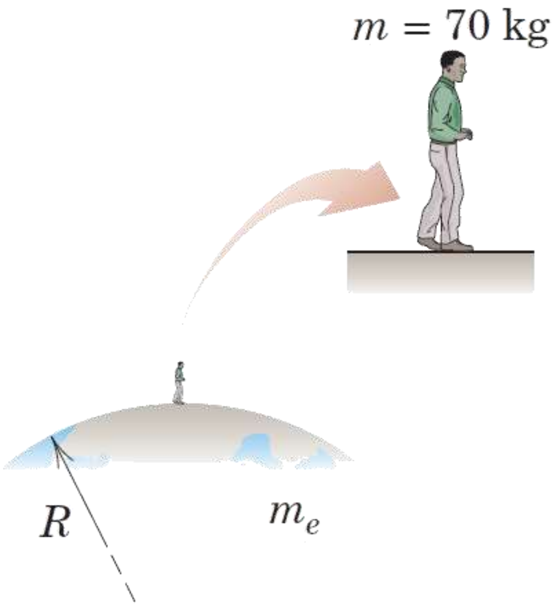
Article 1/9 – Sample Problem 1/2 (2 of 2)

- Solution

$$W = \frac{Gm_e m}{R^2} = \frac{(6.673\cdot 10^{-11})(5.976\cdot 10^{24})(70)}{[6371\cdot 10^3]^2} = 688 \text{ N} \quad ① \qquad \textit{Ans.}$$

$$W = mg = 70(9.81) = 687 \text{ N} \qquad \textit{Ans.}$$

① The effective distance between the mass centers of the two bodies involved is the radius of the earth.

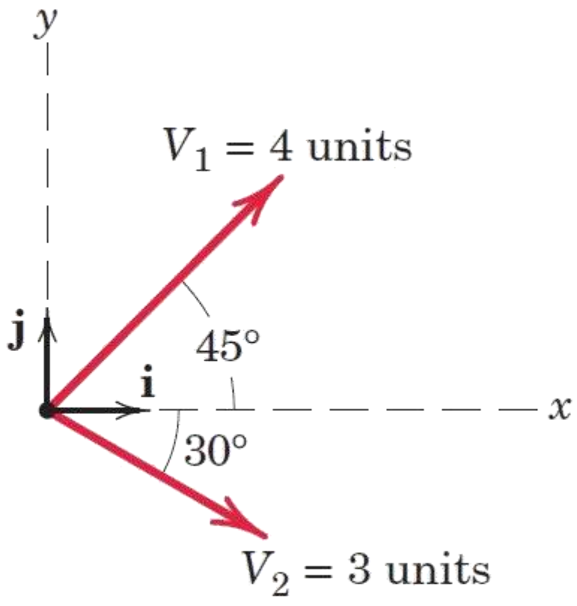


Article 1/9 – Sample Problem 1/3 (1 of 3)

- Problem Statement

For the vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ shown in the figure,

a) determine the magnitude S of their vector sum $\mathbf{S} = \mathbf{V}_1 + \mathbf{V}_2$
b) determine the angle α between $\mathbf{S}$ and the positive x-axis
c) write $\mathbf{S}$ as a vector in terms of the unit vectors $\mathbf{i}$ and $\mathbf{j}$ and then write a unit vector $\mathbf{n}$ along the vector sum $\mathbf{S}$
d) determine the vector difference $\mathbf{D} = \mathbf{V}_1 - \mathbf{V}_2$



Article 1/9 – Sample Problem 1/3 (2 of 3)

- Part *a*) – Magnitude of **S**

$$S^2 = 3^2 + 4^2 - 2(3)(4)\cos 105°$$

$$S = 5.59 \text{ units} \qquad \textit{Ans.}$$

- Part *b*) – Angle Between **S** and *x*-axis

$$\frac{\sin 105°}{5.59} = \frac{\sin(\alpha + 30°)}{4}$$

$$\sin(\alpha + 30°) = 0.692$$

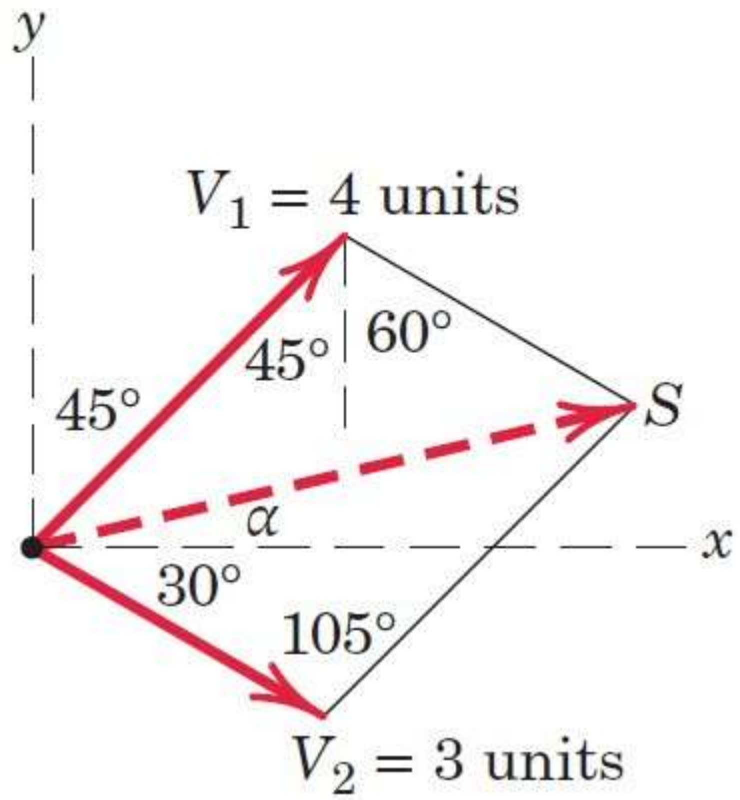
$$(\alpha + 30°) = 43.8° \qquad \alpha = 13.76° \qquad \textit{Ans.}$$

- Part *c*) – Vector Expression for **S** and **n**

$$\mathbf{S} = S[\mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha]$$

$$= 5.59[\mathbf{i}\cos 13.76° + \mathbf{j}\sin 13.76°] = 5.43\mathbf{i} + 1.328\mathbf{j} \text{ units} \quad \textit{Ans.}$$

Then $\quad \mathbf{n} = \dfrac{\mathbf{S}}{S} = \dfrac{5.43\mathbf{i} + 1.328\mathbf{j}}{5.59} = 0.971\mathbf{i} + 0.238\mathbf{j}$ ② $\qquad$ *Ans.*



② A unit vector may always be formed by dividing a vector by its magnitude. Note that a unit vector is dimensionless.

Article 1/9 – Sample Problem 1/3 (of 3)

- Part *d*) – Vector Difference

$$\mathbf{D} = \mathbf{V}_1 - \mathbf{V}_2 = 4(\mathbf{i}\cos 45^\circ + \mathbf{j}\sin 45^\circ) - 3(\mathbf{i}\cos 30^\circ - \mathbf{j}\sin 30^\circ)$$

$$= 0.230\mathbf{i} + 4.33\mathbf{j} \text{ units} \qquad \textit{Ans.}$$