Introduction to Discrete Math

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- ## Mathematical Thinking
  - Convincing Arguments, Find Example, Recursion, Logic, **Invariants**

- ## Probability & Combinatronics
  - Counting, Probability, Random Variables

- ## Graph Theory
  - Graphs (cycles, classes, parameters)

- ## Number Theory & Cryptography
  - Arithmetic in modular form
  - Intro to Cryptography

Mathematical Thinking – Invariants

INVARIANTS

- Double counting

- Homework assignment problem

Double Counting

- It is useful to look at a problem from two different angles

- Get information from two sources

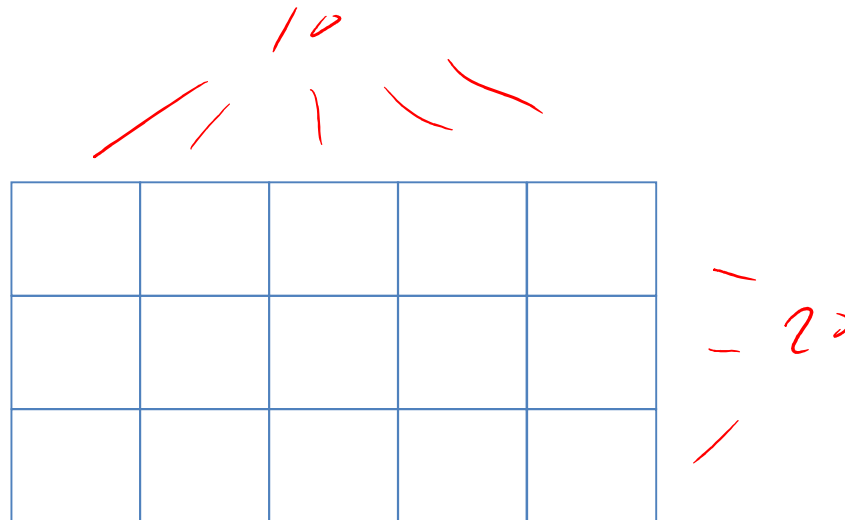- Look at a number from two points of view
  - A standard special case in Math

Double Counting

Problem

- Is it possible to fill a *3 x 5* table with integers so that the sum for each row is equal to *20* while the sum for each column would be *10*?

Let's try:

Double Counting

Problem

- Is it possible to fill a *3 x 5* table with integers so that the sum for each row is equal to *20* while the sum for each column would be *10*?

Let's try:

5	3	4	2	6	20

Double Counting

Problem

- Is it possible to fill a *3 x 5* table with integers so that the sum for each row is equal to *20* while the sum for each column would be *10*?

Let's try:

5	3	4	2	6	20
1	4	3	8	4	20

Double Counting

Problem

- Is it possible to fill a *3 x 5* table with integers so that the sum for each row is equal to *20* while the sum for each column would be *10*?

Let's try:

5	3	4	2	6	20
1	4	3	8	4	20
4					

10

Double Counting

Problem

- Is it possible to fill a *3 x 5* table with integers so that the sum for each row is equal to *20* while the sum for each column would be *10*?

Let's try:

5	3	4	2	6	20
1	4	3	8	4	20
4	3				

10 10

Double Counting

Problem

- Is it possible to fill a *3 x 5* table with integers so that the sum for each row is equal to *20* while the sum for each column would be *10*?

Let's try:

5	3	4	2	6	20
1	4	3	8	4	20
4	3	3			

10 10 10

Double Counting

Problem

- Is it possible to fill a *3 x 5* table with integers so that the sum for each row is equal to *20* while the sum for each column would be *10*?

Let's try:

5	3	4	2	6	20
1	4	3	8	4	20
4	3	3	0		

10 10 10 10

Double Counting

Problem

- Is it possible to fill a *3 x 5* table with integers so that the sum for each row is equal to *20* while the sum for each column would be *10*?

Let's try:

5	3	4	2	6	20
1	4	3	8	4	20
4	3	3	0	0	10

10 10 10 10 10

Double Counting

Problem

- Is it possible to fill a *3 x 5* table with integers so that the sum for each row is equal to *20* while the sum for each column would be *10*?

Let's try:

5	3	4	2	6	20
1	4	3	8	4	20
4	3	3	0	0	10
10	10	10	10	10	

Double Counting

Let us try to prove it is impossible using **reductio ad absurdum.**

					20
					20
					20

10 10 10 10 10

Double Counting

Let us try to prove it is impossible using **reductio ad absurdum.**

<table>
<tr><td></td><td></td><td></td><td></td><td></td><td>20</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>20</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>20</td></tr>
</table>

10 10 10 10 10

- Assume the table is filled-up

Double Counting

Let us try to prove it is impossible using **reductio ad absurdum.**

					20
					20
					20

10 10 10 10 10

- Assume the table is filled-up
- Let's look at the sum of all numbers in the table

Double Counting

Let us try to prove it is impossible using **reductio ad absurdum.**

					20
					20
					20

10 10 10 10 10   60

- Assume the table is filled-up
- Let's look at the sum of all numbers in the table
    - Sum in each row is 20, hence total is 60

Double Counting

Let us try to prove it is impossible using **reductio ad absurdum.**

- Assume the table is filled-up
- Let's look at the sum of all numbers in the table
  - Sum in each row is 20, hence total is 60

Double Counting

Let us try to prove it is impossible using **reductio ad absurdum.**

					20
					20
					20
10	10	10	10	10	50

- Assume the table is filled-up
- Let's look at the sum of all numbers in the table
  - Sum in each row is 20, hence total is **60**
  - But the sum for each column is 10, for a total of 50
- This is a contradiction!

Double Counting

Let us try to prove it is impossible using **reductio ad absurdum.**

					20
					20
					20
10	10	10	10	10	50

- Assume the table is filled-up
- Let's look at the sum of all numbers in the table
  - Sum in each row is 20, hence total is **60**
  - But the sum for each column is 10, for a total of 50
- This is a **contradiction!**

Double Counting

Let us try to prove it is impossible using **reductio ad absurdum.**

					20
					20
					20

10 10 10 10 10 **50**

- Assume the table is filled-up
- Let's look at the sum of all numbers in the table
  - Sum in each row is 20, hence total is **60**
  - But the sum for each column is 10, for a total of 50

Double Counting

Problem

- Is it possible to fill a *3 x 5* table with integers so that the sum for each row is equal to *20* while the sum for each column would be *10*?

Let's summarize

Double Counting

Problem

- Is it possible to fill a *3 x 5* table with integers so that the sum for each row is equal to *20* while the sum for each column would be *10*?

Let's summarize

- We showed that it is impossible by computing some value ("the sum") in two ways

Double Counting

Problem

- Is it possible to fill a *3 x 5* table with integers so that the sum for each row is equal to *20* while the sum for each column would be *10*?

Let's summarize

- We showed that it is impossible by computing some value ("the sum") in two ways

- Key to success??

Double Counting

Problem

- Is it possible to fill a *3 x 5* table with integers so that the sum for each row is equal to *20* while the sum for each column would be *10*?

Let's summarize

- We showed that it is impossible by computing some value ("the sum") in two ways

- Key to success??

  – Find the right value to compute.

- Double counting

- Homework assignment problem

Homework Assignment Problem

Problem

- Each of the 20 students in the class solved three problems from the homework assignment, and each problem was solved by two students. How many problems were in the assignment?

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Let's try:

- We look at some value from two point of view

Homework Assignment Problem

Problem

- Each of the 20 students in the class solved three problems from the homework assignment, and each problem was solved by two students. How many problems were in the assignment?

Let's try:

- We look at some value from two point of view
  - The **total number** of **problems solved** by **all the students**

Homework Assignment Problem

Problem

- Each of the 20 students in the class solved three problems from the homework assignment, and each problem was solved by two students. How many problems were in the assignment?

Let's try:

- We look at some value from two point of view
  - The **total number** of **problems solved** by **all the students**
  - 20 students solved 3 problems each, hence a total of 60 solutions

Homework Assignment Problem

Problem

- Each of the 20 students in the class solved three problems from the homework assignment, and each problem was solved by two students. How many problems were in the assignment?

Let's try:

- We look at some value from two point of view
    - The **total number** of **problems solved** by **all the students**
    - 20 students solved 3 problems each, hence a total of 60 solutions
    - Each problem received 2 solutions

Homework Assignment Problem

Problem

- Each of the 20 students in the class solved three problems from the homework assignment, and each problem was solved by two students. How many problems were in the assignment?

Let's try:

- We look at some value from two point of view
  - The **total number** of **problems solved** by **all the students**
  - 20 students solved 3 problems each, hence a total of 60 solutions
  - Each problem received 2 solutions, hence 60/2 = 30 problems!

Homework Assignment Problem

This problem is similar to the previously discussed one

Problems

	1	2	3
1	1	1	1

Students

- Put "1" in the cell if student solved the problem, "0" if not
  – Recall that 20 students solved 3 problems each and each problem received 2 solutions

Homework Assignment Problem

This problem is similar to the previously discussed one

Problems

	1	2	3	4
1	1	1	1	
2		1	1	1

Students

- Put "1" in the cell if student solved the problem, "0" if not
  - Recall that 20 students solved 3 problems each and each problem received 2 solutions

Homework Assignment Problem

This problem is similar to the previously discussed one

Problems

	1	2	3	4	5	6
1	1	1	1			
2		1	1	1		
3				1	1	1

Students

- Put "1" in the cell if student solved the problem, "0" if not
  - Recall that 20 students solved 3 problems each and each problem received 2 solutions

Homework Assignment Problem

This problem is similar to the previously discussed one

Problems

	1	2	3	4	5	6	7
1	1	1	1				
2		1	1	1			
3				1	1	1	
4					1	1	1

Students

- Put "1" in the cell if student solved the problem, "0" if not
  - Recall that 20 students solved 3 problems each and each problem received 2 solutions

Homework Assignment Problem

This problem is similar to the previously discussed one

Problems

	1	2	3	4	5	6	7	8	9
1	1	1	1						
2		1	1	1					
3				1	1	1			
4					1	1	1		
5							1	1	1

Students

- Put "1" in the cell if student solved the problem, "0" if not
  – Recall that 20 students solved 3 problems each and each problem received 2 solutions

Homework Assignment Problem

This problem is similar to the previously discussed one

Problems

	1	2	3	4	5
1	1	1	1		
2		1	1	1	
3				1	1

	16	17	18	19	20
1					
2					
3					

…

Students

…

	1	2	3	4	5
18					
19					
20					

	16	17	18	19	20
18	1	1			
19		1	1	1	
20			1	1	1

…

- Put "1" in the cell if student solved the problem, "0" if not
  - Recall that 20 students solved 3 problems each and each problem received 2 solutions
  - We come up with 30 columns (problems)

Homework Assignment Problem

This problem is similar to the previously discussed one

Problems

	1	2	3	4	5		26	27	28	29	30
1	1	1	1								
2		1	1	1							
3				1	1						

Students

	1	2	3	4	5		26	27	28	29	30
18							1	1	1		
19								1	1	1	
20	1									1	1

- Put "1" in the cell if student solved the problem, "0" if not
- There are 60 "1's" in the table
- There are 30 columns

Thank you.