a) $\vec{BH} = (1-4,\ 2-5,\ 3-6) = (-3, -3, -3)$

b) $2\vec{HB} - 3\vec{BC} = 2(4-1, 5-2, 6-3) - 3(7-4, 1-5, 3-6)$

$= 2(3, 3, 3) - 3(3, -4, -3) = (6, 6, 6) + (-9, 12, 9) =$

$= (-3, 18, 15)$

c) $\|3\vec{BC} - 2\vec{CH}\| = \|3(7-4, 1-5, 3-6) - 2(1-7, 2-1, 3-3)\|$

$= \|3(3, -4, -3) - 2(-6, 1, 0)\| = \|(9, -12, -9) + (12, -1, 0)\|$

$= \|21, -13, -9\| = \sqrt{21^2 + (-13)^2 + (-9)^2} = 26.28$

e) $\vec{HB} = (3, 3, 3)$ $\quad \vec{BC} = (3, -4, -3)$

$\vec{a}$ $\quad\quad$ $\vec{B}$

$a \cdot b = 9 - 12 - 9 = -12$

$|\vec{a}| = \sqrt{9+9+9} = \sqrt{27}$

$|\vec{B}| = \sqrt{9+16+9} = \sqrt{36}$

$= \cos\theta = \frac{a \cdot b}{|\vec{a}||\vec{B}|} = \frac{-12}{\sqrt{27}\cdot\sqrt{36}} = \frac{-12}{6\cdot 3\sqrt{3}}$

$= -\frac{2\sqrt{3}}{9}$

$\cos\theta = -\frac{2\sqrt{3}}{9}$

c) $\|3\vec{BC}$

$=$

$= \|$

e) $a \cdot b =$

$|\vec{a}| = \sqrt{}$

$|\vec{B}| = \sqrt{}$

$\cos\theta = -\frac{1}{9}$

d) $\vec{AB} \cdot (\vec{CD} - 2\vec{BC})$:

$(3,3,3)\left((-5,2,2) - 2(3,-4,-3)\right) = (3,3,3)\left((-5,2,2) + (-6,8,6)\right)$

$(3,3,3)(-11,10,8) = (-8,30,24)$

(2)

a)
$$\left[\begin{array}{cccc|c} -3 & 2 & 6 & 1 & 5 \\ 0 & 3 & 3 & -5 & 2 \\ 2 & 4 & 4 & -6 & -8 \end{array}\right]$$

③

$$\begin{vmatrix} -26 & 17 & 1 \\ 24 & 21 & 9 \\ -6 & -12 & -4 \end{vmatrix} = (-26)\cdot 21\cdot(-4) + 17\cdot 9\cdot(-6) + 1\cdot 24(-12) - 1\cdot 21\cdot(-6) - (-26)\cdot 9\cdot(-12) - 17\cdot 24(-4) = -72$$

③

$$C = A\cdot B = \begin{pmatrix} -3 & 2 & 2 \\ 1 & 4 & -6 \\ 0 & -2 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{matrix} 1 \\ 9 \\ -4 \end{matrix}$$

$c_{11} = ~~a_{11} b_{11} + a_{12} b~~ = (-3)\cdot 1 + 2\cdot 2 + 2\cdot 0 = -3 + 4 + 0 = \underline{1}$

$c_{21} = 1\cdot 1 + 4\cdot 2 + (-6)\cdot 0 = 1 + 8 + 0 = 9$

$c_{31} = 0\cdot 1 + (-2)\cdot 2 + 2\cdot 0 = 0 - 4 + 0 = -4$

(2)

a) $\left(\begin{array}{cccc|c} -3 & 2 & 6 & 1 & 5 \\ 0 & 3 & 3 & -5 & 2 \\ 2 & 4 & 4 & -6 & -8 \end{array}\right)$

b) $\begin{array}{cccc} -3 & 2 & 6 & 1 \\ 0 & 3 & 3 & -5 \\ 0 & -10 & & \end{array}$

(2) $x_1 = -\frac{16}{3} = \frac{1}{3} \cdot x_4$

$x_2 = \frac{15}{4} + 3x_4$

$x_3 = \frac{-37}{12} + 3x_4$

$x_4 = x_4$

$$x = \begin{pmatrix} -\frac{16}{3} - \frac{1}{3} \cdot x_4 \\ \frac{15}{4} + 3x_4 \\ -\frac{37}{12} - \frac{4}{3} \cdot x_4 \\ x_4 \end{pmatrix}$$

$x_1 = -16/3 - \frac{1}{3} x_4$

$x_2 = 15/4 + 3x_4$

$x_3 = -37/12 - (4/3) x_4$

$x_4 = \text{frei}$