CHAPTER 3
EQUILIBRIUM


Bialobrzeski/laif/Redux Pictures

CHAPTER OUTLINE

3/1 Introduction

SECTION A Equilibrium in Two Dimensions

3/2 System Isolation and the Free-Body Diagram

3/3 Equilibrium Conditions

SECTION B Equilibrium in Three Dimensions

3/4 Equilibrium Conditions

Article 3/1 Introduction

- Equilibrium Conditions (Eq. 3/1)

  - Force Balance: $\Sigma\mathbf{F} = \mathbf{0}$

  - Moment Balance: $\Sigma\mathbf{M} = \mathbf{0}$

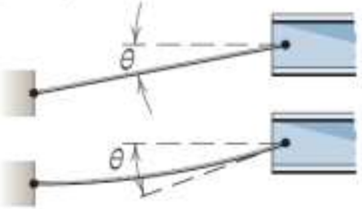
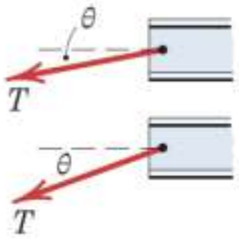
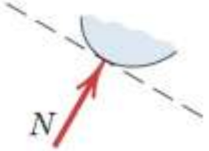
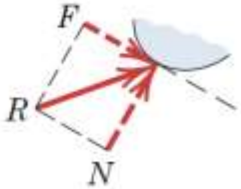
Article 3/2 System Isolation and the Free-Body Diagram

- System Isolation

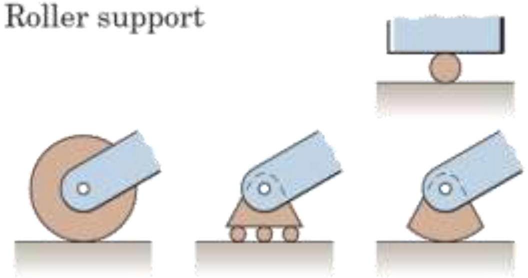
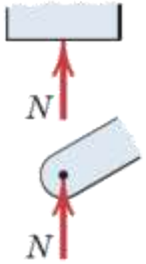
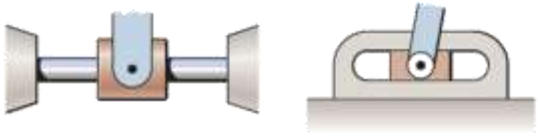
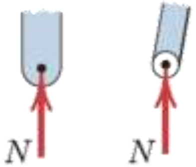
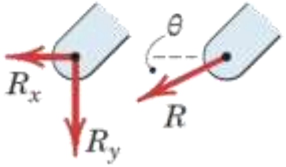
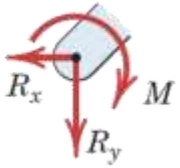
- Free-Body Diagram

  - The free-body diagram is the most important single step in the solution of problems in mechanics.

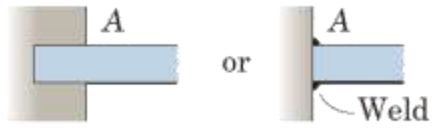
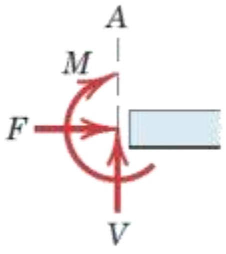
Article 3/2 – Modeling the Action of Forces (1 of 4)

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	**Action on Body to Be Isolated**
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.

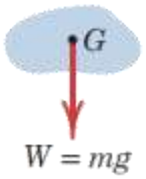
Article 3/2 – Modeling the Action of Forces (2 of 4)

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS		
Type of Contact and Force Origin	Action on Body to Be Isolated	
4. Roller support 		Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 		Collar or slider free to move along smooth guides; can support force normal to guide only.
6. Pin connection 	Pin free to turn  Pin not free to turn 	A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.

Article 3/2 – Modeling the Action of Forces (3 of 4)

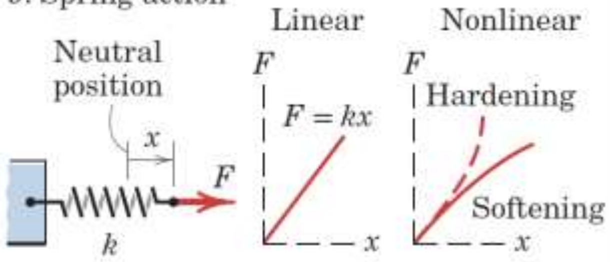
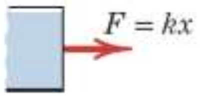
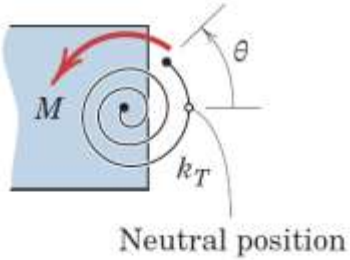
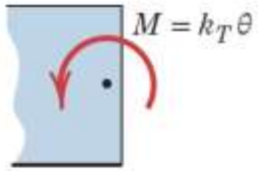
<table>
<tr><th colspan="2">MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS</th></tr>
<tr><th>Type of Contact and Force Origin</th><th>Action on Body to Be Isolated</th></tr>
<tr><td>7. Built-in or fixed support

</td><td>

A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.</td></tr>
<tr><td>8. Gravitational attraction

</td><td>

The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center of gravity G.</td></tr>
</table>

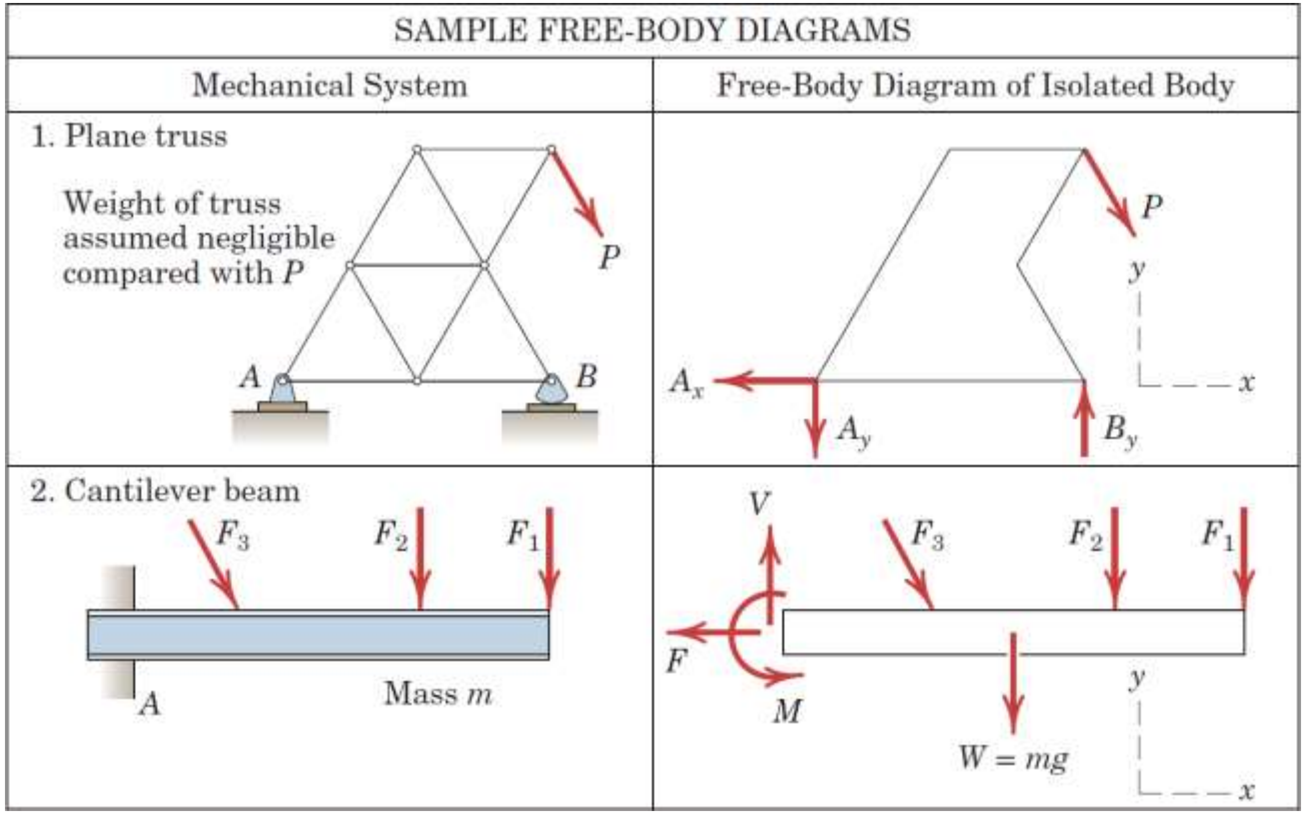
Article 3/2 – Modeling the Action of Forces (4 of 4)

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
9. Spring action 	 Spring force is tensile if the spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.
10. Torsional spring action 	 For a linear torsional spring, the applied moment M is proportional to the angular deflection θ from the neutral position. The stiffness k_T is the moment required to deform the spring one radian.

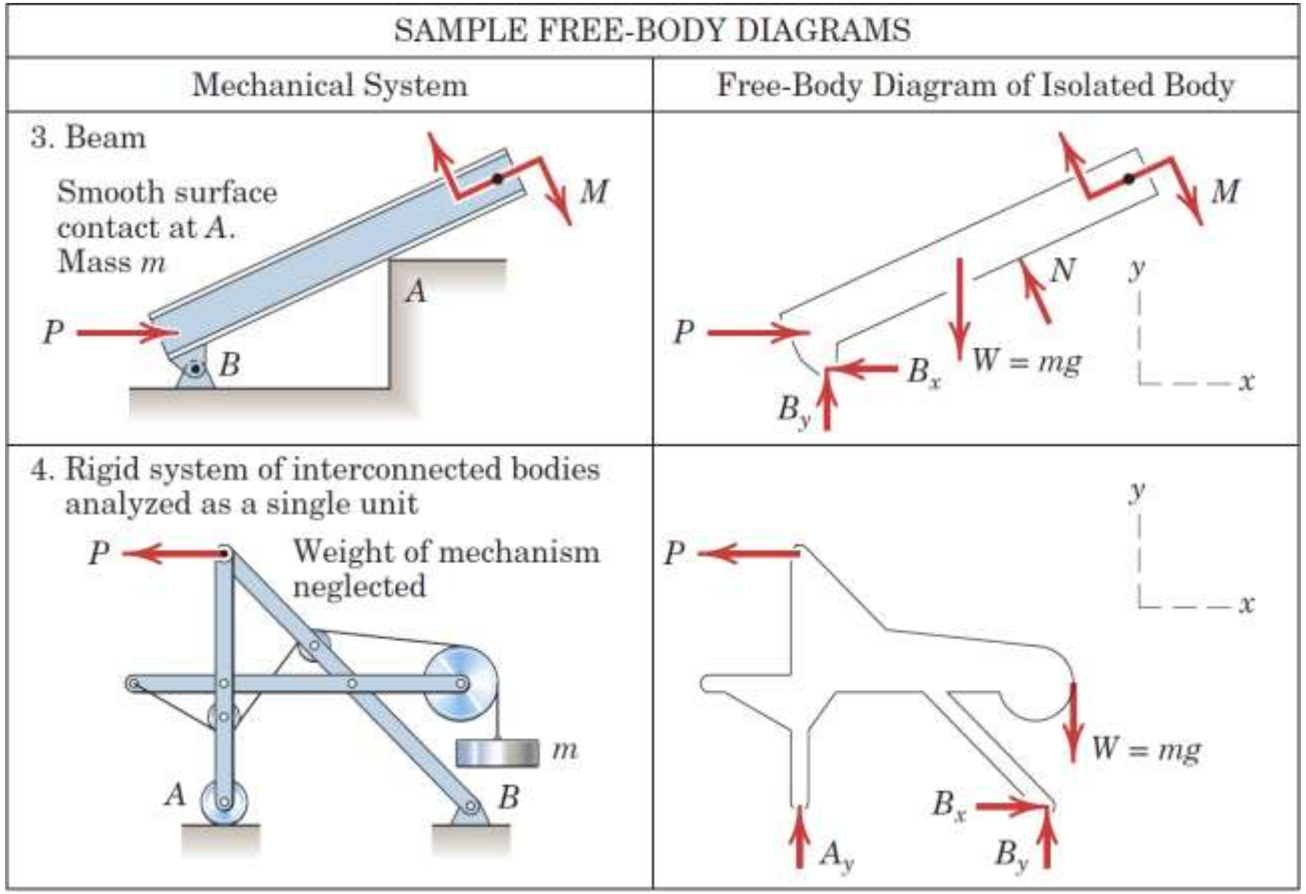
Article 3/2 – Construction of Free-Body Diagrams

- Step 1: Decide which system to isolate.
- Step 2: Isolate the system by drawing a diagram which represents its complete external boundary.
- Step 3: Identify all forces which act on the isolated system as applied by the removed contacting and attracting bodies, and represent them in their proper positions on the diagram of the isolated system.
- Step 4: Show the choice of coordinate axes directly on the diagram.

Article 3/2 – Examples of Free-Body Diagrams (1 of 2)



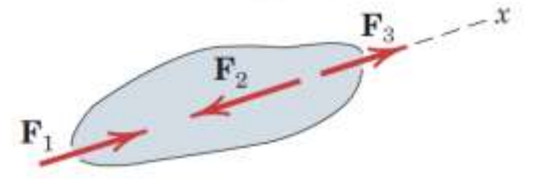
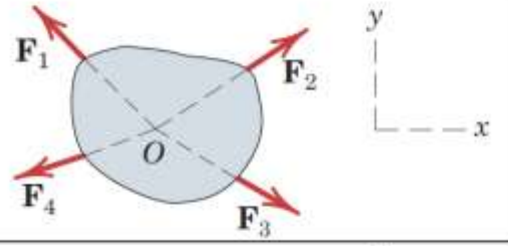
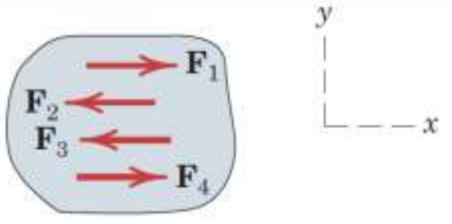
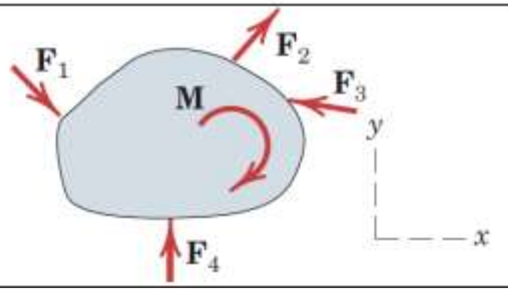
Article 3/2 – Examples of Free-Body Diagrams (2 of 2)



Article 3/3 Equilibrium Conditions

- Scalar Format (Eq. 3/2)
  - $\Sigma F_x = 0$
  - $\Sigma F_y = 0$
  - $\Sigma M_O = 0$

Article 3/3 – Categories of Equilibrium

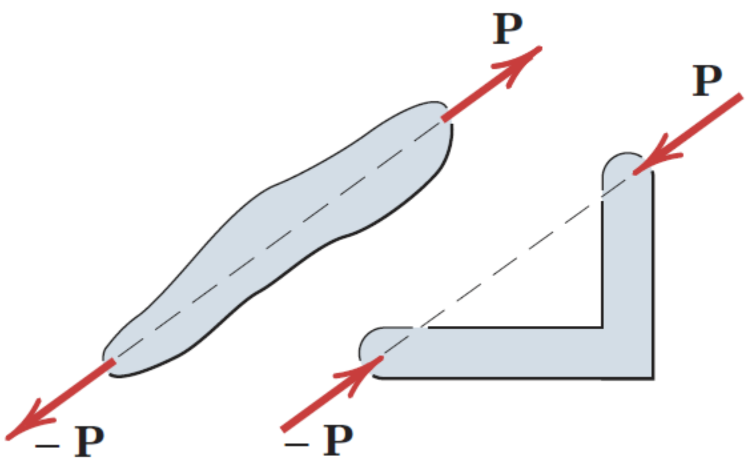
CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_z = 0$ $\Sigma F_y = 0$

Article 3/3 – Two-Force Members

- Definition

  Occurs when a body is in equilibrium under the action of two forces only.

- Forces are…
  - Equal
  - Opposite
  - Collinear
  - Independent of Object Shape



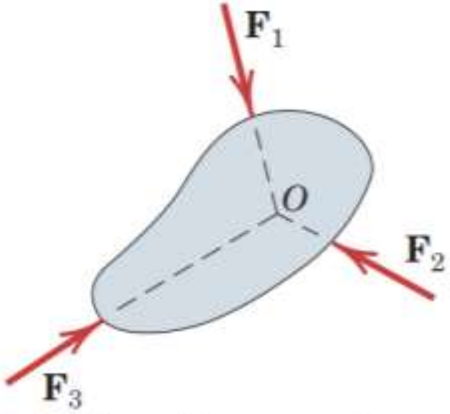
Article 3/3 – Three-Force Members

- Definition

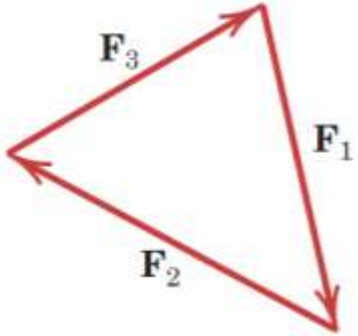
  Occurs when a body is in equilibrium under the action of three forces only.

- Forces are Concurrent

- Case of Parallel Forces



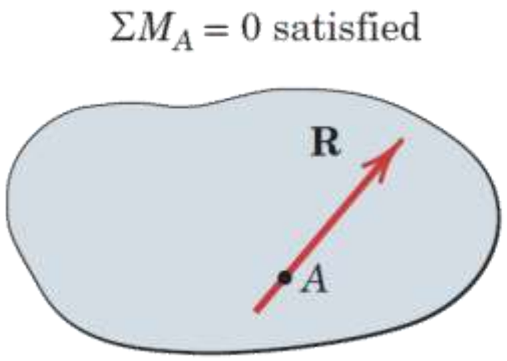
(a) Three-force member



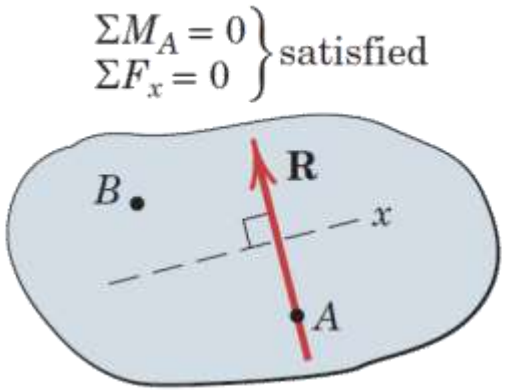
(b) Closed polygon satisfies $\Sigma\mathbf{F} = \mathbf{0}$

Article 3/3 – Alternative Equilibrium Equations (1 of 2)

- Case 1: Use of Two Moment Equations
  - Case (*a*)
    - Resultant Intersects Point *A*
  - Case (*b*)
    - Resultant Intersects Point *A*
    - Resultant is Perpendicular to *x*-axis
  - Equilibrium Conditions
    - $\Sigma F_x = 0$
    - $\Sigma M_A = 0$
    - $\Sigma M_B = 0$



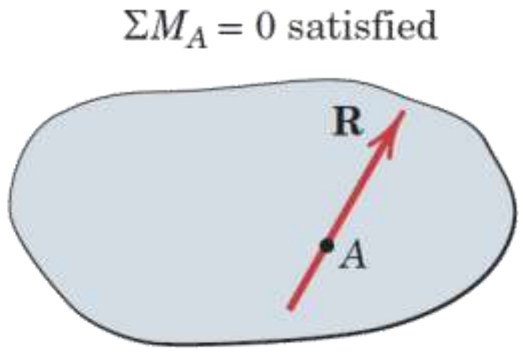
(*a*)



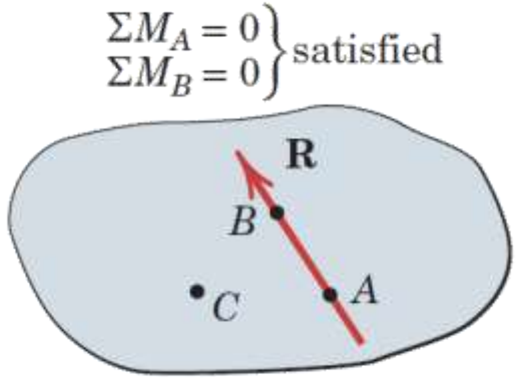
(*b*)

Article 3/3 – Alternative Equilibrium Equations (2 of 2)

- Case 2: Use of Three Moment Equations
  - Case (c)
    - Resultant Intersects Point A
  - Case (d)
    - Resultant Intersects Point A
    - Resultant Intersects Point B
  - Equilibrium Conditions
    - $\Sigma M_A = 0$
    - $\Sigma M_B = 0$
    - $\Sigma M_C = 0$



(c)



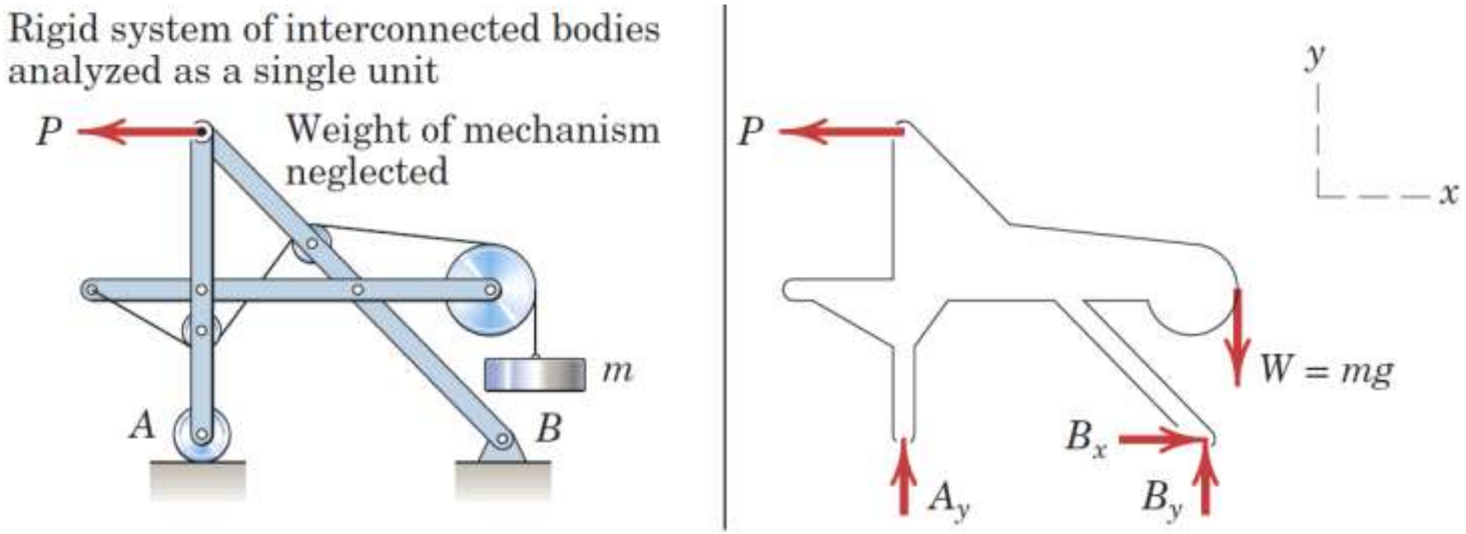
(d)

Article 3/3 – Constraints and Statical Determinacy

- Important Terms
  - Constraint
  - Statically Determinant
  - Statically Indeterminant
  - Redundancy
  - Degree of Statical Indeterminacy

Article 3/3 – Illustration of Determinacy

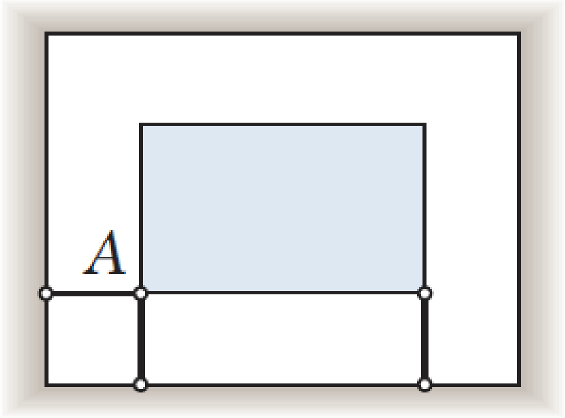
- Statically Determinant System



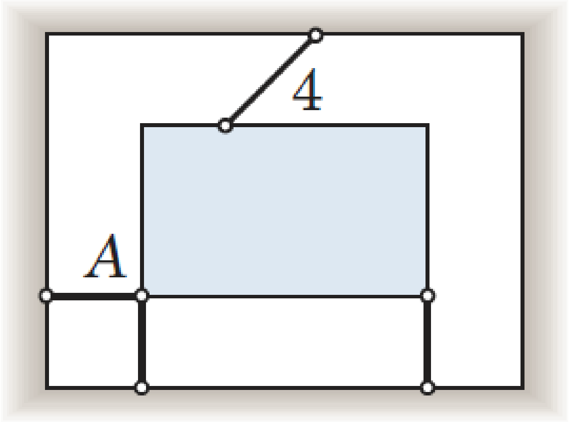
- Statically Indeterminant System – Replace Roller *A* with a Pin

Article 3/3 – Adequacy of Constraints (1 of 2)

- Complete Fixity (Adequate Constraints)

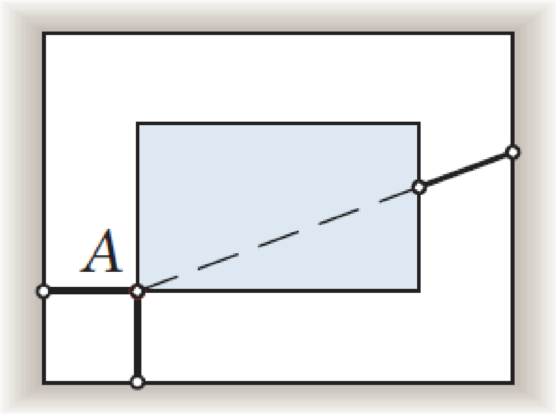


- Excessive Fixity (Redundant Constraints)

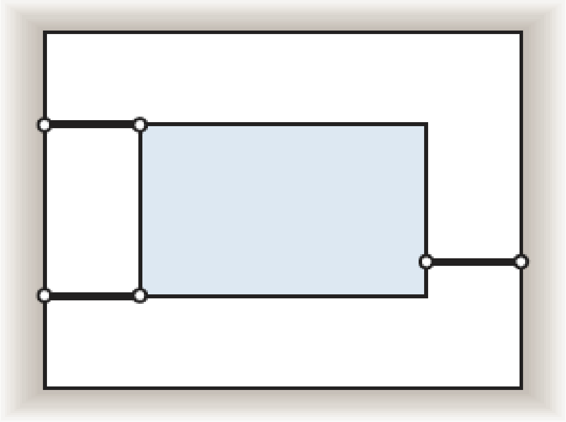


Article 3/3 – Adequacy of Constraints (2 of 2)

- Incomplete Fixity (Partial Constraints)
  - Concurrent Reactions



- Incomplete Fixity (Partial Constraints)
  - Parallel Reactions



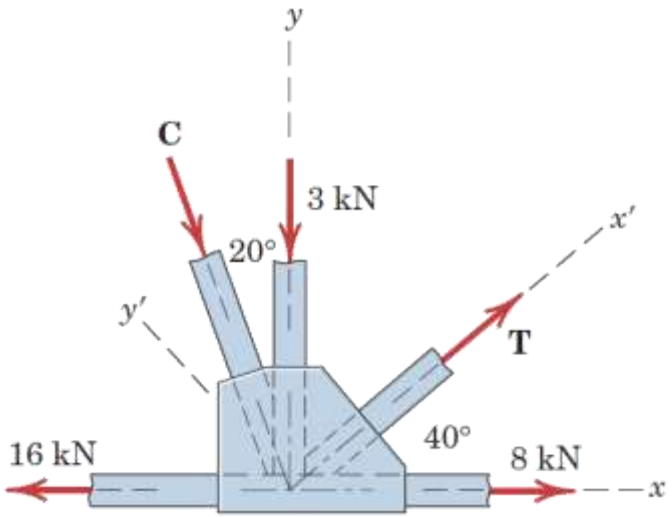
Article 3/3 – Approach to Solving Problems

- Identify clearly the quantities which are known and unknown.
- Make an unambiguous choice of the body (or system) to be isolated and draw its complete free-body diagram.
- Choose a convenient set of reference axes.
- Identify and state the applicable force and moment equations which govern the equilibrium conditions of the problem.
- Match the number of independent equations with the number of unknowns in the problem.
- Carry out the solution and check the results.

Article 3/3 – Sample Problem 3/1 (1 of 4)

- Problem Statement

  Determine the magnitudes of the forces **C** and **T**, which, along with the other three forces shown, act on the bridge-truss joint.



Article 3/3 – Sample Problem 3/1 (2 of 4)

- Solution I (Scalar Algebra) with *x*-*y* axes

$[\Sigma F_x = 0]$ $\qquad 8 + T\cos 40^\circ + C\sin 20^\circ - 16 = 0$

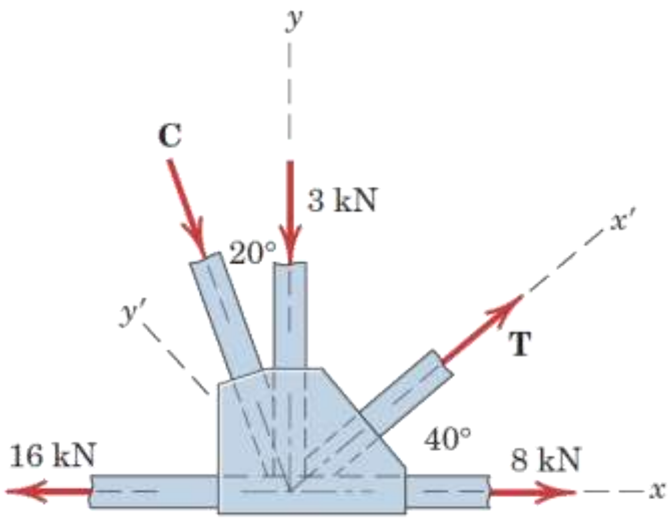
$$0.766T + 0.342C = 8 \qquad (a)$$

$[\Sigma F_y = 0]$ $\qquad T\sin 40^\circ - C\cos 20^\circ - 3 = 0$

$$0.643T - 0.940C = 3 \qquad (b)$$

Simultaneous solution of Eqs. (a) and (b) produces

$$T = 9.09 \text{ kN} \qquad C = 3.03 \text{ kN} \qquad \textit{Ans.}$$



Article 3/3 – Sample Problem 3/1 (3 of 4)

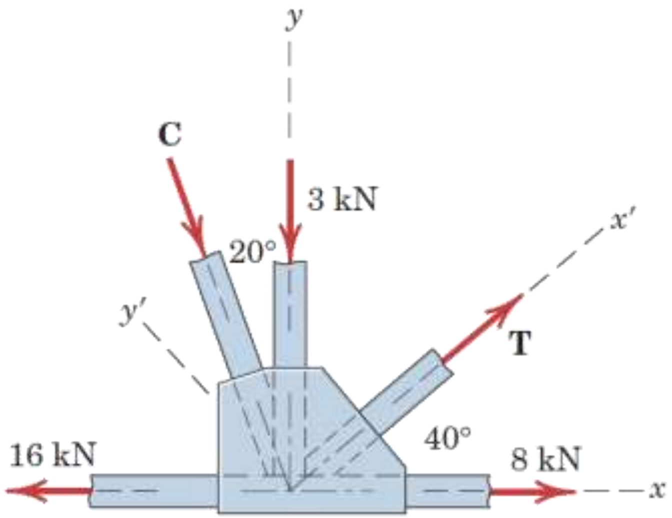
- Solution II (Scalar Algebra) with *x′*-*y′* axes

$[\Sigma F_{y'} = 0]$ $\quad -C\cos 20° - 3\cos 40° - 8\sin 40° + 16\sin 40° = 0$

$C = 3.03\text{ kN}$ *Ans.*

$[\Sigma F_{x'} = 0]$ $\quad T + 8\cos 40° - 16\cos 40° - 3\sin 40° - 3.03\sin 20° = 0$

$T = 9.09\text{ kN}$ *Ans.*



Article 3/3 – Sample Problem 3/1 (4 of 4)

- Solution III (Vector Algebra) with *x*-*y* axes

$$[\Sigma \mathbf{F} = \mathbf{0}] \qquad 8\mathbf{i} + (T\cos 40°)\mathbf{i} + (T\sin 40°)\mathbf{j} - 3\mathbf{j} + (C\sin 20°)\mathbf{i}$$

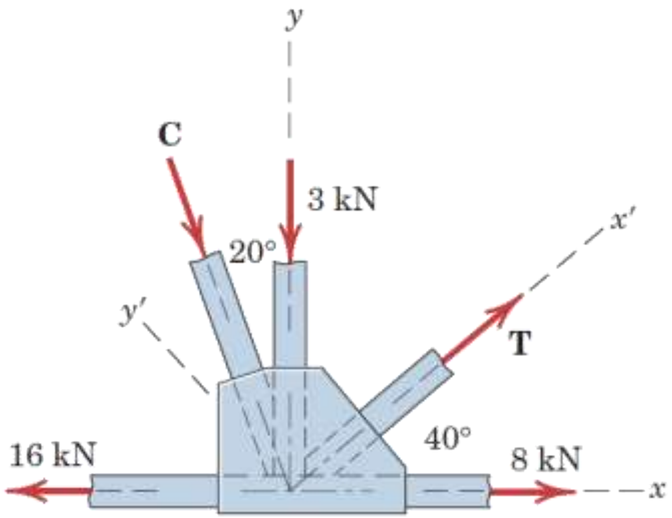
$$- (C\cos 20°)\mathbf{j} - 16\mathbf{i} = \mathbf{0}$$

Equating the coefficients of the **i**- and **j**-terms to zero gives

$$8 + T\cos 40° + C\sin 20° - 16 = 0$$

$$T\sin 40° - 3 - C\cos 20° = 0$$

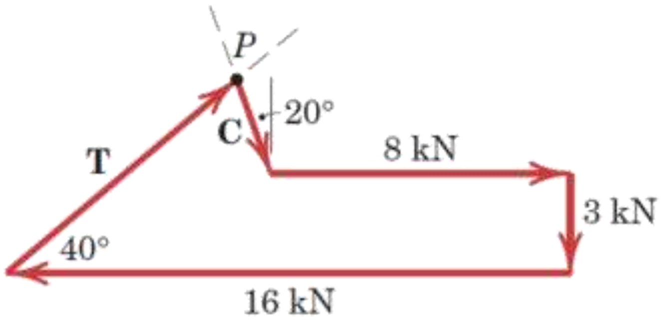
which are the same, of course, as Eqs. (*a*) and (*b*), which we solved above.

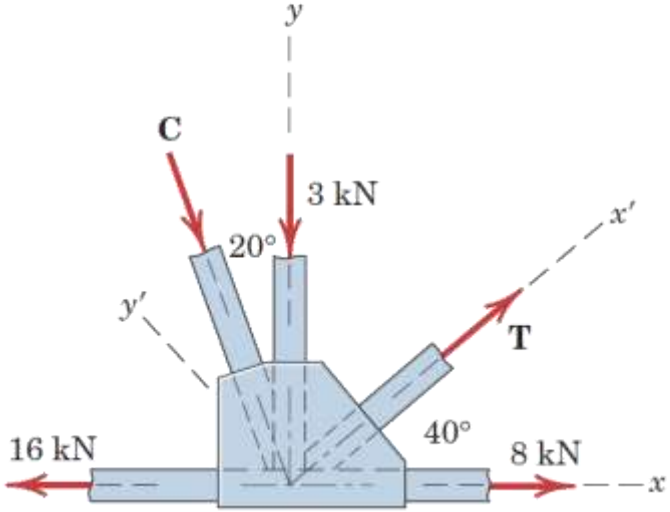


Article 3/3 – Sample Problem 3/1 (4 of 4)

- Solution IV (Geometric)

A graphical solution is easily obtained. The known vectors are laid off head-to-tail to some convenient scale, and the directions of **T** and **C** are then drawn to close the polygon. ③ The resulting intersection at point P completes the solution, thus enabling us to measure the magnitudes of **T** and **C** directly from the drawing to whatever degree of accuracy we incorporate in the construction.



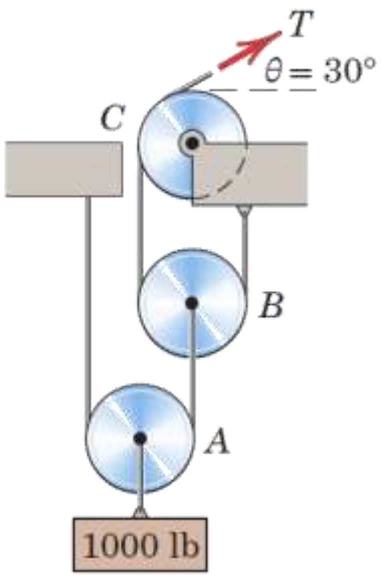


③ The known vectors may be added in any order desired, but they must be added before the unknown vectors.

Article 3/3 – Sample Problem 3/2 (1 of 3)

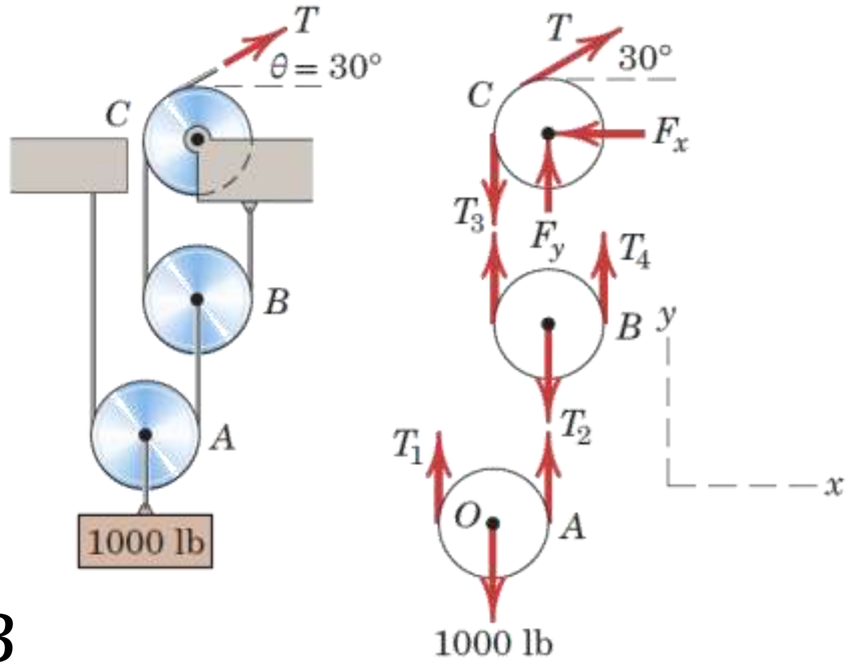
- Problem Statement

  Calculate the tension T in the cable which supports the 1000-lb load with the pulley arrangement shown. Each pulley is free to rotate about its b earing, and the weights of all parts are small compared with the load. Find the magnitude of the total force on the bearing of pulley C.



Article 3/3 – Sample Problem 3/2 (2 of 3)

- Free-Body Diagrams



- Equilibrium Conditions for Pulleys *A* and *B*

$[\Sigma M_O = 0] \qquad T_1 r - T_2 r = 0 \qquad T_1 = T_2$ ①

$[\Sigma F_y = 0] \qquad T_1 + T_2 - 1000 = 0 \qquad 2T_1 = 1000 \qquad T_1 = T_2 = 500 \text{ lb}$

From the example of pulley *A* we may write the equilibrium of forces on pulley *B* by inspection as

$$T_3 = T_4 = T_2/2 = 250 \text{ lb}$$

① Clearly the radius r does not influence the results. Once we have analyzed a simple pulley, the results should be perfectly clear by inspection.

Article 3/3 – Sample Problem 3/2 (3 of 3)

- Equilibrium Conditions for Pulley *C*

For pulley C the angle $\theta = 30°$ in no way affects the moment of T about the center of the pulley, so that moment equilibrium requires

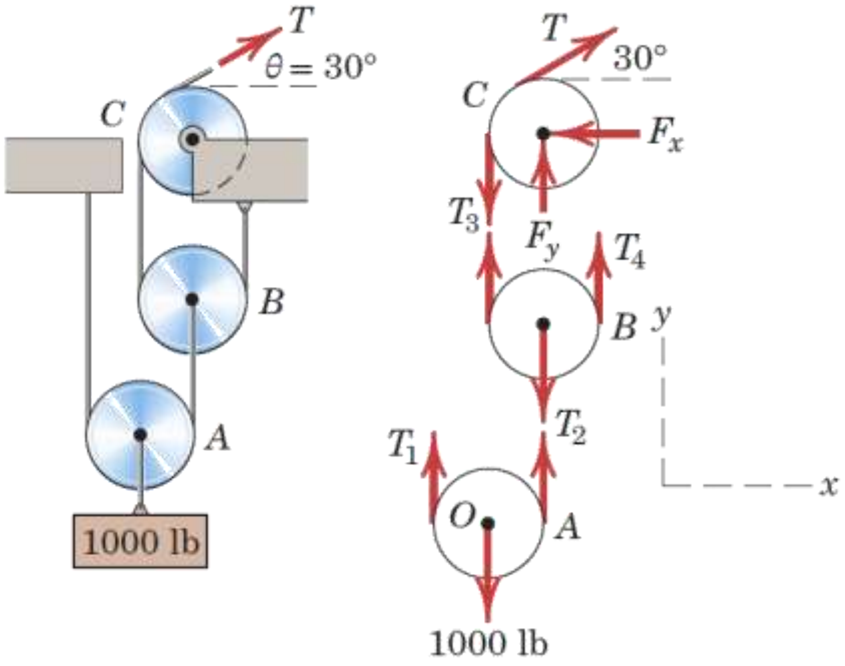
$$T = T_3 \qquad \text{or} \qquad T = 250 \text{ lb} \qquad \textit{Ans.}$$

Equilibrium of the pulley in the x- and y-directions requires

$$[\Sigma F_x = 0] \qquad 250 \cos 30° - F_x = 0 \qquad F_x = 217 \text{ lb}$$

$$[\Sigma F_y = 0] \qquad F_y + 250 \sin 30° - 250 = 0 \qquad F_y = 125 \text{ lb}$$

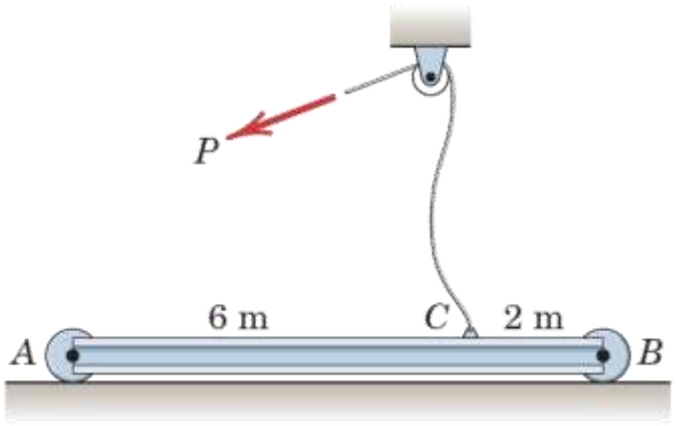
$$\left[F = \sqrt{F_x^2 + F_y^2}\right] \qquad F = \sqrt{(217)^2 + (125)^2} = 250 \text{ lb} \qquad \textit{Ans.}$$



Article 3/3 – Sample Problem 3/3 (1 of 2)

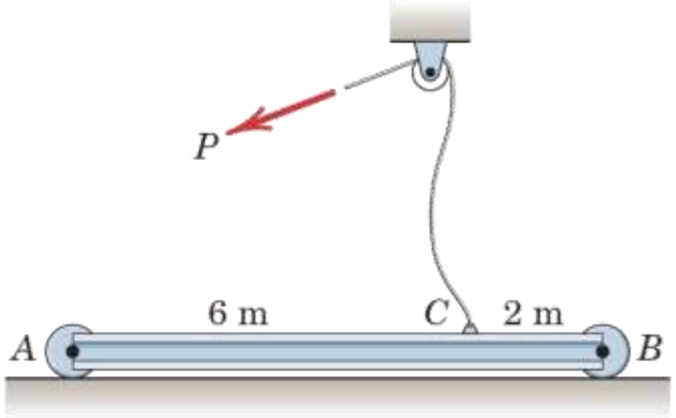
- Problem Statement

  The uniform 100-kg I-beam is supported initially by its end rollers on the horizontal surface at *A* and *B*. By means of the cable at *C*, it is desired to elevate end *B* to a position 3 m above end *A*. Determine the required tension *P*, the reaction at *A*, and the angle θ made by the beam with the horizontal in the elevated position.



Article 3/3 – Sample Problem 3/3 (2 of 2)

- Free-Body Diagram



- Equilibrium Conditions

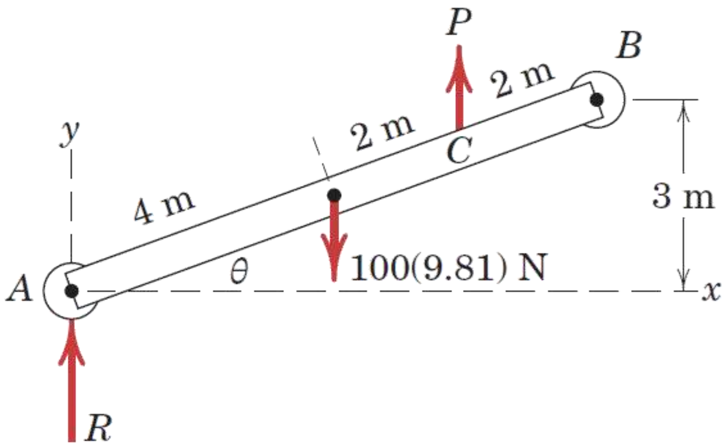
$[\Sigma M_A = 0]$ $P(6 \cos \theta) - 981(4 \cos \theta) = 0$ $P = 654$ N ① *Ans.*

Equilibrium of vertical forces requires

$[\Sigma F_y = 0]$ $654 + R - 981 = 0$ $R = 327$ N *Ans.*

The angle θ depends only on the specified geometry and is

$\sin \theta = 3/8$ $\theta = 22.0°$ *Ans.*

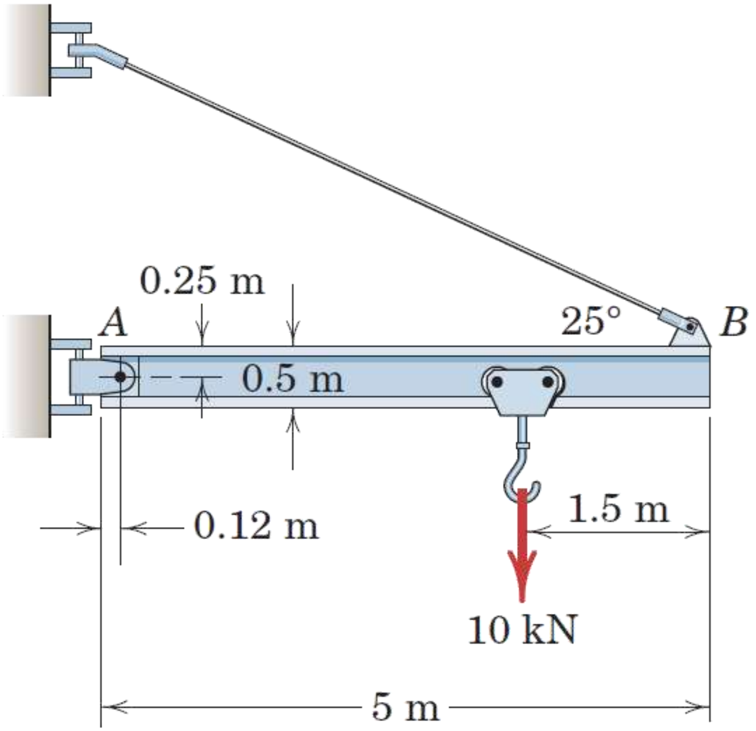


① Clearly the equilibrium of this parallel force system is independent of θ.

Article 3/3 – Sample Problem 3/4 (1 of 3)

- Problem Statement

  Determine the magnitude *T* of the tension in the supporting cable and the magnitude of the force on the pin at *A* for the jib crane shown. The beam *AB* is a standard 0.5-m I-beam with a mass of 95 kg per meter of length.



Article 3/3 – Sample Problem 3/4 (2 of 3)

- Free-Body Diagram

- Equilibrium Conditions

$[\Sigma M_A = 0] \qquad (T\cos 25°)0.25 + (T\sin 25°)(5 - 0.12)$

$\qquad - 10(5 - 1.5 - 0.12) - 4.66(2.5 - 0.12) = 0$ ②

from which $\qquad T = 19.61\text{ kN} \qquad$ *Ans.*

Equating the sums of forces in the x- and y-directions to zero gives

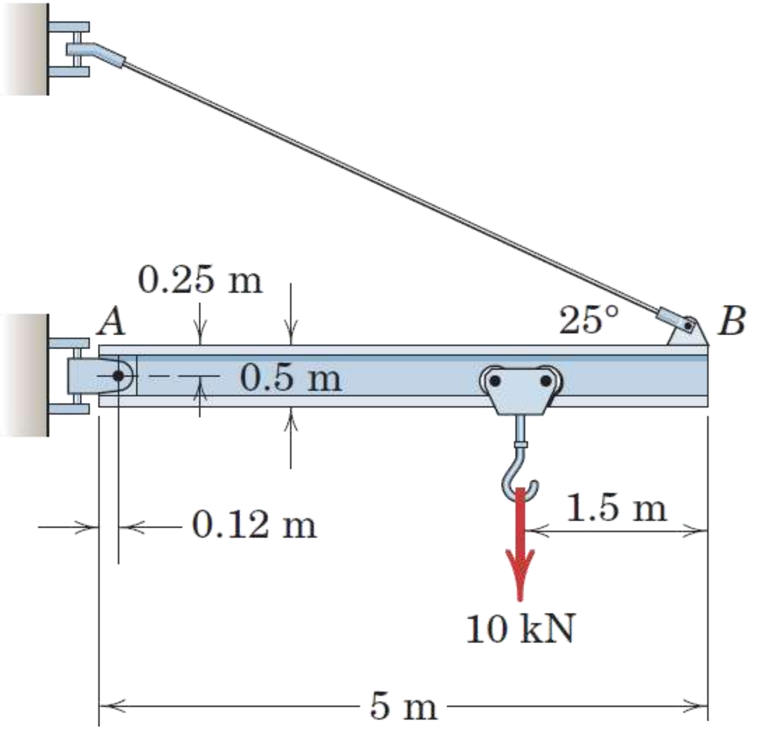
$[\Sigma F_x = 0] \qquad A_x - 19.61\cos 25° = 0 \qquad A_x = 17.77\text{ kN}$

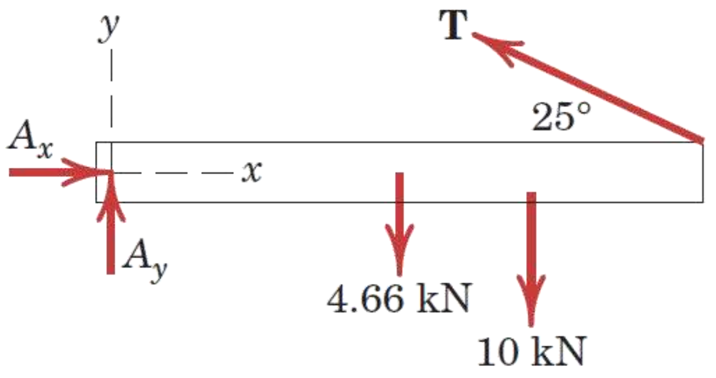
$[\Sigma F_y = 0] \qquad A_y + 19.61\sin 25° - 4.66 - 10 = 0 \qquad A_y = 6.37\text{ kN}$

$[A = \sqrt{A_x^2 + A_y^2}] \qquad A = \sqrt{(17.77)^2 + (6.37)^2} = 18.88\text{ kN}$ ③ $\qquad$ *Ans.*

② The calculation of moments in two-dimensional problems is generally handled more simply by scalar algebra than by the vector cross product $\mathbf{r} \times \mathbf{F}$. In three dimensions, as we will see later, the reverse is often the case.

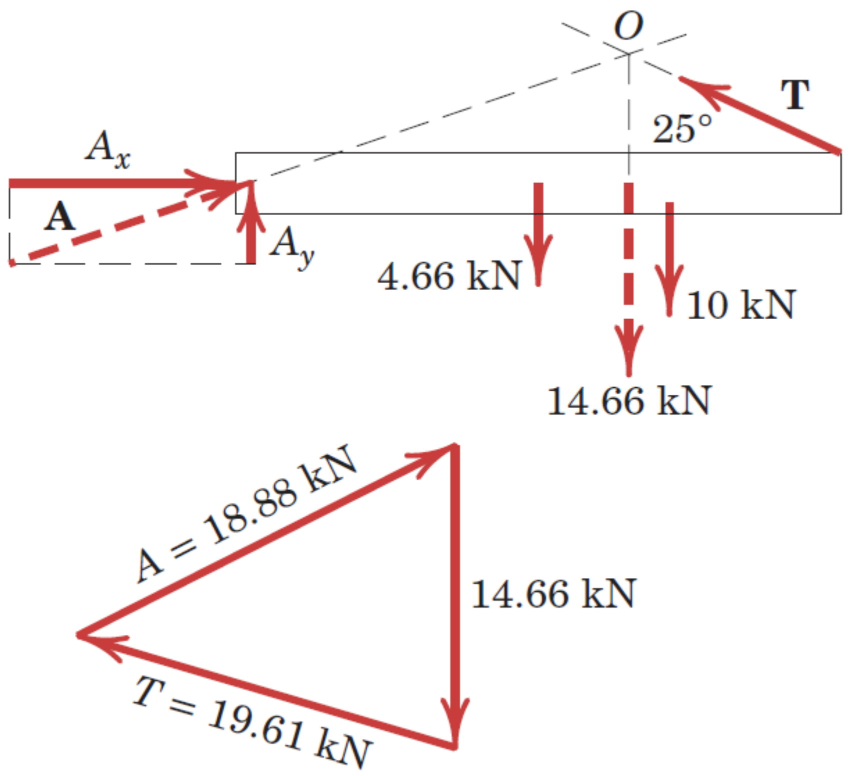
③ The direction of the force at A could be easily calculated if desired. However, in designing the pin A or in checking its strength, it is only the magnitude of the force that matters.





Article 3/3 – Sample Problem 3/4 (3 of 3)

- Graphical Solution



Article 3/4 Equilibrium Conditions (3D)

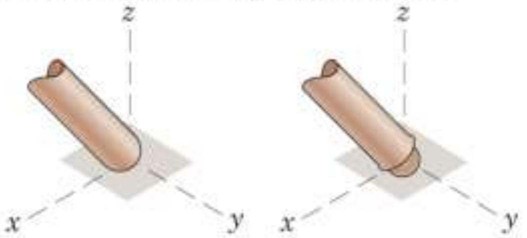
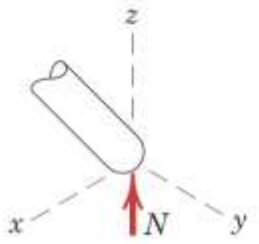
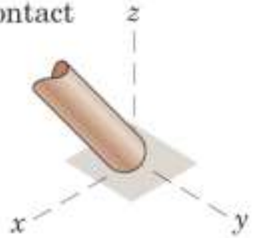
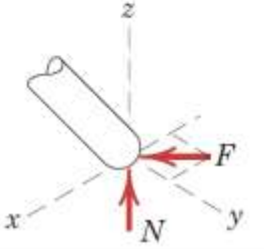
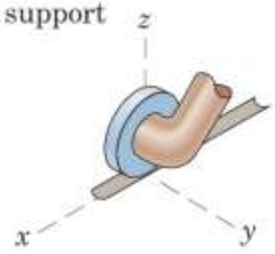
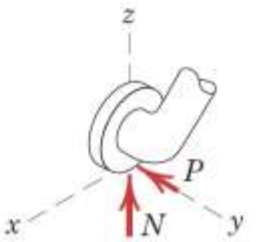
- Equilibrium Conditions (Eq. 3/1 extended)

  - Force Balance: $\Sigma\mathbf{F} = \mathbf{0}$
    - $\Sigma F_x = 0$
    - $\Sigma F_y = 0$
    - $\Sigma F_z = 0$

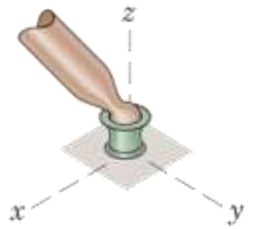
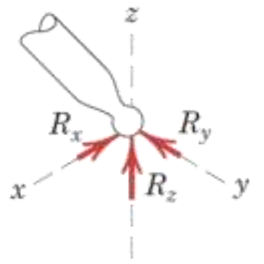
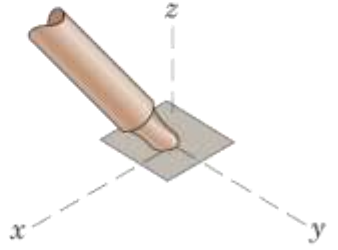
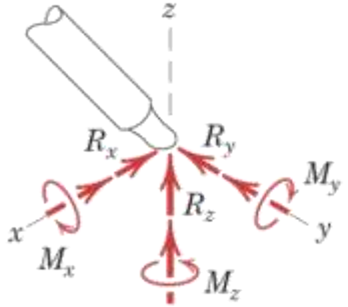
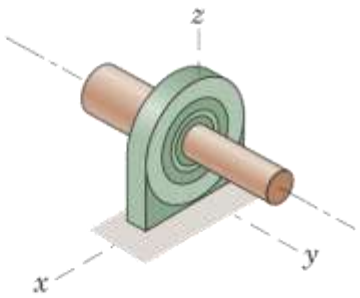
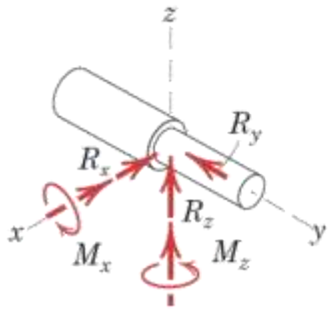
  - Moment Balance: $\Sigma\mathbf{M} = \mathbf{0}$
    - $\Sigma M_x = 0$
    - $\Sigma M_y = 0$
    - $\Sigma M_z = 0$

  - The moment sum is taken about any convenient reference point.

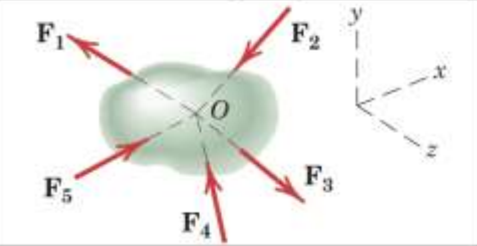
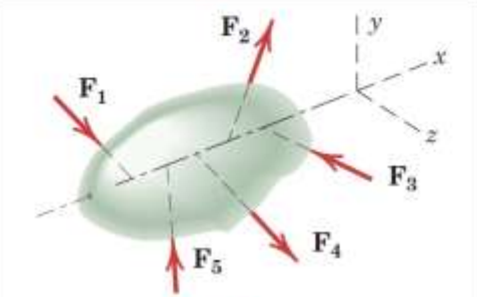
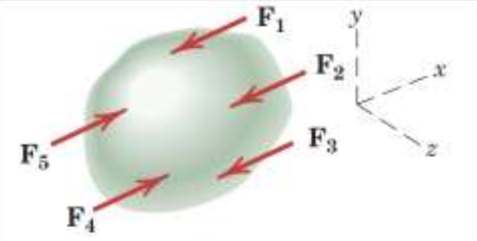
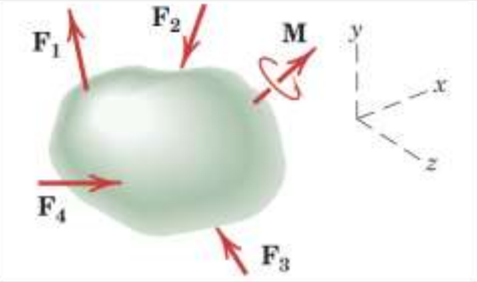
Article 3/4 – Modeling the Action of Forces (1 of 2)

MODELING THE ACTION OF FORCES IN THREE-DIMENSIONAL ANALYSIS		
Type of Contact and Force Origin	**Action on Body to Be Isolated**	
1. Member in contact with smooth surface, or ball-supported member 		Force must be normal to the surface and directed toward the member.
2. Member in contact with rough surface 		The possibility exists for a force F tangent to the surface (friction force) to act on the member, as well as a normal force N.
3. Roller or wheel support with lateral constraint 		A lateral force P exerted by the guide on the wheel can exist, in addition to the normal force N.

Article 3/4 – Modeling the Action of Forces (1 of 2)

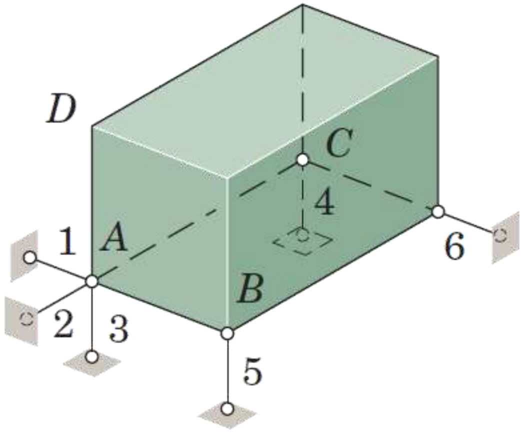
MODELING THE ACTION OF FORCES IN THREE-DIMENSIONAL ANALYSIS		
Type of Contact and Force Origin	**Action on Body to Be Isolated**	
4. Ball-and-socket joint 		A ball-and-socket joint free to pivot about the center of the ball can support a force **R** with all three components.
5. Fixed connection (embedded or welded) 		In addition to three components of force, a fixed connection can support a couple **M** represented by its three components.
6. Thrust-bearing support 		Thrust bearing is capable of supporting axial force R_y as well as radial forces R_x and R_z. Couples M_x and M_z must, in some cases, be assumed zero in order to provide statical determinacy.

Article 3/4 – Categories of Equilibrium

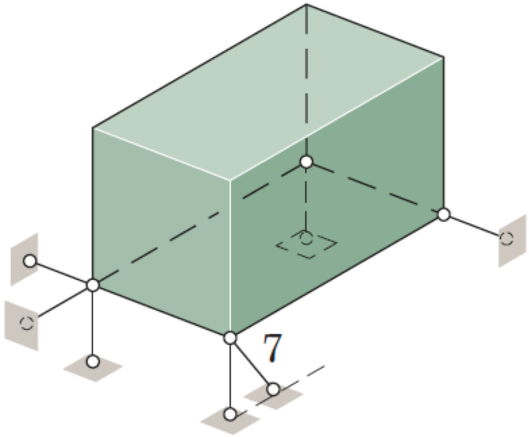
CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$
2. Concurrent with a line		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma F_y = 0$ $\Sigma M_z = 0$ $\Sigma F_z = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_x = 0$ $\Sigma F_y = 0$ $\Sigma M_y = 0$ $\Sigma F_z = 0$ $\Sigma M_z = 0$

Article 3/4 – Constraints and Statical Determinacy

- Complete Fixity (Adequate Constraints)

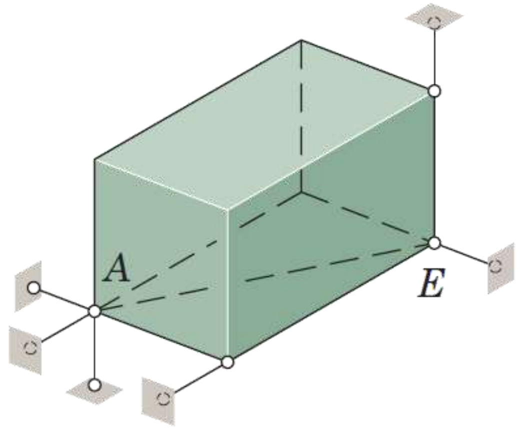


- Excessive Fixity (Redundant Constraints)

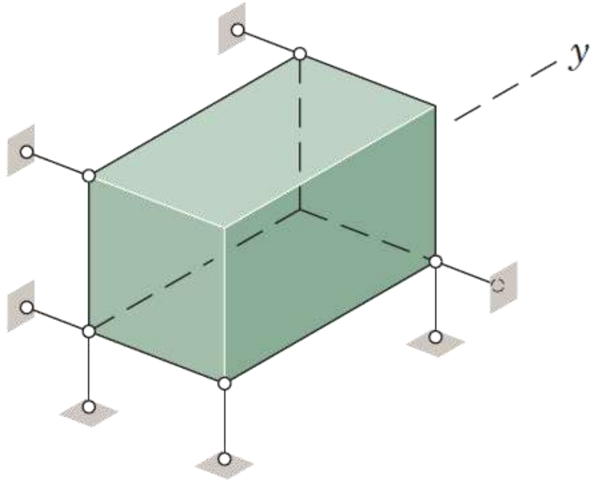


Article 3/4 – Constraints and Statical Determinacy

- Incomplete Fixity (Partial Constraints)
  - No Moment Resistance about Line *AE*



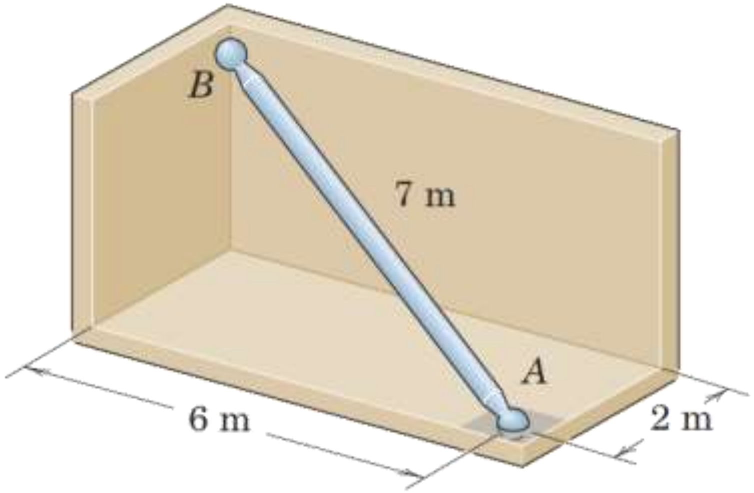
- Incomplete Fixity (Partial Constraints)
  - No Force Resistance along *y*-Axis



Article 3/4 – Sample Problem 3/5 (1 of 4)

- Problem Statement

  The uniform 7-m steel shaft has a mass of 200 kg and is supported by a ball-and-socket joint at *A* in the horizontal floor. The ball end *B* rests against the smooth vertical walls as shown. Compute the forces exerted by the walls and the floor on the ends of the shaft.



Article 3/4 – Sample Problem 3/5 (2 of 4)

- Free-Body Diagram
- Equilibrium Conditions – Moment Balance

Vector Solution We will use A as a moment center to eliminate reference to the forces at A. The position vectors needed to compute the moments about A are

$$\mathbf{r}_{AG} = -1\mathbf{i} - 3\mathbf{j} + 1.5\mathbf{k} \text{ m} \qquad \text{and} \qquad \mathbf{r}_{AB} = -2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k} \text{ m}$$

where the mass center G is located halfway between A and B.

The vector moment equation gives

$$[\Sigma \mathbf{M}_A = \mathbf{0}] \qquad \mathbf{r}_{AB} \times (\mathbf{B}_x + \mathbf{B}_y) + \mathbf{r}_{AG} \times \mathbf{W} = \mathbf{0}$$

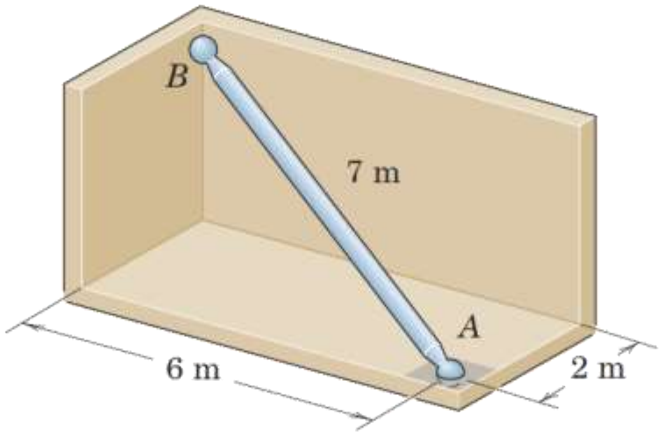
$$(-2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}) \times (B_x\mathbf{i} + B_y\mathbf{j}) + (-\mathbf{i} - 3\mathbf{j} + 1.5\mathbf{k}) \times (-1962\mathbf{k}) = \mathbf{0}$$

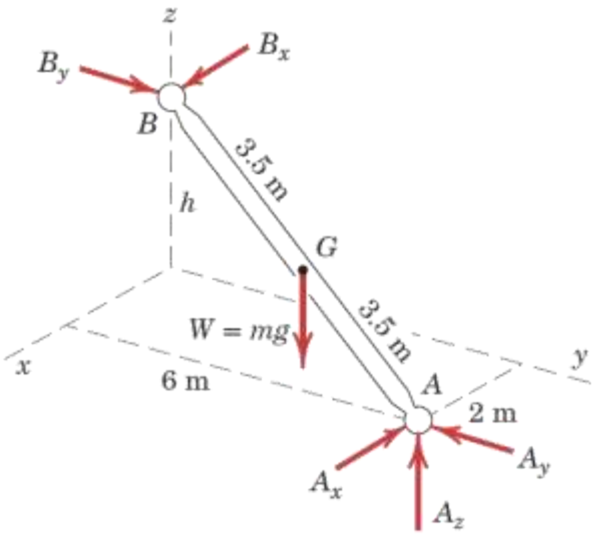
$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & -6 & 3 \\ B_x & B_y & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -3 & 1.5 \\ 0 & 0 & -1962 \end{vmatrix} = \mathbf{0}$$

$$(-3B_y + 5890)\mathbf{i} + (3B_x - 1962)\mathbf{j} + (-2B_y + 6B_x)\mathbf{k} = \mathbf{0}$$

Equating the coefficients of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ to zero and solving give

$$B_x = 654 \text{ N} \qquad \text{and} \qquad B_y = 1962 \text{ N} \quad ② \qquad \textit{Ans.}$$





Article 3/4 – Sample Problem 3/5 (3 of 4)

- Equilibrium Conditions – Force Balance

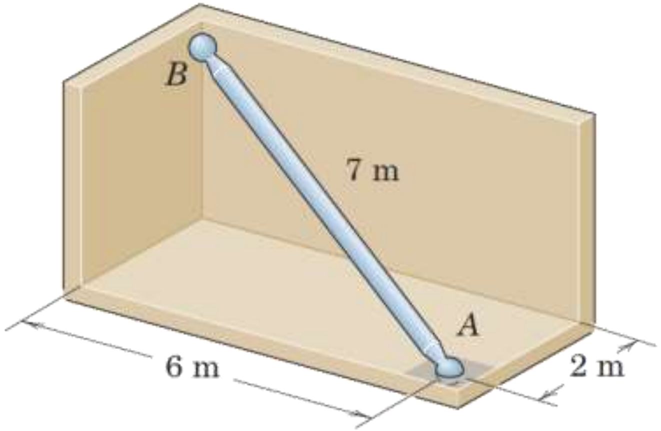
The forces at A are easily determined by

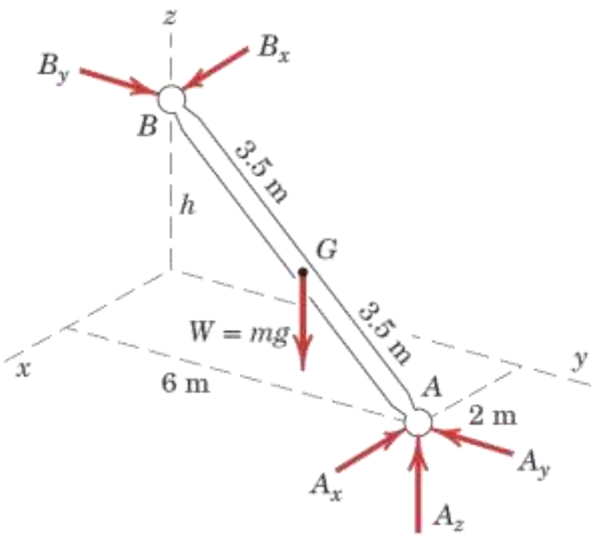
$[\Sigma \mathbf{F} = \mathbf{0}]$ $\qquad (654 - A_x)\mathbf{i} + (1962 - A_y)\mathbf{j} + (-1962 + A_z)\mathbf{k} = \mathbf{0}$

and $\qquad A_x = 654\text{ N} \qquad A_y = 1962\text{ N} \qquad A_z = 1962\text{ N}$

Finally, $\qquad A = \sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2}$

$$= \sqrt{(654)^2 + (1962)^2 + (1962)^2} = 2850\text{ N} \qquad \textit{Ans.}$$





Article 3/4 – Sample Problem 3/5 (4 of 4)

- Equilibrium Conditions

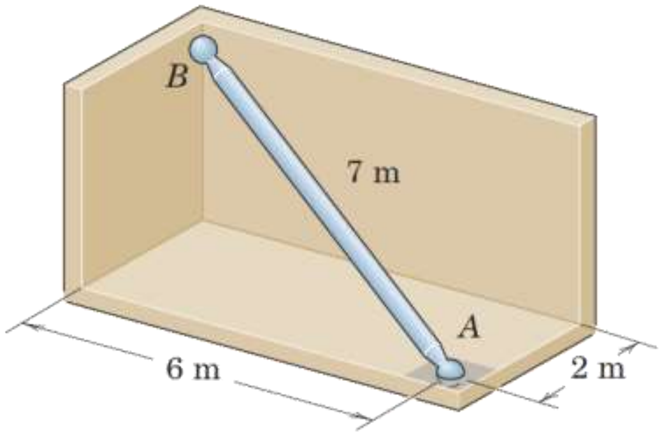
Scalar Solution Evaluating the scalar moment equations about axes through A parallel, respectively, to the x- and y-axes, gives

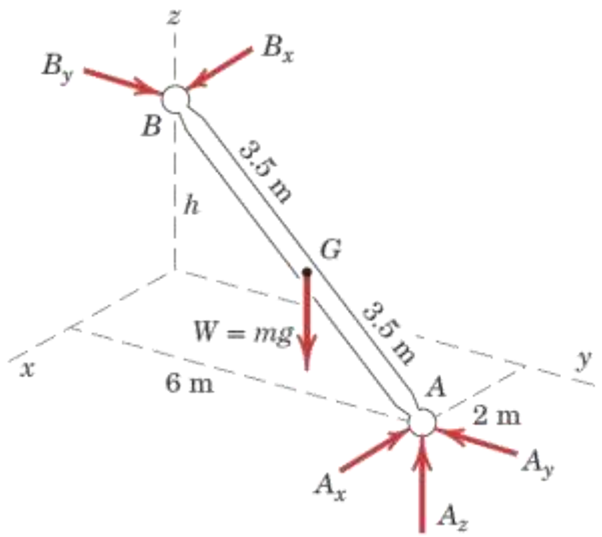
$[\Sigma M_{A_x} = 0]$	$1962(3) - 3B_y = 0$	$B_y = 1962$ N
$[\Sigma M_{A_y} = 0]$	$-1962(1) + 3B_x = 0$	$B_x = 654$ N ③

The force equations give, simply,

$[\Sigma F_x = 0]$	$-A_x + 654 = 0$	$A_x = 654$ N
$[\Sigma F_y = 0]$	$-A_y + 1962 = 0$	$A_y = 1962$ N
$[\Sigma F_z = 0]$	$A_z - 1962 = 0$	$A_z = 1962$ N

③ We observe that a moment sum about an axis through A parallel to the z-axis merely gives us $6B_x - 2B_y = 0$, which serves only as a check as noted previously. Alternatively we could have first obtained A_z from $\Sigma F_z = 0$ and then taken our moment equations about axes through B to obtain A_x and A_y.

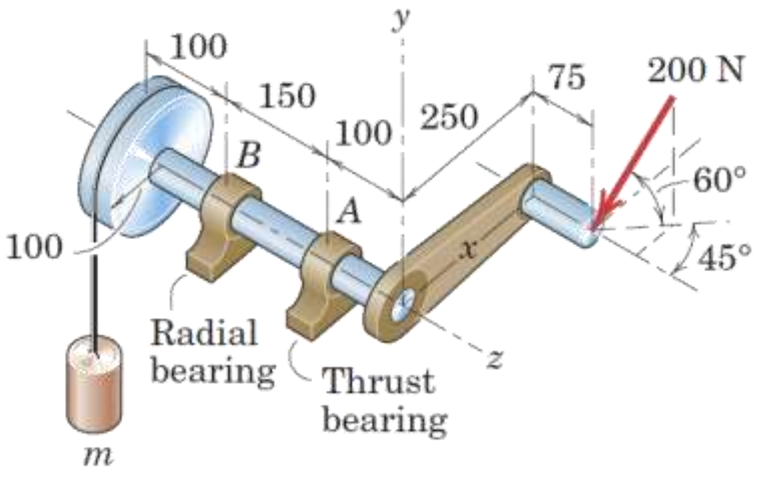




Article 3/4 – Sample Problem 3/6 (1 of 2)

- Problem Statement

A 200-N force is applied to the handle of the hoist in the direction shown. The bearing *A* supports the thrust (force in the direction of the shaft axis), while bearing *B* supports only radial load (load normal to the shaft axis). Determine the mass *m* which can be supported and the total radial force exerted on the shaft by each bearing. Assume neither bearing to be capable of supporting a moment about a line normal to the shaft axis.



Dimensions in millimeters

Article 3/4 – Sample Problem 3/6 (2 of 2)

- Free-Body Diagram (Orthographic Projections)
- Equilibrium Conditions

From the x-y projection: ②

$[\Sigma M_O = 0] \qquad 100(9.81m) - 250(173.2) = 0 \qquad m = 44.1 \text{ kg} \qquad Ans.$

From the x-z projection:

$[\Sigma M_A = 0] \qquad 150B_x + 175(70.7) - 250(70.7) = 0 \qquad B_x = 35.4 \text{ N}$

$[\Sigma F_x = 0] \qquad A_x + 35.4 - 70.7 = 0 \qquad A_x = 35.4 \text{ N}$

The y-z view gives ③

$[\Sigma M_A = 0] \qquad 150B_y + 175(173.2) - 250(44.1)(9.81) = 0 \qquad B_y = 520 \text{ N}$

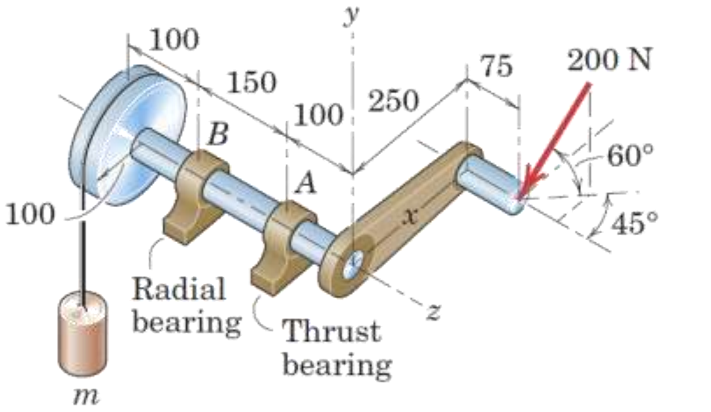
$[\Sigma F_y = 0] \qquad A_y + 520 - 173.2 - (44.1)(9.81) = 0 \qquad A_y = 86.8 \text{ N}$

$[\Sigma F_z = 0] \qquad A_z = 70.7 \text{ N}$

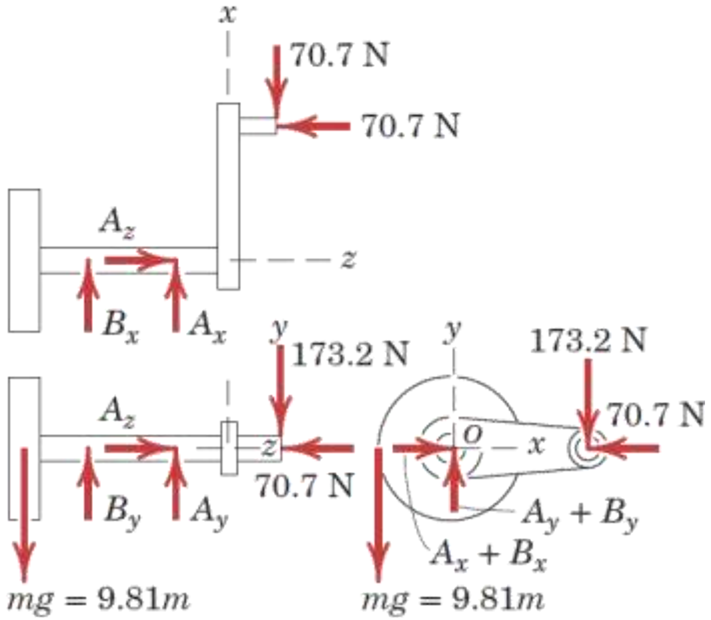
The total radial forces on the bearings become

$[A_r = \sqrt{A_x{}^2 + A_y{}^2}] \qquad A_r = \sqrt{(35.4)^2 + (86.8)^2} = 93.5 \text{ N} \qquad Ans.$

$[B = \sqrt{B_x{}^2 + B_y{}^2}] \qquad B = \sqrt{(35.4)^2 + (520)^2} = 521 \text{ N} \quad ④ \qquad Ans.$



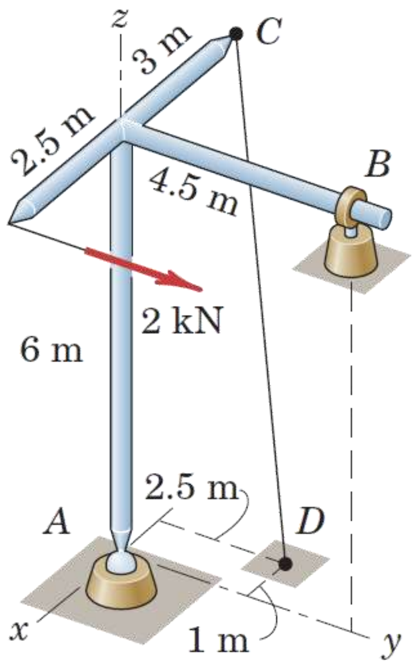
Dimensions in millimeters



Article 3/4 – Sample Problem 3/7 (1 of 3)

- Problem Statement

  The welded tubular frame is secured to the horizontal *x*-*y* plane by a ball-and-socket joint at *A* and receives support from the loose-fitting ring at *B*. Under the action of the 2-kN load, rotation about a line from *A* to *B* is prevented by the cable *CD*, and the frame is stable in the position shown. Neglect the weight of the frame compared with the applied load and determine the tension *T* in the cable, the reaction at the ring, and the reaction components at *A*.



Article 3/4 – Sample Problem 3/7 (2 of 3)

- Free-Body Diagram
- Moment Sum about Line *AB*

$\mathbf{n} = \frac{1}{\sqrt{6^2 + 4.5^2}}(4.5\mathbf{j} + 6\mathbf{k}) = \frac{1}{5}(3\mathbf{j} + 4\mathbf{k})$. The moment of $\mathbf{T}$ about AB is the component in the direction of AB of the vector moment about the point A and equals $\mathbf{r}_1 \times \mathbf{T} \cdot \mathbf{n}$. Similarly the moment of the applied load F about AB is $\mathbf{r}_2 \times \mathbf{F} \cdot \mathbf{n}$. With $\overline{CD} = \sqrt{46.2}$ m, the vector expressions for $\mathbf{T}$, $\mathbf{F}$, $\mathbf{r}_1$, and $\mathbf{r}_2$ are

$$\mathbf{T} = \frac{T}{\sqrt{46.2}}(2\mathbf{i} + 2.5\mathbf{j} - 6\mathbf{k}) \qquad \mathbf{F} = 2\mathbf{j} \text{ kN}$$

$$\mathbf{r}_1 = -\mathbf{i} + 2.5\mathbf{j} \text{ m} \qquad \mathbf{r}_2 = 2.5\mathbf{i} + 6\mathbf{k} \text{ m} \quad ②$$

The moment equation now becomes

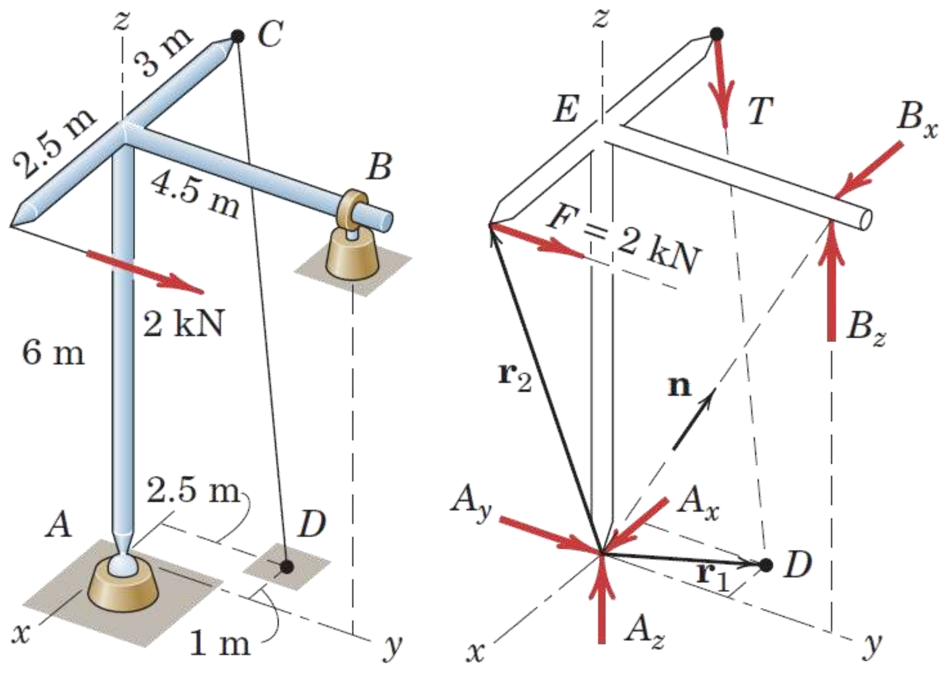
$$[\Sigma M_{AB} = 0] \quad (-\mathbf{i} + 2.5\mathbf{j}) \times \frac{T}{\sqrt{46.2}}(2\mathbf{i} + 2.5\mathbf{j} - 6\mathbf{k}) \cdot \frac{1}{5}(3\mathbf{j} + 4\mathbf{k}) + (2.5\mathbf{i} + 6\mathbf{k}) \times (2\mathbf{j}) \cdot \frac{1}{5}(3\mathbf{j} + 4\mathbf{k}) = 0$$

Completion of the vector operations gives

$$-\frac{48T}{\sqrt{46.2}} + 20 = 0 \qquad T = 2.83 \text{ kN} \qquad \textit{Ans.}$$

and the components of T become

$$T_x = 0.833 \text{ kN} \qquad T_y = 1.042 \text{ kN} \qquad T_z = -2.50 \text{ kN}$$



Article 3/4 – Sample Problem 3/7 (3 of 3)

- Remaining Equilibrium Conditions

$[\Sigma M_z = 0]$	$2(2.5) - 4.5B_x - 1.042(3) = 0$	$B_x = 0.417\text{ kN}$	*Ans.*
$[\Sigma M_x = 0]$	$4.5B_z - 2(6) - 1.042(6) = 0$	$B_z = 4.06\text{ kN}$	*Ans.*
$[\Sigma F_x = 0]$	$A_x + 0.417 + 0.833 = 0$	$A_x = -1.250\text{ kN}$	*Ans.*
$[\Sigma F_y = 0]$	$A_y + 2 + 1.042 = 0$	$A_y = -3.04\text{ kN}$ ③	*Ans.*
$[\Sigma F_z = 0]$	$A_z + 4.06 - 2.50 = 0$	$A_z = -1.556\text{ kN}$	*Ans.*

③ The negative signs associated with the *A*-components indicate that they are in the opposite direction to those shown on the free-body diagram.