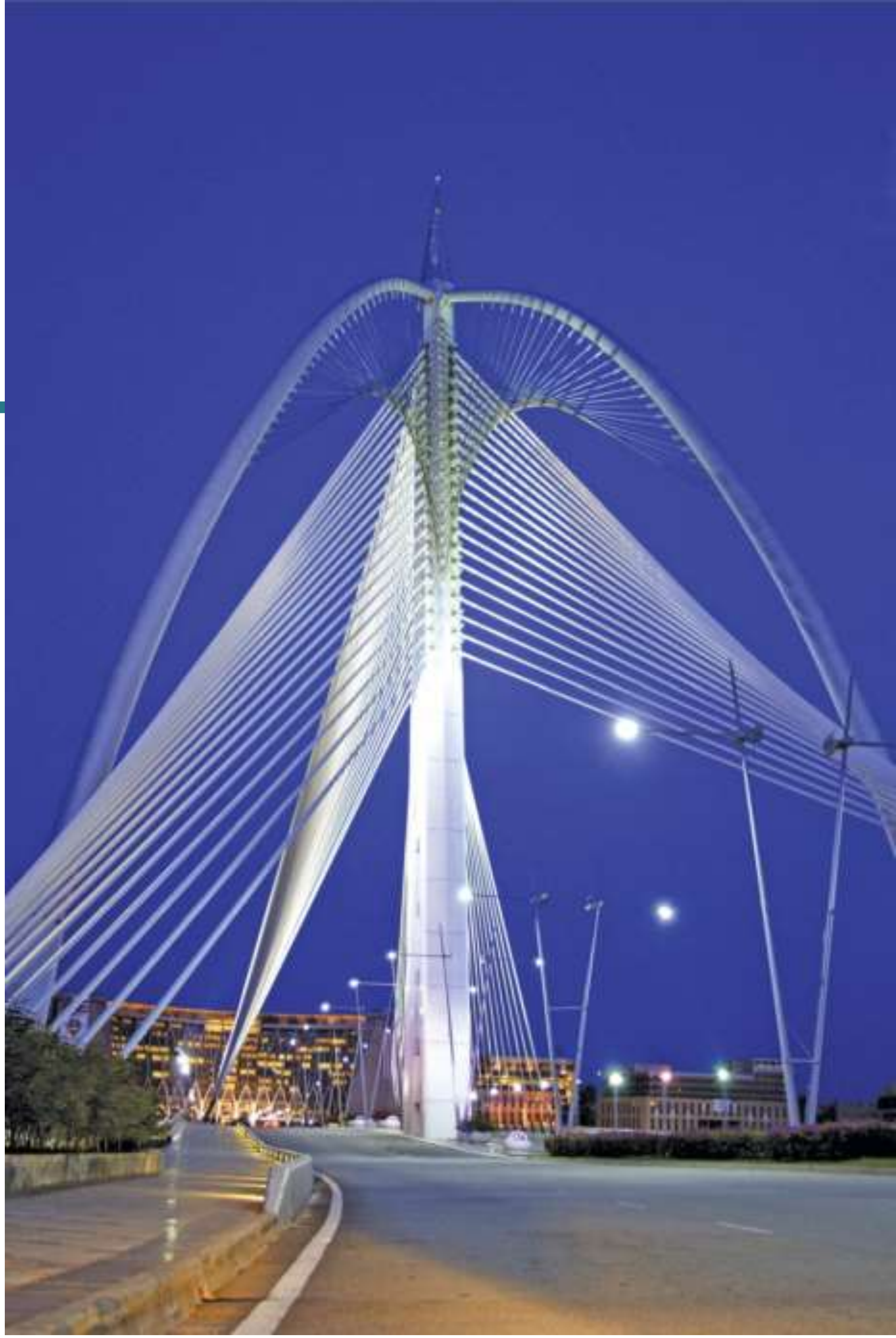
CHAPTER 4
STRUCTURES



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CHAPTER OUTLINE

Article 4/1 Introduction

- Reminders from Chapter 3
  - Equilibrium of a Single Rigid Body
  - Free-Body Diagrams of an Entire Structure
  - Application of Force and Moment Equations of Equilibrium

- Purpose of Chapter 4
  - Equilibrium of Several Rigid Bodies
  - Free-Body Diagrams of Portions of a Structure
  - Application of Force and Moment Equations of Equilibrium
  - Application of Newton’s 3rd Law

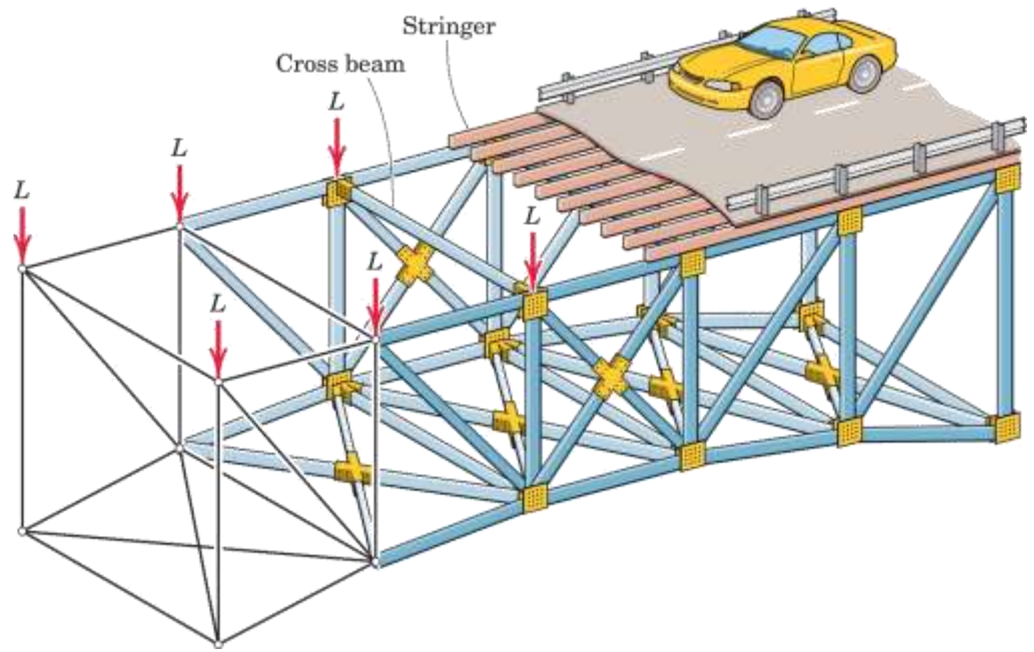
Article 4/2 Plane Trusses

- Introduction

Previously, we have been been concerned with the **external** forces acting on a body. In this section, however, we study the forces **internal** to a structure. Determining the load on each internal member of a truss or frame allows one to then **design** that particular member.

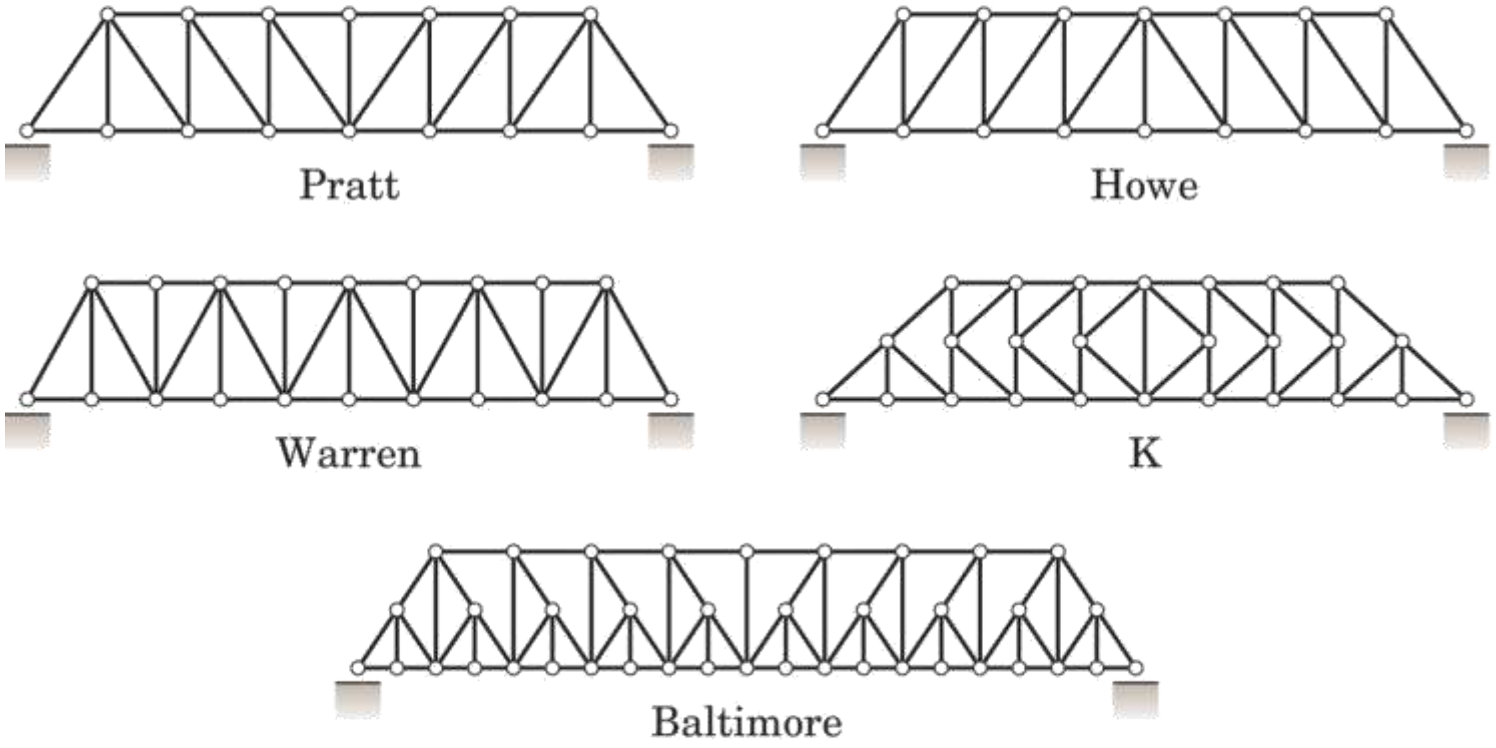
- Example Bridge Truss

Each vertical side of the structure is a plane truss, which is a framework composed of members joined at their ends. A simplified **model** is indicated at the left end of the illustration. The forces *L* represent the portions of the weights of the roadway, vehicles, stringers, and cross beams which are transferred to the truss joints. Note that joints are modeled as simple pins, even though they might actually be gusset plates or welded connections.



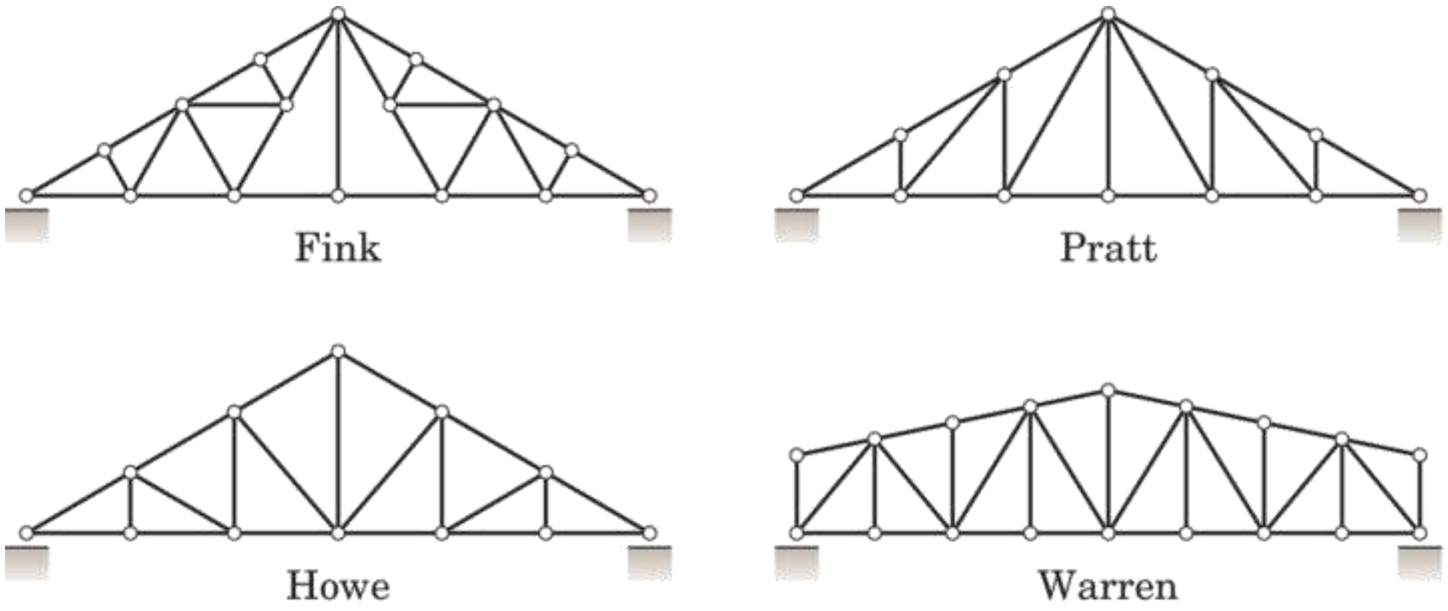
Article 4/2 – Common Truss Shapes (1 of 2)

- Commonly Used Bridge Trusses



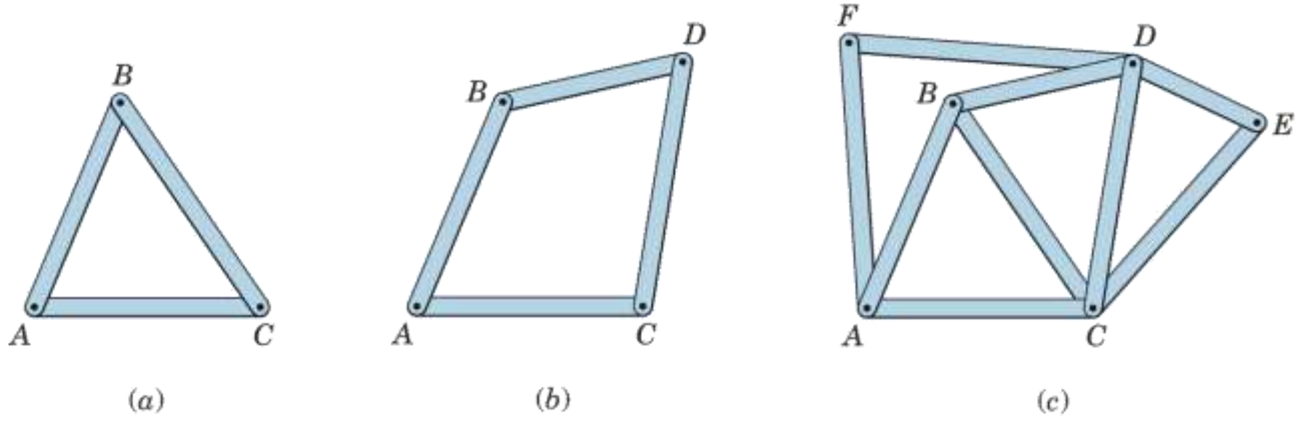
Article 4/2 – Common Truss Shapes (2 of 2)

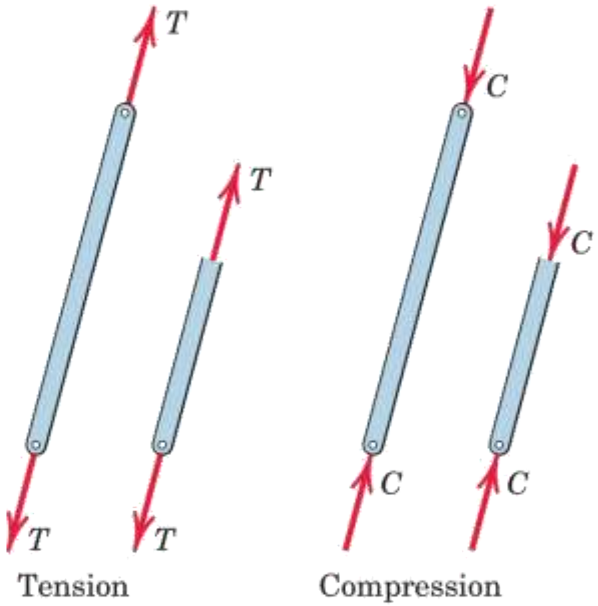
- Commonly Used Roof Trusses



Article 4/2 – Features of Simple Trusses (1 of 2)

- Simple Trusses
  - Rigid
  - Basic Element
  - Two-Force Members
  - Weight of Truss Members
  - Location of Applied Loads

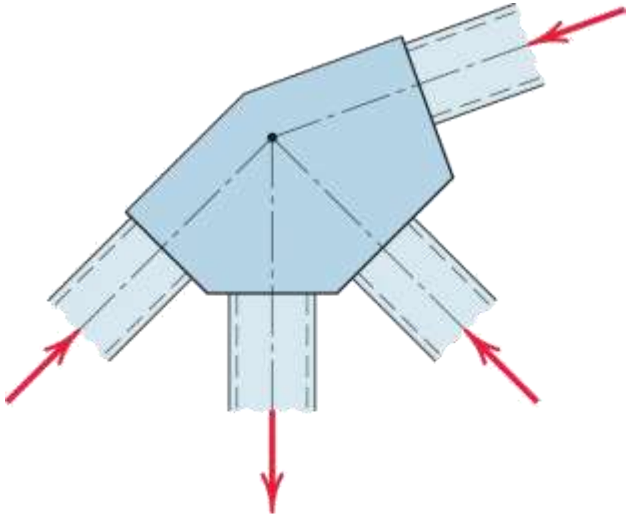


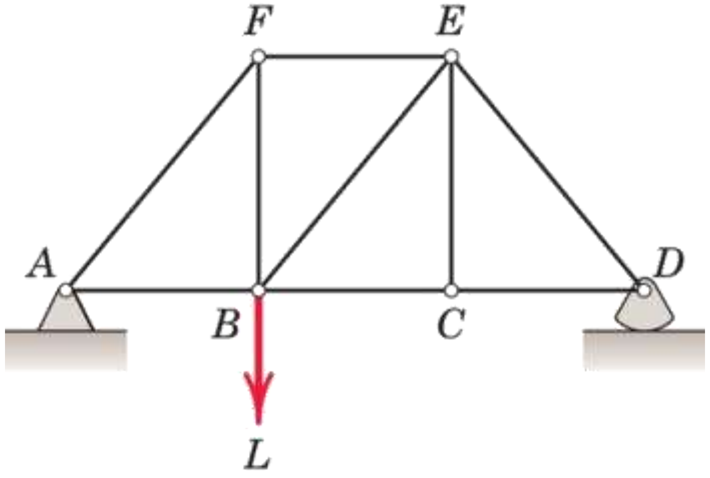


Two-Force Members

Article 4/2 – Features of Simple Trusses (2 of 2)

- Simple Trusses (cont.)
  - Connections between Members
  - Support Representation





The trusses in this text are statically determinant, which means a complete application of Newton's First Law will provide the solution for all unknown reactions and internal forces. Trusses that cannot be solved in this fashion are statically indeterminate, and information regarding the deformation of the members must be included to obtain the solution.

Article 4/3 Method of Joints

- Features of the Method
  - Looks at Equilibrium of Each Joint Connection
  - Limited to Force Summation Equations of Equilibrium at each Joint
  - Limited to Finding Two Unknown Reactions at a Time
  - Proceeds Systematically Across the Truss
  - Very Methodical and Easy to Utilize
  - Very Useful for Finding all of the Truss-Member Forces

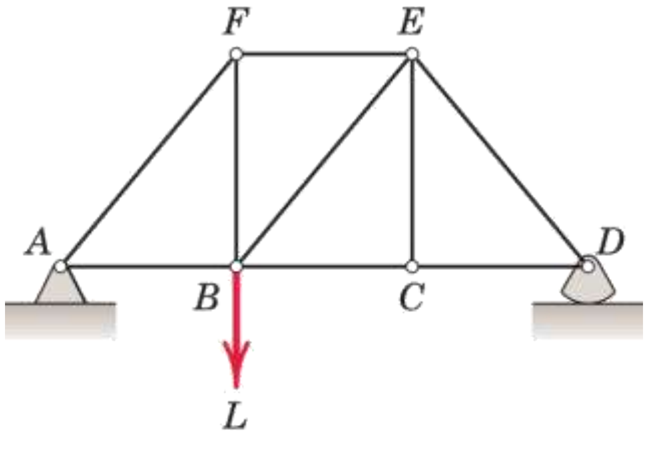
Article 4/3 – Overview of Method of Joints

- General Procedure
  1. In general, find the external reactions first (this may or may not be necessary).
  2. Start by analyzing a joint where at least one known applied load acts, and at most, only two unknown truss forces are present.
  3. Apply a sum of forces in an appropriate *x*- and *y*-direction at this joint.
  4. Draw members in tension or compression, and stay consistent with your choice until you move to the next joint.
  5. Solve the equilibrium equations for the unknown truss forces.
  6. Move to the next adjacent joint and repeat the process.

Article 4/3 – Illustration of Method of Joints (1 of 3)

- Determine the force in each member of the truss.

  1. Determine the external reactions



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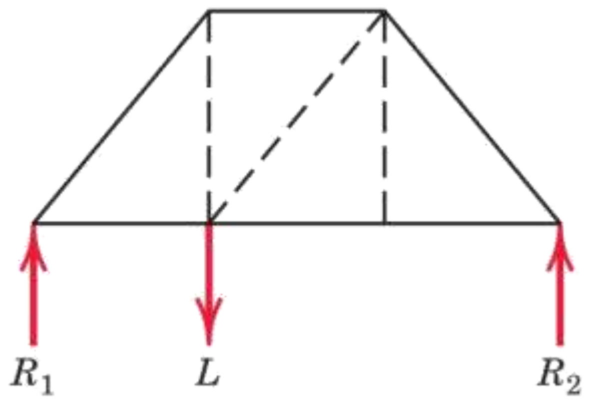

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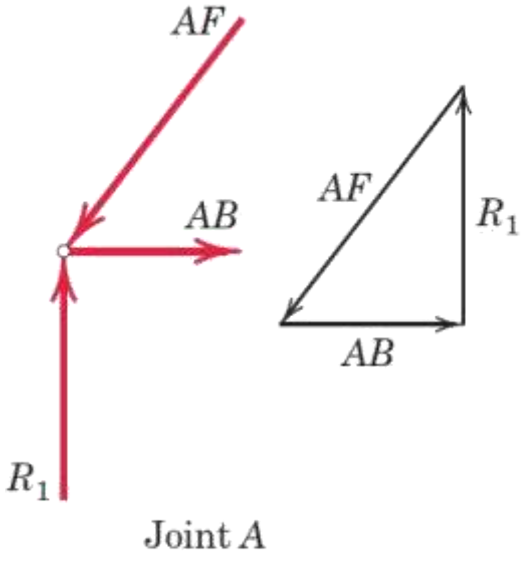
Article 4/3 – Illustration of Method of Joints (2 of 3)

- Determine the force in each member of the truss.

1. Determine the external reactions.

2. Start at a joint with only two unknowns, joint *A*.

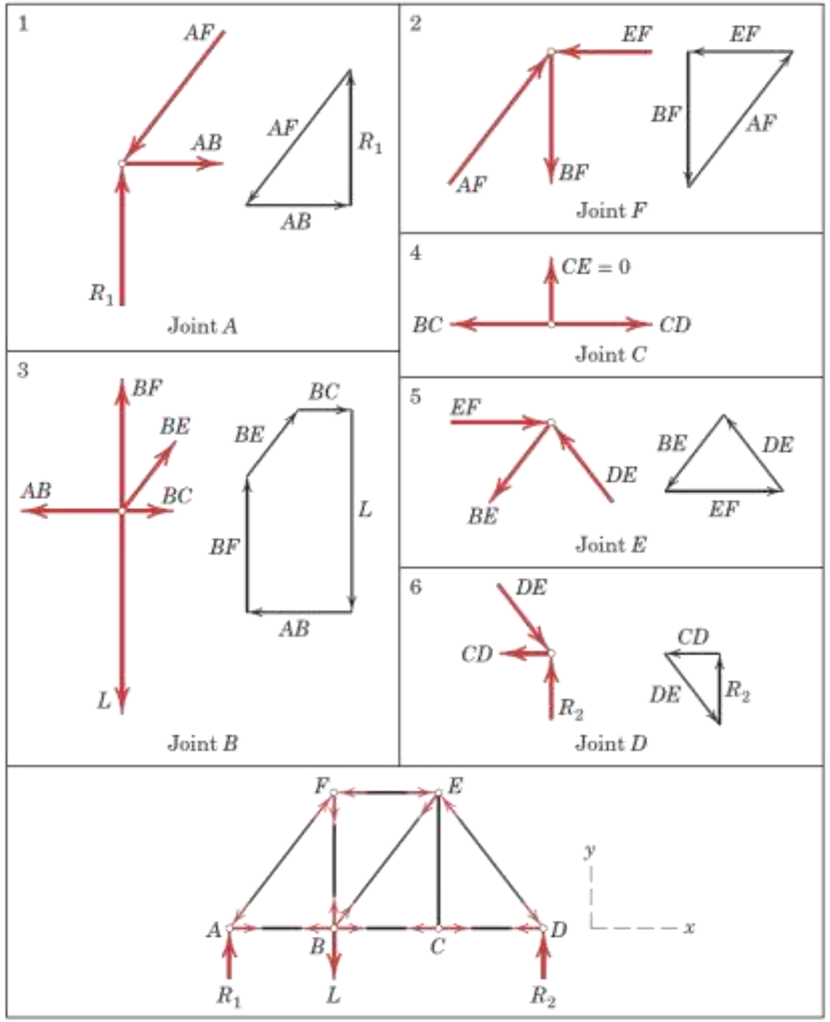
3. Draw the free-body diagram and apply equilibrium equations.



Article 4/3 – Illustration of Method of Joints (3 of 3)

- Determine the force in each member of the truss.

  1. Determine the external reactions.
  2. Start at a joint with only two unknowns, Joint *A*.
  3. Draw the free-body diagram and apply equilibrium equations.
  4. Proceed to the next joint and repeat the process.
  5. Be careful with the signs of the members and indicate if they are in tension or compression.



Article 4/3 – Internal and External Redundancy

- External Redundancy

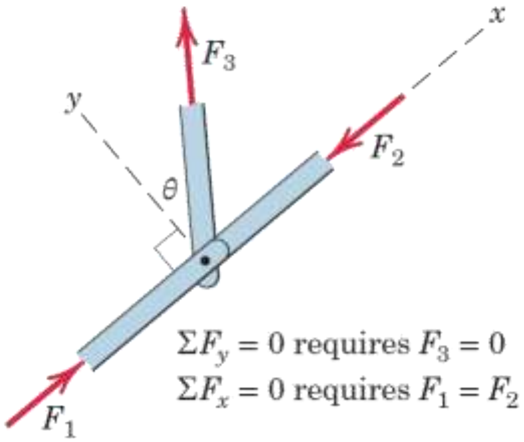
- Internal Redundancy

- Checking for Internal Statical Determinacy
  - Number of Internal Members, m
  - Number of Joint Connections, j
  - Truss is statically determinate internally if… $m + 3 = 2j$

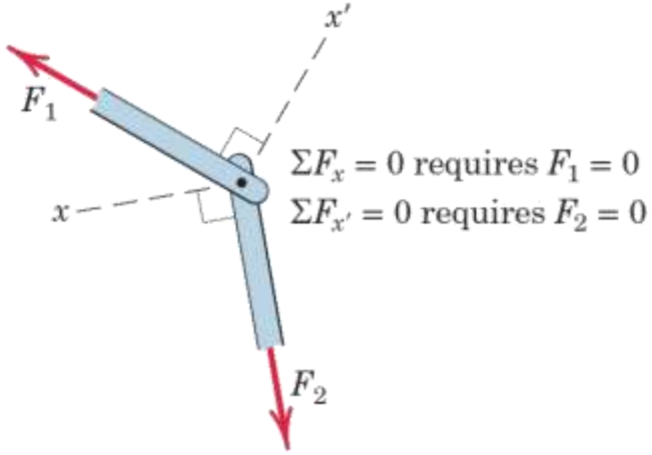
- Comment on Stability

Article 4/3 – Special Conditions in Trusses (1 of 3)

- Condition 1: Three Members at a Joint, Two are Collinear

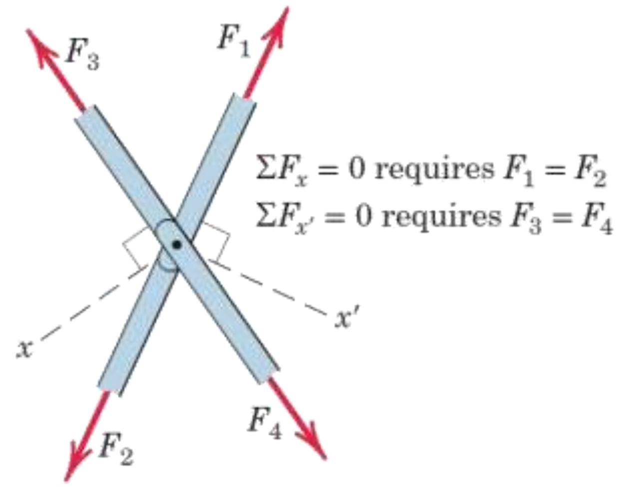


- Condition 2: Two Members at a Joint, Not Collinear

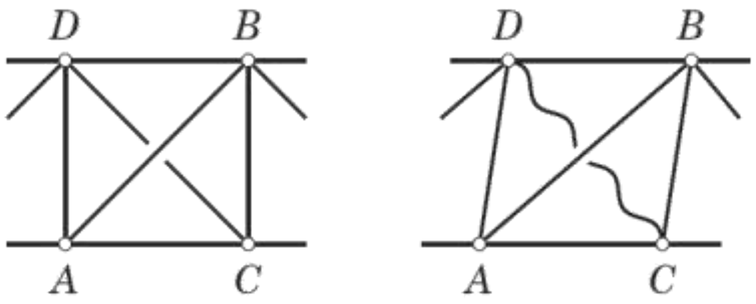


Article 4/3 – Special Conditions in Trusses (2 of 3)

- Condition 3: Two Pairs of Collinear Members at a Joint

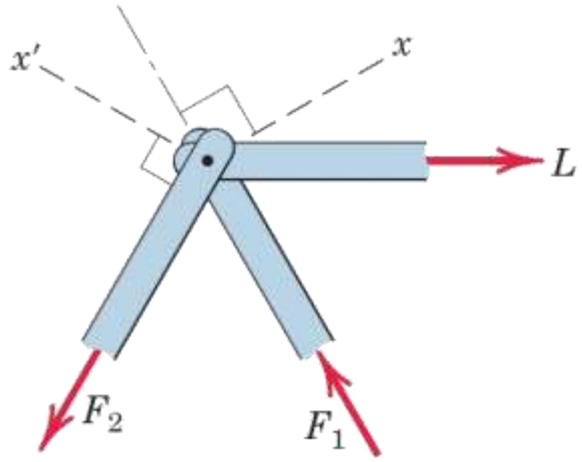


- Condition 4: Cross Bracing



Article 4/3 – Special Conditions in Trusses (3 of 3)

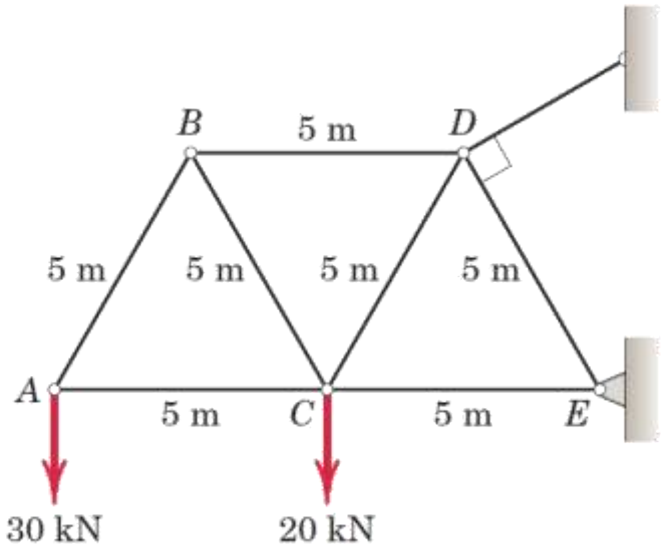
- Condition 5: Avoiding Simultaneous Equations
  - Summing Forces along x-Axis Eliminates F_1
  - Summing Forces along x'-Axis Eliminates F_2



Article 4/3 – Sample Problem 4/1 (1 of 4)

- Problem Statement

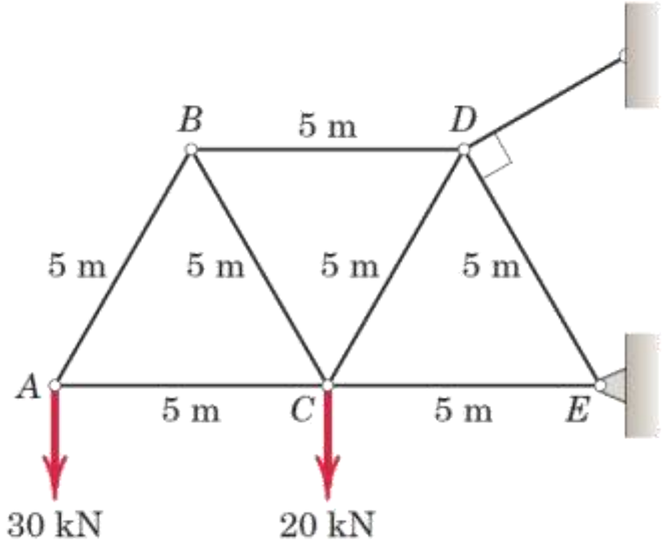
  Compute the force in each member of the loaded cantilever truss by the method of joints.

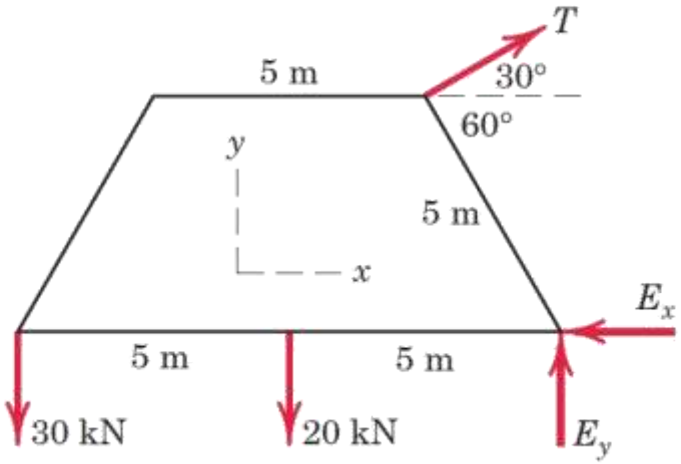


Article 4/3 – Sample Problem 4/1 (2 of 4)

- Find External Reactions

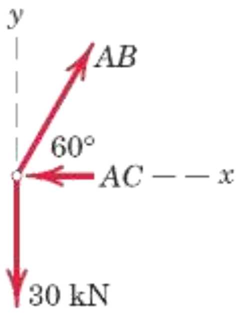
$[\Sigma M_E = 0]$	$5T - 20(5) - 30(10) = 0$	$T = 80 \text{ kN}$
$[\Sigma F_x = 0]$	$80 \cos 30° - E_x = 0$	$E_x = 69.3 \text{ kN}$
$[\Sigma F_y = 0]$	$80 \sin 30° + E_y - 20 - 30 = 0$	$E_y = 10 \text{ kN}$





Article 4/3 – Sample Problem 4/1 (3 of 4)

- Equilibrium of Joint *A*



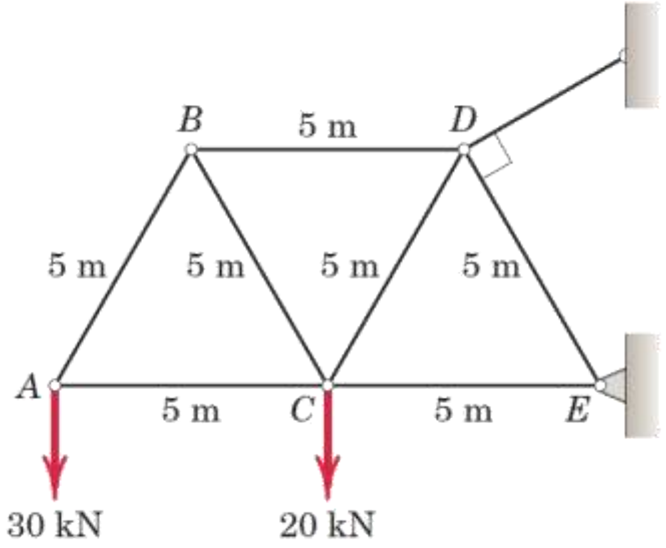
Joint *A*

Equilibrium requires

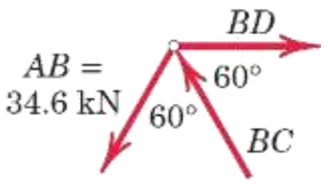
$[\Sigma F_y = 0]$ $\quad 0.866AB - 30 = 0 \quad AB = 34.6 \text{ kN } T \quad$ *Ans.*

$[\Sigma F_x = 0]$ $\quad AC - 0.5(34.6) = 0 \quad AC = 17.32 \text{ kN } C \quad$ *Ans.*

where *T* stands for tension and *C* stands for compression. ①



- Equilibrium of Joint *B*



Joint *B*

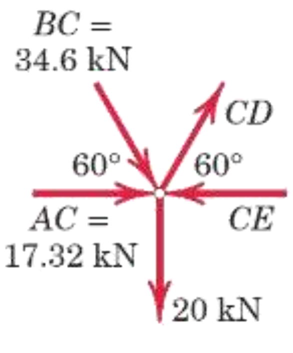
$[\Sigma F_y = 0]$ $\quad 0.866BC - 0.866(34.6) = 0 \quad BC = 34.6 \text{ kN } C \quad$ *Ans.*

$[\Sigma F_x = 0]$ $\quad BD - 2(0.5)(34.6) = 0 \quad BD = 34.6 \text{ kN } T \quad$ *Ans.*

① It should be stressed that the tension/compression designation refers to the *member*, not the joint. Note that we draw the force arrow on the same side of the joint as the member which exerts the force. In this way tension (arrow away from the joint) is distinguished from compression (arrow toward the joint).

Article 4/3 – Sample Problem 4/1 (4 of 4)

- Equilibrium of Joint C



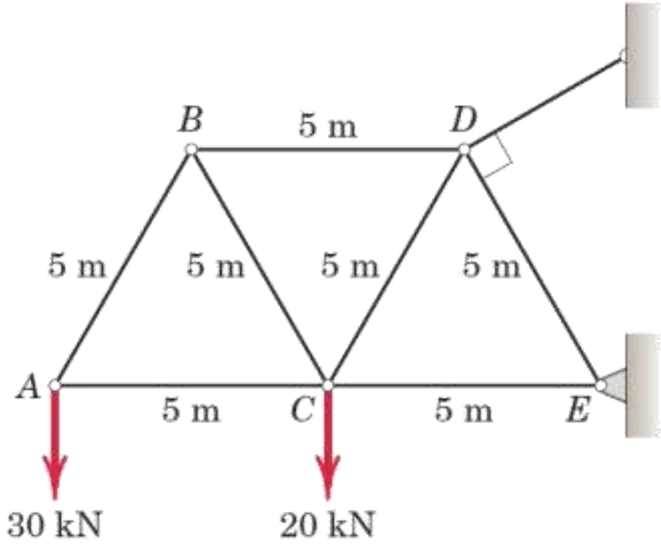
Joint C

$[\Sigma F_y = 0]$ $\quad 0.866CD - 0.866(34.6) - 20 = 0$

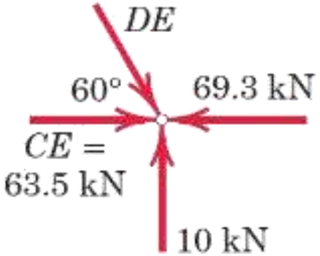
$CD = 57.7 \text{ kN } T$ *Ans.*

$[\Sigma F_x = 0]$ $\quad CE - 17.32 - 0.5(34.6) - 0.5(57.7) = 0$

$CE = 63.5 \text{ kN } C$ *Ans.*



- Equilibrium of Joint E



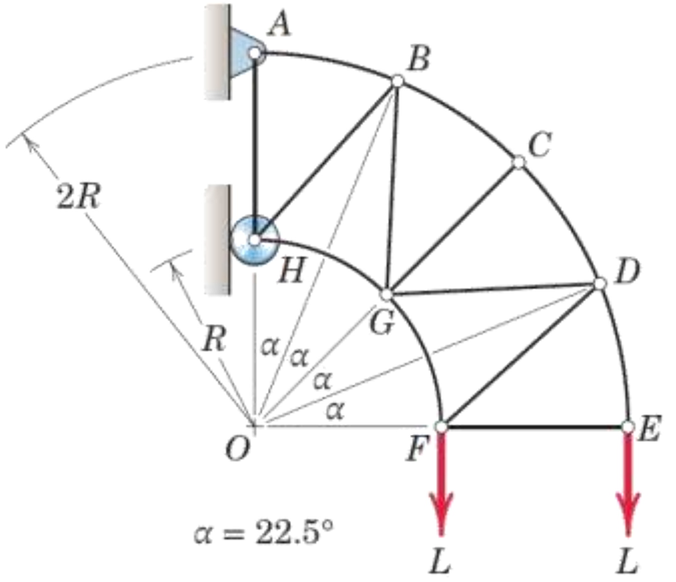
Joint E

$[\Sigma F_y = 0]$ $\quad 0.866DE = 10 \quad DE = 11.55 \text{ kN } C$ *Ans.*

Article 4/3 – Sample Problem 4/2 (1 of 4)

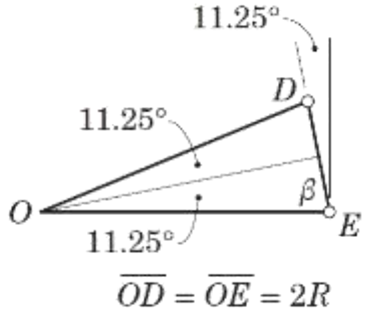
- Problem Statement

  The simple truss shown supports the two loads, each of magnitude *L*. Determine the forces in members *DE*, *DF*, *DG*, and *CD*.

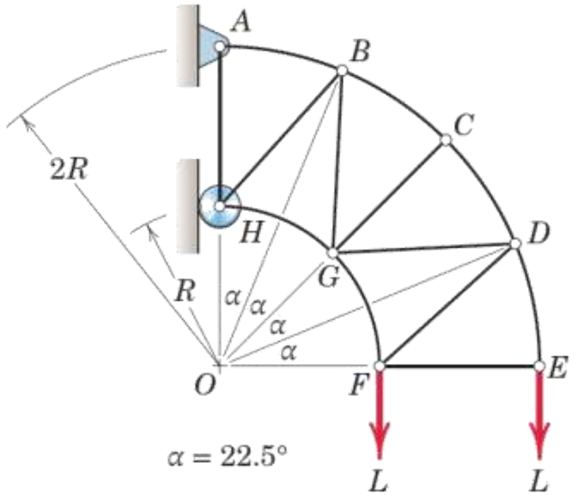


Article 4/3 – Sample Problem 4/2 (2 of 4)

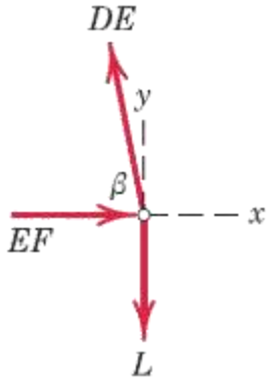
- Geometry of Joint *E*



We can begin with joint E because there are only two unknown member forces acting there. With reference to the free-body diagram and accompanying geometry for joint E, we note that $\beta = 180° - 11.25° - 90° = 78.8°$.



- Equilibrium of Joint *E*



Joint E

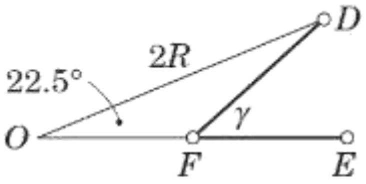
$[\Sigma F_y = 0]$ $\quad DE \sin 78.8° - L = 0 \quad DE = 1.020L\ T$ ① *Ans.*

$[\Sigma F_x = 0]$ $\quad EF - DE \cos 78.8° = 0 \quad EF = 0.1989L\ C$

① Rather than calculate and use the angle $\beta = 78.8°$ in the force equations, we could have used the 11.25° angle directly.

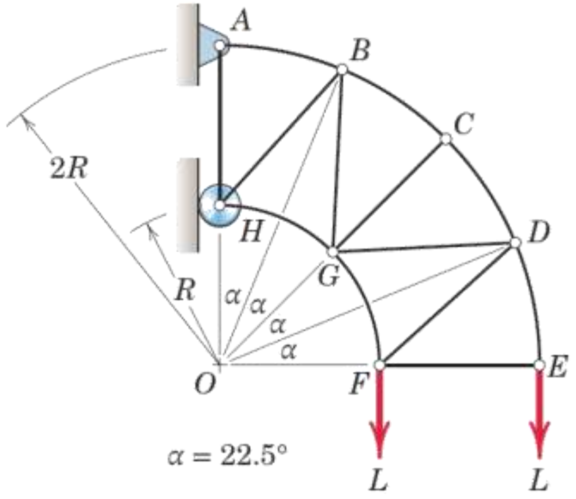
Article 4/3 – Sample Problem 4/2 (3 of 4)

- Geometry of Joint *F*

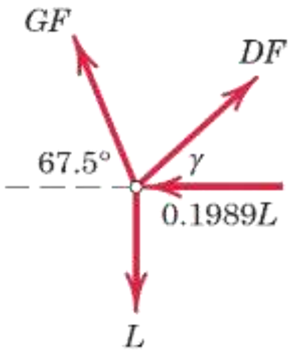


$\overline{OF} = \overline{FE} = R$

$$\gamma = \tan^{-1}\left[\frac{2R \sin 22.5°}{2R \cos 22.5° - R}\right] = 42.1°$$



- Equilibrium of Joint *F*



Joint *F*

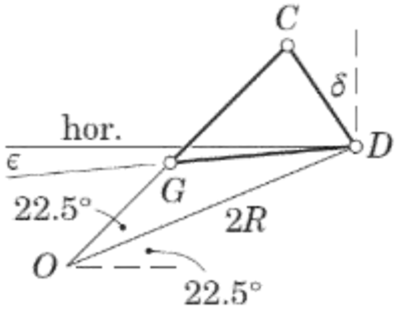
$[\Sigma F_x = 0] \qquad -GF \cos 67.5° + DF \cos 42.1° - 0.1989L = 0$

$[\Sigma F_y = 0] \qquad GF \sin 67.5° + DF \sin 42.1° - L = 0$

Simultaneous solution of these two equations yields

$GF = 0.646L\ T \qquad DF = 0.601L\ T$ *Ans.*

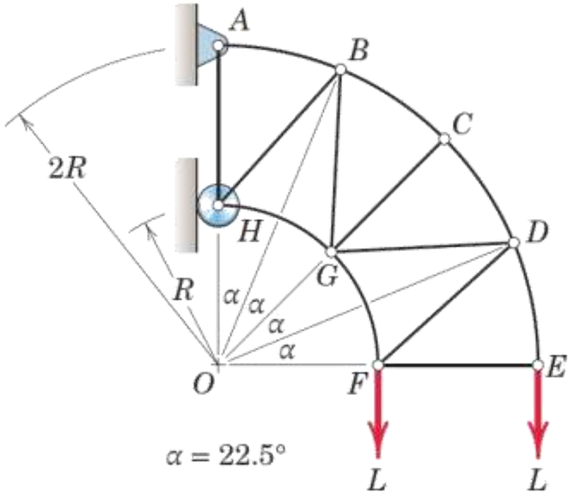
- Geometry of Joint *D*



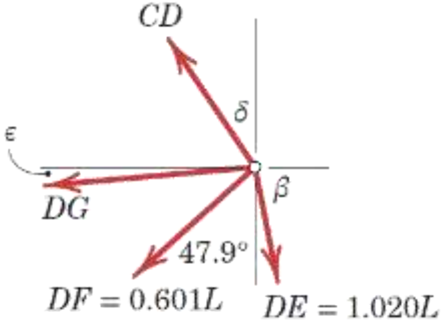
$\overline{OG} = \overline{CG} = R$

$$\delta = \tan^{-1}\left[\frac{2R\cos 22.5^\circ - 2R\cos 45^\circ}{2R\sin 45^\circ - 2R\sin 22.5^\circ}\right] = 33.8^\circ$$

$$\epsilon = \tan^{-1}\left[\frac{2R\sin 22.5^\circ - R\sin 45^\circ}{2R\cos 22.5^\circ - R\cos 45^\circ}\right] = 2.92^\circ$$



- Equilibrium of Joint *D*



Joint *D*

$[\Sigma F_x = 0]$

$$-DG\cos 2.92^\circ - CD\sin 33.8^\circ - 0.601L\sin 47.9^\circ + 1.020L\cos 78.8^\circ = 0$$

$[\Sigma F_y = 0]$

$$-DG\sin 2.92^\circ + CD\cos 33.8^\circ - 0.601L\cos 47.9^\circ - 1.020L\sin 78.8^\circ = 0$$

The simultaneous solution is

$CD = 1.617L\ T \qquad DG = -1.147L$ or $DG = 1.147L\ C \qquad$ *Ans.*

Article 4/4 Method of Sections

- Differences with the Method of Joints
  - Can utilize the moment equation of equilibrium.
  - Can analyze an entire section of a truss.
  - Can determine the force in *almost any* desired member directly.
- A Word of Caution
  - Don’t abandon the method of joints.
  - Usually limited to cutting *at most* three members at one time.

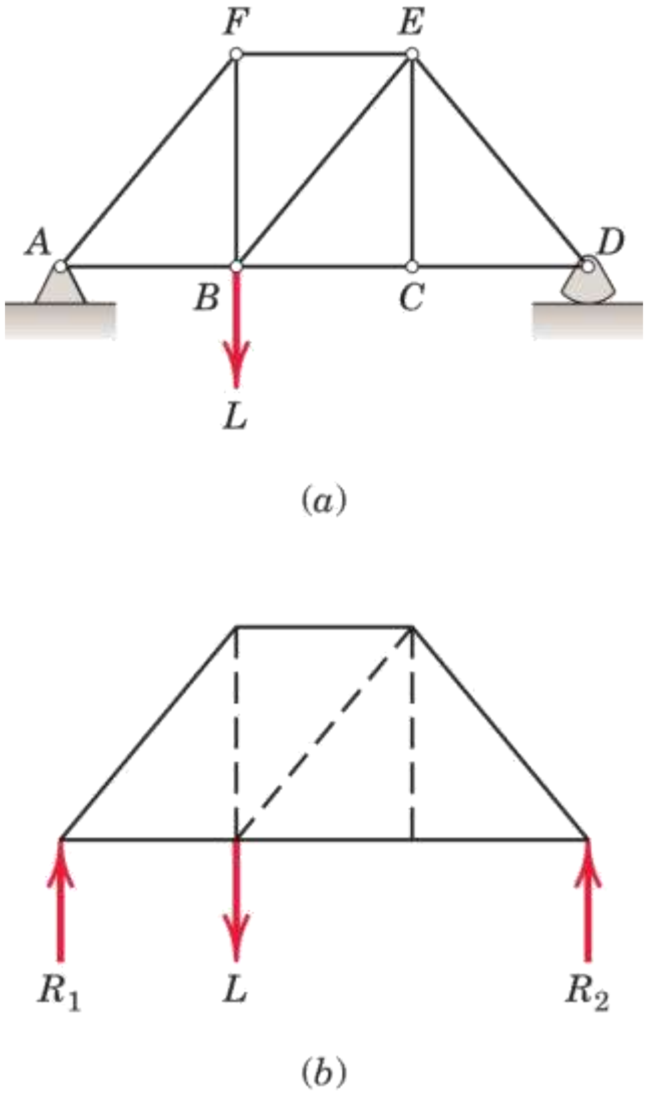
Article 4/4 – Overview of Method of Sections

- General Procedure
  1. In general, find the external reactions first (this may or may not be necessary).
  2. If possible, pass a section (or cut) through the desired member and up to two (2) other unknown members, isolating a portion of the truss.
  3. Apply two-dimensional rigid-body equilibrium equations to the isolated truss portion; e.g., $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma M_O = 0$.
  4. Solve for the unknowns.
  5. Note that the methods of sections and joints may be used in combination.

Article 4/4 – Illustration of Method of Sections (1 of 2)

- Determine the force in member *BE*.

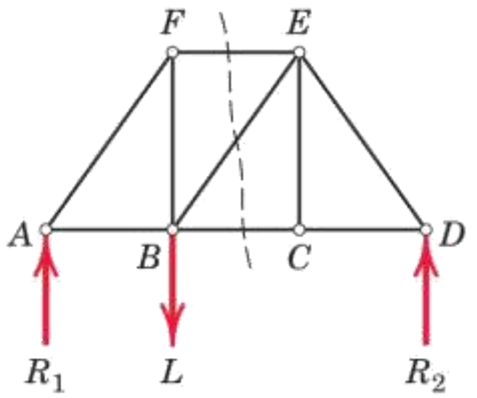
  1. Determine the external reactions.



Article 4/4 – Illustration of Method of Sections (2 of 2)

- Determine the force in member *BE*.

1. Determine the external reactions.
2. Pass a section through member *BE*.
3. Choose either side of the truss.
4. Apply equilibrium equations.



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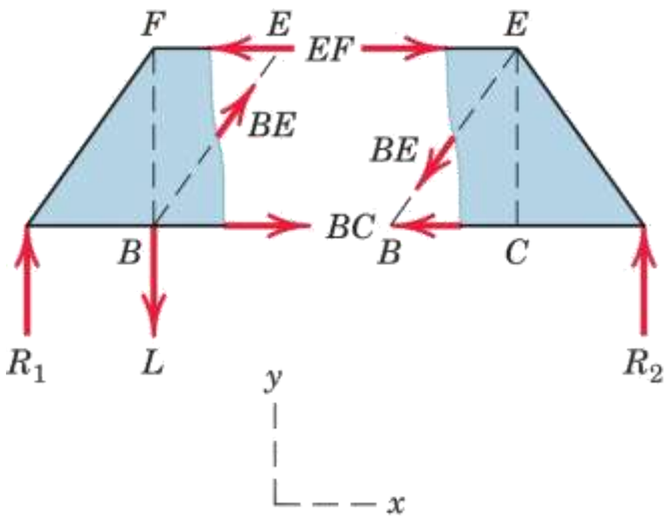

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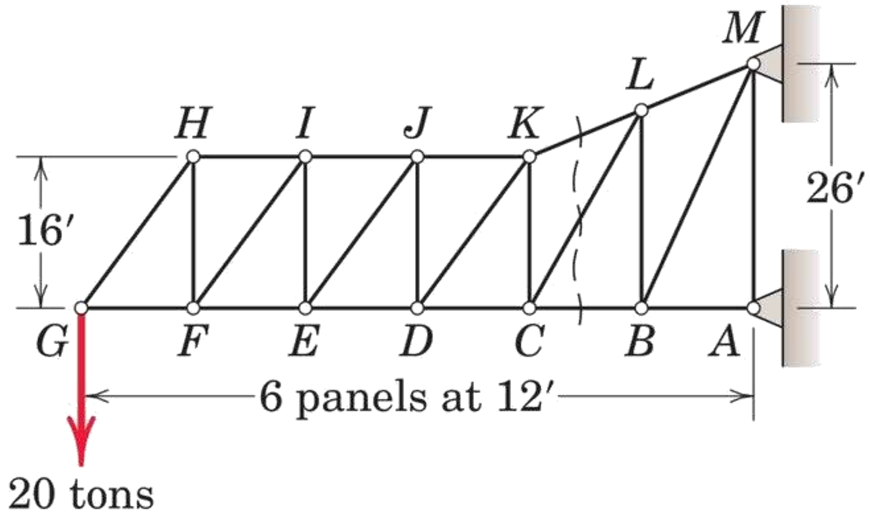
Article 4/4 – Additional Considerations

- Additional Considerations
  - Consider an entire portion of the truss in equilibrium.
  - Forces internal to the section are not considered.
  - Use either portion of the truss after cutting.
  - Combine with method of joints if necessary.
  - Select advantageous moment centers.
  - Stay mindful of assumed directions for unknown forces, tension or compression.

Article 4/4 – Sample Problem 4/3 (1 of 3)

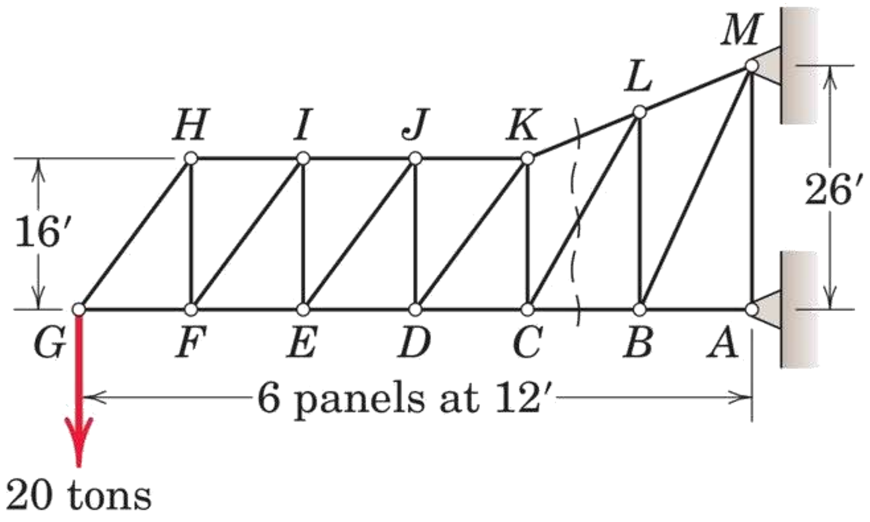
- Problem Statement

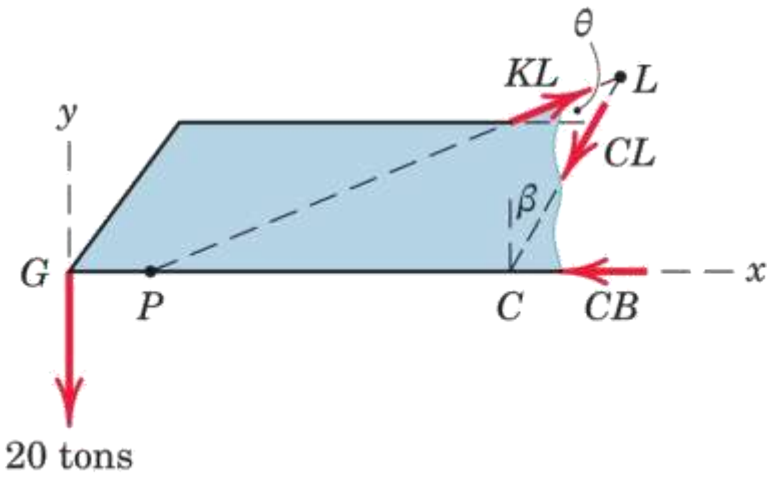
  Calculate the forces induced in members *KL*, *CL*, and *CB* by the 20-ton load on the cantilever truss.



Article 4/4 – Sample Problem 4/3 (2 of 3)

- FBD of Section





- Equilibrium Conditions

Summing moments about L requires finding the moment arm $\overline{BL} = 16 + (26 - 16)/2 = 21$ ft. Thus,

$$[\Sigma M_L = 0] \qquad 20(5)(12) - CB(21) = 0 \qquad CB = 57.1 \text{ tons } C \qquad \textit{Ans.}$$

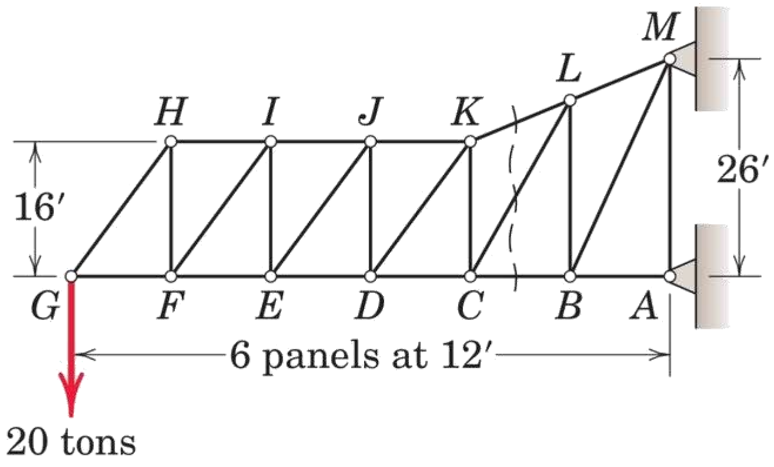
Next we take moments about C, which requires a calculation of $\cos\theta$. From the given dimensions we see $\theta = \tan^{-1}(5/12)$ so that $\cos\theta = 12/13$. Therefore,

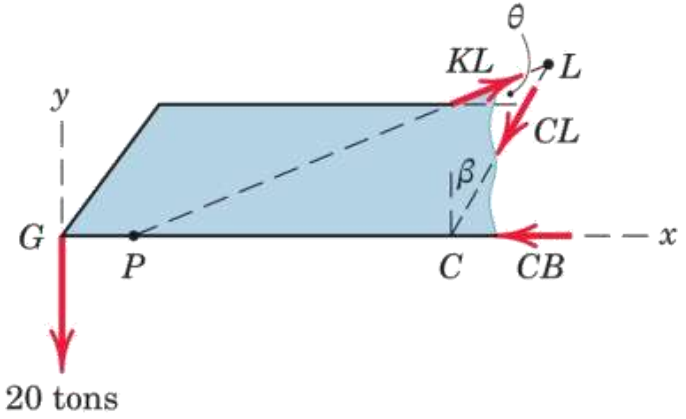
$$[\Sigma M_C = 0] \qquad 20(4)(12) - \frac{12}{13}KL(16) = 0 \qquad KL = 65 \text{ tons } T \qquad \textit{Ans.}$$

Finally, we may find CL by a moment sum about P, whose distance from C is given by $\overline{PC}/16 = 24/(26 - 16)$ or $\overline{PC} = 38.4$ ft. We also need β, which is given by $\beta = \tan^{-1}(\overline{CB}/\overline{BL}) = \tan^{-1}(12/21) = 29.7°$ and $\cos\beta = 0.868$. We now have

$$[\Sigma M_P = 0] \qquad 20(48 - 38.4) - CL(0.868)(38.4) = 0$$

$$CL = 5.76 \text{ tons } C \qquad \textit{Ans.}$$

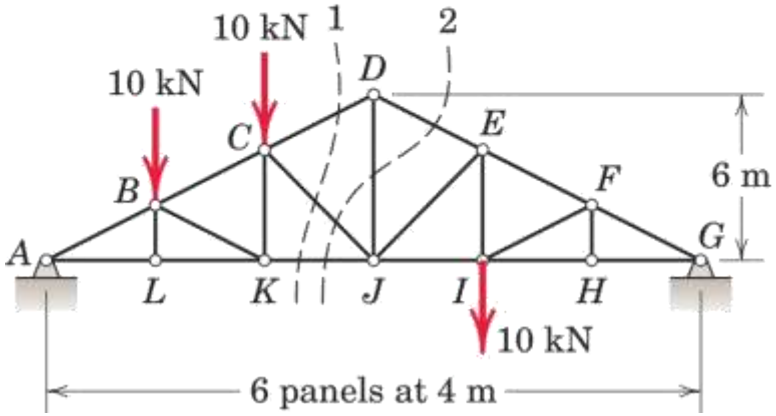




Article 4/4 – Sample Problem 4/4 (1 of 4)

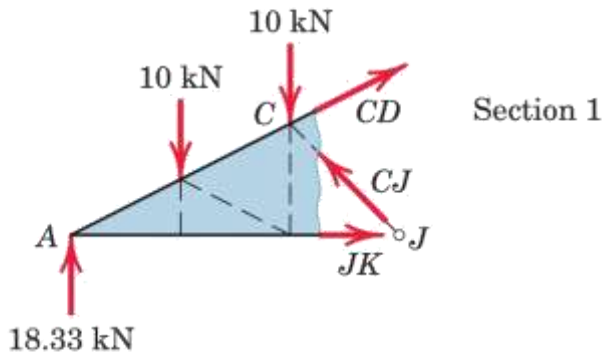
- Problem Statement

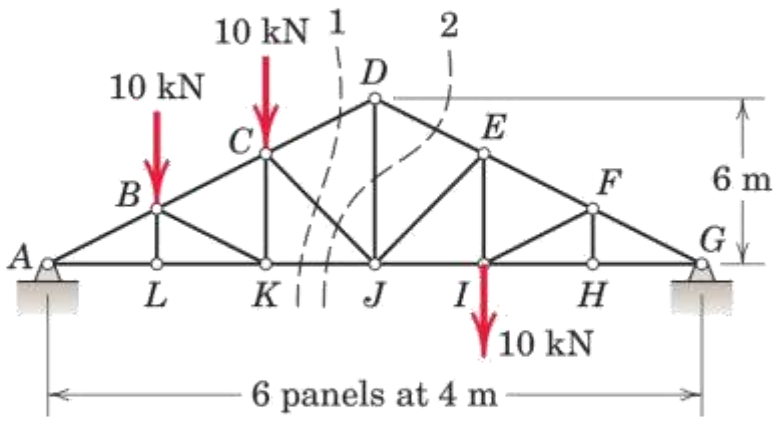
  Calculate the force in member *DJ* of the Howe roof truss illustrated. Neglect any horizontal components of force at the supports.



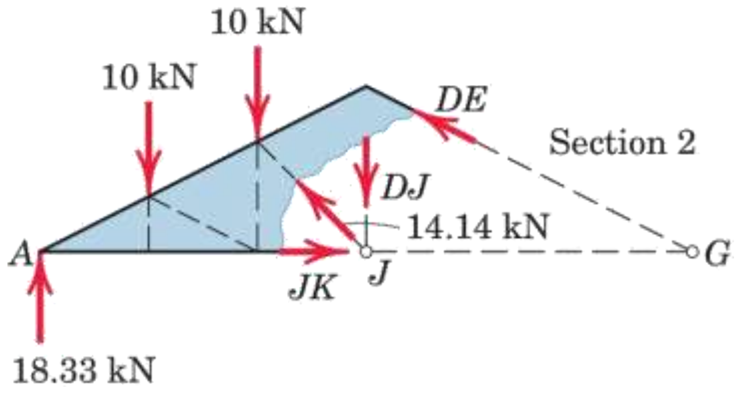
Article 4/4 – Sample Problem 4/4 (2 of 4)

- FBD of Section 1





- FBD of Section 2



Article 4/4 – Sample Problem 4/4 (3 of 4)

- Equilibrium Conditions

By the analysis of section 1, CJ is obtained from

$[\Sigma M_A = 0]$ $\quad 0.707CJ(12) - 10(4) - 10(8) = 0 \quad CJ = 14.14 \text{ kN } C$

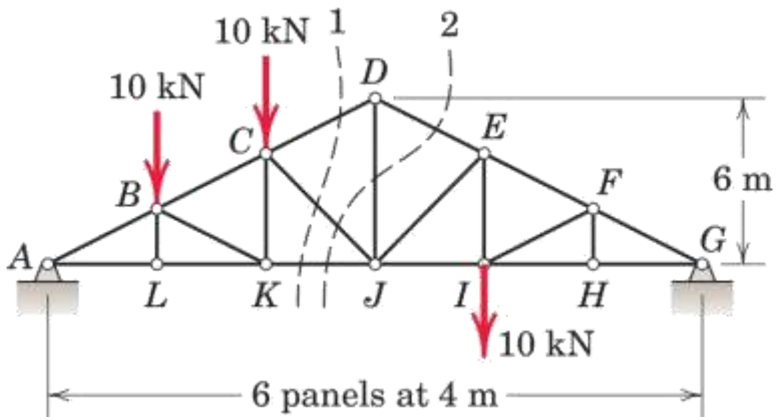
In this equation the moment of CJ is calculated by considering its horizontal and vertical components acting at point J. Equilibrium of moments about J requires

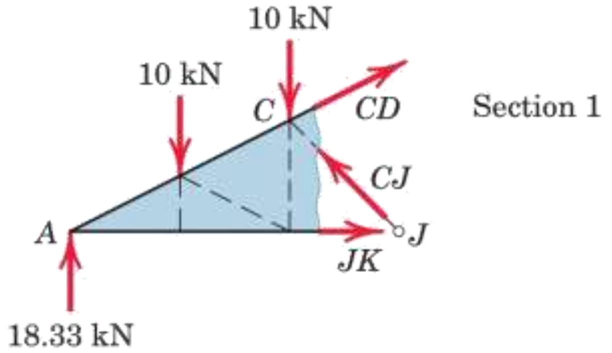
$[\Sigma M_J = 0]$ $\quad 0.894CD(6) + 18.33(12) - 10(4) - 10(8) = 0$

$$CD = -18.63 \text{ kN}$$

The moment of CD about J is calculated here by considering its two components as acting through D. The minus sign indicates that CD was assigned in the wrong direction.

Hence, $\quad CD = 18.63 \text{ kN } C$





Article 4/4 – Sample Problem 4/4 (4 of 4)

- Equilibrium Conditions

From the free-body diagram of section 2, which now includes the known value of CJ, a balance of moments about G is seen to eliminate DE and JK. Thus,

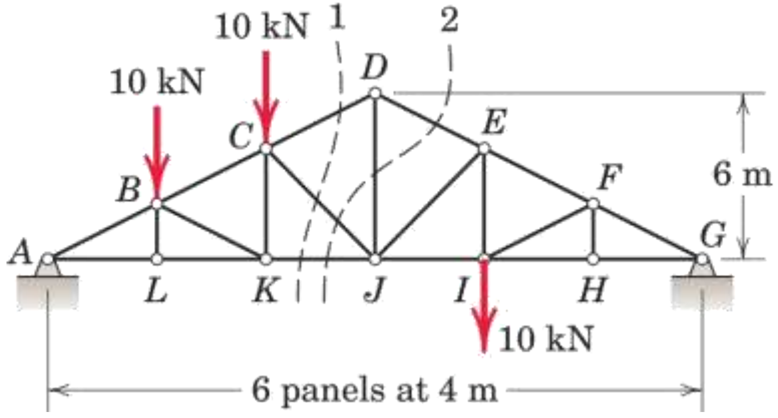
$[\Sigma M_G = 0]$

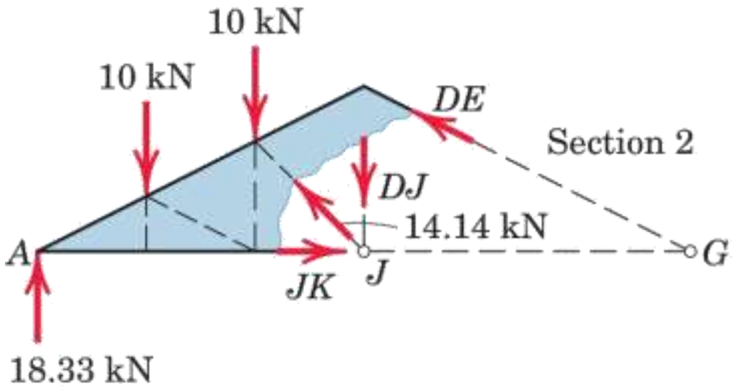
$$12DJ + 10(16) + 10(20) - 18.33(24) - 14.14(0.707)(12) = 0$$

$$DJ = 16.67 \text{ kN } T \qquad \textit{Ans.}$$

Again the moment of CJ is determined from its components considered to be acting at J. The answer for DJ is positive, so that the assumed tensile direction is correct.

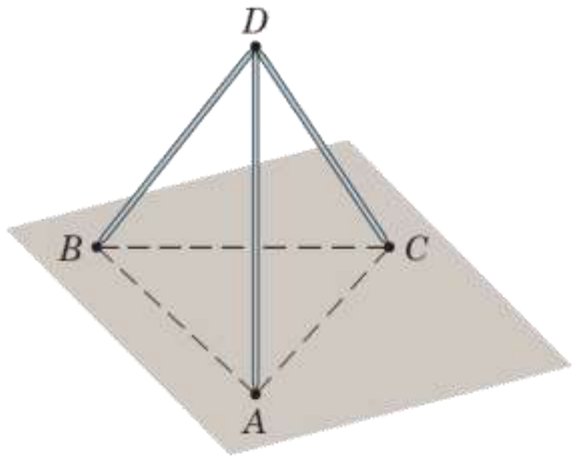
An alternative approach to the entire problem is to utilize section 1 to determine CD and then use the method of joints applied at D to determine DJ.

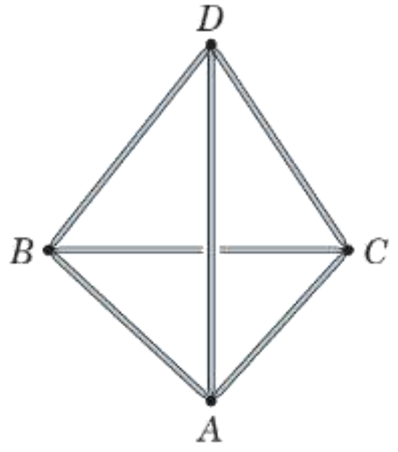




Article 4/5 Space Trusses (1 of 2)

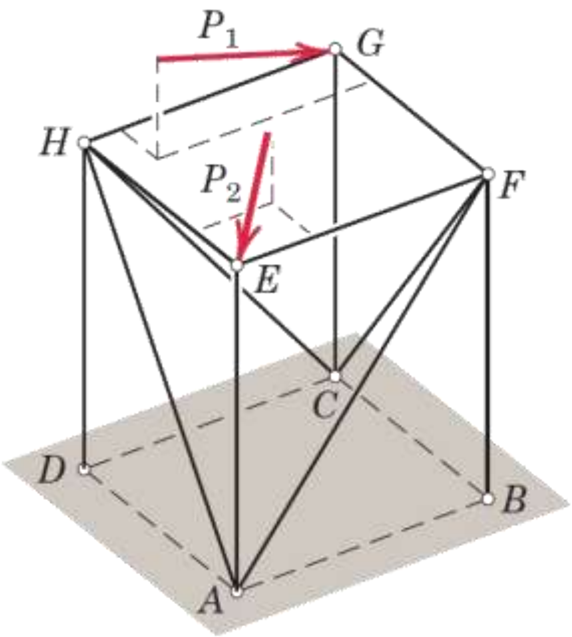
- Three-Dimensional Analogue of Plane Truss
- Basic Shape is a Tetrahedron
- Analysis Utilizes Method of Joints and Method of Sections extended to Three Dimensions
- Statically Determinant if… $m + 6 = 3j$





Article 4/5 Space Trusses (2 of 2)

- Method of Joints for Space Trusses
  - Same Basic Procedure as for Plane Trusses
  - $\Sigma\mathbf{F} = \mathbf{0}$ at each Joint ($\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma F_z = 0$)
  - Limited to Solving Three (3) Unknown Forces at a Time

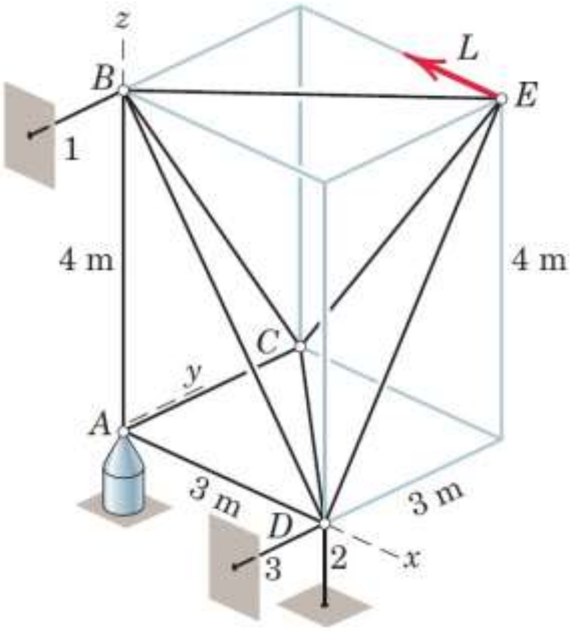


- Method of Joints for Space Trusses
  - Same Basic Procedure as for Plane Trusses
  - $\Sigma\mathbf{F} = \mathbf{0}$ for any Section ($\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma F_z = 0$)
  - $\Sigma\mathbf{M} = \mathbf{0}$ for any Section ($\Sigma M_x = 0$, $\Sigma M_y = 0$, and $\Sigma M_z = 0$)
  - Limited to Solving Six (6) Unknown Forces at a Time

Article 4/5 – Sample Problem 4/5 (1 of 3)

- Problem Statement

  The space truss consists of the rigid tetrahedron *ABCD* anchored by a ball-and-socket connection at *A* and prevented from any rotation about the *x*-, *y*-, or *z*-axes by the respective links 1, 2, and 3. The load *L* is applied to joint *E*, which is rigidly fixed to the tetrahedron by the three additional links. Solve for the forces in the members at joint *E* and indicate the procedure for the determination of the forces in the remaining members of the truss.

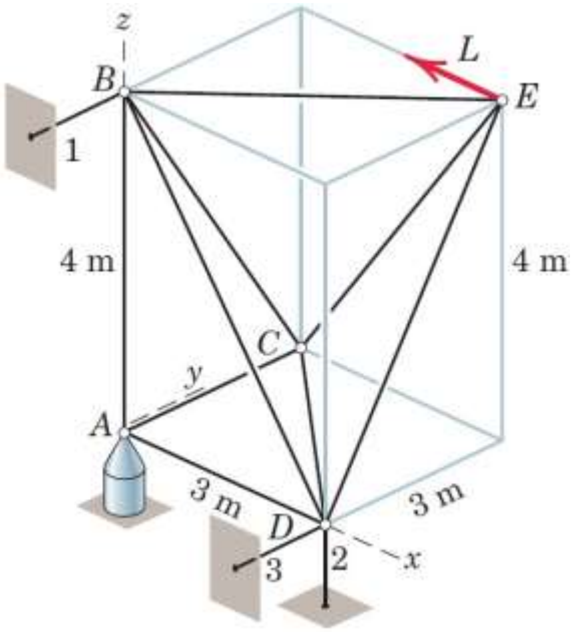


Article 4/5 – Sample Problem 4/5 (2 of 3)

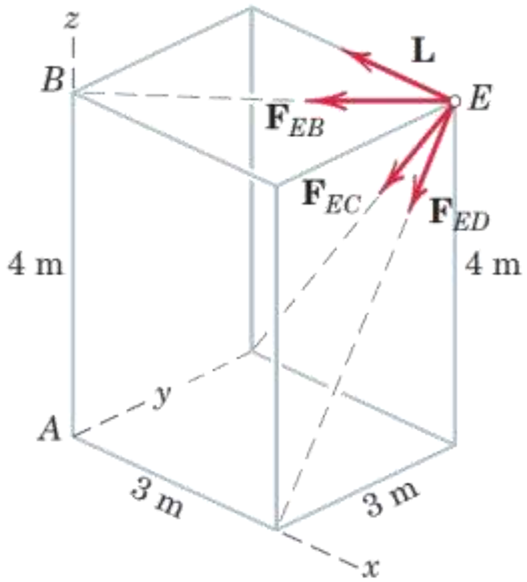
- Check Determinacy

  $m + 6 = 3j$

  $9 + 6 = 3(5)$ True



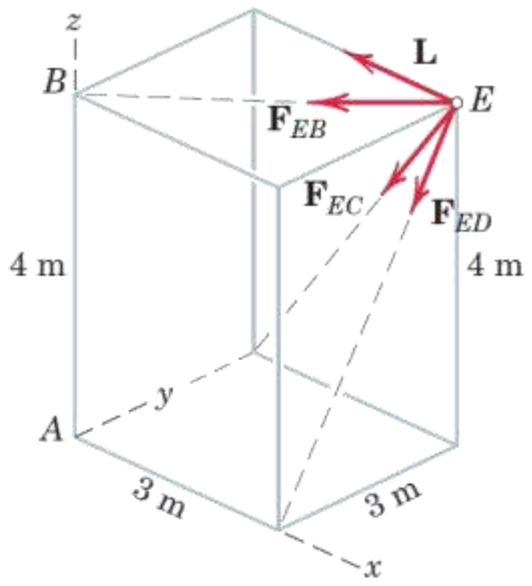
- Free-Body Diagram of Joint *E*



$$\mathbf{F}_{EB} = \frac{F_{EB}}{\sqrt{2}}(-\mathbf{i} - \mathbf{j}), \qquad \mathbf{F}_{EC} = \frac{F_{EC}}{5}(-3\mathbf{i} - 4\mathbf{k}), \qquad \mathbf{F}_{ED} = \frac{F_{ED}}{5}(-3\mathbf{j} - 4\mathbf{k})$$

Article 4/5 – Sample Problem 4/5 (3 of 3)

- Equilibrium of Joint *E*



$[\Sigma \mathbf{F} = \mathbf{0}] \qquad \mathbf{L} + \mathbf{F}_{EB} + \mathbf{F}_{EC} + \mathbf{F}_{ED} = \mathbf{0} \qquad$ or

$$-L\mathbf{i} + \frac{F_{EB}}{\sqrt{2}}(-\mathbf{i} - \mathbf{j}) + \frac{F_{EC}}{5}(-3\mathbf{i} - 4\mathbf{k}) + \frac{F_{ED}}{5}(-3\mathbf{j} - 4\mathbf{k}) = \mathbf{0}$$

Rearranging terms gives

$$\left(-L - \frac{F_{EB}}{\sqrt{2}} - \frac{3F_{EC}}{5}\right)\mathbf{i} + \left(-\frac{F_{EB}}{\sqrt{2}} - \frac{3F_{ED}}{5}\right)\mathbf{j} + \left(-\frac{4F_{EC}}{5} - \frac{4F_{ED}}{5}\right)\mathbf{k} = \mathbf{0}$$

Equating the coefficients of the **i**-, **j**-, and **k**-unit vectors to zero gives the three equations

$$\frac{F_{EB}}{\sqrt{2}} + \frac{3F_{EC}}{5} = -L \qquad \frac{F_{EB}}{\sqrt{2}} + \frac{3F_{ED}}{5} = 0 \qquad F_{EC} + F_{ED} = 0$$

Solving the equations gives us

$$F_{EB} = -L/\sqrt{2} \qquad F_{EC} = -5L/6 \qquad F_{ED} = 5L/6 \qquad \textit{Ans.}$$

Article 4/6 Frames and Machines

- Definition

A structure is called a *frame* or *machine* if at least one of its individual members is a *multiforce member*. A multiforce member is defined as one with three or more forces acting on it, or one with two or more forces and one or more couples acting on it.

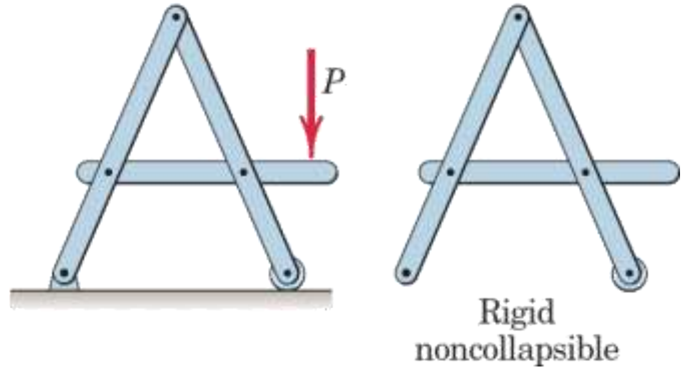
- Example – Jaws of Life

Billy Gadbury/Shutterstock

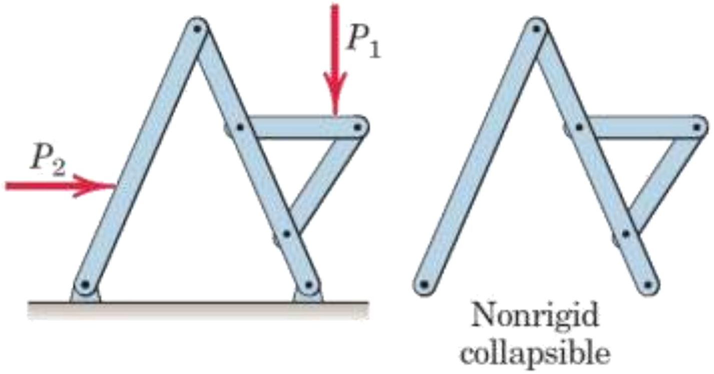
Two devices used by rescuers to free accident victims from wreckage. The "jaws of life" machine shown at the left is the subject of problems in this article and the chapter-review article.

Article 4/6 – Types of Structures

- Interconnected Rigid Bodies with Multiforce Members
  - Rigid Noncollapsible
    - Find external reactions for the entire structure first.
    - Dismember to find internal reactions.
    - Look for two-force members.

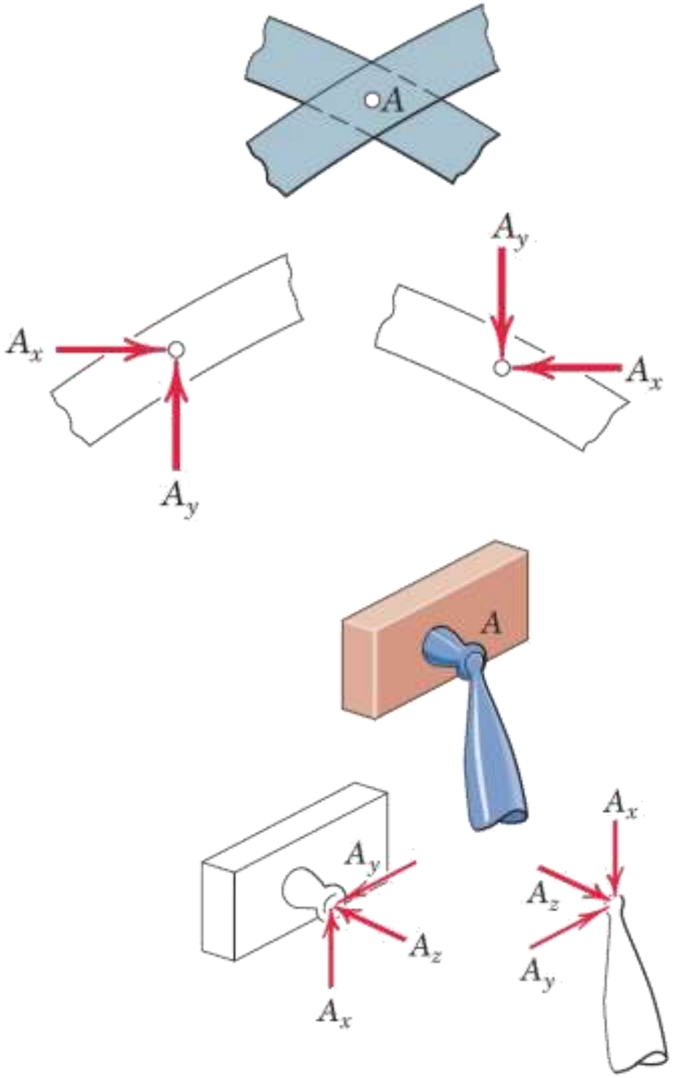


  - Nonrigid Collapsible
    - Dismember the structure first to find internal reactions.
    - Compute external reactions later.
    - Look for two-force members.



Article 4/6 – Force Representation and FBDs

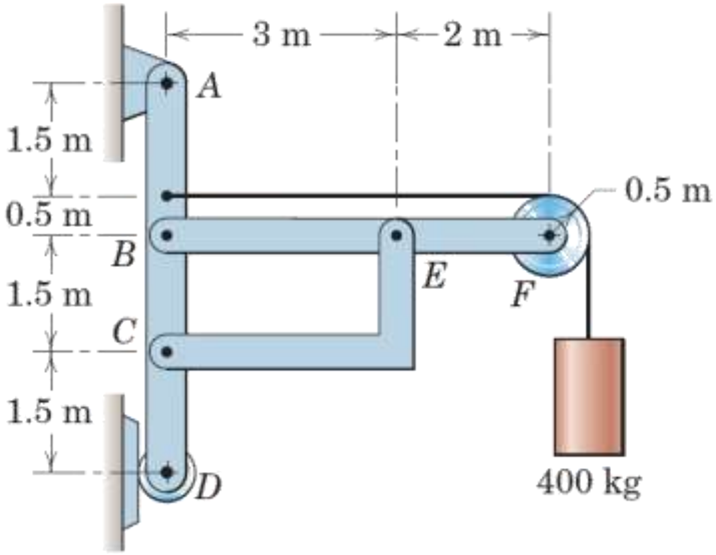
- Importance of Action-Reaction Pairs
- Assumed Force Directions
- Vector Notation
- Simultaneous Equations



Article 4/6 – Sample Problem 4/6 (1 of 3)

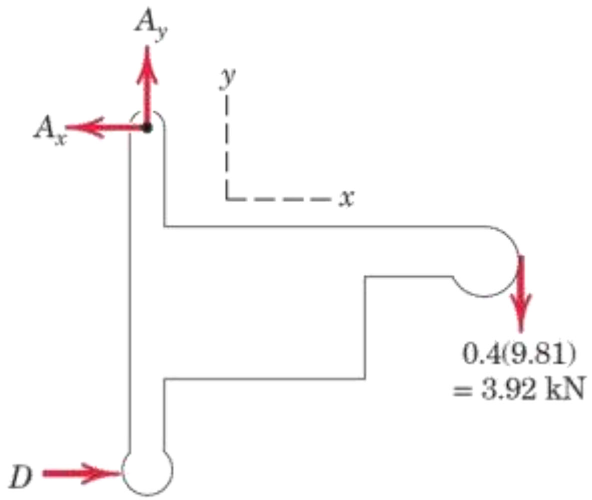
- Problem Statement

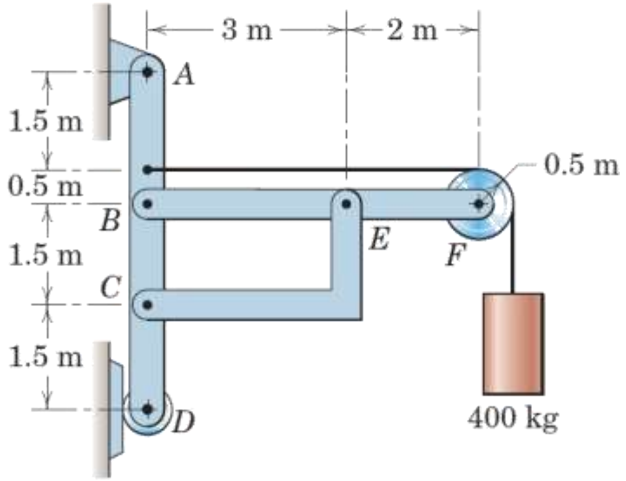
  The frame supports the 400-kg load in the manner shown. Neglect the weights of the members compared with the forces induced by the load and compute the horizontal and vertical components of all forces acting on each of the members.



Article 4/6 – Sample Problem 4/6 (2 of 3)

- Free-Body Diagram of Entire Structure (Rigid)





- Equilibrium Conditions

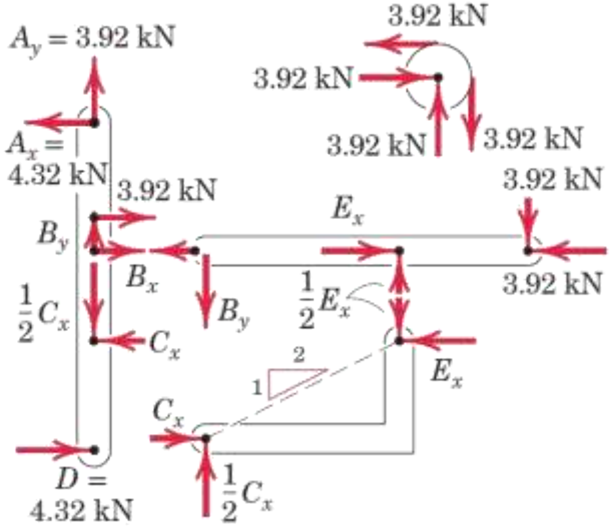
$[\Sigma M_A = 0] \qquad 5.5(0.4)(9.81) - 5D = 0 \qquad D = 4.32 \text{ kN}$

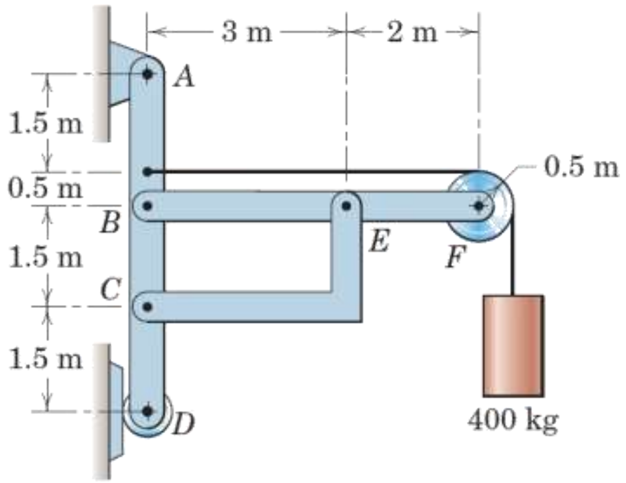
$[\Sigma F_x = 0] \qquad A_x - 4.32 = 0 \qquad A_x = 4.32 \text{ kN}$

$[\Sigma F_y = 0] \qquad A_y - 3.92 = 0 \qquad A_y = 3.92 \text{ kN}$

Article 4/6 – Sample Problem 4/6 (3 of 3)

- Free-Body Diagrams of Individual Pieces





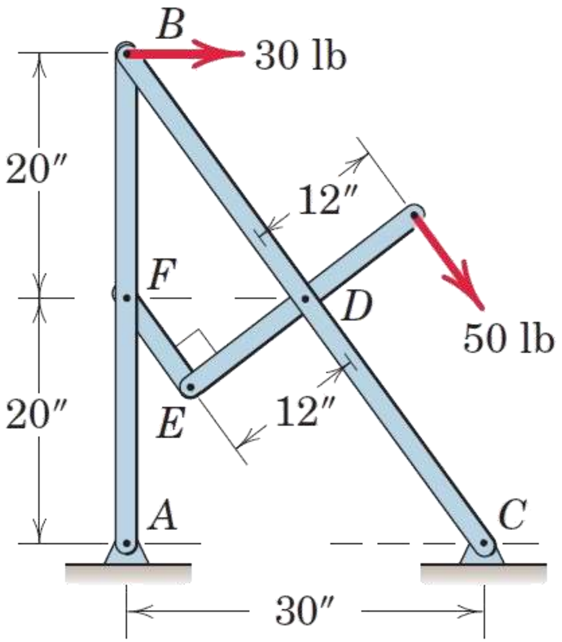
- Equilibrium of Member *BF*

$[\Sigma M_B = 0]$	$3.92(5) - \frac{1}{2}E_x(3) = 0$	$E_x = 13.08$ kN	*Ans.*
$[\Sigma F_y = 0]$	$B_y + 3.92 - 13.08/2 = 0$	$B_y = 2.62$ kN	*Ans.*
$[\Sigma F_x = 0]$	$B_x + 3.92 - 13.08 = 0$	$B_x = 9.15$ kN	*Ans.*

Article 4/6 – Sample Problem 4/7 (1 of 4)

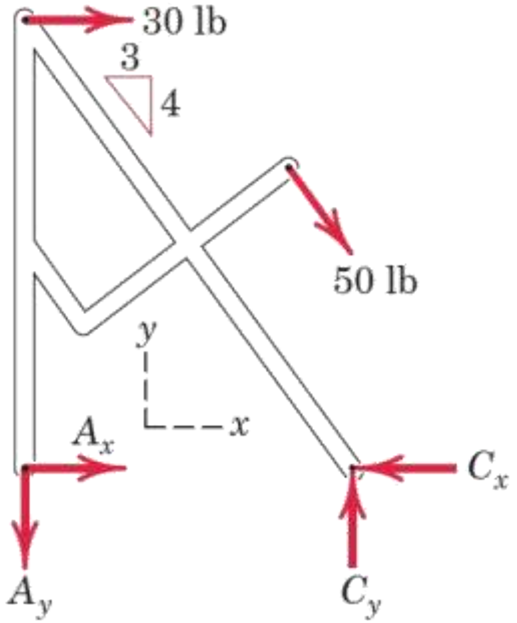
- Problem Statement

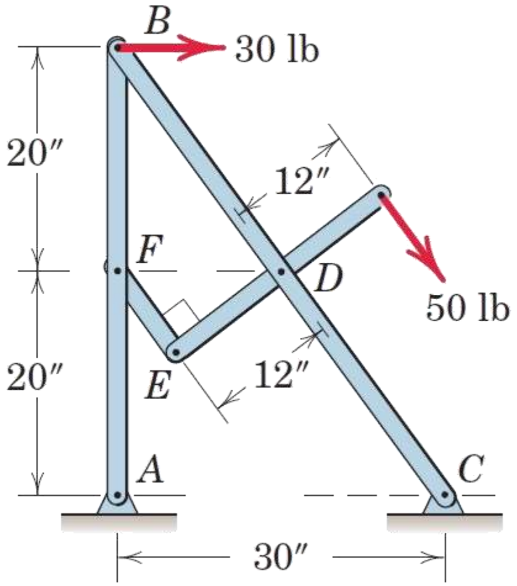
  Neglect the weight of the frame and compute the forces acting on all of its members.



Article 4/6 – Sample Problem 4/7 (2 of 4)

- Free-Body Diagram of Entire Structure (Nonrigid)





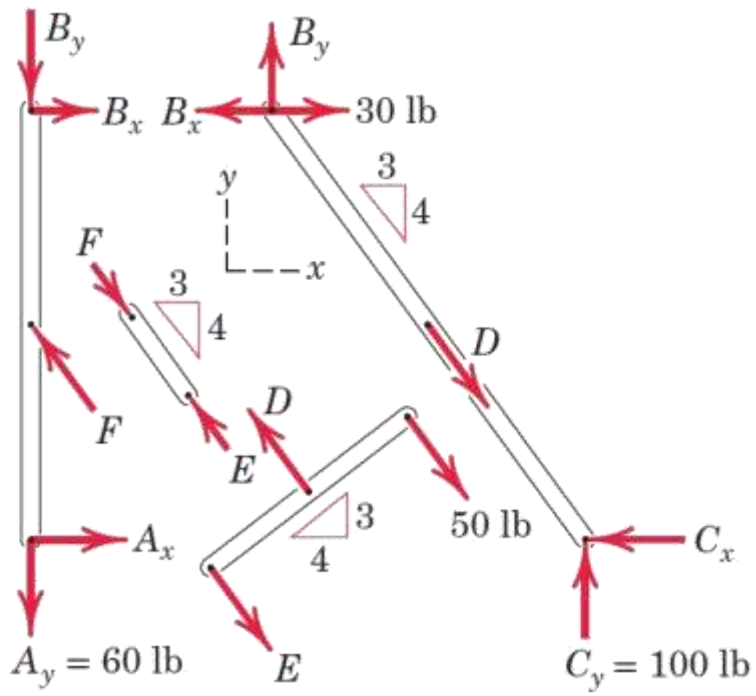
- Vertical Reactions at *A* and *C*

$[\Sigma M_C = 0] \qquad 50(12) + 30(40) - 30A_y = 0 \qquad A_y = 60 \text{ lb} \qquad \textit{Ans.}$

$[\Sigma F_y = 0] \qquad C_y - 50(4/5) - 60 = 0 \qquad C_y = 100 \text{ lb} \qquad \textit{Ans.}$

Article 4/6 – Sample Problem 4/7 (3 of 4)

- Free-Body Diagrams of Individual Pieces



- Equilibrium of Member *ED*

$[\Sigma M_D = 0]$	$50(12) - 12E = 0$	$E = 50$ lb	*Ans.*
$[\Sigma F = 0]$	$D - 50 - 50 = 0$	$D = 100$ lb	*Ans.*

F is equal and opposite to *E* since *EF* is a two-force member.

- Equilibrium of Member *AB*

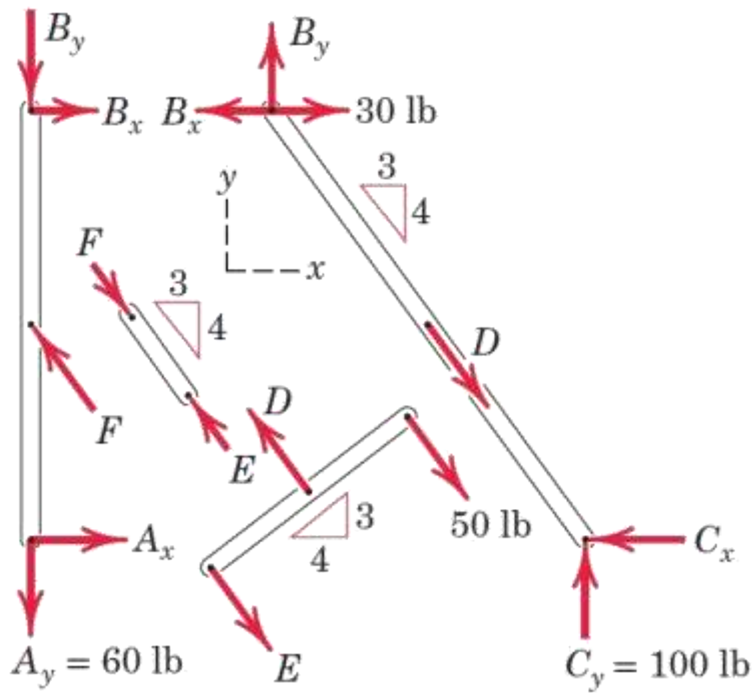
$[\Sigma M_A = 0]$	$50(3/5)(20) - B_x(40) = 0$	$B_x = 15$ lb	*Ans.*
$[\Sigma F_x = 0]$	$A_x + 15 - 50(3/5) = 0$	$A_x = 15$ lb	*Ans.*
$[\Sigma F_y = 0]$	$50(4/5) - 60 - B_y = 0$	$B_y = -20$ lb	*Ans.*

Article 4/6 – Sample Problem 4/7 (4 of 4)

- Free-Body Diagrams of Individual Pieces

- Equilibrium of Member *BC*

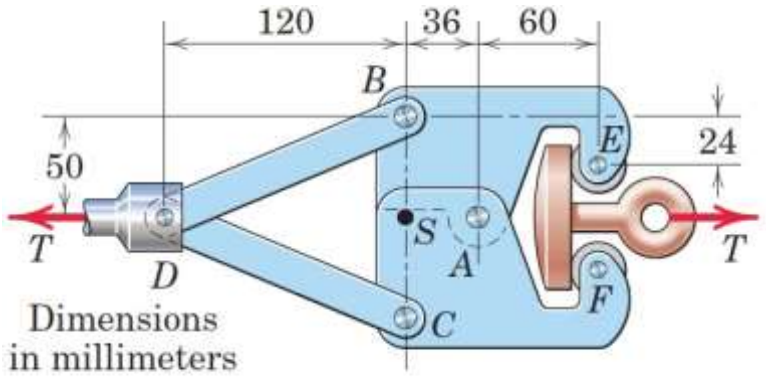
$[\Sigma F_x = 0]$ $\quad 30 + 100(3/5) - 15 - C_x = 0 \quad C_x = 75$ lb ④ *Ans.*

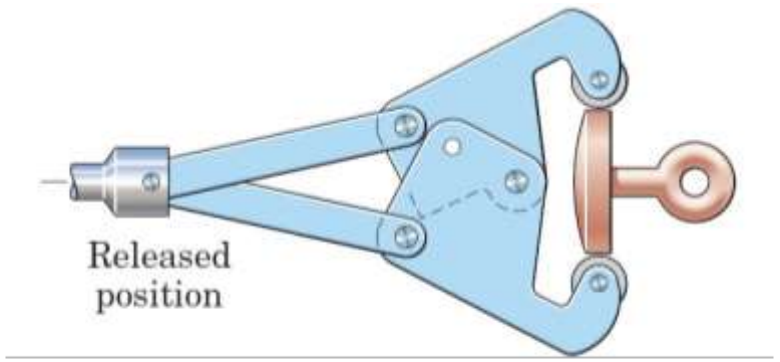


Article 4/6 – Sample Problem 4/8 (1 of 3)

- Problem Statement

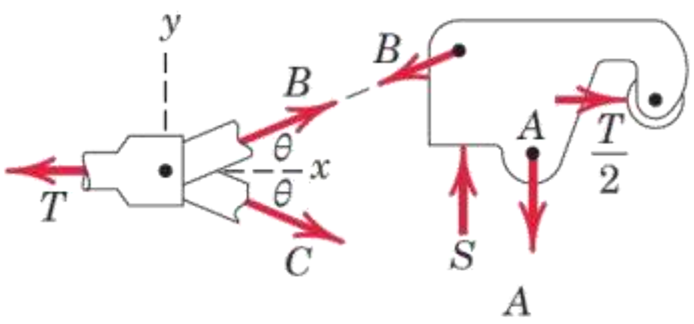
  The machine shown is designed as an overload protection device which releases the load when it exceeds a predetermined value *T*. A soft metal shear pin *S* is inserted in a hole in the lower half and is acted on by the upper half. When the total force on the pin exceeds its strength, it will break. The two halves then rotate about *A* under the action of the tensions in *BD* and *CD*, as shown in the second sketch, and rollers *E* and *F* release the eye bolt. Determine the maximum allowable tension *T* if the pin *S* will shear when the total force on it is 800 N. Also compute the corresponding force on the hinge pin *A*.

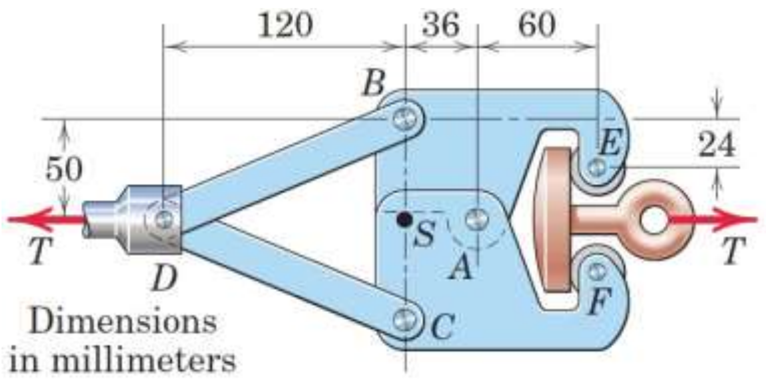




Article 4/6 – Sample Problem 4/8 (2 of 3)

- Individual Free-Body Diagrams





- Equilibrium of the Connection D (Note the Symmetry)

$[\Sigma F_x = 0]$ $\quad B\cos\theta + C\cos\theta - T = 0 \quad 2B\cos\theta = T$

$$B = T/(2\cos\theta)$$

① It is always useful to recognize symmetry. Here it tells us that the forces acting on the two parts behave as mirror images of each other with respect to the x-axis. Thus, we cannot have an action on one member in the plus x-direction and its reaction on the other member in the negative x-direction. Consequently, the forces at S and A have no x-components.

Article 4/6 – Sample Problem 4/8 (3 of 3)

- Equilibrium of the Upper Part

From the free-body diagram of the upper part we express the equilibrium of moments about point A. Substituting $S = 800$ N and the expression for B gives

$[\Sigma M_A = 0]$

$$\frac{T}{2\cos\theta}(\cos\theta)(50) + \frac{T}{2\cos\theta}(\sin\theta)(36) - 36(800) - \frac{T}{2}(26) = 0 \quad ②$$

Substituting $\sin\theta/\cos\theta = \tan\theta = 5/12$ and solving for T give

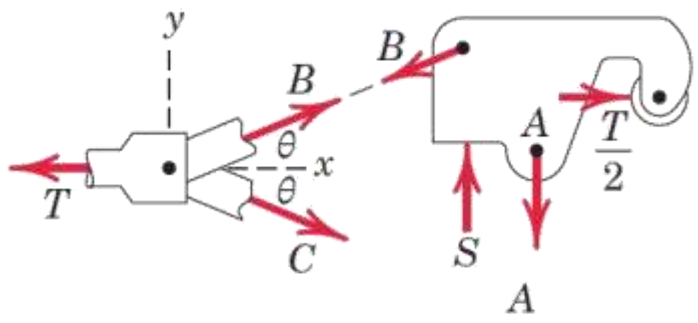
$$T\left(25 + \frac{5(36)}{2(12)} - 13\right) = 28\ 800$$

$$T = 1477 \text{ N} \qquad \text{or} \qquad T = 1.477 \text{ kN} \qquad \textit{Ans.}$$

Finally, equilibrium in the y-direction gives us

$[\Sigma F_y = 0]$ $\qquad S - B\sin\theta - A = 0$

$$800 - \frac{1477}{2(12/13)}\frac{5}{13} - A = 0 \qquad A = 492 \text{ N} \qquad \textit{Ans.}$$

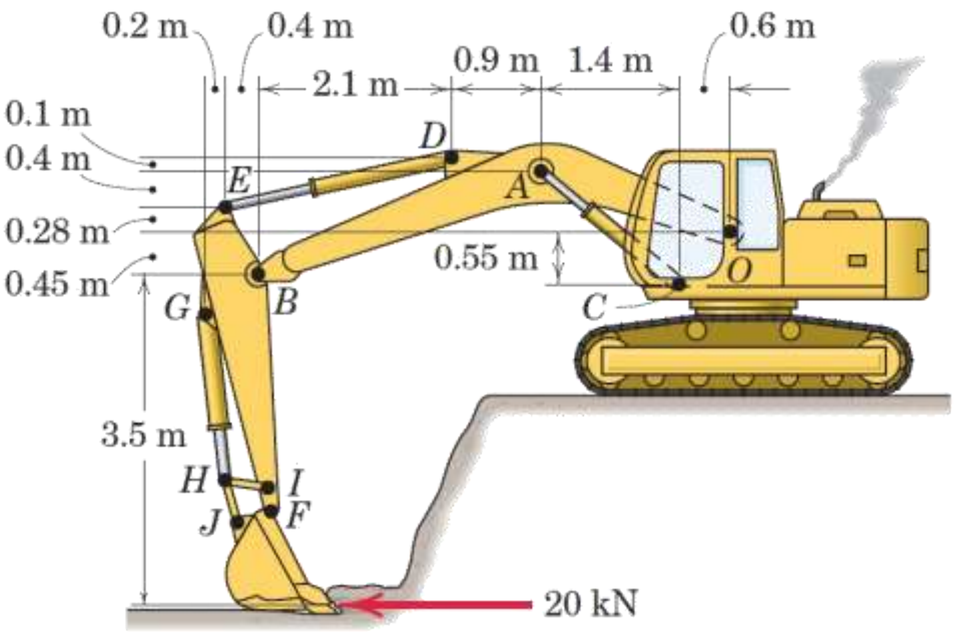


② Be careful not to forget the moment of the y-component of B. Note that our units here are newton-millimeters.

Article 4/6 – Sample Problem 4/9 (1 of 3)

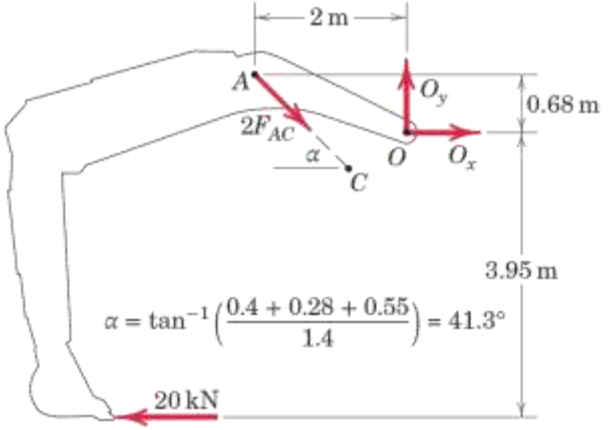
- Problem Statement

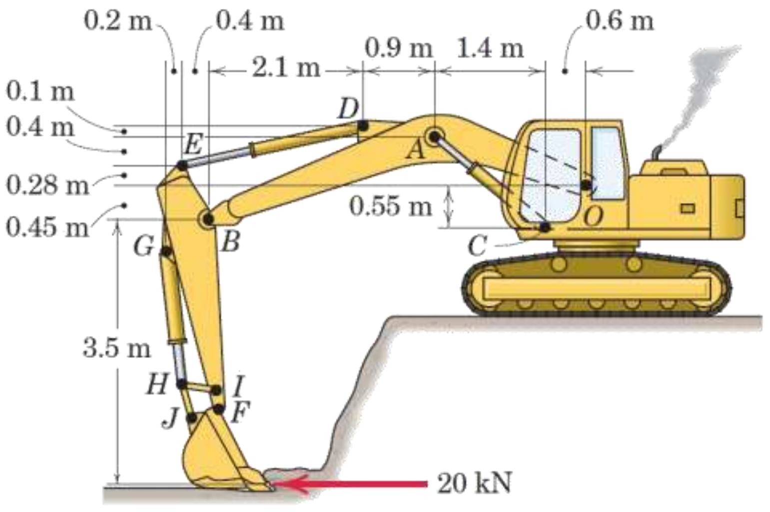
In the particular position shown, the excavator applies a 20-kN force parallel to the ground. There are two hydraulic cylinders *AC* to control the arm *OAB* and a single cylinder *DE* to control arm *EBIF*. (*a*) Determine the force in the hydraulic cylinders *AC* and the pressure p_{AC} against their pistons, which have an effective diameter of 95 mm. (*b*) Also determine the force in hydraulic cylinder *DE* and the pressure p_{DE} against its 105-mm-diameter piston. Neglect the weights of the members compared with the effects of the 20-kN force.



Article 4/6 – Sample Problem 4/9 (2 of 3)

- Free-Body Diagram of Entire Arm Assembly





- Force and Pressure in Cylinder *AC*

$[\Sigma M_O = 0]$

$$-20\,000(3.95) - 2F_{AC}\cos 41.3°(0.68) + 2F_{AC}\sin 41.3°(2) = 0$$

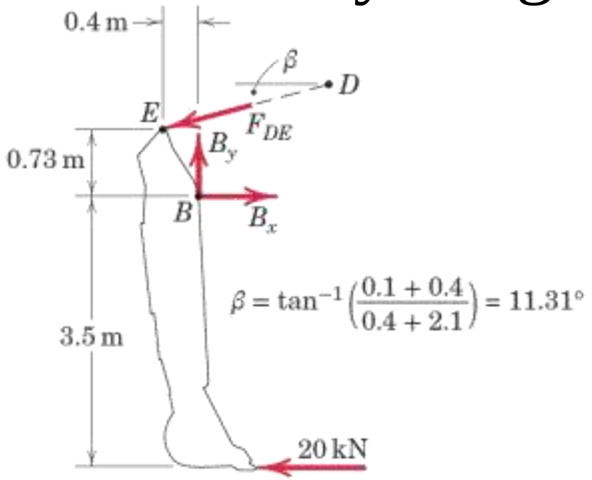
$$F_{AC} = 48\,800 \text{ N or } 48.8 \text{ kN} \qquad \textit{Ans.}$$

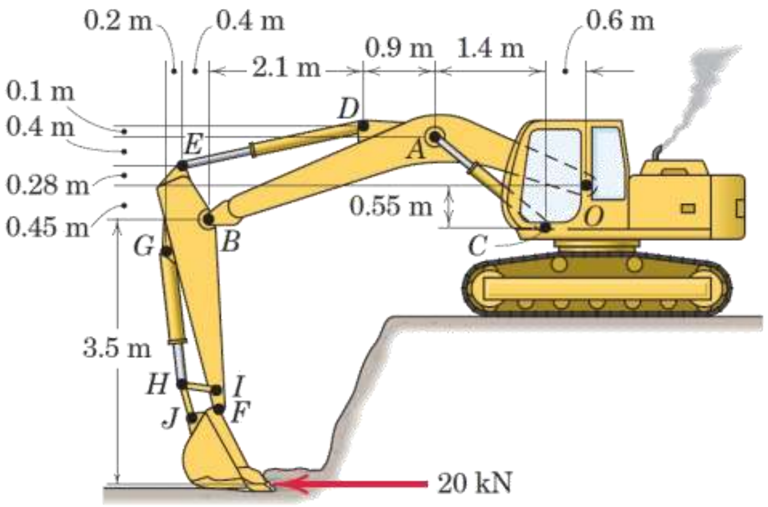
$$p_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{48\,800}{\left(\pi\frac{0.095^2}{4}\right)} = 6.89(10^6)\text{ Pa or } 6.89 \text{ MPa} \quad ① \qquad \textit{Ans.}$$

① Recall that force = (pressure)(area).

Article 4/6 – Sample Problem 4/9 (3 of 3)

- Free-Body Diagram of Lower Arm





- Force and Pressure in Cylinder *DE*

$[\Sigma M_B = 0]$

$$-20\,000(3.5) + F_{DE}\cos 11.31°(0.73) + F_{DE}\sin 11.31°(0.4) = 0$$

$$F_{DE} = 88\,100 \text{ N or } 88.1 \text{ kN} \qquad \textit{Ans.}$$

$$p_{DE} = \frac{F_{DE}}{A_{DE}} = \frac{88\,100}{\left(\pi\dfrac{0.105^2}{4}\right)} = 10.18(10^6) \text{ Pa or } 10.18 \text{ MPa} \qquad \textit{Ans.}$$