Introduction to Discrete Math

Felipe P. Vista IV

Reminder

- Everybody, make sure that your name in ZOOM is in the following format:
  - Ex: 202054321 Juan Dela Cruz

Not changing your name to this format

* you **will** be marked Absent * → absent?

Course Outline

- Mathematical Thinking
  - Convincing Arguments, Find Example, Recursion, Logic, Invariants
- Probability & Combinatronics
  - Counting, Probability, Random Variables
- Graph Theory
  - Graphs (cycles, classes, parameters)
- Number Theory & Cryptography
  - Arithmetic in modular form
  - Intro to Cryptography

Mathematical Thinking – Binomial Coefficients

PRACTICE COUNTING

Number of Hands

Question

What is the number of 5-card hands dealt-off a standard 52-deck card?

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Answer

$$\binom{52}{5} = \frac{52!}{5!47!} = \frac{52\times51\times50\times49\times48}{5\times4\times3\times2\times1} = 2598960$$

Two Hearts & Three Spades

Question

What is the number of 5-card hands with two hearts and three spades?

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Answer

$$\binom{13}{2}\binom{13}{3}=\left(\frac{13!}{2!(13-2)!}\right)\left(\frac{13!}{3!(13-3)!}\right)=\left(\frac{13\times12\times11\times\cdots\times1}{(2\times1)(11\times\cdots\times1)}\right)\left(\frac{13\times12\times11\times\cdots\times1}{(3\times2\times1)(10\times\cdots\times1)}\right)$$

$$=\left(\frac{13\times12}{2\times1}\right)\left(\frac{13\times12\times11}{3\times2\times1}\right)=\left(\frac{156}{2}\right)\left(\frac{176}{6}\right)=22308$$

4-Digit Numbers with "7"

Question

How many non-negative integers with at most 4 digits and at least one of the digit is "7"? i.e **7*

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Answer

Tuple total for 10 elements with 4 digits less tuple of 4-digit number without digit 7

$$= tuple_{total} - tuple_{no\,digit\,7}$$

$$= 10^4 - 9^4 = 3439$$

Code

```
from itertools import product

count = 0
for d in product(range(10), repeat = 4):
    if 7 in d:
        count += 1

print(count)
print(10**4 - 9**4)   #code for 10^4 - 9^4
```

Online python compilers:

- https://repl.it/languages/python3
- https://www.tutorialspoint.com/execute_python_online.php
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4-Digit Numbers with Increasing Digits

Question

What is the most number of non-negative integers with at most four digits that are sorted in increasing order?

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Question

What is the most number of non-negative integers with at most four digits that are sorted in increasing order?

Answer

10 choose 4

$$\binom{10}{4} = 210$$

Code

```
from itertools import product

count = 0
for d in product(range(10), repeat = 4):
    if d[0] <= d[1] and d[1] <= d[2] and d[2] <= d[3] :
        count += 1
        print(d)

print(count)
```

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// OUTPUT
(0,0,0,0)       (0,0,0,3)       ...             (9,9,9,8)
(0,0,0,1)       (0,0,0,4)       (9,9,9,6)       (9,9,9,9)
(0,0,0,2)       (0,0,0,5)       (9,9,9,7)
```

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```

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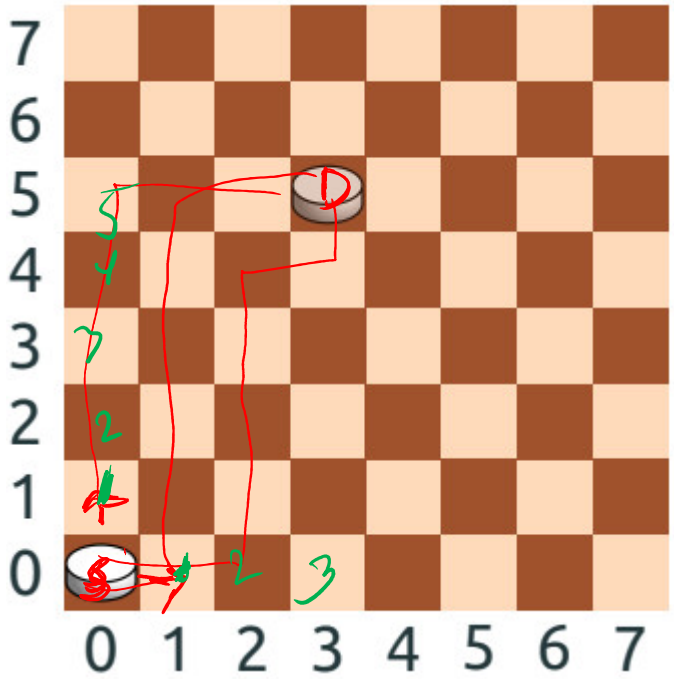
print(count)

// OUTPUT
(0,1,2,3)       (0,1,2,6)       ...             (5,7,8,9)
(0,1,2,4)       (0,1,2,7)       (5,6,7,9)       (6,7,8,9)
(0,1,2,5)       (0,1,2,8)       (5,6,8,9)       210
```

Piece on a Chessboard

Question

A piece can move one step up or one step right. How many number of ways can we go from cell[0,0] to cell [5,3]?



Piece on a Chessboard

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  - Three moves to the right plus five moves going upwards

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  - Note that any combination of three moves to the right and five moves upward is valid path to the goal(cell[5,3])

Piece on a Chessboard

Solution

- There are exactly eight moves to get to the goal
  - Three moves to the right plus five moves going upwards
  - Note that any combination of three moves to the right and five moves upward is valid path to the goal(cell[5,3])
- Therefore, the **answer** is 8 choose 3

$$\binom{8}{3} = 56$$

$$\frac{8!}{3!(8-5)!} = \frac{8!}{3!(5!)} =$$

Piece on a Chessboard

Solution

- There are exactly eight moves to get to the goal
  - Three moves to the right plus five moves going upwards
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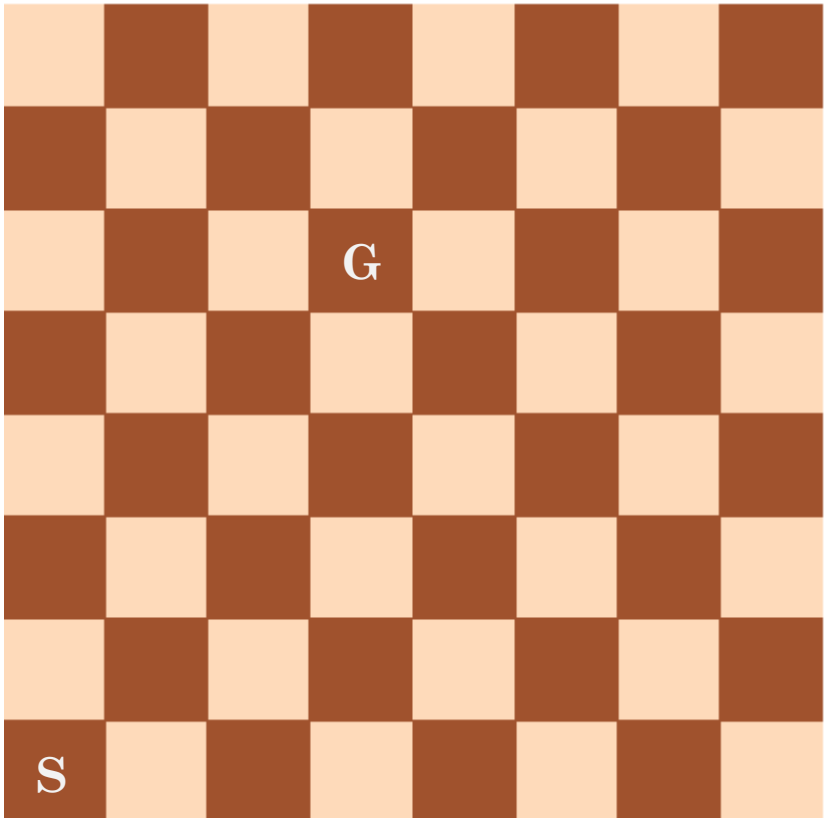
$$\binom{8}{3} = 56 = \binom{8}{5} ???????$$

$$\frac{8!}{3!(8-3)!} = \frac{8!}{5!(8-5)!}$$

Solution using Pascal's Triangle



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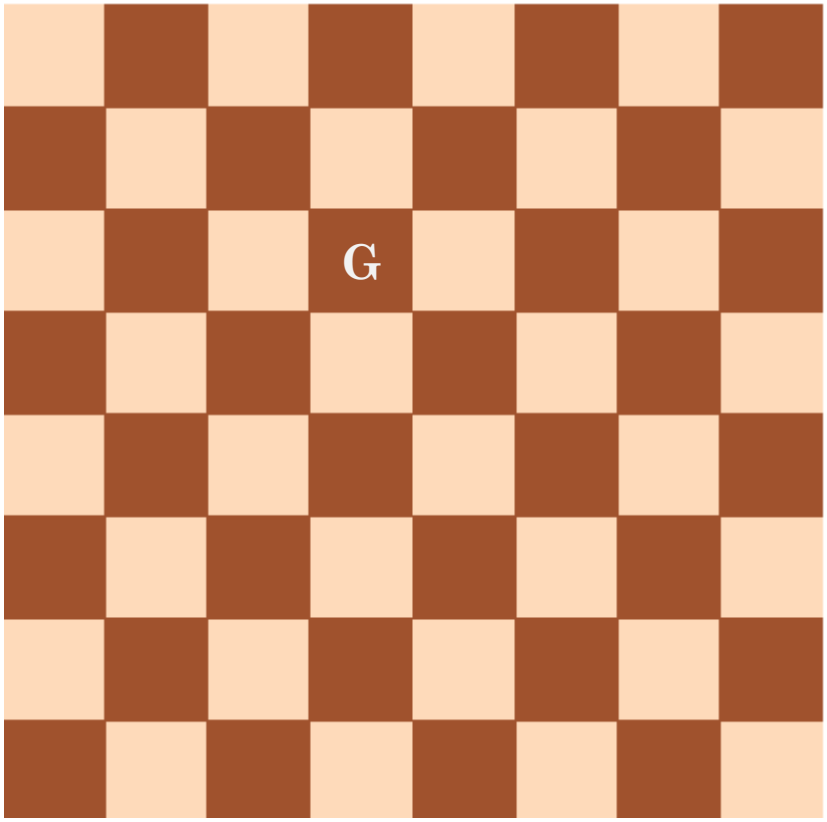

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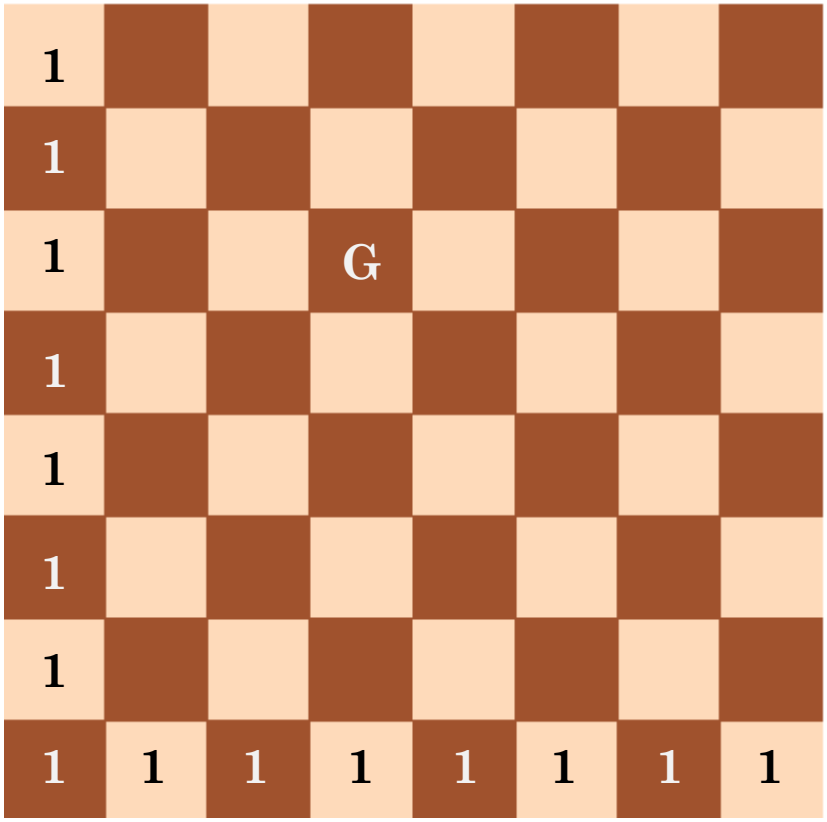

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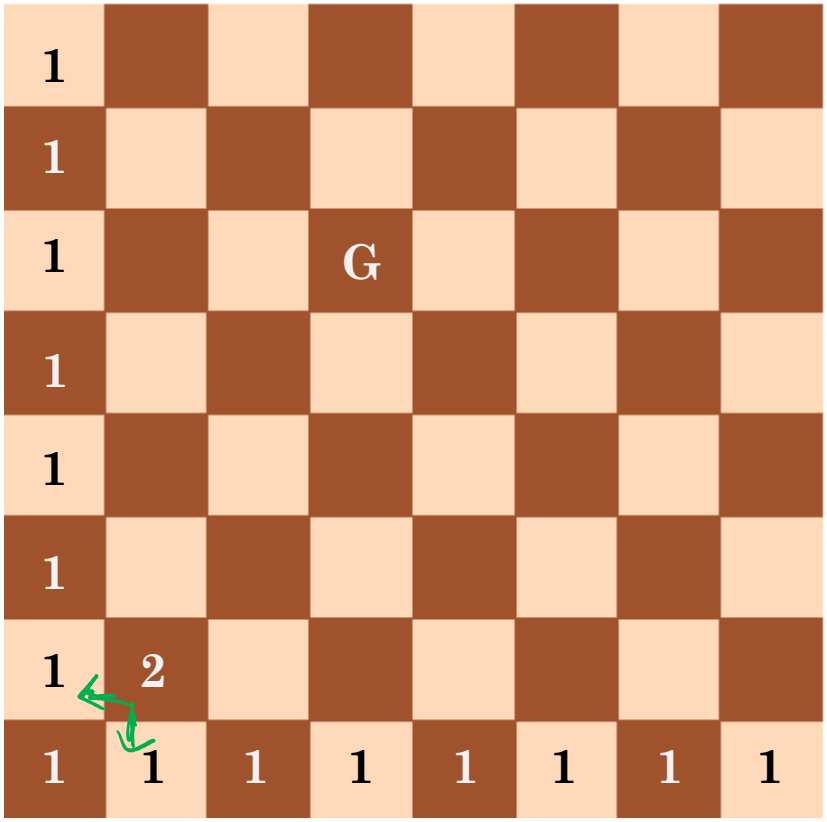

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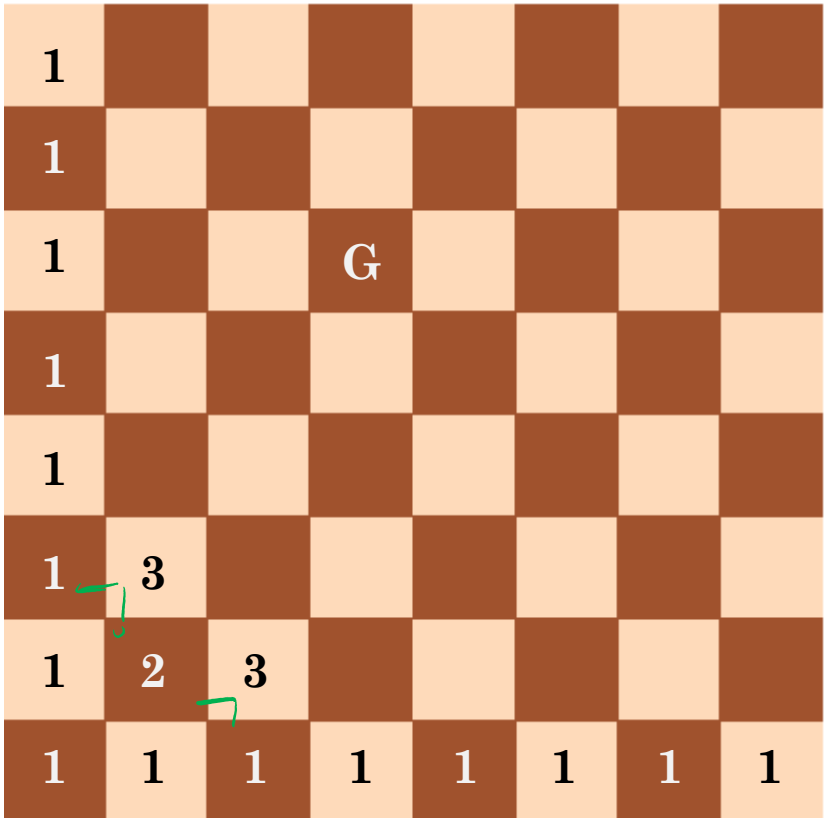
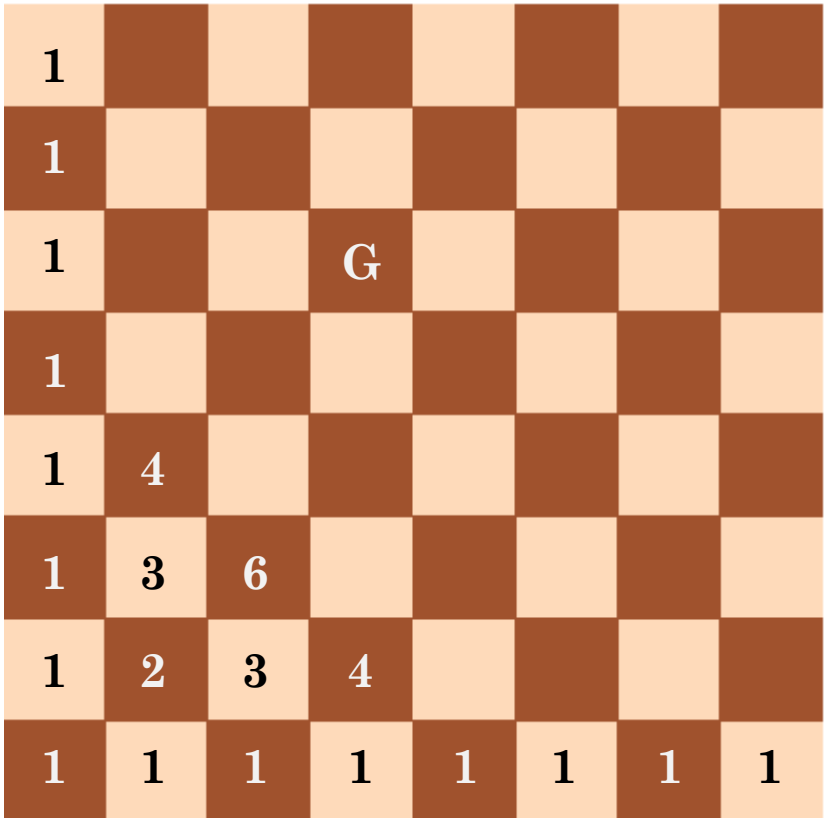


Solution using Pascal's Triangle



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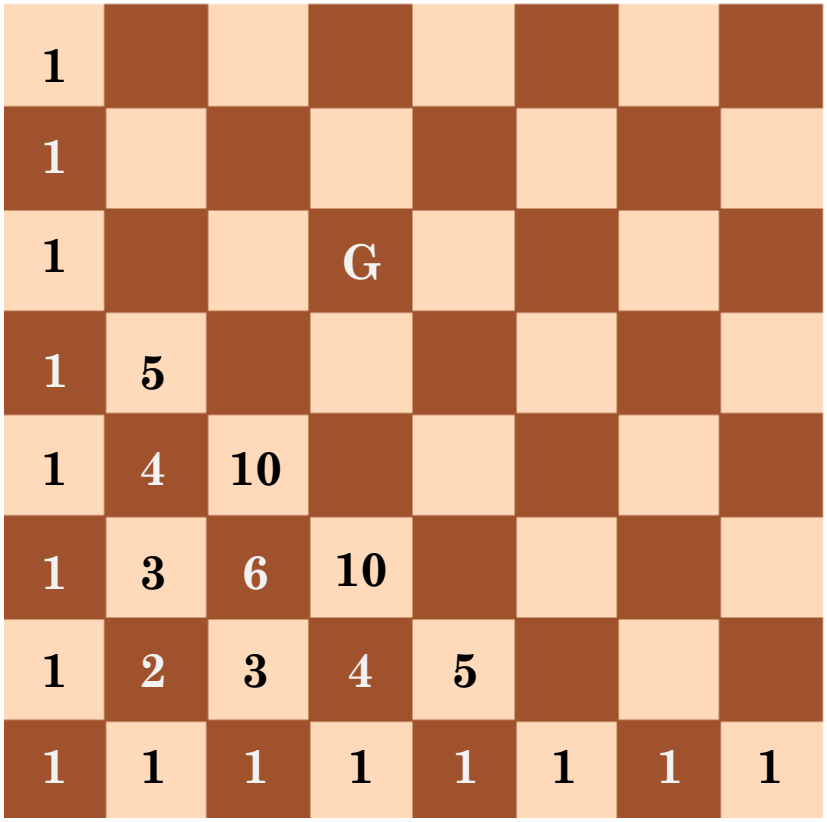

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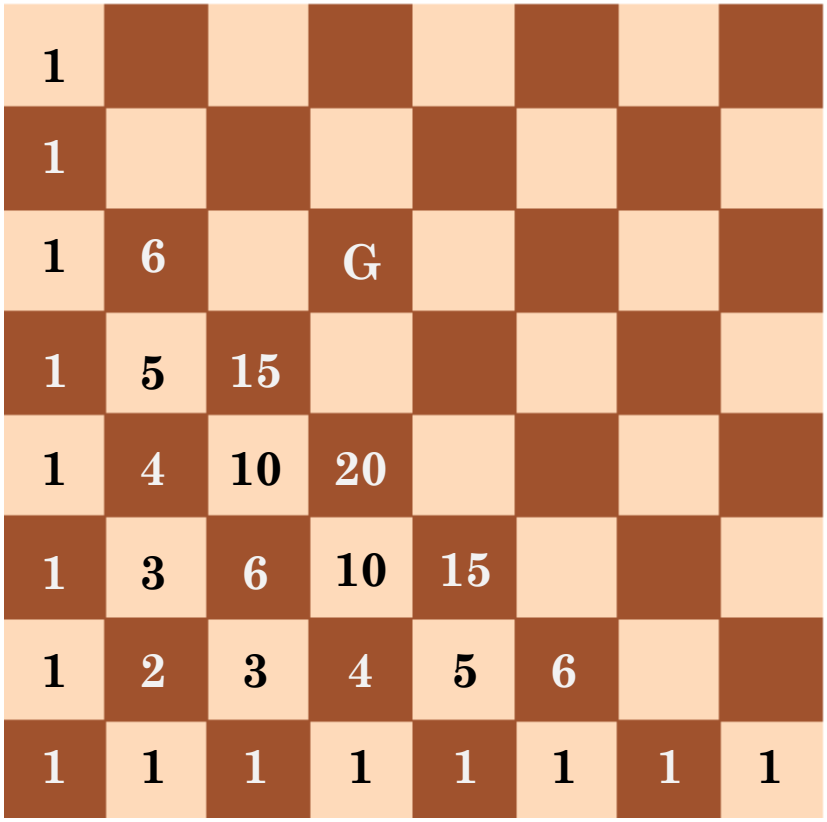

Solution using Pascal's Triangle

1							
1	7						
1	6	21					
1	5	15	35				
1	4	10	20	35			
1	3	6	10	15	21		
1	2	3	4	5	6	7	
1	1	1	1	1	1	1	1

Solution using Pascal's Triangle

1	8						
1	7	28					
1	6	21	56				
1	5	15	35	70			
1	4	10	20	35	36		
1	3	6	10	15	21	28	
1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1

Thank you.