

AEM2(Class 15, Final Exam) 2021.12.13

Department:

Student Id Number:

Student Name:

Important Notice:

1. Send your solution by email to twjeong@jbnu.ac.kr by 10:35, Dec. 13.
 2. File name of your solution:
AEM2_Final_2021-2_YourName_Id-Number.
 3. Show your solutions in detail. No points will be allowed for solutions without intermediate procedure.
 4. Partial points are given for incorrect or partial solution.
 5. Each of ten problems has the same weighting factor.
 6. May use any material, but other person's help is strictly prohibited.
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1. Integrate $\oint_C (\bar{z})^2 dz$, C : clockwise unit circle centered at $(0, 0)$.

2. Determine $\oint_C \frac{z^2 - 1}{z^2 + 1} dz$, C : counterclockwise circle $|z - i| = 3$.

3. Determine $\oint_C \frac{z^2 + 2}{(z + 1)^2(z^2 + 1)} dz$, C : CCW rectangle with vertices $(\pm 2, \pm 2)$, $(\pm 2, \mp 2)$.

4. Determine if $\sum_{n=5}^{\infty} \left[\frac{(-1)^{n+2}}{2n-5} + \frac{2}{(n+1)^2+5} \right]$ converges.

5. Determine the region of convergence for $\sum_{n=0}^{\infty} [3^n + (-3)^n + 3^{-n}](2z - i)^n$

6. Determine the region of convergence for

$$\sum_{n=0}^{100} \frac{n^3 + 2n^2 + 1}{4n^3 + 9} \frac{(z+1)^n}{10^{n+1}} + \sum_{n=101}^{\infty} \frac{2n^2 + 9}{3^n (n^2 + 10n + 5)} (z-i)^{2n+1}.$$

7. Find all Laurent series of $f(z) = \frac{4z-1}{z^2+z-2}$ with center 1.

8. Evaluate $\oint_C \left(\frac{z^2+3}{z^4-2z^2+1} + e^{1/z} \right) dz$, $C: |z-i-1|=2$, CCW.

9. Integrate $\int_0^\pi \frac{10}{4+3\cos\theta} d\theta$.

10. Find the principal value of $\int_{-\infty}^{\infty} \frac{3x+2}{(x^2+9)(x-4)x} dx$.