# Math2(Class 29, Midterm Exam) 2021.10.25

**Department:**

**Student Id Number:**

**Student Name:**

1. **Determine whether the sequence converges or diverges. If it converges, find the limit. Justify your answer.(20points)**

   **(a)** $a_n = \cos n\pi + (-1)^n$

   **(b)** $a_n = \dfrac{2n + (-1)^n 5}{4n - (-1)^n 3}$

   **(c)** $a_n = \dfrac{(\ln n)^2}{n}$

   **(d)** $a_n = n^{1/n}$

   **(e)** $a_n = \dfrac{100^n}{n!}$

2. **Determine whether the series converges or diverges. Justify your answer.(20points)**

   **(a)** $\sum_{n=1}^{\infty} (-5)^n n^{-3}$

   **(b)** $\sum_{n=1}^{\infty} 9^n / (\pi^{2n} + 10)$

   **(c)** $\sum_{n=1}^{\infty} \left[ (-1)^{n+1} / (10n + 3) \right]$

   **(d)** $\sum_{n=1}^{\infty} (-1)^n n^2 3^{-n}$

   **(e)** $\sum_{n=1}^{100} n^2 + \sum_{n=1}^{\infty} 1 / (5n + 3)$

3. **Answer the questions.(20points)**

(a) **Find the ROC(radius of convergence) for** $\displaystyle\sum_{n=0}^{\infty} \frac{(3n)!}{2^{2n}(n!)^3}(x-\pi/2)^n$.

(b) **Find the ROC for** $\displaystyle\sum_{n=0}^{100} \frac{n!}{2^n}(x+3)^n + \sum_{n=101}^{\infty} \frac{5n^2+7n+10}{3n^2+2n+5}(x-5)^{n+1}$.

(c) **Find the ROC(radius of convergence) for** $\displaystyle\sum_{n=0}^{\infty} \frac{n^n}{n!}(x-\pi)^n$.

(d) **Find the sum for** $\displaystyle\sum_{n=1}^{\infty} \left[\frac{1}{n^2+5n+6} - \frac{1}{5^n}\right]$.

(e) **Find the sum for** $\displaystyle\sum_{n=1}^{\infty} 4^{3+n}\pi^{-2n+1}$.

4. **Answer the questions.(20points)**

(a) **Find a Taylor series for** $1/x$ **with center 1.**

(b) **Find a Taylor series for** $\sin x$ **with center** $\pi/2$

(c) **Find a Maclaurin series for** $\sin(2x^2)$

(d) **Find a Maclaurin series for** $\sin^2 x$

5. **Solve the problems.(20points)**

(a) **Sketch the curve** $x=2+\sin t, \ y=\cos^2 t, \ 0 \le t \le 2\pi$.

(b) **Find equations of the tangents to the curve**
$x=t^2+t-1, \ y=t^2+3t-5$ **at (5,5).**

(c) **Find the area enclosed by the curve**
$x=2\cos t, \ y=t-\cos t, \ 0 \le t \le 2\pi$ **and the y-axis.**

(d) **Find the length of the curve**
$x=2\cos^2 t, \ y=2\cos t \sin t, \ \pi/3 \le t \le 3\pi/4$.