

Math2(Class 29, Midterm Exam) 2021.10.25

Department:

Student Id Number:

Student Name:

Important Notice:

1. Send your solution by email to twjeong@jbnu.ac.kr by 17:50, Oct. 25.
2. File name of your solution:
Math2_Midterm_2021-2_YourName_Id-Number.
3. Show your solutions in detail. No points will be allowed for solutions without intermediate procedure.
4. Partial points are given for incorrect or partial solution.
5. May use any material, but other person's help is strictly prohibited.

1. Determine whether the sequence converges or diverges. If it converges, find the limit. Justify your answer.(20points)

(a) $a_n = \cos n\pi + (-1)^n$

(b) $a_n = \frac{2n + (-1)^n 5}{4n - (-1)^n 3}$

(c) $a_n = \frac{(\ln n)^2}{n}$

(d) $a_n = n^{1/n}$

(e) $a_n = \frac{100^n}{n!}$

2. Determine whether the series converges or diverges. Justify your answer.(20points)

(a) $\sum_{n=1}^{\infty} (-5)^n n^{-3}$

(b) $\sum_{n=1}^{\infty} 9^n / (\pi^{2n} + 10)$

(c) $\sum_{n=1}^{\infty} [(-1)^{n+1} / (10n + 3)]$

(d) $\sum_{n=1}^{\infty} (-1)^n n^2 3^{-n}$

(e) $\sum_{n=1}^{100} n^2 + \sum_{n=1}^{\infty} 1 / (5n + 3)$

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3. Answer the questions.(20points)

- (a) Find the ROC(radius of convergence) for $\sum_{n=0}^{\infty} \frac{(3n)!}{2^{2n}(n!)^3} (x - \pi/2)^n$.
- (b) Find the ROC for $\sum_{n=0}^{100} \frac{n!}{2^n} (x+3)^n + \sum_{n=101}^{\infty} \frac{5n^2+7n+10}{3n^2+2n+5} (x-5)^{n+1}$.
- (c) Find the ROC(radius of convergence) for $\sum_{n=0}^{\infty} \frac{n^n}{n!} (x - \pi)^n$.
- (d) Find the sum for $\sum_{n=1}^{\infty} \left[\frac{1}{n^2+5n+6} - \frac{1}{5^n} \right]$.
- (e) Find the sum for $\sum_{n=1}^{\infty} 4^{3+n} \pi^{-2n+1}$.

4. Answer the questions.(20points)

- (a) Find a Taylor series for $1/x$ with center 1.
- (b) Find a Taylor series for $\sin x$ with center $\pi/2$
- (c) Find a Maclaurin series for $\sin(2x^2)$
- (d) Find a Maclaurin series for $\sin^2 x$

5. Solve the problems.(20points)

- (a) Sketch the curve $x = 2 + \sin t$, $y = \cos^2 t$, $0 \leq t \leq 2\pi$.
- (b) Find equations of the tangents to the curve
 $x = t^2 + t - 1$, $y = t^2 + 3t - 5$ at (5,5).
- (c) Find the area enclosed by the curve
 $x = 2\cos t$, $y = t - \cos t$, $0 \leq t \leq 2\pi$ and the y-axis.
- (d) Find the length of the curve
 $x = 2\cos^2 t$, $y = 2\cos t \sin t$, $\pi/3 \leq t \leq 3\pi/4$.