

Hypothesis testing for a proportion

Hypothesis testing for a single proportion

1. Set the hypotheses :

$$H_0 : p = \text{null value}$$

$$H_A : p < \text{or } > \text{or } \neq \text{ null value}$$

2. Calculate the point estimate: \hat{p}

3. Check conditions:

Independence: sampled observations must be independent (random sample/assignment & if sampling without replacement, $n < 10\%$ of population)

Sample size/skew: $np \geq 10$ and $n(1-p) \geq 10$

4. Draw sampling distribution, shade p-value, calculate test statistic

$$Z = \frac{\hat{p} - p_0}{SE}, \quad SE = \sqrt{\frac{p_0(1 - p_0)}{n}}$$

5. Make a decision, and interpret it in context of the research question:

if $p\text{-value} < \alpha$, reject H_0 ; the data provide convincing evidence for H_A

if $p\text{-value} > \alpha$, fail to reject H_0 ; the data do not provide convincing evidence for H_A

CI vs. HT for proportions

Success-failure condition:

- CI: At least 10 *observed* successes and failures
- HT: At least 10 *expected* successes and failures,
calculated using the null value

Standard error: $SE = \sqrt{\frac{p(1-p)}{n}}$

- CI: calculate using observed sample proportion:

$$SE = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

- HT: calculate using the null value:

$$SE = \sqrt{\frac{p_0(1-p_0)}{n}}$$

CI vs. HT for proportions

	confidence interval	hypothesis test
success-failure condition	$n\hat{p} \geq 10$ $n(1 - \hat{p}) \geq 10$	$np_0 \geq 10$ $n(1 - p_0) \geq 10$
standard error	$SE = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$	$SE = \sqrt{\frac{p_0(1 - p_0)}{n}}$

Practice

The GSS found that 571 out of 670 (85%) of Americans answered the question on experimental design correctly. Do these data provide convincing evidence that more than 80% of Americans who have a good intuition about experimental design?

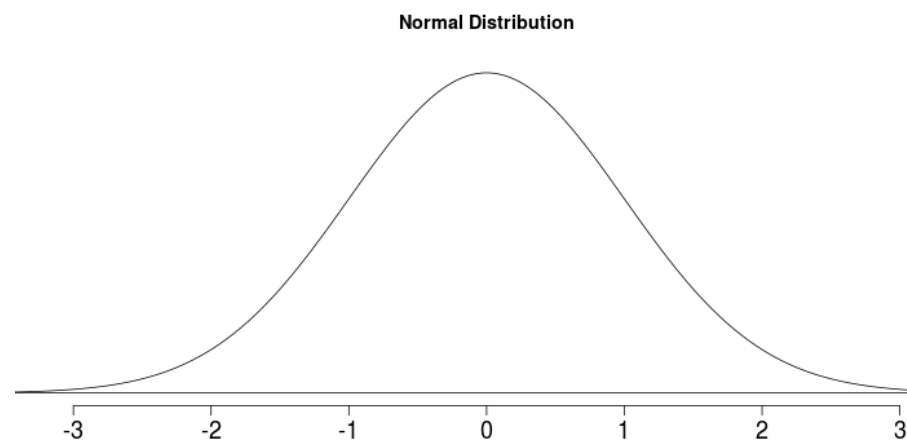
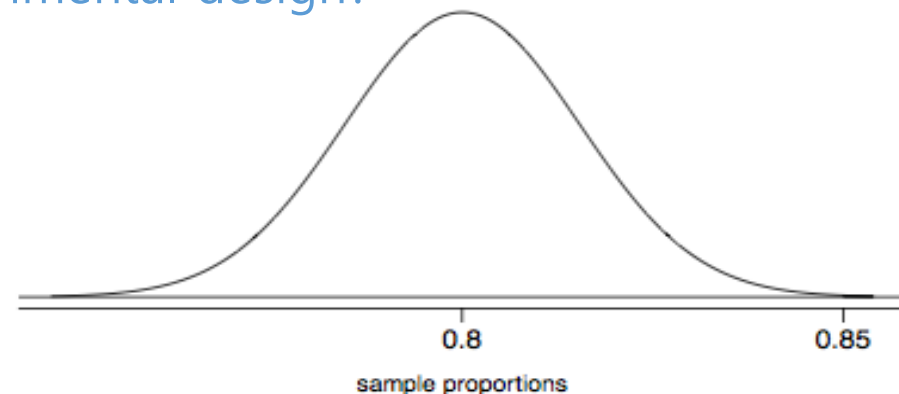
$$H_0 : p = 0.80 \quad H_A : p > 0.80$$

$$SE = \sqrt{\frac{0.80 \times 0.20}{670}} = 0.0154$$

$$Z = \frac{0.85 - 0.80}{0.0154} = 3.25$$

$$p\text{-value} = 1 - 0.9994 = 0.0006$$

Since the p-value is low, we reject H_0 . The data provide convincing evidence that more than 80% of Americans have a good intuition on experimental design.



Recap - inference for one proportion

Population parameter: p , point estimate: \hat{p}

Conditions

- independence
 - random sample and 10% condition
- at least 10 successes and failures
 - if not \rightarrow randomization

Standard error: $SE = \sqrt{\frac{p(1-p)}{n}}$

- for CI: use \hat{p}
- for HT: use p_0

Assignment

Problem 1 11% of 1,001 Americans responding to a 2006 Gallup survey stated that they have objections to celebrating Halloween on religious grounds. At 95% confidence level, the margin of error for this survey is $\pm 3\%$. A news piece on this study's findings states: "More than 10% of all Americans have objections on religious grounds to celebrating Halloween." At 95% confidence level, is this news piece's statement justified?

- (a) Yes
- (b) No
- (c) Can't tell

Assignment

Problem 2 A 2013 Pew Research poll found that 60% of 1983 randomly sampled American adults believe in evolution. Does this provide convincing evidence that majority of Americans believe in evolution? Give an answer to this question by conducting hypothesis test.