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Linear Algebra(Class 6, Midterm Exam) 2020.10.19

1. Let A(1, 2, 3), B(4, 5, 6), C(7, 1, 3), and D(2 3, 5). (20points)
   (a) $\overrightarrow{BA} =$ (b) $2\overrightarrow{AB} - 3\overrightarrow{BC} =$
   (c) $\|3\overrightarrow{BC} - 2\overrightarrow{CA}\| =$ (d) $\overrightarrow{AB} \cdot (\overrightarrow{CD} - 2\overrightarrow{BC}) =$
   (e) Find the angle between $\overrightarrow{AB}$ and $\overrightarrow{BC}$.
   (f) Find the equation of the plane that passes through points A, B, and C.
   (g) Find a normal vector for the plane $2x+6y-7z=10$.

2. Answer the questions for the given nonhomogeneous linear system.(20points)
   (a) Find the augmented matrix.
   (b) Change the augmented matrix into a reduced row echelon form using Gauss-Jordan elimination.
   (c) Find a general solution of the linear system.

$$\begin{aligned} -3x_1 + 2x_2 + 6x_3 + x_4 &= 5 \\ 3x_2 + 3x_3 - 5x_4 &= 2 \\ 2x_1 + 4x_2 + 4x_3 - 6x_4 &= -8 \end{aligned}$$

3. Solve the systems $AX_1=b_1$, $AX_2=b_2$, and $AX_3=b_3$ where A, b_1, b_2, and b_3 are given as follows:(20points=5+10+5)
   (a) Find the augmented matrix.
   (b) Change the augmented matrix into a row echelon form using Gauss-Jordan elimination.
   (c) Find solutions of the linear systems.

$$\mathbf{A} = \begin{bmatrix} -3 & 2 & 2 \\ 1 & 4 & -6 \\ 0 & -2 & 2 \end{bmatrix} \quad \mathbf{b}_1 = \begin{bmatrix} 8 \\ 1 \\ -2 \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} 1 \\ 8 \\ 2 \end{bmatrix} \quad \mathbf{b}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

4. Consider a linear system AX=b where A and b are given below.(20points)
   (a) Factorize A as A=*LU.*
   (b) Factorize A as A=*LDU.*
   (c) Solve the linear system using (a).

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 3 \\ -2 & -2 & 7 \\ -5 & 0 & 20 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}$$

5. Answer the questions for the matrix given below.(20points)
   (a) Determine the minor M_{23} and cofactor C_{23}.
   (b) Determine det(2A).
   (c) Determine the value of b so the linear system AX=b may be consistent.
   (d) Find $\det(A^{-1})$.

$$\mathbf{A} = \begin{pmatrix} -3 & 3 & 9 & 6 \\ 1 & -2 & 15 & 6 \\ 7 & 1 & 1 & 5 \\ 2 & 1 & -1 & 3 \end{pmatrix}$$