

# REPORT ASSIGNMENT 2

The Sudoku problem

**RICKARD SÖRLIN**

### 1a) A representation of a solution answer of the problem

To represent a solution for the Sudoku problem the variables can be represented as follows.  
 $X = \{ X_{ij} : \forall i, j \in \{ 1,2,3,4,5,6,7,8,9 \} \}$  where  $X_{ij}$  is representing the variables in i-th row and j-th column.

The Sudoku is a constraint satisfaction problem, and the domain needs to be defined.  
 $D = \{ D_{ij} : \forall i, j \in \{ 1,2,3,4,5,6,7,8,9 \} \}$  where each variable  $X_{ij}$  has their own domain  $D_{ij}$  that represents what value can be assigned to it to be consistent with the constraints.

*The Sudoku has an initial state for the problem where some of the variables can have a predefined value from their domain that is consistent to the constraints.*

*To solve the problem all empty cell that is represented by a variable on the board needs to be assigned a value from the domain and still be consistent with all the constraints and pass the goal test to reach the goal state of the problem .*

### 1b) The restrictions of the problem (constraints).

The Sudoku have three constraints that the algorithm needs to follow to reach the goal state. The first one is that none of the values from the variable's domain  $D_{ij}$  is allowed to occur more than once in a row and is defined as follow.

#### Row Constraints

$\forall i \in \{1,2,3,4,5,6,7,8,9\}$  all different  $\{x_{i1}, x_{i2}, x_{i3}, x_{i4}, x_{i5}, x_{i6}, x_{i7}, x_{i8}, x_{i9}\}$

The second constraint is that none of the values from the variable's domain  $D_{ij}$  is allowed to occur more than once in a column and is defined as follow.

#### Column Constraints

$\forall j \in \{1,2,3,4,5,6,7,8,9\}$  all different  $\{x_{1j}, x_{2j}, x_{3j}, x_{4j}, x_{5j}, x_{6j}, x_{7j}, x_{8j}, x_{9j}\}$

The third constraint is that none of the values from the variable's domain  $D_{ij}$  is allowed to occur more than once in each 3\*3 subgrid and is defined as follow.

#### 3\*3 sub grid Constraints

$\forall i,j$  all different  $\{ x_{ij}, x_{i(j+1)}, x_{i(j+2)}, x_{(i+1)j}, x_{(i+1)(j+1)}, x_{(i+1)(j+2)}, x_{(i+2)j}, x_{(i+2)(j+1)}, x_{(i+2)(j+2)} \}$

**1c) What is considered as a state?, in addition, explain why.**

A state can be the current layout of the sudoku board for example at the beginning the initial state where algorithm hasn't begun to solve the Sudoku.

When the algorithm has started to solve the Sudoku by exploring different states in the search space by assigning a value from the domain to each variable.

The current state changes to a new state each time a new value is assigned to a variable from the domain that is consistent to the constraints.

When all variables are assigned a value from the domain and is consistent according to the constraints the current state is changed to a goal state.

So there is different type of states, and the state is changing each time a variable is assigned a value and helps the algorithm to progress to the goal state and backtrack if constraints isn't satisfied.

**1d) Which is the initial state? In addition, explain why.**

The initial state is at the beginning the starting point where some of the cells have unsigned values that is represented with zeros and some cells already have numbers assigned in the range 1 to 9 but is consistent to the constraints.

The initial state is where the algorithm hasn't begun to solve the Sudoku problem normally less assigned values gives less information and larger search space of states for the algorithm to reach the goal state from the initial state.

**1e). Which is/are the possible actions? in addition, explain why.**

The possible actions are to assign a value from the domain to the variable to progress to goal state and solve the Sudoku. When the constraints is not met for a variable the algorithm needs to backtrack and the other possible action is to remove the assigned value from the variable so the constraints is met.

**1f) What is the maximum branching factor of the tree (b)? in addition explain why .**

The maximum Branching factor (b) is 9 for an empty cell/variable because it can be assigned a number in the range 1 to 9 in worst case if the branching factor isn't reduced/pruned in the code implementation.

**1g) What is the maximum depth of the search tree (m)? in addition , explain why.**

The Maximum depth (m) of the search tree is depending on the total amount of empty cells variables in the initial state and will increase for each.

Each Sudoku can have different initial states with different number of empty cells and by that will the search tree have different depth. For each of these empty cells a decision needs to be made where a new level is created in the tree and makes the tree deeper

But the maximum depth (m) can be maximum 81 for a 9 column and 9 row Sudoku.