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Project portfolio selection and planning with fuzzy constraints

Fátima Pérez[a], Trinidad Gómez[b], Rafael Caballero[b], Vicente Liern[c,*]

[a] Department of Applied Economics (Statistics and Econometrics), Faculty of Economics and Business Studies, University of Málaga, C/Ejido 6, 29013, Málaga, Spain
[b] Department of Applied Economics (Mathematics), Faculty of Economics and Business Studies, University of Málaga, C/Ejido 6, 29013, Málaga, Spain
[c] Department of Business Mathematics, Faculty of Economics, University of Valencia, Avda. Tarrongers s/n, 46022 Valencia, Spain

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ABSTRACT

Selecting a project portfolio is a complex process involving many factors and considerations from the time it is proposed to the time the project portfolio is finally selected. Given that making a good selection is of crucial importance, it is essential to develop well-founded mathematical models to lead the organization to its final goal. To achieve this, such models have to reflect as closely as possible both the real situation of the organization as well as its targets and preferences.

However, since the process of selecting and implementing project portfolios occurs in real environments and not in laboratories, uncertainty and a lack of knowledge regarding some data is always an important issue due to the strong interdependence between the projects and the political, economic, social, and legal conditions in which they are carried out.

In this work, a mathematical model is proposed which extends the classical approach incorporating the inherent uncertainty to these problems. We have handled this uncertainty, vagueness and/or imprecision through the use of fuzzy parameters, which allow representation of information not fully known by the decision makers. The model combines selecting and planning project portfolios, specifies different relationships between projects (synergies, incompatibilities, time order, etc.) and other important constraints appearing in real situations. Moreover, a resolution procedure is developed which obtains, simultaneously, the optimal portfolio and the range for the confidence levels associated to it. An illustrative example and a real application are given in order to show the potentiality of the approach. The results are complemented with graphical tools, which show the usefulness of the proposed model to assist the decision makers.

1. Introduction

Organizations typically pursue a wide variety of objectives that cannot easily be achieved by a single project. Therefore, groups of projects (i.e. portfolios) that share a limited number of resources over a given period of time have to be selected (Archer and Ghasemzadeh, 1999). A project portfolio is the set of projects selected that can achieve the established objectives (Li et al., 2016).

There is a wealth of literature on the many methods used in the field of project portfolio selection (see for instance Heidenberger and Stummer, 1999; Iamratanakul et al., 2008). One set of widely used techniques focuses on the ranking of the investment required for each proposal with the aim at then distributing the budget until it is fully spent (e.g. financial methods (Silvola, 2006), scoring methods (Lawson et al., 2006), Analytical Hierarchy Process (Feng et al., 2011), and multiple attribute utility theory (Duarte and Reis, 2006)). However, these approaches are not always feasible for three main reasons:

a) They usually only take into account budget constraints. However, organizations have to deal with other constraints related to staff and resources as well as political, social, and environmental factors that also act as constraints (Mavrotas et al., 2008).
b) The dynamic nature of the process is not usually taken into account. Budget constraints usually refer to one period of time with all the selected projects usually starting at the same time. This is quite restrictive because a degree of flexibility regarding implementation or execution time may lead to a better distribution of resources (Jafarzadeh et al., 2015).
c) There may be complementarity and incompatibility relationships as well as synergies between the candidate projects such that they are not independent of each other; thus, the best projects when taken individually may not necessarily form the best set when taken as a group (Chien, 2002)).

Consequently, selecting the projects that best match the needs, requirements, and objectives of an organization is a complex task.

* Corresponding author.
E-mail address: vicente.liern@uv.es (V. Liern).

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Multiple factors have to be taken into account throughout the decision process. All this has led to growing interest in other techniques derived from mathematical programming that are able to better incorporate a greater degree of complexity (Rabbani et al., 2010). Our study is framed within these mathematical programming methods trying to handle the above described limitations of other mathematical programming approaches. In addition, it takes into account the vague or even unknown nature of the data providing an alternative perspective for different levels of uncertainty to the rational (technocratic) solution (Martinsuo, 2013).

Concretely, in this paper, we propose a model to select and schedule, simultaneously, an optimum project portfolio among several proposals, taking into account that some parameters are fuzzy numbers. We have incorporated different interactions between some of the candidate projects, the possibility of transferring cash resources not consumed in one period to the next period, and the different temporal availability of resources or other requirements appearing in real situations. In addition, a theoretical analysis is performed to obtain, simultaneously, all the optimal solutions and the range for the confidence levels associated with each of them. The strength of this approach is to inform decision makers about how variation in confidence levels affects the optimal portfolio allowing them to make the decision on more accurate information.

The paper is structured as follows: a review about the use of mathematical programming to select and schedule an optimal portfolio is presented in Section 2; In Section 3 a description of the project portfolio selection model is provided. Section 4 deals with the parametric analyse of the solution of the problem, and an illustrative example is showed in Section 5. In Section 6, a real application to test the proposed approach is provided. The conclusions are presented in the final section.

2. Review of mathematical programming applied to project portfolio selection

The use of mathematical programming models in project management goes back to the study by Weingartner (1966), who generalized the work carried out by Lorie and Savage (1955) and formalized it as a linear programming model. In addition, he also studied project interdependencies due to incompatibility or complementarity relationships, which can be incorporated in the model as additional constraints. Subsequently, synergies derived from running more than one project simultaneously were also taken into account as this led to a better sharing of costs and/or benefits. These relationships are modelled by including additional terms when assessing a given portfolio, and may have an effect on the objective functions and/or the resource constraints, as described in the studies by Czajkowski and Jones (1986), Schmidt (1993), Dickinson et al. (2001), Zuluaga et al. (2007), Medaglia et al. (2007), Rabbani et al. (2010), Solak et al. (2010), Tofighian and Naderi (2015) among others. These studies only cover synergies between two projects. On the other hand, Santhanam and Kyparisis (1995) and, in a more general sense, Stummer and Heidenberger (2003), Carazo et al. (2010) and Li et al. (2016) have proposed models where the synergies are generalized to sets of projects. The model presented in the next section follows this latter approach.

Organizations seek solutions that enable them to plan their resources over several time periods. In other words, they seek to develop policies that favour stability and continuity allowing them to reach their overall economic, social and environmental objectives in the medium and long term. For this reason, managers face the challenging task of having to simultaneously select projects and plan them within the planning horizon best suited to the organization. The literature on this topic is scarce (Naderi, 2013), probably due to the complexity involved in simultaneously selecting and scheduling the best projects. However, adding flexibility regarding the starting point of the projects (including the time factor in the model) can lead to precedence relationships between some of them. In other words, some projects can only start if their predecessors have already finished or a certain number of time periods have passed since the predecessor project started. This is illustrated in studies by Ghasemzadeh et al. (1999), Rabbani et al. (2010), and Emami et al. (2016). The time factor also enables a better distribution of monetary resources; if some resources are not used up in a given period of the planning horizon they can be transferred to the next period (Zuluaga et al., 2007; Medaglia et al., 2008; Jafarzadeh et al., 2015). It is worth noting that some studies take into account a planning horizon during project selection, but they assume that all the projects begin in the first period (see Dickinson et al., 2001; Stummer and Heidenberger, 2003; Doerner et al., 2004, 2006).

On the other hand, as the projects are selected before they are actually implemented, the information available may be characterized by imprecision and uncertainty. In particular, the budgets or the resources required by each project and the expected benefits may vary considerable, since their value is simply estimated before the projects are running. In this sense, uncertainty regarding certain parameters in the model has to be taken into account in such a way that the solutions obtained are reliable even within contexts characterized by change.

In recent years, there has been an increase in studies on scheduling and selecting project portfolios that use fuzzy techniques to deal with uncertainty. Due to the lack of historical data it has become usual to resort to experts who, based on their own experience, suggest modal values and the variation interval expected regarding unknown parameters (Wang and Hwang, 2007). In fact, many aggregation techniques have been developed to obtain these values when more than one expert is involved or when the expert's opinions have been given at different stages (Yager, 2004). In any case, this leads to the description of each of these values as a fuzzy number where the membership function contains information about the degree of truth of the parameter.

The abovementioned parameters can be found both in the constraints and/or in the objective functions in the projects portfolio selection problem. Some authors propose flexible programming approaches to deal with uncertainty in the constraints (e.g. Pereira, 1988; Kuchta, 2000; Machacha and Bhattacharya, 2000; Mohamed and McCowan, 2001; Carlsson et al., 2007; Ke and Liu, 2007). Other authors apply possibilistic programming techniques (e.g. Wang and Hwang, 2007; Hasuike et al., 2009; Mohagheghi et al., 2015; Liu and Liu, 2017; Tavana et al., 2015).

Other interesting approach which incorporates uncertainty into the projects selection model is due to Chang and Lee (2012). These authors took the model developed by Cook and Green (2000) as their starting point and modified it by using triangular fuzzy numbers. In this way, they obtained a fuzzy data envelopment analysis (FDEA) model in which constraints are added to the objective function as penalties.

Nevertheless, depending on the type of problem addressed, most of these studies have focused on portfolio selection alone and have not included scheduling. In fact, only Ke and Liu (2007) and Bhattacharyya et al. (2011) developed models addressing both project portfolio selection and scheduling. With synergies something similar occurs. As commented before, very few studies have dealt with the issue of potential synergies between projects. For example, Wang and Hwang (2007) included synergies but only between two projects. Fernandez and Navarro (2002) circumvented the issue by arguing that synergies can be included in the model by creating a new project that would combine the information pertaining to the new synergy. Bhattacharyya et al. (2011) included synergies using a polynomial model, which involves a high computational cost. Tabrizi et al. (2016) consider the projects synergy and sourcing options under information ambiguity. Finally, none of the studies previously described include additional constraints in the models, such as precedence relationships, which are of great relevance to this problem.

In this context, our aim was to develop a model that brings together as many features as possible, taking into account different types of constraints and the uncertainty associated with certain parameters.

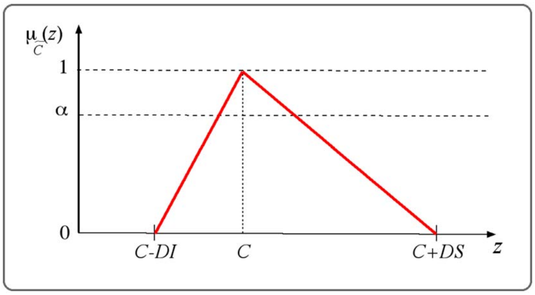

Fig. 1. Triangular fuzzy number provided by the experts.

Regarding uncertainty, we focus on some parameters in constraints related to renewable resources that are expressed as fuzzy numbers with a variability that is represented by the confidence level associated with the alpha level (α-cut) given to each resource. We chose triangular fuzzy numbers not only because they are easy to handle and reflect situations involving uncertainty very well (Zimmermann, 1996; León et al., 2003), but also because they have been successfully used in situations similar to the one we present (for example, see Chang and Lee, 2012).

3. Mathematical model

In this section we will present the proposed mathematical programming model that from now on will be denoted as P1. Let us assume an organization has to select the best group of projects from a set of I project proposals. The organization also needs to know when each project will start within a given planning horizon divided into T periods. Then, the decision variables in our model are denoted by:

$$x_{it} = \begin{cases} 1 & \text{if project } i \text{ starts at } t \\ 0 & \text{otherwise} \end{cases}$$

where $i = 1, 2, \dots, I, t = 1, 2, \dots, T$.

The vector $x = (x_{11}, x_{12}, \dots, x_{1T}, x_{21}, x_{22}, \dots, x_{2T}, \dots, x_{I1}, x_{I2}, \dots, x_{IT})$, with $T\cdot I$ binary variables represents a project portfolio. In addition d_i $(i = 1, 2, \dots, I)$ denotes the duration of each project involved in the process (months, semesters, years, etc).

3.1. Objective function

The organization needs to evaluate the candidate projects, according to a set of attributes at every period k of the planning horizon. Therefore, the objective function is defined as follows:

$$F(x) = \sum_{k=1}^{T} w_k \cdot \left(\sum_{i=1}^{I} \sum_{t=1}^{k} c_{i,k+1-t} \cdot x_{it} + \sum_{j=1}^{s} g_{jk}(x) \cdot a_{jk} \right) \tag{1}$$

where the coefficients $c_{i,k+1-t}$ are the values from the function in the period k, for project i, if this project is selected and started at t (i.e., the project would be at its execution time $k + 1 - t$).

On the other hand, the second summand in the function corresponds to the variation (increase or decrease) generated by the interrelationships or synergies that might exist between the projects in the portfolio. To take these variations into account, we use A_j, $j = 1, 2, \dots, s$, sets of independent projects in which the decision maker sets the minimum (m_j) and maximum (M_j) number of projects that must be active for the variation generated at an amount a_{jk}. Thus, the functions $g_{jk}(x)$ take value 1 if the number of active projects in set A_j, at point in time k is between the values m_j and M_j; otherwise they take the value 0. Finally, w_k are weights which enable the aggregation of time-related information.

3.2. Constraints

3.2.1. Renewable resource constraints

These constraints limit the amount of each type of resource spent at each point in time. To this end, it would be essential to know the exact amount of resources needed for each project at each execution time. However, these amounts are not fully known at the time they are included, and thus we need a procedure to include uncertainty. In this case, we formulate this uncertainty using fuzzy triangular numbers $\widetilde{C} = (C, DI, DS)$ where C denotes the modal[1] value offered by the expert[2] and DI and DS denote the lower and upper tolerance levels assigned by the expert for each parameter. For each parameter, the membership function is given by (see Fig. 1):

$$\mu_{\widetilde{C}}(z) = \begin{cases} 1 - \frac{1}{DI}(C - z) & if \ z \in [C - DI, \ C) \\ 1 - \frac{1}{DS}(z - C) & if \ z \in [C, \ C + DS] \\ 0 & otherwise \end{cases} \tag{2}$$

We will handle fuzzy numbers by means of their alpha-cuts. Given a fuzzy set $\widetilde{C}$ in Z and a real number $\alpha \in [0, 1]$, then the α-cut of set $\widetilde{C}$ is the crisp set:

$$\{z \in Z / \mu_{\widetilde{C}}(z) \geq \alpha\} \tag{3}$$

When a confidence level is set such that $\alpha \in [0, 1]$, we obtain the variation interval for the number associated with a given confidence level within which the decision maker considers that all values are likely.

We assume that $r_{i,u,v}$ represents the modal value for the resources that project i needs at the time of execution v, for the resource category u; and that $\underline{r}_{i,u,v}$ and $\bar{r}_{i,u,v}$ are the left and right spreads these resources can have. We can then represent the fuzzy triangular number associated with the required resources as the triad $\left(r_{i,u,v}, \ \underline{r}_{i,u,v}, \ \bar{r}_{i,u,v}\right)$. Thus, given a confidence level for the resource category $\alpha_u \in [0, 1]$, the variation interval associated with each of these fuzzy numbers is given by:

$$\left[r_{i,u,v} - (1 - \alpha_u)\,\underline{r}_{i,u,v}, \ r_{i,u,v} + (1 - \alpha_u)\bar{r}_{i,u,v} \right] \tag{4}$$

Therefore, the constraint associated with the use of each resource and at each point in time is given by:

$$\widetilde{LR}_{uk} \lesssim \sum_{i=1}^{I} \sum_{t=1}^{k} \tilde{r}_{i,u,k+1-t} \cdot x_{it} \lesssim \widetilde{UR}_{uk}, \quad u \in \{1, 2, \dots, U\}, \ \ k \ \in \{1, 2, \dots, T\} \tag{5}$$

where $\tilde{r}_{i,u,k+1-t}$ represents the fuzzy amount of resources required by project i, for the resource u if this project was selected at t. $\widetilde{UR}_{uk}$ and $\widetilde{LR}_{uk}$ represent, respectively, the maximum and minimum amount of resource u the organisation can spend at time k. These coefficients are also triangular fuzzy numbers with the same characteristics as those described above.

Some resources may not be fully used up and the organization may be interested in transferring them to the next point in time using the corresponding interest rate, $rate_u(k)$:

[1] By the "modal value" we mean "the value that experts consider to be the most likely value".

[2] Should there be more than one expert, each providing a different modal value and variation interval. In this case, we could use the means of these values to create the triangular number.

$$\begin{aligned}\widetilde{LR}_{uk} \lesssim \sum_{i=1}^{I}\sum_{t=1}^{k} \tilde{r}_{i,u,k+1-t}\cdot x_{it} \lesssim \widetilde{UR}_{uk} + (1+rate_u(k))\cdot \\ \left(\widetilde{UR}_{u,k-1} - \sum_{i=1}^{I}\sum_{t=1}^{k-1}\tilde{r}_{i,u,k-t}\cdot x_{it}\right), \\ u \in\{1,2,\ldots,U\},\quad k\in\{1,2,\ldots,T\}\end{aligned} \tag{6}$$

where if k is zero, $\widetilde{UR}_{u,k}$ and $\tilde{r}_{i,u,k}$ are equal to zero.

Furthermore, these resources might be affected if we introduce synergies between projects. For example, these synergies could show that several projects may share a specific resource if the projects are implemented simultaneously. Thus, we establish A_j, $j = s+1, \ldots, \hat{s}$, sets of independent projects in which the decision maker sets the minimum (m_j) and maximum (M_j) number of projects that have to be selected for the synergy to be active. The change produced by the synergy can also be affected by uncertainty at that early stage of the process. Again, this uncertainty can be formulated by means of triangular fuzzy numbers based on the information we have regarding their modal value (h_{juk}) and the deviations this value ($\overline{h}_{juk}$ and $\underline{h}_{juk}$) is allowed. Thus, functions $g_{jk}(x)$ take value 1 if the number of active projects in set A_j at point in time k is between the values m_j and M_j; otherwise, they take value 0. Thus, we have the following constraints:

$$\begin{aligned}\widetilde{LR}_{uk} \lesssim \sum_{i=1}^{I}\left(\sum_{t=1}^{k-1}(\tilde{r}_{i,u,k+1-t} + (1+rate_u(k))\cdot\tilde{r}_{i,u,k-t})\cdot x_{it} + \tilde{r}_{i,u,1}\cdot x_{ik}\right. \\ + \sum_{j=s+1}^{\hat{s}} g_{jk}(x)\cdot\tilde{h}_{juk} + (1+rate_u(k))\cdot\sum_{j=s+1}^{\hat{s}} g_{j,k-1}(x)\cdot\tilde{h}_{j,u,k-1} \lesssim \widetilde{UR}_{uk} \\ + (1+rate_u(k))\cdot\widetilde{UR}_{u,k-1},\quad u \in\{1,2,\ldots,U\},\quad k\in\{1,2,\ldots,T\}\end{aligned} \tag{7}$$

To operate with these constraints, the fuzzy numbers in the constraints have to be compared. The literature provides a vast range of techniques that can be used to make comparisons based on different points of view, thereby offering different options depending on the characteristics of the problem (Verdegay, 1982; Campos and Verdegay, 1989).

Taking into account that the definition of fuzzy numbers and their subsequent use are based on confidence levels set by the decision makers, we perform the comparison by using one of the α-cuts, suggested by Chang and Lee (1994). Although this approach does not use all the information stored in the fuzzy sets, and may sometimes appear to be restrictive, it rapidly provides results and for this reason it is a widely used approach (León et al., 2003). We use the comparison method introduced by Tanaka et al. (1984), which permits fuzzy numbers to be compared using α-cuts without creating elements that cannot be compared or that lead to ambiguities. Given two triangular fuzzy numbers $\tilde{a}, \tilde{b}$ whose associated triads are $\left(\underline{a}, a, \overline{a}\right)$ and $\left(\underline{b}, b, \overline{b}\right)$, respectively, and a confidence level $\alpha\in[0,1]$, the comparison process shows that $\tilde{a} \le \tilde{b}$ at confidence level α, if:

$$a + (1-\delta)\overline{a} \le b + (1-\delta)\overline{b} \tag{8}$$

$$a - (1-\delta)\,\underline{a} \le b - (1-\delta)\,\underline{b} \tag{9}$$

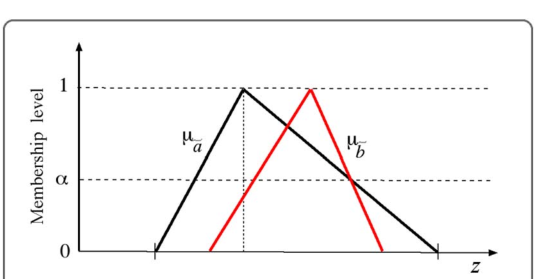

Fig. 2. Comparison of fuzzy numbers $\tilde{a}$ and $\tilde{b}$ ($\tilde{a} \le \tilde{b}$ at confidence level α).

for all $\delta\in[\alpha,1]$.

That is, under this definition, a fuzzy number will be smaller than another at a confidence level $\alpha\in[0,1]$ if the relationship shown in Fig. 2 is verified.

Thus, according to this expression, given the confidence levels $\alpha_u\in[0,1]$, the resource constraints are transformed into the following two blocks:

$$\begin{aligned} & RI_{uk} + (1-\delta_u)\cdot\overline{RI}_{uk} \\ & \le \sum_{i=1}^{I}\left(\sum_{t=1}^{k-1}\left(r_{i,u,k+1-t} + (1-\delta_u)\cdot\overline{r}_{i,u,k+1-t} + (1+rate_u(k))\cdot\left(r_{i,u,k-t}\right.\right.\right. \\ & \quad + (1-\delta_u) \\ & \cdot\,\overline{r}_{i,u,k-t}))\cdot x_{it} + (r_{i,u,1} + (1-\delta_u)\cdot\overline{r}_{i,u,1})\cdot x_{ik} \\ & + \sum_{j=s+1}^{\hat{s}} g_{jk}(x)\cdot(h_{juk} + (1-\delta_u)\cdot\overline{h}_{juk}) + (1+rate_u(k)) \\ & \cdot\sum_{j=s+1}^{\hat{s}} g_{j,k-1}(x)\cdot(h_{j,u,k-1} + (1-\delta_u)\cdot\overline{h}_{j,u,k-1}) \\ & \le RS_{uk} + (1-\delta_u)\cdot\overline{RS}_{uk} + (1+rate_u(k))\cdot(RS_{uk} + (1-\delta_u)\cdot\overline{RS}_{u,k-1}) \end{aligned} \tag{10}$$

$$\begin{aligned} & RI_{uk} - (1-\delta_u)\cdot\underline{RI}_{uk} \\ & \le \sum_{i=1}^{I}\left(\sum_{t=1}^{k-1}\left(r_{i,u,k+1-t} - (1-\delta_u)\cdot\underline{r}_{i,u,k+1-t} + (1+rate_u(k))\cdot\left(r_{i,u,k-t}\right.\right.\right. \\ & \quad - (1-\delta_u) \\ & \cdot\,\underline{r}_{i,u,k-t}))\cdot x_{it} + (r_{i,u,1} - (1-\delta_u)\cdot\underline{r}_{i,u,1})\cdot x_{ik} \\ & + \sum_{j=s+1}^{\hat{s}} g_{jk}(x)\cdot(h_{juk} - (1-\delta_u)\cdot\underline{h}_{juk}) + (1+rate_u(k)) \\ & \cdot\sum_{j=s+1}^{\hat{s}} g_{j,k-1}(x)\cdot(h_{j,u,k-1} - (1-\delta_u)\cdot\underline{h}_{j,u,k-1}) \\ & \le RS_{uk} - (1-\delta_u)\cdot\underline{RS}_{uk} + (1+rate_u(k))\cdot(RS_{uk} - (1-\delta_u)\cdot\underline{RS}_{u,k-1}), \\ & \quad u \in\{1,2,\ldots,U\},\quad k\in\{1,2,\ldots,T\},\quad \delta_u\in[\alpha_u,1] \end{aligned} \tag{11}$$

The main disadvantage of comparing fuzzy numbers in this way is that the starting number of constraints doubles when the transformation is applied. On the other hand, for each group of confidence levels, $\alpha_u, u = 1,2,\ldots,U$, set by the decision makers, we obtain a different feasible set and therefore, the solution to the problem changes. That is, if these levels are slightly modified, the selected portfolio may be a different one, leading to different values in the objective function. For this reason, it is very useful for the organization to obtain information about the strength of the selected portfolio, regarding the confidence levels for which this portfolio is still optimal.

Thus, in Section 3, we present a theoretical analysis of how the solution to a problem changes when the confidence levels vary throughout its whole interval [0,1]. This analysis yields the set of all the solutions associated with any confidence level set by the decision maker, thus providing more and better information about the problem.

3.2.2. Other constraints in the model

The other constraints included in the model are listed below:

a) Constraints related to synergies between the projects

These constraints ensure that the functions $g_{jk}(x)$ take value '1' only if the synergies between projects are activated.

For greater clarity, two functions are defined: $g_{jk}{}^{m}(x)$, $g_{jk}{}^{M}(x)$, associated with each $g_{jk}(x)$, such that the first takes value '1' when the number of activated projects of subset A_j is greater than m_j, while the second takes value '1' when the number of activated projects of such subset A_j is less than M_j; they each take value '0' otherwise. Therefore, $g_{jk}(x) = g_{jk}{}^{m}(x)\cdot g_{jk}{}^{M}(x)$ takes value '1' when the number of active projects is within the bounds set by the organization and '0' otherwise, as defined above.

Consequently, the constraints are as follows:

$$\left(\sum_{i\in A_j}\sum_{t=k-d_i+1}^{k} x_{it}\right) - m_j + 1 \le I\cdot g_{jk}^{m}(x) \le \left(\sum_{i\in A_j}\sum_{t=k-d_i+1}^{k} x_{it}\right) - m_j + I \quad j = 1, 2, \ldots, \hat{s} \tag{12}$$

$$M_j - \left(\sum_{i\in A_j}\sum_{t=k-d_i+1}^{k} x_{it}\right) + 1 \le I\cdot g_{jk}^{M}(x) \le M_j - \left(\sum_{i\in A_j}\sum_{t=k-d_i+1}^{k} x_{it}\right) + I \quad j = 1, 2, \ldots, \hat{s} \tag{13}$$

where, each d_i represents the duration of the i-th project, as before and I is the set of project proposals.

b) Temporary linear constraints

In this case, the decision maker may include some constraints about the active projects included in the portfolio at period k that do not depend on their execution time. Lower ($\underline{b}(k)$) and higher ($\overline{b}(k)$) bounds are defined as well as the coefficient matrix ($B(k)$).

$$\underline{b}(k) \le B(k)\cdot\begin{pmatrix}\sum_{t=k-d_1+1}^{k} x_{1t}\\ \vdots \\ \sum_{t=k-d_I+1}^{k} x_{It}\end{pmatrix} \le \overline{b}(k) \quad k = 1, 2, \ldots, T \tag{14}$$

For example, the number of projects the decision makers wish to have simultaneously active at each period k can be set with these constraints.

c) Global linear constraints

These constraints are the same as the previous ones, but do not depend on the period. This group could include, for example, a constraint to ensure that versions of the same project cannot be part of the same portfolio.

$$\underline{b} \le B\cdot\begin{pmatrix}\sum_{t=1}^{T} x_{1t}\\ \vdots \\ \sum_{t=1}^{T} x_{It}\end{pmatrix} \le \overline{b} \tag{15}$$

where lower ($\underline{b}$) and higher ($\overline{b}$) bounds are defined as well as the coefficient matrix (B).

d) Mandatory and uniqueness constraints

These constraints ensure that each project starts only once, and the mandatory execution of certain projects can be forced by using the lower bound (CL_i) (by setting $CL_i = 1$).

$$CL_i \le \sum_{t=1}^{T} x_{it} \le 1 \quad i = 1, 2, \ldots, I \tag{16}$$

e) Constraints about the starting period

Given a subset E of all projects, these constraints can force certain projects, if selected, to be chosen at periods defined by the decision makers.

$$\underline{\beta}_i\cdot\sum_{t=1}^{T} x_{it} \le \sum_{t=1}^{T} t\cdot x_{it} \le \overline{\beta}_i \quad i \in E \tag{17}$$

where $\underline{\beta}_i$ and $\overline{\beta}_i$ represent the lower and upper time bounds, respectively, for the i-*th* project.

f) Precedence constraints

Given a set P_l of predecessor projects to project l, these constraints ensure that project l cannot be selected if its predecessors were not selected.

$$\sum_{t=1}^{T} x_{it} \ge \sum_{t=1}^{T} x_{lt} \quad i \in P_l \tag{18}$$

Similarly, we can model that project l cannot begin until at least h_i periods have passed since its predecessors began and no more than H_i periods:

$$\sum_{t=1}^{T} x_{lt}\left(\sum_{t=1}^{T} t\cdot x_{it} + h_i\right) \le \sum_{t=1}^{T} t\cdot x_{lt} \le \sum_{t=1}^{T} t\cdot x_{it} + H_i \quad i \in P_l \tag{19}$$

4. Theoretical basis for solving the problem

The resolution of the problem is determined by all the confidence level sets. That is, for each vector $(\alpha_1, \alpha_2, \ldots, \alpha_U)$ provided by the decision makers, an optimal portfolio for the problem is obtained. However, our goal is not to find an optimal portfolio within some specific confidence levels, but to find all optimal portfolios associated with all vectors $(\alpha_1, \alpha_2, \ldots, \alpha_U) \in [0,1] \times [0,1] \times \ldots^{(U} \times [0,1]$. Thus, we consider that the solution to the problem is the set of optimal portfolios associated with different confidence levels.

Each confidence level α_u, $u = 1, 2, \ldots, U$, only appears in the two constraints associated with each category of resources u, so its value is independent of the values taken by the other confidence levels. That is, the feasibility of a solution for confidence level α_1, for example, is determined by the value taken by that level, by the amount of resource 1 needed to execute the projects involved in the solution, and the budget available to this resource. However, since none of these elements appear in the other constraints, or *vice versa*, then the value taken by each of the confidence levels is independent of those taken by the others.

We must know the minimum and maximum values for each α_u, $u = 1, 2, \ldots, U$, for which each solution is optimal. However, according to the fuzzy number comparison carried out in the constraints, if a solution is optimal for a set of given confidence levels, then the solution would be still optimal for any confidence level greater than the values already given. Thus, we can state that the feasible set to the problem is larger when the values of the confidence levels increase. So, for each optimal solution we find, it is sufficient to know the minimum values of the confidence levels for which the solution remains optimal. In this case, and due to independence between the confidence levels, we can obtain the area where the portfolio is optimal.

To do this, based on the problem *P1* defined in Section 2, we propose a multiobjective problem such that by solving it just once we obtain a solution that provides us with this information. As we are seeking the minimum confidence levels for each optimal portfolio, we propose the following definition:

Definition 1. An optimal point x^* of problem *P1* associated with the confidence levels $(\alpha_1{}^*, \alpha_2{}^*, \ldots, \alpha_U{}^*)$, is an extremal vertex of this problem, if for any point x' such that $F(x^*) = F(x')$, we have $\alpha_u{}^* \le \alpha_u{}'$ $\forall u \in \{1, 2, \ldots, U\}$. In this case, this extremal vertex is denoted by (x^*, α^*).

Given this definition, by obtaining the set of extremal vertices of the problem, we also obtain the set of optimal solutions according to the different confidence levels. In addition, we also obtain the minimum values the confidence levels can take for the solutions to remain optimal. Thus, we now calculate the set of extremal vertices of problem *P1*. To do this, we use the following multiobjective problem:

Definition 2. We define the multiobjective problem *P2* associated with *P1* as follows:

$$\begin{aligned} &\text{Min } (\alpha_1,\ \alpha_2, \ldots, \alpha_U) \\ &\text{Max } F(x) \\ &\text{s. t.}: x \ \in D \subseteq \mathbb{R}^{T\cdot I} \\ &\qquad \alpha_u \ \in [0,1] \end{aligned} \tag{20}$$

where D represents the feasible set of problem *P1*.

Henceforth, we denote by $(\alpha'_1, \alpha'_2, \ldots, \alpha_U';x')$ the feasible portfolio x' from *P1* for the set of confidence levels $(\alpha'_1, \alpha'_2, \ldots, \alpha'_U)$. As demonstrated below, calculating the efficient solutions of problem *P2* is equivalent to computing the set of extremal vertices of problem *P1*, which directly leads to obtaining the points we are looking for.

Theorem 1. Given the problems *P1* and *P2*, let x^* be a solution to problem *P1* associated with confidence levels $\alpha^* = (\alpha_1^*,\ \alpha_2^*, \ldots, \alpha_U^*)$. Then, (x^*, α^*) is an extremal vertex of problem *P1* if and only if $(\alpha_1^*,\ \alpha_2^*, \ldots, \alpha_U^*, x^*)$ is an efficient point of problem *P2*.

Proof. ⇐) Let $(\alpha_1^*,\ \alpha_2^*, \ldots, \alpha_U^*,\ x^*)$ be an efficient point of problem *P2* with $\alpha^* = (\alpha_1^*,\ \alpha_2^*, \ldots, \alpha_U^*)$. We assume, by reduction ad absurdum, that (x^*, α^*) is not an extremal vertex of problem *P1*.

We can then find confidence levels $\alpha' = (\alpha_1', \alpha_2', \ldots,\ \alpha_U')$ for point x^*, with some $\alpha_j' \neq \alpha^*_j$, $j \in \{1, 2, \ldots, U\}$, such that (x^*, α') is in fact an extremal vertex of problem *P1*.

By the definition of extremal vertex, it has to be the case that $\alpha_u^* \geq \alpha_u'\ \ \forall u \in \{1, 2, \ldots, U\}$, which, together with the foregoing, gives us $\alpha_j^* > \alpha_j'$ for any $j \in \{1, 2, \ldots, U\}$,

Then $(\alpha_1',\ \alpha_2', \ldots, \alpha_U',\ F(x^*))$ would dominate $(\alpha_1^*,\ \alpha_2^*, \ldots, \alpha_U^*,\ F(x^*))$, which is a contradiction since $(\alpha_1^*,\ \alpha_2^*, \ldots, \alpha_U^*,\ x^*)$ was an efficient point of problem *P2*. ■

⇒) Let (x^*, α^*) be an extremal vertex of problem *P1*. Assume, by reduction *ad absurdum*, that $(\alpha_1^*,\ \alpha_2^*, \ldots, \alpha_U^*,\ F(x^*))$ is dominated by another point. That is, let us assume that $(\alpha_1', \alpha_2', \ldots,\ \alpha_U',\ F(x'))$ exists such that:

➢ $\alpha_u^* \geq \alpha_u' \forall u \in \{1, 2, \ldots, U\}$ (a)
➢ $F(x^*) \leq F(x')$

with some of the inequalities verified in a strict sense.

We know that $(\alpha_1^*,\ \alpha_2^*, \ldots, \alpha_U^*; x^*)$ and $(\alpha_1', \alpha_2', \ldots,\ \alpha_U'; x')$, are feasible points of problem *P1*. Then, if point $(\alpha_1^*,\ \alpha_2^*, \ldots, \alpha_U^*; x^*)$ is feasible, this implies that the doubled resource constraints that are obtained after applying the fuzzy number intervals comparison are verified, that is, x' is a feasible point for any $\mu_u \in [\ \alpha_u', 1],\ \ \forall u \in \{1, 2, \ldots, U\}$.

Specifically, given that $\alpha_u^* \geq \alpha_u' \forall u \in \{1, 2, \ldots, U\}$, then x' is a feasible point for the confidence levels $(\alpha^*_1, \alpha^*_2, \ldots, \alpha^*_U)$, and therefore the point $(\alpha^*_1, \alpha^*_2, \ldots,\ \alpha^*_U; x')$ is a feasible point of problem *P1*.

That is, points $(\alpha^*_1, \alpha^*_2, \ldots,\ \alpha^*_U; x')$ and $(\alpha^*_1, \alpha^*_2, \ldots,\ \alpha^*_U; x^*)$ are both feasible points of problem *P1*. However, we also know that (x^*, α^*) was an extremal vertex of problem *P1*, which means that x^* is an optimal solution when the confidence levels are $(\alpha^*_1, \alpha^*_2, \ldots,\ \alpha^*_U)$. Thus, one of the following can occur:

➢ Either, $F(x^*) > F(x')$, which clearly contradicts (a).
➢ Or, $F(x^*) = F(x')$ in which case, as (x^*, α^*) is an extremal vertex, by definition we have $\alpha_u^* \leq \alpha_u' \forall u \in \{1, 2, \ldots, U\}$ which together with (a), leads to $\alpha_u^* = \alpha_u' \forall u \in \{1, 2, \ldots, U\}$.

That is, we obtain $F(x^*) = F(x')$ and $\alpha_u^* = \alpha_u' \forall u \in \{1, 2, \ldots, U\}$, which contradicts that $(\alpha_1^*,\ \alpha_2^*, \ldots, \alpha_U^*,\ F(x^*))$ was dominated by $(\alpha_1', \alpha_2', \ldots,\ \alpha_U',\ F(x'))$.

Thus, calculating the optimal points obtained by varying the confidence levels within their ranges—as well as the minimum confidence levels for which they remain optimal—is equivalent to solving the multiobjective problem *P2*.

The efficient points for problem *P2* allow us to show decision makers how variation in confidence levels affects the optimal solution for problem *P1*.

The model described is powerful in the sense that it is simple to calculate using any number of resource constraints, the results can be interpreted in depth and, most importantly, the model can be generalised and applied to any fuzzy problem involving the comparison of fuzzy numbers in the constraints. In addition, the equivalence demonstrated strongly simplifies the resolution of the problem. This is because we obtain the set of all solutions for the parametric problem *P1* by resolving a single multiobjective problem *P2*. That is, we only need to resolve problem *P2* once instead of resolving problem *P1* several times to include the full variation interval of parameters $\alpha_u,\ \ u = 1, 2, \ldots, U$.

5. Illustrative example

Let us assume that an organization needs to select, from a set of 10 possible events, which ones it is going to sponsor during the next 4 years. The aim is to maximise the direct expected benefits (in thousands of monetary units). These benefits include the individual contribution of each event, and a positive synergy between projects (events) number 4 and 6, if they are selected to be sponsored jointly in the second year. Then, the objective function is as follows:

$$\text{Max } \sum_{k=1}^{4} \left(\sum_{i=1}^{10} \sum_{t=1}^{k} c_{i,k+1-t} \cdot x_{it} \right) + 74.3 \cdot g_{12}(x) \tag{21}$$

where coefficients $c_{i,k+1-t}$ are given in Table 1, and function $g_{12}(x)$ (it takes value 1 when event numbers 4 and 6 are active in the second year, and 0 otherwise) is determined by the following constraints, where $A_1 = \{4, 6\}$.

$$g_{12}^m(x) + g_{12}^M(x) - g_{12}(x) \leq 1 \tag{22}$$

$$g_{12}^m(x) + g_{12}^M(x) - g_{12}(x) \geq 0 \tag{23}$$

$$\left(\sum_{i \in A_1} \sum_{t=3-d_i}^{2} x_{it} \right) - 1 \leq 10 \cdot g_{12}^m(x) \leq \left(\sum_{i \in A_1} \sum_{t=3-d_i}^{2} x_{it} \right) + 8 \tag{24}$$

$$3 - \left(\sum_{i \in A_1} \sum_{t=3-d_i}^{2} x_{it} \right) \leq 10 \cdot g_{12}^M(x) \leq 12 - \left(\sum_{i \in A_1} \sum_{t=3-d_i}^{2} x_{it} \right) \tag{25}$$

Furthermore, each sponsorship proposal requires two types of renewable resources: investing both in publicity ($u = 1$) and in recruiting the staff required ($u = 2$). Both of them are allocated a maximum budget, but no minimum; if the budget is not fully spent in 1 year, the

Table 1
Values of coefficients c_{ik}.

	k = 1	k = 2	k = 3	k = 4
c_{1k}	66.8	49.65		
c_{2k}	92.49			
c_{3k}	21.24	74.3	51.25	95.41
c_{4k}	79.37	62.3		
c_{5k}	13.12	92.9	80.27	23.73
c_{6k}	6.82			
c_{7k}	4.43	39.46	78.98	16.8
c_{8k}	95.82	73.24		
c_{9k}	11.93	16.24		
c_{10k}	3.2	44.54		

remainder is not subsequently reinvested. In both type of resources, the resources required for each project and the budget limits for the organization are not fully known. Then, modal values and their spreads are given. The constraints on investing in publicity ($u = 1$) are as follows ($k = 1, 2, 3, 4$):

$$0 \widetilde{\leq} \widetilde{84.664}\cdot x_{11} + \widetilde{83.042}\cdot x_{21} + \widetilde{44.688}\cdot x_{31} + \widetilde{42.106}\cdot x_{41} + \widetilde{27.208}\cdot x_{51} + \widetilde{33.228}\cdot x_{61} + \widetilde{74.182}\cdot x_{71} + \widetilde{92.524}\cdot x_{81} + \widetilde{39.09}\cdot x_{91} + \widetilde{22.856}\cdot x_{10,1} \widetilde{\leq} \widetilde{150.25} \tag{26}$$

$$0 \widetilde{\leq} \widetilde{47.554}\cdot x_{11} + \widetilde{84.664}\cdot x_{12} + \widetilde{83.042}\cdot x_{22} + \widetilde{89.132}\cdot x_{31} + \widetilde{44.688}\cdot x_{32} + \widetilde{20.796}\cdot x_{41} + \widetilde{42.106}\cdot x_{42} + \widetilde{85.308}\cdot x_{51} + \widetilde{27.208}\cdot x_{52} + \widetilde{33.228}\cdot x_{62} + \widetilde{24.228}\cdot x_{71} + \widetilde{74.182}\cdot x_{72} + \widetilde{66.848}\cdot x_{81} + +\widetilde{92.524}\cdot x_{82} + \widetilde{88.288}\cdot x_{91} + \widetilde{39.09}\cdot x_{92} + \widetilde{88.46}\cdot x_{10,1} + \widetilde{22.856}\cdot x_{10,2} \widetilde{\leq} \widetilde{138.5} \tag{27}$$

$$0 \widetilde{\leq} \widetilde{47.554}\cdot x_{12} + \widetilde{84.664}\cdot x_{13} + \widetilde{83.042}\cdot x_{23} + \widetilde{78.13}\cdot x_{31} + \widetilde{89.132}\cdot x_{32} + \widetilde{44.688}\cdot x_{33} + \widetilde{20.796}\cdot x_{42} + \widetilde{42.106}\cdot x_{43} + \widetilde{11.35}\cdot x_{51} + \widetilde{85.308}\cdot x_{52} + \widetilde{27.208}\cdot x_{53} + \widetilde{33.228}\cdot x_{63} + \widetilde{3.484}\cdot x_{71} + \widetilde{24.228}\cdot x_{72} + \widetilde{74.182}\cdot x_{73} + \widetilde{66.848}\cdot x_{82} + \widetilde{92.524}\cdot x_{83} + \widetilde{88.288}\cdot x_{92} + \widetilde{39.09}\cdot x_{93} + \widetilde{88.46}\cdot x_{10,2} + \widetilde{22.856}\cdot x_{10,3} \widetilde{\leq} \widetilde{285.98} \tag{28}$$

$$0 \widetilde{\leq} \widetilde{47.554}\cdot x_{13} + \widetilde{84.664}\cdot x_{14} + \widetilde{83.042}\cdot x_{24} + \widetilde{16.474}\cdot x_{31} + \widetilde{78.13}\cdot x_{32} + \widetilde{89.132}\cdot x_{33} + \widetilde{44.688}\cdot x_{34} + \widetilde{20.796}\cdot x_{43} + \widetilde{42.106}\cdot x_{44} + \widetilde{2.402}\cdot x_{51} + \widetilde{11.35}\cdot x_{52} + \widetilde{85.308}\cdot x_{53} + \widetilde{27.208}\cdot x_{54} + \widetilde{33.228}\cdot x_{64} + \widetilde{18.232}\cdot x_{71} + \widetilde{3.484}\cdot x_{72} + \widetilde{24.228}\cdot x_{73} + \widetilde{74.182}\cdot x_{74} + \widetilde{66.848}\cdot x_{83} + \widetilde{92.524}\cdot x_{84} + \widetilde{88.288}\cdot x_{93} + \widetilde{39.09}\cdot x_{94} + \widetilde{88.46}\cdot x_{10,3} + \widetilde{22.856}\cdot x_{10,4} \widetilde{\leq} \widetilde{275.13} \tag{29}$$

The resources required for each project may undergo decreases of up to 25% of the predicted value and increases of up to 30%. In addition, the budget limits on the resources available may also vary, in which case we would assume a variation ranging from a 30% reduction to a 20% increase.

Constraints concerning the investment in recruiting the staff required are formulated in the same way, and the values of coefficients are shown in Table 2.

In addition, some tickets for attending the events will be raffled among customers during the second and third year. Then, the upper and lower limits on this expenditure will be constrained as follow:

$$1 \leq \sum_{t=1}^{2} x_{3t} + \sum_{t=1}^{2} x_{4t} + \sum_{t=1}^{2} x_{5t} + \sum_{t=1}^{2} x_{8t} + \sum_{t=1}^{2} x_{9t} + \sum_{t=1}^{2} x_{10,t} \leq 4 \tag{30}$$

$$1 \leq \sum_{t=2}^{3} x_{1t} + \sum_{t=1}^{3} x_{3t} + \sum_{t=2}^{3} x_{8t} + \sum_{t=2}^{3} x_{9t} \leq 3 \tag{31}$$

On the other hand, constraints will also be established for the budget allocated to flights, accommodation, and food expenses during the entire process

Table 2
Values of coefficients $\widetilde{r_{i2k}}$ and $\widetilde{UR_{2k}}$

	k = 1	k = 2	k = 3	k = 4
$\widetilde{r_{1,2,k}}$	$\widetilde{25.096}$	$\widetilde{94.354}$		
$\widetilde{r_{2,2,k}}$	$\widetilde{41.618}$			
$\widetilde{r_{3,2,k}}$	$\widetilde{24.208}$	$\widetilde{37.436}$	$\widetilde{56.45}$	$\widetilde{88.546}$
$\widetilde{r_{4,2,k}}$	$\widetilde{39.53}$	$\widetilde{31.924}$		
$\widetilde{r_{5,2,k}}$	$\widetilde{1.648}$	$\widetilde{74.204}$	$\widetilde{64.422}$	$\widetilde{10.658}$
$\widetilde{r_{6,2,k}}$	$\widetilde{27.9}$			
$\widetilde{r_{7,2,k}}$	$\widetilde{6.718}$	$\widetilde{76.732}$	$\widetilde{0.116}$	$\widetilde{10.84}$
$\widetilde{r_{8,2,k}}$	$\widetilde{66.404}$	$\widetilde{12.248}$		
$\widetilde{r_{9,2,k}}$	$\widetilde{7.81}$	$\widetilde{24}$		
$\widetilde{r_{10,2,k}}$	$\widetilde{44.272}$	$\widetilde{96.524}$		
$\widetilde{UR_{2k}}$	$\widetilde{104.23}$	$\widetilde{298.69}$	$\widetilde{208.97}$	$\widetilde{201.32}$

$$37 \leq 48\cdot \sum_{t=1}^{4} x_{1t} + 96\cdot \sum_{t=1}^{4} x_{2t} + 83\cdot \sum_{t=1}^{4} x_{3t} + 41\cdot \sum_{t=1}^{4} x_{4t} + 7\cdot \sum_{t=1}^{4} x_{5t} + 14\cdot \sum_{t=1}^{4} x_{6t} + 36\cdot \sum_{t=1}^{4} x_{7t} + 19\cdot \sum_{t=1}^{4} x_{8t} + 10\cdot \sum_{t=1}^{4} x_{9t} + 7\cdot \sum_{t=1}^{4} x_{10,t} \leq 270 \tag{32}$$

$$52 \leq 34\cdot \sum_{t=1}^{4} x_{1t} + 77\cdot \sum_{t=1}^{4} x_{2t} + 99\cdot \sum_{t=1}^{4} x_{3t} + 29\cdot \sum_{t=1}^{4} x_{4t} + 70\cdot \sum_{t=1}^{4} x_{5t} + 22\cdot \sum_{t=1}^{4} x_{6t} + 92\cdot \sum_{t=1}^{4} x_{7t} + 21\cdot \sum_{t=1}^{4} x_{8t} + 67\cdot \sum_{t=1}^{4} x_{9t} + 9\cdot \sum_{t=1}^{4} x_{10,t} \leq 390 \tag{33}$$

$$45 \leq 1\cdot \sum_{t=1}^{4} x_{1t} + 59\cdot \sum_{t=1}^{4} x_{2t} + 5\cdot \sum_{t=1}^{4} x_{3t} + 62\cdot \sum_{t=1}^{4} x_{4t} + 9\cdot \sum_{t=1}^{4} x_{5t} + 63\cdot \sum_{t=1}^{4} x_{6t} + 70\cdot \sum_{t=1}^{4} x_{7t} + 98\cdot \sum_{t=1}^{4} x_{8t} + 9\cdot \sum_{t=1}^{4} x_{9t} + 65\cdot \sum_{t=1}^{4} x_{10,t} \leq 330 \tag{34}$$

It is not considered mandatory to sponsor any particular event, so, following constraints are added to the formulation of the problem:

$$0 \leq \sum_{t=1}^{4} x_{it} \leq 1 \quad i = 1, 2, \ldots, 10 \tag{35}$$

However, there are some constraints regarding the year sponsorship could start for each event:

$$\sum_{t=1}^{4} t\cdot x_{1t} \leq 3;\ 3\cdot \sum_{t=1}^{4} x_{2t} \leq \sum_{t=1}^{4} t\cdot x_{2t} \leq 4;\ \sum_{t=1}^{4} t\cdot x_{3t} \leq 3;\ 2\cdot \sum_{t=1}^{4} x_{4t} \leq \sum_{t=1}^{4} t\cdot x_{4t} \leq 3;$$
$$2\cdot \sum_{t=1}^{4} x_{6t} \leq \sum_{t=1}^{4} t\cdot x_{6t} \leq 3;\ 2\cdot \sum_{t=1}^{4} x_{7t} \leq \sum_{t=1}^{4} t\cdot x_{7t} \leq 3;\ 2\cdot \sum_{t=1}^{4} x_{8t} \leq \sum_{t=1}^{4} t\cdot x_{8t} \leq 4;$$
$$\sum_{t=1}^{4} t\cdot x_{9t} \leq 3;\ \sum_{t=1}^{4} t\cdot x_{10,t} \leq 2 \tag{36}$$

Finally, due to market conditions, event number 4 cannot be sponsored unless event number 5 began to be sponsored at least 2 years before.

$$\sum_{t=1}^{4} x_{4t}\left(\sum_{t=1}^{4} t\cdot x_{5t} + 2\right) \leq \sum_{t=1}^{4} t\cdot x_{4t} \leq \sum_{t=1}^{4} t\cdot x_{5t} + 2 \tag{37}$$

Therefore, once the problem is formulated, two changes in this formulation must be made. Firstly, resource constraints must be transformed, as shown in Section 2. Secondly, the multiobjective problem defined in Section 3 associated to this one must be formulated. In this way, two objective functions are added to the problem.

$$\text{Min } (\alpha_1, \alpha_2) \tag{38}$$

Thus, the mathematical problem *P2* to be solved has three objective functions and 48 constraints.

First, we present the results obtained when we consider that the two resources share the same confidence level. Table 3 shows 5 efficient portfolios obtained for problem *P2* (which are optimal solutions for problem *P1*), ranked according to the value of their objective function. Thus, we can see that as the values obtained for the objective function decrease, the values of the minimum confidence levels also become lower. In this case, the variation interval in the value of the objective function is within the range $[666.65, 776.63]$.

Fig. 3 shows the variations in the objective function values for each target efficient portfolio according to the different confidence levels.

Fig. 3 also shows how the confidence level α decreases as the values obtained in the objective function decrease. This is easily explained by bearing in mind the way we have compared the fuzzy numbers in the resource constraints. Thus, when we pass from a given value of α to a higher value, the new feasible set of the problem is larger than the previous one, since either the portfolio obtained with the new confidence level is the same as in the previous case, or a new portfolio is obtained that is better than the previous one in terms of the value reached for the objective function. This new portfolio was not feasible

Table 3
Efficient portfolios obtained

Point	Objective	α
1	776.63	0.59
2	773.58	0.54
3	736.51	0.39
4	711.08	0.17
5	666.65	0

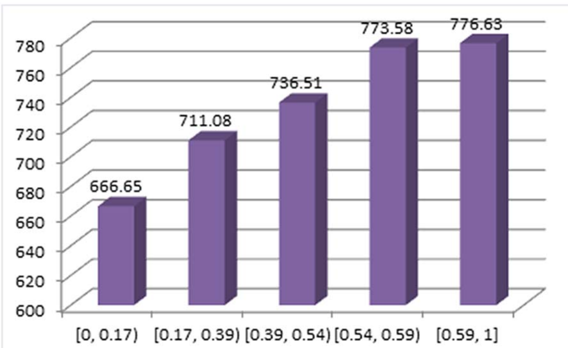

Fig. 3. Objective function values obtained for the portfolios according to the confidence levels

for the previous confidence level because the comparison of the fuzzy numbers at this confidence level was not verified. That is, as the values of α increase the feasible set of the problem is larger.

Table 4 shows the projects included in the portfolios selected, including the points in time in which they would start within the planning period. If we compare the five efficient portfolios obtained for the different values established for the confidence level α, they contain the same number of projects to be selected. Moreover, we can see that three events (1, 5 and 8) are selected in all the portfolios and that event 8 always starts at the same point in time, whereas the starting points of the other two events change. This means that the decision makers can be confident that these projects are strong candidates because, despite potential variations in resource requirements, they are always selected in the solution portfolio for the constraints established. Furthermore, given the results shown in Table 4, projects 9 and 10 are never selected as part of the optimal solution portfolio. Therefore, the decision makers can rule them out because under no confidence level are they sufficiently good or able to fulfil the constraints required to be part of the portfolio of the projects selected.

The results obtained also allow us to analyse the stability of the solution obtained without fuzzy data, that is, when the confidence level α is equal to 1. In our example, the solution obtained ($\alpha = 1$) is stable, at least up to a value of α equal to 0.59. Thus, if we do not include uncertainty in the model, then the optimal portfolio obtained would be the same than one obtained for a confidence level of at least 0.59.

After this, now we examine the results when each resource has a different and independent confidence level. That is, we generalize the previous example to the case in which the resources can be differentiated from each other according to the confidence level provided by its variation.

The efficient points obtained by solving problem *P2* (optimal for *P1*) are the 7 points shown in Table 5, again presented as a list according to the value reached in the objective function.

In this case, it is not possible to create a bar chart similar to the one depicted in Fig. 3, since we now have two different confidence levels with their own variation intervals. Although the information needed to analyse the results is shown in Table 5, this presentation format does not clearly represent the results to the decision maker.

We can construct a chart that helps to analyse the variability of the solutions and shows the solutions according to the values of their associated confidence levels so that it is easier for the user to interpret the results.

As shown in Fig. 4, we begin by placing in the plane the 7 efficient points obtained by solving problem *P2*. The figure shows the efficient points distributed according to the minimum values α_1 and α_2 can take for the project portfolios to be optimal. In addition, the value taken by the objective function for the given portfolio is displayed next to each point.

We can thus appreciate the internal structure of the spatial region delimited by the values of the confidence levels. This can be shown even more clearly by marking the confidence region representing each portfolio. To do this, we only have to delimit the area of the portfolio with the highest objective function value and then continue doing so iteratively for the other regions in descending order according to the objective function value. Furthermore, given the interdependence between the confidence levels, the regions are delimited by lines parallel to the coordinate axes.

This provides a more accurate picture of the internal structure of the region defined by the confidence levels. Thus, Fig. 5 shows the solution space delimited according to the confidence levels selected. The decisions can now be taken based on the risk the decision maker is willing to take regarding the variation permitted to the resource requirements of the projects.

For example, if the variation for the first resource is estimated to be around 70% of the confidence level and 40% for the second resource, we can safely state that portfolio 2, which has an objective function with a value of 773.58, is the ideal portfolio to fulfil the aims of the organisation. In addition, the user can identify the best portfolio for the confidence levels selected. Indeed, a glance at Fig. 5 shows that portfolio 2 is also optimal for a confidence level ranging between 55% and 100% for the first resource, and between 0% and 55% for the second resource. This information can be of vital importance to the user since it provides all the tools needed without having to solve the problem for the other confidence levels.

The analysis of Fig. 5 is also of use in studying the general behaviour of the problem in regions particularly sensitive to minimal changes that

Table 4
Projects of the efficient portfolios obtained

[0, 0.17)		[0.17, 0.39)		[0.39, 0.54)		[0.54, 0.59)		[0.59, 1]	
Obj	666.65	Obj	711.08	Obj	736.51	Obj	773.58	Obj	776.63
Project	t	Project	t	Project	t	Project	t	Project	t
1	1	1	1	1	3	1	1	1	1
2	4	2	4	2	4	2	4	3	3
3	3	3	2	4	3	4	3	4	3
5	2	5	2	5	1	5	1	5	1
6	2	8	3	6	2	7	3	7	3
8	3			8	3	8	3	8	3

Table 5
Efficient portfolios obtained

Point	Objective	α_1	α_2
1	776.63	0.54	0.59
2	773.58	0.54	0
3	760.03	0.25	0.95
4	736.51	0	0.39
5	715.6	0.46	0.14
6	711.08	0.17	0
7	666.65	0	0

could occur in the desired confidence levels. For example, a region in this category is one with a confidence level for the first resource between 40% and 55% and for the second resource below 40%; that is, small changes in confidence levels lead to different portfolio solutions. This figure can help to facilitate the decision makers' work.

However, if the number of resources for the problem we want to solve is greater than two, then it is impossible to construct a chart like Fig. 5. We could only do this in the case of space $\mathbb{R}^3$, but this certainly would lead to technical difficulties when presenting the results to the organization. So, we need to be able to analyze the results when they cannot be displayed in graphical way. Thus, by also using the results shown in Table 5, we can obtain the optimal project portfolio according to the confidence levels (or intervals of confidence levels) established by the decision maker. To do this, we simply "remove" those solutions whose maximum confidence levels are lower than those determined by the user (values strikethrough in Table 6) and then choose within the remaining group the solution that has a higher objective function (bold values in Table 6). For example, let us assume we want a confidence level of around 50% for the first resource and 60% for the second; we first remove from the table those portfolios whose minimum confidence levels are above 0.5 for α_1 or 0.6 for α_2, as shown in Table 6.

Thus, just four possibilities remain: points 4, 5, 6, and 7. However, from these we choose the one with the greatest value in the objective function, that is, point 4 (value = 736.51). And this would be the same solution portfolio selected in Fig. 5. The same occurs if the decision maker chooses any other confidence level.

The projects that compose each of the 7 efficient portfolios are shown in Table 7. The main point of interest is that projects 5 and 8 are always selected regardless of the values taken by the two confidence levels, and therefore they have to be selected in any case. Furthermore, projects 1, 2, and 4 often appear in the efficient portfolios, but not in all of them. For this reason, decision makers can use this information, together with the assessment of confidence levels, if they have a particular interest in selecting or not selecting these projects. In contrast, projects 9 and 10 are never selected, so the user has to know, regardless of the confidence levels chosen, that they will never be part of the optimal portfolio.

Finally, it should be noted that, given the results obtained, the first example, in which the same confidence levels were used for all resources, is a particular case of this example. That is, from Fig. 5 we can obtain the same results as before by simply drawing the bisector of the quadrant to identify the efficient portfolios if the confidence levels are the same. Thus, for the problem just solved, the analysis carried out in the previous example yields the same results shown in Fig. 6.

That is, we obtain 5 potentially optimal portfolios according to the confidence level desired. Moreover, from Fig. 6, we can also obtain the intervals of variation of the confidence level for each of the five efficient portfolios.

With all this study, the decision maker has full and accurate information on which to base their assessment and choice. Although some of the initial information involved a certain degree of uncertainty, by the end of the process not only has the objective function value been obtained for each project portfolio, but also the confidence level at which the portfolio is optimal. This even enables the decision makers to calculate the specific variation interval of the resources required for the portfolio to be optimal.

6. A real-world application

In this section, we apply our model to a real case in a Spanish state university. The university's Department for the Strategic Planning of Infrastructure (DSPI) needs to plan, at the beginning of the period and for a given time horizon, the budget available to fund a project portfolio by choosing from among all the candidate projects. The process takes

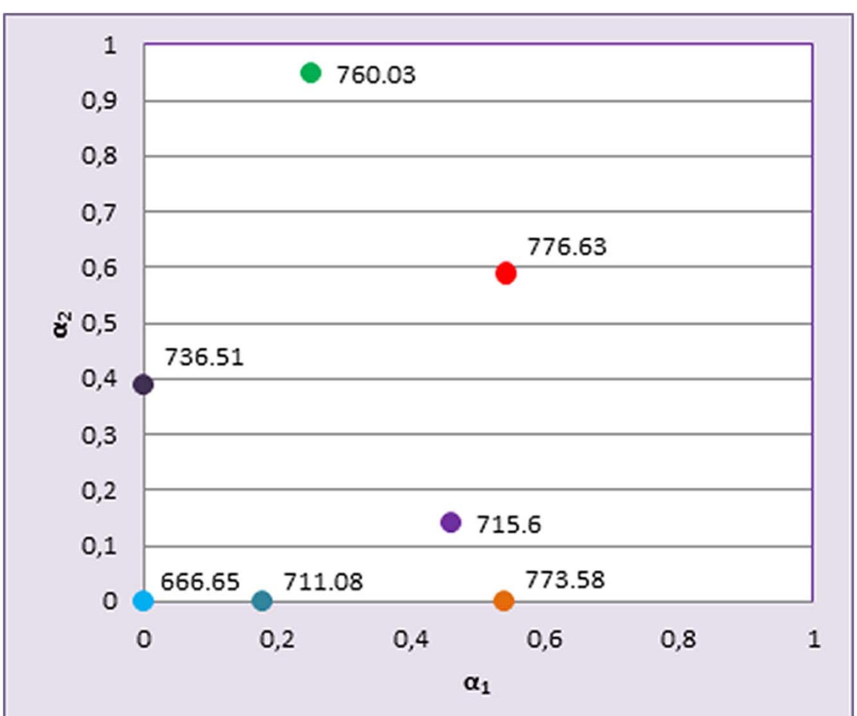

Fig. 4. Minimum confidence levels for each efficient portfolio

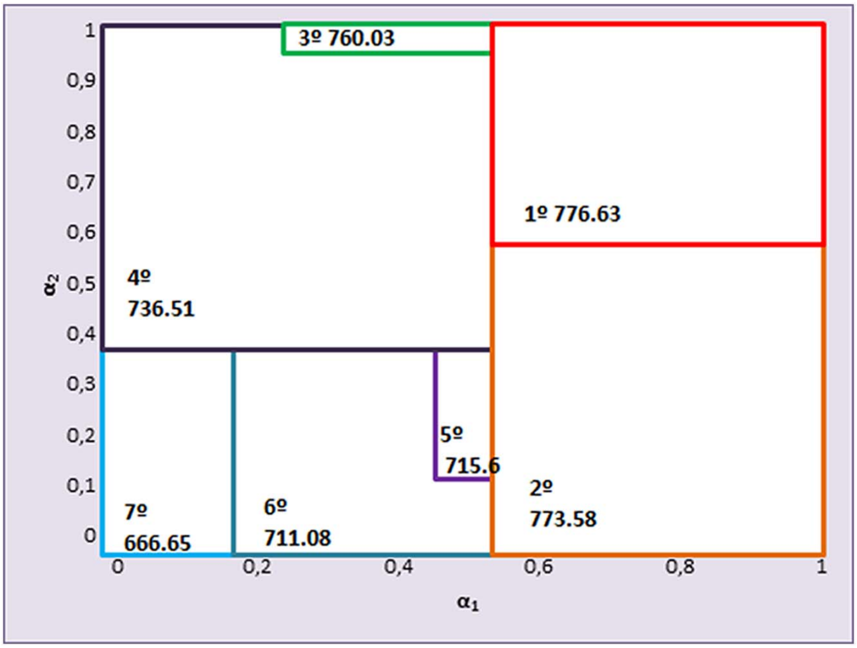

Fig. 5. Solutions associated with the different values of the confidence levels

Table 6
Selecting a portfolio given certain confidence levels

Point	Objective F	α_1	α_2
~~1~~	~~776.63~~	~~0.54~~	~~0.59~~
~~2~~	~~773.58~~	~~0.54~~	~~0~~
~~3~~	~~760.03~~	~~0.25~~	~~0.95~~
4	**736.51**	**0**	**0.39**
5	715.6	0.46	0.14
6	711.08	0.17	0
7	666.65	0	0

Table 7
Projects associated with the efficient portfolios

α1	0.54	α1	0.54	α1	0.25	α1	0	α1	0.46
α2	0.59	α2	0	α2	0.95	α2	0.39	α2	0.14
Obj	776.63	Obj	773.58	Obj	760.03	Obj	736.51	Obj	715.6
Project	t	Project	t	Project	t	Project	t	Project	t
1	1	1	1	2	4	1	3	2	4
3	3	2	4	3	2	2	4	3	3
4	3	4	3	4	3	4	3	4	3
5	1	5	1	5	1	5	1	5	1
7	3	7	3	8	3	6	2	6	2
8	3	8	3			8	3	8	3

α1	0. 17	α1	0
α2	0	α2	0
Obj	711.08	Obj	666.65
Project	t	Project	t
1	1	1	1
2	4	2	4
3	2	3	3
5	2	5	2
8	3	6	2
		8	3

into account the organization's needs, objectives and priorities, and several strategic and political constraints.

The information provided by the university can be summarized as follows:

- There are fifty-two alternative projects (i = 1, 2, …, 52), and the time planning *horizon* consists of 4 consecutive semesters (k = 1, 2, 3, 4).
- The *objective* considered is to maximize the positive impact. This objective function is defined as the sum of positive effects that the selected projects have on the organisation.
- The constraints considered are as follows:
  - There are two renewable resources: monetary resources available (α_1) and risk of the portfolio (α_2).
  - There is one global resource: project 1, 2 and 3 are very similar, and hence only one can be selected in one portfolio.
  - Some projects are mandatory.
  - In addition, there are four precedence relationships.

Therefore, by solving the resulting problem with our method, we obtain the 8 optimal project portfolios shown in Table 8. The variation interval in the value of the objective function is within the range [253, 268].

Fig. 7 shows the solution space delimited according to the confidence levels values. This provides a more accurate picture of the internal structure of the region defined by them. The decisions can now be taken by the DSPI regarding the variation permitted to the monetary resources and the risk of each portfolio. The analysis of Fig. 5 shows that if $\alpha_1 < 0.2$, small changes in confidence levels lead to different portfolio solutions.

The total number of projects as well as the projects that compose each of the 8 efficient portfolios are shown in Table 9. The main point of interest is that there are a total of 33 projects that are always selected regardless of the values taken by the two confidence levels, and therefore they have to be selected in any case. In contrast, there are 11 projects that are never selected, so the DSPI has to know that they will never be part of the optimal portfolio.

With all of this, the DSPI has full and accurate information on which to base their assessment and choice. Although some of the initial information involved a certain degree of uncertainty, by the end of the process not only has the objective function value been obtained for each

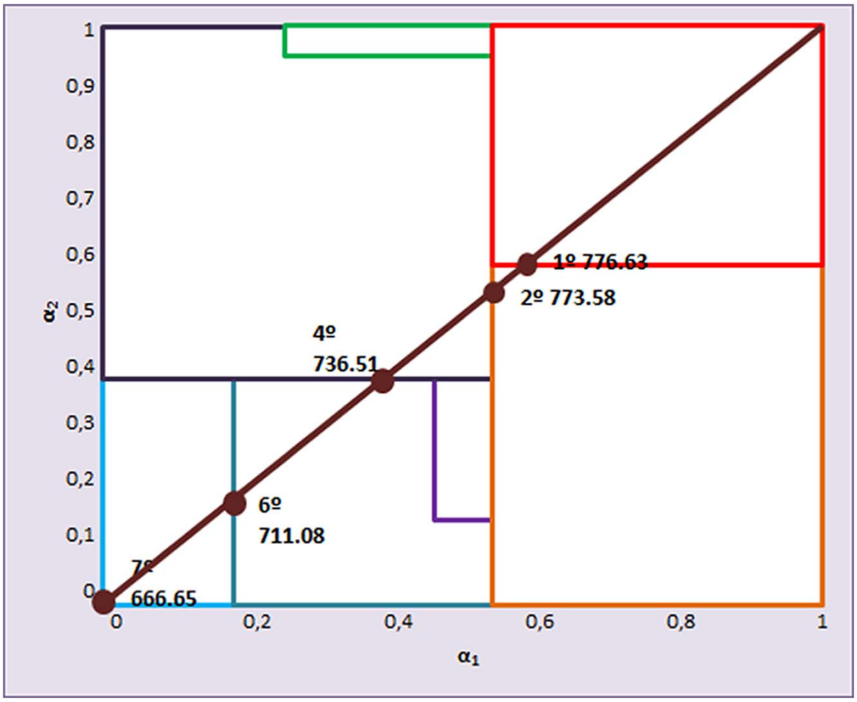

Fig. 6. Comparison of the results obtained with similar and different confidence levels

Table 8
Efficient portfolios obtained

Point	Objective	α_1	α_2
1	268	0	0.75
2	267	0.03	0.59
3	262	0	0.54
4	261	0.07	0.37
5	260	0.4	0.2
6	257	0	0.13
7	254	0.05	0.03
8	253	0	0

project portfolio, but also the confidence level at which the portfolio is optimal. Therefore, the main advantage presented by the results of the model we propose is the stability ensured of these portfolios against any changes that may affect the renewable constraints. However, when the problem is solved using the traditional method, DSPI have no assurance concerning the variations of the coefficients.

All of these results were shown to the DSPI, and they fully appreciated the usefulness of the results offered, since not only do they provide optimal portfolios but they also offer a measure of the stability of each portfolio against variations. Furthermore, they were impressed about the saving of time and effort, because this process takes just a few days and, usually, this process takes those several months of meetings, due to the amount of information to take into account.

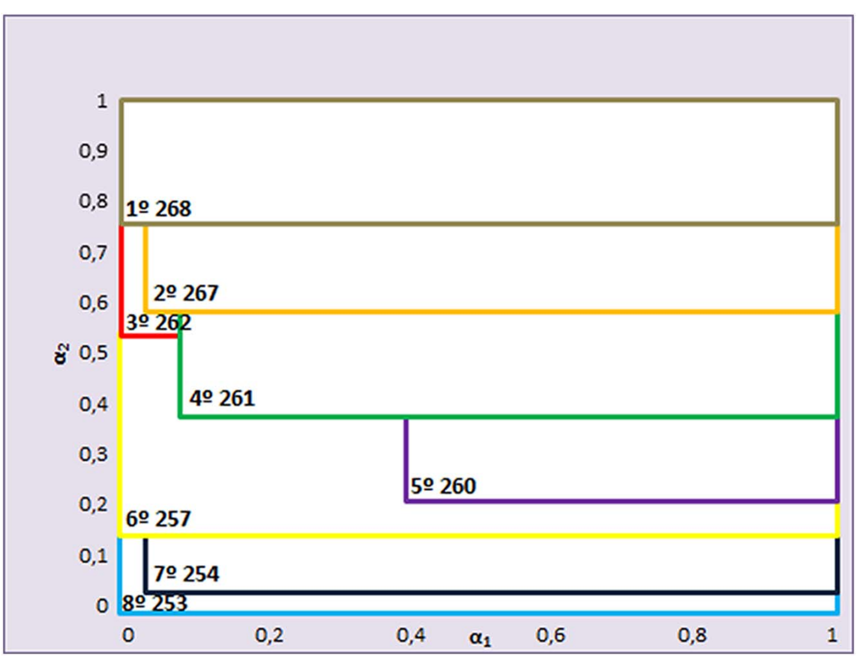

Fig. 7. Solutions associated with the different values of the confidence levels

Table 9
Projects associated with the efficient portfolios

Point	Objective	Projects	Number of projects
1	268	1, 4–15, 20–33, 38–40, 43–44, 46–52	39
2	267	2, 4–11, 13–15, 20–33, 39–40, 42–44, 46–52	38
3	262	2, 4–11, 13–15, 20–33, 39–40, 42–44, 46–52	38
4	261	2, 4–15, 20–24, 26–33, 38–40, 42, 44, 46–52	38
5	260	2, 4–11, 13–15, 20–33, 38–40, 42, 44, 46–52	38
6	257	2, 4–11, 13–15, 20–33, 38, 40, 42, 44, 46–52	37
7	254	2, 4–15, 20–24, 26–33, 38–40, 44, 46–52	37
8	253	2, 4–11, 13–15, 20–33, 38–40, 44, 46–52	37

7. Conclusions

Decision makers usually have to face a budget and other kind of constraints when they have to decide which projects are going to be undertaken (to satisfy their requirements and guarantee profitable growth). The purpose of this work is to assist them in the task of Project Portfolio Selection and Scheduling which is by nature a problem involving uncertainty and/or ambiguity.

In this study, uncertainty has been included in the form of triangular fuzzy numbers in the constraints that limit the amount of each type of resource spent at each point in time. Since these fuzzy numbers are included in the constraints, they have to be ranked in some way, and hence we employed comparison methods based on the confidence levels. The introduction of this comparison method yields a new set of deterministic constraints dependent on the confidence levels selected by the organization.

This means that the feasibility, and therefore the optimality, of a project portfolio is determined by the confidence levels selected, and thus we conducted a theoretical study of the solutions that can be obtained according to variations in the confidence levels. The results from an illustrative example show the equivalence between the solutions obtained for a new multiobjective problem, which was based on the foregoing, and the set of solutions of the original problem. The strength of this approach resides in its potential to be generalized to any problem that, under a fuzzy perspective, makes use of α-cuts in the constraints. In this work, we have applied it to a real problem in a Spanish University. As shown, the results from the theoretical study can be graphically represented, providing the number of dimensions is not too high. Thus, a set of graphical tools that can be easily interpreted by decision makers can be generated. These tools form the basis for the organisation to make the choices that best suit their needs as well as providing information about how good the optimal solution might be bearing in mind the variation present in the initial information, which is characterised by a certain degree of uncertainty.

The main limitation of the proposed model is the use of triangular numbers in the formulation of uncertainty. However, as future work, we will try to replicate the process for another kind of fuzzy uncertainty. Another future development of the presented approach will consist of the proposal of an expert system where the presented model will be integrated, designing thus a useful tool for decision makers in organizations.

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Fátima Pérez is a Professor in the Department of Applied Economics (Mathematics) at the University of Malaga. She has published research papers in reputed international journals in the frame of applied mathematics and applied economics. Her research interests are in the areas of project portfolio selection, multiobjective programming, metaheuristic algorithms, development of computer software and composite indicators.

Trinidad Gómez holds her Degree in Economics and her PhD in Economy and Business Administration. Presently, she is a Senior Professor at the Department of Applied Economics (Mathematics) of the University of Málaga (Spain). Her current research is focused on multiobjective programming techniques from the theoretical, algorithmical and implementations points of view. She is also working on applications in the field of project portfolio selection, in public and private organisations.

Rafael Caballero is a Senior Professor in the Department of Applied Economics (Mathematics), University of Malaga, Spain. He holds his PhD in Mathematics from the University of Malaga. He is interested in the field of multiple objective programming. Presently, his research is on heuristics methods and application to problems in economy and business.

Vicente Liern holds his Degree in Mathematics and his PhD in Theoretical Physics from the University of Valencia. He is a Senior Professor of Financial Economics at the same university. He began his research career in the field of mathematical physics, with the study of the motion of elementary particles. Subsequently, his research field is mathematical optimisation under uncertainty conditions with applications to portfolio management, location and management of human resources.