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A local adaptive learning system for online portfolio selection☆

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ABSTRACT

Online portfolio selection is an important problem in financial trading which has attracted increasing interests from the machine learning and data mining community. Most existing state-of-the-art systems, however, rely on the defensive strategies, which lack adaptivity to some extent and may lose investment opportunities in the real financial market. In this paper, we propose a novel local adaptive learning model which seamlessly integrates both defensive and aggressive strategies to enhance the adaptivity and profitability of the whole portfolio system. Different from some popular portfolio selection systems that assume a predefined price tendency, we set up an evaluation function to predict the tendency and activate the corresponding selection strategies. The total capital is allocated according to the value expectations of different assets. Through taking the bests of the complementary strategies, it can make a good balance between wealth returns and risks. In addition, our system is capable of dealing with financial data in linear time, which is suitable for real-time trading applications. Experimental results on several benchmark datasets show that our model outperforms some state-of-the-art ones both in effectiveness and efficiency.

1. Introduction

Portfolio selection (PS) [1–3] is a fundamental and important research topic in financial data engineering and related areas, which aims to optimize wealth allocation across a set of real-world assets. To achieve certain targets, e.g., maximizing the cumulative wealth, PS plays a key role in numerous financial applications, such as quantitative trading, intelligent wealth management and hedge fund management. Meanwhile, PS for financial trading is very challenging in that the real-world financial market is somewhat uncertain and the information arrives sequentially with some noises. Thus a portfolio manager needs to make decisions carefully without time-consuming considerations, and adjust the capital allocation in order to get acceptable wealth returns.

In recent years, machine learning [4–7] and artificial intelligence techniques [8–12] have greatly pushed forward the progress of PS research. One active research direction is the online portfolio selection (OPS) [13], which aims to receive financial data and make PS decisions through an online manner.

Generally, there are four categories for the OPS problem. The first type of approaches, which is often adopted as the baseline, is called "Benchmark". A trivial strategy is Buy-And-Hold (BAH) [13], which is also called the Market strategy. This strategy invests equally in d assets at the beginning and remains unchanged. On the contrary, Best-Stock (BS) [13] invests all capital on the best asset of the whole investment period. Thus BS is a hindsight strategy. Overall, the Benchmark approaches do not adopt complex or sophisticated models via statistics and machine learning.

The second type of approaches is "Follow the Winner". These approaches assume that more successful assets will continue to perform well in the next period and increase the weights of them. The Universal Portfolio method [14] assigns capital to some good performance experts and finally pools their wealth. The Exponential Gradient method [15] tracks the best asset in the previous period but try to keep previous portfolio information with a regularization term. The Online Newton Step method [16] tracks the best assets in the last few periods while keep some historical portfolio information via online convex optimization.

The third type, "Follow the Loser", is based on the mean-reversion idea that the best-performing assets may not keep successful in the next period and will have the trend to reverse their mean value [17]. Thus the weights of less successful assets will be increased, or more wealth are transferred from winners to losers. The Anti-Correlation [18] method transfers wealth from successful assets to the anti-correlated bad performing ones.

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The Passive-Aggressive Mean Reversion method [19] adopts a well-known online learning algorithm, namely passive-aggressive learning to exploit mean reversion. The Online-Moving Average Reversion [20] predicts asset price changes through moving average while the Robust Median Reversion [21] adopts L^1-median instead to enhance the robustness to noise.

The fourth category is the "Pattern Matching approach", which does not assume explicit directions but make portfolio decisions based on similar historical patterns. The key issues for this type of approach include searching similar samples from historical data in certain time windows and then how to optimize the current portfolio based on the matched data set. Classic algorithms include correlation matching [22], nearest neighbor [23] and kernel method [24].

Despite the diversity of OPS models, it is always important to predict the asset prices trend and make corresponding wealth allocations. However, most OPS models have one limitation in common that they lack some adaptivity to the real-world financial market and may lose some investment opportunities due to their predefined trend assumption. As for the pattern matching based methods, the drawback is obvious in that they call for a huge amount of computation, which goes against the real-time nature of OPS. This is because the real-world financial markets lack apparent cycling features and to find similar historical patterns is quite difficult.

To tackle the drawbacks mentioned above, we propose a novel OPS system called "LOcal ADaptive online portfolio system" (LOAD). The core idea is to enhance the adaptivity of the OPS system for the real-world financial market through the fusion of aggressive and defensive strategies. We establish an evaluation function via local regression analysis to activate corresponding strategies and predict asset prices trend to make PS decisions. The proposed LOAD model is different from most existing OPS systems in two aspects. Firstly, it is a self-adaptive OPS system which effectively fuses complementary strategies and is not a simple ensemble of models with different hyper-parameters. In addition, LOAD does not predefine price trend of the assets. It does not make any probability distribution assumption of the whole financial market, and is also less affected by the changes of the mean and variance of the market. Unlike the pattern-matching based models, LOAD learns a local regression model through the financial data in a small time window, which avoids time-consuming pattern search and ensures the real-time performance for practical trading. Besides, we find that taking both of the long term and short term assets price effect into the price trend prediction can improve the overall performance of the portfolio system.

The experimental results show that the proposed LOAD system outperforms several state-of-the-art and classic OPS models in terms of wealth return as well as risk management. Moreover, LOAD works at a linear time and is proper for real-time financial data applications.

As a summary, this paper makes four contributions as follows:

- we propose a novel OPS system with better adaptivity to real-world financial market through seamless integration of aggressive and defensive strategies.
- we establish an evaluation function to assess the assets and estimate their prices trend using local regression, which is more flexible and efficient than the pattern-matching approaches.
- we take both long term and short term assets price effect into the trend prediction and portfolio decision process.
- we conduct extensive experiments on benchmark datasets to verify the effectiveness and efficiency of the proposed LOAD model in comparison with several state-of-the-art and classic ones.

The rest of the paper is organized as follows. The illustration of the OPS problem setting is introduced in Section 2. A review of related works is presented in Section 3. The LOAD portfolio system is illustrated in details in Section 4. Experimental results of LOAD are presented in Section 5. Finally, conclusions are drawn in Section 6.

2. Problem setting

The OPS problem is a dynamic decision-making process and is fitted to online learning naturally. At the end of the tth trading period, the m assets are represented by a daily close price vector $\mathbf{p}_t = [p_t^1, p_t^2, \ldots, p_t^m]^\mathbf{T}$, $t = 0, 1, 2, \ldots, n$. Based on the price vector, a price relative vector is defined: $\mathbf{x}_t = [x_t^1, x_t^2, \ldots, x_t^m]^\mathbf{T}$, where x_t^i denotes the price relative change of the ith asset in the tth period:

$$\mathbf{x}_t = \frac{\mathbf{p}_t}{\mathbf{p}_{t-1}} \tag{1}$$

At the beginning of the tth trading period, a portfolio manager or an expert system needs to allocate capital to the assets in order to adapt to market changes. The capital allocation is represented by a portfolio vector $\mathbf{b}_t = [b_t^1, b_t^2, \ldots, b_t^m]^\mathrm{T}$, where b_t^i denotes the proportion of the overall capital invested in the ith asset. In general, it is assumed that the portfolio is self-financed and there is no short selling (i.e. there are no debts or divestment), which means $b_t^i \geq 0$ and $\sum_{i=1}^m b^i = 1$.

The performance of a portfolio selection model on the tth period is specified by a daily wealth increasing factor $\mathbf{b}_t^\mathrm{T}\mathbf{x}_t$. Hence the cumulative wealth S_t changes over time and the final cumulative wealth S_n with initial value $S_0 = 1$ can be calculated as:

$$S_n = \prod_{t=1}^{n}(\mathbf{b}_t^\mathbf{T}\mathbf{x}_t) \tag{2}$$

From the interpretation above, the OPS problem is formulated as a dynamic process which learns a sequence of portfolio vectors $\{\mathbf{b}_t\}_{t=1}^n$ with statistical or artificial intelligence techniques, so as to maximize the final cumulative wealth S_n. Different portfolio selection models are featured by how they get the portfolio vectors and mainly evaluated by the final cumulative wealth S_n as well as some risk management metrics.

3. Related work

As one of the most important branches of quantitative finance research, the PS problem has been studied extensively over the past decades [25]. Recently, the OPS issue which leverages more theories and practice from the machine learning community has aroused widespread attention [13,26,27]. Empirical studies have shown that the wealth return of assets is predictable in financial market through certain approaches [28–31]. Thus many models which are based on diverse assumptions and strategies have been proposed to tackle this challenging task. Each OPS model adopts a specific strategy with different update schemes that change the capital allocation among assets dynamically in order to get excess wealth return. We review some representative OPS models in this section. Firstly, we introduce the benchmark model. Then we categorize existing models based on different strategies in terms of capital transfer and give a brief comparison of our model with them.

The benchmark model is *Buy-And-Hold* (BAH) [13], which is also called *market strategy*. The BAH model invests equally in m assets at the beginning and remains unchanged during the next periods, thus the weights of assets change passively with their price variations. A special BAH model, the *Best-Stock*, invests all

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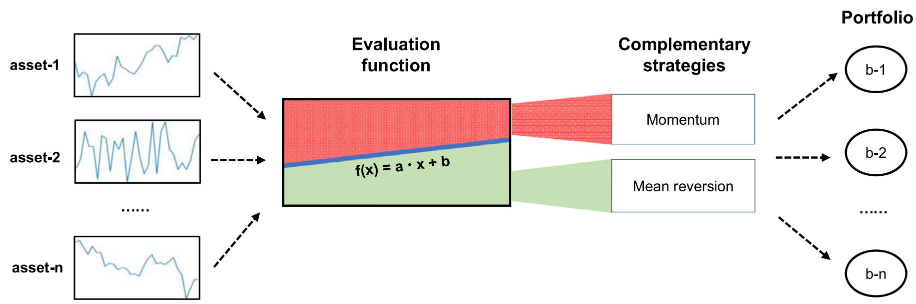

Fig. 1. Illustration of the proposed local adaptive learning system for portfolio selection. Complementary strategies are fused via an evaluation function to enhance the overall adaptivity and profitability (capability to gain acceptable wealth returns).

the capital into the best asset over the periods which is an optimal BAH in hindsight. Overall, the Benchmark PS models are quite simple because they do not adopt complex or sophisticated techniques via statistics and machine learning to explore the patterns within the data. Thus they are often taken as the baselines for performance comparison with new designed models.

Most modern OPS models are based on certain assumptions and explore some investment strategies to allocate capitals to the assets dynamically. One classic group of OPS models adopt predefined trend representations. The momentum principle based models assume that more successful assets will continue to perform well in the next period and thus increase the weights of them to earn more wealth return. For example, the Universal Portfolio model [14] assigns capital to some good performance assets and finally pools their wealth. The Exponential Gradient model [15] tracks the best asset in the previous period while try to keep previous portfolio information with a regularization term. The Online Newton Step method [16] tracks the best assets in the last few periods while it keeps some historical portfolio information via online convex optimization. On the contrary, the mean reversion principle based models believe that the bad-performing assets will stop their declining tendency in the next period and resort to their historical mean values [17]. For example, the Anti-Correlation [18] model transfers wealth from successful assets to the anti-correlated bad performing ones. The Passive-Aggressive Mean Reversion model [19] adopts a well-known online learning algorithm, namely passive-aggressive learning [32] to exploit mean reversion. The Online-Moving Average Reversion model [20] predicts asset price changes through moving average while the Robust Median Reversion [21] adopts L^1-median instead to enhance the robustness to noises. Overall, these models are only effective in limited markets due to their pre-defined trend assumption.

Another type is the pattern-matching based model, which does not assume explicit wealth transfer directions but makes portfolio decisions based on historical pattern analysis. The key issue for these models is to locate similar patterns in a relative long historical time window, which leads to time-consuming computations. For example, the CORN system [22] uses cross window correlation to predict the current trend. The B^{NN} system [23] adopts the nearest neighbor algorithm while the B^{K} system [24] exploits the kernel method to search similar historical patterns for making PS decisions. Although these methods do not predefine price trend and show more flexibility, they often demand for a huge amount of computation which severely reduces the performance for practical trading.

The proposed LOAD model in this paper can be seen as an integration of the above models. Like the pattern-matching methods, the LOAD does not predefine assets price tendency. However, it eliminates time-consuming pattern searching and leverages efficient local regression to fuse different strategies based on the momentum and the mean reversion principles.

4. Local adaptive learning system for portfolio selection

The aggressive strategy [14–16] assumes that more successful assets will continue to perform well in the next period and increase the investment to them. On the contrary, the defensive strategy [17,19,20] holds that assets that perform well in the last period cannot continue to have good performance and will reverse their historical mean value. The real financial markets are not always effective and normally distributed. Thus neither aggressive nor defensive strategies can always have good performance. To fill the gap between them, we establish an evaluation function based on local regression to estimate the price trend for assets. After the price relative has been estimated, a portfolio vector is determined within an online optimization framework.

4.1. Motivation

Empirical studies [28–30] show that the financial markets are predictable. In recent years, some OPS systems explore the mean reversion strategy and achieve state-of-the-art performance [20]. These systems hold the assumption that the asset price will revert to its historical mean values, which is actually a defensive strategy. They can often show some robustness in some aspects, however, the assumption is not always in conformity with the real-world financial market data. In fact, there also coexists another phenomenon, which is also called the “momentum” principle, that the better performing assets will continue its rising trend and further push up the prices. Thus excess returns may also be gained if aggressive OPS systems are developed using the momentum strategy. In our experiments, we find that the mean reversion strategy is effective and can achieve acceptable wealth return in certain markets. Due to its defensive nature, however, it may lose some investment opportunities for some good-performing assets, where the momentum effects play a more important role. For example, the mean reversion methods assume that the price will revert to its historical value, then if the price actually still goes up, this strategy will lose investment opportunity.

To deal with this problem and take the advantages of both strategies, we establish an adaptive model. Some “really good” assets are picked out by an evaluation function and their trend prediction is based on the momentum principle. The “really good” assets are defined as the assets that have an obvious upward trend in a local time window. Note that “an obvious upward trend” is measured by the gradient of the evaluation function which is higher than a threshold value. The evaluation function is based on a regression model that is trained within the local time window. If the gradient of the function is larger than a threshold, then the asset is picked out as a “really good” one.

As for other assets, it is assumed that the mean reversion strategy is more effective. Through this fusion, the overall system can be more adaptive and effective than systems based on single strategies. The overall of our OPS system is illustrated in Fig. 1.

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4.2. Local adaptive system for price trend representation

The first step of the entire LOAD portfolio system is to estimate the relative price $\hat{\mathbf{x}}_{t+1}$. We intend to fuse the momentum and mean reversion strategy to estimate the asset prices via an evaluation function. Specifically, we set up a local regression model as the evaluation function which is characterized by two aspects. Firstly, we assume that building a regression model to depict the entire financial market is difficult and unpractical. Instead, we train a model based on the asset prices in the recent w periods which is used to predict the price in the next period. Secondly, we use *time* as the input variable to train the regression model. This has relaxed some constraints of the assumptions on the distribution of the financial market data. Then for the ith asset, the function is defined as:

$$f_i(t) = a_i t + \gamma_i \tag{3}$$

where a_i denotes the gradient and γ_i is the bias term. The function models the relationship between time and the asset prices in a time window. At the end of the tth trading period, the asset prices in a time window $[p_{t-w+1}, \ldots, p_t]$ are adopted to train the function via the optimization as:

$$\min_a \sum_{\tau=1}^{w} \|p_{t-w+\tau} - f(\tau)\|^2 + \lambda \|a\|^2 \tag{4}$$

where $\lambda \geq 0$ is a parameter controlling the influence of the regulation term.

We use a as the metric to evaluate the performance of different assets. Intuitively, $a > 0$ denotes that the asset price tends to be upward, i.e. has a good performance. However, we find that there should be some threshold η rather than zero which can make the prediction more accurate. Specifically, when $a > \eta$, the asset has a "real" good performance, which follows the momentum principle and will get continuous investment. As for its price in the next period, we use the maximum price in the recent w periods as the estimation which is calculated as:

$$\hat{p}_{t+1} = \max_{0 \leq k \leq w-1} p_{t-k} \tag{5}$$

On the contrary, we believe that the asset has an average performance in the past time window, and it follows the mean reversion principle which estimates the price as:

$$\hat{p}_{t+1}(\alpha) = \mathbf{MA}_t(\alpha) = \alpha p_t + (1-\alpha)\mathbf{MA}_{t-1}(\alpha) \tag{6a}$$

$$\mathbf{MA}_1(\alpha) = p_1 \tag{6b}$$

where α is a decaying factor. Note that this estimation via exponential average takes account of all the historical data, and it performs better than averages in small time windows.

Then the estimated prices of different assets are gathered as a vector $\hat{\mathbf{p}}_{t+1}$ to calculate the predicted price relative $\hat{\mathbf{x}}_{t+1}$ as:

$$\hat{\mathbf{x}}_{t+1} = \frac{\hat{\mathbf{p}}_{t+1}}{\mathbf{p}_t} \tag{7}$$

4.3. Online portfolio optimization

When the price relative $\hat{\mathbf{x}}_{t+1}$ is predicted, the next step is to make the final portfolio decision to maximize the expected wealth return while keeping last portfolio information through a regularization term. Then the final online portfolio can be solved in an optimization equation as:

$$\hat{\mathbf{b}}_{t+1} = \operatorname*{argmin}_{\mathbf{b} \in \Delta_m} \frac{1}{2} \|\mathbf{b} - \mathbf{b_t}\|^2 \quad s.t. \quad \mathbf{b}^{\mathbf{T}} \hat{\mathbf{x}}_{t+1} \geq \epsilon \tag{8}$$

Note that there are also simplex constraints on $\mathbf{b}_{t+1}$ as follows:

$$\mathbf{1}^{\mathbf{T}} \mathbf{b}_{t+1} = 1 \tag{9}$$

Algorithm 1: The LOAD online PS system

Input: The asset prices in the recent time window $\{\mathbf{p}_{t-k}\}_{k=0}^{w-1}$ and the current portfolio $\mathbf{b}_t$.
Output: The portfolio $\mathbf{b}_{t+1}$ of the next period; Cumulative wealth S_t for each period.

```
Initialization: b_1 = (1/m) 1, S_0 = 1, x_1 = 1;
repeat
    Rebalance the portfolio to b_t;
    Receive stock price p_t;
    Calculate the price relative vector x̂_t;
    Calculate daily return and cumulative wealth
    return: S_t = S_{t-1} · (b_t · x_t)
    Calculate the local regression model by Eq. (3);
    if a_i > η then
        Calculate the price prediction p^i_{t+1} by Eq. (5)
    else
        Calculate the price prediction p^i_{t+1} by Eq. (6)
    Calculate the next price relative vector x̂_{t+1};
    Optimization: b̂_{t+1} = b_t + γ_{t+1}(x̂_{t+1} − x̄_{t+1} 1)
    Projection: b̂_{t+1} = argmin_{b∈Δm} ||b − b_{t+1}||^2
until End of trading periods;
```

$$\mathbf{b}_{t+1} \succeq \mathbf{0} \tag{10}$$

According to [33] and [20], the solution of the optimization problem can be solved in an online update manner as:

$$\mathbf{b}_{t+1} = \mathbf{b}_t + \gamma(\hat{\mathbf{x}}_{t+1} - \boldsymbol{\mu}) \tag{11}$$

where $\boldsymbol{\mu}$ is the mean value of the predicted price vector and γ is calculated as:

$$\gamma = \max\left\{0, \frac{\epsilon - \mathbf{b}^{\mathbf{T}} \hat{\mathbf{x}}_{t+1}}{\|\hat{\mathbf{x}}_{t+1} - \boldsymbol{\mu}\|^2}\right\} \tag{12}$$

The whole LOAD portfolio system can be summarized as Algorithm 1.

5. Experiments

5.1. Experimental setup

5.1.1. Datasets

We conduct experiments on six benchmark datasets. The five classic datasets include NYSE(O) [14], NYSE(N) [14], TSE [31], SP500 [31] and MSCI [26]. They contain real-world financial data in terms of **daily close price relatives** from diverse stock and index markets. These markets include the New York Exchange, the Toronto Stock Exchange, Standard & Pool 500, and the MSCI World Index. The NYSE(O) and NYSE(N) are two common datasets from the US stock market for PS research pioneered by Cover. The dataset SP500 is collected from the US market by Borodin et al. The TSE dataset is collected from the Toronto stock market by Borodin et al. The MSCI dataset is a collection of global equity indices which represent the world-wide equity markets from 24 countries. In addition, we also construct a dataset called HS300 from the stock market of China (http://www.csindex.com.cn). It consists of the price relatives of 23 representative stocks in a more recent time. All of these datasets cover a wide range of assets and long trading periods, which enables us to fully evaluate the OPS systems under different events in the stock market. More detailed information of the datasets is shown in Table 1.

It should be noted that we just focus on historical daily closing prices in stock markets, which are easy to obtain and publicly

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Table 1
Information of six benchmark datasets.

Dataset	Region	Time Span	Days	Assets
NYSE(O)	US	3/7/1962–31/12/1984	5651	36
NYSE(N)	US	1/1/1985–30/6/2010	6431	23
SP500	US	2/1/1998–31/1/2003	1276	25
TSE	CA	4/1/1994–31/12/1998	1259	88
MSCI	Global	1/4/2006–31/3/2010	1043	24
HS300	CN	2/1/2018–17/5/2019	332	23

Table 2
Final cumulative wealths of different portfolio selection systems on six benchmark datasets.

Dataset	NYSE(O)	NYSE(N)	MSCI	SP500	TSE	HS300
Market	14.50	18.06	0.91	1.34	1.61	1.09
Best-Stock	54.14	83.51	1.50	3.78	6.28	1.64
ONS	109.19	21.59	0.86	3.34	1.62	1.10
UP	27.10	31.73	0.92	1.66	1.58	1.11
EG	27.09	31.00	0.93	1.63	1.59	1.11
CORN	8.10E+11	2.33E+05	11.26	5.29	9.79	1.57
Anticor	2.04E+07	2.11E+05	2.77	5.61	28.68	1.07
PAMR	5.14E+15	1.25E+06	15.23	5.10	264.86	0.96
CWMR	6.49E+15	1.41E+06	17.28	5.95	332.62	0.96
OLMAR	1.02E+18	4.69E+08	22.51	9.59	732.44	**1.71**
RMR	1.64E+17	3.25E+08	16.76	8.28	181.34	1.12
LOAD	**1.03E+18**	**6.66E+08**	**24.24**	**9.78**	**733.77**	1.65

available to other researchers. Data from high-frequency intraday trading data are either too expensive or difficult to get and process, thus are beyond our scope in this paper.

5.1.2. Evaluation metrics

We adopt six metrics to evaluate the performance of different OPS systems: (1) *Cumulative Wealth* (CW), the most common metric to primarily evaluate the performance of different models; (2) *Mean Excess Return* (MER) [34], the average excess return upon the market strategy; (3) *Sharpe Ratio* (SR) [35], the risk-adjusted average return; (4) *Information Ratio* (IR) [36], risk-adjusted MER. Overall, the former two metrics evaluate the performance of investment while the latter two test the risk-controlling capability which is important in the financial community. Note that these metric values are the higher the better which show the profitability of a OPS system. (5) *CW with Transaction Costs* [37], the cumulative wealth according to the proportional transaction costs. Since transaction cost is the most important issue in practical trading, this metric is the most important one for evaluating an OPS system. A higher value of this metric indicates a better OPS system for practical use. (6) *Computational Time*, the average running time (in seconds) for one trading period. This metric evaluates the running efficiency for an OPS system, and lowering down computational time can improve the practicality for real-life trading.

5.1.3. Comparison approaches

To fully analyze the performance of the proposed LOAD system, we adopt several classic and state-of-the-art OPS models to be tested for performance comparison. These OPS approaches are summarized as follows.

Benchmark approaches

- Market: uniform Buy-and-Hold strategy [13];
- Best-Stock: Buy and hold the best stock in hindsight [13];

Aggressive approaches

- ONS: Online newton step [16];
- UP: Cover's universal portfolio algorithm [14];
- EG: Exponential gradient algorithm [15];

Defensive approaches

- Anticor: Anti-correlation algorithm [18];
- PAMR: Passive-aggressive moving reversion algorithm [19];
- CWMR: Confidence weighted mean reversion algorithm [38];
- OLMAR: Online moving average reversion algorithm [20];
- RMR: Robust median reversion algorithm [21];

Pattern-matching approaches

- CORN: Correlation-driven nonparametric learning algorithm [22];

Among these OPS approaches, Market and Best-Stock are benchmark methods; ONS, UP and EG are aggressive methods based on the momentum principle; PAMR, CWMR, OLMAR and RMR are defensive methods directed by the mean reversion principle; CORN is a state-of-the-art pattern-matching method through historical similar patterns searching. Their parameters are set to the same values as their original papers.

Table 3
Mean excess returns of different portfolio selection systems on six benchmark datasets.

Dataset	NYSE(O)	NYSE(N)	MSCI	SP500	TSE	HS300
Best-Stock	0.0003	0.0003	0.0004	0.0012	0.0016	0.0015
ONS	0.0004	0.0003	0.00001	0.0007	0.0001	0.0001
UP	0.0001	0.0001	0.00002	0.0001	0.00002	0.0001
EG	0.0001	0.0001	0.0001	0.0001	0.00001	0.0001
CORN	0.0049	0.0017	0.0025	0.0014	0.0020	0.0013
Anticor	0.0026	0.0016	0.0011	0.0013	0.0027	−0.00001
PAMR	0.0064	0.0021	0.0029	0.0014	0.0050	−0.0018
CWMR	0.0064	0.0022	0.0030	0.0015	0.0053	−0.0018
OLMAR	0.0075	0.0032	0.0033	**0.0020**	**0.0065**	**0.0016**
RMR	0.0071	0.0032	0.0030	0.0019	0.0053	0.0003
LOAD	**0.0076**	**0.0033**	**0.0034**	**0.0020**	**0.0065**	**0.0015**

5.1.4. Parameter settings

There are three key parameters to be set in our OPS system. As for the momentum strategy, the size of time window is set to 5, since asset prices in most recent trading days can provide more reliable information for the momentum principle and short-term financial analysis and prediction. As for the mean reversion strategy, the parameter α in (6) is set to 0.5 which is a typical setting for mean reversion based OPS models [20]. The parameter η plays a key role in the whole system since it determines which strategy is activated. In our experiments, we find that when η is set to 0.1, the LOAD system performs well on most of the datasets, thus we set it to 0.1 in the experiments. Note that this is an empirical set and all the parameters are kept fixed throughout all the experiments.

5.2. Experimental results

5.2.1. Experiment 1: evaluation of cumulative wealth

The final Cumulative Wealth (CW) is adopted as the main metric to evaluate the investment performance of each OPS system. The final CWs of different OPS systems on the six datasets are shown in Table 2. The proposed LOAD system outperforms the other state-of-the-art methods on most of the datasets except HS300 (ranks the second). Especially, LOAD has a better performance than the methods that just leverage a single strategy.

It should be pointed out that, as for OPS models, even a tiny performance improvement can lead to a huge profit gain. In the OPS research which is within the scope of this paper, all the

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Table 4
Sharpe ratios of different OPS systems on six benchmark datasets.

Dataset	NYSE(O)	NYSE(N)	MSCI	SP500	TSE	HS300
Market	0.0552	0.0457	0.0017	0.0227	0.0505	0.0242
Best-Stock	0.0537	0.0471	0.0375	0.0483	0.0589	0.0638
ONS	0.0791	0.0313	0.0019	**0.0704**	0.0287	0.0255
UP	0.0732	0.0507	0.0031	0.0354	0.0484	0.0281
EG	0.0728	0.0506	0.0033	0.0346	0.0493	0.0280
CORN	0.1635	0.0967	0.1289	0.0582	0.0678	0.0693
Anticor	0.1721	0.0972	0.0607	0.0683	0.1065	0.0206
PAMR	0.2149	0.0863	0.1252	0.0569	**0.1184**	0.0075
CWMR	0.2169	0.0857	0.1307	0.0607	0.1181	0.0076
OLMAR	**0.2237**	0.1049	0.1310	0.0683	0.1157	**0.0776**
RMR	0.2153	0.1032	0.1220	0.0660	0.0982	0.0270
LOAD	**0.2218**	**0.1064**	**0.1331**	**0.1331**	0.1156	**0.0732**

Table 5
Information ratios of different OPS systems on six benchmark datasets.

Dataset	NYSE(O)	NYSE(N)	MSCI	SP500	TSE	HS300
Best-Stock	0.0241	0.0224	0.0334	0.0465	0.0497	0.0576
ONS	0.0424	0.0313	0.0013	0.0582	0.0093	0.0117
UP	0.0358	0.0256	0.0289	0.0373	−0.0074	0.0823
EG	0.0365	0.0258	0.0339	0.0365	−0.0047	**0.0859**
CORN	0.1570	0.0859	0.1835	0.0618	0.0594	0.0735
Anticor	0.1743	0.0899	0.1467	**0.0833**	0.0992	−0.0013
PAMR	0.2120	0.0763	0.1876	0.0573	**0.1130**	−0.0106
CWMR	0.2143	0.0757	**0.1960**	0.0622	0.1128	−0.0099
OLMAR	**0.2218**	0.0969	0.1919	0.0700	0.1110	0.0831
RMR	0.2123	0.0953	0.1788	0.0677	0.0932	0.0156
LOAD	**0.2198**	**0.0986**	**0.1947**	**0.0703**	0.1108	0.0773

prices are in the form of "price relatives", thus all the results in terms of profit gains are all actually in the form of relative value, i.e. ratios. It is a common sense in economics and finance that, in a full and effective financial market, gaining even a small increase in terms of ratios can lead to big money gains due to the extremely huge capital amount. In Table 2, for example, if the initial investment is one dollar on the NYSE(O) market, then the LOAD can gain 1.03E+18 dollars, and the OLMAR can get 1.02E+18 dollars. Although the absolute ratio is 1.01, the LOAD system can actually gain 1.0E+16 dollars more than the OLMAR. This is a huge number of money. In fact, the initial investment is often more than millions in the real market, then the profit gain of LOAD compared with OLMAR is surely much more.

The result has demonstrated that the enhanced adaptivity gained by the LOAD system can make better investment achievements in the real-world financial markets.

One special case is HS300 on which LOAD ranks the second place. This can be attributed to two reasons. First, this result indicates that the financial market of China may be a little different in terms of statistical features from the others. Second, the mean reversion phenomenon plays a highly significant role in this market, thus the aggressive strategy suffers high risks in it.

To show more details in wealth cumulations during online running, we also plot the daily CWs of different OPS systems on two representative datasets in Fig. 2. From the plots, it can be seen that the LOAD system achieves higher wealth return than other OPS systems in most periods, which has demonstrated the advantage of our model when making portfolio decisions.

5.2.2. Experiment 2: mean excess return

The Mean Excess Return (MER) is defined as the average excess return of an OPS system over the market:

$$\mathbf{MER} = \bar{r}_s - \bar{r}_m = \frac{1}{n}\sum_{t=1}^{n}(r_{s,t} - r_{m,t}) \tag{13}$$

where $r_{s,t}$ and $r_{m,t}$ are the **daily returns** of an OPS system and the Market strategy in the tth period respectively. The daily return of

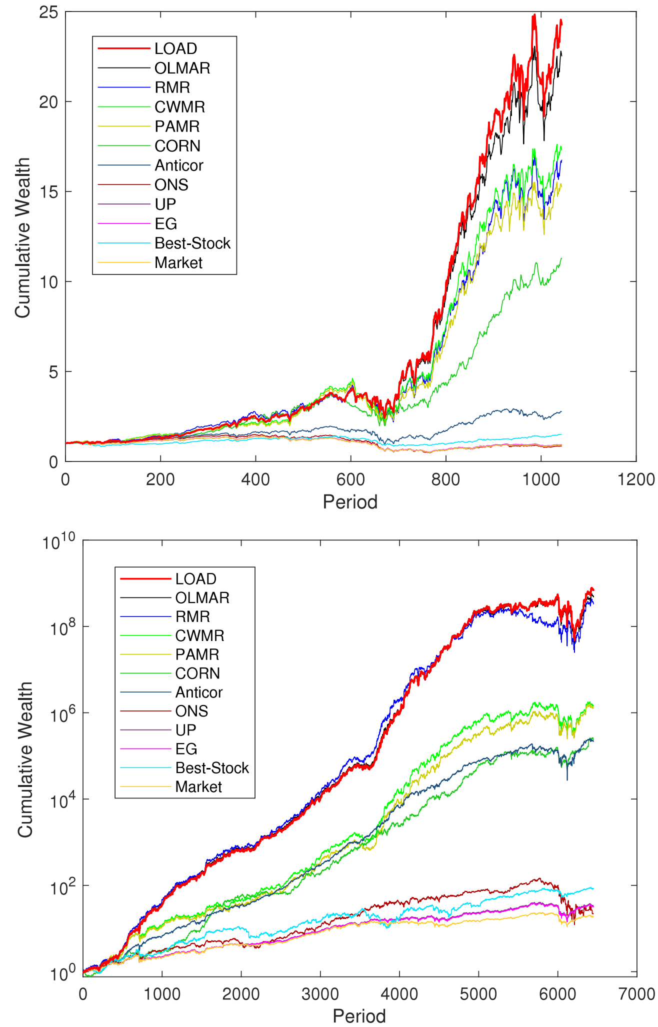

Fig. 2. Daily cumulative wealth of different OPS systems on two real financial datasets: MSCI (upper) and NYSE(N) (bottom).

a specific OPS system in the tth period is calculated as:

$$r_t = \mathbf{b}_t^{\mathrm{T}}\mathbf{x}_t - 1 \tag{14}$$

It is should be noted that the MERs are all small values, nevertheless even a tiny difference of MER may make a wide gap between different OPS systems in terms of final CW. The MERs of different OPS systems are shown in Table 3. The proposed LOAD system achieves the highest MERs on most the six benchmark datasets except HS300, which has demonstrated its profitability in a long term. Also, LOAD ranks the second place in the HS300 dataset. Since MER is highly related with CW, the reasons for this result are the same with that for CW.

The overall good profitability of our LOAD system can be attributed to the fusion of the complementary strategies in our portfolio model. Self-adaptivity via this fusion is able to capture more investment opportunities, thus achieve more daily and cumulative wealth returns.

5.2.3. Experiment 3: evaluation of Sharpe Ratio

In real-world financial trading, higher profits often come with higher risks. Thus it is an important issue for an investment system to make a balance between excess wealth returns and risks. The Sharpe Ratio (SR) is a popular metric to evaluate risk-adjusted

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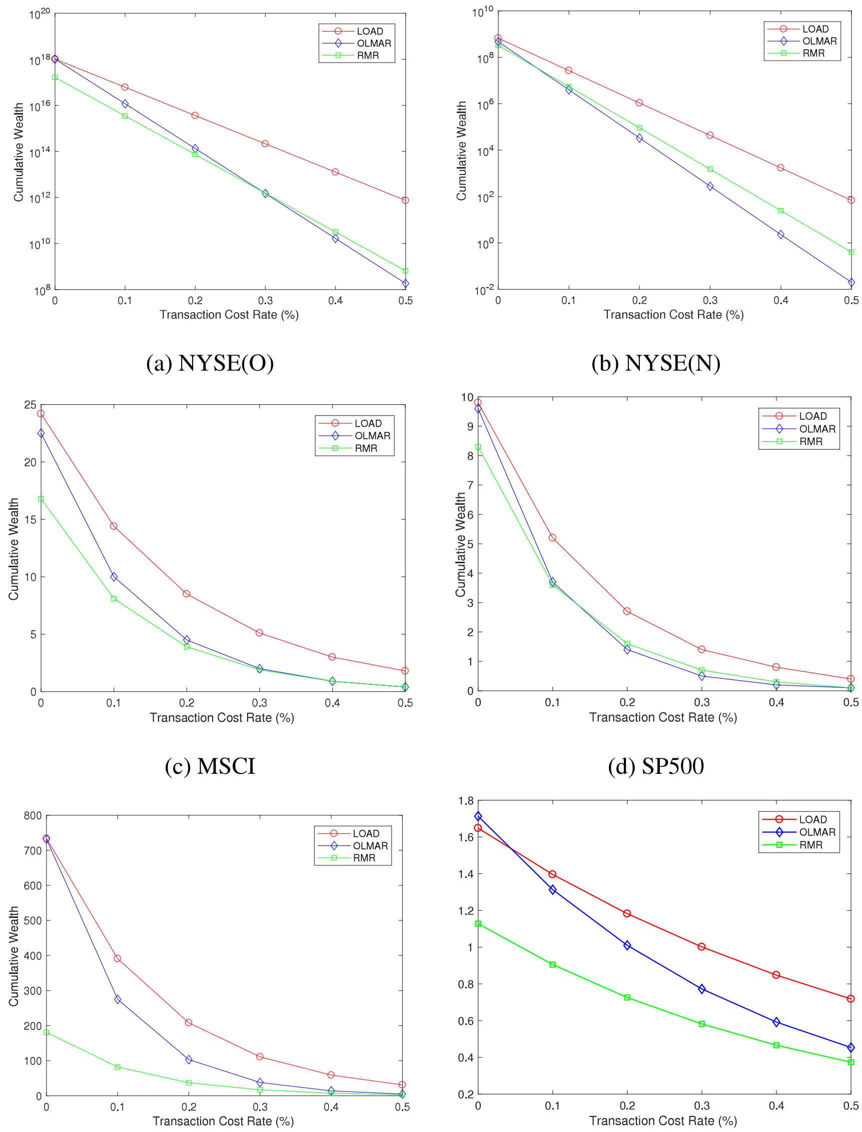


(a) NYSE(O) (b) NYSE(N)

(c) MSCI (d) SP500

(e) TSE (f) HS300

Fig. 3. Cumulative Wealths (CWs) of different systems with transaction cost rate on six benchmarks datasets. When the transaction cost increases from 0 to 0.5%, the proposed LOAD system significantly outperforms the other state-of-the-art methods by a large margin.

returns, which is defined as:

$$SR = \frac{\bar{R}_s - R_f}{\sigma_s} \tag{15}$$

where R_f is the return of a risk-free asset and we set it to 0 since no risk-free assets are considered here. $\bar{R}_s$ and σ_s denote the average return and the standard deviation of a specific OPS system in n trading periods. The higher the SR is, the more risk-adjusted return is achieved.

The SRs of different OPS systems are shown in Table 4. The proposed LOAD system achieves the highest SRs on the NYSE(N) and MSCI datasets, and ranks second in the SP500 dataset. In addition, it also shows competitive performance with the OLMAR and RMR on other datasets. It should be noted that the defensive OPS systems (OLMAR, RMR) are based on the mean reversion principle, which is a strategy to alleviate volatility and avoid risk to a large extent, thus is able to achieve high SRs. Despite this, our LOAD system still shows robust performance with a balance between excess returns and risks.

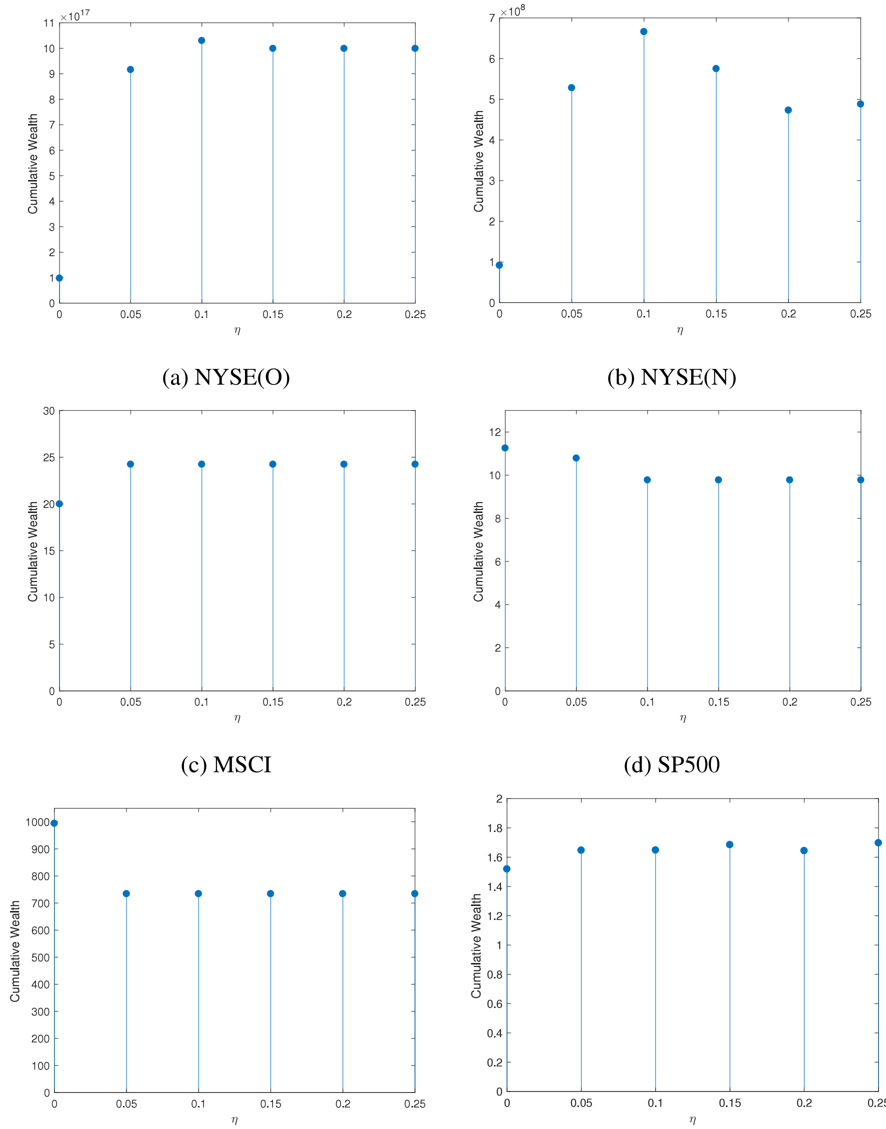
5.2.4. Experiment 4: information ratio

Information Ratio (IR) is a metric to evaluate the excess risk-adjusted returns over the Market strategy, which is defined as:

$$IR = \frac{\bar{R}_s - \bar{R}_m}{\sigma(R_s - R_m)} \tag{16}$$

where R_s and R_m are the returns of n trading periods (in the form of a vector) of an OPS system and the Market strategy respectively, and $\sigma(\star)$ denotes the standard deviation of the excess return of an OPS system over the Market in n trading periods. The higher the IR is, the more risk-adjusted return is achieved.

The IRs of different OPS systems are shown in Table 5. From the results, the proposed LOAD system achieves the best performance on the NYSE(N) dataset, and ranks top 2 on most of the other datasets. In addition, the LOAD system achieves higher IRs than the two state-of-the-art defensive OPS systems OLMAR and RMR on the NYSE(N), MSCI and SP500 datasets respectively. Overall, the results have demonstrated that the LOAD system not only has good profitability but also controls risks effectively.



(a) NYSE(O) (b) NYSE(N)

(c) MSCI (d) SP500

(e) TSE (f) HS300

Fig. 4. Cumulative Wealth (CW) of our LOAD system with respect to η on six benchmarks datasets.

The overall good performance in terms of risk-adjusted return can be attributed to that the trend prediction based on local regression have absorbed more information within the financial data. This can reduce the risks in terms of the uncertainty of asset prices fluctuations.

5.2.5. Experiment 5: transaction costs

Transaction cost is the most important issue in practice for real-world trading. Suppose there is a transaction cost rate $r \in (0, 1)$ for the portfolio re-balance in each trade, then the cumulative wealth according to the proportional transaction cost model [20,37] can be calculated as:

$$S_n^r = S_0 \prod_{t=1}^{n} \left[(\mathbf{b}_t^{\mathbf{T}} \mathbf{x}_t) \times \left(1 - \frac{r}{2} \sum_{i=1}^{m} |b_{t,i} - \tilde{b}_{t-1,i}| \right) \right] \quad (17)$$

where $\tilde{b}_{t-1,i} = \frac{b_{t-1,i} \times x_{t-1,i}}{\mathbf{b}_{t-1}^{\mathbf{T}} \mathbf{x}_{t-1}}$ is the price adjusted portfolio of asset i in the $(t-1)$th period and $\tilde{\mathbf{b}}_0$ is set to $[0, \dots, 0]^{\mathbf{T}}$. The term $\sum_{i=1}^{m} |b_{t,i} - \tilde{b}_{t-1,i}|$ denotes the transaction cost caused by re-balancing the adjusted portfolio $\tilde{\mathbf{b}}_{t-1}$ to $\mathbf{b}_t$.

To verify the performance for real-world trading, we conduct experiments of cumulative wealth with transaction cost rate r varying in 0∼0.5%. The cumulative wealths of LOAD and two state-of-the-art OPS systems with respect to the transaction cost rate on the benchmark datasets are shown in Fig. 3. From the results, the proposed LOAD system significantly outperforms the other systems on all the datasets, which demonstrates that it is applicable for real-world financial trading.

5.2.6. Experiment 6: computational time

Since we are focusing on **online** portfolio selection which is a time-limited application, the running efficiency is an important and practical issue. The OPS models that can make a good balance between wealth accumulation and running efficiency will be the ideal candidates for real-world financial trading. As for the proposed system, it does not rely on complex nonlinear calculation and all the operations are based on linear models with step-wise update.

We run the LOAD system for 5 times on each dataset to evaluate its computational cost. All the experiments are conducted on an Intel Core 2 i7 CPU with 16G RAM. The average running time (in seconds) on one trading period are 0.0010, 0.0006, 0.0003, 0.0003, 0.0007 and 0.0001 on NYSE(O), NYSE(N), MSCI, SP500, TSE and HS300 respectively. The result indicates that the proposed LOAD system is efficient in computing, thus is suitable for large-scale and real-time financial applications.

It should be mentioned that the pattern-matching based approaches are obviously much more time-consuming than other methods. Their computational time is two orders of magnitude than ours, which is unacceptable for practical trading. This is because these methods consist of two steps in general: the sample selection and portfolio optimization steps. A lot of computational time is spent on the sample searching procedure. In addition, different matching methods, e.g., kernel and nearest neighbor algorithms, can also bring different computational burden.

5.2.7. Experiment 7: parameter sensitivity for η

The parameter η plays a crucial role in our OPS system since it determines when to trigger different strategies. In essence, η is used to pick out some outstanding assets and invest them. It makes a balance between wealth returns and risks. We compute the Cumulative Wealth (CW) under different values of η to test its influence on our LOAD system. The results are shown in Fig. 4.

From the results, the improvement is substantial when η is larger than 0 in the NYSE(O) and NYSE(N) datasets. This indicates that in these markets, only investing the assets that have relatively strong upward trend can get significant wealth return.

Overall, the LOAD portfolio system is quite stable to the variation of η on most datasets and does not suffer huge fluctuations, which shows the robustness of our OPS model.

5.2.8. Discussion

The experimental results demonstrate the effectiveness of the proposed LOAD model. In terms of transaction cost, the LOAD model outperforms all the other OPS approaches. This indicates the proposed model is practical for real-life trading since transaction cost is one of the most important issues for trading. As for CW and MER, the LOAD model also performs very well. In the HS300 dataset, the OLMAR performs a little better. This can be explained that in some special market, there is a remarkable mean reversion phenomenon, and significant different statistical features. As for SR and IR, the mean reversion methods behave well due to their defensive property. However, the proposed LOAD model still achieves comparable performance. Overall, the LOAD system has a made good balance between risk and profit.

There are also some limitations of our model. In some special markets (e.g. HS300), the LOAD does not perform the best in terms of cumulative wealth. Thus the financial data from more countries could be collected and explored to further improve the LOAD model.

6. Conclusion

We propose a local adaptive learning system for online portfolio selection in this paper. The key innovation is to leverage local regression model to fuse different and complementary strategies together to enhance the adaptivity and profitability. In real-world trading, the mean reversion strategy based models can often show robust performance. However, robustness comes with loss of investment opportunities due to their defensive nature. We try to solve this problem by picking out some outstanding assets by machine learning techniques and investing them following the momentum principle. The experimental results have demonstrated that the proposed model can take the bests of both defensive and aggressive strategies and make them complementary to each other in the whole portfolio system. Our model has shown that fusion of complementary strategies is practical and effective to enhance the profitability of a trading system, and may be a better way than a simple ensemble of same strategies using different hyper parameters. In the future, we will introduce more advanced machine learning and artificial intelligence techniques to improve the investment performance. Also, we will collect more real-world financial data for further experiments.

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