Programming Languages and Types

Homework 12

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1 Simply-Typed λ-Calculus

1.1 Typing Derivation

Tell whether each of the following terms in the simply-typed λ-calculus with all the extensions introduced in the lecture is well-typed. If it is, give a typing derivation for it; if not, give the reason. For very large terms, you can name their sub-terms and type them individually.

1. **pred** (**succ false**)
2. $\lambda f : \mathbf{Nat} \to \mathbf{Nat}.\lambda n : \mathbf{Nat}.f\ (f\ (\mathbf{succ}\ n))$
3. **if** (**iszero** (**succ 0**)) **then true else 0**
4. $\{tru = \mathbf{succ}\ \mathbf{0}, tru = \mathbf{true}\}$ **as** $\{tru : \mathbf{Bool}, one : \mathbf{Nat}\}$
5. **let** $b = \mathbf{false}$ **in** (**iszero** b)
6. **let** $p = (\mathbf{0}, \mathbf{succ}\ \mathbf{0})$ **in** (**snd** p, **fst** p)
7. **case** (**inl 0**) **of inl** $x \Rightarrow \mathbf{false} \mid \mathbf{inr}\ x \Rightarrow \mathbf{true}$

8.

$$
\begin{array}{l}
\textbf{fix}\ (\lambda\ fise : (\textbf{Nat} \to \textbf{Bool}) \to (\textbf{Nat} \to \textbf{Bool})\ . \\
\quad \lambda\ n : \textbf{Nat}\ . \\
\quad\quad \textbf{if}\ (\textbf{iszero}\ n) \\
\quad\quad\quad \textbf{then true} \\
\quad\quad\quad \textbf{else if}\ (\textbf{iszero}\ (\textbf{pred}\ n)) \\
\quad\quad\quad\quad \textbf{then false} \\
\quad\quad\quad\quad \textbf{else}\ fise\ (\textbf{pred}\ (\textbf{pred}\ n))\ )
\end{array}
$$

1.2 Programming with Extensions

1. Complete the addition function add : **Nat** $\to$ **Nat** $\to$ **Nat** in the simply-typed λ-calculus with base type **Nat** and extension the fixed-point operator **fix**.[1]

   $add =$ **fix** $(\lambda\ fadd : (\textbf{Nat} \to \textbf{Nat} \to \textbf{Nat}) \to (\textbf{Nat} \to \textbf{Nat} \to \textbf{Nat})\ .\ ?)$

2 System-$\mathcal{F}$

2.1 Parametric Polymorphism

2.2 Typing Church-Encodings

[1] During the exercise session, I gave the wrong type (**Nat** $\to$ **Nat** $\to$ **Nat**) to the variable that is to be bound to the fixed point. Please refer to the updated exercise sheet `ex12.pdf`. In this homework exercise, I have given the type for $fadd$, to remind you of the mistake I made.