

# Programming Languages and Types

## Homework 12

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### 1 Simply-Typed $\lambda$ -Calculus

#### 1.1 Typing Derivation

Tell whether each of the following terms in the simply-typed  $\lambda$ -calculus with all the extensions introduced in the lecture is well-typed. If it is, give a typing derivation for it; if not, give the reason. For very large terms, you can name their sub-terms and type them individually.

1. **pred (succ false)**
2.  $\lambda f : \mathbf{Nat} \rightarrow \mathbf{Nat} . \lambda n : \mathbf{Nat} . f \ (f \ (\mathbf{succ} \ n))$
3. **if (iszero (succ 0)) then true else 0**
4.  $\{one = \mathbf{succ} \ 0, tru = \mathbf{true}\} \text{ as } \{tru : \mathbf{Bool}, one : \mathbf{Nat}\}$
5. **let  $b = \mathbf{false}$  in (iszero  $b$ )**
6. **let  $p = (0, \mathbf{succ} \ 0)$  in (snd  $p$ , fst  $p$ )**
7. **case (inl 0) of inl  $x \Rightarrow \mathbf{false}$  | inr  $x \Rightarrow \mathbf{true}$**

8.

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fix ( $\lambda fise : (\mathbf{Nat} \rightarrow \mathbf{Bool}) \rightarrow (\mathbf{Nat} \rightarrow \mathbf{Bool})$  .  
   $\lambda n : \mathbf{Nat}$  .  
    if (iszero  $n$ )  
      then true  
      else if (iszero (pred  $n$ ))  
        then false  
        else  $fise$  (pred (pred  $n$ )) )
```

## 1.2 Programming with Extensions

1. Complete the addition function  $add : \mathbf{Nat} \rightarrow \mathbf{Nat} \rightarrow \mathbf{Nat}$  in the simply-typed  $\lambda$ -calculus with base type  $\mathbf{Nat}$  and extension the fixed-point operator **fix**.<sup>1</sup>

$add = \mathbf{fix} (\lambda fadd : (\mathbf{Nat} \rightarrow \mathbf{Nat} \rightarrow \mathbf{Nat}) \rightarrow (\mathbf{Nat} \rightarrow \mathbf{Nat} \rightarrow \mathbf{Nat}) . ?)$

## 2 System- $\mathcal{F}$

### 2.1 Parametric Polymorphism

### 2.2 Typing Church-Encodings

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<sup>1</sup>During the exercise session, I gave the wrong type  $(\mathbf{Nat} \rightarrow \mathbf{Nat} \rightarrow \mathbf{Nat})$  to the variable that is to be bound to the fixed point. Please refer to the updated exercise sheet [ex12.pdf](#). In this homework exercise, I have given the type for  $fadd$ , to remind you of the mistake I made.