EXAMPLE 10–5 A music wire helical compression spring with infinite life is needed to resist a dynamic load that varies from 5 to 20 lbf at 5 Hz while the end deflection varies from $\frac{1}{2}$ to 2 in. Because of assembly considerations, the solid height cannot exceed 1 in and the free length cannot be more than 4 in. The springmaker has the following wire sizes in stock: 0.069, 0.071, 0.080, 0.085, 0.090, 0.095, 0.105, and 0.112 in.

Solution The a priori decisions are:

- Material and condition: for music wire, $A = 201$ kpsi · in^m, $m = 0.145$, $G = 11.75(10^6)$ psi; relative cost is 2.6
- Surface treatment: unpeened
- End treatment: squared and ground
- Robust linearity: $\xi = 0.15$
- Set: use in as-wound condition
- Fatigue-safe: $n_f = 1.5$ using the Sines-Zimmerli fatigue-failure criterion
- Function: $F_{min} = 5$ lbf, $F_{max} = 20$ lbf, $y_{min} = 0.5$ in, $y_{max} = 2$ in, spring operates free (no rod or hole)
- Decision variable: wire size d

They are three stages:

1- The solution will be done, and all possible parameters will be depicted in the matrix AA.
2- If any of the possible parameters do not match to limitations, it will be replaced with 0.
3- In the last but not least stage, only columns that have no zero remains, and others will be replaced with zeros.

The script:

```
clc
clear all
close all
Fmax=20;
Fmin=5;
Ymax=2;
Ymin=0.5;
Fa=(Fmax-Fmin)/2;
Fm=(Fmax+Fmin)/2;
% Spring supported between flat parallel surfaces Constant aa=0.5
aa=0.5;
% Table 10-4
cost=2.6;
% Table 10-4
A=201000;
% Table 10-5
m=0.145;
% expecting d>0.064in
E=28.5*10^6;
G=11.75*10^6;
% Ends squared and ground
% Safety factor "nf"
nf=1.5;
% Robust linearity "RL"
RL=0.15;
% Music Wire A228
% dd is the matrix of diameter from Table A-28
dd=[0.069,0.071,0.080,0.085,0.090,0.095,0.105,0.112];
% AA is the matrix that shows answer Table
AA=zeros(12,8);
% with the Sines criterion, Sse=Ssa=35kpsi
Sse=35000;
Ssa=Sse;
Fs=(1+RL)*Fmax;
for i=1:8;
    d=dd(i);
% Table 10-6
% Ssy=0.45*Sut
Ssy=(A/d^m)*0.45;
% Ssu=0.67*Sut
Ssu=(A/d^m)*0.67;
a=Sse/nf;
B=(8*Fa)/(pi*(d^2));
C=((2*a-B)/(4*B))+sqrt(((2*a-B)/(4*B))^2-((3*a)/(4*B)));
D=C*d;
KB=(4*C+2)/(4*C-3);
Ts=(8*Fs)/(pi*(d^2))*(KB*D/d);
Ta=(8*Fa)/(pi*(d^2))*(KB*D/d);
Tm=(8*Fm)/(pi*(d^2))*(KB*D/d);
ns=Ssy/Ts;
nf=Ssa/Ta;
OD=D+d;
ID=D-d;
Na=G*(d^4)*Ymax/(8*(D^3)*Fmax);
```

```
% Table 10-1:
Nt=Na+2;
Ls=d*Nt;
Lo=Ls+Fs*Ymax/Fmax;
Locr=2.63*D/aa;
fom=-(cost*(3.14^2)*(d^2)*Nt*D)/4;
% Density*g(g=386)=0.284
W=((pi^2)*(d^2)*D*Na*0.284)/4;
fn=0.5*sqrt(386*Fmax/(Ymax*W));
AA(1,i)=D;
AA(4,i)=C;
AA(3,i)=OD;
AA(2,i)=ID;
AA(5,i)=Na;
AA(6,i)=Ls;
AA(7,i)=Lo;
AA(8,i)=Locr;
AA(9,i)=nf;
AA(10,i)=ns;
AA(11,i)=fn;
AA(12,i)=fom;
end
AA
```

Stage 1

AA =

0.2969	0.3315	0.5123	0.6321	0.7673	0.9187	1.2744	1.5685
0.2279	0.2605	0.4323	0.5471	0.6773	0.8237	1.1694	1.4565
0.3659	0.4025	0.5923	0.7171	0.8573	1.0137	1.3794	1.6805
4.3028	4.6696	6.4032	7.4368	8.5254	9.6710	12.1372	14.0047
127.2182	102.4186	44.7547	30.3535	21.3324	15.4260	8.6253	5.9889
8.9161	7.4137	3.7404	2.7500	2.0999	1.6555	1.1157	0.8948
11.2161	9.7137	6.0404	5.0500	4.3999	3.9555	3.4157	3.1948
1.5617	1.7439	2.6945	3.3250	4.0359	4.8326	6.7034	8.2504
1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000
1.8627	1.8550	1.8231	1.8072	1.7923	1.7783	1.7526	1.7363
87.5108	89.6953	96.8792	99.6687	101.9161	103.7615	106.5988	108.1053
-1.1706	-1.1184	-0.9824	-0.9470	-0.9293	-0.9260	-0.9568	-1.0074

```
for i=1:8;
if AA(4,i)<=4
    AA(4,i)=0;
elseif AA(4,i)>=12
      AA(4,i)=0;
end
if AA(5,i)<=3
    AA(5,i)=0;
elseif AA(5,i)>=15
      AA(5,i)=0;
end
if AA(10,i)<=1.5
    AA(10,i)=0;
end
if AA(6,i)>=1
    AA(6,i)=0;
end
if AA(8,i)<=AA(7,i)
```

```
    AA(8,i)=0;
end
if AA(7,i)>=4
    AA(7,i)=0;
end
if AA(11,i)<=100
    AA(11,i)=0;
end
end
% AA(i,j)=0 means Error!
AA
```

Stage 2

AA =

0.2969	0.3315	0.5123	0.6321	0.7673	0.9187	1.2744	1.5685
0.2279	0.2605	0.4323	0.5471	0.6773	0.8237	1.1694	1.4565
0.3659	0.4025	0.5923	0.7171	0.8573	1.0137	1.3794	1.6805
4.3028	4.6696	6.4032	7.4368	8.5254	9.6710	0	0
0	0	0	0	0	0	8.6253	5.9889
0	0	0	0	0	0	0	0.8948
0	0	0	0	0	3.9555	3.4157	3.1948
0	0	0	0	0	4.8326	6.7034	8.2504
1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000
1.8627	1.8550	1.8231	1.8072	1.7923	1.7783	1.7526	1.7363
0	0	0	0	101.9161	103.7615	106.5988	108.1053
-1.1706	-1.1184	-0.9824	-0.9470	-0.9293	-0.9260	-0.9568	-1.0074

```
for i=1:8;
    for j=1:12;
      if AA(j,i)==0
          for j=1:12;
          AA(j,i)=0;
          end
      end
    end
end
AA
```

Stage 3

AA =

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

The third stage's result indicates that none of the Music Wires satisfies the limitations, but if we ignore the errors that are less than 0.2, the 0.105-inch wire is approximately appropriate.

Table 1. Music Wires Properties

d:	0.069	0.071	0.080	0.085	0.090	0.095	0.105	0.112
D	0.297	0.332	0.512	0.632	0.767	0.919	1.274	1.569
ID	0.228	0.261	0.432	0.547	0.677	0.824	1.169	1.457
OD	0.366	0.403	0.592	0.717	0.857	1.014	1.379	1.681
C	4.33	4.67	6.40	7.44	8.53	9.67	12.14	14.00
N_a	127.2	102.4	44.8	30.5	21.3	15.4	8.63	6.0
L_s	8.916	7.414	3.740	2.750	2.100	1.655	1.116	0.895
L_0	11.216	9.714	6.040	5.050	4.400	3.955	3.416	3.195
$(L_0)_{cr}$	1.562	1.744	2.964	3.325	4.036	4.833	6.703	8.250
n_f	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50
n_s	1.86	1.85	1.82	1.81	1.79	1.78	1.75	1.74
f_n	87.5	89.7	96.9	99.7	101.9	103.8	106.6	108
fom	−1.17	−1.12	−0.983	−0.948	−0.930	−0.927	−0.958	−1.01