10-2: A music wire helical compression spring is needed to support a 20-lbf load after being compressed 2 in. Because of assembly considerations the solid height cannot exceed 1 in and the free length cannot be more than 4 in. Design the spring.

- Music wire, A228; from Table 10–4, $A = 201\ 000$ psi-in^m; $m = 0.145$; from Table 10–5, $E = 28.5$ Mpsi, $G = 11.75$ Mpsi (expecting $d > 0.064$ in)
- Ends squared and ground
- Function: $F_{max} = 20$ lbf, $y_{max} = 2$ in
- Safety: use design factor at solid height of $(n_s)_d = 1.2$
- Robust linearity: $\xi = 0.15$
- Use as-wound spring (cheaper), $S_{sy} = 0.45S_{ut}$ from Table 10–6
- Decision variable: $d = 0.080$ in, music wire gauge #30, Table A–28. From Fig. 10–3 and Table 10–6,

They are three stages:

1- The solution will be done, and all possible parameters will be depicted in the matrix AA.
2- If any of the possible parameters do not match to limitations, it will be replaced with 0.
3- In the last but not least stage, only columns that have no zero remains, and others will be replaced with zeros.

STATIC SPRING DESIGN

Choose d

Over-a-rod

As-wound or set

$$D = d_{\text{rod}} + d + \text{allow}$$

Free

As-wound

$$S_{sy} = \text{const}(A)/d^{m\dagger}$$

$$C = \frac{2\alpha - \beta}{4\beta} + \sqrt{\left(\frac{2\alpha - \beta}{4\beta}\right)^2 - \frac{3\alpha}{4\beta}}$$

$$\alpha = \frac{S_{sy}}{n_s} \qquad \beta = \frac{8(1+\xi)F_{\max}}{\pi d^2}$$

$$D = Cd$$

Set removed

$$S_{sy} = 0.65A/d^m$$

$$D = \frac{S_{sy}\pi d^3}{8n_s(1+\xi)F_{\max}}$$

In-a-hole

As-wound or set

$$D = d_{\text{hole}} - d - \text{allow}$$

$$C = D/d$$

$$K_B = (4C+2)/(4C-3)$$

$$\tau_s = 8K_B(1+\xi)F_{\max}D/(\pi d^3)$$

$$n_s = S_{sy}/\tau_s$$

$$\text{OD} = D + d$$

$$\text{ID} = D - d$$

$$N_a = Gd^4 y_{\max}/(8D^3F_{\max})$$

N_t: Table 10–1

L_s: Table 10–1

L_O: Table 10–1

$$(L_O)_{cr} = 2.63D/\alpha$$

$$\text{fom} = -(\text{rel. cost})\gamma\pi^2 d^2 N_t D/4$$

Print or display: d, D, C, OD, ID, N_a, N_t, L_s, L_O, $(L_O)_{cr}$, n_s, fom
Build a table, conduct design assessment by inspection
Eliminate infeasible designs by showing active constraints
Choose among satisfactory designs using the figure of merit

†const is found from Table 10–6.

```
clc
clear all
close all
Fmax=20;
Ymax=2;
% Spring supported between flat parallel surfaces Constant aa=0.5
aa=0.5;
% Table 10-4
cost=2.6;
% Table 10-4
A=201000;
% Table 10-5
m=0.145;
% expecting d>0.064in
E=28.5*10^6;
G=11.75*10^6;
% Ends squared and ground
% Safety factor "ns"
ns=1.2;
% Robust linearity "RL"
RL=0.15;
% Music Wire A228
% dd is the matrix of diameter from Table A-28
dd=[0.063,0.067,0.071,0.075,0.080,0.085,0.090,0.095];
% AA is the matrix that shows answer Table
AA=zeros(10,8);
for i=1:8;
  d=dd(i);
% Table 10-6
% Ssy=0.45*Sut
Ssy=(A/d^m)*0.45;
a=Ssy/ns;
B=(8*(1+RL)*Fmax)/(pi*(d^2));
C=((2*a-B)/(4*B))+sqrt(((2*a-B)/(4*B))^2-((3*a)/(4*B)));
D=C*d;
KB=(4*C+2)/(4*C-3);
Ts=(8*(1+RL)*Fmax)/(pi*(d^2))*(KB*D/d);
ns=Ssy/Ts;
OD=D+d;
ID=D-d;
Na=G*(d^4)*Ymax/(8*(D^3)*Fmax);
% Table 10-1:
Nt=Na+2;
Ls=d*Nt;
Lo=Ls+(1+RL)*Ymax;
Locr=2.63*D/aa;
fom=-(cost*(3.14^2)*(d^2)*Nt*D)/4;
AA(1,i)=D;
AA(2,i)=C;
AA(3,i)=OD;
AA(4,i)=ID;
AA(5,i)=Na;
AA(6,i)=Ls;
AA(7,i)=Lo;
AA(8,i)=Locr;
AA(9,i)=ns;
AA(10,i)=fom;
end
AA
```

Farzad Azizi Zade 2018

Stage 1

AA =

0.3909	0.4792	0.5782	0.6883	0.8427	1.0166	1.2111	1.4273
6.2048	7.1528	8.1433	9.1777	10.5335	11.9596	13.4564	15.0237
0.4539	0.5462	0.6492	0.7633	0.9227	1.1016	1.3011	1.5223
0.3279	0.4122	0.5072	0.6133	0.7627	0.9316	1.1211	1.3323
38.7347	26.8899	19.3107	14.2496	10.0536	7.2982	5.4251	4.1147
2.5663	1.9356	1.5131	1.2187	0.9643	0.7903	0.6683	0.5809
4.8663	4.2356	3.8131	3.5187	3.2643	3.0903	2.9683	2.8809
2.0562	2.5208	3.0412	3.6206	4.4325	5.3471	6.3703	7.5074
1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000
-0.4050	-0.3983	-0.3981	-0.4032	-0.4166	-0.4377	-0.4668	-0.5048

```
for i=1:8;
if AA(2,i)<=4
   AA(2,i)=0;
elseif AA(2,i)>=12
     AA(2,i)=0;
end
if AA(5,i)<=3
   AA(5,i)=0;
elseif AA(5,i)>=15
     AA(5,i)=0;
end
if AA(9,i)<=1.2
   AA(9,i)=0;
end
if AA(6,i)>=1
   AA(6,i)=0;
end
if AA(8,i)<=AA(7,i)
   AA(8,i)=0;
end
if AA(7,i)>=4
   AA(7,i)=0;
end
end
% AA(i,j)=0 means Error!
AA
```

Farzad Azizi Zade 2018

Stage 2

AA =

0.3909	0.4792	0.5782	0.6883	0.8427	1.0166	1.2111	1.4273
6.2048	7.1528	8.1433	9.1777	10.5335	11.9596	0	0
0.4539	0.5462	0.6492	0.7633	0.9227	1.1016	1.3011	1.5223
0.3279	0.4122	0.5072	0.6133	0.7627	0.9316	1.1211	1.3323
0	0	0	14.2496	10.0536	7.2982	5.4251	4.1147
0	0	0	0	0.9643	0.7903	0.6683	0.5809
0	0	3.8131	3.5187	3.2643	3.0903	2.9683	2.8809
0	0	0	3.6206	4.4325	5.3471	6.3703	7.5074
1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000
-0.4050	-0.3983	-0.3981	-0.4032	-0.4166	-0.4377	-0.4668	-0.5048

```
for i=1:8;
    for j=1:10;
      if AA(j,i)==0
          for j=1:10;
          AA(j,i)=0;
          end
      end
    end
end
AA
```

The final stage

AA =

0	0	0	0	0.8427	1.0166	0	0
0	0	0	0	10.5335	11.9596	0	0
0	0	0	0	0.9227	1.1016	0	0
0	0	0	0	0.7627	0.9316	0	0
0	0	0	0	10.0536	7.2982	0	0
0	0	0	0	0.9643	0.7903	0	0
0	0	0	0	3.2643	3.0903	0	0
0	0	0	0	4.4325	5.3471	0	0
0	0	0	0	1.2000	1.2000	0	0
0	0	0	0	-0.4166	-0.4377	0	0

The final result shows that both wires with 0.85-inch and 0.8-inch diameter are appropriate.

d	0.063	0.067	0.071	0.075	0.080	0.085	0.090	0.095
D	0.391	0.479	0.578	0.688	0.843	1.017	1.211	1.427
C	6.205	7.153	8.143	9.178	10.53	11.96	13.46	15.02
OD	0.454	0.546	0.649	0.763	0.923	1.102	1.301	1.522
N_a	39.1	26.9	19.3	14.2	10.1	7.3	5.4	4.1
L_s	2.587	1.936	1.513	1.219	0.964	0.790	0.668	0.581
L_0	4.887	4.236	3.813	3.519	3.264	3.090	2.968	2.881
$(L_0)_{cr}$	2.06	2.52	3.04	3.62	4.43	5.35	6.37	7.51
n_s	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
fom	−0.409	−0.399	−0.398	−0.404	−0.417	−0.438	−0.467	−0.505