Replicating Human Joint Moments Using Neural Networks

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Legged Locomotion of Robots Labor Project

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1 Replicating Human Joint Moments Using Neural Networks

1.1 Project Description

- The goal of this project is to compare different input combinations:
  1. Only using the joint angle of the joint for which you are creating the controller
  2. All joint angles
  3. Both these options combined with the ground reaction force
- The idea is that, if only ***one joint angle*** is sufficient, then less sensors will be required in a prosthesis or exoskeleton, which makes them cheaper and less prone to failure.

1.2 Scientific Questions to Answer

- We want to answer these questions:
  1. Is analyzing a single feature (e.g. Ankle joint angle) is sufficient to predict the joint moment?
  2. How different features are related to each other?
  3. Do we need all features to simulate and predict the joint moment?
  4. How complex models need to be to have a decent performance?
  5. How many paramethers does a neural network should have to perform well?
  6. Does a deep neural network (non-convolutional model) can perform as well as a convolutional neural network?
  7. As data are time-specific, how does recurrent neural networks perform on these data?
  8. Does more paramethers always result to a better performance in above recurrent neural model?
  9. How does long short-term memory networks (LSTM) models perform?
  10. Does a bi-directional neural network model enhance the performance?

2 Project Implementation

2.1 Importing Necessary Libraries

- scipy to read mat files.
- numpy to work with arrays.
- pandas to convert data into dataframes
- matplotlib for plotting plots and points.
- seaborn to plot heatmaps for correlation matrix.
- sklearn to use machine learning algorithms like LinearRegression and DecisionTreeRegressor
- Tensorflow to work with Keras to create and train neural networks.
- datetime to compute elapsed execution time.

[60]:
```
import scipy
import datetime
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
import tensorflow as tf
from tensorflow import keras
from itertools import combinations
from sklearn import svm
from sklearn import preprocessing
from sklearn.decomposition import PCA
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression
from sklearn.linear_model import Ridge, BayesianRidge
from sklearn.linear_model import Lasso
from sklearn.preprocessing import PolynomialFeatures
from sklearn.tree import DecisionTreeRegressor
from sklearn.ensemble import RandomForestRegressor
from sklearn.metrics import mean_absolute_error
from sklearn.inspection import partial_dependence
from sklearn.inspection import PartialDependenceDisplay
from plot_keras_history import show_history, plot_history
```

2.2 Reading data

- we can use scipy.io.loadmat to read and convert .mat files into readable dict in python.

[12]:
```
# Extracting the ankle data from mat file
data_array = scipy.io.loadmat('data/Data_Ankle.mat')['ankle_data']
```

[13]:
```
# Converts the data array into pandas dataframe with corresponding column names
data = pd.DataFrame(data_array, columns=['time',␣
↪'vertical_ground_reaction_force',
```

```
                                         'trunk_joint_angle', 'hip_joint_angle',
                                         'knee_joint_angle',␣
↪'ankle_joint_angle',
                                         'joint_moment'])
```

[14]:
```
# Check if the data is missing
data.isnull().sum()
```

[14]:
```
time                             0
vertical_ground_reaction_force   0
trunk_joint_angle                0
hip_joint_angle                  0
knee_joint_angle                 0
ankle_joint_angle                0
joint_moment                     0
dtype: int64
```

[15]:
```
data.head()
```

[15]:
```
   time  vertical_ground_reaction_force  trunk_joint_angle  hip_joint_angle  \
0  0.00                      -10.321381          -0.094286         0.292974
1  0.01                       -7.990312          -0.096174         0.293179
2  0.02                       -6.208738          -0.096449         0.294008
3  0.03                       -5.126637          -0.097196         0.295568
4  0.04                       -4.774037          -0.095624         0.297472

   knee_joint_angle  ankle_joint_angle  joint_moment
0          0.825063          -1.492588     -2.011628
1          0.822469          -1.493192     -3.655209
2          0.810997          -1.495867     -5.786433
3          0.785731          -1.501793     -6.190008
4          0.745707          -1.511208     -5.542868
```

2.3 Plotting Features

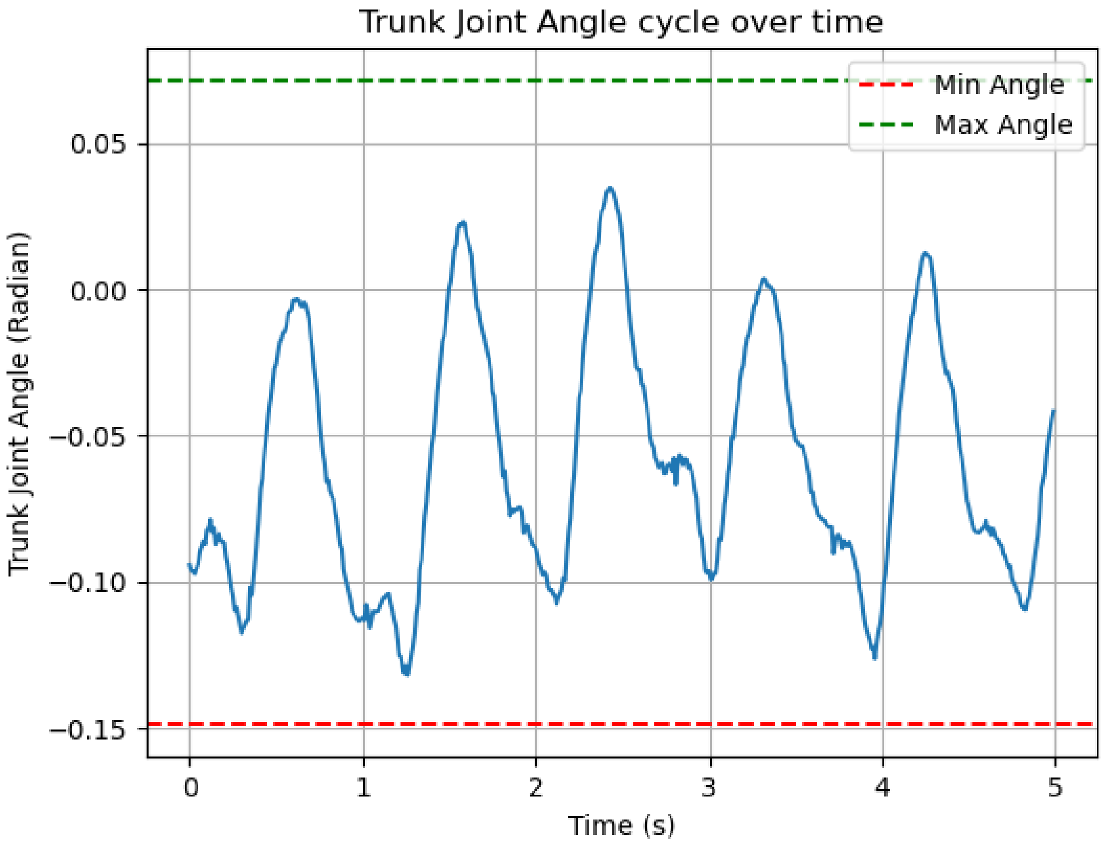
- If we plot the data all together as points, we cannot actually understand the details as there are 48000 dataoints. For this purpose, we should define a time window and plot the data in that time frame.

[16]:
```
def plot_feature_per_time(feature_name, title, label, window_size=500):
    """
    Plots given features with their desired label for y-axis, alongside the␣
↪window-size

    @param feature_name: (str) the feature's name we want to plot w.r.t target␣
↪`Joint Moment`.
    @param title: (str) the plot's title.
```

```
    @param label: (str) the label we want to set for the y-axis.
    @param window_size: (int) the window-size for which we want to plot first␣
↪data.

    @return: None
    """
    plt.axhline(y=data[feature_name].min(), color='r', linestyle='--',␣
↪label='Min Angle')
    plt.axhline(y=data[feature_name].max(), color='g', linestyle='--',␣
↪label='Max Angle')

    plt.plot(data.time[:window_size], data[feature_name][:window_size])
    plt.title(title)
    plt.ylabel(label)
    plt.xlabel('Time (s)')
    plt.legend()
    plt.grid()
    plt.show()
```

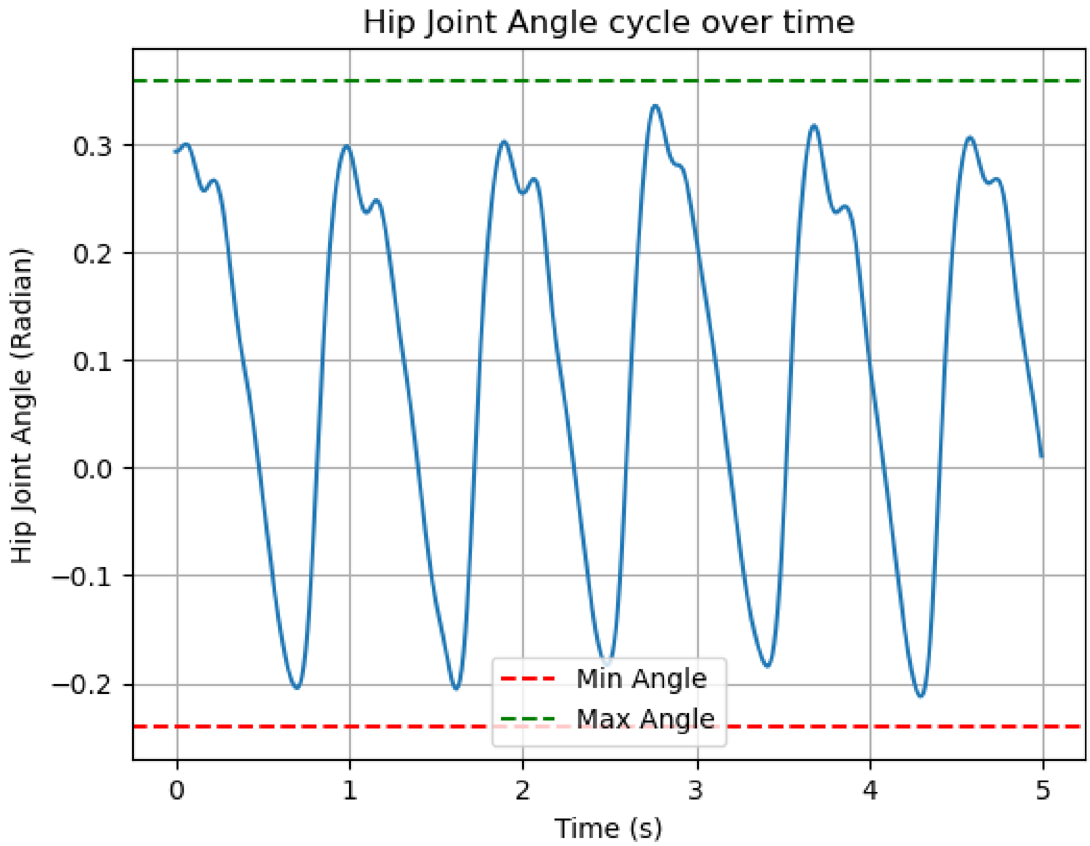
2.3.1 Trunk Joint Angle's Periodic Cycle

[17]:
```
plot_feature_per_time(feature_name='trunk_joint_angle', title='Trunk Joint␣
↪Angle cycle over time', label='Trunk Joint Angle (Radian)')
```



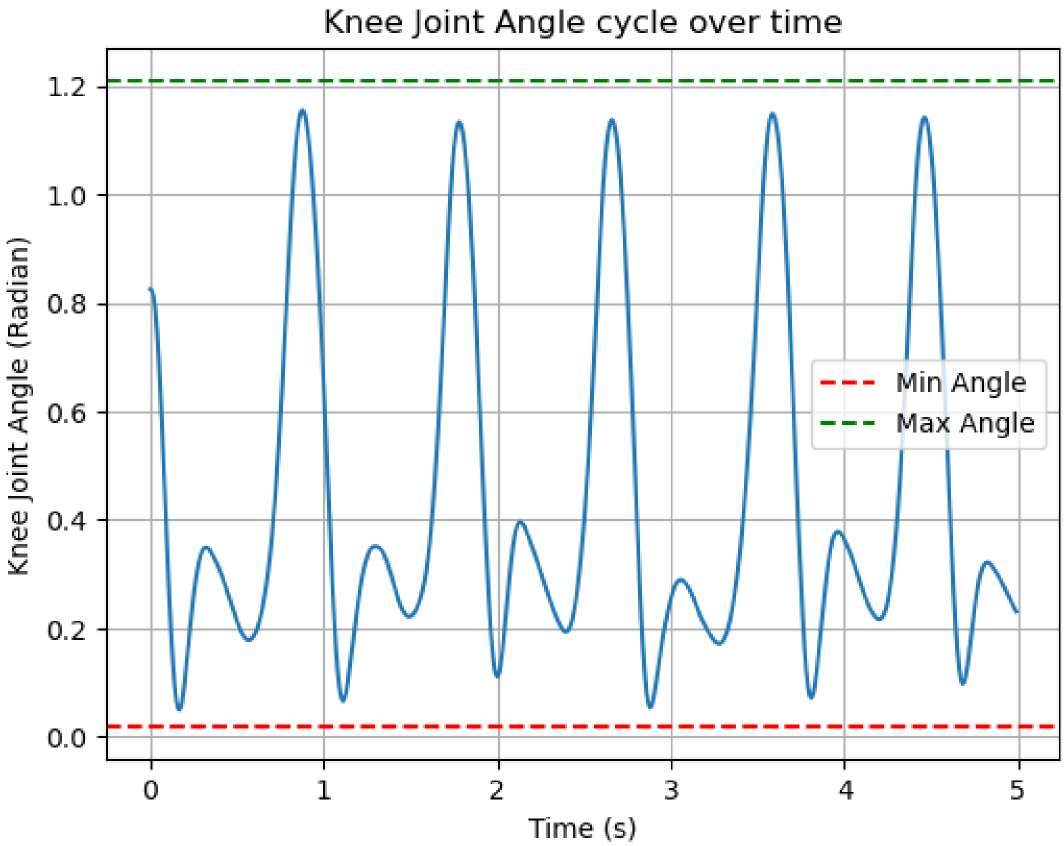
2.3.2 Hip Joint Angle's Periodic Cycle

[18]:
```
plot_feature_per_time(feature_name='hip_joint_angle', title='Hip Joint Angle␣
 ↪cycle over time', label='Hip Joint Angle (Radian)')
```



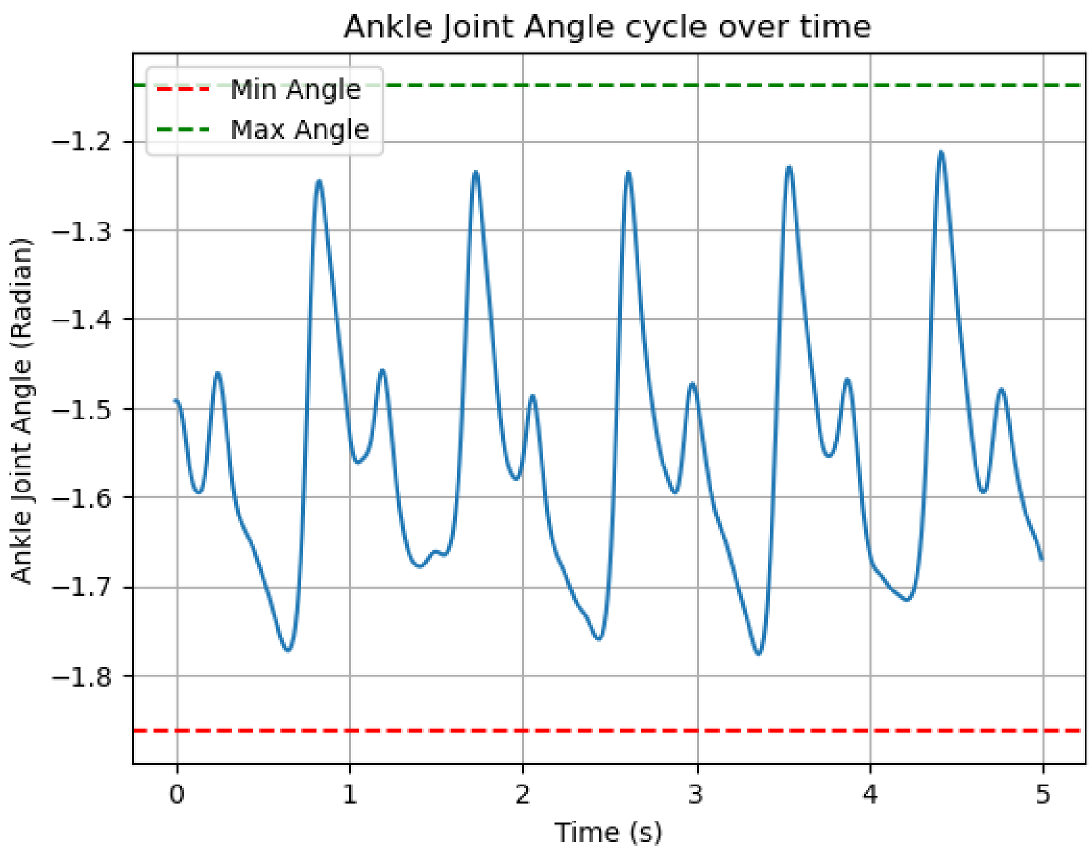
2.3.3 Knee Joint Angle's Periodic Cycle

[19]:
```
plot_feature_per_time(feature_name='knee_joint_angle', title='Knee Joint Angle␣
↪cycle over time', label='Knee Joint Angle (Radian)')
```



2.3.4 Ankle Joint Angle's Periodic Cycle

[20]:
```
plot_feature_per_time(feature_name='ankle_joint_angle', title='Ankle Joint␣
  ↪Angle cycle over time', label='Ankle Joint Angle (Radian)')
```



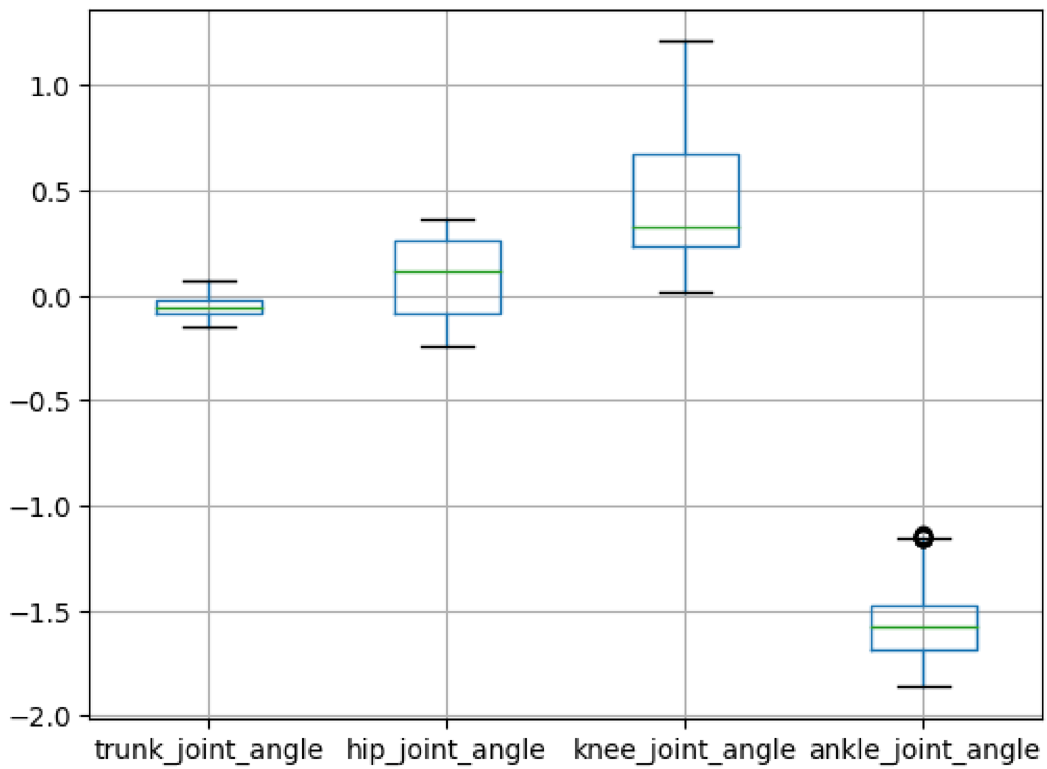
2.3.5 BoxPlot for better understanding

- A Box Plot is the visual representation of the statistical five number summary of a given data set.

  `Five Number Summary includes:`

  1. Minimum
  2. First Quartile
  3. Median (Second Quartile)
  4. Third Quartile
  5. Maximum

[21]:
```
data.boxplot(column=['trunk_joint_angle', 'hip_joint_angle',␣
 ↪'knee_joint_angle', 'ankle_joint_angle'])
plt.show()
```



2.4 Normalizing Data

- we can normalize the data using the `MinMaxScaler` method, to make sure selected features have the same range.

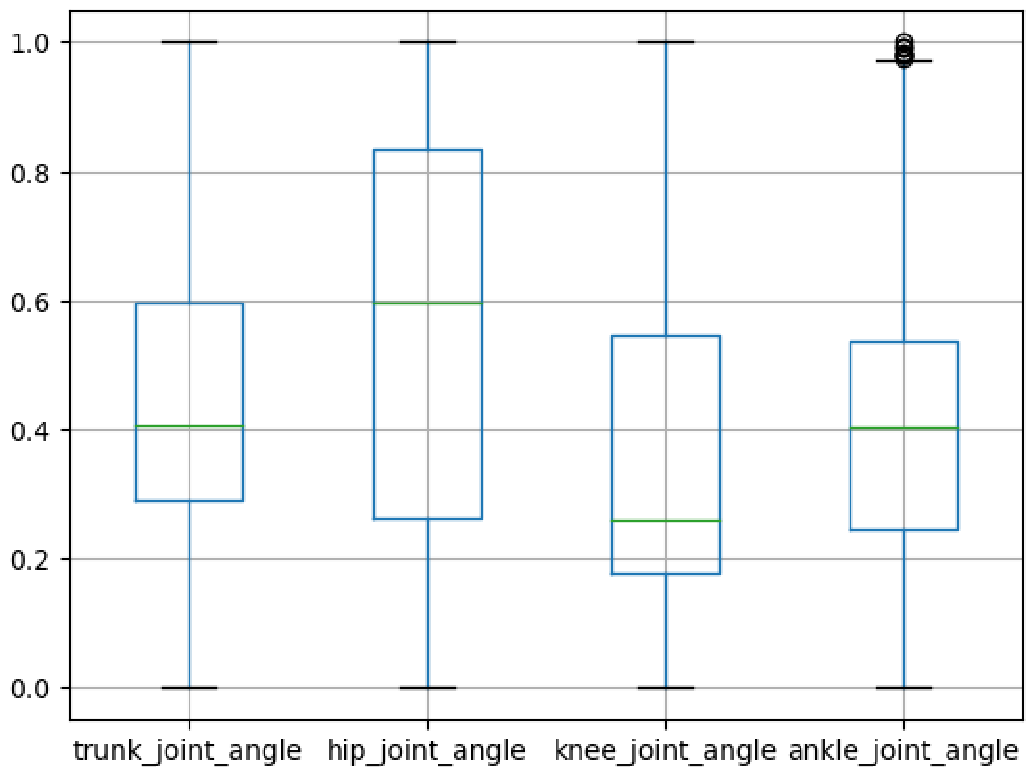
[22]:
```
columns_to_normalize = ['trunk_joint_angle', 'hip_joint_angle',␣
 ↪'knee_joint_angle', 'ankle_joint_angle']

normalized_data = data.copy()

normalized_data[columns_to_normalize] = preprocessing.MinMaxScaler().
 ↪fit_transform(data[columns_to_normalize])
normalized_data.head()
```

[22]:
```
   time  vertical_ground_reaction_force  trunk_joint_angle  hip_joint_angle  \
0  0.00                      -10.321381           0.246712         0.890431
1  0.01                       -7.990312           0.238134         0.890774
2  0.02                       -6.208738           0.236884         0.892159
3  0.03                       -5.126637           0.233489         0.894766
4  0.04                       -4.774037           0.240635         0.897949

   knee_joint_angle  ankle_joint_angle  joint_moment
```

```
0          0.677080            0.510548     -2.011628
1          0.674900            0.509714     -3.655209
2          0.665259            0.506018     -5.786433
3          0.644027            0.497827     -6.190008
4          0.610393            0.484816     -5.542868
```

2.4.1 Normalized BoxPlot

[23]:
```
normalized_data.boxplot(column=['trunk_joint_angle', 'hip_joint_angle',␣
 ↪'knee_joint_angle', 'ankle_joint_angle'])
plt.show()
```

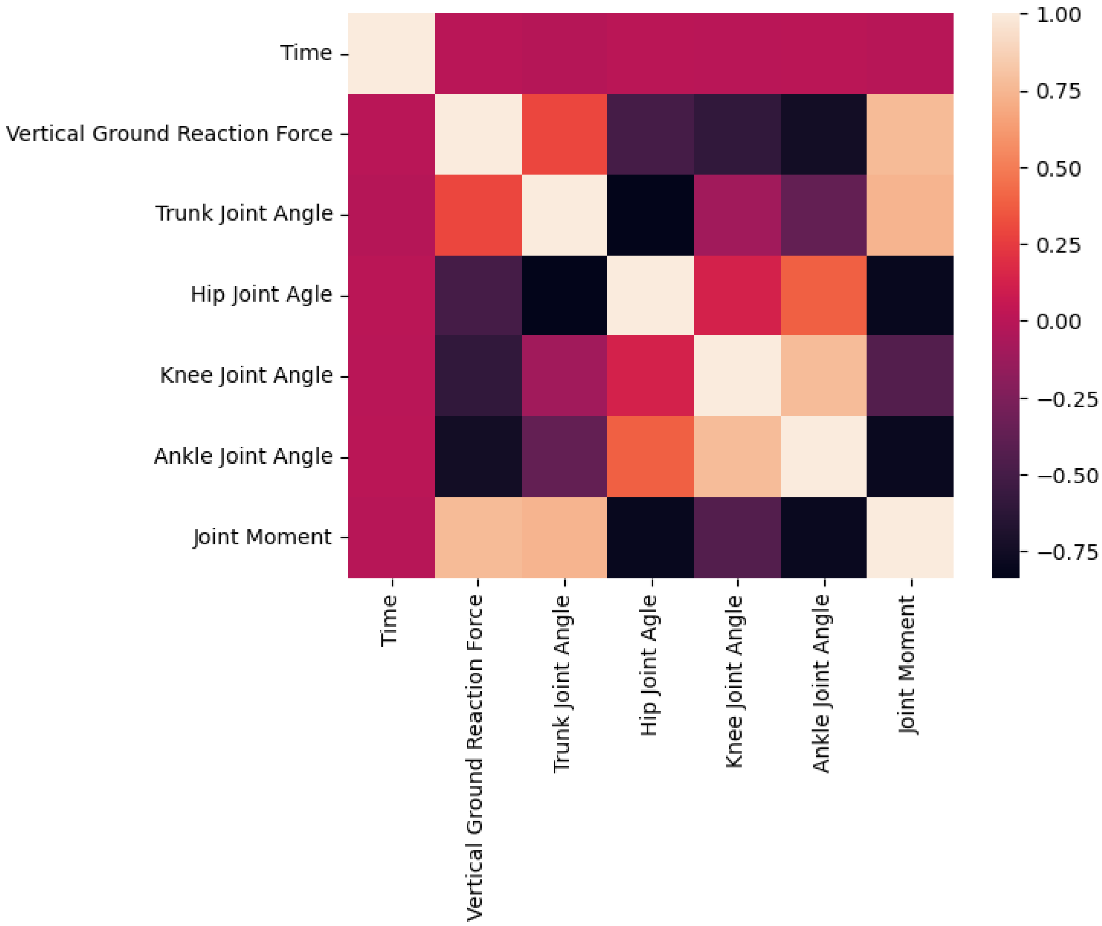


2.5 Correlation Matrix

- Since the the look periodic, we may assume that some features may be correlated somehow. To test this, we can use the `corr()` method to calculate the correlation between selected features.

[24]:
```
corr = data.corr()
corr.style.background_gradient(cmap='coolwarm')
```

```
axis_labels = ['Time', 'Vertical Ground Reaction Force', 'Trunk Joint Angle',␣
↪'Hip Joint Agle', 'Knee Joint Angle', 'Ankle Joint Angle', 'Joint Moment']

sns.heatmap(corr, xticklabels=axis_labels, yticklabels=axis_labels)

plt.show()
```



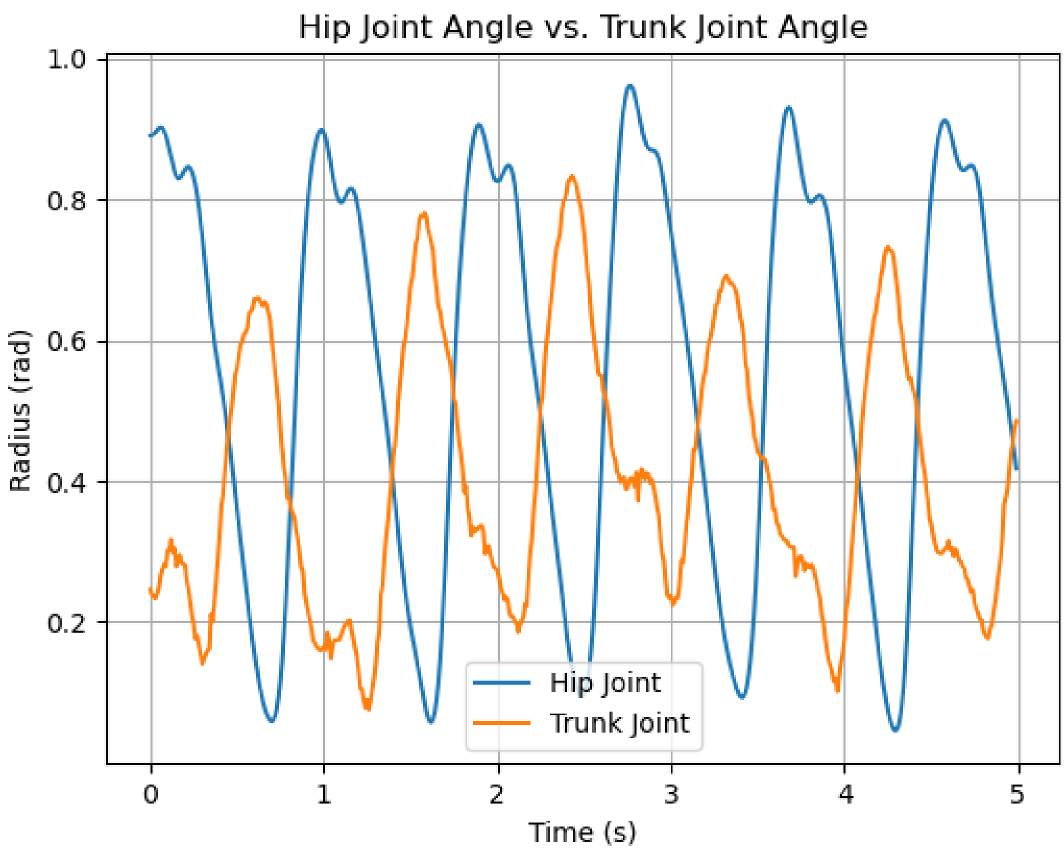
- It can be observed that some features have high correlation with each other (close to 1 or -1). For example, `Trunk Joint Angle` and `Hip Joint Angle` are clearly correlated. To see it better, we can plot them together to see how they behave.

2.6 Plots to Show Correlations

2.6.1 Hip Joint Angle vs. Trunk Joint Angle

[25]:
```
plt.plot(normalized_data.time[:500], normalized_data.hip_joint_angle[:500],␣
↪label='Hip Joint')
```

```
plt.plot(normalized_data.time[:500], normalized_data.trunk_joint_angle[:500],␣
↪label='Trunk Joint')
plt.title('Hip Joint Angle vs. Trunk Joint Angle')
plt.xlabel('Time (s)')
plt.ylabel('Radius (rad)')
plt.legend()
plt.grid()
plt.show()
```

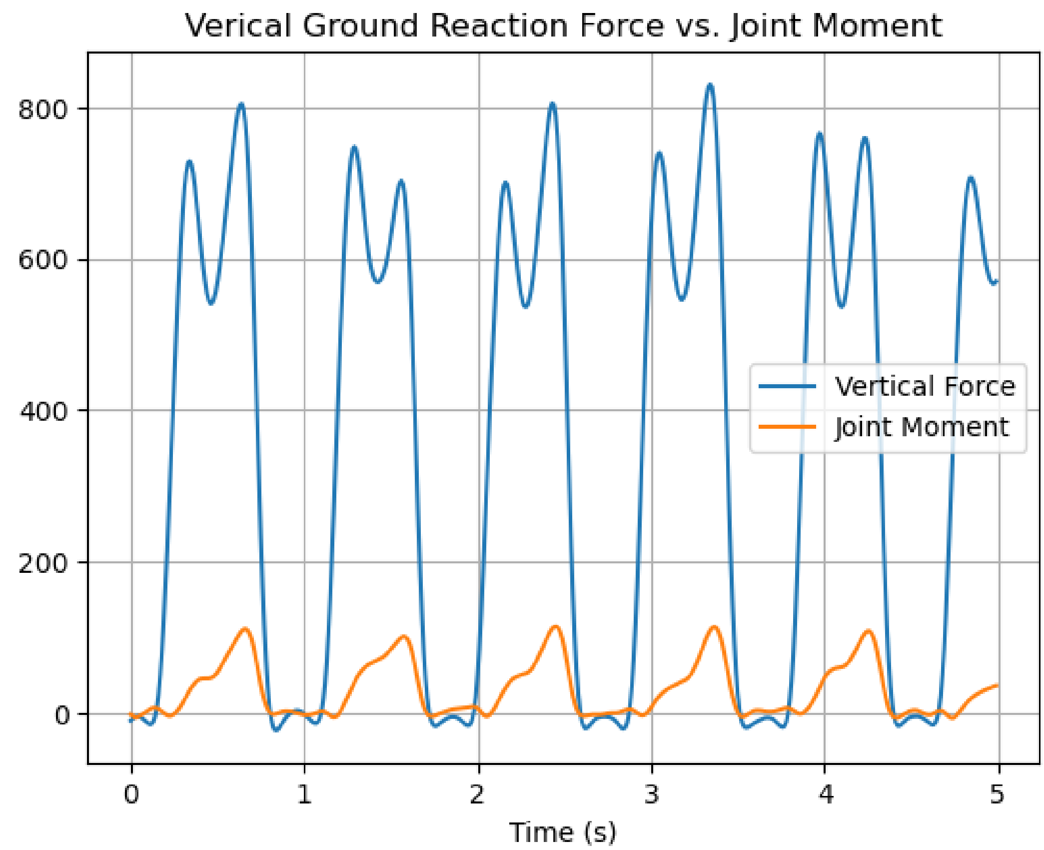


- By taking a look at the above plot, we can easily spot that `Hip Joint Angle` and `Trunk Joint Angle` are behaving somehow contradictory.

2.6.2 Verical Ground Reaction Force vs. Joint Moment

[26]:
```
plt.plot(normalized_data.time[:500], normalized_data.
↪vertical_ground_reaction_force[:500], label='Vertical Force')
plt.plot(normalized_data.time[:500], normalized_data.joint_moment[:500],␣
↪label='Joint Moment')
plt.title('Verical Ground Reaction Force vs. Joint Moment')
plt.xlabel('Time (s)')
```

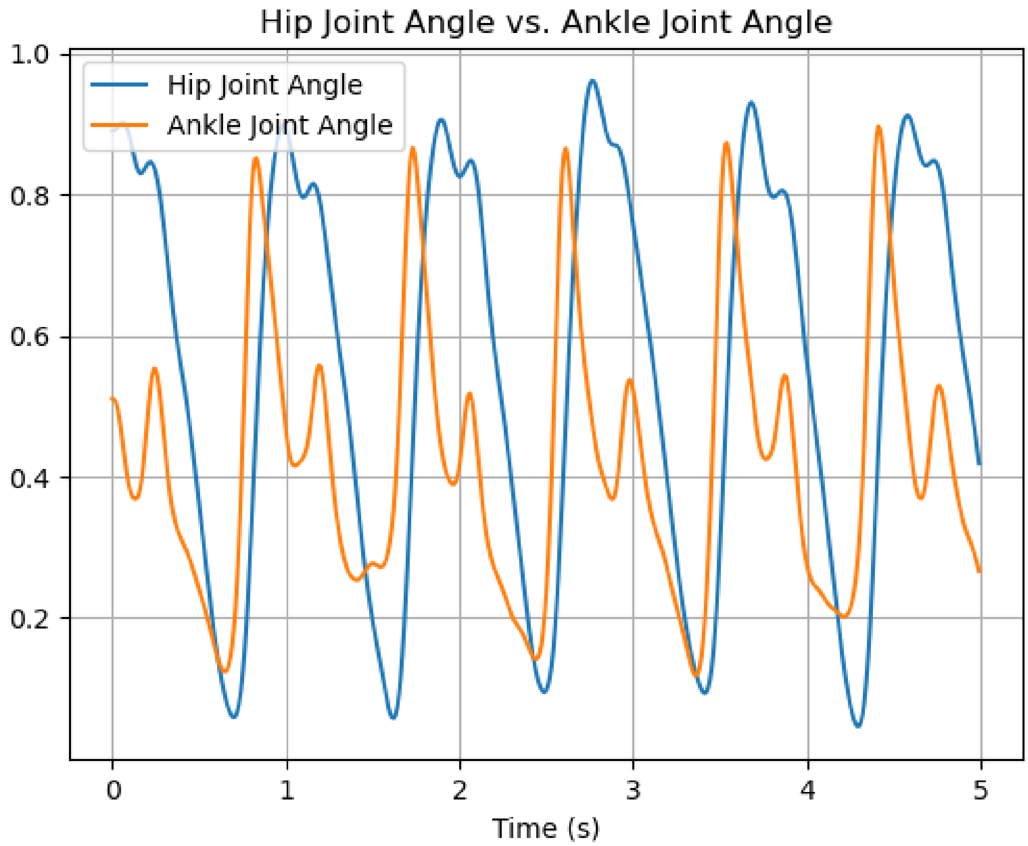
```
plt.legend()
plt.grid()
plt.show()
```



- The same statement can be applied to `Vetical Ground Reaction Force` and `Joint Moment`. They behave the same in the defined time periods, as they rise and fall together.

2.6.3 Hip Joint Angle vs. Ankle Joint Angle

[27]:
```
plt.plot(normalized_data.time[:500], normalized_data.hip_joint_angle[:500],␣
 ↪label='Hip Joint Angle')
plt.plot(normalized_data.time[:500], normalized_data.ankle_joint_angle[:500],␣
 ↪label='Ankle Joint Angle')
plt.title('Hip Joint Angle vs. Ankle Joint Angle')
plt.xlabel('Time (s)')
plt.legend()
plt.grid()
plt.show()
```



- As shown above, there is almost no correlation between `Hip Joint Angle` and `Ankle Joint Angle`.

2.6.4 Correlation Conclusion

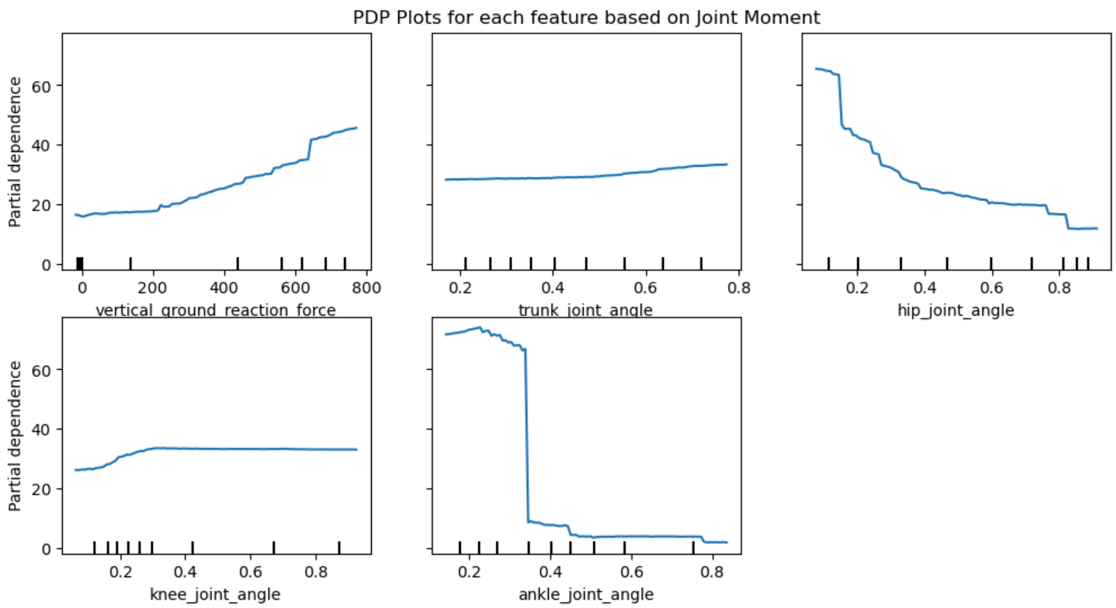
- As shown above, some of the mentioned features are highly correlated and we can omit them for our estimations as they are redundant, and have almost ***no effect*** on the model's performance by training different models with every feature and one time without correlated features.

2.7 PDP Plots

- Partial dependence shows how a particular feature affects a prediction. By making all other features constant, we want to find out how the feature in question influences our outcome.

[28]:
```
X_normalized = normalized_data.drop(columns=['time', 'joint_moment'])
y = normalized_data.joint_moment

features = X_normalized.columns

clf = DecisionTreeRegressor()
```

```
clf.fit(X_normalized, y)

fig, ax = plt.subplots(figsize=(12, 6))

plt.title('PDP Plots for each feature based on Joint Moment')
PartialDependenceDisplay.from_estimator(clf, X_normalized, features, ax=ax)
plt.show()
```

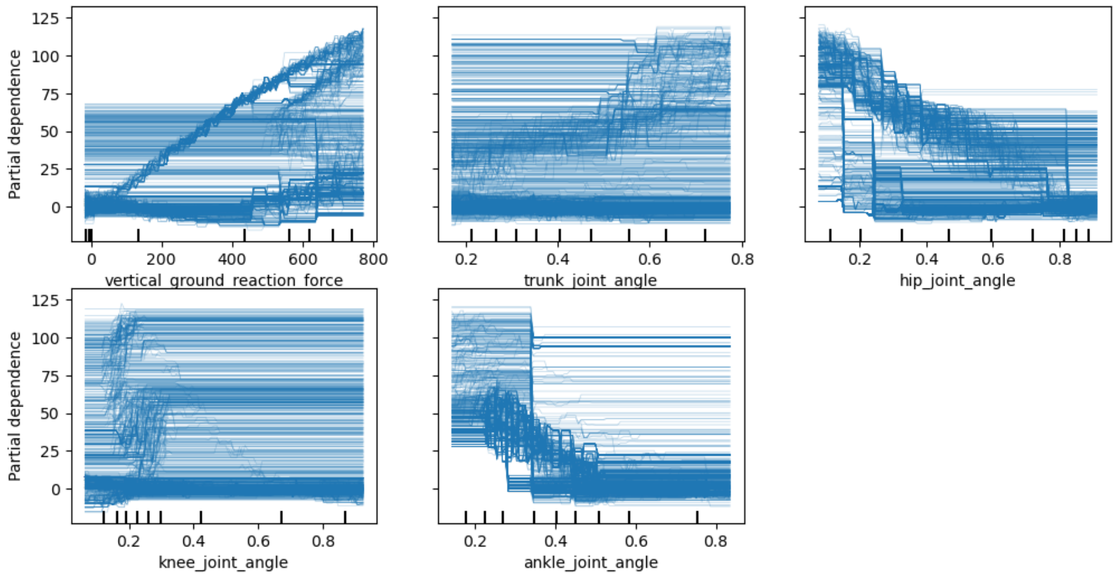


- When Partial Dependence Plots (PDPs) appear almost continuously horizontal, it generally suggests that the relationship between the predictor variable being examined and the target variable is ***weak*** or ***nearly nonexistent***. In other words, the predictor variable ***does not*** have a significant impact on the target variable, at least within the range of values explored in the PDP.

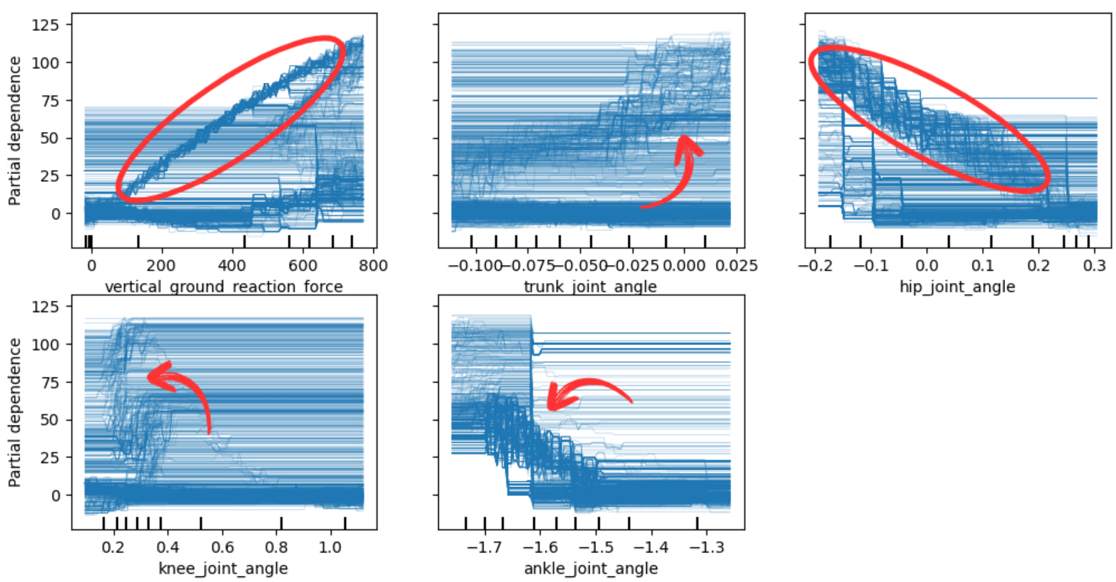
2.8 ICE Plots

- An ICE plot visualizes the dependence of the prediction on a feature for each instance separately, resulting in one line per instance, compared to one line overall in partial dependence plots. A PDP is the average of the lines of an ICE plot.

[29]:
```
fig, ax = plt.subplots(figsize=(12, 6))

PartialDependenceDisplay.from_estimator(clf, X_normalized, features, ax=ax,␣
↪kind='individual')
plt.show()
```



2.8.1 Importnat Notes Regarding the ICE Plots



- As we can see in the above plot, there are some individuals that behave differently with respect to the target feature `Joint Moment`. I interpret these individuals as perturbations in the data.

2.9 Splitting data

[30]:
```
X = data.drop(columns=['time', 'joint_moment'])
X_normalized = normalized_data.drop(columns=['time', 'joint_moment'])

y = data.joint_moment

X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.33,␣
↪random_state=42)

X_train_norm, X_test_norm, y_train, y_test = train_test_split(X_normalized, y,␣
↪test_size=0.33, random_state=42)

print(f"X_train shape: {X_train.shape}")
print(f"y_train shape: {y_train.shape}")

print(f"X_test shape: {X_test.shape}")
print(f"y_test shape: {y_test.shape}")
```

```
X_train shape: (32160, 5)
y_train shape: (32160,)
X_test shape: (15840, 5)
y_test shape: (15840,)
```

2.10 Linear Models

1. LinearRregression
2. DecisionTreeRegressor
3. RandomForestRegressor
4. Lasso
5. Ridge
6. BayesianRidge

2.10.1 Linear Regression

[31]:
```
LR = LinearRegression()

t0 = datetime.datetime.now()

LR.fit(X_train, y_train)

t1 = datetime.datetime.now()

LR_elapsed_time = (t1 - t0).total_seconds()

LR_score = LR.score(X_test, y_test)

LR.fit(X_train_norm, y_train)
```

```
normalized_LR_score = LR.score(X_test_norm, y_test)

print(f"Elapsed Time: {LR_elapsed_time} (s)")
print(f"Linear Regression Score: {LR_score} %, Normalized Data Score:␣
 ↪{normalized_LR_score} %")
```

```
Elapsed Time: 0.007398 (s)
Linear Regression Score: 0.9525636929396039 %, Normalized Data Score:
0.9525636929396034 %
```

2.10.2 Decision Tree Regressor

[32]:
```
regression_tree = DecisionTreeRegressor()

t0 = datetime.datetime.now()

regression_tree.fit(X_train, y_train)

t1 = datetime.datetime.now()

decision_tree_elapsed_time = (t1 - t0).total_seconds()

decision_tree_score = regression_tree.score(X_test, y_test)

regression_tree.fit(X_train_norm, y_train)

normalized_regression_tree_score = regression_tree.score(X_test_norm, y_test)

print(f"Elapsed Time: {decision_tree_elapsed_time} (s)")
print(f"Decision Tree Regressor Score: {decision_tree_score} %, Normalized Data␣
 ↪Score: {normalized_regression_tree_score} %")
```

```
Elapsed Time: 0.217328 (s)
Decision Tree Regressor Score: 0.9825596356429882 %, Normalized Data Score:
0.9824636868955994 %
```

2.10.3 Random Forest Regressor

[33]:
```
random_forest_regressor = RandomForestRegressor()

t0 = datetime.datetime.now()

random_forest_regressor.fit(X_train, y_train)

t1 = datetime.datetime.now()
```

```
random_forest_elapsed_time = (t1 - t0).total_seconds()

random_forest_score = random_forest_regressor.score(X_test, y_test)

random_forest_regressor.fit(X_train_norm, y_train)

normalized_random_forest_score = random_forest_regressor.score(X_test_norm,␣
↪y_test)

print(f"Elapsed Time: {random_forest_elapsed_time} (s)")
print(f"Random Forest Regressor Score: {random_forest_score} %, Normalized Data␣
↪Score: {normalized_random_forest_score} %")
```

```
Elapsed Time: 10.908615 (s)
Random Forest Regressor Score: 0.9911745193127225 %, Normalized Data Score:
0.9911045548717127 %
```

2.10.4 Lasso

[34]:
```
lasso = Lasso()

t0 = datetime.datetime.now()

lasso.fit(X_train, y_train)

t1 = datetime.datetime.now()

lasso_elapsed_time = (t1 - t0).total_seconds()

lasso_score = lasso.score(X_test, y_test)

lasso.fit(X_train_norm, y_train)

normalized_lasso_score = lasso.score(X_test_norm, y_test)

print(f"Elapsed Time: {lasso_elapsed_time} (s)")
print(f"Lasso Regressor Score: {lasso_score} %, Normalized Data Score:␣
↪{normalized_lasso_score} %")
```

```
Elapsed Time: 0.008246 (s)
Lasso Regressor Score: 0.815778636210949 %, Normalized Data Score:
0.8860341610619799 %
```

2.10.5 Ridge Regressor

[35]:
```
ridge = Ridge()

t0 = datetime.datetime.now()

ridge.fit(X_train, y_train)

t1 = datetime.datetime.now()

ridge_elapsed_time = (t1 - t0).total_seconds()

ridge_score = ridge.score(X_test, y_test)

ridge.fit(X_train_norm, y_train)

normalized_ridge_score = ridge.score(X_test_norm, y_test)

print(f"Elapsed Time: {ridge_elapsed_time} (s)")
print(f"Ridge Regressor Score: {ridge_score} %, Normalized Data Score:␣
↪{normalized_ridge_score} %")
```

```
Elapsed Time: 0.010647 (s)
Ridge Regressor Score: 0.9525269914507788 %, Normalized Data Score:
0.952561322978636 %
```

2.10.6 Bayesian Ridge

[36]:
```
baysian = BayesianRidge()

t0 = datetime.datetime.now()

baysian.fit(X_train, y_train)

t1 = datetime.datetime.now()

baysian_elapsed_time = (t1 - t0).total_seconds()

baysian_score = baysian.score(X_test, y_test)

baysian.fit(X_train_norm, y_train)

normalized_baysian_score = baysian.score(X_test_norm, y_test)

print(f"Elapsed Time: {baysian_elapsed_time} (s)")
print(f"Bayesian Ridge Regressor Score: {baysian_score} %, Normalized Data␣
↪Score: {normalized_baysian_score} %")
```

```
Elapsed Time: 0.017475 (s)
Bayesian Ridge Regressor Score: 0.9525638013341299 %, Normalized Data Score:
0.9525636564293843 %
```

2.10.7 Normalization Results

- We can see that normalizing data **mostly** doesn't affect the models' performance except for `Lasso` that increased the score from ~**81.5 %** to ~**88.6 %**.

2.11 Neural Network

2.11.1 CNN Model

[57]:
```python
nn_model = keras.Sequential([
    keras.layers.Conv1D(filters=32, kernel_size=3, activation='relu',␣
 ↪input_shape=(5, 1)),
    keras.layers.MaxPooling1D(pool_size=2),
    keras.layers.Flatten(),
    keras.layers.Dense(64, activation='relu'),
    keras.layers.Dense(1, activation='linear')
], name='CnnModel')
```

[29]:
```python
nn_model.compile(loss='mean_squared_error', optimizer='adam',␣
 ↪metrics=['mean_absolute_error'])
```

[30]:
```python
nn_model.summary()
```

```
Model: "CnnModel"

_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 conv1d (Conv1D)             (None, 3, 32)             128

 max_pooling1d (MaxPooling1  (None, 1, 32)             0
 D)

 flatten (Flatten)           (None, 32)                0

 dense (Dense)               (None, 64)                2112

 dense_1 (Dense)             (None, 1)                 65

=================================================================
Total params: 2305 (9.00 KB)
Trainable params: 2305 (9.00 KB)
Non-trainable params: 0 (0.00 Byte)

_________________________________________________________________
```

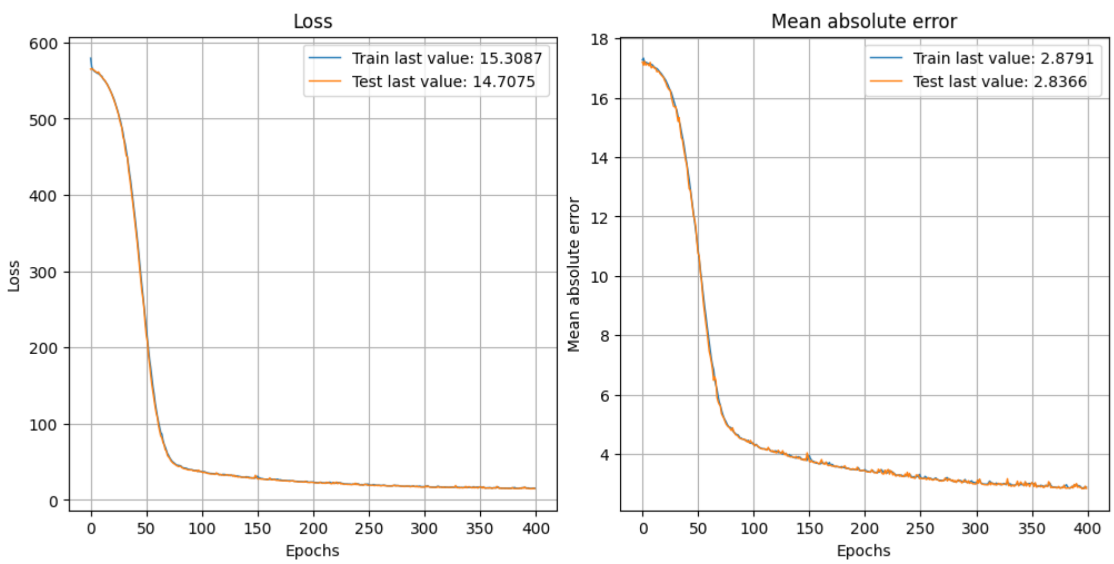
[31]:
```
t0 = datetime.datetime.now()

nn_history = nn_model.fit(X_train, y_train, validation_data=(X_test, y_test),
    epochs=400, batch_size=4096, verbose=0)

t1 = datetime.datetime.now()

cnn_elapsed_time = (t1 - t0).total_seconds()
```

[32]:
```
print(f"Elapsed Time: {cnn_elapsed_time} (s)")
```

```
Elapsed Time: 18.656062 (s)
```

2.11.2 Plotting Loss and MSE for CNN

[33]:
```
show_history(nn_history)
```



2.11.3 RNN Model

[34]:
```
rnn_model = keras.Sequential([
    keras.layers.SimpleRNN(units=64, activation='relu', input_shape=(5, 1)),
    keras.layers.Dense(64, activation='relu'),
    keras.layers.Dense(1, activation='linear')
], name='RnnModel')
```

[35]:
```
rnn_model.compile(loss='mean_squared_error', optimizer='adam',
    metrics=['mean_absolute_error'])
```

[36]:
```python
rnn_model.summary()
```

```
Model: "RnnModel"

_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 simple_rnn (SimpleRNN)      (None, 64)                4224

 dense_2 (Dense)             (None, 64)                4160

 dense_3 (Dense)             (None, 1)                 65

=================================================================
Total params: 8449 (33.00 KB)
Trainable params: 8449 (33.00 KB)
Non-trainable params: 0 (0.00 Byte)
_________________________________________________________________
```

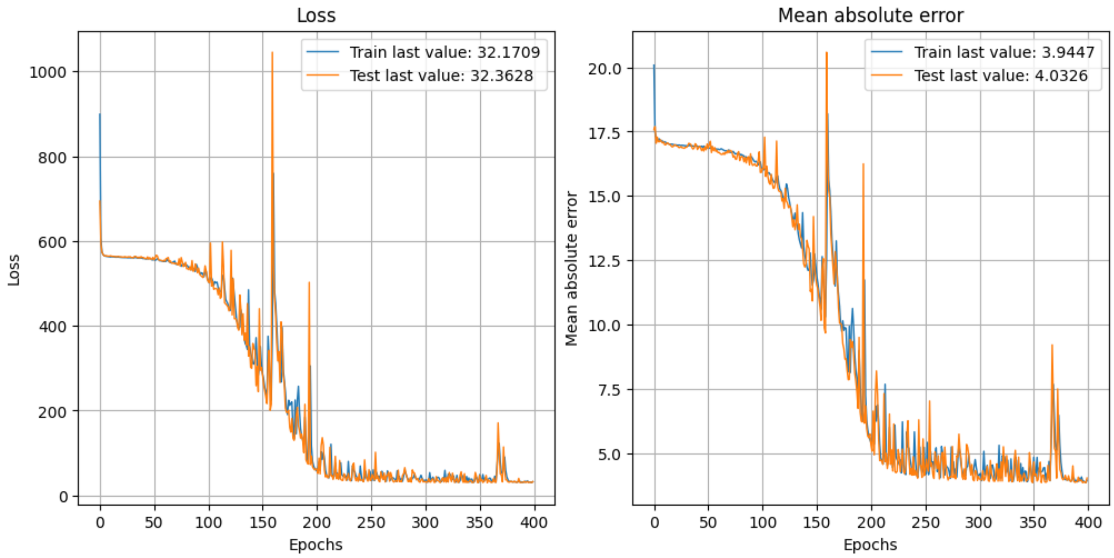
[37]:
```python
t0 = datetime.datetime.now()

rnn_history = rnn_model.fit(X_train, y_train, validation_data=(X_test, y_test),␣
 ↪epochs=400, batch_size=4096, verbose=0)

t1 = datetime.datetime.now()
rnn_elapsed_time = (t1 - t0).total_seconds()
```

[38]:
```python
print(f"Elapsed Time: {rnn_elapsed_time} (s)")
```

```
Elapsed Time: 36.036608 (s)
```

2.11.4 Plotting Loss and MSE

[39]:
```python
show_history(rnn_history)
```



2.11.5 LSTM Model

[40]:
```
lstm_model = keras.Sequential([
    keras.layers.LSTM(units=64, return_sequences=True, input_shape=(5, 1)),
    keras.layers.LSTM(units=64, return_sequences=True),
    keras.layers.TimeDistributed(keras.layers.Dense(64, activation='relu')),
    keras.layers.Flatten(),
    keras.layers.Dense(64, activation='relu'),
    keras.layers.Dropout(0.2),
    keras.layers.Dense(1, activation='linear')
], name='LSTM_Model')
```

[41]:
```
lstm_model.compile(loss='mean_squared_error', optimizer='adam',␣
 ↪metrics=['mean_absolute_error'])
```

[42]:
```
lstm_model.summary()
```

```
Model: "LSTM_Model"

_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 lstm (LSTM)                 (None, 5, 64)             16896

 lstm_1 (LSTM)               (None, 5, 64)             33024

 time_distributed (TimeDist  (None, 5, 64)             4160
 ributed)
```

```
 flatten_1 (Flatten)         (None, 320)               0

 dense_5 (Dense)             (None, 64)                20544

 dropout (Dropout)           (None, 64)                0

 dense_6 (Dense)             (None, 1)                 65

=================================================================
Total params: 74689 (291.75 KB)
Trainable params: 74689 (291.75 KB)
Non-trainable params: 0 (0.00 Byte)
_________________________________________________________________
```

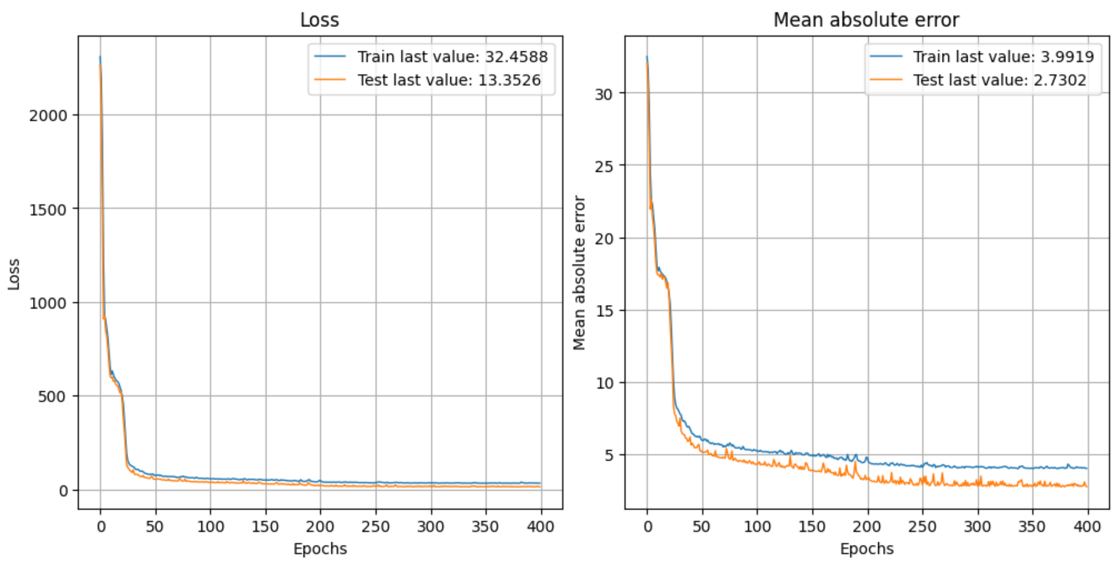
[43]:
```
t0 = datetime.datetime.now()

lstm_history = lstm_model.fit(X_train, y_train, validation_data=(X_test,␣
 ↪y_test), epochs=400, batch_size=4096, verbose=0)

t1 = datetime.datetime.now()
lstm_elapsed_time = (t1 - t0).total_seconds()
```

[44]:
```
print(f"Elapsed Time: {lstm_elapsed_time} (s)")
```

```
Elapsed Time: 264.190752 (s)
```

2.11.6 Plotting Loss and MSE

[45]:
```
show_history(lstm_history)
```



2.11.7 Bi-directional LSTM Model

[46]:
```
bi_directional_lstm_model = keras.Sequential([
    keras.layers.Bidirectional(keras.layers.LSTM(units=64,␣
 ↪return_sequences=True), input_shape=(5, 1)),
    keras.layers.Bidirectional(keras.layers.LSTM(units=64,␣
 ↪return_sequences=True)),
    keras.layers.TimeDistributed(keras.layers.Dense(64, activation='relu')),
    keras.layers.Flatten(),
    keras.layers.Dense(64, activation='relu'),
    keras.layers.Dropout(0.2),
    keras.layers.Dense(1, activation='linear')
], name='LSTM_BidirectionalModel')
```

[47]:
```
bi_directional_lstm_model.compile(loss='mean_squared_error', optimizer='adam',␣
 ↪metrics=['mean_absolute_error'])
```

[48]:
```
bi_directional_lstm_model.summary()
```

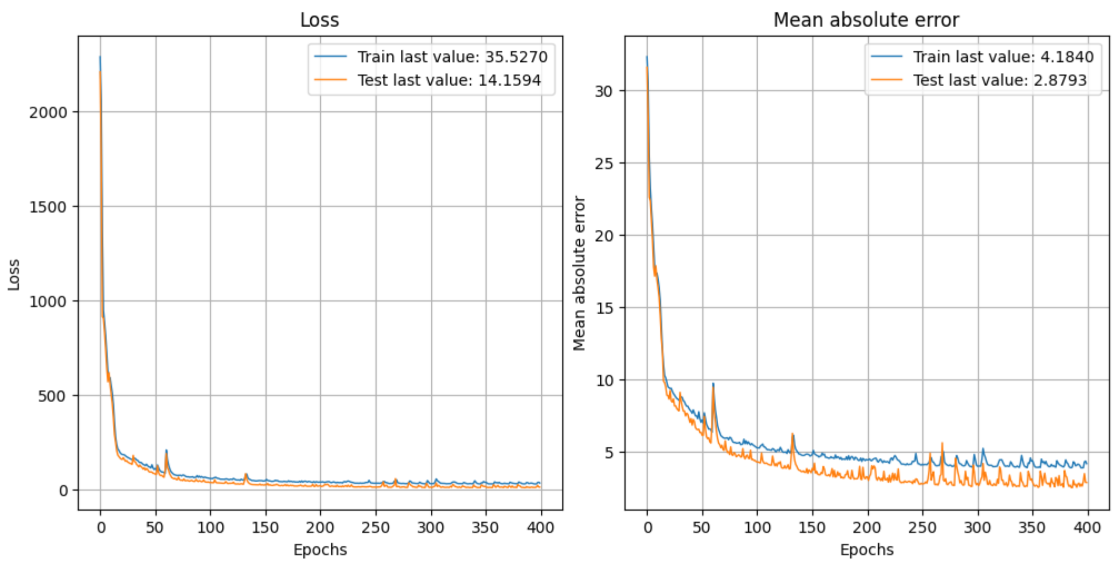
```
Model: "LSTM_BidirectionalModel"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 bidirectional (Bidirection  (None, 5, 128)            33792
 al)

 bidirectional_1 (Bidirecti  (None, 5, 128)            98816
 onal)

 time_distributed_1 (TimeDi  (None, 5, 64)             8256
 stributed)

 flatten_2 (Flatten)         (None, 320)               0

 dense_8 (Dense)             (None, 64)                20544

 dropout_1 (Dropout)         (None, 64)                0

 dense_9 (Dense)             (None, 1)                 65

=================================================================
Total params: 161473 (630.75 KB)
Trainable params: 161473 (630.75 KB)
Non-trainable params: 0 (0.00 Byte)
_________________________________________________________________
```

```
[49]: t0 = datetime.datetime.now()

bi_directional_lstm_history = bi_directional_lstm_model.fit(X_train, y_train,␣
 ↪validation_data=(X_test, y_test), epochs=400, batch_size=4096, verbose=0)

t1 = datetime.datetime.now()
bi_directional_lstm_elapsed_time = (t1 - t0).total_seconds()
```

```
[50]: print(f"Elapsed Time: {bi_directional_lstm_elapsed_time} (s)")
```

```
Elapsed Time: 598.464394 (s)
```

```
[51]: show_history(bi_directional_lstm_history)
```



2.12 Trainings Results Comparison

- Now that we have all the required trained models, we can compute MSE for each of them and compare their performance on the same test set.

```
[56]: clfs = [LR, regression_tree, random_forest_regressor, lasso, ridge, baysian,␣
 ↪nn_model, rnn_model, lstm_model, bi_directional_lstm_model]
elapsed_times = [LR_elapsed_time, decision_tree_elapsed_time,␣
 ↪random_forest_elapsed_time, lasso_elapsed_time, ridge_elapsed_time,␣
 ↪baysian_elapsed_time, cnn_elapsed_time, rnn_elapsed_time, lstm_elapsed_time,␣
 ↪bi_directional_lstm_elapsed_time]
errors = []

for idx, clf in enumerate(clfs):
```

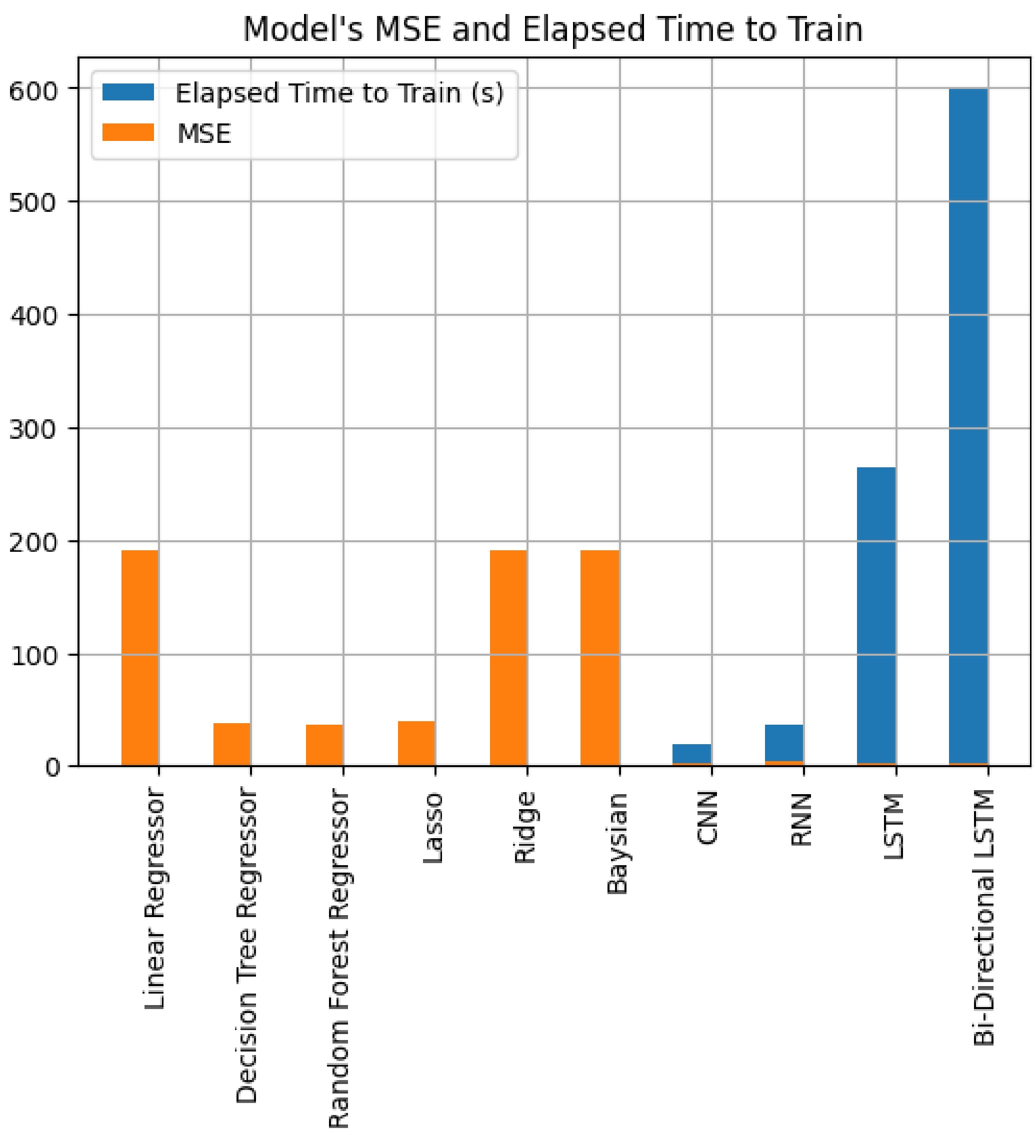
```
    prediction = clf.predict(X_test)

    error = mean_absolute_error(prediction, y_test)
    elapsed_time = elapsed_times[idx]
    errors.append(error)
```

```
495/495 [==============================] - 0s 600us/step
495/495 [==============================] - 1s 1ms/step
495/495 [==============================] - 1s 2ms/step
495/495 [==============================] - 1s 2ms/step
```

Model Name	Mean Square Error	Elapsed Training Time (s)
LinearRegression	191.42694082475876	0.003315
DecisionTreeRegressor	37.489920658237	0.284153
RandomForestRegressor	36.78428096079685	15.339947
Lasso	40.313265306818366	0.003109
Ridge	190.807526262169	0.009265
BayesianRidge	191.41289129527775	0.014366
CNN Model	2.836620958920867	18.656062
RNN Model	4.032565727022468	36.036608
LSTM Model	2.7302180064131942	264.190752
Bi-directional LSTM Model	2.8793313654473787	598.464394

2.12.1 Comparing Training Times and Losses for Every Model

[55]:
```
model_names = ['Linear Regressor', 'Decision Tree Regressor', 'Random Forest␣
↪Regressor', 'Lasso', 'Ridge', 'Baysian', 'CNN', 'RNN', 'LSTM',␣
↪'Bi-Directional LSTM']

X_axis = np.arange(len(model_names))

plt.xticks(X_axis, model_names)
plt.xticks(rotation=90)
plt.bar(X_axis - 0.2, elapsed_times, 0.4, label='Elapsed Time to Train (s)')
plt.title('Model\'s MSE and Elapsed Time to Train')
plt.xticks(X_axis, model_names)
plt.bar(X_axis - 0.2, errors, 0.4, label='MSE')
plt.grid(True)
plt.legend()
plt.show()
```



2.13 Model Performance Conclusion over All Features

- Based on the analysis, we can conclude that Neural Networks (specifically speaking Recurrent Neural Networks) perform significantly better on the data. However, training neural networks, especially LSTM model, takes significantly longer than linear models. For this purpose, below notes can be conducted:

  1. We can use `CNN models`, as they dont't require long training times, and they are precise.
  2. if we want to insist on **importance of time-series** in the data, we can use `RNN` models, which require **short training times**, and perform as well as `LSTM` models, and they don't take long time to train.
  3. If the training time is **important**, and we **don't mind** using **non-neural network** models, we may use `RandomForestRegressor`, `DecisionTreeRegressor`, or `Lasso` ac-

cordingly.
4. Bi-directional LSTM model has no advantage over LSTM model. Furthermore, it's training time is almost **twice** as LSTM model. Consequently, we can accept that the data has **no bi-directional** properties.

- As now we know which model performs the best over all five features, we need to answer the question: **What is the smallest number of features we need to have a decent performance?**
- Smaller feature space means that we need less sensors to gather data, which leads to cheaper prosthesis.

2.14 Feature Space Selection

- Based on the above analysis, we will now select different sub-spaces of our feature space to check which combined features perform as well as all features combined?

2.14.1 Feature space containing only One Feature

Only One Feature with DecisionTreeRegressor

[82]:
```
feature_names = ['vertical_ground_reaction_force',␣
↪'trunk_joint_angle',        'hip_joint_angle', 'knee_joint_angle',␣
↪'ankle_joint_angle']
single_feature_errors = []

for feature_name in feature_names:

    X_one_feature = normalized_data[str(feature_name)].to_numpy().reshape(-1, 1)
    y_one_feature = normalized_data.joint_moment

    X_train_one_feature, X_test_one_feature, y_train_one_feature,␣
↪y_test_one_feature = train_test_split(X_one_feature, y_one_feature,␣
↪test_size=0.33, random_state=42)

    regressor = DecisionTreeRegressor()

    regressor.fit(X_train_one_feature, y_train_one_feature)

    prediction = regressor.predict(X_test_one_feature)

    mse = mean_absolute_error(prediction, y_test_one_feature)

    single_feature_errors.append(mse)
```

Feature Name	Mean Square Error
Vertical Ground Reaction Force	20.745292548520855
Trunk Joint Angle	21.34414962131895
Hip Joint Angle	19.70776012473898

Feature Name	Mean Square Error
Knee Joint Angle	25.92375091658054
Ankle joint Angle	14.111932897082209

2.14.2 Feature space containing Two Feature

2.14.3 Pair Selector Function

- We need a function to return us every pair of features to analyze.

[83]:
```
def generate_pairs(input_array):
    # Initialize an empty list to store pairs
    pairs = []

    # Use nested loops to create pairs of elements
    for i in range(len(input_array)):
        for j in range(i + 1, len(input_array)):
            pair = [input_array[i], input_array[j]]
            pairs.append(pair)

    return pairs
```

[84]:
```
pairs = generate_pairs(feature_names)
pair_errors = []

for pair in pairs:

    X_two_feature = normalized_data[pair].to_numpy()

    y_two_feature = normalized_data.joint_moment

    X_train_two_feature, X_test_two_feature, y_train_two_feature,␣
 ↪y_test_two_feature = train_test_split(X_two_feature, y_two_feature,␣
 ↪test_size=0.33, random_state=42)

    regressor = DecisionTreeRegressor()

    regressor.fit(X_train_two_feature, y_train_two_feature)

    prediction = regressor.predict(X_test_two_feature)

    mse = mean_absolute_error(prediction, y_test_two_feature)

    pair_errors.append(mse)
```

Feature 1	Feature 2	Mean Square Error
Vertical Ground Reaction Force	Trunk Joint Angle	7.178059941631315
Vertical Ground Reaction Force	Hip Joint Angle	5.411830813820987
Vertical Ground Reaction Force	Knee Joint Angle	11.28008864202483
Vertical Ground Reaction Force	Ankle Joint Angle	8.105619084272801
Trunk Joint Angle	Hip Joint Angle	15.16507215852817
Trunk Joint Angle	Knee Joint Angle	9.764600256311368
Trunk Joint Angle	Ankle Joint Angle	7.855781722616615
Hip Joint Angle	Knee Joint Angle	5.388404760784376
Hip Joint Angle	Ankle Joint Angle	6.508475675896628
Knee Joint Angle	Ankle Joint Angle	10.894176677447074

2.14.4 Feature space containing Three Feature

[85]:
```
def generate_triples(input_array):
    # Use itertools.combinations to generate all combinations of 3 elements
    triples = list(combinations(input_array, 3))
    return triples
```

[86]:
```
triples = generate_triples(feature_names)
triples_errors = []

for triple in triples:
    triple = list(triple)

    X_three_feature = normalized_data[triple].to_numpy()

    y_three_feature = normalized_data.joint_moment

    X_train_three_feature, X_test_three_feature, y_train_three_feature,␣
 ↪y_test_three_feature = train_test_split(X_three_feature, y_three_feature,␣
 ↪test_size=0.33, random_state=42)

    regressor = DecisionTreeRegressor()

    regressor.fit(X_train_three_feature, y_train_three_feature)

    prediction = regressor.predict(X_test_three_feature)

    mse = mean_absolute_error(prediction, y_test_three_feature)

    triples_errors.append(mse)
```

Feature 1	Feature 2	Feature 3	Mean Square Error
Vertical Ground Reaction Force	Trunk Joint Angle	Hip Joint Angle	4.851509693009391
Vertical Ground Reaction Force	Trunk Joint Angle	Knee Joint Angle	5.296384786470117
Vertical Ground Reaction Force	Trunk Joint Angle	Ankle Joint Angle	5.610839021495343
Vertical Ground Reaction Force	Hip Joint Angle	Knee Joint Angle	4.30000954161124
Vertical Ground Reaction Force	Hip Joint Angle	Ankle Joint Angle	4.670558354324991
Vertical Ground Reaction Force	Knee Joint Angle	Ankle Joint Angle	5.249182548795679
Trunk Joint Angle	Hip Joint Angle	Knee Joint Angle	4.662134572349286
Trunk Joint Angle	Hip Joint Angle	Ankle Joint Angle	5.394411851185054
Trunk Joint Angle	Knee Joint Angle	Ankle Joint Angle	6.036280521456228
Hip Joint Angle	Knee Joint Angle	Ankle Joint Angle	4.803126949321675

2.14.5 Feature space containing Four Feature

[87]:
```
def generate_quadruples(input_array):
    # Use itertools.combinations to generate all combinations of 4 elements
    quadruples = list(combinations(input_array, 4))
    return quadruples
```

[88]:
```
quadruples = generate_quadruples(feature_names)
quadruples_errors = []

for quadruple in quadruples:
    quadruple = list(quadruple)

    X_four_feature = normalized_data[quadruple].to_numpy()

    y_four_feature = normalized_data.joint_moment

    X_train_four_feature, X_test_four_feature, y_train_four_feature,␣
↪y_test_four_feature = train_test_split(X_four_feature, y_four_feature,␣
↪test_size=0.33, random_state=42)

    regressor = DecisionTreeRegressor()

    regressor.fit(X_train_four_feature, y_train_four_feature)

    prediction = regressor.predict(X_test_four_feature)

    mse = mean_absolute_error(prediction, y_test_four_feature)
```

```
    quadruples_errors.append(mse)
```

Feature 1	Feature 2	Feature 3	Feature 4	Mean Square Error
Vertical Ground Reaction Force	Trunk Joint Angle	Hip Joint Angle	Knee Joint Angle	3.6819554078908934
Vertical Ground Reaction Force	Trunk Joint Angle	Hip Joint Angle	Ankle Joint Angle	4.245834202771391
Vertical Ground Reaction Force	Trunk Joint Angle	Knee Joint Angle	Ankle Joint Angle	4.41575004910883
Vertical Ground Reaction Force	Hip Joint Angle	Knee Joint Angle	Ankle Joint Angle	4.074960649347516
Trunk Joint Angle	Hip Joint Angle	Knee Joint Angle	Ankle Joint Angle	4.184786121519491

2.14.6 Feature space containing All Five Feature

[89]:
```
X_all_feature = normalized_data.drop(columns=['joint_moment']).to_numpy()

y_all_feature = normalized_data.joint_moment

X_train_all_feature, X_test_all_feature, y_train_all_feature,␣
↪y_test_all_feature = train_test_split(X_all_feature, y_all_feature,␣
↪test_size=0.33, random_state=42)

regressor = DecisionTreeRegressor()

regressor.fit(X_train_all_feature, y_train_all_feature)

prediction = regressor.predict(X_test_all_feature)

all_feature_error = mean_absolute_error(prediction, y_test_all_feature)
```

Feature 1	Feature 2	Feature 3	Feature 4	Feature 5	Mean Square Error
Vertical Ground Reaction Force	Trunk Joint Angle	Hip Joint Angle	Knee Joint Angle	Ankle Joint Angle	3.5158019936705047

2.14.7 Features Subset Selection Conclusion

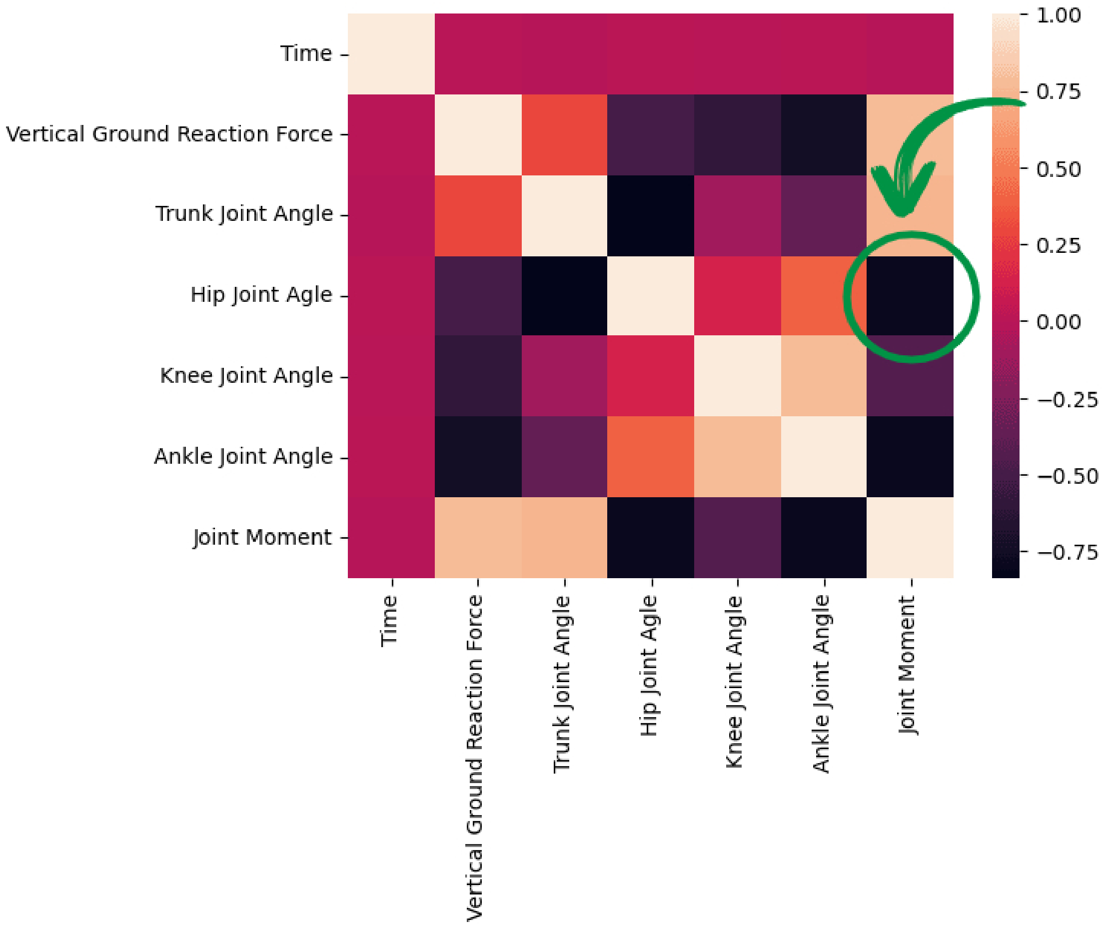
- Best selections in each subset are listed below:

Subset	Mean Square Error
Ankle joint Angle	14.111932897082209
Vertical Ground Reaction Force & Hip Joint Angle	**5.411830813820987**
Hip Joint Angle & Knee Joint Angle	**5.388404760784376**
Vertical Ground Reaction Force & Hip Joint Angle & Knee Joint Angle	4.30000954161124
Vertical Ground Reaction Force & Trunk Joint Angle & Hip Joint Angle & Knee Joint Angle	3.6819554078908934
All Features	3.5158019936705047

- Obviously the subset with all features has the lowest MSE. However, it can be seen that we can reach a great performance by using only **two** features instead of all five.
    1. **`Vertical Ground Reaction Force`** & **`Hip Joint Angle`**
    2. **`Hip Joint Angle`** & **`Knee Joint Angle`**

2.14.8 Note:

- We can see that **`Hip Joint Angle`** appears in best subsets almost everytime. It is related to the fact that according to the correlation matrix, it has a **very high** correlation with `Joint Moment`.


Time
Vertical Ground Reaction Force
Trunk Joint Angle
Hip Joint Agle
Knee Joint Angle
Ankle Joint Angle
Joint Moment
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75