جواب نمونه سوالات میانترم

1

$x_i \in R^p \qquad Y_i \in \{0,1\} \qquad \pi_k = P(Y=k)$

$g_k^{(x)} = P(X=x \mid Y=k)$

a) $f_{bayes}(x) = \arg\max_k \; \pi_k \, g_k(x)$

$P[Y=1 \mid X=x] = q(x) \sim P[Y=0 \mid X=x] = 1 - q(x)$

$P[f(X) \neq Y \mid X=x] = 1 - \max(q(x), 1-q(x))$

$= \boxed{\min(q(x), 1-q(x))}$

Posterior

$$P(Y \mid X) = \frac{\pi_k \, g_k^{(x)}}{P(x)} \propto \pi_k \, g_k^{(x)}$$

Desision Rule

b) $P[F(x) \neq Y \mid X = x, X' = x']$

$\underline{x'}$ is nearest neighbour to $\underline{x}$

$P(Y=1 \mid X = x) = q(x) \qquad P(Y=1 \mid X = x') = q(x')$

$P[F(x) \neq Y \mid X = x, X' = x'] =$

$q(x)(1 - q(x')) + (1 - q(x))\, q(x')$

$= q(x) + q(x') - 2q(x)\,q(x')$

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c) $n \to \infty \qquad q(x') \longrightarrow q(x)$

$R_{1NN}(x) = 2q(x) - 2q(x)^2$

$R_{bayes}(x) = \min\{q(x), 1 - q(x)\}$

$$\to R_{bayes} = E_{2}\left[\min\{q_{(2)}, 1-q_{(2)}\}\right]$$

$$R_{1NN} : E\left[r q_{(2)} - r q^{r}_{(2)} \right]$$

if $q_{(2)} \leq \frac{1}{r}$

$$R_b = E[q_{(x)}] \qquad R_N = r R_b - r E[q^r_{(x)}]$$

$$\rightsquigarrow R_N \geq R_b$$

if $q_{(2)} \geq \frac{1}{r}$

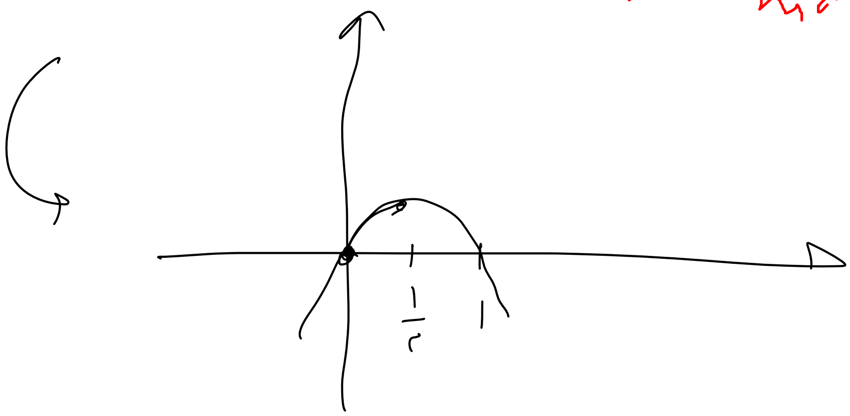
$$R^{(2)}_{bayes} = 1 - q_{(2)} \qquad R^{(2)}_{NN} = \underbrace{r q_{(2)}}_{\geq 1} R^{(2)}_b$$

$$\rightsquigarrow R_{NN} \geq R_b$$

Let $m(x) = \min(q(x),\ 1-q(x))$

$R_b = E[m(x)]$ $\qquad 0 \le m \le \frac{1}{2}$

$$R_{NN} = E\Big[\overset{2q(x)(1-q(x))}{\overset{\|}{2\underbrace{m(x)}_{\min}\underbrace{(1-m(x))}_{\max}}}\Big]$$



$m(1-m)$ is concave

Jensen Inq $\qquad E[h(m)] \le h(E[m])$

$\therefore\ R_{NN} = 2E[h(m(x))] \le$

$h_m = m(1-m)$

$$2h(E[m(x)])$$

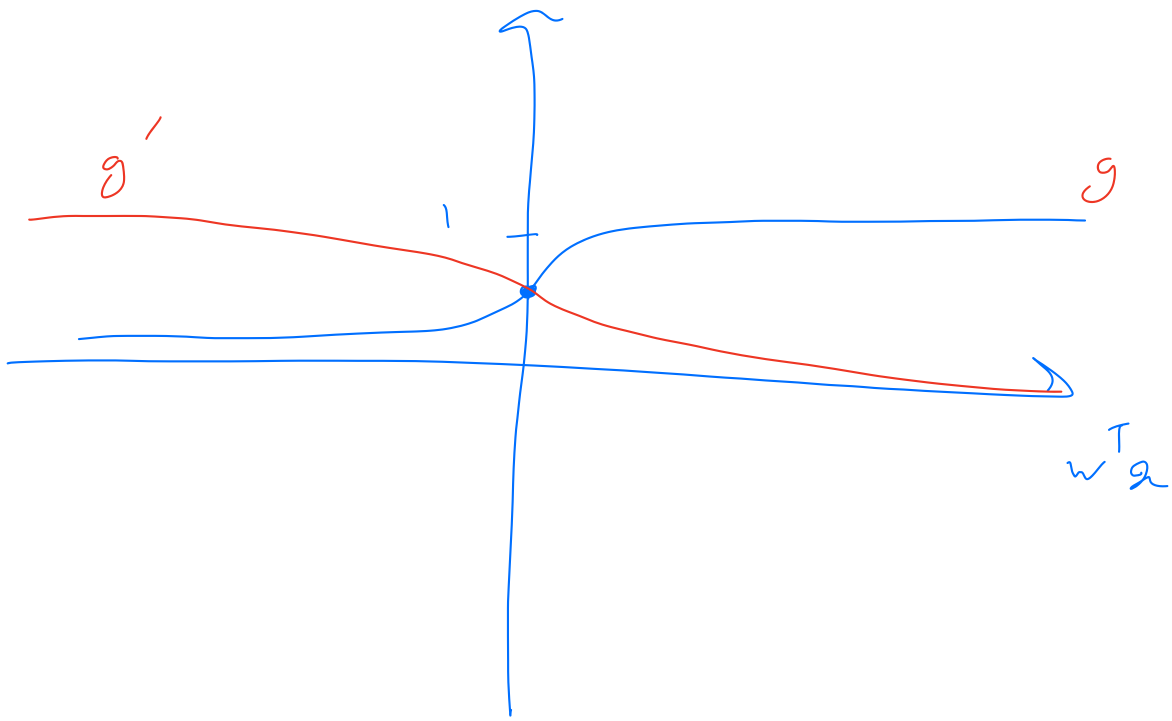
$$= 2h(R_b) = 2R_b(1-R_b)$$

$\rightarrow R_{NN} \leq r R_b (1 - R_b)$

$y = g(w^T x) \qquad g(z) = \dfrac{1}{1+e^{-z}}$

$P(y=1|x) = g(w^T x) = \dfrac{1}{1+e^{-w^T x}}$

Let $g'(z) = \dfrac{1}{1+e^{z}} \rightarrow g'(z) = g(-z)$



$\rightarrow y' = g(-w^T x) = 1 - g(w^T x) = 1 - y$

desicion boundry $\longrightarrow$ cte

$W \longrightarrow -W$

$$Loss = -\underset{\text{Label}}{t} \log \underset{\text{predict}}{(y)} - (1-t)\log(1-y)$$

$$L = -t \log(y) - (1-t)\log(1-y)$$

$$L' = -t \log(1-y) - (1-t)\log(y)$$

$$\frac{\partial L}{\partial w} = (y-t)x$$

$$\frac{\partial L'}{\partial w} = (1-y-t)x$$

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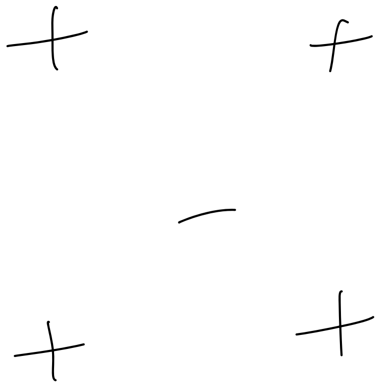
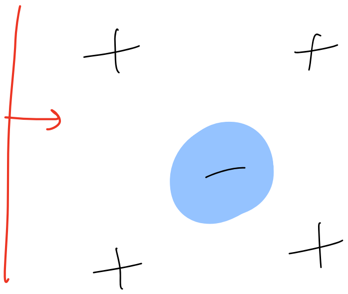
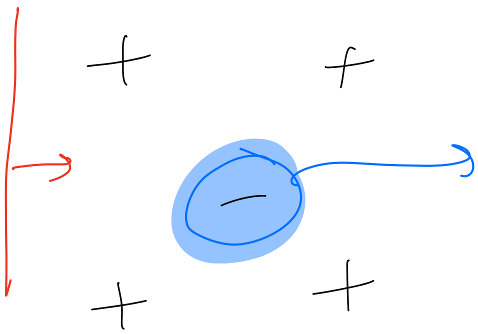


a)

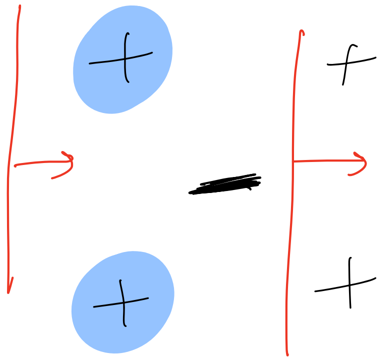


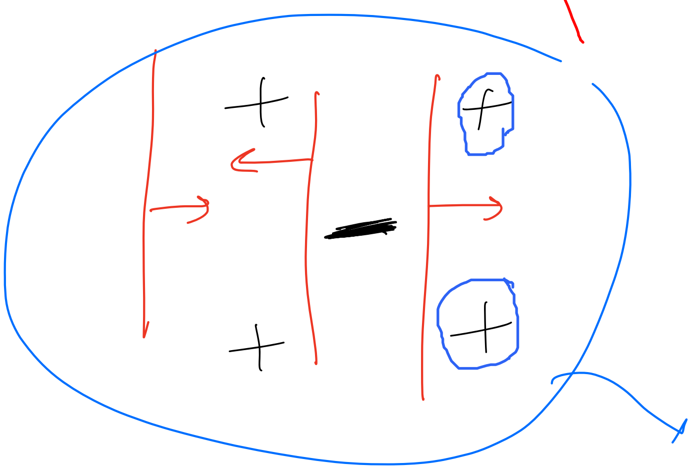
بهترین خط



بزرگ حاشیه

لمس $\frac{۲}{۳}$ درسن.





جواب نا ع

چرا بوستینگ: $\downarrow$ overfit چون مدل های ساده به تعداد

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۴) ببینیم

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۵)

$$\Sigma = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$|\Sigma - \lambda I| = 0$$

$$\rightarrow (2-\lambda)^2 = 1 \rightarrow \lambda_1 = 3 , \lambda_2 = 1$$

$$\Sigma v = \lambda v \rightarrow (\Sigma - \lambda I) v = 0$$

$$\rightarrow \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} v = 0 \rightarrow v_1 = v_2$$

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\rightarrow v_1 = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} v = 0$$

$$\rightarrow v_2 = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{bmatrix}$$

$$\rightarrow \boxed{PC_1 = v_1^T x \quad , \quad PC_2 = v_2^T x}$$

$$\lambda_1 + \lambda_2 = 3 + 1 = 4$$

$$\rightarrow \quad \% PC_1 = \frac{3}{4} \qquad \% PC_2 = \frac{1}{4}$$

$$\boxed{\begin{array}{l} x_1 > x_2 \rightarrow PC_2 > 0 \\ x_2 > x_1 \rightarrow PC_2 < 0 \\ x_1 = x_2 \rightarrow PC_2 = 0 \end{array}}$$

(4)

$$E = \sum_{i=1}^{N} \exp(-y_i f(x_i)) \qquad y \in (-1, 1)$$

a)

$$E_{01} = \sum_{i=1}^{N} 1 \cdot (y_i f(x_i) < 0)$$

$$= \sum_{i=1}^{N} 1 \cdot (-y_i f(x_i) > 0)$$

$$= \sum_{i=1}^{N} 1 \cdot \overset{\ln}{\log} e^{(-y_i f(x_i) > 0)}$$

$$\leq \sum_{i=1}^{N} 1 \cdot e^{(-y_i f(x_i) > 0)} \qquad \begin{matrix} 0 < 1 \\ 1 < e \end{matrix}$$

$$\leq \sum_{i=1}^{N} e^{-y_i f(x_i)} \qquad a > 0 \rightsquigarrow e^a > 1$$

$$= E \checkmark$$

b) $\alpha_t = \frac{1}{2} \ln \left(\frac{1 - e_t}{e_t} \right)$

, $e_t = \sum w_i^t \left(h_t(x_i) \neq y_i \right)$

if $e_t > \frac{1}{2} \rightarrow \boxed{\alpha_t < 0}$

$\rightarrow$ Break ✓

max limit w_i

⇝ ignore out layer

c)

□ +1

○ -1

✓ بد [illegible]