

Chapter: Measures of Central Tendency

Measures of central tendency shows the tendency of some central value around which data tends to converge. For further analysis of the tabular data, measures of central tendency represents the entire mass of data.

Objectives:

- To get one single value that describe the characteristics of the entire data.
- To easily compare the data.

Types:

Different types of central tendency are:

1. Arithmetic Mean
2. Median
3. Mode
4. Geometric Mean
5. Harmonic Mean

Arithmetic Mean:

The arithmetic mean, often simply referred to as mean, is the total of the values of a set of observations divided by their number of observations.

If $x_1, x_2, x_3, \dots, x_N$ represent the values of N items or observations, the arithmetic mean denoted by \bar{x} is defined by

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_N}{N} = \frac{\sum_{i=1}^N x_i}{N}$$

It's also written as $\bar{x} = \frac{\sum x_i}{N}$

In case of frequency distribution

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + f_3 x_3 + \dots + f_N x_N}{f_1 + f_2 + f_3 + \dots + f_N} = \frac{\sum f_i x_i}{\sum f_i} = \frac{\sum f_i x_i}{N}$$

Where N is called total frequency.

Example: The monthly income of 10 employees working in a firm is as follows:

4487	4493	4502	4446	4475	4492	4572	4516	4468	4489
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Find the average monthly income.

Solution: The total income

$$\begin{aligned}\sum x_i &= 4487 + 4493 + 4502 + 4446 + 4475 + 4492 + 4572 + 4516 + 4468 + 4489 \\ &= 44,940\end{aligned}$$

$$\bar{x} = \frac{\sum x_i}{N} = \frac{44940}{10} = 4494$$

Hence the average monthly income is tk 4494

Example: Find the mean of the following data

Class	8	10	15	20
Frequency	5	8	8	4

Solution:

Class (x_i)	Frequency(f_i)	$f_i x_i$
8	5	40
10	8	80
15	8	120
20	4	80
	$\sum f_i = 25$	$\sum f_i x_i = 320$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{\sum f_i x_i}{N} = \frac{320}{25} = 12.8$$

Calculating Mean Using Short-cut Method: The short-cut method is suitable for grouped data. The formula is

$$\bar{x} = A + \frac{\sum f_i d_i}{N} \times h$$

Where

h = The size of class interval.

A = The assumed mean. (It is the middle no of the mid values).

$d_i = \frac{x_i - A}{h}$ = The step deviation from A .

x_i = Mid values of each class.

N = The total frequency.

Example: Calculate mean for the following grouped data using short-cut method.

Class	0-10	10-20	20-30	30-40	40-50
frequency	7	8	20	10	5

Solution: Here $A = 25$ and $h = 10$

Class	Mid value x_i	Frequency f_i	$d_i = \frac{x_i - A}{h}$	$f_i d_i$
0-10	5	7	-2	-14
10-20	15	8	-1	-8
20-30	25 → A	20	0	0
30-40	35	10	+1	+10
40-50	45	5	+2	+10
		$N = 50$		$\sum f_i d_i = -2$

We know mean $\bar{x} = A + \frac{\sum f_i d_i}{N} \times h$

$$\bar{x} = 25 + \frac{-2}{50} \times 10 = 24.6$$

Example: Calculate mean for the following data representing the marks of statistics for 80 students in a class.

Marks	0-20	20-40	40-60	60-80	80-100	100-120	120-140
No of Student	4	26	22	10	9	6	3

Solution: Here $A = 70$ and $h = 20$

Marks	Mid value x_i	No of Student f_i	$d_i = \frac{x_i - A}{h}$	$f_i d_i$
0-20	10	4	-3	-12
20-40	30	26	-2	-52
40-60	50	22	-1	-22
60-80	70	10	0	0
80-100	90	9	+1	+9
100-120	110	6	+2	+12
120-140	130	3	+3	+9
		$N = 80$		$\sum f_i d_i = -56$

We know, Mean $\bar{x} = A + \frac{\sum f_i d_i}{N} \times h$

$$\bar{x} = 70 + \frac{-56}{80} \times 20 = 56$$

Example: Calculate the arithmetic mean of the frequency distribution given below

Height	130-134	135-139	140-144	145-149	150-154	155-159	160-164
No of Students	5	15	28	24	17	10	1

Solution: Here $A = 147$ and $h = 5$

Height	Mid value x_i	No of Students f_i	$d_i = \frac{x_i - A}{h}$	$f_i d_i$
129.5–134.5	132	5	–3	–15
134.5–139.5	137	15	–2	–30
139.5–144.5	142	28	–1	–28
144.5–149.5	147	24	0	0
149.5–154.5	152	17	+1	+17
154.5–159.5	157	10	+2	+20
159.5–164.5	162	1	+3	+3
		$N = 100$		$\sum f_i d_i = -33$

$$\text{Mean } \bar{x} = A + \frac{\sum f_i d_i}{N} \times h = 147 + \frac{-33}{100} \times 5 = 145.35$$

For Practice

1. Calculate the mean of the following data

Height(cm)	65	66	67	68	69	70	71	72	73
No of Plants	1	4	5	7	11	10	6	4	2

ANS: 69.18

2. Find the mean of the following data

Marks	No of Students
0-10	3
10-20	5
20-30	7
30-40	10
40-50	12
50-60	15
60-70	12
70-80	6
80-90	2
90-100	8

ANS: 51.75

Geometric Mean (GM):

- The geometric mean is well defined only for sets of positive real numbers.
- This is calculated by multiplying all the numbers (call the number of numbers n), and taking the nth root of the total.
- A common example of where the geometric mean is the correct choice is when averaging growth rates.
- The geometric mean is NOT the arithmetic mean and it is NOT a simple average.
- Mathematical definition: The nth root of the product of n numbers.

$$GM = \sqrt[n]{x_1 \cdot x_2 \cdot x_3 \dots x_n}$$

For Ungrouped data $GM = AL\left(\frac{\sum \log x}{N}\right)$

For Grouped data $GM = AL\left(\frac{\sum f \log x}{N}\right)$

AL stands for Anti Log.

- **Example:** Find the geometric mean of the following values:
15, 12, 13, 19, 10

Solution:

$$N=5, \sum \log x = 5.648$$

$$\begin{aligned} GM &= AL\left(\frac{\sum \log x}{N}\right) \\ &= AL\left(\frac{5.648}{5}\right) \\ &= 13.48 \end{aligned}$$

<i>x</i>	<i>Log x</i>
15	1.1761
12	1.0792
13	1.1139
19	1.2788
10	1.0000
Total	5.648

Harmonic Mean (HM):

- The harmonic mean is calculated as the number of values N divided by the sum of the reciprocal of the values (1 over each value).

For Ungrouped data $HM = \frac{N}{\sum(\frac{1}{x})}$

For Grouped data $HM = \frac{\sum f}{\sum(\frac{f}{x})}$

- Example:** Calculate the harmonic mean of the numbers: 13.2, 14.2, 14.8, 15.2, and 16.1

Solution:

$$HM = \frac{N}{\sum(\frac{1}{x})} = \frac{5}{0.3147} = 14.63$$

x	$\frac{1}{x}$
13.2	0.0758
14.2	0.0704
14.8	0.0676
15.2	0.0658
16.1	0.0621
Total	$\sum \frac{1}{x} = 0.3147$

- Example:** Calculate the harmonic mean for the given below.

Marks	30-39	40-49	50-59	60-69	70-79	80-89	90-99
F	2	3	11	20	32	25	7

Solution:

$$\begin{aligned} HM &= \frac{\sum f}{\sum(\frac{f}{x})} \\ &= \frac{100}{1.4368} \\ &= 69.60 \end{aligned}$$

Marks	x	f	$\frac{f}{x}$
30-39	34.5	2	0.0580
40-49	44.5	3	0.0674
50-59	54.5	11	0.2018
60-69	64.5	20	0.3101
70-79	74.5	32	0.4295
80-89	84.5	25	0.2959
90-99	94.5	7	0.0741
Total		100	1.4368

- Relation among arithmetic mean, geometric mean, and harmonic mean:**
Geometric mean of a set of positive numbers is less than or equal to their arithmetic mean but is greater than or equal to their harmonic mean.

$$HM \leq GM \leq AM$$

Weighted Mean (WM):

- Weighted mean is the mean of a set of values wherein each value or measurement has a different weight or degree of measurement.
- The weighted mean of observations $x_1, x_2, x_3, \dots, x_n$ having weights $w_1, w_2, w_3, \dots, w_n$ is given by

$$WM = \frac{w_1x_1 + w_2x_2 + w_3x_3 + \dots + w_nx_n}{w_1 + w_2 + w_3 + \dots + w_n} = \frac{\sum w_i x_i}{\sum w_i}$$

- Simple mean is a specific case of weighted mean

- **Example:**

Find the Grade Point Average (GPA) of Paolo for the first semester. Use the table below.

Subjects	Grade (X_i)	Units (w_i)	(X_i) (w_i)
BM 112	1.25	3	3.75
BM 101	1.00	3	3.00
AC 103	1.25	6	7.50
EC 111	1.00	3	3.00
MG 101	1.50	3	4.50
MK 101	1.25	3	3.75
FM 111	1.50	3	4.50
PE 2	1.00	2	2.00
		$\Sigma(w_i) = 26$	$\Sigma(X_i)w_i = 32.00$

Solution:

$$WM = \frac{\sum w_i x_i}{\sum w_i} = \frac{32}{26} = 1.23$$

Properties of the Mean:

- It measures stability. Mean is the most stable among other measures of central tendency because every score contributes to the value of the mean.
- Mean can be calculated for any set of numerical data, so it always exists.
- A set of numerical data has one and only one mean.
- It may easily be affected by the extreme scores.
- It can be applied to interval level of measurement.
- It may not be an actual score in the distribution.
- It is very easy to compute.

Advantages of Mean:

- Sensitive to any change in the value of any observation

Disadvantages of Mean:

- Very sensitive to outlier

Median:

The median is defined as the measure of middle value when set of data are arranged in ascending or descending order.

Calculation of Median (Ungrouped Data)

- First arrange them in ascending or descending order and count number of observation or items N.
- If number of observation N is odd, then $\frac{N+1}{2}$ th observation is median.
- If number of observation N is even, then median is the average of $\frac{N}{2}$ th and $\frac{N}{2} + 1$ th observation.

Example: The weights of 11 mothers in kg were recorded as follows:

47 44 42 41 58 52 55 39 40 43 61

Find the median.

Solution:

Given data in ascending order

39 40 41 42 43 44 47 52 55 58 61

Number of observation $N = 11$, which is odd number.

Median is $\frac{N+1}{2}$ th observation = $\frac{11+1}{2} = 6$ th observation.

6 th observation is 44. Therefore median is 44.

Example: Find the median of the following

20 18 22 27 25 12 15

ANS: 20

Example: The weights of 10 mothers in kg were recorded as follows:

47 44 42 41 58 55 39 40 43 61

Find the median.

Solution:

Given data in ascending order

39 40 41 42 43 44 47 55 58 61

Number of observation $N = 10$, which is even number.

Median is average of $\frac{N}{2} = \frac{10}{2} = 5$ th and $\frac{N}{2} + 1 = \frac{10}{2} + 1 = 6$ th observation.

Therefore median = $\frac{5 \text{ th observation} + 6 \text{ th observation}}{2} = \frac{43 + 44}{2} = 43.5$

Calculation of Median (Grouped Data)

For Grouped data, Median = $L + \frac{\frac{N}{2} - \text{p.c.f}}{f} \times h$

Where

h = The size of class interval.

L = Lower limit of median class. (The class where middle ($\frac{N}{2}$ th) observation lies.)

p. c. f = Preceding cumulative frequency of median class. (Cumulative frequency above median class)

f = Frequency of the median class.

Example: Calculate the median for the distribution of the weights of 150 students from the given below:

Weight	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	18	37	45	27	15	8

Solution:

Weight	Frequency	Cumulative frequency
30-40	18	18
40-50	37	55 → p.c.f
L ← 50-60	45 → f	100
60-70	27	127
70-80	15	142
80-90	8	150
	N = 150	

55 – 100
observation

Median is $\frac{N}{2} = \frac{150}{2} = 75$ th observation. 75 th observation lies in class 50 – 60.

Median class is 50 – 60.

$$\text{Median} = L + \frac{\frac{N}{2} - \text{p.c.f}}{f} \times h = 50 + \frac{\frac{150}{2} - 55}{45} \times 10 = 54.44$$

Example: Following distribution gives the pattern of overtime done by 100 employee.

Calculate the median

Overtime	10-15	15-20	20-25	25-30	30-35	35-40
No of employee	11	20	35	20	8	6

Solution:

Overtime	No of employee	Cumulative frequency
10-15	11	11
15-20	20	31
20-25	35	66
25-30	20	86
30-35	8	94
35-40	6	100
	N = 100	

Median is $\frac{N}{2} = \frac{100}{2} = 50$ th observation. 50 th observation lies in class 20 – 25.

Median class is 20 – 25.

$$\text{Median} = L + \frac{\frac{N}{2} - \text{p.c.f}}{f} \times h = 20 + \frac{\frac{100}{2} - 31}{35} \times 5 = 22.714$$

Hence 50% of the workers doing overtime up to 22.714 hrs and the remaining 50% of the workers doing overtime more than 22.714 hrs.

Example: Calculate the median from the following distribution gives the profit of 125 companies:

Profit (crore)	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No of Companies	4	12	24	36	20	16	8	3

Comment on your result.

Solution:

Profit (crore)	No of Companies	Cumulative frequency
0-10	4	4
10-20	12	16
20-30	24	40
30-40	36	76
40-50	20	96
50-60	16	112
60-70	8	120
70-80	3	125
	N = 125	

Median is $\frac{N}{2} = \frac{125}{2} = 62.5$ th observation. 62.5 th observation lies in class 30 – 40.

Median class is 30 – 40.

$$\text{Median} = L + \frac{\frac{N}{2} - \text{p.c.f}}{f} \times h = 30 + \frac{\frac{125}{2} - 40}{36} \times 10 = 36.25$$

Hence 50% of the companies have profits up to 36.5 crores and the remaining 50% of the companies have profits more than 36.5 crores.

Example: Calculate the median of the frequency distribution given below

Height	130-134	135-139	140-144	145-149	150-154	155-159	160-164
No of Students	5	15	28	24	17	10	1

Solution:

Height	No of Students f_i	Cumulative frequency
129.5–134.5	5	5
134.5–139.5	15	20
139.5–144.5	28	48
144.5–149.5	24	72
149.5–154.5	17	89
154.5–159.5	10	99
159.5–164.5	1	100
	$N = 100$	

Median is $\frac{N}{2} = \frac{100}{2} = 50$ th observation. 50 th observation lies in class 144.5–149.5.

Median class is 144.5–149.5.

$$\text{Median} = L + \frac{\frac{N}{2} - \text{p.c.f}}{f} \times h = 144.5 + \frac{\frac{100}{2} - 48}{24} \times 5 = 144.917$$

Example: Calculate the median from the following distribution

No of days absent	5	10	15	20	25	30	35	40	45
No of Students	29	195	241	117	52	10	6	3	2

Solution:

Class	No of Students	Cumulative frequency
0-5	29	29
5-10	195	224
10-15	241	465
15-20	117	582
20-25	52	634
25-30	10	644
30-35	6	650
35-40	3	653
40-45	2	655
	N = 655	

Median is $\frac{N}{2} = \frac{655}{2} = 327.5$ th observation. 327.5 th observation lies in class 10 - 15.

Median class is 10 - 15.

$$\text{Median} = L + \frac{\frac{N}{2} - \text{p.c.f}}{f} \times h = 10 + \frac{\frac{655}{2} - 224}{241} \times 5 = 12.15$$

For Practice

1. Calculate the median of the following

Marks	0-20	20-40	40-60	60-80	80-100	100-120	120-140
No of Students	4	26	22	10	9	6	3

ANS: 49.09

2. Find the median of the following data

Marks	No of Students
0-10	7
10-20	32
20-30	56
30-40	106
40-50	180
50-60	164
60-70	86
70-80	44

ANS: 47.58

Properties of the Median

- It may not be an actual observation in the data set.
- It can be applied in ordinal level.
- It is not affected by extreme values because median is a positional measure.
- Median exists in both quantitative or qualitative data.

When to Use the Median

- The exact midpoint of the score distribution is desired.
- There are extreme scores in the distribution.

Advantages of the Median:

- Median can be calculated in all distributions.
- Median can be understood even by common people.
- Median can be ascertained even with the extreme items.
- It can be located graphically
- It is most useful dealing with qualitative data.

Disadvantages of the Median:

- It is not based on all the values.
- It is not capable of further mathematical treatment.
- It is affected fluctuation of sampling.
- In case of even no. of values, it may not be the value from the data.

Mode:

Mode is defined as the value which occurs the maximum number of times i.e. having the maximum frequency.

Calculation of Mode (Ungrouped Data)

Example: Six different observations

5 8 10 8 5 8

Find the mode.

Solution:

Since 8 has occurred maximum number of times, i.e. 3 times. So modal value is 8.

Example: Find the mode of the following

0, 1, 6, 7, 2, 3, 7, 6, 6, 2, 6, 0, 5, 6, 0

ANS: 6

Calculation of Mode (Grouped Data)

For Grouped data, $\text{Mode} = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h$

Where

h = The size of class interval.

L = Lower limit of modal class. (The class having maximum frequency.)

Δ_1 = Difference between the frequency of the modal class and the pre-modal class.

Δ_2 = Difference between the frequency of the modal class and the post-modal class.

Example: Calculate the mode for the distribution of the weights of 150 students from the given below:

Weight	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	18	37	45	27	15	8

Solution:

Weight	Frequency	
30-40	18	
40-50	37	} $\Delta_1 = 45 - 37$
L ← 50-60	45	
60-70	27	} $\Delta_2 = 45 - 27$
70-80	15	
80-90	8	

Since highest frequency is 45 which lies in the class 50 – 60.

Modal class is 50 – 60.

$$\text{Mode} = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h = 50 + \frac{8}{8+18} \times 10 = 53.08$$

Example: Find the mode of the following data

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No of Students	7	32	56	106	180	164	86	44

Solution:

Marks	No of Students
0-10	7
10-20	32
20-30	56
30-40	106
40-50	180
50-60	164
60-70	86
70-80	44

Since highest frequency is 180 which lies in the class 40 – 50.

Modal class is 40 – 50.

$$L = 40, \Delta_1 = 180 - 106 = 74, \Delta_2 = 180 - 164 = 16$$

$$\text{Mode} = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h = 40 + \frac{74}{74+16} \times 10 = 48.22$$

Properties of the Mode

- It is a quick approximation of the average.
- It may not be unique.
- It is affected by extreme values.
- It is the most unreliable among the three measures of central tendency because its value is undefined in some observations

When to Use the ~~Median~~ Mode

- When the “typical” value is desired.
- When the data set is measured on a nominal scale.

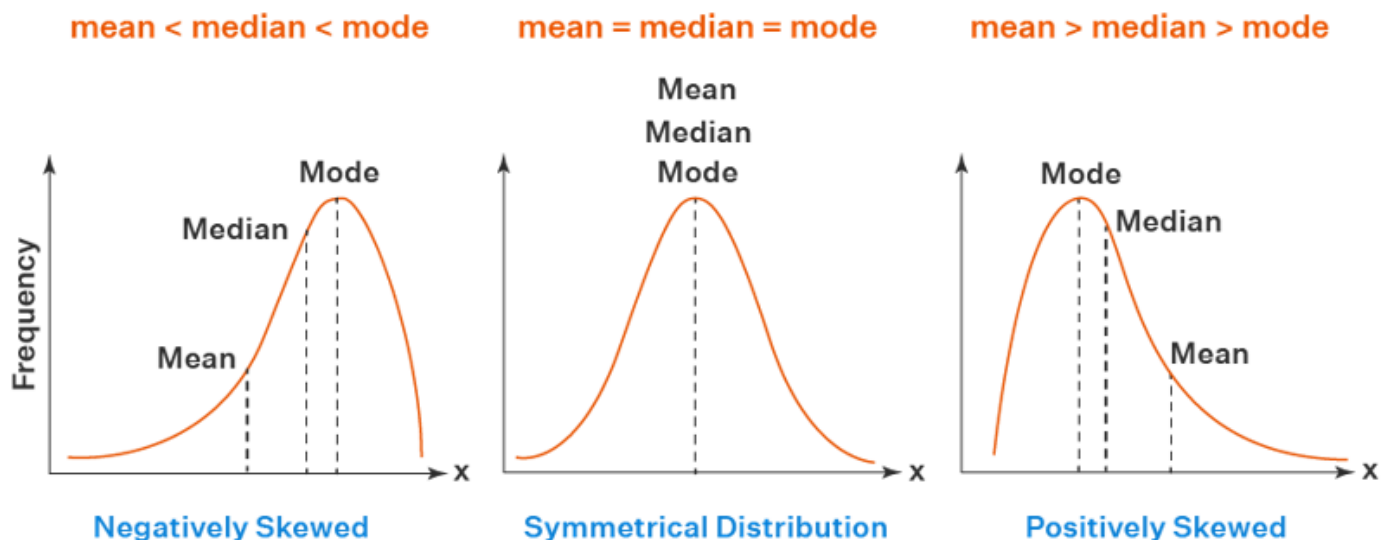
Advantages of the Mode:

- Mode is readily comprehensible and easily calculated
- It is the best representative of data
- It is not at all affected by extreme value.
- The value of mode can also be determined graphically.
- It is usually an actual value of an important part of the series.

Disadvantages of the Mode:

- It is not based on all observations.
- It is not capable of further mathematical manipulation.
- Mode is affected to a great extent by sampling fluctuations.
- Choice of grouping has great influence on the value of mode.

Mean, Median, and Mode



- For normal/symmetrical distributions, mean = median = mode
- In positively skewed distributions, mean > median > mode
- In negatively skewed distributions, mean < median < mode

Empirical Relation between Mean, Median, Mode:

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean}$$

Mean or Median or Mode?

Which one is better???

The choice depends on the nature of the data, the distribution, and our goals.

- Use the mean when the data is symmetrically distributed and does not contain outliers.
- Use the median when the data is skewed or contains outliers.
- Use the mode when you want to identify the most common value in the data.
- The mean is valid only for interval data or ratio data.
- The median can be determined for ordinal data as well as interval and ratio data.
- The mode can be used with nominal, ordinal, interval, and ratio data.
- Mode is the only measure of central tendency that can be used with nominal data.

Example: Calculate the median and mode of the frequency distribution given below.

Hence calculate the mean using empirical relation between them.

Weight	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	18	37	45	27	15	8

Solution:

Weight	Frequency	Cumulative frequency
30-40	18	18
40-50	37	55
50-60	45	100
60-70	27	127
70-80	15	142
80-90	8	150
	N = 150	

Median:

Median is $\frac{N}{2} = \frac{150}{2} = 75$ th observation. 75 th observation lies in class 50 – 60. Median class is 50 – 60.

$$\text{Median} = L + \frac{\frac{N}{2} - \text{p.c.f}}{f} \times h = 50 + \frac{\frac{150}{2} - 55}{45} \times 10 = 54.44$$

Mode:

Since highest frequency is 45 which lies in the class 50 – 60. Modal class is 50 – 60.

$$\text{Mode} = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h = 50 + \frac{8}{8+18} \times 10 = 53.08$$

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean}$$

$$\Rightarrow 2 \text{ Mean} = 3 \text{ Median} - \text{Mode}$$

$$\Rightarrow \text{Mean} = (3 \text{ Median} - \text{Mode})/2$$

$$\therefore \text{Mean} = \frac{3 \times 54.44 - 53.08}{2} = 55.12$$

Example: Calculate the arithmetic mean and median of the frequency distribution given below. Hence calculate the mode using empirical relation between them.

Marks	0-20	20-40	40-60	60-80	80-100	100-120	120-140
No of Student	4	26	22	10	9	6	3

Solution: Here $A = 70$ and $h = 20$

Marks	Mid value x_i	No of Student f_i	$d_i = \frac{x_i - A}{h}$	$f_i d_i$	Cumulative frequency
0-20	10	4	-3	-12	4
20-40	30	26	-2	-52	30
40-60	50	22	-1	-22	52
60-80	70	10	0	0	62
80-100	90	9	+1	+9	71
100-120	110	6	+2	+12	77
120-140	130	3	+3	+9	80
		$N = 80$		$\sum f_i d_i = -56$	

Mean:

$$\text{Mean } \bar{x} = A + \frac{\sum f_i d_i}{N} \times h = 70 + \frac{-56}{80} \times 20 = 56$$

Median:

Median is $\frac{N}{2} = \frac{80}{2} = 40$ th observation. 40 th observation lies in class 40–60. Median class is 40–60.

$$\text{Median} = L + \frac{\frac{N}{2} - \text{p.c.f}}{f} \times h = 40 + \frac{40 - 30}{22} \times 20 = 49.09$$

$$\therefore \text{Mode} = 3 \text{ Median} - 2 \text{ Mean} = (3 \times 49.09) - (2 \times 56) = 35.27$$

Example: Calculate the arithmetic mean and median of the frequency distribution given below. Hence calculate the mode using empirical relation between them.

Height	130-134	135-139	140-144	145-149	150-154	155-159	160-164
No of Students	5	15	28	24	17	10	1

Solution: Here $A = 147$ and $h = 5$

Height	Mid value x_i	No of Students f_i	$d_i = \frac{x_i - A}{h}$	$f_i d_i$	Cumulative frequency
129.5–134.5	132	5	–3	–15	5
134.5–139.5	137	15	–2	–30	20
139.5–144.5	142	28	–1	–28	48
144.5–149.5	147	24	0	0	72
149.5–154.5	152	17	+1	+17	89
154.5–159.5	157	10	+2	+20	99
159.5–164.5	162	1	+3	+3	100
		$N = 100$		$\sum f_i d_i = -33$	

Mean:

$$\text{Mean } \bar{x} = A + \frac{\sum f_i d_i}{N} \times h = 147 + \frac{-33}{100} \times 5 = 145.35$$

Median:

Median is $\frac{N}{2} = \frac{100}{2} = 50$ th observation. 50 th observation lies in class 144.5–149.5. Median class is 144.5–149.5.

$$\text{Median} = L + \frac{\frac{N}{2} - \text{p.c.f}}{f} \times h = 144.5 + \frac{\frac{100}{2} - 48}{24} \times 5 = 144.917$$

$$\begin{aligned}\therefore \text{Mode} &= 3 \text{ Median} - 2 \text{ Mean} = (3 \times 144.917) - (2 \times 145.35) \\ &= 144.051\end{aligned}$$

For Practice

1. Calculate the arithmetic mean and median of the frequency distribution given below.

Hence calculate the mode using empirical relation between them.:

Marks	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No of Students	2	12	15	20	18	10	9	4

ANS: Mean = 58.5, Median = 57.5, Mode = 55.5

2. The median and mode of the following wage distribution are tk 33.5 and tk 34 respectively. However there frequencies are missing. Determine their values:

Wage	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequencies	4	16	?	?	?	6	4	230

ANS: 60, 100, 40