Department of
Computer Science and Engineering

University of Dhaka

# Title: Implementation of the Bisection Method for finding roots of nonlinear equations

CSE 3212: Numerical Analysis Lab

Batch: 28/3rd Year 2nd Semester 2024

# 1 Objective(s)

- To understand the principle of the bisection method.
- To implement the iterative algorithm in a computer program.
- To approximate the root of a nonlinear function to a desired accuracy.
- To analyze the convergence and limitations of the method.

# 2 Theory

The Bisection method is one of the simplest methods to find a zero of a nonlinear function. It is also called the interval halving method. To use the Bisection method, one needs an initial interval that is known to contain a zero of the function. The method systematically reduces the interval. It does this by dividing the interval into two equal parts, performing a simple test, and based on the result of the test, half of the interval is thrown away. The procedure is repeated until the desired interval size is obtained.

If $f(x)$ is continuous on $[a, b]$ and $f(a) \cdot f(b) < 0$, then there exists at least one root $c \in (a, b)$ such that $f(c) = 0$.

## 2.1 Procedure

1. Start with interval $[a, b]$ where the root exists.
2. Compute the midpoint:
$$c = \frac{a+b}{2}$$
3. Check the sign of $f(c)$:
   - If $f(a) \cdot f(c) < 0$, then the root lies in $[a, c]$.
   - Else if $f(c) \cdot f(b) < 0$, then the root lies in $[c, b]$.
4. Repeat until the interval length $|b - a|$ or $|f(c)|$ is less than the desired tolerance.

# 3 Algorithm

**Input:** function $f(x)$, interval $[a, b]$, tolerance $\epsilon$, maximum iterations $N$.
**Output:** approximate root.

Check if $f(a) \cdot f(b) < 0$
**if** condition not satisfied **then**
    Stop (no root guaranteed in $[a, b]$).
**end if**
**for** $k = 1$ to $N$ **do**
    $c = \frac{a+b}{2}$
    **if** $f(c) = 0$ **or** $|b - a| < \epsilon$ **then**
        Stop: root $\approx c$
    **else if** $f(a) \cdot f(c) < 0$ **then**
        $b = c$
    **else**
        $a = c$
    **end if**
**end for**
Output $c$ as approximate root.

# 4 Example Execution

We want to find the root of the function
$$f(x) = x^3 - x - 2$$
in the interval $[1, 2]$.

### Step 1: Initial Interval

$$f(1) = 1^3 - 1 - 2 = -2, \quad f(2) = 2^3 - 2 - 2 = 4$$

Since $f(1) \cdot f(2) < 0$, the root lies in $[1, 2]$.

### Step 2: Iterations

1. **Iteration 1:** Midpoint $c = \frac{1+2}{2} = 1.5$ $f(1.5) = (1.5)^3 - 1.5 - 2 = -0.125$ Sign check: $f(1) \cdot f(1.5) > 0$, so root lies in $[1.5, 2]$.
2. **Iteration 2:** $c = \frac{1.5+2}{2} = 1.75$ $f(1.75) = (1.75)^3 - 1.75 - 2 = 1.609$ Sign check: $f(1.5) \cdot f(1.75) < 0$, so root lies in $[1.5, 1.75]$.
3. **Iteration 3:** $c = \frac{1.5+1.75}{2} = 1.625$ $f(1.625) = (1.625)^3 - 1.625 - 2 = 0.666$ Root lies in $[1.5, 1.625]$.
4. **Iteration 4:** $c = \frac{1.5+1.625}{2} = 1.5625$ $f(1.5625) = (1.5625)^3 - 1.5625 - 2 = 0.252$ Root lies in $[1.5, 1.5625]$.
5. **Iteration 5:** $c = \frac{1.5+1.5625}{2} = 1.53125$ $f(1.53125) = (1.53125)^3 - 1.53125 - 2 = 0.0606$ Root lies in $[1.5, 1.53125]$.
6. **Iteration 6:** $c = \frac{1.5+1.53125}{2} = 1.515625$ $f(1.515625) = (1.515625)^3 - 1.515625 - 2 = -0.032$ Root lies in $[1.515625, 1.53125]$.

### Step 3: Approximate Root

After several iterations, the interval narrows down to around $c \approx 1.5214$.

$$f(1.5214) \approx (1.5214)^3 - 1.5214 - 2 \approx 0$$

**Therefore, the root is approximately $x = 1.5214$.**

# 5 Lab Tasks (Please implement yourself and show the output to the instructor)

## 5.1 Problem 1

Consider the following function

$$f(x) = 225 + 82x - 90x^2 + 44x^3 - 8x^4 + 0.7x^5.$$

Determine the real root of $f(x)$ by applying **bisection method** with a stopping criterion of $\epsilon_s \leq 10\%$ (approximate relative error $e_s \leq 0.10$). Employ initial guesses

$$x_\ell = -1.2, \qquad x_u = -1.0,$$

and iterate until the approximate relative error falls below 0.05%.

Report $x_l$, $x_u$, approximate root $c$ or $x_r$, $f(c)$, True Error $\epsilon_t(\%)$ and absolute error $\epsilon_a(\%)$ after each iterations in tabular format. To avoid round-off error, use 10 places after the decimal point.

Also, generate a graph of absolute error vs the number of iterations to visualize the convergence of error.

## 5.2 Problem 2

Water is flowing in a trapezoidal channel at a rate of $Q = 20\ \text{m}^3/\text{s}$. The critical depth $y$ for such a channel must satisfy the equation:

$$0 = 1 - \frac{Q^2}{gA_c^3 B}$$

where

$$g = 9.81\ \text{m/s}^2, \quad A_c = \text{cross-sectional area (m}^2\text{)}, \quad B = \text{width of the channel at the surface (m)}.$$

For this trapezoidal channel, the width and the cross-sectional area are related to depth $y$ by:

$$B = 3 + y, \qquad A_c = 3y + \frac{y^2}{2}.$$

Solve for the critical depth $y$ using the **Bisection Method**. Use initial guesses of $x_l = 0.5$ and $x_u = 2.5$. Perform iterations until the approximate error falls below 1% or the number of iterations exceeds 10.

Report $x_l$, $x_u$, approximate root $c$ or $x_r$, $f(c)$, True Error $\epsilon_t(\%)$ and absolute error $\epsilon_a(\%)$ after each iterations in tabular format.

Also, generate a graph of absolute error vs the number of iterations to visualize the convergence of error.

# 6 Policy

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