General Two-Stage Stochastic Problem – L-Shaped Method

Consider the problem below.

$$Min\ c^T x + \sum_{\xi} p_\xi {q_\xi}^T y_\xi$$
$$s.t.\ \ Ax = b$$
$$T_\xi x + W_\xi y = h_\xi$$
$$x \geq 0, y \geq 0$$

x are the first stage decisions and y are the second stage decisions. For different scenarios ξ that occurs with probability p_ξ, scenario-dependent parameters change (all parameters with a ξ index). Here, we explained the L-Shaped method to find the optimal decision for x.

Step 0 $\theta \coloneqq -\infty,\ itr \coloneqq 1, w \coloneqq 0$

Step 1 If $itr = 1$:

$$Master\ problem =\ Min\ c^T x$$
$$s.t.\ \ Ax = b$$
$$x \geq 0$$

Else:

$$Master\ problem =\ Min\ c^T x + \theta$$
$$s.t.\ \ Ax = b$$
$$Ex + \theta \geq e$$
$$x \geq 0$$

E and e are explained in the next step.

Solve the master problem for x^* and go to step 2.

Step 2 solve the subproblem given the optimal x^* from step 1.

$$Min \sum_{\xi} p_\xi {q_\xi}^T y_\xi$$
$$s.t.\ \ T_\xi x^* + W_\xi y = h_\xi$$
$$y \geq 0$$

Considering that π^ξ are the dual solutions to the subproblem, calculate:

$E =\ \sum_\xi p_\xi \pi_\xi T_\xi$ Note that E would be matrix with dimensions 1*K

$e = \sum_\xi p_\xi \pi_\xi h_\xi$ Note that e would be a scalar

$w = e - Ex^*$

If $\theta \geq \boldsymbol{w}$ **stop**! You reached the optimal solution. Otherwise, itr+=1 and go back to step 1 by adding the optimality cut below to the master problem.

$Ex + \theta \geq e$ Note that here x means the variable, not the optimal value x^*.

An Example:

$$Min\ Z = 100x_1 + 150x_2 + \mathbb{E}_\xi[q_1y_1 + q_2y_2]$$

$$s.t.\ x_1 + x_2 \leq 120$$

$$6y_1 + 10y_2 \leq 60x_1$$

$$8y_1 + 5y_2 \leq 80x_1$$

$$y_1 \leq d_1\ , y_2 \leq d_2$$

$$x_1 \geq 40, x_2 \geq 20$$

$$y_1 \geq 0\ , y_2 \geq 0$$

For each scenario ξ there are separate d_1, d_2, q_1, q_2 as follows:

$$\xi\ = (d_1, d_2, q_1, q_2) = \begin{Bmatrix} (500,\ 100,\ -24,\ -28),\ pi_1 = 0.4 \\ (300,\ 300,\ -28,\ -32),\ pi_2 = 0.6 \end{Bmatrix}$$

$$c = [100{,}150]\quad W = \begin{bmatrix} 6 & 10 \\ 8 & 5 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, T = \begin{bmatrix} -60x_1 \\ -80x_1 \\ 0 \\ 0 \end{bmatrix}, h = \begin{bmatrix} 0 \\ 0 \\ d_1 \\ d_2 \end{bmatrix}$$