

## **Computer Graphics - Meshes**

To get started: Clone this repository and all its submodule dependencies using:

git clone --recursive https://github.com/alecjacobson/computer-graphicsmeshes.git

**Do not fork:** Clicking "Fork" will create a *public* repository. If you'd like to use GitHub while you work on your assignment, then mirror this repo as a new *private* repository: https://stackoverflow.com/questions/10065526/github-how-to-make-a-fork-of-public-repository-private

## **Background**

Read Section 12.1 of Fundamentals of Computer Graphics (4th Edition).

# Skim read Chapter 11 of Fundamentals of Computer Graphics (4th Edition).

There are many ways to store a triangle (or polygonal) mesh on the computer. The data-structures have very different complexities in terms of code, memory, and access performance. At the heart of these structures, is the problem of storing the two types of information defining a mesh: the *geometry* (where are points on the surface located in space) and the *connectivity* (which points are connected to each other). The connectivity is also sometimes referred to as the topology of the mesh.

The graphics pipeline works on a per-triangle and per-vertex basis. So the simplest way to store geometry is a 3D position  $\mathbf{v}_i \in \mathbf{R}^3$  for each i-th vertex of the mesh. And to store triangle connectivity as an ordered triplet of indices referencing vertices: i,j,k defines a triangle with corners at vertices  $\mathbf{v}_i$ ,  $\mathbf{v}_j$  and  $\mathbf{v}_k$ . Thus, the geometry is stored as a list of n 3D vectors: efficiently, we can put these vectors in the rows of a real-valued matrix  $\mathbf{V} \in \mathbf{R}^{n \times 3}$ . Likewise, the connectivity is stored as a list of m triplets: efficiently, we can put these triplets in the rows of an integer-valued matrix  $\mathbf{F} \in [0, n-1]^{m \times 3}$ .

Question: What if we want to store a (pure-)quad mesh?

#### **Texture Mapping**

Texture mapping is a process for mapping image information (e.g., colors) onto a surface (e.g., triangle mesh). The standard way to define a texture mapping is to augment the 3D geometric information of a mesh with additional 2D *parametrization* information: where do we find each point on the texture image plane? Typically, parameterization coordinates are bound to the unit square.

Mapping a 3D flat polygon to 2D is rather straightforward. The problem of finding a good mapping from a 3D surface to 2D becomes much harder if our surface is not flat (e.g., like a hemisphere), if the surface does not have exact one boundary (e.g., like a sphere) or if the surface has "holes" (e.g., like a torus/doughnut).

Curved surfaces must get *distorted* when flattened onto the plane. This is why Greenland looks bigger than Africa on a common map of the Earth.

The lack or presence of too many boundaries or the presence of "doughnut holes" in surfaces implies that we need to "cut" the surface to lay out it on the plane so all parts of the surface are "face up". Think about trying to flatten a deflated basketball on the ground.

#### **Normals**

For a smooth surface, knowing the surface geometry (i.e., position in space) near a point fully determines the normal vector at that point.

For a discrete mesh, the normal is only well-defined in the middle of planar faces (e.g., inside the triangles of a triangle mesh, but not along the edges or at vertices). Furthermore, if we use these normals for rendering, the surface will have a faceted appearance. This appearance is mathematically correct, but not necessarily desired if we wish to display a smooth looking surface.

Phong realized that linearly interpolating normals stored at the corners of each triangle leads to a smooth appearance.

This raises the question: what normals should we put at vertices or corners of our mesh?

For a faceted surface (e.g., a cube), all corners of a planar face f should share the face's normal  $\mathbf{n}_f \in \mathbf{R}^3$ .

For a smooth surface (e.g., a sphere), corners of triangles located at the same vertex should share the same normal vector. This way the rendering is continuous across the vertex. A common way to define per-vertex normals is to take a weighted average of normals from incident faces. Different weighting schemes are possible: uniform average (easy, but sensitive to irregular triangulations), angle-weighted (geometrically well motivated, but not robust near zero-area triangles), area-weighted (geometrically reasonable, well behaved). In this assignment, we'll compute area-weighted per-vertex normals:

$$\mathbf{n}_v = \frac{\sum_{f \in N(v)} a_f \mathbf{n}_f}{\left\| \sum_{f \in N(v)} a_f \mathbf{n}_f \right\|},$$

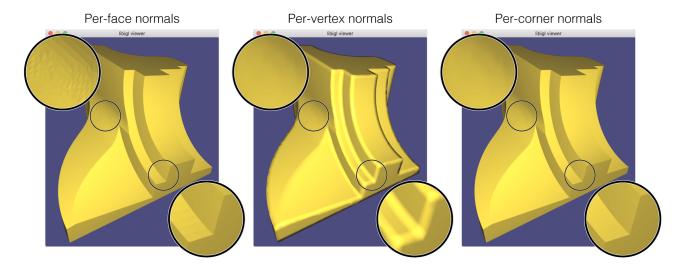
where N(v) is the set of faces neighboring the v-th vertex.

[per-vertex-normal]: images/per-vertex-normal.png height=300px ![Unique triangle normals (orange) are well-defined. We can define a notion of a normal for each vertex (purple) by taking a (weighted) average of normals from incident triangles.][per-vertex-normal]

For surfaces with a mixture of smooth-looking parts and creases, it is useful to define normals independently for each triangle corner (as opposed to each mesh vertex). For each corner, we'll again compute an area-weighted average of normals triangles incident on the shared vertex at this corner, but we'll ignore triangle's whose normal is too different from the corner's face's normal:

$$\mathbf{n}_{f,c} = \frac{\sum_{g \in N(v) \mid \mathbf{n}_g \cdot \mathbf{n}_f > \epsilon} a_g \mathbf{n}_g}{\left\| \left\| \sum_{g \in N(v) \mid \mathbf{n}_g \cdot \mathbf{n}_f > \epsilon} a_g \mathbf{n}_g \right\| \right\|},$$

where  $\epsilon$  is the minimum dot product between two face normals before we declare there is a crease between them.



#### .obj File Format

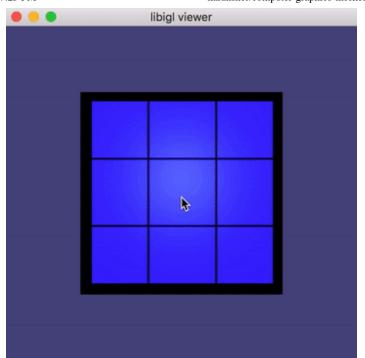
The .obj file format is a face-based representation of a mesh. The connectivity/topological data is stored implicitly by a list of a faces whose corners can *share* geometric information.

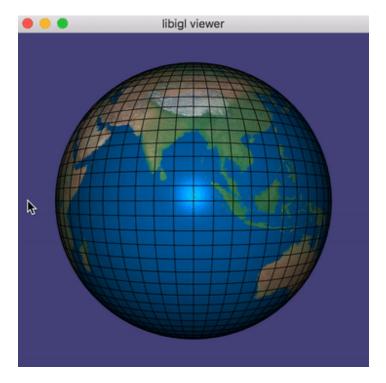
There are three main types of geometric information stored at vertices:

- 3D position information (let's call it v),
- 3D normal vector information (let's call it NV), and
- 2D parameterization information (let's call it UV).

Faces know where to find the position, normal and parameterization information for each corner by following a pointer/index. For a given corner of a given face, the index for position, normal or parameterization information may be different.

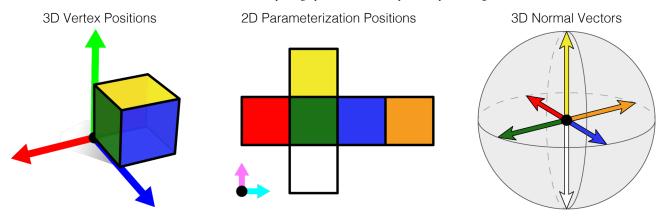
Warning: In C++ indices "start" at 0, but in a .obj file the first element in a list has index 1.





#### Cube example

In this way, the quadrilateral faces of a cube mesh may use 8 unique 3D vertex positions, giving the appearance of a connected surface when visualized in 3D. The same set of faces may use 14 unique 2D parameterization positions, giving the appearance of a cross with a boundary when visualized in 2D. Finally, that same set of faces may use only 6 unique 3D normal vectors.



#### **Eigen Matrices**

Every row of Eigen::MatrixXd can store a mesh vertex position using \_doubles.

Every row of Eigen::MatrixXi can store a list of indices into rows of position matrix using integers as *indices*.

Use .resize(num\_rows,num\_cols) to resize a matrix to be num\_rows by num\_cols. Use X = Eigen::MatrixXd::Zero(num\_rows,num\_cols) to resize and initialize a matrix with zeros.

#### **Subdivision Surfaces**

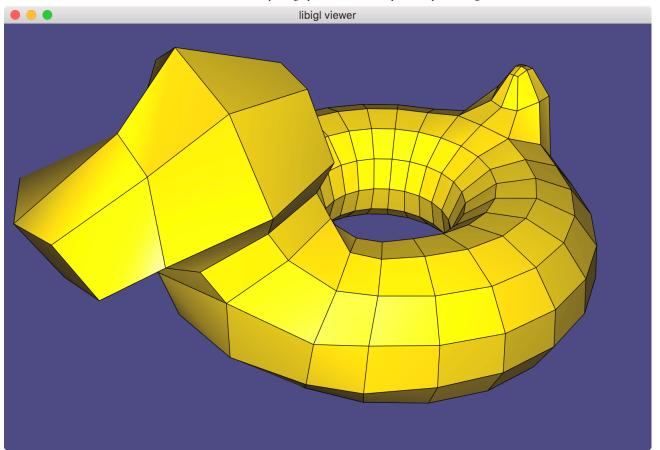
A subdivision surface is a natural generalization of a spline curve. A smooth spline can be defined as the limit of a recursive process applied to a polygon: each edge of the polygon is split with a new vertex and the vertices are smoothed toward eachother. If you've drawn smooth curves using Adobe Illustrator, PowerPoint or Inkscape, then you've used splines.

#### **≔** README.md

Then we smooth vertices toward each other.

The first and still (most) popular subdivision scheme was invented by Catmull (who went on to co-found Pixar) and Clark (founder of Silicon Graphics and Netscape). Catmull-Clark subdivision is defined for inputs meshes with arbitrary polygonal faces (triangles, quads, pentagons, etc.) but always produces a pure-quad mesh as output (i.e., all faces have 4 sides).

To keep things simple, in this assignment we'll assume the input is also a pure-guad mesh.



## **Mesh Viewers**

Mesh Lab is a free mesh-viewer used widely in computer graphics and computer vision research. *Warning*: Mesh Lab does not appear to respect user-provided normals in .obj files.

Autodesk Maya is a commercial 3D modeling and animation software. They often have free student versions.

## **Tasks**

#### White list

You're encouraged to use #include <Eigen/Geometry> to compute the cross product of two 3D vectors .cross .

#### **Black list**

This assignment uses libigl for mesh viewing. libigl has many mesh processing functions implemented in C++, including some of the functions assigned here. Do not copy or look at the following implementations:

igl::per\_vertex\_normals
igl::per\_face\_normals
igl::per\_corner\_normals

igl::double\_area

igl::vertex\_triangle\_adjacency

igl::writeOBJ

## src/write\_obj.cpp

Write a pure-triangle or pure-quad mesh with 3D vertex positions V and faces F, 2D parametrization positions UV and faces UF, 3D normal vectors NV and faces NF to a .obj file.

**Note:** These *two* function overloads represent only a small subset of meshes and mesh-data that can be written to a .obj file.

### src/cube.cpp

Construct the quad mesh of a cube including parameterization and per-face normals.

**Hint:** Draw out on paper and *label* with indices the 3D cube, the 2D parameterized cube, and the normals.

#### src/sphere.cpp

Construct a quad mesh of a sphere with <code>num\_faces\_u × num\_faces\_v faces</code>.

#### src/triangle\_area\_normal.cpp

Compute the normal vector of a 3D triangle given its corner locations. The output vector should have length equal to the area of the triangle.

#### src/per\_face\_normals.cpp

Compute per-face normals for a triangle mesh.

## src/per\_vertex\_normals.cpp

Compute per-vertex normals for a triangle mesh.

## src/vertex\_triangle\_adjacency.cpp

Compute a vertex-triangle adjacency list. For each vertex store a list of all incident faces.

## src/per\_corner\_normals.cpp

Compute per corner normals for a triangle mesh by computing the area-weighted average of normals at incident faces whose normals deviate less than the provided threshold.

## src/catmull\_clark.cpp

Conduct num\_iters iterations of Catmull-Clark subdivision on a pure quad mesh ( V , F ).

#### Releases

No releases published

#### **Packages**

No packages published

#### Languages