CPW resonators
Noise and Decoherence vs Spin Echo
Xmon cQED

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1. Coplanar waveguide resonators
   - Quality factors and effective parameters
   - S-parameters
   - Xmon design peculiarities
   - Simulations
2. Noise and decoherence versus Hahn echo technique
   - Quantum-mechanically treated dephasing
   - Classically treated dephasing
   - Hahn echo
   - Noise PSD
   - Who wins?
3. Xmon cQED
   - Circuit quantization
   - Eigenproblem for an isolated Xmon
   - Strong driving
   - Eigenproblem for a qubit-resonator system
   - Dispersive shifts

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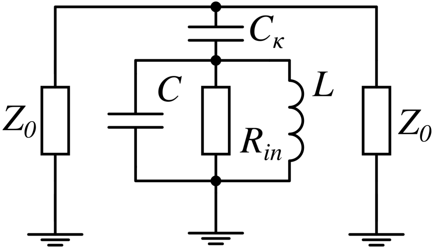
Capacitively coupled CPW resonator as a lumped-element model:



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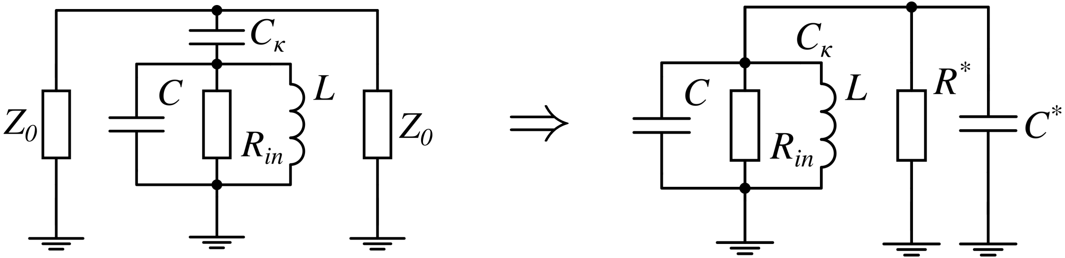

$$R^* = \frac{1+\omega^2 C_\kappa^2 (Z_0/2)^2}{\omega^2 C_\kappa^2 (Z_0/2)}, \quad C^* = \frac{C_\kappa}{1+\omega^2 C_\kappa^2 (Z_0/2)^2} \approx C_\kappa \text{ (for our case).}$$

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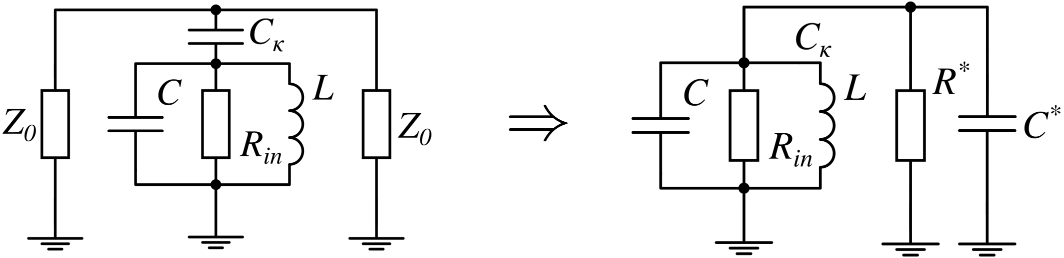
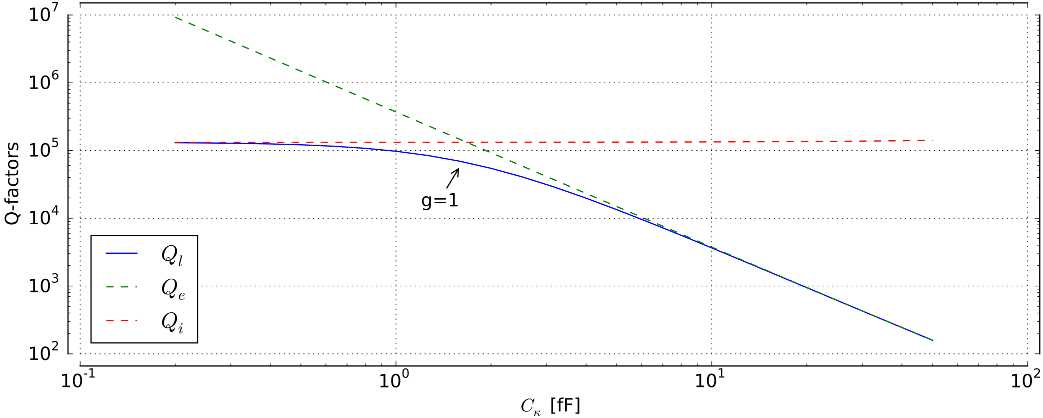
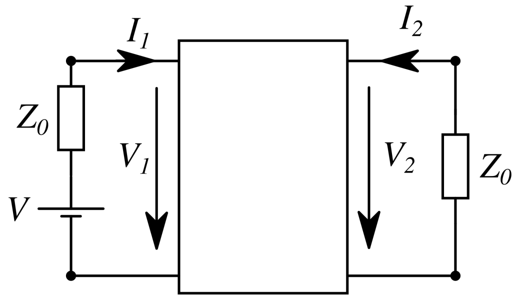


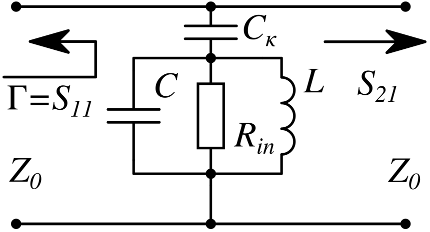
$$Q_i = \omega(C + C^*)R_{in}, \quad Q_e = \omega(C + C^*)R^*, \quad Q_l = \omega(C + C^*)\frac{1}{1/R^* + 1/R_{in}}.$$

Loaded, internal and external quality factors depending on C_κ:



General algorithm for calculating S-parameters of a given device:





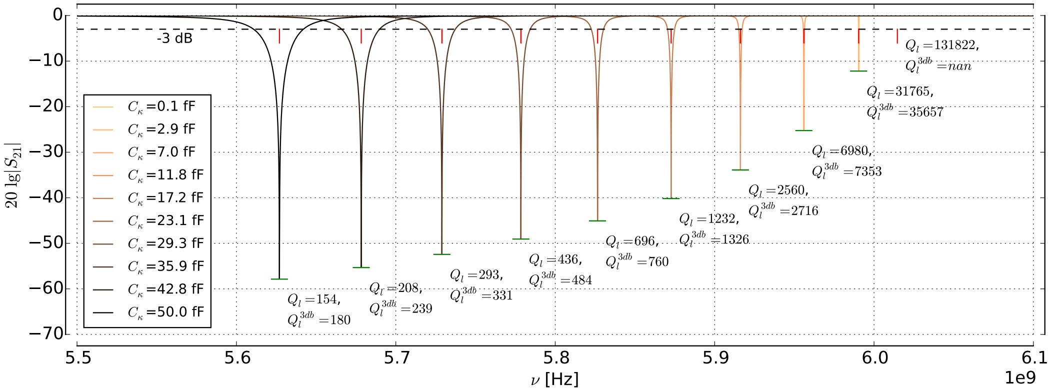
Kirchgoff's laws $\Rightarrow$

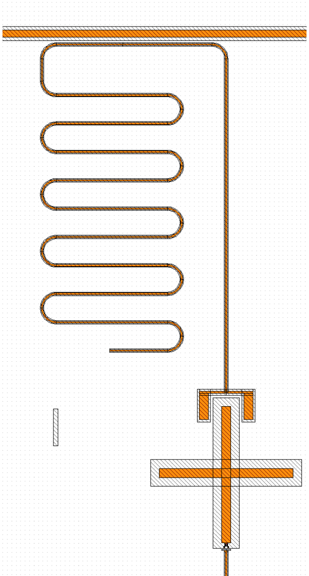
$$V_{1,2} = V_{1,2}^+ + V_{1,2}^-,$$

$$I_{1,2} = \frac{V_{1,2}^+ - V_{1,2}^-}{Z_0}$$

$$\Rightarrow \begin{pmatrix} V_1^- \\ V_2^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \end{pmatrix}.$$

Transmission spectra for the shunting resonator depending on C_κ:





$l = \lambda/4$ coplanar resonators, $W = 4\,\mu m,\ G = 2\,\mu m$.

Unconventional coupling area:

$$C_\kappa^{eff} = C_\kappa \cos\frac{\pi x_\kappa}{2l}\ ?$$

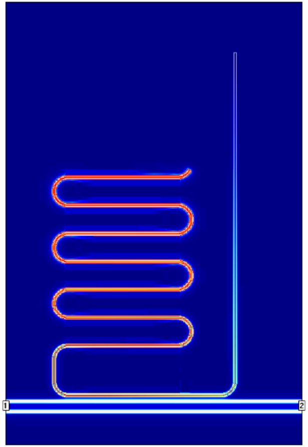
$$M_\kappa = ?$$

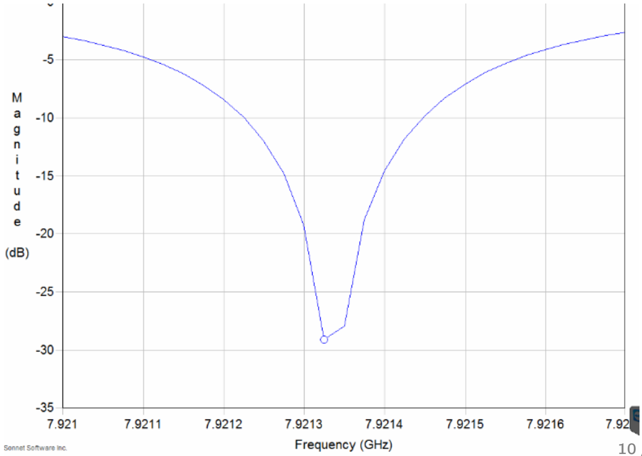
"Claw" coupler at the open end. Adds up some phase $\phi(\omega)$ and can be replaced by

$$\Delta l = \frac{\phi(\omega_r)c}{2\omega_r\sqrt{\varepsilon_{eff}}}$$

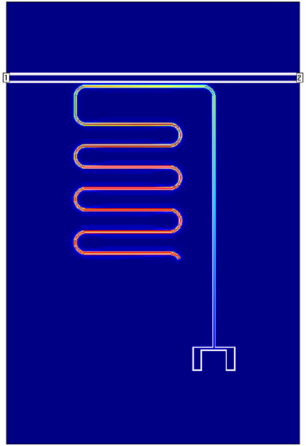
with high accuracy.

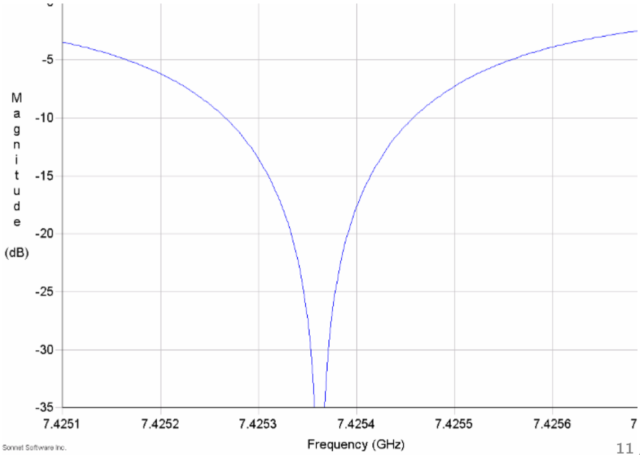
Resonator without a claw. Frequency expected from the length and $C_\kappa \approx 6$ fF (extracted from $Q_L \approx 10^4$) is 7.925 GHz.





Resonator with a claw. Frequency expected from the previous simulation and ϕ (also simulated separately for the given claw) is 7.4255 GHz.





System-bath interaction Hamiltonian:

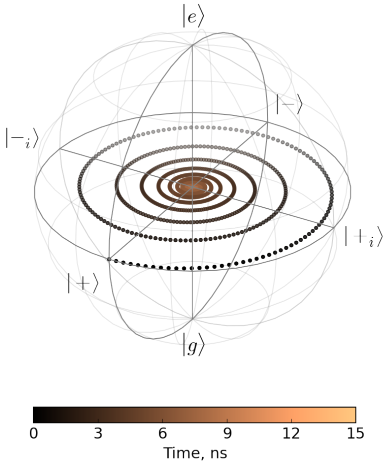
$$\hat{\mathcal{H}}_{sb} = \hat{\sigma}_z \otimes \hat{O}_b,$$

where $\hat{O}_b$ is an arbitrary bath operator. Master equation is then developed:

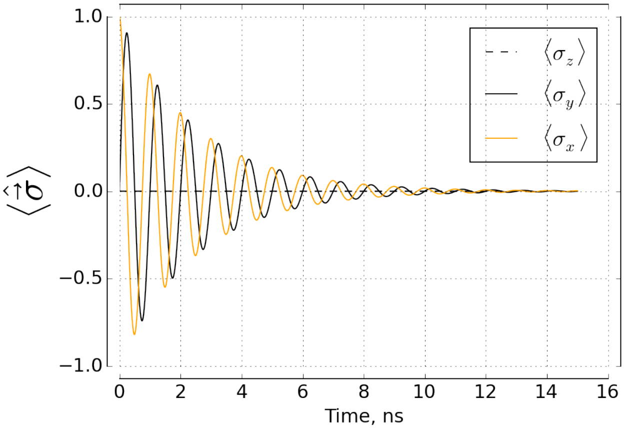
$$\partial_t \hat{\rho}_s = \frac{i}{\hbar}[\hat{\rho}_s, \hat{\mathcal{H}}_s] + \gamma_\phi(\hat{\sigma}_z \hat{\rho}_s \hat{\sigma}_z - \hat{\rho}_s).$$

Steady state is a maximally mixed state:

$$\hat{\rho}_s(\infty) = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}.$$



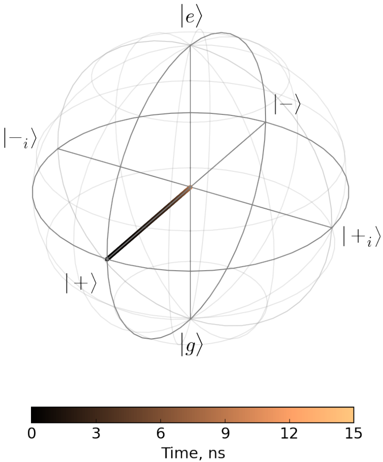
$|e\rangle$
$|-\rangle$
$|-_i\rangle$
$|+_i\rangle$
$|+\rangle$
$|g\rangle$
0
3
6
9
12
15
Time, ns

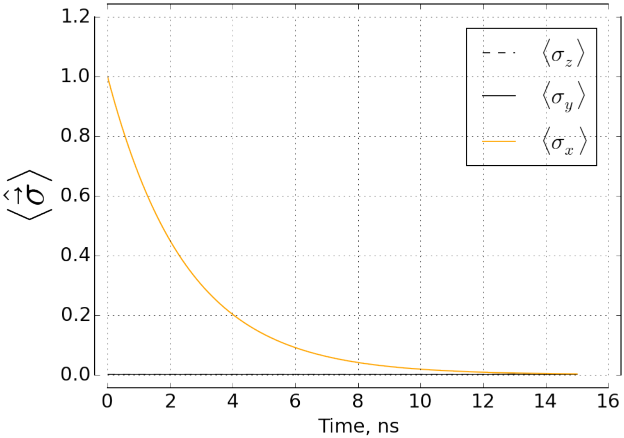


$\langle\vec{\sigma}\rangle$
1.0
0.5
0.0
−0.5
−1.0
0
2
4
6
8
10
12
14
16
Time, ns
$\langle\sigma_z\rangle$
$\langle\sigma_y\rangle$
$\langle\sigma_x\rangle$

Quantum-mechanically treated dephasing

Noise and decoherence versus Hahn echo technique





Unitary evolution under $f(t)\hat{\sigma}_z$, where $f(t)$ is random. In the rotating frame:

$$\hat{\rho}_s(t) = \hat{U}^\dagger(t,0)\ \hat{\rho}\ \hat{U}(t,0), \quad U(t,0) = \hat{T}\exp\left\{-\frac{i}{\hbar}\int_0^t f(\tau)\hat{\sigma}_z\,\mathrm{d}\tau\right\}.$$

$$\hat{\rho}_s(t) = \begin{pmatrix} 1/2 - N(t)/2 & \rho(t) \\ \rho^*(t) & 1/2 + N(t)/2 \end{pmatrix},$$

$$N(t) = N(0),$$

$$\rho(t) = \rho(0)\exp\left\{\frac{2i}{\hbar}\int_0^t f(\tau)\,\mathrm{d}\tau\right\}.$$

Presuming Gaussian instantaneous distribution of $f(t)$ and $x(t) = \frac{2}{\hbar}\int_0^t f(\tau)\,\mathrm{d}\tau$:

$$\langle \rho(t) \rangle = \rho(0) \exp\left\{ -\frac{2}{\hbar^2} \int_0^t \int_0^t \langle f(\tau_1) f(\tau_2) \rangle \,\mathrm{d}\tau_1 \,\mathrm{d}\tau_2 \right\}.$$

Presuming also that $f(t)$ is a wide-sense stationary process:

$$\langle f(\tau_1) f(\tau_2) \rangle = K(\tau_2 - \tau_1) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} S(\omega) e^{i\omega(\tau_2 - \tau_1)} \,\mathrm{d}\omega.$$

Taking the time integrals:

$$\langle \rho(t) \rangle = \rho(0) \exp\left\{ -\frac{2}{\sqrt{2\pi}\hbar^2} \int_{\mathbb{R}} \frac{4\sin^2(\frac{\omega t}{2})}{\omega^2} S(\omega) \,\mathrm{d}\omega \right\}.$$

It can be shown that for Hahn echo sequence the accumulated phase expression reads

$$\rho(t) = \rho(0) \exp\left\{ -\frac{2i}{\hbar} \int_0^{t/2} f(\tau)\,\mathrm{d}\tau + \frac{2i}{\hbar} \int_{t/2}^{t} f(\tau)\,\mathrm{d}\tau \right\}.$$

And after averaging as in the previous slides:

$$\langle \rho(t) \rangle = \rho(0) \exp\left\{ -\frac{2}{\sqrt{2\pi}\hbar^2} \int_{\mathbb{R}} S(\omega) W_t(\omega) \right\},$$

where

$$W_t(\omega) = \tan^2(\omega t/4) \frac{4\sin^2(\omega t/2)}{\omega^2}.$$

Noise PSD

Noise and decoherence versus Hahn echo technique

Who wins?

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