

CPW resonators
Noise and Decoherence vs Spin Echo
Xmon cQED

1 Coplanar waveguide resonators

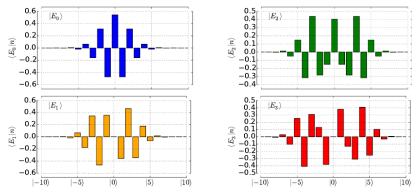
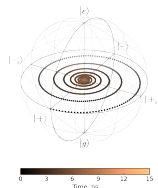
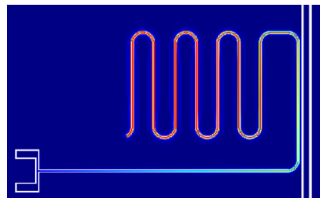
- Quality factors and effective parameters
- S-parameters
- Xmon design features
- Sonnet simulations

2 Noise and decoherence versus Hahn echo technique

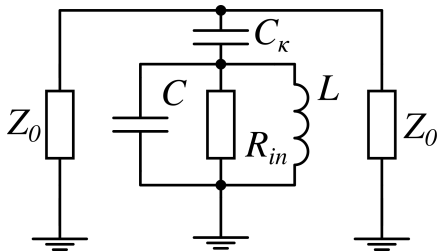
- Quantum-mechanically treated pure dephasing (Ramsey)
- Classically treated pure dephasing (Ramsey)
- Hahn echo
- Noise PSD filtration
- Who wins?

3 Xmon cQED (QuTiP simulations)

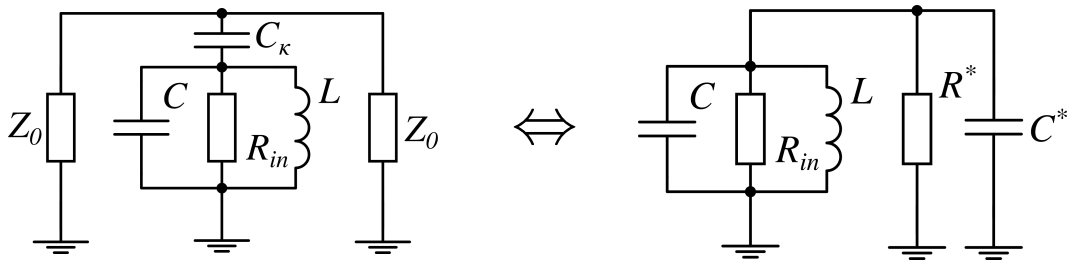
- Circuit quantization
- Cooper pair distribution for an isolated Xmon
- Eigenproblem for a qubit-resonator system
- Dispersive shifts



Capacitively coupled CPW resonator as a lumped-element model:

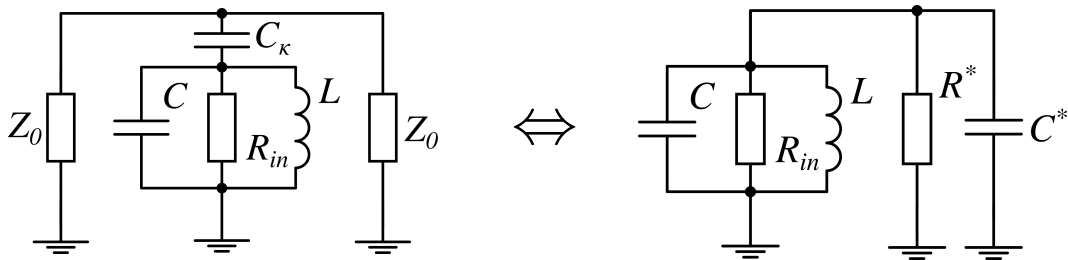


Capacitively coupled CPW resonator as a lumped-element model:



$$R^* = \frac{1 + \omega^2 C_\kappa^2 (Z_0/2)^2}{\omega^2 C_\kappa^2 (Z_0/2)}, \quad C^* = \frac{C_\kappa}{1 + \omega^2 C_\kappa^2 (Z_0/2)^2} \approx C_\kappa \text{ (for our case).}$$

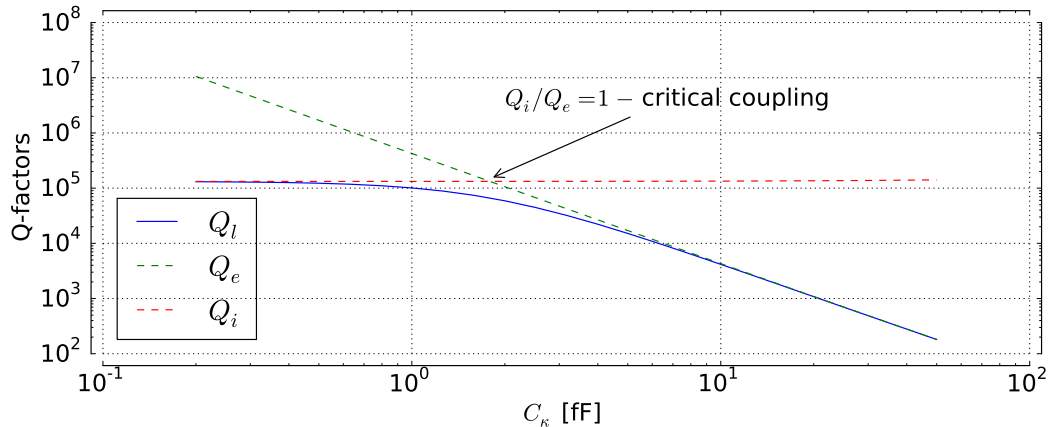
Capacitively coupled CPW resonator as a lumped-element model:



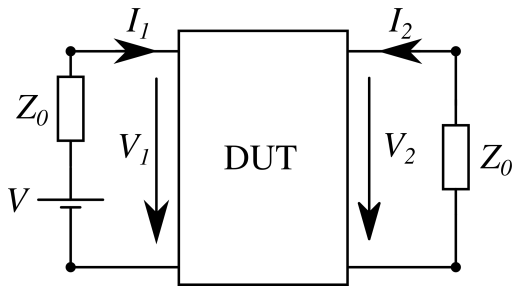
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$$Q_i = \omega(C + C^*)R_{in}, \quad Q_e = \omega(C + C^*)R^*, \quad Q_l = \omega(C + C^*) \frac{1}{1/R^* + 1/R_{in}}.$$

Q-factors depending on C_κ :



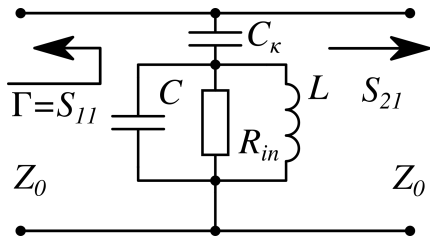
General algorithm for calculating S-parameters of a given device:



$$V_{1,2} = V_{1,2}^+ + V_{1,2}^-$$

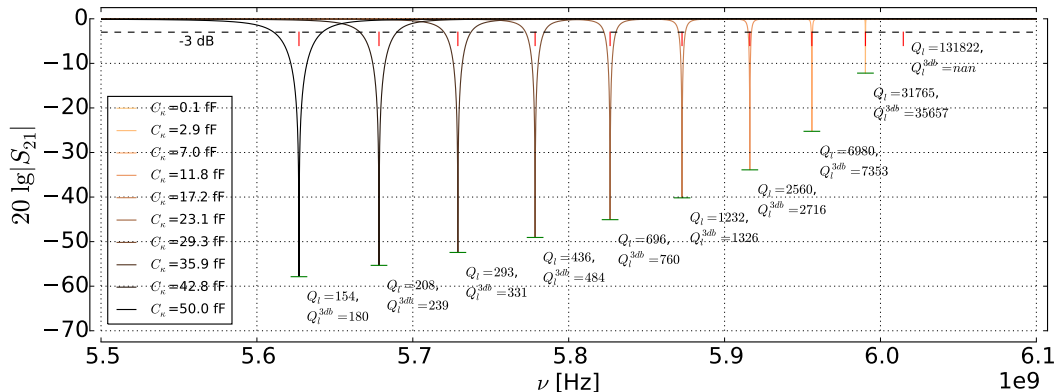
Kirchhoff's laws \Rightarrow

$$I_{1,2} = \frac{V_{1,2}^+ - V_{1,2}^-}{Z_0}$$

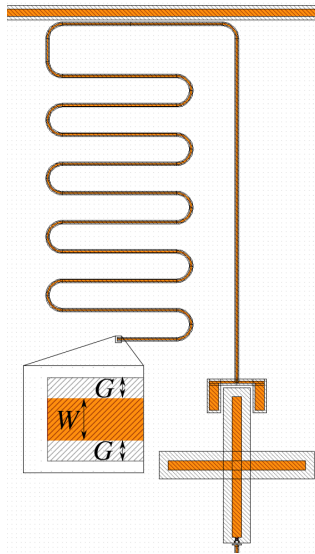


$$\begin{pmatrix} V_1^- \\ V_2^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \end{pmatrix}.$$

Analytical $S_{21}(C_\kappa, \omega) = (1 + Z_0/2Z_{sh})^{-1}$, $Z_{sh} = 1/i\omega C_\kappa + Z_{res}$:



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$l = \lambda/4$ coplanar resonators, $W = 4 \mu m$, $G = 2 \mu m$.

Unconventional coupling to the feedline:

$$C_{\kappa}^{eff} = C_{\kappa} \cos \frac{\pi x_{\kappa}}{2l} \quad ? \quad [1]$$

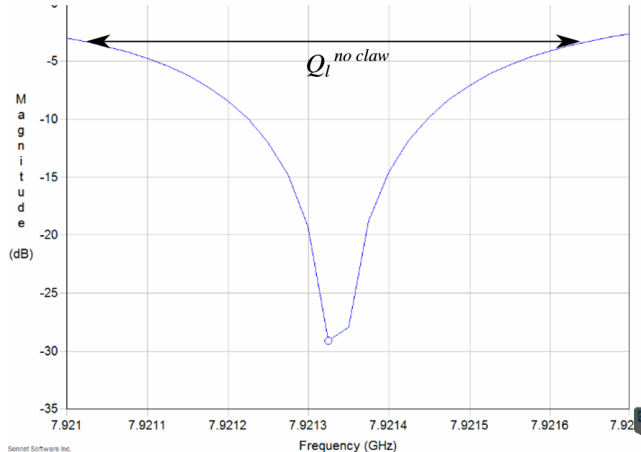
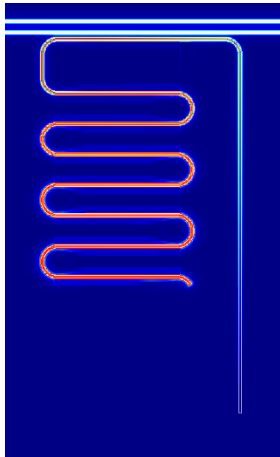
$$M_{\kappa} = ?$$

“Claw” coupler at the open end. Adds up some phase $\phi(\omega)$ and can be replaced [1] with high accuracy by

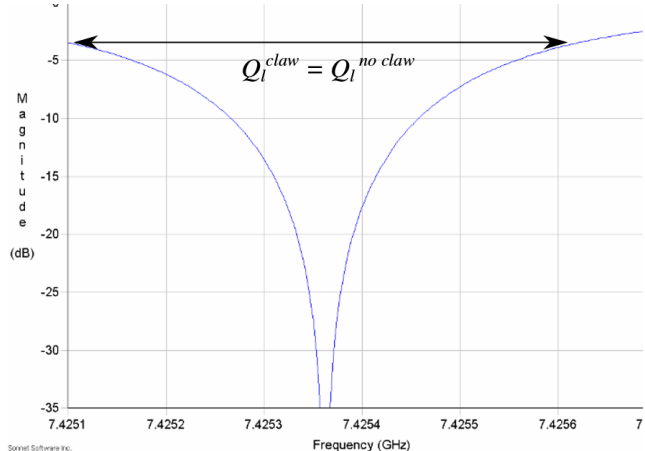
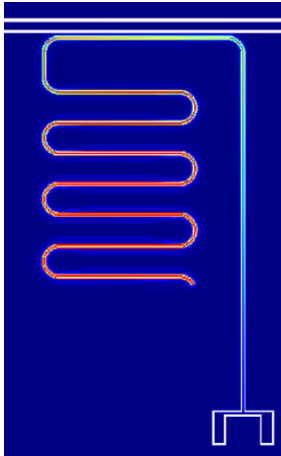
$$\Delta l = \frac{\phi(\omega_r)c}{2\omega_r\sqrt{\epsilon_{eff}}}$$

[1] D. Sank, PhD Thesis, 2014 (J. Martinis group)

Resonator without a claw. Frequency expected from the length and $C_\kappa \approx 6$ fF (extracted from $Q_l \approx 10^4$) is 7.925 GHz.



Resonator with a claw. Frequency expected from the previous simulation and ϕ (also simulated separately for the given claw) is 7.4255 GHz.



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Qubit-environment interaction Hamiltonian:

$$\hat{\mathcal{H}}_{qe} = \hat{O}_e \otimes \hat{\sigma}_z.$$

\hat{O}_e is an arbitrary environment operator, $\hat{\sigma}_z$ – qubit observable.

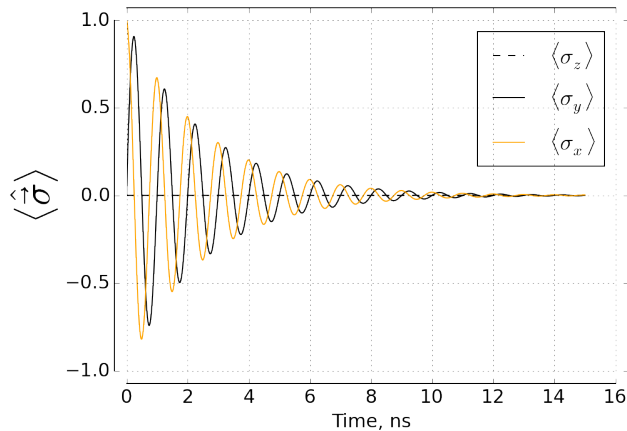
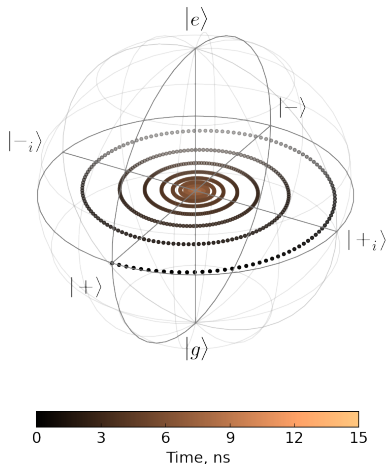
Master equation:

$$\partial_t \hat{\rho}_q = \frac{i}{\hbar} [\hat{\rho}_q, \hat{\mathcal{H}}_q] + \gamma_\phi (\hat{\sigma}_z \hat{\rho}_q \hat{\sigma}_z - \hat{\rho}_q).$$

Steady state:

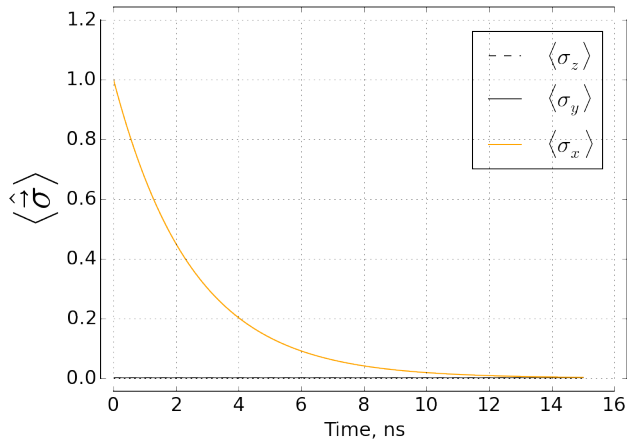
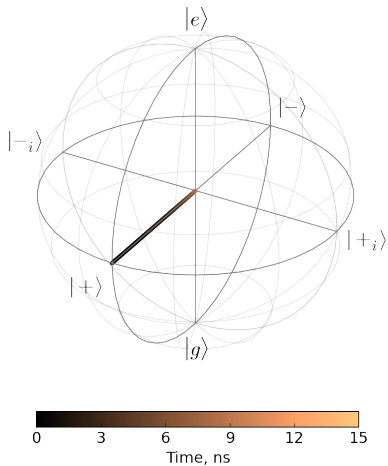
$$\hat{\rho}_q(\infty) = \begin{pmatrix} \rho_{11}(0) & 0 \\ 0 & \rho_{22}(0) \end{pmatrix}.$$

Example: pure dephasing of the $|+\rangle$ state, **lab frame**.



(dynamics simulated with QuTiP)

Example: pure dephasing of the $|+\rangle$ state, **rotating frame**.



(dynamics simulated with QuTiP)

$$\hat{O}_e \otimes \hat{\sigma}_z \rightarrow f(t) \cdot \hat{\sigma}_z, \quad f(t) \text{ is random}$$

Rotating frame evolution (**single run!**):

$$\hat{\rho}_q(t) = \hat{U}^\dagger(t, 0) \hat{\rho}_q(0) \hat{U}(t, 0), \quad \hat{U}(t, 0) = \hat{T} \exp \left\{ -\frac{i}{\hbar} \int_0^t f(\tau) d\tau \hat{\sigma}_z \right\}.$$

$$\hat{\rho}_q(t) = \begin{pmatrix} 1/2 - N(t)/2 & \rho(t) \\ \rho^*(t) & 1/2 + N(t)/2 \end{pmatrix},$$

$$\left. \begin{aligned} N(t) &= N(0), \\ \rho(t) &= \rho(0) \exp \left\{ \frac{2i}{\hbar} \int_0^t f(\tau) d\tau \right\} \end{aligned} \right\} \Rightarrow \text{unitary random walk, no dephasing!}$$

Averaging. For Gaussian instantaneous distribution of $f(t)$:

$$\langle \rho(t) \rangle = \rho(0) \exp \left\{ -\frac{2}{\hbar^2} \int_0^t \int_0^t \langle f(\tau_1) f(\tau_2) \rangle d\tau_1 d\tau_2 \right\}.$$

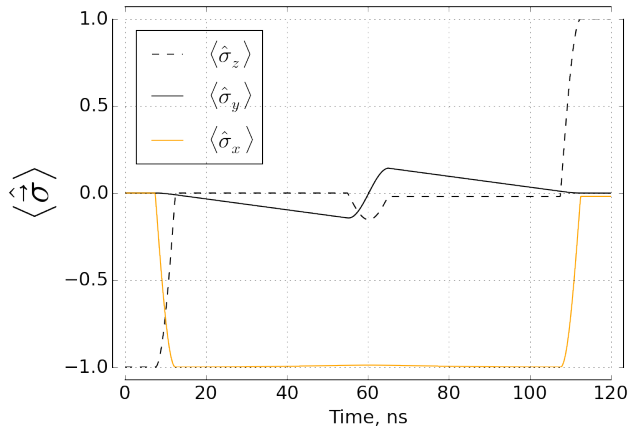
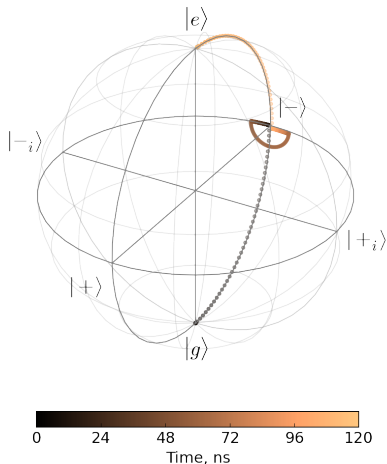
Presuming also that $f(t)$ is a wide-sense stationary process:

$$\langle f(\tau_1) f(\tau_2) \rangle = K_f(\tau_2 - \tau_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} S_f(\omega) e^{i\omega(\tau_2 - \tau_1)} d\omega.$$

$$\langle \rho(t) \rangle = \rho(0) \exp \left\{ -\frac{2}{\sqrt{2\pi}\hbar^2} \int_{-\infty}^{\infty} \frac{4 \sin^2(\frac{\omega t}{2})}{\omega^2} S_f(\omega) d\omega \right\} \equiv \rho(0) e^{-\alpha(t)}$$

\Rightarrow ensemble averaged behaviour of $\rho(t)$ is a decay!

$S_f(\omega) = \delta(\omega)$ ($f(t) = \text{const}$ on the timescale of a single run):



(dynamics simulated with QuTiP)

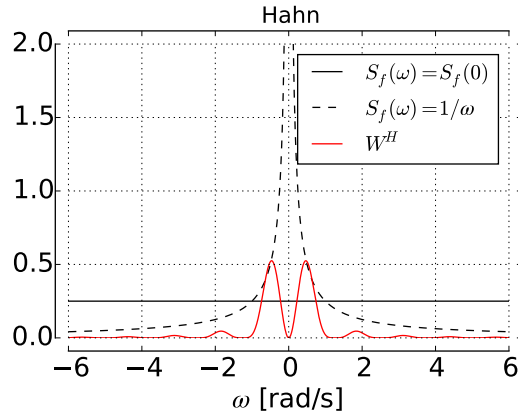
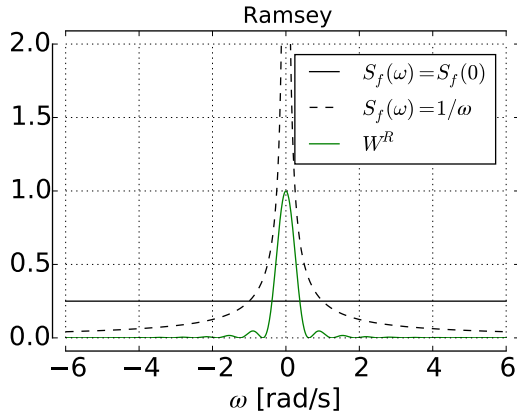
Add one instant π -pulse. Accumulated phase:

$$\rho(t) = \rho(0) \exp \left\{ \underbrace{-\frac{2i}{\hbar} \int_0^{t/2} f(\tau) d\tau}_{\text{before } \pi\text{-pulse}} + \underbrace{\frac{2i}{\hbar} \int_{t/2}^t f(\tau) d\tau}_{\text{after } \pi\text{-pulse}} \right\}.$$

After averaging:

$$\langle \rho(t) \rangle = \rho(0) \exp \left\{ -\frac{2}{\sqrt{2\pi}\hbar^2} \int_{-\infty}^{\infty} S_f(\omega) W_t^H(\omega) d\omega \right\},$$

$$W_t^H(\omega) = \tan^2(\omega t/4) \frac{4 \sin^2(\omega t/2)}{\omega^2}.$$



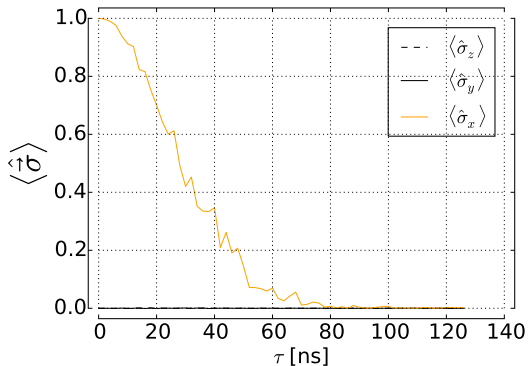
■ Ramsey: $\int \frac{4 \sin^2(\frac{\omega t}{2})}{\omega^2} S_f(\omega) d\omega$

■ Hahn: $\int \tan^2(\omega t/4) \frac{4 \sin^2(\frac{\omega t}{2})}{\omega^2} S_f(\omega) d\omega$

Who wins?

Noise and decoherence versus Hahn echo technique

$S_f(\omega) = 1/\omega$ (pink noise):

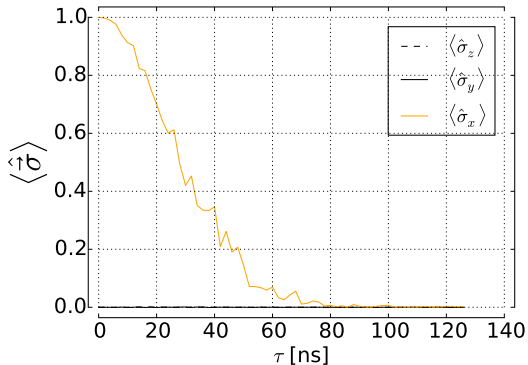


No echo

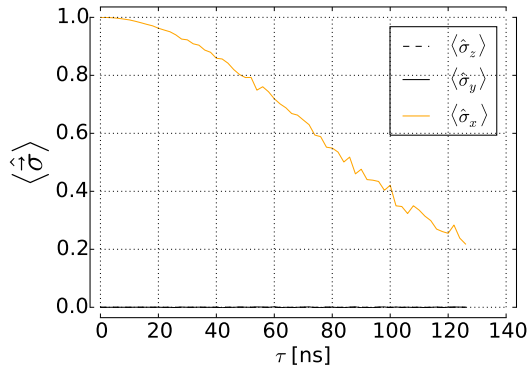
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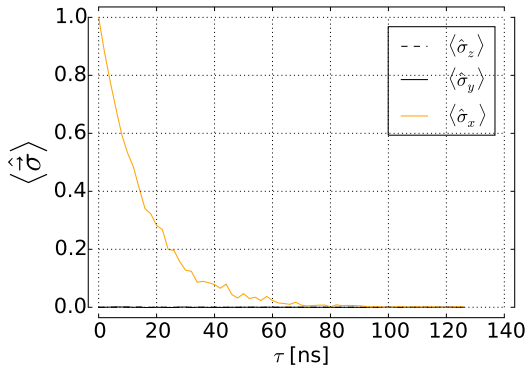
Echo

■ efficient!

Who wins?

Noise and decoherence versus Hahn echo technique

$S_f(\omega) = S_f(0) = \text{const}$ (white noise):

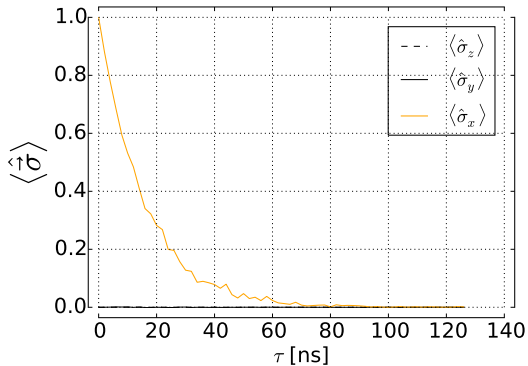


No echo

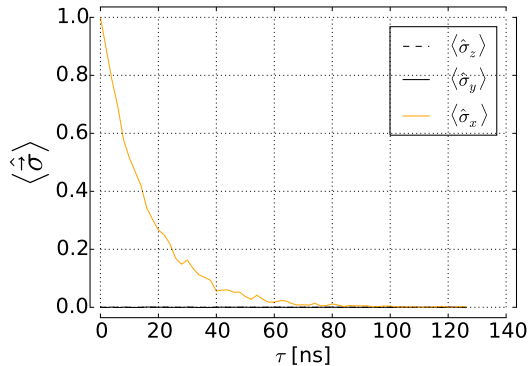
Who wins?

Noise and decoherence versus Hahn echo technique

$S_f(\omega) = S_f(0) = \text{const}$ (white noise):



No echo



Echo

- **totally inefficient**, same for the master equation description

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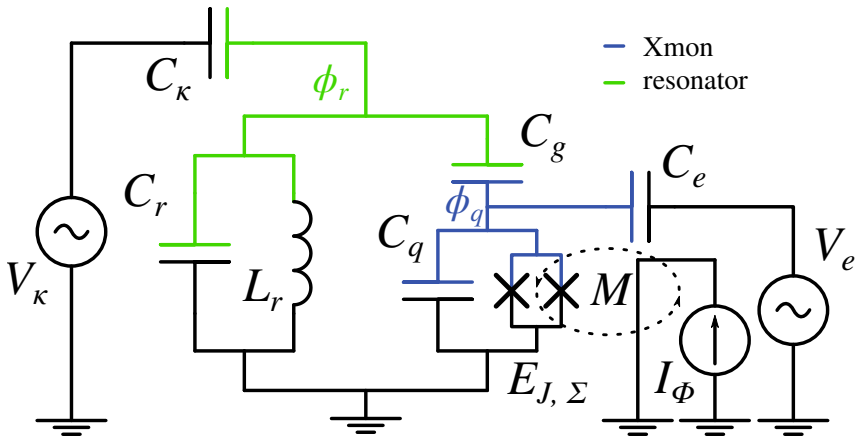
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Equivalent circuit for the transmon-resonator system:



$$\hat{\mathcal{H}} = \underbrace{\frac{\hat{\phi}_r^2}{2L_r} + \frac{(C_q + C_g)\hat{Q}_r^2}{2C_*^2}}_{\text{resonator}} + \underbrace{\frac{(C_g + C_\kappa + C_r)\hat{Q}_q^2}{2C_*^2} - E_J(\Phi_{ext}) \cos \frac{2e}{\hbar} \hat{\phi}_q}_{\text{qubit}} + \underbrace{\frac{C_g \hat{Q}_r \hat{Q}_q}{C_*^2}}_{\text{coupling}} =$$

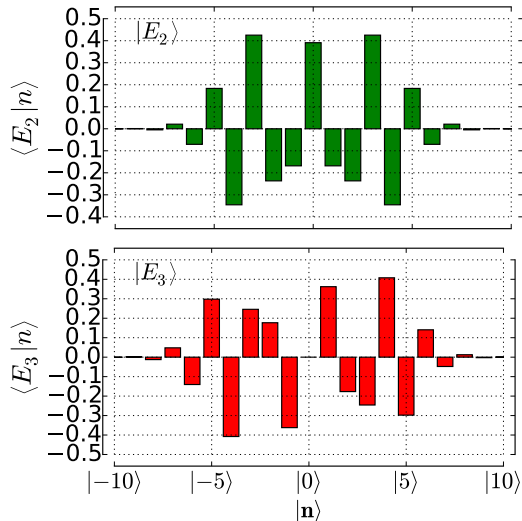
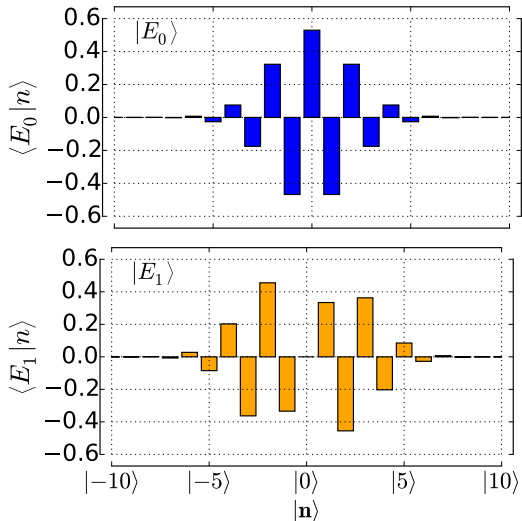
$$= \hbar\omega_r \hat{a}^\dagger \hat{a} \otimes \hat{\mathbb{1}}_q \quad (\hat{\mathcal{H}}_r)$$

$$+ 4E_C \hat{\mathbb{1}}_r \otimes \hat{n}^2 - \frac{E_J(\Phi_{ext})}{2} \hat{\mathbb{1}}_r \otimes \sum_{n=-\infty}^{+\infty} |n+1\rangle\langle n| + |n\rangle\langle n+1| \quad (\hat{\mathcal{H}}_q)$$

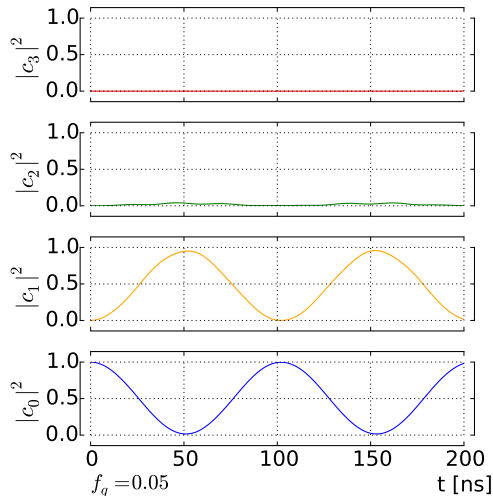
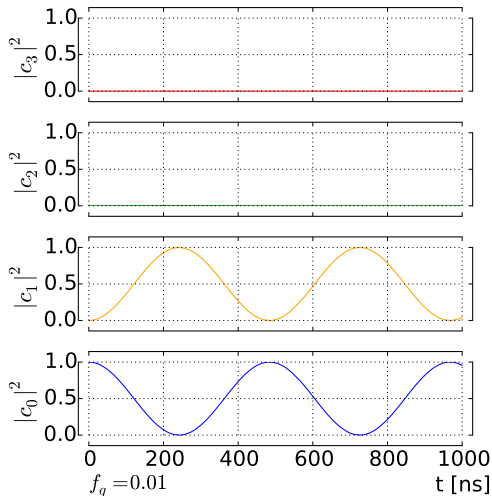
$$- 2e \frac{C_g}{C_*} \sqrt{\frac{\hbar\omega_r}{2(C_q + C_g)}} i(\hat{a}^\dagger - \hat{a}) \otimes \hat{n}, \quad (\hat{\mathcal{H}}_i)$$

Cooper pair distribution for an isolated Xmon

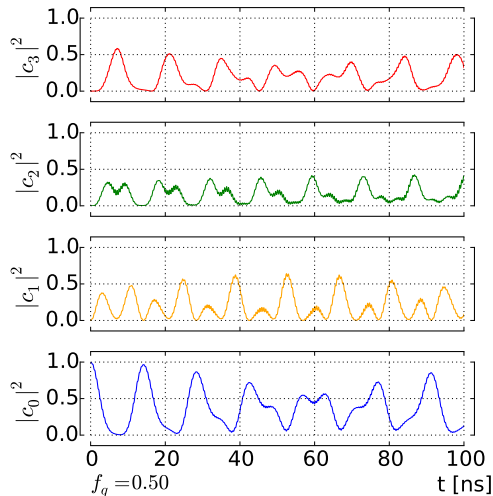
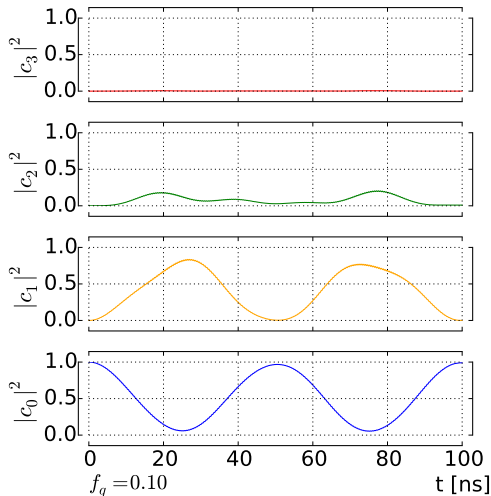
Xmon cQED (QuTiP simulations)



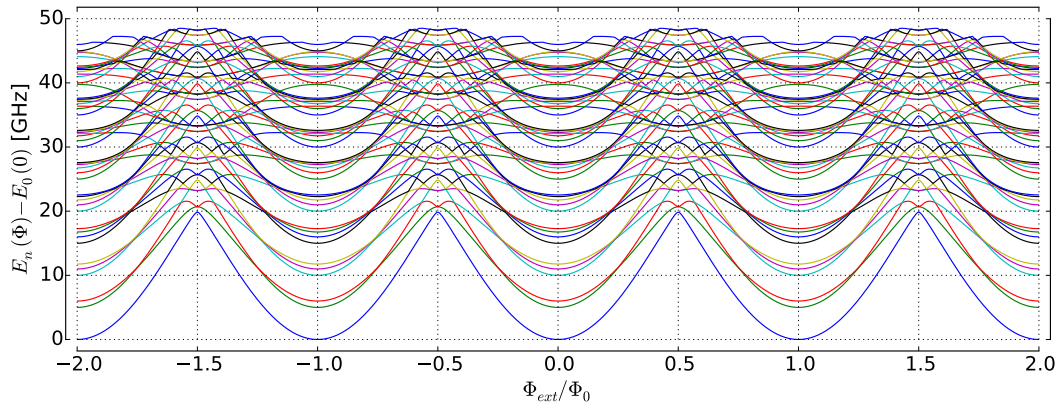
For weak driving the two-level dynamics is preserved ($|\psi(t)\rangle = \sum_k c_k(t)|E_k\rangle$):



But not for the strong drive ($|\psi(t)\rangle = \sum_k c_k(t)|E_k\rangle$):



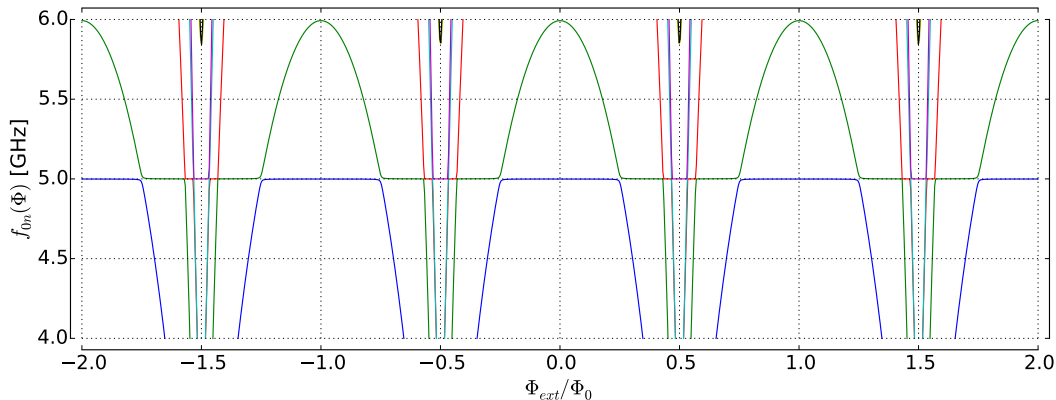
Energy level structure:



Eigenproblem for a qubit-resonator system

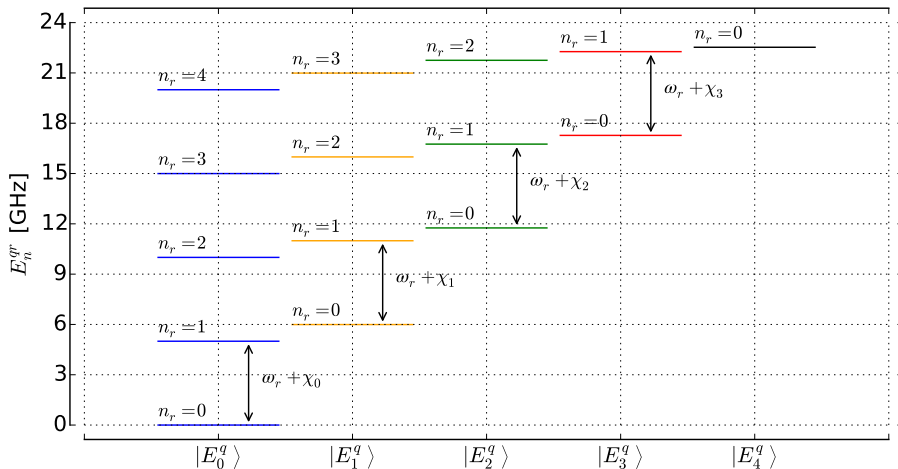
Xmon cQED (QuTiP simulations)

Frequencies $f_{0n} = (E_n - E_0)/h$ for $\omega_q > \omega_r$:



The Grotrian diagram:

Dispersive shifts originate from the level structure of the whole composite system:



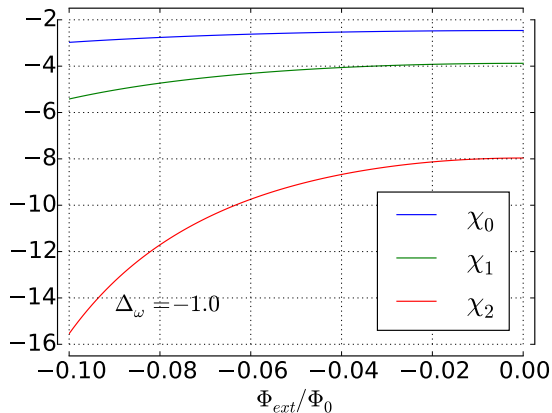
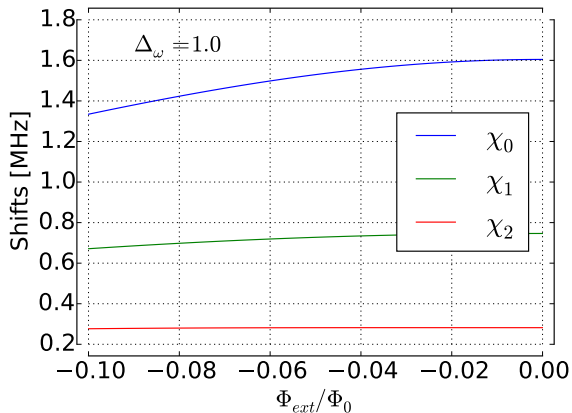
It's possible to determine these shifts with high accuracy using the second order perturbation theory:

$$\chi_0 = g^2 \left[\frac{n_{ge}^2}{\omega_r - \omega_{ge}} - \frac{n_{ge}^2}{\omega_r + \omega_{ge}} \right],$$

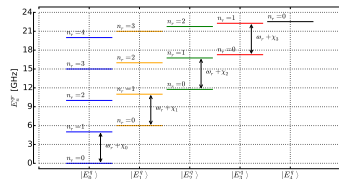
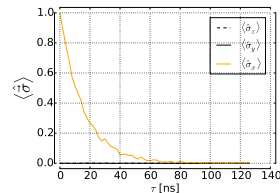
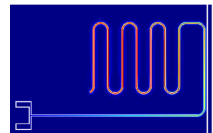
$$\chi_1 = g^2 \left[\frac{n_{ef}^2}{\omega_r - \omega_{ef}} - \frac{n_{ef}^2}{\omega_r + \omega_{ef}} + \frac{n_{eg}^2}{\omega_r + \omega_{ge}} - \frac{n_{eg}^2}{\omega_r - \omega_{ge}} \right],$$

$$\chi_2 = g^2 \left[\frac{n_{fd}^2}{\omega_r - \omega_{fd}} - \frac{n_{fd}^2}{\omega_r + \omega_{fd}} + \frac{n_{fe}^2}{\omega_r + \omega_{ef}} - \frac{n_{fe}^2}{\omega_r - \omega_{ef}} \right].$$

The shifts depend on the detuning $\Delta_\omega = \omega_r - \omega_{ge}$ and Φ_{ext} :



- Quality factors and coupling capacitance calculations, simulations of CPW resonators and extraction of capacitances using Sonnet
- Investigation of classical and quantum noise influence on the qubit, numerical simulations of the pure dephasing under different types of noise
- Quantization of Xmon-resonator circuit to transform capacitances and quality factors to the coupling strengths, driving amplitudes and decay rates



- A. V. Ustinov
- J. Lisenfeld and A. Bilmes

Acknowledgements

Final words



Thank you!