CPW resonators
Noise and Decoherence vs Spin Echo
Xmon cQED

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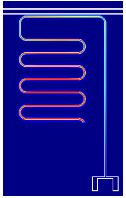
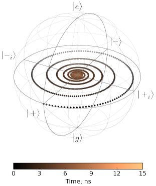
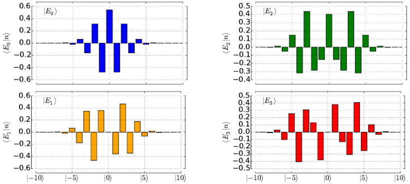
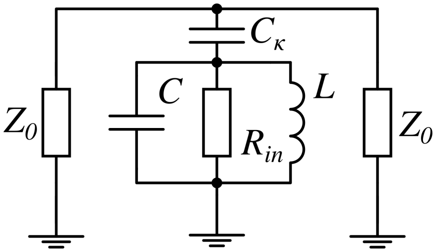






Capacitively coupled CPW resonator as a lumped-element model:



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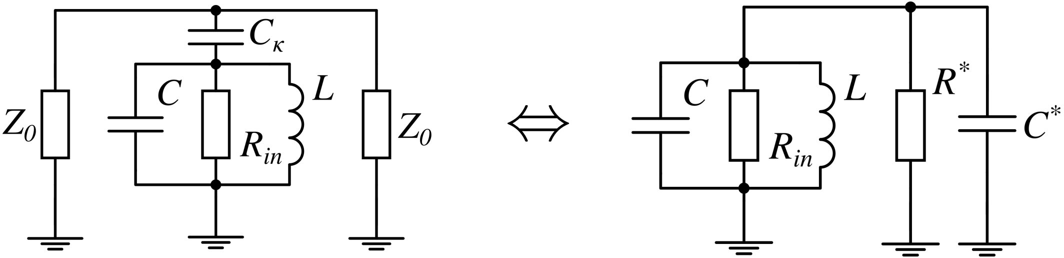
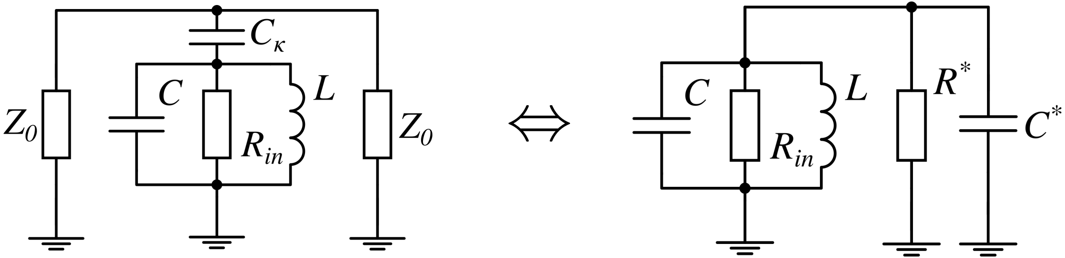


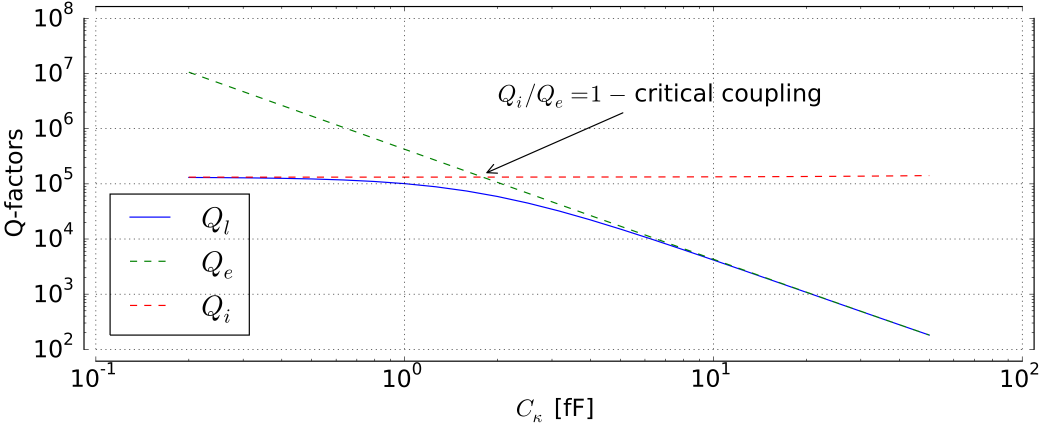
$$R^* = \frac{1+\omega^2 C_\kappa^2 (Z_0/2)^2}{\omega^2 C_\kappa^2 (Z_0/2)}, \quad C^* = \frac{C_\kappa}{1+\omega^2 C_\kappa^2 (Z_0/2)^2} \approx C_\kappa \text{ (for our case).}$$

Capacitively coupled CPW resonator as a lumped-element model:

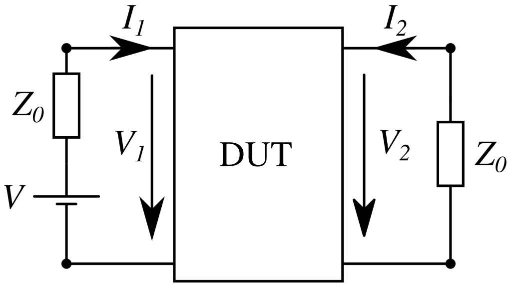


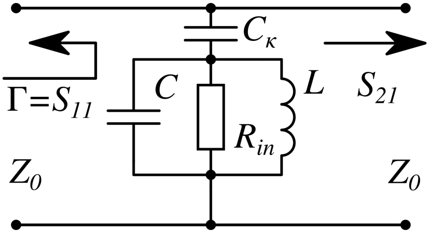
$$Q_i = \omega(C + C^*)R_{in}, \quad Q_e = \omega(C + C^*)R^*, \quad Q_l = \omega(C + C^*)\frac{1}{1/R^* + 1/R_{in}}.$$

Q-factors depending on C_κ:



General algorithm for calculating S-parameters of a given device:





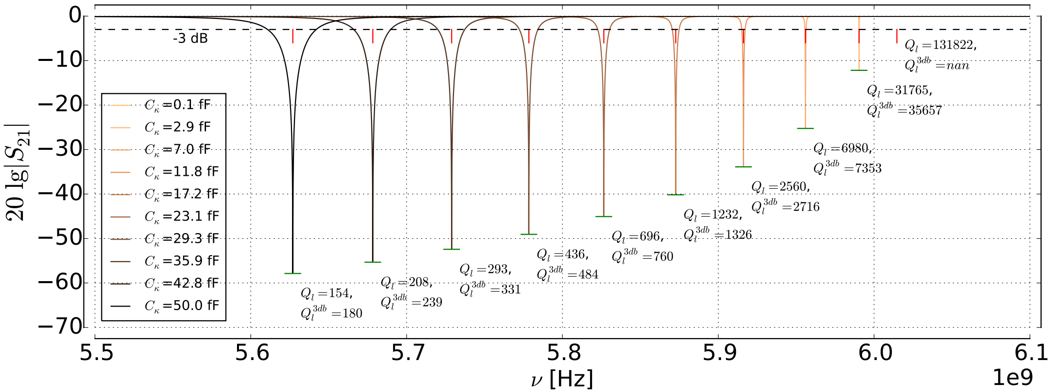
Kirchgoff's laws $\Rightarrow$

$$V_{1,2} = V_{1,2}^{+} + V_{1,2}^{-},$$

$$I_{1,2} = \frac{V_{1,2}^{+} - V_{1,2}^{-}}{Z_0}$$

$$\Rightarrow \quad \begin{pmatrix} V_1^- \\ V_2^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \end{pmatrix}.$$

Analytical $S_{21}(C_\kappa, \omega) = (1 + Z_0/2Z_{sh})^{-1}, \; Z_{sh} = 1/i\omega C_\kappa + Z_{res}$:



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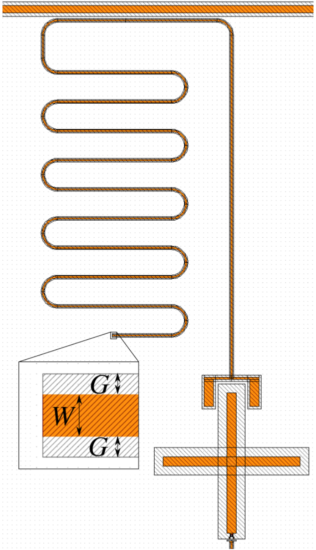


$l = \lambda/4$ coplanar resonators, $W = 4\,\mu m,\ G = 2\,\mu m$.

Unconventional coupling to the feedline:

$$C_\kappa^{eff} = C_\kappa \cos\frac{\pi x_\kappa}{2l}\ ?\ [1]$$

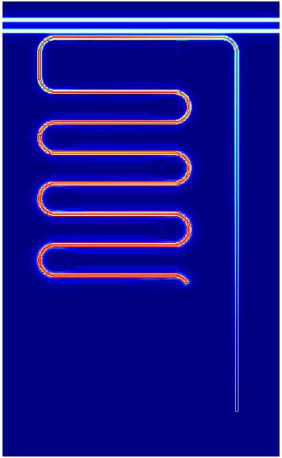
$$M_\kappa = ?$$

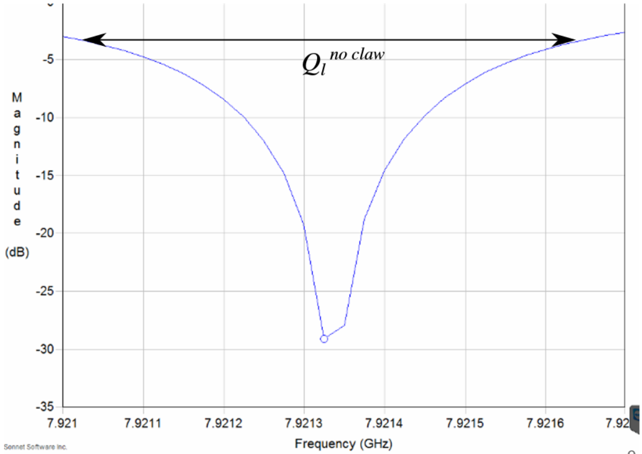
"Claw" coupler at the open end. Adds up some phase $\phi(\omega)$ and can be replaced [1] with high accuracy by

$$\Delta l = \frac{\phi(\omega_r)c}{2\omega_r\sqrt{\varepsilon_{eff}}}$$

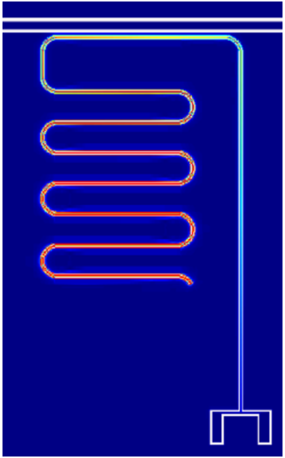
[1] D. Sank, PhD Thesis, 2014

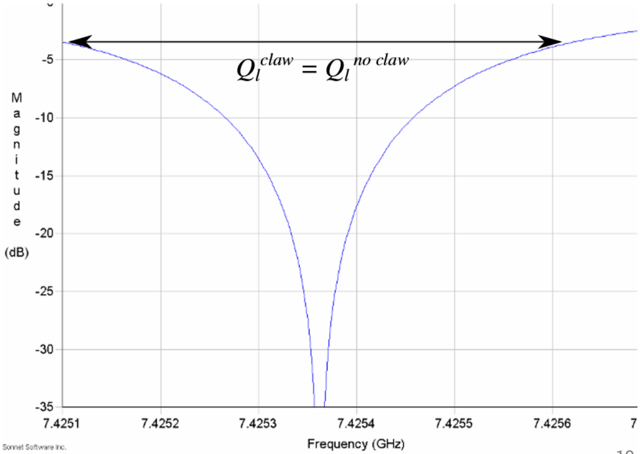
Resonator without a claw. Frequency expected from the length and $C_\kappa \approx 6$ fF (extracted from $Q_L \approx 10^4$) is 7.925 GHz.





Resonator with a claw. Frequency expected from the previous simulation and ϕ (also simulated separately for the given claw) is 7.4255 GHz.





System-bath interaction Hamiltonian:

$$\hat{\mathcal{H}}_{sb} = \hat{\sigma}_z \otimes \hat{O}_b,$$

where $\hat{O}_b$ is an arbitrary bath operator. Master equation is then developed:

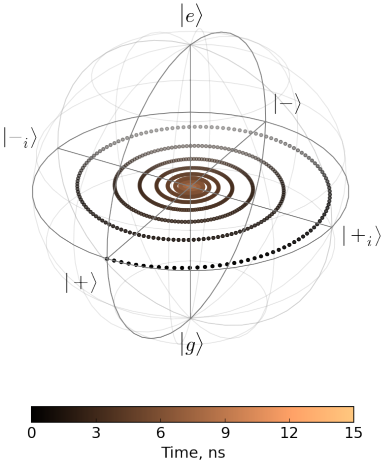
$$\partial_t \hat{\rho}_s = \frac{i}{\hbar}[\hat{\rho}_s, \hat{\mathcal{H}}_s] + \gamma_\phi(\hat{\sigma}_z \hat{\rho}_s \hat{\sigma}_z - \hat{\rho}_s).$$

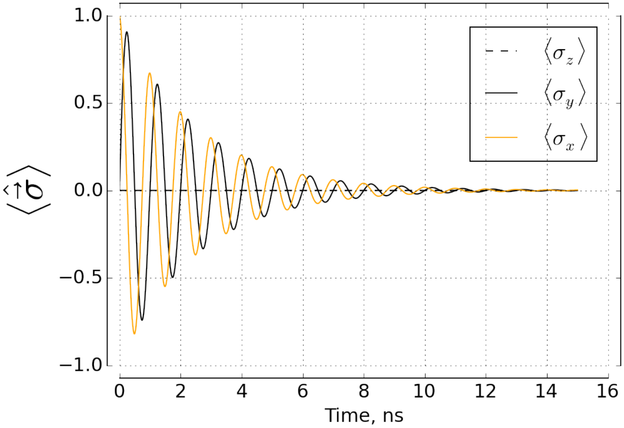
Steady state is a maximally mixed state:

$$\hat{\rho}_s(\infty) = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}.$$

Quantum-mechanically treated dephasing

Noise and decoherence versus Hahn echo technique





Quantum-mechanically treated dephasing

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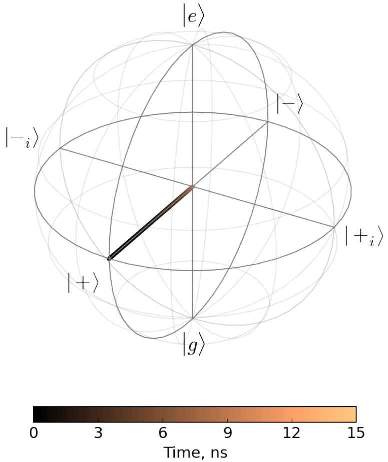

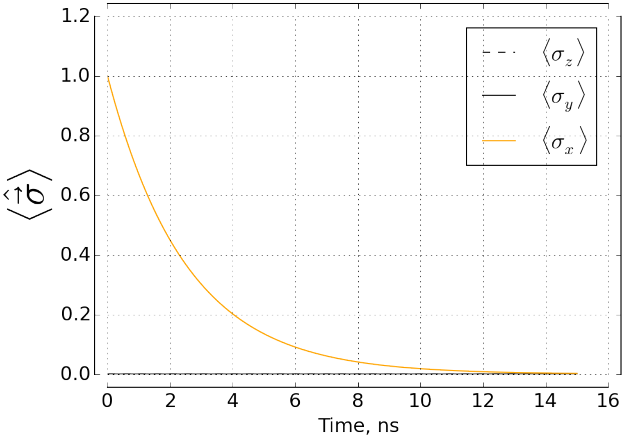

Unitary evolution under $f(t)\hat{\sigma}_z$, where $f(t)$ is random. In the rotating frame:

$$\hat{\rho}_q(t) = \hat{U}^\dagger(t,0)\ \hat{\rho}(0)\ \hat{U}(t,0), \quad U(t,0) = \hat{T}\exp\left\{-\frac{i}{\hbar}\int_0^t f(\tau)\hat{\sigma}_z\,\mathrm{d}\tau\right\}.$$

$$\hat{\rho}_s(t) = \begin{pmatrix} 1/2 - N(t)/2 & \rho(t) \\ \rho^*(t) & 1/2 + N(t)/2 \end{pmatrix},$$

$$N(t) = N(0),$$

$$\rho(t) = \rho(0)\exp\left\{\frac{2i}{\hbar}\int_0^t f(\tau)\,\mathrm{d}\tau\right\}.$$

Presuming Gaussian instantaneous distribution of $f(t)$ and $x(t) = \frac{2}{\hbar}\int_0^t f(\tau)\,\mathrm{d}\tau$:

$$\langle \rho(t) \rangle = \rho(0)\exp\left\{-\frac{2}{\hbar^2}\int_0^t\int_0^t \langle f(\tau_1)f(\tau_2)\rangle \,\mathrm{d}\tau_1\,\mathrm{d}\tau_2\right\}.$$

Presuming also that $f(t)$ is a wide-sense stationary process:

$$\langle f(\tau_1)f(\tau_2)\rangle = K_f(\tau_2-\tau_1) = \frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}} S_f(\omega)e^{i\omega(\tau_2-\tau_1)}\,\mathrm{d}\omega.$$

Taking the time integrals:

$$\langle \rho(t) \rangle = \rho(0)\exp\left\{-\frac{2}{\sqrt{2\pi}\hbar^2}\int_{\mathbb{R}}\frac{4\sin^2(\frac{\omega t}{2})}{\omega^2}S_f(\omega)\,\mathrm{d}\omega\right\}.$$

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It can be shown that for Hahn echo sequence the accumulated phase expression reads

$$\rho(t) = \rho(0) \exp\left\{-\frac{2i}{\hbar}\int_0^{t/2} f(\tau)\,\mathrm{d}\tau + \frac{2i}{\hbar}\int_{t/2}^{t} f(\tau)\,\mathrm{d}\tau\right\}.$$

And after averaging as in the previous slides:

$$\langle\rho(t)\rangle = \rho(0) \exp\left\{-\frac{2}{\sqrt{2\pi}\hbar^2}\int_{\mathbb{R}} S_f(\omega) W_t(\omega)\,\mathrm{d}\omega\right\},$$

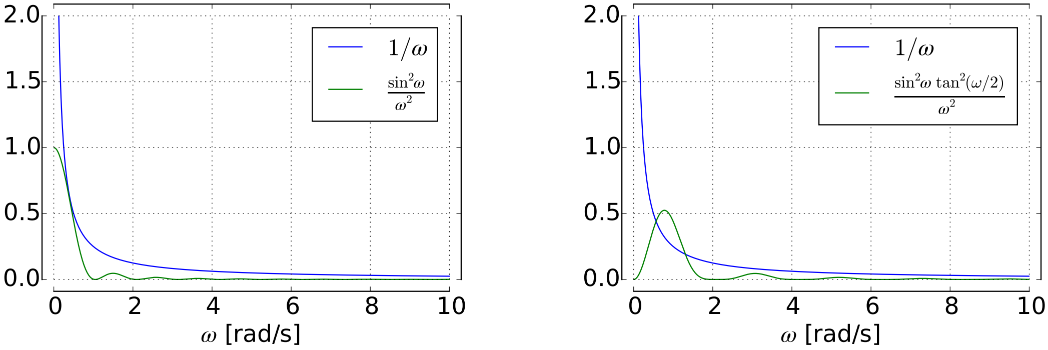
where

$$W_t(\omega) = \tan^2(\omega t/4)\frac{4\sin^2(\omega t/2)}{\omega^2}.$$

Noise PSD filtration

Noise and decoherence versus Hahn echo technique

In both Ramsey and Hahn echo experiments the noise PSD gets convolved with a corresponding filter function $W_t(\omega)$. For $t = 5$:



$S(\omega) = \delta(\omega)$ $(f(t) = const)$:

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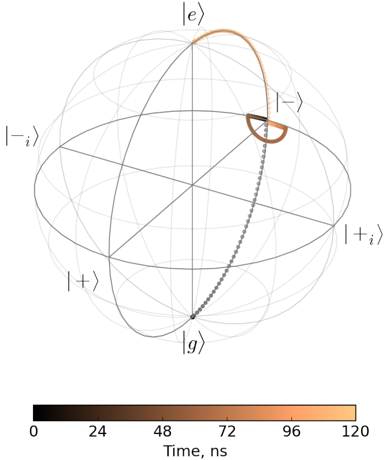

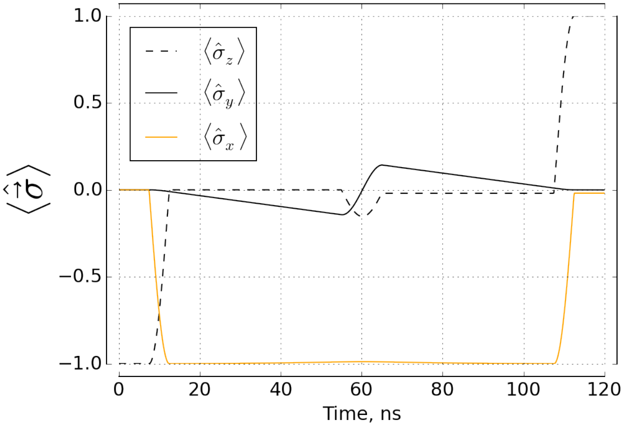

Master equation:

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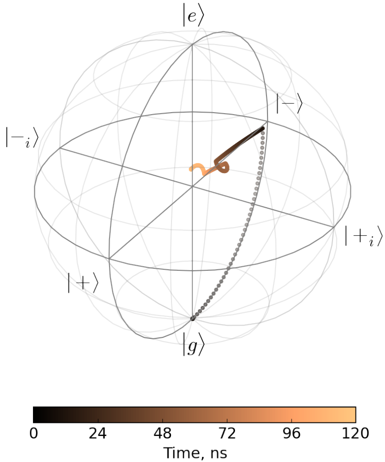
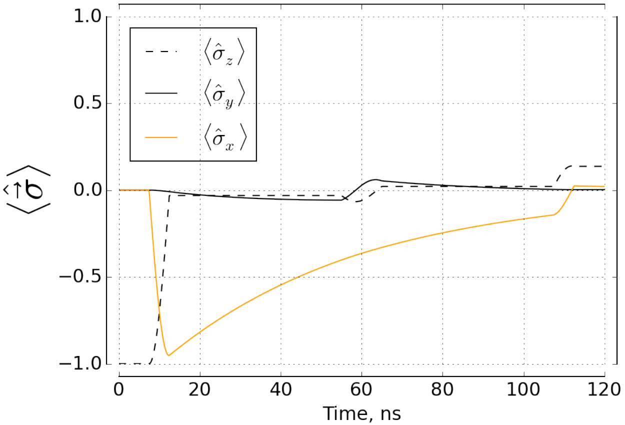
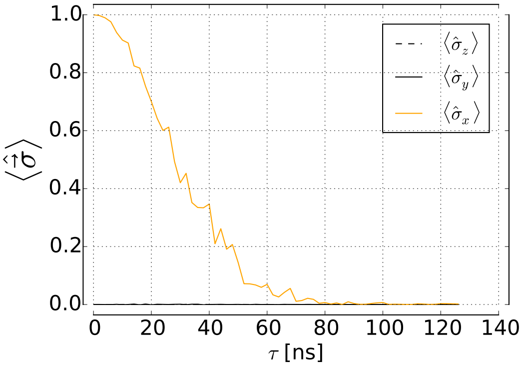




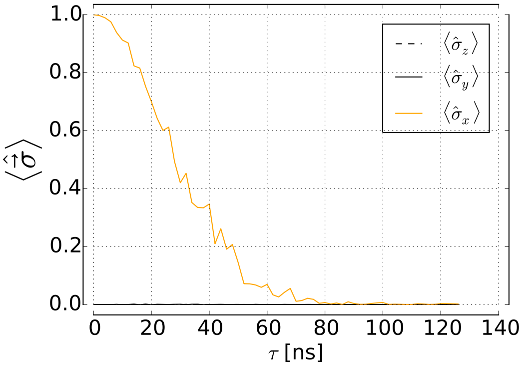
Who wins?
Noise and decoherence versus Hahn echo technique

$S(\omega) = 1/\omega$ (pink noise):

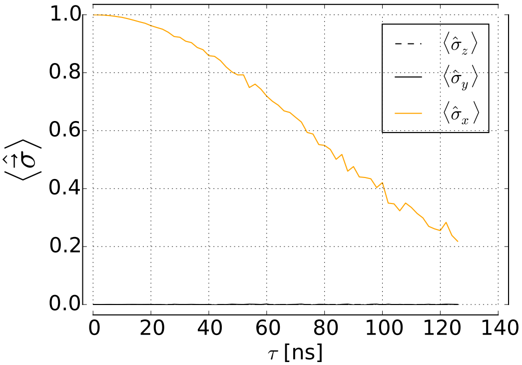


No echo

$S(\omega) = 1/\omega$ (pink noise):



No echo

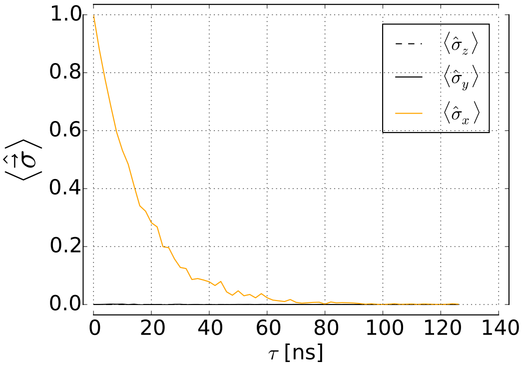


Echo

Who wins?

Noise and decoherence versus Hahn echo technique

$S(\omega) = S(0) = const$ (white noise):



No echo

Who wins?
Noise and decoherence versus Hahn echo technique

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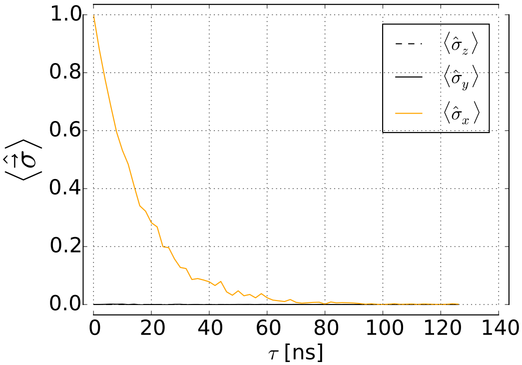
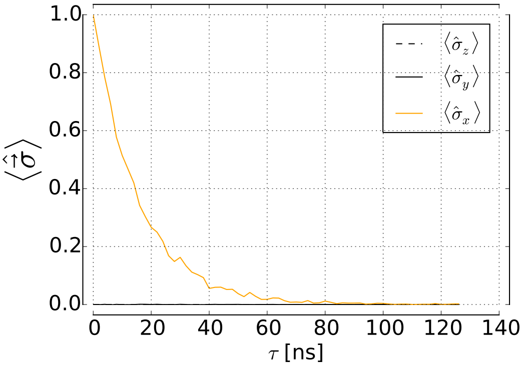
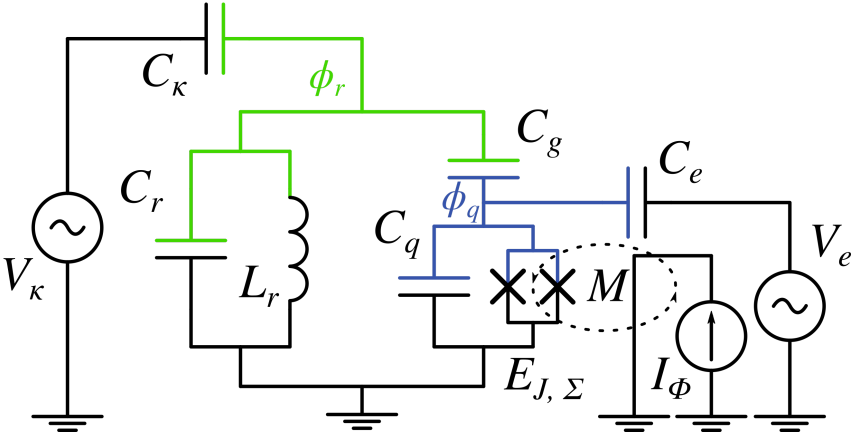


No echo



Echo

Equivalent circuit for the transmon-resonator system:



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Hamiltonian:

$$\hat{\mathcal{H}} = \underbrace{\frac{\hat{\phi}_r^2}{2L_r} + \frac{(C_q + C_g)\hat{Q}_r^2}{2C_*^2}}_{\text{resonator}} + \underbrace{\frac{(C_g + C_\kappa + C_r)\hat{Q}_q^2}{2C_*^2} - E_J(\Phi_{ext}) \cos \frac{2e}{\hbar}\hat{\phi}_q}_{\text{qubit}} + \underbrace{\frac{C_g \hat{Q}_r \hat{Q}_q}{C_*^2}}_{\text{coupling}} =$$

$$= \hbar\omega_r \; \hat{a}^\dagger \hat{a} \otimes \hat{\mathbb{1}}_q \quad (\hat{\mathcal{H}}_r)$$

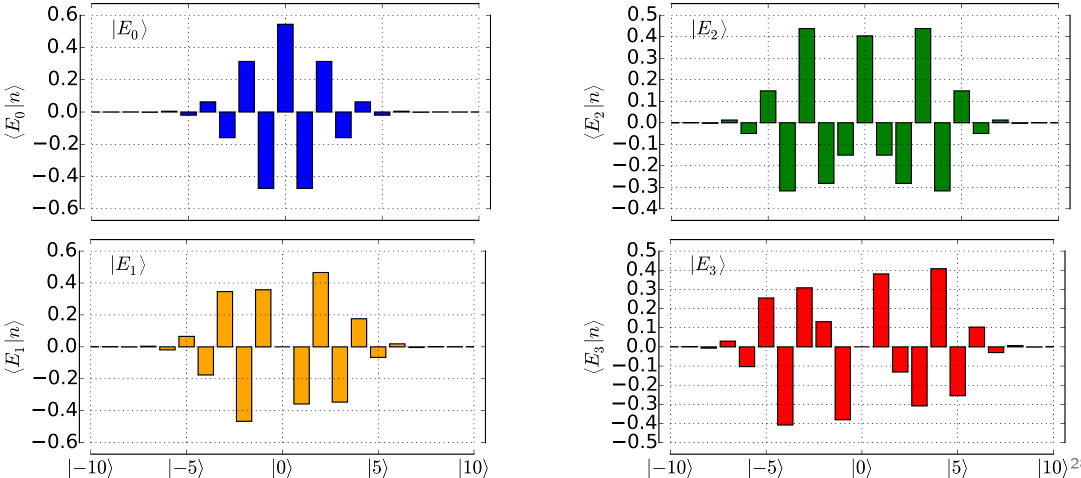
$$+4E_C \; \hat{\mathbb{1}}_r \otimes \hat{n}^2 - \frac{E_J(\Phi_{ext})}{2} \; \hat{\mathbb{1}}_r \otimes \sum_{n=-\infty}^{+\infty} |n+1\rangle\langle n| + |n\rangle\langle n+1| \quad (\hat{\mathcal{H}}_q)$$

$$-2e\frac{C_g}{C_*}\sqrt{\frac{\hbar\omega_r}{2(C_q + C_g)}} \; i(\hat{a}^\dagger - \hat{a}) \otimes \hat{n}, \quad (\hat{\mathcal{H}}_i)$$

Eigenproblem for an isolated Xmon

Xmon cQED

Cooper pairs distribution in the transmon eigenstates:



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Evolution of the transmon wavefunction is defined with

$$|\psi\rangle(t) = \hat{T}\exp\left\{-\frac{i}{\hbar}\left(\hat{\mathcal{H}}t + \int_0^t \hat{\mathcal{H}}_q^d(\tau)\,\mathrm{d}\tau\right)\right\}|E_0^q\rangle = \sum_k c_k(t)|E_k^q\rangle,$$

where

$$\hat{\mathcal{H}}_q^d(t) = \frac{C_e V_e(t)}{C_q + C_e}\hat{Q}_q \propto f_q(t)\ \hat{\mathbb{1}}_r \otimes \hat{n}.$$

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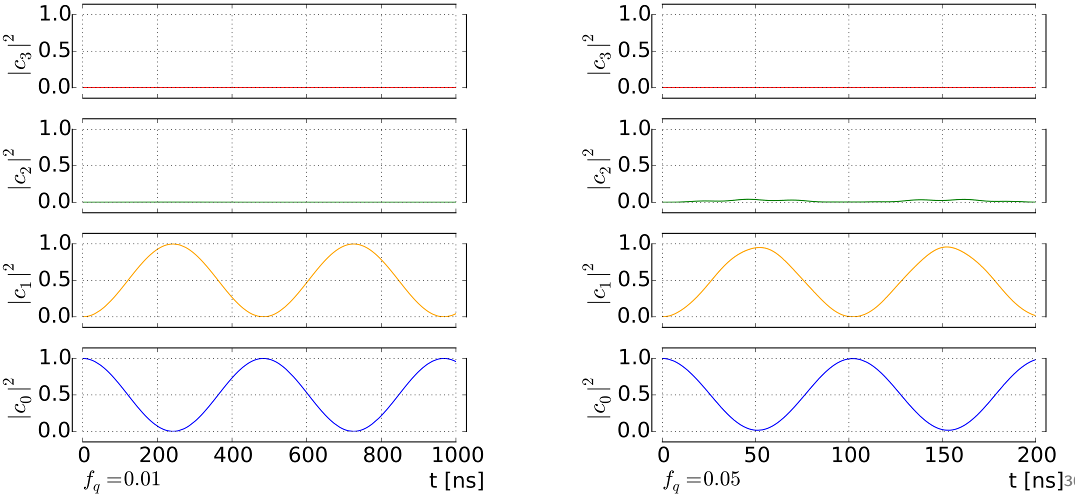
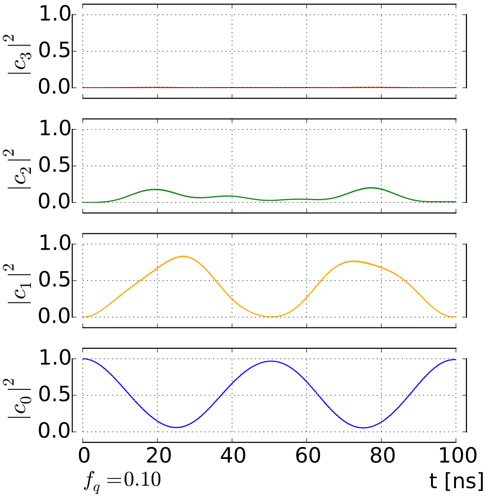
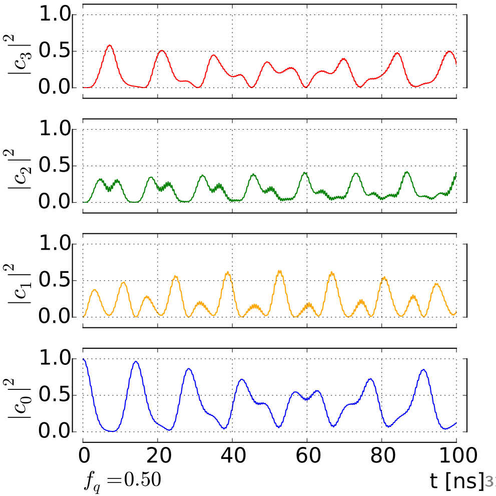
For weak driving the two-level dynamics is preserved:

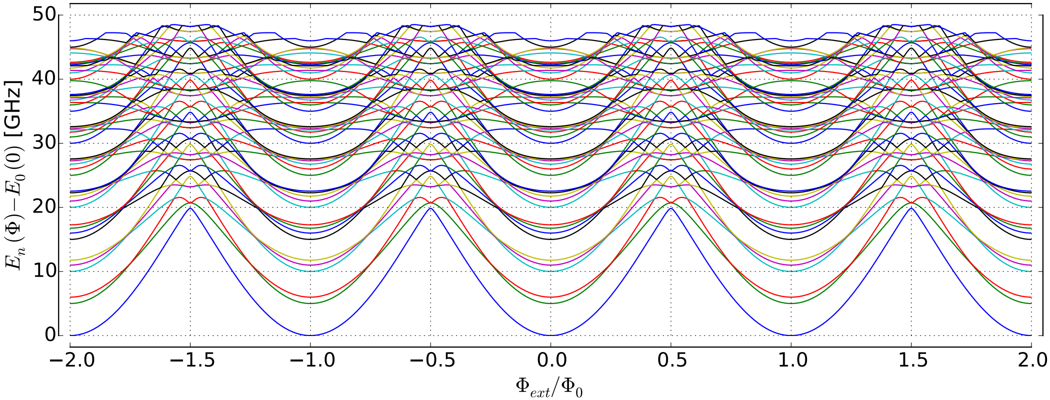


But not for the stronger one:



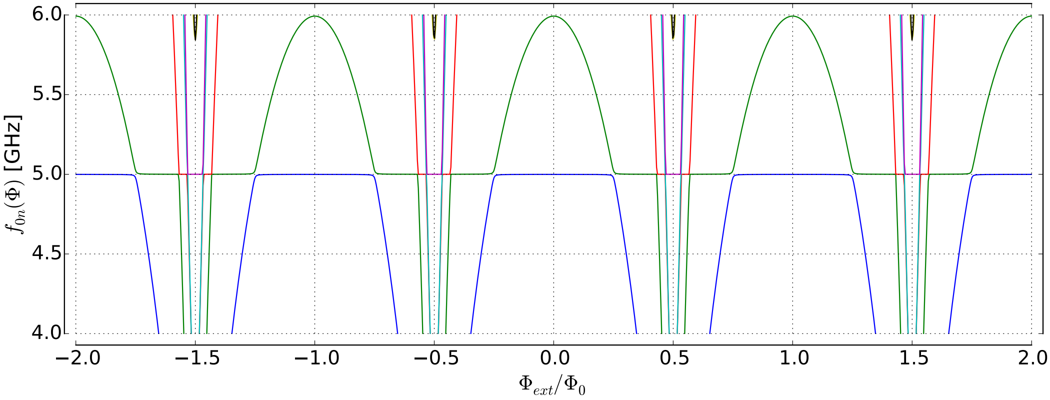


Energy levels:

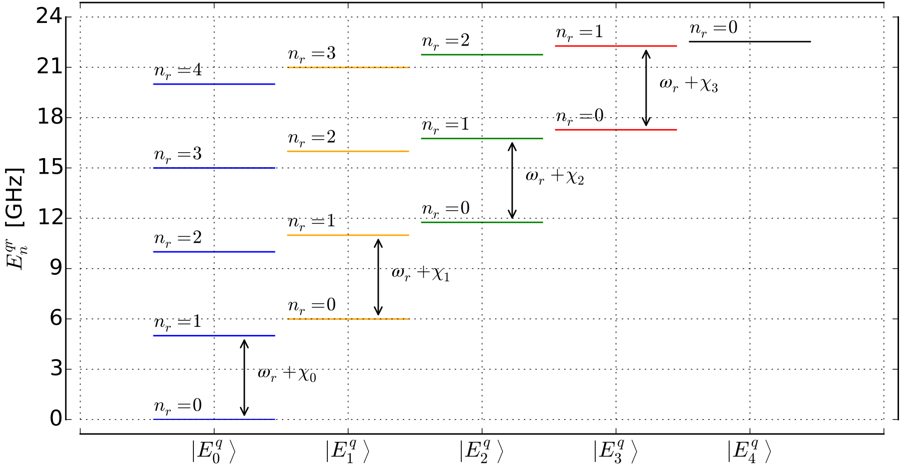


Eigenproblem for a qubit-resonator system
Xmon cQED

Frequencies:



Dispersive shifts originate from the level structure of the whole composite system:



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It's possible to estimate these shifts with high accuracy using the second order perturbation theory:

$$\chi_0 = g^2 \left[\frac{n_{ge}^2}{\omega_r - \omega_{ge}} - \frac{n_{ge}^2}{\omega_r + \omega_{ge}} \right],$$

$$\chi_1 = g^2 \left[\frac{n_{ef}^2}{\omega_r - \omega_{ef}} - \frac{n_{ef}^2}{\omega_r + \omega_{ef}} + \frac{n_{eg}^2}{\omega_r + \omega_{ge}} - \frac{n_{eg}^2}{\omega_r - \omega_{ge}} \right],$$

$$\chi_2 = g^2 \left[\frac{n_{fd}^2}{\omega_r - \omega_{fd}} - \frac{n_{fd}^2}{\omega_r + \omega_{fd}} + \frac{n_{fe}^2}{\omega_r + \omega_{ef}} - \frac{n_{fe}^2}{\omega_r - \omega_{ef}} \right].$$

The shifts depend on the detuning and thus Φ_{ext} ($\Delta_\omega = \omega_r - \omega_{ge}$):