CPW resonators
Noise and Decoherence vs Spin Echo
Xmon cQED

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May 31, 2016

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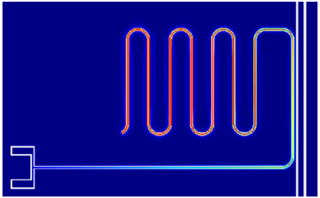
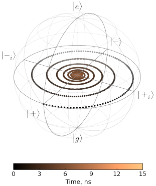
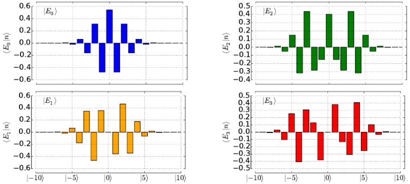


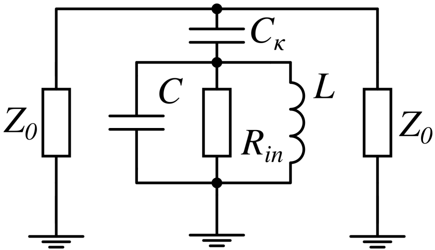


$|e\rangle$
$|-\rangle$
$|-_i\rangle$
$|+_i\rangle$
$|+\rangle$
$|g\rangle$
0 3 6 9 12 15
Time, ns



$|E_0\rangle$
$\langle E_0|n\rangle$
$|E_2\rangle$
$\langle E_2|n\rangle$
$|E_1\rangle$
$\langle E_1|n\rangle$
$|E_3\rangle$
$\langle E_3|n\rangle$
$|-10\rangle$ $|-5\rangle$ $|0\rangle$ $|5\rangle$ $|10\rangle$

Capacitively coupled CPW resonator as a lumped-element model:



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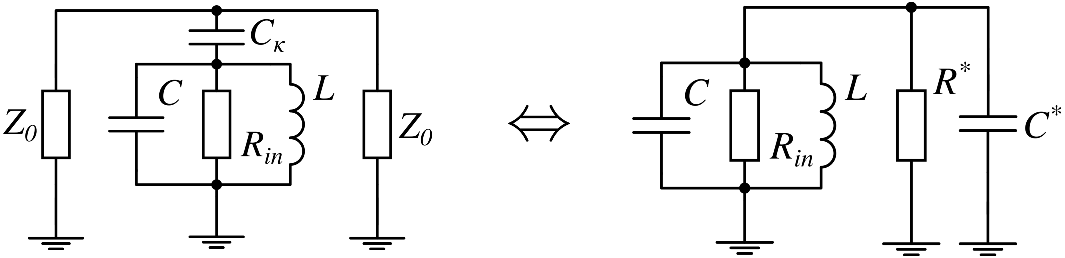
Capacitively coupled CPW resonator as a lumped-element model:



$$R^* = \frac{1+\omega^2 C_\kappa^2 (Z_0/2)^2}{\omega^2 C_\kappa^2 (Z_0/2)}, \quad C^* = \frac{C_\kappa}{1+\omega^2 C_\kappa^2 (Z_0/2)^2} \approx C_\kappa \text{ (for our case).}$$

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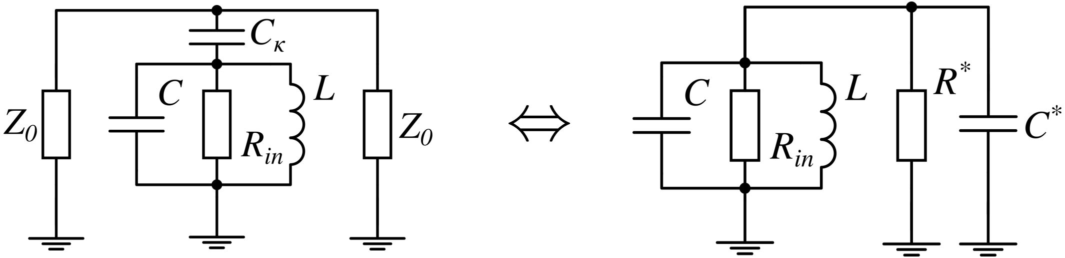
Capacitively coupled CPW resonator as a lumped-element model:



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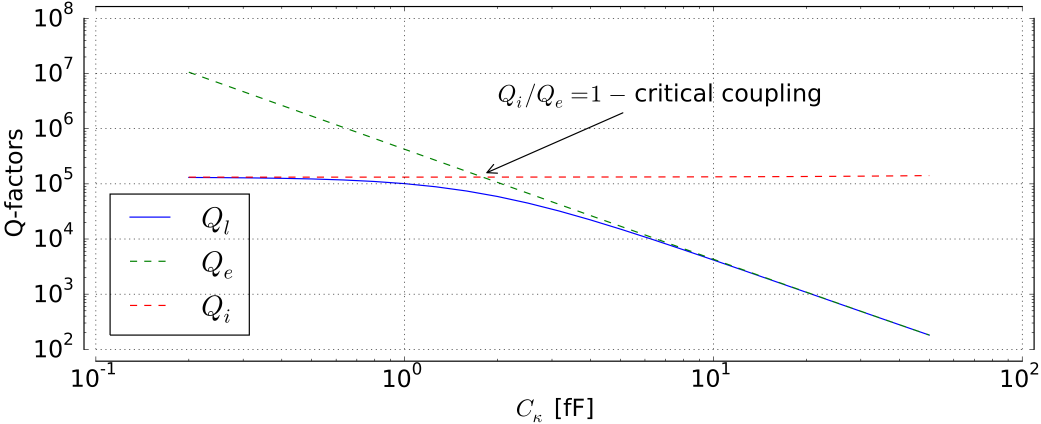
$$Q_i = \omega(C+C^*)R_{in}, \quad Q_e = \omega(C+C^*)R^*, \quad Q_l = \omega(C+C^*)\frac{1}{1/R^* + 1/R_{in}}.$$

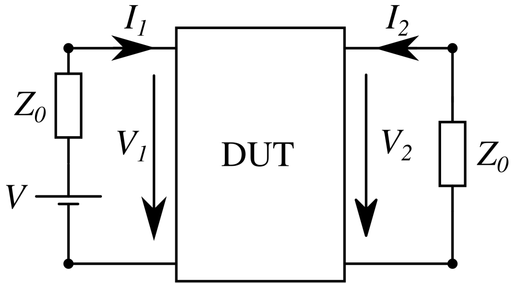
Q-factors depending on C_κ:

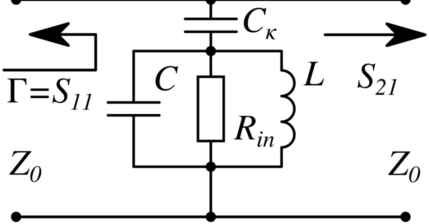


S-parameters

Coplanar waveguide resonators

General algorithm for calculating S-parameters of a given device:





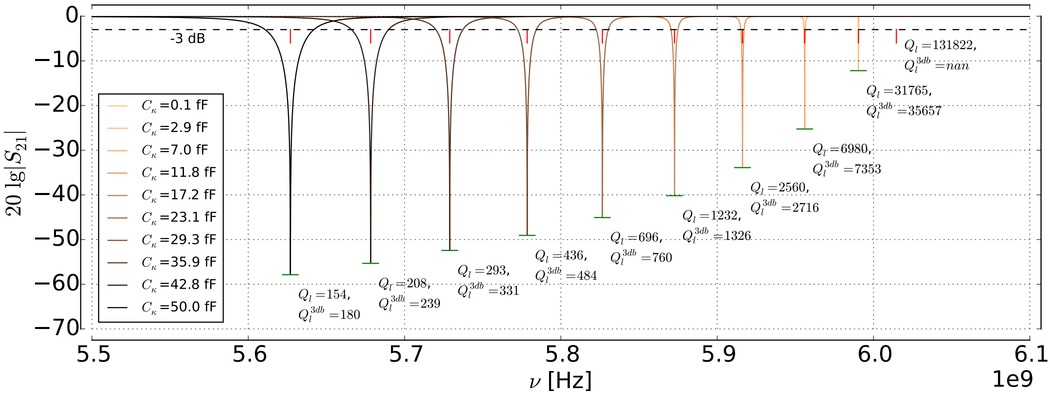
$$V_{1,2} = V_{1,2}^{+} + V_{1,2}^{-},$$

Kirchgoff's laws $\Rightarrow$

$$I_{1,2} = \frac{V_{1,2}^{+} - V_{1,2}^{-}}{Z_0}$$

$$\Rightarrow \quad \begin{pmatrix} V_1^- \\ V_2^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \end{pmatrix}.$$

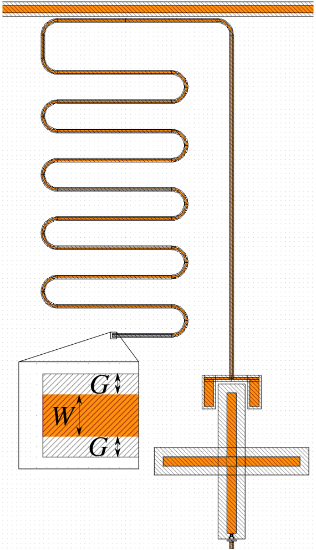
Analytical $S_{21}(C_\kappa, \omega) = (1 + Z_0/2Z_{sh})^{-1},\ Z_{sh} = 1/i\omega C_\kappa + Z_{res}$:



Xmon design features

Coplanar waveguide resonators

Designed in LayoutEditor



$l = \lambda/4$ coplanar resonators, $W = 4\,\mu m,\ G = 2\,\mu m.$

Unconventional coupling to the feedline:

$$C_\kappa^{eff} = C_\kappa \cos\frac{\pi x_\kappa}{2l}\ ?\ [1]$$

$$M_\kappa = ?$$

"Claw" coupler at the open end. Adds up some phase $\phi(\omega)$ and can be replaced [1] with high accuracy by

$$\Delta l = \frac{\phi(\omega_r)c}{2\omega_r\sqrt{\varepsilon_{eff}}}$$

[1] D. Sank, PhD Thesis, 2014 (J. Martinis group)

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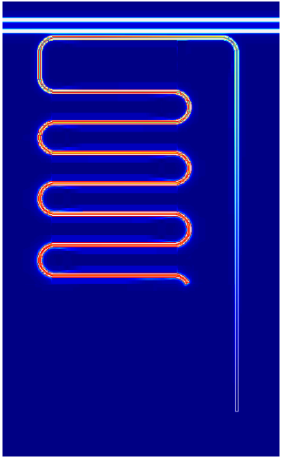
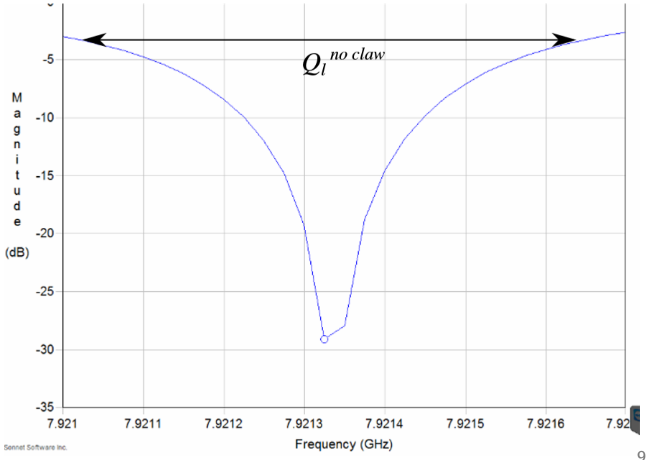
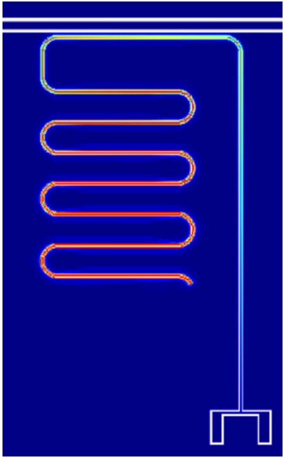
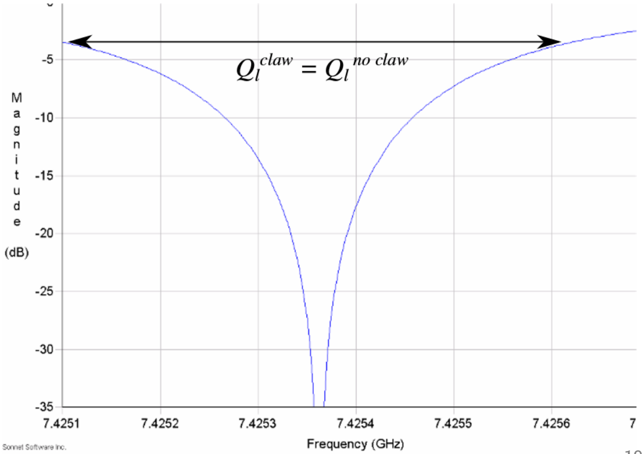
Resonator without a claw. Frequency expected from the length and $C_\kappa \approx 6$ fF (extracted from $Q_l \approx 10^4$) is 7.925 GHz.





Resonator with a claw. Frequency expected from the previous simulation and ϕ (also simulated separately for the given claw) is 7.4255 GHz.





Qubit-environment interaction Hamiltonian:

$$\hat{\mathcal{H}}_{qe} = \hat{O}_e \otimes \hat{\sigma}_z .$$

$\hat{O}_e$ is an arbitrary environment operator, $\hat{\sigma}_z$ – qubit observable.

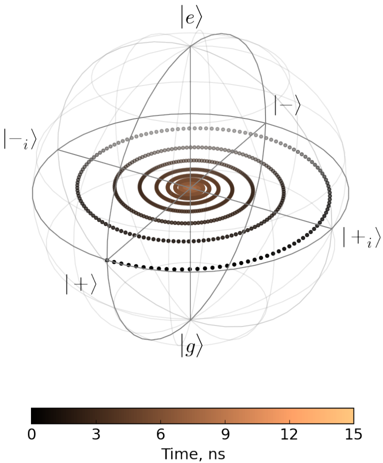
Master equation:

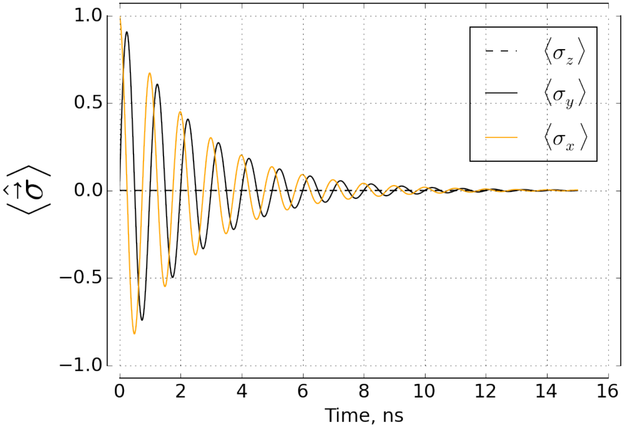
$$\partial_t \hat{\rho}_q = \frac{i}{\hbar}[\hat{\rho}_q, \hat{\mathcal{H}}_q] + \gamma_\phi(\hat{\sigma}_z \hat{\rho}_q \hat{\sigma}_z - \hat{\rho}_q).$$

Steady state:

$$\hat{\rho}_q(\infty) = \begin{pmatrix} \rho_{11}(0) & 0 \\ 0 & \rho_{22}(0) \end{pmatrix} .$$

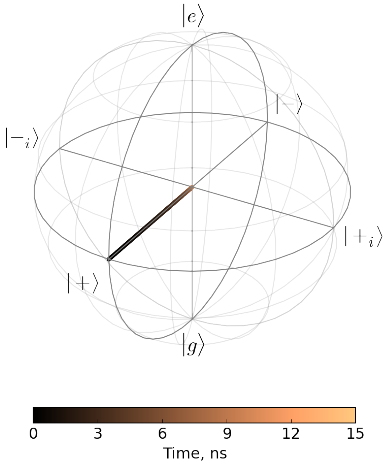
Example: pure dephasing of the $|+\rangle$ state, **lab frame**.

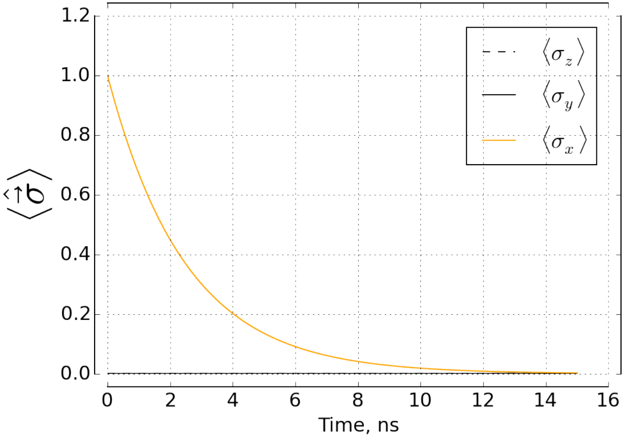




(dynamics simulated with QuTiP)

Example: pure dephasing of the $|+\rangle$ state, **rotating frame**.





(dynamics simulated with QuTiP)

Classically treated pure dephasing (Ramsey)

Noise and decoherence versus Hahn echo technique

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$$\hat{O}_e \otimes \hat{\sigma}_z \to f(t) \cdot \hat{\sigma}_z, \;\; f(t) \text{ is random}$$

Rotating frame evolution (**single run!**):

$$\hat{\rho}_q(t) = \hat{U}^\dagger(t,0)\; \hat{\rho}_q(0)\; \hat{U}(t,0), \quad \hat{U}(t,0) = \hat{T}\exp\left\{-\frac{i}{\hbar}\int_0^t f(\tau)\,\mathrm{d}\tau\; \hat{\sigma}_z\right\}.$$

$$\hat{\rho}_q(t) = \begin{pmatrix} 1/2 - N(t)/2 & \rho(t) \\ \rho^*(t) & 1/2 + N(t)/2 \end{pmatrix},$$

$$\left.\begin{aligned} N(t) &= N(0), \\ \rho(t) &= \rho(0)\exp\left\{\frac{2i}{\hbar}\int_0^t f(\tau)\,\mathrm{d}\tau\right\} \end{aligned}\right\} \Rightarrow$$

unitary random walk, no dephasing!

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Averaging. For Gaussian instantaneous distribution of $f(t)$:

$$\langle\rho(t)\rangle = \rho(0)\exp\left\{-\frac{2}{\hbar^2}\int_0^t\int_0^t \langle f(\tau_1)f(\tau_2)\rangle\,\mathrm{d}\tau_1\,\mathrm{d}\tau_2\right\}.$$

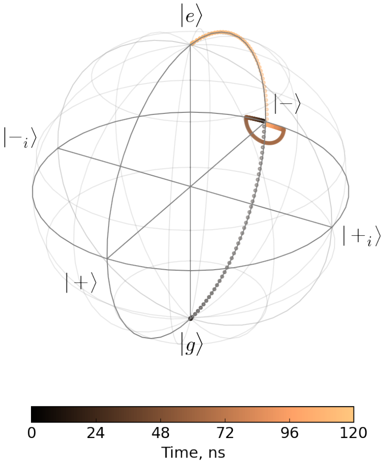
Presuming also that $f(t)$ is a wide-sense stationary process:

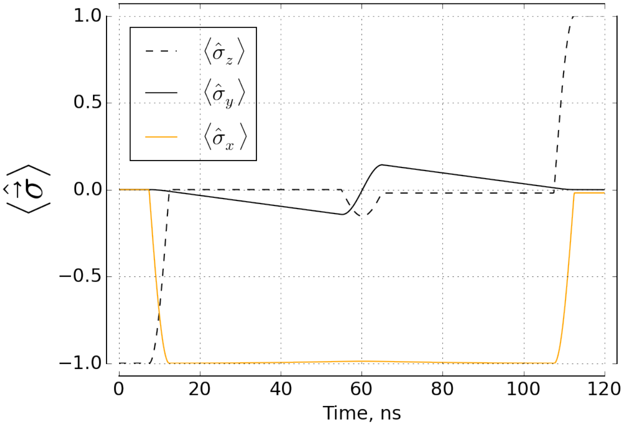
$$\langle f(\tau_1)f(\tau_2)\rangle = K_f(\tau_2-\tau_1) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} S_f(\omega)e^{i\omega(\tau_2-\tau_1)}\,\mathrm{d}\omega.$$

$$\langle\rho(t)\rangle = \rho(0)\exp\left\{-\frac{2}{\sqrt{2\pi}\hbar^2}\int_{-\infty}^{\infty}\frac{4\sin^2(\frac{\omega t}{2})}{\omega^2}S_f(\omega)\,\mathrm{d}\omega\right\} \equiv \rho(0)e^{-\alpha(t)}$$

$\Rightarrow$ **ensemble averaged behaviour of $\rho(t)$ is a decay!**

$S_f(\omega) = \delta(\omega)$ $(f(t) = const$ on the timescale of a single run):





(dynamics simulated with QuTiP)

Add one instant π-pulse. Accumulated phase:

$$\rho(t) = \rho(0)\exp\Big\{\underbrace{-\frac{2i}{\hbar}\int_0^{t/2} f(\tau)\,\mathrm{d}\tau}_{\text{before } \pi\text{-pulse}} + \underbrace{\frac{2i}{\hbar}\int_{t/2}^{t} f(\tau)\,\mathrm{d}\tau}_{\text{after } \pi\text{-pulse}}\Big\}.$$

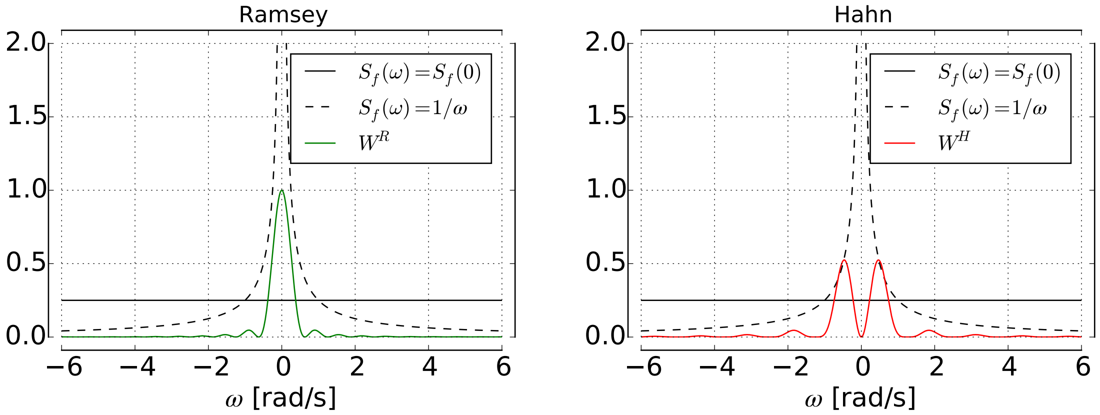
After averaging:

$$\langle\rho(t)\rangle = \rho(0)\exp\Big\{-\frac{2}{\sqrt{2\pi}\hbar^2}\int\limits_{-\infty}^{\infty} S_f(\omega) W_t^H(\omega)\,\mathrm{d}\omega\Big\},$$

$$W_t^H(\omega) = \tan^2(\omega t/4)\frac{4\sin^2(\omega t/2)}{\omega^2}.$$

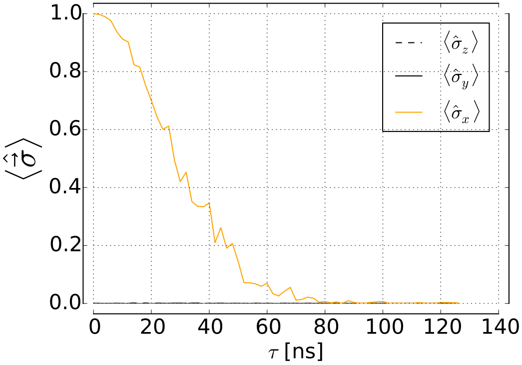
Noise PSD filtration

Noise and decoherence versus Hahn echo technique



- Ramsey: $\int \frac{4\sin^2(\frac{\omega t}{2})}{\omega^2} S_f(\omega)\,\mathrm{d}\omega$
- Hahn: $\int \tan^2(\omega t/4) \frac{4\sin^2(\frac{\omega t}{2})}{\omega^2} S_f(\omega)\,\mathrm{d}\omega$

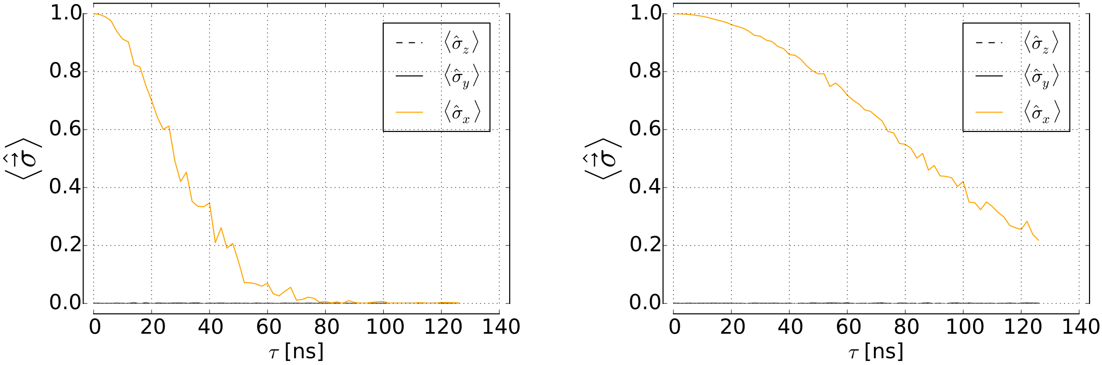
$S_f(\omega) = 1/\omega$ (pink noise):



No echo

Who wins?
Noise and decoherence versus Hahn echo technique

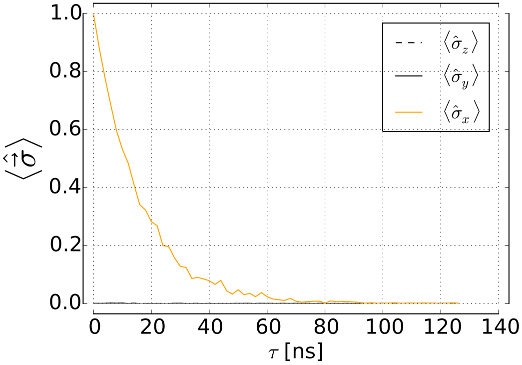
$S_f(\omega) = 1/\omega$ (pink noise):



No echo | Echo

- **efficient!**

$S_f(\omega) = S_f(0) = const$ (white noise):

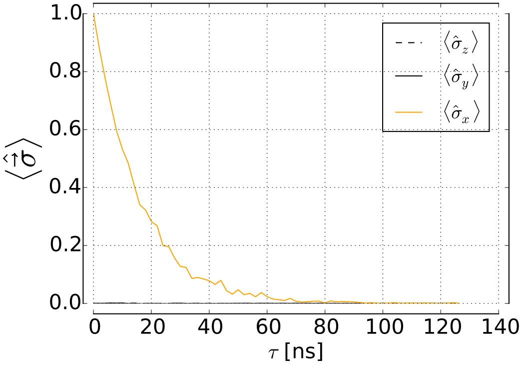


No echo

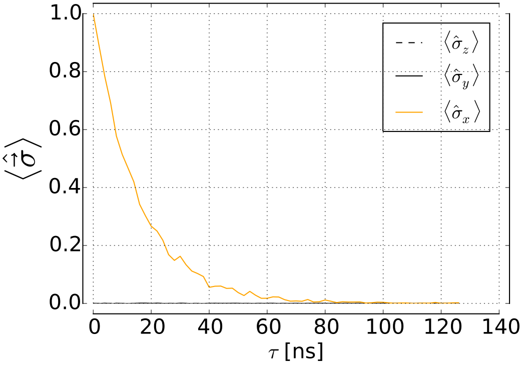
Who wins?

Noise and decoherence versus Hahn echo technique

$S_f(\omega) = S_f(0) = const$ (white noise):



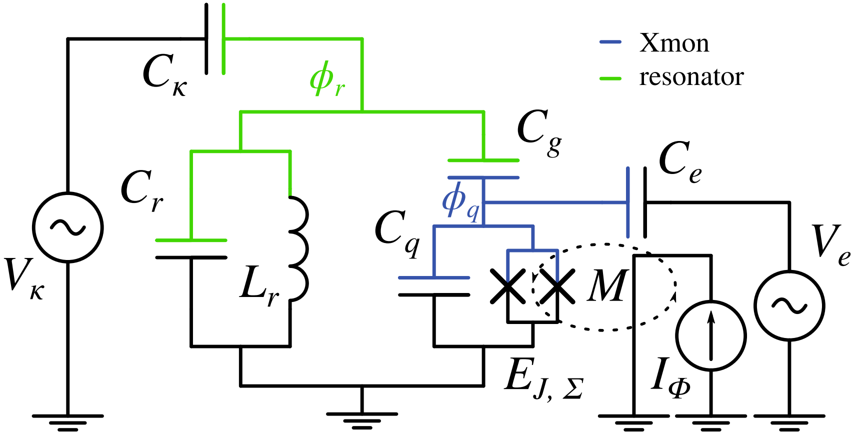
No echo



Echo

- **totally inefficient**, same for the master equation description

Equivalent circuit for the transmon-resonator system:



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$$\hat{\mathcal{H}} = \underbrace{\frac{\hat{\phi}_r^2}{2L_r} + \frac{(C_q + C_g)\hat{Q}_r^2}{2C_*^2}}_{\text{resonator}} + \underbrace{\frac{(C_g + C_\kappa + C_r)\hat{Q}_q^2}{2C_*^2} - E_J(\Phi_{ext}) \cos \frac{2e}{\hbar}\hat{\phi}_q}_{\text{qubit}} + \underbrace{\frac{C_g \hat{Q}_r \hat{Q}_q}{C_*^2}}_{\text{coupling}} =$$

$$= \hbar\omega_r \; \hat{a}^\dagger \hat{a} \otimes \hat{\mathbb{1}}_q \quad (\hat{\mathcal{H}}_r)$$

$$+4E_C \; \hat{\mathbb{1}}_r \otimes \hat{n}^2 - \frac{E_J(\Phi_{ext})}{2} \; \hat{\mathbb{1}}_r \otimes \sum_{n=-\infty}^{+\infty} |n+1\rangle\langle n| + |n\rangle\langle n+1| \quad (\hat{\mathcal{H}}_q)$$

$$-2e\frac{C_g}{C_*}\sqrt{\frac{\hbar\omega_r}{2(C_q + C_g)}} \; i(\hat{a}^\dagger - \hat{a}) \otimes \hat{n}, \quad (\hat{\mathcal{H}}_i)$$

Cooper pair distribution for an isolated Xmon

Xmon cQED (QuTiP simulations)

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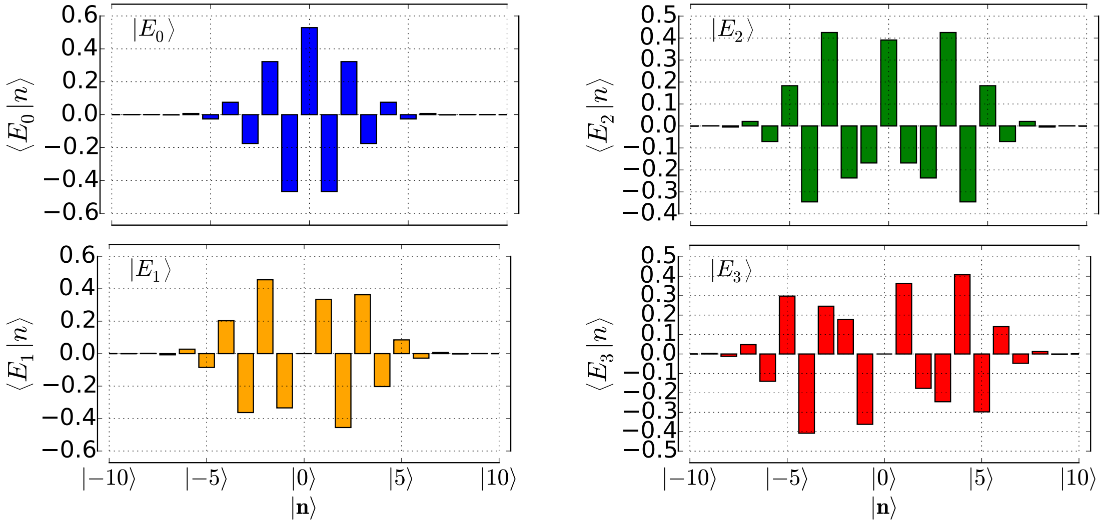
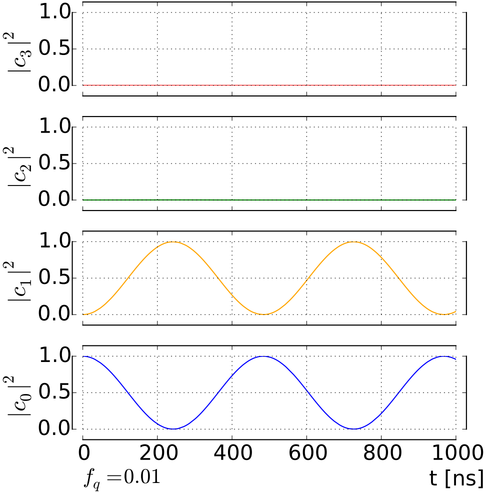
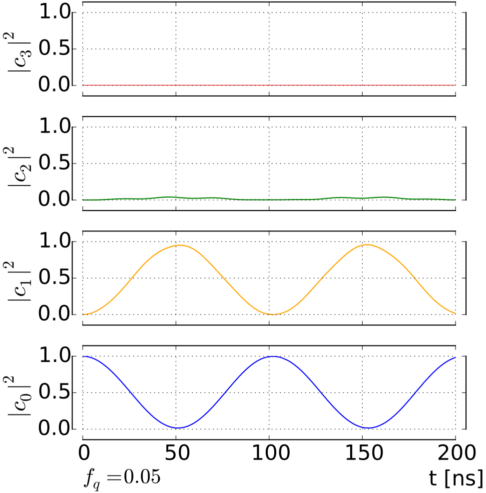


Strong driving

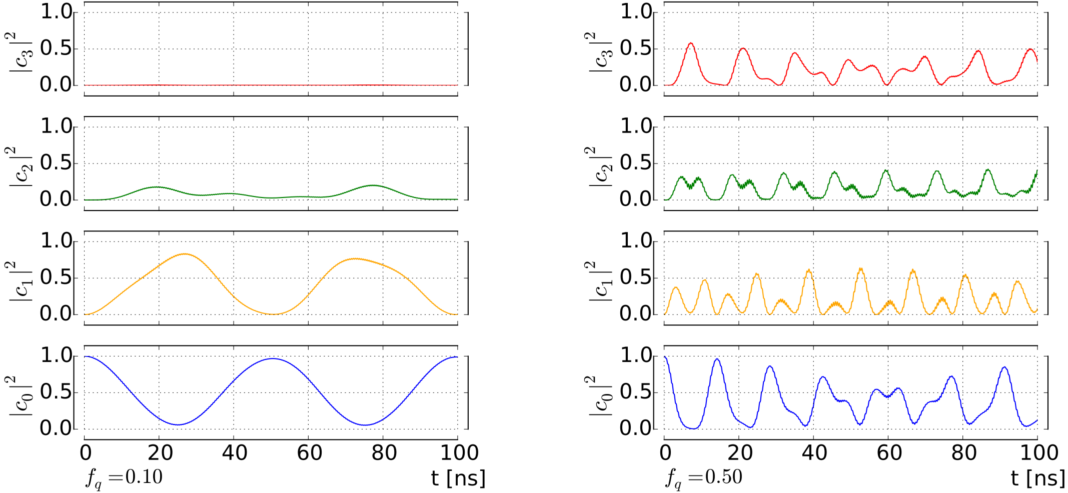
Xmon cQED (QuTiP simulations)

For weak driving the two-level dynamics is preserved ($|\psi(t)\rangle = \sum_k c_k(t)|E_k\rangle$):



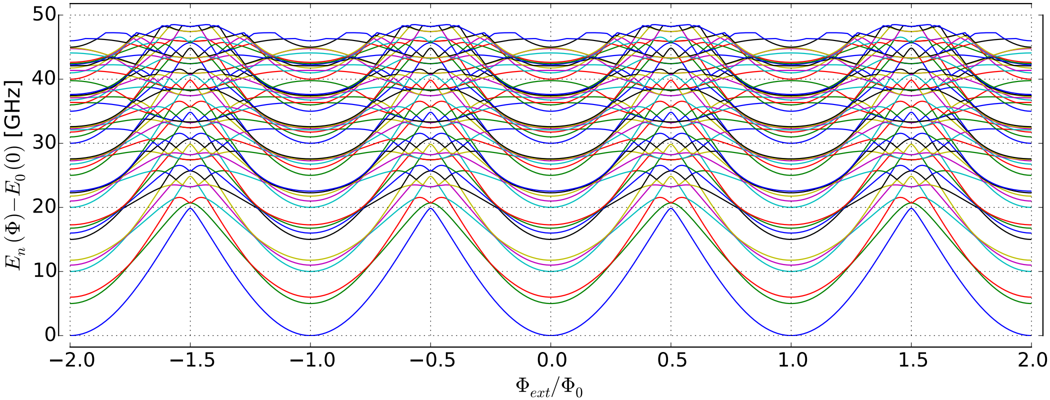


But not for the strong drive ($|\psi(t)\rangle = \sum_k c_k(t)|E_k\rangle$):



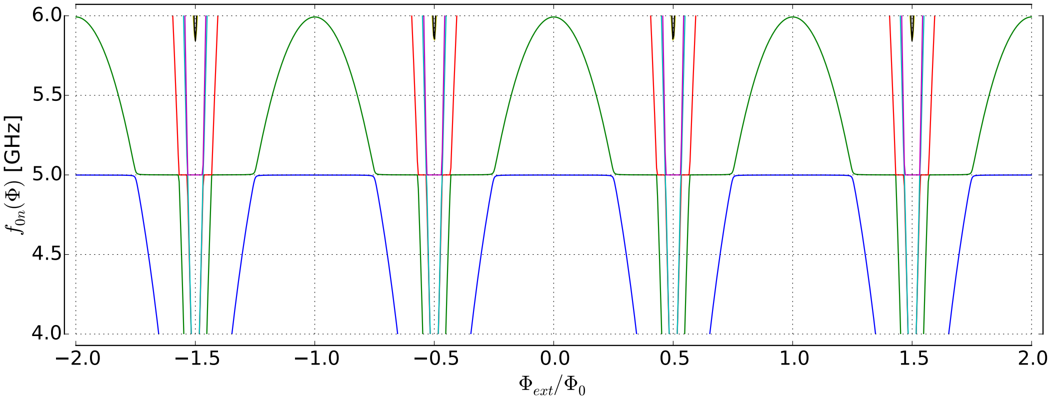
Eigenproblem for a qubit-resonator system
Xmon cQED (QuTiP simulations)

Energy level structure:



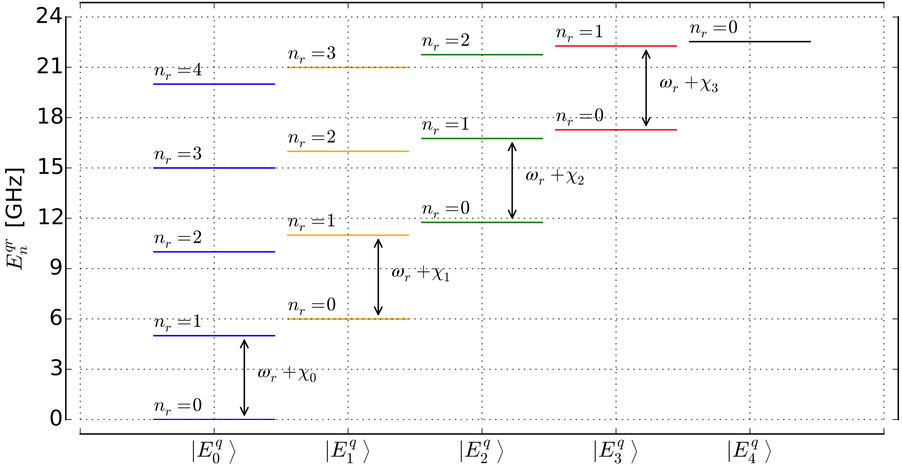
Eigenproblem for a qubit-resonator system
Xmon cQED (QuTiP simulations)

Frequencies $f_{0n} = (E_n - E_0)/h$ for $\omega_q > \omega_r$:



The Grotrian diagram:

Dispersive shifts originate from the level structure of the whole composite system:



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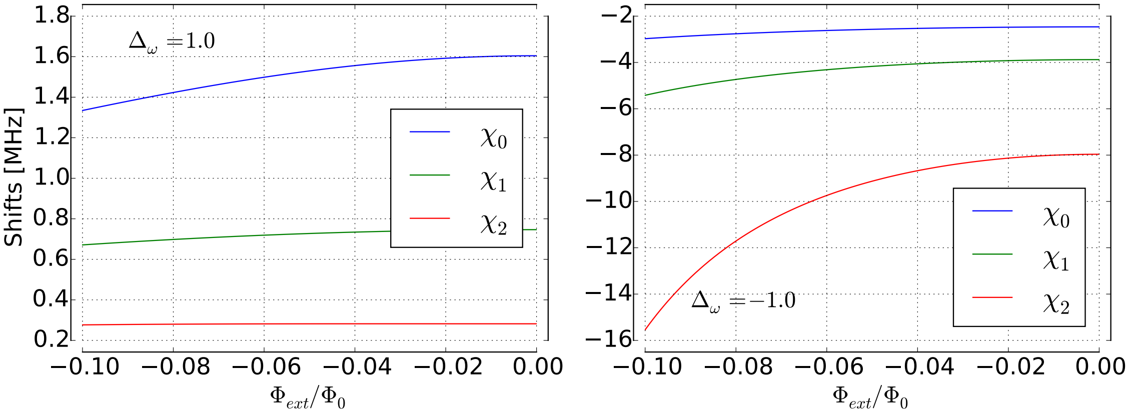
It's possible to determine these shifts with high accuracy using the second order perturbation theory:

$$\chi_0 = g^2 \left[\frac{n_{ge}^2}{\omega_r - \omega_{ge}} - \frac{n_{ge}^2}{\omega_r + \omega_{ge}} \right],$$

$$\chi_1 = g^2 \left[\frac{n_{ef}^2}{\omega_r - \omega_{ef}} - \frac{n_{ef}^2}{\omega_r + \omega_{ef}} + \frac{n_{eg}^2}{\omega_r + \omega_{ge}} - \frac{n_{eg}^2}{\omega_r - \omega_{ge}} \right],$$

$$\chi_2 = g^2 \left[\frac{n_{fd}^2}{\omega_r - \omega_{fd}} - \frac{n_{fd}^2}{\omega_r + \omega_{fd}} + \frac{n_{fe}^2}{\omega_r + \omega_{ef}} - \frac{n_{fe}^2}{\omega_r - \omega_{ef}} \right].$$

The shifts depend on the detuning $\Delta_\omega = \omega_r - \omega_{ge}$ and Φ_{ext}:



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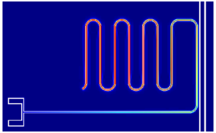
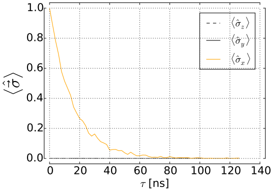
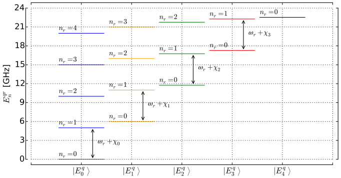
- **Quality factors** and **coupling capacitance** calculations, simulations of **CPW resonators** and **extraction of capacitances** using Sonnet
- Investigation of **classical** and **quantum noise** influence on the qubit, numerical simulations of the **pure dephasing under different types of noise**
- **Quantization of Xmon-resonator** circuit to transform capacitances and quality factors to the **coupling strengths, driving amplitudes** and **decay rates**







- A. V. Ustinov
- J. Lisenfeld and A. Bilmes

Thank you!