



### Module 10-1 Rotational Variables

•1 A good baseball pitcher can throw a baseball toward home plate at 85 mi/h with a spin of 1800 rev/min. How many revolutions does the baseball make on its way to home plate? For simplicity, assume that the 60 ft path is a straight line.

•2 What is the angular speed of (a) the second hand, (b) the minute hand, and (c) the hour hand of a smoothly running analog watch? Answer in radians per second.

••3  When a slice of buttered toast is accidentally pushed over the edge of a counter, it rotates as it falls. If the distance to the floor is 76 cm and for rotation less than 1 rev, what are the (a) smallest and (b) largest angular speeds that cause the toast to hit and then topple to be butter-side down?

••4 The angular position of a point on a rotating wheel is given by  $\theta = 2.0 + 4.0t^2 + 2.0t^3$ , where  $\theta$  is in radians and  $t$  is in seconds. At  $t = 0$ , what are (a) the point's angular position and (b) its angular velocity? (c) What is its angular velocity at  $t = 4.0$  s? (d) Calculate its angular acceleration at  $t = 2.0$  s. (e) Is its angular acceleration constant?

••5  **ILW** A diver makes 2.5 revolutions on the way from a 10-m-high platform to the water. Assuming zero initial vertical velocity, find the average angular velocity during the dive.

••6 The angular position of a point on the rim of a rotating wheel is given by  $\theta = 4.0t - 3.0t^2 + t^3$ , where  $\theta$  is in radians and  $t$  is in seconds. What are the angular velocities at (a)  $t = 2.0$  s and (b)  $t = 4.0$  s? (c) What is the average angular acceleration for the time interval that begins at  $t = 2.0$  s and ends at  $t = 4.0$  s? What are the instantaneous angular accelerations at (d) the beginning and (e) the end of this time interval?

••7 The wheel in Fig. 10-30 has eight equally spaced spokes and a radius of 30 cm. It is mounted on a fixed axle and is spinning at 2.5 rev/s. You want to shoot a 20-cm-long arrow parallel to this axle and

through the wheel without hitting any of the spokes. Assume that the arrow and the spokes are very thin. (a) What minimum speed must the arrow have? (b) Does it matter where between the axle and rim of the wheel you aim? If so, what is the best location?

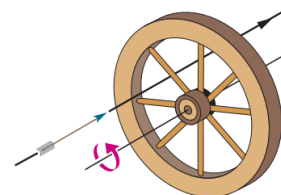


Figure 10-30 Problem 7.

•••8 The angular acceleration of a wheel is  $\alpha = 6.0t^4 - 4.0t^2$ , with  $\alpha$  in radians per second-squared and  $t$  in seconds. At time  $t = 0$ , the wheel has an angular velocity of  $+2.0$  rad/s and an angular position of  $+1.0$  rad. Write expressions for (a) the angular velocity (rad/s) and (b) the angular position (rad) as functions of time (s).

### Module 10-2 Rotation with Constant Angular Acceleration

•9 A drum rotates around its central axis at an angular velocity of 12.60 rad/s. If the drum then slows at a constant rate of 4.20 rad/s<sup>2</sup>, (a) how much time does it take and (b) through what angle does it rotate in coming to rest?

•10 Starting from rest, a disk rotates about its central axis with constant angular acceleration. In 5.0 s, it rotates 25 rad. During that time, what are the magnitudes of (a) the angular acceleration and (b) the average angular velocity? (c) What is the instantaneous angular velocity of the disk at the end of the 5.0 s? (d) With the angular acceleration unchanged, through what additional angle will the disk turn during the next 5.0 s?

•11 A disk, initially rotating at 120 rad/s, is slowed down with a constant angular acceleration of magnitude 4.0 rad/s<sup>2</sup>. (a) How much time does the disk take to stop? (b) Through what angle does the disk rotate during that time?

•12 The angular speed of an automobile engine is increased at a constant rate from 1200 rev/min to 3000 rev/min in 12 s. (a) What is

its angular acceleration in revolutions per minute-squared? (b) How many revolutions does the engine make during this 12 s interval?

**••13 ILW** A flywheel turns through 40 rev as it slows from an angular speed of 1.5 rad/s to a stop. (a) Assuming a constant angular acceleration, find the time for it to come to rest. (b) What is its angular acceleration? (c) How much time is required for it to complete the first 20 of the 40 revolutions?

**••14 GO** A disk rotates about its central axis starting from rest and accelerates with constant angular acceleration. At one time it is rotating at 10 rev/s; 60 revolutions later, its angular speed is 15 rev/s. Calculate (a) the angular acceleration, (b) the time required to complete the 60 revolutions, (c) the time required to reach the 10 rev/s angular speed, and (d) the number of revolutions from rest until the time the disk reaches the 10 rev/s angular speed.

**••15 SSM** Starting from rest, a wheel has constant  $\alpha = 3.0 \text{ rad/s}^2$ . During a certain 4.0 s interval, it turns through 120 rad. How much time did it take to reach that 4.0 s interval?

**••16** A merry-go-round rotates from rest with an angular acceleration of  $1.50 \text{ rad/s}^2$ . How long does it take to rotate through (a) the first 2.00 rev and (b) the next 2.00 rev?

**••17** At  $t = 0$ , a flywheel has an angular velocity of  $4.7 \text{ rad/s}$ , a constant angular acceleration of  $-0.25 \text{ rad/s}^2$ , and a reference line at  $\theta_0 = 0$ . (a) Through what maximum angle  $\theta_{\text{max}}$  will the reference line turn in the positive direction? What are the (b) first and (c) second times the reference line will be at  $\theta = \frac{1}{2}\theta_{\text{max}}$ ? At what (d) negative time and (e) positive time will the reference line be at  $\theta = 10.5 \text{ rad}$ ? (f) Graph  $\theta$  versus  $t$ , and indicate your answers.

**••18** A pulsar is a rapidly rotating neutron star that emits a radio beam the way a lighthouse emits a light beam. We receive a radio pulse for each rotation of the star. The period  $T$  of rotation is found by measuring the time between pulses. The pulsar in the Crab nebula has a period of rotation of  $T = 0.033 \text{ s}$  that is increasing at the rate of  $1.26 \times 10^{-5} \text{ s/y}$ . (a) What is the pulsar's angular acceleration  $\alpha$ ? (b) If  $\alpha$  is constant, how many years from now will the pulsar stop rotating? (c) The pulsar originated in a supernova explosion seen in the year 1054. Assuming constant  $\alpha$ , find the initial  $T$ .

### Module 10-3 Relating the Linear and Angular Variables

**•19** What are the magnitudes of (a) the angular velocity, (b) the radial acceleration, and (c) the tangential acceleration of a spaceship taking a circular turn of radius 3220 km at a speed of 29 000 km/h?

**•20** An object rotates about a fixed axis, and the angular position of a reference line on the object is given by  $\theta = 0.40e^{2t}$ , where  $\theta$  is in radians and  $t$  is in seconds. Consider a point on the object that is 4.0 cm from the axis of rotation. At  $t = 0$ , what are the magnitudes of the point's (a) tangential component of acceleration and (b) radial component of acceleration?

**•21** Between 1911 and 1990, the top of the leaning bell tower at Pisa, Italy, moved toward the south at an average rate of 1.2 mm/y. The tower is 55 m tall. In radians per second, what is the average angular speed of the tower's top about its base?

**•22** An astronaut is tested in a centrifuge with radius 10 m and rotating according to  $\theta = 0.30t^2$ . At  $t = 5.0 \text{ s}$ , what are the magnitudes of the (a) angular velocity, (b) linear velocity, (c) tangential acceleration, and (d) radial acceleration?

**•23 SSM WWW** A flywheel with a diameter of 1.20 m is rotating at an angular speed of 200 rev/min. (a) What is the angular speed of the flywheel in radians per second? (b) What is the linear speed of a point on the rim of the flywheel? (c) What constant angular ac-

celeration (in revolutions per minute-squared) will increase the wheel's angular speed to 1000 rev/min in 60.0 s? (d) How many revolutions does the wheel make during that 60.0 s?

**••24** A vinyl record is played by rotating the record so that an approximately circular groove in the vinyl slides under a stylus. Bumps in the groove run into the stylus, causing it to oscillate. The equipment converts those oscillations to electrical signals and then to sound. Suppose that a record turns at the rate of  $33\frac{1}{3} \text{ rev/min}$ , the groove being played is at a radius of 10.0 cm, and the bumps in the groove are uniformly separated by 1.75 mm. At what rate (hits per second) do the bumps hit the stylus?

**••25 SSM** (a) What is the angular speed  $\omega$  about the polar axis of a point on Earth's surface at latitude  $40^\circ \text{ N}$ ? (Earth rotates about that axis.) (b) What is the linear speed  $v$  of the point? What are (c)  $\omega$  and (d)  $v$  for a point at the equator?

**••26** The flywheel of a steam engine runs with a constant angular velocity of 150 rev/min. When steam is shut off, the friction of the bearings and of the air stops the wheel in 2.2 h. (a) What is the constant angular acceleration, in revolutions per minute-squared, of the wheel during the slowdown? (b) How many revolutions does the wheel make before stopping? (c) At the instant the flywheel is turning at 75 rev/min, what is the tangential component of the linear acceleration of a flywheel particle that is 50 cm from the axis of rotation? (d) What is the magnitude of the net linear acceleration of the particle in (c)?

**••27** A seed is on a turntable rotating at  $33\frac{1}{3} \text{ rev/min}$ , 6.0 cm from the rotation axis. What are (a) the seed's acceleration and (b) the least coefficient of static friction to avoid slippage? (c) If the turntable had undergone constant angular acceleration from rest in 0.25 s, what is the least coefficient to avoid slippage?

**••28** In Fig. 10-31, wheel A of radius  $r_A = 10 \text{ cm}$  is coupled by belt B to wheel C of radius  $r_C = 25 \text{ cm}$ . The angular speed of wheel A is increased from rest at a constant rate of  $1.6 \text{ rad/s}^2$ . Find the time needed for wheel C to reach an angular speed of 100 rev/min, assuming the belt does not slip. (Hint: If the belt does not slip, the linear speeds at the two rims must be equal.)

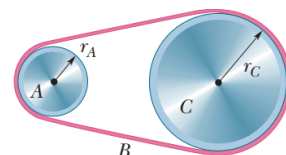


Figure 10-31 Problem 28.

**••29** Figure 10-32 shows an early method of measuring the speed of light that makes use of a rotating slotted wheel. A beam of

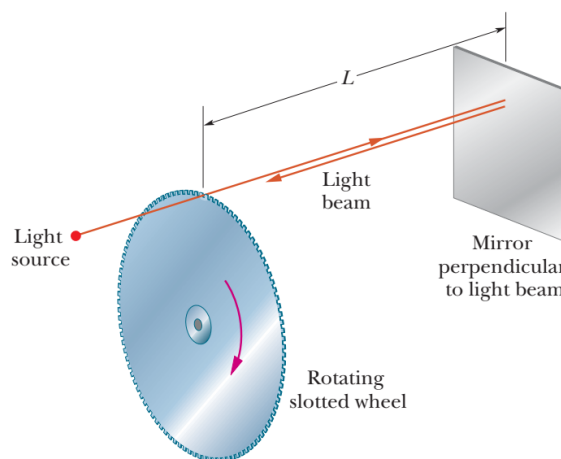


Figure 10-32 Problem 29.



light passes through one of the slots at the outside edge of the wheel, travels to a distant mirror, and returns to the wheel just in time to pass through the next slot in the wheel. One such slotted wheel has a radius of 5.0 cm and 500 slots around its edge. Measurements taken when the mirror is  $L = 500$  m from the wheel indicate a speed of light of  $3.0 \times 10^5$  km/s. (a) What is the (constant) angular speed of the wheel? (b) What is the linear speed of a point on the edge of the wheel?

••30 A gyroscope flywheel of radius 2.83 cm is accelerated from rest at  $14.2 \text{ rad/s}^2$  until its angular speed is 2760 rev/min. (a) What is the tangential acceleration of a point on the rim of the flywheel during this spin-up process? (b) What is the radial acceleration of this point when the flywheel is spinning at full speed? (c) Through what distance does a point on the rim move during the spin-up?

••31 GO A disk, with a radius of 0.25 m, is to be rotated like a merry-go-round through 800 rad, starting from rest, gaining angular speed at the constant rate  $\alpha_1$  through the first 400 rad and then losing angular speed at the constant rate  $-\alpha_1$  until it is again at rest. The magnitude of the centripetal acceleration of any portion of the disk is not to exceed  $400 \text{ m/s}^2$ . (a) What is the least time required for the rotation? (b) What is the corresponding value of  $\alpha_1$ ?

••32 A car starts from rest and moves around a circular track of radius 30.0 m. Its speed increases at the constant rate of  $0.500 \text{ m/s}^2$ . (a) What is the magnitude of its *net* linear acceleration 15.0 s later? (b) What angle does this net acceleration vector make with the car's velocity at this time?

#### Module 10-4 Kinetic Energy of Rotation

••33 SSM Calculate the rotational inertia of a wheel that has a kinetic energy of 24 400 J when rotating at 602 rev/min.

••34 Figure 10-33 gives angular speed versus time for a thin rod that rotates around one end. The scale on the  $\omega$  axis is set by  $\omega_s = 6.0 \text{ rad/s}$ . (a) What is the magnitude of the rod's angular acceleration? (b) At  $t = 4.0$  s, the rod has a rotational kinetic energy of 1.60 J. What is its kinetic energy at  $t = 0$ ?

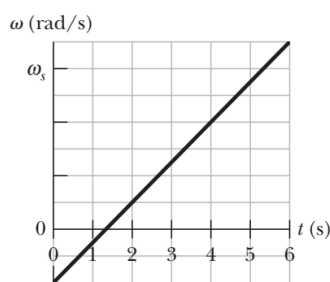


Figure 10-33 Problem 34.

#### Module 10-5 Calculating the Rotational Inertia

••35 SSM Two uniform solid cylinders, each rotating about its central (longitudinal) axis at 235 rad/s, have the same mass of 1.25 kg but differ in radius. What is the rotational kinetic energy of (a) the smaller cylinder, of radius 0.25 m, and (b) the larger cylinder, of radius 0.75 m?

••36 Figure 10-34a shows a disk that can rotate about an axis at

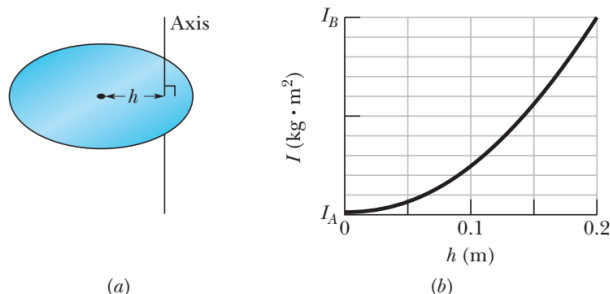


Figure 10-34 Problem 36.

a radial distance  $h$  from the center of the disk. Figure 10-34b gives the rotational inertia  $I$  of the disk about the axis as a function of that distance  $h$ , from the center out to the edge of the disk. The scale on the  $I$  axis is set by  $I_A = 0.050 \text{ kg} \cdot \text{m}^2$  and  $I_B = 0.150 \text{ kg} \cdot \text{m}^2$ . What is the mass of the disk?

••37 SSM Calculate the rotational inertia of a meter stick, with mass 0.56 kg, about an axis perpendicular to the stick and located at the 20 cm mark. (Treat the stick as a thin rod.)

••38 Figure 10-35 shows three 0.0100 kg particles that have been glued to a rod of length  $L = 6.00$  cm and negligible mass. The assembly can rotate around a perpendicular axis through point  $O$  at the left end. If we remove one particle (that is, 33% of the mass), by what percentage does the rotational inertia of the assembly around the rotation axis decrease when that removed particle is (a) the innermost one and (b) the outermost one?

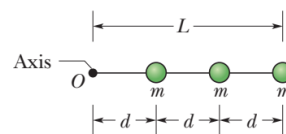


Figure 10-35 Problems 38 and 62.

••39 Trucks can be run on energy stored in a rotating flywheel, with an electric motor getting the flywheel up to its top speed of  $200\pi \text{ rad/s}$ . Suppose that one such flywheel is a solid, uniform cylinder with a mass of 500 kg and a radius of 1.0 m. (a) What is the kinetic energy of the flywheel after charging? (b) If the truck uses an average power of 8.0 kW, for how many minutes can it operate between chargings?

••40 Figure 10-36 shows an arrangement of 15 identical disks that have been glued together in a rod-like shape of length  $L = 1.0000$  m and (total) mass  $M = 100.0$  mg. The disks are uniform, and the disk arrangement can rotate about a perpendicular axis through its central disk at point  $O$ . (a) What is the rotational inertia of the arrangement about that axis? (b) If we approximated the arrangement as being a uniform rod of mass  $M$  and length  $L$ , what percentage error would we make in using the formula in Table 10-2e to calculate the rotational inertia?

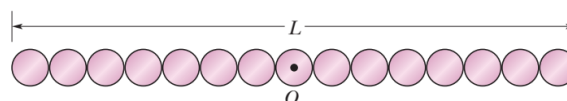


Figure 10-36 Problem 40.

••41 GO In Fig. 10-37, two particles, each with mass  $m = 0.85$  kg, are fastened to each other, and to a rotation axis at  $O$ , by two thin rods, each with length  $d = 5.6$  cm and mass  $M = 1.2$  kg. The combination rotates around the rotation axis with the angular speed  $\omega = 0.30 \text{ rad/s}$ . Measured about  $O$ , what are the combination's (a) rotational inertia and (b) kinetic energy?

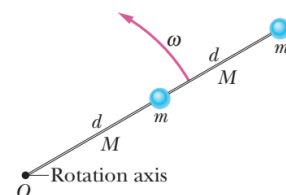


Figure 10-37 Problem 41.

••42 The masses and coordinates of four particles are as follows: 50 g,  $x = 2.0$  cm,  $y = 2.0$  cm; 25 g,  $x = 0$ ,  $y = 4.0$  cm; 25 g,  $x = -3.0$  cm,  $y = -3.0$  cm; 30 g,  $x = -2.0$  cm,  $y = 4.0$  cm. What are the rotational inertias of this collection about the (a)  $x$ , (b)  $y$ , and (c)  $z$  axes? (d) Suppose that we symbolize the answers to (a) and (b) as  $A$  and  $B$ , respectively. Then what is the answer to (c) in terms of  $A$  and  $B$ ?

### Module 11-1 Rolling as Translation and Rotation Combined

•1 A car travels at 80 km/h on a level road in the positive direction of an  $x$  axis. Each tire has a diameter of 66 cm. Relative to a woman riding in the car and in unit-vector notation, what are the velocity  $\vec{v}$  at the (a) center, (b) top, and (c) bottom of the tire and the magnitude  $a$  of the acceleration at the (d) center, (e) top, and (f) bottom of each tire? Relative to a hitchhiker sitting next to the road and in unit-vector notation, what are the velocity  $\vec{v}$  at the (g) center, (h) top, and (i) bottom of the tire and the magnitude  $a$  of the acceleration at the (j) center, (k) top, and (l) bottom of each tire?

•2 An automobile traveling at 80.0 km/h has tires of 75.0 cm diameter. (a) What is the angular speed of the tires about their axles? (b) If the car is brought to a stop uniformly in 30.0 complete turns of the tires (without skidding), what is the magnitude of the angular acceleration of the wheels? (c) How far does the car move during the braking?

### Module 11-2 Forces and Kinetic Energy of Rolling

•3 **SSM** A 140 kg hoop rolls along a horizontal floor so that the hoop's center of mass has a speed of 0.150 m/s. How much work must be done on the hoop to stop it?

•4 A uniform solid sphere rolls down an incline. (a) What must be the incline angle if the linear acceleration of the center of the sphere is to have a magnitude of  $0.10g$ ? (b) If a frictionless block were to slide down the incline at that angle, would its acceleration magnitude be more than, less than, or equal to  $0.10g$ ? Why?

•5 **ILW** A 1000 kg car has four 10 kg wheels. When the car is moving, what fraction of its total kinetic energy is due to rotation of the wheels about their axles? Assume that the wheels are uniform disks of the same mass and size. Why do you not need to know the radius of the wheels?

••6 Figure 11-30 gives the speed  $v$  versus time  $t$  for a 0.500 kg object of radius 6.00 cm that rolls smoothly down a  $30^\circ$  ramp. The scale on the velocity axis is set by  $v_s = 4.0$  m/s. What is the rotational inertia of the object?

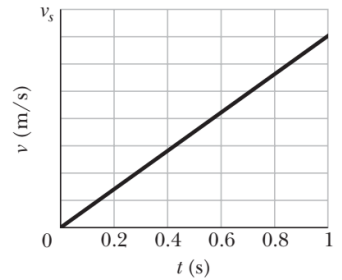


Figure 11-30 Problem 6.

••7 **ILW** In Fig. 11-31, a solid cylinder of radius 10 cm and mass 12 kg starts from rest and rolls without slipping a distance  $L = 6.0$  m down a roof that is inclined at angle  $\theta = 30^\circ$ . (a) What is the angular speed of the cylinder about its center as it leaves the roof? (b) The roof's edge is at height  $H = 5.0$  m. How far horizontally from the roof's edge does the cylinder hit the level ground?

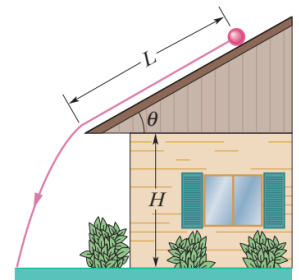


Figure 11-31 Problem 7.

••8 Figure 11-32 shows the potential energy  $U(x)$  of a solid ball that can roll along an  $x$  axis. The scale on the  $U$  axis is set by  $U_s = 100$  J. The ball is uniform, rolls smoothly, and has a mass of 0.400 kg. It is released at  $x = 7.0$  m headed in the negative direction of the  $x$  axis with a mechanical energy of 75 J. (a) If the ball can reach  $x = 0$  m, what is its speed there, and if it cannot, what is its turning point? Suppose, instead, it is headed in the positive direction of the  $x$  axis when it is released at  $x = 7.0$  m with 75 J. (b) If the ball can reach  $x = 13$  m, what is its speed there, and if it cannot, what is its turning point?

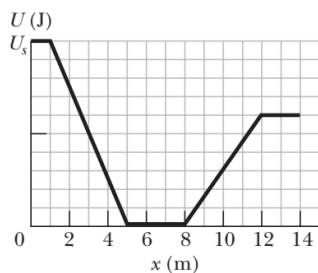


Figure 11-32 Problem 8.

••9 In Fig. 11-33, a solid ball rolls smoothly from rest (starting at height  $H = 6.0$  m) until it leaves the horizontal section at the end of the track, at height  $h = 2.0$  m. How far horizontally from point A does the ball hit the floor?

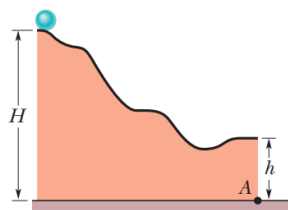


Figure 11-33 Problem 9.

••10 A hollow sphere of radius 0.15 m, with rotational inertia  $I = 0.040 \text{ kg} \cdot \text{m}^2$  about a line through its center of mass, rolls without slipping up a surface inclined at  $30^\circ$  to the horizontal. At a certain initial position, the sphere's total kinetic energy is 20 J. (a) How much of this initial kinetic energy is rotational? (b) What is the speed of the center of mass of the sphere at the initial position? When the sphere has moved 1.0 m up the incline from its initial position, what are (c) its total kinetic energy and (d) the speed of its center of mass?

••11 In Fig. 11-34, a constant horizontal force  $\vec{F}_{\text{app}}$  of magnitude 10 N is applied to a wheel of mass 10 kg and radius 0.30 m. The wheel rolls smoothly on the horizontal surface, and the acceleration of its center of mass has magnitude  $0.60 \text{ m/s}^2$ . (a) In unit-vector notation, what is the frictional force on the wheel? (b) What is the rotational inertia of the wheel about the rotation axis through its center of mass?

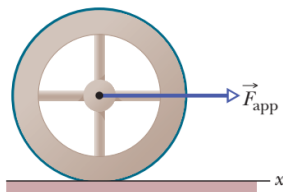


Figure 11-34 Problem 11.

••12 In Fig. 11-35, a solid brass ball of mass 0.280 g will roll smoothly along a loop-the-loop track when released from rest along the straight section. The circular loop has radius  $R = 14.0$  cm, and the ball has radius  $r \ll R$ . (a) What is  $h$  if the ball is on the verge of leaving the track when it reaches the top of the loop? If the ball is released at height  $h = 6.00R$ , what are the (b) magnitude and (c) direction of the horizontal force component acting on the ball at point Q?

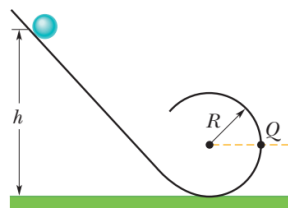


Figure 11-35 Problem 12.

••13 Nonuniform ball. In Fig. 11-36, a ball of mass  $M$  and radius  $R$

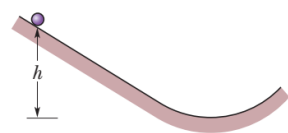


Figure 11-36 Problem 13.

rolls smoothly from rest down a ramp and onto a circular loop of radius 0.48 m. The initial height of the ball is  $h = 0.36$  m. At the loop bottom, the magnitude of the normal force on the ball is  $2.00Mg$ . The ball consists of an outer spherical shell (of a certain uniform density) that is glued to a central sphere (of a different uniform density). The rotational inertia of the ball can be expressed in the general form  $I = \beta MR^2$ , but  $\beta$  is not 0.4 as it is for a ball of uniform density. Determine  $\beta$ .

••14 In Fig. 11-37, a small, solid, uniform ball is to be shot from point P so that it rolls smoothly along a horizontal path, up along a ramp, and onto a plateau. Then it leaves the plateau horizontally to land on a game board, at a horizontal distance  $d$  from the right edge of the plateau. The vertical heights are  $h_1 = 5.00$  cm and  $h_2 = 1.60$  cm. With what speed must the ball be shot at point P for it to land at  $d = 6.00$  cm?

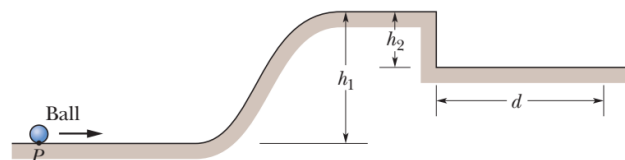


Figure 11-37 Problem 14.

••15 A bowler throws a bowling ball of radius  $R = 11$  cm along a lane. The ball (Fig. 11-38) slides on the lane with initial speed  $v_{\text{com},0} = 8.5 \text{ m/s}$  and initial angular speed  $\omega_0 = 0$ . The coefficient of kinetic friction between the ball and the lane is 0.21. The kinetic frictional force  $\vec{f}_k$  acting on the ball causes a linear acceleration of the ball while producing a torque that causes an angular acceleration of the ball. When speed  $v_{\text{com}}$  has decreased enough and angular speed  $\omega$  has increased enough, the ball stops sliding and then rolls smoothly. (a) What then is  $v_{\text{com}}$  in terms of  $\omega$ ? During the sliding, what are the ball's (b) linear acceleration and (c) angular acceleration? (d) How long does the ball slide? (e) How far does the ball slide? (f) What is the linear speed of the ball when smooth rolling begins?

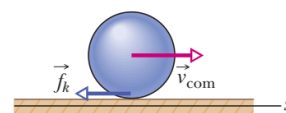


Figure 11-38 Problem 15.

••16 Nonuniform cylindrical object. In Fig. 11-39, a cylindrical object of mass  $M$  and radius  $R$  rolls smoothly from rest down a ramp and onto a horizontal section. From there it rolls off the ramp and onto the floor, landing a horizontal distance  $d = 0.506$  m from the end of the ramp. The initial height of the object is  $H = 0.90$  m; the end of the ramp is at height  $h = 0.10$  m. The object consists of an outer cylindrical shell (of a certain uniform density) that is glued to a central cylinder (of a different uniform density). The rotational inertia of the object can be expressed in the general form  $I = \beta MR^2$ , but  $\beta$  is not 0.5 as it is for a cylinder of uniform density. Determine  $\beta$ .

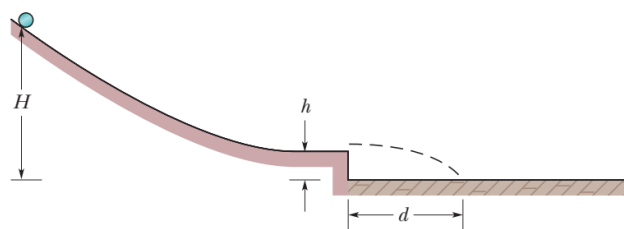


Figure 11-39 Problem 16.