Calculus project:

Q#1: Define the term Calculus in your own understanding and its application in Mathematics and computer science, and justify your reasoning with at least three practical applications

[Real world problems].

Definition of Calculus: Calculus is a branch of mathematics that deals with change. It is primarily used to obtain optimized solutions to practical problems. Calculus helps us understand the relationships between values that are connected by functions, focusing on concepts like differentiation, integration, limits, and functions. These concepts allow us to analyze how things change over time or space.

Applications in Computer Science:

1. **Artificial Intelligence and Machine Learning:** Calculus is essential for understanding the mathematical principles behind machine learning algorithms, particularly for training models like neural networks. In these algorithms, calculus helps optimize and adjust parameters (weights) using techniques like gradient descent, which minimizes errors in predictions.
2. **Data Science:** Calculus plays a crucial role in data science for tasks such as statistical modeling, hypothesis testing, and understanding large datasets. Calculus is used to derive key metrics, model continuous data patterns, and optimize algorithms, all of which are essential for making accurate, data-driven decisions.
3. **Computer Graphics:** In computer graphics, calculus is used to model and manipulate shapes, render scenes, and simulate physical phenomena such as light, motion, and shadows. For example, it is used to calculate the trajectories of objects, simulate realistic animations, and apply transformations to 3D models.

Applications in Mathematics:

1. A region with curved sides has a centre of mass (Centroid).
2. The space that exists between two curves.
3. The area enclosed by a curve.

Conclusion: We can clearly see that CALCULUS almost played its role in every aspect of life .If we talk about Mathematics , Physics , Data Science , Computer Science , Artificial Intelligence calculus not only involved but play very important role in understanding the major and basic concepts and also helps us to think and observe things keenly and in result advancement of science and technology .

Q#2: Identify at least two real world applications, focus must be on single variable optimization.

Real-World Examples of Single Variable Optimization :

Engineering: Building a Bridge Think about designing a bridge. It has to be strong enough to handle cars and trucks but not so overbuilt that it costs a fortune. How calculus helps: Engineers use it to figure out the perfect balance strong enough to last but efficient enough to save materials and money. It's all about finding that middle ground where safety and cost meet.

Economics: Optimizing Production Let's say you run a bakery. If you bake too many loaves of bread, you risk wasting ingredients. If you bake too few, you lose out on potential sales. How calculus helps: It can help you model your profit based on how much bread you make and show you the perfect amount to bake to maximize earnings without waste.

Conclusion : When you break it down, calculus isn't just math for textbooks; it's a tool we use to solve real problems. Whether it's setting the right price, designing better structures, or even perfecting your soccer skills, calculus is behind the scenes, helping us make smarter decisions every day.

Question # 3

(A) -

Given :- $V(r) = C(r_0 - r)r^2 \quad (r_0/2 < r < r_0)$.

To show :- $V(r)$ is maximized at $r = \frac{2}{3}r_0$.

Solution :-

As our function is ;-

$$V(r) = C(r_0 - r)r^2$$

$$V(r) = Cr_0r^2 - Cr^3.$$

Taking derivative w.r.t 'r' ;-

$$\frac{dv}{dr} = C\left(\frac{d}{dr}(r_0r^2) - \frac{d}{dr}(r^3)\right)$$

$$\frac{dv}{dr} = C(2r_0r - 3r^2)$$

Now for critical points ;- put $\left(\frac{dv}{dr} = 0\right)$.

$$C(2r_0r - 3r^2) = 0.$$

$$2r_0r - 3r^2 = 0$$

$$2r_0\cancel{r} = 3r^{\cancel{2}}.$$

$$3r = 2r_0$$

$$\boxed{r = \frac{2}{3}r_0}.$$

Now again taking derivative in order to tell it is maximized or minimized at $\boxed{r = 2/3\, r_0}$

$$\frac{d^2V}{d^2r} = \frac{d}{dr} C(2r_0 r - 3r^2)$$

$$\frac{d^2V}{d^2r} = C(2r_0 - 6r)$$

put $\left(r = \frac{2}{3} r_0\right)$

$$\frac{d^2V}{d^2r} = C(2r_0 - \cancel{6}^{2} \times \frac{2}{\cancel{3}} r_0)$$

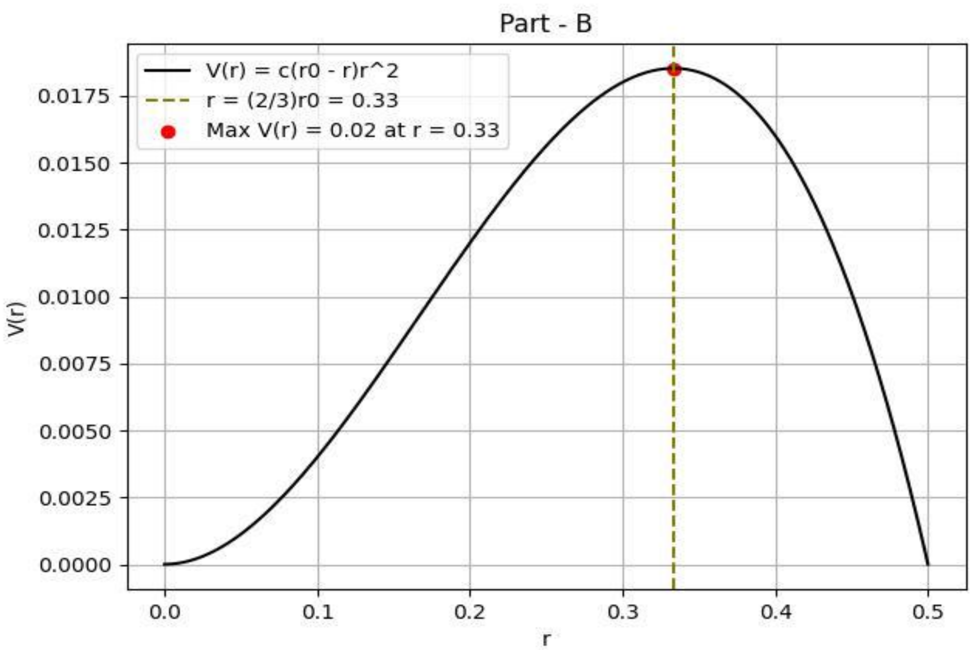
$$\frac{d^2V}{d^2r} = C(2r_0 - 4r_0)$$

$$\frac{d^2V}{d^2r} = -C2r_0$$

$$\boxed{\frac{d^2V}{d^2r} = -C2r_0}$$

The negative sign indicates that at critical point $\left(r = \frac{2}{3} r_0\right)$ the function is being maximized.

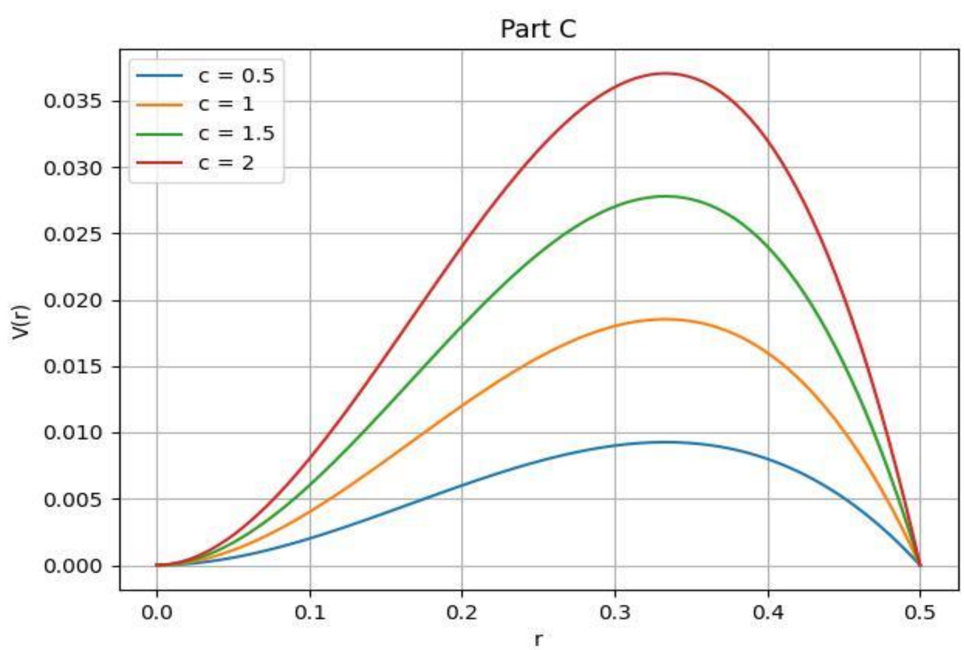
Question no 3(b)



- import numpy as np
- import matplotlib.pyplot as plt
- r0 = 0.5
- c = 1
- def V(r):
- return c * (r0 - r) * r**2
- r = np.linspace(0, r0, 500)
- v = V(r)
- R_Max = (2/3) * r0
- V_Max = V(R_Max)
- # Plot the graph
- plt.figure(figsize=(10, 5))
- plt.plot(r, v, label="V(r) = c(r0 - r)r^2", color="black")
- plt.axvline(R_Max, color="olive", linestyle="--", label=f"r = (2/3)r0 = {R_Max:.2f}")

- plt.scatter(R_Max, V_Max, color="red", label=f"Max V(r) = {V_Max:.2f} at r = {R_Max:.2f}")
- # Labels and title
- plt.title(" Part - B ")
- plt.xlabel("r")
- plt.ylabel("V(r)")
- plt.legend()
- plt.grid(True)
- plt.show()

Question no 3(c)



The command that was used to plot this graph is:

- import numpy as np
- import matplotlib.pyplot as plt
- r_max = 0.5

- def V(r, c):
- return c * (r_max - r) * r**2
- r = np.linspace(0, r_max, 100)
- c_val = [0.5, 1, 1.5, 2]
- plt.figure(figsize=(7, 5))
- for c in c_val:
- v = V(r, c)
- plt.plot(r, v, label=f"c = {c}")
- plt.title("Part C")
- plt.xlabel("r")
- plt.ylabel("V(r)")
- plt.legend()
- plt.grid(True)
- plt.show()