Introduction to Hypothesis Testing and its Application in Real-Life and Data Science

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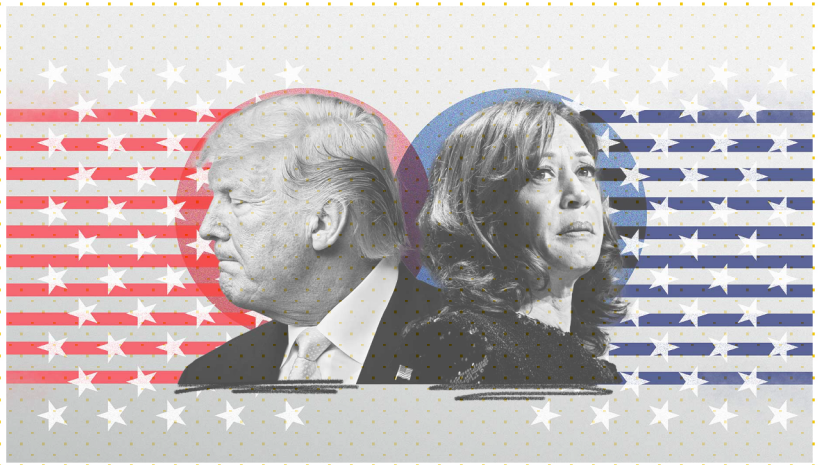


Announcements + schedule + links

- Lectures

  In person, SCHEDULE
- Discussions

  In person, SCHEDULE
- Office Hours

  SCHEDULES
- Website| Piazza (Q&A) | Canvas (recordings + quizzes) | GradeScope (HW submission) | GitHub (code + slides) | Google Colab
- HWs, HW solutions, due dates and exams.

COLAB

Course Plan: Where We Are Now

Week	Date	Topic	Activities
1	Monday – Jan 6	Combinatorics and Counting	HW 1 Programming 1 Exit Tickets 1-3
	Wednesday – Jan 8	Set Theory and Axioms of Statistics	
	Friday – Jan 10	Conditional Probability and Bayes' Rule	
2	Monday – Jan 13	Law of Total Probability, Independence, and Dependence	HW 2 Programming 2 Exit Tickets 4-6
	Wednesday – Jan 15	Conditional Independence, Chain Rule, Random Variables	
	Friday – Jan 17	Probability Mass Function, Cumulative Distribution, Expectation, and Variance	
3	Monday – Jan 20	Bernoulli and Binomial Distributions	Exam 1 Exit Tickets 7-9
	Wednesday – Jan 22	Geometric, Hypergeometric, and Poisson Distributions	
	Friday – Jan 24	Continuous Random Variables, Uniform Distribution, Normal Distribution	
4	Monday – Jan 27	Normal Distribution	HW 3 Programming 3 Exit Tickets 10-12
	Wednesday – Jan 29	Exponential Distribution and Its Memorylessness	
	Friday – Jan 31	Joint Distribution	
5	Monday – Feb 3	Conditional Joint Distribution, LOTUS	HW 4 Programming 4 Exit Tickets 13-15
	Wednesday – Feb 5	Covariance and Correlation	
	Friday – Feb 7	Independent Random Variables	

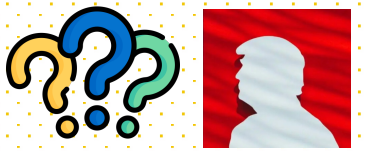
Week	Date	Topic	Activities
6	Monday – Feb 10	Central Limit Theorem	Exam 2 Exit Tickets 16-18 Project proposal
	Wednesday – Feb 12	Markov's Inequality, Chebyshev's Inequality, and Law of Large Numbers	
	Friday – Feb 14	Conditional Expectation, Linearity of Expectation, Inverse Transform Method	
7	Monday – Feb 17	Introduction to Statistics	HW 5 Programming 5 Exit Tickets 19-21
	Wednesday – Feb 19	Point Estimation and Maximum Likelihood Estimator	
	Friday – Feb 21	Interval Estimation, Confidence Interval	
8	Monday – Feb 24	Hypothesis Testing, z-Test, Fisher Test,p-Value	Project phase 1 Exit Tickets 22-24
	Wednesday – Feb 26	Project Phase 1 discussion	
	Friday – Feb 28	Chi-squared Distribution and Test, t-Distribution and t-Test, p-value cont.	
9	Monday – March 3	Nonparametric Tests, Permutation Test, p-Value Adjustment	HW 6 Programming 6 Exit Tickets 25-27
	Wednesday – March 5	Linear Regression	
	Friday – March 7	Bayesian Inference, Maximum a Posteriori Method, and Conjugate Priors	
10	Monday – March 10	Projects presentation and evaluation	Projects presentation and evaluation Final
	Wednesday – March 12	Projects presentation and evaluation	
	Friday – March 14	Final	

Today's Learning Goals

- Prerequisites: probability, distributions, Central Limit Theorem.
- Introduction to Hypothesis Testing (Using statistics to evaluate some hypothesis/statement about a population)
  - What is a Hypothesis?
  - Null and Alternative Hypotheses
  - Decision Making
    - Test Statistics
    - Accept or reject Null Hypothesis
    - P-value
  - Specific Statistical Tests
    - Z-test
    - Fisher Exact Test
  - Errors in Hypothesis Testing
    - Type I and Type II errors
  - More Applications
    - Hypothesis Testing in Data Science

Practical Use Cases of Statistics (Election)

Predicting which of the USA presidency candidates will win the election.
We ask a sample of population and estimate which candidate is more likely to win based on their opinions.



President Trump will earn more electoral votes.

Former Vise-President Harris's will earn more electoral votes.

Sample (poll here) can give us a good **estimate** about who is more probable to win the election.

images sources: first, second

Hypothesis

A **hypothesis** is an assumption (statement) about parameters of a population or distribution that can be True or Not True.

Note: parameter is a numerical or measurable characteristic of a population, e.g., $average\ (\mu), variance\ (\sigma^2), min, max, range, etc.$

Example: If we have a collection of individuals, hypothesize the max, average, etc. (e.g. average height of Americans is more than 5 feet 8 inches).

Example: We can have an assumption that the average GPA of first year students is > 3.0.

Example: We can hypothesize that men play more video games than women.

Example: We have a coin and our hypothesis is that it is unbiased ($P(Head) = \frac{1}{2}$).

Null and Alternative Hypothesis

We aim to evaluate the effectiveness of a medicine on **a group of patients (population)** using a randomized clinical trial. We give the **actual medicine** to half of the participants, while the other half receive **a placebo**.

6 people
took medicine

6 people
took placebo

Medicine is not effective (has no effect on the outcome vs placebo) **Null Hypothesis (H_0)**

Medicine is effective (there will be a difference between two populations).
Alternative Hypothesis (H_a or H_1)

The assumption that is believed to be true until evidence is shown that it is not, is called **Null Hypothesis (H_0).**

The complementary (often) view is called **Alternative Hypothesis (H_a or H_1).**

Null and Alternative Hypothesis

Null Hypothesis (H_0): The assumption that is believed to be true.
Alternative Hypothesis (H_a or H_1): The complementary (often) view of Null hypothesis.

Discuss real-life scenarios of the null (H_0) and alternative hypotheses (H_1).

Example: **Null hypothesis (H_0)** : Men play more video games than women.
Alternative hypothesis (H_a) : Women play more video games than men.

Example: **Null Hypothesis (H_0)** : People like Rock and pop music equally.
Alternative hypothesis (H_a) : More than 50% of people prefer Rock music over Pop.

Example: **Null Hypothesis (H_0)** : The coin used in a soccer game is unbiased ($P(Head) = \frac{1}{2}$).
Alternative hypothesis (H_a) : The coin is biased ($P(Head) \neq \frac{1}{2}$).

Test Statistics

6 people
took medicine

6 people
took placebo

cured , # not cured

after 1 month

cured, # not cured

Which characteristic(s) should we study to measure the medicine's impact?

Test statistics is a function of sample data (mean, standard deviation, count, etc.) that helps us determine whether we should reject the Null Hypothesis or not.

Hypothesis Testing

6 people
took medicine

6 people
took placebo

4 cured , 2 not cured

after 1 month

1 cured, 5 not cured

Medicine is not effective. **Null Hypothesis (H_0)**

Medicine is effective (there will be a difference between two populations).
Alternative Hypothesis (H_a or H_1)

Hypothesis Testing: After collecting data, if the data is consistent with the null hypothesis, we **accept the null hypothesis (H_0)**.
Otherwise, if we have an strong evidence for the alternative hypothesis we reject the Null Hypothesis and **accept the Alternative Hypothesis**.

Hypothesis Testing

6 people
took medicine

4 cured , 2 not cured

6 people
took placebo

1 cured, 5 not cured

after 1 month

Should we allow the pharmaceutical company to release the medicine?

Is it possible to observe 4 cured , 2 not cured in the population who took placebo?
Or is the observed result statistically significant to reject the Null Hypothesis?

Should we set a threshold for rejecting the Null Hypothesis (H_0)?

P-value (probability value)

6 people
took medicine

6 people
took placebo

4 cured , 2 not cured

after 1 month

1 cured, 5 not cured

Let's show number of cured people with variable X.
We want to know how probable it is to observe 4 cured , 2 not cured or more extreme cases (5 or 6 cured) assuming H_0 was True: $P(X \geq 4 \,|H_0)$

Remember: Null Hypothesis (H_0): Medicine is not effective (there is no difference between two populations and any observed difference is due to chance or other factors rather than medicine)

Does the pharmaceutical company want $P(X \geq 4|H_0)$ be high or low?

Significance Level

6 people
took medicine

6 people
took placebo

4 cured , 2 not cured

after 1 month

1 cured, 5 not cured

Let's show number of cured people with variable X
We want to know how probable it is to observe 4 (or more) cured if there was no difference between the two populations (medicine was unimpactful) (H_0 was True):

$P(X \geq 4 \,|H_0)$

$$\boldsymbol{P(X \geq threshold \,|H_0) \leq \alpha}$$

significance level (α) is the probability of mistakenly rejecting H_0.
Note: significance level is often set to be **1%** or **5%.**

P-value

6 people
Took medicine

6 people
Took placebo

4 cured , 2 not cured

after 1 month

1 cured, 5 not cured

If $P(\boldsymbol{X \geq threshold | H_0}) \leq \boldsymbol{\alpha}$, we can reject $Null\ Hypothesis\ (H_0)$ with $\boldsymbol{1-\alpha}$ **confidence**.

$P(X \geq x \mid H_0)$ is called p-value and is the probability of observing value x or more extreme values of x assuming H_0 holds.

Later in the slides: Fisher Exact Test

Testing the Hypothesis - P-value

Null Hypothesis (H_0) : The coin used in a soccer game is unbiased ($P(Head) = \frac{1}{2}$).

Alternative hypothesis (H_a) : The coin is biased ($P(Head) \neq \frac{1}{2}$).

Experiment: We flip the coin **20 times**

Test Statistic: Number of heads (we show it with variable X)

Head Tail

Intuitively we might say if $X = 9, 10\ or\ 11$, we do not reject the **Null hypothesis (H_0)** and we can conclude that the coin is not biased and otherwise we reject the H_0 in favor of H_a

Testing the Hypothesis - P-value

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Experiment: We flip the coin **20 times**

Test Statistic: Number of heads (we show it with variable X)

Head Tail

What **range** of X allows us to conclude that the coin is unbiased (accept H_0) with 99% confidence?

Testing the Hypothesis - P-value

Null Hypothesis (H_0) : The coin used in a soccer game is unbiased ($P(Head) = \frac{1}{2}$).

Alternative hypothesis (H_a) : The coin is biased ($P(Head) \neq \frac{1}{2}$).

$P(X = 20 \mid H_0)$: If the coin was fair, how likely was it to observe 20 heads?

#Heads (X)	$P_{H_0}(X = x) = P(X = x \mid H_0)$
20	
19	
18	
17	
16	
15	
14	

Head Tail

Testing the Hypothesis - P-value

Null Hypothesis (H_0) : The coin used in a soccer game is unbiased ($P(Head) = \frac{1}{2}$).

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# Heads (X)	$P(X = x \mid H_0)$
20, 0	$\binom{20}{20} * \frac{1}{2^{20}} = 0.0001\%$
19, 1	$\binom{20}{19} * \frac{1}{2^{20}} = 0.0019\%$
18, 2	$\binom{20}{18} * \frac{1}{2^{20}} = 0.0181\%$
17, 3	$\binom{20}{17} * \frac{1}{2^{20}} = 0.1087\%$
16, 4	$\binom{20}{16} * \frac{1}{2^{20}} = 0.4621\%$
15, 5	$\binom{20}{15} * \frac{1}{2^{20}} = 1.4786\%$
14, 6	$\binom{20}{14} * \frac{1}{2^{20}} = 3.6964\%$

Head Tail

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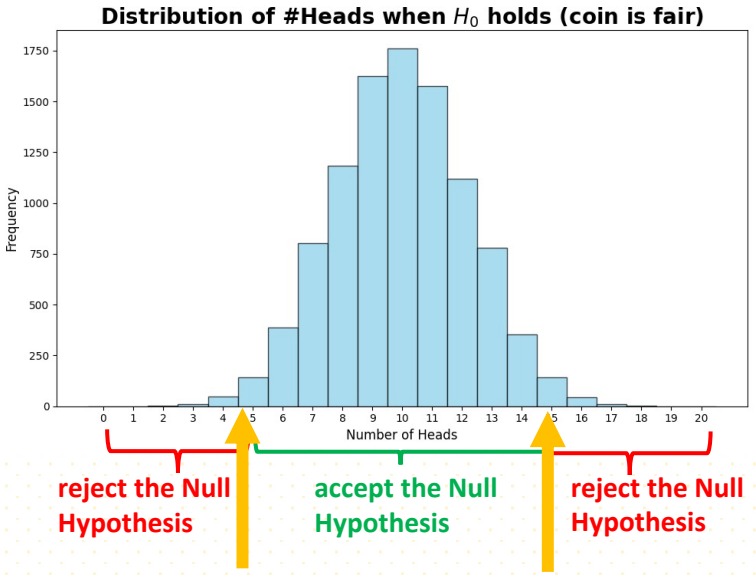

Testing the Hypothesis - P-value

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Alternative hypothesis (H_a) : The coin is biased ($P(Head) \neq \frac{1}{2}$).

# Heads (X)	$P(X = x \mid H_0)$	$P(X \geq x \mid H_0)$
20	$\binom{20}{20} * \frac{1}{2^{20}} = 0.0001\%$	
19	$\binom{20}{19} * \frac{1}{2^{20}} = 0.0019\%$	
18	$\binom{20}{18} * \frac{1}{2^{20}} = 0.0181\%$	
17	$\binom{20}{17} * \frac{1}{2^{20}} = 0.1087\%$	
16	$\binom{20}{16} * \frac{1}{2^{20}} = 0.4621\%$	
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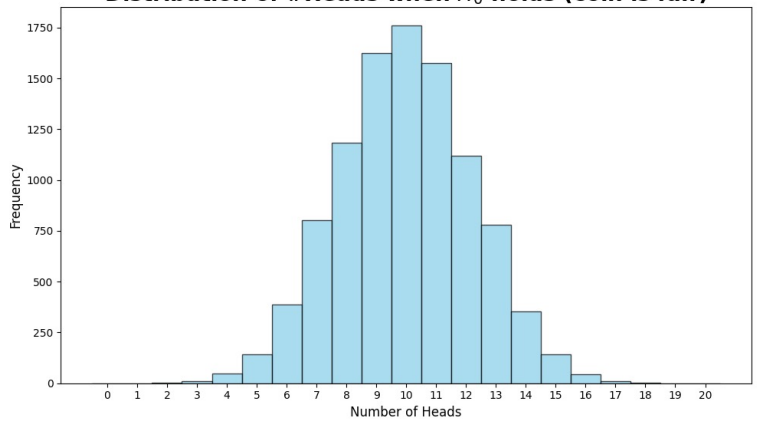
Head Tail



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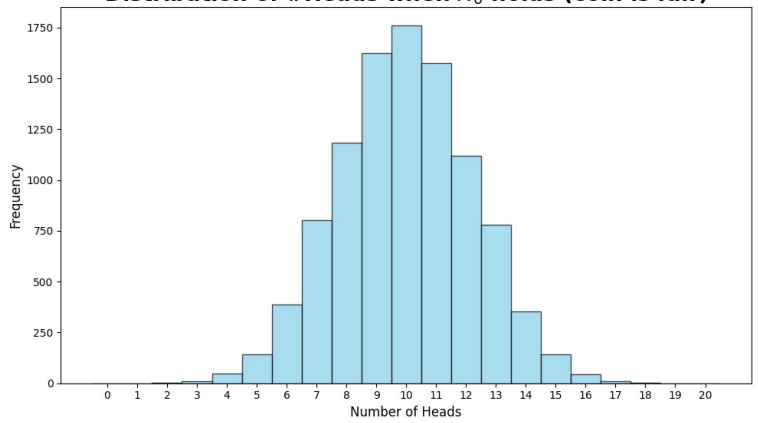
Alternative hypothesis (H_a) : The coin is biased ($P(Head) \neq \frac{1}{2}$).

# Heads (X)	$P(X = x \mid H_0)$	$P(X \geq x \mid H_0)$
20	$\binom{20}{20} * \frac{1}{2^{20}} = 0.0001\%$	0.0001%
19	$\binom{20}{19} * \frac{1}{2^{20}} = 0.0019\%$	0.002%
18	$\binom{20}{18} * \frac{1}{2^{20}} = 0.0181\%$	0.0201%
17	$\binom{20}{17} * \frac{1}{2^{20}} = 0.1087\%$	0.1288%
16	$\binom{20}{16} * \frac{1}{2^{20}} = 0.4621\%$	0.5909%
15	$\binom{20}{15} * \frac{1}{2^{20}} = 1.4786\%$	2.0695%
14	$\binom{20}{14} * \frac{1}{2^{20}} = 3.6964\%$	5.7659%

Head Tail



Testing the Hypothesis - P-value

Null Hypothesis (H_0) : The coin used in a soccer game is unbiased ($P(Head) = \frac{1}{2}$).

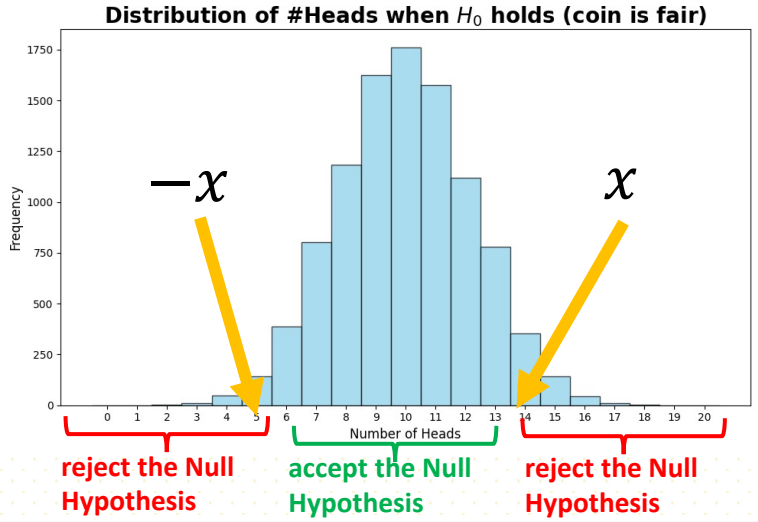
Alternative hypothesis (H_a) : The coin is biased ($P(Head) \neq \frac{1}{2}$).

#Heads (X)	$P_{H_0}(X = x) = P(X = x \mid H_0)$	$P_{H_0}(\lvert X \rvert \geq x)$
20, 0	$\binom{20}{20} * \frac{1}{2^{20}} = 0.0001\%$	0.0001% ×2
19, 1	$\binom{20}{19} * \frac{1}{2^{20}} = 0.0019\%$	0.002% ×2
18, 2	$\binom{20}{18} * \frac{1}{2^{20}} = 0.0181\%$	0.0201% ×2
17, 3	$\binom{20}{17} * \frac{1}{2^{20}} = 0.1087\%$	0.1288% ×2
16, 4	$\binom{20}{16} * \frac{1}{2^{20}} = 0.4621\%$	0.5909% ×2
15, 5	$\binom{20}{15} * \frac{1}{2^{20}} = 1.4786\%$	2.0695% ×2
14, 6	$\binom{20}{14} * \frac{1}{2^{20}} = 3.6964\%$	5.7659% ×2

$$P(|X| \geq x \mid H_0) \leq 5\%$$

What is the confidence interval for accepting the null hypothesis?

$$5 \leq x \leq 15$$



Testing the Hypothesis - P-value

Null Hypothesis (H_0) : The coin used in a soccer game is unbiased ($P(Head) = \frac{1}{2}$).

Alternative hypothesis (H_a) : The coin is biased ($P(Head) \neq \frac{1}{2}$).

#Heads (X)	$1 - P_{H_0}(x \leq X \leq x)$
20, 0	0.0001% ×2
19, 1	0.002% ×2
18, 2	0.0201% ×2
17, 3	0.1288% ×2
16, 4	0.5909% ×2
15, 5	2.0695% ×2
14, 6	5.7659% ×2

Note: If significance level is 5% and P-value is less than that, i.e., $P_{H_0}(|X| \geq x) < 5\%$, we can claim our results as significant results and can reject H_0 with 95% confidence.

Testing the Hypothesis - P-value

Null Hypothesis (H_0) : The coin used in a soccer game is unbiased ($P(Head) = \frac{1}{2}$).

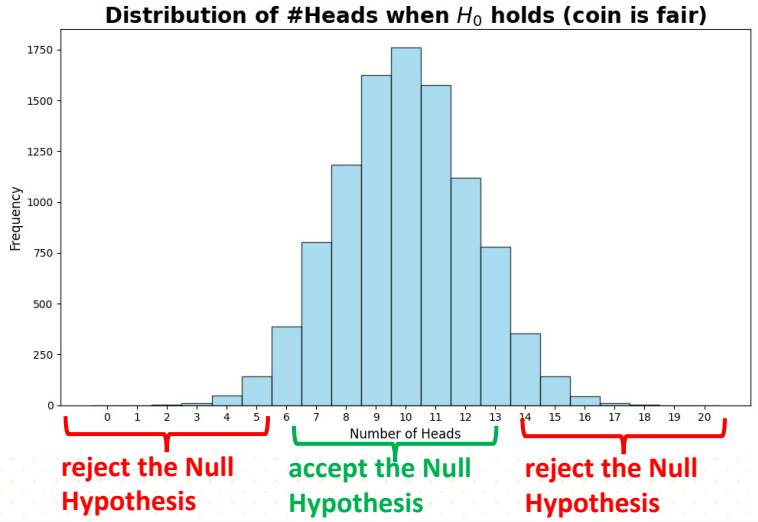
Alternative hypothesis (H_a) : The coin is biased ($P(Head) \neq \frac{1}{2}$).

#Heads (X)	$P_{H_0}(X = x) = P(X = x \mid H_0)$	$P_{H_0}(\lvert X \rvert \geq x)$
20, 0	$\binom{20}{20} * \frac{1}{2^{20}} = 0.0001\%$	0.0001% ×2
19, 1	$\binom{20}{19} * \frac{1}{2^{20}} = 0.0019\%$	0.002% ×2
18, 2	$\binom{20}{18} * \frac{1}{2^{20}} = 0.0181\%$	0.0201% ×2
17, 3	$\binom{20}{17} * \frac{1}{2^{20}} = 0.1087\%$	0.1288% ×2
16, 4	$\binom{20}{16} * \frac{1}{2^{20}} = 0.4621\%$	0.5909% ×2
15, 5	$\binom{20}{15} * \frac{1}{2^{20}} = 1.4786\%$	2.0695% ×2
14, 6	$\binom{20}{14} * \frac{1}{2^{20}} = 3.6964\%$	5.7659% ×2

$$P(|X| \geq x \mid H_0) \leq 5\%$$

What is the confidence interval for accepting the null hypothesis?

$$3 \leq x \leq 17$$



Z-test (when the sample size is large enough)

Null Hypothesis (H_0) : The coin used in a soccer game is unbiased ($P(Head) = \frac{1}{2}$).

Alternative hypothesis (H_a) : The coin is biased ($P(Head) \neq \frac{1}{2}$).

$$Y_i = \begin{cases} 1 & \text{if heads} \\ 0 & \text{if tails} \end{cases} \qquad X = \sum_{i=1}^{n=100} Y_i$$

$$Y_i \sim Bernoulli\left(p = \frac{1}{2}\right) \rightarrow E(Y_i) = p = \frac{1}{2} \quad and \quad Var(Y_i) = p(1-p) = \frac{1}{4}$$

$$E(X) = \mu = np = 100 * \frac{1}{2} = 50 \quad and \quad Var(X) = \sigma^2 = np(1-p) = 100 * \frac{1}{4} = 25$$

$$CLT: X \sim \text{Normal}(\mu = 50, \sigma = 5)$$

Now having a normal variable, we can use a statistics test called Z-test.

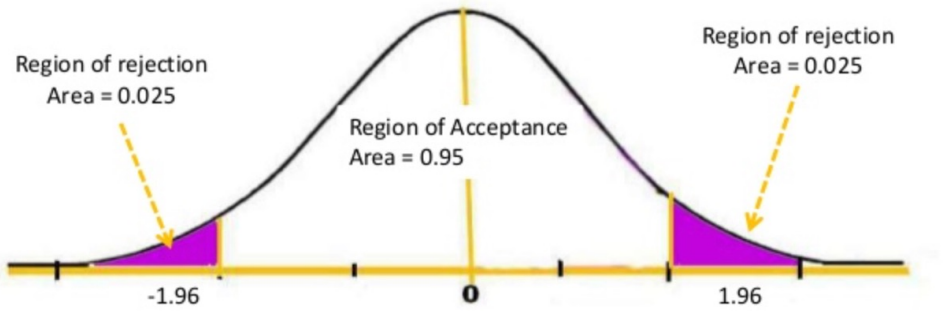
Z-test (when the sample size is large enough)

Null Hypothesis (H_0) : The coin used in a soccer game is unbiased ($P(Head) = \frac{1}{2}$).

Alternative hypothesis (H_a) : The coin is biased ($P(Head) \neq \frac{1}{2}$).

We first convert X to a standard normal variable $Z = \frac{X-\mu}{\sigma} = \frac{X-50}{5}$

$Z \sim Normal(0, 1)$



$P(|Z| < a) = 0.95$ (confidence level is 95%)

$a = |\Phi^{-1}(1 - 0.025)| = 1.96 \rightarrow \quad Confidence\ Interval: [-1.96, 1.96]$

$-1.96 < Z < 1.96$ and $Z = \frac{X-\mu}{\sigma} = \frac{X-50}{5}$. Hence, $-1.96 < \frac{X-50}{5} < 1.96 \rightarrow 40.2 < X < 59.8$

Note: $\Phi(z) = P(Z \leq z)$ and here $\Phi(1.96) = P(Z \leq 1.96) = 0.975$

Fisher's Exact Test

6 people
Took medicine

3 cured , 3 not cured
50% of people got cured

after 1 month

6 people
Took placebo

2 cured, 4 not cured
33% of people got cured

If $P(X \geq threshold|H_0) \leq \alpha$, we can reject H_0 with $1-\alpha$ confidence.

	cured	Not cured	Total
Took medicine	3 = a	3 = b	a + b
Took placebo	2 = c	4 = d	c + d
Total	a + c	b + d	n

$$P(observing\ this\ table) = \frac{\binom{a+b}{a}\binom{c+d}{c}}{\binom{n}{a+c}} = \frac{\binom{3+3}{3}\binom{2+4}{2}}{\binom{3+3+2+4}{3+2}} = 0.37$$

$$\boldsymbol{P(observing\ this\ table\ or\ more\ extreme\ results\ |H_0)} = ???$$

Fisher's Exact Test

6 people
Took medicine

6 people
Took placebo

3 cured , 3 not cured

after 1 month

2 cured, 4 not cured

$$P(observing\ this\ table\ or\ more\ extreme\ results\ |H_0) \gg 0.05$$

insufficient evidence to reject the null hypothesis

	cured	Not cured
Took medicine	**3 = a**	3 = b
Took placebo	2 = c	4 = d

	cured	Not cured
Took medicine	**5 = a**	1 = b
Took placebo	2 = c	4 = d

	cured	Not cured
Took medicine	**4 = a**	2 = b
Took placebo	2 = c	4 = d

	cured	Not cured
Took medicine	**6 = a**	0 = b
Took placebo	2 = c	4 = d

Fisher's Exact Test

6 people
Took medicine

6 people
Took placebo

5 cured , 1 not cured

after 1 month

0 cured, 6 not cured

If $P(X \geq threshold|H_0) \leq \alpha$, we can reject H_0 with $1-\alpha$ confidence.

	cured	Not cured	Total
Took medicine	5 = a	1 = b	a + b
Took placebo	0 = c	6 = d	c + d
Total	a + c	b + d	n

$$P(observing\ this\ table\ or\ more\ extreme\ results) = \frac{\binom{5+1}{5}\binom{0+6}{0}}{\binom{12}{5+0}} + \frac{\binom{6+0}{6}\binom{0+6}{0}}{\binom{12}{6+0}} = 0.0152 < 0.05$$

sufficient evidence to reject the null hypothesis with $1 - 0.05 = 95\%$ confidence.

Type I and Type II Error

$$\boldsymbol{P(X \geq threshold\,|H_0) \leq \alpha}$$

Note: significance level (α) is the maximum probability of making Type I error (mistakenly rejecting H_0) we allow to happen.

	Reality = H_0	**Reality = H_1**
Test result = H_0	Correct (TN)	Type II error FN
Test result = H_1	Type I error FP	Correct (TP)

False Positive

False Negative

Fisher's Exact Test

	Reality $= H_0$	Reality $= H_1$
Test result $= H_0$	Correct (TN)	Type II error FN
Test result $= H_1$	Type I error FP	Correct (TP)

6 people
Took medicine

3 cured , 3 not cured
50% of people got cured

after 1 month

6 people
Took placebo

2 cured, 4 not cured
33% of people got cured

Observation: $P(observing\ above\ or\ more\ extreme\ results) \gg\gg\gg 0.05$
Claim: Reject the Null Hypothesis

What type of error happens in above scenario if we reject the null hypothesis?

A P-Value use-case in Data Science Feature Selection

	R&D Spend	Administration	Marketing Spend	State	Profit
0	165349.20	136897.80	471784.10	New York	192261.83
1	162597.70	151377.59	443898.53	California	191792.06
2	153441.51	101145.55	407934.54	Florida	191050.39
3	144372.41	118671.85	383199.62	New York	182901.99
4	142107.34	91391.77	366168.42	Florida	166187.94

from **50_Startups** dataset

Fit a multiple Linear Regression to the data:

$$\textbf{Profit} = \textbf{R\&D spend} * C_1 + \textbf{Administraion} * C_2 + \textbf{Marketing Spend} * C_3 + \textbf{State} * C_4 + \beta \text{ (intercept)}$$

Which features are unlikely to contribute significantly to the **Profit** prediction?

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Backward Elimination Steps:

1) Start with all features $f1, f2, f3, \dots, fn$, run a multiple Linear Regression to get:

$$\textbf{Profit} = \textbf{R\&D spend} * \mathrm{C}_1 + \textbf{Administraion} * C_2 + \textbf{Marketing Spend} * \mathrm{C}_3 + \textbf{State} * \mathrm{C}_4 + \beta \text{ (intercept)}$$

1) Identify the least significant feature (calculate features' p-values and remove one with highest p-value)
2) Re-run the multiple linear regression with reduced number of features

   e.g. if **State** is insignificant:

$$\textbf{Profit} = \textbf{R\&D spend} * \mathrm{C}_1 + \textbf{Administraion} * C_2 + \textbf{Marketing Spend} * \mathrm{C}_3 + \beta$$

   Perform 2 and 3 iteratively and stop when all the remaining features are significant

How do we calculate P-value for each feature?

	R&D Spend	Administration	Marketing Sp
0	165349.20	136897.80	47178
1	162597.70	151377.59	44389
2	153441.51	101145.55	40793
3	144372.41	118671.85	383199.62
4	142107.34	91391.77	366168.42

COLAB

Exit Ticket – Due Tonight at 23:59 PM

Resources

Fundamentals of Statistics and Probability, Sharif University of Technology, Professor Ali Sharifi Zarchi

Statistics and Data Science, UC San Diego, Professor Alon Orlitsky

Review: What We've Covered So Far

When do we use statistics?

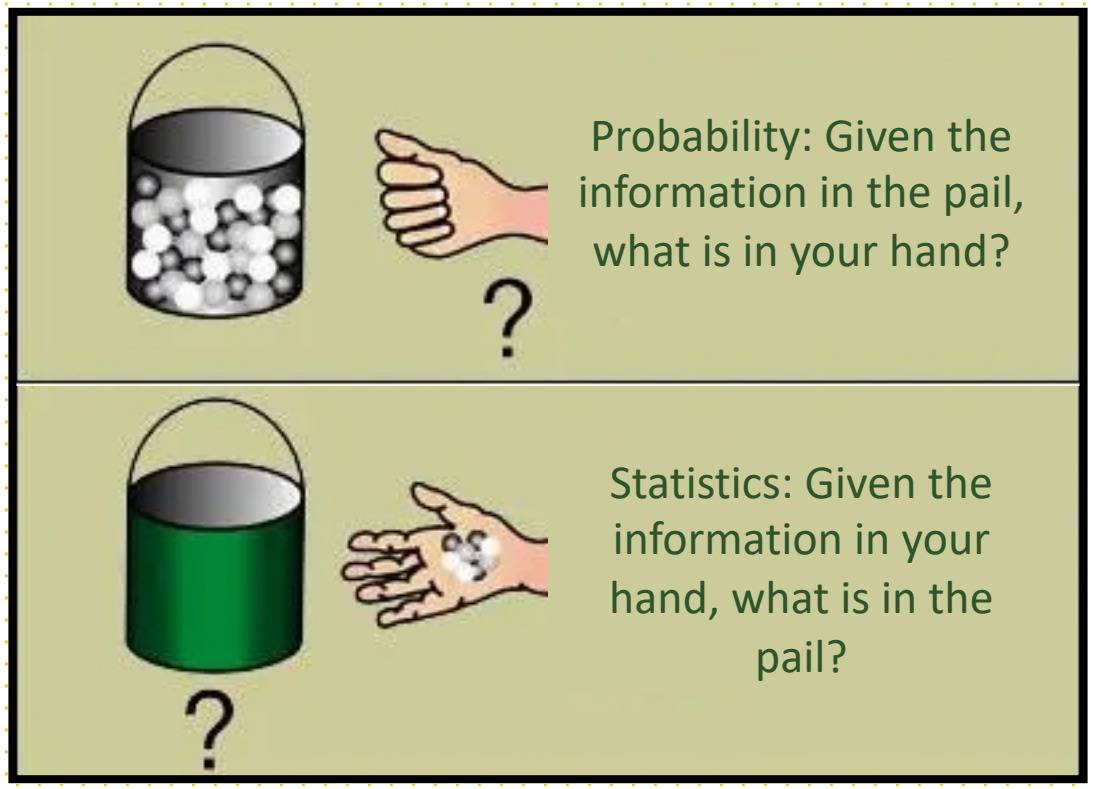

image source: https://teachingstatisticsandprobability.wordpress.com/

A P-Value use-case in Data Science Feature Selection


How do we calculate P-value for each feature?

Null Hypothesis: : The given feature has no effect in calculating the profit.
Alternative Hypothesis: The given feature has some effect in the target dependent variable (profit).

P(observing the given feature coefficient or more extreme values of it| H_0) $\leq \alpha$
If P-value $\leq \alpha$, then we can reject the null hypothesis.
If P-value $> \alpha$, then we can accept the null hypothesis and remove the feature.

	R&D Spend	Administration	Marketing Spend	State	Profit
0	165349.20	136897.80	471784.10	New York	192261.83
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HW bonus question: Testing a hypothesis about a population

Choose a Null Hypothesis and prove/disprove it.

Example: election

Null Hypothesis (H_0) : Harris will win the election (will get more votes).

Alternative Hypothesis (H_a or H_1) : Trump will win the election.

Steps for testing a hypothesis:

1) Design an experiment (Test)
2) Define numerical outcomes (Test statistics to measure election outcomes) (e.g., the number of votes for each candidate, the ratio of votes (e.g. >51%))
3) Gather data (sample of population, please submit you hypothesis by tonight so we can gather data from your classmates)
4) Data consistent with null hypothesis or not?
   A. Calculate P(T=test statistics | H_0) $\leq$ significance value (often 5%, 1%, etc.)
      Yes. H_0 is inconsistent with data (reject Null)
      No. H_0 is consistent with data (accept Null)

HW bonus question example
Practical Use Cases of Hypothesis Testing (A/B Test)

We want to determine which version of our software performs better. We test a sample of users and estimate the preferred version based on their behavior or feedback before the final release.

Null Hypothesis (H_0) : Version A of the software performs better than Version B.

Alternative Hypothesis (H_a or H_1) : Version B of the software performs better than Version A