Hyperbolic growth

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摘　要

When a quantity grows towards a singularity under a finite variation(a "finite-time singularity"), it is said to undergo hyperbolic growth[1]. More precisely, the reciprocal function $\frac{1}{x}$ has a hyperbola as a graph, and has a singularity at 0, meaning that the limit as $x \to 0$ is infinite: any similar graph is said to exhibit hyperbolic growth.

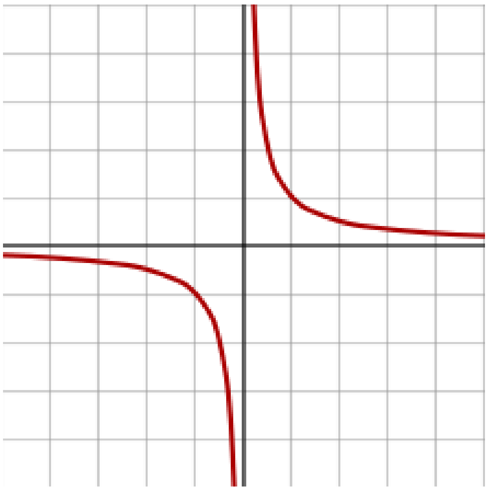

图 **1:** figs:hyperbolic-growth

关键词: hyperbolic growth

1 Description

If the output of a function is inversely proportional to its input, or inversely proportional to the difference from a given value x_0, the function will exhibit hyperbolic growth, with a singularity at x_0.

In the real world, hyperbolic growth is created by certain non-linear positive feedback mechanisms[2].

1.1 Comparisons with other growth

Like exponential growth and logistic growth, hyperbolic growth is highly nonlinear, but differs in important respects. These functions can be confused, as exponential growth, hyperbolic growth, and the first half of logistic growth are convex functions; however their asymptotic behavior(behavior as input gets large) differs dramatically.

- logistic growth is constrained(has a finite limit, even as time goes to infinity),
- exponential growth grows to infinity as time goes to infinity (but is always finite for finite time),
- hyperbolic growth has a singularity in finite time (grows to infinity at a finite time).

2 Mathematical example

The function

$$x(t) = \frac{1}{t_c - t} \tag{1}$$

exhibits hyperbolic growth with a singularity at time t_c: in the limit as $t \rightarrow t_c$, the function goes to infinity.

More generally, the function

$$x(t) = \frac{K}{t_c - t} \tag{2}$$

exhibits hyperbolic growth, where K is a scale factor.

Note that this algebraic function can be regarded as analytical solution for the function's differential[1]:

$$\frac{dx}{dt} = \frac{K}{(t_c - t)^2} = \frac{x^2}{K} \tag{3}$$

This means that with hyperbolic growth the absolute growth rate of the variable x in the moment t is proportional to the square of the value of x in the moment t. Respectively, the quadratic-hyperbolic function looks as follows:

$$x(t) = \frac{K}{(t_c - t)^2} \tag{4}$$

3 See also

- Exponential growth
- Logistic growth
- Mathematical singularity

参考文献

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[3] WIKI. Hyperbolic growth[J]. Wikipedia:https://en.wikipedia.org/wiki/Hyperbolic_growth.