29/4/24 (After Mid)

3- DIFFERENTIATION:

R.C

r_A $\quad\quad$ $r_{inst} \rightarrow$ Slope of tangent

$f'(x)$ or $\frac{dy}{dx}$

$$\boxed{r_{inst} = m = f'(x)}$$

$$r_{inst} = m = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

~~(By definition)~~

(By first Principle Rule).

→ Technique:

$\frac{dy}{dx}$: Slope of tangent line
: Find slope ($f'(x)$)

→ **Explicit → Implicit**

(Explain in terms of x and y) (y on one side only)

(Both side x and y appears) (Separating from mixed equation)

(i) $y = x^2 + 2x + 3$

$y = \sqrt{x}$

(i) $yx + y - x + 1 = 0$

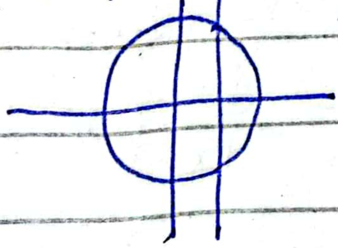
$y(x+1) - (x-1) = 0$

$y = \frac{x-1}{x+1}$

→ Equation of the circle:

$\underbrace{x^2 + y^2 = 1}_{\text{(Implicit)}} \Rightarrow y = \pm\sqrt{1-x^2}$

$\therefore \boxed{\underbrace{y_1 = \sqrt{1-x^2}}_{\text{(upper semi circle)}}, \underbrace{y_2 = -\sqrt{1-x^2}}_{\text{(lower semi circle)}}}$.



Ex #1:

$x = y^2 \Rightarrow y = f(x)$

$y = \pm\sqrt{x}$

$\boxed{f_1(x) = \sqrt{x}, \quad f_2(x) = -\sqrt{x}}$

→ Folium of Descarts:

$$x^3+y^3=3xy$$

$$-x^3+3xy=y^3 \qquad \boxed{y=?}$$

$$x^3=3xy-y^3$$

$$x^3=y(3x-y^2) \quad \times$$

→ "Can not solve it for y in terms of x."

→ Apply Direct differentiation on implicit function or equation.

(i) $xy=1$

$$y=\frac{1}{x}$$

Differentiate w.r.t x

$$\frac{dy}{dx}=\frac{d}{dx}(x^{-1})$$

$$=-x^{-2}$$

(Simplify method)

$$\boxed{\frac{dy}{dx}=-\frac{1}{x^2}}$$

(Direct method)

$xy=1$

Differentiate w.r.t x

$$\frac{d}{dx}(xy)=\frac{d}{dx}(1)$$

$$y+x\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-y}{x}$$

$$=\frac{-\frac{1}{x}}{x} \quad \text{... from (ii)}$$

$$\boxed{\frac{dy}{dx}=\frac{-1}{x^2}}$$

(ii) $5y^2+\sin y=x^2$ find $\frac{dy}{dx}$

Differentiate w.r.t x

$$10y\frac{dy}{dx}+\cos y\frac{dy}{dx}=2x$$

(Again Derivative by $\frac{dy}{dx}$ as not differentiating $\frac{d}{dx}$(...))

$$\frac{dy}{dx}(10y+\cos y)=2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x}{10y+\cos y} \quad \text{Ans}$$

→ **Example:**

$$4x^2-2y^2=9 \quad \ldots (1) \qquad \boxed{\frac{d^2y}{dx^2}=?}$$

Differentiate w.r.t x:

$$8x-4y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{8x}{4y}$$

$$\frac{dy}{dx}=\frac{2x}{y} \rightarrow (2)$$

Again differentiate w.r.t x

$$\frac{d^2y}{dx^2}=\frac{(2)(y)-(2x)\left(\frac{dy}{dx}\right)}{y^2}$$

$$=\frac{2y-2x\left(\frac{2x}{y}\right)}{y^2} \quad \text{(From (2))}$$

$$=\frac{2y^2-4x^2}{y^3}$$

$$=\frac{-(4x^2-2y^2)}{y^3}$$

$$=\frac{-9}{y^3} \quad \text{(From (1))}$$

$$y^2 - x + 1 = 0$$

$$2y\frac{dy}{dx} - 1 + 0 = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2y}$$

$$m = \frac{1}{2y}$$

∵ in $y = f(x)$ Applying Direct differentiation

∵ $\frac{dy}{dx} = f'(x) = m$

- m at $(2,-1)$

$$m\Big|_{(2,-1)} = \frac{1}{2(-1)} = -\frac{1}{2}$$

$$\boxed{m = -\frac{1}{2}}$$

- m at $(2,1)$

$$m = \frac{1}{2y}$$

$$m\Big|_{(2,1)} = \frac{1}{2(1)}$$

$$\boxed{m = \frac{1}{2}}$$

→ **Example #5:**

(a) $x^3 + y^3 = 3xy$

$$\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx}(3xy)$$

$$3x^2 + 3y^2\frac{dy}{dx} = 3\left[x\cdot\frac{dy}{dx} + 1\cdot y\right]$$

$$3x^2 + 3y^2\frac{dy}{dx} = 3\left[x\frac{dy}{dx} + y\right]$$

$$3\left(x^2 + y^2\frac{dy}{dx}\right] = 3\left[x\frac{dy}{dx} + y\right]$$

$$x^2 + y^2\frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$x^2 - y = x\frac{dy}{dx} - y^2\frac{dy}{dx}$$

$$x^2 - y = (x - y^2)\frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{x^2 - y}{x - y^2}}$$

(b) Equation of Tangent line passing through $(3/2, 3/2)$

$$y - y_0 = m(x - x_0) \quad \cdots\cdots (3)$$

Here $x_0 = 3/2$, $y_0 = 3/2$

m at $(3/2, 3/2)$

→ step 2: $m = \dfrac{y - x^2}{y^2 - x}$

$$m\Big|_{(3/2, 3/2)} = \frac{(3/2) - (3/2)^2}{(3/2)^2 - (3/2)} = -1$$

$\therefore$ Eq. 3 becomes:

$$y - \frac{3}{2} = (-1)\left(x - \frac{3}{2}\right)$$

$$y - 3/2 = -x + 3/2$$

$$x + y = \frac{3}{2} + \frac{3}{2}$$

$$x + y = 6/2$$

$$\boxed{x + y = 3}$$

$\rightarrow$ If $m = 0$

$$\frac{dy}{dx} = 0$$

$$\frac{y - x^2}{y^2 - x} = 0$$

$$\Rightarrow \quad y - x^2 = 0$$

$$\boxed{y = x^2}$$

Replace $y = x^2$ in eq. 1

$$x^3 + y^3 = 3xy$$

$$x^3 + (x^2)^3 = 3x(x^2)$$

$$x^3 + x^6 = 3x^3$$

$$x^3 + x^6 - 3x^3 = 0$$

$$x^6 - 2x^3 = 0$$

** $\boxed{\text{Ex } 3.1\ (3\text{-}20)}$

* $(x^2)^3 = x^{2\times3} = x^6$

$x^2 \cdot x^2 = x^{2+2} = x^4$

$x^3(x^3-2) = 0$

$x3 = 0$ $\qquad$ $x3-2=0$

$x = 0$ $\qquad$ $x = (2)^{1/3}$

$(0,0)$ $\qquad$ $(2^{1/3}, 2^{2/3})$ ✓✓

putting in

$y = x2$

6/5/24

Inverse Function Differentiation:

$f^{-1}(x)$

$\boxed{y = f(x)}$

↓

One-to-One function

$f: X \to Y$

$f(x) = y \quad \forall x$

: onto (Horizontal)

: into (Unique Number)

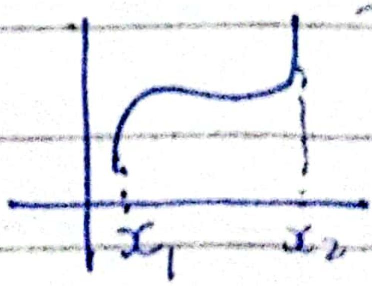
$f(x) = x^2$

$y = x^2$

$\pm y^{1/2} = x$

⇒ one-to-one function ($f^{-1}(x)$ exists)

: $f(x)$ is increasing or decreasing



→ $f'(x) = m > 0$ (Increasing)

$m < 0$ (Decreasing)

$m = 0$ (constant)

→ : Conditions:

→ If increasing or decreasing

→ Slope exist

→ Function one-to-one

→ $f^{-1}(x)$ differentiation possible.

: $f(x)$: Range $\rightarrow f^{-1}(x)$: Domain

(2) $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$

(3) $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}}$

: **Example #2:**

$(-\infty, +\infty)$

$f(x) = x^5 + x + 1$

$f'(x) = 5x^4 + 1$

$f'(x) = 5x^4 + 1 > 0$

$\forall\ x \in (-\infty, +\infty)$

: overall behavior increasing or decreasing

: $5(-1)^4 + 1$

$5+1$

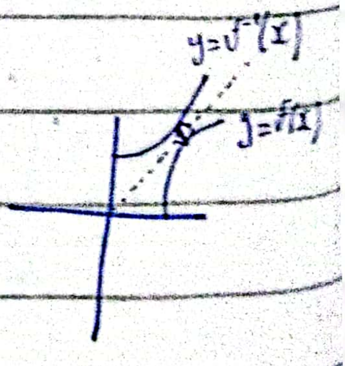
$= 6$

(a) $\Rightarrow f'(x)$ is increasing on $(-\infty, +\infty)$.

$\Rightarrow$ slope of the function $m > 0$ on $(-\infty, +\infty)$.

$\Rightarrow f$ is one to one function on $(-\infty, +\infty)$

(b) $y = f^{-1}(x)$



Applying f on both sides:

$f(y) = f(f^{-1}(x))$

$f(y) = x$

: $f(x) = x^5 + x + 1$

$$\boxed{y^5+y+1=x} \quad (A)$$

Differentiate w.r.t x:

$$\frac{dy}{dx}(5y^4+1)=1$$

$$\frac{dy}{dx}=\frac{1}{5y^4+1} \quad \text{Ans.}$$

(c) $(f^{-1})'(1)=?$

Sol:-

$$\frac{dy}{dx}=\frac{1}{5y^4+1}$$

at $x=1$

Since $y^5+y+1=x$ (From A)

put $x=1$

$y^5+y+1=1$

$y^5+y=0$

$y(y^4+1)=0$

$y=0$, $y^4+1=0$

$y=0$, $y^4=-1$ (Not possible)

$y=0$

$(x=1)(y=0)$

$$\frac{dy}{dx}=\frac{1}{5(0)+1} \Rightarrow \left.\frac{dy}{dx}\right|_{x=1}=1 \quad \text{Ans.}$$

$\because f(3)=(3)^5+3+1$

$f(t)=t^5+t+1$

$f(y)$

$y=x^2$; $y'=2x$

$x=2$

$y'=4$

y' at $x=2$

*. Monday
(Paper show)
*: Next Monday (3.1, 3.2, 3.3(1-5), 3.6)

* (a) $f(x) = 5x^3 + x - 7$

$f'(x) = 15x^2 + 1 > 0 \quad \forall x \in (+\infty, -\infty)$

$\Rightarrow f^{-1}(x)$ exists

(b) Let $y = f^{-1}(x)$

i.e: $f(y) = x$

$5y^3 + y - 7 = x$

Differentiate w.r.t y:

$15y^2 + 1 = \frac{dx}{dy}$

$$\boxed{\frac{1}{15y^2+1} = \frac{1}{\frac{dx}{dy}}} \quad \text{—— (1)}$$

$\therefore$ By (3)

$\Rightarrow \frac{dy}{dx} = \frac{1}{15y^2+1}$ Ans.

- Direct method:

Since $5y^3 + y - 7 = x$

Differentiate w.r.t x

$(15y^2 + 1)\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{15y^2+1}$ —— (2)

From 1 and 2:

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{15y^2+1}$$

→ Q8: $f(x) = \frac{1}{x^2}$ $x > 0$

$f'(x) = (x^{-2})$

$f'(x) = -2x^{-3}$

$f'(x) = -\frac{2}{x^3}$

→ Q: $f(x) = \frac{2}{x+3}$

$y = \frac{2}{x+3}$ $\Rightarrow y = f(x)$
i.e: $f^{-1}(y) = x$

$yx + 3y = 2$

$yx = 2 - 3y$

$x = \frac{2-3y}{y}$: divide by 'y'

$x = \frac{2}{y} - \frac{3y}{y}$

$x = \frac{2}{y} - 3$

$f^{-1}(y) = \frac{2}{y} - 3$:from (i)

Replace y by x:

$f^{-1}(x) = \frac{2}{x} - 3$ Ans

:By Direct Function Method

Differentiate w.r.t x

$\frac{d}{dx}(f^{-1}(x)) = \frac{d}{dx}\left(\frac{2}{x} - 3\right) = \frac{-2}{x^2}$ Ans.

④ Trignometric Functions:

let $y = f^{-1}(x)$

$y = \sin^{-1}x$

or $\sin(y) = x$

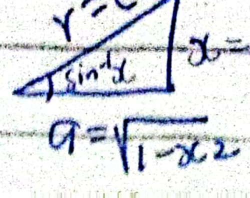
Differentiate w.r.t x:

$\cos y \cdot \frac{dy}{dx} = 1$

or $\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\cos(f^{-1}(x))}$

$\frac{dy}{dx} = \frac{1}{\cos(\sin^{-1}x)}$ ..... ①

→



$:c^2 = a^2 + b^2$

*: Ex: 3.3
(1-58)
(odd Number)

$\sin\theta = \dfrac{x}{1}$

$\cos\theta = \dfrac{\sqrt{1-x^2}}{1}$

$\cos(\sin^{-1}x) = \sqrt{1-x^2}$

putting in (1)

$\dfrac{1}{\cos(\sin^{-1}x)} = \dfrac{1}{\sqrt{1-x^2}}$

: 6 Formulas (pg #201) (slide #225)

Q: $\dfrac{d}{dx}\sin^{-1}x^3 = \dfrac{1}{\sqrt{1-(x^3)^2}}\dfrac{d}{dx}(x^3)$

$= \dfrac{3x^2}{\sqrt{1-x^6}}$ Ans.

Q: $\dfrac{d}{dx}\sec^{-1}(e^x) = \dfrac{1}{|e^x|\sqrt{(e^x)^2-1}}\cdot\dfrac{d}{dx}(e^x)$

$= \dfrac{e^x}{|e^x|\sqrt{e^{2x}-1}}$

$= \dfrac{1}{\sqrt{e^{2x}-1}}$

8/5/24

(1) $\frac{1}{\infty} = 0$

(2) $\frac{0}{1} = 0$

(3) $x^0 = 1$

(4) $0^x = 0$

$: \frac{0}{0} \to 0$, $\to \infty$

$\frac{\infty}{\infty} \to \infty$, $\to 0$

0^0

Indeterminates

⊛ Two solutions from one fraction or rule then it is called Indeterminates.

(1) $y = \frac{f(x)}{g(x)} \quad \left(\frac{0}{0}, \frac{\infty}{\infty}, \frac{-\infty}{-\infty}\right)$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$$

: L'Hopital Rule (Take separate derivation of $f(x)$ and $g(x)$).

⊙ Find $\lim_{x \to 2} \frac{x2-4}{x-2}$

Sol:-

Since $\frac{4-4}{2-2} = \frac{0}{0}$ type

Therefore Apply L' Hopital's Rule:

$$\lim_{x \to 2} \frac{x2-4}{x-2} = \lim_{x \to 2} \frac{\frac{d}{dx}(x2-4)}{\frac{d}{dx}(x-2)}$$

3.1, 3.2, 3.3

** : 3.6 Examples

Quiz on Mondy

$$= \lim_{x \to 2} \frac{2x}{1} = 2(2) = 4 \quad \underline{\text{Ans}}$$

Ex 2(a):

$$\lim_{x \to 0} \frac{\sin 2x}{x}$$

Sol:-

Since $\frac{\sin 2(0)}{0} = \frac{0}{0}$ type

Therefore, by L'Hopital rule:

$$\lim_{x \to 0} \frac{\sin 2x}{x} = \lim_{x \to 0} \frac{\frac{d}{dx} \sin 2x}{\frac{d}{dx} \cdot x}$$

$$= \lim_{x \to 0} \frac{\cos 2x \cdot \frac{d}{dx} 2x}{1}$$

$$\lim_{x \to 0} \frac{\sin 2x}{x} = \lim_{x \to 0} 2\cos 2x$$

$$= 2 \cdot \cos 2(0)$$

$$= 2 \cos 0$$

$$= 2 \cdot 1 = 2 \quad \underline{\text{Ans}}$$

Ex 2: (Last part) $\lim_{x \to +\infty} \frac{x^{-1/3}}{\sin\left(\frac{1}{x}\right)}$

Sol:-

Since $\frac{(\infty)^{-4/3}}{\sin(\frac{1}{\infty})} = \frac{(\frac{1}{\infty})^{4/3}}{\sin 0} = \frac{0}{0}$ type

$$\lim_{x\to+\infty} \frac{x^{-4/3}}{\sin(\frac{1}{x})} = \lim_{x\to+\infty} \frac{\frac{d}{dx}(x^{-4/3})}{\frac{d}{dx}(\sin(\frac{1}{x}))}$$

$$= \lim_{x\to+\infty} \frac{-\frac{4}{3}x^{-7/3}}{(\cos\frac{1}{x})(-\frac{1}{x^2})}$$

$$= \lim_{x\to+\infty} \frac{+\frac{4}{3}x^{-\frac{7}{3}+2}}{\cos(\frac{1}{x})}$$

$\because \frac{d}{dx}(\frac{1}{x}) = \frac{d}{dx}(x^{-1})$

$= (-1)x^{-2}$

$$= \lim_{x\to+\infty} \frac{\frac{4}{3}x^{-1/3}}{\cos\frac{1}{x}}$$

$\because x^{-7/3}.x^2$

$x^{-\frac{7}{3}+2}$

$\because x^{-\frac{1}{3}}$

$$= \frac{\frac{4}{3}(\infty)^{-1/3}}{\cos(\frac{1}{\infty})} = \frac{\frac{4}{3}(0)^{1/3}}{\cos(0)}$$

$$= \frac{0}{1}$$

$= 0$ Ans.

Ex 3: $\lim_{x\to+\infty} \frac{x}{e^x}$

Sol:-

Since $\frac{\infty}{e^\infty} = \frac{\infty}{\infty}$ type:

Therefore by L'Hopital Rule:

$$\lim_{x\to+\infty} \frac{x}{e^x} = \lim_{x\to+\infty} \frac{\frac{d}{dx}x}{\frac{d}{dx}e^x}$$

$$= \lim_{x \to +\infty} \frac{1}{e^x} = \frac{1}{e^\infty}$$

$$= \frac{1}{\infty} = 0 \quad \underline{\text{Ans.}}$$

Imp Q. **Ex:** $\lim_{x \to +\infty} \frac{x^n}{e^x}$

Sol:-

Since $\frac{(\infty)^n}{e^\infty} = \frac{\infty}{\infty}$ type

Therefore by L'Hopital Rule

$$\lim_{x \to +\infty} \frac{x^n}{e^x} = \lim_{x \to +\infty} \frac{\frac{d}{dx} x^n}{\frac{d}{dx} e^x}$$

$$= \lim_{x \to +\infty} \frac{n \cdot x^{n-1}}{e^x}$$

$$= \frac{n \cdot \infty^{n-1}}{e^\infty} = \frac{\infty}{\infty} \text{ type}$$

Again apply L' Hopital Rule :

$$\lim_{x \to +\infty} \frac{x^n}{e^x} = \lim_{x \to +\infty} \frac{\frac{d}{dx}(n \cdot x^{n-1})}{\frac{d}{dx}(e^x)}$$

$$= \lim_{x \to +\infty} \frac{n(n-1)\, x^{n-2}}{e^x} \quad \left(\frac{\infty}{\infty}\right)$$

Again apply L'Hopital Rule

$$= \lim_{x \to +\infty} \frac{n(n-1)(n-2)(x^{n-3})}{e^x} \quad \left(\frac{\infty}{\infty}\right)$$

Applying again and again till n times

$$= \lim_{x \to +\infty} \frac{n(n-1)(n-2)(n-3) \cdots 3.2.1}{e^x}$$

$$= \lim_{x \to +\infty} \frac{n!}{e^x}$$

$$= \frac{n!}{e^\infty} = \frac{n!}{\infty}$$

$= 0$: Ans

⊙. Reverse terms:

$$\boxed{\lim_{x \to +\infty} \frac{e^x}{x^n} = \lim_{x \to +\infty} \frac{e^x}{n!} = \frac{\infty}{n!} = \infty}$$

: Applying till n times

⊙ $y = f(x)^{g(x)}$ ($0^0, \infty^0$ etc... form)

(Take Natural log)

→ Let $y = \lim_{x \to 0} (1 + \sin x)^{1/x}$: 1^∞ form

Taking ln on both sides:

$$\ln y = \lim_{x \to 0} \ln(1 + \sin x)^{1/x}$$

$$= \lim_{x \to 0} \frac{1}{x} \cdot \ln(1 + \sin x)$$

$= \frac{1}{0} \cdot \ln(1+0)$

$= \frac{1}{0} \cdot 0 \qquad \left(\frac{0}{0}\right)$

Applying L' Hopital Rule:

$\Rightarrow \ln y = \lim_{x \to 0} \frac{1}{x} \cdot \ln(1+\sin x)$

$= \lim_{x \to 0} \frac{\frac{d}{dx} \cdot \ln(1+\sin x)}{\frac{d}{dx} \cdot x}$

$= \lim_{x \to 0} \frac{\frac{1}{1+\sin x} \cdot (\cos x)}{1}$

$= \lim_{x \to 0} \frac{\cos x}{1+\sin x}$

$= \frac{\cos 0}{1+\sin 0}$

$= \frac{1}{1}$

$= 1$

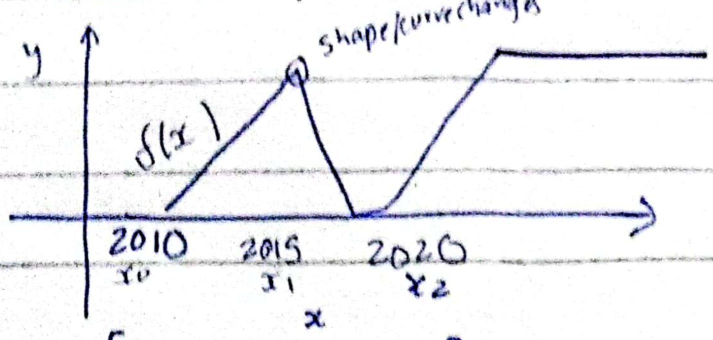
$\ln y = 1$

$\boxed{\ln y = e^{0}}$? : Taking antilog

CHAPTER#4

→ Derivative and its Applications

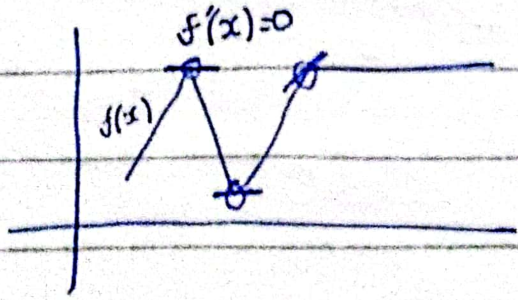
* Business:



[2010 — 2015] → f(x) is increasing

$[x_1 - x_2]$ → f(x) is decreasing

→ Horizontal lines at curve points:



: Stationary points (stay at some point) (Derivative is zero)

→ Function not changing

$$f'(x)=0$$

* Stationary points	Critical Points
$f'(x)=0$	$f'(x)=0$
	$f'(x)=\pm\infty$

→ Every stationary point is critical but inverse is not true.

⊙ Find stationary points

Q # I : $f(x) = x^3 - 3x^2 + 1$

Sol:-

$$f'(x) = 0$$

$$\frac{d}{dx}(x^3 - 3x^2 + 1) = 0$$

$$3x^2 - 6x + 0 = 0$$

$$3x(x-2) = 0$$

$$x(x-2) = 0$$

$$x = 0 \quad , \quad x - 2 = 0$$

$$x = 2$$

Therefore $x = 0$:

$\rightarrow f(0) = 0 - 0 + 1 = +1$

$(0, +1)$

$\rightarrow$ at $x = 2$

$$f(2) = (2)^3 - (3(2)^2 + 1$$

$$= 8 - 12 + 1 = -3$$

$(2, -3)$

Therefore stationary points are $(0, 1)$ and $(2, -3)$

⊙ $f(x) = e^x - 10x$

Sol:- $f'(x) = 0$

$$\frac{d}{dx}(e^x - 10x) = 0$$

$$e^x - 10 = 0$$

$$e^x = 10$$

$$e^x = 10.$$

Taking ln on both sides

$$\ln e^x = \ln 10$$

$$x = \ln 10$$

$$x = 2.302$$

at $x = 2.302$

$$f(2.302) = e^{2.302} - 10(2.302)$$

$$= 9.994 - 23.02 = -13.02$$

⊙ $f(x) = x\ln x \quad ; x > 0$

Sol:-

$$f'(x) = 0$$

$$\frac{d}{dx}(x \cdot \ln x) = 0$$

$$\ln x + x \cdot \frac{1}{x} = 0$$

$$\ln x + 1 = 0$$

$$\ln x = -1$$

$$e^{\ln x} = e^{-1}$$

$$x = e^{-1}$$
$$x = 1/e$$

Now,

At $x = 1/e$

$$f(1/e) = 1/e \cdot \ln e^{-1}$$ $\therefore \ln e^{-1} = l$

$$= -\frac{1}{e}$$ $\because 1/e \cdot (-1) \cdot \ln e$

$\because \ln e = 1$

$$f(x) = x + 2\sin x$$

Sol:-

$$f'(x) = 0$$
$$1 + 2\cos x = 0$$
$$\cos x = -1/2$$
$$x = \cos^{-1}(-1/2)$$
$$x = 2\pi/3$$

$\because$ At $x = 2\pi/3$

$$f(2\pi/3) = (2\pi/3) + 2\sin(2\pi/3)$$

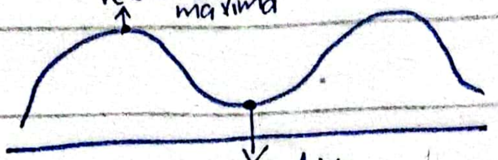
$$\boxed{f(2\pi/3) = 3.83}$$

17/5/24

(i) * Stationary/Critical Points

$f'(x) = 0 \rightarrow \underset{S.P}{\underline{x=a}} \quad \underset{S.V}{\underline{f(a)}}$

(ii) Increasing and Decreasing intervals by Graph



: Minima and maxima exist at stationary points or end

: relative (from neighbour highest)

⊙ Relative Maxima/Minima;

x_1

$f(x_1) \rightarrow$ Function value w.r.t x_1 (relative)

⊙ $f(x) = x^3 - 9x^2 - 48x + 52$ defined on the interval $(-\infty, +\infty)$

(i) Find stationary points:

SOL:-

Step - I - $f'(x) = 0$

$\frac{d}{dx}(x^3 - 9x^2 - 48x + 52) = 0$

$3x^2 - 18x - 48 = 0$

$x^2 - 6x - 16 = 0$

$x^2 - 8x + 2x - 16 = 0$

M : First derivative test and second derivative test (For Exam)

$x(x-8)+2(x-8)=0$

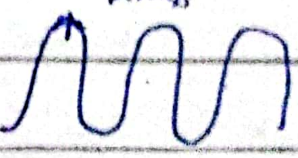
$(x-8)(x+2)=0$

$x=8 \quad , x=-2$

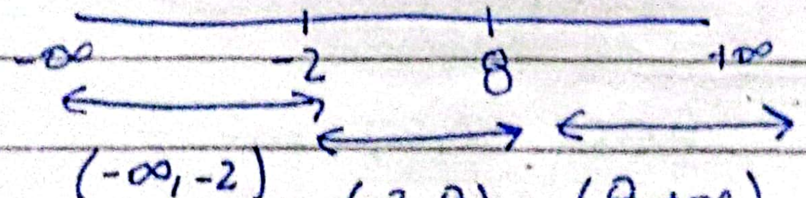
(ii) In which intervals of the domain the function increasing or decreasing.

Sol:-

stationary points



Since stationary points are $x=8,-2$ which divides the number line into three intervals



$(-\infty,-2)$, $(-2,8)$, $(8,+\infty)$

	Intervals	Test Points	$f'(x)=3x^2-18x+48$	Conclusion
Maxima	$(-\infty,-2)$	-3	$f'(x)>0$	Increasing
	$(-2,8)$	0	$f'(x)<0$	Decreasing
Minima	$(8,+\infty)$	9	$f'(x)>0$	Increasing

: Increasing or Decreasing by slope or $f'(x)$

⇒ $f'(x)$ is increasing on the interval $(-\infty,-2]$

⇒ $f'(x)$ is decreasing on the interval $(-2,-8]$

⇒ $f'(x)$ is increasing on the interval $(8,+\infty]$

⊙ At $x=-2$ $f(x)$ moves from increasing to decreasing, Therefore $f(x)$ is relative maxima at $x=-2$ and Relative Maximum value is $f(-2)=(-2)^3-9(-2)^2-48(-2)+52$
$= 104$

$+ \to -$

In → De (Relative Maxima)

D → I (Relative Minima)

⊙ At $x=8$:

(First derivative test)

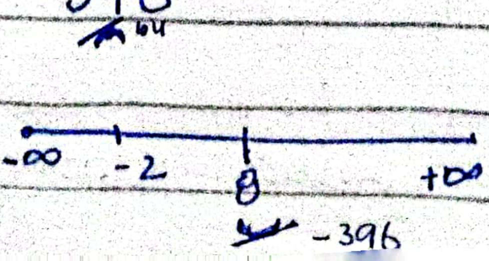
Since $f(x)$ moves from decreasing to increasing (or($-$ to $+$ region)) . So $f(x)$ is relative minima at $x=8$

Relative minima value is:

$$f(8)=(8)^3-9(8)^2-48(8)+52$$
$$= -396$$

Graph

⊙



Activity :

$$h(t) = \frac{1}{3}t^3 - 5t^2 + 21t$$

(i) $h(8) = \frac{1}{3}(8)^3 - 5(8)^2 + 21(8)$

(ii) $f'(x) = 0$

(iii) Increasing or Decreasing interval

(iv) Maxima and Minima. function.

(i) $h(8) = 18.66$

(ii) $\frac{d}{dx}\left(\frac{1}{3}t^3 - 5t^2 + 21t\right) = 0$

$$t^2 - 10t + 21 = 0$$

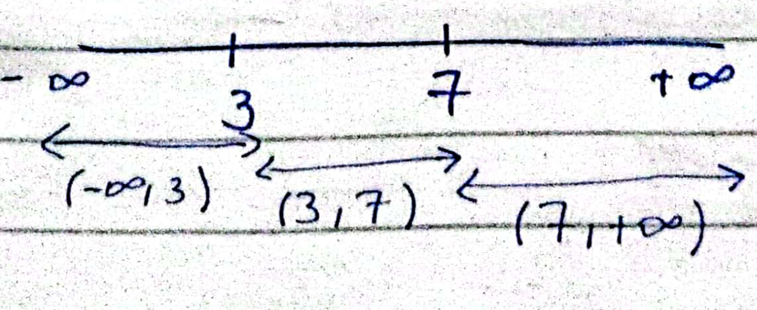
$$t^2 - 7t - 3t + 21 = 0$$

$$t(t-7) - 3(t-7) = 0$$

$$(t-7)(t-3) = 0$$

$$t = 7 \quad , \quad t = 3$$

(iii)



22/5/24

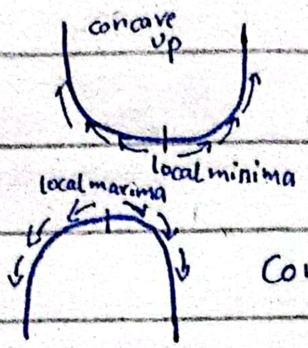
f = Rate of change of f

$(f')'$ = R.C of f' = slopes of f

(second derivative)(Tells slope of f)

⊛ Concavity:

If f is differentiable, then it is said to be concave up.



$f''>0$

: Double Derivative > 0 (concave up) (Point is local minima)

→ Concave down $f''<0$

⊙ Inflection Point:

Second derivative critical points are called inflection points. $(f''(x)=0)$

→ For Critical points : $f'(x)=0$

→ Example 1:

$$f(x) = x^3-9x^2-48x+52$$

Sol:-

⊙ Step-I: Critical Points

$$f'(x)=0$$

$$3x^2 - 18x - 48 = 0$$
$$3(x^2 - 6x - 16) = 0$$
$$x^2 - 8x + 2x - 16 = 0$$
$$x(x-8) + 2(x-8) = 0$$
$$(x-8)\cdot(x+2) = 0$$
$$x = 8 \quad , x = -2$$

C.P's $\boxed{x = 8, -2}$

Step 2: $f''(x) = 6x - 18$

At $x = -2$:

$$f''(-2) = 6(-2) - 18 < 0$$

$\Rightarrow f(x)$ is concave down at $x = -2$

$\Rightarrow f(x)$ has local maxima at $x = -2$ and local maxima at $x = -2$ is:

$$f(-2) = (-2)^3 - 9(-2)^2 - 48(-2) + 52$$
$$f(-2) = 140$$

At $x = 8$:

$$f''(8) = 6(8) - 18 > 0$$

$\Rightarrow f(x)$ is concave up at $x = 8$

$\Rightarrow f(x)$ is local minima at $x = 8$

and local maxima at $x=8$ is:

$$f(8)=(8)^3-9(8)^2-48(8)+52$$
$$=-396$$

⊙ Inflection point:

$$f(x)=x^3-9x^2-48x+52$$
$$f'(x)=3x^2-18x-48$$
$$f''(x)=6x-18$$

For inflection points put $f''(x)=0$

$$\Rightarrow 6x-18=0$$
$$\Rightarrow 6(x-3)=0$$
$$\Rightarrow \boxed{x=3} \text{ Inflection Point}$$

⊙ WORD PROBLEM:

$$q(t)=20(e^{-t}-e^{-2t})$$

(a) At $t=0$:

$$q(0)=20(e^{-0}-e^{-2(0)})$$
$$q(0)=0$$

(b)

$$q(t)=20(e^{-t}-e^{-2t})$$
$$q'(t)=\frac{d}{dt}[20(e^{-t}-e^{-2t})]$$

$$= 20\left[-1\cdot e^{-t} - (-2)(e^{-2t})\right]$$

$$q'(t) = 20(2e^{-2t} - e^{-t})$$

- Critical Point:

put $q'(t) = 0$

$$20(-e^{-t} + 2e^{-2t}) = 0$$

$$20e^{-2t}(-e^{t} + 2) = 0$$

Since $e^{-2t} \neq 0$

$$-e^{t} + 2 = 0$$

$$e^{t} = 2$$

$$\ln e^{t} = \ln 2 \qquad \ln e^{x} = x$$

$$t = \ln 2$$

$$t = 0.693$$

- Step 2:

$$q''(t) = 20(e^{-t} - 4e^{-2t})$$

At $t = 0.693$

$$q''(0.693) = 20(-0.5) < 0$$

$\Rightarrow$ $q(t)$ is concave down at $t = 0.693$

$\Rightarrow$ $q(t)$ has maximum value at $t = 0.693$

Maximum value at $t = 0.693$

is $q(0.693) = 20\left(e^{-0.693} - e^{-2(0.693)}\right)$

$= 4.999 \approx 5mg$

(c) In long run:

$\boxed{\lim_{t \to \infty} q(t) = 0}$: Medicine vanishes or finish at end.

: $\lim_{t \to \infty} q(t)$

For marks $= \lim_{t \to \infty} 20\left(e^{-t} - e^{-2t}\right)$

$= 20\left(e^{-\infty} - e^{-\infty}\right)$

$= 20(0 - 0)$

$= 0$

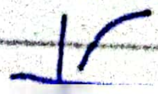
*** ⊛ Integration (Learn formulas till next Lecture)

27/5/24

⊙ Integration:

Σ ↓ Discrete Points

$\int$ ↓ Continuous Points



Integration

Definite integral (Limits are given)

$$\int_a^b f(x)\,dx$$

Area under curve

Indefinite integral (Limits not specified)

$$\int f(x)\,dx$$

Family of curves

→ Slope — $f(x)$: Finding $f(x)$ from slope

(anti-derivative)

$$\int \underbrace{1}_{\text{slope}}\,dx = \boxed{\underbrace{x + \underline{c}}_{\text{curve}}}\ (\text{constant})$$

$\frac{d}{dx}\,x \rightarrow 1$

$(x+1)' \rightarrow 1$

$(x+5)' \rightarrow 1$

So constant is required.

Product Formula

$$\int f(x)^n \cdot f'(x)\,dx = \frac{f(x)^{n+1}}{n+1} + C \quad (n \neq -1)$$

→ $\int x^n \cdot 1\,dx = \frac{x^{n+1}}{n+1} + C$

→ $\int x^{10}\,dx = \frac{x^{10+1}}{10+1} + C = \frac{x^{11}}{11} + C$

$\rightarrow \int (\sin x)^{-2} \cdot \cos x \, dx$: $f(x) = \sin x$, $n = -2$, $f'(x) = \cos x$

$= \dfrac{(\sin x)^{-2+1}}{-2+1} + C$

$= -\dfrac{1}{\sin x} + C$

$\rightarrow \int e^{x} \, dx = e^{x} + C$: Exponential function

⊙ $\int e^{ax} \cdot dx = \dfrac{1}{a} \int a e^{ax} dx$: Missing word divide

$= \dfrac{1}{a} \cdot e^{ax} + C$

$\rightarrow \int \cos x \, dx = \sin x + C$

⊙ $\int \cos (ax) \cdot dx = \dfrac{\sin ax}{a} + C$: Any constant divided

⊙ Formula : $\int \dfrac{f'(x)}{f(x)} dx$ Derivative Function

$= \ln |f(x)| + C$

$\rightarrow \int \dfrac{1}{x} dx = \ln |x| + C$: $\int \dfrac{f'(x)}{f(x)} dx$

⊙ Example:

$\rightarrow \int (3x^6 - 2x^2 + 7x + 1)\, dx$

$= \int 3x^6 dx - \int 2x^2 dx + \int 7x\, dx + \int 1 \cdot dx$

$= 3\int x^6 dx - 2\int x^2 dx + 7\int x\, dx + \int 1 dx$

$= 3\left(\frac{x^{6+1}}{6+1}\right) - 2\left(\frac{x^{2+1}}{2+1}\right) + 7\left(\frac{x^{1+1}}{1+1}\right) + x + C$

$= \frac{3x^7}{7} - \frac{2}{3}x^3 + \frac{7x^2}{2} + x + C$ Ans.

$\rightarrow \int \frac{\cos x}{\sin^2 x}\, dx$

: First log rule not applicable

$= \int (\sin x)^{-2} \cdot \cos x\, dx$

$= \frac{(\sin x)^{-2+1}}{-2+1} + c$

$= -(\sin x)^{-1} + c$

$= -\frac{1}{\sin x} + c$

$= -\operatorname{cosec} x + c$

$$\int \frac{t^2-2t^4}{t^4}\,dt$$

$$= \int \left(\frac{t^2}{t^4} - \frac{2t^4}{t^4}\right)dt$$

$$= \int \left(\frac{1}{t^2} - 2\right)dt$$

$$= \int \frac{1}{t^2}\,dt - 2\int 1\,dt$$

$$= \int t^{-2}\,dt - 2\int 1.\,dt$$

$$= \frac{t^{-2+1}}{-2+1} - 2t + C$$

$$= -\frac{1}{t} - 2t + C.$$ Ans

$$\int \frac{x^2}{x^2+1}\,dx$$

$$= \int \frac{(x^2+1)-1}{x^2+1}\,dx$$

$$= \int \frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}\,dx$$

$$= \int \left(1 - \frac{1}{x^2+1}\right)dx$$

$$= \int 1\,dx - \int \frac{1}{x^2+1}\,dx$$

$$= x - \tan^{-1}x + C$$

Partial Fractions

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

$$\frac{5}{2 \times 3} = \frac{A}{2} + \frac{B}{3}$$

$$\rightarrow \int \frac{5x+1}{(x-1)(x+2)}\,dx$$

Sol:-

$$\frac{5x+1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$

$$: \frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$

$$\frac{5}{(x-1)(x+2)} = \frac{(x+2)A + (x-1)B}{(x-1)(x+2)}$$

$$: \frac{1}{(x^2+2)(x-1)} = \frac{Ax+B}{x^2+2} + \frac{C}{x-1}$$

$$5x+1 = A(x+2) + B(x-1) \quad \ldots\ldots (1)$$

- Put $x-1 = 0$ in (1)

$$x = 1$$

$$5(1)+1 = A(1+2) + B(0)$$

$$6 = 3A$$

$$A = \frac{6}{3}$$

$$\boxed{A = 2}$$

Again put $x = -2$ in ①:

$$5(-2) + 1 = A(0) + B(-2-1)$$

$$-10 + 1 = 0 + (-3B)$$

$$-9 = -3B$$

$$B = \frac{-9}{-3}$$

$$\boxed{B = 3}$$

$$\therefore \frac{5x+1}{(x-1)(x+2)} = \frac{2}{x-1} + \frac{3}{x+2}$$

$$\int \frac{5x+1}{(x-1)(x+2)} = \int \frac{2}{x-1}\,dx + \int \frac{3}{x+2}\,dx$$

$$= 2\int \frac{1}{x-1}\,dx + 3\int \frac{1}{x+2}$$

$$= 2\ln|x-1| + 3\ln|x+2| + c$$

: Applying natural log formula.

⊙ $\int \frac{x-1}{x^2-4}\,dx$

Sol:-

$$\frac{x-1}{x^2-4} = \frac{x-1}{(x-2)(x+2)}$$

$$= \frac{A}{x-2} + \frac{B}{x+2}$$

$$x-1 = A(x+2) + B(x-2)$$

(solved it by yourself)

⊙ $\int \frac{2x+1}{(x-1)^2}\,dx$

: $\frac{1}{(x-a)^n} = \frac{A}{x-a} + \frac{A_2}{(x-a)^2} \cdots + \frac{A_n}{(x-a)^n}$

: $\frac{1}{(x-2)^3} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3}$

: $\frac{1}{x^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3}$

Sol:-

$$\frac{2x+1}{(x-1)^2} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2}$$

$$\Rightarrow 2x+1 = A(x-1) + B \quad \text{—} ①$$

⊙ put $x = +1$ in eq. ①

$$2(1)+1 = B$$

$$\boxed{B=3}$$

⊙ Compare like terms in eq. ①:

$$2x+1 = Ax - A + B$$

For x:

$$2 = A \qquad \therefore -A+B=1$$

$$\boxed{A=2}$$

Since $\dfrac{2x+1}{(x-1)^2} = \dfrac{2}{x-1} + \dfrac{3}{(x-1)^2}$

$$\int \frac{2x+1}{(x-1)^2}\,dx = \int \frac{2}{x-1}\,dx + \int \frac{3}{(x-1)^2}\,dx$$

$$= 2\int \frac{1}{x-1}\,dx + 3\int \frac{1}{(x-1)^2}\,dx$$

$$= 2\ln|x+1| + 3\int (x-1)^{-2}\,dx$$

$$= 2\ln|x-1| + 3\,\frac{(x-1)^{-1}}{-1} + C$$

$$\int \frac{2x+1}{(x-1)^2}\,dx \Rightarrow 2\ln|x-1| - \frac{3}{x-1} + C \quad \text{Ans.}$$

29/5/24

* Integration By Parts:

Multiplication of two functions (IBP).

$$\int f(x)\,g(x)\,dx = f(x)\cdot\int g(x)\,dx - \int\left[f'(x)\cdot\int g(x)\,dx\right]dx$$

⊙ Rules:

① 2nd function → Integration Known   : Not confirmed first and second function

② LIATE → Exponential

(log) Inverse Trignometric, Algebraic, Trignometric

(functions)   : For first and second function

Q: $\int x\cos x\,dx$: LIATE

$x \to A$, $\cos x \to T$

$$\int x\cos x\,dx = x\int \cos x\,dx - \int\left[\frac{d}{dx}\cdot x\cdot\int \cos x\,dx\right]dx$$

$$= x(+\sin x) - \int(1\cdot\sin x)\,dx + C$$

$$= x\sin x - \int \sin x\,dx + C$$

$$= x\sin x - (-\cos x) + C$$

$$= x\sin x + \cos x + C$$

Q: $\int x^2 e^{-x}\,dx$ LIATE

(Already arranged)

$$= x^2\int e^{-x}dx - \int\left[\frac{d}{dx}\cdot x^2\cdot\int e^{-x}\cdot dx\right]dx + c$$

$$= x^2\cdot\frac{e^{-x}}{-1} - \int\left(2x\cdot\frac{e^{x}}{-1}\right)dx + c$$

$$= -x^2e^{-x} + \int 2xe^{-x}dx + c$$

$$= -x^2e^{-x} + 2\int \underline{xe^{-x}}dx + C$$

Again apply this

$$= -x^2e^{-x} + 2\left[x\int e^{-x}dx - \int\left[\frac{d}{dx}\cdot x\cdot\int e^{-x}dx\right]dx + c\right.$$

$$= -x^2e^{-x} + 2\left[x\cdot\frac{e^{-x}}{-1} - \int\left(1\cdot\frac{e^{-x}}{-1}\right)dx + c\right]$$

$$= -x^2e^{-x} + 2\left[-xe^{-x} + \frac{e^{-x}}{-1}\right] + c$$

$$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + c$$

$$= e^{-x}\left[-x^2 - 2x - 2\right] + c$$

Q: $\int \ln x \cdot dx$

(Not defined), So:

$$= \int \underset{A}{1}\cdot \underset{L}{\ln x}\, dx$$

$\underset{①}{L}\,\underset{③}{I}ATE$

$$= \int \ln x \cdot 1 \, dx$$

$$= \ln x \int 1 dx - \int \left[\frac{d}{dx}(\ln x) \cdot \int 1 dx \right] dx + C$$

$$= \ln x \cdot x - \int \left(\frac{1}{x} \cdot x \right) dx + C$$

$$= \ln x \cdot x - \int 1 dx + C$$

$$= x \ln x - x + C \quad \text{Ans..}$$

Q: $\int (\sin x)^n dx$

Sol:-

$$= \int (\sin x)^{n-1} \cdot \sin x \, dx$$

$\because \sin x$ Integration Known $\because$ so 2nd $\sin x$

$$= (\sin x)^{n-1} \cdot \int \sin x dx - \int \left[\frac{d}{dx} (\sin x)^{n-1} \cdot \int \sin x dx \right] dx + C$$

$$= (\sin x)^{n-1} (-\cos x) - \int (n-1)(\sin x)^{n-2} \cos x (-\cos x) dx + C$$

$$= -(\sin x)^{n-1} \cos(x) + (n-1) \int (\sin x)^{n-2} \cos^2 x \, dx + C$$

$$= -(\sin x)^{n-1} \cos x + (n-1) \int (\sin x)^{n-2} (1 - \sin^2 x) dx + C$$

$\boxed{\because \frac{d}{dx}(\sin x)^{n-1} = (n-1) \sin x^{n-1-1} \cdot \cos x \quad \because (n-1) \sin x^{n-2} \cdot \cos x}$

$$= -(\sin x)^{n-1} \cos x + (n-1) \left[\int \sin x^{n-2} dx - \int \sin^n x \, dx \right] + C$$

$$= -(\sin x)^{n-1}\cdot\cos x + (n-1)\int \sin x^{n-2}\,dx - (n-1)\int \sin^n x\,dx + C$$

(Same term as question)

$$(n-1)\int \sin^n x\,dx + \int \sin^n x\,dx = -(\sin^{n-1} x).\cos x + (n-1)\int (\sin x)^{n-2}\,dx + C$$

$$(n-1+1)\int \sin^n x\,dx = -(\sin x)^{n-1}\cos x + (n-1)\int (\sin x)^{n-2}\,dx + C$$

$$\int \sin^n x\,dx = -\frac{1}{n}(\sin x)^{n-1}\cos x + \frac{n-1}{n}\int (\sin x)^{n-2}\,dx + C$$

(Formula)

: cos formula (Before Example 8)

Eg:

$$\int \sin^4 x\,dx = -\frac{1}{4}(\sin x)^{4-1}\cos x + \frac{4-1}{4}\int (\sin x)^{4-2}\,dx + C$$

$$= -\frac{(\sin x)^3}{4}\cos x + \frac{3}{4}\int (\sin x)^2\,dx + C$$

$$= -\frac{(\sin x)^3}{4}\cos x + \frac{3}{4}\left[-\frac{1}{2}(\sin x)^{2-1}\cos x + \frac{2-1}{2}\int \sin x^{2-1}\,dx + C\right]$$

$$= -\frac{(\sin x)^3}{4}\cos x + \frac{3}{4}\left[-\frac{\sin x}{2}\cos x + \frac{1}{2}\int \sin x\,dx + C\right]$$

$$= -\frac{(\sin x)^3}{4}\cos x + \frac{3}{4}\left[-\frac{\sin}{2}\cos x + \frac{1}{2}(-\cos x) + C\right]$$

→ Cos Formula:

$$\int \cos^n x \cdot dx = \frac{1}{n}\cos^{n-1}x \cdot \sin x + \frac{n-1}{n}\int (\cos x)^{n-2} dx$$

Here $n=4$

$$\int \cos^4 x\, dx = \frac{1}{4}\cos^{4-1}x \cdot \sin x + \frac{4-1}{4}\int (\cos x)^{4-2} dx$$

$$= \frac{1}{4}\cos^3 x \cdot \sin x + \frac{3}{4}\int \cos^2 x\, dx$$

$$= \frac{1}{4}\cos^3 x \sin x + \frac{3}{4}\left[\frac{1}{2}\cos^{2-1}x \sin x + \frac{2-1}{2}\int (\cos)^{2-2} dx\right.$$

$$= \frac{1}{4}\cos^3 x \sin x + \frac{3}{4}\left[\frac{1}{2}\cos x \sin x + \frac{1}{2}\int 1\, dx\right] + c$$

$$= \frac{1}{4}\cos^3 x \sin x + \frac{3}{8}\cos x \sin x + \frac{3}{8}x + c$$

⊙ Formula:

→ $\cos^3 x = \cos^2 x \cdot \cos x$

$= (1-\sin^2 x) \cdot \cos x$

If power odd then split it into square form

→ $\cos^4 x \cdot \sin^5 x = \cos^4 x \cdot \sin^2 x \sin^3 x$

$= \cos^4 x \cdot (1-\cos^2 x)\sin^3 x$

→ If both are odd, then anyone can be splitted.

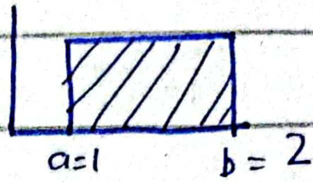
If both are even, then use formula

3/5/24

→ Definite Integral:

(Area Under graph)

- $\int_1^2 x\,dx$



$= \frac{x^2}{2}\Big|_1^2$

$= \frac{(2)^2}{2} - \frac{(1)^2}{2} = \frac{3}{2}$

- $\int_a^b f(x)\,dx = f(b) - f(a)$

- E.g:

$\int_2^7 (2x-8)\,dx$

$= \left(\frac{2x^2}{2} - 8x\right)\Big|_2^7$

$= ((7)^2 - 8(7)) - ((2)^2 - 8(2))$

$= (49-56) - (4-16)$

$= 5$

- $\int_0^\pi \cos x\,dx = \sin\Big|_0^\pi$

$= \sin\pi - \sin 0$

$= 0 - 0$

$= 0$

⊙ Evaluate the integral:

$$\int_1^2 \ln x . dx \qquad : x > 0$$

By parts:

$$= \ln x - x\Big|_1^2 - \int_1^2 \left(\frac{1}{x} \cdot x\right) dx$$

$$= (2\ln 2 - 1.\ln 1) - (x)\Big|_1^2$$

$$= 2\ln 2 - 0 - (2-1)$$

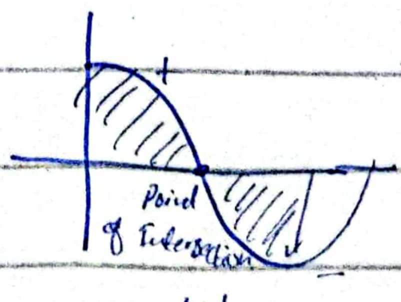
$$= 2\ln 2 - 1 = 0.386$$

⊙ Find the area:

$$\int_0^\pi \cos x dx \qquad : 0 \text{ (zero)}$$

Sol:-

⊙ Step-I:



1:1

$$\cos x = 0$$

$$x = \cos^{-1}(0)$$

$$x = \frac{\pi}{2}$$

: 20 Marks Paper

: Choose $\frac{\pi}{2}$ directly

⊙ Step-2:

$$\int_0^{\pi} \cos x \, dx = \left| \int_0^{\pi/2} \cos x dx \right| + \left| \int_{\pi/2}^{\pi} \cos x dx \right|$$

$$= \sin x \Big|_0^{\frac{\pi}{2}} + \sin x \Big|_{\pi/2}^{\pi}$$

$$= \left| \left(\sin \frac{\pi}{2} - \sin 0\right) \right| + \left| \sin \pi - \sin \frac{\pi}{2} \right|$$

$$= |1-0| + |0-1|$$

$$= 1+1$$

$$= 2$$

(*) Find the area:

$$\int_2^7 (2x-8) \, dx$$

Sol:-

$$2x-8 = 0$$

$$x = 8/2$$

$$\boxed{x = 4}$$

$$\int_2^7 (2x-8) \, dx = \left| \int_2^4 (2x-8) dx \right| + \left| \int_4^7 (2x-8) dx \right|$$

⊙ Find the area:

$$\int_2^3 x^4\,dx$$

Sol:-

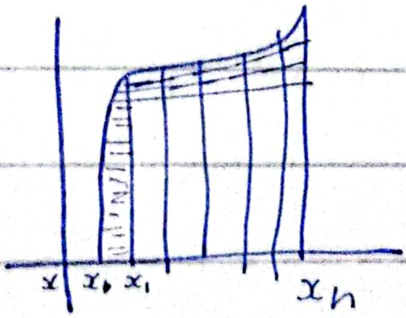
$$\int_2^3 x^4\,dx = \left.\frac{x^5}{5}\right|_2^3$$

$$= \frac{(3)^5}{5} - \frac{(2)^5}{5}$$

$$= \frac{243 - 32}{5}$$

$$= \frac{211}{5}$$

⊙



$$= f(x_1)\,\Delta x_1 + f(x_2)\Delta x_2 + \dots\dots + f(x_n)\Delta x_n$$

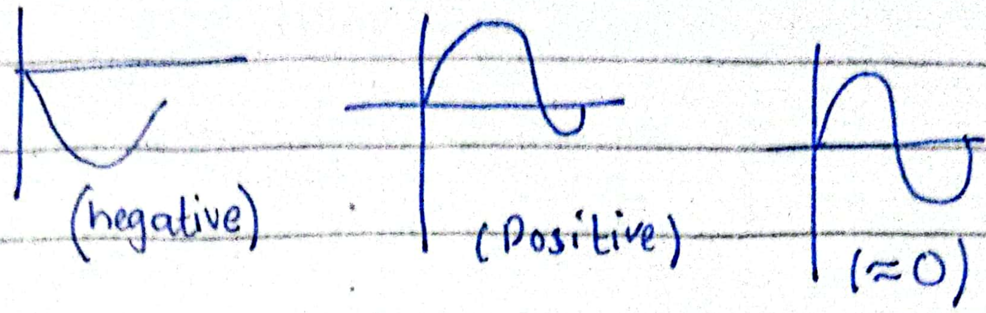
Further part

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i')\cdot\Delta x_i$$

$$= \int_{x_0}^{x_n} f(x)\,dx$$

: Limits applicable.

(4)



⊙ $f(x) = x^3 \qquad [2,3]$

$$\int_2^3 x^3\,dx = \frac{x^4}{4}\Big|_2^3$$

$$= \frac{(2)^4}{4} - \frac{(3)^4}{4}$$

$=$

⊙ $$\int_{-1}^{1} x^4\,dx = \left|\int_{-1}^{0} x^4\,dx\right| + \left|\int_{0}^{1} x^4\,dx\right|$$

$=$

⊙ $$\int_0^{\ln 2} e^{2x}\,dx$$

Sol:-

$$e^{2x} \neq 0 \qquad \forall\, x \in R$$

$$\int_0^{\ln 2} e^{2x}\,dx = \frac{e^{2x}}{2}\Big|_0^{\ln 2} = \frac{e^{2(\ln 2)}}{2} - \frac{e^{2(0)}}{2}$$

$: e^{\ln 2^2} = 2^2 = 4$

$$= \frac{e^{1.38}}{2} - \frac{1}{2} = \frac{4}{2} - \frac{1}{3} = \frac{3}{2}$$

Example:

$N = 1770 + 53t$ — (1)

→ put $t = 0$

$N = 1770 + 53(0)$

$= 1770$ metric tons.

→ In 2010, $t = 20$

$N = 1770 + 53(20)$

$= 2830$

→ 1990 to 2010.

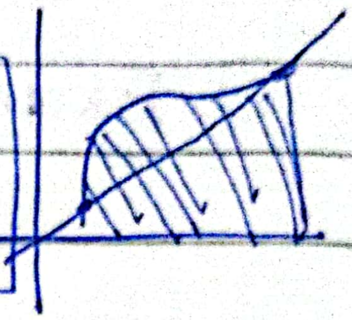
$$\int_0^{20} N\,dt = \int_0^{20} (1770 + 53t)\,dt$$

$$= \left(1770t + \frac{53t^2}{2}\right)\Bigg|_0^{20}$$

$$= \left(1770(20) + \frac{53(20)^2}{2}\right) - \left((1770)(0) - \frac{53(0)^2}{2}\right)$$

⊙ Area between two curves:

$$\boxed{A = \int_a^b [f(x)-g(x)]\,dx}$$



Example: Slide : 36

Imp Question

$f(x) = x+6$

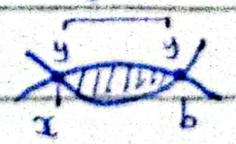
$g(x) = x^2$

Sol:-

$$A = \int_a^b [f(x)-g(x)]\,dx$$

⊙ Limits of integration:

functionsame



$f(x) = g(x)$

$x+6 = x^2$

$x^2 - x - 6 = 0$

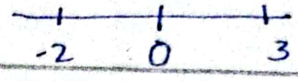
$x^2 - 3x + 2x - 6$

$(x-3)(x+2) = 0$

$x-3=0 \quad , x+2=0$

$x=3 \quad , x=-2$

⊙ Limits of integration is
$a = -2$, $b = 3$, i.e:

$[-2, 3]$: 

⊙ Upper and lower curves:

→ Test $x = 0$: (between interval)

$f(x) = x + 6$; $g(x) = x^2$

$f(0) = 0 + 6$; $g(0) = 0$

$f(0) = 6$

∴ $f(x)$ is greater

$\Rightarrow f(x) > g(x) \quad \forall x \in [-2, 3]$

$$A = \int_a^b [f(x) - g(x)]\, dx$$

$$A = \int_{-2}^{3} [(x+6) - (x^2)]\, dx$$

$$A = \left. \frac{x^2}{2} + 6x - \frac{x^3}{3} \right|_{-2}^{3}$$

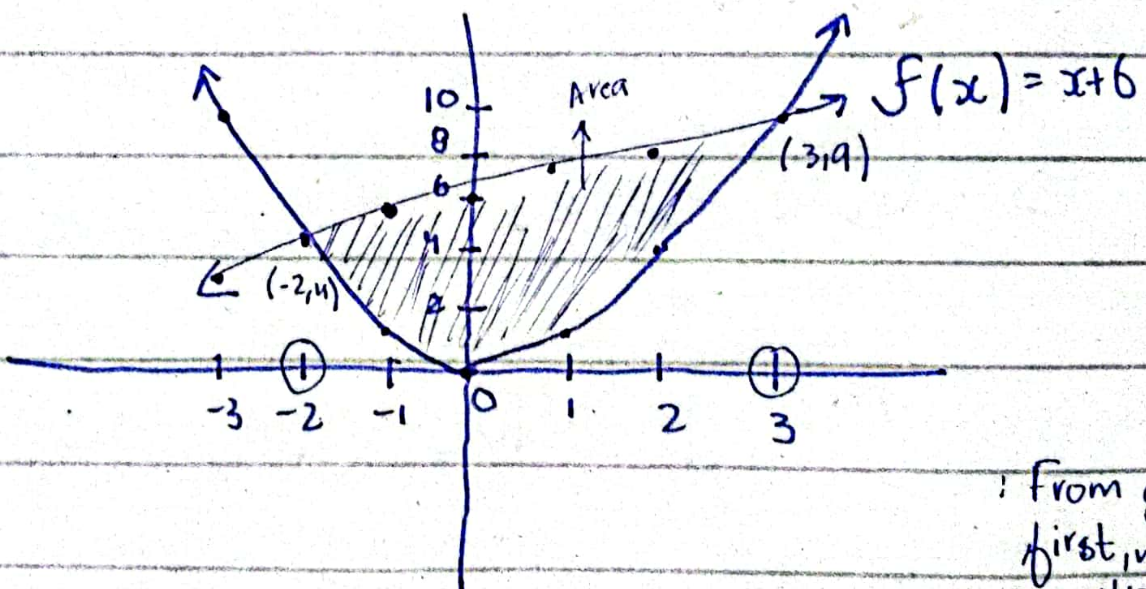
$$A = \frac{(3)^2}{2} + 6(3) - \frac{(3)^3}{3} - \left(\frac{(-2)^2}{2} + 6(-2) - \frac{(-2)^3}{3} \right)$$

$$\boxed{A = 20.83}$$

-

x	$f(x) = x+6$	$g(x) = x^2$
-3	3	9
-2	4	4
-1	5	1
0	6	0
1	7	1
2	8	4
3	9	9

- Graph:



$[-2, 3]$

: From graph first, we can also solve question

(*) Ex: 5.1 (Area → Integral, Limits, Boundary values)