OPERATIONS RESEARCH
BSE : 7th SEMESTER

DAY:

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* General Concept:

- → optimization of cost
- → Minimizing cost and maximizing Profit
- → In short, it is optimization.

* Scenario:

→ 5-week
Lahore → Gujaranwala

→ L → G → L
Monday → Wednesday

→ 400 Rs. (Mon-Fri)
→ 320 Rs. (Sat-Sun)
(Along with Return Ticket)
→ One way Ticket Price : 350 Rs.

→ Optimize Ticket Price.

⊛ Origin of OR:

→ World-War II
→ 1940 - Railways
→ 1920 - Physics
→ Limited resources to be used in world-war II.

⊛ Definition of OR:

→ OR is an art of winning war without actual fight.
→ scientific approach to problems
→ Research on operations
→ Quantitative approach to decision making.

⊛ Phases of OR:

→ Definition ·····> Construction.....

⊛ Limitation of OR:

→ Lack of qualitative factors

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→ Specific category of factors
→ Limited factor consideration
→ Resistance of employees.

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(*) **Research** - To find something.
(*) **Operations** - Simple activities and research to estabish tasks.

⊙ Linear Programming :

→ Highest Power One (Linear)

$x + 2 = 0$ ✓

$\boxed{xy} + 2 = 0$ ✗

→ Systematic approach to achieve some task (Programming)

(*) Definition :

Linear Programming (L.P) is a method to allocate limited resources to competing activities

(*) Object/Goal :

→ Maximize the profit or minimize the cost.

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* Application :

→ Busness (production), Hospitals (staff Scheduling)

→ Transport (Routing) etc....

Example:-

5 kg flour

bread , cake

⇒ Use Resource (flour) to generate maximum profit.

* Properties of LP :-

① → Linearity :

- Relationships between variables and constraints are Linear.

② → Objective function :

- Maximum or Minimum

③ → Constraints :

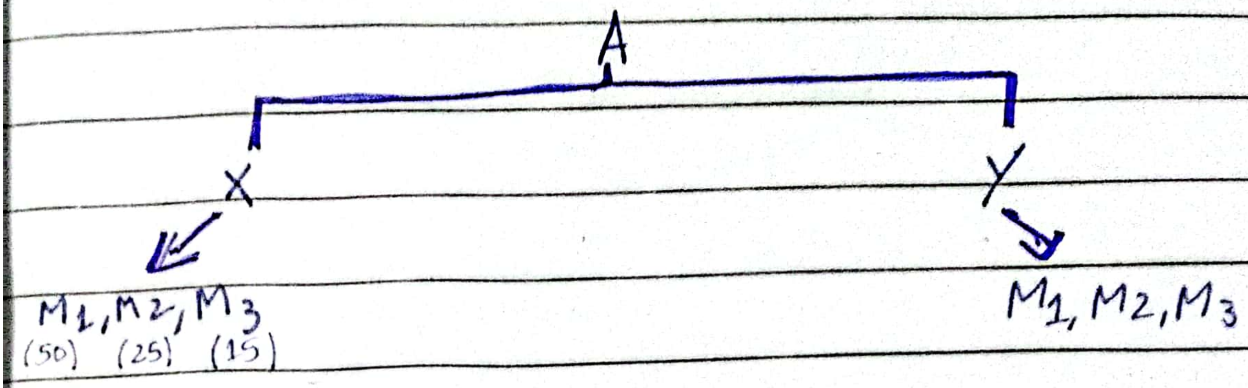
- Limitations on Resources
- E.g $2x + 5y \leq 3$

Constraints

→ **Non-negativity:**

- Decision Variables ≥ 0
- $x, y \geq 0$

Example : 2.1:



- X .....

$M_2 \rightarrow 1$ hour $M_3 \rightarrow 1$ hour

- Y

2 hour $\rightarrow M_1$
2 hour $\rightarrow M_2$
1 hour $\rightarrow M_3$

- Decision - Variable.

let x_1 and x_2 be unit number of X and Y. $(X \rightarrow Y)$

	X	Y	Time (h)
	x_1	x_2	
M1	0	2	50
M2	1	2	25
M3	1	1	15
Profit	5	4	

- Objective function:

Maximize: $Z = 5x_1 + 4x_2$

- Constraints:

$0x_1 + 2x_2 \leq 50$

$1x_1 + 2x_2 \leq 25$

$1x_1 + 1x_1 \leq 15$

$x_1, x_2 \geq 0$

Graphical Method.

X → interior paint

Y → exterior paint

	Exterior paint (x_1)	Interior Paint (x_2)	Maximum daily availability Tons
Raw Material M_1	6	4	24
Raw Material M_2	1	2	6
Profit Per Ton	5	4	

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- **Objective function:**

Maximize : $z = 4x_1 + 5x_2$

- **Constraints:**

$4x_2 + 6x_1 \leq 24$

$2x_2 + 1x_1 \leq 6$

$x_2 \leq x_1 + 1 \Rightarrow x_2 - x_1 \leq 1$

$x_2 \leq 2$

- **Problem 2:**

A company

B company

: Let x_1 and x_2 be number of units for product A & B respectively

	A	B	
	x_1	x_2	Maximum Availability
Machine X	2	2	5 h 30 mins = 330 min
Machine Y	3	2	8h = 480 min
Profit	4	5	

- **Objective function:**

Maximize : $4x_1 + 5x_2$

- **Constraints:**

$2x_1 + 2x_2 \leq 330$

$3x_1 + 2x_2 \leq 480$

: Practice Questions

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① **OR**: The Science of better/optimization

② **LP**: It is a method to allocate limited resources to competing activities.

③ **Properties of LP**:

Basic assumption of LP.

(i) Certainty:- all data known with certainly (fixed)

(ii) Linearity: Proportionality and additively assumed to linear

(iii) Fixed Technology: Requirements remain unchanged.

(iv) Constant Profit contribution: Profit per unit is fixed.

(v) Divisibility: Products can be produced in a fractional unit

(vi) Single Decision Method: Static Model.

(vii) Non-negativity: No regular production

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General Linear Programming Problem (LPP):

A general mathematical way of LPP is as objective function.

$Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$ to be optimized.

⇒ Subject to the conditions:

$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \ (\geq, \leq, =) b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \ (\geq, \leq, =) b_2$

$\vdots$

$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \ (\leq, \geq, =) b_m$

Problem
↓
Model
↓
Solution

$\because x_i \geq 0 \quad i = 1, 2, \ldots, n$

$x_1, x_2, \ldots, x_n \geq 0$

Solution of Linear Programming Problems:

After properly and carefully formulating an LP problem, the next step is to develop a solution for it.

⇒ Solution: A set of decision variables, each having a value is called a solution.

$\because x_1 = 5, \ x_2 = 0.6$

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⇒ Feasible: Proposed Solution to a LP Problem that satisfies all the constraint called feasible.

⇒ Feasible solution Region: The solution / collection of feasible solution is called feasible solution area / region, space etc.....

⇒ Infeasible: Any proposed solution that violates one or more constraints is called infeasible area.

⊙ **Methods to solve LPP:** There are several methods to solve LPP after it has been properly formulated.

(i) Systematic trial and error method.

(ii) The vector Method.

(iii) Graphical Method (when only two variables)

(iv) Simplex Method etc....

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⊙ Graphical Method:

⇒ Problem ①:

Formulation

⊙ **Decision Variables:**

let x_1 (exterior) and x_2 (interior)

be unit numbers of X and Y

X ⟶ Y

⊙ **Objective function:**

: Ex
p :- x_1 ⟶
p :- x_2 ⟶

Maximize : $5x_1 + 4x_2$

⊙ **Constraints:**

⇒ subject:

① $6x_1 + 4x_2 \leq 24$

② $1x_1 + 2x_2 \leq 6$

③: $1x_2 \ngtr 1x_1 + 1$

$x_2 \leq x_1 + 1$

$-x_1 + x_2 \leq 1$

④ : $x_2 \leq 2$

: $x_1, x_2 \geq 0$

Solution:

: let $6x_1 + 4x_2 = 24$

let l_1: $x_1 = 0$, $x_2 = 6$

(0,6)

$x_2 = 0$, $x_1 = 4$

(4,0)

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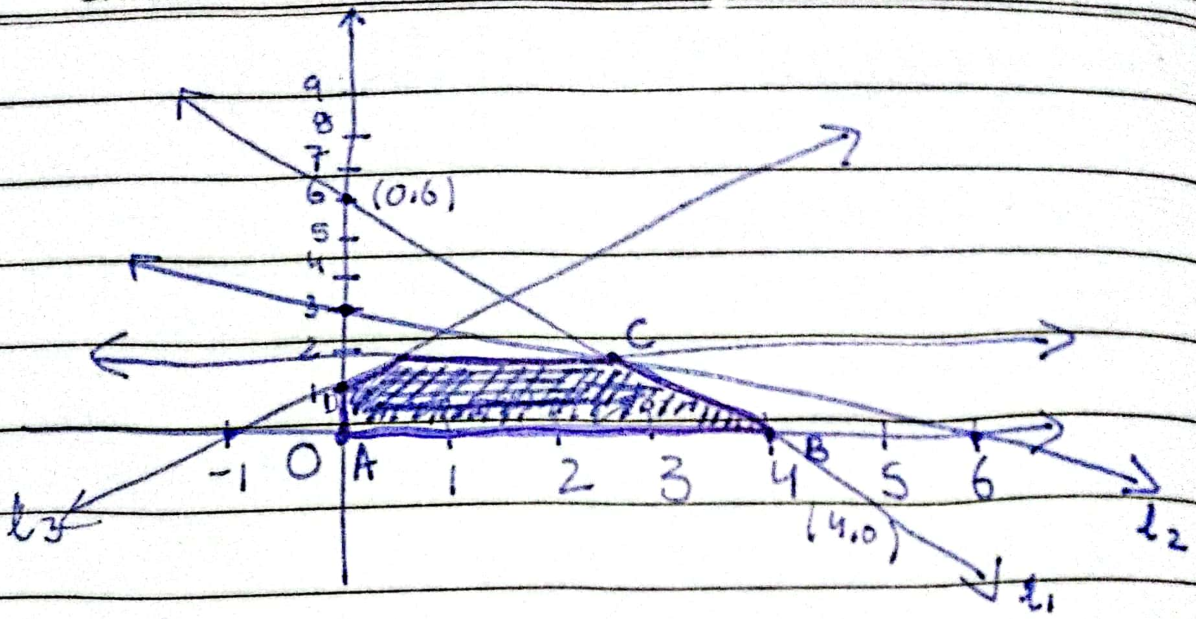

$: l_2 :- \quad x_1 + 2x_2 = 6$

$x_1 = 0, \; x_2 = 3 \quad (0,3)$

$x_2 = 0, \; x_1 = 6 \; (6,0)$

$: l_3 :- \quad -x_1 + x_2 = 1$

$x_1 = 0, \; x_2 = 1 \quad (0,1)$

$x_2 = 0, \; x_1 = -1 \quad (-1,0)$

$\Rightarrow \; 6x_1 + 4x_2 = 24$

$x_1 + 2x_2 = 6$

$: x_1 = 6 - 2x_2$

$\rightarrow 6(6-2x_2) + 4x_2 = 24$

$\rightarrow 36 - 12x_2 + 4x_2 = 24$

$\rightarrow 36 - 8x_2 = 24$

$\rightarrow 36 - 24 = 8x_2$

$\rightarrow \boxed{x_2 = \frac{12}{8}} \quad \Rightarrow \boxed{x_2 = \frac{3}{2} = 1.5}$

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$\Rightarrow x_1 + 2(1.5) = 6$

$x_1 = 6 - 3$

$\boxed{x_1 = 3}$

$\Rightarrow -x_1 + x_2 = 1$

$x_1 + 2x_2 = 6$

$3x_2 = 7$

$x_2 = \frac{7}{3}$

$x_2 = x_1 + 1$

$x_1 + 2(x_1 + 1) = 6$

$x_1 + 2x_1 + 2 = 6$

$3x_1 = 4$

$\Rightarrow \boxed{x_1 = \frac{4}{3}}$

$\Rightarrow 2x_2 = 6 - \frac{4}{3}$

$2x_2 = \frac{14}{3}$

$x_2 = \frac{14}{6} \Rightarrow \boxed{x_2 = \frac{7}{3}}$

$\Rightarrow$ Solution Points : $(0,0)$, $(0,4)$

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Graphical Method:

The graphical method offers an easy way to solve simple Linear Programming with only two decision variables. It is also possible to solve an LP problem with three decision variables because it requires 3 dimensions to illustrate, so the graphical method is not practical when when have more than two variables.

Problem: (Continue....)

→ $A(0,0)$, $B(4,0)$, $C(3,1.5)$, $D(2,2)$, $E(1,2)$ and $F(0,1)$

→ $z = 5x_1 + 4x_2$

$z|_{A(0,0)} = 5(0) + 4(0) = 0$

$z|_{B(4,0)} = 5(4) + 4(0) = 20$

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$Z|_{C(3,1.5)} = 5(3) + 4(1.5) = 21$

$Z|_{D(2,2)} = 5(2) + 4(2) = 18$

$Z|_{E(1,2)} = 5(1) + 4(2) = 13$

$Z|_{F(0,1)} = 5(0) + 4(1) = 4$

$\Rightarrow$ Z is maximum at $C(\overset{x_1}{3}, \overset{x_2}{1.5})$ with maximum value 21. So, we should produce 3 tons of exterior paint and 1.5 tons interior paint to maximize the daily profit

Problem No. 3:

$\rightarrow$ A, B, C :requires (at least)
10, 12, 12

requirements $\geq$

$\rightarrow$

	units/Jar x	Units/carton y	Units Required
A	5	1	10
B	2	2	12
C	1	4	12
Cost	3	2	

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Objective Function:

Minimize (cost) : $Z = 3x + 2y$

Constraints:

Subject to:

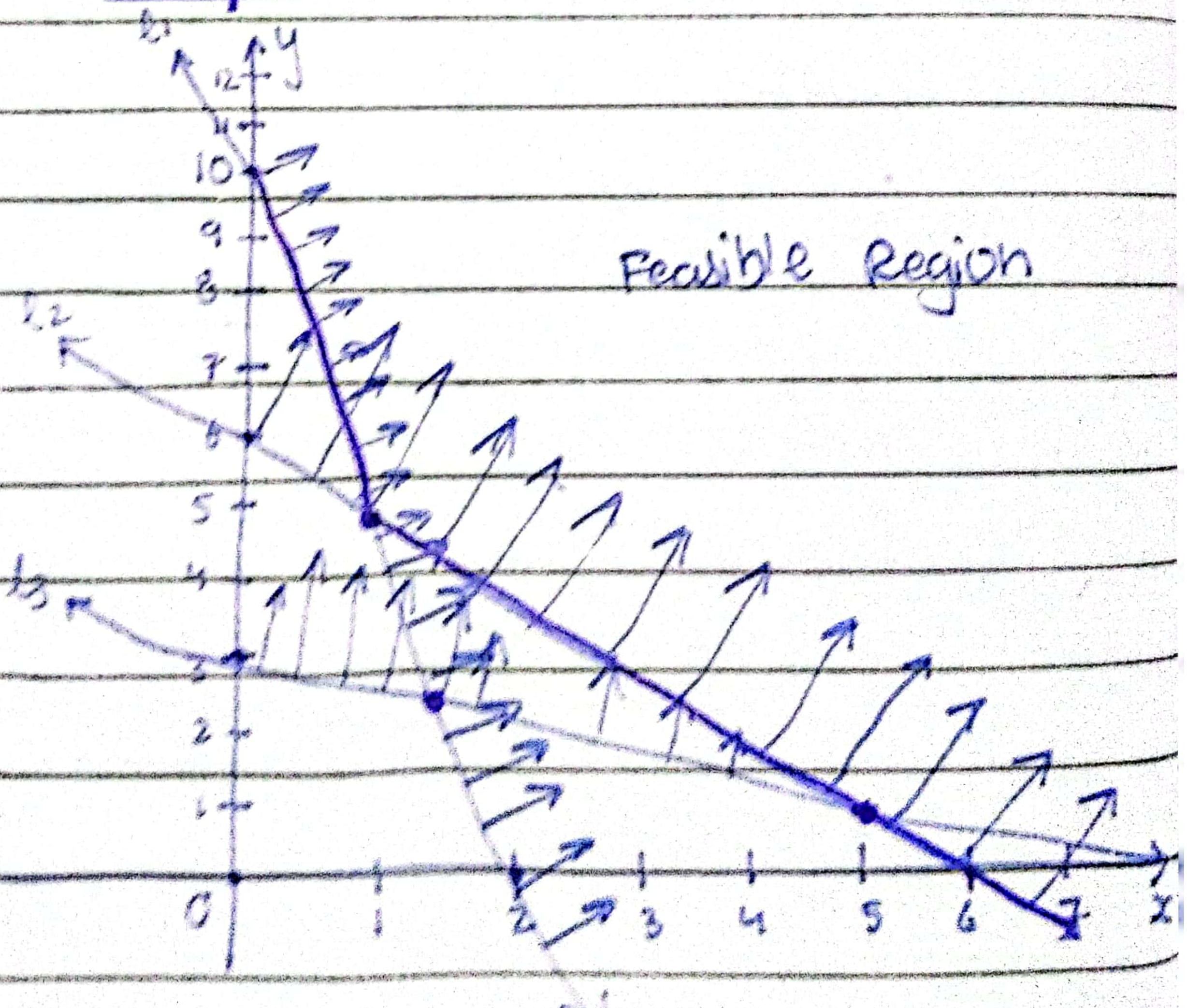
$l_1: 5x + 1y \geq 10$

$l_2: 2x + 2y \geq 12$

$l_3: x + 4y \geq 12$

$x, y \geq 0$

Graph:



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$l_1 : 5x + y = 10$

$x = 0, \; y = 10 \qquad (0, 10)$

$x = 2, \; y = 0 \qquad (2, 0)$

$l_2 : 2x + 2y = 12$

$x + y = 6$

$x = 0, \; y = 6 \qquad (0, 6)$

$y = 0, \; x = 6. \qquad (6, 0)$

$l_3 : x + 4y = 12$

$x = 0, \; y = 3 \qquad (0, 3)$

$y = 0, \; x = 12 \qquad (12, 0)$

⊙ **Points find out:**

→ $l_1 : 5x + y = 10$

$l_2 : 2x + 2y = 12$

$l_1 : y = 10 - 5x$

$2x + 2(10 - 5x) = 12$

$2x + 20 - 10x = 12$

$-8x = -8$

$\boxed{x = 1}$

$y = 10 - 5$

$\boxed{y = 5}$

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→ Find remaining points ....

⊙ $A(12,0)$, $D(0,10)$, $B(4,2)$, $C(1,5)$

$$Z = 3x+2y$$

$\therefore Z|_{A(12,0)} = 3(12)+2(0) = 36$

$\therefore Z|_{B(4,2)} = 3(4)+2(2) = 16$

$\therefore Z|_{C(1,5)} = 3(1)+2(5) = 13$

$\therefore Z|_{D(0,10)} = 3(0)+2(10) = 20$

⊙ **Result:**

→ Z is minimum at $C(1,5)$ with minimum value 13.

→ Here 1 Jar and 5 cartons should be produced to give the minimum value 13

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① The Simplex Method:

→ It is a method to solve Linear Programming Problem.

(i) Entering Variables:

By optimality condition the entering variable in maximizing (minimization) problem is the $\boxed{\text{non-basic}}$ variable having the most negative (positive) coefficient in the z-row. Ties will be broken arbitrarily.

(ii) By feasibility condition:

For both maximization or minimization problems, the leaving variables are associated with smallest non-negative ratio.

Pivot Element:

An element appearing in the intersection of leaving and entering variable row and column we produce pivot element by 1 and all other elements appearing

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in pivot column to zero.

Ending Criterion:

It is the coefficient of the non-basic variables is negative (Positive) in z-row in case of maximization (minimization). Then we stop the last tableau is optimal.

(*) Solve the problem by Simplex Method:

Max: $z = 5x_1 + 4x_2 \Rightarrow z - 5x_1 - 4x_2 = 0$

Subject to:

$$6x_1 + 4x_2 \leq 24$$

$$x_1 + 2x_2 \leq 6$$

$$-x_1 + x_2 \leq 1$$

$$x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

Converting into standard form

: $6x_1 + 4x_2 + \underset{\text{slack variable (Balance)(positive)}}{\gamma_1} = 24$

$x_1 + 2x_2 + \gamma_2 = 6$

$-x_1 + x_2 + \gamma_3 = 1$

; $x_2 + \gamma_4 = 2$

; box of chocolate space left : right variables (positive)

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① Choose entering variable
② Choose leaving variable (smallest)
③ Pivot element

$x_1, x_2, x_3, x_4, \gamma_1, \gamma_2, \gamma_3, \gamma_4 \geq 0$

⊙ Step 1:

(Non-basic)(Decision variables)

Let $x_1 = 0 = x_2 \Rightarrow \gamma_1 = 24, \gamma_2 = 6, \gamma_3 = 1, \gamma_4 = 2$

$(0, 0, 24, 6, 1, 2)$ with $z = 0$

: Put the problem in tableax form:

Basic	x_1	x_2	γ_1	γ_2	γ_3	γ_4	SOL	Ratio
z	-5	-4	0	0	0	0	0	—
(leaving) γ_1	6 pivot	4	1	0	0	0	24	$24/6 = 4$
γ_2	1	2	0	1	0	0	6	$6/1 = 6$
γ_3	-1	1	0	0	1	0	1	—
γ_4	0	1	0	0	0	1	2	—

: $\frac{0}{-5}$ (Not write 0)

: Neg. value not to write

→ By optimality and feasibility condition x_1 is entering and γ_1 is leaving variables.

⊙ Step2:

Basic	x_1	x_2	γ_1	γ_2	γ_3	γ_4	SOL	Ratio
z	$-5+5$ (0)	$-4+(\frac{2}{3}\times 5)$ $(-\frac{2}{3})$	$0+(\frac{1}{6}\times 5)$ $(\frac{5}{6})$	$0+(0\times 5)$ (0)	$0+(0\times 5)$ (0)	$0+(0\times 5)$ (0)	20	—
x_1	1	$2/3$	$1/6$	0	0	0	4	$\frac{4}{2/3} = 4\times\frac{3}{2} =$
γ_2	$1-1$	$4/3$	$-1/6$	1	0	0	2	$3/2$
leaving γ_3	0	$5/3$	$1/6$	0	1	0	5	3
γ_4	0	1	0	0	0	1	2	2

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⊛ Algorithm of Simplex Method OR working Rule of simplex Method.

⊙ **Simplex Method**:

The process of finding optimal solution using simplex method algorith is as follows:

→ **Step 1**: First we convert the problem in standard form by adding the slacks (surplus) variables.

→ **Step 2**: We find basic and non-basic variables and put the problem into tableau form.

→ **Step 3**: Determine a starting basic feasible solution by subtituting non-basic variables equal to zero.

→ **Step 4**: By using optimal condition we select an entering variable with maximum (minimum), most negative (positive) coefficient of non basic variables in object row

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when the problem is of maximum (minimum) type respectively.

- If there is no entering variable, Stop process. The Last tableau is optimal.

→ **Step 5:** Select the leaving variable by using feasible condition, the one corresponding to least ratio determined by column of correspond to entering variable. Neglect ratio with zero or negative denominator. Ties will be broken arbitrarily.

→ **Step 6:** Determine the new basic solution by using appropiate Gauss Jordan computation.

Go to step 5.

Example:

Min : $Z = 8x_1 - 2x_2$

Subject to:

$-4x_1 + 2x_2 \leq 1$, $x_1, x_2 \geq 0$

$5x_1 - 4x_2 \leq 3$

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Sol:-

Max : most negative

Min : positive

$$\boxed{\text{Max } z^* = -\text{Min}(z)}$$

$$\text{Max } z^* = -8x_1 + 2x_2$$

Subject to:

$-4x_1 + 2x_2 \leq 1 \Rightarrow -4x_1 + 2x_2 + r_1 \leq 1$

$5x_1 - 4x_2 \leq 3 \Rightarrow 5x_1 - 4x_2 + r_2 \leq 3$

$x_1, x_2 \geq 0 \Rightarrow x_1, x_2, r_1, r_2 \geq 0$

→ Step 2:

Initial Solution

Let $x_1 = x_2 = 0$ to get

$r_1 = 1, r_2 = 3$ with $z^* = 0$

Starting solution is $(0,0,1,3)$ with $z^* = 0$.

→ Step 3:

Basic	x_1	x_2	r_1	r_2	Sol	Ratio
z	8	-2	0	0	0	-
← r_1	-4	+2	1	0	1	$\frac{1}{2} = 0.5$
r_2	5	-4	0	1	3	-

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⇒ By optimal and feasible condition r_1 is leaving variable and x_2 is entering variable.

Basic	x_1	x_2	r_1	r_2	sol	Ratio
z	4	0	1	0	1	
x_2	-2	1	$\frac{1}{2}$	0	$\frac{1}{2}$	
r_1	-3	0	2	1	5	

$$\therefore x_2 = \frac{1}{2}, \; x_1 = 0 \quad \text{with } z^* = 1$$

$$z = -z^*$$

$$= -(-1)$$

$$z = -1, \; x_1 = 0, \; x_2 = \frac{1}{2} \quad \underline{\text{Ans.}}$$

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⊛ M-Technique or Big-M Method or Method of Penality:

(Greater than condition)

Max : $Z = 4x_1 + 3x_2$

Subject to:

$$2x_1 + x_2 \geq 10$$

$$-3x_1 + 2x_2 < 6$$

$x_1 + x_2 \geq 6$

$x_1, x_2 \geq 0$

Sol:-

let r_1, r_2, r_3 be the slacks variables and R_1, R_2 be the artificial variables.

let $M >> 0$. write the problem into standard form

$$z = 4x_1 + 3x_2 + MR_1 + MR_2$$

: Max (−M)
: Min (+M)
'M' very large number
'R' Artificial Variable

Subject to:

$$2x_1 + x_2 - r_1 + R_1 = 10 \quad \text{.....(i)}$$
$$-3x_1 + 2x_2 + r_2 = 6 \quad \text{.....(ii)}$$
$$x_1 + x_2 - r_3 + R_2 = 6 \quad \text{.....(iii)}$$

$$: x_1, x_2, r_1, r_2, r_3, R_1, R_2 \geq 0$$

- From (i):

$$R_1 = 10 - 2x_1 - x_2 + r_1$$

- From (iii):

$$R_2 = 6 - x_1 - x_2 + r_3$$

put the value of R_1 and R_2 in eq (A) from (i) and (iii)

$$Z = 4x_1 + 3x_2 + M(10 - 2x_1 - x_2 + \gamma_1) + M(6 - x_1 - x_2 + \gamma_3)$$

$$= 4x_1 - 2Mx_1 - Mx_1 + 3x_2 - Mx_2 - Mx_2 + M\gamma_1 + M\gamma_3 + 16M$$

$$Z = x_1(-3M+4) + x_2(3-2M) + M\gamma_1 + M\gamma_3 + 16M$$

$$\Rightarrow Z + x_1(3M-4) + x_2(2M-3) - M\gamma_1 - M\gamma_3 = 16M$$

: let $x_1 = x_2 = \gamma_1 = \gamma_3 = 0$ (non-basic) to get $Z = 16M$

: (Not present γ_2, R_1, R_2 Basic)

Entering ↓ variable

Basic	x_1	x_2	γ_1	γ_2	γ_3	R_1	R_2	Sol	Ratio
Z	$3M-4$	$2M-3$	$-M$	0	$-M$	0	0	$16M$	-
γ_2	-3	2	0	1	0	0	0	6	-
← leaving R_1	Pivot $\boxed{2}$	1	-1	0	0	1	0	10	5
R_2	1	1	0	0	-1	0	1	6	6

: x_1 is entering variable and R_1 is leaving

Basic	x_1	↓ x_2	γ_1	γ_2	γ_3	R_1	R_2	Sol	Ratio
Z	0	$M/2 - 1$	$M/2 - 2$	0	$-M$	$2 - 3/2\,M$	0	$20+M$	-
γ_2	0	$7/2$	$-3/2$	1	0	$3/2$	0	21	6
x_1	1	$1/2$	$-1/2$	0	0	$1/2$	0	5	10
← R_2	0	$1/2$	$1/2$	0	-1	$-1/2$	1	1	2

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Basic	x_1	x_2	γ_1	γ_2	γ_3	R_1	R_2	Sol
z	0	0	-1	0	-2	$-M/2+1$	$-M+2$	22
γ_2	0	0	-5	1	7	5	-7	14
x_1								4
x_2	0	1	1	0	-2	-1	2	2

Rough: x_2 0 1 1 0 -2 -2 2 2

0 $-\frac{1}{2}$ $-\frac{1}{2}$ 0 1 1 -1 -1

+1 $\frac{1}{2}$ $\frac{1}{2}$ 0 -1 $-\frac{1}{2}$ 1 1

1 0 0 0 0

→ Multiply x_2 by $-\left(\frac{M}{2}-1\right)$

0 $-\frac{M}{2}+1$ $-\frac{M}{2}-1$ 0 $M-2$ $M-2$ $-M+2$ $-M+2$

→ 0 $\frac{7}{2}$ $-\frac{3}{2}$ 1 0 $\frac{3}{2}$ 0 21

0 $-\frac{7}{2}$ $-\frac{7}{2}$ 0 7 $\frac{7}{2}$ -7 -7

0 0 -5 1 7 5 -7 14

→ 1 $\frac{1}{2}$ $-\frac{1}{2}$ 0 0 $\frac{1}{2}$ 0 5

→ Since there is no artificial variables in z-column and optimal condition of simplex method satisfied. Hence last tableau

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is optimal with $x_1 = 4$, $x_2 = 2$
and $z = 22$

⊙ Qustion:

Max: $z = x_1 + 2x_2 + x_3$

Subject to:

$x_1 + x_2 + x_3 = 7$

$2x_1 - 5x_2 + x_3 \geq 10$

$x_1, x_2, x_3 \geq 0$

Sol:-

$z = x_1 + 2x_2 + x_3 - MR_1 - MR_2$ —— (A)

Subject to:

$x_1 + x_2 + x_3 + R_1 = 7$ ...(i)

$2x_1 - 5x_2 + x_3 - \gamma_1 + R_3 = 10$ .....(ii)

$\therefore x_1, x_2, \gamma_1, R_1$ and $R_2 \geq 0$

→ let γ_1 be a slack variables and R_1 and R_2 be artificial variables and M be very very large number

: From (i):

$R_1 = 7 - x_1 - x_2 - x_3$

: From (ii):

$R_2 = -2x_1 + 5x_2 - x_3 + \gamma_1$

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→ Put values of R_1 and R_2 in eq. (A) from (i) and (ii)

$$Z = x_1 + 2x_2 + x_3 - M(7 - x_1 - x_2 - x_3) - M(+10 - 2x_1 + 5x_2 - x_3 + r_1)$$

$$= x_1(1 + M + 2M) + x_2(2 + M - 5M) + x_3(1 + M + M) - Mr_1 - 7M - 10M$$

$$= x_1(1 + 3M) + x_2(2 - 4M) + x_3(1 + 2M) - Mr_1 - 17M$$

$$Z + x_1(-1 - 3M) + x_2(4M - 2) + x_3(-2M - 1) + Mr_1 = -17M$$

→ Let $x_1 = x_2 = x_3 = r_1 = 0$ with $Z = -17M$

Basic
Z
R_1
R_2

$\therefore Z = 7$, $x_3 = 4$, $x_1 = 0$

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Infeasible Solution:

→ Big M (if greater than condition)

→ Decision variables non-basic

→ Artificial variables exist in basic column, so no feasible solution (infeasible solution)

Question:

Max : $Z = 8x_2$

Subject to :

$$x_1 - x_2 \geq 0$$

$$2x_1 + 3x_2 \leq -6$$

$$x_1, x_2 \geq 0$$

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* Two methods Phase to solve LPP:

(i) Graphical → (2 DW) , $\leq, \geq, =, >, <$

(ii) simplex Method → (2, 3, ...) , $\leq, <$, (= Slacks (surplus)

(iii) Big-M Method → (2, 3, ...) ($\leq$, + ($\geq$, =) = (slacks / SV) (A.V) , Penalty $M \to \infty$)

: Z (add A.V) → optimize

(iv) Two-Phase Method ($\leq$, ($\geq$, =) (Result more better) (= (slacks / surplus) / A.V)

Z (Remove)

↓

$r_0 = R_1 + R_2 \to r_0 = 0 \to$ Z-optimize

TWO PHASE METHOD TO SOLVE LPP

→ In two phase Method, we always search for the most positive in r_0 phase-1 regardless the minimization or maximization.

Example:

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Q: Minimize : $Z = 2x_1 + 3x_2 - 5x_3$

Subject to :

$x_1 + x_2 + x_3 = 7$: A.V (2) R_1, R_2

$2x_1 - 5x_2 + x_3 \geq 10$ Slack Variables (1)

$x_1, x_2, x_3 \geq 0$

: Write the problem in standard form:

$Z - 2x_1 - 3x_2 + 5x_3 = 0$

Subject to:

$x_1 + x_2 + x_3 + R_1 = 7$

$2x_1 - 5x_2 + x_3 - r_1 + R_2 = 10$

where R_1 and R_2 are artificial variables and r_1 is slack variable

Min : $z - 2x_1 - 3x_2 + 5x_3 = 0$

: $x_1, x_2, x_3, R_1, R_2, r_1 \geq 0$

Phase-1 :

$r_0 = R_1 + R_2$

$\Rightarrow R_1 = 7 - x_1 - x_2 - x_3$

$R_2 = 10 - 2x_1 + 5x_2 - x_3 + r_1$

$r_0 = 7 - x_1 - x_2 - x_3 + 10 - 2x_1 + 5x_2 - x_3 + r_1$

$= x_1(-1-2) + x_2(-1+5) + x_3(-1-1) + r_1 + 17$

$r_0 = -3x_1 + 4x_2 - 2x_3 + r_1 + 17$

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$r_0 + 3x_1 - 4x_2 + 2x_3 - r_1 = 17$

Put the problem into tableau form:

entering

Basic	x_1	x_2	x_3	r_1	R_1	R_2	Sol	Ratio
r_0	3	-4	2	-1	0	0	17	–
R_1	1	1	1	0	1	0	7	7
← R_2 leaving	$\boxed{2}$	-5	1	-1	0	1	10	$\boxed{5}$

$\therefore x_1$ learing R_2 learing

Basic	x_1	x_2	x_3	r_1	R_1	R_2	Sol	Ratio
r_0	0	$7/2$	$1/2$	$1/2$	0	$-3/2$	2	–
R_1	0	$7/2$	$1/2$	$1/2$	1	$-1/2$	2	$4/7$
x_1	1	$-5/2$	$1/2$	$-1/2$	0	$1/2$	5	–

3-3-*

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Basic	x_1	x_2	x_3	r_1	R_1	R_2	Sol
$\boxed{r_0}$	0	0	0	0	-1	-1	0
x_2	0	1	$1/7$	$1/7$	$2/7$	$-1/7$	$4/7$
x_1	1	0	$6/7$	$-1/7$	$5/7$	$-4/7$	$45/7$

$$r_0 = R_1 + R_2$$
$$r_0 = 0$$

Since $r_0 = 0$, so we will go to phase II :-

$$0x_1 + x_2 + \frac{1}{7}x_3 + \frac{1}{7}r_1 = \frac{4}{7}$$

$$\Rightarrow x_2 = \frac{4}{7} - \frac{1}{7}x_3 - \frac{1}{7}r_1$$

$$x_1 + 0 \cdot x_2 + \frac{6}{7}x_3 - \frac{1}{7}r_1 = \frac{45}{7}$$

$$\Rightarrow x_1 = \frac{45}{7} - \frac{6}{7}x_3 + \frac{1}{7}r_1$$

: Now $z - 2x_1 - 3x_2 + 5x_3 + r_1 = 0$

$$z - 2\left(\frac{45}{7} - \frac{6}{7}x_3 + \frac{1}{7}r_1\right) - 3\left(\frac{4}{7} - \frac{1}{7}x_3 - \frac{1}{7}r_1\right) + 5x_3 = 0$$

$$z + \frac{50}{7}x_3 + \frac{1}{7}r_1 = \frac{102}{7}$$

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Basic	x_1	x_2	x_3	r_1	sol	Ratio
z	0	0	$50/7$	$1/7$	$102/7$	–
→ x_2	0	1	$1/7$	$1/7$	$4/7$	$\boxed{4}$
x_1	1	0	$6/7$	$-1/7$	$45/7$	7.5

x_2 is leaving, x_3 is entering

Basic	x_1	x_2	x_3	r_1	sol
z	0	-50	0	$-49/7$	$-98/7$
x_3	0	7	1	1	4
x_1	1	-6	0	-1	$21/7$

Hence no coefficient of non-basic variables is z-row is positive

$\Rightarrow z = -\frac{98}{7}$ with $x_1 = \frac{21}{7}$,

$x_2 = 0$, $x_3 = 4$ Ans.

⊙ Two-Phase Method:

1. Standard form
2. $r_0 = R_1 + R_2$ in x_1, x_2, x_3

Subject to:

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- Simplex Method
  $\delta_0 = 0$, $R_1 = R_2 = 0$

  $z =$
  subject to:
  (Again Simplex Method)

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* Alternative optima:

Alternative optima occurs when a linear programming has more than one optimal solution giving same objective function value (optimal value).

P_1:- $x_1 = 5$, $x_2 = 2$ $z = 10$

P_2:- $x_1 = 2$, $x_2 = 1$ $z = 10$

Optimal Solution Optimal Value

: $x_1 = 0$, $x_2 = ?$

If zero, we can say optimal solution otherwise no ~~optimal solution~~.

: Quiz on Monday

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→ Condition:

In final simplex optimal tableau of any non-basic variable has zero-value ($z-c_j=0$)

Q:

Max: $z = 2x_1 + 4x_2$

Subject to:

$x_1 + 2x_2 \leq 5$

$x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0$

: less (so simplex Method)

(i) Write the problem in standard form.

$z - 2x_1 - 4x_2 = 0$

Subject to:

$x_1 + 2x_2 + r_1 = 5$

$x_1 + x_2 + r_2 = 4$

$x_1, x_2, r_1, r_2 \geq 0$

: Max (Most negative pivot)

(i) **Step 1:**

let $x_1 = x_2 = 0$ with $z = 0$

Basic	x_1	x_2 (Pivot)	r_1	r_2	Sol	Ratio
z	-2	$\boxed{-4}$	0	0	0	—
← r_1	1	2	1	0	5	$5/2$
r_2	1	1	0	1	4	$4/1$

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→ By the condition of optimality and feasibility x_2 is entering variable and r_1 is leaving variable.

Basic	x_1	x_2	r_1	r_2	Sol
z	0	0	2	0	10
x_2	$1/2$	1	$1/2$	0	$5/2$
r_2	$1/2$	0	$-1/2$	1	$3/2$

: Rough

$4\times 1/2$	4×1	$4\times 1/2$	4×0	$4\times 5/2$
2	4	2	0	10
-2	-4	0	0	0
0	0	2	0	10

: Now we check most negative value of x_1 and x_2 which is not present.

→ since no coefficient of non-basic in z-row is negative. So the last tableau is optimal:

$x_1 = 0$, $x_2 = \frac{5}{2}$ with $z = 10$

If x_1, x_2 or any ...

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To Find Alternative optima, let's Proceed further. (we decide by ourselves that x_1 is entering)

Basic	x_1	x_2	r_1	r_2	Sol
z	0	0	2	0	10
x_2	0	1	1	-1	1
x_1	1	0	-1	2	3

$\therefore x_1 = 3,\ x_2 = 1$ with $z = 10$

→ Convex Rule: (If point gives same solution, then range will be within points)

$\gamma \in (0,1)$

$\hat{x}_1 = \gamma x_1 + (1-\gamma) y_1$

$x_2 = \gamma x_2 + (1-\gamma) y_2$

: Start from initial points

→ Since $\gamma \in (0,1)$:

$B(0, 5/2)$

$C(3, 1)$

$: (x, y)$

$\hat{x}_1 = \gamma x_{B_1} + (1-\gamma) x_{C_1}$

$\hat{x}_2 = \gamma x_{B_2} + (1-\gamma) x_{C_2}$

Therefore, $\hat{x}_1 = \gamma(0) + (1-\gamma)(3) = 3 - 3\gamma$

$\hat{x}_2 = \gamma(5/2) + (1-\gamma)(1) = 1 + 3/2\,\gamma$

~~let $\gamma = 0$~~

$\hat{x}_1 = 3,\ \hat{x}_2 = 1 \quad (3,1)$

$\gamma = 1 \rightarrow \hat{x}_1 = 0,\ \hat{x}_2 = 5/2 \quad (0, 5/2)$

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→ let $\gamma = 1/2$

$$3 - 3(1/2) = 3/2$$

$$1 + 3/2(1/2) = 7/4$$

$\hat{x}_1 = 1.5$, $\hat{x}_2 = 1.75$

with $z = 2(1.5) + 4(1.75) = 10$

(Infinite many solutions give $z = 10$)

: $\hat{x}_1 = \alpha x_{B_1} + (1-\alpha) x_{C1}$

$\hat{x}_2 = \alpha x_{B2} + (1-\alpha) x_{C2}$

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Q:

$$Max: z = 2x_1 + 4x_2$$

Subject to:

$$x_1 + 2x_2 \leq 5$$

$$x_1 + x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

Sol:-

Write into standard form:

$$z - 2x_1 - 4x_2 = 0$$

Subject to:

$$x_1 + 2x_2 = 5$$

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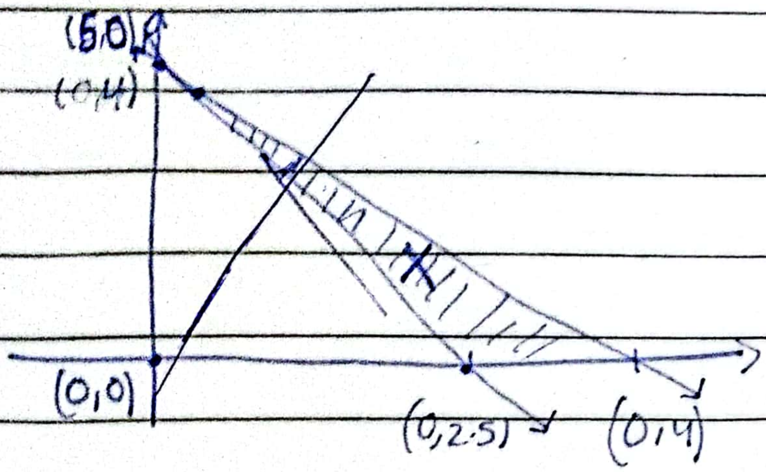
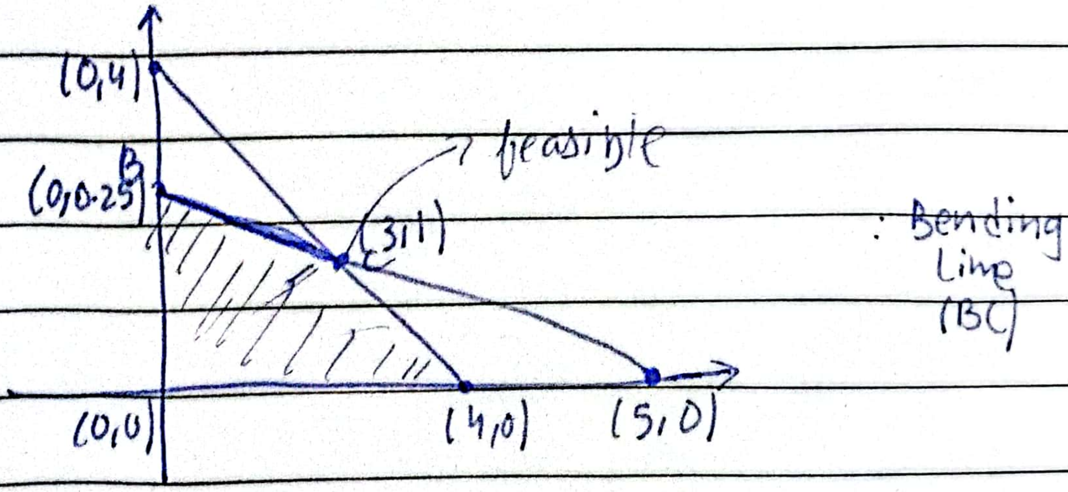
$x_1 + x_2 = 4$

(i) Graph



$(0,0), (0,2.5), (\quad), (4,0)$



$(0,0), (0,2.5), (4,0), (3,1)$

At $(0,0) \Rightarrow z = 0$

$(4,0) \Rightarrow z = 8$

$(0,2.5) \Rightarrow z = 10$ → Alternative Optima (Infinite many solutions)

$(3,1) \Rightarrow z = 10$

(ii) γ (alpha) $\in (0,1)$

$: \hat{x}_3 = \gamma x_{B3} + (1-\gamma) x_{C3}$

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$\hat{x}_1 = \gamma x_{B1} + (1-\gamma) x_{C1}$ B(0, 2.5)

$\hat{x}_2 = \gamma x_{B2} + (1-\gamma) c_2$ C(3, 1)

Therefore $\hat{x}_1 = \gamma(0) + (1-\gamma)(3) = 3 - 3\gamma$

$\hat{x}_2 = \gamma\left(\frac{5}{2}\right) + (1-\gamma)(1) = 1 + \frac{3}{2}\gamma$

$(\hat{x}_1, \hat{x}_2) = \left(3 - 3\gamma, 1 + \frac{3\gamma}{2}\right)$

⊙ Let $\gamma = 0.1 \Rightarrow (\hat{x}_1, \hat{x}_2) = (2.7, 1.15)$
with $z = 2(2.7) + 4(1.15) = 10$

$\gamma = 0.2 \Rightarrow (\hat{x}_1, \hat{x}_2) = (2.4, 1.3)$
with $z = 10$

$\gamma = 0.3 \Rightarrow (\hat{x}_1, \hat{x}_2) = (2.1, 1.45)$
with $z = 10$

⊛ Duality and sensitivity Analysis

⊙ Dual Problem:

→ Different forms but point same.

→ Problem solved with other form easily and give answer accurately.

DATE: DAY:

Q:

Max: $z = 5x_1 + 12x_2 + 4x_3$

Subject to:

$x_1 + 2x_2 + x_3 \leq 10$

$2x_1 - x_2 + 3x_3 = 8$

$x_1, x_2, x_3 \geq 0$

→ Let y_1, y_2 be dual variables then dual of given primal problem is:

Min : $W = 10y_1 + 8y_2$: Constant multiplier

Subject to :

$y_1 + 2y_2 \geq 5$: First column of subject to x_1

$2y_1 - y_2 \geq 12$

$1y_1 + 3y_2 \geq 4$: x_i of Z

: $y_1, y_2 \geq 0$

Q:

Max : $z = 2x_1 + 4x_2$

Subject to:

$x_1 + 2x_2 \leq 5$

$x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0$

Min : $w = 5y_1 + 4y_2$

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Subject to:

$y_1 + y_2 \geq 2$

$2y_1 + y_2 \geq 4$

$y_1, y_2 \geq 0$

Q:

Max : $Z = 5x_1 + 12x_2 + 4x_3$

Subject to:

$x_1 + 2x_2 + x_3 \leq 10$

$2x_1 - x_2 + 3x_3 = 8$

$x_1, x_2, x_3 \geq 0$

Sol:- Write into standard form:

$Z = 5x_1 + 12x_2 + 4x_3$

Subject to:

$x_1 + 2x_2 + x_3 + r_1 = 10$

$2x_1 - x_2 + 3x_3 + r_2 = 8$

$x_1, x_2, x_3 \geq 0$

: $y_1 \geq$ ...

y_2 is unrestricted

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Q:

Min: $z = 5x_1 + 6x_2$

Subject to:

$x_1 + 2x_2 = 5$

$x_1 + 5x_2 \geq 3$

$4x_1 + 7x_2 \leq 8$

x_1 unrestricted , $x_2 \geq 0$

: $x_1 = x_1' - x_1''$

Put the problem into standard form:

Max $z = 5x_1 + 6x_2$

Subject to:

$x_1 + 2x_2 - 0\gamma_1 = 5$ .... ①

$x_1 + 5x_2 - \gamma_2 = 3$ ... ②

$4x_1 + 7x_2 + \gamma_3 = 8$ ..... ③

x_1 is unrestricted so let:

: $x_1 = x_1' - x_1''$ put in ①,②,③, z

$x_1', x_1'' \geq 0$

$(x_1' - x_1'') + 2x_2 - 0\gamma_1 = 5$: $z = 5(x_1' - x_1'') + 6x_2$

$(x_1' - x_1'') + 5x_2 - \gamma_2 = 3$

$4(x_1' - x_1'') + 7x_2 + \gamma_3 = 8$

: $x_1', x_2'', x_2, \gamma_1, \gamma_2, \gamma_3 \geq 0$

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Min:

$W = 5y_1 + 3y_2 + 8y_3$

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* Write the dual of the following primal problem.

Min: $Z = 5x_1 + 2x_2 + x_3$

Subject to:

$2x_1 + 3x_2 + x_3 \geq 20$

$6x_1 + 8x_2 + 5x_3 \geq 30$

$7x_1 + x_2 + 3x_3 \geq 40$

$x_1 + 2x_2 + 4x_3 \geq 50$

Sol:-

$2x_1 + 3x_2 + x_3 - \gamma_1 = 20$

$6x_1 + 8x_2 + 5x_3 - \gamma_2 = 30$

$7x_1 + x_2 + 3x_3 - \gamma_3 = 40$

$x_1 + 2x_2 + 4x_3 - \gamma_4 = 50$

Max: $W = 20y_1 + 30y_2 + 40y_3 + 50y_4$

Subject to:

$2y_1 + 6y_2 + 7y_3 + y_4 \leq 5$

$3y_1 + 8y_2 + y_3 + 2y_4 \leq 2$

$y_1 + 5y_2 + 3y_3 + 4y_4 \leq 1$

$-y_1 \leq 0 \Rightarrow y_1 \geq 0$

$\therefore \; 0 \cdot y_1 - y_2 + 0\, y_3 + 0 \cdot y_4 \leq 0$

$-y_2 \leq 0$

$\Rightarrow y_2 \geq 0$

$y_3 \geq 0$

$y_4 \geq 0$

$$\begin{array}{c|ccccccc} & x_1 & x_2 & x_3 & r_1 & r_2 & r_3 & r_4 \\ \hline y_1 & 2 & 3 & 1 & -1 & 0 & 0 & 0 \\ y_2 & 5 & 8 & 5 & 0 & -1 & 0 & 0 \\ y_3 & 1 & 1 & 3 & 0 & 0 & -1 & 0 \\ y_4 & 1 & 2 & 4 & 0 & 0 & 0 & -1 \end{array}$$

⊛ Transportation Problem:

→ Subclasses of LPPs

→ We overview all scenarios which also include transport service minimize cost.

Source: F_1, F_2, F_3, F_4

Destination: M_1, M_2, M_3

→ Demand and Supply of Goods.

$j \rightarrow$ Destination
$i \rightarrow$ source

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Destinations

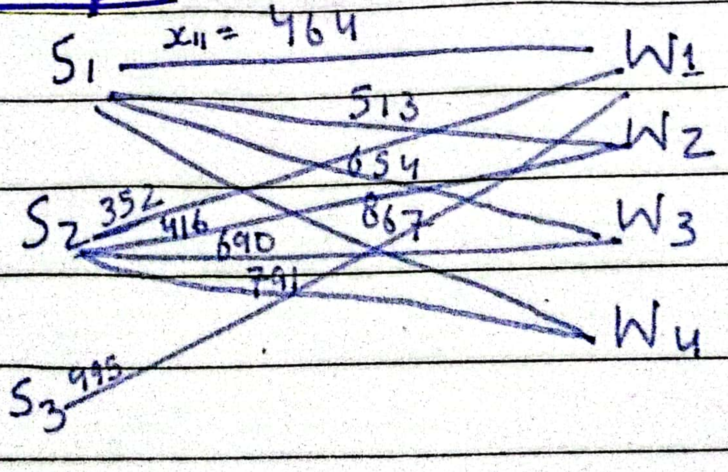
x	1	2	3	......	n	Supply
1	x_{11} $\boxed{C_{11}}$	x_{12} $\boxed{C_{12}}$				a_1
2						a_2
3						a_3
⋮				x_{ij} c_{ij}		⋮
m						

Source

: C_{ij} : Cost of transporting one unit

x_{ij}: Quantity transported

⊛ Example:



: S1: Supply
W1: Demand

S_1	x_{11} 464
S_2	x_{21} 352
S_3	x_{31} 995

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*Transportation Problems (TP):

⊙ Ex : 1

Determine the basic starting solution by means of North-West Corner Method.

Source	Destination 1	2	3	4	Supply
1	10	2	20	11	15
2	12	2	9	20	25
3	4	14	16	18	10
Demand	5	15	15	15	

: ~~select low~~ from demand and supply and write in box

Solution:-

Total Supply = 15+25+10 = 50

Total Demand = 5+15+15+15 = 50

Since the problem is balanced as Total supply is equal to total demand.

S/D	1	2	3	4	Supply
1	10 $\boxed{5}$	2 $\boxed{10}$	20 x	11 x	~~15~~ ~~10~~ 0
2	12 x	7 $\boxed{5}$	9 $\boxed{15}$	20 $\boxed{15}$	~~25~~ ~~20~~ 50
3	4 x	14	16 x	18 $\boxed{10}$	~~10~~ 0
Demand	5	15	15	15	50

→ Total minimum cost of Transportation = $10 \times 5 + 2 \times 10 + 7 \times 5 + 9 \times 15 + 2 \times 5 + 18 \times 10$
(choose Demand and Supply cost and multiply)

$= 520$

Q2:

Total Suppy $= 15 + 25 + 10 = 45$

Total Demand $= 5 + 15 + 15 + 15 = 50$

Since the problem is unbalanced as Total supply is less then total demand.

: For this we add dummy row to balance it.

S/D	1	2	3	4	Supply
1	10 $\boxed{5}$	2 $\boxed{10}$	20 ×	11 ×	~~15~~ ~~10~~ 0
2	12 ×	7 $\boxed{5}$	9 $\boxed{15}$	20 ×	~~20~~ ~~15~~
3	4 ×	14 ×	16 ×	18 $\boxed{10}$	~~10~~
4	0 ×	0 ×	0 ×	0 $\boxed{5}$	~~5~~
Demand	~~5~~ 0	~~15~~ ~~5~~ 0	~~15~~ 0	~~15~~ ~~5~~	50

: Supply less (Add row)

: Demand less (Add column)

→ Total minimum cost of Transportation = $10 \times 5 + 2 \times 10 + 7 \times 5 + 9 \times 15 + 18 \times 10 + 0 \times 5$

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→ Paper:

- Formulation of LPP (Mathematical)
- Graphical Method
- Simplex Method
- Big M or Two Phase Method
- Alternative Optima
- Dual Conversion
- North West and Least cost Method

Q: **Least Cost Method** (Select least)

Source/D	D_1	D_2	D_3	D_4	Supply
S_1	10 ×	2 [15]	20 ×	11 ×	~~15~~
S_2	12 ×	7 ×	9 [15]	20 [10]	~~25~~ ~~10~~ 0
S_3	4 [5]	14 ×	16 ×	18 [5]	~~10~~ ~~5~~ 0
Demand	~~5~~ 0	~~15~~ 0	~~15~~ 0	~~15~~ ~~10~~ 0	50

→ Total Minimum cost $= 2 \times 15 + 4 \times 5 + 9 \times 15 + 18 \times 5 + 20 \times 10$

$=$