Neural Networks: Assignment 2

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1 Introduction

The second assignment of the Neural Networks course consits of three tasks:

- Task 1: Function Optimization
- Task 2: The XOR Problem
- Task 3: Handwritten digit recognition

For the first task, we were given the Rosenbrock's function, and we were asked to test 5 different algorithms for finding the global minimum of this function. The purpose is to get an insight into the limitations of the classical gradient descent algorithm. In the second task we are using a neural network to solve the xor problem. The algorithms of task 1 are used to update the weights between the nodes.

2 Task 1: Function Optimization

2.1 Problem Description

The Rosenbrock function is a function that is used as a performance test for optimization algorithms. The equation can be found in figure 1. It has a global minimum at the point (1,1), where the value of $f = 0$. This is visualized in figure 2.

$$f(x, y) = 100 * (y - x^2)^2 + (1 - x)^2$$

Figure 1: The Rosenbrock's function

We were given the task to optimize the Rosenbrock function using five different algorithms, and subsequently compare their performance, in order to get an insight into the advantages, disadvantages and limitations of the different algorithms.

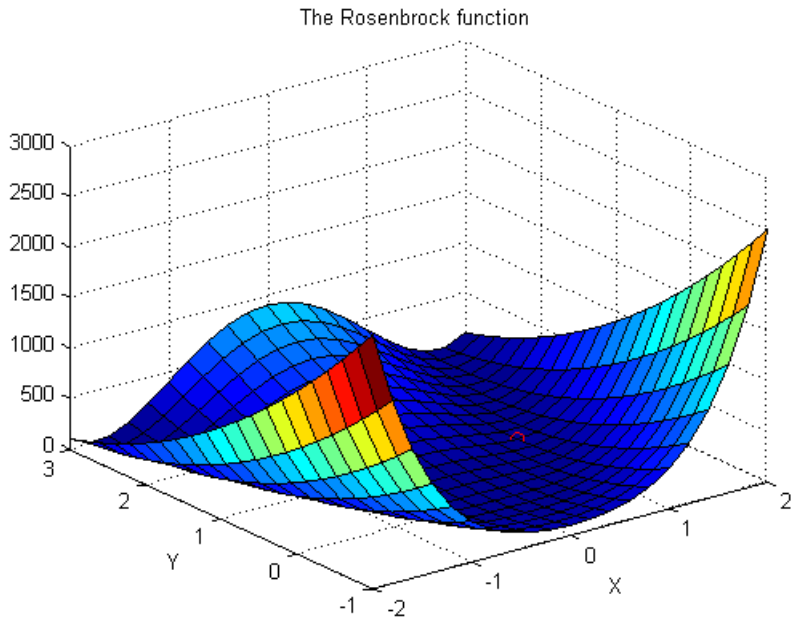


Figure 2: The Rosenbrock function, with minimal point

The 5 different algorithms we tested are:

- Gradient descent
- Gradient descent with line search
- Scaled conjugate gradient
- Conjugate gradient
- Quasi-Newton

To get a good comparison between the algorithms, each algorithm was run 100 times. We first generated 100 random points around (-1,1) as starting points. Each algorithm was started from the same random point.

There were 4 different measures to compare the algorithms with:

- The average number of evaluations of f
- The average number of evaluations of the gradient of f
- The average run time of the algorithm
- The average "success rate".

Together these measures should provide a decent indication which algorithm performs better. Because some functions might be a lot more computationally expensive to evaluate than the Rosenbrock function, the number of evaluations of both the function and the gradient should be as low as possible. A run obviously shouldn't take too long, and it should have a high success rate.

The success rate is measured as reaching the minimum with an accuracy of 0.0001. This means that when the optimal point found by the algorithm is evaluated, the value of f is smaller than 0.0001. Of course we would like to have an optimizer that gets a 100% accuracy every time the algorithm is ran, but that is not always possible.

2.2 Implementation

In figure 3 you can see the pseudo-code of the implemented algorithm.

```
starting_point = repmat([-1,1],100,1) + 0.5*randn(100,2);
options = foptions;         % Standard options
options(1) = -1;            % Turn off printing completely
options(3) = 1e-8;          % Tolerance in value of function
options(14) = 100;          % Max. 100 iterations of algorithm
options(18) = 0.001;        % Learning rate
default_options = options;
functionList = {@graddesc, @graddesc, @scg, @conjgrad, @quasinew};
for i = 1:100
    for j = 1:5
        options = default_options;
        if(j==1 || j==2)
            options(18) = 0.008;
        end
        if(j==2)
            options(7) = 1;
        end
        tic;
        [dump, options, dump, dump] = functionList{j}('rosen', starting_point(i,:), options, 'rosegr
        time = toc;
        results(i,j,1) = options(10);
        results(i,j,2) = options(11);
        results(i,j,3) = time;
        results(i,j,4) = options(8);
    end
end
```

Figure 3: Pseudo-code of task 1.

As described in the assignment, we run the algorithms using 100 randomly generated starting points from the distribution `[-1, 1] + 0.5 * randn(1,2)`.
We set the options of the algorithm to the default options, with a few changes, we set the tolerance of the function value to $1 * 10^{-8}$, the iterations to 100, and we use a different learning rate for the two gradient descent algorithms. The reasoning behind the value is described in section 2.3.

$$\frac{\rho f}{\rho x} = 400x^3 + 2x - 400 * yx - 2$$

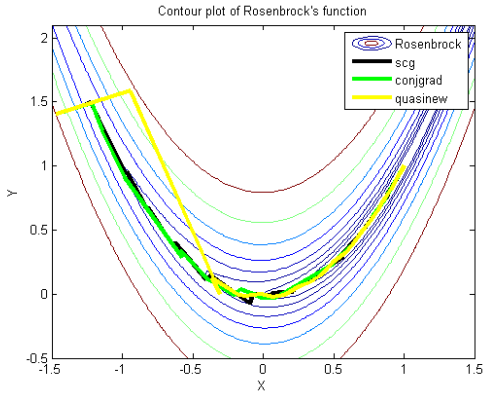
$$\frac{\rho f}{\rho y} = 200y - 200 * x^2$$

Figure 4: Partial derivatives of f

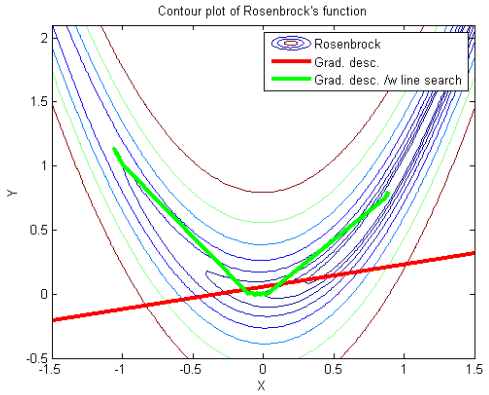
In figure 4, we show the partial derivatives of the Rosenbrock function.

2.3 Experiments

In task 1.3 we were asked to tune both gradient descent algorithms manually. We first kept the number of iterations to 100, and tested different values of the learing rate.
The values are averages over 500 runs. We then tested the best value on 100 runs with 100.000 iterations, and the gradient descent algorithm with line search reached an accuracy of 100%. The standard gradient descent still got 0%. As you can see the optimal error is somewhere around a learning rate of 0.008, which is why we set the learning rate for the experiment to that value.
In order to acquire additional insights into how the algorithms work, we plotted the optimal points found after each iteration in the following plot.



(a) Nadenken



(b) Nadenken

Function	Evaluation	Gradient evaluation	Runtime	Successrate
Gradient descent	101	100	0.00778	0%
Gradient descent /w line search	1816.93	98	0.19121	6%
Scaled conjugate gradient	0.47	0.7	$5.376 * 10^{-5}$	100%
Conjugate gradients	3.64	0.25	0.0003924	100%
Quasi-Newton	0.69	0.22	$7.9669 * 10^{-5}$	100%

Table 1: Results of the 5 algorithms. All values are averages.

In table 1 you can find the results of the different algorithms. All 5 algorithms were run on an Intel Core i5-3470, the values are averages over 100 runs. The parameters used for the algorithms are described in the section 'Implementation'. As you can see the gradient descent algorithm, even after optimization, still achieves a successrate of 0% on the data. We took a look at the output-coordinates of the algorithm, and it shows that this setting of the learning rate is too high, because the coordinates keep growing until they reach infinity. Gradient descent with line search already works a lot better, but it costs more function evaluations. The other three methods clearly achieve better results on all fronts, with less evaluations, a shorter runtime and higher successrates.

2.4 Conclusions

The best algorithm is hard to tell. Both the Scaled Conjugate Gradient approach and the Quasi-Newton algorithm perform well on this particular problem. The Scaled Conjugate Gradient uses less function evaluations than Quasi-Newton, but Quasi-Newton uses less gradient evaluations. The runtime of both algorithms is almost equal, and they both achieve a successrate of 100%. It depends on the computational

difficulty of the problem which algorithms is better. If a function evaluation is costly, SCG might be the better approach, but if gradient evaluations are expensive, Quasi-Newton might be better. The expected number of function evaluations is probably around 1 for both SCG and Quasi-Newton The worst algorithm is clearly gradient descent, since it doesn't find good solutions, even in 100 runs there are no successful solutions. Gradient descent with line search is the slowest algorithm of them all, but because it does produce some useful solutions it is better than the original gradient descent algorithm. The reason for the very poor performance of the gradient descent algorithm is probably that if the learing rate is set too high, the algorithms best coordinates will 'explode' into infinity. If the learning rate is set to a lower rate, 100 iterations are not enough to get to the optimal point. Because the Rosenberg function has such a narrow valley, finding the optimal value is very difficult.

3 Task 2: The XOR Problem

3.1 Problem Description

3.2 Implementation

3.3 Experiments

3.4 Conclusions

4 Task 3: Handwritten Digit Recognition with MLP

4.1 Problem Description

4.2 Implementation

4.3 Experiments

4.4 Conclusions